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Harris Selod, Yves Zenou. Location and Education in South African Cities under and after Apartheid. 1999. halshs-03592681

HAL Id: halshs-03592681

<https://shs.hal.science/halshs-03592681>

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**Location and Education in South African Cities
under and after Apartheid**

Harris SELOD
Yves ZENOU

1999.88



Location and Education in South African Cities under and after Apartheid*

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November 7, 1999

Abstract

We present a model that focuses on the links between location and educational choices in South African cities. By comparing the Apartheid city (in which both schooling and land use are restricted on the basis of race) and the post-Apartheid city (in which there are no such restrictions), we show that the inequality in South Africa between blacks and whites decreases when Apartheid laws are removed. After Apartheid, blacks are better off in spite of fiercer competition in the land market. This is because human capital externalities arise from the removal of Apartheid. Black students are induced to mix with white students, which gives them a chance to improve their human capital. Whites are worse off due to negative human capital externalities and intensified land market competition. In the post-Apartheid context, we show that reducing the commuting costs of black children cannot be Pareto improving but nevertheless reduces inequality.

Key words: Apartheid, South Africa, urban segregation, education externalities, urban land use.

JEL Classification: I2, J7, R14

*We would like to thank Richard Arnott, Jan Brueckner, Pierre-Philippe Combes, Pierre-Henri Derycke and Masahisa Fujita for helpful comments.

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1 Introduction

In June 1998, the South African newspaper *the Star* told the story of Nomakhazi Mdakane, a 46 year-old single mother who lives in the suburbs of Cape Town and commutes everyday to a job located in a central part of the city. Up before dawn, she makes a 1 km walk to a taxi rank and often waits an hour before being able to board her only transport to work, at about 6 am. If she could afford it, she would buy a car but half of her wages are spent on transport already and the rest goes to fees for her daughters who attend an inner-city school.

Such stories are common in South African cities and illustrate some of the major problems that have been inherited from the past policy of Apartheid. Indeed, South Africa's former segregation policy has had a tremendous impact on the workings of South African cities, leading to great inequalities in the distribution of income and human capital among spatially divided population groups. A key feature under Apartheid was that urban non-whites were discriminated on both their residential location and access to schooling: they were forced to reside at peripheral locations with segregated and low quality education, far away from whites and central jobs.

The aim of this paper is to model the South African urban situation during and after Apartheid and to analyze its consequences in terms of education, housing and mobility. It is indeed our contention that the mechanisms of school desegregation (after Apartheid) can only be fully understood at a local and spatial level. *The main question is whether ending the restrictions on residential location and school choices will lead to more equality or larger disparities.*

Keeping this issue in mind, we study two types of situations. In the Apartheid regime, whites live near the city-center where all jobs are located whereas blacks (or non-whites) are forced to reside at the outskirts of the city. Moreover, white children go to the centrally located white school while black children attend a black peripheral school. Because of Apartheid, there is no competition neither in education (black and white children do not mix so there are no human capital externalities) nor in the land market (there are two separate markets).

In the post-Apartheid regime, there are no land-use nor education restrictions so everybody can live or study anywhere in the city. We show that white families still reside at the vicinity of the city-center and bid away black families to the outskirts of the city. This corresponds to the structure of many post-Apartheid South African cities (such as, for instance, Cape Town or Durban). In this context, as illustrated by the Star's article, black parents still have long commuting trips to work in the city-center whereas

whites have much shorter commuting trips. However, in the post-Apartheid regime, there is a certain amount of educational mixing since some blacks are able to study at the central school (the former ‘white school’). If, for black students, there is an integration cost to study in a white school (because, for instance, they have to adapt to the English language or to the new norms), and if blacks are heterogeneous with respect to this cost, we show that, on average, those who attend the central school live relatively closer to the city-center and have a low integration cost. The interesting feature of the post-Apartheid situation is that black families are induced to send their children to the central school, implying educational mixing.

We then compare these two regimes. For whites, it should be clear that they incur a utility loss because they face more competition in the land market (leading to higher land prices) and because their level of education decreases due to their mixing with black students (who inherited a lower initial human capital). Concerning blacks (as a whole), there is a trade-off. On one hand, they gain because of human capital externalities, but on the other hand, they lose because of fiercer competition in the land market. We show that, as a net effect, black families are better off and inequality (as measured by the difference between the utilities of whites and blacks) decreases when restrictions are removed (as in the post-Apartheid regime).

The rest of the paper is as follows. The next section gives some stylized facts about South African cities that fit with our model. Section 3 describes our model while section 4 focuses on the Apartheid equilibrium. In section 5, we determine the post-Apartheid equilibrium and section 6 compares the two types of equilibria. Finally, section 7 concludes.

2 Some facts about South African cities

The aim of this section is to describe the South African situation under and after Apartheid. We focus on different elements that are relevant to our model, namely the structure of cities, transportation costs, inequality and education.

2.1 The structure of South African cities

It is necessary to recall that Apartheid was implemented for almost half a century and resulted in tremendous disparities between communities, one of the main features being the *land-use restrictions* that were imposed on all communities. Under Apartheid, only whites could live close to the city-center where most jobs were located. The non-white labor force (i.e. ‘Asians/Indians’),

‘coloreds’ or ‘blacks/Africans’ according to the former racial classification established by the Apartheid regime) could only live in peripheral locations of cities, sometimes very far from the city-center (see e.g. Smith, 1992). Racially homogeneous *townships* separated by buffer zones were created in order to prevent people from interacting with individuals from other communities. Typically, Asian and coloured *townships* were distant but relatively closer to the city-center whereas black *townships* were located as far away as possible from the center. Even though a certain amount of residential desegregation started to occur at the end of the 1980s, these spatial patterns of segregation still prevail in the 1990s. In Cape town for instance, the black-white index of dissimilarity¹ was above 97% in 1991 (Christopher, 1993) and still exceeded 93% in 1996. As shown in Table 1, there is a very high level of segregation in all South African metropolitan areas and between all communities. These values are extremely high since, even in highly segregated American cities such as Detroit, the dissimilarity index there is ‘only’ in the range of 70% (Cutler, Glaeser and Vigdor, 1999).

Table 1: Indices of dissimilarity for the five major metropolitan areas in 1991

	Johannesburg	Cape Town	Durban	Pretoria	Port Elizabeth
Asian-Black	94.5	98.0	90.9	91.5	99.2
Asian-Colored	88.6	74.0	89.1	96.4	82.1
Asian-White	90.0	90.6	98.3	96.3	93.4
Black-Colored	93.5	97.5	91.0	90.4	95.0
Black-White	89.6	97.3	66.6	87.1	98.2
Colored-White	93.9	96.3	94.7	92.4	97.7

Source: Christopher (1993).

Not surprisingly, one of the main effects of racial zoning and segregation was to break down cities into very contrasted urban zones. In the Cape Metropolitan Area for instance, an urban area that expands beyond 25 km and encompasses over 2.5 million inhabitants, most centers of opportunity are clustered around one edge of the city – the central business district and

¹Considering two communities, blacks and whites for example, the dissimilarity index is equal to $\frac{1}{2} \sum_i \left| \frac{Blacks_i}{Blacks} - \frac{Whites_i}{Whites} \right|$ where i refers to neighborhoods. This index gives the percentage of individuals of a given type who would have to relocate in order to produce a homogeneous distribution of the population within the city. A dissimilarity index of less than 30% is considered as low, between 30% and 60%, as medium, and above 60%, as high (see Cutler, Glaeser and Vigdor, 1999).

its closed surroundings (Mail & Guardian, 1999). Indeed, one can easily distinguish the job rich historically older and central parts of the city from the *townships* and the poor peripheral informal housing areas. The central parts comprise of centers of employment laid out along ‘corridors’ extending outward from the city-center/port. They mainly host middle and higher income people. On the contrary, *townships* are inhabited by middle to lower income people with poor access to activities and services while peripheral informal housing areas mainly consist of high density slums. Needless to say that the main problem caused by the lay-out of South African cities is obviously the *separation of workplace and residence*. In Cape Town, more than 80% of formal employment is located in the CBD or along the ‘corridors’ whereas less than 40% of the population lives there (Cape Metropolitan Council, 1996 and 1998). Therefore, these spatial patterns result in a considerable amount of commuting (Cape Times, 1998, Naude and Crous, 1998).

2.2 Travel to work and the corresponding commuting costs

Mainly because of their respective locations within cities, the different communities experience very distinct commuting patterns. Distances traveled, time costs and transportation modes greatly differ across communities. Indeed, distances to the city-center are uneven across population groups. In South African cities, the average commuting distance for blacks, over 15 km, is twice as long as for whites who travel less than 7 km to go to work (Vines Mikula Associates, 1994). The same remarks apply to time costs. In 1990, in Cape Town for instance, a resident commuting to the city-center from Khayelitsha (one of the city’s black *townships*) had to face 2 hours and 40 minutes of transport, excluding waits at connection points (Urban Problems Research Unit, 1990a and 1990b). Finally, transportation modes also vary with communities. Table 2 shows that, in 1992, 87% of whites and 57% of asians used their cars to commute whereas 79% of blacks and 53% of coloreds resorted to public transportation. Observe that the use of taxis² was quite frequent, especially among black commuters (46%). In 1999, this is even more true for blacks but also for other non-white communities and taxis are used by 65% of urban commuters (Cape Argus, 1999).

²A taxi or minibus-taxi is a cheap and quite unsafe means of transportation that can board up to 15 people. The wide use of taxis among blacks is a recent phenomenon that started in the 1980s as a response to the inadequate public transport from *townships* and informal settlement areas (Cape Argus, 1998b).

Table 2: Mode of transport of metropolitan commuters in 1992

	Whites	Asians	Coloreds	Blacks
Public Transport	7%	33%	53%	79%
bus	4%	22%	10%	20%
taxi	—	8%	8%	46%
train	3%	3%	35%	13%
Car	87%	57%	36%	9%
Walking	2%	4%	7%	11%
Other	4%	6%	4%	1%
Average monthly cost (Rands ³)	221	134	101	67
Average travel time (mn)	25	36	44	51

Source: Vines Mikula Associates (1994).

2.3 Inequality and education

The spatial patterns of South African cities have contributed to create and maintain significant disparities between urban ethnic groups, notably in terms of income and education. In Cape Town, a city in which there are almost as many whites as blacks (23% and 27% of the local population respectively) and where coloreds are in majority (about one half of the local population)⁴, 69% of white households have a monthly income above 3500 Rands whereas 74% of black households have an income inferior to 1500 Rands (see Table 3).

Table 3. Estimated monthly household income distribution in Cape Town in 1995

	Whites	Asians	Coloreds	Blacks
0-1500 Rands	12%	27%	36%	74%
1501-2500 Rands	9%	18%	21%	14%
2501-3500 Rands	10%	13%	15%	5%
Above 3500 Rands	69%	42%	27%	6%

Source: Moving Ahead (1996).

The stark disparities in income can be compared with education imbalances in South African cities. For many years, in accordance with Apartheid's

³1 Rand = 0.1635 US dollar (July 9, 1999).

⁴Asians accounts for the rest of the city's population (3%). See Cape Metropolitan Council (1996).

logic, there was a separation of educational provision on the basis of race. Prior to the 1994 elections that marked the end of Apartheid, education was operated by departments organized by race and geographic location. The system was very unequal, the objective being to limit the post-school opportunities of non-white children, essentially to menial occupations (World Bank, 1995). Table 4 shows the discrepancies in the quality of education that was delivered to the different communities. In the early 1990s, just before the end of Apartheid, while Africans accounted for 75% of the country's population, they only received about 47% of recurrent government expenditure on education. In short, for every 4 Rands spent on a white child, only 3 Rands were spent on an Asian child, 2 Rands on a colored child and 1 Rand on a black child (Thomas, 1996). Moreover, classes in black schools were overcrowded (there were on average 42 students per teacher) and only 14% of black students studied through the end of high school (grade 12) whereas as much as 88% of white children were able to graduate from high school.

Table 4 - Inequality in Education in South Africa in 1991

	Whites	Asians	Coloreds	Blacks
Population share	13%	3%	9%	75%
Share of expenditure	34%	5%	14%	47%
Pupils per teacher	19	22	24	42
Academic survival (up to grade 12)	88%	53%	20%	14%
Average years of education (men)	9.5	7.9	5.5	3.9

Sources: Case and Deaton, 1999, Thomas, 1996 and World Bank, 1995.

In view of these striking figures, one should not be surprised at the resulting discrepancies in the levels of education across population groups. White and black schooling systems were 'two limiting cases in terms of the opportunities that they afforded' (Fedderke, Kadt and Luiz, 1998) and, obviously, the impact of such a system has been strongly discriminating. If we compare with the US, there is the same gap in terms of schooling levels between blacks and whites born in the early 1970s in South Africa and those born in the 1930s in America (Thomas, 1996).

Therefore, after the abolition of Apartheid, the key issue is whether school desegregation can significantly contribute to the reduction of human capital imbalances. It should be noted that, even though limited desegregation began in state schools in the 1980s and the 1990s,⁵ unrestricted formal desegregation was only decided in 1995 and still comes up against many difficulties.

⁵The inefficiency of segregation, the need for qualification, and the *de facto* emergence of 'grey areas' (mixed neighborhoods) boosted desegregation in schools even before the

Sociologists have pointed out the particularities associated with school desegregation in South Africa which boils down to ‘integrating a majority group into a privileged minority culture’ (Penny, Appel, Cutlig and Harley, 1993). In particular, black children bear a certain cost to integration, either because they have to adapt to the language (English or Afrikaans) or because they must adapt to new norms at school (see Naidoo, 1996b, Vally, 1999, Zafar, 1998 and 1998b). In South African cities, these social obstacles to integration also involve spatial considerations. Indeed, parents who care about education may wish to reside close to good (central) schools and thus may be willing to pay higher housing prices (Case and Deaton, 1999). Those who cannot afford moving, might send their children on long and expensive commuting trips as it is currently observed. If moving and commuting costs are prohibitive, then the neighborhood location of schools reinforces the polarization in education by limiting the exposure of pupils to the world beyond their immediate community, and also aggravates inequality in education between rich and poor communities (Smit and Hennessy, 1995).

The facts presented in this section have stressed the crucial role of space in order to understand patterns of education and inequality by race. Therefore, considering the inherited inequality, the problems of access to schooling and the workings of cities, the role of this paper is to gauge the conditions at which school integration will succeed in South African cities.

3 The model

The city is closed, linear with absentee landlords⁶. There are two centers. The first one, the Central Business and Educational District (CBED hereafter), is at the city center (taken as the origin of the line) whereas the other one, the Suburban Educational District (SED hereafter), is exactly at the city fringe. The CBED has all the jobs (and thus all the firms)⁷ as well as one big representative school whereas the SED only has one big representative school and no jobs at all. Observe that the two schools are assumed not to be capacity constrained.

There is a continuum of families uniformly distributed along the linear city and they all consume the same amount of land (normalized to 1). The

abolition of Apartheid laws. For an interesting presentation of school desegregation and subsequent issues in South Africa, see Naidoo (1996), Tikly (1996 and 1996b), Penny, Appel, Cutlig, Harley and Muir (1993).

⁶All these assumptions are very standard in urban economics and relaxing them does not alter our main conclusions (see Fujita, 1989).

⁷Observe that our model could incorporate informal employment since, in South-African cities, a very large part of informal jobs is located in the city-center.

density of residential land parcels is taken to be unity so that there are exactly x units of housing within a distance x from the CBED. The families belong to two different types, non-whites or blacks (type B) and whites (type W) whose respective masses are given by \bar{N}_B and \bar{N}_W (with $\bar{N}_B + \bar{N}_W = \bar{N}$). We assume that $\bar{N}_B > \bar{N}_W$ which is the case in South Africa and in most if not all of its cities (see our comments in section 2.3). The only difference between these two types lies in the initial human capital endowments of adults which is lower for blacks, i.e. $\bar{h}_B < \bar{h}_W$, where \bar{h}_i ($i = B, W$) is the initial amount of human capital of a type i parent. This assumption reflects the fact that, because of South Africa's history, levels of human capital between blacks and whites are unequal (see Table 4 and the discussion in section 2.3).

A family consists of one parent and one child. We use a static model which captures our main message about location and educational choices. Parents are working while children are studying.

After Apartheid, a certain amount of integration is bound to take place within schools. We assume that there is a cost θ to integration (see e.g. Akerlof, 1997, for a discussion of peer pressures in black communities facing integration). In South Africa, this cost reflects the fact that it is the non-white learner who must adjust to the school norms, developing for instance competency in the English language (see Zafar, 1998 and our discussion at the end of section 2.3). Observe that white children being the reference group in the central school, they do not incur any interaction cost with black children other than human capital externalities. We thus assume that only black children bear the cost of integration while trying to integrate into the central former white school. Moreover, all black children do not have the same ability to integrate and interact with the other group. Formally, the integration cost, denoted by θ , is uniformly distributed on $[0, 1]$ among the group of black children.

The timing of the model is as follows. In stage 1, families choose their location in the city. Then, in stage 2, they decide which school to send their children to after revelation of their integration cost.⁸ In stage 3, children participate to the educational process and, in stage 4, families consume.

4 The equilibrium during Apartheid

In this section, we develop a model where blacks are discriminated in both the housing market and the access to the central school. The first hypothesis

⁸Note that under Apartheid, there is no such choice since black and white children are forced to attend separate schools. The fact that the integration cost is revealed after the location decision is discussed at the beginning of section 5.

means that blacks cannot reside in central locations nearby the CBED, a portion of urban space exclusively reserved for whites. This is indeed a well documented fact that under the laws of Apartheid white locations were central whereas those of non-whites were peripheral (see Smith, 1992, and our discussion in section 2.1). The second assumption is in accordance with the first one since under Apartheid blacks did not have access to white (central) schools.

In this context, all workers (black or white parents) commute to the CBED while white students travel to the CBED and black students to the SED. Families endogenously decide their optimal residence within their allocated group area, i.e. between the CBED (normalized to zero) and \bar{N}_W for whites, and between \bar{N}_W and the city fringe $\bar{x}_f \equiv \bar{N}$ for blacks. As a result, there is no competition for land between blacks and whites but families compete within their respective group areas.

Since land consumption is normalized to 1, each parent located at a distance x from the CBED has the following utility function and budget constraint:

$$U_i = \alpha z_i + (1 - \alpha)h_i \quad i = B, W \quad (1)$$

$$\bar{y}_i = z_i + t_i x + \tau_i |x - \hat{x}_i| + R(x) \quad i = B, W \quad (2)$$

where $0 < \alpha < 1$, z_i denotes the consumption of the non-spatial composite good (whose price is taken as the numeraire) of a type i family, h_i is the human capital level of a type i child, \bar{y}_i is the income of a type i parent, t_i and τ_i , are the respective transport costs per unit of distance for a type i parent and a type i child, \hat{x}_i is the location of the school so that, under Apartheid, we have $\hat{x}_W = 0$ and $\hat{x}_B = \bar{N}$. Finally, $R(x)$ denotes the equilibrium land rent at a distance x from the CBED.

The following comments are in order. First, equation (1) assumes that parents care about the human capital of their children, which affects positively their utility. Parents are thus altruist and children' levels of human capital depend on parental choices as, for example, in Glomm (1997). The main difference here is that the present model does not just focus on explicit human capital investment but also on location choices as in Benabou (1993, 1996) to the extent that location choices interact with educational choices. If there were no restrictions on location and educational choices, the preference for a school would depend on a trade-off between quality and proximity. We will elaborate further on this in the next section when we explore the unrestricted market equilibrium.

Second, we assume that the *total* unit commuting costs of blacks are lower than those of whites, i.e. $t_B + \tau_B < t_W + \tau_W$ and that black parents have higher unit transport costs than their children, i.e. $t_B > \tau_B$. The first

assumption can be justified by the fact that blacks and whites do not use in general the same transport modes: whites mainly use cars while blacks resort to public transportation (see Table 2). In Cape Town for instance, a recent study (Cape Argus, 1998a) shows that the commuting cost per kilometer of cars amounts to 1.52 Rands while for public transportation (train, minibus taxi or bus) it is less than 0.15 Rands. Assuming that black families have lower transport cost than white families is specific to South Africa, since it is in general assumed the contrary for North American cities (see e.g., Brueckner and Martin, 1997). Indeed, in South Africa, there has been a huge development of the taxi industry for the poor living in peripheral locations (mainly blacks) and, because of a very fierce competition, taxi fees are very low compared to other means of public transportation. This explains why, in South Africa, black workers generally have lower unit transport costs than whites. The second assumption is easy to understand since many black children do not even use public transportation to go to school but walk instead.

Third, a child's level of human capital is determined by the quality of the school he attends. The general educational output h_j of a school j ($j = C, S$ where C stands for *central school* and S for *suburban school*) is given by:

$$h_j = \frac{N_{Wj}}{N_{Wj} + N_{Bj}} \bar{h}_W + \frac{N_{Bj}}{N_{Wj} + N_{Bj}} \bar{h}_B \quad (3)$$

where N_{ij} is the number of type i children ($N_{iC} + N_{iS} \equiv \bar{N}_i$, $i = B, W$) in school j ($j = C, S$), and \bar{h}_i ($i = B, W$) is the contribution of a type i ($i = B, W$) child to the educational process, as measured by his or her parent's inherited level of human capital. Equation (3) takes into account human capital externalities or spillovers so that each student, regardless of his or her ethnic origin, acquires the school's average human capital contribution. Note that each child's contribution is measured by his or her parent's level of human capital \bar{h}_i (which can be considered as a form of social capital that captures the quality of the home learning environment).

Under Apartheid however, educational choices are restricted and there is no interaction between blacks and whites at school and thus no inter-group education externalities. Then, by using the fact that under Apartheid, $N_{WC} = \bar{N}_W$ and $N_{BS} = \bar{N}_B$, we have :

$$h_W \equiv h_C = \bar{h}_W \quad (4)$$

for white students and

$$h_B \equiv h_S = \bar{h}_B \quad (5)$$

for black students. This means that, under Apartheid, parents and children have the same human capital so that this policy prevents the possibility of

intergenerational improvement for blacks in terms of both location choices and human capital. In other words, under Apartheid, parents' education fully determines the schooling of their children (see e.g. Wilson, 1996).

By using (1) and (2), we obtain:

$$U_B = \alpha [\bar{y}_B - t_B x - \tau_B(\bar{N} - x) - R(x)] + (1 - \alpha) \bar{h}_B, \quad x \in [\bar{N}_W, \bar{N}]$$

$$U_W = \alpha [\bar{y}_W - (t_W + \tau_W)x - R(x)] + (1 - \alpha) \bar{h}_W, \quad x \in [0, \bar{N}_W]$$

The only choice of parents is to determine the family's optimal location by maximizing their utility. In equilibrium, all families of the type $i = B, W$ obtain the same utility levels v_B^A and v_W^A for blacks and whites respectively. We are now able to write the families' bid rents⁹. They are equal to:¹⁰

$$\Psi_B^A(x, v_B^A) = \bar{y}_B - t_B x - \tau_B(\bar{N} - x) + \frac{1 - \alpha}{\alpha} \bar{h}_B - \frac{v_B^A}{\alpha} \quad (6)$$

for blacks and

$$\Psi_W^A(x, v_W^A) = \bar{y}_W - (t_W + \tau_W)x + \frac{1 - \alpha}{\alpha} \bar{h}_W - \frac{v_W^A}{\alpha} \quad (7)$$

for whites. Moreover, we have:

$$\frac{\partial \Psi_B^A(x, v_B^A)}{\partial x} = -(t_B - \tau_B) < 0$$

$$\frac{\partial \Psi_W^A(x, v_W^A)}{\partial x} = -(t_W + \tau_W) < 0$$

Our comments are the following. First, bid rents are always linear and the trade-off is between land rents and commuting costs. Second, the bid rent of whites is always decreasing with distance from the CBED while for blacks this is true only if $t_B > \tau_B$, an assumption that we made and which is compatible with South African cities in which black children have low *unit* transport costs to go to school (see our discussion above). Observe that we allow for cross-commuting between parents and children within the black group area but not between parents or between children (see Figure 1 for an illustration of the Apartheid city).

By normalizing the agricultural land rent (outside the city) to zero, we have the following definition:

⁹Bid rents are functions $\Psi_i(x, v)$ defined as the maximum rent that a family of type i would be willing to pay at a given location x so as to reach a given level of utility v .

¹⁰All variables with superscript A refer to Apartheid.

Definition 1 An Apartheid Equilibrium (AE) is a triple $(v_B^A, v_W^A, R(x))$ such that:

$$R(x) = \begin{cases} \Psi_W^A(x, v_W^A) & \text{for } 0 \leq x \leq \bar{N}_W \\ \Psi_B^A(x, v_B^A) & \text{for } \bar{N}_W < x \leq \bar{N} \\ R_A = 0 & \text{for } x > \bar{N} \end{cases} \quad (8)$$

$$\Psi_W^A(\bar{N}_W, v_W^A) = 0 \quad (9)$$

$$\Psi_B^A(\bar{N}, v_B^A) = 0 \quad (10)$$

By solving (9) and (10), we easily obtain the following equilibrium utilities for blacks and whites:

$$v_B^A = \alpha [\bar{y}_B - t_B \bar{N}] + (1 - \alpha) \bar{h}_B \quad (11)$$

$$v_W^A = \alpha [\bar{y}_W - (t_W + \tau_W) \bar{N}_W] + (1 - \alpha) \bar{h}_W \quad (12)$$

The following comments are in order. First, in the Apartheid equilibrium, the two communities are totally separate so that there are two distinct housing markets (see Figure 1). This urban configuration is typical of South African cities under Apartheid, as initially modelled by Brueckner (1996). Second, an increase in the revenue of one group rises its utility whereas an increase in the transport cost decreases its utility. Moreover, the overall city size negatively affects black families whereas white families are only affected by the size of their own community. In other words, in the Apartheid regime, whites are not affected by the presence of blacks. Lastly, in this context τ_b does not affect the utility of blacks since in equilibrium we determine their utility at a location where distance to school is zero. It is indeed easily verified in Figure 1 that starting from \bar{N} and moving inward any black family experiences an increase in the child's transport cost and a decrease in the parent's commuting cost. Since $t_B > \tau_B$ it causes a decrease in the family's overall transport cost which is nevertheless exactly compensated by a higher land rent.

[Insert Figure 1 here]

The black-white inequality in this economy is equal to:¹¹

$$\begin{aligned} I^A &\equiv v_W^A - v_B^A \\ &= \alpha [\bar{y}_W - \bar{y}_B - (t_W + \tau_W) \bar{N}_W + t_B \bar{N}] + (1 - \alpha) [\bar{h}_W - \bar{h}_B] \end{aligned} \quad (13)$$

so that

¹¹Since the size of each population group is fixed, measuring inequality as a utility difference is equivalent to taking a more complex inequality measure such as the variance of utilities: the marginal variations of both measures have the same sign.

Proposition 1 *In the Apartheid Urban Equilibrium, utilities and inequality are respectively given by (11), (12) and (13). Inequality increases with:*

- *the difference in human capital between black and white children,*
- *the difference in the income of parents,*
- *the black parent's unit commuting cost.*

Inequality decreases with the commuting costs of white parents and white children.

5 The equilibrium after Apartheid

After Apartheid, there are no more spatial nor educational restrictions (due to the removal of Apartheid laws) and workers commute to the CBED while students can either travel to the CBED or to the SED. Families endogenously decide their optimal residence between the CBED and the city fringe $\bar{x}_f \equiv \bar{N}$ and choose the school attended by their child. Because of negative externalities incurred by white learners in our model, we assume that white parents do not send their children to the former black school located in the suburbs (this is truly the case in post-Apartheid South African cities).

It is important to recall the timing of the model. In the first stage, all families choose their location in the city without knowing the type θ of the child (the cost of integration incurred by a black child studying in a former white school) but anticipating (with rational expectations) the number of black children who will attend the central school. In other words, black families base their location decision on expected utility, anticipating the proportion of black children that will go to the central school. In the second stage, types are revealed and black families choose the school depending on their previously determined location. The assumption that types are revealed only after location is chosen takes into account the relative inertia of the housing market compared to educational mobility. Obviously, families and individuals are more mobile in terms of school attendance than in terms of residential location. In stage 3, children obtain different human capital levels and then, in stage 4, families consume the composite good. Therefore, one of the main differences between the Apartheid and the post-Apartheid equilibria is that, in the latter, the initial type of children matters, whereas in the former, children were not given the opportunity to make use of their ability to integrate.

5.1 Housing versus education

The choice of a school for blacks now depends on a trade-off between location, transport costs (distance to the school) and *human capital externalities*. Indeed, as mentioned in the previous section, we assume that there are *local spillovers* or *peer group effects* in the production of education in the sense that studying among students that have a high human capital contribution increases one's education. In particular, since by assumption the inherited human capital of black parents \bar{h}_B is lower than that of white parents \bar{h}_W , equation (3) means that the quality of a school increases with the relative number of white students. This implies that blacks have an incentive to send their children to the central former white school in order to benefit from human capital externalities. Therefore, we can consider two types of black families. The ones who send their child to the central school C (type BC), whose mass is given by N_{BC} , and families whose child remains in the suburban school S after Apartheid (type BS), whose mass is given by N_{BS} ($N_{BC} + N_{BS} \equiv \bar{N}_B$). Assuming that white children only attend the central school, we have the following utility function and budget constraint:

$$U_i = \alpha z_i + (1 - \alpha)h_i \quad i = BS, BC, W \quad (14)$$

$$\bar{y}_i = z_i + t_i x + \tau_i |x - \hat{x}_i| + R(x) \quad (15)$$

where school locations are given by $\hat{x}_{BC} = \hat{x}_W = 0$ and $\hat{x}_{BS} = \bar{N}$, human capital outputs are given by $h_{BS} \equiv h_S$ and $h_{BC} = h_W \equiv h_C$, and transport costs remain unchanged so that $t_{BS} = t_{BC} = t_B$ and $\tau_{BS} = \tau_{BC} = \tau_B$.

By using (14) and (15), and since families do not know their child's type when they decide where to locate, the utility function for white families is given by:

$$U_W = \alpha [\bar{y}_W - (t_W + \tau_W)x - R(x)] + (1 - \alpha) h_C^e \quad (16)$$

where h_C^e is the expected output level of human capital at the central school. For a black family whose child of type θ attends the central school, we have:

$$U_{BC} = \alpha [\bar{y}_B - (t_B + \tau_B)x - R(x)] + (1 - \alpha) h_C^e - \theta \quad (17)$$

Observe that the integration cost of black students takes the form of a disutility when they attend the former white school so that this disutility is increasing in the type θ . Note that θ can also be interpreted as an inverse index of personal learning abilities to the extent that it may be less difficult for a high ability children to be efficient in an environment that he or she is not used to. Lastly, the utility for black families who send their child to the suburban school is equal to:

$$U_{BS} = \alpha \left[\bar{y}_B - t_B x - \tau_B (\bar{N} - x) - R(x) \right] + (1 - \alpha) h_S \quad (18)$$

The expected human capital output at the central school h_C^e is given by:

$$h_C^e = \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} \bar{h}_W + \frac{N_{BC}^e}{\bar{N}_W + N_{BC}^e} \bar{h}_B \quad (19)$$

where N_{BC}^e is the expected number of black children going to the central school. After Apartheid, the human capital output of the suburban school h_S is equal to:

$$h_S = \bar{h}_B \quad (20)$$

Observe that each family can compute all utilities as well as the educational output of each school and will thus be able to decide which school to send its child to once the type is revealed. Therefore, a black family located at a distance x from the CBED decides to send its child of type θ to the central school if the expected payoff U_{BC} is greater than U_{BS} . This is the case when θ is smaller than a threshold value $\tilde{\theta}(x, N_{BC}^e)$ making a black family located at x indifferent between sending its child to either one of the two schools. It is given by:

$$\begin{aligned} \tilde{\theta}(x, N_{BC}^e) &= (1 - \alpha) (h_C^e - h_S^e) - \alpha \tau_B (2x - \bar{N}) \\ &= (1 - \alpha) (\bar{h}_W - \bar{h}_B) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} - \alpha \tau_B (2x - \bar{N}) \end{aligned} \quad (21)$$

We have the following result:

Proposition 2 *Black families are induced to send their children to the central former white school if:*

- *the inherited difference in human capital between blacks and whites is large,*
- *the expected number of black children attending the central school is low,*
- *the number of white students in the city is large.*

Black families living relatively closer to (further from) the CBED have more incentives (less incentives) to send their children to the central school when the unit transport cost of black children is high.

Proof. See the Appendix.

This proposition yields important results that will help us later in the equilibrium analysis. First, the inherited human capital difference between blacks and whites plays an important role in motivating black families to send their children to the central school. In other words, the Apartheid education policy which has led to a dramatic human capital inequality between blacks and whites (see Table 4) now has a crucial impact on the motivation to send black children to the ‘good’ school. The larger this difference, the greater the willingness for blacks to benefit from human capital externalities. Second, the incentives for a particular black family to send its child to the central school depends on the location of this family in the city. More precisely, everything else being equal, people living further away from (closer to) the CBED are less (more) likely to send their child to the central school. This is because, due to the presence of commuting costs, $\tilde{\theta}$ is a decreasing function of x , the distance to the CBED, so that children living far away need to have a very low θ to go to the central school while those living closer to the CBED can have a higher θ and still go to the central school. Figure 2 illustrates this point: a child of a given type θ will decide (ex post) to go to the central school if his/her family lives close to the CBED ($x < \bar{x}$) whereas the same child of type θ will go to the suburban school if his/her family resides far away from the CBED ($x \geq \bar{x}$). Third, the incentives to go to the central school depend negatively on the expected number of black children going to the central school, N_{BC}^e . In other words, there is a *negative group externality* on education incentives since, when N_{BC}^e increases, the expected human capital output of the central school h_C^e decreases, which in turn reduces the incentives of going to the central school. Fourth, when the number of whites \bar{N}_W rises, the incentive to attend the central school increases due to an increase in the expected human capital output. Finally, an increase in the unit transport cost of black children τ_B strengthens the families’ incentives to send their child to the closest school (one school is always relatively closer than the other one). So, since $\bar{N}/2$ is exactly the middle point between the two schools, when indifferent families reside on the right (on the left) of $\bar{N}/2$, a rise of τ_B induces them to send their child to the suburban (central) school.

[Insert Figure 2 here]

We are now able to determine the expected utility of a black family residing at a distance x from the CBED before the revelation of its θ . It is given by:

$$\begin{aligned}
V_B(x) &= \int_0^{\tilde{\theta}} U_{BC} d\theta + \int_{\tilde{\theta}}^1 U_{BS} d\theta \\
&= \int_0^{\tilde{\theta}} [\alpha(\bar{y}_B - (t_B + \tau_B)x - R(x)) + (1 - \alpha)h_C^e - \theta] d\theta \\
&\quad + \int_{\tilde{\theta}}^1 [\alpha(\bar{y}_B - t_B x - \tau_B(\bar{N} - x) - R(x)) + (1 - \alpha)\bar{h}_B] d\theta \\
&= \alpha[\bar{y}_B - t_B x + \tau_B x(1 - 2\tilde{\theta}) - \tau_B \bar{N}(1 - \tilde{\theta}) - R(x)] \\
&\quad + (1 - \alpha) \left[(\bar{h}_W - \bar{h}_B) \tilde{\theta} \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} + \bar{h}_B \right] - \int_0^{\tilde{\theta}} \theta d\theta
\end{aligned}$$

where h_C^e is defined by (19). Observe that the location decision is based on this expected utility since families do not know their type. They know however, that depending on their location, they will choose the central (peripheral) school if their θ is lower (greater) than $\tilde{\theta}$. This decision will be made after the revelation of their type but location will remain unchanged.

We can now obtain the whites' bid rent by inverting equation (16). We thus have:¹²

$$\Psi_W^P(x, v_W^P) = \bar{y}_W - (t_W + \tau_W)x + \frac{(1 - \alpha)}{\alpha} h_C^e - \frac{v_W^P}{\alpha} \quad (22)$$

where v_W^P is the equilibrium utility of white families. Moreover, by inverting blacks' expected utility, we obtain their bid rent:

$$\begin{aligned}
\Psi_B^P(x, v_B^P) &= \bar{y}_B - t_B x + \tau_B x(1 - 2\tilde{\theta}(x)) - \tau_B \bar{N}(1 - \tilde{\theta}(x)) - \frac{v_B^P}{\alpha} \quad (23) \\
&\quad + \frac{1 - \alpha}{\alpha} \left[(\bar{h}_W - \bar{h}_B) \tilde{\theta}(x) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^e} + \bar{h}_B \right] - \frac{1}{\alpha} \int_0^{\tilde{\theta}} \theta d\theta
\end{aligned}$$

Let us now determine the location of all agents in the city (see Figure 3). We have:

Proposition 3 *After Apartheid, the structure of the city remains the same as under Apartheid: black families reside at the outskirts of the city (between \bar{N}_W and \bar{N}) whereas white families locate at the vicinity of the city-center (between 0 and \bar{N}_W).*

¹²All variables with superscript P refer to the post-Apartheid equilibrium.

Proof. See the Appendix.

[Insert Figure 3 here]

This result is quite interesting since, even after Apartheid when blacks and whites are free to choose where to live, we obtain a residential equilibrium (Figure 3) similar to the Apartheid one (Figure 1): whites still live close to the city-center whereas blacks reside in peripheral locations. This urban configuration broadly corresponds to most South African post-Apartheid cities such as Cape Town or Durban. A notable exception is Johannesburg since, after Apartheid, whites *and* jobs have abandoned the city-center to relocate in the suburbs. In fact, our model can also address the case of Johannesburg by flipping the city and thus locating the new CBED in the suburbs. In this particular context, blacks reside close to the historical city-center and whites in the suburbs. However, it should be clear that our results remain valid since what matters is the distance between communities and centers of opportunity.

Observe that, in our model, the main difference between the two equilibria (before and after Apartheid) is that, after Apartheid, the spatial layout of the city is not imposed by law any more but results from a bidding process between blacks and whites. As a consequence, competition in the land market is fiercer and land prices are higher after Apartheid than during Apartheid.

5.2 The equilibrium

As stated above, once location is decided, all types θ are revealed and expected values are observed (recall that all families have rational expectations). Therefore, we have:

Definition 2 *A Post-Apartheid Equilibrium (PAE) with rational expectations is a quadruple $(N_{BC}^P, v_W^P, v_B^P, R(x))$ such that:*

$$R(x) = \begin{cases} \Psi_W^P(x, v_W^P) & \text{for } 0 \leq x \leq \bar{N}_W \\ \Psi_B^P(x, v_B^P) & \text{for } \bar{N}_W < x \leq \bar{N} \\ R_A = 0 & \text{for } x > \bar{N} \end{cases}$$

$$\Psi_W^P(\bar{N}_W, v_W^P) = \Psi_B^P(\bar{N}_W, v_B^P) \quad (24)$$

$$\Psi_B^P(\bar{N}, v_B^P) = 0 \quad (25)$$

$$N_{BC}^P = \int_{\bar{N}_W}^{\bar{N}} \tilde{\theta}(x, N_{BC}^P) dx \quad (26)$$

Equation (24) indicates that, at the border \bar{N}_W , the bid rents of blacks and whites are equal. Equation (25) states that the land rent paid at the city fringe is equal to the outside land rent normalized to 0. Equation (26) says that, under rational expectations, the expected number of black children going to the central school (denoted in equilibrium by N_{BC}^P) has to be the average number of black children that attend the central school. If we denote by $\hat{\theta}$ the equilibrium proportion of black children attending the central school (i.e. $\hat{\theta} \equiv N_{BC}^P/\bar{N}_B$), equation (26) is equivalent to:

$$\hat{\theta} = \frac{1}{\bar{N} - \bar{N}_W} \int_{\bar{N}_W}^{\bar{N}} \tilde{\theta}(x, N_{BC}^P) dx$$

where the RHS is the average proportion of black children whose θ is lower than $\tilde{\theta}(x, N_{BC}^P)$ (when types are randomly distributed across space) and thus attend the central school.

We have:

Proposition 4

$$\text{If } \alpha\tau_B \frac{\bar{N}^2}{\bar{N}_W} < (1 - \alpha) (\bar{h}_W - \bar{h}_B) < 1 - \alpha\tau_B \bar{N}_B \quad (27)$$

holds, then there exists a unique Post-Apartheid Equilibrium (PAE) with rational expectations $(N_{BC}^P, v_W^P, v_B^P, R(x))$.

Proof. See the Appendix.

Observe that condition (27) for existence and uniqueness of this equilibrium (i.e. $\tilde{\theta}(\cdot)$ must be between 0 and 1) is quite intuitive since it means that the difference in inherited human capital between blacks and whites ($\bar{h}_W - \bar{h}_B$) must take a medium value. In other words, if $\bar{h}_W - \bar{h}_B$ were very large, then all black families would send their child to the central school and $\tilde{\theta}(\cdot)$ would tend to be greater than 1. On the contrary, a very low value of $\bar{h}_W - \bar{h}_B$ would discourage all black students from attending the white school and $\tilde{\theta}(\cdot)$ would tend to be less than 0. Also observe that, in the first stage, each black family *randomly* chooses its location between \bar{N}_W and \bar{N} because, whatever the location, the expected utility level is the same and equals v_B^P . However, since θ is a decreasing function of distance x , in equilibrium, low θ children and those living close to the CBED are on average more likely to go to the central school. As previously mentioned, once θ is revealed, each family residing at a particular location x compares its child's θ with the $\tilde{\theta}$ at the same location. But, since $\tilde{\theta}$ decreases with distance to the CBED, if

the family is located close to the city-center, then its child does not need to have a very low θ to go to the central school whereas if the family resides close to the city fringe then the parameter θ must be extremely low for the child to attend the central school. This is why, in equilibrium, families who reside closer to the CBED are on average more likely to send their children to the central school whereas those located closer to the city fringe \bar{N} tend to send their children to the suburban school. In other words, black families must stick to their initial location, which strongly affects the human capital attainment of their children.

We are now able to determine the equilibrium value N_{BC}^P . By using (26), we obtain the following second degree equation:

$$(N_{BC}^P)^2 + [(1 + \alpha\tau_B\bar{N}_B)\bar{N}_W] N_{BC}^P + \bar{N}_W\bar{N}_B [\alpha\tau_B\bar{N}_W - (1 - \alpha)(\bar{h}_W - \bar{h}_B)] = 0$$

There are two solutions to this equation with one obviously being negative so that there exists a unique N_{BC}^P . It is easily verified that this solution is strictly positive by using (27).

In this context, there is a unique equilibrium $\tilde{\theta}^P(x)$ given by (using (21)):

$$\tilde{\theta}^P(x) = (1 - \alpha)(\bar{h}_W - \bar{h}_B) \frac{\bar{N}_W}{\bar{N}_W + N_{BC}^P} - \alpha\tau_B(2x - \bar{N}) \quad (28)$$

and a unique equilibrium human capital output for the central school equal to:

$$h_C^P = \frac{\bar{N}_W\bar{h}_W + N_{BC}^P\bar{h}_B}{\bar{N}_W + N_{BC}^P} \quad (29)$$

with

$$\frac{\partial h_C^P}{\partial N_{BC}^P} = -\frac{(\bar{h}_W - \bar{h}_B)\bar{N}_W}{(\bar{N}_W + N_{BC}^P)^2} < 0 \quad (30)$$

Equation (30) is quite intuitive since it means that when there are more black children who attend the central school, the educational outcome of this school decreases.

Now, since we know the equilibrium values of N_{BC}^P , h_C^P and $\tilde{\theta}^P(x)$, we can derive the equilibrium utility levels of blacks and whites. By using (28), (25) and (24), we get:

$$v_B^P = \alpha(\bar{y}_B - t_B\bar{N}) + (1 - \alpha)\bar{h}_B + \frac{1}{2} [\tilde{\theta}(\bar{N})]^2 \quad (31)$$

$$v_W^P = \alpha \left[\bar{y}_W - (t_W + \tau_W) \bar{N}_W - (t_B - \tau_B) \bar{N}_B + 2\alpha\tau_B^2 \bar{N}_B \bar{N}_W \right] \quad (32)$$

$$+ (1 - \alpha) \left[h_C^P - \frac{2\alpha\tau_B(\bar{h}_W - \bar{h}_B) \bar{N}_B \bar{N}_W}{\bar{N}_W + N_{BC}^P} \right]$$

6 Implications for post-Apartheid policies

In this section, we initiate a welfare analysis, derive some comparative statics results and discuss the implications of education mixing. Let us first determine the changes in the utilities of blacks and whites and the resulting change in inequality between the two equilibria (i.e. during and after Apartheid). It is easily verified that, for blacks, we have:

$$\Delta v_B \equiv v_B^P - v_B^A = \frac{1}{2} \left[\tilde{\theta}(\bar{N}) \right]^2 > 0 \quad (33)$$

This is because, as a net effect, blacks benefit from the removal of restrictions in the land market and the educational system. Indeed, the average human capital of blacks always increases because of peer group effects but land prices are higher due to fiercer competition for central locations. However, the first effect dominates the second one. For whites, we have:

$$\begin{aligned} \Delta v_W &\equiv v_W^P - v_W^A \\ &= \alpha [\Delta z_1 + \Delta z_2] + (1 - \alpha) \Delta h < 0 \end{aligned} \quad (34)$$

where

$$\begin{aligned} \Delta z_1 &= -(t_B - \tau_B) \bar{N}_B < 0 \\ \Delta z_2 &= 2\tau_B \bar{N}_B \bar{N}_W \left[\alpha\tau_B - (1 - \alpha)(\bar{h}_W - \bar{h}_B) / (\bar{N}_W + N_{BC}^P) \right] < 0 \\ \Delta h &= (h_C^P - \bar{h}_W) < 0 \end{aligned}$$

In this equation, Δz_1 is the net revenue loss for whites due to the entry of blacks in the competition for land, Δz_2 is the net revenue loss for whites due to the intensification of the competition for land caused by human capital externalities and Δh denotes the ‘pure’ human capital loss of white children.¹³ It is quite obvious why whites incur a loss in utility since land prices are higher and their human capital is lower ($h_C^P < \bar{h}_W$) after Apartheid. This can easily be seen by looking at (34). The removal of land use restrictions negatively affects the net income of whites ($\Delta z_1 < 0$) due to fiercer competition in the land market. The removal of school segregation leads to a decrease

¹³It is easily verified that, by using (27), $\alpha\tau_B < (1 - \alpha)(\bar{h}_W - \bar{h}_B) / (\bar{N}_W + N_{BC}^P)$.

in their human capital ($\Delta h < 0$) and also has an indirect effect on the price of land (which increases) since the city-center becomes more attractive for blacks who are attracted by the positive human capital externalities, which further reduces the net income of whites ($\Delta z_2 < 0$). Let us denote by $I^P \equiv v_W^P - v_B^P$ the inequality after Apartheid. We have:

$$\Delta I \equiv I^P - I^A = \Delta v_W - \Delta v_B < 0 \quad (35)$$

The following result summarizes our discussion:

Proposition 5 *When Apartheid is removed, whites are worse off whereas blacks are better off and thus the inequality between blacks and whites decreases.*

Let us now continue with the post-Apartheid following result:

Proposition 6 *After Apartheid, the number of black students attending the central former white school increases when:*

- *their unit transportation cost is lower,*
- *the initial human capital difference between blacks and whites is higher,*
- *the number of whites is larger.*

The human capital output of the central school varies in the opposite direction.

Proof. See the Appendix.

The results in Proposition 6 are quite intuitive. Indeed, when the inherited human capital difference between blacks and whites is higher and/or the transport cost for black children is lower, the number of blacks attending the central school (the former white school) rises and the general level of human capital in the central school decreases. We have a similar result for the number of white families. What matters here is the inducement for blacks to go to the central school. If, for example, it is too costly (because of a prohibitive transport cost τ_B), then, on average, black families have less incentives to send their children there.

Since the transport cost of black children τ_B is a key variable affecting N_{BC}^P , it is interesting to examine its impact on the utility levels of blacks. By totally differentiating (31), we obtain:

$$\frac{\partial v_B^P}{\partial \tau_B} = -\tilde{\theta}^P(\bar{N}) \left[\frac{\partial N_{BC}^P}{\partial \tau_B} \frac{(1-\alpha)(\bar{h}_W - \bar{h}_B)\bar{N}_W}{[\bar{N}_W + N_{BC}^P]^2} + \alpha\bar{N} \right] \begin{matrix} > \\ < \end{matrix} 0 \quad (36)$$

Since $\tilde{\theta}^P(\bar{N}) > 0$, $\partial v_B^P/\partial \tau_B$ has the opposite sign of the term in brackets. We can easily identify two opposite forces at work besides the ambiguous effect of land rent. On the one hand, when τ_B decreases, there is a *direct and positive* effect on the utility of blacks since transport costs for children become cheaper and accessibility is improved. On the other hand, this yields a negative group externality: when τ_B decreases, there is an *indirect and negative* effect on the utility of blacks since more (black) children attend the central school (N_{BC}^P increases), which in turn reduces utility (it is easily checked that $\partial v_B^P/\partial N_{BC}^P < 0$). The net effect is thus ambiguous. Next, by differentiating (32), we have:

$$\frac{\partial v_W^P}{\partial \tau_B} = \alpha \bar{N}_B (1 + 4\alpha \tau_B \bar{N}_W) - \frac{(1 - \alpha)(\bar{h}_W - \bar{h}_B)\bar{N}_W}{(\bar{N}_W + N_{BC}^P)^2} \left[\frac{\partial N_{BC}^P}{\partial \tau_B} + 2\alpha \bar{N}_B \left(\bar{N}_W + N_{BC}^P - \tau_B \frac{\partial N_{BC}^P}{\partial \tau_B} \right) \right]$$

which, by using (27), is always positive. Indeed, when τ_B increases, whites experience an increase in their utility since less black families are willing to send their children to the central school. This implies that h_C^P (the human capital of the central school) rises. This effect is dominant and leads to an increase in the utility of whites.

Even though we do not have a complete welfare analysis, the main lesson to be derived from changes in τ_B is that reducing it is clearly not Pareto improving. To summarize,

Proposition 7 *In the post-Apartheid city, reducing the unit transport cost of black children is not Pareto improving since it reduces the utility of white families and does not necessarily increase the utility of black families.*

In order to complete our analysis, we now present some simulations that aim at highlighting the direction and magnitude of utility changes associated with the removal of Apartheid laws (since it is quite cumbersome to determine it analytically). In Table 5, we have reported the results of our analysis. In the base case (see the first column), 60% of the city's residents are non-whites and 40% are whites. The latter have twice as much human capital as blacks. When Apartheid is removed, the mean level of human capital for blacks increases by 7.4%. Among black children, 8.4% attend the central school, which enables them to rise their human capital by 88.8% in comparison with what they would have obtained under Apartheid. For whites, their decreasing human capital (-5.6%) as well as the increasing land rents they face (+59.4% in the CBED) are detrimental to their utility, which is reduced

by 0.8%. As expected, the inequality between blacks and whites decreases by 1.8%. However, the surplus¹⁴ decreases, showing that the utility loss for whites is greater than the utility gain for blacks.

In the second and third columns of Table 5, we have changed the ethnic composition of the city by considering two extreme cases. In the first one, there is a short majority (51%) of non-whites whereas, in the other one, non-whites are much more numerous (80%), as for example in Cape Town. The key lesson is that when the city accommodates a high (low) proportion of blacks, relatively less (more) school integration takes place. Indeed, when blacks represent 80% (51%) of the city population, 7.5% (8.6%) of black families send their children to the central school and the gain in human capital for these children amounts to 77% (91.8%). This is due to the negative group externality previously mentioned. Moreover, land competition becomes fiercer close to the city-center with the proportion of blacks, which leads to a high increase of land rent for the central locations of whites (in the CBED for example, land rent increases by 157.5% when there are 80% of blacks in the city). Finally however, our simulations indicate that the reduction in inequality is greater and the surplus loss is lower in cities which have a very high proportion of blacks.

In the last column, we show that large inherited disparities in human capital strongly induce black children to attend the central school (15.7%) where they can obtain a huge human capital gain (323.8%). This causes a significant reduction in inequality which is nevertheless harmful to the surplus.

¹⁴We define the surplus as the weighted sum of the utilities of blacks and whites. A more general definition could include the rents paid to absentee landlords. However, our focus is primarily on the comparison between the well-being of communities living in the city, excluding absentee landlords.

Table 5: Simulation analysis for changes between the Apartheid and the post-Apartheid equilibria (%)

Base case: $\frac{\bar{N}_B}{\bar{N}} = 60\%$ and $\bar{h}_W = 2\bar{h}_B$

	<i>BASE</i>	$\frac{\bar{N}_B}{\bar{N}} = 51\%$	$\frac{\bar{N}_B}{\bar{N}} = 80\%$	$\bar{h}_W = 5\bar{h}_B$
Δh_B^{mean}	7.4	7.9	5.7	50.8
N_{BC}^P/\bar{N}_B	8.4	8.6	7.5	15.7
Δh_{BC}	88.8	91.8	77.0	323.8
Δh_W	-5.6	-4.1	-11.5	-15.2
$\Delta R(0)$	59.4	41.3	157.5	62.1
$\Delta R(\bar{N}_W^+)$	5.6	5.7	5.0	10.5
Δv_W	-0.8	-0.7	-1.3	-1.5
Δv_B	0.1	0.2	0.1	0.6
ΔI	-1.8	-1.5	-2.7	-3.4
ΔS	-0.5	-0.5	-0.4	-0.9

Base case: $\alpha = 0.5$, $t_B = 0.4$, $\tau_B = 0.1$, $t_W = 0.5$, $\tau_W = 0.3$, $\bar{h}_W = 0.4$, $\bar{h}_B = 0.2$, $\bar{y}_W = 8$, $\bar{y}_B = 4$, $\bar{N}_W = 0.1$, $\bar{N}_B = 0.15$.

Whereas Table 5 focused on the possible gains resulting from the removal of Apartheid, the following table stresses the impact of a variation in the transport cost τ_B on post-Apartheid welfare. Starting from the same base case as used in Table 5, we consider the impact of a 100% increase in τ_B and a 50% decrease in τ_B . First, observe that τ_B is negatively (positively) correlated with blacks' (whites') utility. This indicates that for blacks, when τ_B decreases (see equation (36)), the positive effect (better accessibility) dominates the negative one (negative group externality). Moreover, when there is a twofold increase in τ_B , 5.33% of black children attending the central school will shift to the suburban school. This leads to a higher surplus but more inequality. On the contrary, halving τ_B reduces both the surplus and inequality, suggesting a positive correlation between transport cost for black children and post-Apartheid inequality.

**Table 6: Simulation analysis
for changes in τ_B after Apartheid (%)**

	$\tau_B = 0.2$ (+100%)	$\tau_B = 0.05$ (-50%)
Δv_B^P	-0.04	0.02
Δv_W^P	0.17	-0.08
ΔN_{BC}	-5.33	2.67
ΔI^P	0.37	-0.19
ΔS^P	0.08	-0.04

Base case: $\alpha = 0.5$, $t_B = 0.4$, $\tau_B = 0.1$, $t_W = 0.5$, $\tau_W = 0.3$, $\bar{h}_W = 0.4$,
 $\bar{h}_B = 0.2$, $\bar{y}_W = 8$, $\bar{y}_B = 4$, $\bar{N}_W = 0.1$, $\bar{N}_B = 0.15$.

7 Conclusion

This paper is the first attempt to model education in a spatial urban context in South Africa under and after Apartheid. Though the model may seem quite stylized, we believe that it captures the basic features of most South African post-Apartheid cities. First, we obtain that after Apartheid, even though the land-use restrictions have been removed, the general structure of cities remains the same as under Apartheid: whites still reside close to the centers of opportunity whereas blacks live far away from these centers. Second, when Apartheid racial laws are removed, the inequality between blacks and whites can only decrease because blacks benefit from local human capital externalities by mixing with white students, and because whites are worse off since both their human capital decreases and the land rent increases. Third, we explain the emergence of a new phenomenon widely recognized in post-Apartheid South Africa: the fact that some black students accept to bear very long commutes in order to attend better schools located in white central locations.

Our model also has strong implications for policies that can be implemented in post-Apartheid South African cities, namely busing and transport-to-school subsidies. Indeed, we have shown that a policy aiming at reducing the transport costs of black children has an ambiguous effect on the utility of blacks whereas the impact on the utility of whites is always negative: a reduction in the transport costs of black children induces them to go to the central school, which in turn reduces the educational output of the central school attended by white children. In our simulations, the positive effect on the utility of blacks is dominant: reducing the transport costs of black children always increases the utility of black families and thus reduces post-Apartheid inequality.

From the above discussion, it should be clear that space and education are central to the analysis of South African cities. Space and education were the two main criteria driving discrimination since Apartheid was designed to prevent blacks and whites from interacting, especially in schools, and the spatial separation of communities was the most efficient way to ensure that no such interaction would ever happen. Our analysis shows that post-Apartheid integration can be promoted through education even though the spatial separation between communities remains.

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APPENDIX

Proof of Proposition 2:

By differentiating (21), we easily obtain:

$$\frac{\partial \tilde{\theta}}{\partial(\bar{h}_W - \bar{h}_B)} > 0 \quad , \quad \frac{\partial \tilde{\theta}}{\partial x} < 0 \quad , \quad \frac{\partial \tilde{\theta}}{\partial N_{BC}^e} < 0 \quad , \quad \frac{\partial \tilde{\theta}}{\partial \bar{N}_W} > 0$$

$$\frac{\partial \tilde{\theta}}{\partial \tau_B} > 0 \iff x < \frac{\bar{N}}{2}$$

■

Proof of Proposition 3:

From equations (22) and (23) and using the fact that $t_B > \tau_B$, we have:

$$\frac{\partial \Psi_W^P(x, v_W^P)}{\partial x} = -(t_W + \tau_W) < 0$$

$$\frac{\partial \Psi_B^P(x, v_B^P)}{\partial x} = -t_B + \tau_B [1 - 2\tilde{\theta}(x, N_{BC}^e)] < 0$$

This implies that the bid rent of whites is always linear whereas the bid rent of blacks is strictly convex since:

$$\frac{\partial^2 \Psi_B^P(x, v_B^P)}{\partial x^2} = -2\tau_B \frac{\partial \tilde{\theta}(x, N_{BC}^e)}{\partial x} = 4\alpha\tau_B^2 > 0$$

Moreover, since we have assumed that $t_W + \tau_W > t_B + \tau_B$ and since $0 < \tilde{\theta} < 1$, it is easily verified that,

$$\forall x \in [0, \bar{N}] \quad , \quad \left| \frac{\partial \Psi_W^P(x, v_W^P)}{\partial x} \right| > \left| \frac{\partial \Psi_B^P(x, v_B^P)}{\partial x} \right|$$

so that whites always reside closer to the city center than blacks. In equilibrium, white families live between 0 and \bar{N}_W whereas black families locate between \bar{N}_W and \bar{N} . ■

Proof of Proposition 4:

First, we know that an urban equilibrium always exists and is unique (see Fujita, 1989, ch.4, who shows that the existence and the unicity of an urban equilibrium is guaranteed as soon as bid rents can be ranked in order of relative steepness, here blacks and whites). Second, condition (27) guarantees that $\tilde{\theta}$ is always strictly interior, i.e., $\tilde{\theta}(x, N_{BC}) \in]0, 1[$ for all $(x, N_{BC}) \in [\bar{N}_W, \bar{N}] \times [0, \bar{N}_B]$. Lastly, let us denote by

$$\begin{aligned} \Phi(N_{BC}^P) &\equiv \int_{\bar{N}_W}^{\bar{N}} \tilde{\theta}(x, N_{BC}^P) dx - N_{BC}^P \\ &= \bar{N}_W \bar{N}_B \left[(1 - \alpha) \frac{\bar{h}_W - \bar{h}_B}{\bar{N}_W + N_{BC}^P} - \alpha \tau_B \right] - N_{BC}^P \end{aligned} \quad (37)$$

It is easily verified that this continuous function is always decreasing in N_{BC}^P and that, by using (27), $\Phi(0) > 0$ and $\Phi(\bar{N}_B) < 0$ so that there exists a unique equilibrium. ■

Proof of Proposition 6:

Let us start with the comparative statics on N_{BC}^P . By using (37), simple differentiation provides for any generic exogenous variable k :

$$\frac{\partial N_{BC}^P}{\partial k} = - \frac{\partial \Phi(N_{BC}^P) / \partial k}{\Phi(N_{BC}^P) / \partial N_{BC}^P}$$

which has the sign of $\partial \Phi(N_{BC}^P) / \partial k$. But using the definition (37) of $\Phi(N_{BC}^P)$, it is immediate to see that:

$$\text{sign} \frac{\partial \Phi(N_{BC}^P)}{\partial k} = \text{sign} \int_{\bar{N}_W}^{\bar{N}} \frac{\partial \tilde{\theta}(x, N_{BC}^P)}{\partial k} dx$$

Thus, using Proposition 4, the comparative statics on N_{BC}^P follows immediately for parameters $(\bar{h}_W - \bar{h}_B)$ and \bar{N}_W . Now, for τ_B , it is easily checked that:

$$\int_{\bar{N}_W}^{\bar{N}} \frac{\partial \tilde{\theta}(x, N_{BC}^P)}{\partial k} dx = \alpha \bar{N}_W (\bar{N}_W - \bar{N}) < 0$$

Hence, we have shown that:

$$\frac{\partial N_{BC}^P}{\partial (\bar{h}_W - \bar{h}_B)} > 0 \quad , \quad \frac{\partial N_{BC}^P}{\partial \bar{N}_W} > 0 \quad , \quad \frac{\partial N_{BC}^P}{\partial \tau_B} < 0$$

Concerning the comparative statics on h_C^P , we have:

$$\frac{\partial h_C^P}{\partial k} = \frac{\partial h_C^P}{\partial N_{BC}^P} \frac{\partial N_{BC}^P}{\partial k}$$

so that, by using (30), we have:

$$\frac{\partial h_C^P}{\partial(\bar{h}_W - \bar{h}_B)} < 0 \quad , \quad \frac{\partial h_C^P}{\partial \bar{N}_W} < 0 \quad , \quad \frac{\partial h_C^P}{\partial \tau_B} > 0$$

■

Figure 1. Apartheid Equilibrium (AE)

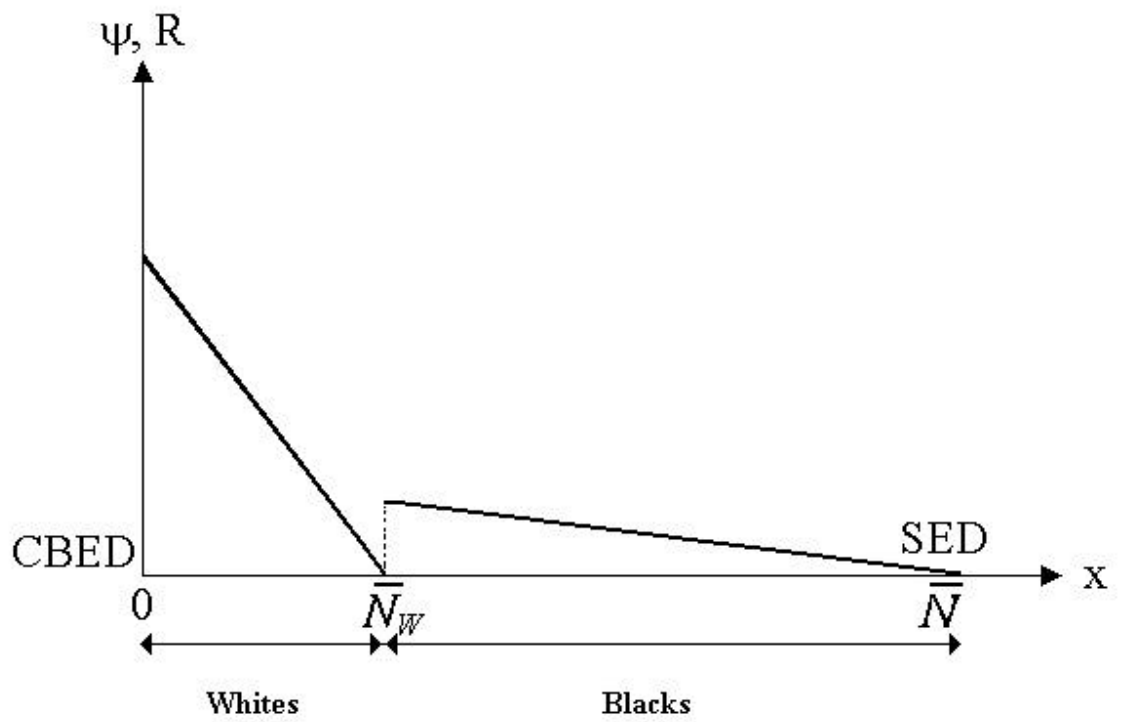


Figure 2. School choices

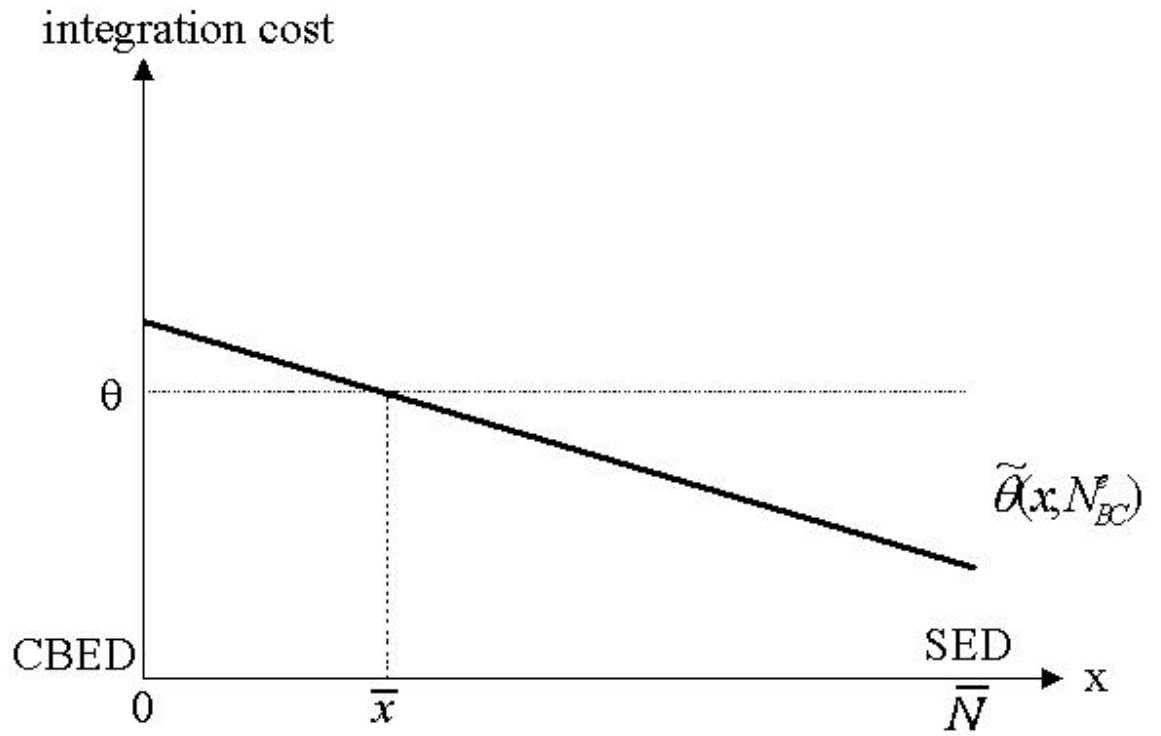


Figure 3. Post-Apartheid Equilibrium (PAE)

