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Abstract

We show that the long-term properties of price and cost in Chamberlin's (1933) monopolistic competition model can be reproduced with a soft-capacity constrained price competition oligopoly model for a homogeneous good with free entry.

Key words: price competition, soft-capacity constraint, free entry, U-shaped cost function, monopolistic competition, Chamberlin.

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1 Introduction.

We revisit the Chamberlinian approach to monopolistic competition under the light of recent progress in the literature on price competition in an oligopoly market when not assuming constant returns to scale. This possibility has for a long time been dismissed because of Edgeworth's 1925 conjecture that there is no equilibrium in this case. Dastidar (1995) has proven the existence of an equilibrium in price competition when the production cost function is convex, paving the way for a new strand of the literature (see Dastidar, 2001, 2011; Hoernig, 2002; Chaudhuri, 1996; Chowdhury, 2002, 2009; Vives, 1999; Chowdhury and Sengupta, 2004; Novshek and Chowdhury, 2003; Bagh, 2010; Routledge, 2010; Yano and Komatsubara, 2018, among others).

Chamberlin developed his pathbreaking theoretical model of monopolistic competition in 1933. This theory has been seminal in many areas of economic theory and has generated an extensive and controversial literature in many fields (industrial organization; international trade; macroeconomics, etc.). In international trade in particular, the modern version of the monopolistic competition model as described by Spence (1976); Dixit and Stiglitz (1977); Krugman (1979); Hart (1985); Parenti et al. (2017), (sees Thisse and Ushchev, 2018, for an extensive survey) has become almost hegemonic, especially in the empirical literature (see Head and Spencer, 2017, for a critical survey).

Chamberlinian monopolistic competition is based on two main ideas: 1. Price setting firms act monopolistically on their differentiated product 2. Free entry drives profits to zero. This framework provided the first ever representation of a market where in the long-run equilibrium, the price is equal to the average cost and is higher than the marginal cost (i.e. the equilibrium is in the increasing returns part of the traditional U-shaped cost function). The model thus accounts for market inefficiency (i.e. markup) and production inefficiency (free entry does not lead to minimal average

costs).[1]

However, the one important limitation of this model is that it does not account for strategic interactions between firms. The traditional assumption to justify this absence of interactions, the "negligibility hypothesis", proposed by Chamberlin himself, is that the number of firms operating in the market is large enough for any interactions to be negligible. It has subsequently been shown that under this assumption, an oligopoly model will converge toward a monopolistic competition equilibrium if consumers have sufficiently different tastes (Hart, 1985) or a *relatively high desire for variety* (Zhelobodko et al., 2012).

This emphasis on consumer preference and the requirement of a sufficiently high appreciation of variety highlights the apparent necessity of product differentiation to obtain the Chamberlinian equilibrium under monopolistic competition. Thus, the conventional wisdom in the literature has been that product differentiation is essential to obtain a Chamberlin-like result. It is well known that for an homogeneous product, Cournot oligopoly tends toward perfect competition when the number of firm tends to infinity, while under Bertrand competition, whatever the number of firms, price equals marginal cost.[2]

In this paper, we propose to reproduce monopolistic competition equilibrium properties within a price competition oligopoly model for a homogeneous good, taking full account of strategic interactions between a finite number of firms.

To do this, we use a model of price competition with soft capacity constraints (Cabon-Dhersin and Drouhin, 2014, 2020) that expands on Dastidar (1995) by assuming that firms rely on two substitutable factors of production chosen sequentially. In the first stage, firms choose a capacity factor. In the second, during which the firms compete on price, a variable factor is adjusted to match incoming demand. Because

[1] Chamberlin (1933) overlooked the fact that within a differentiated goods oligopoly, greater variety is always welfare-increasing (Baumol, 1964; Dixit and Stiglitz, 1977).

[2] Yano (2006) discussed the equilibrium characteristics of various refinements of the price competition game.

the capacity factor and the variable factor are substitutable, above-capacity production is always possible but at an increasing marginal cost (i.e the capacity constraint is "soft").[3] As demonstrated by Cabon-Dhersin and Drouhin (2020) under very general assumptions, this framework produces highly unconventional results, two of which are particularly important to illustrate our point: 1. the non-cooperative equilibrium of the non-repeated game is collusive (and thus can be found as a solution to a profit maximization program). 2. the model has an equilibrium whatever the returns to scales.[4]

To stay as close as possible to the the original Chamberlinian framework, we use a Stone-Geary production function that generates a U-shaped average cost function. Since this function is non-homothetic, this function is particularly interesting to study free-entry equilibria not considered in Cabon-Dhersin and Drouhin (2020).

The paper is organized as follows. Section 2 presents the model. The equilibrium for a given number of firms is presented in section 3. The notion of free-entry equilibrium is introduced in section 4, allowing the Chamberlinian long-term properties of price and cost to be reproduced in an oligopoly model with a single homogeneous good. Section 5 concludes.

2 The model

We consider n identical firms with a two-factor production function, with the factors chosen sequentially. In the first stage, the quantity of a fixed factor, z, is chosen. This is interpreted as the production capacity. In the second stage the firms compete on

[3] It should be emphasized that this notion of a soft capacity constraint is based on rigorous micro-foundations, starting from a general production function, with the only requirement being that it must be quasi-concave. In particular, this generates a cost function that is not necessarily additively separable in terms of the capacity factor (which is fixed in the second stage) and the variable factor.

[4] Provided a firm considers the price as given, be it that of competing firms as in Bertrand competition games, or simply the market price as in perfect competition, the second-order condition of optimality is not satisfied. In our approach, it is the sequential nature of the choices of the capacity and the variable factor that ensure a short-term convex cost function is obtained in the second stage, whatever the returns to scales, thereby guaranteeing the existence of an equilibrium.

price and adjust the level of the variable factor v to match incoming demand.

We assume the production function to be derived from the Stone-Geary class of utility functions:

$$f(z,v) = A \cdot \left((z-z_0)^{1-\alpha} v^{\alpha}\right)^{\rho} \tag{1}$$

with z_0, the minimum level for the fixed factor, interpreted as the "minimum size of the firm", $\rho > 0$, the "asymptotic scale elasticity of production", and, $\rho\,\alpha < 1$ and $\rho\,(1-\alpha) < 1$. Stone-Geary functions have been used extensively in consumer theory to provide a utility representation of consumer preference with subsistence levels for some goods. In consumer theory, because utility is an *ordinal* concept, quasi-concavity is a sufficient condition for there to be an interior solution to the consumer choice program. The Stone-Geary function is quasi-concave over all its range. In producer theory in contrast, the production function is *cardinal* by nature. The notion of "returns to scale" captures the "cardinal" nature of producer decisions. However, essentially because only monopoly models admit an equilibrium when returns to scale are increasing, the possibility of "internal" increasing returns has been progressively dismissed in the modern literature on oligopoly markets. Returning to the Stone-Geary formulation, we can calculate the local scale elasticity of production:

$$\eta(z,v) = \frac{f_z(z,v)z + f_v(z,v)v}{f(z,v)} = (1-\alpha)\rho\frac{z}{z-z_0} + \alpha\rho \tag{2}$$

We can see that η only depends on z, decreasing from $+\infty$, when z tends to z_0, and decreasing down to ρ, the "asymptotic elasticity of production", when z tends to $+\infty$. Thus, when ρ is strictly lower than one, there is a threshold value of z at which the returns to scale changes from being increasing (beneath the threshold) to decreasing (above), leading to an U-shaped average cost function, as illustrated in Beattie and Aradhyula (2015). Because our general model of price competition (Cabon-Dhersin and Drouhin, 2020) disentangles the existence of equilibrium from the nature of the returns to scale, the Stone-Geary production function with a minimum firm size can

rightfully be used to investigate certain industrial organization problems.

We compute $\hat{v}$, the quantity of the variable factor, as an implicit function defined by $y = f(z, v)$:

$$\hat{v}(y, z) = \frac{y^{\frac{1}{\alpha\rho}}}{A^{\frac{1}{\alpha\rho}}(z - z_0)^{\frac{1-\alpha}{\alpha}}} \tag{3}$$

For the sake of simplicity, we assume a linear demand function $D(p) = b \cdot (p_{max} - p)$, with p_{max} a choke price, p, the market price, and $b > 0$.

When n firms operate in the market (at the same price), they share the demand equally and the profit function $\hat{\pi}$ can be written:

$$\hat{\pi}(p, z, n) = p\frac{D(p)}{n} - p_z z - p_v \frac{\left(\frac{D(p)}{n}\right)^{\frac{1}{\alpha\rho}}}{A^{\frac{1}{\alpha\rho}}(z - z_0)^{\frac{1-\alpha}{\alpha}}} \tag{4}$$

with p_z the price of the fixed factor, and p_v the price of the variable factor.

3 Equilibrium for a finite number of firms

For each firm i, with $i = 1 \cdots n$, the strategic variables are: z_i the production capacity, chosen in the first stage, and p_i, the price chosen in the second stage, constrained by the capacity chosen in the first stage. This capacity constraint is *soft* because production can always be increased in the second stage by paying an increasing marginal cost. At the equilibrium, all firms choose the same capacity and price.

To find the equilibrium of the game with a finite number of firms n, we define:

1. $\bar{p}(z, n)$, the price that solves the equation $\hat{\pi}(p, z, n) = \hat{\pi}(p, z, 1)$, the threshold under which firms have no interest in undercutting their rivals in the second stage, when costs are convex (Dastidar, 1995, 2001),

2. $\hat{p}(z, n)$, the price that solves $\hat{\pi}(p, z, n) = -p_z z$, the threshold under which the variable profit is negative in the second stage,

3. $p^*(z,n)$, the price that maximizes $\hat{\pi}(p,z,n)$ for a given z and n, the purely collusive price,

4. $\bar{z}(p,n)$, the level of the fixed factor that maximizes $\hat{\pi}(p,z,n)$ s.t. $p \leq \bar{p}(z,n)$ (i.e. p is a Nash equilibrium in the second stage),

5. $z^*(p,n)$, the level of the fixed factor that maximizes $\hat{\pi}(p,z,n)$.

The equilibrium prediction of the game is given by:

Proposition 1 (Proposition 3 in Cabon-Dhersin and Drouhin (2020)). *The unique outcome in which all n firms choose the same fixed factor level, $z^C(n)$, in the first stage and quote the same price, $p^C(n)$, in the second, with $p^C(n)$ being a solution of the program,*

$$\mathcal{P}_1(n) \begin{cases} & \max_p \hat{\pi}(p,z,n) \\ s.t. & z^C(n) = \bar{z}(p,n) \\ & \hat{\pi}(p,z,n) \geq \hat{\pi}(\hat{p}(z,n), \operatorname{argmax}_{\tilde{z}} \hat{\pi}(\hat{p}(z,n),\tilde{z},1),1) \\ & \hat{\pi}(p,z,n) \geq 0 \end{cases}$$

which is a subgame perfect Nash equilibrium (SPNE) of the game. Moreover, $\hat{\pi}(z^C(n), p^C(n))$ is the payoff dominant SPNE of the game.

The three constraints in Program $\mathcal{P}_1(n)$ avoid potential deviations during strategic interactions. The first constraint ensures that the price corresponds to a Nash equilibrium in the second stage. The second ensures that the fixed factor is high enough to prevent rivals investing massively to trigger a limit pricing strategy (i.e a non-existence of a limit pricing strategy (NELPS) constraint). The third guarantees positive profits, because firms are always free to choose $z = 0$ in the first stage to avoid negative profits.

4 Free-entry equilibrium: a Chamberlin-like model

In this section, we extend the model to endogenize the number of firms operating in the market (free-entry equilibrium). Let us add an initial stage 0 in which firms can choose to leave or enter the market, and then play a soft-capacity constrained price competition game in the subsequent stages. Of course, if $(p^C(n), z^C(n))$ is a solution of program $\mathcal{P}_1(n)$, all the firms in the market make a positive profit and have no incentive to leave. What about outside firms? A firm will decide to enter the market if the market with $n+1$ firms has a profit-making equilibrium (i.e if Program $\mathcal{P}_1(n+1)$ has a solution with positive profits).

Definition 1. *n^e is the free-entry equilibrium number of firms if and only if:*

$$\begin{cases} (z^C(n^e), p^C(n^e)) \text{ is the solution of program } \mathcal{P}_1(n^e) \\ \text{and} \\ \mathcal{P}_1(n^e+1) \text{ has no solution} \end{cases}$$

Proposition 2. *Provided*

i) the production function is a Stone-Geary function such as equation (1), with $\rho < 1$,

ii) there is an equilibrium for at least one $n \geq 2$,

iii) the demand function has a choke price p_{max} (i.e. $\forall p \geq p_{max},\ D(p) = 0$),

there is always a finite free-entry equilibrium value for the number of firms.

Proof: It is sufficient to prove that $n = +\infty$ cannot be an equilibrium value in the general model of price competition with a soft capacity constraint when the production function is a Stone-Geary function. The average cost of production is

$$AC(p, z, n) = p_z z \left(\frac{D(p)}{n}\right)^{-1} + p_v \frac{\left(\frac{D(p)}{n}\right)^{\frac{1}{\alpha\rho}-1}}{A^{\frac{1}{\alpha\rho}}(z - z_0)^{\frac{1-\alpha}{\alpha}}} \tag{5}$$

Because of the choke price assumption, the equilibrium price is always strictly lower than p_{max} and demand is finite and strictly positive. Moreover, when n tends to

infinity, $D(p)/n$ tends to zero and the fixed factor z tends to z_0. We deduce that: $\lim_{n\to+\infty} AC(p,z,n) = +\infty$. (The first term of equation (5) tends to $+\infty$ and the second term is necessarily positive). Thus, when the number of firms tends to infinity, the profit becomes negative, contradicting the conditions of proposition (1). $\square$

Let us examine a numerical example of the properties of the soft-capacity constrained price competition model with a Stone-Geary production function. The results described in Proposition (1) make the equilibrium quite easy to compute because the equilibrium of the non-cooperative game is a solution of the constrained optimization program. In the following, we use the general calibration procedure outlined on page 106 of Cabon-Dhersin and Drouhin (2020).

We assume the parameters in the demand function have the following values: $b = 1$, and $p_{max} = 10$. For the Stone-Geary function, we take $\alpha = .7$, $\rho = .9$ and $A = 1$. Finally, we normalize the factor prices p_z and p_v to one. The results are summarized in Figure 1.

The left panel shows the price and average cost as a function of the number of firms. The price pattern is very similar to the one described by Cabon-Dhersin and Drouhin (2020). When there are few firms in the market (here $n \in \{2,3\}$), the NELPS constraint is binding, meaning that the firms need to account for the possibility of being excluded from the market by a firm that overinvests in the first stage. Between, $n = 4$ and $n = 15$ the "Nash Equilibrium constraint in the second stage" is binding, meaning that $p^C(n) = \bar{p}(z^C, n)$. This constraint becomes less effective as the number of firms increases, meaning that prices also increase. Beyond $n = 15$, neither constraint is binding, such that the equilibrium price is purely collusive and $\bar{z}(p,n) = z^*(p,n)$. Because in our example the returns to scale are slightly decreasing in this range, the price also decreases slightly as the number of firms increases. Finally, the curve marked by red Xs shows the effective average cost for each firm (Equation (5)), illustrating how the average profit goes to zero for $n^e = 30$.

At this point, it is important to note that the average cost for the firms in our

model, described by Equation (5), differs from the average cost function in most standard models. In standard models, the cost function does not depend on the type of competition in the market for output. It is just the cost of the optimal combination of factors. In our model however, firms account for the need to have a Nash Equilibrium in the second stage. Since in the following we use the usual "average cost" function as a benchmark, we refer to it as the "long-term" average cost.

The conditional factor demand $z(y, p_z, p_w)$, $v(y, p_z, p_w)$ and the long-term average cost are easy to calculate for a Stone-Geary production function:

$$z(y, p_z, p_w) = \left(\frac{y}{A}\right)^{1/\rho} \left(\frac{p_v}{p_z}\right)^{\alpha} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + z_0$$

$$v(y, p_z, p_w) = \left(\frac{y}{A}\right)^{1/\rho} \left(\frac{p_z}{p_v}\right)^{1-\alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}$$

$$LTAC(y, p_z, p_v) = y^{1/\rho-1} A^{-1/\rho} p_v^{\alpha} p_z^{1-\alpha} \frac{(1-\alpha)^{\alpha-1}}{\alpha^{\alpha}} + p_z \frac{z_0}{y}$$

The graph on the right of Figure 1 has the same y-axis (price and cost) as the one of the left but the x-axis shows y, the equilibrium output level of each firm for different numbers of firms. Of course, y tends to decrease as the number of firms operating in the market increases. The main interest of this representation is that it allows us to trace out both the equilibrium price/output and average cost/output combinations for any number of firms and the inverse demand function and long-term average and marginal cost functions. We can thus describe the dynamics of entry in the traditional "industrial organization" way.

Below $n = 15$, the average cost is higher than the long-term average cost, because one of the first two constraints of Program $\mathcal{P}_1(n)$ is binding. Above $n = 15$, the outcome of the program is purely collusive. The average cost is equal to the long-term average cost. Firms continue to enter as long as the profit remains positive. For $n^e = 30$, profits are so close to zero that entry becomes unprofitable for outside firms. This is the standard condition of (approximate) tangency between the average revenue

function (i.e. the demand function) and the long-term average cost function. Because the free-entry equilibrium occurs in the decreasing part of the average cost function, the mark-up is positive, a situation that was described for the first time by Chamberlin (1933) (Figure 17 on page 99 in the 7th edition, 1956).

5 Conclusion

In this article, we introduce the notion of free-entry equilibrium into a soft-capacity constrained price competition model with a traditional U-shaped average cost function, as used by Chamberlin himself in keeping with microeconomic theory at the time. This allows us to reproduce the Chamberlinian pattern of monopolistic competition in a homogeneous good oligopoly model, fully accounting for strategic interactions and without relying on the "negligibility hypothesis". The soft capacity constraint that produces a convex cost function in the second stage is essential to achieve this result.

In the case of a homothetic production function, as in Cabon-Dhersin and Drouhin's 2020 textbook example, the free-entry equilibrium involves an infinite number of atomistic firms operating in the market at a price that does not converge to the average cost. The fact that the markup in the free-entry equilibrium is the same as in the collusive case is a Chamberlinian property. In the present article, we use on a non-homothetic production function and prove that the same pattern can be achieved with a finite number of non-atomistic firms. Note that the non-homotheticity of the production functions should be considered when endogeneizing firm size. Crucially, our model can deal with these types of functions, allowing strategic interactions to be fully accounted for in a game theoretic approach.

Product differentiation is undeniably very important for analyzing competition in many markets. Nevertheless, we also believe that our soft capacity constraint is a realistic property that can also be encountered in many markets, especially when production is investment intensive. This article shows that this notion of a soft capacity constraint,

along with rigorous modeling of the resulting free-entry equilibrium, produces results that challenge conventional wisdom and offers a means to tackle previously unexplored problems.

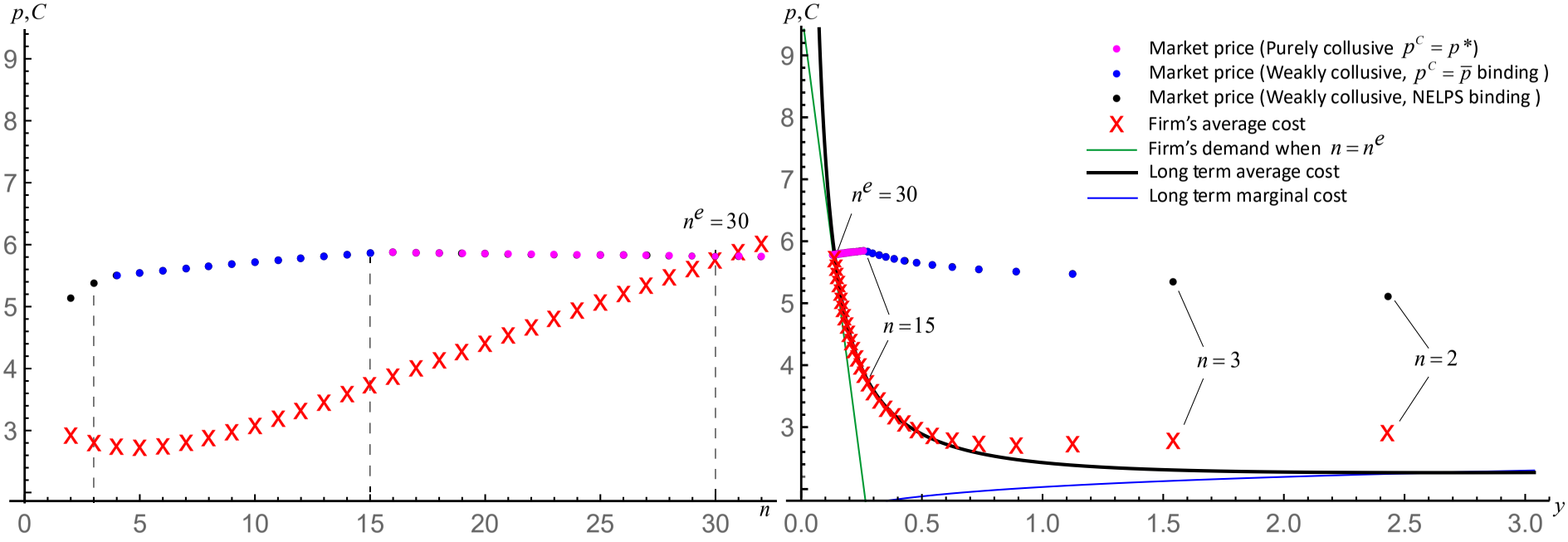

Figure 1: Price and costs as a function of the number of firms and their output level ($\rho = .9$, $\alpha = .7$, $A = 1$, $p_{max} = 10$, $b = 1$, and $p_z = p_v = 1$.)

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