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Dialogical Insights on Structural Rules
A Plea for the New Old Ways

TALK AT THE *WORKSHOP ON GAMES IN LOGIC*
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1 Introduction

Wilfried Hodges's (2001, 2013) criticism of dialogical logic and even more generally of the contribution of the games perspective to logic has been influential and echoes of it are ubiquitous in the literature in the field. His scepticism that seems to adopt a strategy level perspective, wonders on the "story" that might motivate a game theoretical approach. The conclusion of Hodges is that dialogical logic and other game theoretical approaches might have some psychological – if any – rather than a philosophical or formal or mathematical result.

Some defenders of the dialogical approach bit the bullet and reminding us of the fine-grained analysis of Argumentation contained in traditional Rhetoric, expanded on the psychological and cognitive features of the dialogical interpretation of inferences, understood as adding persuasive explanatory power to deductive steps obtained by applying, for example, Gentzen-style rules, rather than only producing a sequence of those steps.

One of most recent forms of such kind of response, called the Built-in-Opponent (BIO) conception of deduction, stresses, among other issues, the contribution of the dialogical interpretation of the structural rules, which yields a table of rankings from most plausibly revisable to least plausibly revisable, a ranking that is not reliant on *ad hoc* paradoxes driven decisions – see Duthil Novaes&French (2018) and Duthil Novaes (2021, pp. 78-84).

BIO also developed into a new brand of the *Dialogical Pluralism* launched by Helge Rückert, Laurent Keiff and myself.[1] BIO-*Pluralism*; if I may call it so, highlighted some important social and cognitive features of the dialogical approach under the background of a general perspective on argumentation.

An alternative response to Hodges's criticism that seems to be compatible with the original perspectives of Lorenzen and Lorenz (1978) is one where structural rules are understood as shaping different forms of human-server interaction – see Blass (1992) or Fermüller (2021). Fermüller's approach is closer to Lorenzen-Lorenz take than to the BIO approach, in the sense that Fermüller's proof-theoretical rules are generated by reflecting on dialogical interaction

[1] See Rahman&Keiff (2004), Keiff (2007), Rückert(2012). For the BIO Pluralism see French (2019), Duthil Novaes (2021, ibidem).

rather than taking the dialogical stance to be an interpretative layer on top of a proof-theoretical setting.

Hodges, who complains about the lack of philosophical foundations but at the same time is sceptic about such foundations – though apparently he did not read the extensive philosophical, historic and systematic, work of Kuno Lorenz[2]–, misses one of the main points of the dialogical approach. Indeed, as pointed out by Per Martin-Löf,[3] the main philosophical tenet of the dialogical approach is that invites to re-examine the usual order between meaning and pragmatics (and knowledge): it is not the case that a pragmatic force is attached to an already given semantic core, but judgemental content amounts to the pragmatic notion of a task to be solved.

The dialogical perspective emphasises the point that the validity of an inference emerges from the fact that knowing how to execute the task attached to the premisses leads to knowing on how to execute the task at stake in the conclusion – Martin-Löf (2019, 2020). Plays, are the locus where these tasks are carried out.

In fact, Paul Lorenzen and Kuno Lorenz, the inceptors of the dialogical approach conceived structural rules as emerging from rules governing the interaction of challenge and defences that constitute a play, from the thesis up to the elementary statements that result from such an interaction – not the other way around. This **bottom up** procedure, which is a crucial trait of the dialogical framework, and which distinguishes it from the **top-down** (axiomatic) construction of Gentzen-style inferences, has been overlooked by many of the criticisms and particularly so in relation to the understanding of the dialectical stand on some structural rules such as reflexivity.

In the present paper I will recall the "old-ways" to structural rules and explore new further developments that integrate some of the instruments of fully interpreted languages in the style of Per Martin-Löf's Constructive Type Theory within the bottom up play level perspective for material and formal dialogues that we call *Immanent Reasoning*.

To avoid misunderstandings, I fully share the view put forward by the BIO understanding of the dialogical stance, as offering a framework for the development of a particular form of social practice were cognition is at work.

However, I rather follow Lorenz's proposal to see both the explanatory and communicative aspects as constituting the double nature of the cultural process in which concepts emerge from a complex interplay of *why* and *how* questions. In this context, the dialogical teaching-learning situation is where *competition*—the I-perspective—and *cooperation*—the You-perspective—interact: both intertwine in collective forms of dialogical interaction that take place at the play level – cf. Lorenz (2010, pp. 140-147).

Let me briefly discuss Martin-Löf's point

2. Revisiting the Semantic-Pragmatic Order:

[2] See the following volumes that collect the main philosophical papers by Kuno Lorenz on *dialogical constructivism*, from Plato through, among others, Herder, Dilthey, Peirce, Wittgenstein, Austin, Reichenbach, Carnap and Husserl: Lorenz (2009, 2010, 2011, 2021).
[3] Martin-Löf (2016, 2017a,b, 2019, 2020). See too Klev (2022).

Despite the fact that I happily endorse many of Martin-Löf's insights on dialogical logic, different to him I claim that the dialogical perspective is an irreducible trait of meaning. In other words, if meaning amounts to solving a task, the rational endeavour that yields an inference involving meaningful expressions is a collective one.

The issue is that, according to the Lorenzen-Lorenz approach, the "task" to be accomplished it to be understood as one to be accomplished in interactive irreducible *Lehr-Lernsituationen*.[4] In other words, the answer to the question on the contribution of the Dialogical framework can be put as follows:

- Dialogical Logic proposes a new concept of content, namely, the content of a statement amounts to knowing how to solve the interactive task involved in a teaching-learning situation where the expression has been stated. They are not only games of knowing why, but knowing how. Moreover, knowing how to solve that task **does not** necessarily amount to knowing how to develop a winnings strategy, but to knowing how to fulfil the commitments prescribed by the **local** rules for this statement, when it occurs in a play governed by some specific rules of **global meaning**.

Let me spend some lines on Martin-Löf's view on the issue:

In his Oslo and Stockholm lectures, Martin-Löf's (2017a; 2017b) delves in the structure of the deontic and epistemic layers of statements within his view on dialogical logic. In order to approach this normative aspect which pervades logic up to its technical parts, let us briefly discuss the following extracts of "Assertion and Request":[5]

> […] we have this distinction, which I just mentioned, between, on the one hand, the social character of language, and on the other side, the non-social […] view of language. But there is a pair of words that fits very well here, namely to speak of the monological conception of logic, or language in general, versus a dialogical one. And here I am showing some special respect for Lorenzen, who is the one who introduced the very term dialogical logic.
> […] Suppose that someone claims a disjunction to be true, asserts, or judges, a disjunction to be true. Then someone else has the right to come and ask him, Is it the left disjunct or is it the right disjunct that is true? There comes an opponent here, who questions the original assertion, and I could write that in this way:
>
> $$? \vdash A \vee B \text{ true}$$
>
> And by doing that, he obliges the original assertor to answer either that A is true that is, to assert either that A is true or that B is true, so he has a choice, and we need to have some symbol for the choice here.
>
> $$\text{(Dis)} \quad \frac{\vdash A \vee B \text{ true} \qquad ? \vdash A \vee B \text{ true}}{\vdash A \text{ true} \mid \vdash B \text{ true}}$$
>
> […] I want to give a non-logical example of this pattern, and the reason is that that can give us a feel for what is really at stake here, what it is about. For this non-logical example, think of a child, of 5 years of age or something, running to his mother saying, Mum, I can swim! Then the mother, maybe somewhat incredulously says, Oh, can you?, in which case it's a question; or she could say, Oh, show me!, in which there would be an exclamation mark instead. So that's the second step, corresponding to the second premiss. And as a result of this the child swims. Now the conclusion is no longer a linguistic act, as all the other acts given here, but the whole inference, if one call it an inference, has the character of what Aristotle called a

[4] cf. Kamlah&Lorenzen (1972), Lorenz (2009, pp. 5-23)
[5] Transcription of (Martin-Löf, 2017a, pp. 1-3 ; 7).

practical syllogism, so we could say practical inference, perhaps, because the conclusion is an action rather than an assertion, or some other kind of linguistic act.

[…] The swimming example clearly brings out that this is something that has to do with knowledge-how: knowledge how to swim

[…] So, what are the new things that we are faced with here? Well, first of all, we have a new kind of speech act, which is performed by […] either speaker and hearer, or else respondent and opponent, or proponent and opponent, as Lorenzen usually says, so that's terminology but the novelty is that we **have a new kind of speech act in addition to assertion**.

[…] So, let's call **them rules of interaction**, **in addition to inference rules in** the usual sense, which of course remain in place as we are used to them.

[…] Now let's turn to the request mood. And then it's simplest to begin directly with the rules, because the explanation is visible directly from the rules. So, the rules that involve request are these, that if someone has made an assertion, then you may question his assertion, the opponent may question his assertion.

$$\text{(Req1)} \qquad \frac{\vdash \mathrm{C}}{?\ \vdash_{\mathrm{may}} \mathrm{C}}$$

Now we have an example of a rule where we have a may. The other rule says that if we have the assertion $\vdash\ C$, and it has been challenged, then the assertor must execute his knowledge how to do C. […]

$$\text{(Req2)} \qquad \frac{\vdash \mathrm{C} \qquad ?\vdash \mathrm{C}}{\vdash_{\mathrm{must}} \mathrm{C}'}$$

The rules Req1 and Req2 do both, they condense the local rules of meaning, and they bring to the fore the normative feature of those rules.

However, a dialogician in the Lorenzen-Lorenz fashion will add players **X** and **Y** to Req2, in order to stress both that the dialogical rules do not involve inference but *interaction*, and that they constitute a new approach to the action-based background underlying Lorenzen's (1955) *Operative Logik*. This would yield the following, where we substitute the horizontal bar for an arrow:[6]

$$\text{(Req2)} \qquad \begin{array}{c} \vdash^{\mathbf{X}} C \qquad\qquad ?\ \vdash^{\mathbf{Y}}_{may}\ C \\ \Downarrow \\ ?\ \vdash^{\mathbf{X}}_{must}\ C' \end{array}$$

Such a rule does indeed condense the rules of local meaning, **but it still does not express the choices while defending or challenging**; yet it is the distribution of these choices that determines for example that the meaning of a disjunction is different from that of a conjunction: while in the former case (disjunction) the defender *must choose* a component, the latter (conjunction) requires of the challenger that, *her right to challenge is bounded to her duty to choose* the side to be requested (though she might further on request the other side). Hence, the rules for disjunction and conjunction (if we adapt them to Martin-Löf's rules) would be the following:

$$\text{(Dis)} \qquad \begin{array}{c} \vdash^{\mathbf{X}} D \qquad\qquad ?\ \vdash^{\mathbf{Y}}_{may}\ \mathrm{D} \\ \Downarrow \end{array}$$

[6] In the context of Operative Logik operations are expressed by means of arrows of the form " ⇒".

$$\vdash^{\mathbf{X}}_{must} D'$$

X must choose one of the components of *D*

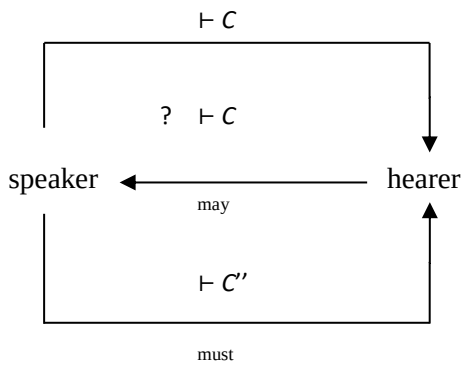
$$\text{(Conj)}\quad \vdash^{\mathbf{X}} C \qquad ?_{left} \vdash^{\mathbf{Y}}_{may} \mathrm{C} \;\Downarrow\; \vdash^{\mathbf{X}}_{must} C'$$

X must state the left component of *C*

$$\text{(Conj)}\quad \vdash^{\mathbf{X}} C \qquad ?_{right} \vdash^{\mathbf{Y}}_{may} \mathrm{C} \;\Downarrow\; ? \vdash^{\mathbf{X}}_{must} C''$$

X must state the right component of *C*

These rules can be considered as inserting in the rules the back and forth movement described by Martin-Löf (2017a, p. 8) with the following diagram:



Notice however that these rules only determine the local meaning of disjunction and conjunction, not their global meaning. For example, while classical and constructive disjunction share the same rules of local meaning, they differ at the global level of meaning: in a classical disjunction the defender may come back on the choice he made for defending his disjunction, though in a constructive disjunction this is not allowed, once a player has made a choice he must live with it.

What is more, from the Lorenzen-Lorenz point of the view these rules are **neither rules of inference** (for example rules of introduction and elimination) nor **deontic rules on top of rules of inferences**: they become rules of inference only when we focus on the choices **P** must take into consideration in order to claim that **he has a winning strategy** for the thesis.

The introduction rules on the one hand establish what **P** has to bring forward in order to develop a winning strategy, when **O** challenges it according to the local and global rules. Thus, in the case of a disjunction, **P** must *choose* and *state* one of the two components. So, **P**'s obligation lies in the fact that he must choose, and so **P**'s *duty to choose* yields the introduction rule. Compare this with the conjunction where it is the *challenger* who has the *right to choose* (and who does not state but request his choice). But in both cases, defending a disjunction and defending a conjunction, only one conclusion will be produced, not two: in the case of a

conjunction, the challenger will ask one after the other (recall that it is an interaction taking place within a dialogue where each step alternates between moves of each of the players).

The elimination rules on the other hand prescribe what moves of **O** must **P** consider in order to develop a winning strategy when **O** states the proposition at stake. So, if **O** stated a disjunction, **P** must be able to win whatever the choices of **O** be.

The rock bottom of the dialogical approach is still the play level notion of dialogue-definiteness of the proposition, whereby the deontic feature of logic is here built directly within the dialogical concept of statements about a proposition.:

> For an expression to count as a proposition A there must exist an individual play about the statement $\boldsymbol{X}\ !\ A$, in the course of which $\boldsymbol{X}$ is committed to bring forward a local reason to back that proposition, and the play reaches a final position with either win or loss after a finite number of moves according to definite local and structural rules. Lorenz (2001, p. 258).

3 The BIO Conception of Deduction: Deductive Systems + Dialogical Reading

The Built-in-Opponent (BIO) conception of deduction often takes the form of Prover-Skeptic games inspired from Sörensen and Urzyczyn (2006). In the BIO, the Prover (or Proponent) is the player who proves that the conclusion of the deduction follows from the premises, and the Skeptic (or Opponent) is the player who doubts each step of the proof and who will raise objections if he can. The game starts with Prover asking Skeptic to grant the premises, which Skeptic accepts for the sake of the argument. Then Prover states what necessarily follows from these premises, and Skeptic's role is to make sure that each new statement made by Prover clearly follows from the previous statements. Prover can thus provide counter-examples to a statement or ask clarifications (why does this follow?"). Skeptic's role is thus to check that the proof is compelling.

From this informal structure, the BIO interface can import rules from some given deductive system and yield its dialogical interpretation. One of preferred deductive systems deployed within the BIO literature is the one of traditional Natural Deduction, with the introduction and elimination rules casted in sequent style. In such a context, Prover then brings forth sequents for Skeptic to accept, and Prover is entitled to state sequents that follow in virtue of the introduction and elimination rules or in virtue of structural rules. In such a context Skeptic's main move now is that of offering sequents for Prover to prove, instead of asking more open-ended 'why' questions (though in practice, the sequents Skeptic offers can be viewed as requests for further clarification).

- Notice that with the BIO framework there is **no dialogical meaning explanation** of neither the logical constants nor of inference, but deductive meaning explanations are imported, in this case, from Gentzen-Style introduction and elimination rules, what BIO adds is an interpretation layer that should provide **perspicuity** on the deductive steps.

Let us illustrate the use of BIO recalling the standard example deployed in (Duthil Novaes & French (2018), French (2021) and Duthil Novaes (2021, pp. 78-84). The informal presentation is the following – I added the number of the moves at left for those of the Skeptic and at the right for those of the Prover, moreover, the numbers at the side of the middle line with an interrogation sign indicate the line challenged

Skeptic	Prover
	I reckon that $p \supset r$ follows from $p \supset q$ and $q \supset r$ 0
1 Yeah? Well if that's so then suppose I grant you $p \supset q$ and $q \supset r$ along with p, how are you meant to get r ? 0?	
	1? If you grant me q I can get r from $q \supset r$ (which you just granted). 2
3 But why should I grant you q? 2?	
	1? Well if you were to grant me p then I could get q from $p \supset q$ which you granted at the start. 4
5 But why should I grant you p? 4?	Because you granted it to me at the start 6

So far so good, but now when this informal dialogue is casted in a game for interpreting an inference carried out in Gentzen-style, we obtain the following presentation which should render the informal requests into moves of requesting and providing sequents

Skeptic	Prover
	$p \supset q, q \supset r \Rightarrow p \supset r,$ 0 *Prover begins by stating the main claim (thesis) to be defended* • Notice that "$p \supset r$," at the right of the arrow is what Prover claims be able to prove, if the statements at the left of the arrow are granted.
1 $p \supset q, q \supset r, p \Rightarrow r$ (?) 0? *Skeptic challenges the thesis by applying the introduction rule for implication to the right.* • Notice that "r" at the right of the arrow is what Skeptic asks Prover to prove, it is **not part** of Skeptic's statements. That is why we added the interrogation mark	
	1? $q \supset r \Rightarrow q \supset r$ 2.1 $p \supset q, p \Rightarrow q$ 2.2 *Prover recalls that Skeptic already granted $q \supset r$ and so it cannot be challenged (**this is expressed by stating the corresponding axiom**), then claims that q, can be obtained from $p \supset q$ and p – by applying the elimination rule for implication.* • Notice that Prover **does not actually infer *r***, but provides the sequents from which the conclusion of the main thesis can be inferred by applying the elimination rule for implication (once to $p \supset q$, and then to $q \supset r$)

	• Notice too that the occurrence of "q" at the right of the arrow is here to be read as what **Prover claims can be proven.**
3 $p \supset q, p \Rightarrow q$ (?) 2.2? Skeptic grants the left of the arrow and requests Prover to infer q In fact this is the only move available to the Skeptic, sinc 2.1 is and axiom.	$p \supset q \Rightarrow p \supset q$ 3.1 $p \Rightarrow p$ 3.2 *Prover recalls that Skeptic already granted not only $p \supset q$ but also p and also neither of them can be challenged (**this is expressed by stating the corresponding axioms**),* *then states (implicitly) that q, can be obtained from $p \supset q$ and p – by applying the elimination rule for implication. Again, Prover does not actually infer q.*

The fact that these form of carrying out dialogical interaction, allow both interlocutors to state the same sequents, make it not that easy to see how this moves match those moves within the informal structure of the argument. Particularly so since Prover does not actually infer the required conclusions. A particular feature of these dialogues is that they are *asymmetric*, i.e. same sequent admits different dialogical interpretations if it is stated by Skeptic or by Prover. So , if Skeptic states $\Gamma \Rightarrow p$, "they are essentially saying, if you grant all formulas in Γ can you demonstrate that p?", whereas if it is Prover who states it, Prover themselves are committed to demonstrate p, if Γ has been granted – cf. Duthil-Novaes&French (2018, p. 136). This asymmetry, indicates that BIO's dialogues adopt a strategy perspective.[7]

Choices not leading to a winning strategy are nevertheless possible, but they are to be considered as suboptimal moves while building a strategy, not as moves of their own triggered by the dialogical meaning explanation of the expressions involved. Suboptimal moves, arise here because of a not optimal use of the given inference rules

Actually, it looks as if there were two different parallel lists of sequents whereby the sequents of the Prover are ultimately grounded on axioms. These axioms, in its final (reflexive) shape, do not keep any trace that manifest that have their origin in statement granted by Skeptic. This will also lead to the result that within BIO, reflexivity is seen as an implausible feature of the consequence relation, when stated by Prover.

We will come back to these points later on which have an incidence when on the dialogical take on structural rules, but for the moment let us recall the old-fashioned Lorenzen-Lorenz style of dialogue that seems to be quite closer to the informal structure. We recall that, the dialogical meaning explanation of an implication amounts to the challenger granting the antecedent and the defender stating the consequent, similar applies to the rule governing the challenge and defence of an arrow. We deploy here one slight modification introduced by Laurent Keiff (2007), whereby the challenge of an implication (and of an arrow) consists on granting the antecedent and requesting the consequent.

I will use the Lorenzen style notation, whereby a challenge is stated at the line below the challenged move, and the defence is stated at the same line where the challenge has been launched. The example follows once more the one of the BIO-literature mentioned above, including the wording used there:

[7] cf. Clerbout&McConaughey (2022, section 3.3)

Skeptic	Prover
	$p \supset q, q \supset r \Rightarrow p \supset r$ 0 I reckon that $p \supset r$ follows from $p \supset q$ and $q \supset r$
1.1 ! $p \supset q$, 0? 1.2 ! $q \supset r$, 1.3 ! p 1.4?r Yeah? Well if that's so then suppose I grant you $p \supset q$ and $q \supset r$ along with p, how are you meant to get r ?	*sic*(9) 8 I fulfil herewith the commitment I undertook by stating the thesis.
9! "r" Fine I fulfil the commitment I undertook when I granted $q \supset r$. I thus grant r.	1.2? ! q 2.1 r? 2.2 If you grant me q I can get r from $q \supset r$ (which you just granted with 1.2).
3 *ut* q? 2.1? But why should I grant you q?	*sic*(3) 8
7! q Fine I fulfil the commitment I undertook when I granted p $\supset$q. I thus grant q indeed.	1.1? ! p 4.1 q? 4.2 Well if you were to grant me p then I could get q from $p \supset q$ which you granted at the start.
5 3 *ut* p? 4.1? But why should I grant you p?	*sic*(1.3) 6 Because you granted it to me at the start (with your move 1.3)

The text in green makes explicit the move that results from carrying out the commitments undertook. These are left implicit in the original formulation given in the BIO-literature.

This last dialogue, is patently close to the informal argument. There is though one "jump" there: the first move should consist in Skeptic granting $p \supset q$, $q \supset r$, and requesting $p \supset r$. In a subsequent move, Skeptic will challenge the implication by granting p and requesting r. The table above follows this leapfrog as present in the original presentation by Duthil Novaes&French (2018).

Be that as it may, the Lorenzen-Lorenz style of dialogical logic, seems at least to have the same the persuasive explanatory power that required by BIO. Moreover, it is neither more nor less adversarial/cooperative than the sequents interchanges displayed by the BIO-dialogues above. If other forms of BIO dialogues admit cooperative rather than adversarial moves, there seems to be nothing in principle that prevents Lorenzen-Lorenz dialogues to be shaped accordingly. With the surplus that the latter are thoroughly interactive: meaning that the

meaning of the logical constants are also governed by challenge and defences rules rather than by rules imported from a given inferential system;

Nevertheless, the cognitive point brought forward by the BIO approach, which highlights the *perspicuity* won by the dialogical setting, is indeed a fundamental trait of the different dialogical frameworks whatever the shape they take. As already mentioned, Hodges limits the fruitfulness of the dialogical framework to this kind of psychological and didactic aims, but, according to his view, it does not have any significative impact on the semantic and/or logical level. .

However, as stressed by Paul Lorenzen and Kuno Lorenz right from the start, the main claim of the dialogical framework, is precisely that the **teaching-learning situation** constitutes the core of the notion of meaning: **it is not an additional (optional) feature** to be added to an already meaningful expression – cf. Lorenzen (1960), Kamlah&Lorenzen (1972), Lorenz (1970, 2001, 2009, 2010).

Let us now explore further insights the BIO perspective on the structural rules might provide.

3 The BIO Ranking of Structural Rules, Dialogical Pluralism and some Insights from the "Old Ways".

Perspicuity and explanatory power won by a dialogical setting, understood as games of knowing why and **how**, become a salient issue when it aims at motivating the choice of some structural rules over others. Indeed, Lorenzen and Lorenz dialogical logic offered a *substructural* response (to use the terminology launched by Peter Schröder-Heister and Kosta Došen (1993) years after the birth of dialogical logic) to Quine's dictum, that when non-classical logics speak of a *deviant* connective they change the subject of conversation.[8]

The point of Quine can be put as positing the question on how to establish a meaningful dialogue on the standards of deductive rationality if those standards differ. The answer of Lorenzen and Lorenz, was to distinguish *local meaning* and *global meaning*, on their view, inherent to *Lehr-Lernsituationen*. Thus, classical and intuitionistic connectives share the same local meaning (*necessarily symmetric* or player independent), but differ only on one rule on global meaning (the play rules that generate the structural rules of the strategy level),[9] namely

[8] Recall the famous paragraph in his *Philosophy of Logic*, but similar can be found in earlier works: *To turn to a popular extravaganza, what if someone were to reject the law of non-contradiction and so accept an occasional sentence and its negation as both true ? An answer one hears is that this would vitiate all science. Any conjunction of the form 'p∧~p' logically implies every sentence whatever ; therefore, acceptance of one sentence and its negation as true would commit us to accepting every sentence as true, and thus forfeiting all distinction between true and false. In an answer to this answer, one hears that such a full-width trivialization could perhaps be staved off by making compensatory adjustments to block this indiscriminate deducibility of all sentences from an inconsistency. Perhaps, it is suggested, we can so rig our new logic that it will isolate its contradictions and contain them. My view of this dialogue is that neither party knows what he is talking about. They think they are talking about negation, '~', 'not' ; but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form 'p∧~p' as true, and stopped regarding such sentences as implying all others. ere, evidently, is the deviant logician's predicament : when he tries to deny the doctrine he only changes the subject*. Quine (1986, second edition, p. 81)

[9] This, as discussed by Rahman (2022), could also be linked to the notion of "focus" in Geach (1967), whereby two interlocutors share the same "focus" concerning their respective intentional acts – rather than reference, but

whereas in classical logic the global ruling allows the Proponent to, so to say, *redo* his response to a challenge (or ignore the last challenge launched by the Opponent and answer to a previous one), such kind of backwards moves are not allowed to the Opponent when in an intuitionistic setting – which generates the rejection of weakening to the right at the proof-theoretical level.

This, among other philosophical challenges, motivated the program for *Dialogical Pluralism* incepted in Saarbrücken and Lille by 1997, which extended the scope of the original project of Lorenzen and Lorenz (limited to classical and intuitionistic logic, and some early proposals towards modal logics), to include a large number of non-classical logics including free-logics, modal logics, temporal logics, connexive logic, paraconsistent logics, relevant logics and linear logic.[10]

Let us see how The BIO stance approaches to such challenges, and how this approach backs their own brand of Dialogical Pluralism – cf. French (2015), Duthil Novaes&French (2018), French (2021).

The BIO proposal is to offer a kind of pluralism that is not the result of ad hoc solutions to some counterintuitive results, such as paradoxes, but rather rooted on general features of argumentative architecture shaped by the different structural rules, whereby the switching from adversarial to cooperative games might provide a parameter for rejecting or endorsing a structural rule. I will start from what is the most plausible in the BIO ranking to the less plausible of structural rules in that ranking.

TRANSITIVITY/CUT:

In relation to transitivity, or more precisely, *Cut*, Duthil-Novaes&French(2018), rightly point out, that *Cut*, allows a concatenation of inferential steps: If *A* offers explanatory power for stating *B*, and *B* for *C*, *A* should have the power to explain *C*. Thus rejecting, *Cut* would amount to deny transmission of explanatory power and perspicuity. So far so good, however, notice that

1. The BIO approach is strategic. Indeed, what *Cut* concatenates are winning strategies: or even better it allows composing winning strategies in an effective way, that is, by construction. If we switch to the play level, the dialogical perspicuity won during a particular play, might not be transmissible, at least not in the same way, since during the play, some particular moves specific to that play (such as order of a challenge), might not be transmissible, because this order might be irrelevant from the strategic point of view. Perhaps we might need to distinguish Play level Perspicuity from Strategy level Perspicuity.

2. The most important proof-theoretical feature of *Cut* within standard logics, is its *admissibility*. Indeed *Zulässigkeit*, another fundamental notion invented by Paul Lorenzen (1955), determines if a new rule can or not be integrated to a given inferential system in an effective way.[11] Ebbinghaus (1964) observed that Lorenzen notion was

differ on the predicates attributed to this focus. In other word, the intentional object of the intentional acts of both interlocutors might be the same, despite the fact that the content of those acts differ.

[10] See Rahman&Keiff (2005), Keiff (2007), Rückert (2011), Redmond&Fontaine(2011).

[11] For a perlucid presentation on the origins of *Zulässigkeit* see Schröder-Heister (2008), where he also explains why Lorenzen abandonned the *Operative Logik* and turned into the dialogical perspective. In that paper Schröder-Heister explains admissibility in the following terms : *R is admissible in a calculus K, if every application of R can be eliminated from every derivation in K+R. The implicational relation between existential statements*

inspired by Aristotle's reduction methods to the "perfect moods" of the first figure.[12] In fact, Ebbinghaus works thoroughly out, how Aristotle's reduction can be casted into Lorenzen's *Operative Logik*, whereby the syllogistic moods are understood as inference rules. In such a context, a rule of a non-perfect syllogism is admissible if its reduction yields the dialectical meaning explanations that constitute the rules of the perfect moods of the first figure. According to this view, Aristotelian reduction is not about reducing, to more "evident moods", but rather by tracking back the dialogical meaning explanations of the quantifiers.[13] Lorenzen himself, before the dialogical turn, thought that admissibility statements as such provide sufficient conditions for meaning and knowledge. Accordingly, it is the admissibility of *Cut*, that provides both meaning and justification of an inference rule for sequents. In a dialogical setting, admissibility amounts to the normalization of winning strategies (at the level of strategies) via the *Socratic Rule* – Rahman et al. (2018, pp. 176-179) – and, at the play level it amounts to reducing the rules for the *Analysis of Local Reasons* to the Rules for the *Synthesis of Local Reasons* – cf. Rahman (2019, pp. 40-41).

Interesting is the fact that Fermüller (2021, p. 136, footnote 5), who proposes to understand propositions, in the context of human-server dialogue, as information-packs piled in *stacks*, develops a sequent calculus whereby *Cut* is not admissible, since once the first layer of a stack has been used, it is available anymore, so the concatenation of sequents might not fulfil *Cut*. This, also assumes, the failure of contraction, which constitutes the subject of our next discussion.

CONTRACTION TO THE LEFT:

Concerning the Dialogical interpretation of Contraction, Duthil Novaes&French (2018), adopt here a logical rather than psychological or cognitive view:

> In other words, discursive commitments created by assertions are prima facie not the kind of resource that is no longer available once 'used' (say, if one of the participants refers back to a commitment previously undertaken, either by herself or by someone else, to draw an inference), and thus the number of 'copies' of such commitments seems irrelevant. This means that, in principle, there is no obvious rationale for restricting contraction from a dialogical perspective, given the nature of discursive commitment as a resource that does not expire or gets 'used up' (unlike bank notes to buy cigarettes, as in Girard's classical example to motivate linear logic). Duthil Novaes&French (2018, pp. 144-145).

It is true that since Girard's (1987) invention of Linear Logic, contraction has been linked to the deductive handling of limited resources. However, *resources bounded reasoning* can also be conceived as rendering some form of radical constructivism whereby *epistemic cognitive constraints* on the transmission of information plays a salient role – see Marion&Sadrzadeh (2004), Rahman (2002). Moreover, since the landmark work of Blass (1992), a dialogical interpretation of linear logic has been developed based on human-server interaction. A recent further development of Blass's interpretation is Fermüller's (2021) work mentioned above. Let me first recall, how contraction is manifested in Lorenzen-Lorenz dialogues before I go to some more general philosophical remarks.

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expressed in $\vdash_{K+R}A$ *implies* $\vdash_{R}A$ *is reduced to the insight that a certain procedure reduces any given derivation in K+R in such a way that the resulting derivation does no longer use R. According to Lorenzen, this is the sort of insight (evidence) on which constructive logic and mathematics is based. It goes beyond the insight that something is derivable in K, but is still something which has a "definite" meaning*.

[12] See Lion&Rahman(2018), Crubellier et al. (2019), McConaughey (2021).

[13] In Ebbinghaus (1964), these are still presented as sequent calculus rules, but at the end of his work he points out that they should be understood in a dialogical setting.

Finiteness, Repetition Rank and Dialogue Definiteness. One important feature of Lorenzen-Lorenz conception of proposition and statement is its finiteness: To paraphrase Lorenz (2001, p. 258): for an expression to count as a proposition *A* there must exist an individual play about the statement **X** ! *A*, in the course of which the interlocutor **X** is committed to bring forward a local reason to back that proposition, in a play which must reach a final position after a **finite number** of moves according to definite particle and structural rules. The inceptors of the dialogical framework observed that if language-games are to be conceived as mediators of meaning carried out by social interaction, these language-games must be games actually playable by human beings: it must be the case that we **can actually perform them.** This is what Lorenz's notion of **dialogue-definiteness** is about.[14] Technically speaking, finiteness is implemented by the explicit introduction of a repetition rank which bounds **the number of times a move can be challenged and/or defended**, rather than producing copies of the moves at stake. As pointed out by Lorenz (2001, p. 260) the bounds fixed by **the repetition rank is the play-level source of the strategy level rule for contraction to the left**. Clerbout (2014a,b) delved into the metalogical consequences of the repetition rank and showed how it is related to the semi undecidability of first-order logic – cf. Clerbout (2014c).
One important general philosophical result of Clerbout work is that though every play must be finite, it does not prevent the existence of an infinite number of them. This also answers to a usual misunderstanding of Lorenzen-Lorenz dialogues: **most of the them, are in principle, 0-sume plays, but winning or loosing a play does not preclude strategic undecidability.** It is again the issue on the play level: many criticisms forget that the 0-sum feature of plays does not necessarily transmit to the strategy level: this is the very point of the dialogical interpretation of constructive logic after all.

Restricting Contraction and What the Dialogical Framework brings to Linear Logic. One result of giving up the unrestricted use of contraction is that two pairs for conjunction and disjunction come to the fore: multiplicative and additive conjunction and disjunction. One of the most important properties of the dialogical local rules for the multiplicatives is that they establish that every component of a conjunction (or a disjunction) should be used – in contrast to the rules for the additives where one component can be left out. Thus, the usual truth-functional distinction between conjunction (both components have to be true) and disjunction (at least one of the components has to be true) is of no help here. In other words, choice, the fundamental trait of dialogues, shapes the meaning of the linear connective. So, for instance, while in the multiplicative conjunction is the challenger who chooses the dialogical context, in the multiplicative disjunction it is the defender who can make that choice – cf. Rahman&Rückert (2011b). Another feature of the dialogical approach to Linear logic is that it proposes plays with some form of imperfect information, whereby one player triggers the switch to another parallel play while keeping the information of the original play, but the second player has at its disposal only the information made explicit in the new parallel play: she does not necessarily know what the first player moved in the first play.

- Summing up, contraction is indeed crucial for the dialogical understanding of what a proposition is. To put it in other terms, a total unrestricted contraction within dialogical plays (i.e. with no explicit or implicit repetition rank) would contravene the very dialogical notion of proposition and its role in a language game that mediates meaning

[14] For a discussion see Rahman et al. (2018, pp. 278-282).

by the means of rules at the reach of human capability. Does an extreme form of restriction make sense, such as in the multiplicative connectives of linear logic? Well, as already mentioned there are plausible contexts where the focus on epistemic constraints might makes such kind of dialogical games a fruitful endeavour.

- Moreover, the different connectives and operators of linear logic allow to switch from more restrictive to looser epistemic constraints – in the dialogical setting such a switch is governed by the choice on the repetition rank. In fact, one of the deep challenges of Girard's approach is to abandon the substructural approach for an operator-driven setting which encodes the structural properties: adding for example a repetition operator to a negation allows switching from classical to intuitionistic negation. Such operators, are to be thought, I suggest as *pragmatic enrichment processes,* in the sense of Recanati: they do not *saturate* an incomplete expression but is added to it, in a way similar to the adding of an adverbial expression to a verb.
- An interesting proof-theoretical feature of arguments with restricted contraction or explicit repetition rank, is that they can show the degree of dependence of a proof upon an axiom. Indeed, assume that for some proof the axiom of choice is used twice, it makes perfect sense to ask if it one can develop a proof with only one use of the axiom of choice – this is a paraphrase of Mares&Paoli (2014, p. 452f). So, it can be very useful, to make the repetition rank at use explicit – to mention just another example, recall that in order to prove the validity of the double negation of third excluded in a dialogical setting for constructive logic, the Proponent needs to challenge twice Opponent's statement $\sim(A\vee\sim A)$: if the repetition rank were one, then Proponent would not be able to win the thesis $\sim\sim(A\vee\sim A)$.

WEAKENING TO THE LEFT:

Weakening to the left is one of the main structural rules the BIO approach is willing to give up or at least in the context of some kind of cooperative games. Given, that Duthil Novaes&French (2018) take it that the discussion of this rule provides important insights on the dialogical cognitive understanding of proofs, allow me quite long quotes – the quotes stem from the early joint paper since in Duthil Novaes (2021) one can find the same formulations:

> Left Weakening is quite revealing in terms of basic structural properties of proofs understood as dialogues. […]? Informally, in our interpretation Left Weakening says that, having granted *A* and *B* as premises, and then *C* as following necessarily from *A* and *B*, an interlocutor cannot retract commitment to *C* if she grants an additional *D*, for any *D*. Thus, the permissibility of Left Weakening follows straightforwardly from the property of necessary truth preservation and its sibling, the property of monotonicity. […]. Prima facie, it seems that Left Weakening is irreproachable from the point of view of the Prover-Skeptic dialogues we are dealing with here: if Skeptic has granted *A* and *B*, and they together entail *C*, then no additional premise *D* will defeat his obligation to concede *C*. In other words, with respect to *Skeptic*'s moves, it seems that no restriction on Left Weakening can be justified. Duthil Novaes&French (2018, p. 143).

What is a bit puzzling here is that in the BIO setting for sequents mentioned above, when Skeptic brings into the play a sequent such as $A, B \Rightarrow C$, according to the intended BIO interpretation, the Skeptic is not really granting the proposition at the right of the sequent, namely *C*, but challenging Prover to do so. This also applies to stating $A, B, D \Rightarrow C$ *aftte having stated before* $A, B \Rightarrow C$,. Perhaps, what is meant, is that if Prover shows that *C* is the case, provided Skeptic grants *A*, *B*, then Skeptic must also grant *C*, given the weakened version of the original sequent, however in the plays described by the BIO setting so far as I understood Skeptic will not explicitly make a move granting the right of an arrow. So, perhaps what the quote above assumes is the implicit granting of Skeptic. This difficulty seems to a be result of

BIO setting that does not really deliver player independent moves for sequents. Let us pursue the discussion further on:

> But what about *Prover*? Can Prover avail herself of the possibilities offered by Left Weakening? At this point, it becomes crucial to consider dialogues that are to different degrees adversarial or cooperative. In purely adversarial dialogues, Prover seeks to force (coerce) Skeptic to grant the final conclusion at all cost, and this is the main or perhaps even the sole goal of such interactions […]. In contrast, in a more cooperative situation, Prover may want Skeptic not to be coerced into granting something, but for Skeptic truly to understand *why* the conclusion follows from the premises. […]. In an adversarial setting, Left Weakening in fact becomes *strategically advantageous* for Prover. If the goal is to force Skeptic to grant something he wants to avoid granting (as this will mean he will lose and Prover will win), then it is advantageous for Prover to use distracting maneuvers such as asking for premises that are in fact not relevant to the main conclusion to be drawn eventually. If there are no restrictions on asking for irrelevant premises, Prover can exploit this feature of the game to her advantage, as this may confuse Skeptic by causing information overload. Duthil Novaes&French (2018, p. 144).

If I rightly understood in cooperative contexts, Prover might wish to elucidate why the premises lead to the conclusion by restricting the steps to those that are relevant for proving that conclusion. So far so good, but notice that weakening, because it is a strategic rule after all, assumes that the conclusion has already been achieved - recall their own formulation of weakening $\Gamma \Rightarrow C \vdash \Gamma, A \Rightarrow C$ – Duthil Novaes (2018, p. 143). So, it is not clear what the *distracting* moves should accomplish, if the conclusion has been already proved.

I think that the origin of the difficulty here is that the BIO setting moves quite loosely between the play and the strategy level. What we need, is to have a global rule that allows at the play level the degree of "irrelevance" that is manifested at the strategy level by the rule of weakening. Let me paraphrase here once more Lorenz himself:

- The source play level source of the strategic rule for weakening to the left – Lorenz calls it *Thinning* - is the global rule that allows the Proponent to challenge any of the statements granted by the Opponent. Lorenz (2001, p. 259 and 260).

Given this global rule, which does not assume that the conclusion has been proved or even granted, we can think of settings where, in some sense, only relevant moves are kept. This was the motivation for the formulation of several forms of dialogical settings for relevant logic, including paraconsistent ones.[15]

The qualification in *some sense*, should indicate that as we will see, content-relevance is not really an issue that is achieved by restricting the use of the global rule that yield weakening, but by making local reasons explicit. I will come to this point in the last section of the paper.

REFLEXIVITY:

According to BIO analysis reflexivity is the less plausible of the structural rules under consideration – Duthil Novaes&French (2018, pp. 146-147). In short, stating the sequent $A \Rightarrow A$, is as logical valid as trivial and uninformative as it can be.

There is also the remark that Arstotle's syllogistic rules are based on an irreflexive consequence relation. The source of this remark is Duncombe's (2014) paper who indeed

[15] Such as in Rahman&Carnielli (2000), Rahman&Rückert (2011a), Beirlaen&Fontaine (2016) and Barrio,Clerbout&Rahman (2020).

claims that Aristotle's consequence relation is irreflexive. Let me stress two points in relation to the use of Reflexivity in a dialogical setting:

1) Bottom up uses of Reflexivity: In general, Reflexivity is in a dialogical setting **the end of an interaction** not the start.[16] More precisely, the very point of the dialectical stance since its beginnings, say Plato's *Gorgias* 472b-c, up to Aristotle's meaning explanations of the Quantifiers in the *Topics* and to the role of *ecthesis* in the syllogism, is that logical grounding is based on what Marion&Rückert (2016) call the *Socratic Rule*, whereby the Proponent grounds his thesis on elementary statements constitutive of the thesis and granted by the Opponent.[17]

In a dialogical setting, the interaction that starts bottom up, **at the play level**, by testing the thesis that launched the game, ends, if successful, by forcing the Opponent to grant what she challenged.

In short what the strategy level Reflexivity sequent expresses is that at the play level the Opponent has been forced to grant *A* and this allows Proponent to state the requested conclusion.[18]

In a dialectical setting, statements such as $A \Rightarrow A$ do not come out of the blue, but are the result of a play on a thesis involving possible initial concessions (premises). I think the problem with the BIO analysis here is that the setting does not keep track of the interactive moves that originated the statements.

More generally, the Socratic Rule, is arguably one of the sources of formal but contentual reasoning, and at the strategy level it is the source of judgmental equality (which is fundamental for normalization procedures). In fact, it makes sense to distinguish the *formal rule* from the *Socratic rule* This brings me to my second remark.

2 The Socratic rule and the formal rule: In dialogical logic Lorenzen and Lorenz distinguished between *material* and *formal dialogues*. In the latter the grounding of elementary statements, is based on what in game theory is called *copy-cat move*: Proponent when asked to ground an elementary statement such as **P** ! 5=4, can show that Opponent has been already forced to grant it, due to possible other previous moves granted before: so, **P** will state, whatever reasons were behind your granting 5=4, I can do the same. Helge Rückert suggested that formal dialogues can be seen as a kind of games of imperfect information, where **O** never makes explicit the reasons behind he elementary statements. In such kind of games, the only possible move for **P** is to overtake the whole statement. These kind of moves, are formal, and perhaps even formalistic, and lead to a notion of validity **independent of any model**, rather than true in every model – Helge Rückert claims that, from a dialogical point of view, *independence* expresses the very logical sense of the German expression *Geltung*. To stress the point of Duthil Novaes&Franch (2018): what about if the thesis is the sequent **P** ! 5=4 ⇒

[16] The bottom up development reminds us Frege's famous passage in the *Grundlagen der Arithmetik*:
Es kommt nun darauf an den Beweis zu finden und ihn bis auf Urwahrheiten zurückzuverfolgen.
(The task now is to find the demonstration and trace it back to basic truths). Frege (1884, section 3, p. 4).

[17] Attributing to Aristotle's syllogistic rules an irreflexive consequence relation is similar to assessing that the elimination rule for implication is (or perhaps should be) irreflexive because instances of an implication where the same proposition occurs as antecedent and consequent makes the application of *modus ponendo ponens* fruitless.

[18] For thorough studies on the issue in Aristotle, see Crubellier et al. (2018) and McConaughey (2021).

5=4 ? – assuming formal dialogues. Well, the response will be based on a purely formalist view on logical validity. Material truth, is not the issue here, thus in contexts where content matters, rather than logical *Geltung*, the use of the formal rule is indeed not at all helpful. So, in the context of a strategy composed out of formal dialogues, qualifying the use of Reflexivity as *truth-preservation*, seems to constitute an abuse of language.
In contrast, if *material* dialogues are at work, the Opponent will be taken to the task to make explicit the reasons behind the granting the elementary statements, and these reasons, can be contested in a formation game. Of course, Opponent can launch such a formation game just before the actual testing of the thesis has been launched. In fact, only after a common ground on the meaning of the statement involved has been achieved, can this testing start – Marion&Rückert (2018) points out that this is in fact the procedure practiced by Aristotle in the dialectical games of his *Topics*, particularly so in *Topics* 157a34–37 and 160b1–6, where he sets the dialectical meaning explanation of the quantifiers – see too Crubellier et (2018), McConaughey (2021, chapter 4.2.2).

But how to keep track of the interactive source of a statement? Well, this requires enriching the standard setting with a language that puts into the object language the task that constitutes the content of a statement. At the play level we call this *local reason*, at the strategy level, it is called *strategic reason*: the latter encode the way how the tasks have to be accomplished in order to produce a valid inference.

This takes us to the next section, but let us briefly come back to the ranking and its criteria.

On our view the ranking is related to going from metalogical to meaning considerations.

- ***Cut*** does definitely require a strategy point of view: it allows to come back to previous winning strategies in order to compose further ones.
- **Contraction**, at the strategy level is linked to important metalogical results. In the dialogical setting of Lorenzen-Lorenz games, the particular form these metalogical results take, stem from the play level requirement the dialogical meaning explanations to be finite – though from the strategy level perspective it might trigger an infinite number of plays.
- **Weakening**, as understood by relevantists, is about inferences preserving content dependences. However, if worries on content dependences come to the fore, then as discussed below, rather than tinkering at the inference level, we should revise before the formation rules of the dialogical meaning explanations at stake. Revisions involving meaning explanations take place foremost at the play level.
- **Reflexivity** requires distinguishing between *material dialogue* and *formal dialogues*. The primary play level source is the *Socratic Rule* at work in material dialogues which governs the dialogical meaning explanations of those elementary statements that result from challenging the thesis of a play. The use of the formal or copy-cat Rule which generates formal dialogues, leads to the dialogical notion of logical validity (*Geltung*) which renders a winning strategy that is independent of any specific content attributed to the elementary statements at work. In fact, formal strategies can be built out of material ones if the content of the latter is abstracted away to the level of function types.

Does the parameter of adversarial versus cooperative games help? So far as I can see, its application focuses on weakening, perhaps this parameter is also at stake when rejecting the plausibility of Reflexivity.

Am not sure if it is very helpful, after all, when Prover restricts to “relevant” moves, this can not only be seen as choosing to be cooperative but it can also be seen as being forced by Skeptic to test the thesis under tougher conditions. Compare with the effect of dropping weakening to the right, which imposes Prover to produce constructive proofs.

Nevertheless, the focus on content formation is crucial and its dynamics is a fascinating issue that requires thorough exploration. In the following section I will suggest a framework, where on my view, such an exploration should be carried out

4 *Immanent Reasoning*: A Plaidoyer for the New Old Ways

4.1 Local and Strategic Reasons

Dialogues are games of giving and asking for reasons; yet, in the standard dialogical framework, the reasons for each statement are left implicit and do not appear in the notation of the statement. Statements of the form

$$\mathbf{X}\ !\ A$$

where A is proposition, are to be read as

Player **X** states A

Clearly, these statements do not give any information on the *reasons* behind this statement. The *Immanent Reasoning* framework imports Martin-Löf’s (1984) Constructive Type Theory (CTT) form of a judgement in order to make these reasons explicit. Fully developed, statements thus have the following form:

$$\mathbf{X}\ a\colon A$$

Where, when stated at the play level, “a” stands for the *local reason* entitles the stating of A .

In this fashion, the reasons one has for making a statement are specified at the object-language level. Notice that in such a setting, *giving a reason*, for stating, say, an elementary proposition, amounts to

a) producing the *local reason*, that is, the task “a” to be accomplished and that provides the content of the statement;
b) Bringing to the fore other statements upon which A might be dependent
   For instance: $b(x)\colon A[x]\ \ (x\colon B)$, which presupposes $A[x]\colon prop\ \ (x\colon B)$

The latter can in a further step, be formulated as involving a quantifier or an implication:

$$\frac{B\colon prop \qquad A[x]\colon prop\ \ (x\colon B)}{(\forall\, x\colon B)\, A[x]\colon prop} \qquad\qquad \frac{B\colon prop \qquad A\colon prop}{A \supset B\colon prop}$$

- Since other frameworks lack such distinctions, quite often *grounding* is mostly reduced to implicative links or logical deductive ones. Robert Brandom, who seems to be aware of the shortcomings of standard frameworks, speaks of *material inferences*.

In a CTT context, "material inferences", are governed by formation and introduction rules that yield the canonical elements that constitute the proposition/set/type at stake. CTT rules, however are always set at the strategy level, the *Immanent Reasoning* (IR) framework distinguishes *local* reasons and *strategic* reasons: as we will see, within the IR setting, revision of meaning concern foremost the play level.

Local reasons are brought forth in particular plays, whereas strategic reasons are a recapitulation of all the possible plays: local reasons provide relevant and sufficient means for winning a play, but in general a local reason does not provide the appropriate elements for constituting a winning strategy.

Strategic reasons entitle to make assertions, i.e., provide relevant and sufficient means for constituting a winning strategy justifying an assertion. The expression "statement" is used for the posits that have not yet reached the level of a justified assertion.

When the reason is not explicit, the exclamation mark "!"marks the presence of an *implicit* reason. According to this view, in the standard dialogical framework, each statement has an implicit reason backing it. This implicit reason may be made explicit in IR (but it does not have to).

Let me now without ado introduce both the IR rules for local meaning those for composing strategies:

Synthesis rules for local reasons

	Move	Challenge	Defence
Conjunction	$\mathbf{X}\,!\ A \wedge B$	$\mathbf{Y}\,?\,L^{\wedge}$ or $\mathbf{Y}\,?\,R^{\wedge}$	$\mathbf{X}\,p_1 : A$ (resp.) $\mathbf{X}\,p_2 : B$
Existential quantification	$\mathbf{X}\,!\,(\exists x : A)B(x)$	$\mathbf{Y}\,?\,L^{\exists}$ or $\mathbf{Y}\,?\,R^{\exists}$	$\mathbf{X}\,p_1 : A$ (resp.) $\mathbf{X}\,p_2 : B(p_1)$
Disjunction	$\mathbf{X}\,!\,A \vee B$	$\mathbf{Y}\,?^{\vee}$	$\mathbf{X}\,p_1 : A$ or $\mathbf{X}\,p_2 : B$
Implication	$\mathbf{X}\,!\ A \supset B$	$\mathbf{Y}\,p_1 : A$	$\mathbf{X}\,p_2 : B$
Universal quantification	$\mathbf{X}\,!\,(\forall x : A)B(x)$	$\mathbf{Y}\,p_1 : A$	$\mathbf{X}\,p_2 : B(p_1)$

Negation	**X** ! $\neg A$ Also expressed as **X** ! $A \supset \bot$	**Y** $p_1 : A$	**X** $p_2 : \bot$

Analysis rules for local reasons

	Move	Challenge	Defence
Conjunction	**X** $p : A \wedge B$	**Y** ? $L^{\wedge}$ or **Y** ? $R^{\wedge}$	**X** $L^{\wedge}(p) \ : A$ (resp.) **X** $R^{\wedge}(p) \ : B$
Existential quantification	**X** $p : (\exists x : A)B(x)$	**Y** ? $L^{\exists}$ or **Y** ? $R^{\exists}$	**X** $L^{\exists}(p) \ : A$ (resp.) **X** $R^{\exists}(p) \ : B(L^{\exists}(p) \ )$
Disjunction	**X** $p \ : A \vee B$	**Y** ?$^{\vee}$	**X** $L^{\vee}(p \ ) \ : A$ or **X** $R^{\vee}(p \ ) \ : B$
Implication	**X** $p : A \supset B$	**Y** $L^{\supset}(p) \ : A$	**X** $R^{\supset}(p) \ : B$
Universal quantification	**X** $p : (\forall x : A)B(x)$	**Y** $L^{\forall}(p) \ : A$	**X** $R^{\forall}(p) \ : B(L^{\forall}(p) \ )$
Negation	**X** $p : \neg A$ Also expressed as **X** $p : A \supset \bot$	**Y** $L^{\neg}(p) \ : A$ **Y** $L^{\supset}(p) \ : A$	**X** $R^{\neg}(p) \ : \bot$ **X** $R^{\supset}(p) \ : \bot$
Falsum	**X** $p \ : \bot$ (given **Y** ! ***C***)	**Y** $you\ gave\ up\ (n) : C$[19]	—

Synthesis of strategic reasons for P:

	Move	Synthesis of local reasons		Synthesis of strategic reasons
		Challenge	Defence	

[19] In fact, since falsum involves an elementary proposition rather than a connective it should be regulated by structural rules. See structural rule SR7 below. The reading of stating **falsum** as giving up stems from (Keiff, 2007).

				Canonical Argumentation form
Conjunction	**P** ! $A \wedge B$	**O** ? $L^{\wedge}$ or **O** ? $R^{\wedge}$	**P** $p_1 : A$ (resp.) **P** $p_2 : B$	**P** $< p_1, p_2 >: A \wedge B$
Existential quantification	**P**! $(\exists x : A)B(x)$	**O** ? $L^{\exists}$ or **O** ? $R^{\exists}$	**P** $p_1 : A$ (resp.) **P** $p_2 : B(p_1)$	**P** $< p_1, p_2 >: (\exists x : A)B(x)$
Disjunction	**P** ! $A \vee B$	**O** ?$^{\vee}$	**P** $p_1 : A$ or **P** $p_2 : B$	**P** $p_1 : A \vee B$ or **P** $p_2 : A \vee B$
Implication	**P** ! $A \supset B$	**O** $p_1 : A$	**P** $p_2 : B$	**P** $p_j^{\mathbf{P}}[\![p_i^{\mathbf{O}}]\!] : A \supset B$ (where $p_j^{\mathbf{P}}$:B and $p_i^{\mathbf{O}}$:A)
Universal quantification	**P** ! $(\forall x : A)B(x)$	**O** $p_1 : A$	**P** $p_2 : B(p_1)$	**P** $p_j^{\mathbf{P}}[\![p_i^{\mathbf{O}}]\!] : (\forall x : A)B(x)$ (where $p_j^{\mathbf{P}}$:$B(p_i^{\mathbf{O}})$ and $p_i^{\mathbf{O}}$:A)
Negation	**P** ! $A \supset \perp$	**O** $p_1 : A$	**P** $you\ gave\ up(n)$: ...	**P** $you\ gave\ up(n)$ $[\![p_i^{\mathbf{O}}]\!] : A \supset \perp$

Remarks:

- For implication (and universal quantifier), the strategy reason encodes the following instruction: *for any choice of* **O** for the antecedent, the is a way to produce strategic object for the consequent – in CTT tis corresponds to lamda-abstract of the function from the antecedent to the consequent. Notice that the strategic object does not constitute an *actual* reason, but a method that shows how to accomplish the task at stake.
- For negation, we must bear in mind that we are considering **P**-strategies, that is, plays in which **P** wins, and we are not providing particle rules with a proper challenge and defence, but we are adopting a strategy perspective on the reason behind a statement; thus the response to an **O**-challenge on a negation cannot be **P** ! $\perp$, which would amount to **P** losing; this statement "**P** $you\ gave\ up(n)$ $[\![p_i^{\mathbf{O}}]\!] : A \supset \perp$" indicates that **P**'s strategic reason for the negation is based on **O**'s move n (where **O** is forced to state $\perp$), move n which is dependent upon **O**'s choice p_i as local reason for the antecedent of the negation. Recall that in the dialogical setting the meaning of the negation amounts to switch of the roles of challenger and defender.

4.2 Brief Remarks on the Dynamics of Disagreements on Content

Let us assume the sequent

$$A \wedge B \Rightarrow A$$

If we make the strategic object explicit we have

$$z: A \wedge B \Rightarrow L\hat{}(z): A$$

$$\text{whereby: } L\hat{}(z) = z_1: A$$

Or keeping trace of the interaction

$$z: A \wedge B \Rightarrow L(z)^{\mathbf{P}}: A$$
$$\text{whereby: } L\hat{}(z)^{\mathbf{P}} = z_1{}^{\mathbf{O}}: A$$

The encoded instruction for the Proponent should be clear: accomplish the task of extracting a strategic reason for the left component of the conjunction in such a way that its result equals the Opponents own strategic object for *A*. Notice, that if we take it that this resulted from weakening to the left, the dependence of the statement to the right of the arrow upon *z* is not affected neither logically (because of weakening) nor contentually, since the explicit indication of the strategic object, remains the same, and this remains the same after further uses of weakening such as:

$$z: A \wedge B \Rightarrow L(z)^{\mathbf{P}}: A$$

$$z: A \wedge B, C \Rightarrow L(z)^{\mathbf{P}}: A$$
$$L\hat{}(z)^{\mathbf{P}} = z_1{}^{\mathbf{O}}: A$$

In fact, if such kind of relevance is what we are looking for, we do need neither to get rid of the strategy level weakening to the left nor do we need to impose at the play level, restrictions on the use of statements granted by the Opponent. In fact, one can use this kind of strategy for solving most of the paradoxes that affect standard deontic logic – see Rahman, Granström& Farjami (2019), Rahman, Zidani&Young (2022).

The following modification to the standard dialogical rules might put the point clearer:

Assume **P**'s thesis is

$$! \, A \wedge B \Rightarrow A$$

Then **O** can challenge it not only by granting the left of the arrow, but also by granting an additional one of her choice, say, y: $\sim(A \wedge B)$. Still, **P** can forget the additional premise for his response

P ! $A \wedge B \Rightarrow A$
O ! z: $A \wedge B$
O ! y: $\sim(A \wedge B)$
O ? A
P ! $L\hat{}(z) = z_1{}^{\mathbf{O}}: A$

P might also use *exfalso* but such a move, from the play level point of view, is more costly and indeed less informative concerning explanatory purposes involving content:[20]

0 **P** ! $A \wedge B \Rightarrow A$
  1.1 **O** ! z: $A \wedge B$
  1.2 **O** ! y: $\sim(A \wedge B)$
  1.3 **O** ? A (?0)
2 **P** ! z: $A \wedge B$ (?1.2)
  3 **O** $\perp$
4 **P** ! *you(gave-up-3)*: A

Thus, in this second response, **P** backs up A, by indicating that **O** gave up, rather than producing the reason for ! A.

Notice that if we add to the framework the global rule that **P** must give priority to local reasons that do not have the form *you(gave-up-n)*: A, then a kind of a*daptive logic* results.

Minimal logic will result from allowing **P** to use the local reason *you(gave-up-n)* only for **P** himself stating **P** ! *you(gave-up-n)*: $\perp$. If more extreme paraconsistent wishes are to be fulfilled then moves as *you(gave-up-n)*: A can be totally forbidden or arranged by a kind of order given by the complexity of the statement under scrutiny.

However, we could go a step deeper and ask for the meaning formation of A and contest for example that z_1 is a suitable local reason of A, or at the strategy level, contest that z_1 is a canonical element of A.

If this takes place, then we switch from the discussion on the structural rules to le level of meaning explanations. As so often, in the context of mathematics, the discussion is simpler or at least clear enough: it amounts to examine the meaning explanations for the set involved. However, in contexts, outside mathematics, the point is how to develop a common ground starting from a disagreement on the content a stake.

An important step in this direction has been accomplished by Clément Lion (2019, 2023), who inspired by Heinzmann (2013) semiotic interpretation, declines Lorenzen *Orthosprache*, as the intertwining of several levels of dialectical interaction from an iconic to a symbolic level, and that might involve misunderstandings, until some "bar" of common ground of meaning has been settled – though this settlement is not permanent and can launch the search for a new bar. Notice that, even if some initial common ground for discussion has been agreed, this does not assume truth-functionality – the underlying logic is shaped by constructivist tenets, at least at its start.

At this point, I would like to suggest that reading Lorenzen-Lorenz dialogues under the lens of Islamicate dialectic is a promising venue for studying the interactive emergence of meaning, that is, for studying the dialectical procedures that give rise to the meaning explanations of the expressions under scrutiny.

[20] I simplified notation and shortened the development of the moves.

In fact, such a path was already initiated by Larry Miller in his Princeton dissertation of 1984 and printed in 2021, particularly in his study of Shams al-Dīn al-Samarqandīs famous *Risālafī ādāb al-baḥth* (*Treatise on the protocols of dialectical inquiry*). However, it went the other way around: Miller deployed dialogues, mainly in the shape given by Lorenz, in order to elucidate the structure of Islamicate dialectic, I think the time is ripe to explore both sides of the coin.

In point of fact, the main aim of Medieval Islamic dialectics, let me recall, was precisely to develop a framework for disputes concerning meaning. The idea behind was, to use Walter E. Young's (2017) words to develop a dynamic system for *forging concepts through interaction*. The place of birth of such dialectical theories was, not surprisingly, the normative field of legal reasoning, where norms must be constantly evolve and adapt to new social situations.

Actually, there remains, a dearth of volumes which pool research from both philosophy of language, jurisprudence and Islamicate Argumentation theory. In our time, significant work is being conducted in argumentation theory, but little of it draws from, or relates to, the rich intellectual traditions of Islam. Exceedingly few historians of logic, let alone modern argumentation theorists, seem to have heard of Islamicate dialectical theory (*jadal*, *munāẓara*, or *ādābal-baḥth*), much less benefited from its millennium of insightful developments – see Young (2021), Rahman&Young(2022).

In such Islamicate dialectical theory, plays on content include both several forms of cooperative moves that might trigger to start **the play again** with a right hint offered by the Questioner or destructive moves that indicate that the meaning at stake does not meet the minimal common round standards for continuing the interaction. Walter E. Young (2017) recorded more than 200 hundred constructive and destructive moves. One interesting point of this framework is that revisions, that in contemporary argumentation theory, are linked to non monotony, concern here plays on the determining the dialogical meaning explanations suitable to the purposes of the interaction: games on logical validity, presupposed that the games on the meaning formation games relevant to the concepts at stake have been settled, at least temporarily.

5 Concluding Remarks

Summing up, though I think that many pressing cognitive and social issues of the dialogical framework have been stressed by the BIO approach, I still think that the strategy perspective adopted by BIO is closer to a monological than to a dialogical setting.

Moreover, I propose, the BIO approach should contain a set of rules that show how strategy rules are generated, instead of importing them from some given deductive system. If not, I think, no independent level of dialogical meaning can be brought to the fore. If no independent level of dialogical meaning can be brought to the fore, then dialogical criticism to inference rules (including the structural rules), does not seem to stand on firm ground. The built-in-Opponent, implicit at the strategy level, corresponding to some inference system, should, on my view, emerge from explicit dialogical meaning explanations independently of the resulting inferential rules.

According to the *old* perspective on the dialogical stance, the play level is the level where meaning is forged. It reduces neither to the (singular) performances that actualize the

interaction-types of the play level, nor to the constitution of the schema that yields a winning strategy.

We call our dialogues *dialogues for immanent reasoning* precisely because *reasons* behind a statement, that are now *explicit* denizens of the object-language of plays, are *internal* to the development of the dialogical interaction itself.

More generally, the emergence of concepts, so we claim, are not only games of giving and asking for reasons (games involving *Why*-questions) they are also games that include moves establishing *how is it that the reason brought forward accomplishes the explicative task*. Dialogues for immanent reasoning are dialogical games of *Why* and *How* constituted by actualizations and schematizations:

However, within the dialogical framework actualizing and schematizing should not be understood as performing two separate actions: through these actions we acquire the competence that is associated to the meaning of an expression by *learning* to *play*.

This feature of Dialogical Constructivism stems from Herder's view[21] that the cultural process is a process of education, in which teaching and learning always occur together: dialogues display this double nature of the cultural process in which concepts emerge from a complex interplay of *why* and *how* questions.

In this sense, as pointed out by Lorenz (2010, pp. 140-147), the dialogical teaching-learning situation is where *competition*—the I-perspective—and *cooperation*—the You-perspective—interact: both intertwine in collective forms of dialogical interaction that take place at the play level.

The insights of Paul Lorenzen and Kuno Lorenz on dialogical logic as re-establishing the historical links of ethics and logic provides a clear answer to Wilfried Hodge's (2008) sceptical view to what the dialogical framework's contribution is. Hodge's criticism seems to target the *mathematical* interest of a dialogical conception of logic, rather than a philosophical interest which does not seem to attract much of his interest after all.

In lieu of a general plaidoyer for the dialogical framework's philosophical contribution to the foundations of logic and mathematics, which would bring us too far – see Lion (2023 forthcoming), let us highlight three points which result from the above remarks:

1) the dialogical interpretation offers a sound venue for the development of inference-based foundations of logic;
2) the dialogical take on the interaction of epistemic and deontic notions in logic, as well as the specification of the play level's role, sets new ways of implementing the interface pragmatics-semantics within logic.
3) the introduction of *knowing how* into the realm of logic is of crucial import.

Obviously, formal semantics in the Tarski-style is blind to the first point, misunderstands the nature of the interface involved in the second, and ignores the third.

[21] See (Herder, 1960 [1772]), Part II.

Of course, this goes towards a radical, irreducible form of dialogicism that I happily profess and promote, and can understand that not every one would like to come on board.

However, the present proposal, can also be seen as an invitation for reflecting together.

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