



**HAL**  
open science

# Integrated scheduling of a multi-site mining supply chain with blending, alternative routings and co-production

Hajar Hilali, Vincent Hovelaque, Vincent Giard

## ► To cite this version:

Hajar Hilali, Vincent Hovelaque, Vincent Giard. Integrated scheduling of a multi-site mining supply chain with blending, alternative routings and co-production. *International Journal of Production Research*, 2023, 61 (6), pp.1829-1848. <10.1080/00207543.2022.2049909>. <halshs-03612471>

**HAL Id: halshs-03612471**

**<https://shs.hal.science/halshs-03612471v1>**

Submitted on 27 Apr 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire HAL, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.



Distributed under a Creative Commons CC BY-NC 4.0 - Attribution - Non-commercial use - International License

# Integrated scheduling of a multi-site mining supply chain with blending, alternative routings and co-production

Hajar Hilali<sup>a,b</sup>, Vincent Hovelaque<sup>a,b</sup>, Vincent Giard<sup>a,c1</sup>

<sup>a</sup> *EMINES - School of Industrial Management, Mohammed VI Polytechnic University, Ben Guerir, Morocco*

<sup>b</sup> *Univ Rennes, CNRS, CREM - UMR 6211, F-35000 Rennes, France*

<sup>c</sup> *Université Paris-Dauphine, PSL Research University, 75016, Paris, France*

[Hajar.Hilali, Vincent.Hovelaque, Vincent.Giard}@emines.um6p.ma](mailto:{Hajar.Hilali, Vincent.Hovelaque, Vincent.Giard}@emines.um6p.ma)

[vincent.hovelaque@univ-rennes1.fr](mailto:vincent.hovelaque@univ-rennes1.fr), [vincent.giard@dauphine.psl.eu](mailto:vincent.giard@dauphine.psl.eu)

---

<sup>1</sup> [ORCID](#): Hajar Hilali: 0000-0003-0303-0937; Vincent Hovelaque: 0000-0002-2492-1758; Vincent Giard: 0000-0003-4818-8279

# **Integrated scheduling of a multi-site mining supply chain with blending, alternative routings and co-production**

*Abstract* - This paper proposes a multi-site global optimisation model of blending operations, alternative routings and order scheduling with a co-production flow. It is performed in a phosphoric supply chain owning three open-pit mines of different geological structures and chemical compositions, each having a dry blending plant. One main challenge of the phosphate industries is to produce merchantable ores (MO) that verify a quality charter (chemical composition) with chemical heterogeneous source ores (SO). Thus, a process of SO blending followed by a treatment is mostly necessary to obtain the required MO. Six alternative routings exist to produce an MO; one of them involving a calcination plant generates a co-product. The model objective is to determine the least costly production program of a set of MO orders to be delivered within precise time windows, in given quantities and compliance with a specific quality charter, by assigning for each MO its blending plant, routing, blend of SOs and production schedule; it also defines the feeding of the blending plants. A real case study illustrates the proposed model.

**Keywords:** Integrated scheduling, multi-site, blending, alternative routings, co-production, mining supply chain.

Multi-sites integrated scheduling problems have received special attention from researchers in supply chain management. This study aims to complete previous works by proposing an integrated multi-site scheduling model applied to the part of a phosphoric supply chain starting from several mines and using multi-process lines, alternative routings and a co-production to produce merchantable ores (MO).

## **1 Introduction**

After a presentation of the scientific position of this paper, a description of the main features of the studied management problem is presented before explaining the paper's objectives.

### ***1.1 Scientific position***

Unlike in discrete production, very few theories embrace supply chain (SC) starting from extraction to yield semi-row material (liquid or granules) produced with continuous processes. Few generic models can be applied and adapted to one link of this kind of SC. For instance, the blending model created by Dantzig in 1947 (Dantzig 1982) would be usable in the context of mixing phosphate ores to get a merchantable ore that fits composition constraints. Azzamouri et al. (2021) consider that the source ore is extracted from a dynamic extraction planning, involving a dynamic problem to solve. In the context of one mine and one blending structure, this model considers the versatility of the ore composition and ore availability to produce by blending the merchantable ores whose specifications range vary depending on customer's needs. In addition, it considers the existence of alternative routings (usually ignored in the blending literature) used in the washing plant and several storage constraints. Therefore, this dynamic blending model may be considered as the first generic model to use in that kind of SC. However, it does not include the co-production routing, which is an alternative to washing plant routings that can be used after the dry blending process (APICS (ASCM, 2019) defines the co-product as "*a product that is usually manufactured together or sequentially because of product or process similarities*"). This new article extends the scope of this previous work by considering: *i*) several mines simultaneously, without pre-affecting MOs orders to mines, their routings, and their production sequences, and *ii*) a co-production using a calcination process (an alternative process to the washing one, performed in an existing calcination plant), never studied in the literature.

### ***1.2 Main features of the studied management problem***

The extended phosphoric SC worldwide is summarised as follows (Steiner et al. 2015): (1) phosphate rock extraction, (2) beneficiation, the most common of which are washing, grinding and flotation (others less usual are calcination or magnetic separation), and (3) chemical

processing into fertiliser. More than 80% of the marketable rock phosphate (post-beneficiation) produced annually is used in chemical fertilisers, mainly via phosphoric acid (Scholz et al. 2014). As with phosphate rock, phosphoric acid is mainly consumed where it is produced. The largest producers of phosphoric acid in the world are China, the United States and Morocco<sup>2</sup>. The OCP<sup>3</sup> Group is a world leader in the market of phosphate and its derivatives. The remote location of phosphate mines in Morocco has led the OCP to structure its productive organisation in three axes known as the North axis, Centre axis and South axis, which encompass various processes of ore extraction, ore blending, phosphoric acid and fertiliser production and export by sea (see Figure 1). The OCP gave us full access to all the Centre axis information necessary to carry out this research. This axis includes three independent mines, from which about 30 source ores (SOs) with different chemical characteristics are extracted and transformed by different processes in Merchantable Ores (MOs), as detailed in Figure 2, which expands Figure 2 of Azzamouri et al. (2021) to several mines, adds the calcination plant and four new routings. Processes and problems encountered in that axis are representative of most phosphoric SCs, as described in Steiner et al. (2015). However, its operational issues are made more difficult by the number of mines and processing plants, especially since it uses marginally a calcination process used to produce jointly two different MOs. It should be noted that all the processes (mining sites and processing units) and technical data introduced in this paper are actual data used in the field. This research aims to test and propose new steering of the Centre axis integrating the three mining sites and six alternative routings simultaneously to better manage the multi-site and multi-routing flow problem while keeping the same existing productive structure.

---

<sup>2</sup> <http://acep.eu/data-and-statistics/phosphoric-acid/>

<sup>3</sup>. The Office Chérifien des Phosphates (OCP) founded in Morocco in 1920, became in 2008, the OCP Group S.A. owned by the Moroccan Government and the Banque Populaire du Maroc (<https://www.ocpgroup.ma/history>).

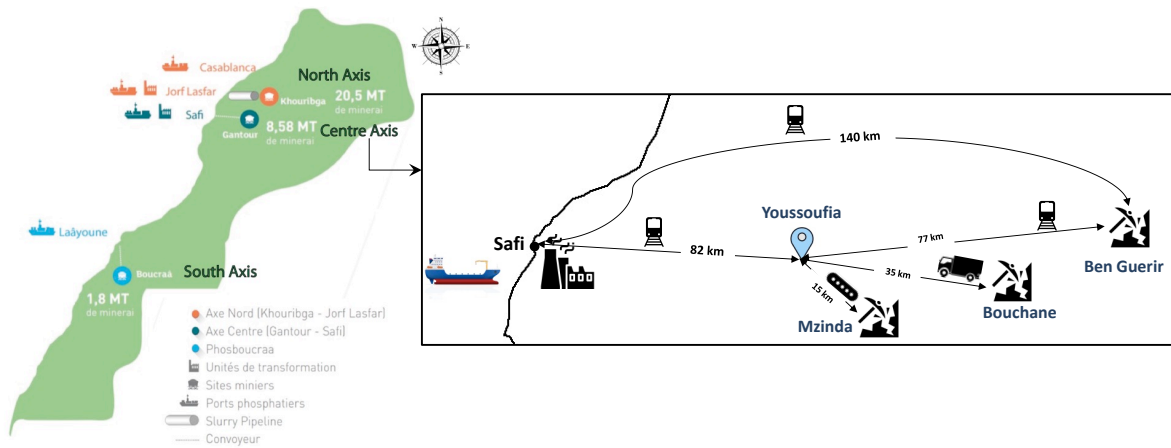


Figure 1. The three Axes of the OCP phosphate supply chain

The management of that phosphoric supply chain must consider the following characteristics.

- An MO order is characterised by its quantity, delivery time window and a required composition defined by a Quality Charter (QC) that specifies the client acceptable ranges of weight percentage of a few chemical components (up to six). Around two dozen QCs meet demands made by internal clients (chemical plant) or external clients (export). This lack of precision is explained by the fact that the customer requirements are quite unstable, except for a shortlist of fertilisers, because the variety of fertilisers that meet the requirement of the couple "plant-soil" is very high (Benhamou et al., 2020a). To be more specific, in 2019, 28 fertilisers (among a list of 50 fertilisers already produced) with a number of components varying from 2 to 6 (Benhamou et al., 2020b). These MOs belong to an order book of 6 to 9 MO orders for dry blending units (in 2020), representing a workload of about one month.
- An SO is defined by its composition and an available quantity at a given date. Due to geological constraints and the used extraction planning, the set of available SOs changes quite rapidly over time. The extraction planning is independent for each mine and can be viewed as a push system disconnected from the short-term MOs demand. At the end of the extraction process (performed in a push-mode), the newly available SOs are stored in a primary stock in the mine in separate locations to avoid mixing. When an order of

a given SO is issued, the requested quantity is transported by truck from the primary stock to the head station of the transportation system to be routed by conveyor to the secondary stock several miles away. There, it is stored in a place where its integrity is preserved. The composition of an SO hardly ever satisfies the QC of an MO, requiring transformation processes.

- The production of an MO, triggered by a customer order (pull system), starts by mixing a selected set of SOs, taken from a secondary stock in a determined proportion. The composition of this intermediate output (dry blending ore) is a linear combination of the SOs compositions. Most of the time, very few intermediate outputs meet the MO quality charter. Most of them need additional treatments performed in a washing plant or a calcination plant, chosen among five routings. These additional treatments have two effects: *i*) an important mass reduction occurs, necessitating a significant increase in the SOs volumes required to obtain the required volume of an MO order; *ii*) they change the composition of the initial SOs blend to allow the QC respect.
- The calcination process results in MOs for export and a co-product blended with an intermediate output produced in the washing plant to obtain an MO used by chemical plants.

Currently, every week, a schedule is defined for a four-week rolling horizon, specifying: *i*) for each MO order: its mine assignment, its routing and blend, and its production schedule in the plants; *ii*) the quantities and transfer periods of the SOs to move from primary stocks to secondary stock. A model of dynamic blending (Azzamouri et al. 2021), currently in use in the Centre axis, yields an optimal solution for each site in a local view perspective.

### ***1.3 Paper's objectives***

Our goal is to propose a new steering approach of the Centre axis, generalisable to other phosphoric supply chains, by addressing, from a production cost perspective, the integrated

process planning problem in a multi-site environment with alternative routings and a co-production. The mathematical model determines for each order: *i*) the blending plant on which it will be executed, *ii*) the start and the end date of the order production in dry blending plants, *iv*) the alternative routing to use (and thus the use or not of a washing or a calcination plant) and the end date of the order production after the treatment units, *vi*) the blends of SOs and the program of feeding secondary stock from primary stock.

This new perspective will move from independent mine management to integrated management and yields two main consequences: *i*) the SOs allocation is improved, as the respect of the quality chart of some MOs may be easier in one mine than in another; *ii*) it may prevent the use of a costly washing process routing, which decreases the production cost (savings of SOs, chemical product, water, and energy).

In addition, the model includes an SC perspective. Chemical plants are internal clients of the system that transform MOs into phosphoric acid. Several successive batches of MOs having the same QC are used in the plant that produces phosphoric acid. All these MOs respect the QC, but the chemical composition of successive MOs differs, enforcing costly adjustments in the chemical plants. For that client, it is more efficient to provide him with successive batches whose compositions are as stable as possible. The model addresses that concern.

The rest of the paper is organised as follows: Section 2 reviews the literature on three areas linked to blending (multi-site scheduling, alternative routings and co-production). The third section develops the main features of this applicative model and the proposed mathematical formulation. In section 4, a numerical test illustrates the interest of the approach, and the last section provides a conclusion and implementation perspectives.

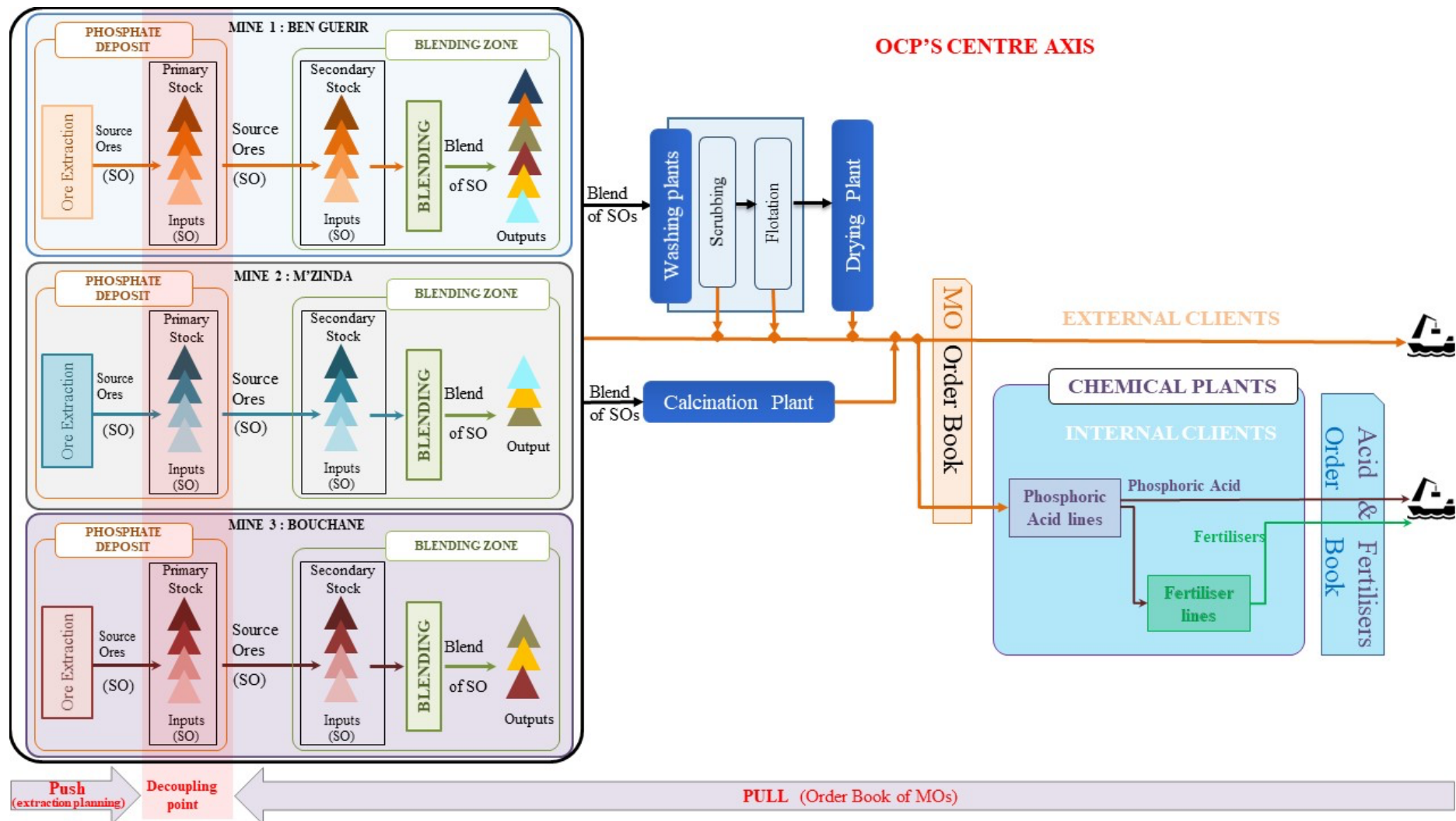


Figure 2. Focus on the Centre Axis of the OCP phosphate SC

## 2 Literature review

Blending optimisation is one of the first linear problems treated with the simplex method (Dantzig, 1982). Numerous and various works have been published since this seminal work. In a recent work close to our present paper, Azzamouri et al. (2021) proposed a detailed literature review of blending problems according to four fundamental criteria: *i*) input supply (sequential, parallel); *ii*) blending process (single-stage, multi-stage); *iii*) MOs characteristics and the *iv*) optimisation criterion. The originality of our proposal lies in three other dimensions: *a*) Multi-site integrated scheduling of blending operations, *b*) Blending and alternative routings, and *c*) Co-production. The impact of these new dimensions on the studied problem is analysed in the three following sub-sections.

### 2.1 *Multi-site scheduling of blending operations literature*

#### 2.1.1 *Multi-site integrated scheduling problem*

Grossmann et al. (2002) stated that planning and scheduling are defined on time horizons of different sizes. On the one hand, scheduling addresses operational decisions levels (sequencing) on short time horizons (days, few weeks). On the other hand, planning deals with high-level decisions (production, inventory levels) with longer time horizons (weeks, months). These two notions were also defined by Badhotiya et al. (2019) as follows: "*Planning deals with "what" and "how", while scheduling answers the questions related to "when" and "whom" in a manufacturing scenario*". In our case, the number of orders is quite stable, on average 7 orders over a period of 4 to 6 weeks for internal and external clients. This makes the borderline between planning and scheduling very tight. The problem of scheduling in a multi-site manufacturing context has been reported in several articles (Bruns and Sauer 1995; Miller and Matta 2003; Lin et al. 2015; Marandi and Fatemi Ghomi 2019; etc.). The multi-site planning, scheduling and integration of planning and scheduling literature in a multi-site context has been

recently analysed by Badhotiya et al. (2019); their main findings can be summarised as follows:

- i*) The developed mathematical models use essentially mixed-integer linear programming (with an optimal or approximate approach depending on the size of the problem);
- ii*) Most of applied works deal with process industries (chemical, pharmaceuticals, etc.) but no work in a mining environment was cited;
- iii*) Research works simplify characteristics of real manufacturing systems such as some costs, time values, etc. Recently, Rakiz et al. (2021) developed a multi-level lot-sizing problem model that integrates production, storage and distribution in a mining environment and minimises costs with predetermined routings and without considering the chemical composition of SOs, and thus the QC of MO orders.

### *2.1.2 Scheduling blending operations literature*

Numerous studies have been conducted in the literature to address combined problems of integrated scheduling and blending operations. Scheduling with blending operations have dealt with either a fixed number of recipes (Joly et al. 2002; Jia and Ierapetritou 2003; Li et al. 2009, 2016; Bengtsson et al. 2013) or a recipe optimisation followed by an order scheduling (Glismann and Gruhn 2001; Jia and Ierapetritou 2004; Mouret et al. 2009; Castillo and Mahalec 2015; Neiro et al. 2019). The multi-period problem has been solved with several solution techniques such as mixed-integer non-linear programming (Joly et al. 2002; Kolodziej et al. 2013; Li et al. 2016; Neiro et al. 2019) and mixed-integer linear programming (Jia and Ierapetritou 2003; Méndez et al. 2006; Li et al. 2009). Most of the analysed articles refer to an economic optimisation criterion, including *i*) minimisation of the operating cost (Joly et al. 2002; Jia and Ierapetritou 2004; Li et al. 2009, 2016; Cerda et al. 2016 b), and *ii*) maximisation of profit (Glismann and Gruhn 2001; Jia and Ierapetritou 2004; Méndez et al. 2006; Bengtsson et al. 2013; Kolodziej et al. 2013; Lotero et al. 2016). Several articles have also carried out case studies with actual data for oil refineries (Joly et al. 2002; Jia and Ierapetritou 2004; Neiro et

al. 2019).

## **2.2 *Blending and alternative routings/operations sequence***

The use of alternative routings (or alternative processes or alternative operations sequence) is mainly linked to scheduling problems. Bader and Bowyer (1973) have explored the value of offering different alternative production routing to improve fibre-reinforced thermoplastics' stiffness and strength. Zhang et al. (2005) developed a multi-objective model for integrated resource selection and operations sequence problems. The objective function used includes the minimisation of the makespan, the balancing of the workload, and the minimisation of the total transition times. Lee et al. (2004) considered the problem to minimise the total cost (sum of the operation processing cost and machine, setup and tool change costs). Recently, Kim and Kim (2020) formulated a mixed-integer programming model for a 3D printer-based parallel machine scheduling problem. The model incorporates multiple processing alternatives and sequence-dependent preparation times for minimising the makespan. An optimisation method based on the coal blend and cooking process characteristics has been proposed in Yuan et al. (2020). The implicit notion of alternative routing, blending of coal followed by the cooking process, partly resembles our problem, excepted that all blends go through the cooking process to produce a single type of product. They develop a multi-constraint optimisation model that minimises the cost of coal blending. Experimental results on actual data show that the proposed approach can significantly reduce the costs of blended coal.

## **2.3 *Co-product***

In the operations management literature, co-production has been widely studied (Gaudreault et al. 2011; Grohman et al. 2013; Bansal and Transchel 2014; Rafiei et al. 2015; Dumetz et al. 2021 ...).

On the one hand, some papers consider that the rate of co-production is not controlled and then considered as a random variable. Öner and Bilgiç (2008) developed two models to study the impact of uncontrolled co-production on the production planning and lot scheduling of multiple products; a numerical study has shown that the cycle time increases with the co-production rate. Recently, Chen et al. (2018) developed a three-stage stochastic model to study operational decisions of a single-input and multiple-output production process with co-production under demand uncertainty. They show that the quality upgrading policy's cost creates value because it reduces total production cost.

On the other hand, several papers have addressed the problem of co-production with fixed (controlled) rates in different industries such as the forest industry (Gaudreault et al. 2011; Rafiei et al. 2015; Dumetz et al. 2017), chemical industry (Grohman et al. 2013; Darmawan et al. 2020), and glass industry (Caner Taşkın and Tamer Ünal 2009). Dumetz et al. (2017) proposed a simulation-based model to evaluate order acceptance policies for divergent production processes with co-production. This model was later extended in Dumetz et al. (2021) to simulate production planning that incorporates the tactical and operational decision levels of North American sawmills, as well as market behaviour, using a rolling horizon. In the same application area (lumber industry) involving co-production, Wery et al. (2018) developed a simulator in combination with a tactical planning model to perform sales and operations planning. Recently, Lin et al. (2020) developed a constrained optimisation model for natural resource companies to design their product line to exploit a co-product value. This optimal design incorporates decisions regarding the quality and the price of the co-product.

Other recent works treat co-production and blending. Dong et al. (2014) studied the problem of a blending unit that blends two co-products (two quality levels of outputs) to make end petroleum products in the presence of price uncertainty. Chen et al. (2018) also addressed the

problem of how to blend the products from the co-production process to meet the demand of each market segment (each market requires a specific quality level).

The specificity in our case is that the co-product has to be blended with other inputs to meet the client's quality requirements. This blend with co-product covers part of the client's order, making the multi-site production scheduling particularly difficult.

So far, we have not found any papers in the literature that deal simultaneously with blending, multi-site scheduling, alternative routings, and co-production.

### **3 Mathematical modelling**

This section proposes a non-linear mathematical model to address blending operations integrated scheduling in a multi-site context with a co-production. Even if some constraints are initially non-linear, the model can be easily linearised in our case (Sioshansi and Conejo 2017). The linearisation protocol is presented in a footnote later on, and the linearised version of the model is provided in the Mendeley file. The model incorporates real and original features such as non-identical blending plants (mining sites), the presence of alternative routings and co-production.

#### **3.1 Main features**

As illustrated in figures 1 and 2, the Centre axis of OCP's SC is structured in three levels: dry blending operations from SOs extracted from three mines, physical treatment operations (washing plants and calcination, involving alternative routings), and chemical operations. Except for the dry blended, physical and chemical operations depend on the client orders of MO, the chosen routing and the SOs availabilities. Nine features characterise the studied part of the phosphate SC. Moreover, all parameters are deterministic because field surveys show very low variability and high robustness to the quality of SOs about to be used and routings characteristics.

- *Feature a)* SOs are not purchased from an international market but are provided by three internal mining sites; an SO is present in a layer; a deposit is a stack of layers. Each mine has its own independent medium-term extraction program (push system). During the phosphate extraction, the passage to a lower layer is conditioned by the preliminary withdrawal of SO that has been previously prepared in the current layer. Thus, the SOs availability changes over time.
- *Feature b)* The extraction process ends with feeding the SO' primary stocks in the mine area. Primary stocks are linked by conveyors to secondary stocks of limited capacity and located in the blending area. Transferring a given quantity of a given SO at a given date to secondary stock is a decision taken by the managers of the dry blending plants (pull system).
- *Feature c)* Each mine has a different geological structure; the chemical composition of an SO varies from one layer to another within the same mine's deposit, and sometimes, from one location to another in a mine's layer. This composition's variation, not always known in advance, involve creating additional SOs and new separate storage areas in primary and secondary stocks. Thus, in addition to SOs availability, chemical composition changes over time, but they are known in the planning horizon.
- *Feature d)* In addition to complying with the quality charter, the MOs feeding the phosphoric acid production lines must try to ensure the chemical composition stability of the successive sent batches around the target values (determined by the phosphoric manager lines) to avoid costly process adjustments.
- *Feature e)* Six possible alternative routings  $h$  are available at Youssoufia: 1) Routing  $h=1$ , dry blending (MO is produced in the blending plant, by mixing SOs, in a given proportion, without additional treatment), 2) Routing  $h=2$ , dry blending followed by a scrubbing treatment in the washing plant, 3) Routing  $h=3$ , dry blending followed

- by a washing treatment including scrubbing and flotation, 4) Routing  $h=4$ , dry blending followed by a drying process (in case of wet SOs), 5) Routing  $h=5$ , dry blending followed by a washing treatment including scrubbing and flotation and a drying process, 6) Routing  $h=6$ , dry blending followed by the calcination unit.
- *Feature f)* Each order relates to an MO and specifies the quantity to deliver in a given time window. The production scheduling assigns MO orders to mines and selects, for each order, the alternative routing to use, the SO quantities to blend in the mine dry blending plant and its start and end date of production in that plant. To better analyse all possible flows of an order production, three categories of MO orders are defined and pointed out in Figure 3. The first subset  $\mathcal{K}_1$  presents the case of MO orders that are not involving co-production. ( $h < 6$ );  $k_1$  will be used to mean  $k \in \mathcal{K}_1$ . The second subset  $\mathcal{K}_2$  includes the MO orders manufactured using the alternative routing  $h=6$  (calcination) and intended for export;  $k_2$  will be used to mean  $k \in \mathcal{K}_2$ . The last subset  $\mathcal{K}_3$ , linked to the previous one, corresponds to MO orders that supply a part of the phosphoric acid production lines, obtained by mixing a calcination co-product and an intermediate product produced by a dry blending plant and using an alternative routing  $h$ ;  $k_3$  will be used to mean  $k \in \mathcal{K}_3$ .
  - *Feature g)* The calcination unit, as presented in figure 2 below, comprises three sub-processes; warming, calcination and cooling.
    - Two streams are merged at the inlet of the warming process, starting the calcination unit: *i*) a dry blend (routing  $h=1$ ) coming from the mine sites and feeding the stock ST1; *ii*) an additional input present in ST2 stock.
    - At the end of the warming process, three outlet-flows can be distinguished, outlet flow (1) refers to rejection stock, outlet flow (2) feeds the calciner and the co-product blend,

and outlet flow (3) is made of fine particles transported by gases produced in the calciner. Flow (3) is constrained by a maximum quantity recovered during the warming process. Any product passing through the calciner undergoes the same weight reduction coefficient. The product recovered after the calciner is cooled to be exported by boat to the customer.

- The co-product blend is made from the outlet flow (3), a part of outlet flow (2), and input from the ST2 stock. The input I present in the ST2 stock is of a moisture nature. A technical constraint is necessary to avoid clogging the installations during a mixture between inputs of different natures (wet, dry). In our case, this constraint is integrated into the blend of input I (ST2) and dry MO (ST1) at the warming inlet and in the co-product blend.
- *Feature h)* In co-production, an order of the subset  $\mathcal{K}_2$  is linked with an order of the subset  $\mathcal{K}_3$ . These two MO orders are synchronised and must be finished on the same date. One can notice that, here, the order book determines: *i)* which order belongs to the set  $\mathcal{K}_2$  and the quantity to produce, the used routing ( $h = 6$ ) being known, but not the mine providing the inputs to use; *ii)* which order of the subset  $\mathcal{K}_3$  is linked to the previous one and the quantity to deliver but not the routing and the mine providing the inputs to use in addition to the co-product input.

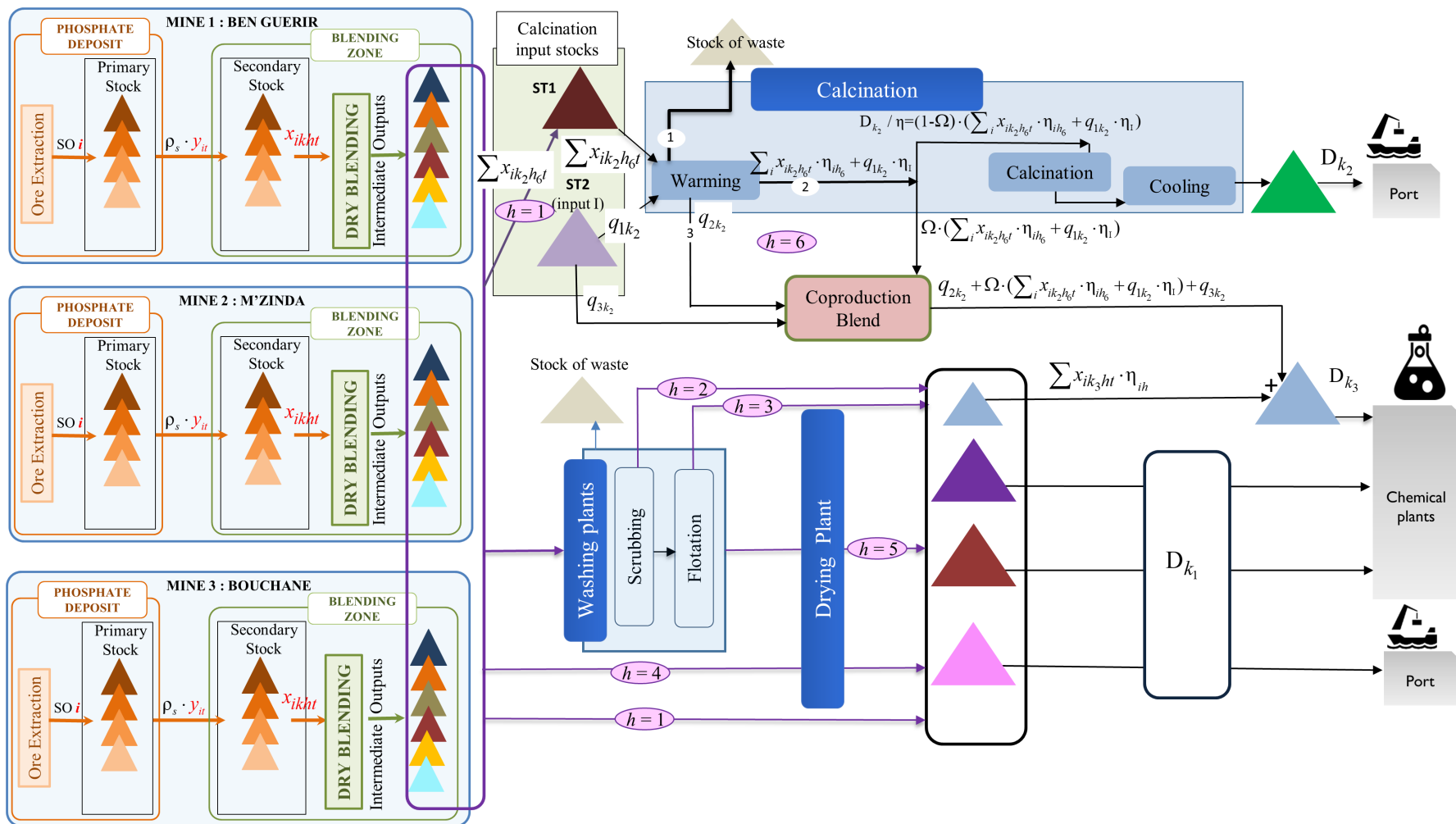


Figure3. OCP Gantour Axis flow diagram

### 3.2 Notations

#### Set of indices

$t$  Index of time periods ( $t = 1, \dots, T$ )

$s$  Index of mining sites ( $s = 1, \dots, S$ )

$i$  Index of inputs ( $i = 1, \dots, I$ )

$i = 1, \dots, I - 1$  refers to inputs existing in a mine (SOs)

$i = I$  refers to input present in the ST2 stock at the entry of the calcination unit

$c$  Index of chemical components ( $c = 1, \dots, C$ )

$j$  Index of MOs ( $j = 1, \dots, J$ )

$h$  Index of alternative routings ( $h = 1, \dots, H$ )

$k$  Index of orders ( $k = 1, \dots, K$ )

Each order  $k$  belongs to one of the 3 subsets  $\{\mathcal{K}_1, \mathcal{K}_2, \mathcal{K}_3\}$

The subset  $\mathcal{F}$  concerns MOs supplying chemical plants, and each order  $k$  belongs to one of the 2 subsets  $\{\mathcal{F}, \mathcal{F}^-\}$

#### General parameters

$\lambda_k$  Index of an MO  $j = \lambda_k$  to be produced for the  $k^{\text{th}}$  order (that relates to a single MO)

$D_k$  Quantity of an MO  $\lambda_k$  to be produced for order  $k$  (tons)

$G_k$  Index  $k' \in \mathcal{K}_2$  from the order linked in co-production to the order  $k \in \mathcal{K}_3$

$M$  High value.

$m$  Minimal percentage of an input weight in a blend obtained by dry blending

#### Technical parameters

$\beta_{c\lambda_k}^{\text{Min}}$  Minimal percentage of the weight of component  $c$  in the total weight of the MO  $\lambda_k$  of order  $k$

$\beta_{c\lambda_k}^{\text{Max}}$  Maximal percentage of the weight of component  $c$  in the total weight of the MO  $\lambda_k$  of order  $k$

$\tau_{cj}$  Target percentage of the weight of component  $c$  in the total weight of MO  $j$

$\alpha_{ci}$  Percentage of component  $c$  in the weight of input  $i$

$\zeta_c$	Percentage of component $c$ in the weight of fine particles recovered during the warming process
$\eta_{ih}$	Reduction weight rate of input $i$ when using routing $h$ ( $\eta_{ih} \leq 1$ )
$\eta$	Reduction weight rate of any MO produced by the calciner
$\eta_I$	Average reduction weight rate of the input I from ST2 stock through warming process
$\gamma_{cih}$	Distortion <sup>4</sup> rate of component $c$ in input $i$ using routing $h$
$\gamma_c$	Distortion rate of component $c$ of any MO produced by the calciner
$\gamma_{ci}$	Distortion rate of component $c$ of the input I from ST2 stock through the warming process
$U_{jh}$	Boolean parameter equal to 1 if alternative routing $h$ technically exists for MO $j$
$\zeta$	Proportion of fines particles recovered during the warming process
$\Psi_1$	Maximum rate of the quantity of ST2 stock to be included in the blend at the inlet of the calcination unit
$\Psi_2$	Maximum rate of the quantity of the ST2 stock to be included in the co-product blend
$\Omega$	Proportion of warming process output (calcination) sent as a co-product

#### Inventory parameters

$A_i$	Site index of input $i$ . An input is related to a single mine site, if two inputs from two different mine sites have the same chemical characteristics, they are nevertheless considered as two different inputs
$S_i^{\text{Max}}$	Maximal stock of input $i$ at the end of any period $t$ in the secondary stock (tons)
$S_s^{\text{Max}}$	Capacity on the global storage in the secondary stock of site $s$ (tons)
$S_{st}^{\text{Min}}$	Total minimum stock present in the secondary stock of the site $s$ at the end of period $t$ (tons)
$S_{i0}$	Initial stock of input $i$ in the blending zone secondary stock (tons)

---

<sup>4</sup> The passage of SOs through one or more treatment units linked to a routing ( $h > 1$ ) induces both a reduction weight of the set of SOs processed in the dry blending plant, and a modification of the composition in chemical components (BPL, CO<sub>2</sub>, MgO, S<sub>2</sub>O<sub>2</sub>, Cd) of each SO of that set: after using routing  $h$ , the initial weight of input  $i$  is decreased by the reduction factor  $\eta_{ci}$  and the initial percentage  $\alpha_{ci}$  of component  $c$  in the weight of input  $i$  must be multiplied by a distortion rate  $\gamma_{cih}$  (which can be greater or lower than 1).

$B_{it}$	Free cumulative availability of input $i$ at the end of period $t$ in the primary stock, regardless of withdrawals of input $i$ , from the initial stock $B_{i0}$ (tons)
$B_{it}^c$	Constrained cumulative availability of input $i$ at the end of period $t$ in the primary stock (tons)
$\rho_s$	Flow rate of the conveyors of site $s$ (tons/day)
$R_{st}$	Number of conveyors available during period $t$ in site $s$
$K_s^{\text{Max}}$	Maximum number of available inputs in the secondary stock at the end of period $T$ in site $s$

### Time parameters

$v_{khs}$	Production time for dry blending of order $k$ in site $s$ with routing $h$ (day)
$u_{khs}$	Production time (physical treatment) of order $k$ produced on site $s$ with routing $h$ ( $h > 1$ ) (day)
$E_k$	Earliest end date of fulfilment of order $k$ (day)
$L_k$	Latest end date of fulfilment of order $k$ (day)

### Cost parameters

$\varphi_k$	Additional costs induced by the alternative routing $h > 1$ , instead of the alternative routing $h = 1$ during the production of the order $k$ (\$/ton)
$\sigma_c$	Penalty cost for an absolute gap (in tons) between the required weight of component $c$ in the quantity of MO $\lambda_k$ to be produced for order $k$ , and the obtained weight obtained by the optimal blend of inputs

### Decision variables

$x_{ikht}$	Quantity of input $i$ (from site $s = A_i$ ) used in the production of order $k$ (that relates to MO $j = \lambda_k$ ) with alternative routing $h$ , whose dry blending ends at $t$ (tons)
$o_{skht}$	Binary equal to 1 if order $k$ is executed in the site $s$ with alternative routing $h$ , the dry blending ending at $t$ . This variable exists only if $U_{\lambda_k h} = 1$
$\omega_{skht}$	Binary equal to 1 if order $k$ is produced in plant $s$ with the alternative routing $h$ , the production in treatment units (post-dry blending) ending at $t$ . This variable exists only if $h > 1$

- $q_{1k}$  Quantity to take from the input present in the ST2 stock and which will feed the warming process (tons)
- $q_{2k}$  Quantity of fine particles; recovered during the warming process and to be included in the co-product blend (tons)
- $q_{3k}$  Quantity to take from ST2 stock and which enters in the co-product blend (tons)
- $y_{it}$  Binary equal to 1 if feeding of input  $i$  in the secondary stock of the blending zone, from the primary stock, occurs during period  $t$ . The existence of the decision variable  $y_{it}$  is subject to free cumulative availability  $B_{it}$

Auxiliary variables

- $\delta_{skht}$  Binary equal to 1 if order  $k$  is produced in site  $s$  using alternative routing  $h$  during period  $t$
- $S_{it}$  Stock of input  $i$  in the secondary stock at the end of period  $t$  (tons)
- $u_i$  Binary equal to 1 if input  $i$  is still present in the secondary stock at the end of the planning horizon  $T$ , that is when  $S_{iT} > 0$
- $\Delta_{c\lambda_k}$  Absolute gap between the required weight of component  $c$  in the quantity of MO  $\lambda_k$  (order  $k$ ) and the weight of this component obtained by the optimal blend of inputs.

Notice that the production of order  $k$  must be ended in the range  $[E_k, L_k]$ , then:

- index  $t$  must belong to that range for  $\omega_{skht}$ , if routing  $h > 1$  is used, and for  $o_{skht}$  if routing  $h = 1$  (dry blending only) is used;
- in the case of routing  $h > 1$ , as the washing or the calcination treatments cannot overlap the dry blending treatment, this latter cannot end after the period  $L_k - v_{khs}$ .

These restrictions may be introduced in the model via constraints, but it can be avoided by using an Algebraic Modelling Language (AML, available in some software like Xpress (used here) or GAMS or AMPL or...; Fourer 2013), which rests on the separation of a generic description of the model, and data to use; after what, an instance of the model combining the generic model and data, which can be submitted to a solver. AML allows the usage of predicates to drive the creation

of an instance of the problem. A predicate is a logical expression that can take a value of "true" or "false" to define the domain of an index, which in turn binds the existence of a decision variable, depending on the parameters' values. Hence, the predicates act like a pre-treatment of the models based on the problem data and reduce the number of binary variables and constraints. Consequently, problems of practical sizes can be solved in a reasonable time using off-the-shelf commercial software. In our case, to avoid unnecessary time-restriction constraints and useless expansion of the time-definition domain, predicates can be used as follows: index  $t$  of  $\omega_{skht}$  is restricted by the predicate  $|h > 1 \wedge (t \geq E_k \wedge t \leq L_k)$ , and index  $t$  of  $o_{skht}$  or  $x_{ikht}$ , by the predicate  $|(h = 1 \wedge (t \geq E_k \wedge t \leq L_k)) \vee (h > 1 \wedge (t \geq v_{khs} \wedge t \leq L_k - v_{khs}))|$ .

To simplify reading the constraints, we represent these predicates by three expressions that must be true:  $P_1 = h > 1 \wedge (t \geq E_k \wedge t \leq L_k)$  used by  $\omega_{skht}$ ,  $P_3 = (t \geq v_{khs}) \wedge (t \leq L_k - v_{khs})$  used by  $o_{skht}$  and  $x_{ikht}$ , in the context where  $h > 1$ , and  $P_2 = (h = 1 \wedge (t \geq E_k \wedge t \leq L_k)) \vee (h > 1 \wedge P_3)$  used by  $o_{skht}$  and  $x_{ikht}$  when routing  $h$  is not specified. Note that other predicates relating to other indexes will be used to limit the existence domain of the decision variables as much as possible. The following two subsections will present the objective function then the 24 constraints. To facilitate the understanding, each relation/constraint is described and explained in one block.

### 3.3 Objective function

The objective function is an economic one. A difference is to make (Bara et al., 2020) between the direct variable costs, which reflect actual charges observed in the income statement, and the penalties used to drive the solution towards what can be considered preferable. The objective-function includes *i*) direct variable costs, whose impact can be observed in the income statement (knowing that these costs are fictitious for confidential reasons) and corresponding to a local efficiency criterion for the management in charge of the transformation of SOs into MOs; *ii*) penalty costs corresponding to an effectiveness criterion for the management in charge of chemical plants. These two points of view are not considered in a multicriteria SC perspective

(Sawik, 2020) but as a way to increase the global SC performance designed in an SC perspective: if one can determine the additional adjustment costs involved in the phosphoric acid plant by deviance more or less important around the target composition of internal MOs, a trade-off can be done between the two SC links to increase the global SC efficiency. That penalty use keeps prima of economic.

$$\text{Min}(\sum_{i,k,t,h|h>1\wedge P_2} \varphi_h \cdot x_{ikht} + \sum_{k|\lambda_k \in \mathcal{F}} \sum_c \sigma_c \cdot \Delta_{ck}) \quad (1)$$

- The first category includes the additional direct cost of using washing or calcination treatments after the dry blending process ( $\sum_{i,k,t,h|h>1\wedge P_2} \varphi_h \cdot x_{ikht}$ ). It incorporates the costs of energy, chemical additives and waste evacuation and, as these routings induce a material reduction, the extraction cost of material lost by using an additional process. When the used routing implies only the dry blending process ( $h = 1$ ), the additional cost is null ( $\varphi_h = 0$ ).
- The second category includes one penalty cost used to favour solutions among those that respect the quality charter. The deviation cost  $\sigma_c$  is used to smooth the composition between successive batches of the same QC ordered by the internal client; a composition target is used to obtain that smoothing ( $\sum_{k|\lambda_k \in \mathcal{F}} \sum_c \sigma_c \cdot \Delta_{ck}$ ). This cost must be considered as a parameter used to analyse the possible tradeoff between a variance decrease (solution effectiveness for chemical plants) against an increase of observable expenses (solution efficiency for the ore transformation plants), as illustrated in the case study.

### 3.4 Description of constraints

Six sets of constraints must be considered.

### 3.4.1 Production in the dry blending plant

The production of an MO order starts with a dry blend process in a unique site  $s$  and uses a unique routing  $h$ , which must exist for MO  $j$  ( $\lambda_k = j$ ), and must end in the dry blending plant at the date  $t$  restrained by the predicate  $P_2$ .

$$\sum_{s,h,t|U_{\lambda_k h}=1 \wedge P_2} o_{skht} = 1, \forall k \quad (2)$$

Then, one must enforce  $x_{ikht}$  to share with  $o_{skht}$  the same dry-blending end date, the same site and the same routing. It is ensured by constraint (2) that enforces  $x_{ikht}$  to be null if  $o_{skht}$  is null; in that relation,  $m$  is a lower limit defined by technical constraints (minimum size of a lot to be taken from stock) and an upper limit  $M$  (maximum SO amount that can be used to produce an MO order (which is  $D_k / \eta_{ih}$ )<sup>5</sup>).

$$\begin{cases} \sum_{i|A_i=s} x_{ikht} \leq M \cdot o_{skht} \\ \sum_{i|A_i=s} x_{ikht} \geq m \cdot o_{skht} \end{cases} \quad \forall s,k,h,t|U_{\lambda_k h}=1 \wedge P_2 \quad (3)$$

### 3.4.2 Demand satisfaction in volume

Constraints (4) to (6) enforce the balance of order  $k$ , between the produced quantity  $D_k$  and the total amount of inputs used to deliver that quantity, considering the order' subset type.

Constraint (4) refers to the general case, where the order  $k \in \mathcal{K}_1$  is not produced by calcination.

$$\sum_{i,h,t|A_i=s \wedge U_{\lambda_k h}=1 \wedge h < 6 \wedge P_2} x_{ikht} \cdot \eta_{ih} = D_k \cdot \sum_{h,t|U_{\lambda_k h}=1 \wedge h < 6 \wedge P_2} o_{skht}, \forall s,k|k \in \mathcal{K}_1 \quad (4)$$

<sup>5</sup>  $M$  can be replaced by  $2 \cdot D_k$ , as the used routing  $h$  is not already known and because 0.5 is slightly above the lowest possible value  $\eta_{ih}$ )

Constraint (5) concerns MO orders of subset  $\mathcal{K}_2$  mobilising routing  $h=6$  (exported MOs produced by calcination). As shown in figure 3, a stable proportion  $\Omega$  of the warming process quantity (flow 2) feeds the co-product blend in the calcination process. Thus, the quantity entering in calcination is equal to  $\left( (1-\Omega) \cdot \sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_3} x_{ikht} \cdot \eta_{ih} + q_{1k} \cdot \eta_I \right)$  and decreased due to the calcination yield rate  $\eta$ .

$$\left( (1-\Omega) \cdot \eta \cdot \sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_3} x_{ikht} \cdot \eta_{ih} + q_{1k} \cdot \eta_I \right) = D_k \cdot \sum_{h,t|h=6 \wedge U_{\lambda_k h}=1 \wedge P_1} \omega_{skht}, \forall s, k | k \in \mathcal{K}_2 \quad (5)$$

Constraint (6) determines, for  $k \in \mathcal{K}_3$ , how the ordered quantity  $D_k$  is produced by combining the SOs blend  $\sum_{i,h,t|A_i=s \wedge U_{\lambda_k h}=1 \wedge (h>1 \wedge h<6) \wedge P_3} x_{ikht} \cdot \eta_{ih}$  produced for  $k \in \mathcal{K}_3$ , with the co-production induced by  $k' \in \mathcal{K}_2$ , linked to order  $k$  such as  $G_k = k'$ . This latter combines the fraction  $\Omega$  of production of the warming process  $\sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_{G_k} h}=1 \wedge P_3} x_{iG_k ht} \cdot \eta_{ih} + q_{1G_k} \cdot \eta_I$  with the outlet flow (fines) of the warming process ( $q_{2G_k}$ , flow 3 in Figure 3) and additional input from the stock ST2 ( $q_{3G_k}$ ).

$$\sum_{i,s,h,t|h<6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_2} x_{ikht} \cdot \eta_{ih} + \Omega \cdot \left( \sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_{G_k} h}=1 \wedge P_3} x_{iG_k ht} \cdot \eta_{ih} + q_{1G_k} \cdot \eta_I \right) + q_{2G_k} + q_{3G_k} = D_k \cdot \sum_{h,t|(h>1 \wedge h<6) \wedge U_{\lambda_k h}=1 \wedge P_1} \omega_{skht}, \forall s, k | k \in \mathcal{K}_3 \quad (6)$$

Some constraints must be considered in the determination of  $q_{1k}$ ,  $q_{2k}$  and  $q_{3k}$  (see Figure 3). The relative importance of the quantity  $q_{2k}$  of fine in the inflows entering the warming plant is constrained by Constraint (7). For technical reasons, the quantity  $q_{1k}$  of input  $i = I$  (from the ST2 stock) is limited in the warming plant's inflows (Constraint 8), and the quantity  $q_{3k}$  of input  $i = I$  is limited in the co-production blend (Constraint 9).

$$q_{1k} \leq \Psi_1 \cdot \sum_{i,s,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_3} x_{ikht}, \forall k | k \in \mathcal{K}_2 \quad (7)$$

$$q_{2k} \leq \zeta \cdot \left( \sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_3} x_{ikht} + q_{1k} \right), \forall k | k \in \mathcal{K}_2 \quad (8)$$

$$q_{3k} \leq \Psi_2 \cdot \left( q_{2k} + \Omega \cdot \left( \sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_3} x_{ikht} + q_{1k} \right) \right), \forall k | k \in \mathcal{K}_2 \quad (9)$$

### 3.4.3 Demand satisfaction in quality

Constraints (10) relate the compliance with the client quality charter requirements in the three subsets  $\mathcal{K}_1$ ,  $\mathcal{K}_2$  and  $\mathcal{K}_3$  respectively. Field studies show that  $\beta_{cj}^{\text{Min}}$  and  $\beta_{cj}^{\text{Max}}$ , the bounds of the range of weight percentage of component  $c$  for MO  $j$ , are independent of alternative routing used  $h$ , and of the site  $s$  where the order is blended. The centre part of each above relation corresponds to the weight of component  $c$  in the blend constituting the order  $k$ , considering the left part of Constraints (10). The minimum (resp. maximum) weight of each component is determined by  $\beta_{cj}^{\text{Min}} \cdot D_k$  (resp.  $\beta_{cj}^{\text{Max}} \cdot D_k$ ).

$$\left\{ \begin{array}{l} \beta_{c\lambda_k}^{\text{Min}} \cdot D_k \leq \sum_{i,h,t|A_i=s \wedge U_{\lambda_k h}=1 \wedge h < 6 \wedge P_2} \alpha_{ci} \cdot \eta_{ih} \cdot \gamma_{cjh} \cdot x_{i\lambda_k ht} \leq \beta_{c\lambda_k}^{\text{Max}} \cdot D_k, \forall c, k | k \in \mathcal{K}_1 \\ \beta_{c\lambda_k}^{\text{Min}} \cdot D_k \leq (1 - \Omega) \cdot \left( \sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_3} \alpha_{ci} \cdot \eta_{ih} \cdot \gamma_{cjh} \cdot x_{ikht} + \alpha_{cl} \cdot \eta_l \cdot \gamma_{cl} \cdot q_{1k} \right) \cdot \eta \leq \\ \beta_{c\lambda_k}^{\text{Max}} \cdot D_k, \forall c, \forall k | k \in \mathcal{K}_2 \\ \beta_{c\lambda_k}^{\text{Min}} \cdot D_k \leq \sum_{i,s,h,t|h < 6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_2} x_{ikht} \cdot \eta_{ih} \cdot \alpha_{ci} + \\ \Omega \cdot \left( \sum_{i,h,t|h=6 \wedge A_i=s \wedge U_{\lambda_k h}=1 \wedge P_3} \alpha_{ci} \cdot \eta_{ih} \cdot \gamma_{cjh} \cdot x_{iG_k ht} + \alpha_{cl} \cdot \eta_l \cdot \gamma_{cl} \cdot q_{1G_k} \right) + \zeta_c \cdot q_{2G_k} + \alpha_{cl} \cdot q_{3G_k} \leq \\ \beta_{c\lambda_k}^{\text{Max}} \cdot D_k, \forall c, \forall k | k \in \mathcal{K}_3 \end{array} \right. \quad (10)$$

### 3.4.4 Upstream flow from mine to SO stock in the blending area

As stated in features  $a$  and  $b$ , SO would be available at a given time on a secondary stock depending on the extraction planning of the mine  $s$ . The production of any order passes first

through the dry-blending plants, where the order's estimated production time in the blending plants is  $v_{khs}$ . The completion date in blending units of this order is  $t_k = \sum_{s,h,t} \mathbb{1}_{U_{\lambda_k h}=1} t \cdot O_{skht}$ ,

then, the dry blending process of order  $k$  occurs in the period's range  $[t_k - v_{khs} + 1; t_k]$ .

Equation (11) defines the execution period of the dry blend of order  $k$ . Recall that  $\delta_{skht}$  is an auxiliary binary variable equal to 1 if order  $k$  is produced in site  $s$  using alternative routing  $h$  during the period  $t$ . Indeed, if  $O_{skht'} = 1$ , then  $\delta_{skht} = 1$  for  $t' \leq t \leq t' + v_{khs} - 1$ , where  $v_{khs}$  is the parameter corresponding to known duration of the blend of order  $k$  with routing  $h$  on site  $s$ .

$$\delta_{skht} = \sum_{t'} \mathbb{1}_{U_{\lambda_k h}=1 \wedge (t' \leq t \wedge t' \geq t - v_{khs} + 1) \wedge P_2} O_{skht'}, \forall s, k, h, t \quad (11)$$

As  $\sum_{k,h} \delta_{skht} = \sum_{k,h} \sum_{t'} \mathbb{1}_{U_{\lambda_k h}=1 \wedge (t' \geq t \wedge t' \leq t + v_{khs} - 1) \wedge P_2} O_{skht'}$ ,  $\forall t, s$ , the disjunctive constraint (12)

prevents a dry blending plant of mine  $s$  to treat more than one order during a given period  $t$ .

$$\sum_{k,h} \sum_{t'} \mathbb{1}_{U_{\lambda_k h}=1 \wedge (t' \geq t \wedge t' \leq t + v_{khs} - 1) \wedge P_2} O_{skht'} \leq 1, \forall t, s \quad (12)$$

The following relations focus on the stock level behaviour. Constraint (13) guarantees that the variable  $S_{it}$  is positive. The flow conservation in the secondary stock is expressed by Equation (14). Due to constraints on the feeding of secondary stock, a given input  $i$  cannot be transported from the mine during period  $t$  by more than one of the  $R_{st}$  conveyors (all the conveyors leaving mine  $s$  having the same transfer rate  $\rho_s$ ). During each dry blending period of this order, the quantity  $x_{i\lambda_k ht} / v_{khs}$  is consumed, as a linear consumption of inputs is assumed during the blending process. Using equation (11), the consumption of input  $i$  in period  $t$  for order  $k$  using routing  $h$  is  $x_{i\lambda_k ht} \cdot \delta_{skht} / v_{khs}$ . Then, the stock  $S_{it}$  depends on its previous level  $S_{i,t-1}$ , increased by supplies  $\rho_s \cdot y_{it}$  ( $y_{it}$  is a binary decision variable expressing a transfer from primary to secondary stock)

and decreased by consuming this input for the blend of order  $k$ . Remark that  $S_{it}$  is non-linear<sup>6</sup> due to the previous definition of  $\delta_{skht}$ .

$$S_{it} \geq 0, \forall i, s | A_i = s, \forall t \quad (13)$$

$$S_{i,t} = S_{i,t-1} + \rho_s \cdot y_{it} - \sum_{k,h | U_{\lambda_k h} = 1 \wedge P_2} x_{ikh t'} \cdot \delta_{skht} / v_{khs}, \forall i, s | A_i = s, \forall t \quad (14)$$

### 3.4.5 Downstream flow from the dry blending plant

Figure 4 illustrates the interdependences between time and routing in the model

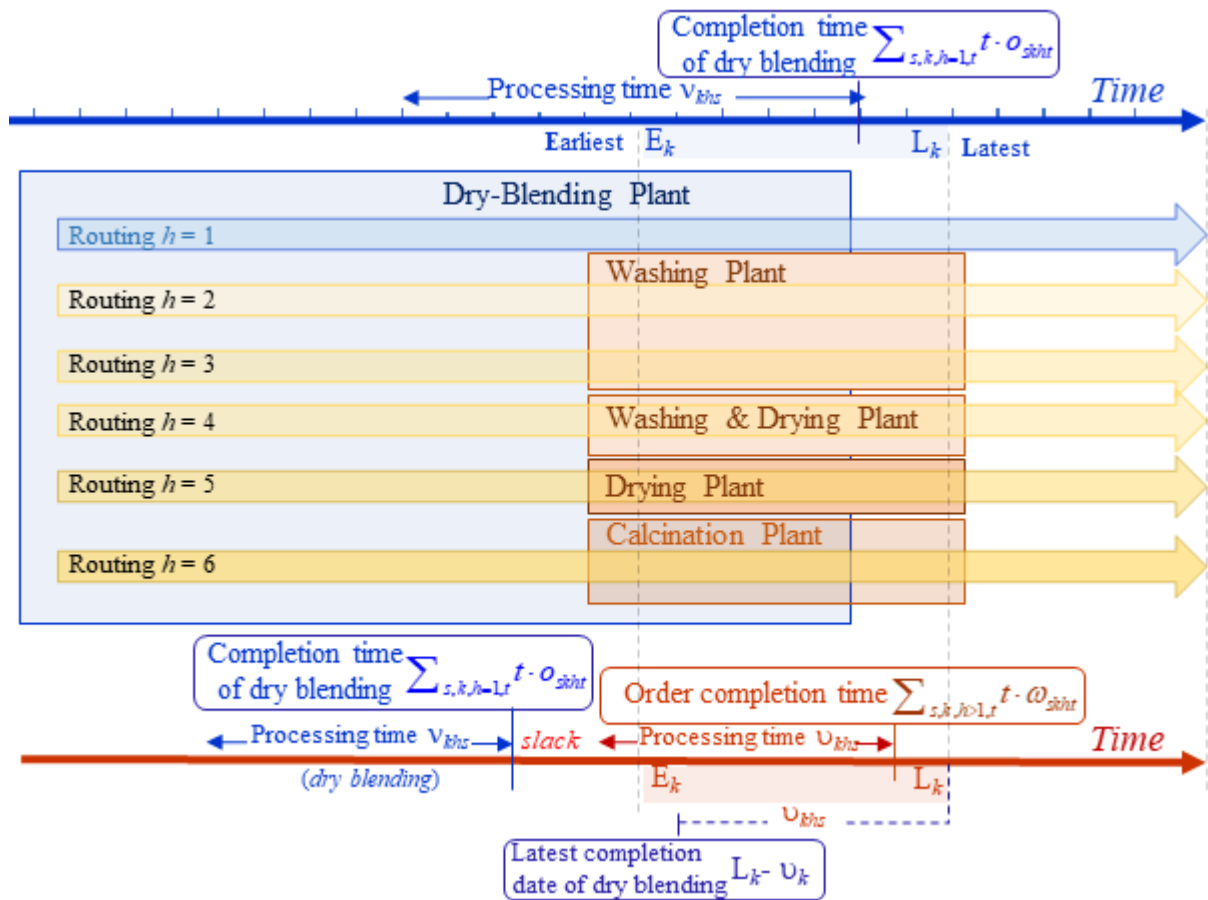


Figure 4. Illustration of the interdependences between time and routing in the model

<sup>6</sup> The product of a binary and a real is linearised using the following property (see Sioshansi & Conejo, 2017, p.149). Consider the real variable  $x$  ( $0 < x < X_{\max}$ ) with  $X_{\max}$  a preset real parameter, and  $y$  a binary variable. The non-linear equation  $z = x \cdot y$  is then equivalent to the following four linear equations:  $z \geq 0$ ,  $z \leq y \cdot X_{\max}$ ,  $z \leq x$  et  $z \geq x - (1 - y) \cdot X_{\max}$ .

An order may be delivered if all the treatments required by its chosen routing are ended. Constraint (15) enforces all the MO orders whose routing is  $h > 1$  to be delivered and share the same routing and site as the dry blending ones.

$$\begin{cases} \sum_{h,t|U_{\lambda_k h}=1 \wedge h > 1 \wedge P_1} h \cdot \omega_{skht} = \sum_{h,t|U_{\lambda_k h}=1 \wedge h > 1 \wedge P_3} h \cdot o_{skht}, \forall k, s \\ \sum_{s,t|U_{\lambda_k h}=1 \wedge h > 1 \wedge P_1} s \cdot \omega_{skht} = \sum_{s,t|U_{\lambda_k h}=1 \wedge h > 1 \wedge P_3} s \cdot o_{skht}, \forall k, h \end{cases} \quad (15)$$

The completion time of the dry blending treatment is  $\sum_{s,t,h|U_{\lambda_k h}=1 \wedge P_2} t \cdot o_{skht}$  and that of washing or calcination treatment  $\sum_{s,t,h|U_{\lambda_k h}=1 \wedge P_1} t \cdot \omega_{skht}$ . Then, constraint (16) prevents the washing (or the calcination) process from starting before the end of the dry blending process, knowing that  $h > 1$ . As the transportation time between the dry blending plants and the washing and calcination plants is low (only a few hours) compared to the plant's production times (several days), the model adopts the assumption of neglecting the transportation times.

$$\sum_{s,t,h|U_{\lambda_k h}=1 \wedge P_1} (t - v_{khs} + 1) \cdot \omega_{skht} > \sum_{s,t,h|U_{\lambda_k h}=1 \wedge P_2} t \cdot o_{skht} \quad \forall k \quad (16)$$

A disjunctive constraint, similar to constraint (12) must be introduced to prevent a washing plant or the calcination plant to treat more than one order during a given period.

$$\sum_{k,h} \sum_{t|U_{\lambda_k h}=1 \wedge P_1} \omega_{skht} \leq 1, \forall t, s \quad (17)$$

Constraint (18) models feature  $j$ . It ensures that the order  $k \in \mathcal{K}_2$  using routing  $h = 6$  (*dry blending + calcination*) and the order  $k' \in \mathcal{K}_3$ , to which the generated co-product is linked ( $G_{k'} = k$ ), has the same order completion date.

$$\sum_{s,h,t|h=6 \wedge U_{\lambda_k h}=1 \wedge P_1} t \cdot \omega_{skht} = \sum_{s,h,t|h < 6 \wedge U_{\lambda_k h}=1 \wedge G_{k'}=k \wedge P_3} t \cdot \omega_{sk'ht}, \forall k | k \in \mathcal{K}_2 \quad (18)$$

### 3.4.6 General feeding constraints

The following constraints are almost identical to those presented in Azzamouri et al. (2021). They have been adapted to include the mining site  $s$ .

Decisions taken upstream of the extraction chain define possible input supplies and withdrawals from the primary stocks (end of the extraction process) to the secondary stocks (feeding the blending plant). As this information is available along with daily output extractions, we can define the free cumulative availability  $B_{it}$  of input  $i$  at the end of period  $t$  in the primary stock, regardless of withdrawals of input  $i$ , from the initial stock  $B_{i0}$ . The existence of the decision variable  $y_{it}$  is subject to free cumulative availability  $B_{it}$  ( $B_{it} = 0 \rightarrow y_{it} = 0$ ). The input withdrawal process from primary stocks to secondary stocks is constrained by the number  $R_{st}$  of conveyors available during period  $t$  in the mining site  $s$  (Constraint 19) and by a sufficient available build-up (Constraint 20).

$$\sum_{i|A_i=s} y_{it} \leq R_{st}, \forall s, t \quad (19)$$

$$B_{it} \geq \rho_s \sum_{t' \leq t} y_{it'}, \forall i, s | A_i = s, \forall t \quad (20)$$

One must consider that, as long as extracted ore is not removed, it is not possible to access lower layers where extracted ore is lying. Thus, the tactical extraction program involves that, at a given time  $t$ , part of available input  $i$  must be removed. This feature can be expressed in terms of cumulative availability  $B_{it}^c$ , which restrains the withdrawal volume of input  $i$  through constraint (21).

$$B_{it} - \rho_s \cdot \sum_{t' \leq t} y_{it'} \leq B_{it}^c, \forall i, s | A_i = s, \forall t \quad (21)$$

Constraint (22) is a constraint on overall storage to keep it below  $S_s^{\text{Max}}$ , and constraint (23) is a constraint on overall storage to keep it above  $S_{st}^{\text{Min}}$  for each period  $t$

$$\sum_{i|A_i=s} S_{it} < S_s^{\text{Max}}, \forall t, s \quad (22)$$

$$\sum_{i|A_i=s} S_{it} \geq S_{st}^{\text{Min}}, \forall t, s \quad (23)$$

The set of inputs changes over time due to possible modifications of component composition from one parcel of the deposit to another. It potentially increases the variety of inputs to be stored in the secondary stock and may prevent appropriate segregation of input stocks in the blending area. To avoid such a drift, it is necessary to limit the number of different inputs stored in the secondary stock in each mine site  $s$  to  $K_s^{\text{Max}}$  at the end of the planning horizon  $T$ . This leads us to introduce constraint (24), where  $M$  is a large number, enforces the limitation of different inputs stored at the end of period  $T$ .

$$\begin{cases} S_{iT} < M \cdot u_i, \forall i \\ \sum_{i|A_i=s} u_i \leq K_s^{\text{Max}}, \forall s \end{cases} \quad (24)$$

Relations (25) enables to determine, for order  $k$  such as  $k \in \mathcal{F}$  supplying the chemical plants on the Safi site, the absolute gap between order weight of component  $c$  of MO  $\lambda_k$ , and its target composition ( $\tau_{c\lambda_k} \cdot D_k$ ).

$$\begin{cases} \Delta_{c\lambda_k} \geq \sum_{i,h,t|U_{\lambda_k h}=1 \wedge A_i=s \wedge P_2} \alpha_{ci} \cdot \eta_{ih} \cdot \gamma_{cjh} \cdot x_{i\lambda_k ht} - \tau_{c\lambda_k} \cdot D_k \\ \Delta_{c\lambda_k} \geq \tau_{c\lambda_k} \cdot D_k - \sum_{i,h,t|U_{\lambda_k h}=1 \wedge A_i=s \wedge P_2} \alpha_{ci} \cdot \eta_{ih} \cdot \gamma_{cjh} \cdot x_{i\lambda_k ht} \end{cases} \quad \forall c, k | k \in \mathcal{F} \quad (25)$$

#### 4 Case study

We tested our model on a sample inspired by actual data of 7 MO orders over 50 periods (the temporal division used here is the day). We first present all the input data in §4.1, the different results are explained in §4.2, and the last part, §4.3, discusses an economic evaluation to illustrate our model's efficiency.



Table 2. Reduction and distortion rates

Routing $h$							
	$h=1$	$h=2$	$h=3$	$h=4$	$h=5$	$h=6$	
<b>Reduction rate <math>\eta_h</math> using routing <math>h</math></b>							
	1	0.73	0.59	0.89	0.5	0.65	
<b>Distortion rate <math>\gamma_{ch}</math> of component <math>c</math> using routing <math>h</math></b>							
Component $c$	$c=1$	1	1.059	1.21	1	1.21	1.055
	$c=2$	1	0.933	0.76	1	0.76	0.87
	$c=3$	1	0.77	0.44	1	0.44	0.75
	$c=4$	1	0.57	0.55	1	0.55	0.73
	$c=5$	1	0.7	0.65	1	0.65	0.85

Table 3. Quality charter

Constraints on the share $\beta_{cj}$ (%) du component $c$ in the weight of output $j$						
MO $j$						
		$j=1$ (Tess)	$j=2$ (Stand)	$j=3$ (Ycci)	$j=4$ (Ycc2)	$j=5$ (G10)
Client		Internal	Internal	External	External	Internal
Component $c$	$c=1$	$65.12 < \beta_{11} < 66.80$	$65.12 < \beta_{12} < 66.80$	$73 < \beta_{13} < 74.5$	$72 < \beta_{14} < 73$	$65.5 < \beta_{15} < 66.80$
	$c=2$	$5.00 < \beta_{21} < 6.50$	$5.00 < \beta_{22} < 6.50$	$2.2 < \beta_{23} < 3$	$2.2 < \beta_{24} < 3$	$6.00 < \beta_{25} < 6.50$
	$c=3$	$5.5 < \beta_{31} < 8.5$	$5.5 < \beta_{32} < 8.5$	$\beta_{33} < 3$	$\beta_{34} < 3$	$\beta_{35} < 6.5$
	$c=4$	$\beta_{41} < 0.65$	$\beta_{42} < 0.65$	$\beta_{43} < 0.75$	$\beta_{44} < 0.75$	$\beta_{45} < 0.75$
	$c=5$	$\beta_{51} < 6.50$	$\beta_{52} < 8.00$	$\beta_{53} < 18$	$\beta_{54} < 18$	$\beta_{55} < 18$

We study an order book of 7 MOs orders for 50 days ( $T = 50$ ) which includes post blending treatment times (Figure 5). The choice of this illustration is based on the need to compare our solution with the one currently used in the company. In this problem, constraints on cumulative availability  $B_{it}^c$  and on minimal stocks are discarded. Table 4 describes the production program.

Table 4. Problem data: Order specification

Order $k$	1	2	3	4	5	6	7
MO $j = \lambda_k$	5	2	3	2	4	1	5
Demand $D_k$	75,000	30,000	20,000	30,000	17,000	30,000	70,000
Earliest delivery date $E_k$	6	5	9	10	20	22	24
Latest delivery date $L_k$	34	16	34	42	49	36	49
Index of order linked to $k$ $G_k$	3	0	1	0	7	0	5

The costs used in the objective function are the following ones:  $\varphi_{h=1}=0$ ;  $\varphi_{h=2}=16$ ;  $\varphi_{h=3}=25$  ,

$\varphi_{h=4}=21$ ,  $\varphi_{h=5}=46$ ;  $\varphi_{h=6}=61$  (note that costs are additional costs involved by routing  $h > 1$  when routing  $h = 1$  is not used). For the deviance cost, a central scenario with an identical penalty for all the components  $\sigma_c = 100, \forall c$  is considered.

## 4.2 Results

The Xpress solver produced the results below, using a PC (Intel® Core™ i7-7600U CPU @ 2.50 GHz - 8 Gb RAM). The computing time is less than one minute (52 seconds on average). The problem parameters are listed in Table 5. The use of predicates (presolved) reduces the size of the initial problem (matrix) considerably. The predicates on variables have allowed a reduction of 48% of the search domain of variables, and the predicates on constraints, a reduction of 43%, but without the use of predicates on variables, additional constraints would be necessary to restrain the variables domains to the pertinent set so the reduction gain of constraints is much more important.

Table 5. Overview of the problem size

	Rows (constraints)	Columns (variables)	Nonzero elements	Global entities
Matrix	37,964	34,217	602,139	7,816
Presolved	21,664	17,850	267,645	3,675

The optimal resolution of this dynamic problem leads to obtain for each order: *i*) the mining site where it will be executed; *ii*) the alternative routing that will follow, *iii*) the start and end date for the execution of the order in dry blending units; *iv*) the sequencing of MO orders at the process unit level (scrubbing, scrubbing + flotation and calcination) with the date of completion of each order; *v*) the optimal blend of SOs; *vi*) the feeding program from primary to secondary stock for each mining site (Table 6).

Table 6. Optimal values for the execution of MO orders in dry blending and treatment plants

$O_{skht}$						$\Omega_{skht}$
Order $k$	MO $j=\lambda_k$	Mining site $s$	Selected operations sequence $h$	$\delta_{skht}$		Production end date $t$ of order $k$ (in treatment plants)
				Production start date (in dry blending plants)	Production end date (in dry blending plants)	
1	5	Mine 2	3	21	25	34
2	2	Mine 1	2	1	6	16
3	3	Mine 3	6	20	27	34
4	2	Mine 1	2	28	33	41
5	4	Mine 3	6	29	34	42
6	1	Mine 1	2	22	27	35
7	5	Mine 2	3	7	12	42

Order  $k=3$  (resp. 5) belonging to the subset  $\mathcal{K}_2$  and order  $k=1$ , (resp. 7) belonging to the subset  $\mathcal{K}_3$ , are linked orders sharing the same end date.

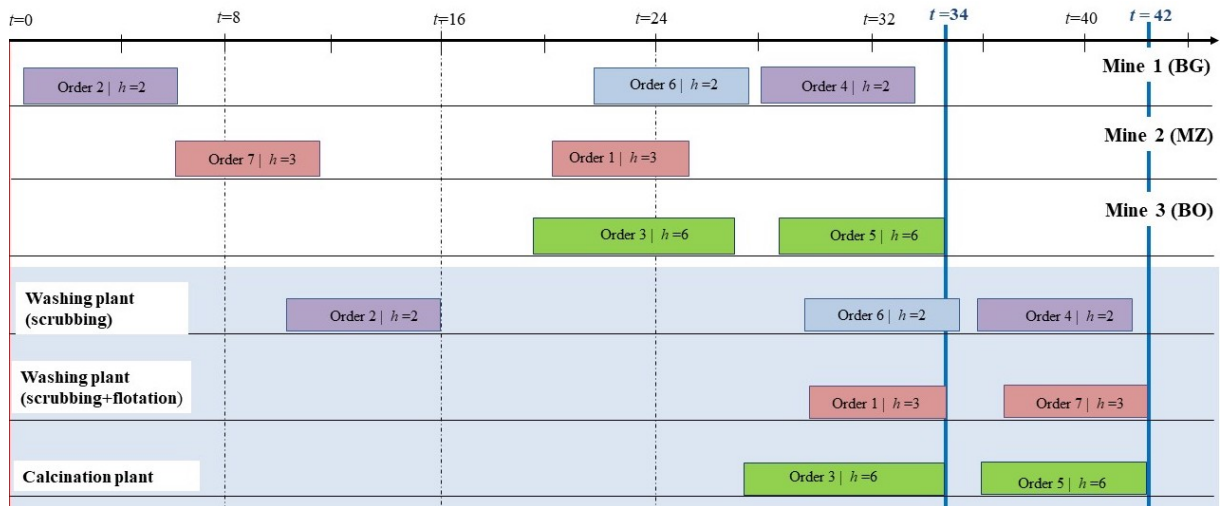


Figure 5. Scheduling of MO orders at the mining sites and at the treatment plants

Figure 6 presents the optimal values of the decision variables  $q_{1k}, q_{2k}, q_{3k}$ , relating to MO orders using routing  $h=6$  (calcination unit) and the total coproduced quantity. Figure 6 also explains the link between the orders of the two subsets  $\mathcal{K}_2$  and  $\mathcal{K}_3$ . For instance, the total demand for order  $k=1$  linked to the order  $k=3$  (using the calcination unit) according to Table 4 is 75,000 tons, of which 61,559 tons was coproduced in the calcination unit considering the results presented in

Figure 6, which means that the complement to be produced to satisfy this order is 13,441 tons.

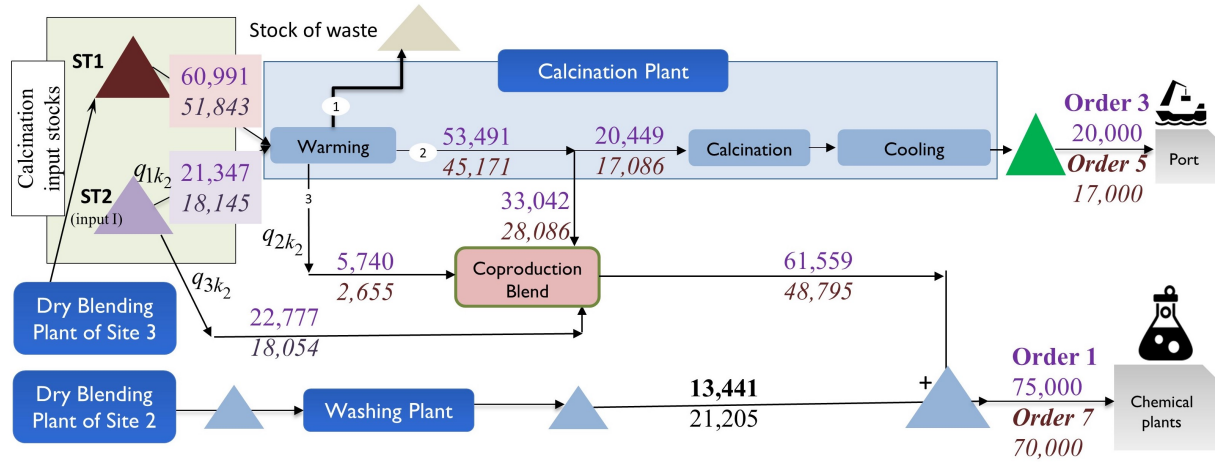


Figure 6. Numerical illustration of the linked orders belonging to subsets  $\mathcal{K}_2$  and  $\mathcal{K}_3$

The model also determines the optimal blends of inputs to be operated for each order (Table 7) and the feeds of inputs from primary stock to secondary stock for each mining site<sup>7</sup>.

Table 7. Optimal blends

$x_{ikht}$	Input (SO) $i$																$\sum_i x_{ikht}$	
	$i=5$	$i=6$	$i=7$	$i=8$	$i=9$	$i=12$	$i=14$	$i=18$	$i=19$	$i=20$	$i=21$	$i=23$	$i=24$	$i=25$	$i=26$	$i=27$		$i=28$
$k=1 \rightarrow j=\lambda_1=5$	0	0	0	0	0	0	0	12070	4361	550	5800	0	0	0	0	0	0	22781
$k=2 \rightarrow j=\lambda_2=2$	5620	9249	0	15823	0	6956	3448	0	0	0	0	0	0	0	0	0	0	41096
$k=3 \rightarrow j=\lambda_3=3$	0	0	0	0	0	0	0	0	0	0	0	0	7429	20704	23259	5100	4500	60991
$k=4 \rightarrow j=\lambda_3=2$	0	0	10264	0	24095	0	6738	0	0	0	0	0	0	0	0	0	0	41096
$k=5 \rightarrow j=\lambda_3=4$	0	0	0	0	0	0	0	0	0	0	0	12070	0	18332	21441	0	0	51843
$k=6 \rightarrow j=\lambda_3=2$	0	10180	0	3937	23717	0	3263	0	0	0	0	0	0	0	0	0	0	41096
$k=7 \rightarrow j=\lambda_3=5$	0	0	0	0	0	0	0	22919	8871	4150	0	0	0	0	0	0	0	35940
$\Sigma$	5620	19429	10264	19760	47811	6956	13449	34989	13232	4700	5800	12070	7429	39035	44700	5100	4500	294843
$x_{ikht} / \sum_i x_{ikht} (\%)$																		
$k=1 \rightarrow j=\lambda_1=5$	0%	0%	0%	0%	0%	0%	0%	53%	19%	2%	25%	0%	0%	0%	0%	0%	0%	100%
$k=2 \rightarrow j=\lambda_2=2$	14%	23%	0%	39%	0%	17%	8%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%
$k=3 \rightarrow j=\lambda_3=3$	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	12%	34%	38%	8%	7%	100%
$k=4 \rightarrow j=\lambda_3=2$	0%	0%	25%	0%	59%	0%	16%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%
$k=5 \rightarrow j=\lambda_3=4$	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	23%	0%	35%	41%	0%	0%	100%
$k=6 \rightarrow j=\lambda_3=2$	0%	25%	0%	10%	58%	0%	8%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	100%
$k=7 \rightarrow j=\lambda_3=5$	0%	0%	0%	0%	0%	0%	0%	64%	25%	12%	0%	0%	0%	0%	0%	0%	0%	100%

### 4.3 Discussion

After presenting a benchmark of the optimal results, compared with the solution currently used in the field, the managerial interest of using the deviance cost in a SC perspective will be

<sup>7</sup> (<https://data.mendeley.com/datasets/mz2f7gxb9w/draft?a=683bb4cc-48bf-4b62-bd19-f1b7152644f3>)

explained.

#### 4.3.1 Benchmark

In the current configuration used in the field: *i*) the order for an MO is assigned to a mine and produced with a predefined routing; and *ii*) the scheduling of orders on the blending and treatment units is also predetermined. These predeterminations are made without taking into account either what is available on the phosphate deposit in real-time at the three mining sites or the variable aspect of the chemical composition of the SOs. This method leads to multiple corrections to meet the client's requirements in terms of quality and delivery time. A benchmark was carried out between the solution proposed by the model and the configuration currently used in the field to test the model's performance.

Concerning the orders assignments, the solution proposed by our model differs from the configuration used in the field on 2 points: *i*) orders  $k=3$  and  $k=5$  have been assigned to mine 1, *ii*) order  $k=6$  has been produced using the routing  $h=3$ . Also, for MO orders of subsets  $\mathcal{K}_2$  and  $\mathcal{K}_3$  (i.e.,  $k=1, k=3, k=5, k=7$ ), the total SOs used to meet all these orders is 230,609 T in the current configuration against 171,555 T of SOs used in our model, saving 25% of raw material. This economy is mainly due to the integrated global optimisation proposed in our model, allowing a better choice of the flows at the calcination unit and, thus, to coproduce more. Of the entire case study orders, more than 65,000T of SOs were used in addition to the existing configuration, compared to the solution proposed by our model. This saving in the amount of SOs used by our model also results in savings in treatment units of approximately 65,000 \$, which attracts the interest of OCP managers

#### 4.3.2 The trade-off between efficiency and effectiveness

The model makes it possible to provide negotiation elements for the two parts of the SC studied:

the client, which is the chemical complex (where deviance can cause additional production costs), and the supplier, which is the mining sites (§ 3.4). In our numerical example based on actual data, for a penalty set to  $\sigma_c = 100$  for all chemical components, the deviation of the MOs destined to produce phosphoric acid is null.

Now, we considered the same data set and added two new orders (internal) for MOs intended for the production of phosphoric acid. The addition of two MO orders is justified by the fact that in the optimal solution provided by our model, the deviance is null for the actual case presented. The purpose of this adjunction is to have a solution respecting the quality charter but with non-zero deviance to study the impact of the variation of the deviance penalty on the optimal solution.

As explained in section §3.4, the objective function aggregates a non-observable cost related to the stability of the composition of the MOs feeding the chemical plants and an observable cost related to the direct costs incurred by using one of the six alternative routings. We will be varying the stability cost (penalty  $\sigma_c$ ), which is identical for all the chemical components, to take the values  $\{0, 0.5, 1, 1.5, 10, 100, 1000\}$ . The different results are summarised in Table 7.

Table 7. Costs of the objective function as a function of the unit cost of deviance

Penalty $\sigma$	0	0.5	1	1.5	10	100	1000	<p style="text-align: center;"><b>Effectiveness/Efficiency</b></p>
Objective function ( $10^3$ )	111	114	116	117	120	146	410	
Deviation (tonne)	5318	5151	3335	2223	314	293	293	
<b>Deviation (%)</b>	<b>1.21%</b>	<b>1.16%</b>	<b>0.73%</b>	<b>0.45%</b>	<b>0.062%</b>	<b>0.058%</b>	<b>0.058%</b>	
Total SOs used ( $10^3$ tonne)	441	443	460	495	507	508	508	
<b>Total production cost (<math>10^3</math>)</b>	<b>111.4</b>	<b>111.5</b>	<b>113</b>	<b>114</b>	<b>116.8</b>	<b>117</b>	<b>117</b>	

The introduction of a deviation penalty  $\sigma \geq 100$  generates a decrease of up to 94% of the deviation in tonnes from the target values compared to a zero cost of stability ( $\sigma = 0$ ). However, this reduction in deviance is only possible by increasing the total amount of SOs used by up to 15% (resp. 10%) compared to  $\sigma = 0$  for  $\sigma \geq 100$  (resp.  $\sigma = 1$ ). This is equal to an increase in the

real cost of production of up to 5% (resp. 4%) compared to  $\sigma = 0$  for  $\sigma \geq 100$  (resp.  $\sigma = 1$ ). This illustration thus provides the basis for negotiation between the two adjacent links of that SC. The loss of efficiency (increase in actual cost) upstream is only acceptable at a higher level of supervision of the SC, as long as the improvement in efficiency downstream (due to the decrease of adjustment costs) results in at least the same savings.

## 5 Conclusion

This model yields some major improvements on the previous model of Azzamouri et al. (2021). It replaces arbitrary used routing, affectation, and schedules of MO orders to the dry blending plants of mines by a more efficient assignment of orders, routings and SOs to use while respecting the MOs quality charts and the best feeding of secondary stocks. It includes a model extension to the calcination blending process that leads to a co-production, never studied in the OR literature. If an SC owns a unique mine, the model remains attractive as it provides the more efficient solution by its optimal joint definition of SOs blends, routings and sequencing of MOs to deliver. This approach is then transposable to numerous other phosphate mining sites where similar problems arise.

This model is interesting from an SC perspective: this part of the phosphate supply chain is connected upward to mines and downward to chemical plants. Looking upstream, the model provides, in a few weeks' horizons, the best choice of SOs to bring to the secondary stocks located at the foot of the dry blending plants. Looking downstream, the model provides information to use to reach a better global efficiency of an enlarged SC perimeter, including acid phosphoric plants, by yielding the loss of efficiency of the studied productive system, induced by an amelioration of the stabilisation of MOs used to produce phosphoric acid, which decreases its production cost (currently in an unknown proportion).

The first tests of the proposed global optimisation model are conclusive and show the interest of

not assigning mines and routings to orders. Several additional tests must be performed before adoption through a Decision Support System (DSS), and this point was acted by OCP management. The motivation of these additional tests is more a matter of winning the support of the operatives to this change in the production management than to show (which is obvious) that the use of an optimal solution is preferable to that of a "tinkered" solution

## 6 Data Availability Statement

The authors confirm that the data supporting the findings of this study are available within the article and its supplementary materials. Data Set and Results are included in the Mendeley data (<https://data.mendeley.com/datasets/mz2f7gxb9w/draft?a=683bb4cc-48bf-4b62-bd19-f1b7152644f3>)

## 7 References

- ASCM. APICS Dictionary, The essential supply chain reference. 2019. 16<sup>th</sup> ed.. <https://www.ascm.org/learning-development/certifications-credentials/dictionary/>
- Azzamouri, A., M. Bamoumen, H. Hilali, V. Hovelaque, and V. Giard. 2021. "Flexibility of dynamic blending with alternative routings combined with security stocks: A new approach in a mining supply chain." *International Journal of Production Research*, 1-18. <https://doi.org/10.1080/00207543.2020.1814443>
- Bader, M.G., and W.H. Bowyer. 1973. "An improved method of production for high strength fibre-reinforced thermoplastics." *Composites 4*, 150–156. [https://doi.org/10.1016/0010-4361\(73\)90105-5](https://doi.org/10.1016/0010-4361(73)90105-5)
- Badhotiya, G. K., G. Soni, and M. L. Mittal. 2019. "Multi-site planning and scheduling: State-of-the-art review and future research directions." *Journal of Global Operations and Strategic Sourcing*, 13(1), 17-37. <https://doi.org/10.1108/JGOSS-02-2019-0014>
- Bansal S, and S. Transchel. 2014. "Managing supply risk for vertically differentiated co-products." *Production Oper. Management* 23(9): 1577–1598.
- Bara, B., F. Gautier, and V. Giard. 2020. "Usage des pénalités dans les modèles économiques de management industriel", MOSIM'20, Agadir, Maroc. <https://hal.archives-ouvertes.fr/hal-03005697v1>
- Bengtsson, J., D. Bredström, P. Flisberg, and M. Rönnqvist. 2013. "Robust planning of blending activities at refineries." *Journal of the Operational Research Society*, 64(6), 848-863. <https://doi.org/10.1057/jors.2012.86>
- Benhamou, L., Giard, V., Khouloud, M., Fenies, P., Fontane, F. 2020a. "Reverse Blending: An economically efficient approach to the challenge of fertilizer mass customization." *International Journal of Production Economics*. 226, 107603 <https://doi.org/10.1016/j.ijpe.2019.107603>
- Benhamou L., Fenies P., Giard V. 2020b. "Potential Benefits of Reverse Blending in the Fertilizer Industry" In: Lalic B., Majstorovic V., Marjanovic U., von Cieminski G.,

- Romero D. (eds) *Advances in Production Management Systems. The Path to Digital Transformation and Innovation of Production Management Systems*. APMS 2020. IFIP Advances in Information and Communication Technology, vol 591. Springer, Cham. [https://doi.org/10.1007/978-3-030-57993-7\\_26](https://doi.org/10.1007/978-3-030-57993-7_26)
- Bruns, R., and J. Sauer. 1995. "Knowledge-based multi-site coordination and scheduling "
- Caner Taşkın, Z., and A. Tamer Ünal. 2009. "Tactical level planning in float glass manufacturing with coproduction, random yields and substitutable products." *European Journal of Operational Research* 199, 252–261. <https://doi.org/10.1016/j.ejor.2008.11.024>
- Castillo, P. A. C., and V. Mahalec. 2014. " Inventory pinch based, multiscale models for integrated planning and scheduling-part I: Gasoline blend planning. " *AIChE Journal*, 60(6), 2158-2178. <https://doi.org/10.1002/aic.14423>
- Cerdá, J., P. C. Pautasso, and D. C. Cafaro. 2016. "Optimizing Gasoline Recipes and Blending Operations Using Nonlinear Blend Models." *Industrial & Engineering Chemistry Research*, 55(28), 7782-7800. <https://doi.org/10.1021/acs.iecr.6b01566>
- Chen, X., P. Kouvelis, and M. Biazaran. 2018. "Value of operational flexibility in coproduction systems with yield and demand uncertainty." *International Journal of Production Research*, 56(1-2), 491-507. <https://doi.org/10.1080/00207543.2017.1402138>
- Dantzig, George B. 1982. "Reminiscences About the Origins of Linear Programming." *Operations Research Letters* 1 (2): 43–48.
- Darmawan, A., M. W. Ajiwibowo, K. Tokimatsu, and M. Aziz. 2020. "Efficient coproduction of power and ammonia from black liquor." *International Journal of Hydrogen Energy* 45, 34437–34448. <https://doi.org/10.1016/j.ijhydene.2020.02.196>
- Dong, L., P. Kouvelis, and X. Wu. 2014. "The Value of Operational Flexibility in the Presence of Input and Output Price Uncertainties with Oil Refining Applications." *Management Science* 60, 2908–2926. <https://doi.org/10.1287/mnsc.2014.1996>
- Dumetz, L., J. Gaudreault, A. Thomas, N. Lehoux, P. Marier, and H. El-Haouzi. 2017. "Evaluating order acceptance policies for divergent production systems with coproduction." *International Journal of Production Research*, 55(13), 3631-3643. <https://doi.org/10.1080/00207543.2016.1193250>
- Dumetz, L., J. Gaudreault, H. Bril El-Haouzi, A. Thomas, N. Lehoux, and P. Marier. 2021. "Tactical-operational coordination of a divergent production system with coproduction: the sawmilling challenge." *INFOR: Information Systems and Operational Research* 1–23. <https://doi.org/10.1080/03155986.2021.1906057>
- Fourer R. (2013) Algebraic Modeling Languages for Optimization. In: Gass S.I., Fu M.C. (eds) *Encyclopedia of Operations Research and Management Science*. Springer, Boston, MA. [https://doi.org/10.1007/978-1-4419-1153-7\\_25](https://doi.org/10.1007/978-1-4419-1153-7_25)
- FICO Xpress Optimisation Suite, <http://www.fico.com/en/products/fico-xpress-optimisation-suite>
- Gaudreault, J., J.-M. Frayret, A. Rousseau, and S. D'Amours. 2011. "Combined planning and scheduling in a divergent production system with coproduction: A case study in the lumber industry." *Computers & Operations Research* 38, 1238–1250. <https://doi.org/10.1016/j.cor.2010.10.013>
- Glismann, K., 2001. "Short-term scheduling and recipe optimisation of blending processes." *Computers and Chemical Engineering*, 8.
- Grohman, K., R. Cameron, Y. Kim, W. Widmer, and G. Luzio. 2013a. "Extraction and recovery of pectic fragments from citrus processing waste for coproduction with ethanol: Pectic Fragments from Citrus Processing waste." *J. Chem. Technol. Biotechnol.* 88, 395–407. <https://doi.org/10.1002/jctb.3859>
- Grossmann, I. E., 2002. "Discrete Optimisation Methods and their Role in the Integration of Planning and Scheduling". *AIChE Symposium Series*, 326, 150-168.

- Jia, Z., and M. Ierapetritou. 2003. "Mixed-Integer Linear Programming Model for Gasoline Blending and Distribution Scheduling." *Industrial & Engineering Chemistry Research*, 42(4), 825-835. <https://doi.org/10.1021/ie0204843>
- Jia, Z., and M. Ierapetritou. 2004. "Efficient short-term scheduling of refinery operations based on a continuous time formulation." *Computers & Chemical Engineering*, 28(6-7), 1001-1019. <https://doi.org/10.1016/j.compchemeng.2003.09.007>
- Kim, J., and H.-J. Kim, 2021. "Parallel machine scheduling with multiple processing alternatives and sequence-dependent setup times." *International Journal of Production Research* 59, 5438–5453. <https://doi.org/10.1080/00207543.2020.1781278>
- Kolodziej, S. P., I. E. Grossmann, K. C. Furman, and N.W. Sawaya. 2013. "A discretisation-based approach for the optimisation of the multiperiod blend scheduling problem." *Computers & Chemical Engineering*, 53, 122-142. <https://doi.org/10.1016/j.compchemeng.2013.01.016>
- Kumar, A., and S. Chatterjee. 2017. "Open-pit coal mine production sequencing incorporating grade blending and stockpiling options: An application from an Indian mine." *Engineering Optimisation*, 49(5), 762-776. <https://doi.org/10.1080/0305215X.2016.1210312>
- Lee, D.-H., D. Kiritsis, and P. Xirouchakis. 2004. "Iterative approach to operation selection and sequencing in process planning." *International Journal of Production Research* 42, 4745–4766. <https://doi.org/10.1080/00207540410001720412>
- Li, J., I. A. Karimi, R. Srinivasan. 2009. "Recipe determination and scheduling of gasoline blending operations." *AIChE Journal*, NA-NA. <https://doi.org/10.1002/aic.11970>
- Li, J., X. Xiao, and C. A. Floudas. 2016. "Integrated gasoline blending and order delivery operations: Part I. short-term scheduling and global optimization for single and multiperiod operations." *AIChE Journal*, 62(6), 2043-2070. <https://doi.org/10.1002/aic.15168>
- Lin, D., C.C. Teo, and C.K. Lee. 2015. "Heuristics for integrated job assignment and scheduling in the multi-plant remanufacturing system." *International Journal of Production Research*, Vol. 53 No. 9, pp. 2674-2689. <https://doi.org/10.1080/00207543.2014.975851>
- Lin, Y.-T., H. Sun, and S. Wang. 2020. "Designing Sustainable Products Under Coproduction Technology." *Manufacturing & Service Operations Management*, 22(6), 1181-1198. <https://doi.org/10.1287/msom.2020.0881>
- Lotero, I., F. Trespalacios, I. E. Grossmann, D. J. Papageorgiou, and M.-S. Cheon. 2016. "An MILP-MINLP decomposition method for the global optimisation of a source based model of the multiperiod blending problem." *Computers & Chemical Engineering*, 87, 13-35. <https://doi.org/10.1016/j.compchemeng.2015.12.017>
- Marandi, F., and S. M. T. Fatemi Ghomi. 2019. "Integrated multi-factory production and distribution scheduling applying vehicle routing approach." *International Journal of Production Research*, 57(3), 722-748. <https://doi.org/10.1080/00207543.2018.1481301>
- Méndez, C. A., I. E. Grossmann, I. Harjunkoski, and P. Kaboré. 2006. "A simultaneous optimisation approach for off-line blending and scheduling of oil-refinery operations." *Computers & Chemical Engineering*, 30(4), 614-634. <https://doi.org/10.1016/j.compchemeng.2005.11.004>
- Miller, T., and R. Matta. 2003. "Integrating production and transportation scheduling decisions between two geographically separated plants." *Journal of Business Logistics*, 24(1), 111-146. <https://doi.org/10.1002/j.2158-1592.2003.tb00034.x>
- Neiro, S. M. da S., V. V. Murata, B. Jahn, R. R. Seixas, E. Horn Hollmann, and C. S. Pereira. 2019. "Dealing with Multiple Tank Outflows and In-Line Blending in Continuous-Time Crude Oil Scheduling Problems." *Industrial & Engineering Chemistry Research*, 58(11), 4495-4510. <https://doi.org/10.1021/acs.iecr.8b03749>
- Öner, S., and T. Bilgiç. 2008. "Economic lot scheduling with uncontrolled coproduction."

- European Journal of Operational Research* 188, 793–810.  
<https://doi.org/10.1016/j.ejor.2007.05.016>
- Rafiei, R., M. Nourelfath, J. Gaudreault, L.A. De Santa-Eulalia, and M. Bouchard. 2015. “Dynamic safety stock in coproduction demand-driven wood remanufacturing mills: A case study.” *International Journal of Production Economics* 165, 90–99.  
<https://doi.org/10.1016/j.ijpe.2015.03.024>
- Rakiz, A., N. Absi, and P. Fenies. 2021. “An integrated production and direct shipment problem in a mining industry.” *International Journal of Production Research*, DOI:10.1080/00207543.2021.1998699
- Rao US., JM. Swaminathan, and J. Zhang. 2004. “Multi-product inventory planning with downward substitution, stochastic demand and setup costs.” *IIE Trans.* 36(1):59–71.
- Sawik, B. (2020), "Selected Multiple Criteria Supply Chain Optimization Problems", Lawrence, K.D. and Pai, D.R. (Ed.) Applications of Management Science (Applications of Management Science, Vol. 20), Emerald Publishing Limited, Bingley, pp. 31-58.  
<https://doi.org/10.1108/S0276-897620200000020003>
- Scholz, R.W., A.H. Roy, and D.T. Hellums. 2014a. “Sustainable phosphorus management: a transdisciplinary challenge.” In: Scholz, R.W., Roy, A.H., Brand, F.S., Hellums, D.T., Ulrich, A.E. (Eds.), *Sustainable Phosphorus Management: A Global Transdisciplinary Roadmap*. Springer, Berlin, pp. 1–113.
- Sioshansi, R., A.J. Conejo. 2017. "Optimisation in Engineering: Models and Algorithms", in vol. 120 of *Springer Optimization and Its Applications*. <https://doi.org/10.1007/978-3-319-56769-3>
- Steiner, G., B. Geissler, I. Watson, M.C. Mew. 2015. “Efficiency developments in phosphate rock mining over the last three decades.” *Resources, Conservation and Recycling* 105, 235–245. <https://doi.org/10.1016/j.resconrec.2015.10.004>
- Wery, J., J. Gaudreault, A. Thomas, and P. Marier. 2018. “Simulation-optimisation based framework for Sales and Operations Planning taking into account new products opportunities in a coproduction context.” *Computers in Industry* 94, 41–51.  
<https://doi.org/10.1016/j.compind.2017.10.002>
- Yuan, Y., Q. Qu, L. Chen, and M. Wu. 2020. “Modeling and optimisation of coal blending and coking costs using coal petrography.” *Information Sciences*, 522, 49-68.  
<https://doi.org/10.1016/j.ins.2020.02.072>
- Zhang, H., M. Gen, and Y. Seo. 2006. “An effective coding approach for multiobjective integrated resource selection and operation sequences problem.” *J Intell Manuf* 17, 385–397. <https://doi.org/10.1007/s10845-005-0012-y>
- Zhao, H., M.G. Ierapetritou, N.K., Shah, and G. Rong. 2017. “Integrated model of refining and petrochemical plant for enterprise-wide optimisation.” *Computers and Chemical Engineering*, Vol. 97, pp. 194-207. <https://doi.org/10.1016/j.compchemeng.2016.11.020>