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On the optimal design of taxes and transfers[*]

Axelle Ferriere   Philipp Grübener   Gaston Navarro   Oliko Vardishvili

PSE & CNRS   Goethe University   Federal Reserve Board   Yale University

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Abstract

We study the optimal joint design of targeted transfers and progressive income taxes. We develop a simple analytical model and demonstrate an optimally negative relation between transfers and income-tax progressivity, due to both *efficiency* and *redistribution* concerns. That is, higher transfers should be financed with lower income-tax progressivity. We next quantify the optimal fiscal plan in a rich dynamic model calibrated to the U.S. economy. Transfers should be generous and financed with moderate income-tax progressivity. To redistribute while preserving efficiency, average tax-and-transfer rates should be more progressive than marginal rates. Transfers, even if lump-sum, precisely allow to disentangle average from marginal rates. Targeted transfers further implement non-monotonic marginal rates, but generate only modest additional gains relative to a lump-sum transfer. Quantitatively, the left tail of the income distribution determines the optimal size of the transfer, while the right tail drives the optimal income-tax progressivity.

Keywords: Heterogeneous Agents, Fiscal Policy, Optimal Taxation, Redistribution.
*JEL:* E21, E62, H21, H23, H53.

1 Introduction

High levels of inequality have made redistributive policies a core topic in recent policy debates. Two key components of redistributive policies are means-tested transfers and progressive income taxes. Both policies can significantly alter the income distribution. In the United States, targeted transfers amount to about 25% of income for the poorest income quintile, while income taxes reduce richest-quintile income by a similar percentage (see Figure 1). The optimal design of these two tools, progressive income taxes and targeted transfers, is thus of paramount importance. A recurring question has been, should transfers be more generous and, if so, should they be financed with more progressive taxes?

In this paper, we study the joint optimal design of targeted transfers and progressive income taxes. We do so in two steps. We first develop a simple analytical model to characterize the optimal relation between transfers and income-tax progressivity. Second, we calibrate a rich dynamic model of the U.S. economy and use it to quantify the findings of the analytical model. We pursue a Ramsey approach and endow a planner with flexible functions for transfers and income taxes, which we refer to as the tax-and-transfer ($t\&T$) system.

There are several advantages to our approach. The functions we use resemble current policies implemented by many countries. Thus, our analysis sheds light on the optimal use of currently available instruments. Furthermore, the Ramsey approach is suitable for a rich quantitative evaluation with empirically realistic efficiency and redistribution concerns. Finally, the instruments we use are simple and characterized by few economically intuitive parameters. Yet, they are flexible enough to generate non-linear, and potentially non-monotonic, overall $t\&T$ schedules. The flexibility of our Ramsey approach allows us to build intuition on the optimal taxation trade-offs generally discussed in the public finance literature.

We present two main findings. First, we demonstrate an optimally negative relation between transfers and income-tax progressivity. That is, given economic fundamentals, higher transfers should be financed with lower income-tax progressivity. Second, optimal transfers are large: redistribution should be achieved via generous transfers, while efficiency should be preserved with moderately progressive income taxes. We present these findings using the analytical framework and then evaluate these trade-offs in the quantitative model.

We build on the work in Heathcote, Storesletten, and Violante (2017) and develop a tractable analytical framework to discuss the optimal negative relation between transfers and income-tax progressivity. We assume a continuum of households subject to idiosyncratic labor risk and endow a planner with a log-linear income tax and a lump-sum transfer. Households face a static consumption-labor supply decision. We use local ap-
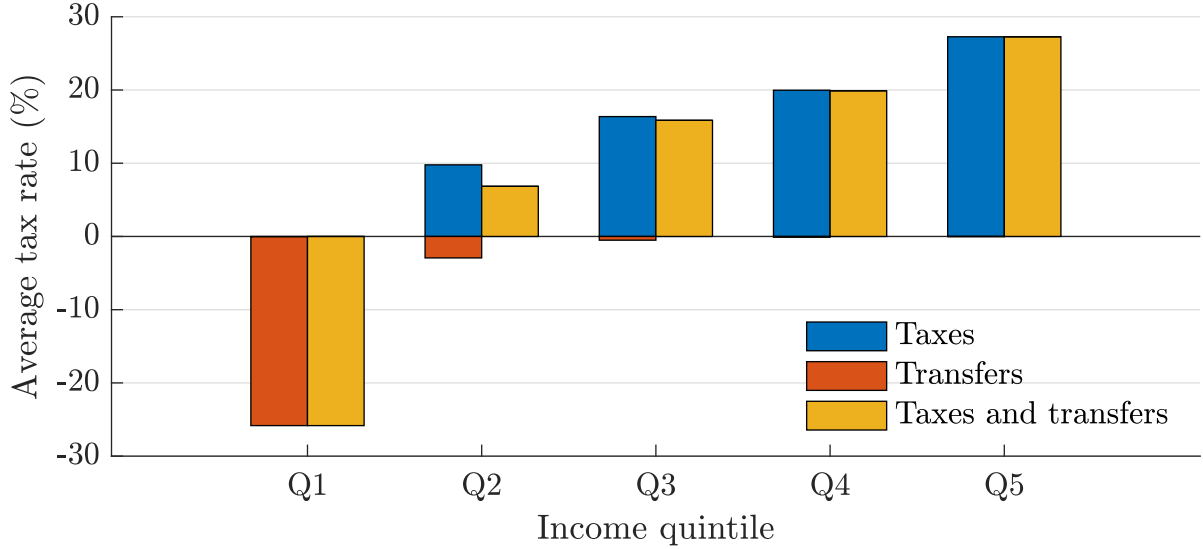
Figure 1: Redistribution in the U.S.

Notes: This figure shows average tax and transfer rates by income quintile for the working-age population in 2013. Data is from the Congressional Budget Office (CBO). Details on the components of taxes and transfers are included in Appendix B.1.

proximations to derive a closed-form formula for welfare that shows the optimal negative relation between transfers and income-tax progressivity.

The welfare formula shows that the optimally negative relation is due to both *efficiency* and *redistribution* concerns. Higher income-tax progressivity and larger transfers both discourage labor supply. In turn, a planner finances larger transfers with lower progressivity to preserve labor supply incentives. This is what we refer to as the *efficiency* concern. In terms of *redistribution*, a planner aims to decrease dispersion in consumption. Higher income-tax progressivity and larger transfers both reduce consumption dispersion. The larger the tax progressivity, the lower the consumption dispersion and the smaller the welfare gain from transfers. Thus, larger transfers are optimally financed with less progressive income taxes because of both *efficiency* and *redistribution* concerns. We then investigate the optimal size of transfers in this simple set-up. We find that transfers should be large, and thus income-tax progressivity should be low. Generous transfers with low income-tax progressivity implement more progressive average than marginal $t\&T$ rates, a feature that we robustly find to be welfare improving.

We confirm these analytical findings in a rich, quantitative Bewley-Huggett-Aiyagari incomplete-market model calibrated to match the U.S. economy. We enrich the set of fiscal instruments and endow the planner with targeted transfers that, as in the data, phase out with both labor and capital income. The phase-out of transfers, combined with progressive income taxes, allows for non-monotonic marginal $t\&T$ rates, a feature often found desirable in the Mirrleesian literature. We also incorporate an empirically realistic process for idiosyncratic labor risk, with a thick right tail of productivity and higher-order

moments of income risk. We consider once-and-for-all changes to the fiscal instruments and incorporate transitions toward the new steady state in our welfare computations.

The optimal $t\&T$ system in the quantitative model is substantially more redistributive than the current system in the United States. Optimal transfers amount to $26,100$ (2013 U.S. dollars) per year for the lowest-income household. This number implies an income floor of 45% of median income. Transfers optimally phase out, albeit at a slow rate, such that a household with median income receives a transfer of about $7,400$. In line with our analytical findings, the large transfers are optimally financed with moderate income-tax progressivity, similar to the current U.S. system. The optimal system implies large welfare gains of 9.64% in consumption equivalent terms. These welfare gains are largely due to better insurance and redistribution. In addition, despite a large drop in aggregate labor, there are small efficiency improvements because of a better allocation of hours worked. While welfare gains primarily accrue to the poor, 79% of households benefit from the reform.

Most of the welfare gains in the benchmark plan can be achieved with either of two common tax proposals: a universal basic income (UBI) or an affine plan. In terms of tax instruments, the UBI plan eliminates the phase-out of transfers but still optimizes the progressivity of income taxes. The affine plan is a UBI financed with flat income taxes. The optimal UBI plan implies a lump-sum transfer of $26,000$ per household, financed with barely progressive income taxes: the income-tax rate averages 60% for the poorest income quintile, and 64% for the richest income quintile. As the optimal UBI plan features almost flat taxes, the optimal affine plan is similar to the UBI one. The affine plan features a lump-sum transfer of $27,200$ and a tax rate of 64%. The welfare gain of the UBI is 9.43% in consumption equivalent terms, while the affine gains are 9.38%, both close to the gains in the benchmark plan. Thus, our framework is supportive of a lump-sum transfer, provided that it is financed with the right income-tax progressivity. Alternatively, if implementing income-tax rates above 60% is infeasible, a planner should favor a phase-out of transfers, as it is associated with lower tax rates.

The UBI and affine experiments point to the importance of disentangling average and marginal rates. Transfers, even if lump-sum, allow to separate the progressivity of average and marginal $t\&T$ rates. In order to redistribute while preserving efficiency, the optimal plan robustly features more progressive average than marginal rates. Additionally, the phasing-out of transfers allows for non-monotonic marginal rates. Most of the welfare gains come from disentangling the progressivity of average and marginal $t\&T$ rates, while the phase-out of transfers allows for lower income-tax rates.

The optimal negative relation between transfers and income-tax progressivity also holds in the quantitative model, and it is shaped by the income distribution. In particular, the right tail of the income distribution shifts this relation: the lower the income

concentration at the top, the lower the income-tax progressivity for each level of transfer. Yet, income concentration at the top barely affects the optimal level of transfers, and only the income-tax progressivity adjusts. In contrast, when bottom-income households are richer, the optimal income-tax progressivity for each transfer does not change, but the optimal transfers decrease. Thus, optimal transfers are driven by the left tail of the income distribution, while income-tax progressivity is determined by the right tail of the distribution.

Related literature. This paper belongs to a large literature on optimal taxation.[1] On the theoretical side, we build on Heathcote, Storesletten and Violante (2014, 2017), which proposes an analytical framework with partial insurance against idiosyncratic shocks to study risk sharing and optimal taxation. We extend this framework to allow for transfers in addition to progressive taxes. In a related static framework, Heathcote and Tsujiyama (2021) studies the optimal Mirrleesian non-linear tax schedule and compares it to simpler tax systems such as affine and log-linear tax schedules. While not solving for the optimal fully non-linear plan, we endow the government with more flexible fiscal instruments and incorporate them into a richer dynamic quantitative model. Chang and Park (2020) derives the Mirrleesian tax formula in a Huggett (1993) economy.[2]

Our quantitative framework relates to several papers interested in optimal tax progressivity in incomplete-market models. An early contribution is Conesa and Krueger (2006), and subsequent work has focused on: transitional dynamics (Bakış, Kaymak, and Poschke 2015), superstars and entrepreneurs (Brüggemann 2021; Kindermann and Krueger 2021), human capital accumulation (Badel, Huggett, and Luo 2020; Krueger and Ludwig 2016; Peterman 2016), and Laffer curves (Guner, Lopez-Daneri, and Ventura 2016; Holter, Krueger, and Stepanchuk 2019). Our paper also relates to recent analyses of universal basic income policies and/or negative income taxes by Lopez-Daneri (2016), Daruich and Fernández (2021), Conesa, Li, and Li (2021), and Luduvice (2021).

Two recent works analyze transfers and progressive taxes in a rich quantitative setup. Boar and Midrigan (2021) focuses more specifically on the role of wealth taxes in a framework with entrepreneurs. Guner, Kaygusuz, and Ventura (2021) develops a rich modeling of the household and a detailed calibration of the current U.S. $t\&T$ system. In line with our quantitative finding, both papers find that an affine plan is welfare improving, but we provide new insights on the optimal trade-off between transfers and progressivity, and also consider non-monotonic marginal rates with transfers that optimally phase out.

Roadmap. Section 2 presents the analytical model. Section 3 contains the quanti-

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[1] Seminal papers in the Mirrleesian tradition include Mirrlees (1971), Diamond (1998) and Saez (2001).
[2] Our paper also relates to advances in New Dynamic Public Finance such as Kapička (2013), Farhi and Werning (2013), Golosov, Troshkin, and Tsyvinski (2016), Findeisen and Sachs (2017), Stantcheva (2017) and Boerma and McGrattan (2020).

tative model and its calibration. Section 4 analyzes the optimal fiscal plan. Section 5 discusses the effect of income distribution on optimal transfers and income-tax progressivity. Section 6 concludes.

2   An analytical model

We start with a simple analytical model to show the optimal relationship between the size of transfers and the income-tax progressivity. We build on Heathcote, Storesletten, and Violante (2017) and propose a tractable heterogeneous-agent model in which a government uses progressive income taxes to finance public spending and a lump-sum transfer. We first analyze the relationship between transfers and income-tax progressivity in the representative-agent case. Next, we consider heterogeneity. We use local approximations to analytically show how the welfare gains of transfers vary with income-tax progressivity and resort to numerical methods to verify the accuracy of the approximations. Our numerical findings also suggest that optimal transfers should be large, a result that we confirm in the quantitative model with endogenous self-insurance in Section 3.

2.1   Environment: a static Bewley-Hugett economy

The economy is populated by a continuum of ex-ante homogenous households, a representative firm, and a utilitarian government. Households are hand-to-mouth, value consumption c and leisure $1 - n$, and their labor productivity z follows a stochastic Markov process. The representative firm uses a linear technology to transform labor into output. The government finances exogenous government spending G and a lump-sum transfer T with labor taxes, which are assumed to be log-linear as in Heathcote, Storesletten, and Violante (2017).[3]

Taxes.—A household with labor income y pays taxes $\mathcal{T}(y) = y - \lambda y^{1-\tau}$, where τ captures the progressivity and λ the level of taxes. For $\tau = 0$, tax rates are flat and equal to $1 - \lambda$. When $\tau > 0$ ($\tau < 0$), marginal and average tax rates are increasing (decreasing) in income. Figure 2 shows the tax function.

Households.—In period t, household i chooses consumption c_{it} and labor n_{it} to maximize utility

$$u(c_{it}, n_{it}) = \ln c_{it} - B \frac{n_{it}^{1+\varphi}}{1 + \varphi} \tag{1}$$
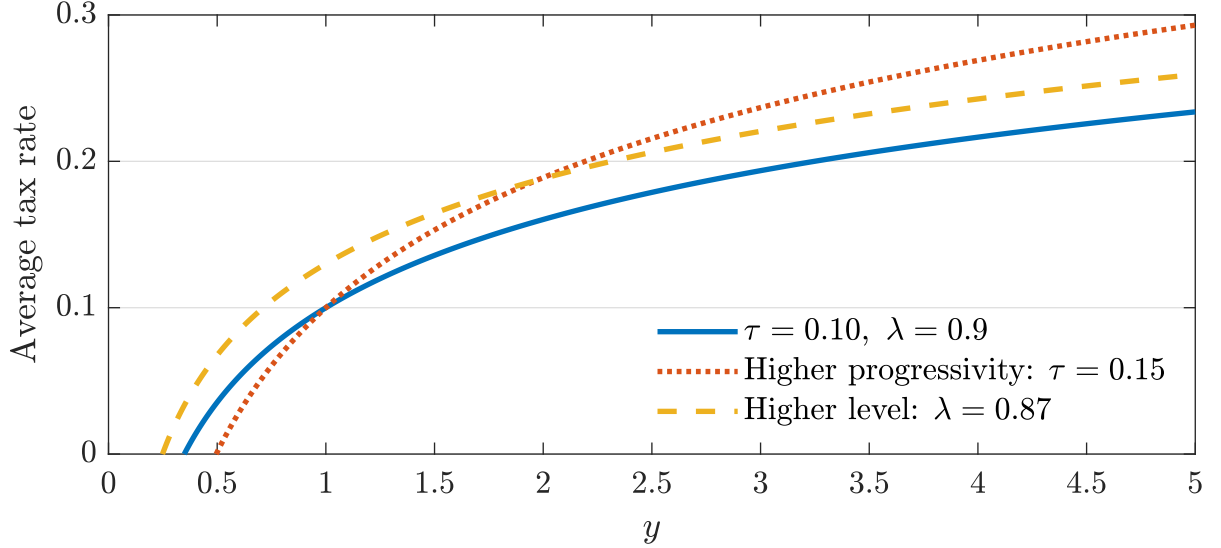
subject to a static budget constraint

$$c_{it} = \lambda \left(z_{it} n_{it} \right)^{1-\tau} + T. \tag{2}$$

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[3]This tax function has been widely used since at least Feldstein (1969) and Benabou (2002).

Figure 2: Log-linear tax function

Notes: This figure shows the average tax rate for various parameterizations of the log-linear tax function.

We assume a log-AR(1) process for z, with v_ω controlling the degree of heterogeneity across households

$$\log z_{it} = \rho \log z_{it-1} + \omega_{it}, \quad \omega_{it} \sim \mathcal{N}\left(-\frac{v_\omega}{2(1+\rho)}, v_\omega\right).$$

Note that log utility, together with hand-to-mouth households, ensures a closed-form solution for labor when transfers are zero, a property that will be useful in deriving the welfare formula.[4]

Government.— The government budget constraint is

$$G + T = \int z_{it} n_{it} di - \lambda \int (z_{it} n_{it})^{1-\tau} di. \tag{3}$$

Technology.— The resource constraint is

$$\int c_{it} di + G = \int z_{it} n_{it} di. \tag{4}$$

2.2 Representative agent

We first abstract from heterogeneity and assume $v_\omega = 0$.

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[4]In the absence of transfers, a no-trade theorem applies with log utility when z follows a random walk, as shown in Heathcote, Storesletten, and Violante (2017). We deviate from the no-trade conditions by including transfers. We further deviate by assuming a log-AR(1) process for z, which simplifies the analytical results. The no-saving assumption is relaxed in the quantitative model of Section 3. We have also derived an analytical relation between optimal transfers and progressivity using the structure of stochastic shocks in Heathcote, Storesletten, and Violante (2017). While the expression gets cumbersome, the negative relationship between progressivity and transfers remains.

First-best.—Let $n^\star(G)$ denote the first-best allocation, which maximizes the utility of the representative agent given the resource constraint (4). It is characterized by

$$Bn^\varphi(n - G) = 1. \tag{5}$$

Second-best.—We turn to the optimal allocation when spending is financed with a progressive income tax and a transfer.

Claim 1 *For all levels of transfers $T < n^\star(G) - G$, the efficient allocation $n^\star(G)$ can be implemented with progressivity $\tau^\star(G, T)$ equal to*

$$\tau^\star(G, T) = -\frac{G + T}{n^\star(G) - (G + T)}. \tag{6}$$

As such, the optimal income tax progressivity τ decreases with transfers T.

All proofs in this section are relegated to Appendix A.

To understand why the first-best allocation can always be implemented regardless of the level of transfers, we first focus on the case when $T = 0$. Maximizing the household's utility (1) given the budget constraint (2) yields an optimal labor policy function in closed form,

$$n = \left(\frac{1 - \tau}{B}\right)^{\frac{1}{1+\varphi}} \equiv n_0(\tau), \tag{7}$$

where the 0 subscript stands for zero transfers. Equation (7) shows a one-to-one negative relationship between n and τ, which ensures that a planner can always pick the progressivity τ to implement $n^\star(G)$. Thus, as the first-best allocation $n^\star(G)$ increases with G, the optimal τ declines with G. This externality is described in Heathcote, Storesletten, and Violante (2017): a planner finances spending with negative progressivity to incentivize labor supply.

When transfers are not zero, the policy function for labor does not admit a closed-form solution. However, it is easy to show that labor supply decreases with transfers T because of a wealth effect. This property explains the optimal negative relationship between τ and T in Claim 1. Progressivity falls as transfers increase, such that labor supply remains at $n^\star(G)$ despite the more generous transfers.[5] In this representative-agent set-up, the negative relationship between transfers and progressivity is entirely driven by *efficiency* concerns.

Figure 3 presents a numerical illustration of the relationship between transfers and progressivity for the representative agent case. The Frisch elasticity φ^{-1} is set to 0.4,

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[5]Note that equation (6) retrieves an old result in the Ramsey literature. When $\tau = 0$, the optimal transfer is $T = -G$. That is, the government finances all spending with a lump-sum tax and sets the distortionary flat tax rate at zero.
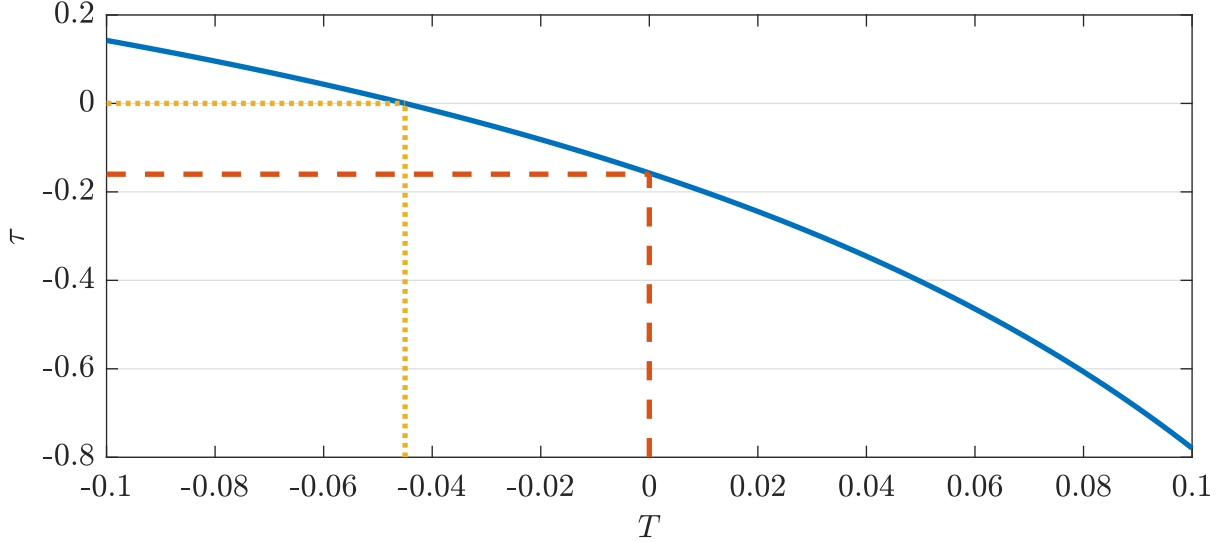
Figure 3: Representative agent: implementing the first-best allocation

Notes: This figure shows the combinations of lump-sum transfer T and income-tax progressivity τ that implement the first-best allocation in the representative-agent case. The dashed line highlights the case of zero transfer, in which progressivity is negative: $\tau = -0.16$. The dotted line marks the scenario of an affine tax system, $\tau = 0$, in which a lump-sum tax finances all government spending: $T = -G$.

and the labor disutility parameter B is chosen such that labor supply $n_0(\tau) = 0.3$. Progressivity is fixed at $\tau = 0.18$, based on the U.S. estimates of Heathcote, Storesletten, and Violante (2017), and government spending is set to 15% of output. As can be seen, optimal progressivity τ declines as transfers T increase.

2.3 Heterogeneity

We next consider the economy with heterogeneous agents. In this case, the second-best allocation is not efficient and so cannot be characterized by the first-best allocation. We thus proceed as follows. First, we derive a formula for welfare as a function of progressivity τ in the tractable case of $T = 0$.[6] Then, we use local approximations around that point to characterize welfare as a function of τ and T.

2.3.1 Heterogeneity and no transfers

When transfers are zero, the optimal labor policy is constant across households and, again, equal to $n_0(\tau) \; \forall z$. Given this simple policy function, we can compute output, $Y_0(\tau) = n_0(\tau)$, and the tax function level parameter $\lambda_0(\tau)$ from the government budget

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[6]This formula is a special case of Heathcote, Storesletten, and Violante (2017) that, beyond the alternative structure of stochastic shocks, also considers skill investment and preference heterogeneity.

constraint. Then, we can derive aggregate welfare as a function of progressivity τ as

$$W(\tau) = \underbrace{\underbrace{\log\left(n_0(\tau) - G\right)}_{\text{Size}} \underbrace{- \frac{1-\tau}{1+\varphi}}_{\text{Labor disutility}}}_{\text{Efficiency}} \underbrace{-(1-\tau)^2 \frac{v_\omega}{2(1-\rho^2)}}_{\text{Redistribution}}. \tag{8}$$

This expression has a straightforward economic interpretation. The first two terms capture the *efficiency* concerns governing the optimal choice of τ. Because progressivity depresses labor supply, a larger τ reduces the *size* of the economy, and thus aggregate consumption $C_0(\tau) = n_0(\tau) - G$. Yet, a larger τ also reduces *labor disutility* $-B\frac{n^{1+\varphi}}{1+\varphi}$, which, using equation (7), equals the second term in the formula. The last term is proportional to the variance of log consumption and captures *redistribution* concerns. Larger progressivity reduces dispersion in consumption, which is welfare improving. As such, heterogeneity increases optimal progressivity. Notice that, when $v_\omega = 0$, the progressivity τ that maximizes (8) implements the first-best allocation, as in Section 2.2.

Figure 4 illustrates the forces determining optimal progressivity τ. We calibrate $\rho = 0.935$ and set $v_\omega = 0.034$ to match a variance of log consumption of 0.18, as also measured in Heathcote, Storesletten, and Violante (2017). Optimal progressivity is zero with a representative agent and no public spending. Positive spending decreases τ to -0.16, reflecting the spending externality discussed earlier. Adding heterogeneity increases τ to 0.29 because of redistribution concerns.

As τ is positive, average and marginal tax rates both increase with income. The presence of transfers will loosen the tight link between average and marginal rates, as we discuss next.

2.3.2 Heterogeneity and transfers

With non-zero transfers, the policy function for labor does not admit a closed-form solution. Instead, we use the implicit function theorem and approximate the policy around the case with $T = 0$:

$$\hat{n}_{it} \approx n_0(\tau) - \frac{T}{1+\varphi} \frac{n_0(\tau)}{n_0(\tau) - G} \exp\left(-\tau(1-\tau)\frac{v_\omega}{2(1-\rho^2)}\right) z_{it}^{-(1-\tau)}. \tag{9}$$

Note that labor supply falls in transfers because of the wealth effect.

Using this approximation, we follow similar steps as in the case without transfers to obtain an expression for welfare as a function of progressivity τ and transfer T:

$$W(\tau, T) = W(\tau, 0) + T\left[\Omega_e^{ra}(\tau) + \Omega_r^{ha}(\tau, v_\omega) + \Omega_e^{ha}(\tau, v_\omega)\right].$$
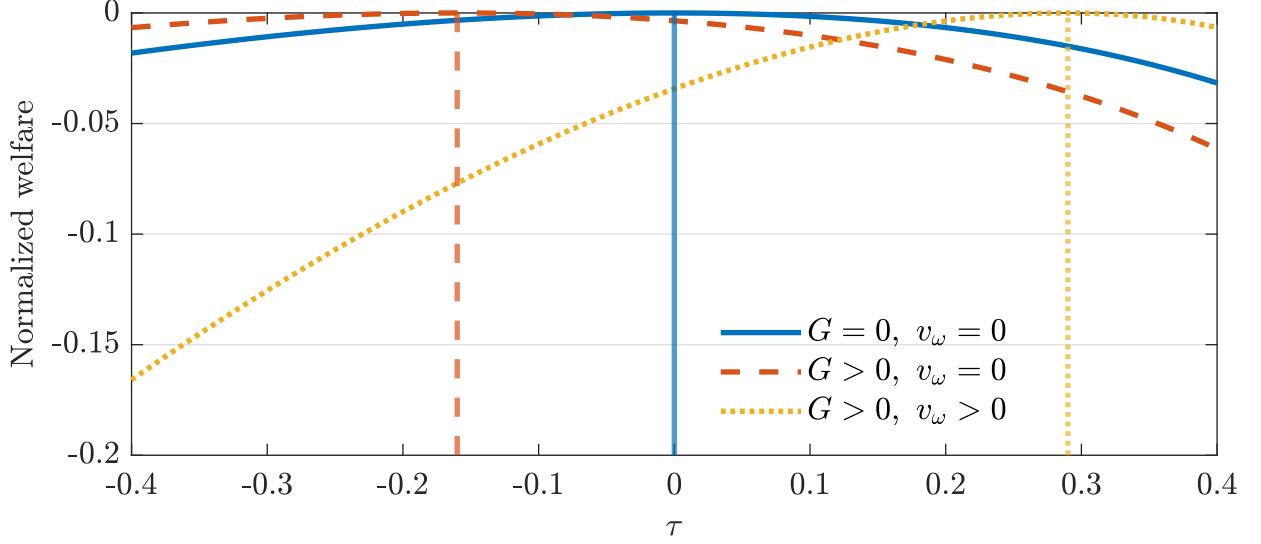
Figure 4: Optimal income-tax progressivity without transfers

Notes: This figure shows welfare as a function of income-tax progressivity when transfers are zero for three cases. Case 1 features no heterogeneity and no exogenous spending requirement. Case 2 features positive exogenous spending, while in case 3 there is also household heterogeneity. Welfare is normalized to zero at the respective optimal progressivity.

The parenthesis captures the marginal effect of transfers T on welfare for a given progressivity level τ. This effect can be decomposed in three terms: an *efficiency* term $\Omega_e^{ra}(\tau)$, which summarizes the trade-offs for a representative agent, and two additional terms coming from the cross section of heterogeneous agents: an *efficiency* term $\Omega_e^{ha}(\tau, v_\omega)$ and a *redistribution* term $\Omega_r(\tau, v_\omega)$. We discuss each term next.

The first term is independent of heterogeneity v_ω and captures the *efficiency* trade-off described in the representative agent case:

$$\Omega_e^{ra}(\tau) \equiv \underbrace{-\frac{n_0(\tau)}{n_0(\tau) - G} \frac{1}{1+\varphi} \frac{1}{n_0(\tau) - G}}_{\text{Size}} + \underbrace{\frac{1-\tau}{1+\varphi} \frac{1}{n_0(\tau) - G}}_{\text{Labor disutility}} \tag{10.a}$$

$$= u_c(C_0(\tau)) \left. \frac{\partial Y^{ra}(\tau, T)}{\partial T} \right|_{T=0} + u_n(n_0(\tau)) \left. \frac{\partial n^{ra}(\tau, T)}{\partial T} \right|_{T=0}$$

where Y^{ra} and n^{ra} denote output and labor in the representative-agent economy. Output decreases with transfers, and the welfare cost of smaller output is evaluated using the marginal utility of consumption. At the same time, lower hours induce a welfare gain, evaluated using the marginal utility of leisure.

To see how efficiency gains of transfers change with τ, we can rearrange $\Omega_e^{ra}(\tau)$ as

$$\Omega_e^{ra}(\tau) = \frac{1}{1+\varphi} \frac{1}{n_0(\tau) - G} \left[\frac{-G}{n_0(\tau) - G} - \tau \right].$$

10

Claim 2 $\partial \Omega_e^{ra}(\tau)/\partial \tau < 0 \; \forall \tau$, and $\Omega_e^{ra}(\tau^\star(G,0)) = 0$.

The intuition behind $\Omega_e^{ra}(\tau)$ is analogous to the one in Section 2.2. Overall, the efficiency gains of using a transfer decrease with progressivity τ, in line with Claim 1. When $\tau = \tau^\star(G,0)$, the first-best is implemented with zero transfers, and there are no efficiency gains from deviating from $T = 0$. Hence, $\Omega_e^{ra}(\tau^\star(G,0)) = 0$. When $\tau < \tau^\star(G,0)$, labor supply is above optimal, so it is welfare improving to use transfers to decrease labor supply and $\Omega_e^{ra}(\tau) > 0$. Analogously, labor is below optimal when $\tau > \tau^\star(G,0)$, so negative transfers increase welfare and $\Omega_e^{ra}(\tau) < 0$.

The second term captures the distribution of wealth effects on the labor supply of heterogeneous agents:

$$\Omega_e^{ha}(\tau, v_\omega) = \frac{\tau(1-\tau)}{n_0(\tau) - G} \frac{v_\omega}{1 - \rho^2} \frac{n_0(\tau)}{n_0(\tau) - G} \frac{1}{1 + \varphi} \qquad \text{(10.b)}$$

$$= u_c(C_0(\tau)) \left[\left. \frac{\partial Y(\tau, T)}{\partial T} \right|_{T=0} - \left. \frac{\partial Y^{ra}(\tau, T)}{\partial T} \right|_{T=0} \right].$$

This term changes non-monotonically with τ when $v_\omega > 0$. Transfers reduce labor supply more in the presence of heterogeneity, as the larger wealth effect of the poor more than offsets the smaller wealth effect of the rich. Higher progressivity reduces this dispersion of wealth effects, bringing labor supply closer to the representative agent case. However, the effect on output depends not only on labor supply but also on the distribution of z, which makes the effect of τ non-monotonic. In our calibrations, $\Omega_e^{ha}(\tau, v_\omega)$ is typically small, and $\partial [\Omega_e^{ra}(\tau) + \Omega_e^{ha}(\tau, v_\omega)]/\partial \tau < 0$ holds quantitatively so that the total efficiency gains of transfers decrease with progressivity.

Finally, the last term reflects *redistribution* concerns associated with transfers when agents are heterogeneous:

$$\Omega_r(\tau, v_\omega) = \frac{(1-\tau)^2}{n_0(\tau) - G} \frac{v_\omega}{1 - \rho^2} = \mathbb{E}\left[u_c(c_0(\tau)) \right] - u_c(C_0(\tau)), \qquad \text{(10.c)}$$

where $c_0(z, \tau)$ is individual consumption as in equation (2).

Claim 3 $\partial \Omega_r(\tau, v_\omega)/\partial \tau \leq 0 \; \forall \tau \in [-1, \hat\tau(G)]$, with $\hat\tau(G) = 1$ for $G = 0$.

This term captures dispersion in consumption and is strictly positive as long as $v_\omega > 0$ and $\tau < 1$. Indeed, transfers are welfare improving as they reduce dispersion in marginal utilities of consumption. However, under some mild conditions, the redistribution gain of using a lump-sum transfer decreases with progressivity; that is, $\partial \Omega_r(\tau, v_\omega)/\partial \tau \leq 0$.[7] This result is intuitive. Larger progressivity already reduces the dispersion in marginal utilities, such that there are fewer gains from reducing consumption dispersion even further

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[7]A formal derivation of the bound $\hat\tau(G)$ can be found in Appendix A. The bound is typically close to 1, and exactly 1 when G=0. In our calibration, it is equal to 0.998.
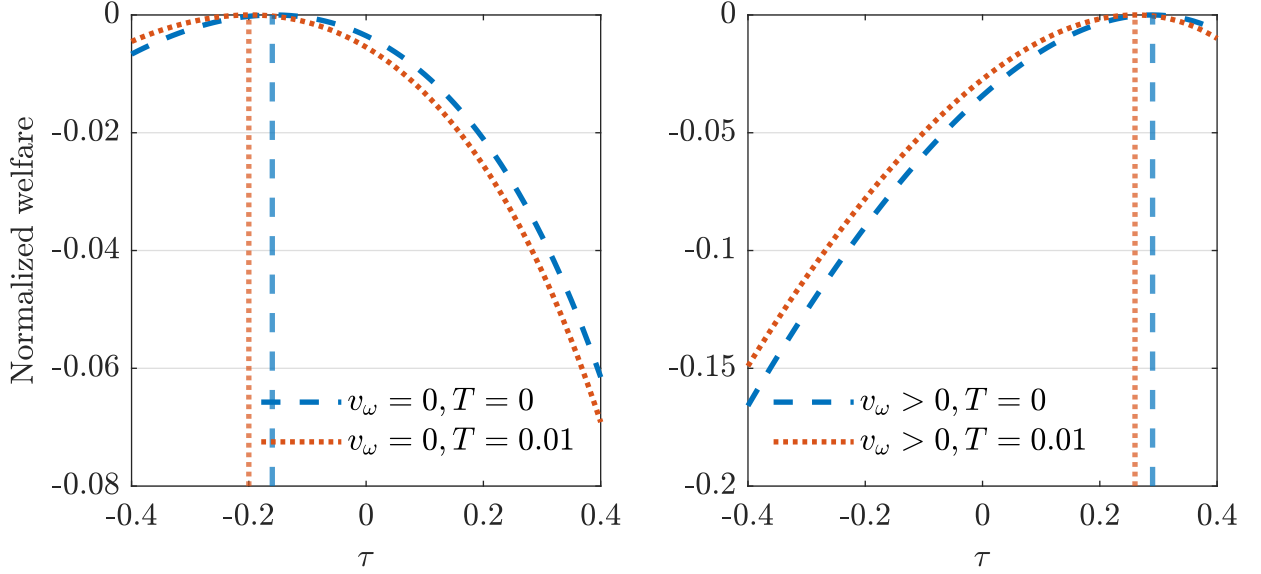
Figure 5: Optimal income-tax progressivity with transfers

Notes: This figure shows welfare as a function of income-tax progressivity, comparing the case of zero transfers and positive transfers for both the representative-agent (left panel) and heterogeneous-agent (right panel) scenarios. Exogenous spending is positive. Welfare is normalized to zero at the respective optimal progressivity.

with higher transfers. Thus, transfers and progressivity act as substitutes to redistribute resources. In the extreme case of $\tau = 1$, after-tax incomes are equalized, and the redistributive gains from having a positive transfer are zero; that is, $\Omega_r(1, v_\omega) = 0$. Overall, *redistribution* concerns strengthen the negative optimal relationship between transfers and tax progressivity driven by *efficiency* concerns: the more generous the transfer T, the smaller the progressivity τ.

Figure 5 illustrates how transfers affect optimal progressivity using our approximation around $T = 0$. Without heterogeneity, spending is optimally financed with negative progressivity when transfers are zero. Optimal progressivity further declines when transfers are positive, solely for *efficiency* reasons. With heterogeneity, positive transfers again reduce the optimal progressivity, because of both *efficiency* and *redistribution* concerns.

2.3.3 Optimal transfers

The welfare formula we derived is insightful on the optimal relationship between T and τ, but it is based on an approximation around $T = 0$. Hence, we now pursue a numerical solution of the model. The goal is twofold: to check the accuracy of the approximation and to compute the optimal level of transfers.

The numerical solution confirms the optimal negative relation between transfers T and progressivity τ around and away from $T = 0$, as Figure 6 shows. In addition, our approximation accurately characterizes the optimal tax progressivity for a wide range of
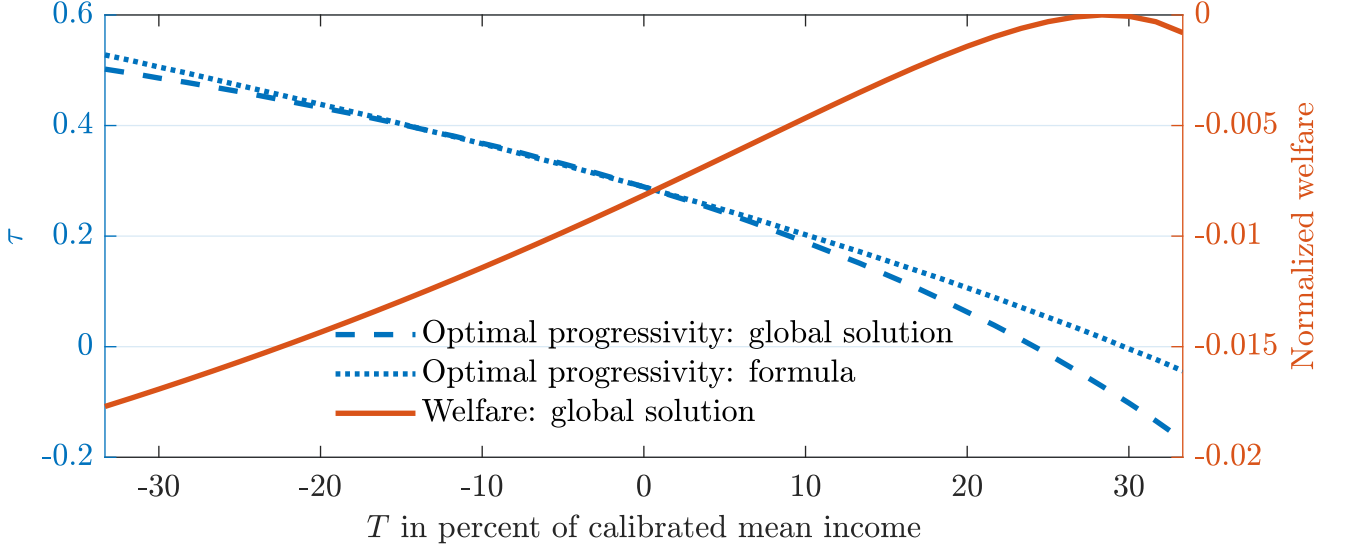
Figure 6: Optimal income-tax progressivity given transfers

Notes: This figure compares, for each transfer, the optimal income-tax progressivity implied by the formula to the global (numerical) solution. It also shows, for each transfer, the welfare associated with the optimal progressivity.

transfers. Figure 6 also plots welfare as a function of transfers. The highest welfare is achieved with a large lump-sum transfer combined with negative progressivity: T amounts to roughly 30% of mean income and income taxes are regressive with $\tau = -0.07$.

The optimal $t\&T$ system features progressivity in average rates due to the optimal large transfers. However, the $t\&T$ system is regressive in terms of marginal rates because of the optimal negative τ. Figure 7 plots the average and marginal $t\&T$ rates implied by the welfare maximizing policy. Larger progressivity in average than in marginal rates is reminiscent of typical results in the Mirrleesian literature.

Figure 7 also includes the average and marginal $t\&T$ rates of the optimal log-linear plan when transfers are constrained to be zero. In this case, progressivity is high, at $\tau = 0.29$, because a positive τ is the only tool available for the planner to redistribute. However, in the absence of transfers, redistribution through increasing average rates implies increasing marginal rates. A lump-sum transfer allows to break the tight link between average and marginal rates imposed by the log-linear function, which is welfare improving, as this simple calibration suggests.

2.4 Taking stock

There are two main takeaways from the analytical part. First, there is an optimally negative relationship between the size of transfers and the income-tax progressivity. This negative relationship is due to both *efficiency* and *redistribution* concerns. Second, adding a transfer to the log-linear tax function allows the fiscal plan to implement more progres-
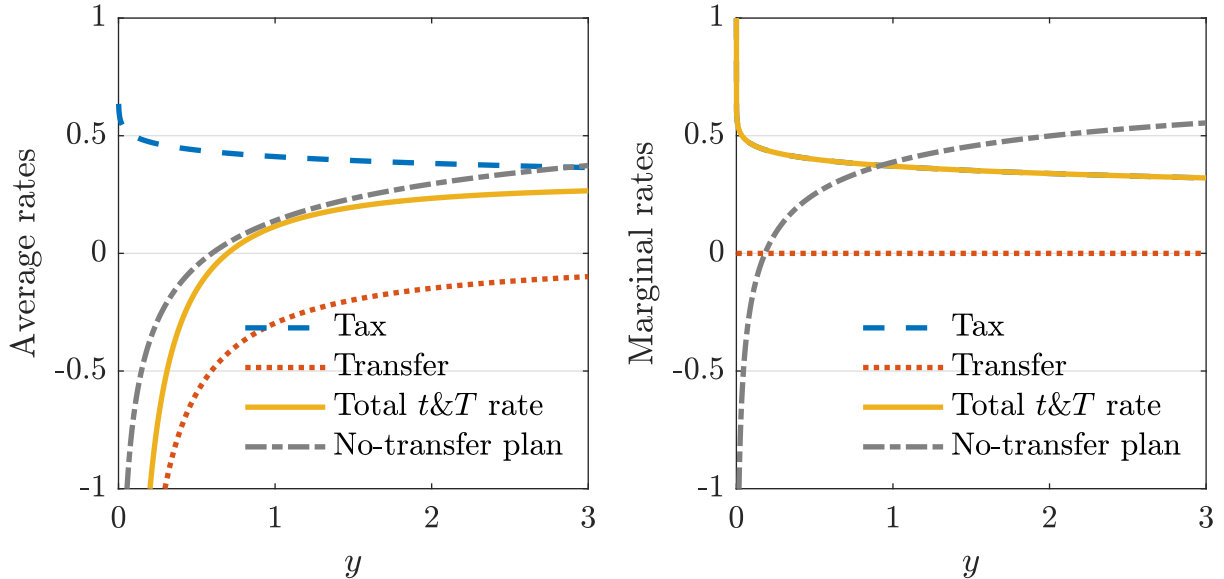
Figure 7: Optimal tax-and-transfer system: tax rates

Notes: This figure shows average (left panel) and marginal rates (right panel) of the optimal tax-and-transfer system. For comparison, the figure also plots optimal average and marginal rates in the no-transfer case.

sive average than marginal $t\&T$ rates, which is welfare improving. In fact, in this simple model, the optimal plan features a large transfer together with regressive income taxes.

While calibrated, the analytical model is too simple to make a truly quantitative statement about the optimal combination of the instruments. To address this question, we move to a more quantitative macro model of the U.S. economy next.

3 A quantitative model

We extend our previous discussion by analyzing the optimal design of taxes and transfers in a rich dynamic quantitative environment with endogenous self-insurance. In particular, we incorporate a flexible tax-and-transfer system into a canonical heterogeneous-agent model (Aiyagari 1994) augmented with realistic labor income risk. We describe the model environment and its calibration in this section and discuss the optimal tax plan and the role of transfers in Section 4.

3.1 Environment

The economy is populated by a continuum of households, a representative firm, and a government. The firm produces output by combining labor and capital, both of which are supplied by households. The government finances transfers and spending by taxing households' labor and capital incomes. We present the economy at its stationary equilibrium but will consider transitions when evaluating tax reforms.

Households.—Households value consumption c and leisure $1 - n$. Their idiosyncratic labor productivity z follows a Markov process with transition probabilities $\pi_z(z', z)$. Labor productivity shocks are uninsurable: households can only trade a one-period risk-free bond to self-insure, subject to a no-borrowing limit. Let $V(a, z)$ be the maximal attainable value to a household with assets a and idiosyncratic productivity z:

$$V(a, z) = \max_{c, a', n} \left\{ \frac{c^{1-\sigma}}{1 - \sigma} - B \frac{n^{1+\varphi}}{1 + \varphi} + \beta \mathbb{E}_{z'} \left[V(a', z') | z \right] \right\}$$

$$\text{s.t.}$$

$$c + a' \leq wzn + (1 + r)a - \mathcal{T}(wzn, ra)$$

$$a' \geq 0,$$

where w and r stand for wages and the interest rate, respectively. Households' taxes and transfers are captured by $\mathcal{T}(wzn, ra)$, which depend on labor income wzn and capital earnings ra. We discuss the shape of $\mathcal{T}(\cdot)$ in detail later. Let $n(a, z), c(a, z)$ and $a'(a, z)$ denote a household's optimal policies.

Representative firm.—The representative firm demands labor and capital in order to maximize current profits

$$\Pi = \max_{K, L} \left\{ K^{1-\alpha} L^{\alpha} - wL - (r + \delta)K \right\}, \tag{12}$$

where δ is the depreciation rate of capital. Optimality conditions for the firm are standard: marginal products are equalized to the cost of each factor.

Government.—The government's budget constraint is given by:

$$G + (1 + r)D = D + \int \mathcal{T}(wzn, ra)d\mu(a, z) \tag{13}$$

where G is government spending, D is government debt, and $\mu(a, z)$ is the measure of households with state (a, z) in the economy.

Stationary equilibrium.—Let A be the space for assets and Z the space for productivity. Define the state space $S = A \times Z$ and let $\mathcal{B}$ be the Borel $\sigma-$algebra induced by S. A formal definition of the competitive stationary equilibrium for this economy is provided next.

A **competitive stationary equilibrium** for this economy is given by value function $V(a, z)$ and policies $\{n(a, z), c(a, z), a'(a, z)\}$ for the household; policies for the firm $\{L, K\}$; government decisions $\{G, D, \mathcal{T}\}$; a measure μ over $\mathcal{B}$; and prices $\{r, w\}$ such that, given prices and government decisions: *(i)* households' policies solve their problems and achieve value $V(a, z)$, *(ii)* the firm's policies solve its problem, *(iii)* the government's budget constraint is satisfied, *(iv)* the capital market clears: $K + D = \int_{\mathcal{B}} a'(a, z)d\mu(a, z)$,

(v) the labor market clears: $L = \int_{\mathcal{B}} zn(a,z)d\mu(a,z)$, *(vi)* the goods market clears: $Y = \int_{\mathcal{B}} c(a,z)d\mu(a,z) + \delta K + G$, and *(vii)* the measure μ is consistent with households' policies: $\mu(\mathcal{B}) = \int_{\mathcal{B}} Q((a,z),\mathcal{B})d\mu(a,z)$ where Q is a transition function between any two periods defined by $Q((a,z),\mathcal{B}) = \mathbb{I}_{\{a'(a,z)\in\mathcal{B}\}}\sum_{z'\in\mathcal{B}}\pi_z(z',z)$.

3.2 A flexible tax-and-transfer function

We endow the government with two fiscal tools capturing the key elements of the U.S. $t\&T$ system: a non-linear labor tax and targeted transfers. Conveniently, the overall $t\&T$ system is characterized by a few parameters only, all of which have a clear economic intuition. Yet, the function allows for flexible–and potentially non-monotonic–shapes of the overall marginal $t\&T$ rates, a feature often found desirable in the optimal taxation literature.

In particular, we divide the $t\&T$ function $\mathcal{T}(\cdot)$ into three components: a flat tax τ_k on capital income y_k, a non-linear tax $\tau(y_\ell)$ on labor income y_ℓ, and a targeted transfer component $T(y)$ on total income $y = y_k + y_\ell$. From these components, we keep the capital tax constant (and purposely simple) and focus attention on labor taxes and transfers.[8]

We assume a labor tax function that is characterized by two parameters, θ and λ, as

$$\tau(y_\ell) = \exp\left(\log(\lambda)\left(\frac{y_\ell}{\bar{y}}\right)^{-2\theta}\right), \tag{14}$$

where $\bar{y}$ is median income. As with the log-linear tax function used in Section 2, our proposed tax function has two interpretable parameters: θ for the progressivity and λ for its level. A positive (negative) θ implies marginal tax rates that increase (decrease) with income. At $\theta = 0$, the tax is flat at λ. For all θ, the tax rate is exactly λ when income is at its median, $y_\ell = \bar{y}$. The left panel of Figure 8 shows how $\tau(y_\ell)$ varies with θ and λ.

This labor tax function is always non-negative, in line with statutory tax rates in the United States. Other than that, our function largely resembles the log-linear tax used in Section 2, as shown in Figure 17 of Appendix C.1. The level of progressivity θ is approximately on the same scale as the progressivity τ of the log-linear tax. The non-negative labor taxes imply that we rely exclusively on income-dependent transfers to generate negative $t\&T$ rates, as we explain next.

We assume a transfer function that is characterized by two parameters: a level m and a phase-out rate ξ. In particular, the transfer given to a household with total income y is given as

$$T(y) = m\bar{y}\frac{2\exp\left\{-\xi\left(\frac{y}{\bar{y}}\right)\right\}}{1 + \exp\left\{-\xi\left(\frac{y}{\bar{y}}\right)\right\}}. \tag{15}$$

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[8]For a recent detailed discussion on capital taxes, see Boar and Midrigan (2021).
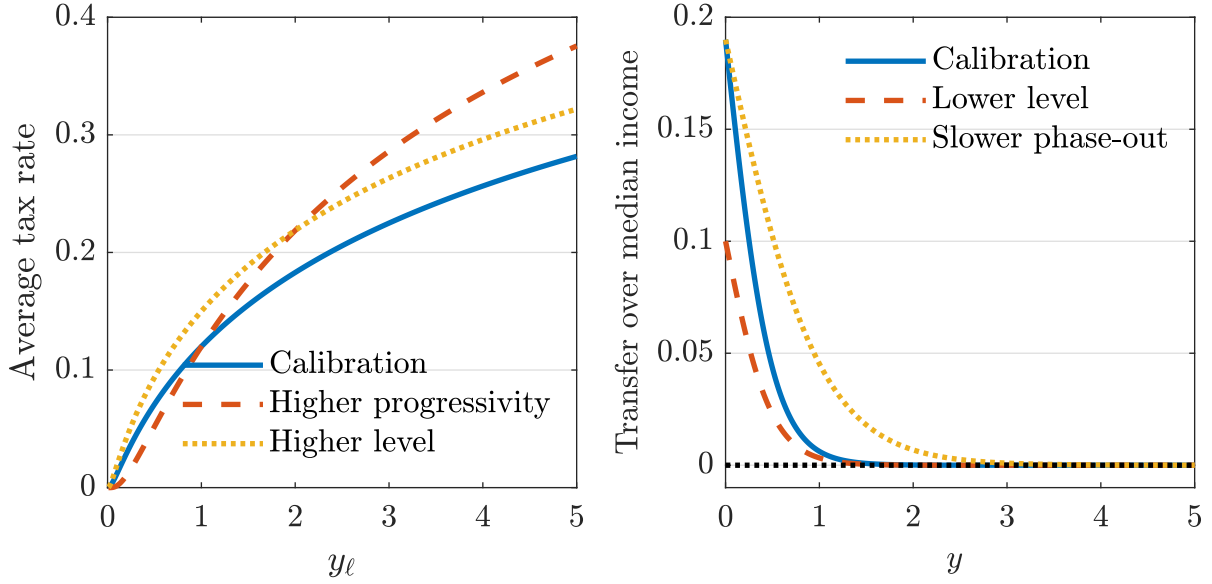
Figure 8: Labor tax function and targeted transfers

Notes: The left panel illustrates the shape of the new labor tax function. It compares the average tax rate in the calibration ($\lambda = 0.12$ and $\theta = 0.16$) to a higher progressivity ($\theta = 0.24$) and a higher level ($\lambda = 0.15$). The right panel plots the new transfer function. It compares transfers normalized by median income in the calibration ($m = 0.19$ and $\xi = 4.1$) to a lower level ($m = 0.1$) and a slower phase-out ($\xi = 2$). Income is normalized by median income.

The parameter m measures transfers to a household with zero income as a multiple of median income $\bar{y}$. The parameter ξ determines how quickly transfers phase-out with total income. When $\xi = 0$, transfers are a lump sum. As ξ becomes larger, transfers phase-out faster. The right panel of Figure 8 shows how transfers vary with m and ξ.

This functional form for transfers is a realistic description of U.S. income security programs, where transfers are means-tested, typically on both labor and capital income. Moreover, the phasing out of the transfer allows for non-monotonic marginal rates of the entire $t\&T$ system, which is not possible with a lump-sum transfer.

Thus, given the capital tax τ_k, a government's policy is characterized by four parameters: the progressivity and level of labor taxes, θ and λ, and the level and phase-out rate of transfers, m and ξ. We refer to a fiscal plan $\boldsymbol{\tau} = \{\theta, \lambda, m, \xi\}$ as a government's policy that satisfies its budget constraint (13).

3.3 Calibration and model fit

We calibrate the model to the U.S. economy in 2013. We take a period in the model to be a year. Several parameters are calibrated within the model, while others are taken from the literature, as we discuss next.

Labor income risk.—Recent empirical work has unveiled key statistics about labor income dynamics, two of which are potentially important for our purposes. First, the

recent work in Guvenen, Karahan, Ozkan, and Song (2021) shows that earnings growth rates are negatively skewed and exhibit excess kurtosis. That is, relative to a normal distribution, there are more individuals with small and large earnings changes, but fewer with medium-sized earnings changes. Often, the large earnings changes are negative. Second, labor income inequality has increased in recent decades.[9] These facts are relevant to the efficiency and redistribution trade-offs we analyze in this paper. Thus, we propose a process for idiosyncratic labor risk that can account for them.

In particular, we assume that a household's productivity z follows a Gaussian Mixture Autoregressive (GMAR) process in logs as,

$$\log z_t = \rho \log z_{t-1} + \eta_t$$

$$\eta_t \sim \begin{cases} \mathcal{N}\left(\mu_1, \sigma_1^2\right) & \text{with probability } p_1, \\ \mathcal{N}\left(\mu_2, \sigma_2^2\right) & \text{with probability } 1 - p_1, \end{cases} \tag{16}$$

with $\mathbb{E}[\eta] = p_1\mu_1 + (1-p_1)\mu_2 = 0$, so that μ_2 is pinned down given μ_1 and p_1.

Most times, households draw an innovation from the first normal that has a low variance. Draws from the second normal occur infrequently, but they have a large variance and a negative mean. In this way, we can generate frequent small and infrequent large negative earnings changes—that is, the negative skewness and excess kurtosis empirically documented for labor income growth. We discretize the productivity process using the method of Farmer and Toda (2017).

Furthermore, to better capture the concentration of incomes at the top, we follow Hubmer, Krusell, and Smith (2020) and make one additional adjustment to the productivity process. We adjust the top 15% states in our productivity grid such that they follow a Pareto distribution with tail $\kappa = 1.6$ as estimated by Aoki and Nirei (2017).

The income process is then characterized by five parameters: $(\rho, \mu_1, \sigma_1, \sigma_2, p_1)$. We pick these parameters to match key statistics of households' labor earnings growth from the Panel Study of Income Dynamics (PSID) as well as income concentration at the top.[10] We target four moments of the labor income growth distribution: the standard deviation (0.33), the difference between the ninetieth and the tenth percentile (0.61), the skewness (-0.35), and the kurtosis (11.43). Additionally, we target a labor income share of 38% for the top 10% of labor income earners, as found in the Survey of Consumer Finances for 2013. Table 1 contains all the parameter values.

Taxes and transfers.—We set capital taxes at 35% (Trabandt and Uhlig 2011) and calibrate the four parameters $\boldsymbol{\tau}$ of the fiscal function to match the tax-and-transfer rates

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[9]See Piketty and Saez (2003).

[10]We do not directly use the statistics reported in Guvenen, Karahan, Ozkan, and Song (2021), because they are computed at the individual level. Instead, we follow De Nardi, Fella, and Paz-Pardo (2019) and use PSID data to compute income growth statistics at the household level, but base estimates on pre-tax income data. See Appendix B.2 for more details.

Table 1: Parameter values

Preferences			Income Process		
β	Discount factor	0.962	ρ	Persistence	0.935
σ	Risk aversion	2.000	p_1	Weight on first normal	0.850
$1/\varphi$	Labor supply elasticity	0.400	μ_1	Mean of first normal	0.016
B	Disutility of labor	85.00	μ_2	Mean of second normal	-0.091
Production			σ_1	Std. dev. of first normal	0.150
δ	Depreciation rate	0.080	σ_2	Std. dev. of second normal	0.630
α	Labor share	0.640	κ	Pareto tail parameter	1.600
Government					
θ	Tax progressivity	0.160	D	Public debt	0.600
λ	Tax level	0.118	G	Government spending	0.126
m	Transfer level	0.190	τ_k	Capital tax rate	0.350
ξ	Transfer phase-out	4.100			

from the CBO shown in Figure 1. We target the distribution of transfer rates across households and calibrate the transfer level at $m = 0.19$ with a rapid phase-out at $\xi = 4.1$. As such, the lowest-income household receives $9,700 in 2013 dollars. For labor taxes, we calibrate the level at $\lambda = 12\%$ and progressivity at $\theta = 0.16$ to target average tax rates in the second and fifth quintile. As Table 2 shows, the model matches reasonably well the distribution of transfers and income-tax rates across households.[11] Figure 9 plots average and marginal $t\&T$ rates as a function of labor income for different values of capital income.

Remaining parameters.—We set the coefficient of relative risk aversion σ to 2, a value that is more standard in quantitative macroeconomics than the log utility used for tractability in Section 2. We fix the Frisch elasticity φ to 0.4, also a common value. We set the production side parameters to standard values, with the labor share $\alpha = 0.64$ and the (annual) depreciation rate $\delta = 0.08$. We calibrate the discount factor β, government debt D, and labor disutility B to jointly match an interest rate of 2%, a government debt-to-output ratio of 60%, and an average labor supply of 0.3. Spending is implied by government budget clearing and results in a spending-to-output ratio of 14%, in line with the data. All parameters are summarized in Table 1.

The model matches well the distribution of income, as Table 3 shows. While the labor-income share of the top 10% is targeted, the model features a remarkable fit for all income quintiles. Importantly, the model captures accurately how poor the poor are, as it matches well the labor income of the tenth, fifth, and even first percentiles of the income distribution (see Appendix B.2). As we show in Section 5, the left and right

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[11]See Appendix B.1 for details on computations of transfers and tax rates in the data.

Table 2: Calibration: taxes and transfers

Tax rates	Q1	Q2	Q3	Q4	Q5
Data	0%	10%	16%	20%	27%
Model	8%	11%	14%	17%	28%
Transfers rates	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**
Data	26%	3%	1%	0%	0%
Model	24%	4%	1%	0%	0%

Notes: Average tax and transfers rates per income quintile. Data: CBO 2013, working-age households.

Table 3: Calibration: income and wealth

Labor income	Q1	Q2	Q3	Q4	Q5	Top 10
Data	4%	9%	14%	21%	53%	38%
Model	4%	9%	14%	20%	52%	38%
Net worth	**Q1**	**Q2**	**Q3**	**Q4**	**Q5**	**Top 10**
Data	-1%	1%	3%	9%	88%	71%
Model	0%	2%	7%	18%	72%	52%

Notes: Labor income shares by labor income quintiles and wealth shares by wealth quintiles. Data: SCF 2013, households aged 25-60.
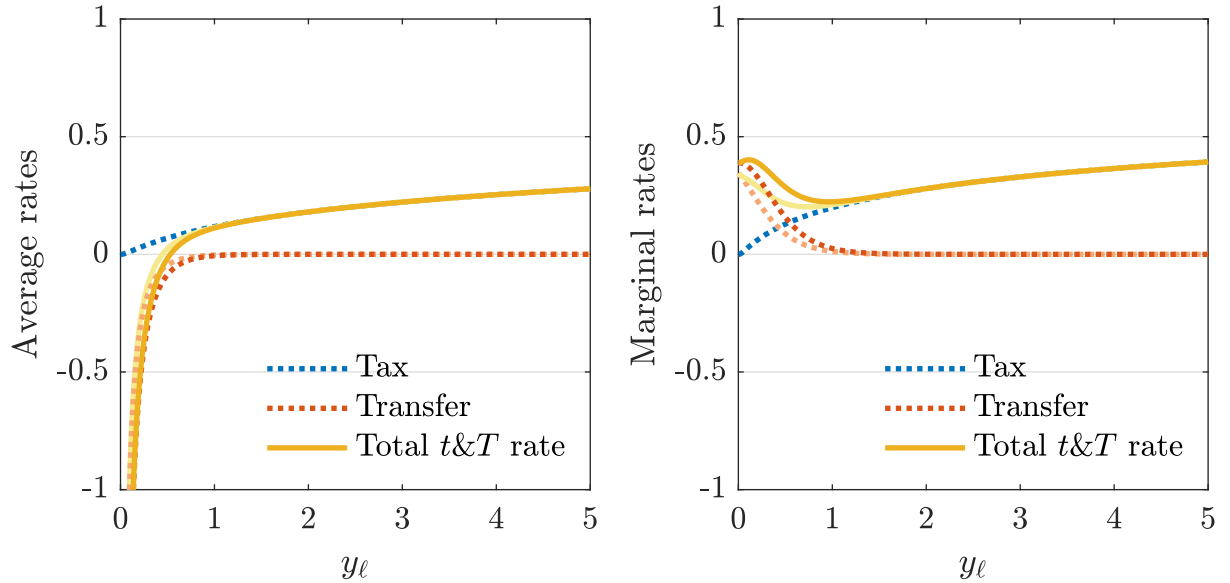
Figure 9: Calibration: average and marginal tax-and-transfer rates

Notes: This figure shows average and marginal $t\&T$ rates in the calibrated steady state along the labor income distribution, normalized by median income. As transfers phase-out with respect to total income, the level of capital income matters for the $t\&T$ rates. We report two cases: zero capital income (legend color) and mean capital income (lighter color).

tails of the income distribution shape the optimal plan. In contrast, the model falls short in generating enough wealth concentration at the top, a common shortcoming of this type of model. Section 5 considers a model extension with heterogeneous stochastic discount factors to improve the wealth distribution fit. Finally, the model generates a reasonable labor elasticity at the top. We conduct a partial-equilibrium experiment in which marginal tax rates increase by 1% unexpectedly and permanently, as in the tax reform we consider next. The implied labor elasticity of the top 1% is 0.20 in the model, a number well within the range of values reported in the literature.[12]

4 Optimal tax-and-transfer plan

In this section, we show that the findings of the analytical model carry over to our quantitative environment: a planner optimally trades higher transfers for lower income-tax progressivity. The optimal fiscal plan features generous transfers and moderate labor tax progressivity. Thus, as in the analytical case, transfers are key to generate more progressive average than marginal $t\&T$ rates. Finally, while a phasing-out of transfers is optimal, a lump-sum transfer comes close in terms of welfare gains.

4.1 A Ramsey approach

A government's plan is fully characterized by $\boldsymbol{\tau} = \{\theta, \lambda, m, \xi\}$, the progressivity and level of labor taxes, and the level and phase-out rate of transfers. We use a utilitarian welfare criterion to evaluate a one-time change in policy $\boldsymbol{\tau}$ and include transitions in the welfare computations.[13]

In particular, let $V_0(a, z; \boldsymbol{\tau})$ be the life-time utility of a household with assets a and productivity z in the period when the policy $\boldsymbol{\tau}$ is implemented. The utilitarian welfare criterion $\mathcal{W}(\boldsymbol{\tau})$ considers the sum of utilities $V_0(\cdot)$ as,

$$\mathcal{W}(\boldsymbol{\tau}) = \int V_0(a, z; \boldsymbol{\tau}) d\mu_0(a, z). \tag{17}$$

Notice that policy $\boldsymbol{\tau}$ affects household life-time utility, but the measure $\mu_0(a, z)$ is given by the initial steady state of the economy.[14]

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[12] See Kindermann and Krueger (2021) for a discussion of the empirical literature and the model counterpart in a related framework.

[13] We actually optimize on three parameters—θ, λ, and ξ—and set m to satisfy the budget constraint. Thus, m varies somewhat along the transition. We report the long-run value of m. See Appendix C.2 for computational details. For optimal time-varying tax systems, see Acikgöz, Hagedorn, Holter, and Wang (2021) and Dyrda and Pedroni (2021).

[14] Appendix C.3 reports the optimal fiscal plan when optimizing on steady-state welfare.

4.2 Optimal tax-and-transfer system

The optimal system is substantially more redistributive than the system currently in place in the United States. Optimal transfers are large, at $26,100 in 2013 U.S. dollars for the lowest-income household. This value implies an income floor of 45% of median income ($m = 0.45$). The optimal phasing-out is slow–at $\xi = 1.85$ compared to $\xi = 4.1$ in the calibration–implying a transfer of $7,400 for a household at calibrated median income. Optimal income-tax progressivity is moderate at $\theta = 0.16$, similar to its calibrated value.

Optimal average $t\&T$ rates are more progressive than marginal rates, as Figure 10 shows. Average rates monotonically increase with income, because of both transfers and progressive labor taxes. Marginal rates, however, are not monotonic. They are high for low-income earners because transfers phase out, lower for medium-income earners, and high again for top-income earners because of progressive labor taxes.

Transfers in the optimal system are substantially more generous than in the status quo. As shown in Table 4, the transfer rate is 170% for the bottom income quintile, compared to only 26% in the data. Transfers also remain generous for the second and third quintiles in the optimal plan, while they are virtually zero in the empirical counterpart. The larger transfers are financed with higher labor taxes, but the optimal income-tax progressivity is similar to the status quo. Thus, tax rates increase roughly uniformly across quintiles. Table 4 also shows average and marginal $t\&T$ rates by income quintile. Average $t\&T$ rates are more progressive than in the status quo: they are equal to -154% for the bottom quintile and monotonically increase with income to reach 44% for the top quintile. Marginal $t\&T$ rates are non-monotonic, at above 60% in the bottom three quintiles and around 50% in the top two quintiles.

Overall, the optimal plan achieves redistribution via large transfers at the bottom. It also preserves efficiency with moderate income-tax progressivity, as to incentivize labor from productive households. This quantitative finding is in line with the analytical model, where transfers are optimally used to disentangle average and marginal progressivity.

4.3 The economy shrinks, but most people benefit

The optimal tax-and-transfer system results in a decline in economic activity, as Figure 11 shows. The higher taxes and more generous transfers lead to lower savings, and thus wages decline while the interest rate increases. The distribution of hours worked in the final steady state also shifts left, as Figure 12 shows. Overall, output in the final steady state is 18% lower than in the initial steady state.

Yet, not only does the optimal tax-and-transfer system increase utilitarian welfare, but it is also favored by a majority of households over the status quo. Aggregate welfare
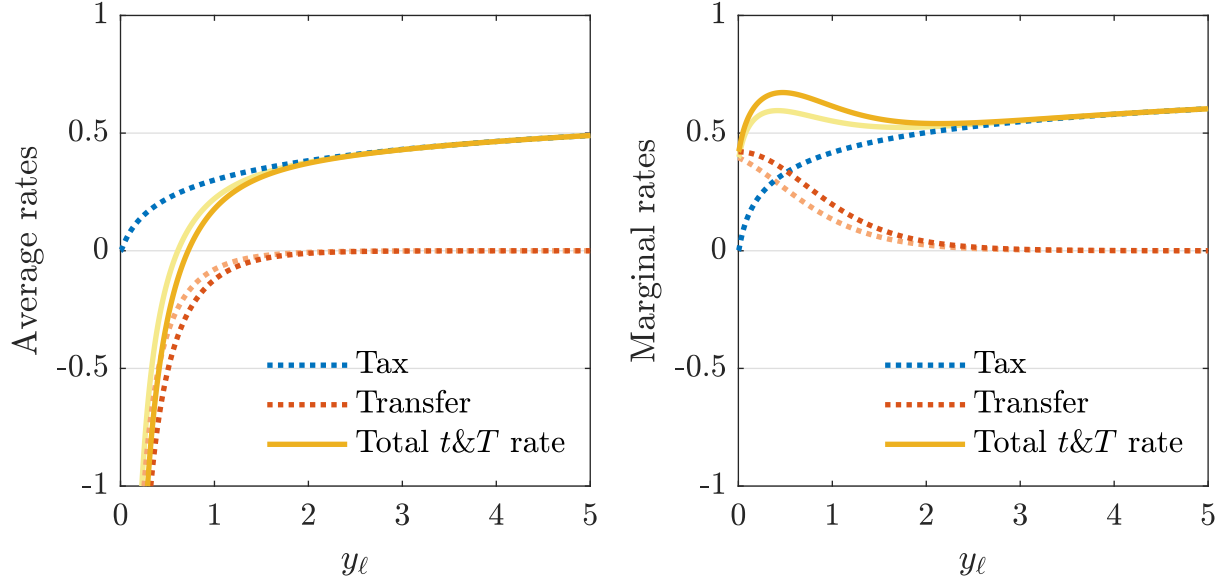
Figure 10: Optimal tax-and-transfer system: average and marginal $t\&T$ rates

Notes: This figure shows optimal average and marginal $t\&T$ rates along the labor income distribution, normalized by calibrated median income. We report two cases: zero capital income (legend color) and mean capital income (lighter color).

Table 4: Optimal tax-and-transfer system

Data	Q1	Q2	Q3	Q4	Q5
Tax rate	0%	10%	16%	20%	27%
Transfer rate	26%	3%	1%	0%	0%
Average $t\&T$ rate	-26%	-7%	15%	20%	27%
Optimal plan	Q1	Q2	Q3	Q4	Q5
Tax rate	16%	22%	27%	32%	44%
Transfer rate	170%	59%	22%	6%	0%
Average $t\&T$ rate	-154%	-37%	5%	26%	44%
Marginal $t\&T$ rate	61%	66%	62%	54%	51%
UBI plan	Q1	Q2	Q3	Q4	Q5
Tax rate	56%	56%	57%	55%	58%
Transfer rate	181%	85%	53%	35%	13%
Average $t\&T$ rate	-125%	-29%	4%	20%	45%
Marginal $t\&T$ rate	60%	61%	62%	63%	64%

Notes: This table shows average tax and transfer rates, as well as average and marginal $t\&T$ rates, by income quintile. It reports the CBO data, the optimal plan with targeted transfers, and the UBI plan with lump-sum transfers. The CBO does not report marginal rates.

gains amount to 9.64% in consumption equivalent terms.[15] As Figure 13 shows, these large welfare gains accrue primarily to the poor, who benefit the most from the generous transfers. Households with high productivity experience welfare losses on average, though those with higher assets may still benefit from the tax reform due to the higher interest rates. Overall, 79% of households benefit from implementing the optimal plan.[16]

We follow Bhandari, Evans, Golosov, and Sargent (2021) and decompose the welfare gains into three components: aggregate efficiency, redistribution, and insurance. Efficiency captures the welfare gains resulting from changes in aggregate resources. Redistribution captures changes in ex-ante shares of consumption and leisure, while insurance captures changes in ex-post utility risk.[17]

About two-thirds of the gains come from insurance, one-fourth from redistribution and the remainder from the efficiency component. The insurance gains reflect lower volatility of consumption due to larger transfers. All households record a gain in the insurance component, but this gain is larger for low-asset households. The redistribution component is driven by lower dispersion in ex-ante consumption shares. This component is heterogeneous across households: it is large for low-asset/low-productivity households, while it is negative for high-productivity but low-asset households, in line with the distribution of consumption equivalents. Overall, the redistribution component masks substantial heterogeneity, which aggregates to a positive but small number. Interestingly, the efficiency component is slightly positive, as the decrease in aggregate consumption is offset by larger leisure. Efficiency gains are driven by a better allocation of hours worked, as shown in the right panel of Figure 12.

## 4.4 Exploring the phase-out of transfers: UBI and affine plans

A positive phase-out of transfers allows to implement non-monotonic marginal $t\&T$ rates, which is a property featured in the optimal plan and is also reminiscent of the optimal U-shaped marginal rates often found in the public finance literature. To evaluate the importance of non-monotonic marginal rates, we compute the optimal plan when transfers do not phase-out ($\xi = 0$). In this case, the shape of marginal $t\&T$ rates is monotonic and determined by the labor tax progressivity.

Eliminating the transfer phase-out allows us to discuss two common tax proposals: a universal basic income (UBI) and an affine plan. The UBI plan eliminates the phase-out of transfers but still optimizes the progressivity of labor taxes θ. The affine plan is a UBI plan financed with flat income taxes–that is, $\theta = 0$.

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[15]See Appendix C.5 for a formal derivation of consumption equivalents in our environment.

[16]Appendix C.4 compares welfare gains to the optimal plan with a log-linear income tax and no transfers. Welfare gains in that case are +5.08% in consumption equivalent terms—large, but lower than in the optimal plan with more flexible instruments.

[17]See Appendix C.5 for more details.
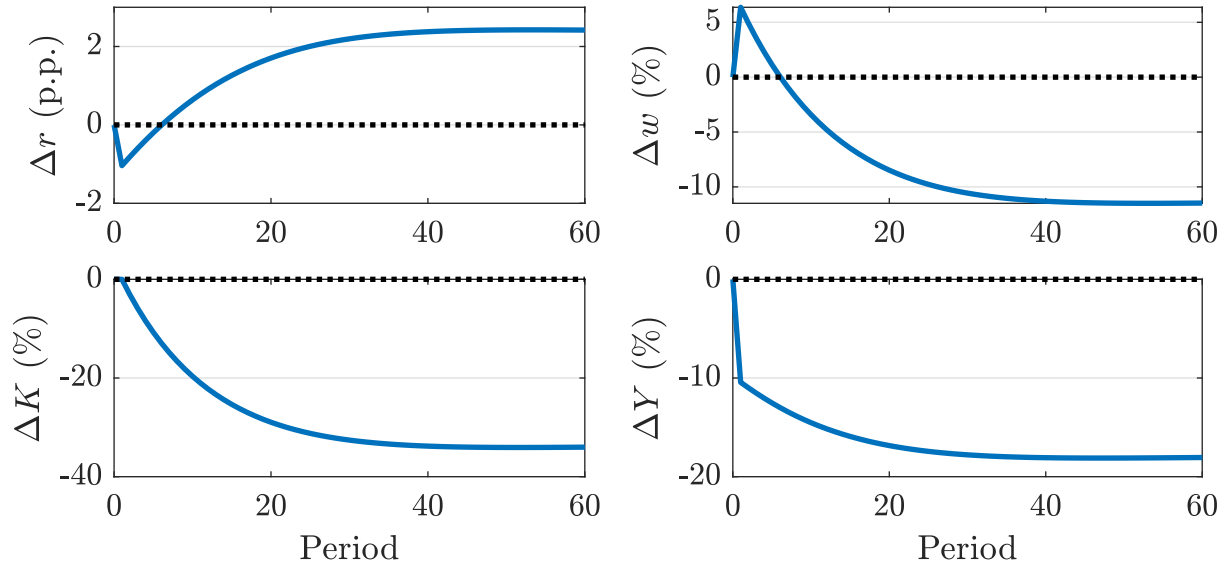
Figure 11: Quantitative model: responses of prices and quantities to the tax reform

Notes: This figure plots the transition path for the interest rate, wages, capital and output after the tax reform is implemented. $t = 0$ shows the calibrated steady state. Responses of wages, output and capital are plotted in percentage deviation from steady state; the interest rate response is plotted in differences.
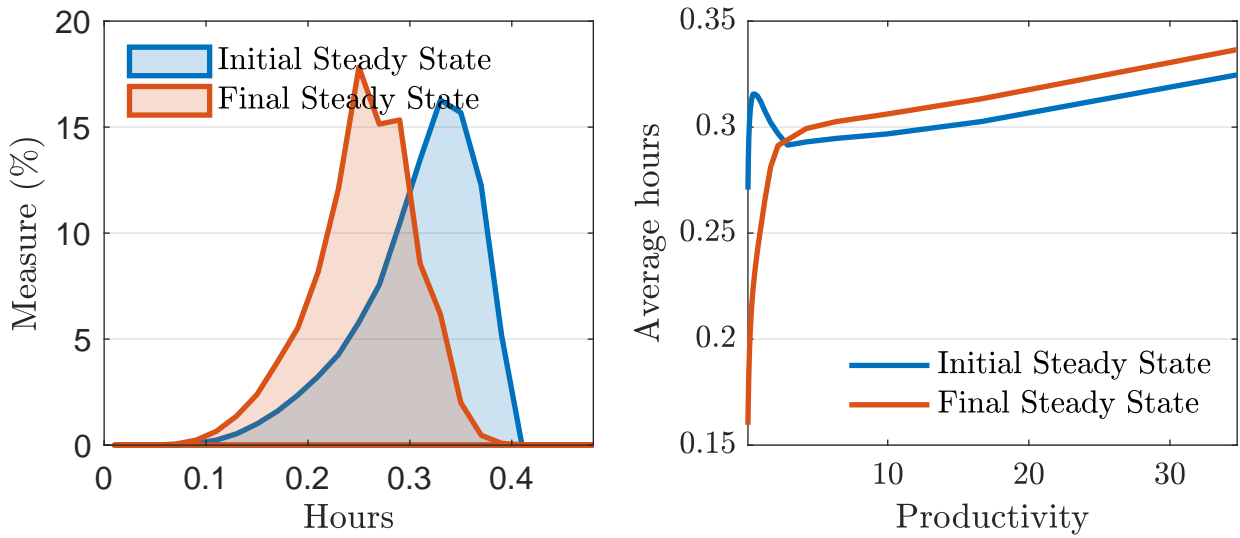
Figure 12: Quantitative model: hours worked in the initial and final steady states

Notes: The left panel plots the distribution of hours worked in the initial and final steady states. The right panel shows average hours worked by productivity level in the initial and final steady states.
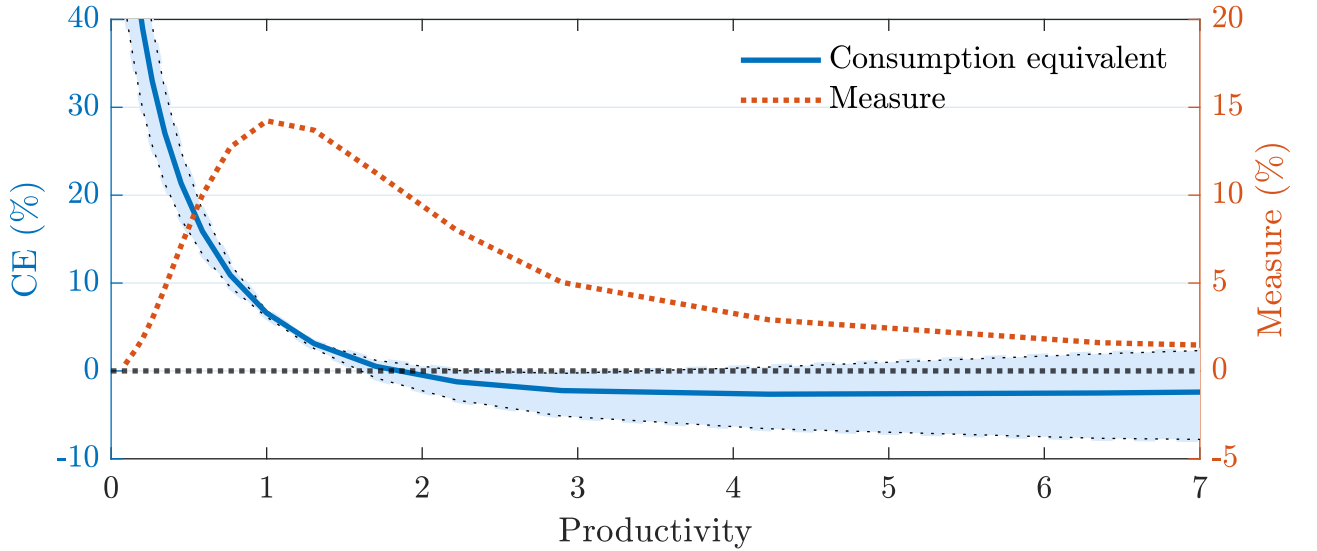
Figure 13: Optimal tax-and-transfer system: consumption equivalent welfare gains

Notes: This figure shows welfare gains, in terms of consumption equivalent (CE), from a tax reform to the optimal system. The solid line plots the average CE by productivity level z, while the two dashed lines show the bottom-20% and the top-80% of the distribution of CE at each z. Both axes are cut for readability; welfare gains peak at 55% for the lowest-productivity level and converge to about -2% for top levels of productivity. The right axis plots the measure of households for each productivity level.

The UBI plan includes large transfers at $m = 0.43$, about $26,000$ for each household, which are optimally financed with almost flat labor taxes at $\theta = 0.03$. Transfers in the optimal UBI amount to a large government outlay, representing 26% of GDP compared to 10% in the benchmark plan. In order to finance the transfers and preserve labor supply incentives, income-tax progressivity falls as to maintain roughly flat marginal $t\&T$ rates. Labor taxes are thus higher than in the plan with phase-out and almost constant across households, spanning from 60% in the first income quintile to 64% in the top income quintile. Although with different tax and transfer rates, the UBI plan achieves about the same level of redistribution as with phase-out, with comparable average $t\&T$ rates across households–see Table 4.

The optimal affine plan is similar to the UBI, as the latter uses almost flat taxes. The affine plan features a lump-sum transfer of $27,200$ to each household and a tax rate of 64%.

The welfare gain of the UBI is 9.43% in consumption equivalent terms, while the affine gains are 9.38%, both close to the gains in the plan with phase-out. Thus, our framework is supportive of lump-sum transfers, to the extent that tax rates above 60% are implementable. Otherwise, a planner should favor a phase-out of transfers, which is associated with lower tax rates.

Overall, the UBI and affine exercises point to the importance of disentangling average and marginal rates. Transfers, even if lump-sum, allow to separate the progressivity of

average and marginal tax rates. The phasing-out of transfers additionally allows for non-monotonic marginal tax rates. The exercises in this section suggest that disentangling the progressivity of average and marginal tax rates can generate most of the welfare gains, while phasing-out of transfers allows for lower income-tax rates.

5 Trading off progressivity and transfers

A key insight of the analytical model is the optimal negative relation between transfers and progressivity. This relation remains valid in the quantitative model: for a given phase-out ξ, larger transfers m are optimally associated with lower progressivity θ. We refer to this relation as the m-θ line. We show that the optimal m-θ line is robustly downward-sloping and explore how it shifts with the distributions of income, wealth, and income risk.

We compute the optimal m-θ line for a given ξ. To ease exposition, we assume no phase-out ($\xi = 0$) throughout this section, but results are robust to using positive ξ.[18]

5.1 Income distribution: a story of two tails

The downward-sloping m-θ line shifts with the left and right tails of the income distribution. To show this, we compute the m-θ line in our benchmark and in another two economies: a "No Pareto" economy and a "Richer Poor" economy. For the "No Pareto" economy, we remove the Pareto tail adjustment to the highest values of the productivity grid. For the "Richer Poor", we increase the lowest 20% of the productivity distribution to equate its 20$^{\text{th}}$-percentile. In turn, the 10$^{\text{th}}$-percentile of the labor-income distribution amounts to 50% of the median labor income, compared to 30% in the benchmark economy. In both cases, we readjust all remaining parameters to match the same calibration targets as in our benchmark, except for the income distribution. Figure 14 shows the optimal m-θ relation for the benchmark and the two additional cases, as well as the productivity distribution in the three cases.

In the "No Pareto" economy, the m-θ line shifts down. That is, for a given level of transfer m, the optimal progressivity θ falls significantly, by about 0.10. This finding confirms Mankiw, Weinzierl, and Yagan (2009) and Heathcote and Tsujiyama (2021), which emphasize the importance of the right tail of the income distribution in determining the optimal level of progressivity. Remarkably, the optimal level of transfers m is almost the same as in the benchmark economy, but the optimal plan features regressive income taxes, as in the simple model of Section 2. Thus, the concentration of income at the top does not significantly affect the size of optimal transfers, but it does change the optimal

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[18]See Appendix C.6 for results with $\xi > 0$.
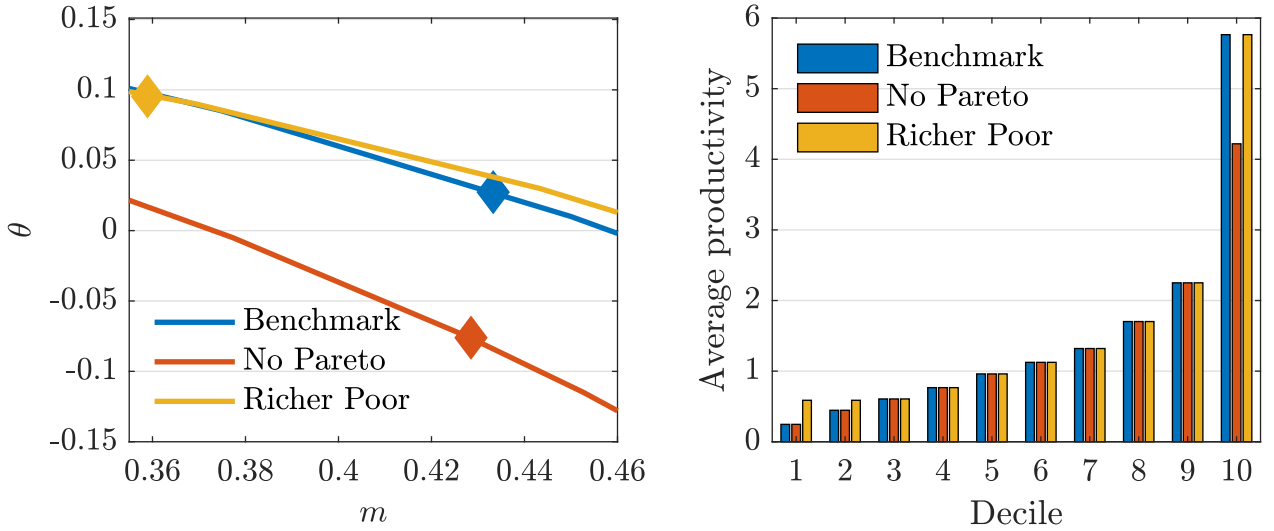
Figure 14: Optimal m-θ line and the income distribution

Notes: The left panel plots the optimal progressivity θ for each level of transfer m, as well as the optimal (m, θ) pair, for three economies: the benchmark, the "No Pareto", and the "Richer Poor" economies. The phase-out parameter ξ is fixed to zero. The right panel shows the average productivity level z by decile in each economy.

way of financing them. With the Pareto tail, a planner is able to raise sufficient revenues from the top and prefers lower tax rates on middle-income households.

In contrast, the m-θ line barely shifts in the "Richer Poor" economy. That is, for a given level of transfer m, the optimal level of progressivity is similar to the benchmark. However, optimal transfers decrease to $m = 0.36$ compared to $m = 0.43$ in the benchmark, a drop of about \$4,200. Thus, the lower the income of the poorest is, the larger the transfers the optimal plan prescribes.

5.2 Income risk distribution: skewness and kurtosis

Higher-order moments of the labor income distribution only moderately affect the optimal fiscal plan. We compute the m-θ line for the case when labor productivity follows an AR(1) process, abstracting from the Pareto tail for simplicity. We label this case as "AR No Pareto" and compare it to the "No Pareto" economy, which features the GMAR process described in Section 3. We keep the persistence of the AR(1) fixed to the calibrated value and set its innovation variance to match the same overall productivity variance as in the GMAR case. The left panel of Figure 15 shows the optimal m-θ line for the "AR No Pareto" and "No Pareto" cases.

As compared to the GMAR, the AR process has fewer large negative shocks but more frequent medium-size shocks, as shown in the right panel of Figure 15. These differences have offsetting effects on optimal taxes: while the less frequent left tail shocks reduce the need for insurance, the more frequent medium-size shocks do the opposite. This results

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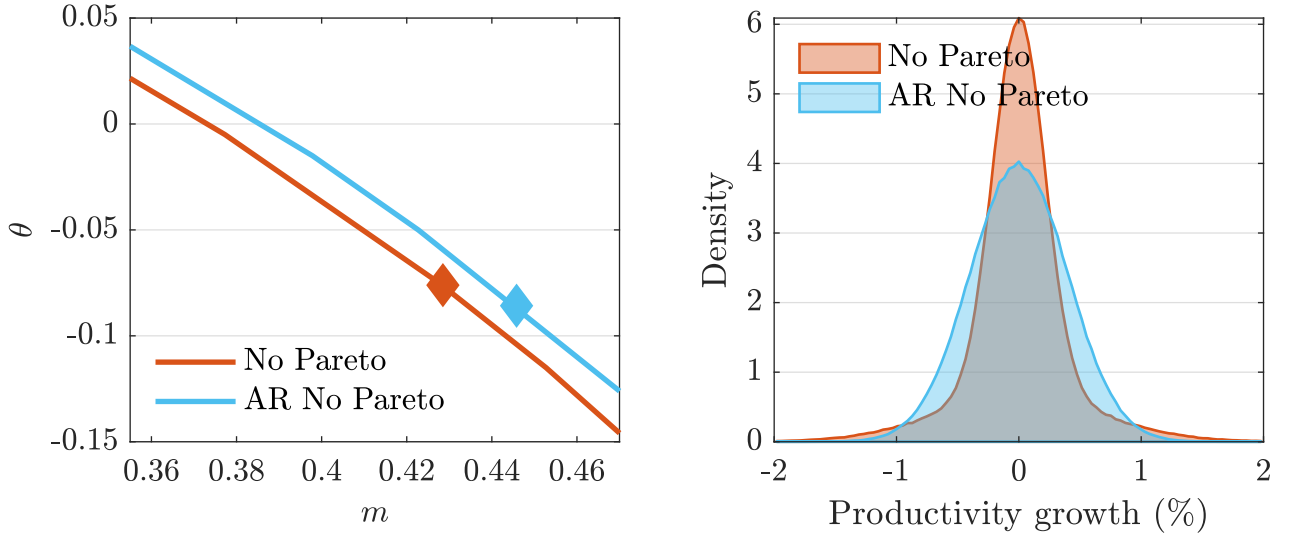
Figure 15: Optimal m-θ line and the distribution of income risk

Notes: The left panel plots the optimal progressivity θ for each level of transfer m, as well as the optimal (m, θ) pair, for two economies: the "No Pareto" economy with the GMAR process, and the "AR No Pareto" with the AR process. The phase-out parameter ξ is fixed to zero. The right panel shows the distribution of productivity changes in both economies.

in a similar progressivity and somewhat larger transfers—from $m = 0.43$ to $m = 0.45$, an increase of about $\$1,000$. Overall, higher order moments of income risk do not drastically change the optimal $t\&T$ plan.[19]

5.3 Wealth inequality

Higher wealth concentration reinforces redistribution concerns. We compute the m-θ line in a case with heterogeneous stochastic discount factors, as in [Krusell and Smith (1998)](#), which we label "Heterogeneous β". We calibrate the process for discount factors to match the concentration of wealth at the top, a moment that the benchmark model could not replicate.[20] Figure [16](#) shows the optimal m-θ line in the "Heterogeneous β" and in the benchmark as well as the wealth distribution in both cases.

The optimal m-θ line remains downward sloping but shifts up in the "Heterogeneous β" case: for a given level of transfers, the optimal progressivity is larger, by about 0.05. With a higher wealth concentration, redistribution concerns lead a planner to finance transfers with more progressive income taxes, as high-wealth households are also likely to be high-income. Optimal transfers increase by $\$3,400$, from $m = 0.43$ to $m = 0.49$, such that optimal progressivity θ remains about the same as in the benchmark.

This exercise should be taken with caution, as wealth inequality is generated with

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[19]This result is consistent with [De Nardi, Fella, and Paz-Pardo (2019)](#), which finds that the welfare cost of idiosyncratic risk is somewhat lower when modeling higher-order moments of income risk.

[20]See Appendix [C.6](#) for more details.
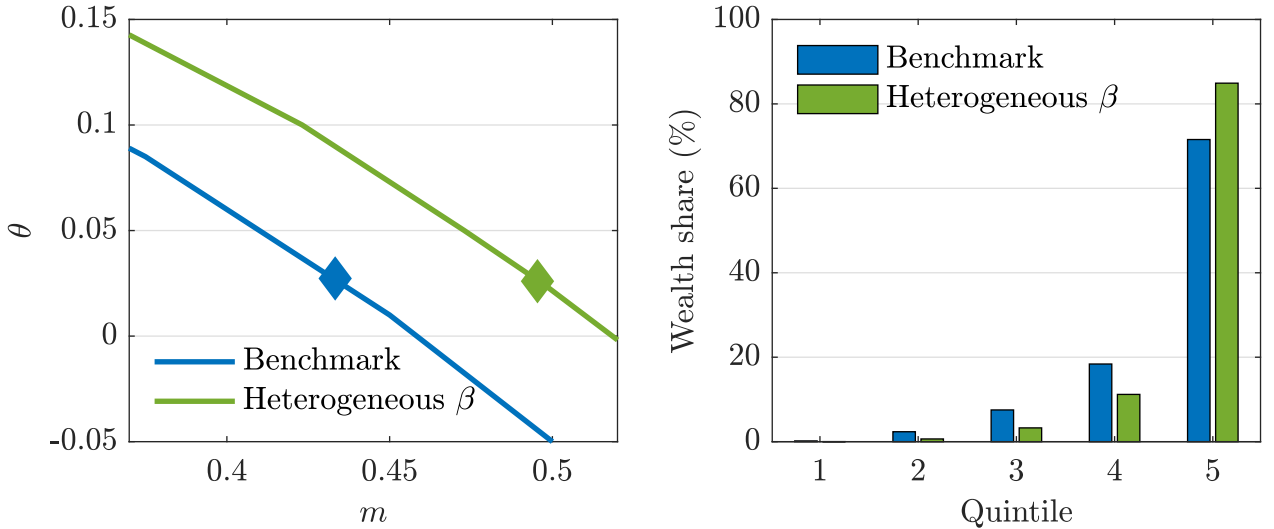
Figure 16: Optimal m-θ line and the wealth distribution

Notes: The left panel plots the optimal progressivity θ for each level of transfer m, as well as the optimal (m, θ) pair, for the benchmark and the "Heterogeneous β" economies. The phase-out parameter ξ is fixed to zero. The right panel shows the distribution of wealth by quintile in both economies.

a simple and, importantly, policy-invariant modeling tool.[21] Yet, it suggests that the benchmark calibration delivers a lower bound for the generosity of the optimal tax plan.

6 Conclusion

In this paper, we studied the optimal design of the tax-and-transfer system. We developed a tractable analytical model demonstrating an optimally negative relation between transfers and income-tax progressivity, due to both *efficiency* and *redistribution* concerns. We then quantified the optimal fiscal plan in a rich dynamic model calibrated to the U.S. economy. We found that optimal transfers should be generous, with a slow phase-out, and financed with moderate income-tax progressivity. That is, the optimal plan features more progressive average than marginal $t\&T$ rates. Furthermore, there are large welfare gains from implementing this plan, that would be supported by a majority of households.

Our Ramsey approach is compatible with a quantitative and dynamic evaluation of efficiency and redistribution concerns. The instruments we used are simple and intuitive, and they resemble current policies implemented by many countries. Yet, they are richer than what is typically used in the Ramsey literature, and flexible enough to generate non-linear non-monotonic $t\&T$ schedules. As such, our analysis contributes to bridging the gap between Mirrlees and Ramsey traditions.

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[21]See Boar and Midrigan (2021) and Brüggemann (2021) for a richer modeling of wealth accumulation due to entrepreneurial activity.

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A Analytical model

A.1 Representative agent

As a special case of the analytical model, we first consider a representative agent. We show that (1) for any level of transfer there exists a progressivity that implements the efficient allocation, and (2) this progressivity is decreasing in the transfer.

Efficient allocation.—The first-best allocation maximizes utility subject to the resource constraint (4):

$$\max_{c,n} \ \log c - B\frac{n^{1+\varphi}}{1+\varphi} \ \text{s.t.} \ c + G = n.$$

The first-order condition of this problem characterizes the efficient labor $n^\star(G)$:

$$Bn^\varphi(n-G) = 1. \tag{18}$$

Equilibrium with taxes and transfers.—Consider a government that has access to a log-linear income-tax function and a lump-sum transfer. The household chooses $\{c,n\}$ to maximize utility (1) given the budget constraint (2). The household first-order condition is

$$Bn^\varphi = \frac{\lambda\,(1-\tau)\,n^{-\tau}}{\lambda n^{1-\tau} + T}. \tag{19}$$

The government budget constraint is

$$n - \lambda n^{1-\tau} = G + T. \tag{20}$$

Implementation of the efficient allocation.—For each transfer T, a planner can implement the first-best allocation $n^\star(G)$ with an income-tax progressivity equal to

$$\tau(G,T) = -\frac{G+T}{n^\star(G) - G - T}. \tag{21}$$

To prove this result, replace (21) in (19) to obtain

$$Bn^\varphi\left(\lambda n^{1-\tau} + T\right) = \left(\frac{n}{n-G-T}\right)\lambda n^{-\tau},$$

and use (20) to obtain (18). Finally, it is easy to see from (21) that $\partial \tau(G,T)/\partial T < 0$, which proves Claim (1).

A.2 Heterogeneity and no transfers

In this section we derive the closed-form expression for welfare when the government uses only the log-linear tax function.

Idiosyncratic productivity.—Define $\alpha_{i,t} \equiv \log z_{i,t}$. One can show that $\mathbb{E}[\alpha_{i,t}] = -\frac{v_\omega}{2(1-\rho^2)}$ and $\mathbb{V}[\alpha_{i,t}] = \frac{v_\omega}{1-\rho^2}$. As $z_{i,t} = \exp(\alpha_{i,t})$, $\mathbb{E}[z_{i,t}] = 1 \ \forall t$.

Household problem.—The consumers solve a static problem: they maximize utility (1) given the budget constraint (2), with $T = 0$. Computing the first-order condition, we get:

$$n_{it} = \left(\frac{1-\tau}{B}\right)^{\frac{1}{1+\varphi}} \equiv n_0(\tau). \tag{22}$$

For the rest of the Appendix, we omit the dependence of n_0 on τ to ease notation.

Computing λ.—Output is given by

$$Y_t = \int y_{i,t} di = \int \exp(\alpha_{i,t}) n_{i,t} di = n_0.$$

To compute λ, we also need to compute

$$\tilde{Y}_t \equiv \int y_{i,t}^{1-\tau} di = \int [n_0 \exp(\alpha_{i,t})]^{1-\tau} di = n_0^{1-\tau} \exp\left(-\tau \frac{(1-\tau)v_\omega}{2(1-\rho^2)}\right).$$

Therefore, using the government's budget constraint (3), we can express λ as

$$\lambda = \frac{Y-G}{\tilde{Y}} = \frac{n_0-G}{n_0^{1-\tau}} \exp\left(\tau \frac{(1-\tau)v_\omega}{2(1-\rho^2)}\right) \equiv \lambda_0. \tag{23}$$

Welfare.—To compute welfare in closed form, we first plug the equilibrium value of consumption and hours worked into the utility function:

$$u_{i,t} = \log \lambda + (1-\tau) \log(\exp(\alpha_{i,t}) n_0) - \frac{B}{1+\varphi} n_0^{1+\varphi}$$

$$= \log \lambda + \frac{1-\tau}{1+\varphi} \log\left(\frac{1-\tau}{B}\right) + (1-\tau)\alpha_{i,t} - \frac{1-\tau}{1+\varphi}.$$

Integrating over the distribution of households, we obtain

$$W(\tau) = \int u_{i,t} di = \log \lambda + \frac{1-\tau}{1+\varphi} \log\left(\frac{1-\tau}{B}\right) - \frac{(1-\tau)v_\omega}{2(1-\rho^2)} - \frac{1-\tau}{1+\varphi},$$

and, using the closed-form solution (23) for λ, we obtain equation (8) in the main text.

A.3 Heterogeneity and transfers

We now derive welfare as a function of progressivity τ and the transfer T. The logic of the derivation is the same as in the previous section. However, we cannot express the policy function for labor in closed form, so we linearize around $T = 0$.

Household problem.—The first-order condition of the household problem reads

$$B n_{it}^{\varphi} = \frac{1}{c_{it}} \left(\exp(\alpha_{it}) n_{it} \right)^{-\tau} \exp(\alpha_{it}) \lambda \left(1 - \tau \right);$$

that is, after rearranging,

$$B n_{it}^{1+\varphi} + \frac{T}{\lambda} B n_{it}^{\varphi+\tau} \exp(-(1-\tau)\alpha_{it}) - (1-\tau) = 0. \tag{24}$$

Equation (24) defines the function $G(T, \lambda, n_{it})$ s.t. $G(T, \lambda, n_{it}) = 0$. At the optimum, the labor decision is such that, for a given T, $G(T, \lambda, n_{it}(T, \lambda)) = 0$.

Linear approximation of labor policy.—At $T = 0 \equiv T_0$, we know that $n_{it}(T_0, .) = n_0$. The implicit function theorem holds, and we can compute the slope of n_{it} in the neighborhood of T_0 as

$$\left. \frac{\partial n_{it}(T, \lambda)}{\partial T} \right|_{(T_0, n_0, \lambda_0)} = - \frac{\left. \frac{\partial G(T,\lambda,n_{it})}{\partial T} \right|_{(T_0,n_0,\lambda_0)}}{\left. \frac{\partial G(T,\lambda,n_{it})}{\partial n_{it}} \right|_{(T_0,n_0,\lambda_0)}}.$$

Let $\eta \equiv \exp\left((1-\tau) \frac{v_\omega}{1-\rho^2} \right)$, where we omit the dependence of η on τ and v_ω to ease notation. We compute the two partial derivatives and obtain a linear approximation around (T_0, n_0, λ_0) of $n_{it}(T)$ denoted $\hat{n}_{it}(T)$:

$$\hat{n}_{it}(T) = n_0 + T \left. \frac{\partial n_{it}(T)}{\partial T} \right|_{(T_0,n_0,\lambda_0)} = n_0 - \frac{T}{1+\varphi} \frac{n_0}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1-\tau)\alpha_{it}). \tag{25}$$

Computing λ.—Again, we need to compute Y and $\tilde{Y}$. We start with approximated output:

$$\hat{Y}_t = \int y_{i,t} di = \int \exp(\alpha_{i,t}) n_{i,t} di = n_0 - \frac{T}{1+\varphi} \frac{n_0}{n_0 - G} \eta^{-\tau}. \tag{26}$$

To obtain $\tilde{Y}$, we first approximate $\hat{n}^{1-\tau}$. Using equation (25) we get

$$\hat{n}_{it}^{1-\tau}(T) = \left[n_0 - \frac{T}{1+\varphi} \frac{n_0}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1-\tau)\alpha_{it}) \right]^{1-\tau},$$

which we can linearize to obtain

$$\hat{n}_{it}^{1-\tau}(T) = n_0^{1-\tau} - \frac{T}{1+\varphi} (1-\tau) \frac{n_0^{1-\tau}}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1-\tau)\alpha_{it}).$$

It follows that

$$\hat{\tilde{Y}}_t = \int \left[n_0^{1-\tau} - \frac{T}{1+\varphi} (1-\tau) \frac{n_0^{1-\tau}}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp(-(1-\tau)\alpha_{it}) \right] \exp\left[(1-\tau)\alpha_{it}\right] d_i$$

$$= n_0^{1-\tau} \eta^{-\frac{\tau}{2}} \left[1 - \frac{T}{1+\varphi} \frac{1-\tau}{n_o - G} \right].$$

Using these expressions, we can compute λ using the government budget constraint (3),

$$\hat{\lambda}(T) = \frac{\hat{Y} - G - T}{\hat{\tilde{Y}}} = \frac{n_0 - \frac{T}{1+\varphi} \frac{n_0}{n_0 - G} \eta^{-\tau} - G - T}{n_0^{1-\tau} \eta^{-\frac{\tau}{2}} \left[1 - \frac{T}{1+\varphi} \frac{1-\tau}{n_0 - G} \right]},$$

and, taking derivatives and linearizing around $T = 0$,

$$\hat{\lambda}(T) = \lambda_0 + \frac{T}{1+\varphi} \frac{1}{\eta^{-\frac{\tau}{2}} n_0^{1-\tau}} \left[-\frac{n_0}{n_0 - G} \eta^{-\tau} - (\varphi + \tau) \right]. \tag{27}$$

Welfare.—We approximate utility around $T = 0$. The utility of an agent is given by

$$u_{it} = \log \left[\lambda \left[\exp\left(\alpha_{i,t}\right) n_{it} \right]^{1-\tau} + T \right] - \frac{B}{1+\varphi} n_{it}^{1+\varphi},$$

which, using our expressions (27) for $\hat{\lambda}(T)$ and (25) for $\hat{n}_{it}(T)$, can be approximated as

$$\hat{u}_{it} = u_{it,0} + T \left\{ \frac{\lambda'(T) \exp\left[(1-\tau)\alpha_{it}\right] n_0^{1-\tau} - \lambda_0 \frac{1-\tau}{1+\varphi} \frac{n_0^{1-\tau}}{n_0 - G} \eta^{-\frac{\tau}{2}} + 1}{\lambda_0 \exp\left[(1-\tau)\alpha_{it}\right] n_0^{1-\tau}} + \frac{B}{1+\varphi} \frac{n_0^{1+\varphi}}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp\left[-(1-\tau)\alpha_{it}\right] \right\}$$

$$= u_{it,0} + T \left\{ \frac{1}{1+\varphi} \frac{1}{n_0 - G} \left[-\frac{n_0}{n_0 - G} \eta^{-\tau} - (\varphi + \tau) \right] - \frac{1-\tau}{1+\varphi} \frac{\eta^{-\frac{\tau}{2}}}{n_0 - G} \exp\left[-(1-\tau)\alpha_{it}\right] \ldots \right.$$

$$\left. + \frac{\eta^{-\frac{\tau}{2}}}{n_0 - G} \exp\left[-(1-\tau)\alpha_{it}\right] + \frac{B}{1+\varphi} \frac{n_0^{1+\varphi}}{n_0 - G} \eta^{-\frac{\tau}{2}} \exp\left[-(1-\tau)\alpha_{it}\right] \right\}.$$

Integrating this equation yields $W(\tau, T) = W(\tau, 0) + \hat{\Omega}(\tau, v_\omega)T$, with $\hat{\Omega}(\tau, v_\omega)$ defined as

$$\hat{\Omega}(\tau, v_\omega) \equiv \frac{1}{1+\varphi} \frac{1}{n_0 - G} \left[-\frac{n_0}{n_0 - G} + 1 - \tau \right] + \frac{1}{1+\varphi} \frac{n_0}{(n_0 - G)^2} \left[-\eta^{-\tau} + 1 \right] + \frac{1}{n_0 - G} \left[\eta^{1-\tau} - 1 \right], \tag{28}$$

and where, again, we omit the dependence of n_0 on τ and of η on τ and v_ω.

Welfare decomposition.—The first term in equation (28), which equates $\Omega_e^{ra}(\tau)$ defined in equation (10.a), can be rearranged as

$$-\frac{1}{n_0 - G} \frac{1}{1+\varphi} \frac{n_0}{n_0 - G} + \frac{1-\tau}{n_0} \frac{1}{1+\varphi} \frac{n_0}{n_0 - G}.$$

It is equal to the marginal utility of aggregate consumption $u_c(C_0) = 1/(n_0 - G)$, multiplied by $\partial \hat{Y}^{ra}(T)/\partial T$, where $\hat{Y}^{ra}(T)$ is defined as the representative-agent version of (26), with $\eta = 1$, plus the marginal utility of aggregate leisure $-u_n(n_0) = Bn_0^\varphi = (1-\tau)/n_0$, multiplied by $\partial \hat{n}^{ra}(T)/\partial T$, where $\hat{n}^{ra}(T)$ is defined as the representative-agent version of (25), with $\eta = 1$.

The second term in equation (28) can be rearranged as

$$\frac{1}{n_0 - G}\left[-\frac{1}{1+\varphi}\frac{n_0}{n_0 - G}\eta^{-\tau} + \frac{1}{1+\varphi}\frac{n_0}{n_0 - G}\right].$$

It is equal to the marginal utility of aggregate consumption $u_c(C_0)$, multiplied by $\partial \hat{Y}(T)/\partial T$ minus $\partial \hat{Y}^{ra}(T)/\partial T$. Approximating further using $\exp(x) \approx 1 + x$ delivers $\Omega_e^{ha}(\tau, v_\omega)$, defined in (10.b).

The third term in equation (28) can be rewritten as

$$\frac{1}{n_0 - G}\eta^{1-\tau} - \frac{1}{n_0 - G}$$

and is equal to the average marginal utility across households, $\int u_c(\lambda_0 (z_{it}n_0)^{1-\tau})di$, minus marginal utility of aggregate consumption $u_c(C_0)$. Approximating further using $\exp(x) \approx 1 + x$ delivers $\Omega_r(\tau, v_\omega)$, defined in equation (10.c).

Derivatives.—Claim 2 states that the first term in equation (28), $\Omega_e^{ra}(\tau)$, is decreasing in τ. To show that, we sign the derivative equal to

$$\frac{1}{1+\varphi}\frac{1}{n_0 - G}\left(\frac{n_0}{n_0 - G}\frac{1}{1+\varphi}\left[-\frac{n_0 + G}{n_0 - G}\frac{1}{1 - \tau} + 1\right] - 1\right).$$

When $\tau > 0$, $\frac{1}{1-\tau} > 1$ and $\frac{n+G}{n-G} \geq 1$ such that $-\frac{1}{1-\tau}\frac{n+G}{n-G} + 1 < 0$ and the derivative is always negative. More generally, we need to show that the parenthesis is negative. As $\varphi > 0$, it is sufficient to show that

$$\frac{n_0}{n_0 - G}\left[-\frac{n_0 + G}{n_0 - G}\frac{1}{1 - \tau} + 1\right] \leq 1$$

is always true. This condition can be rewritten as

$$-(1 - \tau)G^2 \leq n_0(n_0 + \tau G). \tag{29}$$

Equation (29) always holds as the left-hand side is negative $\forall \tau$, while the right-hand side is positive as $\tau \geq -1$ and $G \leq n_0$ by feasibility.

Claim 3 states that $\Omega_r(\tau, v_\omega)$, defined in equation (10.c), is decreasing in τ. The derivative reads

$$(1 - \tau)\frac{v_\omega}{1 - \rho^2}\frac{1}{n_0(\tau) - G}\left(-2 + \frac{1}{1+\varphi}\frac{n_0(\tau)}{n_0(\tau) - G}\right), \tag{30}$$

which is negative $\forall \tau \in [-1; 1]$ when $G = 0$. When $G > 0$, equation (30) is negative on $\tau \in [-1, \hat{\tau}(G)]$, with $\hat{\tau}(G) \equiv 1 - BG^{1+\varphi}\left(\frac{2(1+\varphi)}{2(1+\varphi)-1}\right)^{1+\varphi}$, which is equal to 0.998 in our calibration.

B Data

B.1 Taxes and transfers

We use Congressional Budget Office (CBO) data to compute average tax and transfer rates.

We consider only non-elderly households.[22] As an income concept we use a broad measure of market income including wages, employee's contribution for deferred compensation, employer's contribution for health insurance, employer's share of payroll taxes, federal unemployment tax, corporate tax borne by labor, corporate tax borne by capital, capital gains, tax-exempt interest, taxable interest, positive rent, dividends, and other market income.

For taxes, we consider all taxes reported by the CBO: individual income taxes, payroll taxes, corporate income taxes, and excise taxes. Tax credits such as the Earned Income Tax Credit (EITC) and the Child Tax Credit (CTC) are included in taxes in the data. While these credits are at least partially refundable and therefore could be considered transfers, we cannot isolate them in the CBO data and therefore leave them in taxes. State taxes are not reported.

For transfers, we only consider those that are meant to provide income security. Because our model is an infinite horizon model, we do not consider transfers to the elderly such as social security and Medicare. Also, we do not model health shocks or disability, so we leave Medicaid and Supplemental Security Income (SSI) out of our transfer measure. Our transfer measure includes programs to provide income security such as the Supplemental Nutrition Assistance Program (SNAP; commonly known as food stamps), Temporary Assistance for Needy Families (TANF), housing assistance, and some other smaller means-tested transfer programs.

B.2 Income and wealth

B.2.1 Survey of Consumer Finances

The first source for wealth and income data is the 2013 Survey of Consumer Finances (SCF). The unit of observation is a family, defined as the economically dominant single person or couple (whether married or living together as partners) and all other persons in the household who are financially interdependent with that economically dominant person or couple. We restrict the sample to households where the head's age is between 25 and 60 and only keep observations with labor income (as defined below) above $5,000. Weights are used throughout. Net worth consists of all real and financial assets net of all debts. Labor income consists of income from wages and salaries. Asset positions are for 2013, whereas income refers to 2012.

B.2.2 Panel Study of Income Dynamics

The second data source is the Panel Study of Income Dynamics (PSID), which we use to obtain moments of the earnings growth distribution. We are interested in pre-tax labor earnings at

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[22]As data provided by the CBO is aggregated by quintile over the entire population only, the quintiles do not exactly add up to 20% when excluding elderly households. When comparing tax and transfer rates in the model to the data, we adjust the cutoffs to match the quintile shares in the data: 17.9%, 19.0%, 20.3%, 21.1%, 21.7%.

the household level. The PSID was conducted annually between 1968 and 1997 and biannually since. As the model frequency is annual, we restrict ourselves to the surveys before 1997. Furthermore, 1970 earnings questions refer to labor earnings in the previous year, while they refer to the same year before 1970. Hence, we start in 1969 with income information from the 1970 survey. Finally, we stop in 1992 because income definitions change after that point. Labor income includes "labor part of farm income and business income, wages, bonuses, overtime, commissions, professional practice, labor part of income from roomers and boarders". To obtain household-level income, we add up incomes of the head and wife.[23] We translate incomes to real incomes in 2013 terms using the CPI. Income growth is computed using log differences. We restrict ourselves to households where the head is between ages 25 and 60 and only keep observations with labor income above $5,000 (in 2013 dollars). We use only the representative Survey Research Center sample.

We compute moments of the earnings growth distribution while keeping the household composition fixed; that is, requiring the head and wife to be the same individuals from $t-1$ to $t+1$. We use residualized earnings, taking out year and age effects.

Finally, we report the distribution of labor income in 2013 at the very bottom of the labor income distribution. Labor income at the first, fifth, and tenth percentiles amounts to 10%, 20% and 31% of median labor income.

C Quantitative model

C.1 Tax function

Figure 17 compares average and marginal tax rates implied by the log-linear tax function and the new tax function. In the new tax function, we set $\theta = 0.16$ and $\lambda = 0.12$, as in the calibration. The corresponding parameters in the log-linear tax function are $\tau = 0.16$ and $\lambda = 1 - 0.12$.

C.2 Numerical solution

C.2.1 Steady state

To solve for the steady state of the economy, we need to find the real interest rate r that clears the capital market and the level of transfers m that clears the government budget constraint. We explain next how we do this.

0. Set grids for assets $\vec{a}$ and productivity levels $\vec{z}$. Let $N_a = 300$ and $N_z = 19$ be the number of points in each grid, respectively. Compute the transition matrix of productivities $\pi_z(z', z)$ using Farmer and Toda (2017).[24]

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[23]PSID terminology for this time period.

[24]We use the Matlab package provided at https://sites.google.com/site/aatoda111/discretization.
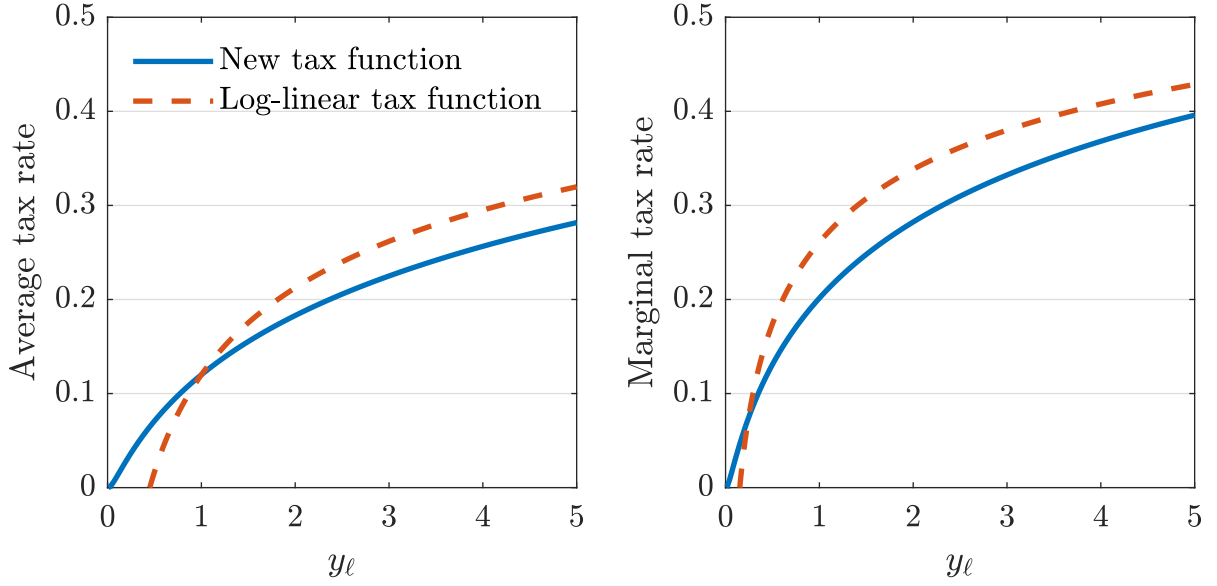
Figure 17: New income tax function: comparison to log-linear function

Notes: This figure compares average and marginal tax rates implied by the log-linear tax function and the new tax function for $\tau = \theta = 0.16$. Labor income is plotted relative to median income.

1. Guess values for the interest rate r and the transfer parameter m, and compute the wage w implied by the guessed r.

2. Solve for household policies by value function iteration. In particular, for a given guess, guess a value function $V(a, z)$ and update the value function as

$$\hat{V}(a, z) = \max_{a' \geq 0, n \in [0,1]} \left\{ \frac{c^{1-\sigma}}{1-\sigma} - B \frac{n^{1+\varphi}}{1+\varphi} + \beta \sum_{z' \in \vec{z}} \pi_z(z', z) V(a', z') \right\}$$

$$\text{s.t. } c + a' \leq wzn + (1+r)a - \mathcal{T}(wzn, ra).$$

Iterate until $\left\| \hat{V} - V \right\| < \varepsilon^V$. We use $\varepsilon^V = 1e - 10$.

3. Compute the stationary measure implied by the optimal policies of step 2. In particular, for a given guess $\mu(a, z)$, compute implied measure $\hat{\mu}(a, z)$ as

$$\hat{\mu}(a_{i'}, z_{j'}) = \sum_{i=1}^{N_a} \sum_{j=1}^{N_z} \mathbb{L}\left\{ a_{i'} = a'(a_i, z_j) \right\} \pi_z(z_{j'}, z_j) \mu(a_i, z_j)$$

where $\mathbb{L}$ computes a linear interpolation: $\mathbb{L}(a_i, a') = \mathbb{I}(a' \in (a_{i-1}, a_i]) \frac{a - a_{i-1}}{a_i - a_{i-1}}$. Iterate until $\| \hat{\mu} - \mu \| < \varepsilon^\mu$. We use $\varepsilon^\mu = 1e - 11$.

4. Compute asset market clearing error: $E^K = A - D - \hat{K}$ where $A = \int a d\mu(a, z)$ is households' asset holdings and $\hat{K} = L \left(\frac{r+\delta}{1-\alpha} \right)^{-1/\alpha}$ is capital demand given interest rate and labor supply $L = \int n(a, z) d\mu(a, z)$. Also compute government budget constraint error $E^G = G + rD - \int \mathcal{T}(wzn(a, z), ra) d\mu(a, z)$. Let $\mathcal{E}(X) \equiv (E^K, E^G)$ collect the two errors

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given the guess $X \equiv \{m, r\}$. An equilibrium can be written as

$$\mathcal{E}(X) = 0. \qquad (31)$$

We solve for X in equation (31) using a quasi-Newton method.

C.2.2 One transition

We assume a once-and-for-all fiscal reform, where the two tax parameters λ and θ and the transfer phase-out ξ jump to their new values and the transfer level $\{m_t\}$ adjusts every period to clear the government budget constraint. We assume that the economy has converged to its new steady state $\bar{T}$ periods after the shock. We first compute the new steady state as described in Section C.2.1 and obtain the value function $\bar{V}(a, z)$ and the equilibrium vector $\bar{X} = (\bar{m}, \bar{r})$ of the new steady state. As the economy has converged in $\bar{T}$, we know that the value function at $t = \bar{T}$ equals its steady-state value $V_{\bar{T}}(a, z) = \bar{V}(a, z)$. We also know that the measure at time $t = 1$ is equal to the initial steady-state value $\mu_1(a, z) = \mu(a, z)$. Then, given a guess for transfers and interest rates $\{m_t, r_t\}_{t=1}^{\bar{T}}$ such that $(m_{\bar{T}}, r_{\bar{T}}) = (\bar{m}, \bar{r})$, we solve the household problem backwards and iterate on the sequence $\{m_t, r_t\}_{t=1}^{\bar{T}}$ using a quasi-Newton algorithm to clear markets.

C.2.3 Global optimum

To find the optimal tax reform, we rely on the TikTak Global Optimization multistart algorithm–see Arnoud, Guvenen, and Kleineberg (2019) for a detailed description.[25]

We optimize on the triplet $\{\theta, \lambda, \xi\}$. For each triplet, we compute the transition as described in Section C.2.2. The algorithm looks for the triplet that maximizes welfare at the implementation of the fiscal reform; that is,

$$\int V_1(a, z) d\mu(a, z).$$

We start local optimizers using the best 30 points in a Sobol sequence of 795 points. The local optimizer is Nelder Mead.

C.3 Optimal steady state

The fiscal plan that optimizes steady-state welfare is: $m = 0.36$, $\theta = 0.03$, and $\xi = 0$. In line with Bakış, Kaymak, and Poschke (2015), the optimal plan is more generous when incorporating transitions: the optimal UBI plan features similar income-tax progressivity but significantly larger transfers, about $\$4,000$ larger than in steady-state. Furthermore, transfers are optimally a lump sum when optimizing steady-state welfare only, as a phase-out implies a capital tax. When considering only steady states, reducing capital taxation generates large gains, a result that is

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[25]We use the Fortran package provided at https://github.com/serdarozkan/TikTak.

not robust to transitions–see Domeij and Heathcote (2004) for a detailed discussion of capital taxes and transitions in incomplete-market economies.

We further investigate the optimal UBI plan that optimizes steady-state welfare keeping income-tax progressivity at its status-quo value, $\theta = 0.16$. Given the negative relation between m and θ, the optimal level of transfers declines to $m = 0.27$, equivalent to \$16, 100. This number is larger than in Guner, Kaygusuz, and Ventura (2021), in which the optimal UBI is \$10, 400 for a household with two children. Their smaller number may be explained by a combination of factors, such as a richer household structure, lower risk-aversion, and a different calibration of the income process both at the top and at the bottom. In particular, our income process features a Pareto tail and a lower income share of the first income quintile. Both elements increase the optimal transfer for a given level of income-tax progressivity. For instance, when removing the Pareto tail, the optimal transfer declines to \$14, 200 when fixing the progressivity to the status quo and optimizing the steady-state welfare.

C.4   Optimal log-linear plan

We compute the optimal log-linear tax plan using a labor tax function as in the analytical section: $\mathcal{T}(y) = y - \lambda(y/\bar{y})^{1-\tau}$. The economy is calibrated in steady state as described in Section 3, and a planner implements a one-time fiscal reform where the benchmark taxes and transfers are replaced by a log-linear function with constant income-tax progressivity τ. As in Section 4, we take into account transitions for welfare computations. The optimal plan features large progressivity, at $\tau = 0.49$, compared to the estimated $\tau = 0.18$ for the current U.S. system. The optimal plan achieves less redistribution, and at the cost of strongly increasing marginal $t\&T$ rates (see Table 5). Thus, welfare gains are large, at $+5.08\%$, but smaller than in the optimal plan with more flexible instruments.

In contrast with Heathcote and Tsujiyama (2021), we also find that the affine plan is preferable to the log-linear plan. This difference may be driven by several factors, including endogenous private insurance, which reinforces the value of insuring consumption fluctuations; capital accumulation, which may lower optimal marginal rates at the top to preserve capital accumulation incentives; and higher risk-aversion, which increases the welfare gains associated with a lump-sum transfer.[26]

C.5   Consumption equivalents and welfare decomposition

C.5.1   Consumption equivalents

Consumption equivalents $\gamma(a, z)$ are computed as the increase in consumption in the status quo, keeping status-quo policies fixed, which would make household (a, z) indifferent between the status quo and the tax reform. More formally, one can write $V(a, z)$, the life-time utility of a

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[26]See Chang and Park (2020) for the effects of private insurance on marginal rates in a dynamic Mirrleesian set-up. See Heathcote and Tsujiyama (2021), Table 2, for the optimal lump-sum in the static Mirrleesian plan as a function of risk-aversion.

Table 5: Optimal log-linear plan

Optimal log-linear plan	Q1	Q2	Q3	Q4	Q5
Average $t\&T$ rate	-46%	-12%	5%	17%	46%
Marginal $t\&T$ rate	10%	32%	45%	51%	65%
Optimal plan	Q1	Q2	Q3	Q4	Q5
Average $t\&T$ rate	-155%	-37%	6%	25%	44%
Marginal $t\&T$ rate	62%	66%	62%	53%	51%

Notes: This table shows average and marginal $t\&T$ rates by income quintile for the optimal log-linear plan, and compares them to the optimal plan with the benchmark tax and transfers function.

household with state (a, z) in the status quo, as

$$V(a, z) = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} - B \frac{n_t^{1+\varphi}}{1+\varphi} \right\} | a, z \right]$$

where expectation $\mathbb{E}_0$ is taken over future paths of shocks given individual states at the moment of the tax reform.

For a given j-triplet (θ, λ, ξ), let $V_1^j(a, z)$ be the utility of household (a, z) when implementing the reform. We compute $\gamma(a, z)$ such that

$$\mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{[(1 + \gamma(a, z))c_t]^{1-\sigma}}{1-\sigma} - B \frac{n_t^{1+\varphi}}{1+\varphi} \right\} | a, z \right] = V_1^j(a, z).$$

Consumption equivalents are thus equal to

$$\gamma(a, z) = -1 + \left[1 + \frac{V_1^j(a, z) - V(a, z)}{\mathcal{C}(a, z)} \right]^{\frac{1}{1-\sigma}}, \tag{32}$$

where $\mathcal{C}(a, z) = \mathbb{E}_t \left[\sum_{t=0}^{\infty} \beta^t \left\{ \frac{c_t^{1-\sigma}}{1-\sigma} \right\} \right]$ can be computed using a simple iteration of policy functions. Finally, we aggregate over the distribution of consumption equivalents using the measure in the status quo.

C.5.2 Welfare decomposition

We follow the new decomposition proposed by Bhandari et al. (2021).[27] Table 6 reports the contribution of aggregate efficiency, redistribution, and insurance to the total welfare gains generated by the tax reform described in Section 4. Each component is further decomposed in terms of leisure and consumption.

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[27]We thank the authors for sharing detailed notes on how to implement this decomposition in a related framework.

Table 6: Welfare decomposition

Source of welfare gains	Total	Consumption	Leisure
Efficiency	7.29%	-92.00%	99.29%
Redistribution	22.16%	20.32%	1.85%
Insurance	70.55%	76.16%	-5.61%

Notes: This table decomposes the welfare gains in three components: efficiency, redistribution, and insurance. Each component is further decomposed in terms of consumption and leisure.

C.6   The optimal m-θ line
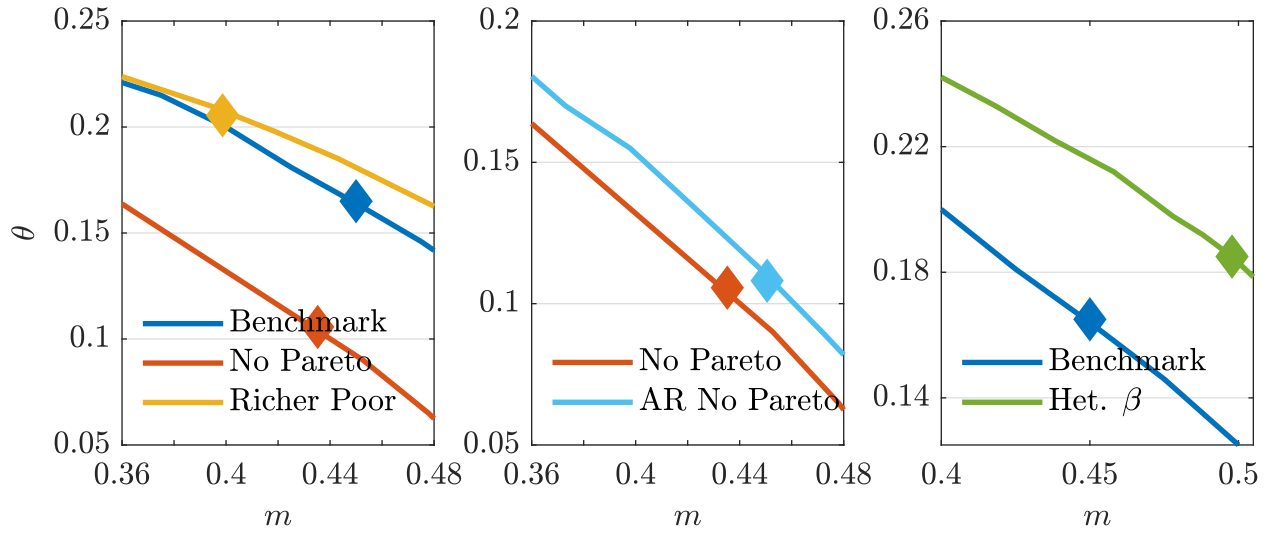
C.6.1   The "Heterogeneous β" economy

We calibrate the stochastic process for β as follows. β can take two values: $\beta_L = 0.943$ and $\beta_H = 0.994$. The probability of switching from low to high β is 0.10%, while the probability of switching from high to low β is 0.90%. The stationary distribution features 90% of households with low β. With this stochastic process, the model matches the wealth share of the top 10% (69%, compared to 71% in the data).

C.6.2   Optimal m-θ line with positive phase-out

Section 5 plotted the optimal negative relation between transfers and progressivity in the benchmark model, as well as in several alternative calibrations. Throughout that section, we kept the transfer phase-out fixed at $\xi = 0$. Figure 18 plots the optimal m-θ line in all economies when the transfer phase-out is at its optimal value, $\xi = 1.85$. As expected, all lines shift up, as progressivity is higher when transfers phase-out. Furthermore, all results discussed in Section 5 are robust to using positive ξ.

Figure 18: Optimal m-θ line with positive phase-out

Notes: This figure plots the optimal progressivity θ for each level of transfer m in the benchmark, the "No Pareto", the "Richer Poor", the "AR No Pareto" and the "Heterogeneous β" economies. The phase-out parameter is fixed at $\xi = 1.85$.