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Class Differences and the Commercial Revolution: An Equilibrium Selection Story

Maurizio Iacopetta*

SKEMA Business School, Université Côte d'Azur (GREDEG)

and

Sciences-Po (OFCE), France

Abstract

The reopening of the Mediterranean routes in the tenth century sparked, in some regions of Europe, a long period of economic boom. It also triggered a social change whereby some members of the nobility, despite their social status, turned to commerce. I explain these events through the lens a Kiyotaki and Wright (1989) model, extended to a two-country world. The response of the elite to a communication shock causes the economy to transit from a low- to a high-production equilibrium, if pre-existing class differences are not too large. Quantitative experiments illustrate the view of economic historians that medieval expansion ensued in regions where the elite enjoyed relatively modest privileges and was slow in places, most notably France, where the elite's preoccupation for preserving the social-status was strong.

Keywords: Equilibrium Selection, Trade, Search.

JEL codes: C73, D83, O11

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1 Introduction

The reopening of the long-distance routes across the shores of the Mediterranean in the tenth century marked a turning point of European history, for it ushered a long period of economic development, usually referred as the Commercial Revolution.¹ The changes in the standards of living, however, have been uneven across European regions. Italy benefited the most, at least initially, from the new trade opportunities. Recent estimates suggest, for instance, that while in Northern and Central Italy average income increased by a factor of about 3.5 between the year 1000 and 1500, in France, as in most other European regions, over the same period it increased only by 50 percent (Bolt and van Zanden 2014; Maddison 2009).²

The social attitudes toward trade and production is a major factor in determining the extent to which a community benefits from the effects of an innovation. According to one view, in places where the social prestige for commerce was low, the wealthy elite focused on earning rents from land or other rent-seeking occupations, whereas trade and industry was relegated to the least fortunate people.³

In this article, I study how the frequency of international encounters and improvements in the production technology can transform a pre-industrial society. The idea that improved communication may induce a larger share of the population to direct its talent toward productive activities has already been suggested (see, for instance, Murphy, Shleifer and Vishny (1991)) but not yet formalized. This paper fills the gap by extending the search model of Kiyotaki and Wright (1989) to a two-country world. Originally constructed to show the emergence of a medium of exchange, this class of models has been applied to a variety of issues related to market frictions (see, among others, Araujo and Minetti, 2011, on the rise of medieval banks), and more recently to the dynamics of inequality (Iacopetta and Minetti, 2019, and Iacopetta, 2019). Differently than previous works, the analysis here focuses on the interaction between two economies that operate under similar Wicksellian triangles but that differ with respect to the degree of social stratification. All individuals produce and trade their products in decentralized markets. Agents can trade both domestically and internationally, albeit at different frequencies. For some, the commoners, trade is the only way of earning income. For others, the nobles, income may also come from rents. An agent can barter his production good against a consumption good. Nevertheless, he can also act as

¹Lopez (1976) dates the medieval Commercial Revolution between 950 and 1350. Cipolla (1993) calls the 1000-1300 period the Great Medieval Expansion.

²The available estimates, which are of a very rough variety, suggest no significant income difference across Europe in the year 1000, with the possible exception of regions under Byzantine or Arabic influence that enjoyed slightly better standards of living. For instance, Maddison (2009) reports about the same income for Italy and France, for the year 1000.

³See Baumol (1990), Murphy, Shleifer and Vishny (1991), and Acemoglu (1995) for an early discussion of this view.

middleman. An important question of the analysis is to understand under which conditions members of the nobility become middlemen and what consequences such development has on the equilibrium of the economy. The two-country set up allows me to be specific about the effects of an intensification of international encounters.

The analysis implies that an amelioration of long-distance routes can ignite a substantial rise of average income if social class differences are not too large. It also shows that a productivity shock can fuel a process of social class convergence and income expansion even in stratified society. These results have implications for more general debates about inequality, trade, and economic development, and all the more so because the literature has so often referred to the European historical paths to draw insights on the subject.

The paper discusses the ramifications of an integration shock through simulations that contrast the pre-industrial growth experiences of France and Italy. The transitional dynamics triggered by the shock are consistent not only with the medieval income divergence between these two countries, but also with their social dynamics. During the Middle Ages the upper classes in Italy gentrified and as a result the class barriers between members of the aristocratic and merchant families went down. Robert Lopez (1976, p. 66) writes, for instance, "The business fever, when it came, left almost no one untouched. Perhaps the most telling change occurred among those noble families that grew too large to live comfortably off their inherited land. Outside Italy, the supernumerary children tried to escape poverty and boredom through an ecclesiastic career, marriage to a noble heiress, or military service at somebody's hire. In the Italian towns, they more often found the same opportunities and thrills by pooling their capital in business ventures, which involved a chance of picking up along the way a fight with pirates, brigands, unfriendly lords, and possibly Infidels. ...Indeed it is usually difficult, in the scanty documents of the tenth and eleventh centuries, to tell apart merchants who had bought real estates with the profits of trade, and were called "honorable" or "noble", from the noblemen who had sold their estate, invested the proceed in trade, and married merchants' daughters."

Conversely, in France the debate about whether the rise of a commercial nobility should be favored or obstructed stretched up to the eve of the Industrial Revolution. Montesquieu, for instance, although favorable to the development of commerce, fiercely opposed the idea of a commercial nobility on the ground that such a transformation would have put in peril the stability of the monarchy. Influential intellectuals of his time, however, preoccupied by the economic and military rise of England, rebuked such a position.⁴

Although the analysis implies that social stratification was an important obstacle for the

⁴Montesquieu (1748) elaborates this point particularly in Tome 2, Book XX, Ch. 21, where he states "It is contrary to the spirit of monarchy to admit the nobility into commerce. The custom of allowing the nobility of England to trade is one of those things which has mostly contributed to weaken its monarchical government." Opponents published the pamphlet "Commercial Nobility" (*La Noblesse Commerçante*) that appeared in 1756 (Adam, 2003).

diffusion of the benefits of the Commercial Revolution, it also suggests that technological advances in the market-oriented sector eventually wins over the initial resistance to trade of at least a fraction of the upper classes. Therefore, the equilibrium in which commoners and nobility converge both in terms of income and commercial endeavours emerges even in a stratified society as long as this is open to adopt new production techniques. The implications of the model with respect to inequality dynamics appear in line with the historical evidence discussed by Alfani (2015, 2017), Alfani and Ammannati (2017), and Alfani and di Tullio (2019).

The following section links this work to the existing literature. Section (3) describes the economic environment. Section (4) shows how privileges affect the emergence of a particular steady state equilibrium. Section (5) proposes quantitative experiments to illustrate the propagation mechanism of a shock to the international matching rate, and to the state of the production technology. Section (6) relates the implications of the experiments to stylized facts of the medieval European expansion and reviews the resistance of the French aristocracy to trade. Section (7) concludes. An Online Appendix contains all the analytical proofs and an extension of the model applied to migration.

2 Literature Review

This work contributes to the literature that views the economic rise of Western Europe interconnected with its social transformation (Baumol 1990, Murphy, Shleifer and Vishny 1991, Acemoglu 1995, Galor and Weil 2000, Clark 2007, Greif 2006, Doepke and Zilibotti 2005 and 2008, Puga and Trefler 2014, Chu, Fan, and Wang, 2020). Greif (2006) has stressed the importance of game-theoretic analysis to interpret the human behavior in trade in a variety of historical settings. In this work, as in Doepke and Zilibotti (2008), but differently than in Galor and Weil (2000) and Clark (2007), the social attitude toward commercial activities is the result of a conscious intertemporal decision. Unlike some of this literature, however, I keep preferences, work habits, and patience fixed from one generation to the next. In addition, differently than Doepke and Zilibotti (2005, 2008) in my theory – motivated by historical episodes prior the Industrial Revolution – the nobility can become an important agent of change.

The analytical framework I use comes from the search literature pioneered by Kiyotaki and Wright (1989). This class of models originally developed to study the emergence of money has been widely used to study frictions in trade transactions, the institutional responses to them (see for instance Araujo and Minetti, 2011, for the rise of banks) and more recently the dynamics of inequality (Iacopetta and Minetti, 2019, and Iacopetta, 2019).

This paper provides a complementary insight to works, inspired by the pioneering study

of Max Weber (1978 [1922]) *The City*, that attribute Europe’s economic rise to the advent of politically autonomous cities. Some have argued that the city-states shielded their citizens from the ambitions of princely rulers (DeLong and Shleifer 1993, Bosker, Buringh and van Zanden 2013, Cantoni and Yuchtman 2014) and that they favored the accumulation of social capital (Guiso, Sapienza, and Zingales, 2016). Others, however, have observed that the presence of guilds in these cities created an important entry barrier to new enterprises that was eventually detrimental to their long-run prosperity (Stasavage 2014, Puga and Treffer 2014; for an overview see Ogilvie 2011).

3 The Environment

The model economy is an extension of Kiyotaki and Wright (1989), henceforth KW, to a two-country world. The extension aims at investigating the repercussion of an intensification of long-distance trade. In the KW framework, an individual may find optimal to participate in *indirect* trade, namely, in trading away the good produced for a commodity other than her preferred consumption good. Indirect trade becomes *speculative* trade when it is done out of liquidity considerations, that is, when it is motivated by the prospect of expanding the flow of consumption through the higher marketability of the commodity acquired through indirect trade. The extension of KW to international trade is useful in formalizing the popular view among economic historians that speculative trade greatly contributed to the fortune of once poor regions, such as Venice. In the two-country environment formalized below, individuals, in addition to being matched with their compatriots, are also matched with foreigners for possible trade.

An important departure from the KW framework is that individuals do not face any storage cost for holding commodities. In deciding whether to engage in indirect trade, in KW an agent compares the liquidity benefits that such a trade may generate, with the difference in storage costs between the commodity she currently holds and the one she is offered. In the current framework, the agent compares the liquidity benefits against the prospect of losing a fraction of the flow of rents. The utility cost of engaging in speculative trade captures a well documented social norm of pre-modern societies: being involved in trade and industry devalued the social-status of the aristocracy (see Baumol, Ch. 5, 2002).⁵ Scorning industry and commerce had the obvious drawback of giving up gains from trade, that, with the intensification of the Mediterranean routes, were growing in importance.

If in KW the difference in storage costs across the three goods conditions the emergence

⁵Several authors have emphasized the role of social-status seeking in development. In a recent work, Chu et al. (2020) study the long run effects of social status in a Schumpeterian growth model. For a recent survey on the subject see Heffetz and Frank (2011).

of a particular equilibrium, here the distribution of the flow of rents across types of agents plays that role. Specifically, there are three types of individuals. Two of them, to whom I refer as nobility or elite, earn a positive flow of rent. A third one, to whom I refer as commoners, does not earn any rent. Depending on technological and market conditions – the matching rate, the marketability of goods, and the size of the market – members of the elite may engage in speculative trade.

Before turning to the description of the model, it is worth mentioning what it is *not* in there: fiat money. Of the two environments studied by KW, with and without fiat money, I extend the one without fiat money. Fiat money had not yet appeared in the Middle Ages, at least not of the kind studied by KW. While inside money, i.e. notes that represented or were backed up by private credit, started to circulate with the rise of medieval banks, outside money, i.e. notes issued by government authority unbacked by assets of the monetary authorities, appeared much later – some argue it did in the 1970s with the abandonment of the gold-standard (see among others, Goetzman 2016, Ch. 22).⁶

3.1 The Model

The economy is populated by a continuum of infinitely-lived agents with unit mass. Agents are divided into two countries, Home and Foreign. The size of the Home population is $n \in (0, 1)$, and that of the Foreign population is $n^* = 1 - n$. There are three types of agents in each country. The measure of Home and Foreign type- i individual is μ_i and μ_i^* , respectively, so that $\sum_{i=1}^3 \mu_i = n$ and $\sum_{i=1}^3 \mu_i^* = 1 - n$. A Home (Foreign) individual of type i enjoys a rent $r_i \geq 0$ ($r_i^* \geq 0$). Specifically, the Home type 1 and type 3 agents earn a positive rent, whereas type 2 agents do not: $r_1 > 0$, $r_3 > 0$, and $r_2 = 0$. A similar specification of rents is assumed for the Foreign country. Sometimes I will refer to type 2 agents as commoners – I think of them as the medieval merchants and artisans living in urban centers, with no feudal obligations and with no economic support from the local lord – and to type 1 and 3 individuals as elite or nobility. While the earnings of type 2 agents derive exclusively from production and trading, type 1 and type 3 agents have also an endowment that generates a constant (and exogenous) flow of rents. Rents capture, in reduced form, a variety benefits

⁶The reader interested in a formal extension of the KW to a two-country world with fiat money can look at Matsuyama et al. (1993). Differently than Matsuyama et al. (1993), in the present work, the patterns of trades in commodities characterize the emergence of particular equilibria. In Matsuyama et al. (1993) there is no indirect trade. An individual first trades her production good for fiat money, and then fiat money for her consumption good. The main objective of Matsuyama et al. (1993) is to understand the circumstances that favor the acceptance of foreign vs. domestic fiat currency. This paper, instead, focuses on the circumstances that lead individuals to engage in indirect trade.

and privileges that the elite enjoyed in virtue of its position, namely compulsory services rooted in legal and social norms that individuals attached to feuds owed to their lords.

In either region a type- i agent consumes good i and produces good $i + 1$, (modulo 3). The unit cost of production, in terms of utility, is d and d^* in the Home and Foreign country, respectively. To keep things simple, any agent can hold one and only one good at a time. Agents do not pay storage costs. Nevertheless, when type- i agent holds good $i + 2$, which is a commodity other than her production or the consumption good, she has a disutility of $\delta_i r_i$ – the social-status cost of being involved in indirect trade.

Time is continuous and extends from zero to infinity. Matching occurs in the usual way it does in a continuous time set up. The matching technology, which is similar to the two country-world of Matsuyama et al. (1993), is summarized in table (1). A pair of agents is randomly and uniformly chosen from the population to meet for a possible trade according to a Poisson process. The matching technology α governs the domestic frequency of matching and, without loss of generality, is assumed to be the same in the two countries. The average waiting time for a Home agent to run into another Home agent is $\frac{1}{\alpha n}$, and to run into a Foreign agent is $\frac{1}{\beta \alpha (1-n)}$. The corresponding expressions for a Foreign agent are $\frac{1}{\alpha (1-n)}$ and $\frac{1}{\beta \alpha n}$, respectively. The parameter $0 < \beta < 1$ measures the relative frequency of international meetings. This means that a pair of agents who live in two different countries meet less frequently than a pair of agents who live in the same country. The parameter β measures the degree of integration between the two economies. Observe that an increase in β does not reduce the odds that a domestic agent meets a compatriot. A greater integration between the Home and the Foreign economy will be represented through a rise in β .

A bilateral trade occurs if, and only if, it is mutually agreeable. Agent i always accepts good i , but never holds it, as he immediately consumes it upon reception. Therefore, agent i always enters the market with either one unit of good $i + 1$, or one unit of good $i + 2$.

An individual i may find it convenient to trade away good $i + 1$ against the commodity $i + 2$, namely to accept in trade a good that he does not consume with the intent to trade it in the future for good i . I refer to such type of trade as indirect trade, and to the individual who takes part into it as middleman. The profitability of such a strategy depends on two considerations. One is a "liquidity" consideration, namely the comparison of the marketability of good $i + 1$ vs. good $i + 2$. If type agent i expects to obtain his consumption good sooner when holding good $i + 2$ than when holding good $i + 1$, he may be tempted to trade good $i + 1$ for $i + 2$ when such an occasion arises. The other is the cost of going through such a trade. Home agent i incurs a per-period cost of δr_i when he is in the position of middleman. A similar situation exists in the Foreign country.

The proportion of Home and Foreign type- i agents that hold good j at time t is denoted

by $p_{i,j}(t)$ and $p_{i,j}^*$ respectively. Since $p_{i,i}(t) = p_{i,i}^*(t) = 0$

$$p_{i,i+1}(t) + p_{i,i+2}(t) = \mu_i, \quad (1)$$

and

$$p_{i,i+1}^*(t) + p_{i,i+2}^*(t) = \mu_i^* \quad (2)$$

for any $t > 0$. Therefore, the state of the economy can be represented in a more parsimonious way by $\mathbf{p}(t) = \{p_{1,2}(t), p_{2,3}(t), p_{3,1}(t), p_{1,2}^*(t), p_{2,3}^*(t), p_{3,1}^*(t)\}$. Assume that Home (Foreign) agents of the same type i make the same choice s_i (s_i^*). I indicate with $s_i(t) = 1$ agents i decision in favor of trading $i+1$ for $i+2$, and with $s_i(t) = 0$ the decision against such a trade. The corresponding choices by the Foreign agents i are $s_i^*(t) = 1$ and $s_i^*(t) = 0$, respectively. The stock of good $i+1$ held by Home agents of type i evolves according to (the time index is dropped):

$$\begin{aligned} \dot{p}_{i,i+1} = & \alpha\{p_{i,i+2}[p_{i+1,i}(1 - s_{i+1}) + p_{i+2,i} + p_{i+2,i+1}(1 - s_i)] - p_{i,i+1}p_{i+1,i+2}s_i\} + \\ & + \beta\alpha\{p_{i,i+2}[p_{i+1,i}^*(1 - s_{i+1}^*) + p_{i+2,i}^* + p_{i+2,i+1}^*(1 - s_i)] - p_{i,i+1}p_{i+1,i+2}^*s_i\}. \end{aligned} \quad (3)$$

The terms inside the brackets before the minus sign in the first row calculate the probability that a Home type- i agent is called for a match with a compatriot while holding good $i+2$ and ultimately holds good $i+1$. Either such an event materializes because of barter or because the agent leaves the meeting with good i , consumes it, and then immediately produces good $i+1$. The expression that follows accounts for the probability that a Home agent of type i , who holds good $i+1$, ultimately has good $i+2$, after meeting with a compatriot. The terms in the second row capture the outcome of international meetings. A similar expression to (3) holds for the Foreign region:

$$\begin{aligned} \dot{p}_{i,i+1}^* = & \alpha\{p_{i,i+2}^*[p_{i+1,i}^*(1 - s_{i+1}^*) + p_{i+2,i}^* + p_{i+2,i+1}^*(1 - s_i^*)] - p_{i,i+1}^*p_{i+1,i+2}^*s_i^*\} + \\ & + \beta\alpha\{p_{i,i+2}^*[p_{i+1,i}^*(1 - s_{i+1}^*) + p_{i+2,i}^* + p_{i+2,i+1}^*(1 - s_i^*)] - p_{i,i+1}^*p_{i+1,i+2}^*s_i^*\}. \end{aligned} \quad (4)$$

The behaviors of $p_{i,i+2}$ and $p_{i,i+2}^*$ are derived through (1) and (2). Observe that (3) and (4) describe the evolution of commodities for any arbitrary set of strategies $\mathbf{s} = (s_1, s_2, s_3, s_1^*, s_2^*, s_3^*)$, not necessarily for an equilibrium one.

The level of production of the ensemble of type- i Home agents, Y_i , is equivalent to number of matches in which type- i agents exit meetings holding commodity i , namely,

$$\begin{aligned} Y_i = & \alpha\{p_{i,i+1}(p_{i+1,i} + p_{i+2,i}s_{i+2}) + p_{i,i+2}[p_{i+1,i}(1 - s_{i+1}) + p_{i+2,i}]\} + \\ & \beta\alpha\{p_{i,i+1}(p_{i+1,i}^* + p_{i+2,i}^*s_{i+2}^*) + p_{i,i+2}[p_{i+1,i}^*(1 - s_{i+1}^*) + p_{i+2,i}^*]\}. \end{aligned} \quad (5)$$

Type- i agents' aggregate income can be calculated by summing their rents to the value of production:

$$I_i = Y_i + p_{i,i+1}r_i + p_{i,i+2}r_i(1 - \delta_i).$$

The Home country's aggregate flow of production and income is $Y = \sum_i Y_i$ and $I = \sum_i I_i$, respectively. Similar production and income expressions, Y_i^* and I_i^* , can be derived for the Foreign country.

The choice of a particular individual i depends on the differential value between holding commodity $i + 1$ and $i + 2$: $\Delta_i \equiv V_{i,i+1} - V_{i,i+2}$.⁷ Denoting with $\tilde{\sigma}_i(\tau; \mathbf{s}(\tau), \mathbf{p}(\tau))$ the best response profile of strategies of a particular Home agent i to other players' strategies $\mathbf{s}(\tau)$, it must be that along the pattern of inventories $\mathbf{p}(\tau)$

$$\tilde{\sigma}_i(\tau; \mathbf{s}(\tau), \mathbf{p}(\tau)) = \begin{cases} 1 & \text{if } \Delta_i(\tau) \leq 0 \\ 0 & \text{if } \Delta_i(\tau) > 0 \end{cases} \quad (6)$$

for any $\tau \geq t$. The best response function $\tilde{\sigma}_i^*(\tau; \mathbf{s}(\tau), \mathbf{p}(\tau))$ in the Foreign country satisfies the same properties of (6). Thus, the formulation of the problem corresponds to a Markov decision process in which the representative agent optimizes over a sequence of functions, $s_i(\cdot)$.

Given an initial distribution of inventories, $\mathbf{p}(0) = \mathbf{p}_0$, a Nash Equilibrium is a path of strategies $\hat{s}(t)$ together with a distribution of inventories $\hat{\mathbf{p}}(t)$ such that for all $t > 0$:

- i. $\hat{\mathbf{p}}(t)$ and $\hat{s}(t)$ satisfy the dynamics equations (3)-(4) with the initial condition $\hat{\mathbf{p}}(0) = \mathbf{p}_0$ and subject to the constraints (1)-(2);
- ii. Every agent maximizes his or her expected utility, given the strategy profiles of the rest of the population; and
- iii. $\tilde{\sigma}_i(t; \hat{\mathbf{s}}(t), \hat{\mathbf{p}}(t)) = \hat{s}_i(t)$, and $\tilde{\sigma}_i^*(t; \hat{\mathbf{s}}(t), \hat{\mathbf{p}}(t)) = \hat{s}_i^*(t)$.

4 Steady States and Stability

This section discusses the existence of steady state Nash Equilibria, their stability properties, and how the emergence of a particular Nash equilibrium depends on the level of rents and on the degree of integration between the two economies. To keep things simple, I focus on the case in which agents are equally divided across the three groups: $\mu_i = \frac{n}{3}$ and $\mu_i^* = \frac{1-n}{3}$.

⁷The Online Appendix, Section A.1, derives the expressions for $V_{i,i+1}(s)$ and $V_{i,i+2}(s)$.

4.1 Steady State Nash Equilibria

The steady state of the distribution of commodities can be derived through Eqs. (3)-(4) by setting $\dot{p}_{i,j} = \dot{p}_{i,j}^* = 0$. For such a steady state to be a Nash equilibrium, the set of strategies \mathbf{s} has to be coherent with the sign of Δ_i and Δ_i^* . From now on, I will often indicate the strategies $\mathbf{s} \equiv (s_1, s_2, s_3, s_1^*, s_2^*, s_3^*)$ in a more intuitive way as $(s_1, s_2, s_3) - (s_1^*, s_2^*, s_3^*)$. After some derivations one obtains that Δ_i and Δ_i^* have the same signs of the following expressions:

$$\tilde{\Delta}_i \equiv s_{i+1}p_{i+1,i} - p_{i+2,i}(1 - s_{i+2}) + \beta[s_{i+1}^*p_{i+1,i}^* - p_{i+2,i}^*(1 - s_{i+2}^*)] + \frac{\delta_i r_i}{u\alpha}, \quad (7)$$

and

$$\tilde{\Delta}_i^* \equiv s_{i+1}^*p_{i+1,i}^* - p_{i+2,i}^*(1 - s_{i+2}^*) + \beta[s_{i+1}p_{i+1,i} - p_{i+2,i}(1 - s_{i+2})] + \frac{\delta_i^* r_i^*}{u^*\alpha}. \quad (8)$$

respectively. The expressions in 7 indicate that there are two kinds of reasons Home type i considers when trading $i + 1$ for $i + 2$. One is the relative marketability of the two goods in the domestic market, $s_{i+1}p_{i+1,i} - p_{i+2,i}(1 - s_{i+2})$, and on the international market, $\beta[s_{i+1}^*p_{i+1,i}^* - p_{i+2,i}^*(1 - s_{i+2}^*)]$. For instance, with a set of strategies (0,1,0)-(0,1,0), type 1's odds of consuming when called for a trade in the domestic market is $p_{2,1}$ if carrying good 2 and $p_{3,1}$ if carrying good 3 and when participating into the international market, $p_{2,1}^*$, and $p_{3,1}^*$ respectively. The other is the cost of engaging in indirect trade, normalized for the matching rate, α and the net utility u . If the difference in marketability is not sufficiently large to compensate for the cost of indirect trade, then the best strategy for type 1 individuals is to set $s_1 = 0$.

Next, I consider the conditions for emergence of a (0,1,0)-(0,1,0) Nash equilibrium – only type 2 individuals (commoners) do indirect trade. Then I look into the conditions that can shift the economy into a new equilibrium in which the other two types (the elite) in the Home or Foreign country do indirect trade. It turns out there are only three Nash equilibria where one of the two types that earn a positive rent engage into indirect trade: (1,1,0)-(0,1,0), (0,1,0)-(1,1,0), and (1,1,0)-(1,1,0). Below is a summary of the conditions for the existence of the four Nash equilibria mentioned above and their stability properties (for more details see the Online Appendix). In the Propositions that follow the $p_{i,j}$ are evaluated at their steady state values.

Proposition 1 *A (0,1,0)-(0,1,0) Nash steady state equilibrium exists if*

$$p_{3,1} - p_{2,1} + \beta(p_{3,1}^* - p_{2,1}^*) \equiv \frac{1}{6}[n + (1 - n)\beta] < \frac{\delta_1 r_1}{\alpha u} \quad (9)$$

and

$$p_{3,1}^* - p_{2,1}^* + \beta(p_{3,1} - p_{2,1}) \equiv \frac{1}{6}(1 - n + n\beta) < \frac{\delta_1^* r_1^*}{\alpha u^*}. \quad (10)$$

The distributions of commodities in the Home and Foreign country are

$$\mathbf{p}^H = \frac{n}{3} [1, \frac{1}{2}, 1] \quad (11)$$

and

$$\mathbf{p}^F = \frac{1-n}{3} [1, \frac{1}{2}, 1], \quad (12)$$

respectively.

Proof. See Appendix ■

Proposition 2 A $(1,1,0)$ - $(1,1,0)$ Nash steady state equilibrium exists if

$$p_{3,1} - p_{2,1} + \beta(p_{3,1}^* - p_{2,1}^*) \equiv [n + \beta(1-n)] \frac{\sqrt{2}-1}{3} > \frac{\delta_1 r_1}{\alpha u} \quad (13)$$

and

$$p_{3,1}^* - p_{2,1}^* + \beta(p_{3,1} - p_{2,1}) \equiv (1-n + \beta n) \frac{\sqrt{2}-1}{3} > \frac{\delta_1^* r_1^*}{\alpha u^*}. \quad (14)$$

The distributions of commodities in the Home and Foreign country are

$$\mathbf{p}^H = \frac{n}{3} [\frac{1}{\sqrt{2}}, \sqrt{2}-1, 1], \quad (15)$$

and

$$\mathbf{p}^F = \frac{1-n}{3} [\frac{1}{\sqrt{2}}, \sqrt{2}-1, 1], \quad (16)$$

respectively.

Proof. See Appendix ■

Proposition 3 A $(1,1,0)$ - $(0,1,0)$ Nash steady state equilibrium exists if

$$p_{3,1} - p_{2,1} + \beta(p_{3,1}^* - p_{2,1}^*) > \frac{\delta_1 r_1}{\alpha u} \quad (17)$$

and

$$p_{3,1}^* - p_{2,1}^* + \beta(p_{3,1} - p_{2,1}) < \frac{\delta_1^* r_1^*}{\alpha u^*}, \quad (18)$$

The distributions of commodities in the Home and Foreign country are

$$\mathbf{p}^H = [p_{1,2}, p_{2,3}, \frac{n}{3}], \quad (19)$$

and

$$\mathbf{p}^F = \frac{1-n}{3} [1, p_{2,3}^*, 1] \quad (20)$$

respectively, where $p_{2,3}^* = \frac{\frac{1-n}{3} + \beta p_{1,2}}{\frac{2(1-n)}{3} + \beta (p_{1,2} + \frac{n}{3})}$, and $p_{1,2}$ and $p_{2,3}$ satisfy

$$p_{1,2} = \frac{n}{3} \frac{\frac{n}{3} + \beta \frac{1-n}{3}}{p_{2,3} + \beta p_{2,3}^* + \frac{n}{3} + \beta \frac{1-n}{3}}$$

and

$$p_{2,3} = \frac{\frac{n}{3} + \beta \frac{1-n}{3}}{\frac{n}{3} - p_{2,3}} p_{2,3} - \beta \frac{1-n}{3}.$$

where $p_{2,3}$ and $p_{2,3}^*$ are evaluated at their steady state values.

Proof. See Appendix ■

The characterization of the (0,1,0)-(1,1,0) mirrors that of Proposition 3.

4.2 Taking Stock: Economic Integration and Rents in Equilibrium Selection

To fix ideas about how economic integration and the level of rents affect the emergence of a fundamental of speculative set of strategies in the Home country, consider first the special case of no integration, with $\beta = 0$. In this scenario the type of equilibria that emerge in the Home country are directly comparable with those of KW, Model A. Setting the matching parameter $\alpha = 1$, the condition (9) of Proposition (1) that yields the (0,1,0) equilibrium in the Home country simplifies to

$$(p_{3,1} - p_{2,1})u = \frac{n}{6}u < \delta_1 r_1,$$

which says that the expected liquidity gains derived from indirect trade are smaller than the cost, captured by the loss of rent $\delta_1 r_1$. Relative to the corresponding KW (p. 936) condition for the fundamental steady state, here the differential inventory cost is replaced by the forfeited flow of rent. Similarly, the distribution of commodities in the Home country, $\mathbf{p}^H = \frac{n}{3}[1, \frac{1}{2}, 1]$, resembles that reported in KW (p. 936) for the fundamental equilibrium. Conversely, Propositions 2 and 3 say that the Home country selects a (1,1,0) strategies, i.e. speculative strategies, if

$$(p_{3,1} - p_{2,1})u = \frac{n}{6}u > \delta_1 r_1$$

with $\mathbf{p}^H = \frac{n}{3}[\frac{1}{\sqrt{2}}, \sqrt{2} - 1, 1]$.

The analysis that follows studies events that induce the economy to switch from fundamental to speculative strategies. The addition of the Foreign country makes it more likely that the Home country selects speculative strategies. In a larger market it is more likely that type 1's odds of meeting agents who want good 3 relative to good 2 is greater. The

additional term $\beta(p_{3,1}^* - p_{2,1}^*)$ that appears on the left hand side of condition (17) of the (1,1,0)-(0,1,0) equilibrium formalizes this intuition.

The role of the flow of rent on the emergence of a speculative strategy is captured by the presence of the term r_1 on the right hand side of (17): The lower r_1 , the smaller is the utility cost of type 1 individuals (one of the two elite groups), the more likely they engage in speculative trade.

Fig. (1) gives a global picture of how the emergence of the fundamental and speculative equilibria in the Home country is conditioned by the degree of economic integration β and the level of rent r_i . Assuming a high level of rents of the privileged groups of the Foreign country, and focusing on the Home country, the (0,1,0)-(0,1,0) equilibrium exists for low degrees of integration and for high level of rent, r_1 , in the Home country. Conversely, when international encounters are intense (high β), or when rental income is small (low r_1), the (1,1,0)-(0,1,0) equilibrium emerges.

To appreciate the income gap across the equilibria studied so far, it is useful to compute to overall level of production of the two countries. In the (0,1,0)-(0,1,0) equilibrium, the Home production of good i is $Y_i = \frac{n}{3} \frac{1}{2} (\frac{n}{3} + \beta \frac{1-n}{3})$, for every i . Therefore, the overall Home production is

$$Y = \frac{n^2}{6} \left(1 + \beta \frac{1-n}{n}\right). \quad (21)$$

Similarly, that of the Foreign country is

$$Y^* = \frac{(1-n)^2}{6} \left(1 + \beta \frac{n}{1-n}\right). \quad (22)$$

Among the four equilibria reviewed, this is the one that yields the lowest output. Conversely, the (1,1,0)-(1,1,0) yields the highest level of production in both countries. This is given by the expressions on the right-hand sides of (21-22) multiplied by the factor $\sqrt{2}$. In the remaining two equilibria, (1,1,0)-(0,1,0) and (0,1,0)-(1,1,0), the level of output is somewhere between the previous two – the higher the level of integration, the closer is production to that of the (1,1,0)-(1,1,0) equilibrium.

Next, I investigate through quantitative experiments the adjustment process of the economy that follows a shock to the integration parameter β , assuming the two countries are on the (0,1,0)-(0,1,0) equilibrium before the transportation shock. The study relies on an analysis of the dynamic properties of the system, that can be found in the Online Appendix, Section A.3. Here it suffices to state the following:

Proposition 4 (*Convergence*) *The equilibria (0,1,0)-(0,1,0), (0,1,0)-(1,1,0), and (1,1,0)-(0,1,0) are locally stable.*

Proof. See Appendix ■

5 Quantitative Experiments

The Introduction mentioned the remarkable social change in the central and northern regions of Italy in connection with the spread of the Commercial Revolution. Cipolla (1993, pp. 92-93) observes that a clear sign of the change of attitudes of the rural Italian nobility toward commerce is to be found in the unique feudal aspect of several Italian cities:

"As father Salimbene da Parma explicitly reported in his Chronicle whereas in France (and, we might add, in Germany and in England too) the nobility remained entrenched in their country castles in a position of more or less overt hostility toward the emerging cities, in northern Italy many members of the feudal nobility, perceiving the new direction in which history was moving, while not abandoning their castles in the countryside, took up residence in the cities and built palaces and towers that gave Italian cities a singular feudal aspect – even at the physical level."

This section proposes applications that shed further lights on the conditions that allow a society to enjoy or miss the benefits of the Commercial Revolution. I select a set of parameters so that the Foreign country stays on a fundamental set of strategies (0,1,0) both before or after the integration shock. The conclusions of the experiments, however, do not change qualitatively if Foreign type 1 individuals respond to the shock by switching to indirect trade, i.e. if s_1^* goes from 0 to 1. One may think of the Home country as a European region, for instance Northern Italy or France, and of the Foreign country as Byzantium.

In the first experiment, the Home economy moves from fundamental to speculative strategies, as result of an integration shock, and type 1 agents (nobles) become also merchants. It shows that although both in the two countries, Home and Foreign, average income goes up, it does more so in the Home country. More importantly, there is a significant reduction of the income gap between commoners and nobles, especially in the short run. In the second experiment, the same shock does not cause any change in the trade patterns; the economy remains stuck into the fundamental equilibrium. The third experiment discusses the consequences of a technological change in production. A fourth experiment, that deals with migration, is discussed in the Online Appendix, Section A.4.

5.1 The Initial State

The initial position of the world is a fundamental steady state equilibrium (0,1,0)-(0,1,0) – as discussed in section 4.2 this yields the lowest level of production. It represents the situation in most regions in Western Europe after the fall of the Roman Empire and before the tenth century. With the exception of those regions under the economic influence of the Arabs and of Byzantium, most notably Spain and Sicily, Europe reached its low point approximately in the period 700-950. The Jews provided a link across the shores of the Mediterranean but their success in trade was often halted by the reaction of the majority of the population that

would drive them out of the community and force them to forfeit their capital (Lopez 1976, p. 62). The monetary system also became primitive. The Carolingian reform of the 8th century improved slightly the situation with the introduction of the monometallism. Yet, only one coin of silver was struck and was used for local transactions. The pound and the shilling were not yet coins but only used as units of accounting (see Cipolla 1993, Ch. 7, and Sargent and Townsend 2009).

5.2 The Integration Shock, β

In the tenth century a significant development occurred. Through the actions of small communities living on the shores of Northern Italian peninsula, the Mediterranean became again a commercial highway. During the late eleventh and twelfth centuries, merchants from particular Italian cities, most notably Venice, Pisa, and Genoa, developed commercial links between the raw materials of the West and the oriental goods of the East.

The new fervor for commerce was not associated with any major developments in naval engineering. The most common Venetian vessel was an improved version of the Roman and later Byzantine galley. Venetian trade was conducted primarily through short-haul navigation with relatively small galleys. Nor was it the result of a policy of a centralized government as, for instance, in Sung China (Kelly 1997). Quite the contrary, it took place in the most politically fragmented part of Europe.

Political events, such as the decline of feudal warfare or the reconquest of the Christians, contributed to expand the initial maritime actions of the Italian cities. A great opportunity for commercial expansion was given, for instance, to Venice in 1081 when the Byzantines requested Venetian naval support against a Sicilian-Norman invasion of the empire. Similarly, Genoa and Pisa, which during the eleventh century had been building up their trade in the western Mediterranean, won mercantile bases in the East because of the conquest of the First Crusade of the Syro-Palestinian coast. Further progress of Genoa's mercantile enterprise in Constantinople was closely connected with Byzantine hegemony over lost portions of its territory (see Day 1977). Constantinople was eager to win Western allies to defend its territory against the aggression on the part the German emperor, Frederick Barbarossa, or King William I of Sicily.

The communication lines across the shores of the Mediterranean were established at a remarkable speed for the time. For instance, between 1274 and 1293 the value of wares imported and exported by sea in Genoa increased more than four times (Cipolla 1993, Ch. 8). Several studies in economic history, demography, and epidemiology observe also that the rapid spread of the Black Death is a vivid, albeit dramatic, evidence of the great intensification of the communication network in Europe and in the Mediterranean (Benedictow, 2004). Epidemiological studies find that the frequency of human contacts must have been

more important than previously thought, if one wants to account for the speed of diffusion of the pandemic (Alfani and Murphy 2017).

5.3 The Commercial Revolution (CR)

Initially, the economy satisfies conditions (9)-(10) in Proposition 1 and is in a fundamental equilibrium (i.e. (0,1,0)-(0,1,0)). In this initial state, commoners (type 2 agents) already practice indirect trade, but the nobility does not. In accordance with Lopez's account about the Jews keeping a tenuous trade link between Europe and the Muslim and Byzantine regions, in the initial state $\beta > 0$, although it is sufficiently small that the economy is below the dashed line of fig. 1.

Consider an integration shock large enough to drive the Home economy away from the low-production equilibrium toward speculative equilibrium (1,1,0)-(0,1,0) – this lies in the region above the solid line of fig. 1. The consequences of the shock on the distribution of goods, on the frequency of trade,⁸ on the level of income, and on income disparity across types, are plotted in fig. (2). In the aftermath of the shock, Home type 1 agents (nobles) find optimal to enter into indirect trade.⁹ Panel A shows that fewer type 1 and type 2 agents hold their production goods. This movement is caused by the strategy switch of type 1 agents. The higher degree of integration causes an acceleration in the frequency of trade of about 50 percent. There are two reasons for the acceleration. One is the greater number of matches that leads to a 20 percent uniform increase of trade frequency for all goods (intensive margin). The other, which accounts for the remaining 30 percent trade acceleration and for the wide variation over the transition, is the expansion of the trade network (extensive margin). The initial trade frequency spike of good 2 is similar to that of good 3, but good 2's frequency declines, whereas that of good 3 keeps increasing as the economy converges to the speculative equilibrium.

These frequency adjustments are useful to interpret the movement of income. Fig. 2.c

⁸The frequency of trade measures the number of times that a good is traded within a unit of time. Let $t_i(v)dv$ be the probability that good i is traded on the market between time v and $v + dv$. The frequency of trade can then be calculated as follows:

$$t_i(v) = \alpha \{ p_{i+2,i} [\mu_i - p_{i,i+1}(1 - s_{i+2}) + s_{i+1} p_{i+1,i+2}] + \beta p_{i+2,i} [\mu_i^* - p_{i,i+1}^*(1 - s_{i+2}^*) + s_{i+1}^* p_{i+1,i+2}^*] \\ (\mu_{i+1} - p_{i+1,i+2}) [p_{i,i+1} + (\mu_i - p_{i,i+1})(1 - s_{i+1}) + (\mu_{i+2} - p_{i+2,i})(1 - s_{i+2})] \\ + \beta (\mu_{i+1} - p_{i+1,i+2}) [p_{i,i+1}^* + (\mu_i^* - p_{i,i+1}^*)(1 - s_{i+1}^*) + (\mu_{i+2}^* - p_{i+2,i}^*)(1 - s_{i+2}^*)] \}.$$

⁹In general, the switch can occur with some delay after the shock. At high discount rates, for instance, the gains associated with current distribution of goods are relatively more important and the switching waiting time is longer.

shows that the income of both commoners and nobles jumps up after the shock: The higher frequency of long distance trade shortens any individual's average waiting time for obtaining his preferred good on the market. And with the creation of new trade links type 1 agents also become middle-men. The resulting change in the composition of goods circulating on the market (see fig. 2.a) affects the gains from trade.

Commoners, are particularly favored by the mutation of the trade network because the demand for the good they produce (good 3) increases considerably, more than that of the other two types of goods (fig. 2.b). Although commoners remain at the bottom of the income distribution scale, their distance from the two privileged groups shrinks, both because a larger proportion of national income is now generated through trade, and because of the greater demand for their products (see fig. 2.d).

People living in the Foreign country also benefit from integration shock for two reasons: the higher frequency of the existing trade routes, and the trade creation induced by the entrance of the Home nobility into indirect trade. Nevertheless, this latter development does not benefit the foreign residents as much as people living in the Home country because a type 1 Home agent meets Foreign residents less frequently than his compatriots. Therefore, the Home country income gap with respect to that of the Foreign country (in the pre-shock fundamental equilibrium, the Foreign country had a higher average income because its type 2 agents enjoyed a greater flow of rents), shrinks – see fig. 3.c. Additional experiments (not shown) indicate that, under a different set of parameters, in the Home country commoners' income can jump above that of the nobility, and that Home country's average income can overtake that of the Foreign country.

Income convergence in the Home country is consistent with the Lopez's observation, reported at the beginning of this section, that in Northern Italy the Commercial Revolution blurred the dividing line between merchants and nobles.

5.4 Missing the Commercial Revolution

A larger gap between the privileged groups and commoners reduces the odds that an integration shock generates an economic boom. Imagine, for instance, that in the Home country the level of rents are as high as that in Foreign country of the previous experiment ($r_i = r_i^* = 0.35$ for $i = 1, 3$). Then Proposition (3) implies that, if the rental loss associated with speculative trade is sufficiently large in relation to the integration parameter β , the speculative equilibrium does not emerge (see fig. 1.b). The integration shock that triggered the behavioral change of the privileged Home agents in the CR experiment may fail to do the same in a Home country with a more privileged nobility. Section (6.3) suggests that this scenario may help explain the French nobility's resistance to trade and France's slower medieval growth relative to that of Northern Italy.

Specifically, consider an economy in the same fundamental equilibrium of the CR experiment. The integration parameter, β , also jumps up from 0.15 to 0.55, as in the CR experiment. Fig. (4) depicts the consequences of such a shock in the Home country – the Foreign country would exhibit the same movements because $n = \frac{1}{2}$, and it has the same parameters' values (see table 2) of the Home country. The higher matching rate does not alter at all the distribution of goods (fig. 4.a) but pushes the frequency of trade of the three goods up by the same proportion (fig. 4.b). The more intense trading activity is also a boon for production. This, however, does not increase as much as in the CR experiment (fig. 4.c). Indeed the Home's income expansion of the current experiment is even lower than that observed for Foreign country in the CR experiment, because in that case Foreign residents benefited indirectly from the new trade link created by type 1 Home agents.

5.5 Technological Shock

The type of speculative equilibrium (1,1,0)-(0,1,0) that emerged in the CR experiment, when Home nobility had a relatively modest rent, can appear also when the nobility is wealthier. If the Home country adopts new production technique, for instance, the prospect of earning income from decentralized markets increases, and with it the liquidity of commodities. Good 3 becomes more valuable for agents of type 1 when u is bigger.

Fig. (5.a) shows how a decline in the production cost, that can be represented through a lower d or equivalently a greater u , affects the partition of the space (r, β) with respect to the equilibria (1,1,0)-(0,1,0) and (0,1,0)-(0,1,0). The region with a commercial nobility expands relative to the one in which only commoners trade. This means that if after an integration shock the Home country nobility maintains its no-indirect trade strategy, a further shock that increases productivity can induce a strategy switch. Fig. (5.b) illustrates the adjustment process. There is a level effect, which can be attributed to process innovation, and a game strategy change effect, to which some refer as Smithian growth. This latter reaction explains the non-linear impulse responses. For instance, the income of the commoners converges to that of the upper classes immediately the productivity shock because its production good becomes instantly more demanded by type 1 agents and these all have good 2 to trade. However, as more and more of type 1 have good 3, the demand for this slows down and type 2's income drops.

Although, in the current model, advances in production technology and rental income are independent from one another, they can be interconnected. Mokyr (2016), for instance, suggested that the Enlightenment paved the way to the Industrial Revolution: The origin of modern growth would be associated with the fact that some members of the European elite in the 18th century turned their attention to the advancement of scientific knowledge, useful for the amelioration of production. From this vintage point, the fact that a minority of the

population was earning a regular rent may have helped devote critical human and physical resources to scientific pursuits.

6 Discussion

This section discusses a few aspects concerning the onset and the dissemination of the late medieval Commercial Revolution related to the experiments. First, it uses some key passages of the seminal works of Cipolla, Lopez, and Bairoch to complement the numerical data on the growth spurt in Central and Northern Italy presented in the Introduction. Second, it discusses recent studies that show a decline in inequality in Italian urban centers between 1300 and 1450 – the final leg of the Commercial Revolution. Third, it briefly contrasts the French nobility's hostility toward commerce with the respectability this was gaining among the nobility in Central and Northern Italy. Finally, it suggests that the shrinking economic, social, and geographical distance between the nobility and merchants could have favored the appearance of contractual innovations, such as the *colleganza* in Venice and the *commenda* in Genoa and in other parts of Italy.

6.1 The Medieval Growth Spurt

The Introduction described the exceptional income growth in Central and Northern Italy over the late medieval period. The use of the GDP per capita statistics in contemporary prices for earlier centuries inevitably relies on strong assumptions. The sense of income acceleration conveyed by the data agree, nonetheless, with the accounts of Cipolla (1993) and Lopez (1976). In describing the 13th century economic situation in Northern and Central Italy, Lopez (1976, p. 106) writes, for example,

[N]ever before had such a large proportion of the population been free from want, or such a variety and abundance of goods been constantly available – not in ancient Rome at its peak, not in Byzantine and Islamic countries at their best .

Cipolla (1993) reports that contemporary observers, when visiting urban centers, were marveled by "the abundance of all goods", "the almost innumerable merchants with their variety of wares," and the fact that "any man, if he is healthy and not a good-for-nothing, may earn his living expenses and esteem according to his station." (Cipolla 1993, Ch. 8).¹⁰ The expansion of maritime trade also created an impulse to trading activities within Europe. The fairs, for instance, that initially were weekly markets to trade local products and man-

¹⁰Cipolla goes further to argue that some morally charged sections of the Dante's Comedy, were clearly motivated by the disruptive forces of the medieval growth.

ufactured goods, during the thirteenth century, most notably the fairs of Champagne (see Braudel 1983, Ch.1), became the crossroads for trading all sorts of local and exotic goods.

Economic historians take often the population growth of a region as a good predictor of more conspicuous consumption in pre-capitalist societies. Bairoch (1988) estimates that the population in Venice went from approximately 45 thousand in the year 1000 to 75 thousand in the year 1200 and reached a medieval peak of 110 thousand in the year 1300. The population explosion in Genoa is even more staggering. The corresponding figures are 15, 30, and 100 thousand, respectively.

6.2 Inequality and the *Colleganza*

Puga and Treffer (2014) contend that the rise of Venice was characterized by high social mobility and low income inequality. Similarly, in Genoa, the dividing line between nobles and merchants was rather blurred. Lopez (1964, p. 447- 48) offers the following account:

"The habit of sharing risk on the decks of the ships and rubbing shoulders in the narrow alleys and shops of the business center created a strong equalitarian tradition. Ever since the origins of the Genoese Commune, the law admitted no privileges of birth and ignored the very word "nobility." Naturally, this does not mean that nobility was unknown, for aristocrats turned businessmen played leading roles in town and at sea; but the first recognition it got was through fourteenth-century decrees that excluded noblemen from public offices. The informal term *bonitas*, good or solid citizens, described the Genoese upper class more accurately."

Outside Italy not only the social distinction but also the wealth gap between the aristocracy and the rest of the population was more marked. France was a region dominated by a rich landed aristocracy until the set in of modern growth. Wickham (2009, pp. 209-10) argues that such a state originates from as far back as Merovingian France, when rich landowners had more than a dozen of landed estates each, and a highly militarized factional politics, and that Bavaria, on smaller scale, resembled France.¹¹ In contrast, the wealth of the aristocracy in Lombard Italy was much more modest: even the richest families owned between five and ten estates, which was close to a minimum for aristocrats in France. (Wickham 2009, pp.145-146).

The experiment on the CR implies that with the onset of the Commercial Revolution there should be a convergence of income between the merchants and the nobility-turned merchants. Dynamic inequality data on medieval times are hard to find. Alfani (2015)

¹¹Wickeman (2009) also reports that the accumulation of conspicuous wealth by the French landed aristocracy continued during the Carolingian and post-Carolingian periods, when more land came under aristocratic control than before and less land was under the control of non-aristocrats. The concentration of land ownership occurred especially Saxony, where Charlemagne's conquest resulted in a rapid takeover of land that had previously been under peasant ownership.

was able to delineate the economic inequality for the Sabaudian State – located on the North-West of the Italian peninsula – from the 14th to the 18th century. Alfani's data show that from the year 1300 to 1450 – in the final leg of the Commercial Revolution period – inequality, measured through the Gini coefficient went systematically down in all the cities of the sample (Alfani 2015, fig. 2.a, p. 1072, and fig. 7, p. 1084). Alfani (2017) and Alfani and Ammannati (2017) also finds a similar downward trend of inequality in the 1300-1450 period when looking at the share of the top 5% rich in the Sabaudian State and in the Florentine State.

We do not know how representative these inequality trends are for the rest of center-north Italy. In the case of Venice, the political shift in the Great Council and in the social extraction of the Doge, suggests that income convergence between merchants and nobility did occur. The first Doges (i.e. from 810 to 1032) were hereditary and members of the traditional nobility. In the year 1032, however, for the first time in Venice history, a merchant becomes Doge in Venice. For Puga and Trefler (2014 p. 764) "The election of Flabanico as Doge was a transformative moment in Venetian history. He was a wealthy silk merchant, and most subsequent Doges over the next many centuries were also merchants involved in long-distance trade. Flabanico's election thus represents the triumph of the merchants." After three centuries, however, social mobility in Venice may have declined due to the long-distance trade restrictions introduced by the Great Council at the beginning of the 14th century (the *Serrata*). The recent work of Alfani and di Tullio (2019) also documents an increase in wealth inequality in several urban centers of the Republic of Venice (*Terrraferma*) after the 14th century (see their fig. 3.1 and table 3.1).

Interestingly, Puga and Trefler (2014) use the decline of *collegantia* contracts, with commoners and nobles as parties in long-distance ventures, as evidence of the lower degree of social mobility. Indeed, one could argue that the contractual innovations of the *collegantia* in Venice, the *commenda* in Genoa and other parts of Italy, and subsequently the *compagnia*, were facilitated by the relative low social barriers between the commoners and the nobles, which permitted them to make mutually beneficial deals: The new winds of commerce were eroding feudal constraints.¹² Otto of Frisingen, uncle of Frederick Barbarossa, observed that:

In the Italian communes they do not disdain to grant the girdle of knighthood or honorable positions to young people of inferior station, and even to workers of the vile mechanical arts, whom other peoples bar like the plague from the more respectable and honorable circles. (Cited by Cipolla 1993, p. 149.)

¹²Under commenda and collegantia contracts, one party provided funds to an another party to carry out a trade as partners in the enterprise, sharing the profit. The party who provided funds, however, would typically bear both sea and commercial risk. Conversely, a compagnia contract was a step toward a modern business partnership as it contemplated both general and limited partnerships as well as joint liability (see Boerner and Ritschl, 2008).

6.3 The Resistance of French Aristocracy to Commerce

Despite France being in a favorable geographical position, it did not benefit from the Commercial Revolution as much as Northern Italy. One indication that pre-revolutionary France did not catch the trade wave is the slow income rise during the Middle Ages – as noted in the Introduction income increased only 50 percent between 1000 and 1500. Another indication of the French slow growth associated with trade is the modest change in population in its seaport cities. This was true not only on the Mediterranean shore, but on the Atlantic one. In the year 1700, the population of Bordeaux, the most important French 18th century port, was approximately equal to that of two centuries earlier and only one-quarter that of Amsterdam (Bairoch 1988).

In the model, it is costly for the nobility to become middleman. This assumption is based on the observations of a number of historians who have argued that the European landed aristocracy was resistant to embracing commerce due to concerns that such a move would cost them privileges – especially property tax exemptions. Such a belief was well grounded. In France, for instance, in 1295, when Philip the Fair exempted the nobles from paying him a certain aid, he specifically excluded those in trade. In 1435, Charles VII refused to exempt from taxation nobles who sold their wines in taverns. Earlier in the 16th century, Francis I voided the tax exemptions of nobles who meddled in commercial activities. The position of the French monarchy with respect to having a commercial nobility was, nevertheless, ambiguous and often reversed its course out of pressure from the same nobility that feared losing its privileges. It is revealing that Louis XI, who was very familiar with the state of affairs in the Italian cities, took steps to establish commercial routes to compete with Venice and Genoa.¹³ In the second half of the 15th century, the monarch prepared legislation to eliminate the separation between the nobility and merchants, in imitation of what was already in place in Italian cities. The law, however, was never implemented. Instead, a few decades later, in 1560, prior legislation and customs that sanctioned the nobility's participation in commerce was codified (the Code of Ordinances of Orleans, see Zeller, 1946). In 1629, under Richelieu, nobles were however allowed to engage in maritime commerce, either directly or by proxy, without fear of losing their status. Although some nobles seized opportunities in overseas trade by participating in trading companies (Chaussinand-Nogaret 1985), by the middle of the 18th century, the debate among French intellectuals on whether to have a commercial nobility remained lively. An example is the controversy stirred by the pamphlet "Commercial Nobility" (*La Noblesse Commerçante*) that appeared in 1756. The author, an influential writer at that time, rebutted Montesquieu's position, elaborated in *The Spirit of the Laws* (1748, Book XX, Ch. 21), that opposed the idea of having a commercial nobility

¹³Lopez (1964) reports that at the end of the 13th century, the taxes Genoa raised from her sea trade amounted to approximately ten times the receipts of the French royal treasury.

(see Adam, 2003, and Grassby, 1960, for details on the controversy).

7 Final Remarks

Recent scholarship has questioned an approach of the growth literature that explains the improvements of the material-well independently from the social dynamics. Some have also argued that to understand the origin of modern growth, it is useful to study the society's attitude with respect to trade and industry. This paper contributes to ongoing efforts to study the interdependence of economic expansion and social changes, by proposing a search model in which the emergence of a specific pattern of exchange depends on the amount of privileges that the upper class loses when moving to commerce. It showed that the response of the upper strata to changes in environment, such as a more efficient communication technology or reduction in the cost of production might direct the economy toward a low or a high production equilibrium. One experiment, inspired by the observation that starting from the tenth century the contacts between the Christian, Orthodox, and Muslims communities became more frequent, revealed that a shock that strengthens the integration between the Home and Foreign region reduces the distance between the commoners and upper classes, particularly in the region where the privileged group switch to indirect trade. A second experiment, motivated by the literature that attributes the rise of Europe to the new scientific thinking that flourished during the Enlightenment period, showed that a country that does not respond to a matching shock may still respond to a technological one.

Even though I focused on episodes prior to the Industrial Revolution, the feedback between social differences and equilibrium selection that the model suggests is rather general and could be employed to interpret more recent events of economic integration. Likewise, future research could investigate how differences in the discount rate across social classes or the endogenous response of the matching rate to demographic changes affect the equilibrium selection mechanism.

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Table 1: The matching technology

	Home agent	Foreign agent
Home agent	αn	$\alpha\beta(1 - n)$
Foreign agent	$\beta\alpha n$	$\alpha(1 - n)$

Table 2: Parameters

	Home Country						Foreign Country				Both Countries			
	$\frac{\mu_i}{n}$	r_1	r_3	r_2	δ_i	n	$\frac{\mu_i^*}{1-n}$	r_1^*, r_3^*	r_2^*	d^*	ρ	α	u	β
Commercial Revolution	$\frac{1}{3}$	0.2	0.35	0	0.5	0.5	$\frac{1}{3}$	0.35	0	0.5	0.03	1	1	0.15
Missing the Commercial Revolution	$\frac{1}{3}$	0.35	0.35	0	0.5	0.5	$\frac{1}{3}$	0.35	0	0.5	0.03	1	1	0.15

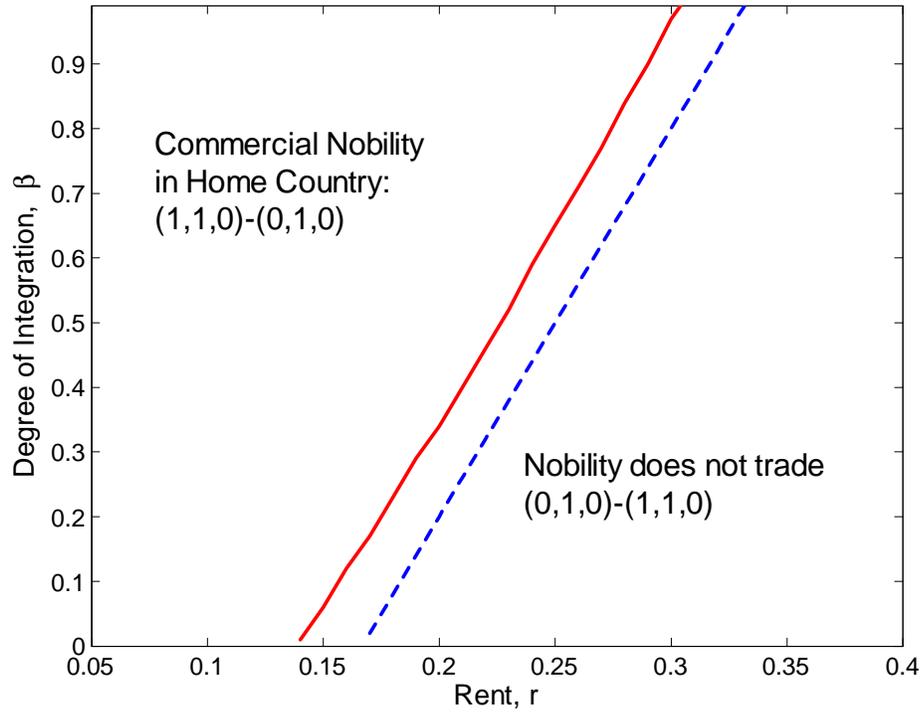
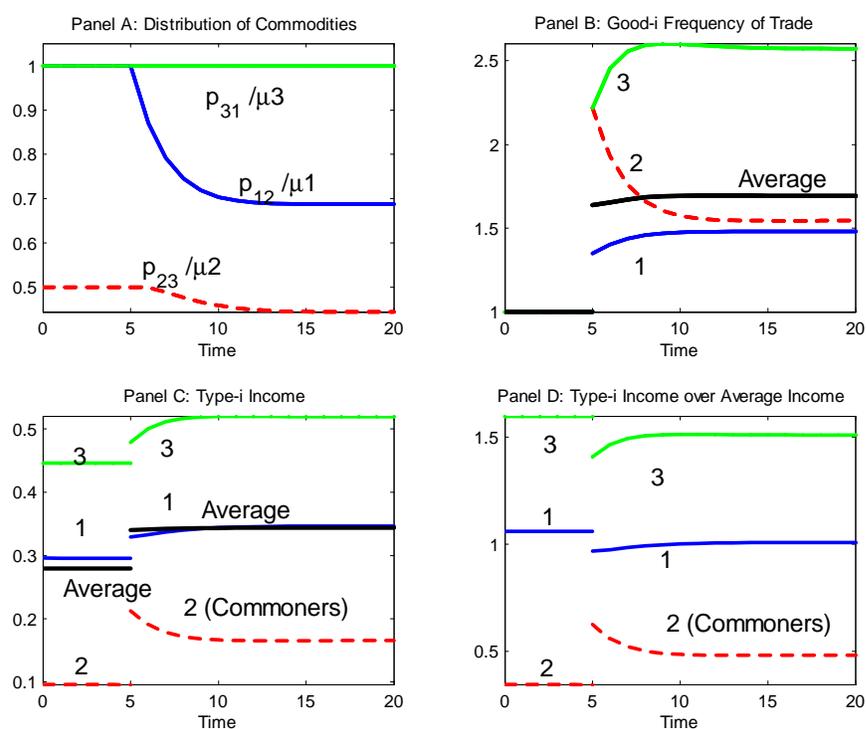


Figure 1: Economic Integration and Privileges

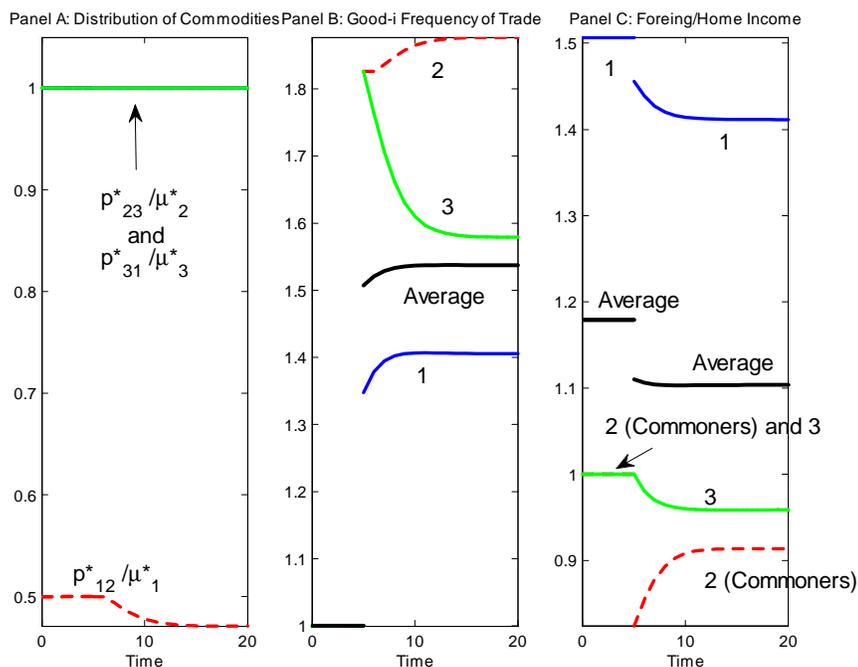
- Note: $r_1 = r_3 = 0.2$, $r_1^* = r_3^* = 0.35$, and $n = 0.5$. For other parameters see table (2). Between the two lines, only mixed strategy equilibria with $s_1 \in (0, 1)$ may exist.

Figure 2: Cross-Class Income Convergence, Home Country



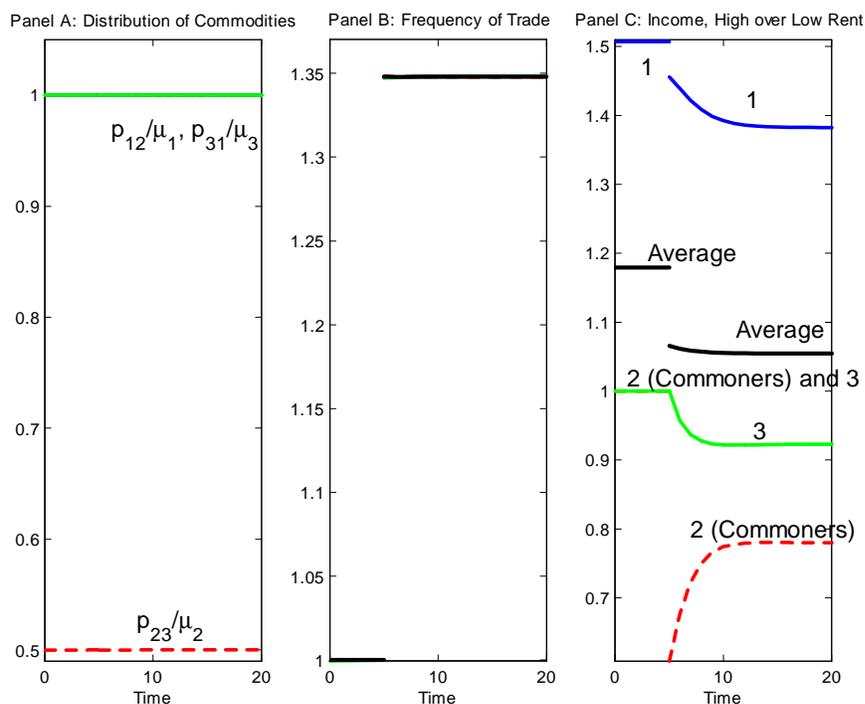
- Note: The numbers 1,2,3 in Panel B and C refer to the type of good and type of agent, respectively. The shock causes β to go from 0.15 to 0.4. The frequency of trade after the shock is a ratio of the pre-shock level. The parameters' values are in Table 2.

Figure 3: Effects of the Commercial Revolution in the Foreign Country



- Note: See note of fig. (2).

Figure 4: Missing the Commercial Revolution (Home Country)



- Note: The shock is the same as in fig. (2). The paths of income in Panel C are ratios of the corresponding paths in Panel C, fig. (2). See table 2 for parameters values.

Online Appendix

Class Differences and the Commercial Revolution: An Equilibrium Selection Story

Maurizio Iacopetta

SKEMA Business School, Université Côte d'Azur (GREDEG)

and

Sciences-Po (OFCE), France

This Appendix provides more details on the determination of the Value Functions (A.1) on the proofs about the existence of the Nash Equilibria discussed in the text (A.2), and the convergence properties of some of these equilibria (A.3). Finally, it illustrates the effects of migration (A.4) on equilibrium selection.

A.1 Value Functions

Consider a Home agent of type i who has to compute her best profile of strategies, given a pattern of inventories, $\mathbf{p}(t)$, and a pattern of strategies for other agents, $\mathbf{s}(t)$ – including those of her own type. Let $V_{i,j}(t)$ be the value function when holding good j at time t . When $j = i + 1$, we have that

$$\begin{aligned}
 V_{i,i+1}(t) = & \max_{\{\sigma_i(v)\}_{v \geq t}} \int_t^\infty \{ \alpha e^{-(\rho + \hat{\alpha})(v-t)} (\sigma_i [p_{i,i+2}(1 - s_i) + p_{i+1,i+2}] V_{i,i+2} + \\
 & + [n - \sigma_i (p_{i,i+2}(1 - s_i) + p_{i+1,i+2})] V_{i,i+1} + [p_{i+1,i} + p_{i+2,i} s_{i+2}] u + \\
 & + \beta \sigma_i [p_{i,i+2}^* (1 - s_i^*) + p_{i+1,i+2}^*] V_{i,i+2} + \\
 & + \beta [1 - n - \sigma_i (p_{i,i+2}^* (1 - s_i^*) + p_{i+1,i+2}^*)] V_{i,i+1} + \beta [p_{i+1,i}^* + p_{i+2,i}^* s_{i+2}^*] u) + \\
 & + \hat{\alpha} e^{-\hat{\alpha}(v-t)} \frac{1 - e^{-(v-t)\rho}}{\rho} r_i \} dv
 \end{aligned} \tag{23}$$

where $\hat{\alpha} \equiv \alpha n + \beta \alpha (1 - n)$. The term after the discount factor is the probability that the Home agent trades $i + 1$ for $i + 2$ – in which case, he is left with $V_{i,i+2}$ as his continuation value. Otherwise, he ultimately has good $i + 1$, either because no trade occurs, or because he acquired her consumption good – an event that occurs with probability $p_{i+1,i} + p_{i+2,i} s_{i+2}$ – and then produces good $i + 1$. The terms that follow captures the similar occurrences when this agent meets a Foreign individual. The last term is the rental income from time t to time v .

The corresponding expression when he hold good $i + 2$ is

$$\begin{aligned}
V_{i,i+2}(t) = & \max_{\{\sigma_i(v)\}_{v \geq t}} \int_t^\infty \{ \alpha e^{-(\rho + \hat{\alpha})(v-t)} ([p_{i+1,i}(1 - s_{i+1}) + p_{i+2,i} + \\
& + (p_{i,i+1}s_i + p_{i+2,i+1})(1 - \sigma_i)]V_{i,i+1} + \\
& + [n - p_{i+1,i}(1 - s_{i+1}) - p_{i+2,i} - (p_{i,i+1}s_i + p_{i+2,i+1})(1 - \sigma_i)]V_{i,i+2} + \\
& + (p_{i+1,i}(1 - s_{i+1}) + p_{i+2,i})u + \\
& + \beta [p_{i+1,i}^*(1 - s_{i+1}^*) + p_{i+2,i}^* + (p_{i,i+1}^*s_i^* + p_{i+2,i+1}^*)(1 - \sigma_i)]V_{i,i+1} \\
& + \beta (p_{i+1,i}^*(1 - s_{i+1}^*) + p_{i+2,i}^*)u + \\
& + \beta [1 - n - p_{i+1,i}^*(1 - s_{i+1}^*) - p_{i+2,i}^* - (p_{i,i+1}^*s_i^* + p_{i+2,i+1}^*)(1 - \sigma_i)]V_{i,i+2} + \\
& + \hat{\alpha} e^{-(\rho + \hat{\alpha})(v-t)} \frac{1 - e^{-(v-t)\rho}}{\rho} (1 - \delta_i)r_i \} dv.
\end{aligned} \tag{24}$$

Taking time derivatives of (23) and (24) and then taking the difference of the resulting $\dot{V}_{i,i+1}$ and $\dot{V}_{i,i+2}$, after some algebra, we have

$$\begin{aligned}
\dot{\Delta}_i = & \alpha (\{ \sigma_i [p_{i,i+2}(1 - s_i) + p_{i+1,i+2}] + [p_{i+1,i}(1 - s_{i+1}) + p_{i+2,i} + \\
& + (p_{i,i+1}s_i + p_{i+2,i+1})(1 - \sigma_i)] \} \Delta_i + \\
& - [p_{i+1,i} + p_{i+2,i}s_{i+2} - p_{i+1,i}(1 - s_{i+1}) - p_{i+2,i}]u) + \\
& \beta \alpha (\{ \sigma_i [p_{i,i+2}^*(1 - s_i^*) + p_{i+1,i+2}^*] + [p_{i+1,i}^*(1 - s_{i+1}^*) + p_{i+2,i}^* + \\
& + (p_{i,i+1}^*s_i^* + p_{i+2,i+1}^*)(1 - \sigma_i)] \} \Delta_i + \\
& - [p_{i+1,i}^* + p_{i+2,i}^*s_{i+2}^* - p_{i+1,i}^*(1 - s_{i+1}^*) - p_{i+2,i}^*]u) \\
& - \delta_i r_i + \rho \Delta_i.
\end{aligned} \tag{25}$$

In steady state $\dot{\Delta}_i = 0$. The terms in the brackets in front of Δ_i are non-negative because they correspond to probabilities. Therefore, in steady state the sign of Δ_i is the same as that of the expression

$$\begin{aligned}
\tilde{\Delta}_i \equiv & p_{i+1,i} + p_{i+2,i}s_{i+2} - p_{i+1,i}(1 - s_{i+1}) - p_{i+2,i} + \\
& + \beta [p_{i+1,i}^* + p_{i+2,i}^*s_{i+2}^* - p_{i+1,i}^*(1 - s_{i+1}^*) - p_{i+2,i}^*] + \frac{\delta_i r_i}{u\alpha}.
\end{aligned} \tag{26}$$

A similar expression holds for the Foreign country:

$$\begin{aligned}
\tilde{\Delta}_i^* \equiv & p_{i+1,i}^* + p_{i+2,i}^*s_{i+2}^* - p_{i+1,i}^*(1 - s_{i+1}^*) - p_{i+2,i}^* + \\
& + \beta [p_{i+1,i}^* + p_{i+2,i}^*s_{i+2}^* - p_{i+1,i}^*(1 - s_{i+1}^*) - p_{i+2,i}^*] + \frac{\delta_i^* r_i^*}{u\alpha}.
\end{aligned} \tag{27}$$

To verify whether a given set of strategies, \mathbf{s} , sustains a Nash equilibrium one needs to check the consistency between \mathbf{s} and the sign of $\tilde{\Delta}_i$ and $\tilde{\Delta}_i^*$.

A.2 Nash Equilibria

Proposition 1. Case (0,1,0)-(0,1,0). Under this set of strategies, following (3) and (4) reported in the section A.3 of this Appendix, the steady state distribution in the Home country is $p_{1,2} = \mu_1$, $p_{2,3} = \frac{\mu_2[\mu_1 + \beta\mu_1^*]}{\mu_3 + \mu_1 + \beta(\mu_3^* + \mu_1^*)}$, and $p_{3,1} = \mu_3$, and in the Foreign country $p_{1,2}^* = \mu_1^*$, $p_{2,3}^* = \frac{\mu_2^*[\mu_1^* + \beta\mu_1]}{\mu_3^* + \mu_1^* + \beta(\mu_3 + \mu_1)}$, $p_{3,1}^* = \mu_3^*$. According to (26) and (27) such a steady state is a Nash equilibrium if

$$p_{3,1} - p_{2,1} + \beta(p_{3,1}^* - p_{2,1}^*) < \frac{\delta_1 r_1}{\alpha u} \quad (28)$$

$$d - p_{1,2} - p_{1,2}^* \beta \leq 0, \quad (29)$$

and

$$0 > -\frac{\delta_3 r_3}{\alpha u}, \quad (30)$$

are verified in the Home country, and

$$p_{3,1}^* - p_{2,1}^* + \beta(p_{3,1} - p_{2,1}) < \frac{\delta_1 r_1^*}{\alpha u} \quad (31)$$

$$-p_{1,2}^* - p_{1,2} \beta \leq 0, \quad (32)$$

and

$$0 > -\frac{\delta_3^* r_3^*}{\alpha u}, \quad (33)$$

hold in the Foreign country. Despite the fact that conditions (29), (30), (32) and (33) are verified for any distribution of $p_{i,j}$, conditions (28) and (31) hold only under some parameters restrictions. Specifically, assume that $\mu_i = \frac{n}{3}$ and $\mu_i^* = \frac{1-n}{3}$; (28) and (31) reduce to

$$\frac{\delta_1 r_1}{\alpha u} - \frac{1}{6}[n + (1-n)\beta] > 0$$

and

$$\frac{\delta_1^* r_1^*}{\alpha u} - \frac{1}{6}(1-n+n\beta) > 0,$$

which correspond to (9) and (10).

Finally, it is easy to verify that $p_{1,2} = p_{3,1} = \frac{n}{3}$, and $p_{2,3} = \frac{n}{6}$, and that $p_{1,2}^* = p_{3,1}^* = \frac{1-n}{3}$, and $p_{2,3}^* = \frac{1-n}{6}$. Hence, the result reported in equations (11)-(12).

Proposition 2. Case (1,1,0)-(1,1,0). The conditions for type 2 and type 3 agents in the Home and Foreign country are still expressed as in (29)-(30), and (32)-(33), respectively. Conversely, to have $\tilde{\Delta}_1 < 0$ and $\tilde{\Delta}_1^* < 0$ inequalities (28) and (31) are replaced with

$$p_{3,1} - p_{2,1} + \beta(p_{3,1}^* - p_{2,1}^*) > \frac{\delta_1 r_1}{\alpha u} \quad (34)$$

and

$$p_{3,1}^* - p_{2,1}^* + \beta(p_{3,1} - p_{2,1}) > \frac{\delta_1^* r_1^*}{\alpha u} \quad (35)$$

These two conditions require β to be small enough for the (1,1,0)-(1,1,0) to be an equilibrium. The steady state values of $p_{i,j}, p_{i,j}^*$, derived in the Convergence section in this Appendix, imply that in the symmetric case in which $\mu_i = \frac{n}{3}$ and $\mu_i^* = \frac{1-n}{3}$, in the Home country, $p_{1,2} = \frac{n}{3} \frac{1}{\sqrt{2}}$, $p_{2,3} = \frac{n}{3}(\sqrt{2} - 1)$, and $p_{3,1} = \frac{n}{3}$, whereas in the Foreign country, $p_{1,2}^* = \frac{1-n}{3} \frac{1}{\sqrt{2}}$, $p_{2,3}^* = \frac{1-n}{3}(\sqrt{2} - 1)$, and $p_{3,1}^* = \frac{1-n}{3}$. Using this result, the above equations yield (13)-(16) of Proposition 2.

Proposition 3. Case (1,1,0)-(0,1,0). Section A.3 of this Online Appendix demonstrates that, under the set of strategies (1,1,0)-(0,1,0), (3) and (4) the steady state commodities in the Foreign country is $p_{1,2}^* = \mu_1^*$, $p_{2,3}^* = \mu_2^* \frac{\mu_1^* + \beta p_{1,2}}{\mu_3^* + \mu_1^* + \beta(p_{1,2} + \mu_3)}$, and $p_{3,1}^* = \mu_3^*$. In the Home country $p_{3,1} = \mu_3$ and $p_{1,2}$ and $p_{2,3}$ satisfy

$$p_{1,2} = \mu_1 \frac{\mu_3 + \beta \mu_3^*}{p_{2,3} + \beta p_{2,3}^* + \mu_3 + \beta \mu_3^*}$$

and

$$p_{1,2} = \frac{\mu_3 + \beta \mu_3^*}{\mu_2 - p_{2,3}} p_{2,3} - \beta \mu_1^*.$$

In the interval $(0, \mu_2)$ there is only one value of $p_{2,3}$ for which the right-hand sides of the above to equations are equal to one another. The above expressions in symmetric case in which $\mu_i = \frac{n}{3}$ and $\mu_i^* = \frac{1-n}{3}$, reduces to expressions reduces to (19)-(20).

The conditions for a Nash equilibrium (29)-(33) still hold with $p_{i,j}, p_{i,j}^*$ evaluated at the steady state (1,1,0)-(0,1,0). The sign of the inequality in (28), however, is reversed. Hence, the the two key conditions to verify are

$$p_{3,1} - p_{2,1} + \beta(p_{3,1}^* - p_{2,1}^*)\beta > \frac{\delta_1 r_1}{\alpha u}, \quad (36)$$

and (31) that becomes

$$p_{3,1}^* - p_{2,1}^* + \beta(p_{3,1} - p_{2,1})\beta < \frac{\delta_1 r_1^*}{\alpha u}. \quad (37)$$

Note that when $\mu_i = \frac{n}{3}$, and $\mu_i^* = \frac{1-n}{3}$ the distribution of commodities is $p_{1,2}^* = \frac{1-n}{3}$, $p_{2,3}^* = \frac{n}{3} \frac{\frac{1-n}{3} + \beta p_{1,2}}{\frac{2(1-n)}{3} + \beta(p_{1,2} + \frac{n}{3})}$ and $p_{3,1}^* = \frac{1-n}{3}$. In the Home country, $p_{3,1} = \frac{n}{3}$ and $p_{1,2}$ and $p_{2,3}$ satisfy

$$p_{1,2} = \frac{n}{3} \frac{[\frac{n}{3} + \beta \frac{1-n}{3}]}{p_{2,3} + \beta p_{2,3}^* + \frac{n}{3} + \beta \frac{1-n}{3}},$$

and

$$p_{1,2} = \frac{\frac{n}{3} + \beta \frac{1-n}{3}}{\frac{n}{3} - p_{2,3}} p_{2,3} - \beta \frac{1-n}{3}$$

Hence the expressions in (19)-(20).

Case (0,1,0)-(1,1,0). This mirrors the previous equilibrium. Foreign type 1 agents play $s_1^* = 1$ and Home type 1 agents play $s_1 = 0$. Therefore, the steady state solution for \mathbf{p} and the conditions for the existence of the Nash equilibrium can be obtained by adapting the expressions obtained for the (1,1,0)-(0,1,0) equilibrium.

A.3 Convergence

This section studies the convergence properties of the system (3)-(4) under a specific set of time-constant strategies. The extended forms of (3) is

$$\begin{aligned}\dot{p}_{1,2} = & \alpha\{p_{1,3}[p_{2,1}(1 - s_2) + p_{3,1} + p_{3,2}(1 - s_1)] - p_{1,2}p_{2,3}s_1\} + \\ & + \beta\alpha\{p_{1,3}[p_{2,1}^*(1 - s_2^*) + p_{3,1}^* + p_{3,2}^*(1 - s_1)] - p_{1,2}p_{2,3}^*s_1\}\end{aligned}$$

$$\begin{aligned}\dot{p}_{2,3} = & \alpha\{p_{2,1}[p_{3,2}(1 - s_3) + p_{1,2} + p_{1,3}(1 - s_2)] - p_{2,3}p_{3,1}s_2\} + \\ & + \beta\alpha\{p_{2,1}[p_{3,2}^*(1 - s_3^*) + p_{1,2}^* + p_{1,3}^*(1 - s_2)] - p_{2,3}p_{3,1}^*s_2\}\end{aligned}$$

$$\begin{aligned}\dot{p}_{3,1} = & \alpha\{p_{3,2}[p_{1,3}(1 - s_1) + p_{2,3} + p_{2,1}(1 - s_3)] - p_{3,1}p_{1,2}s_3\} + \\ & + \beta\alpha\{p_{3,2}[p_{1,3}^*(1 - s_1^*) + p_{2,3}^* + p_{2,1}^*(1 - s_3)] - p_{3,1}p_{1,2}^*s_3\}\end{aligned}$$

Three similar equations, not reported, are implied by (4). Below (3) and (4) are evaluated on a specific set of Nash strategies.

Case (0,1,0)-(0,1,0). The (3) in extended form becomes

$$\dot{p}_{1,2} = p_{1,3}\alpha(\mu_3 + \beta\mu_3^*) \quad (38)$$

$$\dot{p}_{2,3} = \alpha[p_{2,1}(p_{3,2} + p_{1,2}) - p_{2,3}p_{3,1}] + \beta\alpha[p_{2,1}(p_{3,2}^* + p_{1,2}^*) - p_{2,3}p_{3,1}^*] \quad (39)$$

and

$$\dot{p}_{3,1} = p_{3,2}\alpha(\mu_2 + \beta\mu_2^*). \quad (40)$$

The extended form of (4) is

$$\dot{p}_{1,2}^* = p_{1,3}^*\alpha(\mu_3^* + \beta\mu_3) \quad (41)$$

$$\dot{p}_{2,3}^* = \alpha[p_{2,1}^*(p_{3,2}^* + p_{1,2}^*) - p_{2,3}^*p_{3,1}^*] + \beta\alpha[p_{2,1}^*(p_{3,2} + p_{1,2}) - p_{2,3}p_{3,1}] \quad (42)$$

and

$$\dot{p}_{3,1}^* = p_{3,2}^*\alpha(\mu_2^* + \beta\mu_2) \quad (43)$$

Eq. (38) implies that the line $p_{1,2} = \mu_1$ is globally attractive. Similarly, Eq. (40) implies that $p_{3,1} = \mu_3$ is globally attractive. The same considerations applied to (41) and (42) imply that $p_{1,2}^* \rightarrow \mu_1^*$ and $p_{3,1}^* \rightarrow \mu_3^*$. As a result, the system collapses to

$$\dot{p}_{2,3} = \alpha[\mu_2\mu_1 - p_{2,3}(\mu_3 + \mu_1)] + \beta\alpha[(\mu_2 - p_{2,3})\mu_1^* - p_{2,3}^*\mu_3^*] \quad (44)$$

and

$$\dot{p}_{2,3}^* = \alpha[\mu_2^*\mu_1^* - p_{2,3}^*(\mu_3^* + \mu_1^*)] + \beta\alpha[(\mu_2^* - p_{2,3}^*)\mu_1 - p_{2,3}\mu_3] \quad (45)$$

The Jacobian, J , of the system of these two equations is

$$J = \alpha \begin{bmatrix} -(\mu_3 + \mu_1 + \beta\mu_1^*) & -\beta\mu_3^* \\ -\beta\mu_3 & -(\mu_3^* + \mu_1^* + \beta\mu_1) \end{bmatrix}.$$

The determinant is positive, and the trace is negative; therefore, both eigenvalues are negative, and the system is globally stable. By setting the right-hand side of (44) and of (45) to zero, we obtain $p_{2,3} = \frac{\mu_2[\mu_1 + \beta\mu_1^*]}{\mu_3 + \mu_1 + \beta(\mu_3^* + \mu_1^*)}$ and $p_{2,3}^* = \frac{\mu_2^*[\mu_1^* + \beta\mu_1]}{\mu_3^* + \mu_1^* + \beta(\mu_3 + \mu_1)}$.

Case (1,1,0)-(0,1,0). Eq. (3) reduces to

$$\begin{aligned} \dot{p}_{1,2} &= \alpha(p_{1,3}p_{3,1} - p_{1,2}p_{2,3}) + \beta\alpha(p_{1,3}p_{3,1}^* - p_{1,2}p_{2,3}^*) \\ \dot{p}_{2,3} &= \alpha[p_{2,1}(p_{3,2} + p_{1,2}) - p_{2,3}p_{3,1}] + \beta\alpha[p_{2,1}(p_{3,2}^* + p_{1,2}^*) - p_{2,3}p_{3,1}^*] \\ \dot{p}_{3,1} &= \alpha[p_{3,2}(p_{2,3} + p_{2,1})] + \beta\alpha[p_{3,2}(p_{2,3}^* + p_{2,1}^*)] \end{aligned}$$

and eq. (4) to

$$\begin{aligned} \dot{p}_{1,2}^* &= \alpha p_{1,3}^*\mu_3^* + \beta\alpha\{p_{1,3}^*\mu_3 - p_{1,2}^*p_{2,3}\} \\ \dot{p}_{2,3}^* &= \alpha\{p_{2,1}^*[p_{3,2}^* + p_{1,2}^*] - p_{2,3}^*p_{3,1}^*\} + \beta\alpha\{p_{2,1}^*[p_{3,2} + p_{1,2}] - p_{2,3}^*p_{3,1}\} \\ \dot{p}_{3,1}^* &= \alpha\{p_{3,2}^*[p_{1,3}^* + p_{2,3}^* + p_{2,1}^*]\} + \beta\alpha\{p_{3,2}^*[p_{2,3} + p_{2,1}]\}, \end{aligned}$$

Observe that $p_{3,1}$ and $p_{3,1}^*$ converge, respectively, to μ_3 and μ_3^* . Also $p_{1,2}^* \rightarrow \mu_1^*$ as long as $\mu_3^* + \beta\mu_3 > \mu_1^*\beta p_{2,3}$. This condition certainly holds when types are equally distributed in the Home economy. Indeed, in such a case $\mu_3^* + \beta\mu_3 > \mu_1^*\beta\mu_2 > \mu_1^*\beta p_{2,3}$. For $p_{3,1} \rightarrow \mu_3$, $p_{3,1}^* \rightarrow \mu_3^*$, and $p_{1,2}^* \rightarrow \mu_1^*$, the system collapses to

$$\begin{aligned} \dot{p}_{1,2} &= \alpha(p_{1,3}\mu_3 - p_{1,2}p_{2,3}) + \beta\alpha(p_{1,3}\mu_3^* - p_{1,2}p_{2,3}^*) \\ \dot{p}_{2,3} &= \alpha[p_{2,1}(p_{1,2}) - p_{2,3}\mu_3] + \beta\alpha[p_{2,1}(\mu_1^*) - p_{2,3}\mu_3^*] \end{aligned}$$

and

$$\dot{p}_{2,3}^* = \alpha(p_{2,1}^*\mu_1^* - p_{2,3}^*\mu_3^*) + \beta\alpha\{p_{2,1}^*p_{1,2} - p_{2,3}^*\mu_3\}$$

Since this three-equation system cannot be further reduced, its convergence to the steady state is to be studied numerically. The steady state values of $p_{1,2}$, $p_{2,3}$, and $p_{2,3}^*$ are found by setting the right-hand side of the above three equations to zero. This leads to $p_{2,3}^* = \mu_2^* \frac{\mu_1^* + \beta p_{1,2}}{\mu_3^* + \mu_1^* + \beta(p_{1,2} + \mu_3)}$, with $p_{1,2}$ and $p_{2,3}$ that satisfy

$$p_{1,2} = \mu_1 \frac{[\mu_3 + \beta \mu_3^*]}{p_{2,3} + \beta p_{2,3}^* + \mu_3 + \beta \mu_3^*},$$

and

$$p_{2,3} = \frac{\mu_3 + \beta \mu_3^*}{\mu_2 - p_{2,3}} p_{2,3} - \beta \mu_1^*.$$

The case (0,1,0)-(1,1,0) is omitted as it is symmetric to the case (0,1,0)-(1,1,0).

Case (1,1,0)-(1,1,0). The expressions in (3) and (4) imply

$$\begin{aligned} \dot{p}_{1,2} &= \alpha(p_{1,3}p_{3,1} - p_{1,2}p_{2,3}) + \beta\alpha(p_{1,3}p_{3,1}^* - p_{1,2}p_{2,3}^*) \\ \dot{p}_{2,3} &= \alpha[p_{2,1}(p_{3,2} + p_{1,2}) - p_{2,3}p_{3,1}] + \beta\alpha[p_{2,1}(p_{3,2}^* + p_{1,2}^*) - p_{2,3}p_{3,1}^*] \\ \dot{p}_{3,1} &= \alpha[p_{3,2}(p_{2,3} + p_{2,1})] + \beta\alpha[p_{3,2}(p_{2,3}^* + p_{2,1}^*)] \end{aligned}$$

and

$$\begin{aligned} \dot{p}_{1,2}^* &= \alpha(p_{1,3}^*p_{3,1}^* - p_{1,2}^*p_{2,3}^*s_1^*) + \beta\alpha(p_{1,3}^*p_{3,1} - p_{1,2}^*p_{2,3}) \\ \dot{p}_{2,3}^* &= \alpha[p_{2,1}^*(p_{3,2}^* + p_{1,2}^*) - p_{2,3}^*p_{3,1}^*] + \beta\alpha[p_{2,1}^*(p_{3,2} + p_{1,2}) - p_{2,3}^*p_{3,1}] \\ \dot{p}_{3,1}^* &= \alpha[p_{3,2}^*(p_{2,3}^* + p_{2,1}^*)] + \beta\alpha[p_{3,2}^*(p_{2,3} + p_{2,1})], \end{aligned}$$

respectively. For $p_{3,1} \rightarrow \mu_3$ and $p_{3,1}^* \rightarrow \mu_3^*$. Using this result, the system collapses to

$$\begin{aligned} \dot{p}_{1,2} &= \alpha(p_{1,3}\mu_3 - p_{1,2}p_{2,3}) + \beta\alpha(p_{1,3}\mu_3^* - p_{1,2}p_{2,3}^*) \\ \dot{p}_{2,3} &= \alpha(p_{2,1}p_{1,2} - p_{2,3}\mu_3) + \beta\alpha(p_{2,1}p_{1,2}^* - p_{2,3}p_{3,1}^*) \end{aligned}$$

and

$$\begin{aligned} \dot{p}_{1,2}^* &= \alpha(p_{1,3}^*\mu_3^* - p_{1,2}^*p_{2,3}^*) + \beta\alpha(p_{1,3}^*\mu_3 - p_{1,2}^*p_{2,3}) \\ \dot{p}_{2,3}^* &= \alpha(p_{2,1}^*p_{1,2}^* - p_{2,3}^*\mu_3^*) + \beta\alpha(p_{2,1}^*p_{1,2} - p_{2,3}^*p_{3,1}) \end{aligned}$$

Since it cannot easily be reduced to a two-dimensional system, convergence properties are to be verified numerically. (Only in the two-dimensional case the topological restrictions on the structure of the orbits allow us to exploit the Jacobian to make inference on the basin of attraction of the system.)

The solution of the steady state distribution can, nevertheless, be derived analytically. Assume, for simplicity, a symmetric distribution of the three types in the two economies,

so that $\frac{\mu_i}{n} = \frac{\mu_i^*}{1-n}$, or, $\mu_i^* = \mu_i \frac{1-n}{n}$. It is also natural to take the guess that in steady state $p_{2,3}^* = p_{2,3} \frac{1-n}{n}$ and $p_{1,2}^* = p_{1,2} \frac{1-n}{n}$. Therefore, the above two-equation inventory system of the Home country in steady state satisfies

$$0 = \alpha\{(\mu_1 - p_{1,2})\mu_3 - p_{1,2}p_{2,3}\} + \beta\alpha\{(\mu_1 - p_{1,2})\mu_3 \frac{1-n}{n} - p_{1,2}p_{2,3} \frac{1-n}{n}\}$$

$$0 = \alpha\{p_{2,1}(p_{1,2}) - p_{2,3}\mu_3\} + \beta\alpha\{p_{2,1}(p_{1,2} \frac{1-n}{n}) - p_{2,3}\mu_3 \frac{1-n}{n}\}.$$

The solution is $p_{1,2} = \frac{\mu_1\mu_3}{p_{2,3}+\mu_3}$ and $p_{2,3} = \frac{1}{2}[-(\mu_1 + \mu_3) + \sqrt{(\mu_1 + \mu_3)^2 + 4\mu_1\mu_3}]$. By symmetry, the corresponding expressions in the Foreign country are $p_{1,2}^* = \frac{\mu_1^*\mu_3^*}{p_{2,3}^*+\mu_3^*}$ and $p_{2,3}^* = \frac{1}{2}[-(\mu_1^* + \mu_3^*) + \sqrt{(\mu_1^* + \mu_3^*)^2 + 4\mu_1^*\mu_3^*}]$. Note that when $\mu_i = \frac{n}{3}$, $p_{1,2} = \frac{n}{3} \frac{1}{\sqrt{2}}$, $p_{2,3} = \frac{1}{3}n(\sqrt{2} - 1)$, and $p_{3,1} = \frac{n}{3}$. Similarly, with $\mu_i^* = \frac{1-n}{3}$, $p_{1,2} = \frac{1-n}{3} \frac{1}{\sqrt{2}}$, $p_{2,3} = \frac{1}{3}(1-n)(\sqrt{2} - 1)$, and $p_{3,1} = \frac{1-n}{3}$.

A.4 The Commercial Revolution and Migration

From the CR experiment in the main text, one may conjecture that the relative decline of income in the Foreign country (main text, fig. 3.a) prompts some of its residents to migrate to the Home country. Migration can affect the results of the CR experiment through three main channels. One is through the level of the rent: Immigration could erode existing privileges, or, conversely, expand them if lords can get more people under their service. A second channel is through μ_i and μ_i^* : The distribution of the population across the three types could change. A third one is the size of the country. I discuss these channels separately.

Imagine that an equal proportion of each type migrates from the Foreign to the Home country and the level of rents does not change: μ_i , μ_i^* , r_i , and r_i^* are not altered but n goes up. A type i Foreign migrant carries over whatever good he is holding; he can produce the same good $i+1$ and enjoy the same level of rent r_i as his new Home compatriots. This means that Foreign type 1 individual loses parts of his privileges on arriving in the Home country ($r_1^* = 0.35$, but $r_1 = 0.2$). It also means that a Foreign type i , once in the Home country, finds it optimal to use the same strategies of the new compatriots, s_i . Thus, a Foreign type 1 individual becomes merchant in the Home country.

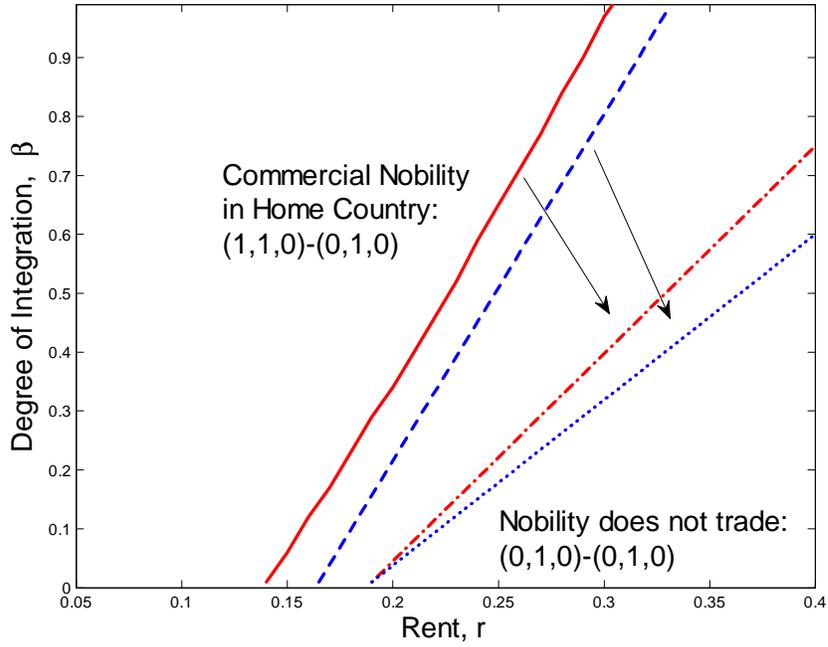
The fig. (6) in this Appendix shows that the qualitative responses to the combined shock to β and n are similar to those observed in the CR experiment. There are, however, two noticeable differences. First, after the shock, a commoner's income is closer to that of the other privileged individuals. Second, the speed of convergence toward the mean is faster. The two phenomena are related. Commoners are the ones who benefit more from the higher fluidity of the market. The arrival of immigrants not only makes the market thicker but also magnifies trade's extensive margin effect.

Reshuffling the population across types in this class of models may have important implications for the type of equilibrium that emerges and on the existence of multiple equilibria.

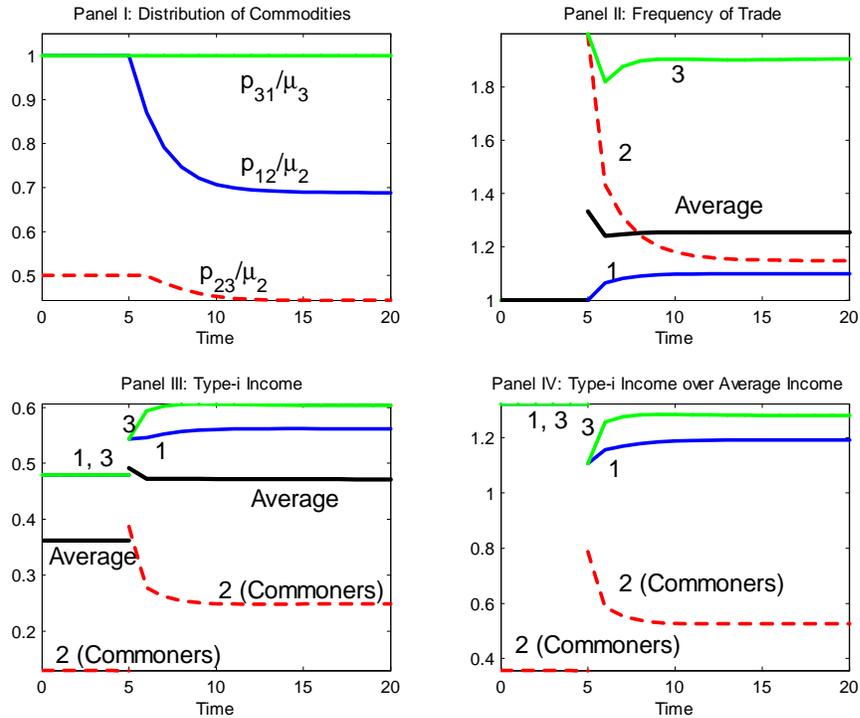
Nevertheless, in the particular scenario in which immigration increases the share of commoners, the equilibria are still those reviewed in section (3). But the conditions that separate the equilibrium $(0,1,0)$ - $(0,1,0)$ from the $(1,1,0)$ - $(1,1,0)$ and $(1,1,0)$ - $(0,1,0)$ equilibria would be different. For instance, in fig. (1.a), in the main text, the $(0,1,0)$ - $(0,1,0)$ region would gain ground from the $(1,1,0)$ - $(1,1,0)$ region. Intuitively, a larger share of commoners-merchants diminishes the nobility's gains from indirect trade. The strategy switch of the nobility observed in the CR experiment would nevertheless occur as long as β is sufficiently large to push the economy in the $(1,1,0)$ - $(1,1,0)$ region.

Finally, if migration lifts the rents in the Home country up, the nobility's switch is less likely. Internal migration from the rural areas to the new urban centers, however, would have the opposite effect of eroding the nobility's privileges, and thus of accelerating the switch.

Figure 5: Technological Shock
Panel A



Panel B



- Note: The technological shock causes u to rise from 1 to 1.5. The numbers 1-3 in Panel B.II and B.III refer to the type of good and type of agent, respectively. The frequency of trade after the shock is a ratio of the pre-shock level. The parameters' values are in Table 2.