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# **Regional Networks in Chinese Mathematics and Astronomy**, 311–618 CE<sup>\*</sup>

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Abstract This article offers a social and geographical network analysis of all attested works, authors, and practitioners in the mathematical sciences in China over the period of disunion and reunification from 311 to 618 CE. Inspired by Karine Chemla's (2009) efforts to distinguish "different mathematical cultures" within the extant corpus of suan 第/算 procedure texts, the goal is to explore a viable framework within which to break down the history of Chinese mathematics along different, pluralistic lines. What I find is that this period is home to distinct regional networks working in isolation from one another, and that situating authors within these networks helps explain continuities and discontinuities in their technical writing. This is evidence of plurality, but one that is incommensurable with Chemla's "mathematical cultures," so I offer it as an alternative means to the same historiographical ends. In examining what our historical subjects said and did about this plurality of traditions, however, we realize that it was as aberrant to them as the political disunion of which it was a productsomething to rectified by "unification" (tongyi 統一), "integration" (tong 通), and, where necessary, force.

<sup>\*</sup> A preliminary version of the present article was first presented in two parts at the conference On Cultures of Scientific Practice in Ancient Mathematical Sciences, Université Paris Diderot, April 10-13, 2019 (https://halshs.archivesouvertes.fr/halshs-02021447) and at the Early Medieval China Workshop, INALCO, November 15-16, 2019 (https://halshs.archives-ouvertes.fr/halshs-02335042), the organizers and participants of which I thank for their feedback. Enormous thanks thereafter go to Catherine Jami, Jean-Pierre Dedieu, and Marcus Bingenheimer for introducing me to the tools of social network analysis, and to Marc Chapuis, my two anonymous reviewers, and Catherine Jami and Christopher Cullen for their detailed corrections and wise suggestions on the final manuscript. I wrote and submitted this article under the first Covid lockdown, in April-May of 2020, and if it is coherent, then the reader has them to thank. Lastly, I only discovered before typesetting that Zhu Yiwen had also turned his attention to the questions of transmission and regional traditions in medieval Chinese mathematics in 2020, Zhu 2021 making some of the same connections that I do in my own work but with different questions, emphases and methods. While it is too late to work his findings into those presented here, I have added a line in a footnote below to highlight one of its most pertinent insights.

In the twentieth century, Chinese astronomy, mathematics and "science" were commonly treated as monoliths with which one could neatly juxtapose some one counterpart in "the West." Suffice it to say that while we are increasingly aware that these monoliths are constructed, part of what sustains them is the structure that we mostly continue to impose upon Chinese sources: that of a single chronological line-a tradition-often segmented by transitions between ruling families. We highlight exceptions, such as the maintenance of parallel Chinese and Muslim astronomical offices from the thirteenth century on, but singular, unbroken tradition remains the general historiographic rule, at least for earlier times.<sup>1</sup> If it is difficult to conceive of an alternative prior to the thirteenth century, it is in part because this structure is inherent in our sources. After all, in the foundational Jiuzhang suanshu 九章算術 (Nine Chapters on Computational Procedures) we have Li Chunfeng's 李淳風 (602-670) subcommentary to Liu Hui's 劉徽 (fl. 263) commentary to a text that Liu Hui attributes to Zhang Cang 張蒼 (253-152 BCE) as having compiled from earlier writings-a text, as it happens, that exhibits extensive parallels with manuscripts recently recovered from Zhang Cang's day.<sup>2</sup> How are we to connect these dots if not into a single tidy line?

One vocal proponent of a pluralist alternative has been Karine Chemla. In the history of mathematics, Chemla (2009) has proposed the concept of "mathematical cultures" to divide texts exhibiting "different ways of doing mathematics" into distinct historical lines. Having divided the

<sup>2</sup> On the *Jiuzhang suanshu*, its history, see the modern critical edition and French translation of Chemla and Guo 2004. In English, see Shen et al. 1999.

<sup>&</sup>lt;sup>1</sup> Perhaps the best example of constructing and juxtaposing monoliths is everything implied in "the Needham Question": "What was it, then, that happened at the Renaissance in Europe whereby mathematised natural science came into being? And why did this not occur in China?" (Needham 1959, p. 154). In terms of continuity, the recent major works of Martzloff 1997, Wu et al. 1998-2004, Chen 2003, and Qu 2005 are titled histories of "Chinese astronomy/mathematics," and while they may in parts tackle sources by theme, elaborate on the biographies, external works, and geographical origins of their subjects, or devote sections to questions of foreign transmission and institutions, their underlying structure is a chronology of a single thing: "Chinese astronomy/mathematics." Late in research and writing of the present article I was made aware of the exemplary study of scholarly networks in astronomy between Yuan  $\pi$  China (1279–1368), the Ilkhanate of Iran (1256–1135), and the Chinese and Muslim experts at the Yuan court in Yang Qiao 2019. The circumstances of her period of study are very different, and as tempting as it is to underline parallels and contrasts between the two, I have restricted myself here to using her work as a source of inspiration for questions and factors to consider in the analysis of my own data.

aforementioned sources into two such lines in 2009, the number of cultures has grown in the expanded sample of Chemla (2018) to no less than five. Nine years and numerous articles in, Chemla remains principally focused on establishing and describing such distinctions, presenting them as challenges to determinism and universalism in the history of mathematics and as a new historiographical way forward. By way of nineteenth-century parallel, she invokes differences between the French research tradition in projective geometry and German number theory, suggesting that this lens allows us to see that it was the choice to adopt one of the former's cultural practices – generality – that led to a specific breakthrough in the latter and to this practice, in turn, "becom[ing] universal."

This is a radical departure from how we are used to thinking about early imperial Chinese sources for mathematics, but it is the next logical step from there that interests me: if China were indeed home to five or more "different mathematical cultures," then one could surely go on to write a cultural history of each, the best antidotes to monoliths and -isms being thick description and complex human storytelling. That is what I would like to see happen, and in this article I will present what I believe to be a necessary first step.

As to what is different, this study will look at a broader array of sources concentrated in a narrower, politically fragmented period, and it will situate procedure texts within a social and geographical network analysis of the people who gave those texts life. In more concrete terms, whereas Chemla's analysis has focused on the close reading of 12 works of *suan* #/# roughly spanning 221 BCE-1300 CE, mine will proceed from a statistical overview of 119 titles and 211 named practitioners in seven related fields from 311 to 618 CE.<sup>3</sup> As to structure, after introducing my terms, methodology, tools, and data, I will provide an outline of the main human and geographical contours of the period 311-618 as I now see it. What I find is that the history of these seven mathematical sciences

<sup>&</sup>lt;sup>3</sup> I give these numbers only to provide the reader a sense of relative scale. Chemla's tally of 12 is based on my count of those extant *suan* procedure texts and commentaries specifically named and analyzed for division into cultures (thus "focused on") in Chemla 2009; 2016; 2017a; 2017b; 2018. As adumbrated in Chemla 2018, pp. 28–30, Chemla and Zhu (forthcoming) will expand this further, including *suan* problems in commentary to the Confucian classics (see below). To this, one might also add the other classics and commentaries in the Tang computational canon that, after 2009, Chemla groups as one culture without singling out for individual analysis (e.g., Chemla 2018, pp. 4–5), bringing the number of individual texts considered to around 20.

can be reduced to that of a handful of regional networks, and that situating texts within the regional networks that produced them helps explain curious things at the level of their "numbers and procedures" (*shu shu* 數 術).<sup>4</sup> These networks, however, are incommensurable with Chemla's "cultures," leading me to ask what "culture"- or "network"-like distinctions our sources themselves recognized as dividing their world. The answer is that none stood out so much as "China versus India," and that there was a conscious, state-sponsored effort to unify the mathematics of the former.

## Terms, Questions, Methodology, and Sources

As the project of decomposing the history of Chinese mathematics into that of distinct local traditions was first proposed in Chemla (2009), allow me to begin with a summary of her groundwork before attempting to lay my own. In short, Chemla introduces the observer's category "culture" to describe "a *way* of doing mathematics that is *specific*" and that, far from being "personal," is attested across "several centuries" and multiple sources (p. 105). As to sources, she compares the *Jiuzhang suanshu* (first century CE), its aforementioned commentaries (263 and 656 CE), and the manuscript *Suanshushu* 筭數書 (*Writings on Calculating Numbers*; Zhang-jiashan tomb 247, Jingzhou 荊州, sealed in or shortly after 186 BCE), the four of which, she tells us, "bear witness to at least two very distinct mathematical cultures, even if they present some commonalities and undoubtedly share certain of their references" (p. 108). As to how the observer is to divide four texts self-labelled *suan* into two distinct "cultures":

Of the number of components that seem pertinent in grasping the specificity of a culture, and that therefore demand our full attention, I have identified the following elements: [1] *Problems*, i.e. the situations and the numerical values that their statements imply; [2] *Numbers*, which are subject to "procedures"; [3] *Procedures*,

<sup>&</sup>lt;sup>4</sup> In actors' terms, "numbers" (*shu* 數) and "procedures" (*shu* 術) are the fundamental building blocks of the practice and literature of *li* 曆 and *suan*. One notes that "Shushu" is also a bibliographical category used in the first-century *Hanshu*, covering these, as one, along with five other sub-categories of technical knowledge devoted principally to divination. This term—*shushu*—would go on to mean different things in different contexts, and, as a bibliographic category, would exclude *li*, *suan*, and the other computational sciences discussed in this article by the seventh century. On *shushu* as a bibliographic category and "the mantic arts," see Kawahara 1996, pp. 54–79, Kalinowski 2004, and Lackner 2020.

which, in modern technical terminology, we designate by the term "algorithms"; [4] *Text types*, as we have previously evoked with the "classics" and "commentaries"; [5] *Instruments of calculation*; [6] *Figures*; [7] *Proofs*; [8] *Epistemological values*; [9] *Types of institutions* and of *social groups*, of *individual profiles*, etc. (p. 108; numbers added, all emphasis original).

Of these, Chemla then retracts "institutions, ... social groups, [and] individual profiles," citing that:

As regards this last dimension, we lack the documentary evidence necessary for a detailed examination of the context and the social organization proper to mathematical activities in ancient China, and thus we cannot go into this in depth, however essential this aspect may be to any culture, in the sense that I intend the term (p. 108).

Having defined her terms, Chemla goes on to elaborate upon specific differences across criteria 1-8 between the Suanshushu and Jiuzhang suanshu, on the one hand, and Liu Hui and Li Chunfeng's commentaries, on the other. "Classic" and "commentary" are different text types, and unlike the "classics," for example, commentaries are concerned with intention, proof, and drawing connections, thus employing problems and procedures differently than do their base texts.5 From this and the mention of instruments, figures, and so on, she reconstructs features of "the 'work setting' in which the mathematical activity attested by our sources unfolded" and, in this setting outside of the text, the "complexes of practices... essential to identify and describe to understand precisely how an intellectual labor is collectively undertaken" (p. 150). It is differences across these, she argues, that marks these four texts as products of two distinct cultures, offering in conclusion that "the examination of cultures understood in this sense provides us, I believe, with important data for furthering our understanding of the local contexts within the framework of which scientific activities are undertaken" (p. 151). In later publications, Chemla extends this same analysis to other works of *suan*, distinguishing "canons" (i.e., "classics") from "the manuscripts" (2016) and "early" from "second period" sources (2017b). Then, building on Zhu (2016a), Chemla (2018, pp. 28-30) and Chemla and Zhu (forthcoming) extend this to clas-

<sup>&</sup>lt;sup>5</sup> Note that Chemla 2009, p. 144, clarifies that she is speaking of "the practice of proof (*démonstration*) attested in ancient China," which is "aimed at establishing the correctness of the procedure," not to be confused with the modern or Euclidean notion of "mathematical proof."

sics and commentaries versus parallel mathematical problems in Kong Yingda 孔穎達 (574-648) and Jia Gongyan's 賈公彦 (fl. 637-650) commentaries to the Confucian classics.

What interests me in this now decade-long project are its aims: to situate texts and practices in local contexts, and to use the latter to rethink the history of the former. I share these aims, but I would like to propose a different way of going about them in terms of categories, sources, and scope.

First, Chemla and Zhu's use of the term "mathematics" in relation to Confucian commentary highlights a point of terminological confusion that merits clarification. Namely, there is an old habit in Chinese studies to use "mathematics" both as a broad observer's category and as a strict one-to-one translation of the word suan 第/算, nominally reducing "the history of Chinese mathematics" to the history of the Suanjing 算經 (Computational Classics). Suan is an instrument ("calculating rods"), a verb ("to calculate"), a practice ("calculation"), and a label applied to texts of a coherent genre from at least the second century BCE on.<sup>6</sup> As a field, suan is one of seven sister sciences, next to *lü* 律 (tono-metrology, or "harmonics"), li 曆 (computational astronomy), pu 譜 (chronology), tianti 天體 (cosmology), and related diagram- (tu  $\blacksquare$ ) and instrument-construction (yi 儀, etc.).7 The contents of these fields are interrelated and overlapping, and some individual works defy classification, but they all comprise Chemla's "numbers" and "procedures," and, confusingly, they all involve the instrument, verb, and practice of suan.8 The nature of their entanglement is such that any self-proclaimed "history of Chinese mathematics" such as Martzloff (1997) and Wu et al. (1998-2004) is obligated to treat problems of *lü*, *li*, etc., featuring in the Suanjing and to turn beyond the *Suanjing* as concerns such topics as  $\pi$ , trigonometry, and polynomial

<sup>&</sup>lt;sup>6</sup> For a fuller survey of the semantic field of *suan*, see Chemla and Guo 2004, pp. 988–989.

<sup>&</sup>lt;sup>7</sup> On the emic taxonomy of these and other sciences, see Sivin 1990. For the Han, see Cullen 2009. For my own evolving take on these categories, see Morgan 2017, pp. 10–25, Morgan and Goodman 2019, and the entry "Astronomes et mathématiciens" in Martin and Chaussende 2020.

<sup>&</sup>lt;sup>8</sup> For example, the contents of the *Zhoubi* 周髀 (*Gnomon of Zhou*) overlaps with works labelled *suan* (computation), *tianlun* 天論 (i.e. *tianti* cosmology), *li* (computational astronomy), *tu* (diagrams), and *yi* (instruments). The dynastic histories cite the *Zhoubi* in relation to all five, and their bibliographical treatises disagree on where to file it until the Song (960–1279), after it had been canonized within the genre of *suan* (compare *Suishu*, *j*. 34, p. 1018, *Jiu Tangshu*, *j*. 47, 2036, *Xin Tangshu*, *j*. 59, pp. 1443–1444, and *Song shi*, *j*. 207, p. 5271).

interpolation.<sup>9</sup> This is standard operating procedure, and Chemla and Zhu take this one step further in comparing "the mathematics" of the *Suanjing* to those of texts that are not labelled *suan* nor by the name of *any* of its sister sciences.

The simple point that I would like to make is that they are right to do so, but if we are to be comprehensive, the search for "different ways of doing mathematics" must consider "problems," "numbers," and "procedures" across all of these fields. Indeed, any "cultural differences" we might find between what Sivin (1990) calls "the quantitative sciences" would provide a control by which to assess those that Chemla identifies within texts labelled *suan* alone. More importantly, I would like to emphasize that insofar as these sciences are entangled, we need to be able to label and treat them as an ensemble for the purposes of analysis.<sup>10</sup> I generally prefer "the exact/mathematical sciences" to this end, but this term emphasizes their plurality as fields whereas what I should like is to focus on their common core of "problems," "numbers," "procedures," and practices of *suan*.

Christopher Cullen's solution to what is essentially the same problem is to focus on the people. Rejecting the anachronistic and often unquestioned application of the term "mathematics" to ancient China, Cullen follows the procedure suggested by Wittgenstein, asking:

Can we identify an activity in ancient China [...] a self-conscious and publicly recognized group of people in ancient China with a family resemblance to what would be called nowadays "mathematicians"? What did these people call themselves? What did they consider their defining skill-set, or their common obsession to be? (Cullen 2009, p. 593).

After an initial prosopographical enquiry, he concludes that it is possible to identify such a group based on a common renown in *suan*, which is often said have been exercised in relation to the fields of *li* and *lü*. It is this underlying practice of *suan* that I understand to be the principal target of Chemla and Zhu's translation of "mathematics."<sup>11</sup> However, to

<sup>&</sup>lt;sup>9</sup> Alternatively, Qu 2005 titles a monograph of what is essentially the same scope "Chinese Astronomy and Mathematics" (*Zhongguo lifa yu shuxue* 中國曆法 與數學).

<sup>&</sup>lt;sup>11</sup> I say this based on Chemla's gloss of the word *suan* as "mathematics" in Chemla and Guo 2004, pp. 988–989, her longstanding emphasis on distinguishing text from practice, and Chemla and Zhu's application of "mathematics" to computational contents in works outside the bibliographical category of *suan*.

clear the slate, I will translate this *suan* as "computation," I will speak of the fields *lü*, *li*, *suan*, etc., as "the computational sciences," and I will reserve "mathematics" to a brief example, further down, of how "mathematical cultures" does not apply to the distinctions that I will be describing.

Second, like Cullen, I must insist that Chinese sources for institutions, social groups, and individual profiles in "computation" are in fact prolific and should be at the center of any discussion of context. As to institutions, there are no less than two monograph-length studies of the Imperial Astronomical Bureau, for example, not to mention the wealth of accounting documents recovered from administrative sites such as the state granary at Zoumalou 走馬樓, Changsha 長沙.12 As to individual profiles, the dynastic histories provide us with whole bibliographies and histories devoted to these fields as well as the biographies of many of those named therein. These biographies are detailed, often providing an individual's dates, place of origin, education, friends, family, collaborators, and acquaintances, movements, posts and noble titles held, competencies, hobbies, research activities, publications, and, sometimes, the person's height.<sup>13</sup> Needless to say, the individual practitioner's interactions are also revelatory of the "social groups" to which that person belonged. Starting from what looks like a simple database search of the word suan in the first three of these histories, Cullen (2009) was able to compile a list of twenty-two named practitioners active under the Han (206 BCE-220 CE) and weave their stories into a coherent prosopography. There is much more to be done in this vein, particularly looking outwards at other keywords and periods.

Third, if we are looking for plurality, and if we can localize authored works in time, space, and social networks thanks primarily to biographies, then it strikes me that we had best choose a particularly messy period to look at – one in which practitioners were divided between warring courts, headquartered in different capitals, whilst foreigners and foreign knowledge poured in from every direction. The messiest in this regard is the period from 311 to 618 CE, covering the so-called Sixteen

<sup>&</sup>lt;sup>12</sup> On the Astronomical Bureau, see Deane 1989 and Chen and Zhang 2008. On the granary at Zoumalou, see Taniguchi 2016.

<sup>&</sup>lt;sup>13</sup> For an idea of what can be done with these materials in early imperial China, see the biographical collections *Chouren zhuan* and Chen 2008, the relevant entries in Martin and Chaussende 2020, and the monograph-length biographies of early Chinese numerati in Xu 1999, Xu 2005, Goodman 2010, and Lien 2011. In medicine, see the formidable recent dissertation of Philippon 2019.

Kingdoms of the Five Barbarians (304–439) and Northern and Southern Dynasties (386–589), so that is where we will look.

To this end, I used the main online text databases<sup>14</sup> to perform database-wide searches for: (1) the authors and co-authors of written works, *extant and lost*, in the aforementioned fields; (2) authors of unwritten work in observation, experiment, testing, instrument-building, or theory;<sup>15</sup> (3) meaningful contributors to public debate; (4) people identified as having learned, taught, or been "good at" one of the aforementioned fields or a specific work therein; and (5) people holding specialist posts such as "computational astronomy worker" (*zhili* 治曆) or "student of computation" (*suansheng* 算生). Conservatively, I exclude those who simply called for a reform or held an administrative post in the Astronomical Bureau or the Ministries of Agriculture and Revenue.<sup>16</sup>

Once I knew who I was looking for, I ran searches on their names, gathering the surrounding text of every unique occurrence thereof into a master file. With everything that had ever been written about these individuals (up to the eleventh century), I then mined this file for personal information and interpersonal relations. Where known, I collected each person's dates, political allegiance(s), place of origin, positions, noble titles, and known activity in each of the aforementioned fields. I logged their known family, friends, and co-authors, etc., as well as every interaction they had with another named living person (e.g., "he held his hand and said" 執手曰). I did not include grandfathers and grandsons, rival generals, etc., unless the two were described as directly interacting, and I purposefully excluded emperors, as almost everyone serves and receives orders from them. For the Buddhist sources retrieved in my searches, I used Marcus Bingenheimer's dataset as a reference for the relational anal-

<sup>&</sup>lt;sup>14</sup> Namely, Scripta Sinica (http://hanchi.ihp.sinica.edu.tw/), the Chinese Text Project (https://ctext.org/), the Kanseki Repository (https://www.kanripo.org/), and CBETA (https://www.cbeta.org/).

<sup>&</sup>lt;sup>15</sup> For example: the physical examination and comparison of unearthed metrological standards via seed-based measurements and geometrical ratios and in *lü* (tono-metrology); the physical measurement of a circle's circumference and diameter by string to determine a better ratio for  $\pi$  in *suan* (computation); the comparison of predicted and observed times and positions to determine a new procedure texts' accuracy in *li* (computational astronomy); constructing a gnomon or observational armillary sphere sufficient for the precise data collection required by *li*; or first proposing (after significant observation, experiment, and testing) that the apparent speed of the sun in fact varies according to its distance from a given point.

<sup>&</sup>lt;sup>16</sup> My approach here is similar to that used in Philippon 2019, pp. 8-10, in circumscribing his corpus of "biographies of doctors."

#### Table 1. Prosopographic sample

		Mathematical competencies						
	D 1.	體	圖	儀	律	曆	算	譜
	People	C	D	I	1	Α	IVI	G
1	Diao Chong 刁沖 (fl. 515–520)	-	-	-	-	-	х	-
2	Diao Rou 刁柔 (501-556)	-	-	-	-	x	-	-
3	Jia Gongyan 賈公彥 (fl. 637-650)	-	-	-	-	-	x	-
4	Kong Yingda 孔穎達 (574-648)	x	-	-	-	x	x	-
5	Li Chongzu 李崇祖 (fl. 539-550)	-	-	-	-	-	x	-
6	Li Yexing 李業興 (484-549)	-	-	-	-	x	-	-
7	Li Zunzu 李遵祖 (fl. 532–550)	-	-	-	-	x	-	-
8	Liu Xuan 劉炫 (c.546–c.613)	-	-	-	-	x	x	-
9	Liu Zhuo 劉焯 (543–610)	x	-	-	x	x	x	-

NOTE: C = *tianti* cosmology, D = *tu* diagrams, I = instruments (clepsydra, armillary sphere, etc.), T =  $l\ddot{u}$  tono-metrology, A = li mathematical astronomy, M = *suan* computation, G = *pu* chronology.

-sis of relevant figures.<sup>17</sup> Lastly, in terms of "weight," I distinguished known and plausible vectors of transmission from other relations.<sup>18</sup>

A glance at the sample gathered in Table 1 suffices to illustrate how much these people typically share in common and yet how difficult it is to label them. This is not to mention the fact that behind each x is evidence of a different type, level, and duration of activity in said field. Conservatively, we might say that what these people shared in common is that they all went down in history, and that they did so, some exclusively but most only in part, for their learning in some aspect or field of "computation." This makes them a "community" in the sense of:

Community: 5b. A group of people who share the same interests, pursuits, or occupation, esp. when distinct from those of the society in which they live (*Oxford English Dictionary*, hereafter OED).

<sup>&</sup>lt;sup>17</sup> Marcus Bingenheimer, "Social network data for the study of Chinese Buddhist history", ver. 2018-08, <u>https://github.com/mbingenheimer/</u> ChineseBuddhism\_SNA.

<sup>&</sup>lt;sup>18</sup> That is, as all known transmissions occurred between friends, family members, or from teacher to student, I have highlighted all such relations as plausible vectors. For examples of each type of transmission in computational astronomy, see Morgan 2015, pp. 566–577.

This paper asks whether these people also formed one or more "communities" in the sense of:

Community: 12. The fact of being in communion; social intercourse; fellowship, amity (OED).

As to their self-identification as a community, to return to Cullen's question, two things make it difficult to extract a singular actor's category to apply to all the people in Table 1, let alone the master list. The first is that, in Chinese, group labels tend to use vague synecdoche, such that *lisuan* 曆算, for example, may include the sciences of *li*, *suan*, timekeeping instruments, and *lü*.<sup>19</sup> The second is that people are complicated and identity is contextual. As we will learn below, for example, the men gathered in Table 1 were also members or family of the same regional school of ritual studies, and how they were presented in the sciences depended upon the science and the specific problem under discussion. What is telling, as Cullen notes, is that our sources often present competency in *suan* as relevant to work in one of these other fields, and not vice versa, suggesting that it is with "computation" that they all identified at some level.<sup>20</sup> But these were polymaths, and a numerate gentleman was foremost a gentleman, let alone a numerate monk, nobleman, or hermit.<sup>21</sup>

As to what to call these people for the purposes of analysis, both Cullen's list and my own largely coincide with the figures gathered in Ruan Yuan's 阮元 (1764–1849) *Chouren zhuan* 疇人傳 (*Biographies of* Chouren, 1799) and Huang Zhongjun's 黃鍾駿 (n.d.) *Chouren zhuan si bian* 疇人傳四 編 (*Fourth Supplement to the Biographies of* Chouren, 1898). Ruan Yuan describes his work as "gathering the big names in *suan* and chronicling the authorities in pacing the heavens" 綜算氏之大名,紀步天之正規.<sup>22</sup> The term *chouren* 疇人, on the other hand, is a reference to a line in the *Shiji* 史記 (*Records of the Grand Historian*, 109/91 BCE), which commentators variously gloss as "hereditary practitioners" 家業世世相傳, "people who knew the stars in olden times" 昔知星人, and "people of the same kind who were brilliant in *li* (computational astronomy)" 同類之人明曆者

<sup>&</sup>lt;sup>19</sup> This is the case of the bibliographical heading "Lisuan" as used in *Sui shu*, *j*. 34, pp. 1022–1026.

 $<sup>^{20}</sup>$  In my own sample, I note that this is the case in *li* (computational astronomy) as concerns for example Li Xiu 李修 (fl. 275/280) and Bu Xian 卜顯 (fl. 275/280) in *Jin shu, j.* 18, p. 564, and Gautama Zhuan 瞿曇譔 (fl. 732–762) in *Xin Tang shu, j.* 27A, p. 587.

<sup>&</sup>lt;sup>21</sup> On the polymathy typical in the technical arts of this age, see Goodman 2005 and 2010.

<sup>&</sup>lt;sup>22</sup> Chouren zhuan, p. 4.

也.<sup>23</sup> Sadly, none of these labels are appropriate for the sort of people listed in Table 1: few of our "hereditary practitioners" actually inherited their knowledge or office, and most "mathematicians and astronomers" were both, in terms of expertise, and neither, in terms of profession. They are all objectively "students of," and they were all, to varying degrees, "experts" and "practitioners," but the question is *of what*? As to observer's categories, we could coin a term that is some combination of awk-ward, unfamiliar, and ambiguous in relation to "the history of mathematics," such as "numerati" or "numbers men." Or, since we commonly use "mathematics" to designate "computation" (*suan*) across genres and sister sciences, we could also use:

Mathematician: 1. A person who is skilled in mathematics; an expert in or student of mathematics (OED).

There is a strong argument to be made for using "mathematician," but I do not want my choice of words to fight with the modern instinct that any such qualifier of skill equates to a socio-professional caste at the center of the subject's identity. Instead, I will variously refer to the people on my list as "experts," "practitioners," "numerati," and "numbers men" depending on which is the most appropriate in context.<sup>24</sup>

As to numbers, within "the messy period" of 311–618 CE, I have found a total of 211 named experts like those in Table 1, 133 of which have biographies, 149 of which have known places of origin, and 103 of which appear as the authors/co-authors of a total of 119 titles in the aforementioned fields, a third of which are extant in whole or in substantial quotation.25 Of these 211 experts, 10 are "singletons" with no known connection to any other living person. The other 201 are part of something bigger. Namely, they are a sub-set of data that I have gathered for the entire historical period up to the end of the Tang (618–907 CE), which as of the date of writing covers 429 experts and circa 9000 separate historical relations and interactions. The years 670–907 are still a work in progress. From Zhang Cang to Li Chunfeng, however, and but for a handful

<sup>&</sup>lt;sup>23</sup> *Shiji, j.* 26, pp. 1258–1259.

<sup>&</sup>lt;sup>24</sup> For a more detailed discussion of such actors' core socio-political identities and their identification of one another by expertise and engagement in a particular form of knowledge in this period, see Cullen 2009. In the later history of science and technology, Song–Qing, particularly as relates to questions of regional and capital-centered knowledge, see the special volume of *Revue de synthèse* including Lamouroux 2010a, Lamouroux 2010b, Mau 2010, and Jami 2010.

<sup>&</sup>lt;sup>25</sup> All 211 are men; one can read about the two earlier exceptions of Ban Zhao 班昭 (44/49–118/121) and Deng Sui 鄧綏 (81–121) in Cullen 2009, p. 605.

of singletons and gaps, this work has revealed a single network of circa 2800 actors all connected by one degree of separation (that is, "a knows c," versus the second-degree "a knows b, and b knows c"). There is little that can be done about the singletons, but the gaps are easily filled with a handful of emperors, second-degree relations, and reasonable inferences for a more complete picture of where everyone stands.26 I mapped this network using Gephi (Figure 1), and I used the China Historical Geographic Information System27 and QGIS to study patterns in the geographical distribution of places of origin by court, period, etc.

Henceforth, I will refer to the aggregate of social relations and interactions connecting a set of individuals as a "network." I distinguish this from a "tradition" (chuan 傳, zong 宗, etc.), which entails the transmission of a body of knowledge across time and, thus, a one-way relationship with the dead. At the intersection of the two, I will use the term "community." To elaborate on the two definitions given above, what I mean by this is a network of individuals that is attested as having (1) directly researched, written, worked, studied, debated, and/or fraternized with one another in life and (2) shared common interests, traditions, areas of expertise, social backgrounds, educational and career patterns, and/or geographic or family ties in such combination and degree as I feel merits the label. However I choose to qualify (or quantify) this intersection of shared characteristics and social ties, what matters for the purposes of this article is simply that a primary source identifies someone as having been involved with some facet of "computation" and with another named, living person.

As I plan to release my data at a later date, this places a considerable burden of proof on me here when introducing someone as, for example, "an expert of a and b, the student of c, father of d, and co-author of e with f and g," each element of which is potentially cobbled from a different source. In the interest of space, I will provide the bare minimum of citations required to evidence such claims, and I will habitually direct the

<sup>&</sup>lt;sup>26</sup> The most hypothetical of my inferences involves inserting missing links in the Hexi Corridor transmission, discussed below. As to filling gaps, where an expert's only documented interaction is with an emperor, usually in the context of giving orders and sending back a memorial, I have included said emperor's interactions with this and other experts, the purpose being to situate the former vis-à-vis his contemporaries at the same court. The same goes for exceptional second-order relations. Another exception is made for emperors reported as having "studied" or "been fond" (*hao* 好) of one or another field: in this case, I include them but exclude their interactions with laymen so as to avoid overpopulating the network and violently altering its centers of gravity.

<sup>&</sup>lt;sup>27</sup> http://sites.fas.harvard.edu/~chgis/



Figure 1. Experts and their relations, 206 BCE-670 CE

NOTE: Figure generated with Gephi v.0.9.2. Known practitioners of the computational sciences ( $l\ddot{u} \not\equiv$ ,  $li \not\equiv$ ,  $suan \not\equiv$ / $\not\equiv$ , etc.) are indicated in black, and laymen in grey, their size determined by connectedness. Edges (lines) vary in thickness and color according to the nature of the relationship: normal vs. known and plausible transmission vector; practitioner-practitioner, practitioner-other, and, to bridge several gaps, other-other.

reader to a figure's place in general histories of astronomy and mathematics such as Chen Meidong (2003) and Wu Wenjun et al. (1998–2004) as proof of that person's credentials. What is more, I ask the reader to keep in mind that there is little that we can do about what our sources omit, fabricate, or confuse in the absence of some other text as proof, thus my emphasis throughout on "known practitioners," "attested relations," and positive evidence. Where I do attempt to derive meaning from absence, I offer a caveat ("so far as we can tell") or supporting evidence ("there is no apparent link between *a* and *b*, and *c* reports that the two never met"). Be aware that the figures offered in this article are by no means perfect and complete maps of the entirety of these men's overlapping social circles and geographical origins but instead a patchwork of what we can ascertain.

Lastly, allow me to clarify several final points of convention. Throughout, I will prioritize the Romanization of the sometimes difficult to translate actor's categories of *lü*, *li*, *suan*, etc., insofar as they represent historical, actor-defined fields and genres. Of these, "a li" or "li procedures" (lishu 曆術) can also refer to a "procedure [text]" on that subject, by which is meant a text comprising algorithms. Following the Jesuit astronomer Antoine Gaubil's (1689-1759) unencumbered, expert instinct when faced with translating this word, in titles I will render a *li* as "an astronomy" (une Astronomie), e.g. "the Triple Concordance Astronomy" (Santong li 三統曆; cf. Gaubil's "l'Aftronomie San Tong") rather than the inaccurate "calendar" or lengthy "astronomical system."28 I now prefer "tono-metrology" over the more common "harmonics" to describe *lü*, which amounts to the study of interrelated metrological standards of pitch, length, capacity, and weight. For ease of reading, I will also relax the use of square quotes around such observer's categories as "network" and "community" from here forward.

#### Human and Geographical Contours

Insomuch as the histories of *lü*, *li*, *suan*, etc., are typically presented as singular chronological lines, to perform a social network analysis of these fields is to smash those lines into a myriad pieces and to watch them rearrange themselves by magic into a single, shifting cat's cradle of which Figure 1 is but a freeze frame. For now, allow me to summarize what

<sup>&</sup>lt;sup>28</sup> Gaubil 1732, *passim*. On the exact contents and semantic range of *li*, appropriate translations, and the problem of reducing *li* to "calendars," see Sivin 2011, Martzloff 2016, pp. 12–24, Cullen 2017, pp. 7–25, and Morgan 2017, pp. 10–26.

emerged to me as the most important contours of this strange new thing formed of individual human relations and anchored in geography.

Whatever the field, almost all the experts of which we know seem to have belonged to a single tight-knit community during times of political unity (221 BCE-220 CE, 280-311 CE, and 589-670 CE). Take for instance (and for context) Kong Yingda and Li Chunfeng, active in Chang'an 長安 several decades after the period in question.<sup>29</sup> Kong Yingda sided with Li Chunfeng in a debate on computational astronomy (*li*).<sup>30</sup> The two both worked on the *Suishu* 隋書 (*History of Sui*).<sup>31</sup> They had at least seventeen associates in common, and one of them—Wang Zhenru 王真儒 (fl. 653-656)—worked on both of their respective canonization projects, concerning the Confucian and computational classics (*Suanjing*).<sup>32</sup> These two men belonged to the same community, by any standard definition, and between them and their immediate colleagues, this community had its fingers in every field and genre of the computational sciences.

There are exceptions, and there is a pattern to those exceptions. Namely, there are hermits who were active in the computational sciences without, to the best of our knowledge, any direct (first-degree) ties to a community of like-minded people. In third- to fourth-century Kuaiji 會稽, for example, we see a loose line of hermits and loners that would seem to go back to the experts of the former Sun-Wu 孫吳 court (222-280). The most famous among these are Ge Hong 葛洪 (283-343) and the grandfa-ther-grandson pair of Yu Song 虞聳 (fl. c.265) and Yu Xi 虞喜 (fl. 307-

<sup>&</sup>lt;sup>29</sup> For a biography of Li Chunfeng, see Goodman 2019; on his place in the history of astronomy and mathematics, see Chen 2003, pp. 350–357, and Wu et al. 1998–2004, vol. 4, pp. 219–249. On the mathematical contents in Kong Yingda's commentaries, see Zhu Yiwen 2016a; 2017a and his earlier articles cited therein.

<sup>&</sup>lt;sup>30</sup> Xin Tangshu, 25.536.

<sup>&</sup>lt;sup>31</sup> Xin Tangshu, 58.1457, places Kong Yingda among the authors of the Suishu's "Monographs in 30 juan" 志三十卷 (cf. Jiu Tangshu, j. 71, p. 2549; j. 189A, p. 4954). On Li Chunfeng's authorship of the "Lüli zhi" 律曆志 (Monograph on Tonometrology and Computational Astronomy), "Tianwen zhi" 天文志 (Monograph on Heavenly Patterns), and "Wuxing zhi" 五行志 (Monograph on Five Agents [Terrestrial Omenology]), see Goodman 2019, pp. 30-31.

<sup>&</sup>lt;sup>32</sup> Namely, Kong Yingda's Wujing zhengyi 五經正義 (Correct Meaning of the Five [Confucian] Classics), compiled 638–653, and Li Chunfeng's computational canon, finished in 656. For Wang Zhenru as a co-author in Kong Yingda's Shangshu zhengyi 尚書正義 (Correct Meaning of the Book of Documents), see Xin Tangshu, 57.1428. As a member of Li Chunfeng's project to "commentate the ten computational classics Wucao, Sunzi, [etc.]" 注五曹、孫子十部算經, see Jiu Tangshu, j. 79, p. 2719; Xin Tangshu, j. 204, p. 5798.

346).<sup>33</sup> There is a similar pattern among the majority of Daoist masters and Buddhist monks: though the person in question might have studied or written on a given subject of computation, he seems to be the only one worth remembering for it in his immediate circle. This pattern may well be an artifact of preservation bias, of course, but so too can we assume that our sources leave much about capital-based socialites unsaid.

Shortly after the reunification of the empire in 280 CE, the tight-knit capital-centered community that went back at least four centuries to the Western Han (206 BCE-9 CE) abruptly disappears. Like Xun Xu's 荀勗 (c.220-289) son Fan 藩 (245-313), some of its members are still alive by the turn of the century, but they seem to have been swept up in the collapse of the Western Jin (265-316): the War of the Eight Princes in (291-306), the revolt of their foreign mercenaries (304), the sack of Luoyang 洛陽 (311), and the resultant exodus to the southern banks of the Yangtze.<sup>34</sup> The result for the history of astronomy and mathematics is a hole—a "dark age"—spanning 150 years from 290 to 440 CE. Geopolitically, the North was divided by continuous warfare between non-Chinese forces, the Sichuan Basin broke off to form a radical, half-barbarian Daoist theocracy, and what was left of the Jin along the Yangtze struggled to integrate populations and defend itself from external conquest and internal unrest.

I have found a total of nineteen practitioners alive and/or active during this dark age. Among these, there is a disproportionate number of hermits, Daoists, and Buddhist monks (9/19), all of the solitary amateur variety. Of the capital-elite-types, most are congregated at the very beginning or end of this period, and those in the middle are relatively undocumented, appearing isolated in their time and place. However, across the darkness runs a single, curious line of transmission linking the period before to the period after.<sup>35</sup>

<sup>&</sup>lt;sup>33</sup> On Yu Song, Yu Xi, and Ge Hong's credentials in *tianti* (cosmology), see Chen 2003, pp. 237–238, 241–245. For Yu Xi's contribution to *li* (computational astronomy), see below.

 $<sup>^{34}</sup>$  On Xun Xu's biography, his work in *lü* (tono-metrology), and his son's continuation of that work, see Goodman 2010, esp. pp. 161–214.

<sup>&</sup>lt;sup>35</sup> Obviously, some practice (and study) of *li* and *suan* must have continued through this "dark age," otherwise there would have been no civil calendar or public financing of infantry armies in these years. Rather, it is that the "monograph" (*zhi*) histories dedicated to such subjects start skipping generations instead of years, presenting the works of single authors without word of collaborators, reviewers, competitors, or supporters. At the same time, mention of these subjects and related works becomes suddenly rare in biographies, and one finds an identically sized hole in bibliographies. Lastly, in discussing immediate precedents, standards, and inspiration, the first works after the 430s either cite the

As it happens, the three most consequential authors of this period all appear in or from the Hexi 河西 Corridor: Zhao Ying 趙嬰 (fl. 315), district chief of Lanchi 蘭池 and commentator of the *Zhoubi* 周髀 (*Gnomon of Zhou*);<sup>36</sup> Zhao Fei 趙敏 of Dunhang 敦煌 (fl. 412), astronomer royal of the Northern Liang 北涼 (397–439) and author of numerous astronomies and the *Zhaofei suanjing* 趙敏算經 (*Zhao Fei's Computational Classic*);<sup>37</sup> and, in between, Jiang Ji 姜岌 of Tianshui 天水 (fl. 384), born under the Former Liang 前涼 (320–376) and the author of an astronomy, a cosmology, and an important breakthrough in the measurement of the sun's position under the Later Qin 後秦 (384–417).<sup>38</sup> Not only do they all hail from the same far-flung borderland, what survives of the Zhaos' work points to a shared tradition, if not "culture."

Specifically, Zhao Fei's astronomy uses an "obscuration [cycle]" (bu 蔀), which was unique to the resonance periods of a quarterremainder astronomy (*sifen li* 四分曆)—a framework such as was used prior to 104 BCE, readopted in 85 CE, and such as features in the *Zhoubi*.<sup>39</sup>

<sup>37</sup> On Zhao Fei, see Chen 2003, pp. 254–255, and Wu et al. 1998–2004, vol. 4, pp. 106–107. The lost *Zhao Fei suanjing* appears in *Suishu*, *j*. 34, p. 1025.

<sup>38</sup> On Jiang Ji, see Chen 2003, pp. 252–254, and the relevant entry in Martin and Chaussende 2020

<sup>39</sup> On the "obscuration" and "quarter-remainder" framework, see Cullen 1996, pp. 20–35, 197–200; cf. Cullen 2017, pp. 138–234. As to Zhao Fei's work, the *Weishu* mentions the implementation in 452 of his *Xuanshi* li 玄始曆 of *circa* 412, referred to on the next page as "Zhao Fei of Dunhuang's *jiayin*<sub>51</sub>-origin astronomy' 敦煌趙歐甲寅之曆 (*Weishu*, *j*. 107A, pp. 2659–2660). Later bibliographies record a *Hexi Jiayin*<sub>51</sub>-origin Astronomy (*Hexi jiayin yuan li* 河西甲寅元曆) and *Hexi Renchen*<sub>29</sub>-origin Astronomy (*Hexi renchen yuan li* 河西壬辰元曆) in *Suishu*, *j*. 34, p. 1022; *Jiu Tangshu*, *j*. 47, p. 2038; and *Xin Tangshu*, *j*. 59, p. 1547. Turning to *Kai-yuan zhanjing*, *j*. 105, p. 5b, we find the resonance periods of a *jiayin*<sub>51</sub>-origin (Yahoo of the Liang Astronomy" (*Liang Zhao li* 梁趙曆), using an "obscuration," listed chronologically between those of Jiang Ji (384) and He Chengtian (443). Having apparently written a commentary to Zhao's *Jiayin*<sub>51</sub>-origin Astronomy (see *Jiu Tangshu*, *j*. 47, p. 2038), Li Chunfeng notes in his history of the period that, "in the Western Liang, for their part, they worked with an obscuration divisor, but

works of the following Northwestern figures or skip straight back to the third century.

<sup>&</sup>lt;sup>36</sup> Also known as Zhao Shuang 趙爽, the commentator's name is systematically given as Zhao Ying in early bibliographies (*Suishu, j.* 34, p. 1018; *Jiu Tangshu, j.* 47, p. 2036; *Xin Tangshu, j.* 59, p. 1543). The same peculiar name (Ying 嬰/Shi 奭, whence the later Shuang 爽) appears in 315 as the district chief of Lanchi in *Shiliuguo chunqiu, j.* 71, pp. 1b-2a; cf. *Cefu yuangui, j.* 232, p. 3b, and *Jinshu, j.* 86, p. 2227, which matches the commentator's traditional dates, e.g. Cullen 1996, pp. 147–148.

As Li Chunfeng explains, the quarter remainder saw continued government use in the State of Shu 蜀 (221–263), but everywhere else, "[Liu] Hong's 劉洪 (fl. 167–206) procedures were the exemplar of calculation for the subsequent age" 洪術為後代推步之師表.<sup>40</sup> The one exception is Li Xiu 李修 and Bu Xian's 卜顯 *Qiandu li* 乾度曆 (*Supernal Measure Astronomy*) of 275/280 CE, "whose procedures matched the quarter-remainder numbers for solar motion, with a slight increase in lunar motion" 其術合日行 四分數而微增月行.<sup>41</sup> As it happens, emperor Yuan of [the Southern] Liang 梁元帝 (r. 552–554) cites the following as one of the reasons why "We do not drink alcohol" 余不飲酒:

凉國太史令趙歐造乾度歷三十年,以心疾卒。

The Astronomer Royal Zhao Fei of the State of Liang worked on constructing [his own?] Qiandu li for thirty years and died of a heart attack.<sup>42</sup>

In short, Zhao Fei's "obscuration" and Zhao Ying's *Zhoubi* are related, and they are as out of place in the fifth century as is a typewriter in the twenty-first. Add to this the "*Hermit Zhao's Quarter Remainder Astronomy* in one volume" 趙隱居四分曆一卷 recorded in the *Suishu* "Jingji zhi" 經籍 志 (Monograph on Classics and [Other] Writings),<sup>43</sup> and one suspects that this curious return to the past is a regional, if not family phenomenon.

Following the collapse of the Later Qin, the Northern Wei (386-535) began swallowing its rivals in the North until only the Liang of the Hexi Corridor remained. Alone, the Xiongnu king of the Northern Liang courted the Chinese emperor of the Liu-Song 劉宋 (420-479) with a book exchange in 437, sending him 19 works in 154 volumes as "a tribute of regional items" 獻方物. Included among histories of the region and the works of local authors were "the *Zhoubi* in one volume" 周髀一卷 and "*The Biography of Zhao Fei and [his] Jiayin51-origin Astronomy* in one volume"

the historical evidence is contradictory, so [I] am unable to record it in detail [here]" 至於西涼, 亦為蔀法, 事迹糾紛, 未能詳記 (*Suishu, j.* 17, p. 416). Taken together, it would appear that Liang 梁 and "Western Liang" 西涼 refer to the Northern Liang 北涼 (in the West), and that the aforementioned *Xuanshi li* and *jiayin*<sub>51</sub>-origin astronomies are one and the same. On *jiayin*<sub>51</sub>, *renchen*<sub>29</sub>, and "origins," see Note 44 below.

<sup>&</sup>lt;sup>40</sup> *Jinshu, j.* 17, p. 503.

<sup>&</sup>lt;sup>41</sup> Jinshu, j. 18, p. 564.

<sup>&</sup>lt;sup>42</sup> Jinlouzi, j. 6, pp. 21b-22a.

<sup>43</sup> Suishu, j. 34, p. 1022.

趙歐傳并甲寅元歷一卷.<sup>44</sup> Two years later, in 439, the Northern Wei (386-535) then conquered the Northern Liang, moving its human and library resources to their capital at Pingcheng 平城.<sup>45</sup>

To reiterate, while North and South went dark, the Hexi Corridor emerged as the only active hub of Chinese computational science *circa* 290–440 CE; then, in the space of two years, its written production was reinjected into the Yellow and Yangtze River heartlands. What happened next? North and South, the lights came back on.

Tellingly, the first subject of the Northern Wei to write an original work in the computational sciences was Cui Hao 崔浩 (d. 450), the court's former ambassador to the Northern Liang and the man at the head of integrating its scholarly community with the Northern Wei's. In the 440s, Cui Hao wrote an *Wuyin yuan li* 五寅元曆 (*Five Yin Origin Astronomy*), which he memorialized for official use. The reform effort stalled, Cui's clan was put to death for an unrelated matter, and the court instead adopted Zhao Fei's *Xuanshi li* 玄始曆 (*Astronomy of the Dark Beginning [Era]*) of *circa* 412 CE.<sup>46</sup> The next few decades saw a frenzy of activity in *li*, and, as it happens, every Northern work thereafter uses an "obscuration" until 597.<sup>47</sup>

There were three Southerners working on *li* prior to the book exchange of 437: Wang Shuozhi 王朔之 (fl. 352), Minister of Agriculture Xu Guang 徐廣 (351–425), and Xu's nephew He Chengtian 何承天 (370–447). He Chengtian was orphaned at the age of five and raised by his maternal uncle, presumably during the latter's multi-decade work on his lost *Ji*-

<sup>&</sup>lt;sup>44</sup> Song shu, j. 96, p. 2416; cf. Cullen 1996, p. 162. As presented from the Northern Liang perspective, see *Shiliuguo chunqiu*, j. 95, pp. 4a-b. Note that "*jiayin*<sub>51</sub>" represents the 51<sup>st</sup> place in the sexagenary cycle, from *jiazi*<sub>01</sub> 甲子 to *guihai*<sub>60</sub> 癸亥, and that in the title of a *li* "astronomy" or "procedure text" (see above), it represents the year chosen as the zero-point for calculation (i.e., the "origin," *yuan* 元).

<sup>&</sup>lt;sup>45</sup> On the 437 book exchange, the position of Dunhuang as a bastion of Han culture in this period, and the role of Cui Hao in bringing it to Pingcheng, see Trombert 2005. I thank Éric Trombert for making me aware of this article in a personal communication of October 25, 2018.

<sup>&</sup>lt;sup>46</sup> Weishu, j. 35, pp. 825–826; j. 107A, pp. 2659–2660.

<sup>&</sup>lt;sup>47</sup> Namely, the Zhengguang li 正光曆 (Astronomy of the Orthodox Glory [Era]; 522), Xinghe li 興和曆 (Astronomy of the Ascendant Harmony [Era]; 539), Tianbao li 天保曆 (Astronomy of the Heavenly Preservation [Era]; 550), Tianhe li 天和曆 (Astronomy of the Heavenly Harmony [Era]; 566), Bingyin yuan li 丙寅元曆 (Bingyin<sub>03</sub>-origin Astronomy; 579), and Kaihuang li 開皇曆 (Astronomy of the Opening Sovereignty [Era]; 584). See the list of resonance periods in Kaiyuan zhanjing, j. 105, pp. 8a–12a, and for the Xinghe li, Weishu, j. 107B, p. 2700.

wang qiyao li 既往七曜曆 (Astronomy of the Seven Luminaries of the Past).<sup>48</sup> As it happens, Wang Shuozhi was also an acquaintance of an acquaintance of Xu Guang's brother, Miao 邈 (343–397).<sup>49</sup> Anyhow, six years after the book exchange, in 443, He Chengtian would memorialize his *Yuanjia li* 元嘉曆 (Astronomy of the Epochal Excellence [Era]), initiating the first astronomical reform that the South had seen since Yang Wei's 楊偉 Jingchu li 景初曆 (Astronomy of the Luminous Inception [Era]) of 237. He makes no reference to Zhao Fei or "obscurations." Nevertheless, the timing is curious, and it is worth noting that his Southern successor, Zu Chongzhi 祖沖 之 (429–500), makes open use of Jiang Ji and Zhao Fei's innovations.<sup>50</sup>

What of the *Zhoubi* and Zhao Ying's commentary? In the North, the text rose to immediate prominence. Indeed, it was while struggling with its calculations that Cui Hao's friend Kou Qianzhi 寇謙之 (365-448) is said to have had the revelation that would lead him to found the Northern Celestial Masters' Church of Daoism.<sup>51</sup> In a similar vein, one of the new "students of computation," Yin Shao 殷紹 (fl. 458), reports having learned the *Zhoubi* from a Daoist mystic in the mountains of the Shandong Peninsula.<sup>52</sup> In the sixth century, the Northern numbers men Xindu Fang 信都芳 (d. 543/550) and Zhen Luan 甄鸞 (fl. 535-570) would go on to write commentaries, the latter of which is extant through Li Chunfeng (from Chang'an).<sup>53</sup> Lastly, the *Zhoubi* was on the curriculum of Liu Zhuo 劉煒 (543-610) and Liu Xuan's 劉炫 (c.546-c.613) private school in the

<sup>50</sup> Zu Chongzhi mentions using Jiang Ji's method of extrapolating the position of the sun from lunar eclipse in his memorial of 462, cited in *Songshu*, *j*. 13, p. 290. On his "breaking the rule" as per Zhao Fei, see below.

<sup>51</sup> Weishu, j. 114, pp. 3049–3050; cf. Cullen 1996, pp. 162–163. Concerning Cui Hao and Kou Qianzhi's relationship, see *Weishu*, j. 35, pp. 814–815. On Kou Qianzhi and his church, see Mather 1979.

<sup>52</sup> Weishu, j. 91, pp. 1955–1956.

<sup>53</sup> Xindu Fang is variously attributed with a *Zhoubi zong* 周髀宗 (*The Gnomon of Zhou Tradition; Bei Qishu, j.* 49, p. 675), with "commentating the four procedures of the *Zhoubi*" 注周髀四術 (*Suishu, j.* 19, p. 526), and a *Sishu Zhoubi zong* 四術周髀 宗 (*The Gnomon of Zhou Tradition in Four Procedures; Beishi, j.* 89, p. 2934).

<sup>&</sup>lt;sup>48</sup> Songshu, *j*. 12, p. 261; *j*. 64, p. 1701. On He Chengtian, see Chen 2003, pp. 261–266, and Wu et al. 1998–2004, vol. 4, pp. 108–113.

<sup>&</sup>lt;sup>49</sup> Namely, "Wang Shuozhi of the Eastern Jin [once] asked Fan Ning 范寗 (c.339-401)" 東晉王朔之問范寗 about imperial mourning rites (*Tongdian*, *j*. 80, p. 2177, and "Miao [once] wrote to [Fan] Ning" 邈與甯書 about the local government in Yuzhang 豫章 (*Jinshu*, *j*. 91, p. 2356). Wang Shuozhi authored the lost *Tongli* 通曆 (*Universal Astronomy*) of 352 (*Songshu*, *j*. 12, p. 261; *Jinshu*, *j*. 18, pp. 565-566) and practiced "watching the *qi*" (Yang Jiong 楊炯, "Huntian fu" 渾天 賦, cited in *Yang Yingchuan ji*, *j*. 1, p. 5a; on "watching the *qi*," see Note 101).

North China Plain, attended by locals such as Kong Yingda and the teacher of Jia Gongyan, and it is Liu Zhuo that Li Chunfeng attributes with having laid the text's "shadow rule" to rest.<sup>54</sup>

In the South, the *Zhoubi* fared rather differently. The only direct mention of the *Zhoubi* in a Southern work comes in the "Tianwen zhi" 天文志 (Monograph on Heavenly Patterns) of Shen Yue's 沈約 (441-513) *Songshu* 宋書 (*History of the [Liu-]Song*), which Shen Yue reworked from He Chengtian's earlier history by the same title.<sup>55</sup> By way of introduction, Shen Yue rejects the *Zhoubi*'s "false attribution" (*jiatuo* 假託) to the Duke of Zhou 周公 (r. 1042-1036 BCE) and, after a brief summary of its cosmology, dismisses the text as "the creation of people fond of oddities" 好異者之所作也.<sup>56</sup> This would have been heresy to Zhao Ying, for one, who was so confident in the Duke of Zhou's authorship that he used it as the basis of identifying later insertions.<sup>57</sup>

It is from Cui Hao and He Chengtian, respectively, that the history of the computational sciences can be said to begin in the Northern and Southern Dynasties. As adumbrated by their different reactions to the Hexi Corridor transmission in 437–439, these histories are quite distinct.

Cui Hao and He Chengtian interacted with other experts, but Cui Hao and He Chengtian did not know one another. Neither did the numbers men in Cui Hao's circle know those in He Chengtian's, and so on. As such, whatever the algorithm used in a program like Gephi, two distinct networks emerge from their various interactions and peel away in opposite directions (see Figure 2). Later, each of these networks splits in two

<sup>&</sup>lt;sup>54</sup> On the Lius' school and curriculum, see *Suishu*, *j*. 82, p. 2762, as well as the transmission network illustrated in Figure 4 below. On the "shadow rule," which states that one *cun*  $\neg$  of difference in shadow, at the observer, corresponds to a thousand *li*  $\blacksquare$  at the scale of heaven and earth, see Cullen 1996, p. 111-115. For Liu Zhuo's critique of the shadow rule, see *Suishu*, *j*. 19, pp. 521-522.

<sup>&</sup>lt;sup>55</sup> Songshu, j. 11, pp. 205–206.

<sup>&</sup>lt;sup>56</sup> Songshu, j. 23, p. 679.

<sup>&</sup>lt;sup>57</sup> For example, "Rong Fang and Chenzi were men who came after the Duke of Zhou, so this does not belong to the original text of the *Zhoubi*" 榮方陳子是周公 之後人,非『周髀』之本文 (*Zhoubi suanjing*, *j*. 1, p. 20b, comm.). In the same vein, "This does not belong to the original text of the *Zhoubi* [but is] probably the words of someone asking a teacher, because, [in the introduction], he (the Duke of Zhou) only wanted to know about the division of the *du* and the origin of the method's procedure" 非『周髀』本文, 蓋人問師之辭, 其欲知度之所分、法術之所生耳 (*Zhoubi suanjing*, *j*. 2, p. 52b, comm.). For an example of the influence that this argument continues to hold on the question of the text's composite nature and authentic core, see Chemla 2013, p. 181, n. 17, and Chemla 2014, p. 259; p. 267, n. 37.



NOTE: This figure combines practitioners of the computational sciences known to be living and/or active in 400–462 and their interactions and presumably ongoing relations with others falling within the same period (e.g., brothers remain brothers until one of them dies), excluding normal emperor-subject relations and the book exchange of 437. The basic building block is the individual expert and his immediate circle of known acquaintances (e.g., the six people surrounding Kong Shanshi  $\mathbb{R} \cong [d. 450]$ , a father, a brother, an uncle, a collaborator, a patron, and an accuser). Where these circles overlap, the number and weight of such ties draws practitioners together, revealing a more complex web of interpersonal relations (e.g., Cui Hao 崔浩 [d. 450], Kou Qianzhi 寇謙之 [365-448], and Gao Yun 高允 [390-487], connected by a combination of friendship, teaching, coauthoring, and mutual acquaintances). We know that this one network extends between the courts at Pingcheng 平城 and Guzang 姑臧 through diplomatic contact, conquest, and the integration of surrendered scholar-officials at Pingcheng, but there is no evidence that it extends to Jiankang. Presumably, Zhao Fei 趙敏 (fl. 412) also had friends, family, and colleagues - a "circle" - linked to those at the same court who would go on to work with Cui Hao and Gao Yun. Likewise, He Chengtian was no doubt connected to Cui Hao by some degree of separation via an acquaintance. However, insofar as the two principal socio-geographic networks illustrated here align with the exclusive sets of precedents and authors that their respective members reference past 412 CE, it is reasonable to conclude that they were functionally working in isolation. Lastly, situating these networks within Figure 1 above, one notes that it is only after 462 that the North surpasses the South in terms of the number of known practitioners and works.

following the east-west schism of both the Wei and Liang 梁 (502–557) in the 530s and 540s, only to be reunited by military conquest in 554–589. As I will discuss the integration of these expert communities below, let me simply say two words about the few lines that do run North–South at the end of the period 439–554.

First, the only obvious line of transmission across the Yangtze in this period is that from the Southern expert Zu Geng 祖暅 (fl. 504-525) to the Northern expert Xindu Fang. In short, Zu Geng was a general of the Liang, and *circa* 520 he was captured in battle by the Northern Wei prince Yuan Yanming 元延明 (484-530) – a comrade in computation – who held him prisoner at his home and "had Geng make [for him] a leaning vessel and a water clock inscription" 使暅作欹器、漏刻銘.<sup>58</sup> As Yuan Yanming's client, Xindu Fang "remonstrated the prince to treat him with ritual propriety" 諫王禮遇之, and upon his release, "[Zu Geng] left behind his methods, teaching them to Fang" 留諸法授芳.<sup>59</sup> Coincidentally, the first discussion of computational works by Southern authors in the North – those of He Chengtian and Zu Geng's father, Zu Chongzhi–occurs a decade later, in 539, in a debate between Xindu Fang and Li Yexing 李業 興 (484–549).<sup>60</sup>

Second, the majority of cross-Yangtze interactions in these years involve emissaries sent to rival courts, particularly Northerners sent to the South. The most curious of these diplomatic missions comes in 537, when the same Li Yexing, as Senior Recorder for Comprehensive Duty, is dispatched by the breakaway Eastern Wei 東魏 (534–550) to establish diplomatic ties with the Southern court of emperor Wu of Liang 梁武帝

<sup>&</sup>lt;sup>58</sup> Liangshu, j. 36, p. 524. In instrumentation, Yuan Yanming "gathered various matters on the ingenuity of armillary spheres, tilting vessels, earthquake [detectors], wind vanes, clepsydrae, and wind-watchers, which he combined with illustrations in [his work] *Instrument Standards*" 又聚渾天、欹器、地動、銅烏、漏刻、候風諸巧事, 并圖畫為器準 (*Weishu, j.* 91, p. 1955). In *lü* (tono-metrology), he also worked on adjusting standards of pitch and length via textual scholarship and physical experimentation (*Suishu, j.* 16, pp. 393, 405).

<sup>&</sup>lt;sup>59</sup> Beishi, j. 89, p. 2933.

<sup>&</sup>lt;sup>60</sup> Weishu, j. 107B, pp. 2697-2698; cf. Tang and Wan 2018. In a contemporaneous study that I discovered at the stage of typesetting, Zhu 2021, p. 84-85, also stresses the importance of the Zu Geng-Xindu Fang transmission as concerns the history of computation in Confucian commentary, drawing an additional connection between the southern exegete Huang Kan 皇侃 (488-545) and the Northern school of Xu Zunming 徐遵明 (475-529), of which Kong Yingda and Jia Gongyan were a product. I had originally missed Huang Kan in my list of experts and have yet to map or assess how he connects to the bigger picture presented in this article.

(r. 502–549). He is sent with Senior Recorder Li Xie 李諧 (496–544) and Gentleman of the Ministry of Personnel Lu Yuanming 盧元明 (fl. 528–539), and at court they first discuss North–South differences in classical ritual exegesis with their host and his Senior Recorder, Zhu Yi 朱异 (483–549).<sup>61</sup> I say this exchange is curious, because everyone involved was an expert in one or another branch of computation,<sup>62</sup> making this, technically speaking, an international cabal of mathematicians conspiring to establish a new world order. I say this in jest, but given this and Cui Hao's ambassadorship with the Northern Liang, it would be interesting to collect data on the total proportion of diplomatic missions that were entrusted to the numerati.

Otherwise, these Northern and Southern networks seem to have remained isolated from one another, and the major difference between them is best summarized by Yan Zhitui 顏之推 (531–591), a Southerner writing to his sons from the Northern capital of the Sui 隋 (581–618):

算術亦是六藝要事;自古儒士論天道,定律歷者,皆學通之。然可 以兼明,不可以專業。江南此學殊少,唯范陽祖晅精之,位至南康 太守。河北多曉此術。

Calculation (*suan*) is also an important subject among the six arts [of classical gentlemanly education]. Through the ages, all scholars who have participated in discussions on the way of Heaven and fixed tono-metric and astronomical [policy] have had to master it.<sup>63</sup> With that said, it is better to learn it on the side than make it one's career. South of the Yangtze, the study of this [subject] is exceptionally rare, and the only one to have shone in it was Zu Geng of Fanyang, who arrived at the position of Governor of Nankang. [By contrast], many north of the Yellow River know this art.<sup>64</sup>

<sup>&</sup>lt;sup>61</sup> Weishu, j. 84, pp. 1862–1864. Titles: Senior Recorder for Comprehensive Duty = *tongzhi sanji changshi* 通直散騎常侍, Senior Recorder = *sanji changshi* 散騎常侍, Gentleman of the Ministry of Personnel = *libu langzhong* 吏部郎中.

<sup>&</sup>lt;sup>62</sup> As concerns their credentials, Li Yexing, Li Xie, and Lu Yuanming were all co-authors on the *Xinghe li* 興和曆 (*Astronomy of the Ascendant Harmony [Era]*) of 539 (*Weishu, j.* 107B, pp. 2695–2697; cf. Chen 2003, pp. 285–292; Tang and Wan 2018). On emperor Wu of Liang, see Chen 2003, pp. 274–277. Lastly, Zhu Yi's biography tells us that "gambling, chess, writing, and calculation were all his strengths" 博弈書算,皆其所長 (*Nanshi, j.* 62, p. 1515).

<sup>&</sup>lt;sup>63</sup> I add "policy," because "fixing the tono-metrology and computational astronomy" (*ding lü li* 定律曆), or one or the other alone, is a standard phrase used to describe policy reform concerning governmental standards.

<sup>&</sup>lt;sup>64</sup> Yanshi jiaxun jijie, j. 7, p. 587; tr. modified from Teng 1968, p. 205.

# Add to this Li Chunfeng's one-line history of tono-metrology $(l\ddot{u})$ under Yan Zhitui's former Southern masters:

陳氏制度,亦無改作。 The Chen dynasty's (557–589) system, for its part, saw no modification.<sup>65</sup>

Or the sum of their output in computational astronomy (*li*):

陳氏因梁,亦用祖冲之曆,更無所創改。 The Chen dynasty followed the Liang (502–557), using Zu Chongzhi's astronomy (written in 462) without further innovation or reform.<sup>66</sup>

Likewise, Shen Yue tells us that when Zu Chongzhi first memorialized his work in 462:

世祖下之有司, 使內外博議, 時人少解曆數, 竟無異同之辯。 The Generational Ancestor (emperor Wu of the Southern Qi 南齊武 帝; r. 440-493) passed it down to the relevant officials and had them hold a broad debate [open to those] inside and outside [said office]. Few people of the time understood *li* numbers, [so] in the end there were no arguments for or against.<sup>67</sup>

This is not to mention that Fanyang—whence Zu Geng and Zu Chongzhi—is situated in the outskirts of modern-day Beijing.

From 311 to 589, one finds 180 experts in the computational sciences among the Northern courts and 62 in the South, with those who switched sides counting twice. As concerns switching sides, several interesting patterns emerge when you cross-reference "citizenship" with place of origin (Figure 3). One is that the North was self-sufficient in producing talent, while the South was heavily reliant on *émigrés*. Indeed, the majority of experts in this period came out of the North China Plain, particularly the northern end of it, around the commanderies of Bohai 渤海, Fanyang, Qinghe 清河, and Xindu 信都. This was the mathematical breadbasket, if you will, and it is perhaps not surprising that the courts that held it also report the greatest number of experts – the Northern Wei, Eastern Wei, and Northern Qi 北齊 (550–577). Interestingly, if one compares this to the situation in 221 BCE–311 CE, one sees that the centrality

<sup>65</sup> Suishu, j. 16, p. 391.

<sup>&</sup>lt;sup>66</sup> Suishu, j. 17, p. 417.

<sup>&</sup>lt;sup>67</sup> Songshu, j. 13, p. 304.



Figure 3. Distribution of experts by court and place of origin

NOTE: Dots represent the places of origin of numerati active under a given court over the period in question, the smallest representing a locality that produced 1 known expert, and the largest 9.

of this region is more a product of the period than it is of any geographical determinism.

Experts moved around, but Figure 3 is nevertheless reflective of something important. It is reflective in part of clans, whose members naturally shared the same place of origin. With that said, hereditary practitioners and office-holders were still the minority in this period, at least as concerns the computational sciences, and few father-son or uncle-nephew pairs were as consequential as Zu Chongzhi–Zu Geng or Xu Guang–He Chengtian.<sup>68</sup> More importantly, what we are looking at are the places that a motley collection of individuals received their education in childhood, and the places to which many of them returned to teach in retirement. Many of those who grew up in neighboring commanderies studied together under the same teachers, as illustrated in Figures 4 and 5, and the relationships that they formed there structured their relationships at the capital, as I will illustrate below.

The South would appear to have had nothing like the educational network in the computational sciences particular to the North China Plain, and that might explain the lack of experts and innovations reported by historians and contemporaries alike. It might also explain a striking difference between the two communities: why Southern experts like He Chengtian, Zu Chongzhi, and Zu Geng worked alone, while their Northern counterparts typically worked with a host of co-authors and independent reviewers.<sup>69</sup> Indeed, it is difficult to speak of "the Southern numerati" as much of a community at all, and that is precisely this community's most defining trait.

As to institutions, while North and South shared most of the same structures of government involved with numbers, it is important to note that the Northern Wei expanded the educational mission of the state to include the computational sciences shortly after its conquest of the Northern Liang. Namely, it appointed professors (*boshi* 博士) to the Astronomical Bureau, and it attached "students of computation" to the Im-

<sup>&</sup>lt;sup>68</sup> Setting aside those who "left the family" as monks, the 197 lay practitioners alive and/or active in 440–618 have 90 different surnames; only 25 look to belong to some sort of hereditary line, and those 25 are divided among 9 different families. Other noteworthy father-son transmissions include Li Yexing and his sons Chongzu and Zunzu (see Figure 4) and Yu Shen 庾詵 (fl. 502)–Yu Manqian 庾曼 倩–Yu Jicai 庾季才 (516–603).

<sup>&</sup>lt;sup>69</sup> See for example the dozens of people involved in the creation of the *Zhengguang li* in *Weishu, j.* 107A, pp. 2659–2663, or Yuan Yanming's extensive collaboration with Xindu Fang in *Weishu, j.* 20, p. 530.



#### Figure 4. A Local transmission network

NOTE: This transmission network is reconstructed from known family and teacher-student relations around the Xu Zunming-Xiong Ansheng lineage of classical ritual scholarship (*Beishi, j.* 26, pp. 948–950; *j.* 81, p. 2708; *j.* 81, p. 2721; *j.* 81, p. 2725; *j.* 82, pp. 2762–2763; *Jiu Tangshu, j.* 73, p. 2601; *j.* 189A, 4949). The study of the computational sciences and the ritual classics often went hand in hand (Zhu Yiwen 2015; 2016b; 2017b), and the spread and combination of fields here is typical of the period. As concerns place of origin, the district, where known, is followed by the commandery, all of which are mapped in Figure 5.



Figure 5. A Local transmission network, continued

NOTE: In his youth, Xu Zunming of Huayin "went East of the [Taihang 太行] Mountains for his studies" 詣山東求學 (*Beishi*, *j*. 81, p. 2720) and, as an adult, "taught East of the Mountains to a multitude of students" 教授山東, 生徒甚盛 (*Beishi*, *j*. 33, p. 1232). Those of his lineage in the ritual classics who were involved in the computational sciences (Figure 4) originated from the localities in the North China Plain and Taihang Mountains indicated here. For reference, Changting 昌亭 and Jingcheng 景城 Districts were close enough for Liu Zhuo of Changting to have, "in childhood, sworn an oath of friendship with Liu Xuan of [Jingcheng District,] Hejian [Commandery]" 少與河間劉炫結盟為友 (*Beishi*, *j*. 82, p. 2762). To this list I have also added Qinghe 清河 Commandery, whence Zhang Zixin, and Tiao 蓨 District, whence Zhang Zhouxuan, who we will encounter below.

perial Secretariat and to Buddhist Temples.<sup>70</sup> The institutionalization of this area of education in the North would go on to have an important effect on the social organization of these fields by the Tang (618–907). Namely, we see the formation of official canons, schools, and exams leading, in the astral sciences, to the growth of hereditary clans and a culture of secrecy.<sup>71</sup> In the short term, however, it is difficult to identify what specific effect it may have had under the Northern Dynasties.

<sup>&</sup>lt;sup>70</sup> Wei shu, j. 113, p. 2976ff.

<sup>&</sup>lt;sup>71</sup> On *suan* (computation) education, see Keller and Volkov 2014. On the culture of secrecy in the astral sciences from the Tang forward, see for example Chen 2007, pp. 23–31.

#### Placing Texts in Human Networks

There is more to be said about networks and the institutions, social groups, and individual profiles of 311–618 CE, but I would like to turn, in three examples, to the question of what they might explain about the contents of the technical literature with which they have left us. As outlined at the beginning of the article, this is precisely what Chemla intends her framework of "mathematical cultures" to do, so it behooves me to highlight where our approaches differ. To this end, I will structure my first example in terms of six of her nine criteria for identifying "different cultures": (1) problems, (2) numbers, (3) procedures, (4) text types, (7) proofs, and (8) epistemological values.

My first example concerns "the rule" (*zhang* 章) that there are 235 synodic months in 19 solar years (or 19 × 12 normal calendar months plus 7 intercalations). Also known as the Metonic cycle, this ratio sat at the foundation of computational astronomy since its beginning. The "rule" was not entirely accurate, and as every astronomical constant was built in interlocking ratios thereupon, the rule made it such that a careful adjustment of one rippled arbitrarily through all the others in an unhappy compromise. Nevertheless, it was "the rule," so it persisted, and it was something of a paradigm shift when someone first thought to "break the rule" (*po zhang* 破章).<sup>72</sup>

That person was Zhao Fei, whose *jiayin*<sup>51</sup>-origin *Xuanshi li* of 412 CE used the ratio 7421:600, or 31 × 235 + 136 lunations to 31 × 19 + 11 years. Next, Zu Chongzhi's *Daming li* 大明曆 (*Astronomy of the Great Enlightenment [Era]*) of 462 CE used 4836:391, or 20 × 235 + 136 lunations to  $20 \times 19 + 11$  years. After that were Li Yexing et al., whose *Zhengguang li* 正光曆 (*Astronomy of the Orthodox Glory [Era]*) of 522 CE uses 6246:505, or  $26 \times 235 + 136$  lunations to  $26 \times 19 + 11$  years. That all three add the precise values of 136 months and 11 years to the respective sides of some multiple of "rules" cannot be a coincidence, which tells that Zu and Li et al. not only inherited the problem of "breaking the rule" from Zhao Fei but also a specific set of numbers and procedures for doing so.<sup>73</sup> Combined with what we learned in the previous section, moreover, we realize that we are dealing here with members of two distinct regional networks working in isolation within a single paradigm inherited from a third.

A century later, Zu Chongzhi's same value for the rule appears in the *Tianhe li* 天和曆 (*Astronomy of the Heavenly Harmony [Era*]), written for the

 $<sup>^{72}</sup>$  On the history of "the rule" and intercalation periods, see Qu 2005, pp. 167–204.

<sup>&</sup>lt;sup>73</sup> For an explanation of Zhao Fei's adjustment, see Qu 2005, pp. 172–174.



Figure 6. Zhen Luan 甄鸞 (fl. 535-570)

NOTE: Like the authors of most extant works of *suan* up to this point, Zhen Luan has almost no attested relation to any named contemporary other than an emperor, in this case Yuwen Yong 宇文邕 (emperor Wu of Zhou 周武帝; r. 543–578), who commissioned work from Zhen Luan (then mayor of the capitol) and received and acted upon his submissions. Exceptionally including this emperor and his interactions with other numbers men at court, we can situate Zhen Luan *next* to (if not for lack of evidence within) a hub of interactions, shared contacts, and plausible transmission vectors between men active in the same fields in the same city at the same time. This may help provide context to Zhen Luan's work beyond the written traditions with which he engages. (Again, black indicates known experts, and the thick lines indicate known and plausible transmission vectors).

Northern Zhou 北周 (557–581) court in Chang'an in 566 by Zhen Luan of Wuji 毋極 (Figure 6).<sup>74</sup> Looking over the whole of his oeuvre, we realize that Zhen Luan also uses the ancient "rule" of 235:19 both in his sub-commentary to *Zhoubi* [suanjing] and in his own suan classic, the Wujing suanshu 五經算術 (Computational Procedures for the Five [Confucian] Classics). Zhen Luan is difficult to categorize: he was an author in both *li* and suan; he was born in the North China Plain and managed to serve the rival courts of the Liang (S), Northern Qi (NE), and Northern Zhou (NW);

<sup>&</sup>lt;sup>74</sup> *Tianhe li*, cited in *Kaiyuan zhanjing*, 105.11a–11b. On Zhen Luan, see Kohn 2008 and Wu et al. 1998–2004, vol. 4, pp. 78–81, 169–172.

and his *Tianhe li* dates to after Zu Chongzhi's work had made its way north via the Zu Geng-Xindu Fang transmission of *circa* 520. We know that he "knew better" in *li*, but it is clear from context why he resorts to the ancient "rule" in *suan*: in his subcommentary, he is explaining (or "proving") these numbers' use in the base text and in Zhao Ying's commentary; in his "classic," the topic is "The Method of Fixing Intercalations in the *Book of Documents*" 『尚書』定閏法, so he understandably uses the old quarter-remainder framework commonly presumed to have been used in high antiquity.<sup>75</sup> In terms of individual profiles, therefore, what we have here is one author writing different text types in different fields and who uses a different set of problems, numbers, procedures, proofs, and epistemological values according to the specific task at hand.

Together with a broadened sample of "mathematics," the work of social network analysis better helps us to understand the history of "the rule" and of the texts in which it appears by placing them in a larger, pluralistic context of separate communities and individual polymathy. For the sake of comparison, where this differs from Chemla's "cultural analysis" is that the latter starts in the opposite direction and deals with incommensurate ontologies: it has us begin by deducing features of a "work setting" and "complex of practices" from a text's "problems," "numbers," etc., and then has us use *that* – the "culture" – to explore the "local context" of the text's production. In other words, what Chemla is describing are differences of what I would call "genre" or "tradition" such as they reflect the sort of differences in setting and practices that distinguish the "culture" of a medical textbook from that of a surgical checklist.

As such, "cultural analysis" might work within the framework of "regional networks" to elaborate upon further differences within the two expert communities in Jiankang and Luoyang, but it does not itself work to delineate local communities at that scale. At that scale, any differences in "local context" thus deduced in a vacuum from the aforementioned procedure texts would not map coherently onto both a single author like Zhen Luan and the separate communities in Guzang, Jiankang, and Luoyang. Everything considered, there are too many factors at play to draw a binary distinction between, say, the world pre- and post-*Xuanshi li*, classic versus commentary, or sources labelled *li* versus *suan*. Labelling all of these texts "the same culture," on the other hand, would have us set aside important, evidenced distinctions in socio-geographical context in favor of, essentially, an imaginary cultural monolith of the sort that Chemla's "cultures" are constructed to deconstruct. It is for this reason

<sup>&</sup>lt;sup>75</sup> Zhoubi suanjing, j. 2, p. 29a, subcomm.; Wujing suanshu, j. 1, pp. 1a-2a.

that what I am proposing is perhaps more of a *preliminary* than *alternative* to "cultures," but let us focus instead on what networks do allow us to explain.

Consider the precession of the equinoxes. As a problem, the precession of the equinoxes is an unobservably slow drift in coordinates that compounds into significant errors in the prediction and retrodiction of celestial phenomena more than a couple centuries out. Historically, the solution was to posit an "annual difference" (sui cha 歲差): a minute discrepancy between the number of days that it takes the sun to return to winter solstice and the number of  $du \not \in$  in one circuit of heaven (the mean sun travels 1 du per day) aimed to account for the westwards "retreat" of the position of the sun at successive returns to winter solstice (modern value about 0.014° per year). Yixing 一行 (673/683-727) describes this as "making heaven heaven and the year the year" 使天為天, 歲為歲.76 The introduction of the annual difference marked a revolution in computational astronomy (*li*), opening it to new problems, new data sets, new numbers, and a new epistemological and historical outlook. Equally important, the idea first came out of chronology (pu), and the breakthrough that it inspired in computational astronomy fed back into its sister science's archaeoastronomical aspirations.

Reduced to a simple chronological line, the history of the annual difference is somewhat baffling, particularly from a positivist perspective. In short, the annual difference was introduced by Yu Xi in the fourth century and ignored by all subsequent astronomers like Jiang Ji and Zhao Fei until He Chengtian. To be clear, He Chengtian did not use it in his *Yuanjia li* of 443 either, but Yixing reports that, elsewhere, he doubled Yu Xi's rate for 1 *du* (≈ 1°) of retreat from 50 to 100 years.<sup>77</sup> The first to employ the annual difference in an astronomical procedure text was Zu Chongzhi, in 462, followed by Yu Kuo 虞劌, in 544, who used the rates of 45.92 and 183 years, respectively. However, Li Yexing and all other astronomers before and after continued to ignore this until the turn of the seventh century, when the problem entered common practice through the work of Liu Zhuo and Zhang Zhouxuan 張冑玄 (d. 605/617).<sup>78</sup> While the range in values signals that the annual difference remained experi-

<sup>&</sup>lt;sup>76</sup> "Ridu yi" 日度議, cited in Xin Tangshu, j. 27A, p. 600.

<sup>&</sup>lt;sup>77</sup> "Ridu yi," cited in *Xin Tangshu*, *j*. 27A, p. 600.

<sup>&</sup>lt;sup>78</sup> On the precession of the equinoxes and the history of "the annual difference," see Chen 1995, pp. 261–270, and Qu 2008, pp. 167–195. On the resultant change in epistemological and historical outlook, see Morgan 2017, pp. 183–185. For an example of the archeoastronomical work that it permitted in chronology, see Zhang Peiyu 1999.

mental, what I have always found curious about these three centuries is the utter lack of curiosity: the majority of experts have nothing to say about the matter, not even a word of criticism or disbelief.

If we combine the history of the annual difference with what we know about regional networks, however, everything makes sense. To start, those who used it were all active within the same 120 km radius in the South, while those we might accuse of ignoring them were on the other side of the Yangtze. In philosophy, we know that Wang Chong 王充 of Kuaiji's (27-c.100) Lunheng 論衡 (Balanced Discourse) saw only limited circulation in the South under the Eastern Han (25-220).79 That was in relatively good times, and it is easy to imagine that another Southern work might see the same in times of more marked political division and upheaval (especially as the movement of experts tended to be from North to South in these years). What is more, Yu Xi was a hermit, and not only was he working in isolation on a problem speaking to a capital-centered expert community that had disappeared, his only other contribution to the computational sciences-the Antian lun 安天論 (Discourse on Secure Heaven [Cosmology]) – was universally dismissed as unserious.<sup>80</sup> It is no wonder therefore that this idea took a while to catch on in computational astronomy, that it first caught on in the South, or that it took another century from there to see universal acceptance.

Lastly, let us consider the history of what was probably the single greatest paradigm shift in this period: the work of Zhang Zixin 張子信 of Qinghe (d. 577). According to Li Chunfeng's histories, Zhang Zixin fled to an island to avoid the Ge Rong 葛榮 Rebellion in 526–528, and he stayed there for three decades making armillary sphere observations that would lead to the discovery of the inequality of solar and planetary motion and the seasonal variation in planetary visibility at the horizon.<sup>81</sup> Zhang Zixin did not author a procedure text or political reform of his own, but his discoveries are integrated into Zhang Zhouxuan of Xindu's

<sup>&</sup>lt;sup>79</sup> According to Yuan Shansong's 袁山松 (d. 401) lost history of the Eastern Han, by the time that Cai Yong 蔡邕 (133–192) arrived in Wu 吳 in 179, "[Wang] Chong's *Lunheng* had yet to see transmission in the central lands" 充所作『論衡』,中土未有傳者 (cited in *Hou Hanshu*, *j*. 49, p. 1629, comm.).

<sup>&</sup>lt;sup>80</sup> In their respective "Tianwen zhi," Shen Yue (South) calls the lost Antian lun "whimsical chatter that misses the mark by a great distance" 好異之談, 失之遠矣 (Songshu, j. 23, p. 680), and Li Chunfeng (North) calls it a "whimsical and fantastical theory that would discuss heaven (cosmology) without an understanding of the numbers" 好奇徇異之說, 非極數談天者也 (Jinshu, j. 11, p. 280; Suishu, j. 19, p. 508). On Shen Yue and Li Chunfeng's tidying of the history of cosmology, see Morgan 2019, pp. 146–155.

<sup>81</sup> Suishu, j. 20, p. 561.

Daye li (Astronomy of the Great Patrimony [Era]) 大業曆 of 597 and Liu Zhuo of Xindu's Huangji li 皇極曆 (Astronomy of the Sovereign Pole [Era]) of 604-texts that look like no li that came before them and that include important breakthroughs in *suan* concerning polynomial interpolation. Qu Anjing (2008) marks this as the beginning of a new age in the history of astronomy, but, once again, there is a half-century between Zhang Zixin's discoveries and Zhang Zhouxuan's reform during which most experts ignored and, then, actively fought against these innovations.

Here again, geography and social networks help explain the coexistence of "different mathematics." Zhang Zixin may not have authored an astronomy, but he taught Zhang Mengbin 張孟賓 (fl. 576–579) of Guangping 廣平, and Zhang Mengbin appears with Liu Xiaosun 劉孝孫 of Guangping (d. 594/597) in a completive trial at the Northern Qi court at Ye 鄴 in 576.<sup>82</sup> In 577, before the trial was finished, Ye fell to the Northern Zhou, and the conquerors instituted a new astronomy to celebrate their victory in 579. In 581, the Zhou abdicated, and the new emperor of the Sui 隋 (581–618) likewise called for a celebratory reform. Not surprisingly, both the Northern Zhou and Sui commissioned new astronomies from experts within their own ranks in Chang'an, e.g. Ma Xian 馬顯 (fl. 579– 581), Zhang Bin 張賓 (fl. 568–584), and Liu Hui 劉暉 (fl. 581–594). In short, if Zhang Zixin was "ignored" from *circa* 558 to 581, it was probably because his work was unpublished, unproven, and as yet unknown beyond his immediate circle of local disciples.

Liu Xiaosun resurfaces in Chang'an in 584, immediately after the testing, debate, and unanimous approval of a new astronomy fit to inaugurate the Sui—the *Kaihuang li* 開皇曆 (*Astronomy of the Opening Sovereignty [Era]*). Liu trashes the *Kaihuang li*, demanding that the court throw it out in favor of his own, and he allies himself with Zhang Zhouxuan, then Liu Zhuo, in petitioning the court. What follows is something of a soap opera, the short of which is that the three are continuously thwarted by political obstruction from the *Kaihuang li*'s second author, Astronomer Royal Liu Hui 劉暉, and his allies within the leadership.<sup>83</sup> There are many factors that explain their resistance, but one of them is that this is in fact a clash between regional factions: Liu Xiaosun, Zhang Zhouxuan, and Liu Zhuo were not only partisans of the same strange new mathematics, they were

<sup>&</sup>lt;sup>82</sup> Suishu, j. 17, p. 418.

<sup>&</sup>lt;sup>83</sup> See *Suishu*, *j*. 17, pp. 417–*j*. 18, p. 461, and the summary of these events in Li 2019.

all compatriots from the North China Plain who had come to rain on their conquerors' parade.<sup>84</sup>

Again, however we articulate these regional networks with "mathematical cultures," the former gets us to something like the goal that Chemla envisages for the latter. Namely, mapping people and institutions in time, space, and social networks, we can, in this period, write parallel histories of Chinese mathematics as they unfolded in different cities, following the same texts and ideas as they evolved in different, undetermined directions. We can trace how practice unique to one community was transmitted to and received by another, be it rejected, put to a different use, or made "universal." And we can place these different mathematics in the broader context of each community's specific dynamics, composition, polymathic interests, and political, religious, and philosophical engagements. What is more, unlike "cultures" - an observer's category with no equivalent in the contemporary language-these networks are simply the aggregate of things as historical, concrete, and prolifically documented as the bonds between "friends" (you 友), "family" (jia 家), and "neighbors" (tongjun 同郡). Unlike "cultures," therefore, this means that we can ask what actors thought and did about the divides between them.

## Identifying and Dealing with Diversity

The experts of this period clearly distinguished between fields, genres, and text types within the computational sciences:  $l\ddot{u}$  was different to li, as was a procedure text (*shu* 術) to a history (*zhi* 志) and a "classic" (*jing* 經) to a commentary (*zhu* 注). These things have names, and unless the point is simply to relabel them, the question should be how our historical subjects conceived of two communities working in isolation with different communal approaches to the same field, genre, text type, or problem. As a baseline, let us consider how Chinese experts spoke about their analogues in India.

According to his biography in Huijiao's 慧皎 (497-554) Gaoseng zhuan 高僧傳 (Biographies of Eminent Monks; c.530), the Southern monk Huiyuan 慧遠 (334-416) was once asked by He Chengtian "what *li* do they use in Buddhist country/ies" 佛國將用何曆? Huiyuan provides a variety of trivia about gnomon shadows, the civil calendar, and the superiority of

<sup>&</sup>lt;sup>84</sup> As illustrated in Figure 5 above, their hometowns of Guangping (Zhang Mengbin and Liu Xiaosun), Qinghe (Zhang Zixin), Changting 昌亭 District, Xindu Commandery (Liu Zhuo), and Tiao 蓨 District, Xindu Commandery (Zhang Zhouxuan) all fall within 175 km of one another.

predictive models and the coordinate system in Tianzhu 天竺 (India), which is later independently confirmed by a man from the Kingdom of Poli 婆利國 (on the island of Borneo).<sup>85</sup> In Jizang's 吉藏 (549–623) version, Huiyuan is introduced as "good at explaining the *li suan* of foreign countries" 善解外國曆算.<sup>86</sup> In summary, the understanding here is that *li* and *suan* are universal categories, that India and Borneo are all vaguely the same place, and that "they" do *li suan* differently.

Compared to whom? Consider how the translator Liu Ping 劉憑 (fl. 587-595) frames the matter in the preface to his Wainei bangtong bijiao shufa 外內傍通比校數法 (Comprehensive Comparison of Foreign and Domestic Numerical Methods):

……華夏數法自有三等之差,天竺所陳何無異端之例。然,先譯經 並以大千稱為百億,言一由旬為四十里,依諸算計悉不相合。竊疑 翻傳之日彼此異音,指麾之際於斯取失。故錄眾經算數之法,與華 夏相參……

... the Hua-Xia  $\overline{\mp}$  (Chinese) method of numeration features [numbers that can stand for] three different magnitudes, while that elaborated in Tianzhu (India) shows no examples of such heterodoxy. As such, the first translations of the sūtras all took the Great Thousand (i.e., the World of 1000<sup>3</sup> Worlds) to be "a hundred *yi* 億" (*yi* = 10<sup>5</sup> or 10<sup>8</sup>), and they said that one *yojana* was forty *li*  $\pm$ , neither of which match by any account. It is my humble suspicion that, on the day that they interpreted the [oral] transmission, there was a confusion of pronunciations that led to a mistake at the moment of dictation. Thus shall I record the method(s) of computing numbers in the multitude of sūtras and compare them to [those of] the Hua-Xia (Chinese)...<sup>87</sup>

<sup>&</sup>lt;sup>85</sup> Gaoseng zhuan, T. no 2059, j. 7, p. 368a. According to Liangshu, j. 54, p. 796, the Kingdom of Poli "is located on an island in the middle of the sea southeast of Guangzhou, two months' travel from Guangzhou" 在廣州東南海中洲上, 去廣州 二月日行. Later geographical monographs specify that one passes through areas in modern Vietnam and Malaysia to get there, suggesting that Poli refers to a kingdom near modern Brunei.

<sup>&</sup>lt;sup>86</sup> Zhongguan lunshu, T. no 1824, "Xu" 序, p. 4c1. For more on this exchange, see Zheng and Jiang 2007.

<sup>&</sup>lt;sup>87</sup> Cited in *Lidai Sanbao ji*, *T*. no 2034, *j*. 12, p. 107a. The problem that, in Chinese, "[numbers that can stand for] three different magnitudes" 三等之差 is a reference to the three concurrent systems of numeration beyond a *wan* 萬 (10,000), where an *yi* 億 is either ten *wan* (10 × 10,000) or *wan wan* (10,000<sup>2</sup>), and a *zhao* 兆 can be ten *yi* (10<sup>2</sup> × 10,000), *wan yi* (10,000<sup>3</sup>), or *yi yi* (10,000<sup>4</sup>). This leads to some

In sum, the world of computation is divided into two blocs: Chinese and foreigners, Hua-Xia and Tianzhu. Furthermore, one notes that we are speaking in terms of peoples and regions but not of *languages*, presumably because it was so rare to hear or read about the computational sciences of Tianzhu in Fan 梵 (Sanskrit/Indic) that it was not the first association to leap to mind. In a similar vein, the *Suishu* "Jingji zhi" associates foreign translations with a particular group e.g. the "*Brahmin Computational Methods* in 3 rolls" 婆羅門算法三卷 and "*Brahmin Yin-Yang Astronomical Calculations* in 1 roll" 婆羅門陰陽算曆一卷.<sup>88</sup>

Where at all, the same bibliography tends to identify the Chinese works of this period by court/dynasty and political era, the exception being "*The Hexi jiayin<sub>51</sub>-origin Astronomy* in 1 roll, written by Zhao Fei, astronomer royal of the Liang"河西甲寅元曆一卷, 涼太史趙歐撰.<sup>89</sup> The same goes elsewhere when speaking about people from different regional networks. As to the exceptions, the Southern *Songshu* introduces Zhao Fei as "a man from Hexi (i.e., west of the Yellow River)"河西人, while the account of his capture in the *Beishi* 北史 (*History of the North[ern Dynasties]*) introduces Zu Geng as "a man from Jiangnan (i.e., south of the Yangtze)" 江南人.<sup>90</sup> We know that this is distinct from introducing someone by his place of origin—"Zu Geng of Fanyang" 范陽祖暅—because the Zu clan's place of origin was north of the Yangtze.<sup>91</sup> It is also extremely rare. In general, Chinese numbers men are simply spoken of as individuals so long as it is not a question of Hua-Xia versus Tianzhu, or "foreign" versus "domestic."

The clearest indicator of how these individuals conceived of independent regional traditions in the computational sciences is therefore what they did about them. As a rule, they avidly absorbed their human, material, and textual resources as soon as they became available, erasing any differences within a generation. At an individual level, this would seem to be the natural product of curiosity in what were mostly experiential, competitive, and result-oriented fields. In the North, however, it was

confusion when dealing with large numbers in Buddhist sūtras, such as the exact number of worlds in the "Great Thousand" multiverse.

<sup>&</sup>lt;sup>88</sup> Suishu, j. 34, p. 1026. Alternatively, Jiang and Niu (2001: 108) argue that this *Poluomen* 婆羅門 refers to the Brāhmapakṣa school of Indian astronomy.

<sup>89</sup> Suishu, j. 34, p. 1022.

<sup>&</sup>lt;sup>90</sup> Songshu, j. 98, p. 2416; Beishi, j. 89, p. 2933.

<sup>&</sup>lt;sup>91</sup> Zu Geng appears as "Zu Geng of Fanyang" 范陽祖暅 in Yan Zhitui's (South) *Yanshi jiaxun jijie, j.* 7, p. 587, and his son appears as "Zu Hao of Fanyang" 范陽祖 皓 in *Liangshu, j.* 29, p. 429; cf. "Zhao Fei of Dunhuang" 敦煌趙歐 in *Weishu, j.* 107A, p. 2660.

also the product of deliberate government action – action, for context, by the very governments that purged the Buddhist faith in 446-452 and 574-577 for the sake of unity in matters of religion.

The Hexi Corridor seems to have become the major hub of mathematical learning following the sack of Luoyang in 311. After the Northern Wei armies were done with it in 439, however, it would only produce another four numerati worth remembering, the most important of whom being one of Li Yexing's minor co-authors, Xin Shu 辛術 of Longxi 隴西 (500–559).<sup>92</sup> Whoever survived of Zhao Fei's cohort at Guzang saw the center of activity shift to Pingcheng, where the state immediately set up its own schools and welcomed a flood of talent from the North China Plain. As to their textual tradition, the *Zhoubi* and Zhao Fei's astronomy were not commemorated as "regional items" from Hexi but instead appropriated as the symbols of its conquerors: the *Zhoubi* as the catalyst for Kou Qianzhi's new state church, and the *Jiayin<sub>51</sub>-origin Astronomy* as the embodiment of the Wei's heavenly mandate from 452 to 522.

Within a decade of the *Jiayin*<sub>51</sub>-origin Astronomy's retirement, the Northern Wei was split in two by civil war, and most of the numbers men followed the regent Gao Huan 高歡 (496-547) to the new eastern capital at Ye. Frustrated by their progress, the Eastern Wei general Hou Jing 侯景 (d. 552) rebelled, defected, and rebelled again against the Liang, leaving the South divided between rival capitals at Jiankang 建康 and Jiangling 江陵. By 552, there were now four separate imperial courts, each with their own community of experts, which Chang'an would swallow one after the other over the following decades (Figure 7).

The first to go was Jiangling, which the Western Wei entered in 555 to replace emperor Yuan of Liang with a puppet. Of their experts, Pei Zheng 裴政 (fl. 554-581) was "sent to the capital together with all the court officers within the city" 與城中朝士俱送于京師, as were Ming Kerang 明克讓 (525-594), Xiao Ji 蕭吉 (d. 606), and Yu Jicai 庾季才 (516-

<sup>&</sup>lt;sup>92</sup> The other three were Fan Shao 范紹 of Dunhuang (d. 528), Duan Rong 段榮 of Wuyuan 五原 (478–539), and Xin Yanzhi 辛彦之 of Longxi (d. 591). Of these, one notes that Duan Rong "was fond of *li* procedures in his youth" 少好曆術, and since his grandfather hailed from the old Northern Liang capital at Guzang, there may well have been some connection (*Bei Qishu*, *j*. 16, p. 207). For the others, see *Weishu*, *j*. 107B, p. 2696; *Beishi*, *j*. 46, p. 1701; *j*. 50, pp. 1822–1824; *j*. 82, pp. 2752–2753; *Suishu*, *j*. 16, pp. 393.

603).<sup>93</sup> Once there, the regent Yuwen Tai 宇文泰 (507-556) transferred

## Figure 7. Four courts, 572 CE



Astronomer Royal Yu Jicai to the head of his own Astronomical Bureau and appointed him to the Unicorn Hoof Academy (Linzhi dian 麟趾殿) with Ming Kerang. Not only were they absorbed into the civil service and scholarly community of Chang'an, after the Yuwens assumed the Western Wei throne in 557, the two transplants were made to author a *Zhou li* 周曆 to commemorate the new dynasty—one which would "select from Zu Geng's old discussions and synthesize the procedures of South and North" 採祖暅舊議,通簡南北之術.<sup>94</sup> Yuwen Tai showered Yu Jicai with gifts, which the latter sold to buy his friends and family out of slavery in the ruins of Jiangling.<sup>95</sup>

The next was Ye, which the Zhou took and held in 577, definitively reunifying the North. Reunification came as a blow to what was brewing

<sup>&</sup>lt;sup>93</sup> Suishu, j. 66, p. 1549. In the case of Ming Kerang and Xiao Ji, we simply read that each "surrendered/was surrendered" (*gui* 歸) to Chang'an after the fall of Jiangling (*Suishu*, *j*. 78, p. 1774; *j*. 83, p. 2808).

<sup>94</sup> Suishu, j. 17, pp. 418-419.

<sup>95</sup> Beishi, j. 89, pp. 2947-2948.

in the North China Plain: the visionary astronomer Zhang Zixin died in the fighting,<sup>96</sup> as did any hope of victory by public trial for the work of his local disciples Zhang Mengbin and Liu Xiaosun (see above). Understandably, it was the old guard that Chang'an wanted, and it was the old guards Li Delin 李德林 (532–592) and Zheng Yuanwei 鄭元偉 (fl. 576–584) that would end up working under their local counterparts four years later in the metrological and astronomical reforms to commemorate the Sui.<sup>97</sup> We do not know how Zheng Yuanwei got there, but Li Delin was one of eighteen high officials carted away in the conquering emperor's wagon train.<sup>98</sup> Liu Xiaosun and company would eventually emerge from the weeds to push their own reforms in the name of accuracy and technical progress, but their efforts were actively suppressed until 597.

That left Jiankang, which the Sui "pacified/levelled" (*ping* 平) in 589, taking everything back with them to Daxing 大興 (Chang'an). This included at least five numbers men: Cai Ziyuan 蔡子元 (fl. 589), Geng Xun 耿洵 (fl. 589–611), Mao Shuang 毛爽 (fl. 589), Yu Puming 于普明 (fl. 589), and Yuan Chong 袁充 (544–618). Yuan Chong was transferred to a post in the Astronomical Bureau, of which he would later be named director.<sup>99</sup> Geng Xun, by contrast, was sent there as an "official slave" (*guannu* 官奴), then gifted to Prince Yang Xiu 楊秀 (570–618) in a domestic capacity.<sup>100</sup> It is unclear how the other three arrived in Chang'an, but they were immediately put to work there on yet another state project aimed at assimilating their regional tradition:

遇平江右,得陳氏律管十有二枚,並以付弘。遣曉音律者陳山陽太 守毛爽及太樂令蔡子元、于普明等,以候節氣,作律譜。時爽年老, 以白衣見高祖,授淮州刺史,辭不赴官。因遣協律郎祖孝孫,就其 受法。弘又取此管,吹而定聲。既天下一統,異代器物,皆集樂 府......

During the pacification of Right Bank (the South), [the Sui] obtained [a set of] ten and two [tonometric] regulator pipes from the

<sup>&</sup>lt;sup>96</sup> Bei Qishu, j. 49, p. 680.

<sup>&</sup>lt;sup>97</sup> Zheng Yuanwei appears as a contestant in the Northern Qi astronomical trial of 576–577, then, in 581, as the sixth of sixteen co-authors on the *Kaihuang li* (*Suishu, j.* 17, pp. 417–421). Li Delin is also attributed with a *Kaihuang li* in *Jiu Tangshu, j.* 47, p. 2038, and he appears around the same time as Liu Xuan's superior whilst the latter was "working on heavenly patterns, tono-metrology, and computational astronomy with the various technicians" 又與諸術者修天文律曆 (*Suishu, j.* 75, p. 1719).

<sup>&</sup>lt;sup>98</sup> Beishi, j. 47, pp. 1727–1728.
<sup>99</sup> Suishu, j. 69, p. 1610.
<sup>100</sup> Suishu, j. 78, p. 1770.

[defeated] Chen dynasty, which were handed over to [Niu] Hong 牛弘 (545-610). Experts of sound and pitch regulation, Shanyang Governor Mao Shuang, Prefect Grand Musician Cai Ziyuan, and Yu Puming of the [former] Chen were dispatched to watch the nodal *qi* and make a register of the [captured] regulators.<sup>101</sup> [Mao] Shuang was old in years at the time, and he presented himself to the High Ancestor (Emperor Wen of Sui 隋文帝; r. 581-604) in white clothing; he was granted the Inspectorship of Huaizhou, but he declined and did not go to his post. Thereupon, [the court] dispatched Tono-metrology Gentleman Zu Xiaosun 祖孝孫 (fl. 589-628) to go to him to learn his methods, and [Niu] Hong then took these pipes (that Zu Xiaosun had created) and blew them to fix the tones [of the imperial government's official musical scale]. Now that the world under Heaven had been unified, the instruments and items of other dynasties were all collected at the Music Bureau...<sup>102</sup>

A generation later—in Li Chunfeng's day—there was but a single community of Chinese numerati so far as we can tell. Adding to the aforementioned projects, Li Chunfeng's histories and computational canon (*Suanjing shishu* 算經十書, the *Ten Computational Classics*) would furthermore ensure that that community shared a single past.<sup>103</sup>

#### Conclusion

Interested in its questions and aims, this study began as an experiment to push Karine Chemla's search for "different mathematical cultures" in China to its limits by looking at the broadest possible array of sources within the messiest span of political history. To this end, I extended the search from computation (*suan*) to its sister sciences of tono-metrology ( $l\ddot{u}$ ), computational astronomy (li), chronology (pu), cosmology (*tianti*), and related diagram- (tu) and instrument (yi) construction. I combined

<sup>&</sup>lt;sup>101</sup> "Watching the *qi*" (*houqi* 候氣) refers to the practice of installing pitch-pipes of the twelve notes of the chromatic scale in the ground, in a sealed room, to observe if they spontaneously sound in at the correct time, in the correct order, by force of the seasonal fluctuation of the earth's *qi* at the twelve "nodal *qi*" into which the solar year is divided. See Huang and Chang 1996.

<sup>&</sup>lt;sup>102</sup> Suishu, j. 16, p. 391.

<sup>&</sup>lt;sup>103</sup> On Li Chunfeng's histories of *tianti* (cosmology), *lü* (tono-metrology), and *li* (computational astronomy) in the "Tianwen zhi" and "Lüli zhi" of the current *Jinshu* and *Suishu*, see the various contributions to the first half of Morgan and Chaussende 2019. On the *Suanjing shishu* canon and its role in fixing a national curriculum for *suan* (computation), see Keller and Volkov 2014, pp. 58–63.

extant procedure texts with bibliography and biography, gathering information on hundreds of titles and practitioners to submit to social and geographical network analysis. Lastly, I chose to look at the Sixteen Kingdoms (304–439) and Northern and Southern Dynasties (386–589), during which Chinese experts first came into contact with Indian knowledge whilst presumably divided into isolated expert communities serving different courts. The initial data gathering was done blind, but it was admittedly motivated by the expectation that it would reveal a new, more compelling divide than those argued from comparing the features of *suan* procedure texts alone.

What I did find was evidence of a number of coherent regional networks working in isolation, notably that of the Hexi Corridor (315-439) and the concurrent networks, North and South, which arose upon inheriting the former's resources in 437-439. These networks differed in terms of their size, productivity, institutions, talent pool, educational framework, tendency towards collaboration, and reception of given texts. In terms of their written production, they also differed systematically on a variety of numbers, problems, procedures, and epistemic values, pointing to a divergence of written traditions. I found that these networks explain curious gaps and overt resistance in the history of certain innovations such as "the annual difference" and polynomial interpolation, and they do so without obfuscating the plurality of regional centers or having to maintain that each of someone like Zhen Luan's works is the product of a different "work setting" or "local context." Moreover, insofar as these networks are simply aggregates of concrete actors' categories, historical relations, and transmissions, they move the discussion from questions of observer's categories and their definition onto solid textual footing, no one or twenty data points that one may wish to contest having much impact on a statistical picture this large.

This is an important step towards writing, say, a social history of mathematics in the North China Plain. More than anything, however, what this work has impressed upon me is the degree to which the people and the fields involved reflect one monolithic (if evolving) tradition.

One way in which "cultures" does not work here is that, to meet Chemla's definition, these collective differences in institutions, social groups, and individual profiles would need to persist for "several centuries," which no regional network in this period does. At that scale, the hundreds of numerati from Zhang Cang to Li Chunfeng's day in Figure 1 all belonged to a single tight-knit community. This community split, but while experts North and South may not have had access to one another's work for several generations, they all shared the same greater written tradition going back to the Han. Within the blink of an eye, these break-

away communities and splinter traditions then merged back into a seamless whole. In part, they were forced to merge through military conquest, relocation, and collective state-sponsored projects designed to unify each field in its technical contents and ideological purpose. Individually, however, one sees numbers men like Xindu Fang and Zu Chongzhi eagerly absorb knowledge from the other side of the Yangtze for the simple sake of learning and improving their own work.

Viewed in this light, the tendency in the Chinese computational sciences in this period was towards unity, and this is particularly evident in comparing how our subjects speak of India and Indian sources in translation. On the one hand, our subjects treat li, suan, etc. as universal categories, they juxtapose China and India as monolithic blocs, and while they highlight that *li* and *suan* are done differently in India, they are not so different that they cannot be compared or evaluated on their specific advantages. On the other hand, our subjects make no explicit effort to adopt such perceived advantages as Tianzhu numerals, let alone "integrate" (tong) or "unify" (tongyi) the two traditions. That they eagerly did so with regional traditions within China reveals that our subjects thought of them differently – that they thought of them as *belonging* – and there is perhaps no greater indicator of this than how rarely they label their comrades "men of Hexi" or "men of Jiangnan." Even in diplomatic exchanges and at war, when such demarcations most matter, the numerati spoke of one another as individuals, i.e. as one would members of the same ingroup.

Social and geographical network analysis has fundamentally altered the way that I see the history of the exact sciences in this period, and I would like leave the reader with several questions with which this has left me grappling. Christopher Cullen and Zhu Yiwen have revealed the degree to which early imperial astronomy and mathematics are embedded in the institutions and exegesis of classical ritual (li 禮),<sup>104</sup> and my own findings is that their transmission also apparently goes hand in hand—how deep does this go beyond the one transmission network illustrated in Figure 4 above? What does it change when we realize that

<sup>&</sup>lt;sup>104</sup> It is no secret that the Astronomical Bureau was a subsidiary of the Ministry of Rites (see Deane 1989), nor that tono-metrology (*lii*) was closely tied to ritual music and measures (see Goodman 2010). Somewhat more surprising (to me) is the link that Cullen 1993 and 2007, pp. 240–241, has drawn between computational astronomy and specific imperial ceremonies as well as Zhu Yiwen's work on the preponderance of mathematical contents in Kong Yingda and Jia Gongyan's sub-commentaries to the ritual classics (e.g. Zhu Yiwen 2015; 2016b; 2017b).

the history of astronomy, mathematics, metrology, and instrumentation is the history of the same handful of people and that, within a given dynasty/region, most of these people all knew one another? What do we do when our primary sources reflect assumptions reminiscent of midtwentieth-century history of science that are currently out of fashion positivism, universalism, cultural essentialism—and why would we expect (or project) something closer to our own post-modern sensibilities?<sup>105</sup> To be fair, we may well see things that they do not, but how to we articulate the effects of invisible forces like "networks" and "cultures" with those of the forces that they tell us do and should govern their world? Lastly, what if, instead of eternally rehashing the problems and anachronism of "science," we started over from a statistical analysis of practitioners and such basic questions as what fields go together, what do these practitioners collectively reject, and what are their specific bases and mechanisms for adjudicating truth claims?

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<sup>&</sup>lt;sup>105</sup> On the importance of ideas resembling progress and empiricism in the history of early imperial Chinese astronomy, see Morgan 2017.

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