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Formal insurance and altruism networks*

Tizié Bene[†], Yann Bramoullé[†] and Frédéric Deroïan[†]

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Abstract

We study how altruism networks affect the adoption of formal insurance. Agents have private CARA utilities and are embedded in a network of altruistic relationships. Incomes are subject to both a common shock and a large idiosyncratic shock. Agents can adopt formal insurance to cover the common shock. We show that *ex-post* altruistic transfers induce interdependence in *ex-ante* adoption decisions. We characterize the Nash equilibria of the insurance adoption game. We show that adoption decisions are substitutes and that the number of adopters is unique in equilibrium. The demand for formal insurance is lower with altruism than without at low prices, but higher at high prices. Remarkably, individual incentives are aligned with social welfare. We extend our analysis to CRRA utilities and to a fixed utility cost of adoption.

Keywords: Formal Insurance, Informal Transfers, Altruism Networks

JEL: C72, D85

[†] Aix-Marseille University, CNRS, Aix-Marseille School of Economics

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1 Introduction

The poor in poor countries generally face large risks, especially when it comes to health (illnesses, accidents) and livelihood (climate events), see [Banerjee and Duflo \(2011\)](#). These risks are a major source of stress and reduced well-being, as well as a likely cause of poverty traps. Many such risks could, in principle, be covered by formal insurance, like public universal health coverage and the production insurance provided by insurance companies or NGOs. In the past 40 years, governments and development institutions have worked hard to make formal insurance accessible to households in need. Disappointingly, however, these efforts have generally encountered low take-up, e.g. [Cole et al. \(2013\)](#). One likely explanation for such limited adoption of formal insurance in high-risk contexts is informal safety nets, which may act as barriers to formal insurance.¹ There is widespread evidence that social networks help individuals and households cope with negative shocks, through informal financial transfers and gifts in kind. These transfers and assistance are motivated, to a large extent, by altruism, as individuals give to others they care about. How do effective altruism networks, then, affect the demand for formal insurance? Does altruism always reduce the adoption of formal insurance? These questions have, so far, been neglected; we review the scant literature on the interaction between formal and informal insurance below.

This paper provides the first analysis of how altruism networks affect the demand for formal insurance. We consider a community of agents who care about each other. Agents face both a common and an idiosyncratic risk and can buy formal insurance to cover the common risk. Once shocks are realized, agents make private transfers to each other to support friends in need. We find that altruism networks have a profound impact on demand for formal insurance. Under altruism, an agent anticipates that her own insurance decision will affect the outcomes of others she cares about and that the insurance decisions of others who care about her will affect her outcome. *Ex-post* altruistic transfers thus induce interdependence in *ex-ante* decisions to buy formal insurance. Under standard assumptions on preferences toward risk, we show that the insurance game involves strategic substitutes: an agent is less

¹Other explanations include price effects, liquidity constraints, mistrust, and psychological costs of adopting an unfamiliar product. These explanations are not mutually exclusive, and we study below how price effects and a utility cost of adoption interact with altruism networks in determining the demand for formal insurance.

likely to adopt formal insurance when the number of other adopters increases. We show that this yields a unique number of adopters in equilibrium. We then contrast demand for formal insurance with and without altruism. We find that formal insurance and altruism networks are substitutes at relatively low prices and complements at relatively high prices. Altruism networks thus reduce adoption when the price of insurance is not too high, and increase adoption under high prices. Overall, our analysis demonstrates that an appropriate description of the way informal safety nets operate is key to understanding the determinants and impacts of formal insurance adoption.

We introduce formal insurance into the model of altruism in networks studied in Bourlès, Bramoullé and Perez-Richet (2017, 2021). Agents are embedded in a fixed altruism network, describing the structure of social preferences in the community. An agent’s altruistic utility is a linear combination of her private utilities and the private utilities of others she cares about. We consider a connected altruism network: any agent can be reached from any other agent through a directed path of caring relationships. In our benchmark model, we assume that private utilities display Constant Absolute Risk Aversion (CARA). We assume that the common and the idiosyncratic shocks are binary and independent, and that the idiosyncratic shock is large and only affects one agent at a time. This guarantees that a directed path of transfers flows from any other agent to the affected agent in equilibrium, a key simplifying assumption (Assumption 1). We develop our analysis in several stages.

We first obtain an explicit characterization of the Nash equilibria of the insurance game (Theorem 1). We find that there is a generically unique number of adopters in equilibrium. Moreover, any profile with this equilibrium number is a Nash equilibrium, implying that agents’ positions in the altruism network do not affect adoption.² We build on this characterization and analyze comparative statics with respect to the main parameters of the model. We find that the equilibrium number of formal insurance adopters increases weakly following an increase in the magnitude or likelihood of the common shock, or a decrease in insurance price. Moreover, the number and proportion of adopters is higher in larger communities.

Second, we compare the demand for formal insurance with and without altruism. We

²Still, equilibrium payoffs depend on the network positions of adopters, and network positions also affect adoption in the presence of a fixed utility cost from adopting.

show that there exists a price threshold such that formal insurance adoption is lower under altruism when the insurance price is below the threshold and higher when the price is above the threshold (Theorem 2). This result confirms and qualifies the intuition that informal safety nets can curtail the adoption of formal insurance. We uncover two countervailing effects at play, both induced by effective altruistic transfers. On the one hand, altruism networks help smooth consumption, which reduces the demand for formal insurance. On the other hand, altruism networks enable the costs of formal insurance to be shared, which increases the demand for formal insurance. Substitution effects dominate for low prices, while complementarities induced by cost-sharing dominate for high prices.

Third, we analyze welfare and we show that the Nash equilibria of the insurance game are constrained Pareto efficient (Proposition 3). Conditional on the constraint that agents cannot be fully insured on both risks, individual incentives to adopt formal insurance are thus aligned with social welfare. This remarkable feature relies on a property of multiplicative separability, which guarantees that payoffs all move in the same direction. This provides a new context where a counterpart to the first welfare theorem holds in the presence of strategic interactions.

Fourth, we extend our benchmark analysis in two ways. In the first extension, we assume that private utilities display Constant Relative Risk Aversion, CRRA, rather than CARA. We show that key properties of equilibrium behavior (strategic substitutes, generic uniqueness, Pareto efficiency) hold under CRRA utilities. One main difference is the emergence of wealth effects. Under CRRA, the demand for formal insurance under altruism depends on aggregate wealth, while the demand in the absence of altruism depends on the full wealth distribution.³ The relationship between these two demand curves is more complex, and we provide a simple example where the two curves cross three times in the interior domain. In the second extension, we introduce a fixed utility cost from adopting. We characterize Nash equilibria in that case and find that adoption may now depend on network position. We show that agents with more links, who are closer to others in the network and whose neighbors are also closer to others are, in a sense, more likely to adopt.

³Under CARA, by contrast, there is no wealth effect and the demand for formal insurance with or without altruism does not depend on wealth levels.

Our analysis contributes, first, to a literature studying the impact of informal risk sharing arrangements on formal insurance.⁴ Several studies look at index insurance, an innovative financial product where an agent receives transfers depending on an objective index, such as the amount of rainfall measured at a weather station. Index insurance carries basis risk, i.e., the risk an agent suffers the shock but does not receive transfers.⁵ These studies find empirical evidence that the demand for index insurance rises with increased informal insurance, see [Mobarak and Rosenzweig \(2012\)](#), [Mobarak and Rosenzweig \(2013\)](#), [Dercon et al. \(2014\)](#), [Berg, Blake, and Morsink \(2020\)](#). These studies also develop models where individuals informally share risk in a group, and the complementarity between index insurance and informal risk-sharing arises because informal insurance helps cover the basis risk. By contrast, we consider standard indemnity insurance and agents make informal transfers to each other through an altruism network. We analyze how the demand for formal insurance is affected by the altruism network and show that complementarities between formal insurance and altruism can appear even in the absence of basis risk.

[De Janvry, Dequiedt, and Sadoulet \(2014\)](#) look at incentives to contract full formal insurance when individual utility depends on individual and aggregate wealth. They highlight the strategic interactions and free-riding emerging in individual decisions to adopt formal insurance. While our setup differs in important ways, our analysis confirms the key insight that in the presence of informal transfer arrangements, individual decisions to adopt formal insurance are interdependent.⁶ We show that these strategic interactions do not necessarily lead to free-riding, however. In our context, while individual decisions to adopt formal insurance involve strategic substitutes, Nash equilibria coincide with the constrained Pareto optima.

[Kinnan and Townsend \(2012\)](#) analyze data on formal and informal loans in rural Thailand. They find evidence of large network spillovers: having an indirect connection to a household with a formal loan has the same, strong impact on consumption smoothing for a household

⁴A recent literature looks at how the introduction of formal insurance affects existing informal arrangements, see, e.g., [Takahashi, Barrett, and Ikegami \(2019\)](#), [Boucher, Delpierre, and Verheyden \(2016\)](#).

⁵According to accounting standards, an adverse effect on the policyholder should be a contractual precondition for payment in an insurance contract. According to these standards, index insurance should then be classified as a derivative contract rather than an insurance contract, see [Clarke \(2016\)](#).

⁶In our setup, formal insurance only covers the common shock rather than overall wealth fluctuations and individual utility does not depend on individual and aggregate wealth.

than having a formal loan. These findings are consistent with our theoretical results. When one agent adopts formal insurance, every agent indirectly connected in the altruism network benefits. Overall, we provide the first analysis of demand for formal insurance when agents make informal transfers through networks.

Our analysis also contributes to a literature on informal transfers and networks.⁷ [Ambrus, Mobius, and Szeidl \(2014\)](#) characterize Pareto-constrained risk-sharing arrangements when transfers flow through networks and links can be used as social collateral. [Ambrus, Gao, and Milán \(forthcoming\)](#) analyze Pareto-constrained risk-sharing arrangements under local informational constraints. [Bouhlès, Bramoullé, and Perez-Richet \(2017\)](#) consider a network of altruistic relationships and characterize the Nash equilibria of the game of transfers for non-stochastic incomes. [Bouhlès, Bramoullé, and Perez-Richet \(2021\)](#) look at the impact of altruism networks and transfers when incomes are stochastic. None of these studies consider formal insurance, however. We introduce formal insurance into this literature, and provide the first analysis of the interplay between formal insurance and informal transfers through networks. We find that altruism networks have a first-order impact on demand for formal insurance.

Finally, our analysis contributes to a large literature on the interactions between formal and informal institutions.⁸ [Gagnon and Goyal \(2017\)](#) develop a model where agents choose a network and a market binary action. They assume that the two actions are either substitutes or complements, and analyze equilibria, welfare, and inequality. By contrast, we look at how altruism networks affect the adoption of formal insurance.⁹ We show that the demand for formal insurance is lower under altruism when insurance is low-priced and higher when insurance is high-priced.

The remainder of the paper is organized as follows. We introduce our framework in Section 2. We analyze the game of formal insurance adoptions in Section 3. We extend our analysis to CRRA utilities and to a fixed utility cost of adoption in Section 4 and conclude in Section 5.

⁷One branch of the literature analyzes the stability of risk-sharing networks, see e.g. [Bloch, Genicot, and Ray \(2008\)](#), [Bramoullé and Kranton \(2007\)](#).

⁸See, e.g., [Arnott and Stiglitz \(1991\)](#), [Kranton \(1996\)](#), [Montgomery \(1991\)](#).

⁹While the market action - adopting or not adopting - is binary, as in [Gagnon and Goyal \(2017\)](#), network actions - transfers - are continuous and multidimensional in our setup.

2 Framework

We introduce formal insurance into the model of altruism in networks analyzed by Bourlès, Bramoullé and Perez-Richet (2017, 2021). Consider a community of $n \geq 2$ altruistic agents. Incomes are stochastic, and subject to a community-level shock and to an individual-level shock. Formal insurance covering the community-level shock is available. Each agent decides, *ex-ante*, whether to buy formal insurance. Once incomes are realized, altruistic agents make informal transfers to each other. The model thus has 3 stages. In stage 1, agents decide whether to buy formal insurance. In stage 2, income shocks are realized. In stage 3, agents make private transfers, conditional on realized incomes and on formal insurance decisions.

Stochastic Incomes. Agents have potentially different baseline income, or wealth, levels and face both a common and an idiosyncratic shock. Formally, agent i has stochastic income

$$y_{i0} - \tilde{\mu}_c - \tilde{\lambda}_i \tag{1}$$

where y_{i0} represents i 's baseline non-stochastic income, $\tilde{\mu}_c$ represents a common random shock, and $\tilde{\lambda}_i$ an individual random shock.

For simplicity, we consider binary shocks. More precisely, $\tilde{\mu}_c$ is equal to $\mu > 0$ with probability $q_c \in (0, 1)$ and to 0 with probability $1 - q_c$. This shock affects all agents, for instance, a local weather event, such as heavy rainfalls or a drought. In contrast, the idiosyncratic shock $\tilde{\lambda}_i$ affects only one agent at a time: $\lambda_i = \lambda > 0$ with probability $q_i \in (0, 1)$ and 0 with probability $1 - q_i$, and $\lambda_i = \lambda \Rightarrow \lambda_j = 0$ for $j \neq i$. Since one agent is affected, $\sum_i q_i = 1$. We will assume below that the individual shock, λ , is relatively large, a serious adverse event such as an accident or an illness. Probabilities may differ across agents, capturing potential heterogeneities in agents' exposure to the idiosyncratic shock. Finally, we assume that common and idiosyncratic shocks are independent.

Formal insurance. An external institution offers insurance contracts that fully cover the common risk $-\tilde{\mu}_c$. This formal insurance is available at price $p \geq 0$. We assume that this price is exogenous and focus on the demand for formal insurance in our analysis. Formally,

$p \in [0, \mu]$.¹⁰ The price level of course reflects supply side features. The *actuarial price* $p = q_c \mu$ is an important benchmark, reflecting a competitive insurance market with no frictions and no administrative costs. However, the price could be lower, $p < q_c \mu$, for instance if formal insurance is subsidized by the government or by a non-governmental organization. It could also be higher, $p > q_c \mu$, in the presence of market power or administrative costs.

If an agent buys formal insurance, the stochastic shock $-\tilde{\mu}_c$ is replaced by the non-stochastic price $-p$. Let $x_i \in \{0, 1\}$ denote the insurance decision of agent i , where $x_i = 1$ when agent i buys formal insurance and $x_i = 0$ otherwise. Let $\mathbf{x} \in \{0, 1\}^n$ denote the profile of insurance decisions. Agent i 's stochastic income at the end of period 1, after formal insurance decisions but before informal transfers, is thus equal to $\tilde{y}_i^1 = y_i^0 - p - \tilde{\lambda}_i$ if $x_i = 1$ and to $\tilde{y}_i^1 = y_i^0 - \tilde{\mu}_c - \tilde{\lambda}_i$ if $x_i = 0$.

Informal transfers. Once insurance decisions are taken and shocks are realized, altruistic agents make informal transfers to each other. We now describe how these informal transfers are determined. For this third stage, we adopt the framework of Bourlès, Bramoullé and Perez-Richet (2017). Let y_i denote the final income level, or consumption, of agent i after informal transfers are realized. Let \mathbf{y}_{-i} denote the profile of final incomes of the other agents. Agents may care about each other. Preferences have a private and a social component. Agent i 's private preferences are represented by utility function $u_i : \mathbb{R} \rightarrow \mathbb{R}$ displaying Constant Absolute Risk Aversion (CARA):

$$u_i(y) = -e^{-Ay}. \quad (2)$$

Agent i may be altruistic towards others and her preferences are represented by the social utility function $v_i : \mathbb{R}^n \rightarrow \mathbb{R}$ such that

$$v_i(y_i, \mathbf{y}_{-i}) = u_i(y_i) + \sum_{j \neq i} \alpha_{ij} u_j(y_j) \quad (3)$$

where $\alpha_{ij} \in [0, 1]$ represents the strength of the altruistic relationship between i and j . By convention, $\alpha_{ii} = 0$. The *altruism network* is represented by the matrix $\boldsymbol{\alpha} = (\alpha_{ij})_{i,j}$,

¹⁰If $p > \mu$, agents have no incentives to buy the insurance.

describing the structure of social preferences in the community.

Agent i can give $t_{ij} \geq 0$ to agent j . By convention, $t_{ii} = 0$. The collection of bilateral transfers defines a *network of transfers*, represented by the matrix $\mathbf{t} \in \mathbb{R}_+^{n^2}$. Agent i 's final income level is equal to

$$y_i = y_i^1 + \sum_{j \neq i} (t_{ji} - t_{ij}) \quad (4)$$

Since there is no transfer cost, informal transfers redistribute aggregate income among agents:

$$\sum_i y_i = \sum_i y_i^1.$$

In this third stage, agents play a non-cooperative transfer game. Each agent makes informal transfers to others in order to maximize her altruistic utility, conditional on transfers made by others. We assume that the network of informal transfers is a Nash equilibrium of the transfer game. The transfer network is therefore characterized by the following conditions, see Bourlès, Bramoullé and Perez-Richet (2017) for details. If $\alpha_{ij} > 0$, define $c_{ij} = -\ln(\alpha_{ij})$ as a virtual cost associated with the link between i and j . Stronger links have lower virtual cost. Then, \mathbf{t} is a Nash equilibrium of the transfer game if and only if

$$\forall i, j, y_i \leq y_j + \frac{c_{ij}}{A} \text{ and } t_{ij} > 0 \Rightarrow y_i = y_j + \frac{c_{ij}}{A} \quad (5)$$

An agent does not let the consumption of someone she cares about fall too much below her own consumption.

For all profiles of incomes before transfers, a Nash equilibrium exists and the profile of equilibrium incomes after transfers is unique. This yields a well-defined mapping from incomes before transfers \mathbf{y}^1 to incomes after transfers \mathbf{y} . With CARA utilities, this mapping has a complex piecewise linear shape which generally depends on details of the structure of the altruism network.

For tractability, and since we are interested here in how operative informal transfers affect formal insurance take-up, we make the following simplifying assumption. Say that agent i_0 receives informal support from the full community if for any $i \neq i_0$, there exists a path of informal transfers connecting i to i_0 , i.e., a set of distinct agents $j_1 = i, j_2, \dots, j_l = i_0$ such that for any $s < l$, $t_{j_s j_{s+1}} > 0$. While amounts transferred are not necessarily large, every

other agent in the community is involved in transfers eventually reaching agent i_0 .

Assumption 1. *For any realization of income shocks and any profile of formal insurance decisions, the agent hit by the idiosyncratic shock receives informal support from the full community.*

We show in the Appendix that for any connected altruism network α , there exists a threshold level on the magnitude of the idiosyncratic shock, $\bar{\lambda}$, such that Assumption (1) holds if $\lambda \geq \bar{\lambda}$. This threshold may be quite low when altruistic ties are strong and baseline nonstochastic incomes are homogeneous. It may be quite high, by contrast, when ties are weak or under baseline income heterogeneity. In any case, Assumption (1) holds (1) when the altruism network is connected and the magnitude of the individual shock is large enough. In our analysis, we thus focus on situations where the individual shock always induces informal support from the full community.

A key implication of Assumption (1) is that we can simply express how incomes after transfers depend on incomes before transfers. Let \hat{c}_{ij} denote the virtual cost of a least-cost path connecting i to j in α and let $\hat{\alpha}_{ij} = -\ln(\hat{c}_{ij})$. Transfers must flow through such least-cost paths in a Nash equilibrium. Note that when the altruism network is connected, there is a path connecting any two agents in it and these least costs are well defined for any pair of agents. Let $\bar{y}^1 = \frac{1}{n} \sum_i y_i^1$ denote the average income before transfers in the community.

Lemma 1. *Suppose that agent i_0 suffers from the individual shock and receives informal support from the full community. Then, for all i ,*

$$y_i = \bar{y}^1 + \frac{\hat{c}_{i i_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{c}_{j i_0}}{A}$$

We provide the proof of Lemma 1 and of all other results in the Appendix. Lemma 1 shows that income after transfers is equal to the sum of average income before transfers and of an amount that only depends on positions in the altruism network. In particular, income after transfers is lower for agents who are “closer” to i_0 in the altruism network. To see why, consider a binary altruism network where all links have the same strength, $\alpha_{ij} \in \{0, \alpha\}$. Then \hat{c}_{ij} is simply proportional to the network distance between i and j in the altruism network,

i.e., the number of links in a shortest path between them. In that case, y_i is higher when i is more distant from i_0 .

The insurance game. In the first stage, each agent decides whether to adopt formal insurance, anticipating how informal transfers will operate *ex-post*. A profile of insurance decisions \mathbf{x}^* is a Nash equilibrium of the insurance game if $\mathbb{E}v_i(x_i^*, x_{-i}^*) \geq \mathbb{E}v_i(x_i, x_{-i}^*)$ for all $i \in \mathcal{N}$ and $x_i \in \{0, 1\}$. The expected utility is computed over all possible realizations of common and individual shocks. In what follows, our main objectives are to characterize the Nash equilibria of the insurance game and to analyze their main properties (comparative statics, welfare).

3 Analysis

3.1 Equilibrium characterization

We start by characterizing the Nash equilibria of the insurance game. In our first main result, we show that the insurance game involves strategic substitutes: individual incentives to adopt formal insurance decrease with the number of adopters in the community. Further, an agent's decision to adopt does not depend on her position in the altruism network. We find that this yields a unique number of adopters in equilibrium and we derive an explicit formula relating the equilibrium number of adopters to the model's primitives.

To prove this result, consider some profile of insurance decisions $\mathbf{x} \in \{0, 1\}^n$. Denote by $x = \sum_i x_i$ the number of insurance adopters in profile \mathbf{x} . Similarly, let $y_0 = \sum_i y_{i0}$ denote the aggregate baseline income. Average income in the community at the end of stage 2, following insurance decisions and shocks but before informal transfers, is equal to

$$\bar{y}^1 = \frac{1}{n}(y_0 - \lambda - xp - (n - x)\mu_c). \quad (6)$$

Average income before transfers is thus a simple linear function of the number of insurance adopters.

Next, we express how expected private utility $\mathbb{E}u_i$ and expected altruistic utility $\mathbb{E}v_i$ depend on \mathbf{x} . The expectations are taken over the 2 possible realizations of the common

shocks and over the n possible realizations of the individual shock. To do this, we combine Lemma 1 on incomes after transfers and equation (6) and obtain (see Appendix)

Lemma 2. *There exist $V_i > 0$ such that*

$$\mathbb{E}v_i(\mathbf{x}) = -V_i(1 - q_c + q_c e^{\frac{A}{n}\mu(n-x)})e^{\frac{A}{n}xp}.$$

Lemma 2 shows that an agent's expected utility can be decomposed as the product of two terms: one that depends on network positions and other parameters, but not on insurance decisions, and one that depends on insurance decisions through the overall number of adopters, but not on network positions. This result captures a form of multiplicative separability. Let $\lceil x \rceil$ denote the smallest integer higher than or equal to x if x is not an integer and either one of the two possible values $\{x, x + 1\}$ if x is an integer. Introduce

$$x^* = n \left[1 - \frac{1}{A\mu} \left(\ln \frac{1 - q_c}{q_c} + \ln(e^{\frac{Ap}{n}} - 1) - \ln(1 - e^{-A\frac{\mu-p}{n}}) \right) \right]. \quad (7)$$

We can now state our first main result.

Theorem 1. *An individual is less likely to adopt formal insurance when the number of other adopters increases. A profile of insurance decisions is a Nash equilibrium of the insurance game if and only if the overall number of adopters is equal to $\min(\max(0, \lceil x^* \rceil), n)$.*

Theorem 1 has several noteworthy implications. It shows, first, that *the insurance game involves strategic substitutes*. When an agent adopts formal insurance, stochastic $-\tilde{\mu}$ is replaced by non-stochastic $-p$ in her income before transfers. Altruistic transfers then redistribute incomes among agents. Under Assumption (1), another agent's income after transfer is equal to average income before transfer plus a stochastic term that only depends on the altruism network and on who suffers from the individual shock, see Lemma 1. When an agent adopts formal insurance, stochastic $-\frac{\tilde{\mu}}{n}$ is then replaced by non-stochastic $-\frac{p}{n}$ in other agents' income after transfers. This reduces income variability and the incentives to also adopt formal insurance.

Second, as long as the altruism network stays connected and sufficiently strong and Assumption (1) holds, *Nash equilibria are unaffected by the network's structure and by agents'*

network positions. Any profile where the number of adopters is equal to the equilibrium value is a Nash equilibrium, regardless of who adopts. And these Nash equilibria do not change following changes in α that respect Assumption (1).¹¹ These unexpected properties follow from the multiplicative separability identified in Lemma 2.

Third, *the equilibrium number of adopters is generically unique.* The only exception is when x^* is an integer, in which case the equilibrium number of adopters can be x^* or $x^* + 1$. This notably implies that the demand for formal insurance is well defined: for any price of formal insurance p , there is a generically unique level of overall adoption $x(p)$. In the next Section, we study how this demand for insurance varies with price and with the other parameters of the model.

3.2 Comparative statics

We now focus on comparative statics. We obtain three results. We first analyze how the demand for formal insurance varies with price, both in the absence of altruism and in an altruistic community. We then contrast the demand for formal insurance in the two cases. This allows us to establish when formal and informal insurance are substitutes and when they are complements. Third, we study how the demand for formal insurance under altruism varies with respect to risk aversion, group size, and features of the common shock.

Consider first the demand for formal insurance when agents are not altruistic. In the absence of altruism, an agent's decision to buy formal insurance does not depend on others' decisions. Agent i 's stochastic income is then equal to $y_{i0} - p - \tilde{\lambda}_i$ if she buys insurance and to $y_{i0} - \tilde{\mu} - \tilde{\lambda}_i$ if she does not. Introduce the following price level.

$$p_N = \frac{1}{A} \ln(1 - q_c + q_c e^{A\mu}) \quad (8)$$

We show in the Appendix that price p_N makes a nonaltruistic agent indifferent between buying or not buying formal insurance. In addition, it is higher than the actuarial price, $p_N > q_c \mu$. The demand for formal insurance in the absence of altruism is therefore all or nothing: every agent adopts if $p < p_N$, while no agent adopts if $p > p_N$.

¹¹For instance, we can show that if Assumption (1) holds for α , it also holds for α' where $\alpha'_{ij} \geq \alpha_{ij}$. Nash equilibria thus do not change following increases in the strength of altruistic ties.

Consider next an altruism network under Assumption (1). In our next result, we establish that the demand for formal insurance under altruism is well-behaved, that is, downward sloping. Introduce the following two threshold values for the price:

$$\underline{p} = \frac{n}{A} \ln(1 - q_c + q_c e^{\frac{A\mu}{n}})$$

$$\bar{p} = \frac{n}{A} \ln\left(\frac{1 - q_c + q_c e^{A\mu}}{1 - q_c + q_c e^{A(1-\frac{1}{n})\mu}}\right)$$

Proposition 1. *The equilibrium demand for formal insurance $x(p)$ decreases weakly with price and is such that $x = n$ iff $p < \underline{p}$ and $x = 0$ iff $p > \bar{p}$. At the actuarial price, $p = q_c \mu < \underline{p}$ and all agents adopt formal insurance in equilibrium.*

We can now assess the impact of altruism networks on demand for formal insurance.

Theorem 2. *We have: $\underline{p} < p_N < \bar{p}$. If $p \in (\underline{p}, p_N)$, adoption of formal insurance is lower under altruism. If $p \in (p_N, \bar{p})$, adoption of formal insurance is higher under altruism.*

Theorem 2 clarifies the interactions between formal insurance and altruism networks. When the price of formal insurance is not too high, altruism networks reduce the take up of formal insurance. Formal and informal insurance are then substitutes. In contrast, when the price of formal insurance is high, altruism networks increase its take-up. Formal and informal insurance are then complements. To understand why these interactions depend on price, note that altruistic transfers play two roles here: they help reduce income variability, thus providing a source of informal insurance, and they also lead agents to share non-stochastic incomes, such as the price of formal insurance. When this price is not too high, the first effect dominates: since altruistic agents share risk informally, they have lower incentives to adopt formal insurance. In contrast, the second effect dominates when the price of insurance is high. Since the cost of buying formal insurance is shared with the community, adopting becomes relatively more attractive under high price.

We illustrate Proposition 1 and Theorem 2 in Figure 1. This Figure depicts the demands of formal insurance as a function of price for the following parameters: $n = 10$, $A = 1$, $q_c = 0.2$ and $\mu = 6$. The demand when agents are not altruistic (dashed) is 10 if $p < p_N = 4.2$ and 0

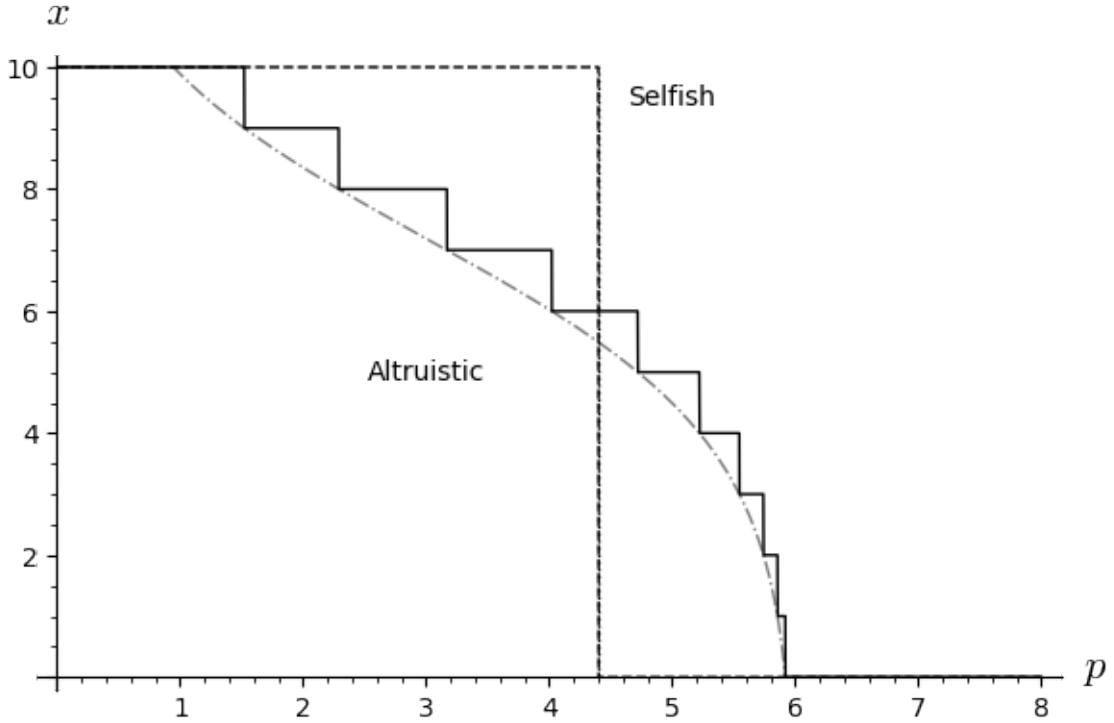


Figure 1: Demand for formal insurance

if $p > p_N$. The demand for formal insurance under altruism (plain) is 10 if $p < \underline{p} \approx 1.5$ and then decreases unit by unit, becoming 0 if $p > \bar{p} \approx 5.9$. The dashed gray curve depicts how x^* from equation (7) varies with p . Consistently with Theorem 2, we see that the demand for formal insurance is lower under altruism when $\underline{p} < p < p_N$ but higher when $p_N < p < \bar{p}$.

We now analyze how formal insurance adoption depends on the other parameters of the model. Building on Theorem 1, we analyze how x^* and x varies with the parameters.

Proposition 2. *The equilibrium number of formal insurance adopters increases weakly following an increase in common shock probability q_c , common shock magnitude μ , absolute risk aversion A , or community size n . In addition, $\frac{x^*}{n}$ increases weakly following an increase in community size n .*

The effects of q_c and μ on the number of adopters are intuitive, and qualitatively similar to effects on insurance demand for non-altruistic agents.¹² In the absence of altruism, however, insurance demand becomes all or nothing. Contrastingly, under altruism, insurance demand

¹²Indeed from equation (8), we can verify that $\frac{\partial p_N}{\partial A} > 0$, $\frac{\partial p_N}{\partial q_c} > 0$ and $\frac{\partial p_N}{\partial \mu} > 0$. Thus, the demand for formal insurance for non-altruistic agents increases weakly in A , q_c and μ .

takes intermediate values and Theorem 1 can be leveraged to measure the precise quantitative impacts of changes in parameters.

Without altruism, demand for formal insurance does not depend on community size. By contrast, Proposition 2 shows that demand for formal insurance under altruism is greater in larger communities, and this demand grows more than proportionally with community size. These effects provide another illustration of the impacts of altruistically induced interactions in adoption decisions.

Finally, the comparative statics with respect to absolute risk aversion A turn out to be difficult to assess, see Appendix. Without altruism, parameter A plays a single, unambiguous role and more risk-averse agents demand more insurance. By contrast, parameter A plays two roles under altruism, since the curvature of the utility function affects both preferences toward uncertainty and altruistic transfers in the absence of uncertainty. By Lemma 1, we see that for any incomes before transfers, income dispersion after transfers drops when A increases, since $|y_i - \bar{y}^1| = \frac{|\hat{c}_{ii_0} - \frac{1}{n} \sum_j \hat{c}_{ji_0}|}{A}$. In this sense, an increase in A yields more informal support, and this can lead to a decrease in the demand for formal insurance. We show in the Appendix that these this second effect is dominated and that the demand for formal insurance under altruism is also weakly increasing in absolute risk aversion.

3.3 Welfare

Finally, we analyze the welfare properties of the insurance game. By Lemma 2, we know that there exists a common function, $w(\cdot)$, such that for every agent i , $\mathbb{E}v_i(\mathbf{x}) = V_i w(\mathbf{x})$ with $V_i > 0$. This implies that agents' interests are aligned. When one agent takes a decision which increases her utility, the utility of all other agents also increases. As a consequence, individual incentives are fully aligned with social welfare. More precisely, say that $\mathbf{x} \in \{0, 1\}^n$ is a Pareto optimum of the insurance game if there is no other profile $\mathbf{x}' \in \{0, 1\}^n$ such that $\forall i, \mathbb{E}v_i(\mathbf{x}') \geq \mathbb{E}v_i(\mathbf{x})$ and $\exists i, \mathbb{E}v_i(\mathbf{x}') > \mathbb{E}v_i(\mathbf{x})$.

Proposition 3. *The Pareto optima of the insurance game coincide with its Nash equilibria.*

In general, informal transfers generate externalities in decisions to take up formal insurance. When an agent adopts formal insurance, her income stream changes; through informal

transfers, this affects others' income streams and utilities. Under our assumptions, however, and quite remarkably, individual incentives are aligned with social welfare. This is due to the multiplicative separability identified in Lemma 1. Proposition 3 can thus be viewed as a form of *second-best* welfare theorem. Note that the Nash equilibria of the insurance game are not first-best efficient. Since agents are risk-averse, first best outcomes would involve perfect insurance over both risks at actuarial prices. However, Proposition 3 shows that, conditional on the fact that the idiosyncratic risk is imperfectly insured by altruistic transfers, the Nash equilibria of the insurance game are constrained Pareto-efficient.

4 Extensions

In this Section, we analyze two extensions of our benchmark framework. We first consider utilities displaying Constant Relative Risk Aversion (CRRA) and then introduce a fixed utility cost of adopting formal insurance.

4.1 CRRA utilities

Assume that preferences toward risk are now represented by CRRA utility functions with coefficient of relative risk aversion $\gamma > 0$:

$$u_i(y) = \ln(y) \text{ if } \gamma = 1 \text{ and } u_i(y) = \frac{y^{1-\gamma}}{1-\gamma} \text{ if } \gamma \neq 1. \quad (9)$$

CRRA and CARA utilities both represent benchmark preferences towards risk. A distinctive, well-known feature of CRRA utilities is that they display decreasing absolute risk aversion: richer agents are less risk-averse. By contrast, risk aversion does not depend on wealth, or baseline income, under CARA.

To analyze the adoption of formal insurance under CRRA utilities, we retrace the different steps of our analysis. We first derive the counterpart to Lemma 1, relating incomes after transfers to incomes before transfers under Assumption 1. We then extend Lemma 2 and show that, remarkably, a form of multiplicative separability also holds under CRRA. This allows us to extend Theorem 1 and Proposition 3 to CRRA utilities. By contrast, the presence

of wealth effects complicates the extension of Theorem 2, as the demand for formal insurance without altruism now depends on the entire wealth distribution.

As with CARA, Assumption (1) allows us to simply express incomes after transfers.

Lemma 3. *Consider CRRA utilities. Suppose that agent i_0 suffers from the individual shock and receives informal support from the full community. Then,*

$$y_i = \frac{n\hat{\alpha}_{ii_0}^{-\frac{1}{\gamma}}}{\sum_j \hat{\alpha}_{ji_0}^{-\frac{1}{\gamma}}} \bar{y}^{-1}$$

Under CRRA and Assumption (1), income after transfers is proportional to average income before transfers. This proportion depends on agents' positions in the altruism network, relative to the agent in need. Even though expressions for incomes after transfers differ for CRRA and CARA utilities,¹³ we show next that they both induce multiplicative separability in decisions to adopt formal insurance.

Lemma 4. *Under CRRA utilities, there exist $V_i > 0$ and V'_i such that*

$$\mathbb{E}v_i(\mathbf{x}) = V_i[(1 - q_c)u(y_0 - \lambda - px) + q_c u(y_0 - \lambda - px - (n - x)\mu)] \text{ if } \gamma \neq 1 \text{ and}$$

$$\mathbb{E}v_i(\mathbf{x}) = V_i[(1 - q_c)u(y_0 - \lambda - px) + q_c u(y_0 - \lambda - px - (n - x)\mu)] + V'_i \text{ if } \gamma = 1.$$

Therefore, the altruistic expected utilities of all agents are affected by the number of adopters through a common function. We show in the Appendix that, as under CARA, this common function is concave. As a consequence, Theorem 1 and Proposition 3 directly extend. The insurance game involves strategic substitutes, Nash equilibria are characterized by a generically unique number of adopters, adoption does not depend on network position, and Nash equilibria coincide with the Pareto optima of the insurance game. Under CRRA, however, the equilibrium number of adopters cannot be expressed as a simple explicit function of the parameters. Moreover, in contrast to CARA and due to wealth effects, Nash equilibria also depend on overall baseline income y_0 and on the size of the individual shock λ .

Formally, let $w(x) = (1 - q_c)u(y_0 - \lambda - px) + q_c u(y_0 - \lambda - px - (n - x)\mu)$. We show in the Appendix that $w(\cdot)$ is strictly concave, and hence the function $X \rightarrow w(X + 1) - w(X)$

¹³Note that under CARA, the *difference* between income after transfers and average income before transfers depends on relative network positions.

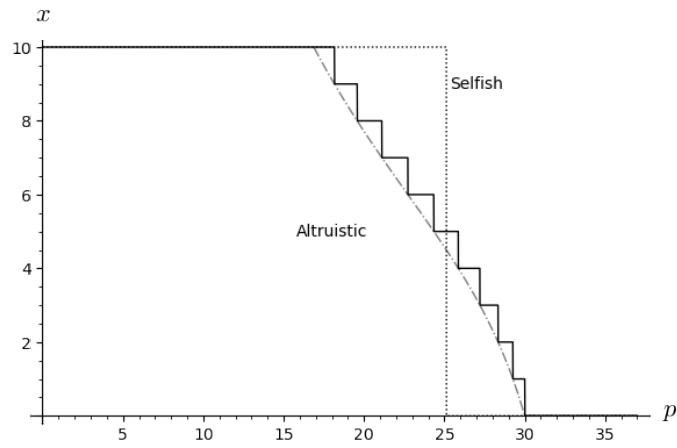
is strictly decreasing. Define $x_{CRR A}^*$ as follows: $x_{CRR A}^* = 0$ if $w(1) - w(0) < 0$, $x_{CRR A}^* = n$ if $w(n) - w(n - 1) > 0$ and otherwise $x_{CRR A}^* = \lceil X \rceil$ where X is the unique solution of the equation $w(X + 1) - w(X) = 0$.

Theorem 3. *Consider CRRA utilities. An individual is less likely to adopt formal insurance when the number of other adopters increases. An insurance profile is a Nash equilibrium if and only if the number of adopters is equal to $x_{CRR A}^*$. The Pareto optima of the insurance game coincide with its Nash equilibria.*

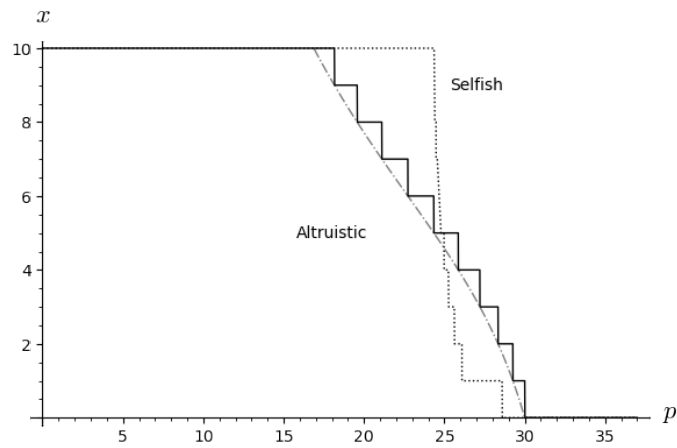
Theorem 3 shows that key features of equilibrium behavior uncovered in Section 3 under CARA also hold under CRRA. By contrast, analysis of the demand for formal insurance is complicated by the presence of wealth effects. Under CARA, regardless of altruism, demand for insurance does not depend on initial levels of wealth, y_{i0} . Under CRRA, Theorem 3 shows that the demand for insurance under altruism depends on the aggregate level of initial wealth, $y_0 = \sum_i y_{i0}$. By contrast, the demand for insurance of agent i in the absence of altruism depends on her initial wealth y_{i0} . Since absolute risk aversion is decreasing with wealth, the threshold price below which an individual buys insurance is higher for poorer agents. Overall demand for insurance in the absence of altruism then depends on the full distribution of wealth levels, \mathbf{y}_0 .

We illustrate three possible outcomes in Figure 2. We depict the demand for formal insurance with and without altruism under three scenarios, and for the following parameter values: $n = 10, \gamma = 1, \mu = 35, q_c = 0.5, q_i = 0.1$ and $y_0 = 415$. In the Upper panel, all agents have the same initial wealth. Demand for insurance without altruism has the same all or nothing shape as under CARA. Theorem 2 extends: demand for formal insurance under altruism is lower at low prices and higher at high prices. In the Middle panel, agents have different wealth levels but wealth variance is low.¹⁴ Demand for formal insurance in the absence of altruism now decreases step by step. However, Theorem 2 still extends, as the demand without altruism crosses the demand with altruism once from above in the intermediate price range. In the Lower panel, agents are either poor or rich, and wealth variance is high. We now see that Theorem 2 does not extend. The two demand curves cross

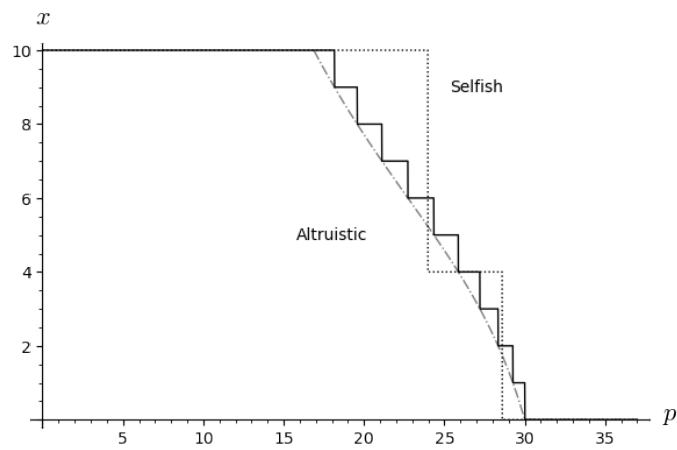
¹⁴We describe in the Appendix the wealth levels in scenarios 2 and 3.



(a) No wealth variance



(b) Low wealth variance



(c) High wealth variance

Figure 2: Demand for formal insurance under CRRA utilities

three times in the intermediate range. As the insurance price increases, formal and informal institutions are, successively, substitutes, complements, substitutes, and complements again.

4.2 Fixed utility cost of adoption

In this second extension, we introduce a fixed utility cost from adopting formal insurance, aiming to capture psychological or cognitive costs entailed in adopting a new, unknown insurance product. The presence of such costs could help explain the documented low adoption rates, see Introduction.

Formally, assume that adopting formal insurance entails a utility cost $C > 0$ and otherwise maintain the assumptions of our benchmark model. By Lemma 2, the expected altruistic utility earned by agent i when the adoption profile is (x_i, \mathbf{x}_{-i}) is now equal to $\mathbb{E}v_i(x_i, \mathbf{x}_{-i}) = V_i w(\sum_j x_j) - Cx_i$ with $V_i > 0$ and $w(x) = -(1 - q_c + q_c e^{\frac{\lambda}{n}\mu(n-x)})e^{\frac{\lambda}{n}xp}$.

In our next result, we extend Theorem 1. We show that the number of adopters is still uniquely pinned down - and this number of course decreases weakly with C . We also show that network positions now matter. In particular, agents with the highest V_i 's always adopt while agents with the lowest V_i 's never adopt. Formally, order agents through decreasing values of V_i : $V_1 \geq V_2 \geq \dots \geq V_n$. Define k^* as follows: $k^* = 0$ if $V_1(w(1) - w(0)) < C$, $k^* = n$ if $V_n(w(n) - w(n-1)) > C$ and otherwise, $k^* = \max\{k \in N : V_k(w(k) - w(k-1)) \geq C\}$.

Theorem 4. *Consider the model with a fixed utility cost of adoption. Order agents through decreasing values of V_i 's. An agent is less likely to adopt if adoption by other agents increases. The equilibrium number of adopters is generically unique and equal to k^* . Moreover, there exist $k_1 \leq k_2$ such that a profile is a Nash equilibrium if and only if agents 1 to k_1 adopt; of the $k_1 + 1$ to k_2 agents, $k^* - k_1$ adopt; and agents $k_2 + 1$ to n do not adopt.*

As Theorem 4 demonstrates, a key implication of the utility cost is that the decision to adopt may now depend on the individual's position in the altruism network. To illustrate, consider the following particular case. Assume that every agent has the same probability of facing the idiosyncratic shock, $q_i = \frac{1}{n}$, and that every altruistic link has the same intensity, $\alpha_{ij} > 0 \Rightarrow \alpha_{ij} = \alpha$. Let d_{ij} be the network distance between i and j , i.e., the number of links in a shortest path from i to j , with the convention that $d_{ii} = 0$. In this case, the virtual cost

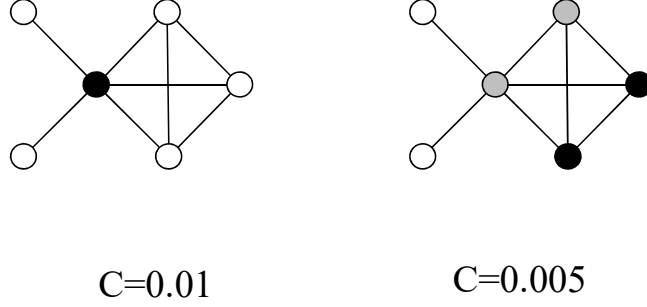


Figure 3: Adoption with a utility cost

of a least cost path connecting i to j is equal to $\hat{c}_{ij} = cd_{ij}$ with $c = -\ln(\alpha)$. Let $\bar{d}_i = \frac{1}{n} \sum_j d_{ij}$ be the average distance from i to others in the network.

From the computations leading to Lemma 2, in the Appendix, we can show that there exists $U > 0$ such that $V_i = U_i + \sum_j \alpha_{ij} U_j$ and $U_i = U \sum_j e^{-c(d_{ij} - \bar{d}_j)}$. Therefore, an agent has a higher value of V_i , and is more likely to adopt formal insurance, when she has more altruistic links, when she is closer to others in the network, and when her neighbors are also closer to others. An agent at a shorter distance from others is more involved in informal support and obtains a lower income after transfers, on average. By concavity, this yields greater variations in expected private utility. Through altruism, being connected with an agent at a shorter distance from others also yields greater variations in expected altruistic utility. This leads to a higher incentive to adopt, in the presence of a fixed utility cost.

We illustrate Theorem 4 in Figure 3. We consider a network with $n = 6$ agents. Agent 1 is central and connected to every other agent, agents 2, 3 and 4 are connected to agent 1 and to each other, while agents 5 and 6 are peripheral and only connected to agent 1. We consider the following parameter values: $A = 0.21$, $\lambda = \mu = 10$, $q_c = 0.5$, $q_i = 1/n$, $\alpha = 0.9$ and $y_0 = 100$. This yields $V_1 > V_2 = V_3 = V_4 > V_5 = V_6$. In the Left panel, $C = 0.01$ and we obtain $k^* = k_1 = 1$ while $k_2 = 4$. At these values, the utility cost is relatively high and the center alone adopts. This is a case where adoption is fully determined by network positions. In the Right panel, there is a drop in utility cost, with $C = 0.005$. In this case, $k^* = 2$ while $k_1 = 0$ and $k_2 = 4$. There are now 2 adopters, who can be any pair of agents 1 to 4. Agents

5 and 6 do not adopt, however. Adoption is now partly determined by network positions.

5 Conclusion

We provide the first analysis of the introduction of formal insurance into a community connected through altruistic ties. Agents face both a community-level and an individual-level shock and can adopt formal insurance to cover the community shock. We assume that the altruism network is connected and that every member is involved in informal support. Under altruism, the decisions to adopt formal insurance become interdependent and we characterize the Nash equilibria of the insurance game. Adoption decisions involve strategic substitutes and lead to a generically unique number of adopters. The demand for insurance under altruism is lower at relatively low prices, but higher at relatively high prices due to the fact that insurance costs end up being shared with others. Nash equilibria are constrained Pareto efficient. We then consider CRRA utilities and a utility cost of adoption.

There are a number of natural directions that future research would take. Incomes could have a more complex stochastic structure, with non-binary and non-independent shocks. Informal support may not always involve the full community. Agents could also, at a cost, reduce the risks faced, giving rise to endogenous risk-taking and moral hazard, as in [Belhaj and Deroian \(2012\)](#) and [Alger and Weibull \(2010\)](#). Developing a full-fledged analysis of the demand for formal insurance under complex income shocks, general altruism networks, and moral hazard would be potentially fruitful and certainly challenging.

6 Appendix A: Proofs

Proof of statements on Assumption (1). From Proposition 4 in Bourlès, Bramoullé and Perez-Richet (2020), we know that if the altruism network is connected, then for any realization of the idiosyncratic shock i_0 , any realization of the common shock and any profile of formal insurance, there exists a threshold level such that Assumption (1) holds if the size of the idiosyncratic shock is greater than this threshold level. Then, define $\hat{\lambda}$ as the maximum of these threshold levels over the finite realizations of shocks and insurance profiles. \square

Proof of Lemma 1. From Assumption (1) and Bourlès, Bramoullé and Perez-Richet (2017), we know that $u'(y_i) = \hat{\alpha}_{ij}u'(y_{i_0})$ for every $i \neq i_0$. This is equivalent to

$$y_i - y_{i_0} = \frac{\hat{c}_{ii_0}}{A}$$

for every $i \neq i_0$. Conservation of income then implies that

$$\sum_i y_i = ny_{i_0} + \sum_{i \neq i_0} \frac{\hat{c}_{ii_0}}{A} = \sum_i y_i^1$$

Therefore, $y_{i_0} = \bar{y}^1 - \frac{1}{n} \sum_{i \neq i_0} \frac{\hat{c}_{ii_0}}{A}$ while $y_i = y_{i_0} + \frac{\hat{c}_{ii_0}}{A}$. \square

Proof of Lemma 2. By Lemma 1 and equation (6), the income after transfers of agent i is equal to

$$y_i = \frac{1}{n}(y_0 - \lambda - xp - (n-x)\mu_c) + \frac{\hat{c}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{c}_{ji_0}}{A}$$

Taking the expectation over realizations of the common shock and the individual shocks yields

$$\mathbb{E}u_i = (1-q_c) \left(- \sum_{i_0} q_{i_0} e^{-A(\frac{1}{n}(y_0 - \lambda - xp) + \frac{\hat{c}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{c}_{ji_0}}{A})} \right) + q_c \left(- \sum_{i_0} q_{i_0} e^{-A(\frac{1}{n}(y_0 - \lambda - xp - (n-x)\mu) + \frac{\hat{c}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{c}_{ji_0}}{A})} \right)$$

which can be rewritten

$$\mathbb{E}u_i = \left[- \sum_{i_0} q_{i_0} e^{-A(\frac{1}{n}(y_0 - \lambda) + \frac{\hat{c}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{c}_{ji_0}}{A})} \right] e^{\frac{A}{n}xp} [1 - q_c + q_c e^{\frac{A}{n}(n-x)\mu}]$$

And define $U_i = \sum_{i_0} q_{i_0} e^{-A(\frac{1}{n}(y_0 - \lambda) + \frac{\hat{c}_{ii_0}}{A} - \frac{1}{n} \sum_j \frac{\hat{c}_{ji_0}}{A})} > 0$ such that $\mathbb{E}u_i = -U_i e^{\frac{A}{n}xp} [1 - q_c + q_c e^{\frac{A}{n}(n-x)\mu}]$. Next, we have

$$\mathbb{E}v_i = \mathbb{E}u_i + \sum_j \alpha_{ij} \mathbb{E}u_j$$

Define $V_i = U_i + \sum_j \alpha_{ij} U_j > 0$. This yields

$$\mathbb{E}v_i = -V_i e^{\frac{A}{n}xp} [1 - q_c + q_c e^{\frac{A}{n}(n-x)\mu}]$$

□

Proof of Theorem 1. By Lemma 2, we have:

$$\mathbb{E}v_i(0, \mathbf{x}_{-i}) = -V_i e^{\frac{A}{n}x_{-i}p} [1 - q_c + q_c e^{\frac{A}{n}(n-x_{-i})\mu}]$$

and

$$\mathbb{E}v_i(1, \mathbf{x}_{-i}) = -V_i e^{\frac{A}{n}x_{-i}p} e^{\frac{A}{n}p} [1 - q_c + q_c e^{\frac{A}{n}(n-x_{-i})\mu} e^{-\frac{A}{n}\mu}]$$

Agent i adopts formal insurance iff $\mathbb{E}v_i(1, \mathbf{x}_{-i}) \geq \mathbb{E}v_i(0, \mathbf{x}_{-i})$. Simplifying and rearranging, this is equivalent to

$$e^{\frac{A}{n}(n-x_{-i})\mu} \geq \frac{1 - q_c}{q_c} \frac{e^{\frac{A}{n}p} - 1}{1 - e^{-\frac{A}{n}(\mu-p)}}$$

Note that the left hand side is a decreasing function of x_{-i} , which shows that agent i is less likely to adopt as x_{-i} increases. Moreover, let $x^* \in \mathbb{R}$ denote the unique value for which equality holds. This shows that $\mathbb{E}v_i(1, \mathbf{x}_{-i}) \geq \mathbb{E}v_i(0, \mathbf{x}_{-i}) \Leftrightarrow x_{-i} \leq x^*$.

If $x^* > n - 1$, then this condition is always satisfied. All agents adopt and a Nash equilibrium is such that $\sum_i x_i = n$.

If $x^* < 0$, then this condition is never satisfied, no agent adopts and a Nash equilibrium is such that $\sum_i x_i = 0$.

If $x^* \in [0, n - 1]$, a Nash equilibrium \mathbf{x} is such that $x_i = 1 \Rightarrow x_{-i} \leq x^*$ and $x_i = 0 \Rightarrow x_{-i} \geq x^*$. Since $x_i = 1 \Rightarrow x_{-i} = x - 1$ and $x_i = 0 \Rightarrow x_{-i} = x$, this is equivalent to $x^* \leq x \leq x^* + 1$. □

Proof of Proposition 1. Compute the derivative of x^* with respect to p :

$$\frac{\partial x^*}{\partial p} = -\frac{1}{\mu} \left(\frac{e^{\frac{A}{n}p}}{e^{\frac{A}{n}p} - 1} + \frac{e^{\frac{A}{n}(p-\mu)}}{1 - e^{\frac{A}{n}(p-\mu)}} \right)$$

and we see that $\frac{\partial x^*}{\partial p} < 0$. Therefore, x^* is decreasing in p and hence equilibrium demand is weakly decreasing in p .

The two threshold values of prices are defined by $x^*(\underline{p}) = n - 1$ and $x^*(\bar{p}) = 0$. Substituting into the formula for x^* and solving for the price levels yields the result.

Finally, note that $q_c \mu < \underline{p}$ is equivalent to

$$e^{\frac{A}{n}q_c \mu} < (1 - q_c)e^0 + q_c e^{\frac{A}{n}\mu}$$

and since $(1 - q_c)(0) + q_c(\frac{A}{n}\mu) = \frac{A}{n}q_c \mu$, this holds by strict convexity of the exponential function. □

Proof of Theorem 2. A nonaltruistic agent adopts formal insurance if and only if

$$-e^{-Ay_{i0}}[(1-q_c)(1-q_i+q_i e^{A\lambda})+q_c((1-q_i)e^{A\mu}+q_i e^{A(\lambda+\mu)})] \leq -e^{-Ay_{i0}}[(1-q_i)e^{Ap}+q_i e^{A(p+\lambda)}]$$

Simplifying and rearranging, this is equivalent to

$$p \leq \frac{1}{A} \ln(1-q_c+q_c e^{A\mu}) = p_N$$

In addition, $p_N > q_c \mu$ is equivalent to $1-q_c+q_c e^{A\mu} > e^{Aq_c \mu}$, which holds by strict convexity of the exponential function.

Next, show that $\underline{p} < p_N$. This inequality is equivalent to

$$\frac{n}{A} \ln(1-q_c+q_c e^{\frac{A\mu}{n}}) < \frac{1}{A} \ln(1-q_c+q_c e^{A\mu})$$

and hence to

$$(1-q_c+q_c e^{\frac{A\mu}{n}})^n < 1-q_c+q_c e^{A\mu}$$

This holds if the function

$$f(n) = (1-q_c+q_c e^{\frac{A\mu}{n}})^n$$

is decreasing with n . Treating n as a continuous variable, it is sufficient to show that $\frac{\partial f(n)}{\partial n} < 0$. Taking the derivative and simplifying, this amounts to show that

$$\ln(1-q_c+q_c e^{\frac{A\mu}{n}}) < q_c \frac{A\mu}{n} \frac{e^{\frac{A\mu}{n}}}{1-q_c+q_c e^{\frac{A\mu}{n}}}$$

Let $y = e^{\frac{A\mu}{n}}$, so that $y > 1$. The previous inequality is equivalent to

$$\phi(q_c) = (1-q_c+q_c y) \ln(1-q_c+q_c y) - q_c y \ln y < 0.$$

The function $\phi(\cdot)$ is a strictly convex function of q_c , since $\phi''(q_c) = \frac{(y-1)^2}{1-q_c+q_c y} > 0$. Moreover, $\phi(0) = \phi(1) = 0$ and therefore $\phi(q_c) < 0$ for $q_c \in]0, 1[$. This implies that $\underline{p} < p_N$.

Finally, we show that $\bar{p} > p_N$. This inequality is equivalent to

$$n \left[1 - \frac{\ln(1-q_c+q_c e^{\frac{n-1}{n} A\mu})}{\ln(1-q_c+q_c e^{A\mu})} \right] > 1$$

and hence to

$$(1-q_c+q_c e^{A\mu})^{\frac{n-1}{n}} > (1-q_c+q_c e^{\frac{n-1}{n} A\mu})$$

Introduce the function ψ such that

$$\psi(\mu) = (1-q_c+q_c e^{A\mu})^{\frac{n-1}{n}} - (1-q_c+q_c e^{\frac{n-1}{n} A\mu})$$

This function satisfies $\psi(0) = 0$ and $\psi'(\mu) > 0$. Indeed,

$$\psi'(\mu) = \frac{n-1}{n} q_c A e^{A\mu} (1 - q_c + q_c e^{A\mu})^{-\frac{1}{n}} - \frac{n-1}{n} q_c A e^{\frac{n-1}{n} A\mu}$$

and $\psi'(\mu) > 0$ if $e^{A\mu} > 1 - q_c + q_c e^{A\mu}$. This last condition is always met. Thus $\psi(\mu) > 0$ which implies $\bar{p} > p_N$. \square

Proof of Proposition 2. Compute the derivatives of x^* in equation (7) with respect to the various parameters and under the assumption that $0 \leq x^* \leq n$. Recall, also, that $0 \leq p \leq \mu$.

$$\begin{aligned} \frac{\partial x^*}{\partial q_c} &= \frac{n}{A\mu} \left(\frac{1}{1 - q_c} + \frac{1}{q_c} \right) > 0 \\ \frac{\partial x^*}{\partial \mu} &= \frac{1}{\mu} (n - x^*) + \frac{1}{\mu} \frac{e^{-A\frac{\mu-p}{n}}}{1 - e^{-A\frac{\mu-p}{n}}} > 0 \\ \frac{\partial \frac{x^*}{n}}{\partial n} &= -\frac{1}{A\mu} \left[-\frac{Ap}{n^2} \frac{e^{\frac{Ap}{n}}}{e^{\frac{Ap}{n}} - 1} - \frac{A(p-\mu)}{n^2} \frac{e^{\frac{A(p-\mu)}{n}}}{1 - e^{\frac{A(p-\mu)}{n}}} \right] \\ \frac{\partial \frac{x^*}{n}}{\partial n} &= \frac{e^{\frac{Ap}{n}}}{\mu n^2 (e^{\frac{Ap}{n}} - 1) (1 - e^{\frac{A(p-\mu)}{n}})} (p - \mu e^{\frac{A(p-\mu)}{n}} - (p - \mu) e^{-\frac{A\mu}{n}}) \end{aligned}$$

The first part of the right hand side is positive. Introduce $f(n) = p - \mu e^{\frac{A(p-\mu)}{n}} - (p - \mu) e^{-\frac{A\mu}{n}}$. Since $f'(n) = \frac{A\mu(\mu-p)}{n^2} [e^{-\frac{A\mu}{n}} - e^{\frac{A(p-\mu)}{n}}] < 0$ and $\lim_{n \rightarrow \infty} f(n) = 0$, this implies that $f(n) > 0$.

Therefore, $\frac{\partial \frac{x^*}{n}}{\partial n} > 0$ and hence $\frac{\partial x^*}{\partial n} > 0$.

Finally, we consider the impact of parameter A . Rewrite x^* as follows:

$$x^* = n - 1 + \frac{p}{\mu} - \frac{n}{A\mu} \left(\ln \frac{1 - q_c}{q_c} + \ln \frac{e^{\frac{Ap}{n}} - 1}{e^{\frac{A(\mu-p)}{n}} - 1} \right)$$

For convenience, we denote $\beta = \frac{1 - q_c}{q_c}$, $\Delta = \ln \beta + \ln \frac{e^{\frac{Ap}{n}} - 1}{e^{\frac{A(\mu-p)}{n}} - 1}$, $f(x) = \frac{x}{1 - e^{-x}}$, and note that f is increasing over $]0, +\infty[$. Consider the following three conditions:

First, $x^* \geq 0$ if and only if

$$\Delta \leq \frac{Ap}{n} + \frac{A\mu(n-1)}{n} \tag{10}$$

Second, $x^* \leq n - 1$ if and only if

$$\Delta \geq \frac{Ap}{n} \tag{11}$$

Third,

$$\frac{\partial x^*}{\partial A} = \frac{n}{A^2 \mu} \Delta - \frac{n}{A \mu} \left(\frac{p}{n} \frac{e^{\frac{Ap}{n}}}{e^{\frac{Ap}{n}} - 1} - \frac{\mu - p}{n} \frac{e^{\frac{A(\mu-p)}{n}}}{e^{\frac{A(\mu-p)}{n}} - 1} \right)$$

which yields $\frac{\partial x^*}{\partial A} < 0$ if and only if

$$\Delta < f\left(\frac{Ap}{n}\right) - f\left(\frac{A(\mu-p)}{n}\right) \quad (12)$$

We next show that (10), (11) and (12) cannot hold simultaneously. For this, we partition the set of parameter possible values in three regions, and we prove our statement for each of the three cases.

Case 1: $\mu - p > p$. Then $\Delta < 0$ by (12), but $\Delta > 0$ by (11), which is impossible.

Case 2: $\mu - p < p$ and $q_c \leq \frac{1}{2}$. Here we have $\beta \geq 1$. Equation (12) can also be written, using the exponential function:

$$\beta \frac{e^{\frac{Ap}{n}} - 1}{e^{\frac{A(\mu-p)}{n}} - 1} < e^{f\left(\frac{Ap}{n}\right)} e^{-f\left(\frac{A(\mu-p)}{n}\right)}$$

That is, denoting $h(x) = (e^x - 1)e^{-f(x)}$,

$$\beta h\left(\frac{Ap}{n}\right) < h\left(\frac{A(\mu-p)}{n}\right) \quad (13)$$

We observe that function h is increasing: indeed, $h > 0$ and $(\ln(h))' = \frac{xe^{-x}}{(1-e^{-x})^2} > 0$. Hence, (13) does not hold since $\beta \geq 1$ and $\frac{A(\mu-p)}{n} < \frac{Ap}{n}$, and thus $\frac{\partial x^*}{\partial A} > 0$.

Case 3: $\mu - p < p$ et $q_c > \frac{1}{2}$. We now have $0 < \beta < 1$ and $\frac{A(\mu-p)}{n} < \frac{Ap}{n}$. It is sufficient to show that

$$\frac{h\left(\frac{A(\mu-p)}{n}\right)}{h\left(\frac{Ap}{n}\right)} < e^{\frac{Ap}{n}} \left(\frac{e^{\frac{A(\mu-p)}{n}} - 1}{e^{\frac{Ap}{n}} - 1} \right) \quad (14)$$

By equation (14), equations (11) and (12) cannot hold simultaneously. Indeed, equation (11) is equivalent to $\beta \geq e^{\frac{Ap}{n}} \left(\frac{e^{\frac{A(\mu-p)}{n}} - 1}{e^{\frac{Ap}{n}} - 1} \right)$, while equation (12) is equivalent to $\beta < \frac{h\left(\frac{A(\mu-p)}{n}\right)}{h\left(\frac{Ap}{n}\right)}$.

Now, given function h , (14) is equivalent to:

$$e^{-f\left(\frac{A(\mu-p)}{n}\right)} < e^{-f\left(\frac{Ap}{n}\right)} e^{\frac{Ap}{n}}$$

Taking the logs, this is equivalent to

$$f\left(\frac{Ap}{n}\right) - \frac{Ap}{n} < f\left(\frac{A(\mu-p)}{n}\right) \quad (15)$$

Note that $f(x) - x = \frac{xe^{-x}}{1-e^{-x}}$. Let $0 < y = x - \delta < x$. Equation (15) holds if

$$f(x) - x < f(y)$$

which is equivalent to

$$\frac{x}{e^x - 1} < \frac{(x - \delta)e^{(x-\delta)}}{e^{(x-\delta)} - 1}$$

I.e.,

$$\frac{e^x - e^\delta}{x - \delta} < \frac{e^x - 1}{x}e^x$$

Let $\varphi(\delta) = \frac{e^x - e^\delta}{x - \delta}$. We have $\varphi(0) = \frac{e^x - 1}{x}$, φ increasing and $\lim_{\delta \rightarrow x} \varphi(\delta) = e^x$. At the limit, which is sufficient, the equation requires $e^x - x - 1 > 0$, which holds. Thus equation (15) - and hence equation (14) - holds. In this case, we also have $\frac{\partial x^*}{\partial A} > 0$. \square

Proof of Proposition 3. Show, first, that Pareto optima are the maxima of the function $w(\cdot)$. If \mathbf{x} is not a Pareto optimum, there exists \mathbf{x}' and i such that $\mathbb{E}v_i(\mathbf{x}') > \mathbb{E}v_i(\mathbf{x})$. This implies that $V_i w(\mathbf{x}') > V_i w(\mathbf{x})$ and hence \mathbf{x} is not a maximum of $w(\cdot)$. Reciprocally, suppose that \mathbf{x} is not a maximum of $w(\cdot)$ and let \mathbf{x}' be such that $w(\mathbf{x}') > w(\mathbf{x})$. Then, for every i , $V_i w(\mathbf{x}') > V_i w(\mathbf{x})$ and hence $\mathbb{E}v_i(\mathbf{x}') > \mathbb{E}v_i(\mathbf{x})$ and \mathbf{x} is not a Pareto optimum.

Next, show that the Nash equilibria are also the maxima of the function $w(\cdot)$. Consider a Nash equilibrium where $0 < x < n$. Agent i who plays $x_i = 1$ plays a best-response if $\mathbb{E}v_i(1, \mathbf{x}_{-i}) \geq \mathbb{E}v_i(0, \mathbf{x}_{-i})$. By Lemma 1, this is equivalent to $w(x) \geq w(x - 1)$. Similarly, agent i who plays 0 plays a best response if $\mathbb{E}v_i(0, \mathbf{x}_{-i}) \geq \mathbb{E}v_i(1, \mathbf{x}_{-i})$, which is equivalent to $w(x) \geq w(x + 1)$. Therefore x is a maximum of $w(\cdot)$ over $\{x - 1, x, x + 1\}$, i.e., a local maximum. Finally, note that $w(\cdot)$ is the sum of two strictly concave functions and hence is strictly concave. Its local maxima therefore coincide with its global maxima. \square

Proof of Lemma 3. Recall, under Assumption 1, $u'(y_i) = \hat{\alpha}_{ij} u'(y_{i_0})$ for every $i \neq i_0$. This is equivalent to $y_i^{-\gamma} = \hat{\alpha}_{ij} y_{i_0}^{-\gamma}$ and hence $y_i = \hat{\alpha}_{ij}^{-\frac{1}{\gamma}} y_{i_0}$. Conservation of incomes then implies that $\sum_j y_j = \sum_j \hat{\alpha}_{ji_0}^{-\frac{1}{\gamma}} y_{i_0} = n\bar{y}^1$, which yields the result. \square

Proof of Lemma 4. The income after transfers of agent i is thus equal to

$$y_i = \frac{\hat{\alpha}_{i i_0}^{-\frac{1}{\gamma}}}{\sum_j \hat{\alpha}_{j i_0}^{-\frac{1}{\gamma}}} (y_0 - \lambda - px - (n - x)\mu)$$

If $\gamma \neq 1$, taking expectations over realizations of shocks yields

$$\mathbb{E}u_i(x) = \left(\sum_{i_0} q_{i_0} \left(\frac{\hat{\alpha}_{i i_0}^{-\frac{1}{\gamma}}}{\sum_j \hat{\alpha}_{j i_0}^{-\frac{1}{\gamma}}} \right)^{1-\gamma} \right) [(1 - q_c)u(y_0 - \lambda - px) + q_c u(y_0 - \lambda - px - (n - x)\mu)]$$

Setting $U_i = \sum_{i_0} q_{i_0} \left(\frac{\hat{\alpha}_{i_0}^{-\frac{1}{\gamma}}}{\sum_j \hat{\alpha}_{j i_0}^{-\frac{1}{\gamma}}} \right)^{1-\gamma}$ and $V_i = U_i + \sum_j \alpha_{ij} U_j$ yields the result.

If $\gamma = 1$, we have:

$$\mathbb{E}u_i(x) = \sum_{i_0} q_{i_0} \ln \left(\frac{\hat{\alpha}_{i_0}^{-1}}{\sum_j \hat{\alpha}_{j i_0}^{-1}} \right) + (1 - q_c) \ln(y_0 - \lambda - px) + q_c \ln(y_0 - \lambda - px - (n - x)\mu).$$

Setting $U_i = \sum_{i_0} q_{i_0} \ln \left(\frac{\hat{\alpha}_{i_0}^{-1}}{\sum_j \hat{\alpha}_{j i_0}^{-1}} \right)$, $V_i = U_i + \sum_j \alpha_{ij} U_j$ and $V'_i = 1 + \sum_j \alpha_{ij}$ yields the result. \square

Proof of Theorem 3. In what follows, let $w(x) = (1 - q_c)u(y_0 - \lambda - px) + q_c u(y_0 - \lambda - px - (n - x)\mu)$. Compute the derivatives of w with respect to x .

$$w'(x) = (1 - q_c)(-p)(y_0 - \lambda - px)^{-\gamma} + q_c(\mu - p)(y_0 - \lambda - px - (n - x)\mu)^{-\gamma}$$

$$w''(x) = -\gamma[(1 - q_c)p^2(y_0 - \lambda - px)^{-\gamma-1} + q_c(\mu - p)^2(y_0 - \lambda - px - (n - x)\mu)^{-\gamma-1}]$$

Therefore, $w''(x) < 0$ and w is strictly concave.

Next, a profile \mathbf{x} is a Nash equilibrium if and only if $x_i = 1 \Rightarrow \mathbb{E}v_i(1, \mathbf{x}_{-i}) \geq \mathbb{E}v_i(0, \mathbf{x}_{-i})$ and $x_i = 0 \Rightarrow \mathbb{E}v_i(0, \mathbf{x}_{-i}) \geq \mathbb{E}v_i(1, \mathbf{x}_{-i})$. By Lemma 3, this is equivalent to $x_i = 1 \Rightarrow w(x) \geq w(x - 1)$ and $x_i = 0 \Rightarrow w(x) \geq w(x + 1)$. Therefore, a Nash equilibrium is interior if and only if the overall number of adopters x is a local maximum of the function $w(\cdot)$ over the set $\{x - 1, x, x + 1\}$.

By strict concavity of $w(\cdot)$, the function $X \rightarrow w(X + 1) - w(X)$ is strictly decreasing. This implies that an agent is less likely to adopt when the number of other adopters increases. There are 3 mutually exclusive cases. (1) If $w(1) - w(0) < 0$, then $\mathbf{x} = \mathbf{0}$ (no adoption) is the only Nash equilibrium. (2) If $w(n) - w(n - 1) > 0$, then $\mathbf{x} = \mathbf{1}$ (full adoption) is the only Nash equilibrium. (3) There exists a unique $x^* \in [0, n - 1]$ such that $w(x^* + 1) - w(x^*) = 0$. Then a profile is a Nash equilibrium if and only if $x^* \leq x \leq x^* + 1$. Generically, x^* is not an integer and then x is the smallest integer larger than or equal to x^* . By contrast if x^* is an integer, the number of adopters in equilibrium can be equal to x^* or $x^* + 1$.

Thank to Lemma 3 and to the strict concavity of $w(\cdot)$, the proof of Proposition 3 directly extends to the CRRA case: Nash equilibria and Pareto optima coincide, and are such that the overall number of adopters is a maximum of the function $w(\cdot)$ over $\{0, 1, \dots, n\}$. \square

Wealth levels underlying Figure 2. Wealth levels in the low variance scenario (Middle Panel) are equal to $y_1 = 37$; $y_2 = y_1 + 2.4 + 1/3$; $y_3 = y_1 + 3.2 + 1/3$; $y_4 = y_1 + 3.9 + 1/3$; $y_5 = y_1 + 4.5 + 1/3$; $y_6 = y_1 + 5 + 1/3$; $y_7 = y_1 + 5.4 + 1/3$; $y_8 = y_1 + 5.7 + 1/3$; $y_9 = y_1 + 5.9 + 1/3$; $y_{10} = y_1 + 6 + 1/3$.

Wealth levels in the high variance scenario (Right Panel) are equal to $y_1 = \dots = y_4 = 37$; $y_5 = \dots = y_{10} = 44.5$. \square

Proof of Theorem 4. A profile \mathbf{x} is a Nash equilibrium iff $x_i = 1 \Rightarrow V_i(w(x) - w(x - 1)) \geq C$ and $x_i = 0 \Rightarrow V_i(w(x + 1) - w(x)) \leq C$.

Suppose first that $V_1(w(1) - w(0)) < C$. Then, $w(1) - w(0) < \frac{C}{V_1} \leq \frac{C}{V_i}$ for any i . If $x > 0$, then $w(x) - w(x - 1) \geq \frac{C}{V_i}$ for some i . However, $w(x) - w(x - 1) \leq w(1) - w(0) < \frac{C}{V_i}$ a contradiction. Therefore, the profile where no one adopts is the only Nash equilibrium. Similarly, if $V_n(w(n) - w(n - 1)) > C$, the profile where everyone adopts is the only Nash equilibrium.

Assume, then, that $V_1(w(1) - w(0)) \geq C$ and $V_n(w(n) - w(n - 1)) \leq C$. Let $k^* = \max\{k \in N : V_k(w(k) - w(k - 1)) \geq C\}$ and $\bar{k} = \max\{k \in N : w(k) - w(k - 1) \geq 0\}$. The function $k \rightarrow V_k(w(k) - w(k - 1))$ is decreasing over $\{0, 1, \dots, \bar{k}\}$ since it is the product of one function which is positive and weakly decreasing and another which is positive and strictly decreasing. Then, it crosses zero precisely between \bar{k} and $\bar{k} + 1$, and stays negative above \bar{k} . Moreover, $V_k(w(k) - w(k - 1)) \geq C \Rightarrow w(k) - w(k - 1) \geq 0$. This implies that $k^* \leq \bar{k}$ and, moreover, $k \leq k^* \Rightarrow V_k(w(k) - w(k - 1)) \geq C$ while $k \geq k^* + 1 \Rightarrow V_k(w(k) - w(k - 1)) < C$.

Suppose that $V_{k^*}(w(k^*) - w(k^* - 1)) > C$, which is generically true. Let us show that the equilibrium number of adopters $x = \sum_i x_i$ is equal to k^* . Suppose that $x < k^*$. Then, there exists $i \leq k^*$ such that $x_i = 0$. This implies that $V_i(w(x + 1) - w(x)) \leq C$. Since $x + 1 \leq k^*$, $w(x + 1) - w(x) \geq w(k^*) - w(k^* - 1) > 0$ and hence $V_i(w(x + 1) - w(x)) \geq V_i(w(k^*) - w(k^* - 1)) \geq V_{k^*}(w(k^*) - w(k^* - 1)) > C$, a contradiction. Suppose that $x > k^*$. Then there exists $i \geq k^* + 1$ such that $V_i(w(x) - w(x - 1)) \geq C$. Since $i \geq k^* + 1$, $V_i \geq V_{k^*+1}$ and since $x \geq k^* + 1$, $w(k^* + 1) - w(k^*) \geq w(x) - w(x_1) \geq 0$. Therefore, $V_{k^*+1}(w(k^* + 1) - w(k^*)) \geq V_i(w(x) - w(x - 1)) \geq C$ which contradicts the definition of k^* . Thus, all Nash equilibria must have size k^* .

When is a profile with k^* adopters a Nash equilibrium? Define k_1 such that $k_1 + 1 = \min\{k \leq k^* + 1 : V_k(w(k^* + 1) - w(k^*)) \leq C\}$. Note that, conditional of the profile having k^* adopters, if $i \leq k_1$, then i cannot play $x_i = 0$ while any $i > k_1$ can potentially play $x_i = 0$. Define k_2 such that $k_2 = \max\{k \geq k^* : V_k(w(k^*) - w(k^* - 1)) \geq C\}$. Then, any $i \leq k_2$ can potentially play $x_i = 1$ while if $i > k_2$, then i cannot play $x_i = 1$. □

References

- I. Alger and J. W. Weibull. Kinship, incentives, and evolution. *American Economic Review*, 100(4):1725–58, 2010.
- A. Ambrus, M. Mobius, and A. Szeidl. Consumption risk-sharing in social networks. *American Economic Review*, 104(1):149–82, 2014.
- A. Ambrus, W. Y. Gao, and P. Milán. Informal risk-sharing with local information. *Review of Economic Studies*, forthcoming.
- R. Arnott and J. E. Stiglitz. Moral hazard and nonmarket institutions: Dysfunctional crowding out of peer monitoring? *American Economic Review*, pages 179–190, 1991.
- A. V. Banerjee and E. Duflo. *Poor economics: a radical rethinking on the way to fight global poverty*. Public affairs, 2011.
- M. Belhaj and F. Deroïan. Risk taking under heterogenous revenue sharing. *Journal of Development Economics*, 98(2):192–202, 2012.
- E. Berg, M. Blake, and K. Morsink. Risk sharing and the demand for insurance: Theory and experimental evidence from ethiopia. *Centre for the Study of African Economies, University of Oxford Working Paper*, 2020.
- F. Bloch, G. Genicot, and D. Ray. Informal insurance in social networks. *Journal of Economic Theory*, 143(1):36–58, 2008.
- S. Boucher, M. Delpierre, and B. Verheyden. The impact of index-based insurance on informal risk-sharing arrangements. *Luxembourg Institute of Socio-Economic Research (LISER) Working Paper*, 2016.
- R. Bournès, Y. Bramoullé, and E. Perez-Richet. Altruism in networks. *Econometrica*, 85(2):675–689, 2017.
- R. Bournès, Y. Bramoullé, and E. Perez-Richet. Altruism and risk sharing in networks. *Journal of the European Economic Association*, 2021.
- Y. Bramoullé and R. Kranton. Risk-sharing networks. *Journal of Economic Behavior & Organization*, 64(3-4):275–294, 2007.
- D. J. Clarke. A theory of rational demand for index insurance. *American Economic Journal: Microeconomics*, 8(1):283–306, 2016.
- S. Cole, X. Giné, J. Tobacman, P. Topalova, R. Townsend, and J. Vickery. Barriers to household risk management: Evidence from india. *American Economic Journal: Applied Economics*, 5(1):104–35, 2013.
- A. De Janvry, V. Dequiedt, and E. Sadoulet. The demand for insurance against common shocks. *Journal of Development Economics*, 106:227–238, 2014.

- S. Dercon, R. V. Hill, D. Clarke, I. Outes-Leon, and A. S. Taffesse. Offering rainfall insurance to informal insurance groups: Evidence from a field experiment in ethiopia. *Journal of Development Economics*, 106:132–143, 2014.
- J. Gagnon and S. Goyal. Networks, markets, and inequality. *American Economic Review*, 107(1):1–30, 2017.
- C. Kinnan and R. Townsend. Kinship and financial networks, formal financial access, and risk reduction. *American Economic Review, Papers and Proceedings*, 102(3):289–93, 2012.
- R. E. Kranton. Reciprocal exchange: a self-sustaining system. *American Economic Review*, 86(4):830–851, 1996.
- A. M. Mobarak and M. R. Rosenzweig. Selling formal insurance to the informally insured. *Yale University Working Paper*, 2012.
- A. M. Mobarak and M. R. Rosenzweig. Informal risk sharing, index insurance, and risk taking in developing countries. *American Economic Review, Papers and Proceedings*, 103(3):375–80, 2013.
- J. D. Montgomery. Social networks and labor-market outcomes: Toward an economic analysis. *American Economic Review*, 81(5):1408–1418, 1991.
- K. Takahashi, C. B. Barrett, and M. Ikegami. Does index insurance crowd in or crowd out informal risk sharing? evidence from rural ethiopia. *American Journal of Agricultural Economics*, 101(3):672–691, 2019.