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A Dynamic Analysis of Criminal Networks

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Abstract

The paper presents a novel approach based on differential games to the study of criminal networks. We extend the static crime network game (Ballester et al., 2004, 2006) to a dynamic setting. First, we determine the relationship between the Markov Perfect Equilibrium (MPE) and the vector of Bonacich centralities. The established proportionality between the Nash equilibrium and the Bonacich centrality in the static game does not hold in general in the dynamic setting. Next, focusing on regular networks, we provide an explicit characterization of equilibrium strategies, and conduct comparative dynamic analysis with respect to the network size, network density, and implicit growth rate of total wealth in the economy. Contrary to the static game, where aggregate equilibrium increases with network size and density, in the dynamic setting, more criminals or more connected criminals can lead to a decrease in total crime, both in the short run and at the steady state. We also examine another novel issue in the network theory literature, i.e., the existence of a voracity effect, occurring when an increase in the implicit growth rate of total wealth in the economy lowers economic growth. We do identify the presence of such a voracity effect in our setting.

JEL Classification: C73, D85, K42.

Keywords: differential games; Markov Perfect Equilibrium; social networks; criminal networks; Bonacich centrality.

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1 Introduction

It is natural to think about criminal and delinquent activities in terms of networks, and more specifically, social networks. Indeed, as argued in Lindquist and Zenou (2019), social network analysis can be quite useful for understanding more about the root causes of crime and delinquency, and for designing crime prevention policies. Not surprisingly, there exists a vast literature devoted to crime and networks (see, e.g., Lindquist and Zenou, 2019, for an overview). However, dynamic considerations in the context of criminal networks have not received enough attention so far. The present paper contributes to the networks and crime literature, by proposing a novel (dynamic) approach to the study of criminal networks, i.e., a differential game approach.

The natural fit between crime and social networks comes particularly from the fact that crime is primarily considered as a group activity and that social interactions heavily affect criminal behavior. Indeed, the importance of social networks and peer influences in criminal activities has been acknowledged for a long time in the criminology and sociology literature (e.g., Sutherland, 1947; Haynie, 2001; Sarnecki, 2001; War, 2002). Also the economic literature is very active in the study of peer and network effects in crime. Sah (1991) and Glaeser et al. (1996) were the first to develop economic models of social interactions and crime, and were followed by others that proposed various theoretical foundations on peer and network effects in criminal activities (e.g., Calvó-Armengol and Zenou, 2004; Ballester et al., 2006, 2010; Cortés et al., 2019). In parallel to theoretical investigations, there is also strong empirical evidence of peer effects in crime (e.g., Ludwig et al., 2001; Kling et al., 2005; Patacchini and Zenou, 2012; Bayer et al., 2009; Damm and Dustmann, 2014).

There exists a sizeable literature on applications of differential games in the field of crime and crime control (e.g., Feichtinger, 1983; Dawid and Feichtinger, 1996; Dubovik and Parakhonyak, 2014; Faria et al., 2019), government corruption (e.g., Kemp and Long, 2009; Ngendakuriyo and Zaccour, 2013, 2017), counterfeiting (e.g., Crettez et al., 2020) and terrorism (e.g., Nova et al., 2010; Wrzaczek et al., 2017). This literature has been able to shed light on a number of important issues related to the dynamics of illegal activities carried out by individuals, firms, and governments. However, it has abstracted from the widely recognized fact that criminals are embedded in social networks (see Ballester et al., 2010). In this paper, we aim to fill this gap in the literature by merging two so far disjoint strands of research, namely, the research on the dynamics of crime without social networks, and the research on social networks without dynamics. Indeed, to the

best of our knowledge, ours is the first analysis of criminal networks in a full-fledged dynamic game.

Our analysis is conducted in terms of a differential game (see Başar and Olsder, 1995, Dockner et al., 2000, Haurie et al., 2012, and Long, 2010 for concepts and applications). As is well known, differential games are particularly useful for modeling economic problems which involve both dynamics and strategic behavior. We propose and analyze an infinite horizon differential game based on the seminal paper by Ballester et al. (2006). At each point in time, criminals embedded in a social network decide how much effort to make, taking as given the efforts of the other criminals. The sum of efforts by all criminals negatively affects the evolution of the state variable (total wealth in the economy). We assume that players use Markovian strategies, i.e., they condition their crime efforts only on the current state variable, which summarizes the entire history of the game, and derive analytically a Markov Perfect Equilibrium.

In the static game by Ballester et al. (2006), the Nash equilibrium is proportional to the Bonacich centrality (Bonacich, 1987). In our dynamic setting, this proportionality does not hold in general. However, we do recover the result by Ballester et al. (2006) as a particular case, when the shadow price of total wealth in the economy is zero, and therefore the dynamic constraint plays no role, leading criminals to behave as in the static game. Moreover, while in the static setup total crime is always strictly positive, in the dynamic setting a corner solution can arise implying a zero level of total crime.

Also the results of our comparative dynamic analysis with respect to the network size and density suggest that some of the conclusions reached in the static literature on criminal networks do not necessarily hold in a Markovian environment, where total crime negatively affects the evolution of total wealth in the economy. Specifically, focusing on regular networks, we show that conditions exist such that more criminals or more connected criminals induce lower crime in the economy. This holds true both in the short run and at the steady state. We also show that a faster growing economy (in the absence of crime) may cause an increase in total crime, which, in the end, may dampen economic growth. This is related to the so called voracity effect (see Tornell and Lane, 1999), which, to the best of our knowledge, has not been studied in the network literature so far.

The rest of the paper is organized as follows. In Section 2, we present the model, first by recalling the static setting of Ballester et al. (2006) in Section 2.1 and then by introducing our dynamic framework in Section 2.2. Section 3 is devoted to the MPE and Bonacich centrality. In

two subsequent sections, we focus on regular networks. In Section 4, an explicit characterization of equilibrium strategies is provided. In Section 5, we conduct a comparative dynamic analysis with respect to the network size (Section 5.1) and the network density (Section 5.2). In Section 5.3, we investigate whether a voracity effect arises in our model. Some concluding remarks are stated in Section 6. All the proofs are presented in the Appendices A till H.

2 Model

2.1 Static Setting

Our point of departure is the framework of Ballester et al. (2006), i.e., a finite population non-cooperative game with linear-quadratic interdependent utility functions, where players decide how much effort to exert. We use this setting in the context of criminal networks (see also Ballester et al., 2004). Henceforth, in our criminal network game, we use criminals for players, and interpret effort levels as crime levels.

Consider criminals embedded in a network of social connections. Each criminal $i = 1, \dots, n$ selects a crime effort $x_i \geq 0$ and obtains a utility payoff (see Ballester et al., 2006)

$$u_i(x_1, \dots, x_n) = \alpha_i x_i + \frac{1}{2} \sigma_{ii} x_i^2 + \sum_{j \neq i} \sigma_{ij} x_i x_j, \quad (1)$$

with $\sigma_{ii} < 0$ to ensure strict concavity in own effort. Net of bilateral influences, criminals have the same payoffs: $\alpha_i = \alpha > 0$ and $\sigma_{ii} = \sigma$ for all $i = 1, \dots, n$.

Let $\Sigma = [\sigma_{ij}]$ be the square matrix of cross-effects. When $\sigma_{ij} > 0$ ($\sigma_{ij} < 0$), then i 's and j 's crime efforts are strategic complements (strategic substitutes) from i 's perspective. For instance, complementarity may arise when criminals take part of a network organization with a common (terrorist) goal, while substitutability may be present when criminals compete against each other for the same victims.

Let $\mathbf{G} = [g_{ij}]$ be the adjacency matrix of the network \mathbf{g} of relative payoff complementarities across pairs, with $0 \leq g_{ij} \leq 1$ and $g_{ii} = 0$ for all $i = 1, \dots, n$. Moreover, let \mathbf{I} and \mathbf{U} denote the n -square identity matrix and the n -square matrix of ones, respectively. By setting $\sigma = -\beta - \gamma$, with $\beta > 0$ and $\gamma \geq 0$, Σ can be decomposed as follows:

$$\Sigma = -\beta \mathbf{I} - \gamma \mathbf{U} + \lambda \mathbf{G},$$

with $\lambda > 0$. In other words, Σ can be decomposed into an idiosyncratic concavity component $(-\beta\mathbf{I})$, a global (uniform) substitutability component $(-\gamma\mathbf{U})$, and a local complementarity component $(\lambda\mathbf{G})$. Hence, (1) can be rewritten as

$$u_i(x_1, \dots, x_n) = \alpha x_i - \frac{1}{2}(\beta + \gamma)x_i^2 - \gamma \sum_{j=1}^n x_i x_j + \lambda \sum_{j=1}^n g_{ij} x_i x_j. \quad (2)$$

Definition 1 Consider a network \mathbf{g} with adjacency n -square matrix \mathbf{G} and a scalar $a > 0$ such that $\mathbf{M}(\mathbf{g}, a) = [\mathbf{I} - a\mathbf{G}]^{-1}$ is well defined and nonnegative. (i) The vector of Bonacich centralities of parameter a in \mathbf{g} is $\mathbf{b}(\mathbf{g}, a) = \mathbf{M}(\mathbf{g}, a)\mathbf{1}$.¹² (ii) The vector of **weighted** Bonacich centralities of parameter a in \mathbf{g} is $\mathbf{b}_\omega(\mathbf{g}, a) = \mathbf{M}(\mathbf{g}, a)\boldsymbol{\omega}$, with $\boldsymbol{\omega} = (\omega_1, \dots, \omega_n)^T$.

We focus on symmetric matrices such that $\sigma_{ij} = \sigma_{ji}$ for all $j \neq i$. Then the largest eigenvalue $\mu_1(\mathbf{G})$ of \mathbf{G} is well defined, with $\mu_1(\mathbf{G}) > 0$ as long as $\sigma_{ij} \neq 0$, for some $j \neq i$. Ballester et al. (2006, Theorem 1) show that the matrix $[\beta\mathbf{I} - \lambda\mathbf{G}]^{-1}$ is well defined and nonnegative if and only if $\beta > \lambda\mu_1(\mathbf{G})$. Then there exists a unique Nash equilibrium which is interior and given by

$$\mathbf{x}^* = \frac{\alpha}{\beta + \gamma b(\mathbf{g}, \lambda^*)} \mathbf{b}(\mathbf{g}, \lambda^*),$$

where $\lambda^* = \lambda/\beta$ is the strength of local interactions (relative to own concavity), $b(\mathbf{g}, \lambda^*) = \sum_{i=1}^n b_i(\mathbf{g}, \lambda^*)$, with $b_i(\mathbf{g}, \lambda^*)$ denoting the Bonacich centrality of node i , and $\mathbf{b}(\mathbf{g}, \lambda^*) = \mathbf{M}(\mathbf{g}, \lambda^*)\mathbf{1}$ is the vector of Bonacich centralities of parameter λ^* .

2.2 Dynamic Setting

We extend the static game previously described to a dynamic setting. Time is continuous and denoted by $t \in [0, \infty)$. Let $y(t) \geq 0$ denote the aggregate level of wealth which is legally produced and $X(t) = \sum_{i=1}^n x_i(t)$ the aggregate level of crime in the economy at t . The intertemporal relationship between $y(t)$ and $X(t)$ is captured by the following differential equation:

$$\dot{y}(t) = \theta y(t) - X(t), \quad y(0) = y_0 \geq 0, \quad (3)$$

with $\theta > 0$ denoting the *implicit* growth rate of total wealth and y_0 the initial level of total wealth in the economy. Crime is assumed to be wealth-reducing. The idea behind (3) is that criminal

¹² $\mathbf{1}$ denotes the n -dimensional vector of ones.

² More precisely, $\mathbf{b}(\mathbf{g}, a)$ is obtained from Bonacich's measure (Bonacich, 1987) by an affine transformation, and $\mathbf{b}(\mathbf{g}, a) = \mathbf{1} + \mathbf{k}(\mathbf{g}, a)$ with $\mathbf{k}(\mathbf{g}, a)$ being Katz's prestige measure (Katz, 1953).

activities such as robberies and tax evasion have a negative impact on the accumulation of total wealth in the economy. Clearly, the growth rate of total wealth in the economy is strictly positive in the absence of crime; otherwise, it can become negative (or nil).

Criminal i 's objective functional is given by

$$J_i = \int_0^{\infty} e^{-rt} u_i(x_1, \dots, x_n) dt,$$

with $u_i(x_1, \dots, x_n)$ defined in (2) and $r > 0$ being the discount rate. Criminal i seeks to maximize J_i w.r.t. x_i subject to (3).

At each t , criminals, after observing $y(t)$, decide how much effort to make, taking as given the efforts of the other criminals. Our equilibrium concept is Markov Perfect Equilibrium (MPE). Specifically, we adopt the closed-loop (feedback) Nash equilibrium: criminals condition their crime effort only on the current state variable, which summarizes the entire history of the game. This restriction captures the notion that bygones are bygones (see Başar and Olsder, 1995; Dockner et al., 2000; Maskin and Tirole, 2001).³ Note that, *a priori*, equilibrium crime efforts can be either increasing or decreasing in y . Formally, strategies are of the form $x_i(t) = \phi_i(t, y(t))$, where ϕ_i is a decision rule specifying a level of crime effort for criminal i for any t and observed y .

The restrictions imposed on closed-loop strategies are given in the following definition.⁴

Definition 2 A n -tuple of closed-loop strategies (ϕ_1, \dots, ϕ_n) is said to be admissible if $(i = 1, \dots, n)$

- (i) $x_i(t) = \phi_i(t, y(t))$ is well defined for all $t \geq 0$
- (ii) the function $t \rightarrow x_i(t) = \phi_i(t, y(t))$ is measurable
- (iii) $\phi_i(t, 0) = 0$
- (iv) the initial value problem $\dot{y}(t) = \theta y(t) - \sum_{i=1}^n \phi_i(t, y(t))$, $y(0) = y_0 > 0$, has a unique solution.

Property (iii), in particular, requires that criminal i makes zero effort if wealth in the economy is nil. Indeed, a necessary condition for crime to exist is that wealth in the economy is strictly positive.

Let ϕ^* be a n -tuple of admissible closed-loop strategies, and ϕ_{-i}^* be the $(n-1)$ -tuple of admissible closed-loop strategies ϕ_j^* , with $j = 1, \dots, n$, $j \neq i$.

³By definition, history dependent strategies, such as trigger strategies, are ruled out.

⁴Similar restrictions are common in the differential game literature (e.g., Dockner and Sorger, 1996; Benckroun, 2003, 2008; Colombo and Labrecciosa, 2015).

Definition 3 The n -tuple ϕ^* constitutes a Markov Perfect Equilibrium (MPE) if, for every possible initial condition (y_0, t_0) :

$$J_i(\phi^*) \geq J_i(\phi_i, \phi_{-i}^*) \text{ for all } i = 1, \dots, n$$

for any closed-loop strategy ϕ_i such that (ϕ_i, ϕ_{-i}^*) is an admissible n -tuple of closed-loop strategies.

Within the set of closed-loop strategies, in line with the bulk of the literature, we restrict attention to strategies of the symmetric and stationary type. The game is symmetric as the discount rate and the time horizon are common to all criminals, their ability to affect the evolution of the state is identical, and criminals' instantaneous payoffs and feasible sets take the same form. Strategies are of the stationary type due to the structure of the game: the equation of motion is autonomous, and the instantaneous payoffs as well as the feasible sets do not explicitly depend on time.⁵

3 MPE and Bonacich Centrality

In this section, in the spirit of Ballester et al. (2006), we study the relationship between the MPE and the vector of Bonacich centralities. In order to derive closed-loop equilibrium strategies, we adopt the value function approach. Let $V_i(y)$ denote criminal i 's value function, which represents the discounted value of the stream of utilities (2) for a game that starts at y . By standard arguments, closed-loop equilibrium strategies must satisfy the following Hamilton-Jacobi-Bellman (HJB) equations ($i = 1, \dots, n$):

$$rV_i(y) = \max_{x_i \geq 0} \left\{ u_i(x_i, \phi_{-i}^*) + V_i'(y) \left[\theta y - x_i - \sum_{j=1, j \neq i}^n \phi_j^*(y) \right] \right\},$$

where $V_i'(y) = \partial V_i(y) / \partial y$ denotes the shadow price of total wealth for criminal i . Assuming that $\beta > \lambda \mu_1(\mathbf{G})$, with $\mu_1(\mathbf{G}) > 0$ denoting the largest eigenvalue of \mathbf{G} , maximization of the RHS of the above HJB implies that

$$[\beta \mathbf{I} + \gamma \mathbf{U} - \lambda \mathbf{G}] \phi^* = \alpha \mathbf{1} - \mathbf{V}',$$

where $\mathbf{V}' = (V_1', \dots, V_n')^T$. Since $\mathbf{U} \phi^* = \Phi^* \mathbf{1}$, where $\Phi^* = \sum_{i=1}^n \phi_i^*$, then

$$\beta[\mathbf{I} - \lambda^* \mathbf{G}] \phi^* = [\alpha - \gamma \Phi^*] \mathbf{1} - \mathbf{V}',$$

⁵Note that stationarity alone is not sufficient to rule out equilibria involving non stationary strategies. However, as pointed out in Dockner *et al.* (2000), non stationary equilibria are of less interest and therefore they are generally not considered in economic applications.

and

$$\beta\phi^* = [\alpha - \gamma\Phi^*] [\mathbf{I} - \lambda^*\mathbf{G}]^{-1} \mathbf{1} - [\mathbf{I} - \lambda^*\mathbf{G}]^{-1} \mathbf{V}'.$$

Using Definition 1, we obtain

$$\beta\phi^* = [\alpha - \gamma\Phi^*] \mathbf{b}(\mathbf{g}, \lambda^*) - \mathbf{b}_{\mathbf{V}'}(\mathbf{g}, \lambda^*),$$

implying that

$$\phi^* = \frac{\alpha\beta + \gamma b_{\mathbf{V}'}(\mathbf{g}, \lambda^*)}{\beta[\beta + \gamma b(\mathbf{g}, \lambda^*)]} \mathbf{b}(\mathbf{g}, \lambda^*) - \frac{\mathbf{b}_{\mathbf{V}'}(\mathbf{g}, \lambda^*)}{\beta}.$$

The above discussion leads to the following proposition.

Proposition 1 *Assume that $\beta > \lambda\mu_1(\mathbf{G})$. There exists a MPE,*

$$\phi^* = \frac{\alpha\beta + \gamma b_{\mathbf{V}'}(\mathbf{g}, \lambda^*)}{\beta[\beta + \gamma b(\mathbf{g}, \lambda^*)]} \mathbf{b}(\mathbf{g}, \lambda^*) - \frac{\mathbf{b}_{\mathbf{V}'}(\mathbf{g}, \lambda^*)}{\beta},$$

for any given $\mathbf{V}' \geq \mathbf{0}$ such that $\phi^ \geq \mathbf{0}$.*

Two remarks are in order. First, the relationship between the MPE and the vector of Bonacich centralities is more nuanced than in the static setting, in which the Nash equilibrium is proportional to the vector of Bonacich centralities (see Ballester et al., 2006). Indeed, in our dynamic setting, such a proportional relationship is not observed in general. Second, conditions on the shadow price of total wealth exist such that a corner solution arises implying a zero level of total crime. In Ballester et al. (2006), instead, total crime is always strictly positive.

If $V'_i = V'$ for all $i = 1, \dots, n$, then $\mathbf{b}_{\mathbf{V}'}(\mathbf{g}, \lambda^*) = V' \mathbf{b}(\mathbf{g}, \lambda^*)$. From ϕ^* given in Proposition 1, it follows that

$$\beta\phi^* = [\alpha - \gamma\phi^* - V'] \mathbf{b}(\mathbf{g}, \lambda^*),$$

which implies that

$$\phi^* = \frac{\alpha - V'}{\beta + \gamma b(\mathbf{g}, \lambda^*)} \mathbf{b}(\mathbf{g}, \lambda^*).$$

We can then state the following corollary.

Corollary 1 *When $V'_i = V'$ for all $i = 1, \dots, n$, ϕ^* given in Proposition 1 becomes*

$$\phi^* = \frac{\alpha - V'}{\beta + \gamma b(\mathbf{g}, \lambda^*)} \mathbf{b}(\mathbf{g}, \lambda^*).$$

Note that, if $V' = 0$, ϕ^* given in Corollary 1 corresponds to \mathbf{x}^* in Ballester et al. (2006, Theorem 1, p.1408). In this case, ϕ^* is proportional to the vector of Bonacich centralities. Intuitively, when the shadow price of total wealth in the economy is zero, the dynamic constraint plays no role, and each criminal behaves as in the static game.

4 Equilibrium Characterization

In this section, in order to provide an explicit characterization of equilibrium strategies, we focus on regular networks. We consider the set of strategies which are linear in the state variable and look for a linear MPE (LMPE).⁶ Throughout the analysis, we make the following assumptions on the parameters of the model:

A1. $\beta > \widehat{\beta}$, with

$$\widehat{\beta} = \max \{2d\lambda - \gamma(2n + 1), \gamma\}.$$

A2. $r < \widehat{r}$, with

$$\widehat{r} = \min \left\{ \frac{\theta(\beta + \gamma - 2d\lambda + 2\gamma n)}{n(\beta - d\lambda + \gamma n)}, \frac{\theta(\beta - \gamma)}{d\lambda - \gamma + n[\beta - d\lambda + \gamma(n - 1)]} \right\}.$$

The above assumptions are required for strategies to be admissible, continuous, and global (defined over the entire state space), and for steady states to be positive.

Proposition 2 *Let*

$$\phi^*(y) = \begin{cases} 0 & \text{for } y \leq \underline{y} \\ \frac{\alpha - Ay - B}{\beta + n\gamma - d\lambda} & \text{for } \underline{y} < y \leq \bar{y} \\ \frac{\alpha}{\beta + n\gamma - d\lambda} & \text{for } y > \bar{y} \end{cases}$$

where $\underline{y} = (\alpha - B)/A > 0$, $\bar{y} = -B/A > \underline{y}$, and $A < 0$ and $B > 0$ are constants that depend on the parameters of the model (given in the proof). The n -tuple of strategies (ϕ^*, \dots, ϕ^*) constitutes a LMPE.

Proposition 2 establishes that the equilibrium strategy is piece-wise linear.⁷ If $y < \underline{y}$, criminals abstain from committing crime and wait for total wealth in the economy to grow until reaching the "maturity" level, \underline{y} . The intuitive explanation is that, for low levels of total wealth, the shadow price of total wealth is so high that it exceeds the benefit to criminals from committing crime, α . Consequently, $\phi^* = \mathbf{0}$. In the static game, instead, being $V' = 0$, corner solutions are ruled out.

⁶Linear strategies are widely considered in applications of differential games (e.g. Fershtman and Kamien, 1987; Tornell and Velasco, 1992; Jun and Vives, 2004; Benckroun, 2008; Colombo and Labrecciosa, 2015). On applications of nonlinear strategies see Tsutsui and Mino (1990), Dockner and Long (1993), and Colombo and Labrecciosa (2015), *inter alia*.

⁷Within the class of linear strategies considered, the MPE is unique.

For levels of wealth between \underline{y} and \bar{y} , total crime is strictly positive and increasing in y , implying that more wealth triggers more crime. The fact that total crime is increasing in y implies that there exists *intertemporal strategic substitutability*: an increase in efforts by criminal i today leads to a decrease in total wealth in the economy which, in turn, induces a decrease in efforts by all the other criminals in the future. Hence, for intermediate levels of total wealth, a positive relationship between total wealth and total crime arises in equilibrium. Finally, if $y > \bar{y}$, criminals play a constant strategy (a degenerate Markovian strategy) corresponding to the static Nash equilibrium. In this case, total wealth is so abundant that the dynamic constraint is no longer binding.

Corollary 2 *There exist two steady-state equilibria of total crime, \hat{X}_{ss} and $\tilde{X}_{ss} > \hat{X}_{ss}$, with*

$$\hat{X}_{ss} = \frac{\theta n (\alpha - B)}{An + \theta (\beta - d\lambda + \gamma n)},$$

and

$$\tilde{X}_{ss} = \frac{n\alpha}{\beta - d\lambda + n\gamma}.$$

If $y_0 \in (0, \tilde{y}_{ss})$, with

$$\tilde{y}_{ss} = \frac{n\alpha}{\theta (\beta - d\lambda + n\gamma)},$$

total crime converges to \hat{X}_{ss} as $t \rightarrow \infty$; if, instead, $y_0 > \tilde{y}_{ss}$, total crime is equal to \tilde{X}_{ss} at all $t \in [0, \infty)$.

The static Nash equilibrium derived in Ballester et al. (2006), which corresponds to \tilde{X}_{ss} , can be sustained either temporarily or *ad infinitum*, depending on initial conditions. Starting from any $y_0 \in (\bar{y}, \tilde{y}_{ss})$, criminals play the static Nash equilibrium only *temporarily*. This is so because once y reaches the threshold \bar{y} , criminals revert to the non degenerate Markovian strategy, which is increasing in y , and total crime decreases until reaching the stationary level \hat{X}_{ss} . Therefore, the static Nash equilibrium cannot be sustained as a subgame perfect Nash equilibrium. It is worth noting that a degenerate Markovian strategy corresponding to the Nash equilibrium strategy for all levels of total wealth would not be admissible, since it would violate property (iii) in Definition 2. Moreover, the value function associated to this constant strategy would not converge to zero as total wealth approaches zero. Therefore, for levels of total wealth below \tilde{y}_{ss} , the equilibrium must be constructed in such a way to satisfy the usual transversality condition, $\lim_{t \rightarrow \infty} e^{-rt} V(y^*(t)) = 0$, with $y^*(t)$ denoting the equilibrium trajectory of total wealth in the economy. The equilibrium strategy given in Proposition 2 does exactly that. The static Nash equilibrium can be sustained

as a subgame perfect equilibrium only when $y_0 \geq \tilde{y}_{ss}$, in which case criminals play the static Nash equilibrium forever and total crime remains constant over time.

Corollary 2 is illustrated in the below figure.

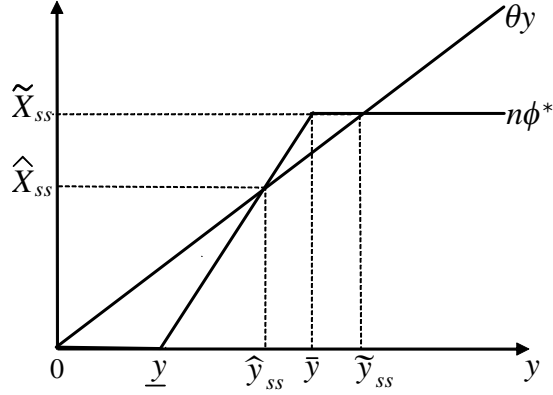


Fig. 1: Steady-state equilibria

As can be seen from Figure 1, total wealth in the economy increases for all $y_0 < \hat{y}_{ss}$ or $y_0 > \tilde{y}_{ss}$ and decreases for all $y_0 \in (\hat{y}_{ss}, \tilde{y}_{ss})$, while it remains constant for $y_0 = \hat{y}_{ss}, \tilde{y}_{ss}$, with $\hat{y}_{ss} = \hat{X}_{ss}/\theta$. However, while \hat{y}_{ss} is locally stable, \tilde{y}_{ss} is unstable.

5 Comparative Dynamics

In this section, keeping the focus on regular networks, we conduct a comparative dynamic analysis with respect to the network size, n , and the network density⁸, via the degree of a node, d , and the implicit growth rate of total wealth in the economy, θ . In order to obtain clear-cut results, we make the following assumption.

A3. $\gamma > d\lambda$.

In Sections 5.1 and 5.2, we proceed as follows. First, we evaluate how equilibrium strategies respond to an increase in the parameter of interest in the neighborhood of a given initial level of total wealth (short-run impact). Second, we investigate how steady-state levels of total crime are

⁸The density of a network is a relative fraction of possible links that are present in the network. In other words, it is the average degree of all n nodes in the network divided by $n - 1$, which for a regular network is equal to $d/(n - 1)$.

affected by an increase in the parameter of interest (long-run impact). In Section 5.3, we derive the equilibrium growth rate of total wealth in the economy and evaluate how it is affected by an increase in θ .

5.1 Network Size

In the static game studied in Ballester et al. (2006), total crime is always increasing in n . To see this, it suffices to differentiate \mathbf{x}^* with respect to n and check that the sign of the derivative is positive (once taken into account that $\beta > d\lambda$ for the matrix $[\beta\mathbf{I} - \lambda\mathbf{G}]^{-1}$ to be nonnegative). In the dynamic game at hand, instead, things are more involved.

5.1.1 Short-run impact

In Lemma 1, we establish how an increase in n affects the thresholds \underline{y} and \bar{y} and the slope of the increasing segment of $n\phi^*$. In Lemma 2, we derive the condition under which equilibrium strategies before and after the change in n cross. Finally, in Lemma 3, we establish how an increase in n affects the flat part of $n\phi^*$ (i.e., \tilde{X}_{ss}).

Lemma 1 (i) $\underline{y}|_{n_2} < \underline{y}|_{n_1}$, (ii) $\bar{y}|_{n_2} > \bar{y}|_{n_1}$, and (iii) $\partial(n_2\phi^*|_{n_2})/\partial y < \partial(n_1\phi^*|_{n_1})/\partial y$, with $n_2 > n_1$.

Lemma 2 Take $y = \underline{y}|_{n_1}$. There exists \tilde{r} such that $n_2\phi^*|_{n_2} \geq (<)n_1\phi^*|_{n_1}$ for $r \geq (<)\tilde{r}$, with $n_2 > n_1$.

As a direct consequence of the fact that $\partial\tilde{X}_{ss}/\partial n = \alpha(\beta - d\lambda)/(\beta - d\lambda + \gamma n)^2 > 0$ (since $\beta > d\lambda$), we have the following lemma.

Lemma 3 $\tilde{X}_{ss}|_{n_2} > \tilde{X}_{ss}|_{n_1}$, with $n_2 > n_1$.

The short-run impact of an increase in n from n_1 to n_2 is illustrated in the below figure (for the three cases $r > \tilde{r}$, $r = \tilde{r}$, and $r < \tilde{r}$).

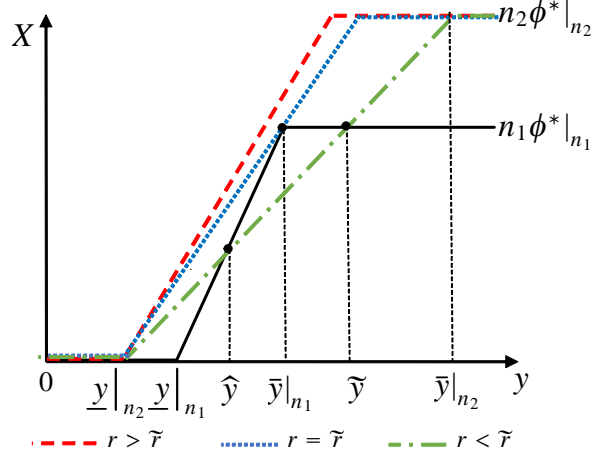


Fig. 2: The short-run impact of an increase in n

Interestingly, only in the case $r \leq \tilde{r}$ there exists at least one point where total crime before and after the change in n cross (see Figure 2). For $r < \tilde{r}$, in particular, total crime before and after the change in n cross twice, and there exists an interval of initial levels of total wealth in the economy ($y_0 \in (\hat{y}, \tilde{y})$) such that short-run total crime decreases as a result of an increase in n .

Lemmas 1-3 yield the following proposition.

Proposition 3 Consider an increase in n from n_1 to n_2 . (i) Take $r > \tilde{r}$. Short-run total crime (henceforth, SRTC) increases for $y_0 > \underline{y}|_{n_2}$, and remains unchanged for $y_0 < \underline{y}|_{n_2}$. (ii) Take $r = \tilde{r}$. SRTC increases for $y_0 > \underline{y}|_{n_2}$ and $y_0 \neq \underline{y}|_{n_1}$, and remains unchanged for $y_0 < \underline{y}|_{n_2}$ or $y_0 = \underline{y}|_{n_1}$. (iii) Take $r < \tilde{r}$. SRTC decreases for $y_0 \in (\hat{y}, \tilde{y})$, increases for either $y_0 \in (\underline{y}|_{n_2}, \hat{y})$ or $y_0 > \tilde{y}$, and remains unchanged for $y_0 < \underline{y}|_{n_2}$.⁹

There are two important differences with respect to the static analysis. First, an increase in the number of criminals induces a short-run decrease in total crime, provided that $y_0 \in (\hat{y}, \tilde{y})$ and that $r < \tilde{r}$, i.e., criminals are sufficiently patient. Second, an increase in the number of criminals leaves

⁹Note that when $y_0 = \underline{y}|_{n_2}$, $y_0 = \underline{y}|_{n_1}$ (with $r = \tilde{r}$), or $y_0 = \hat{y}, \tilde{y}$ (with $r < \tilde{r}$), we have $n_1 \phi^*|_{n_1} = n_2 \phi^*|_{n_2}$, i.e., total crime is the same before and after the change in n . This implies that initial conditions exist such that the instantaneous response of total crime to a change in the number of criminals is nil.

total crime unchanged, at least temporarily, provided that the initial total wealth in the economy is low (i.e., $y_0 < \underline{y}|_{n_2}$). In the absence of dynamics, instead, total crime is always strictly increasing in the number of criminals. The key difference between the static and the dynamic analysis is represented by the shadow price of total wealth in the economy, V' (which can be interpreted as the intertemporal marginal cost of committing crime). While in the static setting, by definition, $V' = 0$, in the dynamic setting, $V' \geq 0$ (with $V' > 0$ for all $y < \bar{y}$). As can be seen from Appendix A, V' is a function of the parameters of the model, including n . Using ϕ^* given in Proposition 2, the impact of n on total crime (for a given y_0) can be decomposed into the sum of two effects, a static effect and a dynamic effect:

$$\frac{\partial n \phi^*}{\partial n} = \underbrace{\frac{(\beta - d\lambda) \alpha}{(\beta + n\gamma - d\lambda)^2}}_{\text{static effect}} + \underbrace{\left[\frac{(d\lambda - \beta) V'}{(\beta + n\gamma - d\lambda)^2} - \frac{n}{\beta + n\gamma - d\lambda} \frac{\partial V'}{\partial n} \right]}_{\text{dynamic effect}}.$$

The static effect is always positive. The dynamic effect, instead, is *a priori* ambiguous: the first term between square brackets is always negative, whereas the second term between square brackets can take either sign, depending on $\partial V'/\partial n$. Proposition 3 establishes that the overall impact of n on total crime crucially depends on the discount rate, r . When r is large, therefore criminals are rather myopic, the static effect dominates, and total crime increases as result of an increase in n . When, instead, r is small, therefore criminals are rather forward looking, the dynamic effect is negative and outweighs the positive static effect for all $y_0 \in (\hat{y}, \tilde{y})$.

In the remainder of this subsection, we are going to show that conditions exist under which the relationship between total crime and the number of criminals can be negative also in the long run.

5.1.2 Long-run impact

In Lemma 4, we show how an increase in n affects \hat{X}_{ss} .

Lemma 4 $\hat{X}_{ss}|_{n_2} \leq (>) \hat{X}_{ss}|_{n_1}$ for $r \geq (<) r_1$, with $n_2 > n_1$, where r_1 is the lowest of the two roots of $\partial \hat{X}_{ss}/\partial n$.

The long-run impact of an increase in n from n_1 to n_2 is illustrated in the below figure (for the three cases $r > r_1$, $r = r_1$, and $r < r_1$).¹⁰

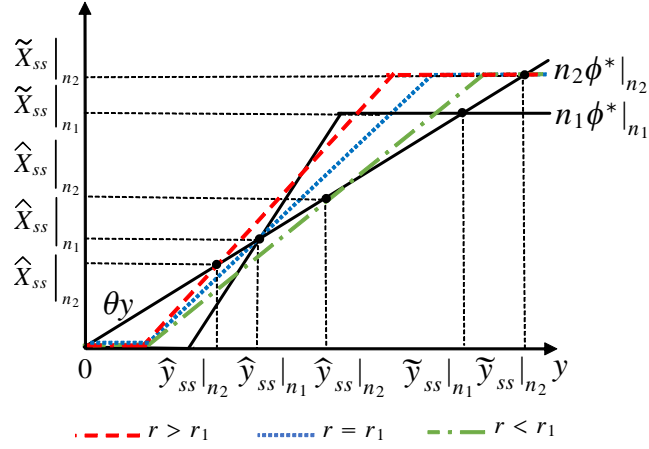


Fig. 3: The long-run impact of an increase in n

As can be seen from Figure 3, the long-run impact of an increase in the number of criminals on total crime depends not only on the initial level of total wealth in the economy, but also on the rate at which criminals discount future utilities from committing crime. In particular, there exist an interval of initial total wealth in the economy ($y_0 < \tilde{y}_{ss}|_{n_1}$) and a level of the discount rate ($r = r_1$) such that, rather surprisingly, the number of criminals has no long-run impact on total crime, i.e., total crime converges to $\hat{X}_{ss}|_{n_1}$, which is the same as $\hat{X}_{ss}|_{n_2}$. Moreover, there exists an interval of initial total wealth in the economy ($[\tilde{y}_{ss}|_{n_1}, \tilde{y}_{ss}|_{n_2})$) such that long-run total crime decreases as a result of an increase in the number of criminals, irrespective of the level of the discount rate. Furthermore, conditions on the parameters of the model exist such that $r_1 < 0$ (see the red dashed line in Figure 3), which implies that an increase in the number of criminals triggers a decrease in long-run total crime for any $y_0 < \tilde{y}_{ss}|_{n_2}$. From Figure 3, we can see that both *undershooting* and *overshooting* are possible. Take, for instance, $y_0 = \hat{y}_{ss}|_{n_1}$. When $r > r_1$, total crime increases in the short run, since the red dashed line evaluated at $\hat{y}_{ss}|_{n_1}$ lies above $n_1 \phi^*|_{n_1}$, and decreases in the long run, since $\hat{X}_{ss}|_{n_2} < \hat{X}_{ss}|_{n_1}$. Hence, there is overshooting. When instead $r < r_1$, total crime

¹⁰Figure 3 is drawn for $r < \tilde{r}$. When $r = \tilde{r}$, the increasing segment of total crime depicted with the red dashed line crosses $n_1 \phi^*|_{n_1}$ once at the kink point. When $r > \tilde{r}$, it never crosses $n_1 \phi^*|_{n_1}$. The impact of n on \hat{X}_{ss} does not depend on \tilde{r} .

decreases in the short run, since the green dash-dotted line evaluated at $\hat{y}_{ss}|_{n_1}$ lies below $n_1 \phi^*|_{n_1}$, and increases in the long run, since $\hat{X}_{ss}|_{n_2} > \hat{X}_{ss}|_{n_1}$. In this case, there is undershooting. By similar arguments, overshooting always occurs when $y_0 = \tilde{y}_{ss}|_{n_1}$. Total crime may decrease both in the short and the long run as a result of an increase in n . Take, for instance, $r > r_1$ and y_0 in the left neighborhood of the intersection between the flat part of $n_1 \phi^*|_{n_1}$ and the red dashed line (see Figure 3). Clearly, total crime initially decreases, then converges towards a lower steady state.

Lemmas 1-4 yield the following proposition.

Proposition 4 *Consider an increase in n from n_1 to n_2 . (i) Take $r > r_1$. Long-run total crime (henceforth, LRTC) decreases for $y_0 < \tilde{y}_{ss}|_{n_2}$, and increases for $y_0 \geq \tilde{y}_{ss}|_{n_2}$. (ii) Take $r = r_1$. LRTC decreases for $y_0 \in [\tilde{y}_{ss}|_{n_1}, \tilde{y}_{ss}|_{n_2})$, increases for $y_0 \geq \tilde{y}_{ss}|_{n_2}$, and remains unchanged for $y_0 < \tilde{y}_{ss}|_{n_1}$. (iii) Take $r < r_1$. LRTC decreases for $y_0 \in [\tilde{y}_{ss}|_{n_1}, \tilde{y}_{ss}|_{n_2})$, and increases for $y_0 \geq \tilde{y}_{ss}|_{n_2}$ or $y_0 < \tilde{y}_{ss}|_{n_1}$.*

As in the short run, in contrast with the static analysis, total crime may decrease or remain unchanged as a result of an increase in the number of criminals, depending on the discount rate and the initial level of total wealth in the economy.

5.2 Network Density

In the static game studied in Ballester et al. (2006), total crime is always increasing in d . This can be seen by differentiating \mathbf{x}^* with respect to d and by verifying that the sign of the derivative is positive. In our dynamic game, instead, whether or not total crime is increasing in d crucially depends on the parameters of the model.

5.2.1 Short-run impact

In Lemma 5, we establish how an increase in d affects the thresholds \underline{y} and \bar{y} and the slope of the increasing segment of $n\phi^*$, while in Lemma 6, we establish how an increase in d affects the flat part of $n\phi^*$.

Lemma 5 (i) $\underline{y}|_{d_2} > \underline{y}|_{d_1}$, (ii) $\bar{y}|_{d_2} > \bar{y}|_{d_1}$, and (iii) $\partial(n\phi^*|_{d_2})/\partial y < \partial(n\phi^*|_{d_1})/\partial y$, with $d_2 > d_1$.

As a direct consequence of the fact that $\partial\tilde{X}_{ss}/\partial d = n\alpha\lambda/(\beta - d\lambda + \gamma n)^2 > 0$, we have the following lemma.

Lemma 6 $\tilde{X}_{ss}|_{d_2} > \tilde{X}_{ss}|_{d_1}$, with $d_2 > d_1$.

The short-run impact of an increase in d from d_1 to d_2 is illustrated in the below figure.

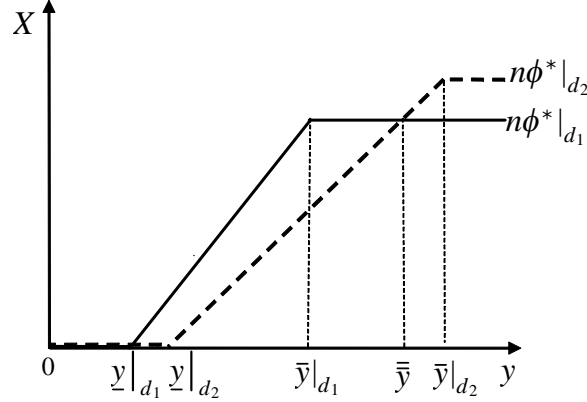


Fig. 4: The short-run impact of an increase in d

As can be seen from Figure 4, there exists an interval of initial total wealth in the economy ($y_0 \in (y|_{d_1}, \bar{y})$) such that an increase in network density leads to a decrease in SRTC.

Lemmas 5 and 6 yield the following proposition.

Proposition 5 *Consider an increase in d from d_1 to d_2 . SRTC increases for $y_0 > \bar{y}$, decreases for $y_0 \in (y|_{d_1}, \bar{y})$, and remains unchanged for $y_0 \in (0, y|_{d_1}]$ or $y_0 = \bar{y}$.*

In contrast with the static analysis, for $y < \bar{y}$ an increase in network density lowers total crime, at least temporarily. As in Section 5.1, the overall impact of the parameter of interest on total crime can be decomposed into two effects, a static effect and a dynamic effect:

$$\frac{\partial n\phi^*}{\partial d} = \underbrace{\frac{\alpha\lambda n}{(\beta - d\lambda + \gamma n)^2}}_{\text{static effect}} + \underbrace{\left[\frac{n}{(d\lambda - \beta - \gamma n)} \frac{\partial V'}{\partial d} - \frac{n\lambda V'}{(\beta - d\lambda + \gamma n)^2} \right]}_{\text{dynamic effect}}.$$

The static effect is always positive. The dynamic effect, instead, can take either sign. Proposition 5 establishes that the static effect dominates for $y_0 > \bar{y}$, whereas for $y_0 < \bar{y}$ the dynamic effect, which is negative, outweighs the static effect, implying that an increase in network density leads to a decrease in SRTC.

5.2.2 Long-run impact

In Lemma 7, we show how an increase in d affects \hat{X}_{ss} , and we compare $\tilde{y}_{ss}|_{d_1}$ and $\hat{y}_{ss}|_{d_2}$ in order to establish where θy crosses $n\phi^*|_{d_2}$ (either below or above $n\phi^*|_{d_1}$).

Lemma 7 (i) $\hat{X}_{ss}|_{d_2} > \hat{X}_{ss}|_{d_1}$, and (ii) $\hat{y}_{ss}|_{d_2} < \tilde{y}_{ss}|_{d_1}$, with $d_2 > d_1$.

The long-run impact of an increase in d from d_1 to d_2 is illustrated in the below figure.

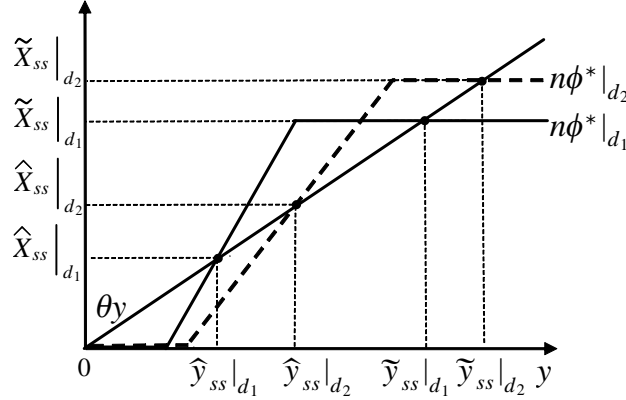


Fig. 5: The long-run impact of an increase in d

Unlike in the analysis of the impact of network size (see Section 5.1.2), now the order relationship between LRTC before and after the change in d does not depend on r . For $y_0 < \tilde{y}_{ss}|_{d_1}$, total crime converges to $\hat{X}_{ss}|_{d_1}$ when the degree of a node is d_1 , and to $\hat{X}_{ss}|_{d_2}$ when the degree is d_2 . For $y_0 \in [\tilde{y}_{ss}|_{d_1}, \tilde{y}_{ss}|_{d_2})$, LRTC remains constant at $\tilde{X}_{ss}|_{d_1}$ when the degree of a node is d_1 , and decreases to $\hat{X}_{ss}|_{d_2}$ when the degree is d_2 . Finally, for $y_0 \geq \tilde{y}_{ss}|_{d_2}$, LRTC remains constant at $\tilde{X}_{ss}|_{d_1}$ when the degree is d_1 , and remains constant at $\tilde{X}_{ss}|_{d_2} > \tilde{X}_{ss}|_{d_1}$ when the degree of a node is d_2 .

Lemmas 5-8 yield the following proposition.

Proposition 6 Consider an increase in d from d_1 to d_2 . LRTC decreases for $y_0 \in [\tilde{y}_{ss}|_{d_1}, \tilde{y}_{ss}|_{d_2})$, and increases otherwise.

For levels of total wealth between $\tilde{y}_{ss}|_{d_1}$ and $\tilde{y}_{ss}|_{d_2}$, we then have a new result compared with the static analysis. This is due to the impact that d has on total crime through V' (which, by definition, is nil in the static setting).

5.3 Implicit Growth Rate and Voracity Effect

In this subsection, we investigate the possibility that an increase in the implicit growth rate of total wealth in the economy (e.g. a productivity gain) lowers economic growth, i.e. whether a *voracity effect* (see Tornell and Lane, 1999) arises in our model. Formally, a voracity effect exists when $\partial(\dot{y}(t)/y(t))/\partial\theta < 0$. Note that, in the absence of crime, $\dot{y}(t)/y(t) = \theta$, therefore a voracity effect never arises. The question addressed in this subsection is new in the network theory literature. In this respect, the focus of this subsection is different from that of the previous subsections, which was on the comparison between the static and the dynamic impact of an increase in either n or d .

Let $g^*(t) = \dot{y}^*(t)/y^*(t)$. In order to compute $g^*(t)$, first, we derive the equilibrium trajectory of total wealth in the economy resulting from (ϕ^*, \dots, ϕ^*) , with ϕ^* given in Proposition 2. For $y_0 \in (\underline{y}, \bar{y})$, we obtain

$$y^*(t) = e^{t\left(\frac{A\tilde{X}_{ss}}{\alpha} + \theta\right)} (y_0 - \hat{y}_{ss}) + \hat{y}_{ss},$$

with $A < 0$ given in Appendix A. Next, we differentiate $y^*(t)$ w.r.t. time:

$$\dot{y}^*(t) = e^{t\left(\frac{A\tilde{X}_{ss}}{\alpha} + \theta\right)} (y_0 - \hat{y}_{ss}) \left(\frac{A\tilde{X}_{ss}}{\alpha} + \theta \right).$$

We can then write the rate of growth of $y^*(t)$ as follows:

$$g^*(t) = \frac{e^{t\left(\frac{A\tilde{X}_{ss}}{\alpha} + \theta\right)} (y_0 - \hat{y}_{ss}) \left(\frac{A\tilde{X}_{ss}}{\alpha} + \theta \right)}{e^{t\left(\frac{A\tilde{X}_{ss}}{\alpha} + \theta\right)} (y_0 - \hat{y}_{ss}) + \hat{y}_{ss}}.$$

Proposition 7 *Take $y_0 \in (\underline{y}, \bar{y})$. If t and r are sufficiently close to zero then $\partial g^*(t)/\partial\theta < 0$, i.e., there exists a voracity effect.*

We have verified numerically that a voracity effect may arise even when t and r are large. The intuitive explanation for the occurrence of a voracity effect is that the indirect effect of an increase in total crime, which, given the wealth-reducing nature of crime, is negative, outweighs the direct positive effect of an increase in θ . Consequently, an increase in the implicit growth rate of total wealth in the economy depresses economic growth.¹¹

¹¹A similar result can be found in a number of related papers (e.g., Tornell and Lane, 1999; Long and Sorger, 2006; Van der Ploeg, 2011). However, to our knowledge, it has never been derived in the context of criminal networks.

6 Concluding Remarks

In this paper, we have taken a novel approach, namely, a differential game approach, to the study of criminal networks, with the aim to reconsider some comparative statics results derived in the static literature, and answer a new set of questions related to the network structure and its impact on the evolution of crime. We have also shed some light on the relationship between the MPE and the Bonacich centrality, and shown that the established proportionality between the Nash equilibrium and the Bonacich centrality does not hold in general in a dynamic setting.

The existing literature on criminal networks abstracts from dynamic considerations. Both the benefits and the costs of crime for criminals are assumed to be static, thus precluding the analysis of important topics such as the impact of network structure on the evolution of crime and the relationship between productivity shocks, crime and growth. Besides theoretical interest, these topics have real-world relevance and their understanding is of paramount importance for designing effective policies. One of the key lessons that can be drawn from the static literature on criminal networks is that networks with a higher number of criminals are associated with higher levels of total crime, and similarly for networks with a higher density. This lesson is valid as long as time does not play any role. Indeed, our dynamic analysis, which, to our knowledge, is novel in the network theory literature, has shown that more criminals or more connected criminals may lead to the counterintuitive opposite result, i.e., a decrease in total crime. This holds true not only in the short run, but also at the steady state. Intuitively, the intertemporal cost of committing crime, which our dynamic framework is able to capture, may increase as a result of an increase in either network size or network density to such an extent that total crime is reduced. Conditions exist under which forward looking criminals anticipate that an increase in network size will lead to an increase in crime by all the other criminals, and, therefore, to a decrease in total wealth in the economy. Consequently, given symmetry, each criminal will find it optimal to decrease their own crime efforts (since equilibrium crime efforts are increasing in total wealth), leading to an equilibrium in which total crime is lower. Similar arguments apply to the case in which network density is increased.

In this paper, we have also highlighted the presence of a voracity effect, occurring when the implicit growth rate of total wealth in the economy is increased and, as a consequence of that, economic growth is reduced. This finding points to the counterintuitive conclusion that, in the presence of crime, positive productivity shocks may have a detrimental effect on economic growth.

Appendix A. Proof of Proposition 2

By standard arguments, closed-loop equilibrium strategies must satisfy the following HJB equations ($i = 1, \dots, n$):

$$rV_i(y) = \max_{x_i \geq 0} \left\{ u_i(x_i, \phi_{-i}^*) + V_i'(y) \left[\theta y - x_i - \sum_{j=1, j \neq i}^n \phi_j^*(y) \right] \right\}, \quad (\text{A.1})$$

where $V_i'(y) = \partial V_i(y)/\partial y$. Maximization of the RHS of the above HJB yields (assuming that $\beta > d\lambda$, which is required for $[\beta \mathbf{I} - \lambda \mathbf{G}]^{-1}$ to be well defined and nonnegative (see Ballester et al., 2006)):

$$\phi^* = [\beta \mathbf{I} + \gamma \mathbf{U} - \lambda \mathbf{G}]^{-1} [\alpha - \mathbf{V}'].$$

For generic n and *regular* networks (d denoting the degree) we get:

$$\phi^* = \frac{\alpha - V'}{\beta + n\gamma - d\lambda}. \quad (\text{A.2})$$

This solution represents a maximum since the expression in curly brackets in (A.1) is concave in x_i . We look for a symmetric LMPE. Consider the following value function:

$$V(y) = \begin{cases} \left(\frac{y}{\underline{y}}\right)^{\frac{r}{\theta}} Z(\underline{y}) & \text{for } y < \underline{y} \\ Z(y) & \text{for } y \in [\underline{y}, \bar{y}] \\ \frac{\alpha^2(B - \gamma)}{2r(\beta - d\lambda + \gamma n)^2} & \text{for } y > \bar{y}, \end{cases} \quad (\text{A.3})$$

where

$$Z(y) = A \frac{y^2}{2} + By + C,$$

with

$$\begin{aligned} A &= -\frac{(r - 2\theta)(\beta - d\lambda + n\gamma)^2}{\beta + \gamma + 2d\lambda(n - 1) - 2n[\beta + \gamma(n - 1)]}, \\ B &= \frac{\alpha(r - 2\theta)\{-\gamma + d\lambda + n[\beta - d\lambda + \gamma(n - 1)]\}}{\theta\{2d\lambda(n - 1) - \beta(2n - 1) - \gamma[2n(n - 1) - 1]\}}, \end{aligned} \quad (\text{A.4})$$

and

$$C = \frac{(B - \alpha)\{\alpha(\gamma - \beta) + B\{-2d\lambda(n - 1) + \beta(2n - 1) + \gamma[2n(n - 1) - 1]\}\}}{2r(\beta - d\lambda + \gamma n)^2},$$

and \underline{y} and \bar{y} given in Proposition 2. Note that Assumptions A1 and A2 guarantee that $B > \alpha > 0$ and that $\underline{y} > 0$.

Proof that $A < 0$. We have $\partial\phi^*(y)/\partial y = -A/(\beta + n\gamma - d\lambda)$. Since it must be $\beta > d\lambda$ for the matrix $[\beta\mathbf{I} - \lambda\mathbf{G}]^{-1}$ to be nonnegative then $\beta + n\gamma - d\lambda > 0$. It follows that $\partial\phi^*(y)/\partial y > 0$ for $A < 0$. We know that $r < 2\theta$ since $r < \hat{r} < 2\theta$. Therefore, from (A.4), $A < 0$ for $\Gamma = \beta + \gamma + 2d\lambda(n-1) - 2n[\beta + \gamma(n-1)] < 0$. Call β_0 the value of β solving $\Gamma = 0$. The expression Γ is decreasing in β . Hence $\beta > \beta_0$ implies that $\Gamma < 0$. Since $\beta_0 < d\lambda$ then $A < 0$, implying that $\phi^*(y)$ is increasing in y .

Proof that $\bar{y} > \underline{y}$. Immediate since $\bar{y} - \underline{y} = -\alpha/A$ and $A < 0$.

Next, we need to prove that $V(y)$ is continuously differentiable, and that the function $\phi^*(y)$ given in Proposition 2 is solution to the problem (A.1).

Proof that $V(y)$ is continuously differentiable. The function $V(y)$ is continuously differentiable over $[0, \underline{y})$, (\underline{y}, \bar{y}) and (\bar{y}, ∞) with

$$\frac{\partial V(y)}{\partial y} = \begin{cases} \frac{r}{\theta \underline{y}} \left(\frac{y}{\underline{y}}\right)^{\frac{r}{\theta}-1} Z(\underline{y}) & \text{for } y < \underline{y} \\ \frac{\partial Z(\underline{y})}{\partial y} & \text{for } y \in [\underline{y}, \bar{y}] \\ 0 & \text{for } y > \bar{y}. \end{cases} \quad (\text{A.5})$$

We first check that the function $V(y)$ is continuous at \underline{y} and \bar{y} :

$$\lim_{y \rightarrow \underline{y}, y < \underline{y}} V(y) = Z(\underline{y}) = \frac{\alpha^2 (nr(\beta - \gamma - d\lambda) + d\lambda r + \gamma n^2 r - \gamma r + \theta(\gamma - \beta))}{r(r - 2\theta)(\beta - d\lambda + \gamma n)^2} = \lim_{y \rightarrow \underline{y}, y > \underline{y}} V(y),$$

and

$$\lim_{y \rightarrow \bar{y}, y < \bar{y}} V(y) = Z(\bar{y}) = \frac{\alpha^2 (B - \gamma)}{2r(\beta - d\lambda + \gamma n)^2} = \lim_{y \rightarrow \bar{y}, y > \bar{y}} V(y).$$

Therefore $V(y)$ is continuous at \underline{y} and \bar{y} . We now verify that $\partial V(y)/\partial y$ is continuous at \underline{y} and \bar{y} :

$$\lim_{y \rightarrow \underline{y}, y < \underline{y}} \frac{\partial V(y)}{\partial y} = \frac{rZ(\underline{y})}{\theta \underline{y}} = \frac{\partial Z(\underline{y})}{\partial y} = \alpha = \lim_{y \rightarrow \underline{y}, y > \underline{y}} \frac{\partial V(y)}{\partial y},$$

implying that $\partial V(y)/\partial y$ is continuous at \underline{y} . Since $\lim_{y \rightarrow \bar{y}, y < \bar{y}} \partial V(y)/\partial y = 0$, $\partial V(y)/\partial y$ is continuous at \bar{y} . Thus the function $V(y)$ is continuously differentiable.

Proof that $\phi^*(y)$ given in Proposition 2 is a solution to the problem (A.1). For $y \geq \underline{y}$ (A.1) admits an interior solution. The first-order condition is given by (A.2). Substituting $\partial V(y)/\partial y$ from (A.5) into (A.2) yields $\phi^*(y)$ given in Proposition 2. For $y < \underline{y}$ problem (A.1) admits the corner solution $\phi^*(y) = 0$. It can be easily checked that the function $V(y)$ given in (A.3) satisfies the differential equation obtained after substituting (A.2) into (A.1).

Appendix B. Proof of Corollary 2

The stationary levels of total wealth in the economy are given by

$$\dot{y} = \theta y - n\phi^*(y) = 0. \quad (\text{B.1})$$

(B.1) admits two positive roots, \hat{y}_{ss} and \tilde{y}_{ss} , with

$$\hat{y}_{ss} = \frac{n(\alpha - B)}{An + \theta(\beta - d\lambda + \gamma n)},$$

and \tilde{y}_{ss} given in Corollary 2. For all $y \in (0, \hat{y}_{ss})$ we have $\dot{y} = \theta y - n\phi^*(y) > 0$, while for all $y \in (\hat{y}_{ss}, \tilde{y}_{ss})$ we have $\dot{y} = \theta y - n\phi^*(y) < 0$. Hence, $\phi^*(y)$ induces a trajectory of total wealth that converges asymptotically to \hat{y}_{ss} for any $y_0 \in (0, \tilde{y}_{ss})$. The steady-state level of total crime associated to \hat{y}_{ss} is \hat{X}_{ss} . For all $y > \tilde{y}_{ss}$, we have $\dot{y} = \theta y - n\phi^*(y) > 0$. Therefore, starting from any $y_0 > \tilde{y}_{ss}$, total crime remains at \tilde{X}_{ss} at any $t \in [0, \infty)$.

Appendix C. Proof of Lemma 1

As to (i), we have

$$\frac{\partial \underline{y}}{\partial n} = \frac{\alpha \left\{ r \left[(\beta - d\lambda)^2 + \gamma [\beta(n-1) - d\lambda(n+1) + \gamma(n+2)] \right] + 2\gamma\theta(\beta - \gamma) \right\}}{\theta(r - 2\theta)(\beta - d\lambda + \gamma n)^3}.$$

The denominator of $\partial \underline{y} / \partial n$ is negative since $r < \hat{r} < 2\theta$ and $\beta > d\lambda$. The numerator of $\partial \underline{y} / \partial n$ is positive since $\gamma(n+2) > d\lambda(n+1)$ by Assumption A3.

As to (ii), we have

$$\frac{\partial \bar{y}}{\partial n} = \frac{\alpha \left[(\beta - d\lambda)^2 + \beta\gamma(n-1) + \gamma[\gamma(n+2) - d\lambda(n+1)] \right]}{\theta(\beta - d\lambda + \gamma n)^3}.$$

The denominator of $\partial \bar{y} / \partial n$ is positive since $\beta > d\lambda$. The numerator of $\partial \bar{y} / \partial n$ is positive as well since $\gamma(n+2) > d\lambda(n+1)$ by Assumption A3.

As to (iii), we have

$$\frac{\partial^2(n\phi^*)}{\partial y \partial n} = \frac{(r - 2\theta) \left\{ \beta^2 + 2d^2\lambda^2 + \beta[(2n+1)\gamma - 3d\lambda] + \gamma[2n(n+1)\gamma - (4n+1)d\lambda] \right\}}{[\beta + \gamma + 2d\lambda(n-1) - 2\beta n - 2\gamma n(n-1)]^2}.$$

The denominator of $\partial^2(n\phi^*) / \partial y \partial n$ is positive. The numerator of $\partial^2(n\phi^*) / \partial y \partial n$ is negative since $r < \hat{r} < 2\theta$ and the expression between curly brackets is positive. To see this, observe that $(2n+1)\gamma - 3d\lambda > 0$ and $2n(n+1)\gamma - (4n+1)d\lambda > 0$ by Assumption A3.

Appendix D. Proof of Lemma 2

Take $y = \underline{y}|_{n_1}$. The expression $n_2 \phi^*|_{n_2} - n_1 \phi^*|_{n_1}$ is linear in r . Let \tilde{r} be the value of r solving $n_2 \phi^*|_{n_2} = n_1 \phi^*|_{n_1}$. It can be checked that $\tilde{r} > 0$ and that $n_2 \phi^*|_{n_2} - n_1 \phi^*|_{n_1}$ is increasing in r . We can then conclude that $n_2 \phi^*|_{n_2} - n_1 \phi^*|_{n_1} \geq (<)0$ for $r \geq (<)\tilde{r}$.

Appendix E. Proof of Lemma 4

It can be checked that the denominator of $\partial \hat{X}_{ss}/\partial n$ is positive, and that the numerator of $\partial \hat{X}_{ss}/\partial n$ is quadratic in r , with the coefficient of r^2 being equal to

$$\Gamma = \alpha n^2 (\beta - d\lambda + \gamma n) \left\{ (\beta - d\lambda)^2 + \beta \gamma (n - 1) + \gamma [(n + 2) \gamma - d\lambda (n + 1)] \right\}.$$

Γ is positive by Assumption A3 and the fact that $\beta > d\lambda$. Hence, $\partial \hat{X}_{ss}/\partial n < (>)0$ for $r \in (\notin)(r_1, r_2)$, where r_1 and r_2 are the roots of $\partial \hat{X}_{ss}/\partial n$, with $r_1 < r_2$. It can also be checked that $r_2 > \hat{r}$ and that r_1 can be either positive or negative (or nil), with $r_1 < \hat{r}$. We can then conclude that $\partial \hat{X}_{ss}/\partial n < (>)0$ for $r > (<)r_1$ and that $\partial \hat{X}_{ss}/\partial n = 0$ for $r = r_1$.

Appendix F. Proof of Lemma 5

As to (i), we have

$$\frac{\partial y}{\partial d} = \frac{\alpha \lambda \{r \{ (n + 1) [\beta + \gamma (n - 2)] - d\lambda (n - 1) \} + 2\theta (\gamma - \beta) \}}{\theta (r - 2\theta) (\beta - d\lambda + \gamma n)^3}.$$

The denominator of $\partial y/\partial d$ is negative. The numerator of $\partial y/\partial d$ is increasing in r given that the coefficient of r , $(n + 1) [\beta + \gamma (n - 2)] - d\lambda (n - 1)$, is positive. To see this, take $n = 2$: the coefficient of r becomes $3\beta - d\lambda$, which is positive given that $\beta > d\lambda$. For $n \geq 3$, we have $(n + 1) (n - 2) \gamma > d\lambda (n - 1)$ by Assumption A3. The numerator of $\partial y/\partial d$ is nil at $r = \bar{r}$, with

$$\bar{r} = \frac{2\theta (\beta - \gamma)}{(n - 1) [\beta + \gamma (n - 2)] - d\lambda (n - 1)}.$$

It can be checked that $\bar{r} > \hat{r}$. We have

$$\text{sign} \left\{ \bar{r} - \frac{\theta (\beta - \gamma)}{d\lambda - \gamma + n [\beta - d\lambda + \gamma (n - 1)]} \right\} = \text{sign} \{ \Lambda \},$$

with

$$\Lambda = \frac{(n - 1) (\beta - d\lambda + \gamma n)}{[\beta + d\lambda + \gamma (n^2 - n - 2) + n (\beta - d\lambda)] [d\lambda + \gamma (n^2 - n - 1) + n (\beta - d\lambda)]}.$$

The numerator of Λ is positive. The denominator of Λ is also positive given that the two terms between square brackets are positive for $n \geq 2$. It follows that $\bar{r} > \hat{r}$, which implies that $r < \bar{r}$ and consequently that the numerator of $\partial y / \partial d$ is negative. Given that the denominator of $\partial y / \partial n$ is also negative, we have $\partial y / \partial d > 0$.

As to (ii), we have

$$\frac{\partial \bar{y}}{\partial d} = \frac{\alpha \lambda \{ (n+1) [\beta + \gamma (n-2)] - d \lambda (n-1) \}}{\theta (\beta - d \lambda + \gamma n)^3}.$$

The denominator of $\partial \bar{y} / \partial d$ is positive. The numerator is also positive, as proven in (i). Therefore, $\partial \bar{y} / \partial d > 0$.

As to (iii), we have

$$\frac{\partial^2 (n\phi^*)}{\partial y \partial d} = \frac{\lambda n (r - 2\theta) (\beta - \gamma)}{[\beta + \gamma + 2d\lambda (n-1) - 2\beta n - 2\gamma n (n-1)]^2},$$

which is clearly negative since $r < 2\theta$ and $\beta > \gamma$.

Appendix G. Proof of Lemma 7

The proof of (i) follows directly from Lemma 5.

As to (ii), *a priori*, there are two cases to consider, depending on the order relationship between $\hat{y}_{ss}|_{d_2}$ and $\tilde{y}_{ss}|_{d_1}$. In the former, θy crosses $n\phi^*|_{d_2}$ below $n\phi^*|_{d_1}$ and, consequently, $\tilde{y}_{ss}|_{d_1} > \hat{y}_{ss}|_{d_2}$, while in the latter θy crosses $n\phi^*|_{d_2}$ above $n\phi^*|_{d_1}$ and, consequently, $\tilde{y}_{ss}|_{d_1} < \hat{y}_{ss}|_{d_2}$. We are going to show that only the case in which $\tilde{y}_{ss}|_{d_1} > \hat{y}_{ss}|_{d_2}$ is admissible. This case is depicted in Figure 5.

We have

$$\hat{y}_{ss}|_{d_2} - \tilde{y}_{ss}|_{d_1} = \frac{n\alpha\Gamma}{\theta},$$

where

$$\Gamma = \frac{nr(-\beta + \gamma + d_2\lambda) - r(\gamma - d_2\lambda) - r\gamma n^2 - \theta(\beta - \gamma)}{(\beta - d_2\lambda + \gamma n)[n(d_2\lambda r - \beta r + 2\gamma\theta) + \theta(\beta + \gamma - 2d_2\lambda) - \gamma r n^2]} - \frac{1}{\beta - d_1\lambda + \gamma n}.$$

Moreover,

$$\frac{\partial \Gamma}{\partial r} = \frac{-\theta[\gamma(n+1) - d_2\lambda]\{(2n-1)\beta - 2(n-1)d_2\lambda + \gamma[2n(n-1) - 1]\}}{(\beta - d_2\lambda + \gamma n)[nr(\beta - d_2\lambda) - \theta(\beta + \gamma - 2d_2\lambda) + \gamma n(nr - 2\theta)]^2}.$$

The denominator of $\partial \Gamma / \partial r$ is positive. The numerator of $\partial \Gamma / \partial r$ is negative since $\gamma(n+1) > d_2\lambda$ by Assumption A3, $(2n-1) > 2(n-1)$, and $2n(n-1) > 1$. It follows that $\partial \Gamma / \partial r < 0$. Furthermore, $\Gamma = 0$ at

$$\bar{r} = \frac{\theta\{-\lambda\{d_1(\gamma-\beta)+d_2[3\beta+\gamma(1+4n)]\}+2d_2^2\lambda^2+2\gamma(n+1)(\beta+\gamma n)\}}{-\lambda\{d_1\{\gamma-n[\beta+\gamma(n-1)]\}+d_2(n+1)(\beta+\gamma n)\}+d_2\lambda^2[d_1+n(d_2-d_1)]+\gamma(n+1)(\beta+\gamma n)},$$

and Γ has a vertical asymptote, given by

$$r_\infty = \frac{\theta [\beta + \gamma + 2(\gamma n - d_2 \lambda)]}{n(\beta - d_2 \lambda + \gamma n)},$$

such that $\Gamma < 0$ for $r < r_\infty$ and $\Gamma > 0$ for $r \in (r_\infty, \bar{r})$ and $\Gamma \leq 0$ for $r \geq \bar{r}$. To see this, we have

$$\text{sign} \{ \Gamma|_{r=0} \} = -\text{sign} \{ \Psi \},$$

where

$$\Psi = [d_1 \lambda (\beta - \gamma) + \gamma (2\beta - d_2 \lambda) + \beta (2\gamma n - 3d_2 \lambda) + 2\gamma^2 n] + 2(\gamma n - d_2 \lambda)^2.$$

By Assumption A3, $2\gamma n > 3d_2 \lambda$ implying that $\Psi > 0$ and that $\Gamma|_{r=0} < 0$. It can be checked that $\lim_{r \rightarrow r_\infty, r < r_\infty} = -\infty$ and $\lim_{r \rightarrow r_\infty, r > r_\infty} = \infty$. The fact that $\Gamma > 0$ for $r \in (r_\infty, \bar{r})$ and that $\Gamma \leq 0$ for $r \geq \bar{r}$ are derived from $\partial \Gamma / \partial r < 0$.

Next, we prove that $\bar{r} > 2\theta > \hat{r}$. We have

$$(\bar{r} - 2\theta)|_{\gamma=\beta} = -\frac{2\lambda\theta(n-1)(d_1 - d_2)}{d_1\lambda(n-1) + \beta(n+1) - nd_2\lambda},$$

which is positive since $(n+1) > n$ implies that $\beta(n+1) - nd_2\lambda$. It can be checked that \bar{r} is decreasing in γ , therefore $\bar{r} > 2\theta > \hat{r}$.

By direct inspection, we can see immediately that $r_\infty \geq \hat{r}$. Hence, $\Gamma < 0$.

Appendix H. Proof of Proposition 7

We have

$$\left. \frac{\partial g^*(t)}{\partial \theta} \right|_{t=0} = \frac{1}{1-n} + \frac{n\Upsilon}{y_0\theta^2},$$

where

$$\Upsilon = \frac{\alpha r}{\beta - d\lambda + \gamma n} + \frac{y_0\theta^2(\beta - \gamma) - \alpha r(n-1)^2}{(n-1)\{-2d\lambda(n-1) + \beta(2n-1) + \gamma[2n(n-1) - 1]\}}.$$

It can be checked that the denominator of $\partial g^*(t)/\partial \theta|_{t=0}$ is positive and that the numerator of $\partial g^*(t)/\partial \theta|_{t=0}$ is decreasing in r (in the admissible parameter range) and nil at \hat{y}_0 , with

$$\hat{y}_0 = \frac{\alpha n r \{-\gamma + d\lambda + n[\beta - d\lambda + \gamma(n-1)]\}}{\theta^2(\beta + \gamma - 2d\lambda + 2\gamma n)(\beta - d\lambda + \gamma n)},$$

which converges to 0 as $r \rightarrow 0$. Since

$$\lim_{r \rightarrow 0} \underline{y} = \frac{\alpha(\beta - \gamma)}{2\theta(\beta - d\lambda + \gamma n)^2} > 0,$$

then $\lim_{r \rightarrow 0}(\underline{y} - \hat{y}_0) > 0$ implying that $\partial g^*(t)/\partial \theta < 0$ for t and r sufficiently small.

References

- [1] Ballester, C., Calvó-Armengol, A., Zenou, Y., 2004, Who's who in crime network. Wanted the key player, IUI Working Paper, No 617, The Research Institute of Industrial Economics (IUI), Stockholm.
- [2] Ballester, C., Calvó-Armengol, A., Zenou, Y., 2006, Who's who in networks. Wanted: the key player, *Econometrica* 74, 1403-1417.
- [3] Ballester, C., Calvó-Armengol, A., Zenou, Y., 2010, Delinquent networks, *Journal of the European Economic Association* 8, 34-61.
- [4] Başar, T., Olsder, G.J., 1995, *Dynamic Noncooperative Game Theory*, San Diego, Academic Press.
- [5] Benchekroun, H., 2003, Unilateral production restrictions in a dynamic duopoly, *Journal of Economic Theory* 111, 214-239.
- [6] Benchekroun, H., 2008, Comparative dynamics in a productive asset oligopoly, *Journal of Economic Theory* 138, 237-261.
- [7] Bonacich, P., 1987, Power and centrality: a family of measures, *American Journal of Sociology* 92, 1170-1182.
- [8] Calvó-Armengol, A., Zenou, Y., 2004, Social networks and crime decisions. The role of social structure in facilitating delinquent behavior, *International Economic Review* 45, 939-958.
- [9] Colombo, L., Labrecciosa, P., 2015, On the Markovian efficiency of Bertrand and Cournot equilibria, *Journal of Economic Theory* 155, 332-358.
- [10] Cortés, D., Friebe, G., Maldonado, D. 2019, Crime and education in a model of information transmission, *Annals of Public Cooperative Economics* 91, 71-93.
- [11] Crettez, B., Hayek, N., Zaccour, G., 2020, Non-deceptive counterfeiting and consumer welfare: A differential game approach, In: Ramsey D.M., Renault J. (eds) *Advances in Dynamic Games. Annals of the International Society of Dynamic Games*, vol 17. Birkhäuser, Cham.
- [12] Dawid, H., Feichtinger, G., 1996, Optimal allocation of drug control efforts: A differential game analysis, *Journal of Optimization Theory and Applications* 91, 279-297.

- [13] Dockner, E.J., Long, N.V., 1993, International pollution control: Cooperative versus non-cooperative strategies, *Journal of Environmental Economics and Management* 24, 13-29.
- [14] Dockner, E.J., Sorger, G., 1996, Existence and properties of equilibria for a dynamic game on productive assets, *Journal of Economic Theory* 171, 201-227.
- [15] Dockner, E.J., Jørgensen, S., Long, N.V., Sorger, G., 2000, *Differential Games in Economics and Management Science*, Cambridge, Cambridge University Press.
- [16] Dubovik, A., Parakhonyak, A., 2014, Drugs, guns, and targeted competition, *Games and Economic Behavior* 87, 497-507.
- [17] Faria, J.R., Mixon, F.G.J., Upadhyaya, A., Upadhyaya, K.P., 2019, Gang rivalry and crime: A differential game approach, *Review of Law and Economics* 15, 1-30.
- [18] Feichtinger, G., 1983, A differential game solution to a model of competition between a thief and the police, *Management Science* 29, 686-699.
- [19] Fershtman, C., Kamien, M., 1987, Dynamic duopolistic competition with sticky prices, *Econometrica* 55, 1151-1164.
- [20] Glaeser, E., Sacerdote, B., Scheinkman, J.A., 1996, Crime and social interactions, *Quarterly Journal of Economics* 111, 507-548.
- [21] Haurie, A., Krawczyk, J.B., Zaccour, G., 2012, *Games and Dynamic Games*, World Scientific.
- [22] Haynie, D.L., 2001, Delinquent peers revisited: does network structure matter?, *American Journal of Sociology*, 106, 1013-1057.
- [23] Jun, B., Vives, X., 2004, Strategic incentives in dynamic duopoly, *Journal of Economic Theory* 116, 249-281.
- [24] Katz, L., 1953, A new status index derived from sociometric analysis, *Psychometrika* 18, 39-43.
- [25] Kemp, M.C., Long, N.V., 2009, Foreign aid in the presence of corruption: Differential games among donors, *Review of International Economics* 17, 230-243.
- [26] Kling, J.R., Ludwig, J., Katz, L.F., 2005, Neighborhood effects on crime for female and male youth: evidence from a randomized housing voucher experiment, *Quarterly Journal of Economics* 120, 87-130.

- [27] Lindquist, M.J., Zenou, Y., 2019, Crime and networks: 10 policy lessons, *Oxford Review of Economic Policy* 35(4), 746-771.
- [28] Long, N.V., 2010, *A Survey of Dynamic Games in Economics*, World Scientific Publishing, Singapore.
- [29] Long, N.V., Sorger, G., 2006, Insecure property rights and growth: the role of appropriation costs, wealth effects, and heterogeneity, *Economic Theory* 28, 513-529.
- [30] Ludwig, J., Duncan, G.J., Hirschfield, P., 2001, Urban poverty and juvenile crime: evidence from a randomized housing-mobility experiment, *Quarterly Journal of Economics* 116, 655-679.
- [31] Maskin, E., Tirole, J., 2001, Markov perfect equilibrium I. Observable actions, *Journal of Economic Theory* 100, 191-219.
- [32] Ngendakuriyo, F., Zaccour, G., 2013, Fighting corruption: To precommit or not?, *Economics Letters* 120, 149-154.
- [33] Ngendakuriyo, F., Zaccour, G., 2017, Should civil society organizations cooperate or compete in fighting a corrupt government?, *Mathematical Social Sciences* 85, 30-36.
- [34] Nova, A.J., Feichtinger, G., Leitmann, G., 2010, A differential game related to terrorism: Nash and Stackelberg strategies, *Journal of Optimization Theory and Applications* 144, 533-555.
- [35] Patacchini, E., Zenou, Y., 2012, Juvenile delinquency and conformism, *Journal of Law, Economics, and Organization* 28, 1-31.
- [36] Sah, R., 1991, Social osmosis and patterns of crime, *Journal of Political Economy* 99, 1272-1295.
- [37] Sarnecki, J., 2001, *Delinquent Networks: Youth Co-offending in Stockholm*, Cambridge, Cambridge University Press.
- [38] Sutherland, E.H., 1947, *Principles of Criminology*, 4th Edition, Chicago, J.B. Lippincott.
- [39] Tornell, A., Velasco, A., 1992, The tragedy of the commons and economic growth: Why does capital flow from poor to rich countries?, *Journal of Political Economy* 100, 1208-1231.

- [40] Tornell, A., Lane, P.R., 1999, The voracity effects, *American Economic Review* 89, 22-46.
- [41] Tsutsui, S., Mino, K., 1990, Nonlinear strategies in dynamic duopolistic competition with sticky prices, *Journal of Economic Theory* 52, 136-161.
- [42] Van der Ploeg, F., 2011, Natural resources: curse or blessing?, *Journal of Economic Literature* 49, 366-420.
- [43] Warr, M., 2002, *Companions in Crime: The Social Aspects of Criminal Conduct*, Cambridge, Cambridge University Press.
- [44] Wrzaczek, S., Kaplan, E.H., Caulkins, J.P., Seidl, A., Feichtinger, G., 2017, Differential terror queue games, *Dynamic Games and Applications* 7, 578-593.