From Heaven to Earth: Circles and the Construction of the Imperial Order in Late Ming and Early Qing China

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Biography

Catherine Jami is a Senior Researcher at the French CNRS. A historian of science, she has published extensively on the mathematical sciences in seventeenth- and eighteenth-century China, and on the circulation of scientific knowledge between China and Europe during the same period, and on the role of Jesuit missionaries in the construction of the Qing state. She also maintains an active interest in Science and Empire studies, and in the spatial dynamics of knowledge, as well as in digital humanities. Since 2017 she has been the editor of the journal *East Asian Science, Technology, and Medicine*.

Abstract

The Jesuit missionaries who worked in China in the seventeenth and eighteenth centuries taught the mathematical sciences to some Chinese literati, reformed the Chinese calendar, and applied European surveying methods to the mapping of the Qing empire. Their activities induced profound changes in the techniques used by the imperial state to manage Heaven and Earth, in an empire where the sovereign was viewed as the intermediary between the cosmos and the human realm. The new techniques in great part relied on mathematical knowledge about the circle. Looking at the definitions of the circle found in three Jesuit textbooks on geometry published in China at the time, and at the context in which these definitions are found thus sheds light in the extent to and ways in which the mathematics that underlay European astronomy at the time were put to use in China to reinforce the cosmological legitimacy of emperors as well as their control over their territories. Thus, one of the outcomes of the Jesuit missionaries' work in China was that similar mathematical knowledge

came to underlie the two very different cosmologies that dominated at the two ends of the Eurasian continent.

Keywords

Astronomy, calendar reform, cartography, circle, definition, Euclidian geometry, Jesuits, Manchu, map, translation

The late Ming and early Qing period (late sixteenth to early eighteenth century) saw an unprecedented circulation of goods, persons, and knowledge between the two ends of the Eurasian continent. Most of the Europeans who actually settled in China at the time were Catholic missionaries. Among the various religious orders present there, it was the Jesuits who had the greatest influence on Chinese elites, as they introduced some of the elements of their own academic training, including the sciences, through the medium of printed books written in classical Chinese, in addition to personal contacts.

The Jesuits' scientific activity in China can be divided into three periods. During the first decades of their mission, their main interlocutors were literati, with whom they produced a number of translations of books from European languages, and some books composed by Jesuits for a specifically Chinese audience. When they worked on the calendar reform starting in 1629, and then implemented it at the Astronomical Bureau (Qintianjian 欽天監) after the advent of the Qing (1644–1911), their mathematical output took a more technical turn. Finally, when they served the Kangxi 康熙 emperor (r. 1662–1722) personally as court savants, they contributed to the production of maps, and also of an imperial textbook of mathematics. To these three periods corresponded three stages in the way they taught the sciences in China. At each stage new elements were introduced, resulting not only in an accumulation, but also in a reshaping of what was presented as "Western knowledge" (*xixue* 西學) relevant to the sciences.

In imperial China, the calendar was more than an administrative matter: in the worldview that prevailed among the elite, the emperor acted as an intermediary between the cosmos and the human realm, and was charged with ensuring that the latter conformed to the order and pace of the former. Imperial ability to promulgate an accurate calendar every year was regarded as a token of dynastic legitimacy. The Jesuits' work on calendar reform thus turned them into crucial technical actors in the upholding of the cosmological imperial order. Moreover, the reform they proposed was not a mere change in astronomical constants or in the algorithms hitherto used to compute the calendar every year. Instead, they introduced a geometrical model as a fundamental explanatory mechanism for observed solar, lunar, and planetary motion. Thus, in the eyes of some scholars of the time, whereas in the Chinese tradition astronomical systems were normative, prescribing 'what must be so' (*suodangran* 所當然), the Jesuits explained 'why it must be so' (*suoyiran* 所以然), providing imperial astronomy with a new kind of foundations.[1]

Although modern scholarship has tended to separate the history of science from the history of religion, sometimes presenting the Jesuits' use of science as a bait on the hook of their religion, the Jesuits themselves understood what they brought to China as forming a single body of knowledge. It was commonly held in Europe at the time that the sciences, as tools for better understanding of nature, were a way that led one closer to God, who had

[1] Jami, *The Emperor's New Mathematics*, 16, 328.

created the cosmological order that underlay nature.[2] In particular, this belief underlay the setting up of the teaching of mathematics in the Society of Jesus during the second half of the sixteenth century. The founder of this teaching, Christoph Clavius (1538–1612), was also the author of the Gregorian calendar (1582). Both as a teacher and as an astronomer, he provided a role model for Jesuits in China, all the more so as several of those who worked there in the early seventeenth century had been his students at the Roman College.

In the early modern European understanding of the word, mathematics encompassed several fields that later came to be regarded as separate disciplines. This was a legacy from Greek antiquity: throughout the Middle-Ages, mathematics was represented in the curriculum by the *quadrivium*, which comprised arithmetic, geometry, astronomy, and harmonics. Clavius proposed a modified definition, which took into account fields that had gained importance in his time. For him, mathematics was divided into pure mathematics, which consisted of arithmetic and geometry, and mixed mathematics, which included six branches: natural astrology (in modern terms astronomy), perspective, geodesy (the representation and measurement of the earth), music, practical arithmetic, and mechanics.

The astronomical reform on which the Jesuits worked in Beijing relied on the Tychonic, system which placed the Earth at the center of the universe, and posited that the Sun and Moon rotated around the Earth, while the Five Planets rotated around the Sun. The mathematical object that lay at the heart of these new foundations was the circle, which, in the scientific tradition inherited by the Jesuits, was regarded as central to cosmology: the motion of heavenly bodies, regarded as perfect, was assumed to result from a combination of uniform circular motions. Therefore, the definition and status of the circle in the mathematical knowledge which underlay the Jesuits' astronomy in China are worthy of attention. They presented the circle as an object defined within the Euclidian tradition of geometry, and which

[2] Cullen and Jami, "Christmas 1668," 4.

was furthermore measured by means of trigonometry, two fields which they introduced into China early on. The spherical shape of the Earth, also introduced in China by the Jesuits during the first decades of their mission, long remained controversial there.[3] This shape and the surveying techniques used to map the Earth in general and later the Qing empire in particular, were also premised on circles, and therefore on the mastery of both Euclidian geometry and trigonometry.

In the present contribution I will focus on the circle, examining how it was defined in three printed geometrical treatises in Chinese, each corresponding to one of the three periods of Jesuit activity in China mentioned above. Also relying on the sources of these treatises, written in Latin, Chinese, French, and Manchu, I will show how these definitions as well as the content, structure and style of the treatises in which they are found changed as mathematics became first a tool for astronomers, and then an imperially sponsored field of learning closely connected to cartographic techniques.

Unlike some geometric objects such as points and angles, that were first introduced into China by the Jesuits, the circle was present in Chinese mathematical texts as represented by the founding classic of the first century CE, the *Nine chapters on mathematical procedures* (*Jiuzhang suanshu* 九章算術). This being said, it is well known that Chinese mathematical texts prior to the late Ming period do not by and large define the objects on which they work.[4] However, the most widely read mathematical treatise of the late imperial period, Cheng

[3] Chu, "Trust, Instruments".

[4] The *Mojing* 墨經 (fifth century BCE) gives the following definition:

圜，一中同長也。, which A. C. Graham rendered as: "*Yüan* (circular) is having the same lengths from one centre." Graham, *Later Mohist Logic*, 307, 309. There is no evidence however that this or any other definition of the circle formed part of the early Chinese mathematical corpus.

Dawei's 程大位 (1533–1606) *Suanfa tongzong* 算法統宗 (Unified lineage of mathematical methods, 1592), provided "Notes on the characters used" (*Yongzi fanli* 用字凡例) at the beginning of its first chapter. They include the following glosses:

> 周 外圍也 。
>
> Circumference: external enclosure.
>
> ...
>
> 徑 周中之弦。
>
> Diameter: the hypotenuse/chord through the middle of a circumference. [5]

The first term mentioned here does not refer to a two-dimensional area, but to a line that can be measured. There is no gloss on *yuan* 圓, the character commonly used to refer to circles in the *Suanfa tongzong*. This term occurs in chapter 3; there one finds figures illustrating it in most of its occurrences. Like most other shapes given there, *yuan* 圓 is followed by the suffix *tian* 田 (field) in the text to refer to a circular field, of which the area is being discussed.[6] This usage is representative of the Chinese mathematical tradition.

The First Jesuit Treatise on Geometry and Its Definition of the Circle

[5] ZKJDT, vol. 2, 1230; *xian* 弦 has been explained shortly before, both in the context of *gougu* 勾股 and in that of *hu* 弧 (arc).

[6] ZKJDT, vol. 2, 1265–1266.

Matteo Ricci (1552–1610), the founder of the Jesuit mission in China, had been one of Clavius's students at the Roman College who worked in China. He translated a number of his works into Chinese. The most famous one, *Jihe yuanben* 幾何原本, is a translation of the first six books of Euclid's *Elements of Geometry*, done from Clavius's Latin edition and completed in 1607 by Ricci and Xu Guangqi 徐光啓 (1562–1633), a high official and Catholic convert. This first documented introduction of Euclidian geometry had a lasting impact on mathematics in China and beyond. The fact that the modern term for "geometry" in Chinese, Japanese, and Korean (幾何學, Ch. *jihexue*) was derived from the title of this translation bears witness to this. In their respective prefaces, the two translators both stressed the importance of mathematics as providing a foundation for Western knowledge.[7]

The *Jihe yuanben* contains the earliest known definition of the circle in the Chinese mathematical literature. Found in Book I, this definition reads as follows:

> 第十五界
>
> 圜者。一形於平地居一界之間。自界至中心作直線俱等[8]
>
> Definition 15
>
> A circle is a figure on even ground, occupying the space [within] a boundary; the straight lines drawn from the boundary to the middle are all equal.

Let us compare this to Clavius's definition in Latin:

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[7] Engelfriet*, Euclid in China*, 454–460, 291–297.

[8] *Jihe yuanben* 1607, 5b. I reproduce the punctuation of this edition, in which there are no punctuation marks at the end of paragraphs.

XV

Circulus est, figura plana sub vna linea comprehensa, quæ peripheria appellatur; ad quam ab vno puncto eorum, quæ intra figuram sunt posita, cadentes omnes rectæ lineæ inter se sunt æquales.[9]

15

A circle is a plane figure contained within a single line, which is called circumference; towards which all the straight lines from one of the points inside the figure are equal.

While the Chinese text conveys the same meaning as the Latin one, it is not in strict conformity with it according to the Euclidian style, in which technical and ordinary words are by no means interchangeable. In this respect, we can point to three discrepancies. Firstly, whereas the notion of plane surface (Latin *plana superficies*; Chinese *pingmian* 平面) is found in definition 7, the translators have chosen to characterize the circle as being on even ground (*pingdi* 平地). Secondly, in the same sentence, they have preferred the use of "boundary" (*jie* 界) to that of "line" (*xian* 線).[10] Thirdly, in the second sentence, they have used the non-technical term "middle" (*zhongxin* 中心), instead of a clause that would have rendered "one of the points of the figure". The point in question is defined as the center of the circle (*yuanxin* 圜心) in the next definition. This being said, it is interesting to note that the Chinese definition uses the character *yuan* 圜 (also read *huan*, a variant of 圓 attested since

[9] Clavius, *Euclidis Elementorum libri XV*, 12.

[10] The fact that *jie* 界 is mostly used in *Jihe yuanben* as an abbreviation of *jieshuo* 界說, "definition," must not have made the reader's task easier.

early texts), rather than the character *yuan* 圓 found in the *Suanfa tongzong* and elsewhere in Chinese texts to name the circle. As we shall see below, this choice most likely resulted from the translators’ wish to emphasize the fact that the object defined here was not merely the intuitively obvious circular shape represented in these texts. This contributes to placing the definition in the Euclidian realm, where geometric objects are defined abstractly by their properties, without any reference to the material world.

The definition given in the Chinese text quoted above is followed by three sentences of clarification, which are taken from Clavius’s lengthy commentary:

> 若甲乙丙為圜。丁為中心。則自甲至丁、與乙至丁、丙至丁。其線俱等[11]
>
> If ABC is a circle, and D is the middle, then the lines from A to D, B to D, and C to D are all equal.
>
> 外圓線為圜之界。內形為圜[12]
>
> The external round line is the circle’s boundary. The figure inside is the circle.
>
> 一說。圜是一形。乃一線屈轉一周、復于元處所作。如上圖甲丁線轉至乙丁。乙丁轉至丙丁。丙丁轉至甲丁。復元處。其中形即成圜[13]

[11] Here and in what follows cyclical characters are rendered by capital letters.

[12] Later editions do not all take up the distinction between 圜 and 圓; see e.g. the 1865 edition reproduced in ZKJDT, vol. 5, 1162, where this sentence reads: 外圜線為圜之界。內形為圜。

[13] *Jihe yuanben* (1607), *juan* 1, 5b-6a.

Explanation: The circle is a figure, which is constructed by a line rotating all the way around until it returns to its initial position. As in the figure above, line AD rotates to BD; line BD rotates to CD; line CD rotates to AD. When it returns to its initial position, the figure within is a circle.

Both the Latin and the Chinese text are accompanied by a figure in the Euclidian tradition to which the commentary refers (fig. 1). These figures are not identical. Points A, B, C, and *jia* 甲, *yi* 乙, *bing* 丙 are not situated in the same position on the circle; moreover, they have been named clockwise in Latin, and anti-clockwise in Chinese. One might nonetheless say that, as with the definitions themselves, the figures are similar enough. In the Chinese text, the figure is explained in the commentary using both the characters *yuan* 圜 (circle) and *yuan* 圓 (round) in a non-interchangeable way. This enables readers to ascertain that the *yuan* 圜 defined here is the same object as the one they may have been familiar with, called *yuantian* 圓田 (lit. round field) by Cheng Dawei. This being said, even the final explanation (*shuo* 說) remains abstract: the rotation of a geometric object (namely a straight line) generates a circle, without any agency of a material object or human action.

[Place figure 1 here]

Whereas in the Latin work Clavius's commentary is printed in italics to distinguish it from the Euclidian text, in the Chinese translation the different authorship of the additions to this text is not made visible by the use of small characters, or by the mention of the commentator's name, as was common in the Chinese textual tradition. Nevertheless, the translation distinguished the Euclidian text: any addition to it is made in separate paragraphs.

This definition of the circle was written at a time when mathematics, rather than astronomy, lay at the core of the Jesuits' teaching of the profane sciences in China.

Mathematics for Astronomers

The 1607 translation of Euclid's *Elements*, however important it might seem in the context of the global history of mathematics, was hardly accessible without the help of a teacher. In the decades that followed its publication, another textbook on Euclidian geometry was composed. It focused on methods rather than on definitions, theorems, and proofs. This second textbook would eventually be integrated in the mathematical knowledge to be used by astronomers.

Completed in 1623, it was entitled *Jihe yaofa* 幾何要法 (Essential methods of geometry). It was the result of collaboration between another Jesuit missionary, Giulio Aleni (1582–1649) and another convert, Qu Shigu 瞿式穀 (1593– ?). At the time of the translation of Euclid's *Elements*, Xu Guangqi, who had just passed the metropolitan examination, was in the capital awaiting his first appointment. By contrast Qu Shigu was a rather obscure literatus; the *Jihe yaofa* was composed in his native town, Changshu 常熟, in Zhejiang province, where Aleni spent some time. The work, first printed in 1631 in Fuzhou 福州, may well be more representative of the Jesuit missionaries' teaching of geometry to Chinese literati than the 1607 translation. It is constructed around extracts from the *Jihe yuanben*, focusing on geometric constructions.[14] The circle is defined at the beginning of chapter 2, which is devoted to it:

[14] Jami, "Giulio Aleni's contribution."

第一界

圜形於平地、居一界之間、為圜。

Definition 1

A round figure on even ground, occupying the space [within] a boundary, is a circle.

第二界

外圓線為圜之界。[15]

Definition 2

The external round line is the boundary of the circle.

Whereas one recognizes the kinship of these definitions with what we read in Book I of *Jihe yuanben*, the distinction between the Euclidian text and Clavius's commentary has been blurred: definition 1 is taken from the former, whereas definition 2 is taken from the latter. Regarding syntax, the convention followed in the definitions of the *Elements* and in the 1607 translation was to make the defined term the topic of the sentence, while the explanation of the term constituted the comment. This convention has been abandoned in the *Jihe yaofa*. One may also note that the distinction between the characters *yuan* 圜 (circle) and *yuan* 圓 (round) has now been incorporated into the definitions themselves, whereas the characterization of the circle by the equidistance of the points of its circumference to its center is simply omitted.

Moreover, the context of the definitions is quite different from that of the *Jihe yuanben*. Thus, the first four methods (*fa* 法) given in chapter 2 of *Jihe yaofa* are devoted not to geometric

[15] *Jihe yaofa* (1631), *juan* 2, 1b.

constructions, but to the making of compasses (*zaogui* 造規); the four different patterns of compasses given are illustrated just like geometric constructions.[16] Only once equipped with the right instrument can the reader tackle the various geometric constructions in which compasses are used. A similar pattern is followed in other chapters of the *Jihe yaofa*, in which the instruments of geometry related to the main topic of each chapter are described in detail (see Table 1). Thus, the *Jihe yaofa* includes not only geometrical knowledge and methods, but also illustrated descriptions of the tools necessary for drawing geometric figures: possessing the right tools is a prerequisite for the practice of geometry.

[Place table 1 here]

In 1629, when he proposed to the Chongzhen 崇禎 emperor (r. 1628–1644) that a calendar reform should be prepared using the Jesuits' methods, Xu Guangqi again emphasized the status of mathematics as a fundamental field. In agreement with him, the Jesuits who set out to prepare the reform relied on a number of European treatises, using the Tychonic astronomical model as a basis for their calculations.[17] This was a major departure from the Ptolemaic model found in their earlier works. It should be noted, however, that Euclidian geometry is a foundation for both models, as both systems use circles in the modelling of orbits for the purposes of calculation.

Xu Guangqi also argued that the series of works written by the Jesuits to prepare this reform, which formed the *Chongzhen lishu* 崇禎曆書 (Astronomical works of the Chongzhen

[16] *Jihe yaofa* (1631), *juan* 2, 4a–6b.

[17] Hashimoto, *Hsü Kuang-ch'i*.

reign, 1629–1634), did more than just set out a new system for calculating the calendar (that is, a set of algorithms and constants). Instead, they provided quantified circle-based geometrical models for solar, lunar, and planetary motion. This meant that if the calendar needed further reform in the future, one could return to this geometrical model and adjust constants according to observations. Thus, the circles of Euclidian geometry were explicitly placed at the foundation of calendrical astronomy. But at the same time, the methods introduced by the Jesuits had to be adapted to Chinese calendar-making: Xu Guangqi advocated "melting their material and substance to cast them into the *Datong* mould."[18] Firstly, this can be understood to refer to the need for a luni-solar calendar, much more complex to compute than the solar calendar used in Europe before and after Clavius's reform. Secondly, this also meant that knowledge imported from Europe was to contribute to reinforce imperial power and dynastic stability by ensuring that the emperor was in a position to effectively perform his function of intermediary between the cosmos and the human realm, by providing his people with an accurate calendar. In other words, whereas the Jesuits' original purpose in teaching mathematics and astronomy in China was to put mathematics in the service of evangelization, by working on an astronomical reform they effectively put their sciences in the service of a radically different cause: the upholding of the cosmology that underlay the imperial order.

The calendar reform proposed by Xu Guangqi was never implemented under the Chongzhen emperor. In 1644, when the Manchus took Beijing during their conquest of the Ming empire, they accepted the offer of one of these Jesuits, Johann Adam Schall von Bell (1592–1666), to calculate a new calendar for their new dynasty, and put him in charge of the

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[18] 熔彼方之材質，入大統之型模. *Datong* is the name of the astronomical system in use during the Ming dynasty (1368–1644); see Hashimoto and Jami, "From the Elements to calendar reform," 274–275.

Astronomical Bureau. At the same time as his Calendar of Timely Modelling (*Shixian li* 時憲曆) was issued, a slightly modified version of the collection of works presented to the Chongzhen emperor was reprinted under the title *Xiyang xinfa lishu* 西洋新法曆書 (Works on astronomy according to the new Western methods).

Among the Jesuits' writings included in the *Chongzhen lishu*, there are five that present mathematical knowledge: *Chousuan* 籌算 (Calculating rods, 1628), which describe the making and use of Napier's bones;[19] *Bili guijie* 比例規解 (Explanation of the proportional compass, 1630), devoted to the instruments devised by Galileo; *Dace* 大測 (Great measurement, 1631); *Celiang quanyi* 測量全義 (Complete meaning of measurement, 1631), which deals with various geometric objects, including spherical triangles, and closes with a chapter on astronomical instruments; and *Geyuan baxian biao* 割圓八線表 (Eight trigonometric tables, 1635), which provides a major calculation aide for astronomy.[20] If one compares these works to the *Jihe yuanben* and other mathematical works by Ricci, they offer a very different picture of mathematics. It might be said that with the astronomical reform, mathematics had become instrumental in two senses. Firstly, it included instruments into the written corpus of knowledge. Secondly, it provided a set of tools for the astronomer.

[19] On this manually operated calculating device and its introduction into China, see Cervera, *Las Varillas de Napier*.

[20] Jami, "Mathematical knowledge."

This interpretation is strengthened by the fact that, when the Qing decided to implement the calendar reform initiated by Xu Guangqi, the geometrical treatise that was chosen for inclusion in the augmented version of the *Chongzhen lishu* renamed *Xiyang xinfa lishu* 西洋新法曆書 (Books on astronomy according to the new Western method) instead of the *Jihe yuanben* was the *Jihe yaofa*, despite the fact that in the other works of the *Xiyang xinfa lishu* there are fourteen references to *Jihe yuanben* and none to *Jihe yaofa*. As we have seen, the latter work's approach to geometry can be characterized as instrumental in the double meaning evoked above. Thus, the astronomical reform resulted in a shift in the mathematics practiced and taught by the Jesuits, who had first presented Euclidian geometry as the foundation of all their profane sciences. Whereas some of the building blocks of Clavius's version of Euclidian geometry can still be identified in the Jesuit's new texts, they are now used to construct a corpus in which the main purpose of mathematics was to use material objects (instruments) to perform calculations and measurements of other material objects (celestial bodies). This is in stark contrast with the mathematics introduced by Ricci, in which geometry and arithmetic are characterized as "discussing them (magnitude and number) in the abstract, casting off material objects" (*tuo yu wu qi er kong lun zhi* 脱於物體而空論之).[21] In keeping with this, beside the *Jihe yuanben*, Ricci also co-authored a work on calculation, the *Tongwen suanzhi* 同文算指 (Instructions for calculation in common script, 1614), which introduced written calculation for the first time. At a time when the abacus, on which the *Suanfa tongzong* rests, was universally used in China, this new technique enabled scholars to perform calculation using only the "four treasures of the study" (*wenfang sibao* 文

[21] Jami, *The Emperor's New Mathematics*, 26; see ZKJDT, vol. 5, 1151.

房四寶), namely brush, inkstone, ink stick, and paper, their distinctive tools, rather than the abacus, the tool of merchants.[22]

This change in the role and status of instruments in Western mathematics as practiced in China corresponds to another change, which concerns the status of mathematics among the various fields to which it is related. When Xu Guangqi promoted the astronomical reform, he upheld Ricci's view on the matter: "magnitude and number" (*dushu* 度數, as he and Ricci had rendered the two instances of quantity that defined the two branches of mathematics) was a fundamental field of knowledge. It could be applied to ten other fields: astronomy as well as astrology and meteorology; surveying for the purpose of water conservancy works; music and harmonics; military engineering and equipment; finances and taxes; building and civil engineering; mechanical devices; cartography; medical hemerology; and time measurement, especially by means of clepsydras and dials.[23] This list reflects Xu Guangqi's concern with statecraft. On the other hand, when it came to structuring the body of knowledge contained in the *Chongzhen lishu*, Xu Guangqi proposed a fivefold classification: bases of the methods (*fayuan* 法原), numbers of the methods (*fashu* 法數), calculations of the methods (*fasuan* 法算), instruments of the methods (*faqi* 法器), and intercommunication (*huitong* 會通). This last category refers to the conversion of units. The treatise on the proportional compass pertains to the "instruments of the methods", as does the last chapter of the *Celiang quanyi*, whereas the first nine chapters of this work belong to the "bases of the methods", together with the *Dace*. Trigonometric tables as well as Napier's bones pertain to the "numbers of the

[22] On this point see Jami, *The Emperor's New Mathematics*, 356–357; Jami, "La carrière de Mei Wending." 31.

[23] Xu Guangqi, *Xu Guangqi ji*, vol. 2, 337–338. See Engelfriet, *Euclid in China*, 349–350.

methods".[24] It is remarkable that this last device belongs with tables rather than with instruments. But the most important point here is that what we regard as a single field, namely mathematics (for which the field called *dushu* by Xu Guangqi can be regarded as one among many definitions found in historical sources), is no longer a relevant entity when it comes to organizing knowledge on the basis of its role in the astronomical compendium. Instead, as stated above, the function of mathematics is now merely to provide tools for constructing the astronomical knowledge that underlay the 1629 calendar reform.

Imperial Mathematics: Measuring the Heavens, but Also the Earth

When the Jesuits began to work on their reform project in 1629, their astronomy and the mathematics that underlay it were seen as highly technical fields. Only a few Chinese scholars studied them; at the same time, the field had little prestige compared to classical studies, which were the prerogative of the literati class. This began to change under the Kangxi emperor (r. 1662–1722), the second Manchu sovereign to rule China, as he took a personal interest in these sciences. New mathematical textbooks were written for him by the Jesuits, and eventually revised and incorporated into the *Shuli jingyun* 數理精蘊 (Essence of numbers and their principles, 1723).[25] The production of this imperial textbook is closely linked to efforts to summarize, and, where relevant, to update the methods expounded in the *Xiyang xinfa lishu*, efforts which resulted in the *Lixiang kaocheng* 曆象考成 (Thorough investigation of astronomical phenomena, 1723). The period of compilation of these two works also overlapped with another major imperial project involving Western learning: the famous geodesic survey of the Qing empire that resulted in a complete map of the its territory,

[24] Xu Guangqi, *Xu Guangqi ji*, vol.2, 377–378 ; see Jami, "Mathematical Knowledge." 667–668.

[25] Jami, *The Emperor's New Mathematics*, 315–384.

the *Huangyu quanlan tu* 皇輿全覽圖 (Overview Maps of the Imperial Territories, completed in 1717). Since the *Shuli jingyun* was intended as a textbook for all scholars and officials, it is worthwhile revisiting it in the light of recent work on this map,[26] in order to understand how concern with measuring the Earth, as well as the Heavens shaped Chinese mathematics in the early eighteenth century.

Whereas the imperial appropriation of the Jesuits' astronomy was motivated by the fact that it was crucial for the dynasty to be able to issue an accurate calendar, the emergence of mathematics as an imperially sponsored field of knowledge stemmed mainly from the personal taste of the Kangxi emperor. Towards the end of his sixty-year reign, during which he had studied mathematics as well as astronomy and music with the Jesuits, he commissioned a compendium, entitled *Yuzhi Lüli yuanyuan* 御製律曆淵源 (Origins of the pitchpipes and the calendar imperially composed, 1723), made up of three treatises representing the three fields. Beside the *Shuli jingyun* mentioned above, these treatises, included the *Lixiang kaocheng* 曆象考成 (Thorough investigation of astronomical phenomena), and the *Lülü zhengyi* 律曆淵源 (Correct interpretation of [standard] pitchpipes). Whereas the association between astronomy and harmonics is as old as dynastic histories — some of which contain a monograph entitled *Lüli zhi* 律曆志 (Monograph on the pitchpipes and the calendar)— mathematics had not been the subject of an imperially-sponsored editorial project for more than a millennium, since the compilation of the *Suanjing shishu* 算經十書

[26] Cams, *Companions in Geography*.

(Ten mathematical classics) by Li Chunfeng 李淳風 (602–670) in 656. Kangxi's project further raised the status of mathematics by associating it with astronomy and harmonics, two fields that were deemed crucial in harmonizing the human realm with the cosmos.

Starting in 1708, that is, five years before the *Yuzhi Lüli yuanyuan* was commissioned, another imperial project was launched: the cartographic survey of the Qing empire, an enterprise on a scale unprecedented worldwide, which resulted in the complete map of the empire entitled *Huangyu quanlan tu*. As with mathematics, astronomy, and harmonics, this project was the outcome of the emperor's reign-long interest in cartography. Since the 1680s, he had taken steps to ensure that he had maps of the Chinese provinces of his empire, as well as of the territories that lay beyond the Great Wall. For the former, he mainly relied on local officials. For the latter, various mapping expeditions were sent; moreover he ordered several Jesuit missionaries to carry out measurements during their travels in his retinue or with various expeditions.[27] Comparing the maps collected in this way led to the realization that there was a discrepancy between the results yielded by traditional surveying methods and those derived by the Jesuits, who used French instruments and techniques to make observations in order to determine the latitude of places through which they travelled. Therefore, in 1702, the emperor sent a team led by one of his sons to measure one degree of a meridian, in order to fix a new standard for the *li* 里.[28] The first expedition sent out in 1708 carried out a survey along the Great Wall. Between 1709 and 1716, six expeditions then surveyed the Inner Territories, while other teams were sent to the Manchu homeland and to the Mongol frontier. Data on Korea were requested from its king, whom the Manchus

[27] Cams, *Companions in Geography*, 44–48.

[28] Cams, *Companions in Geography*, 76–81. This new standard was defined so that there were 200 *li* in a degree of meridian, which gives 1 *li* = 444 m; Jami, *The Emperor's New Mathematics*, 393.

regarded as their tributary. A first version of the copperplate engraved *Huangyu quanlan tu* was completed in 1717. An expedition to Tibet, which set out in the same year, brought back data that was included in a revised version completed in 1719.[29] The surveying teams included at least two missionaries, one official from the Inner Court, a military officer, a civil service official, and a specialist from the Astronomical Bureau.[30] Monitored directly by the emperor and managed from the Inner Court, these expeditions received support from local officials wherever they went. Given the scale of this project, and the fact that the compilation of the *Yuzhi Lüli yuanyuan* was also carried out within the Inner Court, one may ask whether this gigantic survey had any impact on mathematics, in a way similar to the impact that astronomical reform had from the 1630s on. No treatise devoted specifically to surveying and mapmaking was published in China at the time. Therefore, in what follows I would like to assess the ways in which imperial mathematics as found in the *Shuli jingyun* incorporated "mathematics for surveyors," as the *Chongzhen lishu* had incorporated "mathematics for astronomers."

As far as we know, the *Yuzhi Lüli yuanyuan* was compiled by a large team of Chinese scholars, and it is difficult to ascribe the authorship of specific passages to one or more of them in particular.[31] The *Shuli jingyun* is largely based on the lecture notes prepared for the emperor by the Jesuits mentioned above. It is the result of considerable rewriting of these notes: here again no single authorship can be distinguished. The work presents a synthesis of mathematical knowledge available at the time. It is the largest mathematical treatise ever printed in imperial China.[32] Its fifty-three chapters are organized into three parts of very

[29] Cams, *Companions in Geography*, 102–124, 201.

[30] Cams, *Companions in Geography*, 139–140.

[31] Jami, *The Emperor's New Mathematics*, 373–378.

[32] Jami, *The Emperor's New Mathematics*, 315.

different lengths and layout. The first part, devoted to "Establishing the structure to clarify the substance" (*Ligang mingti* 立綱明體), contains five chapters; its text is divided into numbered items (*jie* 節). The second part, concerned with "Dividing into groups to put to use" (*Fentiao zhiyong* 分條致用), contains forty chapters, which consist of problems, each accompanied by a method (*fa* 法) for deriving the solution. The third part contains eight chapters of tables (*biao* 表).

The first part of the work includes three chapters entitled "Elements of geometry" (*Jihe yuanben*). This is not the 1607 translation. Instead, it is based on one of the many textbooks of Euclidian geometry written in seventeenth-century Europe that have the same title, Ignace Gaston Pardies's (1636–1674) *Elémens de géométrie* (1671).[33] It is followed by a chapter on the "Elements of calculation" (*Suanfa yuanben* 算法原本), mainly derived from Book VII and the beginning of Book VIII of Euclid's *Elements*. The second part of the *Shuli jingyun* is organized according to headings taken from both the Jesuits' teaching and the Chinese mathematical tradition, bringing out links between the two wherever relevant.[34] Thus, chapter 18, entitled "Measurement" (*Celiang* 測量), on which we will now focus, bears little resemblance to the *Celiang quanyi* mentioned above, as all the triangles discussed in

[33] Pardies, *Elémens de géométrie*. This book's lengthy title, which can be rendered as *Elements of geometry, in which by a short and easy method one may learn what must be known of Euclid, Archimedes, Apollonius, and the most beautiful inventions of the ancient and modern geometers*, states the author's intention quite clearly.

[34] Jami, *The Emperor's New Mathematics*, 317–339.

chapter 18 are plane. The chapter opens with a short introduction, which, among others, points to the broad applications of the techniques presented in it:

> 不特凡物之高深廣遠。可得而推。即七政之躔度。天地之形體。俱可得而測也。[35]
>
> Not only can the height, depth, width and distance of things be derived, but also the trajectories of the Seven Governors (Sun, Moon, and Planets), and the bodies of Heaven and Earth can all be measured.

The problems that follow, however, are solely concerned with measuring the height or distance of objects that are earthly, whether manmade or natural. They include flagpoles, pavilions, pagodas, walls, trees, mountains, rivers, and rocks: these are the features that surveyors measured and used as landmarks when surveying the empire. The problems share a feature that distinguishes them from the majority of problems found in the *Shuli jingyun*: the method for solving them starts with one or more measurements that yield numerical data not given in the problems at the onset; these data are then entered into the calculations that finally result in the magnitude sought. Altogether there are eighteen problems in the chapter, nine on “measurement by base and altitude” (*gougu celiang* 勾股測量) and nine problems on “trigonometric measurement by magnitude and number” (*sanjiao dushu celiang* 三角度數測量). These require different instruments, as noted in small characters:[36]

[35] *Shuli jingyun xia juan* 18, 1b; ZKJDT, vol. 3, 607.

[36] In the quotations below, what is in small characters in the original text is put between brackets both in Chinese and in the English translation.

勾股測量（凡用矩度。或立表杆。必用垂線。取其與地平成直角。以為準則。若地不平。須記取某處與人目所看相平為記。）[37]

Measurement by base and altitude (In general one uses a quadrant, or one erects a pole and needs to use a plumb line to get it at right angles with the horizontal, so as to have a standard. If the ground is not even, one must take note of a place level with one's line of sight to use as a reference.)

三角度數測量（度數測量。必取資於儀器。全圜儀、半圜儀、象限儀、雖為體不同。其為用則一。以九十度為準。以定表遊表為二視線。其相距之度。即為所測之角。）[38]

Trigonometric measurement by magnitude and number (for measurement by magnitude and number, one must rely on instruments. Although the circle, the semi-circle and the quadrant have different shapes, the method for using them is the same. One takes 90 degrees as the standard, and one takes the fixed pinnules and moving pinnules as the two lines of sight. The [number of] degrees that separate them is the angle one is measuring.)

The first problem of the chapter contains a note that further describes the quadrant one is to use, and then refers the reader back to item XII.16 of *Jihe yuanben*, found in the first part of the *Shuli jingyun*. This item gives a "method for making a surveying instrumentgraduated into divisions" (*zuo fenshu bili celiang yiqi fa* 作分數比例測量儀器法), in this case a semi-circle (fig. 2). The instructions are given in five successive steps:

1. Divide the semi-circle ACB into 180 degrees, and each of these degrees into 60 minutes.

[37] *Shuli jingyun xia juan* 18,1b; ZKJDT, vol. 3, 607.

[38] *Shuli jingyun xia juan* 18, 25a; ZKJDT, vol. 3, 618.

2. Divide the half-square inscribed in this semi-circle into small squares by drawing parallel lines.

3. Mount two fixed pinnules at both ends of the diameter.

4. Mount a moving pinnule at the end of alidade with 100 divisions.

5. Mount a plumb line.[39]

[Place figure 2 here]

This item is followed by two others that explain respectively how to make measurements and to use them to draw a map, and how to reproduce a map on a smaller scale:[40] here one is undoubtedly in the realm of cartography. In other words, the *Shuli jingyun* version of the *Jihe yuanben*, unlike the 1607 translation, includes the technical description and use of surveying instruments. Whereas one could characterize the mathematical knowledge found in the *Chongzhen lishu* as "mathematics for astronomers," the *Shuli jingyun* includes "mathematics for surveyors" as part of the knowledge it presents.

Defining the Circle in Imperial Mathematics

Did this integration of surveying methods, which Pardies included in his work as part of "Practical geometry" (Book IX of his *Elémens de géométrie*), have any impact on the way Euclidian geometry itself was presented? In order to answer this question, let us examine the definition of the circle in the *Shuli jingyun*. It is item I.5 of the *Jihe yuanben*, just after the definitions of point, line, surface, solid and angle.

[39] *Shuli jingyun shang juan* 4, *Jihe yuanben* XII, 16; ZKJDT, vol. 3, 135–136.

[40] *Shuli jingyun shang juan* 4, *Jihe yuanben* XII, 17 & 18; ZKJDT, vol. 3, 136–137.

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凡有一線。以此線之一端為樞。復以此線之一端為界。旋轉一周。即成一圜。如甲乙一線。以甲端為樞。乙端為界。旋轉復至乙處。即成乙丙丁戊之圜。此圜線謂之圜界。圜界內所積之面度。謂之圜面。[41]

5.

Let there be a line. Take one of its ends as a pivot; again take [the other] end as boundary. Turning it through a complete rotation makes a circle. Thus, with line AB, take the A end as a pivot; take the B end as boundary. Turning it until it returns to the [original] place of B makes the BCDE circle. This circular line is called circumference of the circle. The area accumulated inside the circumference is called the area of the circle.

[Place figure 3 here]

There is no trace here of the Euclidian definition, and, although the circle is generated by the rotation of a line, in a way similar to that in Clavius's explanation translated in 1607 by Ricci and Xu, the style is somewhat different (fig. 3). Whereas that "Explanation" is a description of a line rotating around one of its ends, here it might be said that the repeated use of *yi* ... *wei* 以……為 (lit. take... as) introduces a form of agency which involves the reader in a procedure. In order to explore this matter further, let us compare this definition to the one given in Pardies's textbook:

10. Si nous imaginons une ligne ab attachée par le bout au milieu de la ligne dc, & que de plus nous fassions mouvoir cette ligne autour du point a ; quand elle sera revenuë au lieu d'où elle avoit

[41] *Jihe yuanben* I.5, in *Shuli jingyun* II, 3b; ZKJXDT 3, 22.

commencé à se mouvoir, l'extrémité b aura décrit une ligne courbe qui s'appelle Cercle, ou plûtost Circonférence du cercle : car à proprement parler, le Cercle est tout l'espace renfermé dans cette circonférence.[42]

10. If we imagine a line *ab* tied by its end *a* to the midpoint of the line *dc*, and that moreover we make this line move around point *a*; when it has returned to the point from which it had started to move, the end *b* will have described a curved line called a Circle, or rather Circumference of a circle: for strictly speaking the Circle is the whole space enclosed in this circumference.

What is proposed here is not a definition of the circle that characterizes it by its properties, but a mental operation akin to a thought experiment that rests on imagining oneself exerting a concrete action on material objects, effectively causing the rotation of line *ab* that will generate a circle. This concrete aspect is strengthened by the figure that accompanies this definition: the rotating line is represented as a material object very much resembling an alidade such as the ones fixed on surveying instruments described further in the work.[43] Compared to this visual representation, the *Shuli jingyun*'s figure is much more abstract: no material device enabling a rotation of the line is required.

[Place figure 4 here]

Two intermediary texts are known between the French one and the Chinese one quoted above: Pardies's textbook was translated into Manchu in 1690; this formed part of an attempt by the Kangxi emperor to create a body of writings on the sciences in Manchu.[44] The following year, the Manchu text was in turn translated into Chinese, relying heavily on the terminology coined by Ricci and Xu in the 1607 translation. The editors of the *Shuli jingyun*

[42] Pardies, *Elémens de géométrie*, 4–5.

[43] Pardies, *Elemens de géométrie*, 105. In the *Shuli jingyun*, a similar surveying instrument is described; it is also used in some problems (ZKJXDT 3, 135–136 and 618 sq).

[44] Jami, "Science in Manchu."

worked from this last version. Let us now look at the Manchu translation of Pardies's definition:

> *jakuci*
>
> *yaya emu jijun bifi . tere jijun i emu dube ba horgikv obume hadafi . jai emu ube be šurdehei sucungga aššame deribuhe bade isinaha manggi . uthai emu mu* <u>*uheren jijun banjinambi*</u> *.. duibuleci . giya i seme emu jijun i giya seme babe orgikv obume hadafi . i seme dube be guribume . hadaha babe moselame ucungga aššame deribuhe i sere bade isinaha manggi . uthai emu i bing ding u seme muheliyen jijun banjimbi .. ere i bing ding u i muheliyen jijun be muheren i jecen sembi .. jecen de horiha babe muheren sembi ..*[45]
>
> 8.
>
> For any line, nail one end of this line to make a pivot. After you revolve the second end to where it first started, then a line in the shape of a ring is generated. For example, if we nail the endpoint called *jia* of a line *jia yi* to make a pivot moving the end called *yi*; after arriving back to the initial *yi* point, by grinding the nailed place, then a round line called *yi bing ding wu* is generated. This *yi bing ding wu* line is called circle border; the place enclosed by this border is called circle.

This Manchu text retains and amplifies the material character of the French original. The object and action are no longer a product of one's imagination. Moreover, the absence of mathematical terminology in Manchu, to which this translation was doubtless intended as a remedy,[46] prompted the translators to use verbs such as "nail" and "grinding",[47] which I have

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[45] *Giho yuwanben bithe*, fasc. 1, 3–4. I am grateful to Wang Qianjin 汪前進 for his help accessing this source, and to Mark Elliott for his correction of my translation.

[46] As witnessed by the fact that Chinese characters are added next to geometrical terms and the names of points on the manuscript.

[47] *Moselambi* (to mill, to grind) is used to indicate turning in the way a millstone does as it grinds.

chosen to render literally. Let us now look at the Chinese translation of this Manchu definition:

第八

凡有一線。以線之一頭為樞。以線之一頭為界。旋轉復還原處。即成一圜。設如若甲乙線. 以甲為樞。以乙為界。旋轉復還乙處。即成乙丙丁戊之圜。此圜線謂之圜界。界線積處為圜面。[48]

8.

Let there be a line. Take one of its ends as a pivot, and one of its ends as boundary. Turning it through a complete rotation makes a circle. Thus, with line AB, take the A end as a pivot; take the B end as boundary. Turning it until it returns to the [original] place of B makes the circle BCDE. This circular line is called circumference of the circle. The area accumulated inside the circumference is called the area of the circle.

This only differs from the *Shuli jingyun*'s version by a few characters. The action of nailing has been replaced by the more abstract phrase *yi ... wei* 以……為 mentioned above; that of milling has been replaced by the rotation of the line. It seems that concrete images are no longer needed once one has safely landed into a language that already accommodates Euclidian geometry, namely classical Chinese. As a result, the "material quality" of the original French definition, preserved and even emphasized in its Manchu translation, has been largely discarded. On the other hand, the figures illustrating the definitions are similar in the Manchu and Chinese versions (see fig. 4).

Even though the visual reference to the alidade, an essential element of surveying instruments, has been lost in the first translation, it remains that all four versions of this

[48] *Jihe yuanben* 1691, 2b.

definition of the circle are constructions in a mechanical rather than geometric sense of the term, as they involve movement (fig. 5). This is an instance of the fact that Euclidian geometry as presented in the *Shuli jingyun* aims at being pedagogically effective. Like the *Jihe yaofa*, it integrates the making and use of instruments. However, the instruments integrated are no longer those of geometry itself (ruler, pen, compasses and try square), but surveying instruments. As a whole, the *Shuli jingyun* includes mathematics for surveyors. And Pardies's geometry, with its emphasis on application to earthly measurements, met the needs of the Qing state for surveying techniques.

[Place figure 5 here]

Conclusion

By focusing on the definitions of the circle in three mathematical treatises and on the way in which each of them relates to the body of knowledge of which they form part, we have provided an example of the major changes undergone by the content and style of geometry in the Euclidian tradition produced by Jesuit missionaries in China, with the changes in the institutional context in which they were produced, as well as in the main fields of expertise that these Jesuits put in the service of emperors.

I have argued that institutional changes were echoed in the way in which mathematics was written at two junctures. The first one was the calendar reform initiated in 1629, which reduced Euclidian geometry to a prerequisite for astronomy, giving it an instrumental character. The second one came with the appropriation of another field of Western learning, cartography, by the Kangxi emperor. This new appropriation, occurring at a time when mathematics was gaining the status of an independent field of knowledge under imperial patronage, entailed the integration of instruments imported in China for the purpose of surveying the empire into the corpus of mathematical knowledge.

This has implications not only for the history of mathematics, but also for the history of cartography: the techniques used by cartographers, although they were not presented as such in a separate treatise, were widely available in the *Shuli jingyun*. As with astronomy, the key to the imperial state's monopoly on cartography lay not in its retention of knowledge relevant to the field, but in the exclusive possession of the instruments needed to practice cartography to the standards set by the imperial project. For measuring Heaven and Earth required not only the mastery of knowledge, but also the ability to make, use and maintain instruments.

The three definitions of the circle as an object of Euclidian geometry correspond to three different ways of ordering the world to which this field of mathematics was associated. In the first one, mathematics is part of Heavenly knowledge, taken to be the prerogative of the Christian worldview as construed by the Jesuits. In the second one, mathematics is one of the foundations of the knowledge that enables the emperor to act effectively as the intermediary between Heaven and Earth, by harmonizing the rhythm of human life with that of the cosmos. This might be called the temporal dimension of the political cosmology of imperial China. Finally, the third definition of the circle, in a context where mathematics fulfils the needs of imperial cartography, evokes what one might call the spatial dimension of this political cosmology. The survey of his dominions gave Kangxi an intimate knowledge of them, in a way evocative of the mythical sage king Yu 禹 the Great, founder of the Xia 夏 dynasty of Chinese antiquity, whose familiarity with his kingdom resulted from his lifelong work to control the floods.

This appropriation of Euclidian geometry and astronomical knowledge imported from Europe by China's emperors in the seventeenth and eighteenth centuries thus provides an example of the ways in which the scientific and technical knowledge underlying the cosmology prevailing in one region of the world can be put in the service of another cosmology in another region of the world. The circulating knowledge then constitutes the overlap between two cosmologies that remain radically different.

Acknowledgments

I am grateful to Christopher Cullen for reading through this article.

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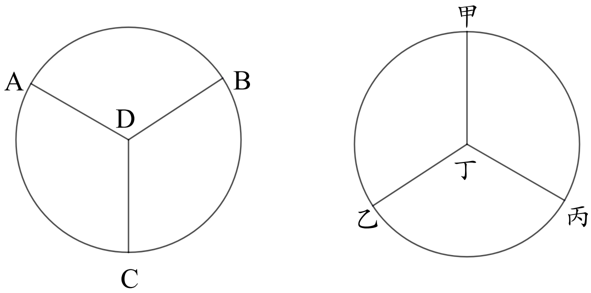

Figure 1

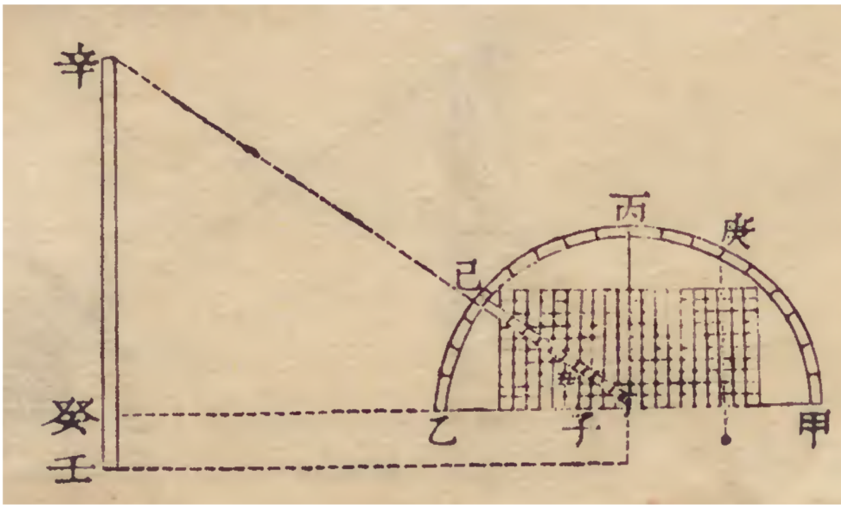

Figure 2

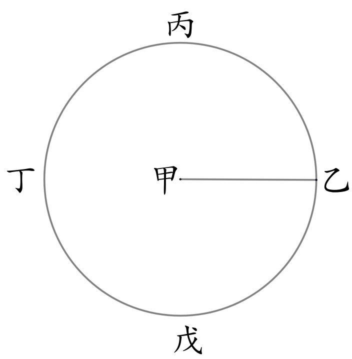

Figure 3

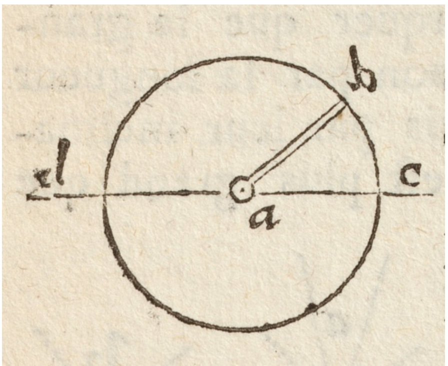

Figure 4

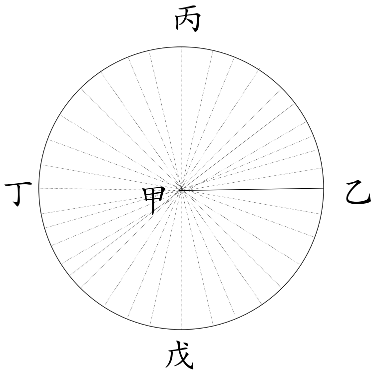

Figure 5

Chapter (卷)	**Topic**	**Instruments**
1	line (線)	ruler, iron/copper pen (尺， 鐵／銅鑄筆)
2	circle (圜)	compasses (規)
3	angle, triangle (角，三角形)	three-legged compass (三髀規)
4	quadrilateral (四邊形)	try square (矩)

Table 1