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Insights into Dedekind and Weber's edition of Riemann's *Gesammelte Werke**

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Abstract

In this paper, I present and analyse Dedekind's and Weber's editorial work which led to the edition of Riemann's Gesammelte Werke in 1876. With several examples, I suggest that this editorial work is to be understood as a mathematical activity in and of itself and provide evidence for it.

1 Introduction: Dedekind reader of Riemann

After the untimely death of the mathematician Bernhard Riemann (1826-1866), according to his wishes, his colleague and friend Richard Dedekind (1831-1916) was given the responsibility of the edition his *Gesammelte Werke*. Dedekind decided (and convinced the other editors) to include, in the edition, unpublished texts extracted from Riemann's *Nachlass*. Dedekind first paired up with A. Clebsch who died unexpectedly in 1872. In November 1874, Dedekind contacted Heinrich Weber (1842-1913) to take charge of the rest of the edition. Riemann's *Werke* were finally published in 1876, with 22 unpublished manuscripts after years of protracted labor.

Dedekind and Riemann were students and *Privatdozenten* together in Göttingen during the 1850s. Both were students of Gauss, Wilhelm Weber and Lejeune-Dirichlet. They defended their *Habilitation* the same year, in 1854. Dedekind often expressed his deep admiration for Riemann's works. Upon meeting Dirichlet, Dedekind turned his attention towards algebraic number theory and focused on it for most of his career. Nevertheless, during their common years as *Privatdozent*, Dedekind followed Riemann's lectures on function theory and was profoundly influenced by Riemann. As is well-known, Dedekind often stated how he relied on the same epistemological and methodological principles as those which guided Riemann:

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It is preferable, as in the modern theory of functions, to seek to infer proofs immediately from fundamental characteristics, rather than from calculations, and indeed to construct the theory in such a way that it is able to predict the results of calculation. ([2], 92. Transl. slightly altered in [23])

Despite these epistemological affinities, Dedekind and Riemann worked in significantly different ways. Indeed, according to Dedekind, Riemann had a "brillant and profound thought, and an anticipatory imagination which led him to frequently take great steps which others could not follow so easily" ([20], 518-19). This characteristic of Riemann's works (also mentioned by Weber in [25]) means that while they were extremely powerful, they were also extremely difficult for Riemann's readers, especially because they could seem mathematically cryptic or vague. Dedekind, on the other hand, insisted on the importance of carefully and explicitly unfolding each step of the mathematical reasoning. This idea is effectively put into action in his works,¹ and is also applied to his reading of other mathematicians' works, in particular of Riemann's In 1854-58, Dedekind wrote dozens of pages of analytical researches on Riemann's *Habilitationsvortrag* ([18]), so as to better understand it.² To Weber, at the beginning of their correspondence (11.11.1874), Dedekind wrote:

I am not the profound expert on Riemann's work that you take me to be. I certainly know those works, and I believe in them, but I do not master them, and I will not master them until having overcome in my way, with the rigor that is customary in number theory, a whole series of obscurities. ([21], 50)

Such a difficulty, already significant in Riemann's published writings, becomes all the more pregnant in his unpublished manuscripts. Indeed, as Weber wrote in [25], "most parts of his *Nachlass* contain, in addition to formulae, only very few indications for establishing the context" ([25], 146). Thus, Dedekind and Weber's editorial work involved the tedious process of deciphering, clearing up and reconstructing Riemann's manuscripts. Most of the editorial work was made through letters, which have (for most of them) been preserved and recently published ([21]). Many of the notes written by Dedekind are also preserved in his *Nachlass.*³ Thus, we have a great amount of material to understand the editors' work, and to truly grasp the nature and extent of their editorial work. In particular, we can observe that during his work on Riemann's manuscript for the edition, Dedekind followed the same approach. Both his discussions with

¹In a letter to Cantor, from 29.08.1899, Dedekind confessed his "*Treppen-Verstand*" ([8], 261). The most striking of Dedekind's writings from that viewpoint is [4]. See also [7] and [13]. ²These researches were published in [22].

³The letters published by Scheel, and Dedekind's *Nachlass* are presently in the Niedersächsische Staats- und Universitätsbibliothek Göttingen (Cod. Ms. Riemann and Cod. Ms. R. Dedekind) and in the Archiv der UB Braunschweig (G 98:11-13). However, it has so far been impossible to locate Weber's *Nachlass* or ascertain what happened to it ([21], 16 and personal communications).

Weber and the notes left in his *Nachlass* show that he carefully unfolded the (often hidden) steps of Riemann's writings.

In the rest of this paper, I will show that Dedekind's and Weber's work as editors of Riemann was indeed an important part of their mathematical practice, and involved a deep mathematical work which went far beyond what is usually thought of as editorial duties.

2 Dedekind and Weber's epistolary editorial work for Riemann's *Gesammelte Werke*

In [25], Weber explains that Dedekind and he "only corrected a few slight imprecisions" which could be "regarded as certain", and that a few "additions, according to Riemann's hand-written remarks and necessary clarifications have been placed in finale notes" ([25], 145). For Dedekind and Weber, the preliminary works for the edition of Riemann's Werke lasted two years.⁴ The vast majority of Dedekind and Weber's editorial work can be found in the letters they exchanged from 01.11.1874 to the end of 1876 ([21]).⁵ In addition to the letters themselves, Dedekind and Weber also sent each other Riemann's manuscripts, their notes on the manuscripts, and as time passes, they start sending each other some of their personal works.⁶ These letters offer an invaluable testimony and an impressive, albeit not exhaustive, insight into the editorial work. Indeed, Dedekind and Weber exchange on all the aspects of the edition: selection of the manuscripts, any practical aspects related to Teubner, Riemann's widow, the publicity and royalties, their difficulties working with Riemann's manuscripts and their (not always successful) attempts to bypass them, the changes made on the texts - even typographical ones. In particular, Dedekind and Weber had to work with unpublished manuscripts, some of which were rough drafts. With such texts, their own very different ways of working play an important role in the editorial process. Indeed, Dedekind and Weber made a point of reading and understanding every manuscript in Riemann's Nachlass, and to publish all the writings they consider complete and interesting enough – and, for some of them, write explicative notes. Their editorial work thus implied not only a thorough selection of texts, but also proofreading Riemann's published works and understanding, clarifying and reconstructing his manuscripts.

 $^{^{4}}$ If one counts the work done by Clebsch and Dedekind earlier, the overall edition of Riemann's *Werke* lasted around five years.

 $^{{}^{5}}$ [21] also provides us with letters between the editors and the publisher Teubner, Elise Riemann, and the other scholars to whom Dedekind and Weber sent Riemann's texts. One counts 110 letters during this time. Among these 110 letters, 30 are written by Teubner, Elise Riemann, Hattendorff and Schwarz (most of them sent to Weber), 20 are written by Dedekind, 50 by Weber and 10 by his wife.

 $^{^{6}}$ These elements are not reproduced in [21], but Riemann's manuscripts and a good part of the material for the edition of his *Werke* are kept in Göttingen's university archive.

2.1 Exploration and selection of elements of Riemann's Nachlass

The edition of those of Riemann's writings published while he was still alive was finished by the time of Clebsch's death. By then, the main remaining task was the edition of the *Nachlass*. In his first letter to Weber, on 01.11.1874, Dedekind explains how he tried to work his way through Riemann's *Nachlass* and the edition of some unpublished texts. Dedekind was not able to identify and edit more than three of Riemann's manuscripts ("on trigonometric sequences, on the hypotheses of geometry and on electrodynamics"). The rest, he wrote, remained "in such a chaotic state, and only with great efforts could [he] reassemble a few fragments which, [he] believe[d], [were] worth publication" ([21], 43). When Weber accepts to become the main editor of Riemann's *Werke*, he receives the entirety of Riemann's *Nachlass* and starts "a precise examination of the manuscripts, including the ones which had been entrusted to other scholars." Weber's exploration of Riemann's *Nachlass* is more successful than Dedekind's. Weber nevertheless refers to Dedekind for advice on his choices.

The process of choice of the texts to be published is (sometimes lengthily) discussed in letters, allowing to understand the criteria. For Riemann's manuscripts to be published, they should be interesting enough, of course. They should be scientifically sound (and as correct as possible). They should be understandable - possibly after appropriate modifications. They should be representative of Riemann's researches and have a recognizable place in his overall intellectual work, that is, if a work does not present important new results but is significantly representative of Riemann's ideas or mathematical development, it should be considered for publication.⁷. Moreover, the texts published should offer a complimentary representation of Riemann – a point particularly important for the choice of the philosophical fragments -, and it is important to ensure that the texts published be well-received. Indeed, Dedekind and Weber pay attention to the fact that a text can be outdated, or on the contrary be welcome in the scientific or philosophical context of the publication (more than in Riemann's time). In this process, Dedekind and Weber impose their own criteria (of worthiness, interestingness, etc) on Riemann's texts, and their own idea of the image of Riemann that should be sent by the Werke. In fact, in many letters, as they work through Riemann's mathematical drafts, they correct mistakes or details which they consider should be corrected. Of course, many corrections are not about the result being wrong properly speaking. Nevertheless, they are noticeable (if not important) corrections. The letters provide a testimony which allows to see the work put into the edition, how important their contact with the texts was and how much they indeed modified the texts. Thus, we can get an idea of the extent to which their deep editorial work led to publish an adapted or even rewritten version of Riemann's texts.

 $^{^{7}}$ It is the case of [16]. See [21], 52.

2.2 Correcting Riemann's texts

The exchanges between Weber and Dedekind show which kind of corrections they discussed, and which corrections made it to print. From the necessity to abide by spelling reforms⁸ to the correction of possible (often small) mathematical mistakes, Dedekind and Weber discuss each modification they wish to inflect upon the texts. It is thus possible, through their letters, to follow the steps of their revisions and understand the grounds of their decisions

Let me just give one short but striking example chosen mainly for its clarity and the possibility to understand the changes made by Weber without entering too deeply into the mathematical content of the text. In letters from November 1875, Dedekind and Weber discuss Riemann's "Zur Theorie der Nobili'schen Farbenringe" ([19]) published in 1855 in the Annalen der Physik und Chemie. On 13.11.1875, Dedekind sends a letter to Weber in which he explains that he retrieved the offprint of the paper which was given to him by Riemann's widow in 1866. Having not looked at the paper for a long time, he was surprised to "find inside not only corrections, but also [some] sheets from Riemann's hand" ([21], 83). But Dedekind lacked time and a good knowledge of the subject to actually compare the two versions – in fact, he confessed to Weber, he never studied deeply this text for he was "frightened by the terrible formulae" ([21], 83). In his answer, on 14.11.1875, Weber confirmed the corrections made on the paper. A comparison between the 1855 and the 1876 versions shows about ten changes in the mathematical formulae. Following Riemann's indications in Dedekind's offprint of the paper, Weber corrected several mistakes, but decided not to include all adjustments proposed by Riemann. Indeed, in 1855, Riemann's gives formulae such as⁹

$$f(q) = e^{-2q} \sqrt{\frac{\pi}{4q}} \sum_{0,\infty} (-1)^m \frac{(1.3...\overline{2m-1})^2}{m!(16q)^m}.$$

In the revisions incorporated by Weber, the indexation was to be replaced by m < 4q + 1, stipulating an upper limit for m, about which Weber wrote:

The notation $\sum_{m < 4q+1}$ contains a stipulation on the limit for the convergence of the semi-convergent sequence, which I haven't had the time to prove rigorously yet.(...) I have transcribed this determination of the limit only in the formules for f(q), $\phi(q)$, while for

⁸Dedekind and Weber discuss, for example, wether they should keep Riemann's spelling of "willkhürlich" or use the up-to-date spelling write "willkürlich".

⁹For a point of the fluid layer, one denotes its horizontal distance from the cathode by r, and the height over the lower boundary surface by z. Riemann wants to determine u, the elevation of its voltage from voltage at the upper side of this boundary layer. One takes $z = \alpha$ at the lower surface and $z = \beta$ at the upper surface and $-\infty < r < \infty$. Sums are taken over m for the expansions into powers. In the course of the paper, Riemann considers b_n the general solution of an equation allowing to determine u''. He takes f(q) and $\phi(q)$ such that $b_n = c_n f(q) + \gamma_n \phi(q)$ (c_n , γ_n are constants) and considers the expansion by increasing powers of q for f and ϕ . To calculate f and ϕ , he proposes to use the half-converging series stated here.

the longer formulae which frightened you so much, I have suppressed [it] completely. ([21], 88)

In the version reproduced in Riemann's Werke, the formulae for f(q) and $\phi(q)$ are indeed transformed with the indexation indicated by Riemann. However, some of the following formulae, which required the same modification, become devoid of any indication about the values of m: neither the 0 to ∞ indicated in 1855, nor the m < 4q + 1 suggested later by Riemann.¹⁰ Weber decided to modify Riemann's formulae because "[i]n general, it is not necessary to write this formula in such a detailed manner" ([21], 88). He adds, to underline again the unnecessary character of this stipulation, that he "suspect[s] that Riemann did this for the physicists".¹¹

3 Riemann's "pale manuscript"

The correspondance between Dedekind and Weber shows us that the edition of *Fragmente über die Grenzfälle der elliptischen Modul-functionen* ([17]), which was a rough draft and was edited by Dedekind, was probably the most difficult one for Dedekind. The paper investigates certain properties of Jacobi series in elliptic function theory when the module of q converges towards 1. In a dozen of letters, Dedekind describes and complains about his difficulties. He even alludes to nightmares given by the manuscript, which he consistently refers to as the "pale manuscript" because the ink faded to the point of being barely readable. For Dedekind, the exchanges with Weber are, in 1876, the occasion to unfold his own work on Riemann's text and to ask Weber for advice. Dedekind is working with several pages, which include :

(...) a (four pages) retranscription of an extremely vague memoir (the other in-folio sheets contain partly only some known formulae of the theory of elliptic functions, partly something that I do not understand, maybe a research on the theory of series in which the logarithm of the general member of a function of the third degree is the index position?). (...) [I]t is the one that attracted my interest because the method found in it allows for an application of the transformation of first degree of the θ -function. I have, by chance, found the same afternoon, after many vain researches the days before, the theorems on continuity still missing by then $(\sum \frac{a_v}{v} = \sum \frac{a_v^0}{v})$.¹² ([21], 101)

 $^{^{10}}$ In particular, the formula for u does not contain any stipulation for the index.

 $^{^{11}}$ In [15], I discuss a few additional examples, notably one in which Dedekind and Weber argue over the possibility of a mistake in Riemann's doctoral dissertation.

 $^{^{12}}$ It is likely that Dedekind does not mean that he found the theorems in Riemann's notes, i.e. he does not mean that Riemann found these theorems before anyone else and never published them. Rather, Dedekind was able to derive, from Riemann's researches, the theorems which were unknown to Riemann.

Riemann's draft only contains formulae and no indications about them. Dedekind attempts to retrace Riemann's original work, understand his "original intent" and set up his own commentary for the Werke ([1]). While doing so, he is led to develop his own approach to the questions, and to rewrite some of Riemann's ideas in such a way that he is able to verify the formulae and trace Riemann's path - and maybe, it seems, the steps omitted by Riemann. In Dedekind's Nachlass, Cod. Ms. Dedekind XI 11 1-2 and XII 4 contain several hundreds of notes from Dedekind's hand for the edition of [17] and for his commentaries in both the 1876 and 1892 editions. Dedekind's notes reveal the mathematical exploration he conducted and illustrate in a strikingly clear fashion the mathematical practice engaged in the editorial process. In particular, for his own approach, he introduces what he calls "elliptic numbers".¹³ Dedekind undertakes many computations and comparisons between his approach and Riemann's so as to better verify the formulae. Hence, for example, in the second part of the paper, the third, fourth and eighth formulae given by Riemann correspond to what Dedekind calls "[his] third case":

Riemann's formula 3. (My case III.) $\mathscr{N}=2n$; $q_0^{2n}=-1$

$$\frac{q+q_0}{q-q_0} \cdot \frac{\pi^2}{16n^2} + \frac{1}{2n} \log \frac{q_0^{2n} - q^{2n}}{q_0^{2n} + q^{2n}} = \frac{\pi^2}{4\mathcal{N}^2} \left(-\frac{48\mathcal{N}^2}{\pi^2} \mathscr{A} \right) - \frac{1}{\mathcal{N}} \left(\log \frac{2}{\pi} - 2\mathscr{B} \right)$$

Does not correspond. For me : $-12\frac{\pi i}{24\mathcal{N}^2(\omega-\omega_0)} + 2\log 2$.

Riemann's formula 4. (My case III). Corresponds.

(...)

Riemann's formula 8. (My case III). $2n = \mathcal{N}$.

$$\log \frac{q+q_0}{q-q_0} = \log \frac{2\mathcal{N}}{\pi} - 2\mathcal{B}$$

For me only : $-2\mathscr{B}$. (Cod. Ms. Dedekind XI 11-1, 19)

It happened that the complete verification of Riemann's formulae turns out to be more complicated – if not impossible. Dedekind then computes numerical examples and verifies that the result found is indeed the same in all cases. Dedekind's researches on Riemann's formulae take many different forms: pages of computations, several versions of the drafts for his commentaries, summaries for each of Riemann's formula and summaries for each page of Riemann's text, unfolding of justifications, explanations, and in some cases clarifications on the context of each formula or step of the manuscript. Dedekind's archive also show how his researches to understand Riemann's manuscript, deepen his interpretation of the formulae and develop his commentary were extended into his own research. While Dedekind almost never published on elliptic functions,¹⁴ he did

¹³These numbers are used in the characterization of Θ_1 and verify a certain number of remarkable properties.

 $^{^{14}}A$ notable exception is [3].

mention to Weber that his work for the commentary was a fruitful one and that he "did not brag too much about the rightness of the elliptic numbers" ([21], 113). Dedekind's *Nachlass* also shows that the study of the "elliptic numbers" were deeply studied and that he did not stop at applying them to explaining aspects of [17]. Indeed, Dedekind studied many other possibilities offered by these numbers, and in particular their link to quadratic residues. This led Dedekind to try to transfer his researches to number theory. In Cod. Ms. Dedekind XI 14, for example, Dedekind re-writes Gauss' [10] by transferring the notions and notations used for and in Riemann's texts. He is able to find again a certain number of Gauss' results on quadratic residues. These researches are completely personal to Dedekind, and clearly show the extent and magnitude of the work displayed, and the depth of the mathematical study in which the editorial work is embedded.

4 Conclusion

Dedekind and Weber's editorial work was a very careful one – devoted, said Riemann's widow. It was essential for them to publish works which would be good for Riemann's standing, which would be interesting for the readers, relevant to his published works, true to his thought (and obviously correct). The choice of the texts to publish was thus a very crucial step. The second essential – and maybe even more crucial – step was that of the proofreading and, if necessary, correction of the texts to be published. As such, the edition of Riemann's *Werke* became an integral part of their mathematical practice during these two years. By 12.02.1876, Emilie Weber, Weber's wife, wrote to Dedekind:

I also believe that [Weber] cannot express how sincerely he thanks you for [the work done together] and I can't either. But I can at least tell you that he already has often endlessly spoken out on the fact that he could not have finished the work without your help, and could not say how much knowledge he has won through the edition, for which we also have you and you only to thank. ([21], 105)

This first collaboration around Riemann's works was the foundation of Dedekind and Weber's collaboration and their common reading of Riemann. Indeed, in [6], Weber and Dedekind propose an arithmetical rewriting of Riemann's function theory based on the transfer of the concepts and methods introduced by Dedekind in number theory,¹⁵ thus accomplishing Dedekind's wish to overcome the difficulties in Riemann's works "with the rigor that is customary in number theory".

 $^{^{15}}$ Dedekind and Weber transfer Dedekind's ideal theory as it is presented in [2]. See [12], [24], and [14].

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