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JEL Codes: D74, H56, P16.

Keywords: Social unrest; Revolution dynamics; Optimal switching time.



People Get Ready: Optimal timing of Revolution *

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Abstract

We study here whether and when a social movement can turn into a successful revolution and modify the production system. To do so, the dynamics of social unrest is defined as a function of the number of discriminated workers, and their organizational skills, wage inequalities, and retaliation. Then, taking the evolution of social unrest into account, we obtain the optimal time for a revolution as the moment which maximizes discriminated workers' lifetime welfare. It is proven that if unrest increases with time and if a revolution does improve the discriminated's welfare, then a revolution arises independently of the initial state and the characteristics of the economy. Whenever the gains are high enough, then a revolution arises immediately. Worth noting, we show that even if social movement loses momentum, a revolution arises if the initial mass of discontent is sizeable or the system is so repressive that it becomes necessary to revolt.

Keywords: Social unrest; Revolution dynamics; Optimal switching time.

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1 Introduction

A revolution can be political, social, economic, commercial, of gender or religious, among many others. Here, we build on the definition of revolution provided by Huntington (1968),

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Skocpol and Theda (1979) and Goodwin (2001), adapting it to describe a mass mobilization, which changes more or less rapidly and fundamentally the social and economic structure of a nation. Additionally, we connect each revolution to the social movement underneath it, which we understand as a “collective challenge to elites, authorities, other groups or cultural codes by some significant number of people with common purposes and solidarity in sustained interaction with elites, opponents and authorities” (pp. 3-4 in Tarrow, 1994). Hence a revolution is the successful end of a powerful social movement. This is precisely the question we raise in this paper: When will a dynamic social movement turn into a successful revolution? Given the strength of the established system, technology and the number of discriminated workers, what is the optimal time to revolt against the unjust system?

Rebel’s organization and retaliation find support through the ages. The year 1848 saw a popular uprising in Europe, virtually every part of Europe between Britain and Russia experienced revolutionary movements. Historians and contemporaries consider the 1848 revolutions as a failure (Parker, 2002; Dowe and Langewiesche, 2001). The social unrest may or may not turn into a successful revolution depending on the organization of the revolutionaries and the strength of the system. For instance, in France, undoubtedly the uprising inspired actions across Europe, but the revolution was not successful because of having no unified set of aims, lack of unity, and coordination problem of rebels (Furtado, 2020). Despite a popular uprising all across Europe in 1848, England did not experience a serious threat of a revolution. In the literature, historians have pointed out that Chartists were suppressed by severe punishments and exile from the state. The transportation of prisoners to the colonies was a key strategy by the British state to show its power and threaten possible future rebels (Saville, 1990). Moreover, governments after the 1840s reduced the burden of taxes on the working class and improved overall living standards (Jones, 1983).

Revolutions continue to surprise and reshape our world. In 2011, a series of pro-democratic protests and uprisings occurred in more than dozens of Arab countries which toppled the regimes of long-serving dictators of Tunis, Libya, Egypt, and Yemen. Other monarchs were able to survive because of monetary and political concessions as well as repression in some countries. Arab spring was fueled by high unemployment, poverty, lack of economic opportunities, and the absence of an efficient public sector (Malik and Awadallah, 2013). The strength of the regime was critical, Brownlee and Masoud (2013) argue that regimes that lacked oil wealth and heredity failed to defend themselves during the Arab spring.

People consider revolting when they suffer from systematic, social, economic or political inequality, like the lack of political rights, relative poverty, social injustice, or poor working and living conditions. There are many approaches and strands of different kinds of literature in human sciences addressing revolts and revolutions. In the following, we shall comment on the most relevant work related to this paper in various literature. Let us start with grievance-based

theories of revolution, which consider perceived grievance as the main cause of social unrest and rebellion.¹ Following grievance-based theories, we shall assume in our model that inequality is the main driver behind revolutions. In this regard, MacCulloch (2005) tests whether inequality is indeed a driver for revolution. Using data from two surveys on revolutionary preferences in a random sample made of a quarter-million individuals, he found that a 1-standard deviation increase in the Gini coefficient explains up to 38% of the standard deviation in the support of the revolution. Besides, he also proved that income has a negative effect on the taste for revolt. We believe that MacCulloch’s results support our approach in which the wealthy do not revolt. On this matter, Stewart (2011) and Cederman et al. (2011) also underline social, political and economic inequality as a trigger behind conflicts and civil wars.

Revolutions have been deeply studied in the Public Choice literature. It goes without saying that people who participate in a revolution bear a personal cost and put themselves at risk to face violence from the regime if the movement fails. However, as the Public Choice literature underlines, if the revolution is successful then everyone will benefit from this public good, regardless of their participation. This is a clear free-riding problem. Indeed, every individual’s best move is to not participate in the protests and let others devote their time and efforts to the cause. Tullock (1971) argues that private potential personal gains are more important than the collective benefits of participation in a revolution. In Tullocks’ view, one needs to consider private gains or losses as the main driver to join a revolution. Revolutions can also be analyzed as volunteer-type public goods. In volunteer-type public goods, the quantity of public goods is the maximal of personal contributions of the individuals. Hirshleifer (1983) calls such public good “Best-shot”: if someone shoots at us then whoever is the “Best-shot” amongst us should end this threat, instead of every one of us shooting and wasting resources. Although these theses in Public Choice are enlightening, our simplified model cannot explain revolutions in terms of a public good, but rather as a revolt of the mass workers against the exploitative system.

Although all revolutions share their grievance origin, they differ in the source of grievance, their magnitude, length, outcomes and the wide variety of consequences they entail. Even if we focus here on social and economic revolutions, which we illustrate in the following paragraph, political revolutions illustrate at best our argument on the question of the optimal timing for a revolution. Indeed, one can first distinguish between failed and successful revolutions. In our specific context, and for simplicity reasons, a failed revolution is a revolution in which discriminated workers do not increase their power share after the coup. However, in the literature, the definition of a failed revolution is more open. It includes social movements that never give rise to a revolution, but also revolutions that do not improve the living conditions

¹ See for instance among many relevant references Davies (1962), Gurr (1968) and (2015), Muller and Seligson (1987), Schock (1996), Klandermans (1997), Buechler (2004) and Boucekine et al. (2019).

of the deprived. Among the failed revolutions, Goodwin (2001) counts Malaya, the Philippines and Honduras as failed revolutionary movements. On the successful side, there is plenty of examples, which differ in their timing and retaliation, just to mention two relevant points to the present paper. For instance, it took Mao Zedong thirty years to prepare and trigger a successful revolution, while Egypt and Tunisian revolutions were able to topple the regimes within months. Regarding retaliation, some revolutions are peaceful (the Velvet Revolution of Czechoslovakia) whereas others generate retaliation and produce civil wars (Libya and Syria).

Regarding the theoretical literature in the economics of social conflict, we would like to put forward first some papers on general equilibrium, which differ in the source of conflict. Let us begin with Grossman (1991), (1994) and (1995), where peasants allocate their time between production, soldering and rebellion. Peasant's income depends on the probability of a successful revolution against landlords, and the interaction between peasants and landlords generates an equilibrium allocation of labour time. In Dal Bo and Dal Bo (2011), there exists an appropriation sector, which is the source of social conflict and which makes all agents in the economy worse off. Finally, let us also mention two other papers on the general equilibrium of revolutions, like Lichbach (1987) and Moore (2000), where dissidents rebel against a government.

At the crossroads between the theoretical economics literature and economic history, Acemoglu and Robinson (2000, 2008) model the conflict between two classes, namely the elite (rich) and workers (poor), which are identical but for their socio-economic class. The authors argue that elites extended the franchise to the non-elite when they fear social unrest and revolution in the economy. Like them, we also consider in the present paper that a group of workers is exploited. Here, discrimination takes the form of a wage gap between the elite and the discriminated workers, which is the main cause of social unrest and revolution. The origin of this wage gap is not economic but social since all individuals have identical skills and abilities. Hence discrimination could be racial, ethnic or of gender, among others. However, for Acemoglu and Robinson (2000) if a revolution is initiated, then it always succeeds. Unlike them, the outcome of the revolution in our model depends on the technology of discriminated workers, the redistribution structure, and the strength of the established system. Throughout history, there have also been revolutions with peaceful closures. Schnytzer (1994) explains the revolutions of Eastern Europe, and models regime changes through peaceful revolutions. In Schnytzer's model, the unrest reaches a point at which the political system collapses and is replaced by a system that provides more freedom.

There is also a vast literature that uses game theory to explain the complex phenomenon of social movements and revolutions. For a successful revolution, dissidents must coordinate and organize themselves. The social movement is more likely to turn into a revolution if sufficiently

many people take this collective action against the government.² Besides, information has also been introduced in revolution game theoretical models. Indeed, note that information plays a key role in social movements since it allows rebels to coordinate, organize and communicate effectively. In reaction to it, the regime can control this flow of information by imprisoning the revolutionary leaders, spreading false propaganda against the movement, or banning the media outlets, for instance.³

Taking a slightly different approach, conflicts have also been treated as evolutionary games, where the people involved in a social movement may change over time (see Ross, 2001, Arce and Sandler, 2003, and Olsson, 2012). Finally, let us mention agent-based models, which use computational simulations to analyze conflicts and social movements (see Epstein, 2002, Bristow et al., 2013, and Lemos et al., 2014).

There are obviously other mathematical approaches to model conflict and social movements. For example, Lang and de Sterck (2014) develop a compartmental model for the dynamics of the Arab revolutions, and Levy and Faria (2007) who extend Ramsey's model to analyze the internal conflict among rival groups living on the same piece of land. Worth noting, ours is also a Ramsey set-up in which exploited workers maximize their lifetime welfare. The novelty of our study is that workers calculate the optimal time of revolution, that is, they choose whether or not to trigger a revolution, and the exact moment in time which maximizes their overall welfare.

This paper proposes a simple model to describe the evolution of social unrest and how social unrest leads to a system change at a specific date. In our economy, there are two types of workers: elites and discriminated workers. Although they work for the same firm, they are perfect substitutes in production and discriminated workers' salary is just a fraction of the elites'. In our one-sector model, workers do not have alternative employment and despite earning wages below their marginal product, they are compelled to work for the elites.⁴ As already mentioned in this introduction, we understand social unrest as a function of both social and economic inequality. For the sake of modelling, we have condensed inequality into a few indicators. Social unrest increases with the wage gap between elites and discriminated workers, with the number of discriminated workers, and with their organizational skills. On the downside, social unrest decreases with the retaliation strength of the system. Taking all

²See among many others Granovetter (1978), Kuran (1989), (1991), (1995), Lohmann (1993), (1994a), (1994b), (2000), Chwe (1999) and Tyson and Smith (2018).

³See for instance Egorov et al. (2009), de Mesquita (2010), Kricheli et al. (2011), Edmond (2013), Guttman and Reuveny (2014), Chen et al. (2016), Little (2016), Shadmehr et al. (2017) or Battaglini (2017).

⁴People can still be discriminated against on the basis of sex, caste, and colour even with the same skills and abilities. Coleman (2003) empirically investigates the presence of racial discrimination in the US labor market. He finds that despite having the same skills, measured by employer's competitive performance, there is a wage gap between black and white men. See also Arabsheibani et al. (2018) for the case of India.

these elements into account, we find that social unrest increases with time when discriminated workers are sufficiently well-organised and when wage inequality is sufficiently large.

Regarding the modeling of optimal timing, we are building on the seminal works of Tomiyama (1985), Tomiyama and Rossana (1989) and Makris (2001). This technique has traditionally been used in two-stage optimal control problems to study optimal technology adoption when there is embodied technical change (Boucekkine et al., 2004) or when firms can shift from brown to green technologies (see Boucekkine et al., 2011, Acemoglu et al. 2016, among many others). Here, we adopt these techniques to find the optimal time to trigger a revolution from the discriminated worker's perspective. Fortunate enough, our problem reduces to a static problem, so although we keep the spirit of the optimal switching time, our problem is much simpler from a technical point of view. We find that if prejudice against discriminated workers is hard enough so that social unrest increases with time, then a revolution does arise immediately if the perceived gains from a revolution are high. Otherwise, if gains are not high enough at time 0, then a revolution will still arise later on when social unrest becomes sufficiently important. In this case, the social movement is not mature enough to make worth a revolution. Worth to notice, we also show that even if social unrest decreases with time, a revolution does arise if the perceived gains from the revolution are large enough, if retaliation is strong or if deprived workers are initially in large numbers. One can think of situations in which the system is strong and it discourages discriminated workers from joining any revolutionary movement. However, precisely because of the strength of the system or of an unbearable wage gap, a revolution becomes necessary and optimal, and there exists a point in time at which a revolution breaks out sustained by a social movement that has matured with time.

This paper is structured as follows: Section 2 describes how social unrest evolves in time. Section 3 presents our economy, and the optimal time for a revolution is obtained and studied in Section 4. Finally, Section 5 presents our conclusions.

2 A simple dynamic description of social unrest

Among the many causes behind revolutions, we assume in this paper that revolutions are triggered by the stock of social unrest, which we denote by R . In turn, social unrest is a function of some social and economic factors that we will distill and reduce into two main driving factors. Namely, social and economic inequality. Society is divided into two distinct groups that for generality we denote as the elites and the discriminated workers. As mentioned in the Introduction, the distinction between elites and discriminated workers is unfounded, meaning that it is not based on any individual productive or legal characteristic. Both types of workers have identical skills, knowledge and could a priori have identical productivity. The

distinction between the two groups is merely social.

We assume first that social unrest depends on social polarity, that we model making social unrest a function of M , the number of discriminated workers, that is, individuals out of the elite. Second, social unrest also depends on economic inequality. In particular, social unrest increases with the inequality in salaries between elites and discriminated workers, which we denote here by W . Third, social unrest is also affected by the downside of social movements against the system. Workers participating in any of these social movements may be punished. We capture the strength of the system by a parameter $\phi \geq 0$. The more powerful the system is, the higher ϕ , the stronger the punishment will be and the quieter will become discriminated workers, reducing R , social unrest. In particular, if $\phi = 0$, then there is no retaliation and one could infer that the system actually agrees with revolutioneers.

Finally, the evolution of a social movement depends on its technology, that is, the effectiveness of strikes, clarity of purpose, and how well-organized the dissident group is against the system. Hence, we are assuming here that R increases with the organization technology of the discriminated workers, and we denote this technology with parameter $B \geq 0$. Taking all these elements into account, we describe the evolution of social unrest by the following dynamic equation

$$\dot{R}(t) = M (BW - \phi) R(t). \quad (1)$$

According to (1), social unrest increases with the number of discriminated workers, the revolution technology and the wage gap between discriminated and elite workers. Solving (1), we obtain the exact evolution of social unrest in time

$$R(t) = R_0 e^{M(BW - \phi)t}, \quad (2)$$

where R_0 is the initial value of social unrest.⁵ Obviously, if discriminated workers are in large numbers, and if inequality in salaries is sufficiently high so as to overcome eventual punishments, i.e. if $BW - \phi > 0$, then social unrest will increase with time. On the contrary, if punishment is sufficiently severe, then social unrest will diminish with time and tend to zero.

We consider that a revolution succeeds when it shifts the standing system even if slightly. In our model a revolutionary movement cannot change the social structure of the economy, meaning that discriminated workers can not become elites or the opposite. However, a revolutionary movement can modify the production system and workers' salaries as a result. Besides, revolutions are here modelled as stochastic processes, that is, their exact outcome is a priori unknown. In particular, the probability that a revolution succeeds at a given time t will be a function of $R(t)$ as follows

$$P[R(t)] = 1 - \frac{1}{R(t)} = 1 - \frac{1}{R_0} e^{-M(W - \phi)t}. \quad (3)$$

⁵See Appendix A for all computational details.

Note that this probability density function has already been used in the context of civil violence in Epstein (2002) and Lemos (2014).

When the system's punishment is strong and consequently ϕ high, discriminated workers do not mobilize enough, $P(R)$ is low and revolution can hardly succeed. Indeed, note that $P(R)$ tends to zero as ϕ increases.

3 The economy

As mentioned, our economy suffers from social and economic inequality in the form of a wage gap. As a result, social unrest is generated and nourished. As we shall show, if some conditions are favorable, then a revolution will be triggered with certainty. The revolution may or may not improve discriminated workers' situation in the firm, but whatever the outcome, the economy enters into a second phase after the revolution. It is important to recall that the outcome of a revolution is a stochastic process, meaning that revolutioneers are uncertain about the final result. Still, if the expected gains from a revolution are high enough, then discriminated workers will attempt their coup and start a revolution. Let us describe all the technical details.

At the initial date, our economy is made of two types of workers: elites, L , and discriminated workers, M . There is one production sector where both types work, and we assume that M and L are perfect substitutes. This means that both elites and discriminated workers have the same abilities and capacities. Besides, the two groups work the same number of hours and have the same set of skills. Nevertheless, discriminated workers are relegated to less advanced tasks within the firm making them less productive and as a result, their wage is less than the elites' by a factor $\alpha \in (0, 1)$. Accordingly, we assume that production is described by a standard Cobb-Douglas production function:

$$Y = A(\alpha l_M + l_L)^\nu, \quad (4)$$

where l_M and l_L are the amounts of labor provided by the elites and the discriminated workers, respectively. $A \in \mathbb{R}^+$ is a parameter measuring total factor productivity and $\nu \in (0, 1)$. According to (4), production is an increasing and concave function of total labor $\alpha l_M + l_L$. Besides, labor is essential to production, meaning that the firm cannot produce without labor.

The firm maximizes profits at every time t by hiring the optimal amounts of discriminated and elite workers, solving the following problem:

$$\max_{l_L, l_M} A(\alpha l_M + l_L)^\nu - w_L l_L - w_M l_M, \quad (5)$$

where w_L and w_M stand for the wages of the elite and the discriminated workers, respectively. Solving (5) requires finding the amounts of l_M and l_L , which will maximize the profits of the

firm. Profit is a concave function of both l_M and l_L . Thanks to concavity, it suffices to take the first-order condition of the objective function with respect to l_M and l_L , and equate them to zero in order to solve the firm's problem. Then, taking the first-order conditions with respect to l_L and l_M we obtain that $w_M = A\alpha\nu[\alpha l_M + l_L]^{\nu-1}$ and $w_L = A\nu[\alpha l_M + l_L]^{\nu-1}$.

•**A measure of inequality.**

Using the gap between w_M and w_L , we obtain a measure of inequality between the elites and discriminated workers:

$$\frac{w_L - w_M}{w_L} = 1 - \frac{w_M}{w_L} = 1 - \alpha, \quad (6)$$

where $1 - \alpha$ corresponds to W , the relative wage gap between the elite and the discriminated workers in equation (2). Using this result, we can write $R(t)$ as

$$R(t) = R_0 e^{M[B(1-\alpha)-\phi]t},$$

and

$$P[R(t)] = 1 - \frac{1}{R(t)} = 1 - \frac{1}{R_0} e^{-M[W(1-\alpha)-\phi]t}.$$

•**The consequences of a revolution for production.**

Suppose that a revolution arises at time T . If the revolution succeeds, then discriminated workers gain some additional power within the firm. This additional power is translated into a higher recognition of their labor. From T onwards, the role of discriminated workers in the firm will increase from α to γ , with $\gamma \leq 1$. As a result, production after the revolution becomes

$$Y = A(\gamma l_M + l_L)^\nu.$$

Salaries obtain as in the first phase by maximizing the firm profits with respect to l_M and l_L , $w_M^s = A\nu\gamma(\gamma l_M + l_L)^{\nu-1}$ and $w_L^s = A\nu[\gamma l_M + l_L]^{\nu-1}$. The wage gap is reduced after a successful revolution since we have now $1 - \frac{w_M^s}{w_L^s} = 1 - \gamma < 1 - \alpha$, and although the power of the elites remains unchanged in absolute terms, they do lose some power in relative terms with respect to discriminated workers.

We say that a revolution fails if discriminated workers do not increase their power share after the coup. In this case, $Y = A(\beta l_M + l_L)^\nu$, with $\beta \leq \alpha < \gamma$. Obviously, if the discriminated workers' role in the firm decreases after the coup, i.e. β is strictly smaller than α , then there is retaliation from the firm. Wages in case of failure are $w_M^f = A\nu\beta(\beta l_M + l_L)^{\nu-1}$, and $w_L^f = A\nu[\beta l_M + l_L]^{\nu-1}$.⁶

⁶Appendix B presents an extension of this problem, where the discriminated workers' bargaining power is

4 Optimal timing for revolution

In what follows we study the optimal timing for revolution, focusing on the decision-making process of discriminated workers, who are ultimately the only ones responsible for a revolution. One can also interpret the optimal timing of a revolution as the maturity point, that is, the exact time at which the social movement is mature enough to lead to a successful revolution. In order to focus on the mechanisms behind a revolution, we assume that workers take actions in order to maximize their lifetime welfare, measured as the discounted aggregate of instantaneous utility provided by consumption. For simplicity reasons, it is also assumed that workers consume all their earnings at every period, that is, they do not save and do not make any intertemporal decisions. Accordingly, consumption of discriminated and elite workers are $c_M = w_M$ and $c_L = w_L$, respectively.

Taking into account the possibility of a revolution, a representative discriminated worker will maximize her lifetime utility by choosing whether and when to initiate a revolution at a time $T \geq 0$, taking her revenue as given. Let us assume that discriminated workers' welfare at any time can be measured using a standard utility function u , which depends solely on consumption and which has the following properties:

Assumption 1. *The utility function $u(\cdot) : \mathbb{R}^+ \rightarrow \mathbb{R}^+$ is a positive, increasing and concave function of consumption.*

Each worker is endowed with one unit of time, and they supply their labor inelastically. Note that by assumption they do not have any other activity that can provide them with utility. Hence, the representative discriminated worker will work full time devoting her unit of time allocation to labor. As a consequence, at equilibrium labor demand for discriminated and elite workers equal supply, that is $l_M = M$ and $l_L = L$. The representative discriminated worker will maximize her lifetime overall discounted welfare by solving the following problem

$$\text{Max}_{\{T, c_M, c_M^s, c_M^f\}} \int_0^T u(c_M) e^{-\rho t} dt + P[R(T)] \int_T^\infty u(c_M^s) e^{-\rho t} dt + (1 - P[R(T)]) \int_T^\infty u(c_M^f) e^{-\rho t} dt. \quad (7)$$

Note that the agent's welfare is discounted in time using the exponential function $e^{-\rho t}$, with $\rho > 0$. The discount function encompasses two phenomena. First, the fact that individuals privilege the present and discount the future; and second, the survival probability, that is, the agent knows that the probability to remain alive decreases steadily with time.

c_M is the discriminated worker consumption before the revolution, and as such, $c_M = w_M =$

endogenous. In particular, we assume that their power is a function of R . Unfortunately, the problem does not have an analytical solution even using the simplest production functions.

$A\nu\alpha(\alpha M + L)^{\nu-1}$. After the revolution, c_M^s stands for the discriminated worker consumption if the revolution succeeds. Since all earnings are consumed also after the revolution, we have that $c_M^s = w_M^s = A\nu\gamma(\gamma M + L)^{\nu-1}$. Similarly, c_M^f is the discriminated worker consumption if the revolution fails, so that $c_M^f = w_M^f = A\nu\beta(\beta M + L)^{\nu-1}$, where $\beta \leq \alpha$, as already mentioned. Hence, $c_M^s > c_M^f$ and $u(c_M^s) > u(c_M^f)$.

Since all three expressions for consumption are constant in time, we can write the household objective function (7) as

$$\frac{1}{\rho} (1 - e^{-\rho T}) u(c_M) + P[R(T)] u(c_M^s) \frac{e^{-\rho T}}{\rho} + (1 - P[R(T)]) u(c_M^f) \frac{e^{-\rho T}}{\rho}. \quad (8)$$

The discriminated worker problem becomes then to choose the time T of a revolution that maximizes (8). We can prove that the discriminated worker problem has a unique solution T :

Proposition 1. *Under the model assumptions, there exists an optimal time T to trigger a revolution*

$$T = \frac{1}{M[B(1 - \alpha) - \phi]} \ln \left(\frac{u(c_M^s) - u(c_M^f) \rho + M[B(1 - \alpha) - \phi]}{u(c_M^s) - u(c_M)} \frac{\rho R_0}{\rho R_0} \right) \geq 0. \quad (9)$$

We can distinguish two cases depending on whether social unrest increases or decreases with time:

- 1) If social unrest is an increasing function of time, that is, if $M[B(1 - \alpha) - \phi] > 0$, and that a revolution always brings gains, i.e. $u(c_M^s) > u(c_M)$, then a revolution always arises. The revolution will arise in finite time if and only if

$$\frac{u(c_M^s) - u(c_M)}{u(c_M^s) - u(c_M^f)} < \frac{\rho + M[B(1 - \alpha) - \phi]}{\rho R_0}, \quad (10)$$

where $u(c_M^s) - u(c_M)$ is the gain in utility from the revolution if it succeeds. If condition (10) does not hold, meaning that the perceived gains are sufficiently high, then revolution is triggered immediately, that is, $T = 0$.

- 2) If $-\rho < M[B(1 - \alpha) - \phi] < 0$ and social unrest decreases with time, then a revolution is triggered in finite time, i.e. $T > 0$ if and only if

$$\frac{u(c_M^s) - u(c_M)}{u(c_M^s) - u(c_M^f)} > \frac{\rho + M[B(1 - \alpha) - \phi]}{\rho R_0}. \quad (11)$$

If condition (11) does not hold, then there will never be a revolution.

Proof. See Appendix D. □

In order to interpret Proposition 1 results, let us read $M[B(1 - \alpha) - \phi]$ as total experienced inequality. Proposition 1 distinguishes between two radically different social situations. In the first situation, social unrest increases with time because total inequality is larger than the expected punishment so the revolutionary movement will gain force with time. In this case, whether a revolution arises immediately or later on depends on the relative gains, measured by $u(c_M^s) - u(c_M)$. If relative gains are sufficiently large already at time 0, then the revolution breaks out immediately, as soon as discriminated workers gather and initiate a social movement against the system.

The second point in Proposition 1 shows that even if social unrest is slowing down with time, a revolution can still arise when the relative perceived gain from revolution is high enough. If gains from the revolution are not large enough, all the odds linked to a revolution will win over the gains, and the revolution will never break out. Note that the condition for revolution in (11) is easily verified when retaliation is harder and ϕ increases. As mentioned in the Introduction, this astonishing case of a revolution breaking out despite a social movement that loses its impulse fits forceful systems with powerful repression apparatuses. With time, the situation of discriminated workers becomes unbearable, and they trigger a revolution when the movement can ensure some gains, despite retaliation.

It makes sense to think that the more powerful the system, i.e. the higher ϕ , the lower c_M^f . In this regard, let us consider that c_M^f is a function of ϕ , with $c_M^{f'}(\phi) < 0$. We obtain the following corollary:

Corollary 1. *Under the model assumptions, if c_M^f is a function of ϕ , with $c_M^{f'}(\phi) < 0$, and denoting by T^e the optimal time for revolution in this case, then*

$$\frac{\partial T^e}{\partial \phi} < \frac{\partial T}{\partial \phi}.$$

Proof. See Appendix E. □

Corollary 1 shows that in case the system had some economic power, and it could reinforce discrimination by lowering the wages of the discriminated, then an increase in the system's power will make the revolution arise sooner than if the system did not have any economic power.

To finish our analysis, we would like to deepen our analysis by underlining the role of gains from revolution. According to the following corollary, revolutions are postponed when there is not enough to gain:

Corollary 2. *Let us assume $M[B(1 - \alpha) - \phi] > 0$ so that social unrest increases with time. The larger R_0 , that is, the initial social unrest, the sooner the revolution arises. If $u(c_M^s)$ tends to $u(c_M)$, then T tends to infinite and revolution never takes place.*

Proof. The corollary can directly be proven by taking partial derivatives of T with respect to R_0 first and then taking the limit when $u(c_M^s)$ tends to $u(c_M)$. \square

Corollary 2 proves that even if unrest increases steadily in time, revolutions do not always occur. In particular, revolutions do not arise if they do not bring substantial gains.

5 Conclusion

Revolutions are defined as a shift (even if slight) in the economic production system so that discriminated workers get better, fairer remuneration. Revolutions are defined as a shift (even if slight) in the economic production system so that discriminated workers get better and fairer remuneration in accord with their marginal products. We have built a simple yet comprehensive model to study the optimal timing of revolutions. Worth noting, this optimal time also corresponds to the moment at which an anti-establishment social movement supported by discriminated workers is mature enough. We first describe the evolution of social unrest mathematically as a function of wage inequality between elites and discriminated workers, the rule of power, dissident organization techniques as well as the number of discriminated workers. Here it has been assumed that discriminated workers and elites have the same skills and that they are equally productive. All wage differences emanate from social discrimination. Besides, revolutions have been understood as stochastic processes, and the larger the social unrest, the higher the probability that a revolution will succeed. We prove that if wage inequality is large enough and social unrest increases with time, then a revolution does arise immediately. Otherwise, if they are not high enough at time 0, then a revolution arises later on when social unrest becomes sufficiently high.

Let us finally put forward that even if social unrest decreases with time, a revolution can still arise if the perceived gains from the revolution are large enough, if retaliation is strong or if deprived workers are initially in large numbers. Indeed, repressive systems make revolutions worth it even if revolutionary movements lose momentum due to possible punishment.

We leave for future research the extension of our model to explain early feminist movements and current racial discrimination. Over simplifying, we can say that one of the main sources of grievance in early feminist movements was the unequal opportunities faced by men and women, and more recently the wage gap between men and women with the same skills, working the same number of hours for the same firm. Our model could actually serve to analyze feminist social movements and their possible uprising against the established system. In this regard, Castagnetti and Giorgetti (2019) find a lower but still significant gender wage gap in the public sector with respect to the private one, and the gender wage gap increases along with the wage distribution in both sectors. Unfortunately, there exist other types of discrimination. Isabelle

et al. (2017) find empirical evidence that like in other OECD countries women in New Zealand earn substantially less than men even with similar observable skills.

References

- Acemoglu, D., Akcigit, U., Hanley, D., and Kerr, W. (2016), “Transition to clean technology,” *Journal of Political Economy*, 124(1), 52-104.
- Acemoglu, D., Robinson, J. A. (2000). “Why did the West extend the franchise? Democracy, inequality, and growth in historical perspective”. *The quarterly journal of economics*, 115(4), 1167-1199.
- Arabsheibani, G., Gupta, P., Mishra, T., Parhi, M. (2018), “Wage differential between caste groups: Are younger and older cohorts different?”, *Economic Modelling*, 74, 10-23.
- Arce M, D. G., and Sandler, T. (2003), “An evolutionary game approach to fundamentalism and conflict,” *Journal of Institutional and Theoretical Economics*, 132–154.
- Battaglini, M. (2017), “Public protests and policy making,” *The Quarterly Journal of Economics*, 132 (1), 485–549.
- Boucekkine, R., Saglam, C., and Vallee, T. (2004), “Technology adoption under embodiment: a two-stage optimal control approach,” *Macroeconomic Dynamics*, 8(2), 250-271.
- Boucekkine, R., Pommeret, A., and Prieur, F. (2013), “Technological vs. ecological switch and the environmental Kuznets Curve,” *American Journal of Agricultural Economics*, 95(2), 252-260.
- Boucekkine, R., Desbordes, R., and Melindi-Ghidi, P. (2019), “Social divisiveness and conflicts: Grievances matter!”
- Bristow, M., Fang, L., and Hipel, K. W. (2013), “Agent-based modeling of competitive and cooperative behavior under conflict,” *IEEE Transactions on Systems, Man, and Cybernetics: Systems*, 44 (7), 834–850.
- Brownlee, J., Masoud, T., Reynolds, A. (2013), “Tracking the Arab Spring: Why the Modest Harvest?,” *Journal of Democracy*, 24(4), 29-44.
- Buechler, S. M. (2004), “The strange career of strain and breakdown theories of collective action,” *The Blackwell companion to social movements*, 47–66.
- Castagnetti, C., Giorgetti, M. L. (2019). “Understanding the gender wage-gap differential between the public and private sectors in Italy: A quantile approach”, *Economic Modelling*, 78, 240-261.
- Cederman, L.E., Weidmann, N. B., and Gleditsch, K. S. (2011), “Horizontal inequalities and ethnonationalist civil war: A global comparison,” *American Political Science Review*, 478–495.
- Chen, H., Lu, Y. K., and Suen, W. (2016), “The power of whispers: a theory of rumor, communication, and revolution,” *International economic review*, 57 (1), 89–116.
- Charles, K. K., Guryan, J. (2008). “Prejudice and wages: an empirical assessment of Becker’s The Economics of Discrimination”, *Journal of political economy*, 116(5), 773-809.

- Chwe, M. S.Y. (1999), "Structure and strategy in collective action," *American journal of sociology*, 105 (1), 128–156.
- Coleman, M. G. (2003), "Job skill and black male wage discrimination", *Social Science Quarterly*, 84(4), 892–906.
- Dal Bo, E., and Dal Bo, P. (2011), "Workers, warriors, and criminals: social conflict in general equilibrium," *Journal of the European Economic Association*, 9 (4), 646–677.
- Davies, J. C. (1962), "Toward a theory of revolution," *American sociological review*, 5–19.
- De Mesquita, E. B. (2010), "Regime change and revolutionary entrepreneurs," *American Political Science Review*, 104 (3), 446–466.
- Dowe, H. and Langewiesche, S. (2001). *Europe in 1848: Revolution and reform*, Berghahn Books.
- Edmond, C. (2013), "Information manipulation, coordination, and regime change," *Review of Economic studies*, 80 (4), 1422–1458.
- Egorov, G., Guriev, S., and Sonin, K. (2009), "Why resource-poor dictators allow freer media: A theory and evidence from panel data," *American political science Review*, 103 (4), 645–668.
- Epstein, J. M. (2002), "Modeling civil violence: An agent-based computational approach," *Proceedings of the National Academy of Sciences*, 99 (suppl 3), 7243–7250.
- Furtado, P. (Ed.). (2020). *Revolutions: How They Changed History and what They Mean Today*, Thames Hudson.
- Goodwin, J. (2001), *No other way out: States and revolutionary movements, 1945-1991*, Cambridge University Press.
- Granovetter, M. (1978), "Threshold models of collective behavior," *American journal of sociology*, 83 (6), 1420–1443.
- Grossman, H. I. (1991), "A general equilibrium model of insurrections," *The American Economic Review*, 912–921.
- Grossman, H. I. (1994), "Production, appropriation, and land reform," *The American Economic Review*, 84 (3), 705–712.
- Grossman, H. I. (1995), "Robin hood and the redistribution of property income," *European Journal of Political Economy*, 11 (3), 399–410.
- Gurr, T. (1968), "A causal model of civil strife: A comparative analysis using new indices," *The American Political Science Review*, 62 (4), 1104–1124.
- Gurr, T. R. (2015), *Why men rebel*, Routledge.
- Guttman, J. M., Reuveny, R. (2014), "On revolt and endogenous economic policy in autocratic regimes", *Public Choice*, 159(1), 27–52.
- Hirshleifer, J. (1983), "From weakest-link to best-shot: The voluntary provision of public goods", *Public choice*, 41(3), 371–386.

- Huntington, S. P. (1968), *Political Order in Changing Societies*, New Haven, CT, and London: Yale University Press.
- Jones, G. S. (1983). *Languages of class: Studies in English working class history 1832-1982*, Cambridge University Press.
- Iacopetta, M. (2021), “Class differences and the Commercial Revolution: An equilibrium selection story”, *Economic Modelling*, 100, 105508.
- Klandermans, P. G. (1997), *The social psychology of protest*.
- Kricheli, R., Livne, Y., and Magaloni, B. (2011), “Taking to the streets: Theory and evidence on protests under authoritarianism,” *Apsa 2010 annual meeting papers*.
- Kuran, T. (1989), “Sparks and prairie fires: A theory of unanticipated political revolution,” *Public choice*, 61 (1), 41–74.
- Kuran, T. (1991), “Now out of never: The element of surprise in the east european revolution of 1989,” *World politics*, 44 (1), 7–48.
- Kuran, T. (1995), “The inevitability of future revolutionary surprises,” *American Journal of Sociology*, 100 (6), 1528–1551.
- Lang, J. C. and De Sterck, H. (2014), “The arab spring: A simple compartmental model for the dynamics of a revolution,” *Mathematical Social Sciences*, 69 , 12–21.
- Lemos, C., Lopes, R. J. and Coelho, H. (2014), “An agent-based model of civil violence with imprisonment delay and legitimacy feedback,” in 2014 second world conference on complex systems (wccs) (pp. 524–529).
- Levy, A. and Faria, J. R. (2007), “Ramsey in dual-population lands: internal conflict and utility-maximizing consumption,” *Defence and Peace Economics*, 18 (4), 339–352.
- Lichbach, M. I. (1987), “Deterrence or escalation? the puzzle of aggregate studies of repression and dissent,” *Journal of Conflict Resolution*, 31 (2), 266–297.
- Little, A. T. (2016), “Communication technology and protest,” *The Journal of Politics*, 78 (1), 152–166.
- Lohmann, S. (1993), “A signaling model of informative and manipulative political action,” *American Political Science Review*, 87(2), 319–333.
- Lohmann, S. (1994a), “The dynamics of informational cascades: The Monday demonstrations in Leipzig, East Germany, 1989–91,” *World politics*, 47(1), 42–101.
- Lohmann, S. (1994b), “Information aggregation through costly political action,” *The American Economic Review*, 518–530.
- Lohmann, S. (2000), “Collective action cascades: An informational rationale for the power in numbers,” *Journal of Economic Surveys*, 14(5), 655–684.
- MacCulloch, R. (2005), “Income inequality and the taste for revolution”, *The Journal of Law and Economics*, 48(1), 93-123.

- Malik, A., Awadallah, B. (2013), "The economics of the Arab Spring", *World Development*, 45, 296-313.
- Makris, M. (2001), "Necessary conditions for infinite horizon discounted two-stage optimal control problems", *Journal of Economic Dynamics and Control* 25, 1935-1950.
- Moore, W. H. (2000), "The repression of dissent: A substitution model of government coercion," *Journal of conflict resolution*, 44(1), 107-127.
- Muller, E. N. and Seligson, M. A. (1987), "Inequality and insurgency," *The American Political Science Review*, 425-451.
- Olsson-Yaouzis, N. (2012), "An evolutionary dynamic of revolutions," *Public Choice*, 151(3), 497-515.
- Parker, D. (Ed.).(2002). "Revolutions and the revolutionary tradition: in the West 1560-1991", Routledge.
- Ross, M. H. (2001), "Contributions of evolutionary thinking to theories of ethnic," *Evolutionary theory and ethnic conflict*, 71.
- Saville, J. (1990). *1848: the British state and the Chartist movement*. Cambridge University Press
- Schock, K. (1996), "A conjunctural model of political conflict: The impact of political opportunities on the relationship between economic inequality and violent political conflict," *Journal of Conflict resolution*, 40(1), 98-133.
- Shadmehr, M. and Bernhardt, D. (2017), "When can citizen communication hinder successful revolution?", *Quarterly Journal of Political Science*, 12(3), 301-323.
- Schnytzer, A. (1994), "An economic model of regime change: Freedom as a public good." *Public Choice* 79, no. 3 : 325-339.
- Sin, I., Stillman, S., Fabling, R. (2020). "What drives the gender wage gap? Examining the roles of sorting, productivity differences, bargaining and discrimination", *The Review of Economics and Statistics*, 1-44.
- Skocpol, T. and Theda, S. (1979), *States and social revolutions: A comparative analysis of France, Russia and China*. Cambridge University Press.
- Stewart, F. (2011), "Horizontal inequalities as a cause of conflict: A review of CRISE findings".
- Tarrow, S. (1994), *Power in Movement: Social Movements, Collective Action and Politics*, Cambridge, UK: Cambridge University Press.
- Tomiyaama, K. (1985), "Two-stage optimal control problems and optimality conditions," *Journal of Economic Dynamics and Control* 9, 317-337.
- Tomiyaama, K. and R. Rossana (1989), "Two-stage optimal control problems with an explicit switch point dependence: Optimality criteria and an example of delivery lags and investment," *Journal of Economic Dynamics and Control* 13, 319-337.
- Tullock, G. (1971). The paradox of revolution. *Public Choice*, 89-99.

Tyson, S. A. and Smith, A. (2018), “Dual-layered coordination and political instability: Repression, co-optation, and the role of information,” *The Journal of Politics*, 80(1), 44–58.

Appendices

A Solving for R

Equation (1) can be rewritten as

$$\frac{\dot{R}(t)}{R(t)} = M[B(1 - \alpha) - \phi].$$

Integrating both sides with respect to t

$$\int \frac{\dot{R}(t)}{R(t)} dt = \int M[B(1 - \alpha) - \phi] dt,$$

we obtain that

$$\ln R(t) = M[B(1 - \alpha) - \phi]t + c,$$

with $c \in \mathbb{R}$. Then, $R(t) = e^{M[B(1 - \alpha) - \phi]t} e^c$. Since $R(0) = R_0$ is known, (2) obtains.

B Endogenous bargaining power of the discriminated workers

As an alternative to our assumption, we could have instead assumed that after the revolution, the discriminated workers' bargaining power

$$Y = A(P[R(T)]M + L)^\nu.$$

In this case, the objective function of a discriminated worker is

$$\frac{1}{\rho} [1 - e^{-\rho T}] u(c_M) + P[R(T)] u(c_M^s) \frac{e^{-\rho T}}{\rho} + (1 - P[R(T)]) u(c_M^f) \frac{e^{-\rho T}}{\rho}, \quad (12)$$

where c_M and c_M^f are as in the main text, and c_M^s is now $c_M^s = A\nu P[R(T)] (P[R(T)]M + L)^{\nu-1}$.

Maximizing (12) is a hard problem. Assuming that both production and utility are linear functions, we have that $u(c_M) = A\alpha$, $u(c_M^s) = AR(T)$ and $u(c_M^f) = A\beta$. Let $x = M[B(1 - \alpha) - \phi]$.

The first-order condition of (12) with respect to T is

$$1 + \alpha + R_0 e^{xT} \left(\frac{x}{\rho} - 1 \right) - \frac{\beta}{R_0} e^{-xT} \left(\frac{x}{\rho} + 1 \right) = 0, \quad (13)$$

which cannot be solved analytically, although it can be proven that a solution exists.

C Derivation from equation (7) to (8)

We show next how we obtain (8) from (7). Let us denote by I the objective function in (7)

$$I = \int_0^T u(c_M) e^{-\rho t} dt + P[R(T)] \int_T^\infty u(c_M^s) e^{-\rho t} dt + (1 - P[R(T)]) \int_T^\infty u(c_M^f) e^{-\rho t} dt.$$

Since c_M , c_M^s and c_M^f are constant, we can solve each of the three integrals in I

$$I = u(c_M) \left. \frac{e^{-\rho t}}{(-\rho)} \right|_0^T + P[R(T)] u(c_M^s) \left. \frac{e^{-\rho t}}{(-\rho)} \right|_T^\infty + (1 - P[R(T)]) u(c_M^f) \left. \frac{e^{-\rho t}}{(-\rho)} \right|_T^\infty,$$

$$I = u(c_M) \left[\frac{-e^{-\rho T}}{\rho} + \frac{1}{\rho} \right] + P[R(T)] u(c_M^s) \frac{1}{(-\rho)} \left(\lim_{t \rightarrow \infty} e^{-\rho t} - e^{-\rho T} \right) + (1 - P[R(T)]) u(c_M^f) \frac{1}{(-\rho)} \left(\lim_{t \rightarrow \infty} e^{-\rho t} - e^{-\rho T} \right)$$

Since $\lim_{t \rightarrow \infty} e^{-\rho t} = 0$, then

$$I = \frac{1}{\rho} (1 - e^{-\rho T}) u(c_M) + P[R(T)] u(c_M^s) \frac{e^{-\rho T}}{\rho} + (1 - P[R(T)]) u(c_M^f) \frac{e^{-\rho T}}{\rho},$$

which is exactly (8) .

D Proof of Proposition 1

Taking the first order condition of (8) with respect to T we obtain:

$$u(c_M) e^{-\rho T} + P'[R(T)] R'(T) u(c_M^s) \frac{e^{-\rho T}}{\rho} - P[R(T)] u(c_M^s) e^{-\rho T} = 0,$$

or

$$-P'[R(T)] R'(T) u(c_M^f) \frac{e^{-\rho T}}{\rho} - (1 - P[R(T)]) u(c_M^f) e^{-\rho T} = 0.$$

Taking $e^{-\rho T}$ common, and re-arranging terms:

$$u(c_M) - u(c_M^f) - P[R(T)] [u(c_M^s) - u(c_M^f)] + \frac{1}{\rho} P'[R(T)] R'(T) [u(c_M^s) - u(c_M^f)] = 0.$$

Dividing both sides by $u(c_M^s) - u(c_M^f)$

$$P[R(T)] = \frac{1}{\rho} P'[R(T)] R'(T) + \frac{u(c_M) - u(c_M^f)}{u(c_M^s) - u(c_M^f)}.$$

Substituting $R(T)$, $P[R(T)]$ and $P'[R(T)]$ by their expressions, that is $R(T) = R_0 e^{M[B(1-\alpha)-\phi]T}$, $P[R(T)] = 1 - \frac{1}{R_0} e^{-M[B(1-\alpha)-\phi]T}$ and $P'[R(T)] = \frac{1}{R(T)^2} = \frac{1}{R_0^2} e^{-2M[B(1-\alpha)-\phi]T}$:

$$1 - \frac{1}{R_0} e^{-M[B(1-\alpha)-\phi]T} = \frac{1}{\rho} \frac{1}{R_0} e^{-M[B(1-\alpha)-\phi]T} M [B(1-\alpha) - \phi] + \frac{u(c_M) - u(c_M^f)}{u(c_M^s) - u(c_M^f)}$$

We can write

$$e^{-M[B(1-\alpha)-\phi]T} = R_0 \frac{u(c_M^s) - u(c_M)}{u(c_M^s) - u(c_M^f)} \frac{\rho}{\rho + M [B(1-\alpha) - \phi]},$$

so that

$$e^{M[B(1-\alpha)-\phi]T} = \frac{u(c_M^s) - u(c_M^f)}{u(c_M^s) - u(c_M)} \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0},$$

and

$$T = \frac{1}{M[B(1-\alpha) - \phi]} \ln \left(\frac{u(c_M^s) - u(c_M^f)}{u(c_M^s) - u(c_M)} \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0} \right).$$

Then the two cases arise:

1. If $M[B(1-\alpha) - \phi] > 0$, then T is a positive number if and only if

$$\frac{u(c_M^s) - u(c_M^f)}{u(c_M^s) - u(c_M)} \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0} > 1,$$

or equivalently, if and only if

$$\frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0} > \frac{u(c_M^s) - u(c_M)}{u(c_M^s) - u(c_M^f)},$$

$$\text{If } \frac{u(c_M^s) - u(c_M)}{u(c_M^s) - u(c_M^f)} = \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0}, \text{ then } T = 0$$

2. If $M[B(1-\alpha) - \phi] < 0$, then revolution can still arise if the logarithm is negative, which arises if and only if

$$\frac{u(c_M^s) - u(c_M^f)}{u(c_M^s) - u(c_M)} \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0} < 1.$$

E Proof of Corollary 1

We know that

$$T = \frac{1}{M[B(1-\alpha) - \phi]} \ln \left(\frac{u(c_M^s) - u(c_M^f)}{u(c_M^s) - u(c_M)} \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0} \right).$$

Then

$$\frac{\partial T}{\partial \phi} = -\frac{1}{M[B(1-\alpha) - \phi]^2} \ln \left(\frac{u(c_M^s) - u(c_M^f)}{u(c_M^s) - u(c_M)} \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0} \right) + \frac{1}{M[B(1-\alpha) - \phi]} \frac{(-M)}{\rho + M[B(1-\alpha) - \phi]}.$$

$$\begin{aligned} \frac{\partial T^e}{\partial \phi} &= -\frac{1}{M[B(1-\alpha) - \phi]^2} \ln \left(\frac{u(c_M^s) - u(c_M^f)}{u(c_M^s) - u(c_M)} \frac{\rho + M[B(1-\alpha) - \phi]}{\rho R_0} \right) \\ &+ \frac{1}{M[B(1-\alpha) - \phi]} \left[\frac{-u'(c_M^f) c_M^{f'}(\phi)}{u(c_M^s) - u(c_M^f)} - \frac{M}{\rho + M[B(1-\alpha) - \phi]} \right], \end{aligned}$$

hence

$$\frac{\partial T^e}{\partial \phi} = \frac{\partial T}{\partial \phi} - \frac{u'(c_M^f) c_M^{f'}(\phi)}{(u(c_M^s) - u(c_M^f)) M[B(1-\alpha) - \phi]}.$$

Since $c_M^{f'}(\phi) < 0$ and $B(1-\alpha) - \phi < 0$ in case 2, then

$$\frac{\partial T^e}{\partial \phi} < \frac{\partial T}{\partial \phi}.$$