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# Uncertainty and Information Sources' Reliability 

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#### Abstract

This paper studies the impact of the reliability of information sources on choices under ambiguity. Using the Ellsberg's (1961) framework it studies two conjectures. First, the conditions of appearance of the Ellsberg paradox when the information source offers two probable proportions of red and black balls in two urns. Second, the consequence on choices of a non-reliable information source. This source proposes a unique proportion of red and black balls against an unknown one (inside box 1). Thus, either the proportion is correct, or it is unknown (ambiguous). We show that despite le information source unreliability, the decision-maker will trust the given information.


Keywords: Uncertainty theory, decision theory, ambiguity aversion, Information.
JEL : I1, I18, I19, D80, D81, D83.

[^0]
## 1. Introduction

Many decisions that involve irreversible commitments are made in an uncertain nonprobabilistic (or ambiguous) environment (Knight (1921)). Obviously, ambiguity comes from the events whose issues are unknown, but also through the sources of information themselves. Indeed, the latter can be either reliable or unreliable or, still, a mix between them, sometime right, sometime wrong. It follows that, constantly, the decision-maker (DM) must (or should) assess the reliability of the information's source, and the trust he must (or should) place in it. For example, at the outset of a sudden unknown pandemic, its characteristic (virus or bacteria genome, mode of spread, best preventive measures to immediately settle, etc.) is unidentified (see Berger et al. (2013)). Then, progressively, medical research brings details on the causes, the virus or bacteria virulence, the recommended prophylaxis, the medications, the vaccines, their effectiveness compared to variants, etc. Gradually, the academic information media improve the understanding of the phenomena even if, initially, they do not always fully agree one with each other. For example, in another scientific field as the Climate Change question, before reaching a consensus on the anthropogenic nature of current global warming, several decades passed, but finally, climate-skeptic scientists and their media were no longer able to argue against scientific advances (Weart (2011), Pachauri, et al. (2014)).

The long controversies between experts and scientists delay the taking of effective decisions. However, these are indispensable to know a phenomenon and reach a consensus on its determinants and consequences. In this respect, information quality plays a key role. Indeed, decisions are made on the basis of the arrival of new data that can reduce or increase uncertainty. However, the information accuracy also depends on the source itself, which may be perfectly or partially reliable.

Some forms of uncertainty are amenable to verification and learning, others are not. About the former, Blackwell (1951) compares information systems where differentiated information called 'experiences' are evaluated (i.e. marginal gains linked to an action) and classified. This structure allows an option ranking from the most to the least optimistic. These situations are called monotonic decision problems. Thus, Blackwell's theorem states that the value of experiment I is weakly superior to that of experiment II for all expected utility maximizers and all sets of actions if and only if experiment I is more "Blackwell informative" than experiment II. One experiment is more informative than another if the former can define the deformation grid of the latter. Thus, for example, the internet and the telegraph can send
the same message. The accuracy rate of the first means can be $99.9 \%$ and only $10 \%$ for the other, for instance. In both cases, ambiguity remains, but its order of magnitude is radically different: «A communication channel with a smaller error probability is more Blackwellinformative. For example, a message sent via email, with an error probability of $1 / 100$, is more informative than a message sent via telegraph, with an error probability of $1 / 10$.» ( Li and Zhou, 2016 p. 20).

Numerous extensions aimed at either improving Blackwell's criterion deemed too demanding (Lehmann (1988)) or extending it to consider agents with utility functions that are not Von Neumann-Morgenstern-like (vNM), (Li and Zhou, 2016). These authors show that once a partial classification of information can be established, then Blackwell's criterion can be applied to a broad spectrum of ambiguity theories. They extend their results to include the Lehmann criterion for non-vNM utility functions and find similar results to the Blackwell conjecture. These approaches are related to situations where information systems can be classified as a result of experimentation or signal interpretation as, for instance, revenues linked to financial securities, marketing data for innovative products from databases, information resources, etc.

Such rankings are not possible in other situations, where decision-making is based on cursory and unique information. The learning phenomena are reduced to their simplest expression, contrary, for instance, to Epstein and Schneider's (2007) where learning involves repetition. Indeed, considering the Ellsberg's framework and potential paradox, repetition induces decision makers to avoid irrational choices: "At the same time, given the ex-ante view that sampling is from an unchanging urn, it is intuitive that learning should resolve ambiguity in the long run. Asymptotically, the agent should behave as if she knew the fraction of black balls were equal to their empirical frequency. In the limit, for any common sample history across the two urns, she should be indifferent between betting on the next draw from the risky as opposed to the ambiguous urn. " Epstein and Schneider (2007, 1279).

The Epstein-Schneider methodology led to several contributions in the financial theory field (see the bibliography of (Ju and Miao 2012)). Both streams we briefly mentioned assume data sets and/or repetition of experiments. This is not the direction this paper takes here. Instead, we consider conjectures where both the information source is weak and the information questionable. For example, in the Ellsberg's (1961) framework, the information concerning the balls proportion in each box is given and certain. In the Ellsberg's example of two urns and two colors, in the first box, the proportion of red and black balls is known with certainty, while in the second box this proportion is unknown. The decision maker does not
question the information accuracy. However, in the real world, information is not as accurate, and data may be missing. This is especially true in emergency or critical situations characterized by flawed information sources.

This paper therefore studies how new information changes agents' priors in a world of radical uncertainty when the information sources are imperfectly reliable. In the uncertainty related literature, information leads to changes in agents' beliefs when new events occur as (Pires 2002 p.13) notes it: " One issue which is not addressed in the previous models is the updating of beliefs and preferences as new information arrives". A large body of contemporaneous literature shows how complex is decision-making facing radical uncertainty. Indeed, this process cannot be achieved using the Savage process based on a Bayesian conditional process.

Indeed, under radical uncertainty where a decision-maker's choice can be described by a subjective expected utility function, the preferences revision, conditional on the knowledge of a certain event realization is achieved simply by revising Bayes' rule and by leaving this function unchanged (Ghirardato (2002)). However, under radical uncertainty, revising preferences is more complex and can take different forms. The literature distinguishes at least four of them. Three were identified by Eichberger, Grant and Kelsey (2010) and a fourth by Dominiak and Lefort (2013). These are the following rules:

- The optimistic revision rule (Gilboa and Schmeidler (1993)),
- The pessimistic revision (Dempster (1968) and Shafer (1976)),
- The generalized Bayesian revision rule (Jaffrey (1992)),
- The neo-Bayesian constant revision rule by Dominiak-Lefort (2013).

Following Eichberger, Grant, Kelsey (2010) ${ }^{1}$, the first three rules affect the decision maker's perception of ambiguity. The first two affect the degree of optimism and the perception of ambiguity, while the third rule, the Bayesian generalized revision, keeps the degree of optimism constant. The fourth rule assumes the constancy of these two variables. Our approach, however, will not push further in this direction, that necessitate a preliminary better knowledge of the implications of uncertain information produced by unreliable sources. Therefore, this article will limit itself to analyzing the consequences of the uncertainty generated by these sources following the Ellsberg's (1961) example.

Sometimes, new information generates more uncertainty than it removes doubts. Then, when the source is undisputable, uncertainty may decrease (or increase) without questioning

[^1]the information reliability. However, when the source is questionable the information itself may generate further ambiguity even if the genuine information object were to reduce uncertainty. When, facing multiple information sources, new information can lead to increase the uncertainty level.

An updating of beliefs involves using updating rules, "that specify for any preference relation and for any event of a measurable partition of the set of states of the world, what is the preference relation once that event is known to have occurred" following Pires (2002 p. 139) words. The object of this paper is then to show that the lack of information sources reliability may negatively influence the agents' beliefs and add ambiguity. Another result is that the decision makers can be induced to prefer information coming from insecure information source rather than facing radical uncertainty.

To address this issue, we retain Ellsberg's (Ellsberg (1961)) example of two urns and two colors. There, the information about the distribution of balls in the first urn is undisputable. In this paper, we just add to the basic framework that the information source may be defective.

Several causes make information unreliable. Indeed, inaccurate information can arise either because the phenomenon is insufficiently known or from the source itself. In the first case, reliable sources can provide contradictory information. Sometimes, it is the information medium itself which is questionable (information sources financed by industrial or political lobbies for instance). Consequently, under some conjectures, the source is unable to provide an accurate information but only probable ones. Under others, even if the information is unambiguous, the source's inaccuracy leads to adding uncertainty to the existing ambiguity.

In this paper, this information will be, first, only probable (Section 2), and second, uncertain or ambiguous (Section 3). Regarding the first point under the Ellsberg scheme, in the first urn (urn 1) the proportion of balls is known with a given probability: the DM must arbitrate between two probable proportions. In the third section, the source of information itself that gives a probable distribution of balls is not fully reliable. Consequently, the information about the proportion of balls inside box 1 is either true with a certain probability or false. In this latter case, the ball the distribution is as unknown as in urn 2. The information source creates ambiguity. We show that in this situation, the DM may prefer to behave as if the information were certain. Thus, the DM prefers situations where the information comes from a low-quality source to no information at all. Ellsberg's paradox can also appear under this conjecture.

## 2. Arrival of contradictory information in the two urns Ellsberg's example.

### 2.1.Information and the standard two urns, two colors Ellsberg's example

Information reduces uncertainty. Nevertheless, sometimes it can be neutral of no help to solve ambiguity and even increase it. For example, consider a situation similar to Ellsberg's (1961) framework where the DM wins if she draws a red ball from one of two urns containing red and black balls. The difference with the usual example is that the proportion of red and black balls is unknown when considering each urn. Then, winning is random issue. The only sure information is that each box contains 100 balls. Now, suppose that the organizer gives information that the proportion in urn 1 is such that red and black balls are in equal proportion, $\left(R^{1}=B^{1}=50\right)$ where, respectively, $\left(R^{1}, B^{1}\right)$ stand for red and black balls in urn 1 The proportion is unknown for the second urn let $(\tilde{R}, \tilde{B})$ where the "tilde $\simeq$ " stands for unknown number (with $\left(R^{1}+B^{1}=\tilde{R}+\tilde{B}=100\right)$ ), all this as Ellsberg (1061).

In round one, drawing a red ball results in a $\$ 100$ payoff, and in the second one, drawing a black ball is winning. Ellsberg paradox appears because, in each turn, the DMs prefers choosing the box with the known proportion (urn 1) rather the one with the unknown proportion (urn 2). She does not associate her first choice with the next and she prefers to choose the urn with the known proportion. She thinks that the winning probability is higher than with urn 2, then, $\left(\operatorname{pr}\left(R^{1}\right)>\operatorname{pr}(\tilde{R})\right.$ with the issue that $\operatorname{pr}(\tilde{R})<1 / 2$. Here, $\operatorname{pr}\left(R^{j}\right)$ is the probability of drawing a red ball in the j -urn $(j=1, \check{\sim})$, and, the same applies for the black ball ( $\operatorname{pr}\left(R^{j}\right), j=1, \tilde{\sim}$ ). However, changing the bet terms (i.e. winning with the drawing of the black ball in the second run), leads the DM to also choose urn 1. This involves that she thinks that $\operatorname{pr}\left(B^{1}\right)>\operatorname{pr}(\tilde{B})$ then consequently, $\operatorname{pr}(\tilde{B})<1 / 2$. Putting together both choices, leads to consider that, $\operatorname{pr}\left(R^{1}\right)+\operatorname{pr}\left(B^{1}\right)>\operatorname{pr}(\tilde{R})+\operatorname{pr}(\tilde{B})$, with $\operatorname{pr}\left(R^{1}\right)+\operatorname{pr}\left(B^{1}\right)>1$ and $\operatorname{pr}(\tilde{R})+\operatorname{pr}(\tilde{B})<1$. The DM does not make the mental relationship that her first choice involves that, logically, she should admit that $\operatorname{pr}\left(B^{1}\right)<1 / 2$. In fact, the DM plays without memory, while the game allows to keep records.

In this example, information does not reduce uncertainty. Indeed, it guides the DM's first choice, but the second round reveals his aversion to ambiguity. So, as Ellsberg shows, uncertainty is added to uncertainty, and this is not considered by standard expected utility theory where the "Sure Thing" principle does not apply. Then, in spite of the information safeness about the proportions of balls in each box, aversion to ambiguity appears because the
first round brings no information. After the drawing of a ball in one of the urns, the game repeats with an interpolation in the winning conditions where a black ball must be drawn to win.

Obviously, if the information changes, such as "the proportion of red balls in urn 1 turns out to be 60 out of 100 instead of 50 ", the bet changes but, if the organizers keep the same game (two rounds with one draw and the same options each time), its nature remains equivalent to the previous setting. If the game is offered to several DMs, considering the second round, the change in proportion (40 black balls in urn 1) may lead to a decrease in the payoff compared to the first round with the draw of a red ball (60/100). Except for a drastic change in proportion, basically the game remains the same. The DM is confident in the reliability of the source and agrees to play the new rules.

### 2.2.Contradictory information from a reliable source

As before, suppose the organizers keep the same set, but now the information about the proportion in Box 1 is no longer univocal. Therefore, the organizers propose several proportions concerning Box 1 . We shall limit it to two.

### 2.2.1. Some example

For a better understanding, we propose a common framework corresponding to a decision-tree (Figure 1). This tree represents the two stages of the DM's choice under the Ellsberg's two colors example. The rounds are indexed by $k=1,2$. There, the payoffs written as $(100(0))$ means that in round $k$ the DM wins $\$ 100$ while, in turn $k^{\prime}$, the payoff is 0 when keeping the same choice.
$R_{j}^{1 k}, B_{j}^{1 k}$ are, respectively, the number of red, black balls in urn 1 , the index $j, j=1,2$, considering round $k, k=1,2$ refers to the number of balls proposed by the information source. For example, the information may say "Box 1 contains either $R_{1}^{1 k}$ red balls with a probability $\gamma$ or $R_{2}^{1 k}$ with a probability $(1-\gamma)$.
$\widetilde{R}, \tilde{B}$ are, respectively, the number of red, black balls in urn 2 , and their number is unknown (with $R_{j}^{1 k}+B_{j}^{1 k}=\widetilde{R^{\prime}} \widetilde{B}, j, j=1,2 ; k, k=1,2$.

## Insert Figure 1

## Example 1 (trivial):

The received information tells that urn 1 contains red balls either in proportion 50/100 (black balls $50 / 100$ ) with a probability of 0.8 or $60 / 100$ (black balls $40 / 100$ ) with a probability of 0.2 The conditions of the game are similar to the 1.1 case. This may be
admitted without further explanation. Indeed, the expected payoff $X$ in round 1 drawing one red balls in this urn is then:

$$
E\left(X \mid U^{1}\right)=0.8\left(\frac{50}{100}\right) \cdot 100+0.2\left(\frac{60}{100}\right) 100=52
$$

Here, the uncertainty bears on the arms of the information concerning the proportions, however, implicitly, the source is considered as safe even if it conveys some uncertainty.

Conclusion: As in the standard Ellsberg experiment, the DM chooses urn 1 (option 1) rather than urn 2 at round 1 . In round 2, in spite of the reduced number of black ball (interval between 40 and 50) most people should also choose urn 1 . Consequently, the Ellsberg's paradox can also appear.

## Example 2 (mixed information).

Now consider that the organizers are unable to tell whether urn 1 contains 100 red balls or 0 balls, on a fifty percent chance basis. So, if the DM chooses option 1 (urn 1), she may encounter either 100 red balls and win, or 100 black balls and lose. The proportion remains unknown until the second round. Obviously, this first round is informative. Indeed, she knows that winning lets no room to still choose the first urn when called to play for the second round. Indeed, there are no black balls in this urn. If she loses, then, in the second round she will choose urn 1 , being sure that the number of black balls is 100 . Here, contrary to the standard example of Ellsberg, the DM is compelled to keep the memory of his past choice. It is easy to see that, considering both rounds, the probability to win at least $\$ 100$ is equal to 1 .

## Example 3

Do things change, if the organizers, keeping the same proportion (100 red balls or 0 in urn 1) considers that the probability of having $\operatorname{Pr}\left(R^{1}=100\right)$ is no longer $(1 / 2)$ but $10 \%$ ? Then, in the first round, the DM must choose either urn 1 with a $10 \%$ chance of winning or take a ball from urn 2 with an unknown chance of winning. Again, the correct strategy is to play urn 1 in the first round, and then, in the second round, either urn 1 if it lost previously, or urn 2 if it won. In this case too, the sequence is informative.

## Example 4

In another scenario, we keep the $10 \%$ chance of winning in the first round by drawing a red ball, but with the information that their number could be 90 out of 100 with a probability of 0.1111 and 0 with a probability of $(1-0.1111=0.8889)$. Choosing the option of playing in urn 1 is less informative than before. Indeed, if the DM wins, she knows that 90 balls are red. Then, in the second round, she deduces that in urn 1, only 10 balls are black. Her chances of
winning are now only $10 \%$ in the face of unknown knowledge about urn 2 . Is the level of urn 1 sufficient to avoid moving to urn 2 ? This depends on subjective factors. Choosing urn 1 means that the probability of drawing a black ball in urn 2 is $P(\tilde{B})<P\left(B^{1}\right)=10 / 100$. Then, here, the question of where to locate aversion to ambiguity is raised.

Things are no better if she had lost. Thus, having drawn a black ball in the first round, she may wonder whether this is due to the $10 / 100$ black ball configuration or to the $100 \%$ black ball possibility in urn 1 . Obviously, this influences her second-round choice. Note that, here, there is an $8 / 9$ chance that urn 1 contains 100 black balls. So, choosing urn 1 seems fair. This should be compared to the winning situation in the first round where, after playing, she learned that $90 / 100$ balls are red, leaving $10 / 100$ black balls in urn 1 .

In conclusion, winning in the first round implies knowing that the probability of winning in the second round is only $10 \%$ of the chance by playing urn 1 . Losing, in this first round, means a strong incentive to play urn 1 for the reasons developed above.

### 2.2.2. Formal analysis

To deal with new information, we give a more formal argument.
Let $B_{j}^{1}, R_{j}^{1}$ be, respectively, the number of black and red balls in urn 1 following the information assumptions, $(j, j=1,2)$, then, $B_{j}^{1}+R_{j}^{1}=K$, (Note that the index $j$ means that inside urn 1, two possible proportions are noted $j=1,2$ ). Concerning urn 2 , this proportion is absolutely unknown.

Considering round 1 , the DM has the option to draw a ball in either urn 1 or urn 2 . She wins when the ball is red. In urn 2, the proportion between black and red balls is unknown while in the urn 1 this proportion is known. However, this knowledge is incomplete because there is a probability $\alpha$ that the proportion of black and red balls in urn 1 is, respectively:

$$
\frac{B_{1}^{1}}{K}, \frac{R_{1}^{1}}{K} \text { with } \frac{B_{1}^{1}+R_{1}^{1}}{K}=1
$$

And a probability $\beta$ (where $1 \geq \alpha \geq 0,1 \geq \beta \geq 0$ and $\alpha+\beta=1$ ) that the distribution of balls in urn 1 is different

$$
\frac{B_{2}^{1}}{K}, \frac{R_{2}^{1}}{K}, \frac{B_{2}^{1}+R_{2}^{1}}{K}=1 ; \frac{B_{1}^{1}}{K} \neq \frac{B_{2}^{1}}{K}, \frac{R_{1}^{1}}{K} \neq \frac{R_{2}^{1}}{K}
$$

We define the payoff expectations according turn $T_{z}, \quad z=1,2 \mathrm{E}\left(\mathrm{X} \mid T_{z}\right)$. As, in round 1 , drawing a red ball is a winning print, then, the expectation to win choosing urn 1 is then:

$$
\mathrm{E}\left(\mathrm{X} \mid T_{1}\right)=\alpha\left(\frac{R_{1}^{1}}{K} \cdot 1+\frac{B_{1}^{1}}{K} \cdot 0\right)+\beta\left(\frac{R_{2}^{1}}{K} \cdot 1+\frac{B_{2}^{1}}{K} 0\right)=
$$

$$
\begin{equation*}
\mathrm{E}\left(\mathrm{X} \mid T_{1}\right)=\frac{R_{2}^{1}}{K}+\alpha\left(\frac{R_{1}^{1}}{K}-\frac{R_{2}^{1}}{K}\right) \tag{1}
\end{equation*}
$$

By the same argument we can study the second round, keeping in mind that now, it is drawing a black ball that gives the payoff.

Then, as $(\beta=1-\alpha)$ :

$$
\begin{equation*}
E\left(\mathrm{X} \mid T_{2}\right)=\frac{B_{2}^{1}}{K}+\alpha\left(\frac{B_{1}^{1}}{K}-\frac{B_{2}^{1}}{K}\right) \tag{2}
\end{equation*}
$$

On the basis of (1) and (2) we compute the conditions that lead to the Ellsberg paradox, that means that conditions on $\alpha$ such that the DM does not get information from her drawing.

Proposition 1: Considering the Ellsberg frame with two balls and two colors, if after two rounds, the choice of the DM verifies the following two inequalities:

$$
E\left(X \mid T_{1}\right)>\frac{\tilde{R}}{K} \text { and } E\left(X \mid T_{2}\right)>\frac{\tilde{B}}{K}
$$

(Where $\tilde{R}, \tilde{B}$ are respectively the number of red and black balls in urn 2), then, these inequalities generate an Ellsberg paradox.

Proof: See appendix 1
These examples and Proposition 1 show that the information quality strongly influences the agents' beliefs. This uncertainty highlights that Ellsberg's two urns example can involve sequential learning (as for example in Epstein and Schneider (2007)) when information is uncertain but also, obviously, that it can give rise to the well-known Ellsberg paradox. Note that here, this section has only consisted of a refinement of the standard Ellsberg example. We have considered that the information source provides the proportion between red and black balls on a probable basis and not with certainty. If the paradox emerges on a strong information basis (the probability of the quality of the source is one), it is obvious that a weak information source cannot cancel out its probable occurrence.

## 3. Unreliable information source

Now we deal with the unreliable information source question. As before, the proportion of red and black balls remains unknown in urn 2 , this constitutes the only sure information. However, concerning urn 1, the information source is reliable with probability $\gamma,(1>\gamma \geq 0)$. The information that this unsure source reveals is that inside Box 1 , red balls
are in quantity $R_{1}$ and black balls $B_{1}$. This means that with a probability of $(1-\gamma)$ the proportion remains unknown in a similar way to urn 2.

### 3.1. Changes in the choice conditions

The information source notifies the DM of a specific proportion of red and black balls corresponding to a lottery $L_{1}$ such that ( $L_{1}=\left\{\$ 100, p=\frac{R_{1}}{K} ; 0,1-p=\frac{B_{1}}{K}\right\}$ ). However, as this source is reliable with a probability $\gamma,(1>\gamma \geq 0)$, and faulty with a probability of ( $1-$ $\gamma)$ ), this lottery's dual is a range of alternatives such that if $K$ is the total number of balls in urn 1 then, $\tilde{R}_{1}$ and $\tilde{B}_{1}$ are respectively the unknown number of red and black balls in urn 1 if the source is wrong $\left(\tilde{R}_{1}+\tilde{B}_{1}=K\right)$. Thus, the probability of drawing a red ball is unknown and is included in a set of possibilities corresponding to the potential lotteries $L_{k}$. Hence, saying that the information source is safe with a probability of $\gamma$ giving the lottery $L_{1}$ comes to say that this challenges the lottery $L_{k}$ that corresponds to $(K-1)$ potential loteries with a probability of $(1-\gamma)$. Concerning urn 1 , this comes to say that the probability to find a similar uncertainty to the one of urn 2 is $(1-\gamma)$. This situation is described in Figure 2 below that take the same notation that figure 1 , but concerning Urn 1, the alternative takes uncertainty rather than an alternative proportion.

## Insert Figure 2

Then, it follows that before choosing either urn 1 or urn 2), the DM must tell organizer whether she prefers randomly selecting an urn or if, on the contrary, she will not. Then, if the choice is random, this means that the DM will draw the ball following a kind of "head and tail" process. The non-random choice is the result of a conscious selection that involves that the DM is convinced that the information source credible.

Let us assume that the source tells the DM that inside box $1, R_{1}>B_{1}$. The reliability of the source is assessed to $\gamma,(1>\gamma \geq 0)$.

## Round 1 :

i) The DM considers the information as credible, then she chooses urn 1 and expects to win $\$ 100$ by drawing a red ball.
a. Suppose she wins.

What can infer the DM from this result for round two?
b. If she loses (drawing a black ball), may this draw tell her that the source of the information is wrong?
(Both situations i-a) and i-b) are studied with round 2 below).
ii) If the DM does not believe in the information source, she will choose randomly between urn 1 and 2 .

Round 2: Once again, the DM can choose between the two boxes. The previous draw may or may not have taught him lessons depending on the results obtained in the first round. These are as follows:

## Case i) a.

Getting a red ball does not mean that the information is correct. Indeed, inside the box, another proportion than the one given by the information could prevail, and the learning effect is null. Following Savage's sure thing principle, the DM must nevertheless consider that the information is correct and choose box 2 , because the information says that $R_{1}>B_{1}$, and logically, the chance to win are lesser than for the first round. Indeed, another choice amounts to invalidating the first one by assuming that the information is correct. Can the DM deny himself from one draw to the next? This would also be a violation of the continuity principle.

Case i) b. Drawing a black ball when a red ball was expected with a higher probability, does not put into question the belief that the information source is not credible. Then, for the second round, by the continuity axiom, the DM should choose urn 2 , because $R_{1}>B_{1}$.

Case ii) Here, the information was not considered credible at the outset and, therefore, the first draw was random. By continuity, the second draw will also be random.

### 3.2. Analysis in preference terms

We deal with this question by considering the DM's preferences formed on lotteries $L_{i},(i \in I)$ where $I$ is the number of lotteries got by the drawing of red and black balls in the conditions defined above and $\mathcal{L}$ is this set of lotteries, $\left(L_{i} \in \mathcal{L}\right)$. Consider then the preference relation " $\gtrsim$ " where " $\sim$ " is indifference and " $>$ " strict preference. We will assume that this relationship meets the following standard axioms:

Axiom 1 (weak-order): i), $\left(\forall i, j, L_{\mathrm{i}} \gtrsim L_{j}\right.$ or $L_{\mathrm{j}} \gtrsim L_{i}$ ), ii) reflexivity, $L_{\mathrm{i}} \gtrsim L_{i}$, iii) transitivity, $\left(\forall i, j, s, L_{\mathrm{i}} \gtrsim L_{j}, L_{\mathrm{j}} \gtrsim L_{s}\right.$ involves that $L_{\mathrm{i}} \gtrsim L_{s}$.

Axiom 2 (continuity, or the sure thing principle): $\gtrsim$ is continue, if for all lotteries, $L_{i}, L_{j}, L_{s}$ such that $L_{i}>L_{j}>L_{s}, \forall \alpha, \beta$ such that $\alpha, \beta \in[0 ; 1], \alpha L_{i}+(1-\alpha) L_{s}>L_{j}>$ $\beta L_{i}+(1-\beta) L_{s}$

Axiom 3 (independence): " $\gtrsim "$ is independent, if for lotteries $L_{i}, L_{j}, L_{s}, \quad L_{\mathrm{i}} \gtrsim L_{j}$ involves that $\forall \alpha, \alpha \in[0 ; 1], \alpha L_{i}+(1-\alpha) L_{s} \gtrsim \alpha L_{j}+(1-\alpha) L_{s}$

These axioms are standard here. Let also be $\gamma$ the probability of the information source reliability $(\gamma \in[0 ; 1]$ and $(1-\gamma)$ with $\gamma>1-\gamma)$. Then, if the DM considers that the information source is credible, she considers the lottery $L_{1}$ in her calculation as the basis of her present and further calculus. If, on the contrary, she thinks that the information is not credible, choosing urn 1 comes to a random choice and conventionally, this will correspond to the lottery $L_{k}$ where $L_{k} \sim L_{k^{\prime \prime}}$. If she considers the information accurate and if the lottery $L_{1}$ is favorable, then, for the first run, $L_{1}>L_{k}$, consequently, $\gamma L_{1}>(1-\gamma) L_{k}$, for $\gamma>1-\gamma$. We establish the proposition 2 :
Proposition 2: If the DM is confident in the information source, then, considering each round, she will choose each urn in a non-random way.

Proof: (see appendix 2)
The distrust in the information source is expressed in the following proposition where, by convention, $L_{k^{\prime \prime}} \cup L_{k}$ means that the DM apply a random choice between both options.
Proposition 3: If the DM prefers a random choice, this means that for each round:
(a) Round 1: $L_{k^{\prime \prime}} \cup L_{k} \gtrsim \gamma L_{1}+(1-\gamma) L_{k}$ with $L_{k^{\prime \prime}} \cup L_{k} \succ L_{1}$,
(b) Round 2: $L_{k^{\prime \prime}} \cup L_{k} \gtrsim \gamma L_{2}+(1-\gamma) L_{k}$ with $L_{k^{\prime \prime}} \cup L_{k} \succ L_{2}$.
(Where $L_{k^{\prime \prime}} \cup L_{k}$ means that the DM indicates to choose randomly between both urns for both rounds).

Then the DM considers the information source as unreliable.
Proof (see appendix 3)
Proposition 3 shows the conditions leading to accepting or rejecting the relevance of an information source based on the DM's preferences. The question is then whether there is a level of source reliability probability that determines the acceptance or rejection of the information source.

Proposition 4: A DM will choose to abide by the information from a given source if she believes its credibility to be greater than the level given by a prior probability $\gamma^{*}$. This comes saying that the information source reliability has a probability $\gamma, \gamma \in\left[\gamma^{*}, 1\right]$.
Proof: This results from the two propositions 2 and 3.
The last proposition means that the DM can lower the threshold $\gamma^{*}$ to the level that she considers as credible. The idea is that even a low-quality information source is better than no information at all. Consequently, the DM can define a subjective threshold from which she considers that the information provided by an unreliable source helps her forming her choice. Let $\gamma^{0}>0$, be the threshold of acceptable information quality for the DM with ( $\gamma^{0} \leq 1-$
$\left.\gamma^{*}\right)$. The DM will consider that the source does not provide sufficiently robust information compared to a random draw. These psychological factors go beyond the scope of the example given.

From proposition 4 we can then show that the individual who is averse to ambiguity will prefer information from an unreliable source to no information at all.
Proposition 5: According to axioms 1-3, a DM will always prefer the choices offered by a source of information with a low level of reliability to no information at all.

## Proof:

The proof needs considering a DM who distrusts an information source where the probability of its reliability is $\gamma^{0} \in\left[0, \gamma^{*}\right]$ and then, she prefers to choose randomly between both urns. Let us assume that the information source says that inside urn $1: \frac{R_{1}}{K}>\frac{B_{1}}{K}$. The proof follows several three steps:

Step 1: As the information is not considered as credible, from proposition 3:

$$
\begin{aligned}
& L_{k^{\prime \prime}} \cup L_{k}>L_{1}, \\
& L_{k^{\prime \prime}} \cup L_{k}>L_{2} .
\end{aligned}
$$

Choosing randomly can be made according to a device that follows a probability distribution $(1>\beta>0),(\beta, 1-\beta)$. Then, it follows that:

$$
\beta L_{k^{\prime \prime}}+(1-\beta) L_{k}>L_{1}, \text { and } \beta L_{k^{\prime \prime}}+(1-\beta) L_{k}>L_{2} .
$$

We know that when the information source is reliable :

$$
L_{1} \gtrsim \gamma L_{1}+(1-\gamma) L_{k}
$$

Because the source is not consistent, then:
For round 1: $\beta L_{k^{\prime \prime}}+(1-\beta) L_{k}>\gamma L_{1}+(1-\gamma) L_{k}$,
For round 2: $\beta L_{k^{\prime \prime}}+(1-\beta) L_{k} \succ \gamma L_{2}+(1-\gamma) L_{k}$
As $L_{k} \sim L_{k^{\prime \prime}}$, it follows that ( $L_{k^{\prime \prime}} \cup L_{k} \sim L_{k} \sim L_{k^{\prime \prime}}$ ), and:

$$
\begin{aligned}
& \beta L_{k}+(1-\beta) L_{k}>\gamma L_{1}+(1-\gamma) L_{k} \text { then, } \\
& \quad \beta L_{k^{\prime \prime}}+(1-\beta) L_{k^{\prime \prime}} \succ \gamma L_{2}+(1-\gamma) L_{k^{\prime \prime}} .
\end{aligned}
$$

Consequently :

$$
L_{k} \succ L_{1} \text { and } L_{k^{\prime \prime}} \succ L_{2} \text {. (A) }
$$

Step 2: Note that the result in (A) is reached without specifying the probability distributions about the information source accuracy. It means that the DM will always choose an urn randomly, whatever the urn considered.

However, such a situation raises questions because, if we consider the two complementary lotteries $L_{1}$ (in round 1) and $L_{2}$ (in round 2) (with ( $L_{1}=\left\{\$ 100, \frac{R_{1}}{K} ; 0, \frac{B_{1}}{K}\right\}$,
$L_{2}=\left\{0, \frac{R_{1}}{K} ; \$ 100, \frac{B_{1}}{K}\right\}$. The given information states that in urn $1: \frac{R_{1}}{K}>\frac{B_{1}}{K}$, then, the simultaneous preferences expressed by $L_{k}>L_{1}$ and $L_{k^{\prime \prime}}>L_{2}$ are not consistent.

Indeed, considering round 2 , the DM knows that the black balls proportion in urn 1 is lower than the red balls number. It follows that choosing randomly means that she considers that $L_{k^{\prime \prime}} \cup L_{k}>L_{2}$ or still, as ( $L_{k^{\prime \prime}} \cup L_{k} \sim L_{k} \sim L_{k^{\prime \prime}}$ ), $L_{k^{\prime \prime}}>L_{2}$. This comes to say that for round 2 , as $\frac{R_{1}}{K}>\frac{B_{1}}{K}$, then choosing urn 2 is a rational choice. However, this involves initially not having selected the random choice option.

Step 3: Furthermore, coming back to round 1, from (A) holding that $L_{k}>L_{1}$ constitutes a contradiction. Indeed, we know that the result in A, is reached independently from any probability consideration about the information accuracy. Then, as the information is that $\frac{R}{K}>\frac{B}{K}$, it follows that $L_{1} \succ L_{k}$. Indeed, a rational agent informed that $\frac{R}{K}>\frac{B}{K}$ in urn 1 will prefer it to urn 2 . This is translated by the following relationship

$$
L_{1}>\gamma L_{1}+(1-\gamma) L_{k^{\prime}}>L_{k^{\prime \prime}}
$$

Then, as $L_{k} \sim L_{k^{\prime \prime}}$, it follows that:

$$
\gamma L_{1}+(1-\gamma) L_{k}>L_{k} \text { and } L_{1}>L_{k}
$$

Step 4: Conclusion
It follows from step 2 and 3 and (A) that whatever the lack of reliability of the information source, the DM has interest to not opt for a random choice and follow the information source. QED

Proposition 5 means that, facing uncertainty, the DMs may prefer to dispose of information than no information at all. The credibility of the source is at stake, but even if the latter is disputable, the DM should behave as if this information was secure. This involves not choosing randomly between both urns.

Note 1: Note that if the proportions were reversed $\frac{B}{K} \geq \frac{R}{K}$, then, $L_{2} \succ L_{k^{\prime \prime}}$ and $L_{k^{\prime \prime}} \gtrsim L_{1}$.
Note 2: If $\frac{R}{K}=\frac{B}{K}$, then, some decision makers can express their aversion for ambiguity comforting the Ellsberg paradox with $L_{s}>L_{k^{\prime \prime}}$ et $L_{s^{\prime}}>L_{k^{\prime \prime}} . s, s^{\prime}=1,2$. This means that the lack of reliability of an information source is not a criterion for inducing people to prefer ambiguity.
Note 3: For $\frac{R}{K} \neq \frac{B}{K}$, it seems difficult to make the Ellsberg paradox appear.
Indeed, consider for instance $\frac{R}{K}>\frac{B}{K}$ and that drawing a red ball wins in the first round. Given this inequality, the DM will choose urn 1 . Whether the draw was a winner or a loser, for the
second round, the information indicates that the probability of drawing a black ball is less than $\left(\frac{1}{2}\right)$ i.e., $\left(\frac{B}{K}<\frac{1}{2}\right)$. Since the source of information is considered unreliable, the total probability of drawing a black ball in box 1 is $\left(\gamma \frac{B}{K}\right)<\frac{1}{2}$. The incentive to keep up choosing box 1 is then weak, and the DM will prefer to try urn 2 the one whose lottery is $L_{k^{\prime \prime}}$.

## 4. Conclusion

Acquiring information is fundamental to reduce uncertainty. However, this process does not guarantee that ambiguity vanishes, as the Ellsberg paradox shows. Indeed, information may constitute the disclosing factor of uncertainty aversion. It can both helping to know better a phenomenon (in its simplest version the proportion of balls in a box in one version of the Ellsberg paradox) without solving the uncertainty (the unknown proportion in the other one), and, however, this fact reveals the DM's attitude towards ambiguity.

This article studies the impact of information quality and specifically the consistency of its source, on the decision makers' choice. The reliability of information about the source is fundamental to making correct choices. The paper shows that, facing total uncertainty, individuals prefer to comply with information provided by an unreliable source than with absence of information. This result may help to understand the situations of total uncertainty when risks of significant damage (accident, illness, stress ) may occur or when there is no clear explanation of a given phenomenon. Thus, individuals in distress may rely on information whose origin may be particularly dubious, thus displaying aversion to uncertainty. This paper is built on the assumption that the source of information reliability is estimated at a known probability distribution. The question that arises is whether decisionmakers, when faced with a dubious source of information if confronted with an information source whose reliability cannot be estimated by a probability would consider it in her choice process. This question requires stronger hypotheses and another analytical framework than the one of paper and will be dealt further.

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## Appendix 1

## Proof of proposition 1:

1) By developing $E\left(\mathrm{X} \mid T_{1}\right)>\frac{\tilde{R}}{K}, E\left(\mathrm{X} \mid T_{2}\right)>\frac{\tilde{B}}{K}$

$$
\left\{\begin{array}{l}
\frac{R_{2}^{1}}{K}+\alpha\left(\frac{R_{1}^{1}}{K}-\frac{R_{2}^{1}}{K}\right)>\frac{\tilde{R}}{K} \\
\frac{B_{2}^{1}}{K}+\alpha\left(\frac{B_{1}^{1}}{K}-\frac{B_{2}^{1}}{K}\right)>\frac{\tilde{B}}{K}
\end{array}\right.
$$

And by summing both inequalities:

$$
\frac{R_{2}^{1}}{K}+\alpha\left(\frac{R_{1}^{1}}{K}-\frac{R_{2}^{1}}{K}\right)+\frac{B_{2}^{1}}{K}+\alpha\left(\frac{B_{1}^{1}}{K}-\frac{B_{2}^{1}}{K}\right)>\frac{\tilde{R}}{K}+\frac{\tilde{B}}{K}
$$

Note that $R_{1}^{1}+B_{1}^{1}=B_{2}^{1}+R_{2}^{1}=K$, (obviously, the same remark applies for the black balls), then:

$$
\frac{K}{K}+\alpha\left(\frac{K}{K}-\frac{K}{K}\right)>\frac{K}{K}
$$

And $1>1$ which is a contradiction.
We find again, the condition for getting the Ellsberg paradox when the information is uncertain.

## Appendix 2

## Proof of proposition 2

a) Beforehand, note that considering urn 1, if $L_{1} \sim L_{k}$, then $\gamma L_{1} \gtrsim(1-\gamma) L_{k}$, for $\gamma \geq 1-\gamma$. It follows that considering information as credible involves that $L_{1} \gtrsim$ $\gamma L_{1}+(1-\gamma) L_{k}$.
If $L_{k^{\prime \prime}}$ represents the unknown lottery of urn 2 (undefined proportions), then $L_{k}, L_{k^{\prime \prime}}$ corresponds to the lotteries of total uncertainty, then $L_{k} \sim L_{k^{\prime \prime}}$, and then by substitution:

$$
L_{1} \gtrsim \gamma L_{1}+(1-\gamma) L_{k^{\prime \prime}}
$$

It follows $L_{1}>L_{k^{\prime \prime}}$. Thus, the DM by choosing urn 1 expresses his aversion to ambiguity.
b) Now, if we consider the second round, $L_{2}$ represents the "known" lottery of urn 1 proposed by the same information source the reliability probability of which remains unchanged, $\gamma . L_{2}$ expresses that we change the winning color without changing the proportions of red and black balls in box 1 , the index 2 of $L_{2}$ refers to the second round. By the same argument, with $\gamma>1-\gamma$ where $(1-\gamma), \gamma, \in$ [ 0,1 ], then:
$L_{2} \gtrsim \gamma L_{2}+(1-\gamma) L_{k}$ and still : $L_{k} \sim L_{k^{\prime \prime}}$. This involves $L_{2} \gtrsim \gamma L_{2}+(1-$ $\gamma) L_{k^{\prime \prime}}$ and then, $L_{2} \succ L_{k^{\prime \prime}}$.

Thus, when the information comes from an unreliable source, if, however, the DM considers the latter as reliable, then we can find the conditions of the Ellsberg paradox. These correspond to the situation expressed above, i.e. :
$L_{2}>L_{k^{\prime \prime}}$ and $L_{1}>L_{k^{\prime \prime}}$
This writing allows us to characterize the conditions of information reliability on the two rounds. Indeed, if it turns out that :
i) For the first round, $L_{1} \gtrsim \gamma L_{1}+(1-\gamma) L_{k}$ with $L_{1}>L_{k^{\prime \prime}}$ and,
ii) For the second round, $L_{k} \gtrsim \gamma L_{2}+(1-\gamma) L_{k}$, with $L_{k^{\prime \prime}} \succ L_{2}$. This means that the DM preferred option 1 in the first round, and, informed by the drawing of a ball, he will prefer box 2 knowing that he has a better chance of winning in this case.
The same reasoning applies to the exact opposite situation:
For the first round, $L_{k} \gtrsim \gamma L_{1}+(1-\gamma) L_{k}$ with $L_{k^{\prime \prime}}>L_{1}$ and for the second one, $L_{2} \gtrsim \gamma L_{2}+(1-\gamma) L_{k}$, with $L_{2} \succ L_{k^{\prime \prime}}$. We find the Ellsberg's paradox as soon as :
i) Considering the first round, $L_{1} \gtrsim \gamma L_{1}+(1-\gamma) L_{k}$ with $L_{1}>L_{k^{\prime \prime}}$ et,
ii) And the second one: $L_{2} \gtrsim \gamma L_{2}+(1-\gamma) L_{k}$, with $L_{2}>L_{k^{\prime \prime}}$.

## Appendix 3

## Proof of proposition 3

Proof is provided in two steps:

1) Suppose that the DM is convinced that the source of information is reliable but that the option of choosing urn 1 is not favorable. Under these conditions, he will comply with the choice expressed by (a) above for the first round. If the choices are made for $(\gamma>1-\gamma)$ where $(1-\gamma), \gamma, \in[0,1]$, then, as $L_{k^{\prime \prime}}>L_{1}$, in the second round, if the DM considers that $L_{k^{\prime \prime}} \succ L_{2}$, this means that no alternative is profitable. This constitutes a contradiction. Indeed, if the information is credible, not choosing an option such as either $L_{1} \succ L_{k^{\prime \prime}}$ or $L_{2} \succ L_{k^{\prime \prime}}$, or both, is a breach of the independence axiom. It follows then that, for the DM, $\gamma$ is such that ( $\gamma \leq 1-$ $\gamma$ ).
2) If, the DM arbitrarily assigns to the number of balls in the urn 2 the thresholds $\frac{\tilde{R}^{1}}{K}$ and $\frac{\tilde{B}^{1}}{K}$, then a profitable alternative means that, for the first round, $\gamma \frac{R_{1}}{K}>$ $(1-\gamma) \frac{\tilde{R}^{1}}{K}$ or that, for the second round, $\gamma \frac{B_{1}}{K}>(1-\gamma) \frac{\tilde{B}^{1}}{K}$. Then, either $L_{1} \succ L_{k^{\prime \prime}}$, or $L_{2}>L_{k^{\prime \prime}}$, or ( $L_{1}>L_{k^{\prime \prime}}$ and $L_{2}>L_{k^{\prime \prime}}$ ). If it turns out that none of the three relations is verified, it means that $L_{k^{\prime \prime}} \succ L_{1}$ for the first round and $L_{k^{\prime \prime}} \succ L_{2}$ for the second one. This means that the DM will prefers relying on a random choice ( $L_{k^{\prime \prime}} \cup L_{k}$ ) marking a preference for ambiguity. This choice implies that the DM chooses randomly either box 1 or box 2 (for example by flipping a coin). This last choice means that the DM prefers to rely on a random choice rather than on the source of information, which is thus not considered reliable and ( $\gamma>0, \gamma \leq 1-$ $\gamma)$.

Figure 1: Decision Tree with known probable proportions in Urn 1


Figure 2 : Decision Tree with known probable or unknow proportions in Urn 1


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[^1]:    ${ }^{1}$ A more complete analysis can be found in Eichberger, Grant, and Kelsey (2007).

