The Hypothetical Nature of Legal Liability and its Temporal Dimension Towards a Dialogical Approach to Causation in Law (First Explorations)

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Abstract: Inferential approaches to legal liability (including cases of pre-emption), embedded in a dialogical framework yield, so we suggest, a quite intuitive implementation of Armgardt's proposal to respond to Moore's (2010) study of cases of causal overdetermination in Law, provided the deployment of a fully interpreted language as the one of Martin-Löf's (1984) Constructive Type Theory is casted in a dialogical framework – rather than in a possible-world one – , whereby (1) hypothetical judgements (involving non-actualized tokens) can be explicitly distinguished from categorical conditional judgements, (2) tokens of actions can be introduced as explicit denizens of the object-language, (3) actual harm/tort events are analysed as tokens dependent upon tokens of their (putative) causes – i.e. as (multi) functions over tokens of their causes, (4) tokens of actions can be "enriched" with a timing function. The paper will be divided in two main parts. . The first one follows Armgardt's strategy to start by establishing collective Liability and proceed afterwards with the identification of individual Liability. The second main part, proposes a framework for the attribution of individual responsibility with regard to the determination of individual Causation. This second way is led by the rationale that legally liable is the one who caused the damage, individually or in association with others.

Introduction

In his challenging paper, *Causation in Law, Overdetermination and Normative Ideal Worlds (*2019), Armgardt pleads for a hypothetical and normative approach to legal liability for causing harm in Penal and Tort Law.

Armgardt's paper is a response to Moore's (2010, pp. 410-425) study of overdetermination, as counterexample to the alleged utility of counterfactual reasoning in order to determine legal liability. Moore'e study is based on the following cases involving a group of arsonists (varying in number),

1. A group of arsonists whose actions are independent and sufficient to produce the damage – burning the house.
2. A group of arsonists such that the action of one of them, sufficient for the total damage, comes together with one that is not sufficient but when it joins to the first the total damage results, and even might accelerate the effect.
3. A group of arsonists, such that the action of two out of three of them suffice for causing the total damage.
4. A group of arsonists such that the action of one of them comes "too late" since the action of another in the group, already produced the total damage

Armgardt's (2019) main idea; is to reformulate the counterfactual reasoning underlying the use of the rule *Conditio Sine Qua Non* of Roman Jurists, for these and other cases as follows:

> [I]n *legal cases* the *counterfactual conditional* should have the following structure:
> *If all involved agents* ***had acted according to their legal duties****, would then the* ***harm*** *have* ***not occurred****?*
> In addition to this, we have to respect the *temporal order* of a case. If harm is done before the second tortfeasor starts to act, he is not liable according to private law (and concerning penal law he can only be punished for an attempt). Only if harm develops over time, the second action can lead to liability according to private law. Armgardt (2019, p. 708).

In that paper a possible-worlds framework is suggested and further developed in Andreas, Armgardt and Gunther (2022) – in short AAG.

In the latter paper two main worries of John Woods (2019, pp. 709-714) concerning Armgardt's (2019) proposal have been addressed, namely:

(1) if a possible-world framework is assumed, it looks as some criteria of similarity between worlds is needed
(2) how to determine the individual contribution to the caused harm in cases involving collective actors.

In relation to the first of Wood's worries, the authors of the follow-up paper introduce a notion of relevance order.

To address the second problem, iterated counterfactual conditionals are proposed.

For instance, given the independent actions a_1, a_2, a_3 and the tort event e (the burning of a house) – all in the actual world –, the following iteration of counterfactuals is applied

- Were the disjunction of the three actions absent (in some counterfactual world), then the event e would not have happened. Moreover, were the disjunction of the three actions absent, then if the presence of one of those actions, say a_3, validates the iterated counterfactual $a_3 \Box\!\!\rightarrow \sim e$, then this action is the one of someone at the wrong place and time, not of an arsonist. In other words, if a_3 has no causal impact on the damage e, following holds:

$$\sim(a_1 \vee a_2 \vee a_3) \Box\!\!\rightarrow \sim e,$$
$$\sim(a_1 \vee a_2 \vee a_3) \Box\!\!\rightarrow (a_3 \Box\!\!\rightarrow \sim e).$$

In contrast, and assuming again that

- Were the disjunction of the three actions absent (in some counterfactual world), then the event e would not have happened. However, were the disjunction of the three actions absent, then, if the presence of independent actions, say, a_1 and a_2, validate the iterated counterfactuals $a_1 \Box\!\!\rightarrow e$, $a_2 \Box\!\!\rightarrow e$, then those actions are said to have causal impact in relation to the tort event e – thus, a_1 and a_2 are indeed the actions of the arsonists:

$$\sim(a_1 \vee a_2 \vee a_3) \Box\!\!\rightarrow \sim e,$$
$$\sim(a_1 \vee a_2 \vee a_3) \Box\!\!\rightarrow (a_1 \Box\!\!\rightarrow e),$$
$$\sim(a_1 \vee a_2 \vee a_3) \Box\!\!\rightarrow (a_2 \Box\!\!\rightarrow e).$$

An alternative to iteration is to substitute iteration with the following condition:

- If the disjunction constituted by any proper subset of the original set of putative cases were absent then the event would have taken place anyway. In our example, one of those proper subsets is one that only has as elements actions one and two. Clearly, if the negation of the disjunction constituting this proper subset, does not back the consequent, that is, if it is would be false that e would have happened anyway, then, as before, a_3 can be said to be causally irrelevant in relation to e.

If $\not\models \sim(a_1 \vee a_2) \Box\!\!\rightarrow e$, then a_3 did not cause e
Whereby
$(a_1, a_2) \subset (a_1, a_2, a_3)$,
$\sim(a_1 \vee a_2 \vee a_3) \Box\!\!\rightarrow \sim e$, and
the three actions and event e actually happened

Thus, under these assumptions, the set (a_1, a_2, a_3) fails to satisfy the iteration or its alternative proper subset condition

So far so good, however, in principle this seems to make the use of Armgardt's Normative Ideal Worlds (NIW) redundant. However, in sections 5 and 6 AAG reintroduce NIW as third condition in order to tackle cases of omissions and promises or engagements. For instance, if some individual ***i*** promised to perform some action *a*, but omitted to do so – i.e. if ~*a* is the case–, the individual is causally responsible for the event *e*, since in every NIW ***i*** would keep her/his promise of not omitting to perform *a*, and the two other conditions are also satisfied. Some other actor ***j***, who did not make such a promise, may omit *a*. Thus, **j**'s action/omission is causally irrelevant with regard to the harm *e*.

As pointed out by AAG (2022, section 7) themselves, the logical analysis of legal liability reasoning does not need a possible-worlds framework. Indeed, the most powerful current approach to causation, namely the one of Halpern and Pearl (2005) – based on an inferential conception of causation by Pearl (2000) –, does not deploy possible worlds at all. However, according to AAG, Halpern-Pearl's theory is, from the point of view of computational complexity, quite demanding and perhaps even too difficult to put into practice by lawyers who have to decide on some concrete cases.

The main claim in the present paper, is that inferential approaches to legal liability (including cases of pre-emption), embedded in a dialogical framework – rather than in a possible-world one –, yield a quite intuitive implementation of Armgardt's proposal of reasoning under the hypothesis of ideal agents, provided

The deployment of a fully interpreted language as Ranta's Type Theoretical Grammar (GTS) – based on Martin-Löf's (1984) Constructive Type Theory (CTT) – casted in a dialogical framework (such as in Rahman et al. (2018)), whereby

1. hypothetical judgements (involving non-actualized tokens) can be explicitly distinguished from categorical conditional judgements,
2. tokens of actions can be introduced as explicit denizens of the object-language,
3. actual harm/tort events are analysed as tokens dependent upon tokens of their (putative) causes – i.e. as (multi) functions over tokens of their causes,
4. tokens of actions can be "enriched" with a timing function that can be put into work in order to sort out actions that came "too late" – i.e. every token of a putative causal action must have a timing anterior to the token of the tort event. This also allows the introduction of spans of time.

These points back the following proposal

- Armgardt's (2019) liability counterfactual, is encoded as a hypothetical verified by a multifunction from the tokens of the collective actions of the presumed wrong doers to the token of relevant harm/tort event. We call it *Normative Ideal Hypothetical* (NIH).
- Individual liability of agent g_i can be defined as a relation (of varying arity), which relates agent g_i, with a token of his own action (and possible too with tokens of the actions of other agents), and the damage attributed to that agent. Individual liability presupposes that NIH includes g_i as one of the agents described in the hypothetical.
- actions causally "irrelevant" to the harm at stake, and which extend the antecedent of the hypothetical, are analysed as involving tokens that do not contribute to the constitution of the (multi)function that define the cause of harm at stake – i.e. irrelevant actions introduce tokens by weakening.
- omissions leading to harm/tort events, are understood as tokens dependent on tokens of actions leading to harm – i.e., as composed functions over such tokens,

N.B. The term "enrichment" has been borrowed from Recanati's (2007a,b) pragmatist understanding of those cases of temporalization that are not obtained by studying events as propositional functions over time, but as dynamic adverbials required by the context. Enrichment processes can be given a logical analysis by deploying a dialogical rendering of Ranta's (1994, p. 108) timing function – see Rahman (2021).

Given this framework different formulations are possible. We will here follow a dialogical meaning explanation, which prescribes how causation claims can be challenged and how they can be defended. In this first exploration Armgardt's (2019) main ideas will constitute a kind of guideline for addressing the main problems to be solved.

The framework, attempts to follow closely the legal reasoning that leads to Armgardt's (2019) proposal. Making all this explicit involves some notational bureaucracy. However, so we claim, this brings the legal insights to the fore.

In fact, the paper is divided in two main sections.

a. The first one follows Armgardt's strategy to start by establishing collective liability and proceed afterwards with the identification of individual liability.

b. The second main part, proposes a framework for the attribution of individual responsibility with regard to the determination of individual Causation. This second way is led by the rationale that legally liable is the one who caused the damage, individually or in association with others.

The latter, will be grounded on a reconstruction of an ancient test for legal causation developed within medieval juristic dialectics, called "co-presence and co-absence" (*al-ṭard wa'l-ʿaks*) or "concomitance" (*dawarān*) – Young (2019)

This second way, which combines the test of causation with a recently developed reconstruction of medieval deontic imperatives, seems to offer a simple way for determining individual liability out of causation – see Rahman&Zidani&Young (2022).

A

From Collective To Individual Liability

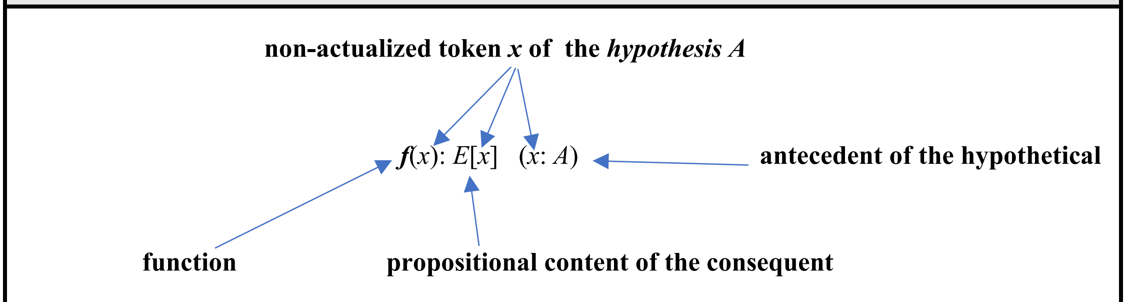
A.I Token, Types of Actions and Hypotheticals

Rahman and Iqbal and Rahman, Farid, Young (2022, pp. 152-159),[1] deploying the expressivity of the fully interpreted language of Per Martin-Löf's Constructive Type Theory (CTT) and its further development by Ranta (1994) into a *Type Theoretical Grammar* (TPG), proposed an analysis of imperatives in legal and ethical contexts whereby those imperatives are expressed as hypothetical judgments constituted by action-types and/or action-events.

The verifier of such a hypothetical judgment is a function that takes tokens (the performances of the actions types) of the hypothesis or hypotheses, (i.e the antecedent of the judgment) and yields tokens of the consequent of the hypothetical. In the afore mentioned literature hypotheses were interpreted as constituting the material legal cause, of the legal effect – i.e. of consequent of the hypothetical –, and the function as the legal efficient cause.

A simple hypothetical involving just one type of actions *A* performed by one actor, and the resulting event *E*, as in a CTT-framework the following structure

SIMPLE HYPOTHETICAL
Event E would would be actualized under the hypothesis that tokens of action-type A are.



Now we might wish to make it explicit that the type of action *D* – performing duty *D*, is constituted by tokens carried out by some agent. This yields the set of all those agents how are included in the hypothetical as performing the type of action *D,* for *duty*:

$$\{ y: G \mid D(y) \}$$

This set is called a Σ-type and it is what an existential amounts to. So, we can write

$$(\exists y: G)D(y)$$

If we interpret the Σ-type as standing *for the set of all those agents that carry out tokens of the action type performing-duty-D, duty* the following **first** approach to Armgardt's *Normative Ideal Hypothetical* (NIH) results:

[1] See Rahman and Iqbal (2018, 80-84); further developed in Rahman, Iqbal, & Soufi (2019, 31-40), and Iqbal (2022).

NIH-1
~*E* would be actualized provided any agent g_i involved performs duty *D*

More precisely, ~*E* would be actualized provided any element of the set of all those agents that carry out tokens of the action type of performing-duty *D* is actualized.

$$f(x): \sim E[x] \quad (x: (\exists y: G)D(y))$$

set of all those agents performing *D*

function

consequent of the hypothetical

N.B. With *involved*, we mean here those actors performing actions at a place and time that include the location and period where the harm *E* occurred. In fact, a pragmatic component plays here an important role: the actors included in the hypothetical are those that are *presumed* to be relevant for causing the event *E* . As pointed out by Armgardt (2019, section 5.5) if a group of agents perform some kinds of actions at the same time, and some harm seems to result, in principle there will be presumption of liability for the whole group. In other words, the constituted of the group will be led by some assumptions on the relevance of their actions for the event at stake. It is now to each actor to defend their case for non-liability.

Notice that we did not use a universal quantifier in order to avoid involving an unrestricted number of agents. Moreover, duty *D* is some specific duty relevant for the event *E* not happening.

However, it seems to be desirable to distinguish the duty of each individual agent. For instance, the duty of a fireman in the event of the burning of a house should be distinguished of the duty of some other kind of actor present during that burning. So, assuming that the number of relevant actors can be enumerated we obtain:

NIH-2
~*E* would be actualized provided any agent g_i involved performs her/his *own* duty D_i.

More precisely, ~*E* would be actualized provided any member of the conjunction of all those agents g_i performing their *own* duty D_i is actualized.

COMPACT NOTATION

$$f(x): \sim E[x] \quad (x: D_1(g_1) \wedge, \ldots, \wedge D_n(g_n))$$

antecedent: conjunction of all those g_i performing their *own* duty D_i

function **propositional content of the consequent**

This assumes:

$D_i(y)$: *prop* $(y: G)$, and $\sim E[x]$: *prop* $(x: D_1(g_1) \wedge, \ldots, \wedge D_n(g_n))$
$g_1, \ldots, g_n$: G

NIH-3
$\sim E$ would be actualized provided any agent g_i involved performs her/his *own* duty D_i.

More precisely, $\sim E$ would be actualized provided any member of the conjunction of all those agents g_i performing their *own* duty D_i is actualized.

EXPLICIT NOTATION

$$f(\pi_1(x), \ldots, \pi_n(x)): \sim E[\pi_1(x), \ldots, \pi_n(x)] \quad (x: D_1(g_1) \wedge, \ldots, \wedge D_n(g_n))$$

function **propositional content** **antecedent**

Whereby, $\pi_i(x)$ is the projection operator that "selects" within x the token **i** of the **i**-member of the conjunction.

This assumes:

$D_i(y)$: *prop* (y: G), and $\sim E[x]$: *prop* (x: $D_1(g_1) \wedge, \ldots, \wedge D_n(g_n)$)
$g_1, \ldots, g_n$: G

- The multi-function implements Armgardt's (2019) request that the not-happening of E is the result of the collective performances of the agents involved taken together.

  N.B. In the standard CTT-notation, each of the tokens, i.e. the proof-objects, of a conjunction are extracted by means of the left-projection function p, and the right-projecton function q. Thus if c: $A \wedge B$, we have c=<$p(c)$, $q(c)$>: $A \wedge B$. If, as in our case the conjunction has more than two members, we need compositions such as c=<$p(c)$, <$p(q(c)$, $q(q(c))$>: $A \wedge (B \wedge D)$. In order to avoid such bracektings we introduced the projection operators π_i.

We might also wish to specify the time and a locus at which E took place, such that there is both a timing and location function, which determines the constitution of G In other words, such a specification yields those actors present at some precise time and locus. In order to avoid notational complexity, we will make these parameters explicit dynamically, i.e., they will enrich an assertion when the required by the legal context.

The notation of hypothetical judgments used above, with the hypotheses to the right, follows the original notation of Per Martin-Löf (1984), which focus on the consequent: *B would be the case provided the Hypotheses* $H_1, \ldots, H_N$. However, sometimes it is useful to use the more familiar back-to-front turnstyle-notation. In the case of formulating NIH we obtain:

NIH-4. Turnstyle
$\sim E$ would be actualized provided any agent g_i involved performs his/her *own* duty D_i.

More precisely, $\sim E$ would be actualized provided any member of the conjunction of all those agents g_i performing their *own* duty D_i is actualized.

COMPACT NOTATION

$$x: D_1(g_1) \wedge, \ldots, \wedge D_n(g_n) \vdash f(x): \sim E[x]$$

antecedent **function**

propositional content

NIH-5. Turnstyle

$\sim E$ would be actualized provided any agent g_i involved performs his/her *own* duty D_i.

More precisely, $\sim E$ would be actualized provided any member of the conjunction of all those agents g_i performing their *own* duty D_i is actualized.

EXPLICIT NOTATION

$$x: D_1(g_1) \wedge, \ldots, \wedge D_n(g_n) \vdash f(\pi_1(x), \ldots, \pi_n(x)): \sim E[\pi_1(x), \ldots, \pi_n(x)]$$

antecedent **function** **propositional content**

Whereby

$\pi_i(x)$ is the projection operator that "selects" within x the token **i** of the **i**-member of the conjunction

Which assumes

$D_i(y)$: *prop* (y: G), and $\sim E[x]$: *prop* (x: $D_1(g_1) \wedge, \ldots, \wedge D_n(g_n)$)

$g_1, \ldots, g_n$: G

A.II Time Dimension

A. II.1 Timing a Token

As mentioned in the introduction the idea is to deal with temporal conditions as contextual parameters that can be made explicit in order to *enrich* a proposition that has already content,[2] rather than in order to complete the meaning of a propositional function. Thus if "*Time*" stands for some set of instants, "τ" for a timing function that takes tokens of some action type and yield instants we obtain:

Timing a Token of an Action Type

$$\frac{A_i(g_i): prop}{x: A_i(g_i) \vdash \tau(x): Time}$$

token x of $A(g_i)$ **timing function**

The assertion *g performs an action of the type A at* t_i – say, *lighten the fire* –, will be first encoded as

$$(\exists x: A(g))\ \tau(x) =_{Time} t_i$$

In this context it is useful to recall Ranta's (1994, pp. 7-9) notion of *sugaring*, a procedure by the means of which the fully explicit formal encoding undergoes some transformations until the natural language expression results. On our view, sugaring processes have to be built upon some specific contextual background: in this case argumentative interaction, that we call *dynamic encoding*, by the means of which *enriching* will result as an answer to a request for precising the time involving some particular token. Enriching has the same effect as adding in natural grammar an adverbial expression, in this case a temporal one.

[2] We owe the expression *enrichment* to Recanati (2017).

Enriching Propositions with Instants of Time
$A(g)$: *prop* t: *Time* -------------------- $A(g)@t$: *prop* Whereby $x: G \vdash A(x)$: *prop* and $g: G$ Thus, "$A(g)@t$ true" stands for a sugaring of $(\exists x: A(g))\ \tau(x) =_{Time} t_i$ true In other words, "$@t$" occurring in $A(g)@t$ stands for an operator, the formal counterpart of an adverbial, that can be added to a proposition.[3]

Thus, the sugaring process that yields this enrichment can be (roughly) described as follows:

$$(\exists x: A(g)\ \tau(x) =_T t_i \Rightarrow A(g)@t_i \Rightarrow g \textit{ lights the fire at } t_i$$

A.II.2 Spans of Time[4]

An interesting feature of Ranta's (1994, section 5.1) Type Theoretical Grammar is that it also allows to deal time spans and intervals with a beginning and an end. This is particularly important because actions like moving, running, etc. do not happen in a moment. Rather they should be considered as extended events which happen in temporal intervals. The category of the spans of a time scale T can be defined as the Cartesian product of T and the set of natural numbers $\mathbb{N}$.[5] More precisely:

$$span(T) = T \times \mathbb{N}$$

To make it clearer, a span of the time scale T is a pair whose first element refers to the beginning point of that span in T and whose second element refers to the number of temporal units (of the scale T) which must be added to the beginning point to form the span under discussion. Stated differently, the second element determines the length of the span. So if $d = \langle t_0, n \rangle : span(T)$, d is a span of the time scale T which begins at t_0 and ends at $t_0 + n$. The span d can also be represented as $[t_0, t_0 + n] : span(T)$. The following functions are also useful:

$$left(d) = begin(d) = t_0 : T$$
$$end(d) = t_0 + n : T$$
$$right(d) = length(d) = n : \mathbb{N}$$

As an example of the spans of time in the time scale *Day*, consider the following span:

$$\langle 9 \text{ January } 1991, 365 \rangle : span(Day)$$

This span of time begins 9 January 1991 and extends for 365 days.[6]

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[3] In our context "$@t$" is not to be read as related to notion of nominal of *Hybrid logic*, since in the latter nominals do not time tokens but rather are to be understood as propositional terms, i.e. they are terms that refer to a proposition such *it is 11 o'clock* – see Blackburn (1993).

[4] This section is based on Rahman & Zarepour (2021, pp. 421-422).

[5] For a detailed technical definition of time spans, see Ranta (1994, p. 115).

[6] It is noteworthy that since 0 is a member of $\mathbb{N}$, every singular moment of the time scale T can be considered as a span of the length 0. In other word, every t of the time scale T corresponds to $\langle t, 0 \rangle$ which is a member of $span(T)$. This shows that everything expressible by the terminology of singular moments of time is also expressible by the terminology of time spans, though the other way around does not hold.

Enriching Propositions with Spans

$A(g)$: *prop*
t: *Time*

$A(g)@<t_0, n>$: *prop*

Action by *g*

starting unit t_0 of the span e.g., day, hour, minute

natural number *n* expressing the length of the span - *span*(*T*)= *T* × ℕ

Whereby $x: G \vdash A(x)$: *prop*, $g: G$, and t_0: *Time-unit* and $n: \mathbb{N}$

Thus, "$A(g)@ <t_0, n>$ true" stands for a sugaring of

$(\exists x: A(g))\ \delta(x) =_{Span(T\text{-}unite)} <t_0, n>$ true

A.III The Dialogical Dynamics of Legal Liability

Armgardt's (2019) main strategy was to determine first liability and then tackle the issue of personal responsibility. In a personal email Armgardt informed me, that this, so to say two main stages procedure coincides with usual legal practice. That is why, in his first paper Armgardt starts with a counterfactual having as antecedent everyone one acting according to his duty.

In this context, the claim that the action of an agent is innocent or irrelevant in relation to the harm at sake, but who has been presumed to be liable, is to be proven by the defendant not by the plaintiff.

The dynamic inherent to the dialogical point of view allows quite naturally to implement this two stages procedure. Moreover, the explicit introduction of temporal parameters and so on can be introduced dynamically during the defence: this is what we mean *with dynamic encoding*.

Before describing this let discuss the first of Woods's (2019) worries, namely the problem of similarity-assumption between counterfactual worlds.

A.III.1 Weakening of NIH

The inferential counterpart to the similarity-assumption required by counterfactuals conceived in a possible world-semantics is weakening. Indeed, many of the well-known paradoxes to standard deontic logic come from the fact that weakening to the left, can extend the initial set of premisses to a new set composed with some premise irrelevant to the conclusion – similar can be said to weakening to the right.

In our example, weakening to the left will extend the antecedent with a new conjunct. Now, since our framework the verifiers – the tokens – are explicit, this extension is harmless. Indeed, the new member of the conjunction will have a different verifier to the original one. Let me exemplify this with a simple extension of an obvious hypothetical.

Given the, hypothetical

$$x: A \vdash x: A$$

the following extension (applying weakening and conjunction introduction) holds

$$x: A \wedge B\ \vdash \boldsymbol{f}(\pi_1(x)): A$$

What the consequent of the hypothetical expresses is that the hypothetical verification of A, is a method that selects the first member of the hypothetical verifier of the antecedent – that is the verification of the first member of the conjunction $A \wedge B$.[7]

In other words, the irrelevant extension will be ignored for the justification of the consequent. Similarly, if we have a conjunction of 3 members , say three arsonists

$$x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$$

and extend it to 4 the consequent of the hypothetical can still be verified by the first 3 members

$$x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \wedge D_4(g_4) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$$

However, if there is a case for considering the extension as a necessary addition for the verification of the consequent, then the new hypothetical can be asserted

$$x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \wedge D_4(g_4) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x), \pi_3(x), \pi_4(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x), \pi_4(x)]$$

Now, perhaps the defendant will claim that the extended hypothetical includes an irrelevant member. This will take the case to plaintiff as discussed further on

A.III.2 Individual Responsibility without Temporal Parameters

As mentioned in the introduction Armgardt's NI-counterfactual offers in principle a good solution to Moore's cases and to some others he adds, mainly involving time and one where the presumed liability of one of the actors is contested. However, without refinement the original solution does not distinguish individual responsibility, since it based precisely on the collective liability of all actors.

In the present framework, it looks as if individual liability can be made explicit, by simply singling out the tokens of the action of each agent, and if required enriching them with temporality.

A particular case is omissions the liability of which, should in some cases distinguished from the action "directly" linked to the damage – e.g. the delay of the firemen in the case of the burning house. The idea is to analyse omissions as actions dependent on others.

A.III.2.1 Sufficient and Independent Acts of Harm

Let us start with the simplest case, the one involving independent and sufficient actions.[8] This give us too the opportunity to introduce some notational conventions that have been developed in *Immanent Reasoning (IR)* and that will be deployed in our analysis.

X ! …, stands for the interlocutor **X** claims ….

Claims commit to further moves, where the claim is backed by some token (in *IR*) such tokens are called *local reasons*), verifying that claim

[7] Certainly, we can also produce the assertion $x: A \wedge B, \vdash right(\boldsymbol{f}(\pi_2(x))): A \vee B,$ which selects the hypothetical verifier of the second member of the conjunction and yields a hypothetical verifier for the right of the disjunction. However, notice that the verifier of the disjunction is then then B and A is ignored.

[8] Cf. Moore (2010, p. 410).

Thus, if **X** claims for example that g_1 performed action A_1, then the antagonist, might require **X** to produce some verification. **X** must then produce such a verifier (when the verification has been produced the claim becomes a justified assertion and so the exclamation mark drops out)

Claim	Request	Response
X ! $A_1(g_1)$	**Y** ?	**X** $b_1: A_1(g_1)$

Coming back to our example. Let us assume that it is agreed by both, Plaintiff and Defendant, that

Three tokens e_1, e_2 and e_3 of the total damage of the type E, have taken place
Three tokens of independent actions b_1, b_2 and b_3 by three agents g_1, g_2 and g_3 have been performed respectively.

Let us further assume that based on his formulation of the NIH, the Plaintiff claims that each of the three agents are legally collectively liable from the point of view of Penal Law (*PL*), and/or Tort Law (*TL*). Furthermore,

- in order to prove that each agent g_i is individually liable for the total damage produced, the Plaintiff is committed to assert that both, a performance b_i and a token e_i, sufficient for the total damage E, can be attributed to the agent g_i, , provided the NIH (which involves g_i),

In other words, verifying the assertion on the legal liability of each agent g_i in relation to E, has the form of a function that associates

the agent g_i,
the function $\boldsymbol{f}$: NIH
his performance b_i, of the action $A_i(g_i)$, and
the token e of the event E (sufficient for verifying E)

with the legal liability of that agent in relation to the event E.

We will not include $\boldsymbol{f}$ in the propositional content of the assertion, but only in the function that verifies the liability. The rationale behind this decision is not simply notational but it seeks to to stress the fact that though verifying an assertion of liability requires the four components, the **concept of personal liability relates an agent with his own action and the token of the (total) damage claimed to be of his own responsibility.**

Assuming i=3, the encoding of the verifier of liability has the following form:[9]

[9] The notation for Personal Liability above is a short-cut, which is closer to the actual dialogical practice, whereby once it is agreed that an assertion is dependent upon an assumption, then the latter is left implicit. The full notation has the form :

Asserting Agent's g_3 Personal Liability ***Explicit Notation***
$g_i: G, x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3), \boldsymbol{f}(x): \sim E[x], b_i: A(g_i), z: E \vdash \boldsymbol{w}(g_3/g_i, \boldsymbol{f}, b_3/g_i, e): L(g_3/g_i, b_3/b_i, e)$. Alternatively, using the abbreviation NIH for the antecedent: $! \, NIH \vdash \boldsymbol{w}(g_3/g_i, \boldsymbol{f}, b_3/g_i, e): L(g_3/g_i, b_3/b_i, e)$.

Asserting Agent's g_3 Personal Liability	
Agent	g_3: G
Verifier of NIH	f: NIH
Token of agent's action	b_3: $A(g_3)$
Token of the damage E associated to g_3	e_3: E

$$\underbrace{w(g_3, f, b_3, e_3)}_{\textit{verifier}}: \underbrace{L(g_3, b_3, e_3)}_{\textit{relation of personal liability}}$$

Whereby NIH has the form

$$x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$$

.

Such an analysis yields the following general encoding

	PERSONAL LIABILITY OF AGENTS OF INDEPENDENT SUFFICIENT ACTIONS OF HARM
Agreed Assertions	**Token of total damage** e: E **Performances of the Agents** b_1: $A_1(g_1)$, b_2: $A_2(g_2)$, b_3: $A_3(g_3)$
Plaintiff's Claims	**NIH** **X** $x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash f(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$ **Personal Liability** **X** ! $PL(g_1, b_1, e) \wedge PL(g_2, b_2, e) \wedge PL(g_3, b_3, e)$ **X** ! $TL(g_1, b_1, e) \wedge TL(g_2, b_2, e) \wedge TL(g_3, b_3, e)$ } **X** ! $L(g_1, b_1, e) \wedge L(g_2, b_2, e) \wedge L(g_3, b_3, e)$
Plaintiff's Comitments	**X** ! NIH $\vdash w(g_1, f, b_1, e): L(g_1, b_1, e)$ **X** ! NIH $\vdash w(g_2, f, b_2, e): L(g_2, b_2, e)$ **X** ! NIH $\vdash w(g_3, f, b_3, e): L(g_3, b_3, e)$ Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.

If we wish to make explicit a further step that describes how claims of personal liability determine the distribution of the costs of the damage among the agents, a new relation, say the Payment-Duty relation, can be formulated. But we will leave this out for the moment. However, in the following cases such a relation beyond the one of personal liability does not seem to be necessary

A.III.2.2 Collective Acts of Harm

A.III.2.2.1 Mixed Cases Involving Acts of Harm

In such form of mixed collective acts of harm called *asymmetrically overdetermined concurrent-cause cases*, both a sufficient and an insufficient cause come together, e.g. when a sufficient fire merges with another fire that is not individually sufficient to cause total harm.[10]

Some of the examples for mixed cased brought up by Moore (2011, p. 411) suggest that time considerations can make a difference since the merging of two actions might cause the harm to be produced faster, e.g. two wounds, a small and major one, such that the victim would have died from the major one alone but the merging of the smaller one had an *accelerating* effect on the death. Accelerating effects, will be discussed when we include temporal parameters, though there is an argument to make for the claim that accelerating effects might have an impact when assessing the individual liability of the agent who performed an action insufficient for producing total damage.

Following Armgardt (2019) we start by establishing the collective liability of all the agents involved by means of a suitable formulation of the NIH. The NIH has the same structure as in the case before. What changes is the distribution of Personal Liability that amounts to the conjunction of

1. the liability of the first agent in relation to his (sufficient) performance b_1 and the token *e* of total damage *E*
2. the liability of the second agent in relation to his (insufficient) performance b_2 and the token *e'* of partial damage *E;* and
3. the liability of the second agent in relation to his (insufficient) performance b_2, g_1's performance b_2 and the token *e* of total damage *E*

Such a conjunction determines the commitments of the Plaintiff, which amount to verifying each of the assertions constituting it. Clearly,

the first member establishes the liability of the first agent in respect to the total damage *E*,
the second the liability of the second agent in relation to the partial damage associated to his performance,
and the third of the liability, again of the second agent, but this time in relation to the total damage associated to the merging of his own performance with the one of the first agent.

The latter could be seen as a first approximation to claims of accelerating effect. So, whereas the second assertion associates the second agent with liability to a lesser damage, the latter can be seen as indicating that the merging of both performances makes the second agent liable for accelerating the process that led to total damage. This is displayed in the following table.

	PERSONAL LIABILITY MIXING SUFFICIENT AND INSUFFICIENT ACTIONS OF HARM	
Agreed Assertions	**Token of total damage**	$e: E$
	Tokens of partial damage	$e': E_1$
	Performances of the agents	$b_1: A_1(g_1)$ $b_2: A_2(g_2)$
	NIH $\mathbf{X}\, x: D_1(g_1) \wedge D_1(g_2) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$	

[10] Cf. Moore (2010, p. 411).

Plaintiff's Claims	**Distribution of Personal Liability** $\mathbf{X} ! \ L(g_1, b_1, e) \wedge L(g_2, b_2, e') \wedge L^*(g_2, b_2, b_1, e', e)$ Partial Liability Accelerating Liability
Plaintiff's Comitments	$\mathbf{X} !$ NIH $\vdash \boldsymbol{w}(g_1, \boldsymbol{f}, b_1, e)$: $L(g_1, b_1, e)$ $\mathbf{X} !$ NIH $\vdash \boldsymbol{w}(g_2, \boldsymbol{f}, b_2, e')$: $L(g_2, b_2, e')$ $\mathbf{X} !$ NIH $\vdash \boldsymbol{u}(g_2, \boldsymbol{f}, b_2, b_1, e)$: $L^*(g_2, b_2, b_1, e)$ Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.

A.III.2.2.2 The Merging of the Actions of Subgroups of Agents

In this case, the performance of each individual is insufficient for producing the total damage, however some subgroups build a sufficient cause of harm. For example, three equally sized fires join to burn the plaintiff's house, when any two of them would have been sufficient to produce the total damage – Moore (2010; p. 412)

The NIH has the same structure as in the case before and as pointed out by Armgardt (2019, section 5.2). Indeed, according the NIH here too all of the three tortfeasors are liable, because if all involved agents had acted according to their legal duties, the harm – i.e. the burning of the house–, would have not occurred.

What changes is the distribution of Personal Liability that amounts to the conjunction of

1. the liability of the first agent in relation to his (sufficient) performance b_1 and the token *e'* of partial damage *E;*
2. the liability of the second agent in relation to his (insufficient) performance b_2 and the token *e''* of partial damage *E;* and
3. the liability of the third agent in relation to his (insufficient) performance b_2 and the token *e'''* of partial damage *E;* and
4. the liability of some agent g_i in relation to his own performance b_i, the performance of b_j of one of the other two agents, and the token *e* of total damage *E*.

Thus, each of the agents is Liable with regard to the combination of his/her own action with the action of any other of the agents.

	PERSONAL LIABILIY I MERGING SUBGROUPS OF INDIVIDUALLY INSUFFICIENT ACTIONS OF HARM			
Agreed Assertions	**Token of total damage**	e: E		
	Partial Damages E_i	$E \equiv E_i \wedge E_j$		
	Tokens of partial damage	e': E_1	e'': E_2	e''': E_3
	Performances of the agents	b_1: $A_1(g_1)$ b_2: $A_2(g_2)$ b_3: $A_3(g_3)$		

Plaintiff's Claims	**NIH** $\mathbf{X}\ x$: $D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x), \pi_3(x))$: $\sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$ **DISTRIBUTION OF PERSONAL LIABILITY** $\mathbf{X}$! $L^*(g_i, b_i, b_j, e)$ $j \neq i$ Whereby in the last member of the conjunction "g_i" stands for one of the three agents, "b_i" for the token of the action of that agent, and "b_j" for the token of the action of one of the two other agents.

Since the actions of any pair of agents out of the three produces the total damage at stake, each of the agents is personally liable, for the total damage with regard to merging of his own action with the one of the others.

	PERSONAL LIABILIY II **MERGING SUBGROUPS OF INDIVIDUALLY INSUFFICIENT ACTIONS OF HARM**
Plaintiff's Comitments	$\mathbf{X}$! NIH $\vdash \boldsymbol{u}(g_i, \boldsymbol{f}, b_i, b_j, e)$: $L^*(g_i, b_i, b_j, e)$ $j \neq i$ Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.

A.III.2.2.3 The Merging of the Actions of all Agents

This case, discussed in AAG (2022, section 6) describes a scenario where the majority of the members of the executive board of company votes in favour of producing and selling one of their products, despite the fact that each member of the board knows for certain that the product is harmful.

Strictly speaking the collective liability that comes out from the NIH is sufficient, the liability in the context is collective after all. However, we might wish to make the inner structure of the distribution of this collective liability over each agent apparent. In principle this distribution it is quite close to the preceding case of subgroups, however in such a scenario it does not seem to make sense to single out some kind of partial damage, of each of the involved agents. Rather it each agent is responsible for the total damage caused by the her/his vote. Thus, we can capture personal liability of each agent with one general form for the liability relation, which relates

(for any) g_i his own performance b_i, the performances of $b_j, \ldots b_k$ of **all** the other agents, and the token e_i of total damage E,

For the sake of simplicity, in the table below, we assume only three agents, each of them, voted in favour of producing the damage. If we wish to add say one or two voters that voted against continuing the production, the case amounts to the one of weakening described above.

COLLECTIVE MERGING OF ACTIONS OF HARM

Agreed Assertions	**Tokens of total damage** e: E **Performances of the agents** b_1: $A_1(g_1)$ b_2: $A_2(g_2)$ b_3: $A_3(g_3)$
Plaintiff's Claims	**NIH** $\mathbf{X}\ x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x), \pi_3(x)): \sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$ **DISTRIBUTION OF PERSONAL LIABILITY** $\mathbf{X}\ !\ L^{**}(g_1, b_1, b_2, b_3, e) \wedge L^{**}(g_2, b_2, b_1, b_3, e) \wedge L^{**}(g_3, b_3, b_1, b_2, e)$
Plaintiff's Comitments	$\mathbf{X}\ !\ \text{NIH} \vdash \boldsymbol{v}(g_1, \boldsymbol{f}, b_1, b_2, b_3, e): L^{**}(g_1, b_1, b_2, b_3, e)$ $\mathbf{X}\ !\ \text{NIH} \vdash \boldsymbol{v}(g_2, \boldsymbol{f}, b_2, b_1, b_3, e): L^{**}(g_2, b_2, b_1, b_3, e)$ $\mathbf{X}\ !\ \text{NIH} \vdash \boldsymbol{v}(g_3, \boldsymbol{f}, b_3, b_1, b_2, e): L^{**}(g_3, b_3, b_1, b_2, e)$ Provided an NIH relevant to these agents, tokens of actions and tokens of harm, holds.

A.III.2.2.4 Being at the Wrong Time and Place

The case where some innocent agent has been included in the liability claim takes an important place in AAG (2022). In this context, Armgardt (2019, section 5.5) describes a scenario where there is uncertainty on the legal liability, concerning harm E, of some of the members of a group of agents performing kinds of actions at the same time, presumed to contribute to that harm.

- Let assume that it is agreed by both, Plaintiff and Defendant, that a token e of the harm-type E has taken place and tokens of the agents g_1, g_2 and g_3 performing respective actions at the same time
- Let us further assume that based on his formulation of the NIH, the Plaintiff claims that that three are legally liable. But then the Defendant claims the non-liability of, say, agent three. This commits the Defendant ought to produce some verifier for his claim.

There are two strategies, perhaps even two moments both of them amount to the Defendant's claim of the irrelevance of his duties and actions in relation to the attributed damage. So, the first defence, that underlies Armgardt's focus on the NIH, is the simplest one and is based on the irrelevance of his own duties, on the irrelevance of his (perhaps dangerous actions).

First defence

The defendant claims that his own duties are irrelevant. In other words, the defendant claims that the NIH has the following form, which coincides with a case of weakening – we deploy once more our main example involving three agents (arsonists):

Defendant $x: D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$

Notice that the point is here that defendant is claiming that carrying out his own duties is irrelevant for the absence of damage E. More precisely, the defendant points is that the meaning constitution of the $\sim E$ is NOT dependent upon the meaning of $D_3(g_3)$. Thus, we have the following

Meaning Constitution of Plaintiff's NIH	**Meaning Constitution of Defendants's NIH First Defence**
$D_1(g_1) \wedge D_1(g_2)\ ) \wedge D_1(g_3)$: *prop* $\vdash \sim E$: prop Which yields x: $D_1(g_1) \wedge D_1(g_2) \wedge D_3(g_3) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x), \pi_3(x))$: $\sim E[\pi_1(x), \pi_2(x), \pi_3(x)]$	$D_1(g_1) \wedge D_1(g_2)$: *prop* $\vdash \sim E$: prop Which yields x: $D_1(g_1) \wedge D_1(g_2) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x))$: $\sim E[\pi_1(x), \pi_2(x)]$ and by weakening x: $D_1(g_1) \wedge D_1(g_2) \wedge D_1(g_3) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x))$: $\sim E[\pi_1(x), \pi_2(x)]$

Second defence

- If the actions under examination are considered to be independent and sufficient actions, the Defendant's claim on the irrelevance of g_3-actions is to be justified by showing that it is not the case that a token of E is *dependent upon* any token of action $A_3(g_3)$.

- If the actions at stake are considered to collectively cause E, such that only through the joint action of each agent harm E results, then, the Defendant's claim on the non-liability of agent three is to be justified by showing that it is not the case that E is *dependent upon the pairs of actions* $(A_1(g_1), A_3(g_3))$, $(A_2(g_2), A_3(g_3))$ and $(A_1(g_1), A_2(g_2), A_3(g_3)$. This yields the following table
Commitment

Second Defence **Irrelevance of g_3-Performances in the Context of Independent and Sufficient Actions of Harm**	**Second Defence** **Irrelevance of g_3-Performances in the Context of Collective Actions of Harm**
Defendant's Commitment d: $\sim(\exists x: A_3(g_3))E(x)$	**Defendant's Commitments** d_1: $\sim(\exists x: A_3(g_3) \wedge A_1(g_1))E(\pi_1(x), \pi_2(x))$ d_2: $\sim(\exists x: A_3(g_3) \wedge A_2(g_2))E(\pi_1(x), \pi_2(x))$ d_3: $\sim(\exists x: A_3(g_3) \wedge A_1(g_1) \wedge A_2(g_2))E(\pi_1(x), \pi_2(x), \pi_3(x))$

Notice that this can be combined with the first defence.

A.III.2.2.5 Omission as not Preventing

The authors of the AAG paper take the view that omissions are some special kind of process rather than pure negativity. True, many philosophers think of negative acts in parallel to negative facts and since the latter is usually denied so the former.

However, I think that in the realm of actions, there is conceptually and logically a point to make in favour of considering omissions to constitute some kind of positive actions.

Generally speaking, the agent of an omission is legally liable, since the agent had the choice between preventing some action/event to happen or not doing so. According to this view, there are three main components in an omission, namely

- The object of an omission act is another act or act or event
- The content of an omission act is to not preventing this other act/event from happening
- It is in the agent's *capacity* to prevent that other act from happening

In relation to the third component, the proviso on the agent's capacity to prevent, the point is that the agent had the choice between preventing or not preventing. This is the source of the agent's liability.

In relation to the second point, from a purely deductive point of view, a negation is conceived as a process that leads to the abortion of another process.[11] If we focus on actions rather than on processes, the negation of an action is seeing to it that the action does not succeed. Furthermore, according to our view, the agent's preventing an action from being carried out, presupposes the agent, having the choice between seeing to it that this action does or not succeed.

In relation to the first point, and in the context of the other two, an omission amounts to not to prevent another action (the object of the omission act) from succeeding.

The Preventing Relation
• If preventing, is preventing the action of *someone else*, we need to pass from the connective level to a relation. (with connective level, we mean the negation understood as an implication such as $A \supset \bot$, that admits the interpretation A is aborted) • Thus, the meaning of the Prevention relation is shaped on one hand by the i. notion of "blocking", which characterizes the deductive meaning of negation; on the other ii. it is shaped by the pragmatist stance of being an action which aims at blocking or aborting another event or action. • **Omissions** are then defined as instances of **not Preventing**.

For the sake of simplicity let us consider only two agents,

Our known arsonist g_1, and t
the firemen (all collectively) g_2, who omitted to come (on time), after the arsonist started the fire and before the total damage was produced.

Let us render the token of the arsonist action as

b: $A(g_1)$, and the omission of the firemen as

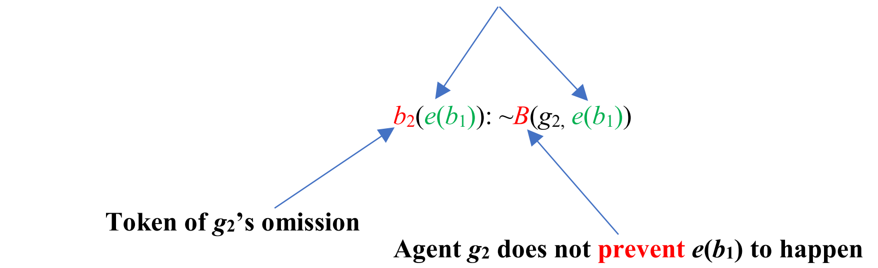
$b_2(e(b_1)$: $\sim B(g_2, e(b_1))$, whereby "$B(g_2, e(b_1)$", stands for "g_2 preventing damage e dependent upon g_2's performance b ":

[11] From the deductive point of view a negation amounts to the implication $A \supset \bot$ true. The proof object of this implication is an operation on a function from A to *Falsum*, such that, when executed, for any proof-object that attempts to proof A a *halt* is produced: $\lambda(x)\boldsymbol{u}(x)$: $A \supset \bot$

Moreover, in order to implement the idea that Preventing is a capacity, we can think of it as the result of a choice, in our example, between preventing or not the house from the burning triggered the arsonist's action. The source of the liability of the firemen is indeed this choice. Since in section B, we will develop a general form for liability that result from choices, I will here simply add as an assumption for the assertion of liability by omission, that, the choice has been already drawn for not Preventing.

OMISSIONS

Performances of agent g_1	$b_1: A(g_1)$
Token of damage E produced by g_1's performance b_1	$e(b_1): E$



Clearly, not preventing the burning (induced by the arsonist to happen) from happening is not the same as actually setting the fire to the house: the firemen's omission is not identical to the arsonist action: the liability of the firemen is dependent upon the arsonist's action after all.

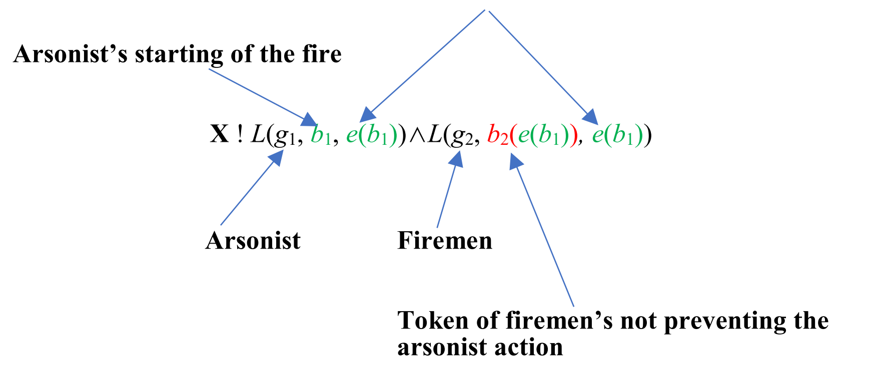
PERSONAL LIABILITY
OF OMISSIONS

NIH

$$\mathbf{X}\ x: D_1(g_1) \wedge D_1(g_2) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$$

DISTRIBUTION OF PERSONAL LIABILITY OF OMISSIONS

Token of the damage produced by the Arsonist's setting the fire to the house



PLAINTIFF'S COMMITMENTS

$$\mathbf{X}\ !\ \mathrm{NIH} \vdash \boldsymbol{w}(g_1, \boldsymbol{f}, b_1, e(b_1)): L(g_1, b_1, e(b_1))$$

X ! NIH, ! $\sim B(g_2, e(b_1)) \vdash \boldsymbol{w}(g_2, \boldsymbol{f}, b_2(e(b_1)), e(b_1))$: $L(g_2, b_2(e(b_1)), e(b_1))$ **Firemen's choice for omitting to prevent the house from burning**

Clearly, an even finer analysis would include time parameters this leads to the next section

A.IV Individual Responsibility and its Temporal Dimension

Strictly speaking, one might think that time parameters are relevant for all the cases discussed above, not only for pre-emptive ones. We might also add locative parameters. But we follow here a dynamic approach according to which temporal and locative parameters are made explicit when required by the context.

A.IV.1 Pre-emption: Liable for Penal Law but Too Late for Private/Tort Law

A pre-emptive case of causation is one where two performances poised to do some damage, each being individually sufficient to do so, are temporally ordered in such a way, that one of both performances, when executed first, pre-empts the capacity of the other one to cause the harm. For instance, two fires independently set, each sufficient to burn the plaintiff's house, do not join. One fire reaches the house first and burns it to the ground, and then the second fire arrives but there is no house to burn – cf. Moore's (2010, p. 412).

As pointed out by Armgardt (2019), both are liable according to *Penal Law* – which does not require the final object of the harmful action to be achieved, but, the second, the *too late* tortfeasor, is not liable according to Private/Tort Law.

In our framework the reasoning is quite straightforward if we decline it in three main steps:

1. it is agreed that according to NIH, would both agents have done their duty, no harm would have resulted
2. it is claimed (we assume, by the defendant), that the second agent is not liable according to Private/Tort Law, since the token of the second agent's actions, arrived after the time acknowledged to correspond to the damage
3. if the second step has been agreed, then it is asserted that the first agent is liable according to both Penal and Private/Tort Law, but the second only according to Penal Law

Shared Assertions	**First Main Step** **NIH**	**Second Main Step** **Defendant Y**	**Third Main Step** **Concluding Assertions**
$e: E@t_j$ $b_1: A_1(g_1)$ $b_2: A_2(g_2)$	**X** $x: D_1(g_1) \wedge D_1(g_2) \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x))$: $\sim E[\pi_1(x), \pi_2(x)]$	**Y** ! $\sim TL(g_2, b_2, e)$ **X** ? **Y** ! $b_2: A_2(g_2)@t_{k>}t_j$	**X, Y** ! NIH $\vdash \boldsymbol{w}(g_1, \boldsymbol{f}, b_1, e_1)$: $PL(g_1, b_1, e)$ **X, Y** ! NIH $\vdash \boldsymbol{w}(g_1, \boldsymbol{f}, b_1, e_1)$: $TL(g_1, b_1, e)$ **X, Y** ! NIH $\vdash \boldsymbol{w}(g_2, \boldsymbol{f}, b_2, e_2)$: $PL(g_2, b_2, e)$

A. IV.2 Physis Strikes. Liable for Penal Law, but Partially Liable for Private/Tort Law?

Armgardt (2019, section 5.4) discusses an interesting additional case involving temporality, assume that the token of an action and a token of a natural event, say, a lightning bolt, are each being individually sufficient to set the fire to the house. Is the agent liable according to Private/Tort Law?

Armgardt suggests in these paragraphs, that since it seems sensible to grant the insurance company that insured the house, to claim some form of liability of the arsonist (according to Privat/Tort Law), the Duty hypothesis requires some form of reformulation.

In our framework, the reasoning, declined again in three main steps, leads to the conclusion that the arsonist partially liable for the damage according to private Law, provided there is some significant span of time between the agent's setting of the fire to the house and the lightning bolt – if both the natural event and the agent's start of the fire are either simultaneous or quite close, it seems that no liability according to Private/Tort Law can be established. The main steps leading to such a conclusion are the following

a. it is agreed that according to (the **modified**) NIH, if it is the case that the agent has been done his duty, and it is also the case that **the natural event (ϕ:Φ) did not happen**, then no harm would have resulted.
b. it is claimed (by the insurance company), that the second agent is liable according to Private/Tort Law for the for the partial damage e': E' occasioned during the span of time between his starting of the fire and the instant at which the natural event happened
c. if the second step has been agreed, then it is asserted that the first agent is liable according to both Penal and Private/Tort Law. Moreover, according to Private/Tort Law the agent is liable for the period established in the previous step.

For the sake of simplicity, we will not explicitly indicate the time when the final damage took place but we assume that it happened at the same time as or immediately after the lightning struck. We further assume that the unity of the span of time are hours.

Shared Assertions	**First Main Step** **NIH**	**Second Main Step** **Insurance's Claim Y**	**Third Main Step** **Concluding Assertions**
Total Damage $e: E$ **Partial Damage** e': E' **Arsonist's Action** b_1: $A_1(g_1)$ **Natural Event** ϕ: Φ	**X** $x: D_1(g_1) \wedge \sim\Phi \vdash \boldsymbol{f}(\pi_1(x), \pi_2(x)): \sim E[\pi_1(x), \pi_2(x)]$	**X** ! $TL(g, b, e')_{@<ti,\ ti+(n=\mathrm{k}-1)>}$ Amount of time passed before the natural event occurred. **Y** ? **X** ! ϕ: $\Phi_{@tk}$ Time at which the natural even happened Whereby x: $TL(g, b, e') \vdash \delta(x)$: $Span(hour)$	**X, Y** ! NIH $\vdash$ $\boldsymbol{w}(g_1, \boldsymbol{f}, b_1, e)$: $PL(g_1, b_1, e)$ **X, Y** ! NIH $\vdash \boldsymbol{w}(g_1, \boldsymbol{f}, b_1, e')$: $TL(g, b, e')_{@<ti,\ ti+(n=\mathrm{k}-1)>}$

A. IV.3 Some More Cases Involving Temporal Order

In fact, Armgardt (2019, section 5.7) records two more cases where temporality is involved.

> In *case* 1, *A* bumps into the car of *C* and destroys the car's left front light. An hour later, *B* bumps into the same car and destroys it completely. What about the liability of *A* according to Private Law?
>
> In *case 2*, *A* causes an accident by which *B* loses his eyesight. Since *B* has a serious disease, he would have lost his eyesight anyway within the next three years. What about the liability, according to Private Law, of *A* if *B* is not able to work anymore? Does he or she have to pay a disability pension in favour of *B* for 3 years only or does he or she have to pay until *B* dies?

In relation to the first case, Armgardt (2019, ibidem) suggests that the analysis must involve the temporal order of both actions. According to our own view, this is indeed the case and the argument runs as follows:

1. If both would have done their duty, no damage would have happened (NIH)
2. The first agent (in Armgardt's example, agent *A*) is liable for the partial Damage that happened *before* the second agent's (in Armgardt's example, agent *B*) bumping into the car. Thus, agent *A* is liable for the car's front light, but not for the total damage of the car.
3. The second agent is liable for the total Damage that happened *before* the second agent's (in Armgardt's example, agent *B*) bumping into the of the car.

Perhaps one could think of a more complicated situation, for instance, if the second agent bumps into the car because it was night and because of the damage produced by the first agent made it impossible to see the car. In such a case, in our framework, the second agent would not be liable. Or perhaps even only by some kind of liability applied to involuntary actions of harm. The logical point is that the second action is dependent upon the first one:

$$x: A_1(g_1)_{@t'<t} \vdash b(x): A_2(g2)_{@t}$$

In case 2, Armgardt (2019, ibidem), concludes that the tortfeasor has to pay only for the three years that "accelerated" the loss of the ey. In our framework, the conclusion coincide with the one of Armgardt, and is lead by an argument similar to the case of the strike of nature. This argument requires also a modification of the NIH. In other words, in our framework, the conclusion follows from the following strategy:

1. If the tortfeasor would have done his/her duty, and **no disease would have affected the victim's eye**, no damage would have happened (NIH)
2. The tortfeasor is liable for the loss of the eye, but this liability is restricted to the span that starts at the time the tortfeasor started the tort and ends, the time at which the disease would have started

The difference to the case of the strike of nature is that in the case of the disease, the damage produced by the disease will not be actualized – assuming the disease is restricted only to the eye. More precisely, g_2's disease *D* – whereby g_2 is the victim, would not have started at, say, instant *t*, if the tortfeasor g_1 would not have carried out action $A_1(g_1)$ before *t*:

$$x: \sim(A_1(g_1)_{@t'<t}) \vdash c(x): D(g_2)_{@t}.$$

N.B. as before, this assumes that all weakening's are irrelevant for the start of the disease

B

From Causes to Individual Liability

A Plaidoyer for the Fruitfulness of Historic Studies For the Development of Contemporary Legal Reasoning

B.I Recovering an Ancient Test for Legal Causation[12]

In Islamic legal theory a set of methodological tools for legal reasoning were developed, whose elements, valid modes, and proper applications were the focus of continual argument and refinement by Sunnī jurists.[13] This centuries-long discourse constitutes a highly developed contribution of the argumentative—more precisely, dialectical—approach to legal reasoning within Islamic Law. A particularly lucid, early output was systematic work of renowned Shāfiʿī dialectician and legal theorist Abū Isḥāq al-Shīrāzī (1003-1083 CE).

A cardinal feature of al-Shīrāzī's take on legal reasoning is his particular notion of efficiency (*taʾthīr*) which tests whether the property *P* purported to be efficient in occasioning the juridical ruling at stake is indeed so. For al-Shīrāzī, *taʾthīr* consists of two complementary procedures:

- co-presence (*ṭard*): whenever the property is present, the ruling is also present, and
- co-absence (*ʿaks*): whenever the property is absent, the ruling is also absent.

While co-presence examines whether ruling *H* is present along with property *P*, co-absence examines whether ruling *H* is absent along with property *P*.[14]

This test of a property's causal efficiency is elsewhere and more commonly called "co-presence and co-absence" (*al-ṭard wa-l-ʿaks*) or "[causal] concomitance" (*dawarān*), and listed among the "modes of causal justification" (*masālik al-taʿlīl*) in works of legal theory (*uṣūl al-fiqh*).[15] Extensive discussions on this causality test—though it remained a debated technique (especially when considered in isolation from other methods)—evolved both before and after al-Shīrāzī in the legal theoretical literature.[16] Concomitance was a key consideration, and it remained, along with a handful of others—especially the tests of "suitability" (*munāsaba*) and "analytical disjunction and exclusion" (*al-sabr wa-l-taqsīm*)—among the most commonly (and thoroughly) treated rational modes of causal justification.

In later legal and dialectical theory especially, co-presence and co-absence were expressed as concomitance "in existence" (*wujūdan*) and "in nonexistence" (*ʿadaman*), and no operative distinction between natural and normative causality appears to have been maintained.[17] In brief, the steps of this test can be generalized as verifying

> (1) concomitance "in existence", if it amounts to verifying if any token of the occasioning type triggers a token of the effect type, and
> (2) concomitance "in nonexistence" if it amounts to verifying if any evidence for the absence of the occasioning type triggers a token of the negation of the effect type.

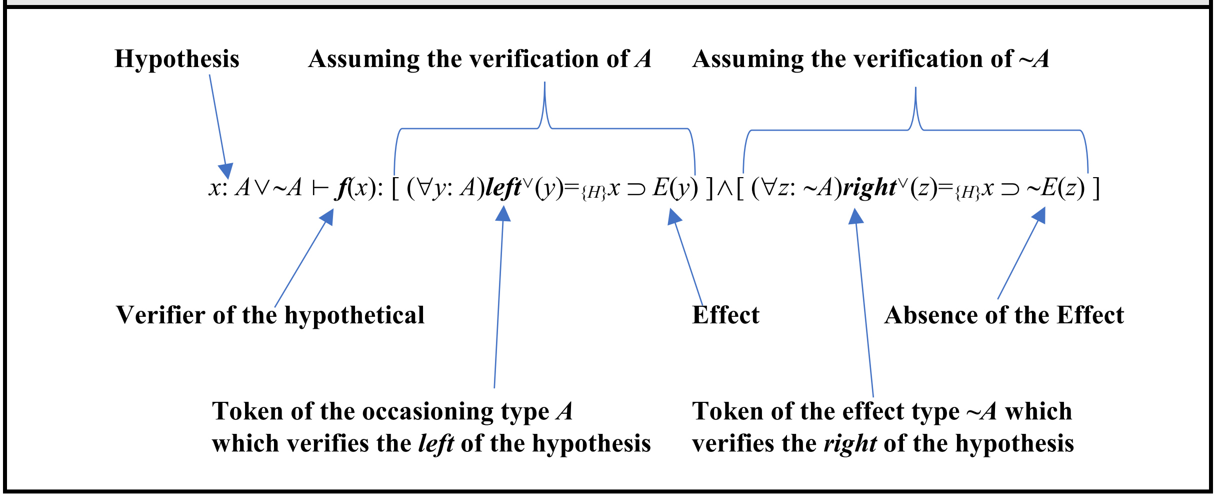
---

[12] The present summary of the Islamic causation test and its formal encoding below, relies heavily on Rahman, S. & W. Young (2022a,b).
[13] That is, what might be called mainstream Sunnīs (excluding, e.g., the Ẓāhirīs and certain Ḥanbalīs).
[14] See Rahman, Iqbal, & Soufi (2019, preface).
[15] See Young (2019).
[16] See Young (2019, Forthcoming A, B).
[17] See Young (2019, esp. 268 ff.)

When both of the occasioning types are formulated as two members of a disjunction that constitutes the antecedent of a hypothetical, this analysis, when developed in the framework of *Immanent Reasoning*, amounts to the following:

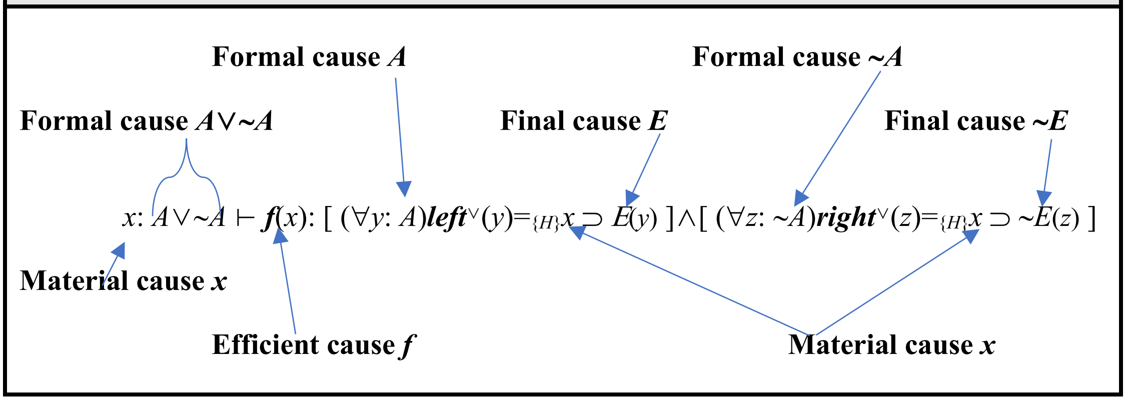
> Given the **token *x*** that witnesses either the presence of ***A*** or of ***~A***:
>
> If any token *y* of *A* is identical to a verifier of the ***left side*** of the disjunction)—viz. ***left***(*y*)= *x*— then (under such assumption) then a token of the effect type *E* is triggered.
>
> If any token *z* of *~A* is identical to a verifier of the ***right side*** of the disjunction)—viz. ***right***(*z*)= *x*— then (under such assumption) then a token of the negation of the effect type *E* is triggered.

THE CAUSATION TEST



In terms of the Aristotelian theory of causality we obtain:

THE CAUSATION TEST AND THE ARISTOTELIAN CAUSES



B.II THE TEST OF CAUSATION AND A MEDIEVAL THEORY OF IMPERATIVES[18]:

The test can be nicely combined with a recently developed theory of Medieval legal Imperatives, by adjusting the hypothesis to the cases under scrutiny – see Rahman&Zidani&Young(2022), see too Rahman&Granström&Farjami (2019)[19].

The idea is to apply a two stages procedure that in some way goes the other way around with regard to the procedures discussed in the section **A** of the present paper.

1) The legal procedure starts by testing individual causation. This requires adjusting the disjunction in the hypothesis to the structure of the actions under study.

2) Then individual Liability is determined lead by the rationale: an agent if Liable at all, is Liable to the degree of damage his/her action produce.

BII.1 Sufficient and Independent Acts of Harm (SIH)

BII.1.1 The Individual Causation Test (ICT) for SIH

For the sake of simplicity let "A_i" stand for the Action of agent **i.** So, the general form of the test for this case has the form

The General Form of the Causation Test
$x: A(g_i) \vee \sim A(g_i) \vdash \boldsymbol{f}(x): [\ (\forall y: A(g_i))\boldsymbol{left}^{\vee}(y) =_{\{H\}} x \supset E_i[y]\ ] \wedge [\ (\forall z: \sim A(g_i))\boldsymbol{right}^{\vee}(z) =_{\{H\}} x \supset \sim E_i[z]$

So, in the case of two independent arsonists we have to run the test for each of the arsonists:

The Causation Test for SIH
$x: A(g_1) \vee \sim A(g_1) \vdash \boldsymbol{f}(x): [\ (\forall y: A(g_1))\boldsymbol{left}^{\vee}(y) =_{\{H\}} x \supset E_1[y]\ ] \wedge [\ (\forall z: \sim A(g_1))\boldsymbol{right}^{\vee}(z) =_{\{H\}} x \supset \sim E_1[z]$
$x: A(g_2) \vee \sim A(g_2) \vdash \boldsymbol{f}(x): [\ (\forall y: A(g_2))\boldsymbol{left}^{\vee}(y) =_{\{H\}} x \supset E_2[y]\ ] \wedge [\ (\forall z: \sim A(g_1))\boldsymbol{right}^{\vee}(z) =_{\{H\}} x \supset \sim E_2[z]$
• Total damage E is defined as the disjunction of its partial damages $E_1 \vee E_2$. $\sim E$ is defined as $\sim E_1 \wedge \sim E_2$.

[18] The present summary of the Islamic causation test and its formal encoding below, relies heavily on Rahman, S. & W. Young (2022a,b).

[19] Though Rahman&Granström&Farjami (2019), published in the same volume as Armgardt's (2019) paper, appeared before the paper by Rahman&Zidani&Young (2022), the former relies heavily on the main philological and systematic work developed in the latter. In fact, Rahman&Granström&Farjami (2019) applies Rahman&Zidani&Young (2022) to solve standard paradoxes of contemporary deontic logic. John Woods (2019), was not that impressed by these kind of application, I hope the present one will work as a kind of plaidoyer for studying Islamic dialectic and logic for legal reasoning.

BII.1.2 Individual Liability and Absences: NIH revisited?

BII.1.2a Individual Causation of Damage

Notice that, according to the formulation of the test mentioned above, the damage E_i is the product of agent g_i's action $A(g_i)$ – similar holds for $\sim E_i$:

The Meaning Explanation of Damages Occasioned by Agents

Verifier for the damage produced by Arsonist g_i's Action

Damage produced by Arsonist g_i's Action

$$g_i: G,\ y(g_i): A(g_i) \vdash e_i[y]: E_i[y]: prop$$

Arsonist **Token of Arsonist's Action**

- Thus, the damage produced is the damage produced by **that** agent. Similar for negation. So the absence of the action of one of the agents does not prevent the damage in **general** to happen, but **the damage so far as it has been occasioned by this very agent**

Let us have a closer look at absences in this context.

BII.1.2b The Positivity of Absences and Duties: NIH revisited?

In the case of absences, it is the token of an active (self) blocking of an action. One way to understand the positive feature of absences is to understand them as indeed following the duty.

Thus, we might substitute in the hypothesis of the causation test, the negation with **following the duty.** From this point of view, the active part of a negation can be turned into pure positiveness. So, it is the fact that the agent followed his duty, rather than a token of the absence of his action, that occasions a putative token of the absence of Damage:

Individual Duties and the Causation Test

$$x: A(g_i) \vee D(g_i) \vdash \boldsymbol{f}(x): [\ (\forall y: A(g_i))\boldsymbol{left}^{\vee}(y) =_{\{H\}} x \supset E_i[y]\ ] \wedge [\ (\forall z: D(g_i))\boldsymbol{right}^{\vee}(z) =_{\{H\}} x \supset \sim E_i[z]]$$

Damage produced by Agent **i**

Absence of Damage associated to agent **i** following his Duty

Given,
$g_i: G,\ y(g_i): A(g_i) \vdash e_i[y]: E_i[y]: prop$

We might even come closer to Armgardt's (2019) proposal and add the duty-following of all the involved agents, but make the damage dependent upon each individual agent.

Collective Duties and the Individual Causation Test
x: $A(g_i) \vee (D(g_i) \wedge D(g_k)) \vdash \boldsymbol{f}(x)$: [$(\forall y$: $A(g_i))$***left***$^{\vee}(y) =_{\{H\}} x \supset E_i[y]$] $\wedge$ [$(\forall z$: $D(g_i) \wedge D(g_k))$***right***$^{\vee}(z) =_{\{H\}} x \supset \sim E_i[\pi(z)]$]   Given, g_i: G, $y(g_i)$: $A(g_i) \vdash e_i[y]$: $E_i[y]$: *prop*

This, in all of its variants, allows to stipulate that if there is a token that verifies E_i, then, agent i is individually liable for E.

BII.1.2c Individual Liability for SIH

Individual Liability for SIH
x: $E_i \vee \sim E_i \vdash \boldsymbol{f}(x)$: [$(\forall y$: $E_i)$***left***$^{\vee}(y) =_{\{H\}} x \supset L_i(g_i)$] $\wedge$ [$(\forall z$: $\sim E_i)$***right***$^{\vee}(z) =_{\{H\}} x \supset \sim L_i(g_i)$ Given, g_i: G, $y(g_i)$: $A(g_i) \vdash e_i[y]$: $E_i[y]$: *prop*   This admits the gloss,   Under the hypothesis that either damage E_i or it absence is verified   If any token y of damage E_i, produced by a token of g_i's action $A(g_i)$, is identical to a verifier of the (left-side of) hypothesis, then g_i is liable   If any token z of absence of damage E_i, produced by a token of an absence of g_i's action $A(g_i)$, is identical to a verifier of the (right-side of) hypothesis, then g_i is not liable

BII.1.3 Other Cases without Temporal Parameters

BII.1.3a Mixed Cases

In the case of the merging of one sufficient action by an arsonist with the action of one whose action is insufficient, the formulation of the causation test and of the liability hypothetical is as exactly the same as the one of the precedent case, but

total damage E is defined as the damage E_1 occasioned by the first arsonist and
partial damage is defined as the damage E_2 occasioned by the second arsonist.

BII.1.3b Damage Occasioned by the Merging of Insufficient Collective Actions

In the case of the collective merging of insufficient actions leading to total tort– recall the case of the executive board of a company that voted in favour of continuing to sell a product, known by the voters to be harmful to health – each E_i is identical to the total damage E. For the sake of simplicity let us assume that each of the votes is necessary and sufficient for the final outcome – either because unanimity is required or because a subset of necessary and sufficient votes has been singled out. Thus, the absence of one suffices to nullify the outcome of the votes.

In such a case, the hypothesis of test of causation requires including the actions of all other agents. Thus, the hypothesis for running the causation test has the form

$$A_1(g_1) \wedge, \ldots, \wedge A_k(g_k) \vee \sim(A_1(g_1) \wedge, \ldots, \wedge A_k(g_k))$$

The total damage produced has then the form

- $E[\pi_1(x), \ldots, \pi_k(x)]$

Which indicates that the total tort (the outcome of the vote), is constituted by the actions of each agent. This all yields:

The Test for Collective Causation
$x: A_1(g_1) \wedge, \ldots, \wedge A_k(g_k) \vee \sim(A_1(g_1) \wedge, \ldots, \wedge A_k(g_k)) \vdash$ $\boldsymbol{f}(x): [(\forall y: A_1(g_1) \wedge, \ldots, \wedge A_k(g_k))\boldsymbol{left}^{\vee}(y)=_{\{H\}}x \supset E[\pi_1(x), \ldots, \pi_k(x)]] \wedge$ $[(\forall z: \sim(A_1(g_1) \wedge, \ldots, \wedge A_k(g_k)))\boldsymbol{right}^{\vee}(z)=_{\{H\}}x \supset \sim E[\pi_1(z), \ldots, \pi_k(z)]$

If we are looking for pinpointing at the individual liability of each of the voters, the point is to restrict the liability to the token of the tort of the agent to be individualized:

The Test for Liability in Cases of Collective Causation
$x: E[\pi_1(z), \ldots, \pi_k(z) \vee \sim E[\pi_1(z), \ldots, \pi_k(z) \vdash$ $\boldsymbol{f}(x): [(\forall y: E[\pi_1(z), \ldots, \pi_k(z))\boldsymbol{left}^{\vee}(y)=_{\{H\}}x \supset L[\pi_i(x)]] \wedge [(\forall z: \sim E[\pi_1(z), \ldots, \pi_k(z))\boldsymbol{right}^{\vee}(z)=_{\{H\}}x \supset \sim L[\pi_i(x)]]$

BII.1.3c Omissions

Omissions will be based on the idea that if the agent prevented the tort from happening then this agent is not liable and liable otherwise. This requires the hypothesis of the causation test to be formulated as:

$$B(g_i, e(b_j)) \vee \sim B(g_i, e(b_j))$$

Then, individual liability has the same form as the general one: the agent **i** is liable if he did not prevent **j** to produce damage E_j, and not liable if he prevented the tort from happening

Individual Liability for Omisions
$x: B(g_i, e(b_j)) \vee \sim B(g_i, e(b_j)) \vdash \boldsymbol{f}(x): [(\forall y: B(g_i, e(b_j)))\boldsymbol{left}^{\vee}(y)=_{\{H\}}x \supset \sim L_i(g_i)] \wedge [(\forall z: \sim B(g_i, e(b_j))_i))\boldsymbol{right}^{\vee}(z)=_{\{H\}}x \supset L_i(g_i)$

BII.2 The Temporal Dimension

Interesting is that in cases of pre-emption and in those of the strike of nature or start of disease, from the purely penal point of view, liability does not require time considerations, beyond those that establish the presence of the action of a presumed tortfeasor.

It is the determination of liability within Private Law that requires a finer temporal analysis

Thus, in the case of pre-emption the test for causation follows the general pattern, without adding temporal parameters. However, in the case of liability according to Private Law, the hypothesis for liability must include some time instant, such that if the damage attributed to the second tortfeasor happens at some time after the one of the first agent, then the second tortfeasor is not liable.

The Test for Tort-Liability in Cases of Pre-emption
x: $E_{2@t_i>t_k} \vee E_{2@t_i\leq t_k} \vdash \boldsymbol{f}(x)$: $[\ (\forall y: E_{2@t_i>t_k})\boldsymbol{left}^{\vee}(y)=_{\{H\}}x \supset {\sim}TL_{\mathrm{i}}(g_2)\ ] \wedge [\ (\forall z: E_{2@t_i\leq t_k})\boldsymbol{right}^{\vee}(z)=_{\{H\}}x \supset TL_2(g_2)$ which considers that there are two tortfeasors, that the tort E_2 attributed to the agent g_2 may come either after of before the time t_k at which the tort E_1 attributed to the agent g_2 happened.

With regard to cases of the strike of nature, according to the analysis discussed above the argument is that if the tort attributed to the agent happened before the strike of nature, then the agent is liable for the span of time that starts with his action and ends just before the nature struck.

The Test for Tort-Liability in Cases of Pre-emption
x: $E_{1@<t_i,\ t_i+(n=\ \mathrm{k}-1)>} \vee E_{1@t_i\geq t_k} \vdash \boldsymbol{f}(x)$: $[\ (\forall y: E_{2@<t_i,\ t_i+(n=\ \mathrm{k}-1)>t_k})\boldsymbol{left}^{\vee}(y)=_{\{H\}}x \supset TL_{\mathrm{i}}(g_2)_{@<t_i,\ t_i+(n=\ \mathrm{k}-1)>}\ ] \wedge [\ (\forall z:$ $E_{1@t_i\geq t_k})\boldsymbol{right}^{\vee}(z)=_{\{H\}}x \supset {\sim}TL_2(g_2)$ which considers that the tort E_1 attributed to the agent g_1 may have started either before or after the time t_k at which the damage E_ϕ occasioned by some strike of nature Φ happened.

Similar analysis admits the case of the eye and the cars.

Concluding Remarks and Work Ahead

The main idea of the present paper is to focus on tokens of actions and make them explicit at the object language level.

We followed two main strategies, the first closer to the original proposal by Armgardt (2019) and a second one closer to the AAG (2022) paper.

As mentioned above the second strategy, is based on a recent reconstruction of a dialectical pattern of legal reasoning incepted by Islamic jurists and logicians. Now, such dialectical approaches were implemented by means of a whole arsenal of hundred of auxiliary moves that prescribe how to provide reliability to the outcome of a causation test – see Young (2017, 2019, 2021) and Forthcoming A and B)

Moreover, the framework also allows to integrate degrees of liability, dependent upon the strength of the causal link between the so-called occasioning factor, (in our case the actions of the agents) and the damage presumed.

In the context of omissions, a degree of liability can be introduced in the following terms:

if the harmful action to be prevented takes place, then if the agent does not prevent it is liable and not liable if s(h)e prevents it, but it is neither Liable nor not Liable if the harmful action does not take place.

In the context of mixed cases, the following degree of liability seems to be natural:
if the action of an agent, sufficient produce the total damage, joins to the one of a second harmful action, that only can produce a smaller damage, the latter is liable, not only for his responsibility for the partial damage, but also for a possible acceleration effect.
But if the first action does not take place at all, then the second agent is only liable to the degree of the partial damage occasioned

- Interesting is that this kind of degrees are degrees of obligation or interdiction, rather than of permissibility – the latter are known in Stoic Logic; Ethics and Law,, such as *commendable, abhorrible* and *neutral*. A thorough development should constitute the subject of future interdisciplinary work.

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