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**Spatial Cournot Competition and Heterogeneous  
Production Cost Across Locations**

Thierry MAYER

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# Spatial Cournot Competition and Heterogeneous Production Costs Across Locations

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## Abstract:

The model developed in this paper extends the strategic location framework under Cournot competition in order to allow for different production costs across locations. The subgame perfect equilibria where two firms choose first a location and then quantities is analyzed under general production cost distributions. It appears that central agglomeration (the equilibrium under the uniform production costs distribution) only arises in the particular case where the center of the segment yields the minimal production cost. If the production cost distribution is globally convex, an agglomerated equilibrium exists at an intermediary point between the locations minimizing production cost and transport cost respectively. The conditions are also derived for the existence of symmetric dispersed location equilibria. Two specific production cost distributions are analyzed: The linear and the inverted U one. It is demonstrated that the unique equilibrium in the linear distribution case is an agglomerated equilibrium and that the inverted U distribution yields a symmetric location of firms in equilibrium.

Keywords : Strategic location choice, Cournot duopoly, Spatial Competition, Agglomeration

## Résumé :

Cet article étend le cadre d'analyse des problèmes de localisation stratégique avec concurrence à la Cournot en introduisant une hétérogénéité spatiale des coûts de production. Le résultat d'agglomération centrale des firmes obtenu avec une distribution uniforme des coûts de production n'est plus qu'un cas particulier de notre modèle. Lorsque la distribution des coûts de production est globalement convexe, il existe un équilibre aggloméré en un point intermédiaire entre le centre et la localisation qui minimise le coût de production. Les conditions d'existence d'équilibres dispersés sont également étudiées. Deux distributions spécifiques des coûts de production font l'objet d'une analyse particulière : la distribution linéaire et la distribution en U inversé. Dans le cas de la distribution linéaire, l'agglomération des firmes est un équilibre unique du jeu. Dans le cas de la distribution en U inversé, les firmes se localisent symétriquement de chaque côté du segment.

Mots-clés : Localisation stratégique, Duopole de Cournot, Concurrence spatiale, Agglomération

JEL Classification : L13, R3

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## Introduction

The motivation of this paper is to provide a theoretical framework for strategic location decisions when production costs vary in space. The influence of production costs and particularly labor-related costs on the location decision of firms has been a central issue in recent debates about the determinants and impacts of foreign investment inflows in many OECD countries. Additionally, the location choices underlying those debates were usually made by large Multinational Enterprises (MNEs) having non negligible market share in their industry and likely to be conscious of the mutual interactions of their choices. A "natural" model for the theoretical analysis of those issues would hence consider the behavior of firms faced with production cost differentials across locations and engaged in oligopolistic competition.

The analytical setting of the paper is a strategic location game with Cournot competition over a continuum of locations. This framework was initiated by Hamilton et al. (1989) and Anderson and Neven (1991). Two firms are assumed to choose first a location and then quantities in a geographical zone where consumers in several locations are to be served. The most important modification made to this basic framework is to consider that locations offer different production costs to firms. This model applies in principle to any problem of location, irrespective of the geographical scope of the study (national, regional, worldwide) : when choosing where in a town or where in the world to locate a new plant, the investor should be interested in the production cost differentials among alternative locations<sup>1</sup>.

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<sup>1</sup> While the example of country choice within a defined zone seems to fit well the model presented here where markets are segmented and costs of production vary across locations, it should be noted that the continuous space assumed here might hinder very important "discrete" aspects of international geography related to the existence of political borders between countries. Differences in regulations and labor markets for instance might yield

In the large literature on strategic location, Bertrand competition is often assumed. The traditional result of endogenous location choice with price competition is that firms isolate from each others in order to soften competition (d'Aspremont et al., 1979). Locating far apart from competitors is a way of differentiating products, which raises profits when firms face Bertrand competition. Agglomeration can only occur when an additional differentiation dimension is allowed : firms might agglomerate if the non-spatial product heterogeneity is sufficiently large (de Palma et al., 1985 ; Anderson et al. 1992).

A more recent and relatively smaller set of papers focused on location choice under Cournot competition<sup>2</sup> (Anderson and Thisse, 1988 ; Hamilton et al., 1989 ; Anderson and Neven, 1991; Gupta et al., 1997). These models have two empirically attractive features in their conclusions. First, whereas Bertrand competition yields *exclusive* sales territories for firms (consumers in each location being served only by the most efficient firm there), competition in quantities exhibits *market overlapping* with intra-industry trade which fits reality better. Second, the general result is central agglomeration for a uniform consumer distribution (Anderson and Neven, 1991). More generally an agglomerated equilibrium exists in an area where the population density is sufficiently important (Gupta et al., 1997). This result of

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important discontinuities in the production costs distribution that will not be dealt with here. Furthermore, recent empirical work in international trade has shown that borders were still a very significant impediment to trade between countries therefore also denoting of discontinuities in international trade patterns. McCallum (1995) showed that, in 1988, a representative Canadian province traded about 20 times more with another Canadian province than with an American state of equal size and distance. Head and Mayer (1999) also found evidence of strong border effects for the European Union even after the completion of the single market program. The continuum of locations assumed here therefore applies better to *intra-national* location choices.

<sup>2</sup> While price setting firms seem more realistic than quantity setting firms, the use of the Cournot model has often been justified in the industrial organization literature on the basis of the results of Kreps and Sheinkman (1983) showing that capacity commitment followed by price competition yielded the Cournot outcome. In our spatial framework, however, Anderson and Neven (1991) point out that there is an additional difficulty arising from the fact that firms serve several locations from a single plant. For the Cournot competition to be a good approximation of real economic decision-making, we thus need the capacity of the plant to be inflexibly determined *and* the allocation of output to each location to be less flexible than price setting. This assumption that can be justified by the existence of transportation costs making reallocation of output across locations more difficult than price adjustments.

geographical concentration of the economic activity is of course very attractive since it corresponds to results in many related fields of the economic theory (see Fujita and Thisse, 1996 for a survey on agglomeration economics). Furthermore spatial clustering of firms has been observed widely in empirical studies both at the intra-national level (Ellison and Glaeser 1997, Henderson and Kuncoro, 1996) and for international location choice (Wheeler and Mody, 1992 ; Head et al. 1995).

To our knowledge, the only comparable attempt of accounting for production costs heterogeneity in location choice is the study of location dynamics under Cournot competition by Combes (1997). Production costs differentials are however not central in this paper which furthermore limits the analysis to two locations. The present contribution can also be viewed as a generalization of the latter that captures a wide range of spatial cost distributions. Sarkar et al. (1997) also have different production costs in a model where Cournot oligopolists choose location over a network of separated locations but they focus on the conditions of existence of a Nash equilibrium in this game.

The main results of the present paper challenge the central agglomeration result obtained by Anderson and Neven (1991) in two ways. First, it is shown that agglomeration occurs at the center only for specific cases: that is, either for the uniform production cost assumed in the original framework or when production costs are minimized in the center. Second, firms do not agglomerate under a production cost distribution often referred to as very frequent in reality : a globally concave distribution with highest production costs in the center. This paper is not the first to question the central agglomeration result of spatial Cournot competition. Pal (1998) recently demonstrated that agglomeration of Cournot competitors was crucially dependent on the type of geography supposed. Two firms playing Cournot on a circular set of

locations locate equidistant from each other while they would agglomerate at the center of a line. This result emphasizes the importance of the existence of a central place in the geography assumed for the agglomeration result to hold. Gupta et al. (1997), introducing spatial heterogeneity in the distribution of consumers also show that the center would host an agglomeration of firms only for very specific cases. They additionally provide examples of spatial differentiation for convex distributions of consumers.

The remainder of the paper is as follows : Section 1 develops the basic model of strategic location with different costs of production across locations. The second section studies agglomerated equilibria and more particularly states the conditions for the existence of a central agglomeration equilibrium. The central agglomeration result appears as a particular case arising with production costs being either uniform across space or minimized at the center of the segment. Section 3 studies the possibility of dispersed equilibria and the conditions under which the agglomerated equilibrium is unique.

## **1. A simple model of strategic location with heterogenous production costs**

The geography assumed in this paper and defining the set of possible locations is a continuous line of length  $l$ , each point representing a market for the product of the duopolists<sup>3</sup>. The model is a two stage game where two firms (labeled 1 and 2) first choose simultaneously locations  $z_1$  and  $z_2$  for their respective plant (assumed to be unique). In the second stage of the game, firms compete in quantities on each of the locations. The subgame perfect Nash equilibrium is found by backward induction: The equilibrium quantities and profits are solved in each

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<sup>3</sup> We will use the term location throughout the paper to designate a point on the line. Each of these points is also a market where the firms sell their product.

location for given  $z_1$  and  $z_2$ . Those profits are then used to find the first stage location equilibrium.

Markets are assumed segmented with firms using discriminatory pricing. This hypothesis has been said to be specially relevant for the international location case where arbitrage costs are likely to be high<sup>4</sup> and was largely used in international trade theory, notably by Brander and Krugman (1983), Smith (1987) or Horstman and Markusen (1992). Anderson and Neven (1990) and Gupta et al. (1997) among many others used this assumption in strategic location theory. Market segmentation enables profit maximizing behavior to be considered separately for each market. It will appear below that, in equilibrium, there is no possibility of arbitrage across markets despite discriminatory pricing practiced by firms.

When serving a particular location  $z$  from  $z_1$ , firm 1 will incur a linear transport cost  $t|z - z_1|$ . The quantity  $q_1(z; z_1, z_2)$  sold on location  $z$  also depends on the unit cost of production of firm 1,  $c(z_1)$ , with  $c(\cdot)$  representing the distribution function of unit costs of production over the line. When choosing a location, a firm hence chooses its distance from *each location* and its production cost for *all locations*. The crucial assumption is made that the location of the firm in either location does not change the production cost conditions of this location<sup>5</sup>. The underlying hypothesis is thus that firms are large relative to the industry but that the industry is small relative to the entire economy.

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<sup>4</sup> In a recent report on the effects of European integration (European Commission, 1996 p 135), the European Commission stated that, despite an observed convergence, the price differentials remained substantial across European countries even in highly traded industries.

<sup>5</sup> For a different assumption where the location of firms drives up income and wages of the location hosting the firms, see Combes (1997).



Assuming that  $z$  is characterized by an inverse demand function  $p(z; z_1, z_2) = 1 - Q(z; z_1, z_2)$  where  $Q(z; z_1, z_2) = q_1(z; z_1, z_2) + q_2(z; z_1, z_2)$  denotes total quantity sold and consumed on location  $z$ , we can write the profit of firm 1 in  $z$  as a function of the locations of the two firms:

$$(1.1) \pi_1(z; z_1, z_2) = [1 - Q(z; z_1, z_2) - t|z - z_1| - c(z_1)]q_1(z; z_1, z_2)$$

A similar expression holds for firm 2 and standard calculations yield the equilibrium quantity and profit of firm 1 in  $z$  :

$$(1.2) q_1(z; z_1, z_2) = \frac{1 - 2t|z - z_1| + t|z - z_2| - 2c(z_1) + c(z_2)}{3}$$

$$(1.3) \pi_1(z; z_1, z_2) = \left[ \frac{1 - 2t|z - z_1| + t|z - z_2| - 2c(z_1) + c(z_2)}{3} \right]^2$$

The market clearing price in  $z$  is

$$p(z; z_1, z_2) = \frac{1 + t|z - z_1| + t|z - z_2| + c(z_1) + c(z_2)}{3}$$

Delivered price is thus an increasing function of total transport cost. We suppose without loss of generality throughout the paper that  $z_1 \leq z_2$ . For all  $z \in [z_1, z_2]$ , there is uniform delivered pricing and its level is decreasing with the proximity of firms as  $p(z; z_1, z_2) = [1 + t(z_2 - z_1) + c(z_1) + c(z_2)]/3$ . For all locations outside the firms' interval, say  $z \in [0, z_1]$ , the price increases with distance from firms as

$p(z; z_1, z_2) = [1 - 2tz + t(z_1 + z_2) + c(z_1) + c(z_2)]/3$ . As underlined by Hamilton et al. (1989) there is therefore no arbitrage opportunity in this setting. This is obvious in the area where the delivered price is uniform; but buying products from this area to sell them in the remote ones is not profitable either because the price charged by firms there rises by only two thirds of the additional transport cost.

As usual in Cournot competition, equilibrium market shares and profits are decreasing functions of own costs and increasing functions of the competitor's costs. Strategic location models are concerned with firms choosing their distance to each location and therefore minimizing overall transport cost of shipping their product to consumers in order to maximize overall profits. This is the key reason why firms agglomerate in a central location when no other costs are considered. This behavior is complicated here by geographical variations in the cost of production. Intuitively, firms should try to minimize both costs in equilibrium. However, we will see below that the two costs do not enter symmetrically in the location decision of the firm.

We wish to focus on situations where all locations can be chosen in equilibrium, so that there are no locations so remote (with respect to transport cost) or with such a high production cost that a firm located there would find it unprofitable to serve consumers living in some other location. The simplest conditions insuring that firms in every location serve all locations are that:

the transport cost is sufficiently low  $tl < \frac{1 - 2c(z_1) + c(z_2)}{2}$ ,

and  $tl > 0$  which implies that the cost distribution needs to be sufficiently "flat":

$$\frac{1 + c(z_2)}{2} > c(z_1) > 2c(z_2) - 1 \quad \forall z_1, z_2$$

We assume that these two conditions are met throughout the rest of the paper which proves particularly important for specification of equilibria. This standard assumption is not innocuous. First, it implies that a rival could not be preempted through location choice in a sequential setting of the game, as profitable entry would always be possible for a follower. Second, the complete overlapping of firms' markets reduces the incentives to differentiate spatially. Here, contrary to models with Bertrand competition, firms cannot secure monopoly positions by locating at the endpoints, which increases the incentive to agglomerate.

The overall profit of firm 1 is simply the sum of its profits on each location:

$$(1.4) \quad \Pi_1(z_1, z_2) = \int_0^l \pi_1(z; z_1, z_2) dz$$

Firm 1 chooses a location on  $[0, l]$  in order to maximize expression (1.4) and we search for a subgame perfect equilibrium where  $z_1$  is optimal given the following quantity subgame.

The first order condition for firm 1 to choose a profit maximizing location is:

$$\begin{aligned} \frac{\partial \Pi_1}{\partial z_1} = & -\frac{4(t + c'(z_1))}{9} \int_0^{z_1} 1 - 2t(z_1 - z) + t(z_2 - z) - 2c(z_1) + c(z_2) dz \\ (1.5) \quad & + \frac{4(t - c'(z_1))}{9} \int_{z_1}^{z_2} 1 - 2t(z - z_1) + t(z_2 - z) - 2c(z_1) + c(z_2) dz \\ & + \frac{4(t - c'(z_1))}{9} \int_{z_2}^l 1 - 2t(z - z_1) + t(z - z_2) - 2c(z_1) + c(z_2) dz = 0 \end{aligned}$$

Which after integration and manipulation yields:

$$(1.6) \quad \frac{9}{4t} \frac{\partial \Pi_1}{\partial z_1} = \left[ -\frac{(1-2c(z_1)+c(z_2))l}{t} - z_2^2 + 2z_1^2 - l \left( 2z_1 - z_2 - \frac{l}{2} \right) \right] c'(z_1) \\ + \left[ (1-2c(z_1)+c(z_2))(l-2z_1) + t \left( (z_1 - z_2)^2 + l(2z_1 - z_2 - \frac{l}{2}) \right) \right] = 0$$

The second order condition is:

$$(1.7) \quad \frac{9}{4t} \frac{\partial^2 \Pi_1}{\partial z_1^2} = 2 \left( \frac{c'(z_1)l}{t} - 2(l-2z_1) \right) c'(z_1) \\ + \left[ -\frac{(1-2c(z_1)+c(z_2))l}{t} - z_2^2 + 2z_1^2 - l \left( 2z_1 - z_2 - \frac{l}{2} \right) \right] c''(z_1) \\ - 2 \left[ (1-2c(z_1)+c(z_2)) + t(z_2 - z_1 - l) \right] < 0$$

Similar expressions hold for firm 2. The second term of equation (1.6) is exactly the same term found when developing the Anderson and Neven (1991) model with linear transport cost. If the production cost is identical across locations and firms, (1.6) reduces to the second term and Anderson and Neven (1991) demonstrated that the unique equilibrium is agglomeration of firms at the center of the segment. The first thing to study is thus the potential persistence of this equilibrium in the present setting.

The second term may be called the "transport cost effect" separated from the "production cost effect" of the first term. We will see below that the effect of this term on the central agglomeration result depends crucially on whether a move towards the center of the segment lowers or raises the production cost.

## 2. Agglomerated equilibria

In order to show the tendency of firms to choose the same location, one needs to study the different incentives faced by firm 1 holding the location of its rival fixed. So as to compare our results with the preceding ones, let us suppose that firm 2 is located at the center of the segment. The first order condition of firm 1 when firm 2 is located at  $l/2$  simplifies to:

$$(2.1) \quad \frac{9}{4t} \frac{\partial \Pi_1}{\partial z_1} \bigg|_{z_2=l/2} = A \cdot c'(z_1) + B \cdot (l - 2z_1) = 0$$

$$\text{Where } A = \left[ -l \left( \frac{(1 - 2c(z_1) + c(l/2))}{t} + 2z_1 - \frac{3}{4}l \right) + 2z_1^2 \right],$$

$$\text{and } B = \left[ (1 - 2c(z_1) + c(l/2)) - \frac{t}{4}(3l + 2z_1) \right]$$

It can be checked that, if the assumptions guaranteeing complete overlapping of firms' market areas are satisfied,  $A$  is negative<sup>6</sup> and  $B$  is positive over the whole range  $[0, l/2]$ . It is instructive to decompose this result in terms of transport cost and production cost effects.

Suppose first that production costs have a uniform distribution such that  $c(z_1) = c(z_2) = c \quad \forall z_1, z_2$ . This leaves us with the Anderson and Neven (1991) model, which is the transport cost effect here, and (2.1) becomes:

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<sup>6</sup> In fact,  $A$  is negative over the whole range in a general way. It is shown in the appendix that the corresponding expression for  $A$  in the general form of the first order condition (1.6) is negative whichever locations firms choose.

$$(l - 2z_1) \left[ 1 - c - \frac{t}{4} (3l + 2z_1) \right] = 0$$

This can only be obtained when firm 1 also locates at  $l/2$ . What does the introduction of non-uniform production costs change to this equilibrium? The transport cost effect remains virtually unchanged but is now counterbalanced by the production cost effect. The coefficient  $A$  being negative, this latter effect gives an incentive to locate at the minimal production cost.

The transport cost effect in (2.1) therefore unambiguously pushes firms to agglomerate at the center as  $B \cdot (l - 2z_1)$  is positive until  $l/2$  and negative thereafter. Additionally to that tendency to reduce transport costs, firms also try to locate as close as possible to the point where production costs are minimized. Consider the simple production costs distribution function in figure 1, which exhibits a single minimum. For the remainder of Section 2, we focus on production costs distribution having this property.

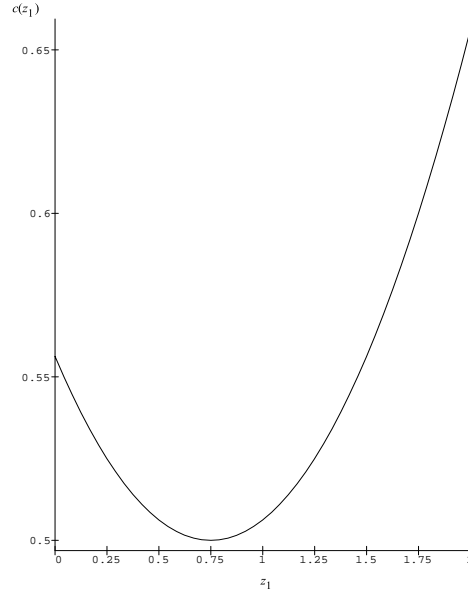


Figure 1:  $c(z_1) = 1/2 + 1/10(z_1 - 3l/8)^2$ ,  $l = 2$

From 0 to  $3l/8$  ( $3l/8 = 0.75$  is the location minimizing production costs in figure 1 since we choose  $l=2$ ), both effects are positive because neither cost is minimized. Between  $3l/8$  and  $l/2$ , the effects conflict with a trade-off between the two costs. Between  $l/2$  and  $l$ , both effects are negative. The reasoning is symmetric for a production cost minimizing location to the right of the central location. Hence, the central agglomeration equilibrium is here conditioned by the production cost prevailing at this particular location. Proposition 1 states that two firms contemplating location in the centre would have an incentive to move towards the location with the minimal production cost.

**Proposition 1 :** *For non-uniform production cost distributions, central agglomeration can only be sustained if  $l/2$  is the locally cost minimizing location. If the production cost distribution is globally convex with  $l/2$  yielding the minimum production cost, central agglomeration is the unique equilibrium.*

Proof : i) Evaluating (2.1) when firm 1 also locates at  $l/2$  gives:

$$\frac{9}{l} \frac{\partial \Pi_1}{\partial z_1} \Big|_{z_1=z_2=l/2} = c'(l/2)(tl - 4[1 - c(l/2)]) = 0$$

which can only be satisfied when  $c'(l/2) = 0$ , given the assumption on transport costs<sup>7</sup>.

ii) It appears that, with a production cost distribution having its global minimum in the center, (2.1) is uniformly positive between 0 and  $l/2$  and uniformly negative between  $l/2$  and  $l$ . ■

This result states that the *central* agglomeration of Cournot oligopolists is very sensitive to the assumption (made in all preceding contributions using that theoretical framework) that production costs are uniform across space:  $c'(z) = 0, \forall z$ . It is shown here that, when the production costs distribution is convex, any distribution where  $c'(l/2) \neq 0$  cannot sustain central agglomeration as an equilibrium. Although agglomeration remains a possible equilibrium under more general distributions, the *central* agglomeration result is thus strongly dependent on the restrictive hypothesis that production costs are identical in all locations. It is to be noted that Gupta et al. (1997) also find that under non - uniform consumers distribution, central agglomeration only arises as a very specific case. The more general conditions for an agglomerated equilibrium are given in proposition 2.

**Proposition 2 :** *For globally convex production cost distributions, an agglomerated equilibrium exists at a location between the minimum production cost and the center. Furthermore, there exists only one agglomerated equilibrium for this type of production cost distribution.*

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<sup>7</sup> Of course  $c'(l/2) = 0$  could also mean that production costs are maximized at  $l/2$  for globally concave production costs distributions. This case is treated in section 3 of the paper.



Proof: The first order condition for firm 1 when both firms are located in the same location  $\bar{z}$  is:

$$(2.2) \quad \frac{9}{4t} \frac{\partial \Pi_1}{\partial \bar{z}} \Big|_{z_1=z_2=\bar{z}} = \left[ \bar{z}^2 - l \left( \frac{1-c(\bar{z})}{t} + \bar{z} - \frac{l}{2} \right) \right] c'(\bar{z}) + (l-2\bar{z}) \left( 1 - c(\bar{z}) - \frac{tl}{2} \right) = 0$$

and the second order condition is:

$$(2.3) \quad \begin{aligned} \frac{9}{4t} \frac{\partial^2 \Pi_1}{\partial \bar{z}^2} &= 2 \left( \frac{c'(\bar{z})l}{t} - 2(l-2\bar{z}) \right) c'(\bar{z}) \\ &+ \left[ -\frac{(1-c(\bar{z}))l}{t} + \bar{z}^2 - l \left( \bar{z} - \frac{l}{2} \right) \right] c''(\bar{z}) \\ &- 2[1 - c(\bar{z}) - tl] < 0 \end{aligned}$$

Here again, the coefficients of  $c'(\bar{z})$  and  $(l-2\bar{z})$  in (2.2) are respectively negative and positive over all possible locations. This ensures that firms cannot agglomerate where  $c'(\bar{z}) = 0$ . Firms then choose to settle their plant in the same location  $\bar{z}$  located between  $l/2$  and the location where production cost is minimized for globally convex production cost distributions, such as the one in figure 1. A necessary condition for firms to agglomerate in  $\bar{z}$  is:

$$(2.4) \quad c'(\bar{z}) = - \frac{t(l-2\bar{z}) \left( 1 - c(\bar{z}) - \frac{tl}{2} \right)}{t\bar{z}^2 - l \left( 1 - c(\bar{z}) + t\bar{z} - \frac{tl}{2} \right)}$$

The profit function is increasing until condition (2.4) is met and decreasing thereafter, ensuring that there is only one agglomerated equilibrium. It is not guaranteed that this

agglomerated equilibrium is the unique equilibrium of the game for any globally convex distribution of production costs. We however prove in the following section that the agglomerated equilibrium described here is the unique equilibrium of the game for at least one distribution covered by proposition 2: the linear one. ■

It can be verified in (2.4) that firms agglomerate at  $l/2$  only if this point minimizes production costs. Another interesting feature arises in the extreme case when the minimum value of the production cost over the set of possible locations is at an endpoint of the segment, say 0, with  $c'(0) > 0$ . Then the firms agglomerate at 0 if  $c'(0) \geq t$  that is, if the increase in production cost incurred from moving towards the center is higher than the unit transport cost.

While agglomeration of firms is still possible in this setting, heterogeneous production costs make central agglomeration a very specific case. In fact, equilibrium agglomeration can occur far from the center, as far as an endpoint of the line, when production costs are not uniform.

Proposition 2 therefore states that firms can agglomerate at some intermediate point between the location minimizing transport costs and the location minimizing production costs. Two questions then arise: 1) Are total unit costs minimized in the agglomerated equilibrium? In other words, how does the trade-off between the two costs work? 2) How does the location chosen by duopolists in the agglomerated equilibrium compare with the location that a monopolist would choose under the same conditions?

1) When they are faced with uniform production costs, firms agglomerate at the center of the segment which is the cost minimizing location under the linear transport cost assumption. As demonstrated in the appendix, the conclusion is different here. The location chosen by both

firms does not minimize total costs. In fact, firms agglomerate nearer to the minimal production cost than necessary to minimize total unit cost. This means implicitly that production costs are more important than transport costs in the location chosen by firms. To see why this is the case, consider an hypothetical (non-equilibrium) situation where both firms are located at  $3l/8$  in figure 1. Then, moving to the right of this point for a firm raises production costs on *all* locations. By contrast, the effect of this move on the transport cost to each location depends on whether the firm gets closer or more distant from it. Being near the production cost minimizing location is therefore relatively more important than being in a central place.

2) The question of the monopoly location and how it relates to the trade off between the two costs is treated in Proposition 3:

**Proposition 3:** *The agglomerated equilibrium stated in Proposition 2 occurs at the same location that would be chosen by a discriminating monopoly. Under uniform costs of production, agglomeration occurs at the median of the quantities sold. Under globally convex production costs distribution, the agglomerated equilibrium is located closer to the point minimizing production costs.*

Proof: In the present model, a monopoly discriminating between locations would maximize the following profits :

$$\Pi_m = \int_0^{z_m} \left[ \frac{1 - t(z_m - z) - c(z_m)}{2} \right]^2 dz + \int_{z_m}^l \left[ \frac{1 - t(z - z_m) - c(z_m)}{2} \right]^2 dz$$

which once solved for the optimal location yields exactly the same expression as (2.4) when looking for the optimal location  $z_m$ . Agglomeration of oligopolists occurs at the precise location a monopolist would choose. This result is in fact robust to various specifications. The models by Anderson and Neven (1991) and Gupta et al. (1997) which are particularly related to the present work exhibit the same property (proof available upon request). The derivative of the monopoly profit equation gives a clear insight of the trade off between the two costs:

$$\begin{aligned} \frac{\partial \Pi_m}{\partial z_m} = & t \left\{ - \int_0^{z_m} \left[ \frac{1-t(z_m-z)-c(z_m)}{2} \right] dz + \int_{z_m}^l \left[ \frac{1-t(z-z_m)-c(z_m)}{2} \right] dz \right\} \\ & - c'(z_m) \left\{ \int_0^{z_m} \left[ \frac{1-t(z_m-z)-c(z_m)}{2} \right] dz + \int_{z_m}^l \left[ \frac{1-t(z-z_m)-c(z_m)}{2} \right] dz \right\} = 0 \end{aligned}$$

It is straightforward in this equation that with uniform production costs, the optimal location choice of the monopoly (and hence of the duopolists) is the median of the quantities sold<sup>8</sup>. For a globally convex distribution of production costs, it can be seen from the respective coefficients on  $t$  and  $c'(z_m)$  in  $\partial \Pi_m / \partial z_m$  that the firms locate closer to the point where production costs are minimized. ■

The derivative of a monopolist's profits with respect to its location shows that the production costs are given a higher weight because they apply to the total quantity sold by the firm. By contrast, the transport cost effect of a firm moving along the line from location 0 to location  $l$  is negative for all locations on its left and positive for all locations on its right.

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<sup>8</sup> For a comparison between the optimal location of a monopoly under different pricing policies, see Anderson et al. (1992) section 8.4

Getting back to the duopoly, an additional way to see the interaction between the two costs is to look at the effect of an increase in transport costs on the "centrality" of  $\bar{z}$ , the location where the two firms agglomerate. Although the precise analytical expression for  $\bar{z}$  is quite intractable, it is possible to identify more precisely the effect of the trade-off between transport and production costs on the distance between the center and the location where firms agglomerate. Expression (2.2) is a function of  $\bar{z}$  and  $t$  which can be defined as  $g(\bar{z}, t) = 0$ . Differentiating  $g$  gives the influence of the unit transport cost on the location where the agglomeration equilibrium occurs :

$$(2.5) \frac{d\bar{z}}{dt} = - \frac{\frac{\partial g(\bar{z}, t)}{\partial t}}{\frac{\partial g(\bar{z}, t)}{\partial \bar{z}}}$$

Provided the second order condition is satisfied, the sign of (2.5) only depends on the sign of the expression :

$$\frac{\partial g(\bar{z}, t)}{\partial t} = \frac{4}{9} \left[ \left( \bar{z}^2 - l \left( \bar{z} - \frac{l}{2} \right) \right) c'(\bar{z}) + (l - 2\bar{z})(1 - c(\bar{z}) - tl) \right]$$

which is unambiguously positive as long as the first order condition is satisfied which guarantees that  $c'(\bar{z}) > 0$  and that  $\bar{z} < l/2$  for the convex distribution of figure 1. The intuition behind this result is very simple : an increase in transport costs, leaving the production cost distribution unchanged raises the need for firms to be near final demand and hence displaces the equilibrium towards the center.

### 3. Dispersed equilibria

Before proceeding to cases where firms might disperse in equilibrium, we give an example of production costs distribution for which the agglomerated equilibrium described in Proposition 2 is also the unique equilibrium of the game.

#### *Uniqueness of equilibrium*

The preceding section has already ruled out several equilibria under globally convex distributions of production costs. It has been demonstrated in particular that dispersed equilibria where one firm located in the center and the other in the lowest production cost location does not exist. The following proposition states that when production costs increase linearly from one end of the segment to the other, agglomeration of firms at the point defined in proposition 2 is the unique equilibrium.

**Proposition 4 :** *With a linear production cost distribution such that  $c(z) = a + bz$ , agglomeration in the location satisfying (2.4) is the unique equilibrium.*

Proof : the uniqueness of the agglomerated equilibrium is derived from an analysis of the reaction functions. For uniqueness, it is sufficient to show that the slopes of the reaction functions have the same sign and that one is everywhere steeper than the other, ensuring that they intersect only once. First note that the first order condition can only be satisfied between the location with the minimal production cost and  $l/2$ , which means that both firms must be located to the left of  $l/2$  and in a location where the slope of the production cost function is

positive in equilibrium when the minimum of the production cost distribution is on the left of  $l/2$  (see the analysis of the range of reaction functions in the appendix). Then, all distributions exhibiting a single conflict range are good candidates for a unique agglomerated equilibrium as we know that such an equilibrium exists in this range and that the reaction functions are restricted to be within this range. This obviously restricts the number of other possible intersections of the reaction functions. However, this property becomes fairly complex to demonstrate with a general distribution, so we restrict ourselves to the simpler although less general linear case where  $c(z_1) = cz_1$ .

The first order condition (1.6) implicitly defines the reaction function of firm 1 :  $z_1 = R_1(z_2)$  which can be studied using the implicit function theorem. We know that:

$$(3.1) R'_1(z_2) = - \frac{\frac{\partial^2 \Pi_1}{\partial z_1 \partial z_2}}{\frac{\partial^2 \Pi_1}{\partial z_1^2}}$$

Provided that the second order condition is satisfied<sup>9</sup>, the slope of the reaction function therefore depends on the sign of the cross partial derivative and hence on whether locations are strategic substitutes or complements (Tirole, 1988). The general form of the cross partial derivative is :

$$(3.2) \frac{9}{4t} \frac{\partial^2 \Pi_1}{\partial z_1 \partial z_2} = \left( l - 2z_2 - \frac{c'(z_2)l}{t} \right) c'(z_1) + \left( c'(z_2)(l - 2z_1) + t(2z_2 - 2z_1 - l) \right)$$

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<sup>9</sup> Which is guaranteed as :  $\frac{9}{4t} \frac{\partial^2 \Pi_1}{\partial z_1^2} = 2 \left( \frac{cl}{t} - 2(l - 2z_1) \right) c - 2[ (1 - 2cz_1 + cz_2) + t(z_2 - z_1 - l) ] < 0$ . Indeed, this expression is uniformly increasing in  $z_1$  and decreasing in  $z_2$  which means that if it is to be positive in the relevant range where  $z_1 < z_2$ , it has to be positive at a point where  $z_1 = z_2$  (that is, at an agglomerated

Assuming a linear production cost distribution  $c(z_1) = cz_1$ , expression (3.2) becomes:

$$(3.2') \quad \frac{9}{4t} \frac{\partial^2 \Pi_1}{\partial z_1 \partial z_2} = (t - c) \left[ l \left( \frac{c}{t} - 1 \right) + 2z_2 \right] - 2(t + c)z_1$$

Recall that to have a solution inside the segment here we need to set  $t > c$ . The only positive element in (3.2') is then  $2z_2$  and inspection of this expression shows that the cross partial derivatives are generally negative (they can be positive if  $z_2$  is very near from  $l/2$  and  $z_1$  very near from 0) and always have the same sign as the cross partial derivative of firm 2 is :

$$(3.2'') \quad \frac{9}{4t} \frac{\partial^2 \Pi_2}{\partial z_1 \partial z_2} = \left( l - 2z_1 - \frac{c'(z_1)l}{t} \right) c'(z_2) + (c'(z_1)(l - 2z_2) + t(2z_2 - 2z_1 - l))$$

It is thus clear that cross partial derivatives have the same value. The relative steepness of the reaction functions will depend only on the difference between the second derivatives, which is:

$$\frac{\partial^2 \Pi_1}{\partial z_1^2} - \frac{\partial^2 \Pi_2}{\partial z_2^2} = \frac{56tc}{9} (z_1 - z_2) < 0 \quad \forall z_1 \leq z_2$$

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equilibrium). We know from the appendix that the agglomerated equilibrium always satisfies the second order condition, hence this is also true for the linear distribution.



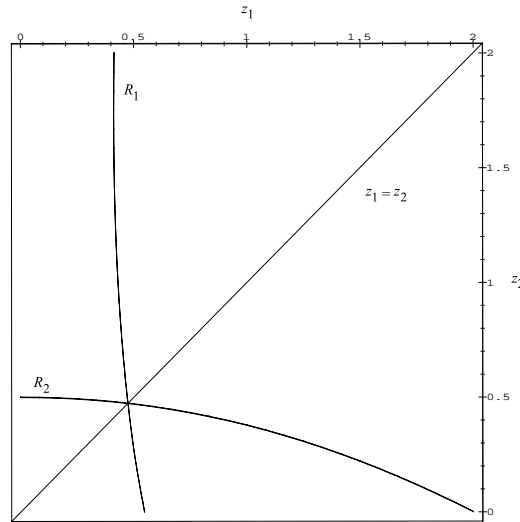


Figure 2 : A unique agglomerated equilibrium with a linear production cost distribution

A reaction function being everywhere steeper than the other and both reaction functions being always of the same sign, the agglomerated equilibrium is unique when the distribution of production costs is linear. Figure 2 illustrates the uniqueness of equilibrium in the linear distribution case. Reaction functions are drawn with parameters values of  $c = 1/16$  and  $t = 1/8$ .

■

#### *The symmetric case*

Proposition 2 states that for globally convex production cost distributions, there exists one (and only one) agglomerated equilibrium. This relies on the fact that the transport and the production cost effects of (2.2) conflict on a single interval. There are distributions of production costs where the two effects conflict on more than a single interval exhibiting possibility of multiple agglomerated equilibria. Consider the following hypothetical globally concave production cost distribution :

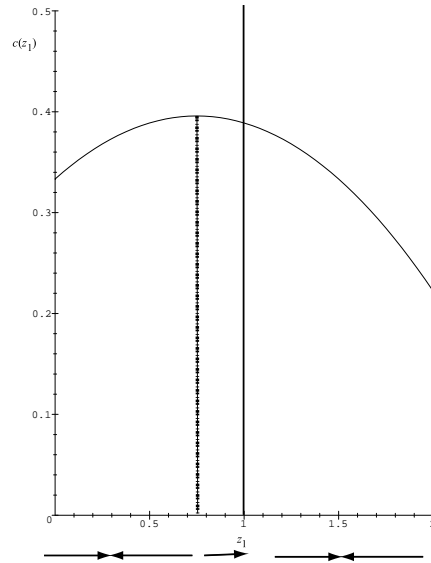


Figure 3 : Possibility of multiple equilibria

There are here two distinct intervals where (2.2) can be verified. Consequently, the two “external” parts of the segment are possible areas for an agglomerated equilibrium. It is however also possible that under this concave distribution, firms choose to differentiate spatially, each of them locating in a different part of the segment where the two effects conflict. We will now study these two possibilities of location equilibrium under a simple globally concave distribution: an inverted U distribution symmetric around  $l/2$ .

If the production cost distribution function is strictly globally concave and symmetrically decreasing around  $l/2$ , then a natural equilibrium to study is a symmetric one with each firm locating on one side of the segment. We also study the potential multiplicity of equilibria with coexisting symmetric and agglomerated equilibria.

**Proposition 5:** For a strictly globally concave distribution of production costs symmetric around  $l/2$ , there exists a single symmetric equilibrium defined by (3.4).

Proof: The first order condition for such an equilibrium is given by :

$$(3.3) \quad \left. \frac{9}{4t} \frac{\partial \Pi_1}{\partial z_1} \right|_{z_2=l-z_1} = \left[ z_1^2 - l \left( \frac{1-c(z_1)}{t} + z_1 - \frac{l}{2} \right) \right] c'(z_1) + (l-2z_1) \left( 1-c(z_1) - \frac{t}{2}(l+4z_1) \right) = 0$$

Over the relevant range ( $0 < z_1 < l/2$ ), the first term is negative and the second positive, which yields a single symmetric dispersed equilibrium satisfying the condition :

$$(3.4) \quad c'(z_1) = - \frac{t(l-2z_1) \left( 1-c(z_1) - \frac{t}{2}(l+4z_1) \right)}{tz_1^2 - l \left( 1-c(z_1) + tz_1 - \frac{tl}{2} \right)} \quad \blacksquare$$

for this inverted U distribution, there are several location configurations depending on the value of the unit transport cost. The following proposition summarizes these configurations :

**Proposition 6 :** For a strictly globally concave distribution of production costs symmetric around  $l/2$ ,

- i) if  $c'(0) > t$  : firms locate at the endpoints of the segment
- ii) if  $t^* > t > c'(0)$  : firms locate symmetrically

iii) if  $t > t^* > c'(0)$  : 1 unstable agglomerated equilibrium coexist with a stable symmetric one

Proof :

i) if  $c'(0) > t$ , expressions (2.4) and (3.4) show that firms locate at the endpoints of the segment both in the agglomerated and symmetric equilibrium configurations. However firms will always prefer to be separated in this case. The intuition to this result is very simple : as unit transport costs are very low compared to production cost, firms want to be the closest possible to the minimal production cost which in this case is *either* endpoint of the segment. However if firm 1 were to be located with firm 2 in  $l$ , it could improve its profits by locating in 0 as this would lower the overall level of competition without raising its production cost. Formally,

$$\Pi_1(0, l) - \Pi_1(l, l) = \frac{1}{9} \left( \int_0^l [1 - t(l - z) - c(l)]^2 dz - \int_0^l [1 + t(l - 3z) - c(l)]^2 dz \right) = \frac{1}{9} \left( \frac{2}{3} t^2 l^3 \right) > 0$$

Agglomeration at  $l$  is therefore not an equilibrium but it is still a local maximum of the profit function. Consider the general form of a symmetric globally concave distribution function :  $c(z) = \alpha z[l - z]$ . When firm 2 is located at  $l$ , the first order condition of firm 1 is

$$\frac{9}{4t} \frac{\partial \Pi_1}{\partial z_1} \Big|_{z_2=l} = (l - 2z_1)(1 - 2\alpha z_1(l - z_1)) \left( 1 - \frac{\alpha l}{t} \right) + \frac{\alpha(l - 2z_1)^3 - t(l^2 - 2z_1^2)}{2} = 0$$

The expressions (2.4) and (3.4) respectively give us the conditions for the existence of an agglomerated equilibrium at  $l$  and a symmetric equilibrium with firm 1 at 0 and firm 2 at  $l$ .

The conditions are the same (it is here  $\alpha l = t$ ) and we can see that the first order condition is verified under this condition either with the agglomeration or maximal differentiation. It is also very easily verified that the second order condition is verified in both cases. Hence, if firm 2 is located at  $l$ , firm 1 satisfies *locally* its first and second order conditions by agglomerating at  $l$ , but it can always do better by locating at the other endpoint.

ii) if  $t^* > t > c'(0)$ , the existence of the symmetric equilibria inside the segment has already been demonstrated in Proposition 5. Following the preceding paragraph, the non-existence of the agglomerated equilibria can be demonstrated. The symmetric distribution gives an incentive to firms agglomerated on one side of the segment to unilaterally relocate to the other half of the segment in order to lower the competition level while keeping their production cost constant<sup>10</sup>. In the paragraph just above it is seen that this force drives firms to prefer maximal differentiation to agglomeration at one of the endpoints. If the agglomerated equilibria are to exist, it must be the case that, at some point nearer to  $l/2$ , this relation is no longer true. The precise point  $\bar{z}$  would be defined such that this expression becomes positive:

$$\Pi_1(l - \bar{z}, l - \bar{z}) - \Pi_1(\bar{z}, l - \bar{z}) = -\frac{1}{9} \frac{2t^2}{3} (l + 4\bar{z})(l - 2\bar{z})^2$$

It is thus verified that firms always prefer symmetric differentiation to agglomeration in the case of a symmetric distribution. Hence agglomerated Nash equilibria do not exist on either side of the segment in that case as firms always have an incentive to deviate.

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<sup>10</sup> The author is grateful to Keith Head for raising this point at an early stage of the paper.

iii) The critical value  $t^*$  defines the value of the unit transport cost for which firms might agglomerate at the *center* of the segment. With a symmetrically decreasing distribution of production costs around  $l/2$ , the first order condition is of course verified for both firms if they locate at  $l/2$ . We thus need to study the situation where the competitors locate in the central and most costly location. Furthermore, we already know from the end of section 2 that an increase in the unit transport cost drives the agglomerated equilibria towards the center. For that point to be a maximum of the profit function, the unit transport cost has to be so high that the second order condition is verified at  $l/2$  which is :

$$(3.5) \left[ -\frac{(1-c(l/2))l}{t} + \frac{l^2}{4} \right] c''(l/2) - 2[1 - c(l/2) - tl] < 0$$

which when taking the relevant root involves the following value for  $t$  :

$$t > t^* = \frac{-c''\left(\frac{l}{2}\right)l^2 + 8 - 8c\left(\frac{l}{2}\right) - \sqrt{64 + c''\left(\frac{l}{2}\right)^2 l^4 + 112c''\left(\frac{l}{2}\right)l^2 \left(1 - c\left(\frac{l}{2}\right)\right) - 128c\left(\frac{l}{2}\right) + 64c\left(\frac{l}{2}\right)^2}}{16l}$$

Is this value of  $t$  ever compatible with our assumption on  $t$  being low enough to guarantee complete overlapping? Substituting for the maximum value of  $t$  ( $t = [1 - c(l/2)]/2l$ ) in (3.5) gives :

$$(3.6) c''(l/2) < -\frac{4}{7l^2} [1 - c(l/2)]$$

which means that the production function distribution needs to be sufficiently flat. To see that, consider the same symmetric distribution  $c(z) = \alpha_z[l - z]$ . Substituting in (3.6) gives

$\alpha < \frac{4}{15}l^2$ . Hence, for  $l/2$  to be an equilibrium,  $t$  has to be large enough and the cost distribution function flat enough. However in this case, the agglomerated equilibrium is unstable as shown by the inspection of the reaction functions. Consider the following symmetric production cost distribution :  $c(z) = \frac{1}{20}z[l-z]$  with  $l=2$ . Then the two last cases of proposition 6 give the following profit functions for firm 1 when firm 2 is located at  $l/2$  (they are graphed only on the relevant range : from 0 to  $l/2$ ):

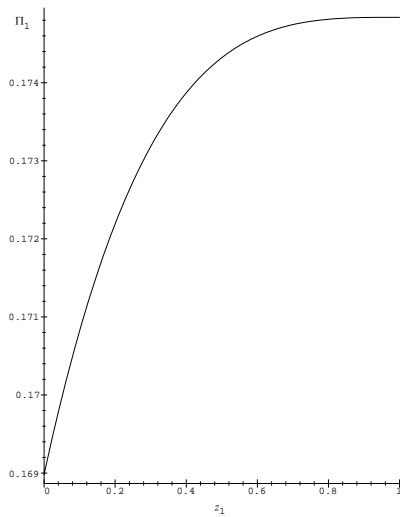


Figure 4 :  $t = t^*$

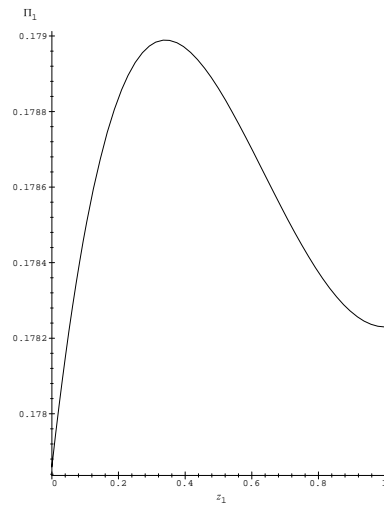


Figure 5 :  $t^* > t > c'(0)$

The agglomerated central equilibrium therefore exists when the transport cost is high enough. In this case, two equilibria coexist : the symmetric and the agglomerated one as can be seen by the two intersections of the reaction functions on figure 6.

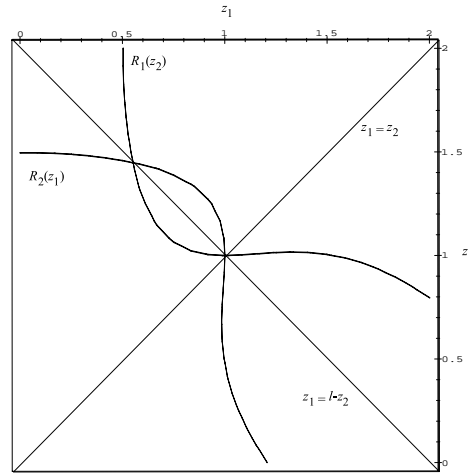


Figure 6 :Reaction functions when  $t = t^*$

Recall that only the upper left half of the reaction function space is relevant as we concentrate here on the case where<sup>11</sup>  $z_1 \leq z_2$ . In figure 6,  $t = t^*$  so that two equilibria exist ; one with the two firms located at the center where production costs are at their peak and one where the firms are symmetrically located near the endpoints. However it appears that the agglomerated equilibrium is unstable in the sense of standard Cournot dynamics as a slight move from one firm away from the center would generate a sequence of relocation towards the symmetric equilibrium. ■

We hence have the result that for this inverted U distribution of production costs, firms will differentiate spatially. How dispersed will they be depends on the transport cost of the specific product they sell; an increase in transport costs tending to increase the geographical proximity of firms. On the other hand, we have seen in proposition 4 that when production costs increase linearly from one end of the segment to the other end, firms agglomerate. We thus have different predictions on equilibrium location patterns depending on the geography of production costs. We can relate both the inverted U and the linear cases to more general



equilibrium models that have been proposed in the literature still assuming that the industry we focus on is sufficiently small so that location decisions do not influence production costs. More precisely, what is the type of model that can generate endogenously the different distributions envisioned in this paper (and considered exogenous by the firms here)?

The type of inverted U distribution we proposed here is usually seen as evidence of agglomeration economies that yield declining land rents or real wages with respect to distance to the central place (see Fujita and Thisse 1996, section 2 for a survey). In that sense, our results could be seen as the description of the location chosen by firms of a small oligopolistic industry taking the shape of labor cost or land prices distribution determined by positive externalities affecting the other sectors as given. This distribution is often referred to as common in economic reality<sup>12</sup>. It is therefore important to consider the fact that this type of production cost geography yields dispersion as opposed to the Anderson and Neven (1991) result of central agglomeration.

The linear distribution can be related to a paper by Rauch (1991) describing an economy where each country would be consisting of a single port of entry being the mouth of a river running perpendicular to the coast. Suppose that the duopolists of our small oligopolistic industry use as unique input an imported good that needs to be shipped from the port at a transport cost increasing linearly with distance. The firms will then face the production cost distribution described on proposition 4 and consequently agglomerate somewhere between the port and the geographical center of the country.

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<sup>11</sup> It must be noted that all symmetric equilibria studied in proposition 6 have in fact their counterpart for the case where  $z_1 \geq z_2$ . Therefore each symmetric equilibrium is in fact two equilibria, one for the case where firm 1 is located to the left of firm 2, one for the opposite case.

<sup>12</sup> Urban economists in particular relate empirically the land price with distance to the center of cities finding strong evidence of negative relationships for many cities. Atack and Margo (1996) for instance provide long period evidence of this relationship for the city of New York.

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## Appendix

### i) Second order condition for the agglomerated equilibrium

The second order condition for an agglomeration to occur in  $\bar{z}$  is:

$$\begin{aligned} \frac{9}{4t} \frac{\partial^2 \Pi_1}{\partial \bar{z}^2} &= 2 \left( \frac{c'(\bar{z})l}{t} - 2(l - 2\bar{z}) \right) c'(\bar{z}) \\ &+ \left[ -\frac{(1-c(\bar{z}))l}{t} + \bar{z}^2 - l \left( \bar{z} - \frac{l}{2} \right) \right] c''(\bar{z}) \\ &- 2[1 - c(\bar{z}) - tl] < 0 \end{aligned}$$

Suppose that the production cost distribution corresponds to the one graphed in figure 1. Then the point where the production costs are minimized is located to the left of  $l/2$ , and we know from (2.2) that  $\bar{z}$  will be between these two locations. The two last terms are negative provided that the assumption on  $t$  is satisfied, noting that the production cost distribution function is necessarily convex at  $\bar{z}$ .

Substituting the equilibrium value of  $c'(\bar{z})$  in  $\frac{c'(\bar{z})l}{t} - 2(l - 2\bar{z})$  gives:

$$\{D - 2\}(l - 2\bar{z}) \text{ where } D = \left[ -\frac{\left(1 - c(\bar{z}) - \frac{tl}{2}\right)l}{t\bar{z}^2 - l\left(1 - c(\bar{z}) + t\bar{z} - \frac{tl}{2}\right)} \right],$$

which is negative as  $D$  takes a maximum value of 1 either at  $\bar{z} = 0$  or  $\bar{z} = l$  (the denominator in  $D$  being convex in  $\bar{z}$ ).

### ii) Minimization of the total unit cost

When the game results in an agglomerated equilibrium, does the chosen location minimize total cost? We know from the general features of Cournot competition that firms maximize their market share and their profit on each location by minimizing their total cost of serving that location. A natural candidate for the agglomerated equilibrium location is therefore the location minimizing the unit cost of serving the whole line, which is :

$$MC = \int_0^l t|\tilde{z} - z| + c(\tilde{z}) dz = t\tilde{z}^2 - t\tilde{z}l + c(\tilde{z})l + \frac{tl^2}{2}$$

with

$$\frac{\partial MC}{\partial \tilde{z}} = c'(\tilde{z})l - t(l - 2\tilde{z}) = 0 \text{ for } c'(\tilde{z}) = \frac{t(l - 2\tilde{z})}{l}$$

Note that the second order condition for the cost to be minimized is easily satisfied under the globally convex curve we assumed for the agglomerated equilibrium. Next, the second term in the first order condition is the same that arises when considering only the transport cost. This confirms that in the Anderson and Neven (1991) framework, the equilibrium minimizes transport costs. However this is not the case here as the condition defining the cost minimizing location differs from (2.4). In fact, with the same argument used above, it can be shown that the maximum value of the slope defined in (2.4) corresponds to the one of the cost minimizing location. It means that, in the present setting, firms always agglomerate nearer to the production cost minimizing location than needed to minimize total unit cost.

### iii) Range of the reaction functions

The general form for the first order condition of firm 1 is :

$$\begin{aligned} \frac{9}{4t} \frac{\partial \Pi_1}{\partial z_1} = & \left[ -\frac{(1 - 2c(z_1) + c(z_2))l}{t} - z_2^2 + 2z_1^2 - l \left( 2z_1 - z_2 - \frac{l}{2} \right) \right] c'(z_1) \\ & + \left[ (1 - 2c(z_1) + c(z_2))(l - 2z_1) + t \left( (z_1 - z_2)^2 + l(2z_1 - z_2 - \frac{l}{2}) \right) \right] = 0 \end{aligned}$$

Suppose that the point where the production cost are minimized is located to the left of  $l/2$ , a sufficient condition for the best response to be located between  $c'(z_1) = 0$  and  $l/2$  is that the first term is negative between  $c'(z_1) = 0$  and  $l$  and that the second term is positive between 0 and  $l/2$  and negative afterwards. Define  $E(z_1, z_2)$  as :

$$E(z_1, z_2) = \left[ -\frac{(1 - 2c(z_1) + c(z_2))l}{t} - z_2^2 + 2z_1^2 - l \left( 2z_1 - z_2 - \frac{l}{2} \right) \right]$$

It is possible to show that  $E(z_1, z_2)$  is strictly negative over all possible locations of both firms:

$$\frac{\partial E}{\partial z_1} = \frac{2c'(z_1)l}{t} - 2(l - 2z_1) \text{ and } \frac{\partial^2 E}{\partial z_1^2} = \frac{2c''(z_1)l}{t} + 4$$

$$\frac{\partial E}{\partial z_2} = -\frac{c'(z_2)l}{t} - 2z_2 + l \text{ and } \frac{\partial^2 E}{\partial z_2^2} = -\frac{c''(z_2)l}{t} - 2$$

$E(z_1, z_2)$  being convex in  $z_1$  and concave in  $z_2$ , we hence want to prove that the maximum value of  $E$  is negative. This maximum is attained for  $z_1 = 0$  or  $l$  and for  $z_2^* = \frac{l}{2} - \frac{c'(z_2)l}{2t}$

Substituting in  $E$  gives :

$$-l \left[ (1 - 2c(z_1) + c(z_2^*)) - \frac{3tl}{4} + \frac{lc(z_2^*)^2}{4t} \right]$$

which is negative provided the condition for complete overlapping of market is verified.

Define  $F(z_1, z_2)$  as :

$$F(z_1, z_2) = (1 - 2c(z_1) + c(z_2))(l - 2z_1) + tl(2z_1 - z_2 - \frac{l}{2}) + t(z_1 - z_2)^2$$

We are interested in the sign of  $F$  depending on the location of firm 1. We want to prove that the signs of  $E$  and  $F$  conflict on a unique and limited range, that is that  $F$  passes from a positive sign to a negative one when  $z_1$  approaches some point near  $l/2$ .

Keeping in mind that we restrict attention to the case where  $z_1 < z_2$ , when  $z_1$  lies in the  $[0, l/2]$  range, the only negative term in  $F$  is  $(2z_1 - z_2 - \frac{l}{2})$ . Furthermore, we know that  $tl < \frac{1 - 2c(z_1) + c(z_2)}{2}$ . Thus for  $F$  to be negative when  $z_1 \in [0, l/2]$ , the following relation must be verified :

$$2(l - 2z_1) < -(2z_1 - z_2 - \frac{l}{2}), \text{ that is : } \frac{3}{2}l - 2z_1 - z_2 < 0$$

Hence, when both firms lie within the  $[0, l/2]$  range,  $F$  is necessarily positive.

When firm 1 (and hence firm2) lie within the  $[l/2, l]$  range, only  $t \left[ l(2z_1 - z_2 - \frac{l}{2}) + (z_1 - z_2)^2 \right]$  can be positive.

With the restriction on  $t$ , we have in this range :

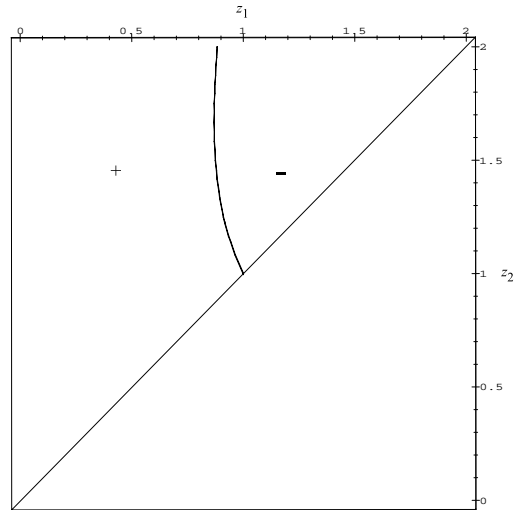
$$F(z_1, z_2) < (l - 2z_1) + \frac{1}{2l} \left[ l(2z_1 - z_2 - \frac{l}{2}) + (z_1 - z_2)^2 \right]$$

The relevant root for this expression is :  $z_1(z_2) = l + 2z_2 - \sqrt{l \left( 3z_2 - \frac{l}{2} \right)}$

in which it can be checked that  $F$  is strictly negative when both firms lie in the  $[l/2, l]$  range.

Thus  $F$  is positive when both firms are located to the left of the central location and negative when both firms are located to the right of the center. This is close from saying that this effect pushes firm 1 towards the center whatever the location of firm 2, but there might be instances where  $F$  becomes negative slightly before  $l/2$ . Indeed, when transport cost are specially high and  $z_2$  close to  $l$ ,  $F$  might be negative at  $l/2$ .

What this means is that, in most of the cases,  $E$  and  $F$  have opposite signs only within the range defined by the location where  $c'(z_1) = 0$  and the location  $l/2$  and hence that reaction functions always lie within this interval. There is a very specific setting where reaction functions can lie in a slightly wider interval and this is best illustrated graphically. The following figure represents  $F$  for the cost distribution of figure 1 :



Then, as we know that  $E$  can only be equal to 0 at point  $3/8$ , the reaction function lies within the mentioned range. Theoretically, the lowest production cost location can be so near to the center that the first order condition is very weakly negative at that point. It can therefore be concluded that, if the minimum of the production cost distribution is not “too” close from  $1/2$ , the reaction functions always lie in the same range defined by the lowest production cost location and the central location.