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Why Is Exclusivity in Broadcasting Rights Prevalent and Why Does Simple Regulation Fail? *

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Abstract

Pay-TV firms compete both downstream to attract viewers and upstream to acquire broadcasting rights. Because profits inherited from downstream competition satisfy a convexity property, allocating rights to the dominant firm maximizes the industry profit. Such an exclusive allocation of rights emerges as a robust equilibrium outcome but may fail to be welfare maximizing. We analyze whether a ban on resale and a ban on package bidding may improve welfare. These corrective policies have no impact on the final allocation but lead to profit redistribution along the value chain.

JEL CODES. L13, L42.

KEYWORDS. Broadcasting rights; upstream and downstream competition; exclusivity.

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1. Introduction

MOTIVATION. The pay-television (pay-TV henceforth) market is characterized by fierce competition all along the value chain. In the upstream market for content, TV firms compete through bidding procedures to obtain must-have content (sports events or premium movies) from content owners (sport leagues, movie producers, etc.). In the downstream market for viewers, TV firms compete in price and quality to attract viewers.

Upstream and downstream competition are deeply intertwined. Downstream, viewers choose which channels to purchase by comparing the price and quality of their respective offers, with the understanding that quality depends on the distribution of broadcasting rights inherited from the upstream stage. Upstream, TV firms express a willingness to pay for contents that depends on how those contents might help them gain market shares downstream.

A remarkable feature of those markets is that most often, a single TV firm ends up obtaining all the broadcasting rights for a class of premium contents. That such exclusivity emerges as a robust outcome raises a number of important questions. First, on the positive side, we may wonder what the fundamental economic forces are that drive such concentration. Why would a content owner choose to pick a single winner and not spread out such rights across rivals to *in fine* touch a larger viewership? Is exclusivity a property that prevails across all equilibrium outcomes? Is it robust to the auction format? Second, and taking a more normative stance, we may also ask whether such exclusivity harms welfare and, under those circumstances, whether simple institutional constraints might have a role in improving this outcome.

To answer these questions, we present a simple model of two-sided competition that blends key features of the upstream and downstream markets. Building such an integrated model is a necessary step to assess the origins and consequences of exclusive agreements in those markets. On the one hand, a partial focus on the upstream bidding market would not suffice to understand bidding strategies since TV firms evaluate their willingness to pay for broadcasting rights with an eye on how the downstream allocation of viewers across channels impacts downstream profits. Acquiring exclusive rights certainly gives one a competitive wedge downstream by attracting more viewers. It also hurts one's competitors by reducing their own market shares, which is a standard feature of auctions with externalities. On the other hand, focusing on the downstream market alone would abusively reduce the analysis to a simple two-stage game with firms choosing the price and quality of their own programming without taking into account that the quality of contents¹ actually depends on how the upstream market is cleared, which itself depends on downstream profits and thus on the quality of programs.

Convexity of Profit Functions. This circularity is at the core of our analysis. Its key implication is that there are strong reasons why exclusive agreements emerge and dominate other nonexclusive modes of distribution from the industry's viewpoint. The key property beyond this result is the *convexity* of the profit functions in the downstream market. A dominant firm downstream gains more from obtaining exclusive rights upstream than what it would lose if those rights were instead given to a weaker competitor.

To understand this property in more detail, it is useful to sketch how the downstream market is segmented. Adopting a modeling of downstream differentiation across TV

¹See Shaked and Sutton (1982, 1983) for some contributions on this front.

channels, which is now well established in the literature, we consider two pay-TV firms located at the extreme points of a Hotelling segment.² Viewers are uniformly distributed along this line and face constant marginal transportation costs, which is a standard metaphor for the preferences bias that consumers may a priori have for each TV channel. When offering a better quality of content, a firm becomes more attractive and increases its market share. This firm may then charge a higher price and increase its profit. At the same time, its lower-quality competitor views its market share as shrinking, and thus reduces its own price and earns lower profits. Starting from a hypothetical situation where both suppliers would offer the same quality of content, for instance, because broadcasting rights are distributed evenly, an (exogenous) increase in the quality provided by one firm redistributes viewers, but the price increase for that firm just equals the price decrease for its rival. As a result of these joint changes in prices and market shares, the increase in profit of the high-quality firm more than offsets the loss in profit for the low-quality supplier. Henceforth, profit functions in the downstream market are convex.

Constrained Efficiency and Monotonic Bidding Equilibria. This convexity property ensures that granting exclusivity for broadcasting rights is *constrained-efficient*; *i.e.*, it maximizes the industry's overall profit. Moreover, if firms are *a priori* different, because, for instance, one of them is better able to market premium programs or has already benefited from a captive viewership, then all rights should be given to this firm, which might reinforce its dominance in a dynamic context.

To avoid making any restrictions on the auction procedure, we view competition in this market as a menu auction in the spirit of Wilson (1979) and Bernheim and Whinston (1986a). Downstream firms submit nonnegative bidding schedules that stipulate bids for all possible distributions of rights that could be chosen by their upstream owner. To illustrate, this owner could choose to evenly allocate rights or to opt for exclusivity depending on the bids he or she may collect. Thanks to the convexity of profit functions, granting exclusivity on broadcasting rights is constrained-efficient; i.e., it maximizes the industry's overall profit. That the equilibrium allocation is constrained-efficient is a robust finding. This finding holds throughout the whole set of equilibria of the bidding procedure that can be reached by means of monotonic bidding schedules.³

Of course, those equilibria differ in terms of the distribution of profits they induce across the industry. Although the weaker firm always obtains the same payoff, profits may be redistributed across equilibria from the dominant firm to the upstream seller of rights. Among all possible equilibria, the highest profit to the dominant firm is achieved by means of truthful bidding schedules (Bernheim and Whinston, 1986a). Those bidding schedules are undominated strategies that perfectly reflect the preferences of firms across the various possible allocations of rights. In such a truthful equilibrium, the dominant firm bids a Vickrey-Clarke-Groves payment that compensates the seller for the foregone opportunity of not having sold to the weaker firm. In other equilibria (based on weakly dominated strategies for the weaker firm), the dominant firm may end up paying even more to obtain exclusivity. This property of the truthful equilibrium thus echoes the celebrated Chicago School argument, which posits that, if any exclusive agreement is

²Armstrong (1999), Gabszewicz et al. (2001, 2002, 2004), Harbord and Ottavianni (2001), Dukes and Gal-Or (2003), Gal-Or and Dukes (2003), Anderson and Coate (2005), Peitz and Valletti (2008) and Stenneck (2014).

³Monotonicity means that a firm should bid more for an allocation that increases its profit, which is quite a weak restriction.

ever signed, the seller has certainly been compensated for the foregone opportunity of not having chosen a more even distribution of rights across downstream competitors.

Is Regulation Warranted? In this respect, an important issue is to determine whether those gains that pertain to the industry can be redistributed to viewers. The presumption often made in related antitrust cases is that they are. In our context, the fact that viewers are charged by TV firms a price that is independent of their location means that those firms are unable to grasp all consumer surplus and, as a result, that the industry's most preferred allocation of rights may not maximize welfare. In this scenario, exclusive agreements might also be considered, perhaps more pessimistically, as reducing downstream competition for viewers and thus hindering consumer surplus. That the industry equilibrium may fail to be welfare maximizing thus a priori calls for some sort of public intervention.

As a result of this tension between the possible efficiency gains of exclusivity for the industry and its negative consequences on viewers, competition authorities throughout the world have taken different postures. In North America, case law specific to sports media views exclusive agreements as a priori legal. In the recent Spinelli v NFL case, the Court held that "because the benefits of exclusive licensing agreements are well-recognized", the arrangements were presumptively legal.⁴ In 2015, the Canadian Competition Bureau approved a 12-year exclusive distribution agreement between the NHL and Rogers Broadcasting, arguing that the deal would not foreclose competition and that pro-competitive gains in the form of quality investments would benefit viewers.⁵ The attitude of the EU competition authority toward such exclusivity agreements differs, as exemplified by the landmark UEFA Champions League decision.⁶ There, the joint-selling arrangement initially notified by the UEFA implied that all broadcasting rights for this elite soccer club competition were sold to a single broadcaster in each Member State and on an exclusive basis for periods up to four years. The European Commission negotiated several important changes to the initial agreement, including that open, transparent and nondiscriminatory tenders must be used to sell those rights, that several packages have to be offered and no single bidder can acquire all packages exclusively, and that exclusivity is limited in time (up to three years as a general rule).

To the extent that corrective policies are certainly called for, policy recommendations would be at risk of being based on a particular selection of the best-response correspondence and thus nonrobust without having at hand a full characterization of equilibria payoffs. An important take-away of our analysis is that such characterization actually shows that the light-handed tools available to competition authorities to prevent dominance have no impact on the allocation of rights and very minor consequences on the distribution of profits across the industry. Thus, exclusivity seems hard to avoid.

LIGHT-HANDED REGULATORY CONSTRAINTS. To show this result, we analyze the impact of several regulatory constraints that might be imposed on the bidding game in

⁴Spinelli v NFL, 96 F Supp 3d 81 (SDNY 2015), available at https://law.justia.com/cases/federal/appellate-courts/ca2/17-0673/17-0673-2018-09-11.html.

⁵See Bachelor et al. (2020).

⁶See 2003/778/EC: Commission Decision of 23 July 2003 relating to a proceeding pursuant to Article 81 of the EC Treaty and Article 53 of the EEA Agreement (COMP/C.2-37.398 – Joint selling of the commercial rights of the UEFA Champions League), available at https://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX%3A32003D0778.

⁷See Toft (2003) for a detailed account of the negotiations between UEFA and the European Commission.

view of improving welfare. Typically, those constraints may be a ban on the resale of the rights or a ban on package bidding when multiple rights might be allocated. To motivate a regulatory view on resale and package bidding, the case of the French soccer league (LFP) is in order. In 2018, broadcasting rights for the next four seasons were on sale. These rights were divided into seven packages, and participants could not make any global offer (i.e., a joint bid for several lots). Mediapro ended up getting the lion's share with five lots, which made an overall very attractive package. beIN and the telecom operator Free each obtained a package. Last but not least, the traditional distributor of those programs over the last couple of decades, Canal+, returned empty-handed. Mediapro's price was quickly judged by many observers as being excessive, and industry experts also reported that its initial plan was to resell some of the rights acquired earlier on to its main competitor, namely, Canal+.8 However, after the tender, Canal+ and beIN promptly sealed an exclusive distribution and sublicensing deal for the broadcasting of the lot obtained by beIN. Mediapro subsequently failed to fulfill its payment obligations to the French soccer league, thereby putting their agreement and the financial health of the league itself at risk. The rights initially obtained by Mediapro were then reallocated in a separate tender, with Amazon obtaining all the rights against a consortium of bidders formed by Canal+ and beIN at a much discounted price relative to Mediapro's initial offer.9

Resale of Rights. Pay-TV markets often feature resales of rights, sometimes even between firms that were earlier competitors at the bidding stage. Resale has often been viewed by practitioners as a means for a dominant firm to increase its monopoly power by buying back rights sold earlier on (maybe wrongly so) to a competitor.

Resale comes with pros and cons. First, any constrained-inefficient allocation of rights can be renegotiated with resale so that, absent informational problems in bargaining and any further transaction costs, the final allocation is necessarily constrained-efficient. Henceforth, there is no doubt that resale may be unattractive from a social welfare viewpoint and should be considered with caution by competition authorities. Second, when it is allowed, the possibility of resale is anticipated by bidders earlier on. They actually bid for getting an *interim* allocation of rights that might be renegotiated. Such renegotiation transforms the bidding game into a constant-sum game. Whoever finally gets the good ends up paying whoever gets it at the *interim* stage. Bidding for an *interim* allocation becomes a fierce head-to-head auction that erodes much of the dominant firm's profit. These profits are now pocketed by the initial owner of rights. Henceforth, any ban on resale has a redistributive impact along the supply chain, although it does not change the final allocation of rights. With or without resale, the final allocation is necessarily constrained-efficient.

Package Bidding and Multiple Rights. We next consider a setting where multiple rights can be sold simultaneously. As alluded to in the above example of the French soccer league, this scenario fits well with most procedures taking place for sports events. In those circumstances, competition authorities have sometimes suggested that the dominance

⁸See, for instance, https://www.lequipe.fr/Football/Article/Droits-tv-la-strategie-ratee-demediapro-pour-rivaliser-avec-canal-et-bein-sports/1186223.

⁹For a detailed account of the tender and the legal disputes following the subsequent withdrawal of Mediapro, see Décision no. 21-D-12 du 11 juin 2021 relative à des pratiques mises en oeuvre par la Ligue de Football Professionnel dans le secteur de la vente de droits de diffusion télévisuelle de compétitions sportives, available at https://www.autoritedelaconcurrence.fr/sites/default/files/integral_texts/2021-06/21d12_0.pdf.

of key players could be undermined by splitting upstream auctions into smaller tender procedures in which broadcasting rights of lesser size would be for sale. ¹⁰ Again, the costs and benefits of such procedures and their welfare consequences should be assessed with a careful look at outcomes that follow from splitting and a comparison with the scenario of a package auction where all rights are on sale at once.

Our analysis starts with the case of package bidding, where firms are allowed to freely bid for any combination of the rights for sale. Again, convexity of the downstream profit functions prevails when multiple rights are at stake. Again, constrained efficiency calls for giving all rights to the dominant firm. This outcome is achieved in all monotonic equilibria, although different distributions of profits can be sustained. Among those possible distributions, the one that most favors the dominant firm is achieved with truthful bidding schedules. The incremental value for the dominant firm of obtaining a second set of rights, when already holding one set of rights, is large. This dominant firm is thus eager to make an aggressive bid for the whole package of exclusivity rights.

We then consider the outcome when auction markets for each set of rights are split. To this end, we first borrow from the vertical restraints literature and define a pairwise-proof allocation as the result of a market-by-market bidding equilibrium. Bidding schedules are found for a given set of rights with the expectation that a constrained-efficient allocation of those rights that would favor the dominant firm arises for other rights. Focusing on (undominated) truthful outcomes on each market, in order to favor the dominant firm, we characterize such allocation. Of course, it is again constrained-efficient. From the logic of Vickrey-Clarke-Groves payments, but now applied market by market, the dominant firm needs only to compensate the owner for the foregone opportunity of not selling rights on a single market. These opportunities correspond to the weaker firm's willingness to pay for exclusivity in only one market, expecting that the dominant firm already gets exclusivity in the other. Due to the convexity of the weaker firm's profit function, this willingness to pay is quite low. In a pairwise-proof allocation, the dominant firm ends up paying little for exclusivity on all rights.

One could a priori conclude that competition authorities would, perhaps contrary to their intention, stack the deck in favor of the dominant firm by banning package bidding. This intuition is wrong. Indeed, we demonstrate that pairwise-proof allocation is not immune to multilateral deviations in both markets at the same time.¹² Intuitively, and it again follows from the convexity of profit functions, the weaker firm could outbid the dominant firm, which pays too little, and obtain exclusivity in both markets by making a more aggressive bid for the package. Policy recommendations relying on pairwise-proofness are thus misleading.

Rather than imposing *pairwise-proofness*, we thus require that bidding schedules should be additively separable across auctions when the latter are split. Robustness to multilateral deviations is then ensured for free with this requirement. This constraint a *priori* seems to limit the expression of the willingness to pay for bundles, which are so attractive in this environment (thanks again to the convexity of profit functions). As

¹⁰In the above-mentioned 2018 tender organized by the LFP, seven lots were allocated sequentially.

¹¹Crémer and Riordan (1987), Horn and Wolinsky (1988), O'Brien and Shaffer (1992), Hart and Tirole (1990) and McAfee and Schwartz (1994). See also Collard-Wexler et al. (2019) for an up-to-date overview of the literature.

¹²In the literature on vertical restraints, this possibility is well known, at least since McAfee and Schwartz (1995), Segal and Whinston (2003), and Rey and Vergé (2004).

such, one may expect that thi would make it difficult for the dominant firm to gain exclusivity in all markets. This intuition is again wrong. We demonstrate that *all* distributions of profits available in any monotonic equilibrium with package auctions remain feasible when the additivity restriction is imposed. This conclusion holds because, with the discrete allocation of rights (each set of rights being allocated to either downstream firm), there is much leeway in finding bids that force the dominant firm to pay what it is worth obtaining exclusivity on all rights while also preventing the initial owner from choosing other allocations.

ORGANIZATION OF THE PAPER. Section 2 gives a brief account of the relevant literature. Section 3 presents the different players (sellers of rights in the upstream market, pay-TV firms, viewers) and describes the upstream and downstream markets in which they interact. This section also discusses the structure of downstream profits and their important convexity property. We also present the structure of the game whose equilibria determine the contractual forms observed. In Section 4, we characterize monotonic equilibria and the corresponding distributions of profits and surplus. Section 5 considers the possibility of resale. Section 6 extends our framework to allow for multiple broadcasting rights and the possible complementarities between those rights from the point of view of downstream firms. Section 7 briefly recaps our findings.

2. Literature Review

This paper touches on several trends in the literature. On the more applied side, the model of pay-TV downstream competition adopted in this article borrows from the Hotelling-based approach established by Gabszewicz et al. (2001, 2002, 2004), Dukes and Gal-Or (2003), Gal-Or and Dukes (2003), Anderson and Coate (2005) and Peitz and Valletti (2008). These contributions examine a number of issues, including content differentiation, advertising intensities and comparisons of welfare under pay-TV and free-to-air (advertising-financed) television. However, none of these articles considers content exclusivity.

More specifically, the literature on exclusivity in the pay-TV market is thin, to say the least. Earlier works were mainly informed by British experiences on the subject. For instance, Armstrong (1999) discusses the possibility that two pay-TV networks could be implicitly colluding by exchanging content. This author analyzes incentives to sign exclusive agreements in this context. Harbord and Ottaviani (2001) analyze contractual agreements, particularly in the context of the resale of broadcasting rights. The fine structure of these contracts (fixed payments or more complex schedules depending on market shares) plays a crucial role in assessing their anti-competitive properties. Stenneck (2014) shows how exclusive distribution can stimulate specific investments by distributors and benefit all viewers, even those who a priori do not have access to the premium offer so distributed. Sonnac (2012) explains the strategic nature of exclusivity clauses in the pay-TV market. Weeds (2016) examines incentives for exclusivity in a dynamic setting and argues that switching costs confer benefits to a vertically integrated operator, which may thus forego the static benefits of selling contents to a rival competitor. In our paper, exclusivity arises because it maximizes industry profit, and (unrestricted) upstream auction procedures have the property of reaching this outcome.

Taking a broader theoretical perspective, this paper contributes on several fronts. The idea that exclusivity is an equilibrium outcome when it is constrained-efficient is already

present under various forms in the vertical contracting literature. Aghion and Bolton (1987), Choné and Linnemer (2015) and Calzolari and Denicolò (2015) develop this point but also stress some of its limits under various technological and informational scenarios. Closer to our perspective is certainly the framework proposed by Bernheim and Whinston (1998). In a model where two firms bid to attract an agent's services, those authors also show that in the absence of frictions, exclusive dealing, when it arises, maximizes the profit of the grand coalition.

To assess the possible consequences of constraints on the auction formats, we need a full characterization of equilibria in such bidding games. This step, although much reminiscent, goes beyond the analysis of menu auction games performed in Bernheim and Whinston (1986a), who instead focus on truthful equilibria. To do so, we rely on some general techniques developed by Martimort and Stole (2012) in their study of aggregate games, *i.e.*, a more general class of games that includes menu auctions and other contracting scenarios.¹³ In our set up, those menu auctions also entail downstream externalities, and our paper contributes to the relevant literature as well (Jéhiel and Moldovanu, 2000; Varma, 2002; Brocas, 2013; Assef and Chade, 2008; Molnar and Virag, 2008; Martimort and Pouyet, 2020; among many others).

Last, our paper, in regard to developing a comparative analysis of package bidding and split auctions for multiple rights, also speaks to the literature on vertical restraints (Crémer and Riordan, 1987; Horn and Wolinsky, 1988; O'Brien and Shaffer, 1992; Hart and Tirole, 1990; McAfee and Schwartz, 1994; Collard-Wexler et al., 2019) and borrows from the notion of pairwise-proofness. In our context, we show the limit of this approach that fails to account for simultaneous deviations in all markets at once. This possibility has been well known since at least the works of McAfee and Schwartz (1995), Segal and Whinston (2003), and Rey and Vergé (2004). To restore the existence of equilibria under all circumstances, we impose an additivity constraint on bidding schedules that capture the idea that bidding takes place market by market.

3. Markets, Players, Preliminary Results

STRUCTURE OF THE INDUSTRY. Two pay-TV suppliers, F_0 and F_1 , compete in the downstream market for viewers. They offer program packages that include some basic service (free-to-air TV or classic movies, for instance) plus possibly some premium programs (sports events or blockbusters). Competition in the downstream market thus entails both a quality (the content of those packages) and a price component. In the upstream market, F_0 and F_1 bid to acquire premium broadcasting rights from a supplier A. Those rights may entail exclusivity or the joint distribution of the premium contents and induce some vertical differentiation between service providers.

There is a continuum of mass one of potential customers for pay-TV services. Those customers are identical in terms of their willingness to pay for basic services. Let us denote by v > 0 this willingness to pay. However, viewers differ in terms of their innate bias toward each supplier. Competition between TV firms takes place à la Hotelling on a [0,1] segment that stands for the service space. The two firms are horizontally differentiated, F_0 and F_1 being located at 0 and 1, respectively. To simplify, viewers are uniformly distributed on that segment. Let x (resp. 1-x) denote the distance between

 $^{^{13}}$ Bernheim and Whinston (1986b) already pointed out that common agency games have this aggregate property.

a given viewer and F_0 (resp. F_1). This distance stands for a measure of the consumer's bias for F_0 (resp. F_1). Traveling a distance x (resp. 1-x) toward F_0 (resp. F_1) "costs" tx (resp. t(1-x)) to a customer, where t>0 is the per unit transportation cost. Viewers choose their favorite supplier based on their location on the [0,1] segment, the prices and the content of the TV packages offered by downstream firms.

Each viewer also has an extra willingness to pay for premium services. This extra valuation depends on how those programs are customized by the supplier. The supplier's promotional efforts stand for better advertising, a better description of the content or even a better match between tastes and services. Since suppliers may be heterogeneous in terms of their ability to improve their service, this extra valuation depends on who owns the broadcasting rights for premium programs and, in particular, whether owners have exclusivity rights. Let $\alpha_i \geq 0$ denote this extra valuation when a viewer purchases premium services from F_i if this firm owns the corresponding rights. Accordingly, the viewer's willingness to pay for F_i 's package is written as $v + \alpha_i$. The values of α_0 and α_1 depend on the allocation of rights across firms, which is a dependence that we will make precise below. For the time being, we provide a result that characterizes how the suppliers' profits depend on the differences of their quality.

LEMMA 1. Fix any configuration (α_0, α_1) . Assuming that v is large enough so that the downstream market is always fully covered, F_i 's (i = 0, 1) profit can be expressed as follows:

(3.1)
$$\widetilde{\Pi}_i(\alpha_i, \alpha_{-i}) = \pi(\alpha_i - \alpha_{-i}),$$

where
$$\pi(x) = \frac{t}{2} \left(1 + \frac{x}{3t}\right)^2$$
. Therefore, $\widetilde{\Pi}_i(\alpha_i, \alpha_{-i})$ is convex in $\alpha_i - \alpha_{-i}$.

We can view the difference $\alpha_i - \alpha_{-i}$ as the incremental quality that F_i 's services bring to viewers in comparison with what F_{-i} would bring to them. That F_i 's profit is convex in that difference actually plays a significant role in our analysis below. The intuition for this convexity property comes from a careful look at how the quality difference $\alpha_i - \alpha_{-i}$ impacts both prices and market shares. If both firms offer the same content, namely, $\alpha_i = \alpha_{-i}$, then they equally share the market. Starting from there, consider increasing the quality difference $\alpha_i - \alpha_{-i}$. So doing allows F_i to charge a higher price for its own customers, namely, the following:¹⁴

$$P^*(\alpha_i - \alpha_{-i}) = t + \frac{\alpha_i - \alpha_{-i}}{3}.$$

A contrario, F_{-i} charges a lower price $P^*(\alpha_{-i} - \alpha_i)$ since it offers a package of a lower relative quality. Had market shares been kept constant, those price changes would just compensate each other to keep the overall profit of the industry constant. However, increasing the quality difference $\alpha_i - \alpha_{-i}$ also allows F_i to attract more viewers, expand its own market share and reduce that of its rival by an equal amount when the market is fully covered. Henceforth, such an increase in the quality difference $\alpha_i - \alpha_{-i}$ also increases F_i 's profit more than it reduces F_{-i} 's profit. The convexity property follows. This property is key to understanding the equilibrium allocation of rights that prevails.

ALLOCATIONS OF RIGHTS AND CONSEQUENCES ON DOWNSTREAM PROFITS. To determine the precise expressions of α_0 and α_1 , several cases must be considered depending

¹⁴See the Proof of Lemma 1 for a derivation of this expression.

on the allocation of broadcasting rights across downstream firms.

Exclusive Distribution. Suppose first that F_0 has obtained exclusivity on premium services. To capture the fact that F_0 enjoys a comparative advantage in improving viewers' valuation of premium programs, we assume that $\alpha_0 = \alpha + \Delta > \alpha_1 = 0$, where Δ and α are both positive parameters.¹⁵ If instead F_1 had exclusivity on the broadcasting rights, we assume that $\alpha_0 = 0 < \alpha_1 = \alpha$. In other words, if it obtains exclusivity, F_1 is not able to increase the viewers' valuation as much as what the dominant firm F_0 would. Under those conditions, F_0 gains more market shares, charges a higher price and makes more profit than what its rival would do with exclusivity.

Joint Distribution. Suppose now that there is no such exclusivity granted. In other words, A sells broadcasting rights to both F_0 and F_1 . In that case, we posit $\alpha_0 = \alpha_1 = \alpha$. Of course, in that scenario, the market is equally shared between both suppliers, and firms charge the same price and make identical profits.

For future reference, let the set of feasible allocations of rights be denoted by $\mathcal{A} = \{0, 1, c, \emptyset\}$, where 0 (resp. 1) stands for exclusivity to F_0 (resp. F_1), c stands for joint distribution, and \emptyset stands for no allocation of rights at all, which is an issue that is of course weakly dominated but included in our description for the sake of completeness. With those notations, we may also define profits in terms of the allocation $a \in \mathcal{A}$ as $\Pi_i(a)$. From our previous description of quality increments, those profits are expressed as $\Pi_0(0) = \widetilde{\Pi}_0(\alpha + \Delta, 0)$, $\Pi_0(1) = \widetilde{\Pi}_0(0, \alpha)$, $\Pi_i(c) = \Pi(c) = \widetilde{\Pi}_i(\alpha, \alpha)$, $\Pi_1(0) = \widetilde{\Pi}_1(\alpha + \Delta, 0)$ and $\Pi_1(1) = \widetilde{\Pi}_1(0, \alpha)$.

BIDDING STRATEGIES. TV firms approach A and bid for acquiring broadcasting rights. Following a methodology that was initiated by Bernheim and Whinston (1998), F_0 and F_1 compete by means of bidding schedules. Those schedules are commitments that stipulate how much the firm is ready to leave to A for each configuration of rights. Formally, a bidding schedule is thus a nonnegative mapping T_i on A. The stands for the share of F_i 's profit that is left to A when an allocation of rights A is chosen by A. Hence, whether exclusivity or a joint distribution emerges is actually an equilibrium outcome.

TIMING. The overall game of bidding and downstream competition unfolds as follows:

- 1. Upstream market. F_0 and F_1 simultaneously offer bidding schedules T_0 and T_1 to A. However, since bidding schedules are nonnegative, A accepts those offers. ¹⁸
- 2. Allocation of rights. A chooses a distribution of rights $a \in \mathcal{A}$ and accordingly pockets the bids $T_i(a)$ (i = 0, 1) from downstream firms.
- 3. Downstream market. F_0 and F_1 simultaneously and noncooperatively choose their prices for the TV-offers they deliver, respectively.

¹⁵An alternative interpretation is that F_0 is a dominant firm that enjoys a captive clientele.

¹⁶Hagiu and Lee (2011) adopt a similar approach in the context of competition between platforms.

¹⁷Nonnegativity of the bidding schedules is without loss of generality. Indeed, A would never accept a bidding schedule from F_i with some negative payments and choose an allocation a corresponding to a negative payment, *i.e.*, $T_i(a) < 0$. A could also increase its payoff by simply choosing the same allocation and excluding F_i from the auction. Beyond this, no further restrictions are made on the bidding strategies.

¹⁸A could a priori accept or refuse each of those offers. The refusal of an offer would then amount to excluding the corresponding bidder from the bidding procedure.

The equilibrium concept is subgame-perfect equilibrium. We will sometimes rely on refinements (based on monotonicity, truthfulness or/and dominance) to pin down a unique outcome.

EFFICIENCY NOTIONS. In the following, we are interested in the efficiency properties of the equilibrium allocations of rights. A constrained efficiency criterion only considers the industry's viewpoint, thus maximizing the total profit of the grand coalition $A - F_0 - F_1$. An efficiency criterion instead assesses the consequences of such an allocation on overall welfare, including consumer surplus, which is an objective that is in line with that of competition authorities.

LEMMA 2. The allocation $a^* = (0)$ is the constrained-efficient allocation, whereas a joint allocation is the worst scenario from the industry's viewpoint:

(3.2)
$$\sum_{i=0,1} \Pi_i(0) > \sum_{i=0,1} \Pi_i(1) > 2\Pi(c).$$

Suppose that t is large enough; more specifically,

(3.3)
$$\alpha \ge \Delta + \frac{5}{18t}(\alpha + \Delta)^2.$$

Then, $a^{**} = (c)$ is the efficient allocation.

To understand the ranking of aggregate profits in (3.2), we must first return to the convexity of profits stressed in Lemma 1. Granting exclusivity to either firm is an extreme allocation that thus dominates a joint allocation from the point of view of the industry. This explains the second inequality in (3.2). Moreover, this effect is more important when the dominant F_0 has exclusivity since the quality differential with its rival is then greater. This explains the first inequality in (3.2).

When t is not too small, *i.e.*, competition is rather weak in the downstream market, welfare is greater with a joint distribution of rights, thereby inducing equal prices and market shares. Under those circumstances, a competition authority might want to intervene in the procedures that rule the upstream market to avoid exclusivity by the dominant firm emerging as an equilibrium outcome, which is an issue that we shall investigate in detail below.

Even when exclusivity is better from the perspective of (static) welfare, a competition authority may still take measures to ensure a nonexclusive outcome to level the playing field in the downstream market. Protecting competition may indeed promote welfare from a dynamic perspective.

4. Equilibria Characterization

4.1. Generalities

At equilibrium, F_i chooses its own bidding schedule T_i to induce an allocation \overline{a} that maximizes its individual profit, *i.e.*, $\Pi_i(a) - T_i(a)$. Given a pair of offers (T_0, T_1) , A implements such allocation if it also maximizes its own payoff, *i.e.*, $\sum_{i=0,1} T_i(a)$. For future reference, we shall denote F_i 's (resp. A's) equilibrium profit by $\overline{\Pi}_i$ (resp. $\overline{\Pi}_a$).

Definition 1. An equilibrium of the continuation game for stage 2 onward is a triplet $(\overline{T}_0, \overline{T}_1, \overline{a})$ that satisfies the following conditions.

- Profit maximization for A. A chooses an allocation within the best-response correspondence $\overline{\mathcal{A}}(\overline{T}_0, \overline{T}_1)$ defined as follows:

(4.1)
$$\overline{\mathcal{A}}(T_0, T_1) = \arg \max_{\tilde{a} \in \mathcal{A}} \sum_{i=0,1} T_i(\tilde{a}) \quad \forall (T_0, T_1).$$

- Profit maximization for F_i , i = 0, 1. \overline{T}_i satisfies the following:

(4.2)
$$\Pi_{i}(\overline{a}) - \overline{T}_{i}(\overline{a}) \in \arg \max_{\substack{T_{i} \geq 0 \\ \widetilde{a} \in \overline{\mathcal{A}}(T_{i}, \overline{T}_{-i})}} \Pi_{i}(\widetilde{a}) - T_{i}(\widetilde{a});$$

thus,
$$\overline{a} \in \overline{\mathcal{A}}(\overline{T}_0, \overline{T}_1)$$
.

We can draw a few immediate consequences from conditions (4.1) and (4.2). First, as a best response to its rival's strategy \overline{T}_{-i} , F_i always minimizes its own payment $\overline{T}_i(\overline{a})$ to induce its most preferred allocation \bar{a} . To illustrate, suppose that F_0 wants to induce exclusivity. For its profit to be maximum in such a configuration, F_1 should not wish to induce another allocation by increasing its own payment to A.

Second, this bidding game appears to be a delegated common agency game, as defined by Bernheim and Whinston (1986a). Martimort and Stole (2012) showed that those games are aggregate games. More specifically, A's behavior on the equilibrium path only depends on the aggregate bidding schedule $\sum_{i=0,1} \overline{T}_i$. This property, together with the fact that payoffs are quasi-linear, then allows us to sum up the equilibrium conditions for each firm to obtain a more compact set of necessary conditions that must be satisfied by any equilibrium allocation. Such an allocation is actually a solution to a self-generating optimization problem in the vocable coined by Martimort and Stole (2012). Taking stock of these properties, the next proposition provides a set of necessary conditions that allows us to restrict the search for equilibrium allocations in a sharp way. 19

PROPOSITION 1. At any equilibrium $(\overline{T}_0, \overline{T}_1, \overline{a})$, the following conditions hold:

(4.3)
$$\overline{a} \in \arg\max_{a \in \mathcal{A}} \sum_{i=0,1} \Pi_i(a) + \sum_{i=0,1} \overline{T}_i(a),$$

(4.4)
$$\overline{a} \in \arg\max_{a \in \mathcal{A}} \sum_{i=0,1}^{i=0,1} \overline{T}_i(a),$$
(4.5)
$$\max_{a \in \mathcal{A}} \overline{T}_i(a) + \overline{T}_{-i}(a) = \max_{a \in \mathcal{A}} \overline{T}_{-i}(a), \quad \forall i = 0, 1,$$

(4.5)
$$\max_{a \in A} \overline{T}_i(a) + \overline{T}_{-i}(a) = \max_{a \in A} \overline{T}_{-i}(a), \quad \forall i = 0, 1,$$

(4.6)
$$\overline{T}_i(a_{-i}) = 0, \quad \forall i = 0, 1,$$

where a_{-i} is A's best choice if F_i were to offer a null payment; i.e.,

(4.7)
$$a_{-i} \in \arg\max_{a \in \mathcal{A}} \overline{T}_{-i}(a).$$

¹⁹These conditions are not necessarily sufficient because the aggregate game is not bijective in the sense defined in Martimort and Stole (2012). More specifically, because bids must remain nonnegative, not all aggregate bids can be undone by a given firm with its own offer when this offer is restricted to remain nonnegative as requested in a delegated common agency game.

The optimality condition (4.3) showcases the self-generating nature of equilibria. An equilibrium allocation maximizes the sum of the bidders' payoff $\sum_{i=0,1} \Pi_i(a)$ plus the seller's profit $\sum_{i=0,1} \overline{T}_i(a)$, which is an equilibrium object. Condition (4.4) simply characterizes the fact that this equilibrium allocation also belongs to the seller's best-response correspondence. More interesting are conditions (4.5) and (4.6) that offer an anchor for A's payment and F_i 's bid schedule, respectively. The first one expresses the fact that A should be indifferent between choosing the equilibrium allocation given the aggregate bid $\overline{T}_i + \overline{T}_{-i}$ and its next best option, which is to choose an optimal allocation for only one bidder. The second condition shows that F_i 's bid is zero had this off-path allocation been chosen by A. Formally, everything happens as if F_i had to make its own bidding schedule attractive enough to A to avoid being excluded from the market.²⁰

We now specialize this general approach to determine what types of equilibria may emerge and discuss their constrained-efficiency properties. To this end, we first restrict the set of bidding strategies in a quite natural way, focusing on *monotonic bidding schedules* such that a bidder bids more for an action that yields a greater payoff, namely,

$$\Pi_i(a) \ge \Pi_i(a') \Rightarrow T_i(a) \ge T_i(a') \quad \forall (a, a') \in \mathcal{A}^2.$$

Accordingly, an equilibrium is *monotonic* if it is implemented by means of *monotonic* bidding schedules. This focus has important implications that we now present.

Proposition 2. The following properties hold at any monotonic equilibrium:

- 1. The equilibrium allocation is constrained-efficient, i.e., $\bar{a} = (0)$.
- 2. The set Σ of feasible profits $(\overline{\Pi}_0, \overline{\Pi}_1, \overline{\Pi}_a)$ in monotonic equilibria is defined by the following conditions:

(4.8)
$$\Pi_0(1) \leq \overline{\Pi}_0 \leq \Pi_0(0) + \Pi_1(0) - \Pi_1(1),$$

$$\overline{\Pi}_1 = \Pi_1(0),$$

$$\Pi_1(1) - \Pi_1(0) \leq \overline{\Pi}_a \leq \Pi_0(0) - \Pi_0(1).$$

The distribution of payoffs that arise at equilibrium can be readily explained. First, since F_0 is a dominant firm in the market, the constrained-efficient allocation $a^* = (0)$ is also the most preferred outcome for that firm, while at the same time, it remains the worst for F_1 . Overall, and even though A could choose to sell rights nonexclusively to both downstream firms, everything happens as if those firms were bidding head-to-head for exclusivity.

From condition (4.6), F_1 bids zero for the equilibrium allocation $\overline{a} = (0)$ since giving rights to F_0 also minimizes F_1 's own payment. However, A can improve its bargaining position vis-à-vis F_0 by threatening to sell exclusive rights to F_1 . F_1 could thus be ready to pay up to $\overline{T}_1(1) = \Pi_1(1) - \Pi_1(0)$ to acquire such exclusivity. This gives the

 $^{^{20}}$ Readers accustomed to the literature might have recognized the similarity between Proposition 1 and Lemma 2 in Bernheim and Whinston (1986a). However, there are a couple of differences that deserve to be stressed. First, the aggregate optimality condition (4.3) replaces their Condition iii), which captures the fact that an equilibrium allocation maximizes the bilateral payoff of a coalition made of any one bidder and the seller. Second, conditions (4.5) and (4.6) altogether replace and supersede their Condition iv) but are more precise, thereby making it explicit for which allocation a bidder's bidding schedule is zero, while Condition iv) only provides the existence of an allocation with a zero bid.

lowest bound on A's equilibrium profit. In fact, F_1 might bid even more for exclusivity, say $\overline{T}_1(1) > \Pi_1(1) - \Pi_1(0)$, as long as, at such an equilibrium, A still chooses to sell exclusively to F_0 . Such a strategy is weakly dominated since F_1 's payoff $\Pi_1(1) - \overline{T}_1(1)$ is less than the payoff $\Pi_1(0)$ obtained when bidding zero. However, such a large bid also improves A's bargaining position vis-à-vis F_0 since it makes switching to F_1 more attractive. To defeat such a bid, F_0 should pay at least $\overline{T}_0(0) = \overline{T}_1(1) > \Pi_1(1) - \Pi_1(0)$, and the highest such payment that F_0 is willing to make is thus $\Pi_0(0) - \Pi_0(1)$ to gain exclusivity. With such large bids, A grasps a larger share of the overall industry's profit obtained by granting exclusivity to F_0 .

Finally, we also stress that an immediate corollary of Proposition 2 is that a joint distribution of rights, *i.e.*, a = (c), never arises at equilibrium. Proposition 2 thus showcases the need for some kind of corrective intervention aiming to implement an allocation that is socially optimal, namely, joint distribution, even though this allocation is never implemented by free competition.

4.2. Truthful Equilibrium

In their general investigation of menu auctions, Bernheim and Whinston (1986a) exhibited an important class of equilibria sustained with so-called *truthful strategies* in the following form:

$$T_i(a) = \max\{\Pi_i(a) - \Pi_i; 0\} \quad \forall a \in \mathcal{A}.$$

These strategies have attractive properties. First, those schedules are monotonic. From Proposition 2, truthful equilibria thus necessarily implement a constrained-efficient allocation, i.e., $\bar{a} = (0)$. Second, F_i keeps a constant profit Π_i over all possible allocations for which it makes a positive bid. In other words, truthful bidding schedules align the preferences of F_i and A over possible allocations that are found attractive for F_i . Third, each downstream firm always has a truthful strategy in its best-response correspondence. Focusing on truthful bidding schedules thus amounts to imposing a refinement in the equilibrium correspondence. This refinement allows a sharp characterization of payoffs in truthful equilibria. In our context, this refinement even pins down a unique outcome. 22

PROPOSITION 3. There exists a unique truthful equilibrium with equilibrium profits given by the following:

(4.9)
$$\overline{\Pi}_{0}^{t} = \Pi_{0}(0) + \Pi_{1}(0) - \Pi_{1}(1) > 0,
\overline{\Pi}_{1}^{t} = \Pi_{1}(0) > 0,
\overline{\Pi}_{a}^{t} = \Pi_{1}(1) - \Pi_{1}(0) > 0.$$

Together with Proposition 2, Proposition 3 shows that the constrained-efficient truthful equilibrium corresponds to an extremal point of the set of possible equilibrium profits. This equilibrium is actually sustained with strategies that are not weakly dominated. Indeed, at any other equilibrium described in Proposition 2 above, F_1 would possibly pay more for acquiring rights than its incremental benefit of doing so, since $\overline{T}(1) > \Pi_1(1) - \Pi_1(0)$. This dominance criterion thus provides another argument to focus on the unique truthful allocation.

²¹See Bernheim and Whinston (1986a, Theorem 1).

²²Laussel and Le Breton (2001) provide general conditions for the uniqueness of a truthful equilibrium.

For future reference, it is worth noting that, in this unique truthful equilibrium, what F_0 bids for exclusivity, namely, $\overline{T}^t(0) = \Pi_1(1) - \Pi_1(0)$, is the Vickrey-Clarke-Groves payment, *i.e.*, what it takes to avoid A choosing his or her next best option, namely, giving exclusivity rights to F_1 .

5. Resale

Resale is an important feature of the pay-TV market. From a theoretical viewpoint, resale nevertheless remains a double-edged sword. First, the renegotiation of any allocation of rights between downstream firms F_0 and F_1 could, a priori, correct any constrained inefficiency that might arise in the downstream market and improve the industry's overall profit. Given that constrained efficiency conflicts with social efficiency, the possibility of resale could go counter welfare maximization. It should thus be viewed with an eye of caution by competition authorities.

In fact, this negative stance is incomplete. When anticipated, the possibility of resale may actually change the downstream firms' bidding strategies, which may in turn lead to payoffs rather different than those found in Proposition 3. In other words, banning resale certainly or not has no impact on whoever in fine holds the rights. The dominant firm F_0 should always get those rights. However, it might impact the distribution of profits and how much the dominant firm pays for exclusivity.

To study how it can be so, consider adding a renegotiation stage in between stages 2 and 3 of the game form thus far studied. Suppose that A has already chosen an arbitrary allocation \tilde{a} and assume that the corresponding bids $T_0(\tilde{a})$ and $T_1(\tilde{a})$ have been sunk. F_0 and F_1 can renegotiate away from the allocation \tilde{a} toward another allocation, say a. We model such renegotiation as a Nash bargaining game with outside options being determined by the chosen allocation \tilde{a} , with parties having equal bargaining power.

Renegotiation determines a final allocation a^* , as well as a compensatory payment z^* , between F_0 and F_1 that altogether solve the Nash-bargaining problem, *i.e.*,

$$(a^*, z^*) \in \arg\max_{(a,z)} \Big(\Pi_0(a) - z - \Pi_0(\widetilde{a})\Big) \Big(\Pi_1(a) + z - \Pi_1(\widetilde{a})\Big).$$

Renegotiation thus implements the constrained-efficient allocation $a^* = (0)$ since it indeed maximizes the joint profit of F_0 and F_1 once bids are sunk. Inserting the value of the compensatory payment $z^*(\widetilde{a}) = \frac{1}{2}(\Pi_0(0) - \Pi_1(0) - \Pi_0(\widetilde{a}) + \Pi_1(\widetilde{a}))$ obtained into the firms' payoffs allows us to rewrite the firms' net profits once renegotiation is taken into account as $\widetilde{\Pi}_i(\widetilde{a}) - T_i(\widetilde{a})$, where

(5.1)
$$\widetilde{\Pi}_i(\widetilde{a}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} \left(\Pi_i(\widetilde{a}) - \Pi_{-i}(\widetilde{a}) \right).$$

The possibility of resale has modified the game between F_0 and F_1 . Remarkably, it is now a constant-sum game. By its choice of a preresale allocation, A determines how much should be transferred from one firm to the other through $ex\ post$ bargaining.²³

²³Readers accustomed with the incomplete contracts literature may have recognized a familiar feature of implementation when renegotiation is allowed. See Maskin and Moore (1999) for instance.

We may now apply the general methodology of Proposition 1 to the new payoff functions so defined. It is easy to check that any *interim* allocation \tilde{a} can be part of an equilibrium of the game form extended by renegotiation. The two downstream firms could just offer the nil contracts $T_i = 0$, let A randomly choose an initial allocation \tilde{a} and then renegotiate away any constrained inefficiency so-obtained by trading rights to move toward the constrained-efficient allocation $\bar{a} = (0)$.

Among all possible equilibria that may arise with resale, there is an interesting class in which A chooses upfront a constrained-efficient allocation that is thus not renegotiated. Those equilibria are in a sense resale-proof. As soon as resale entails some (even slightly) positive transaction costs, such resale-proof equilibria may be particularly attractive.

Following a logic that is by now familiar, the constrained-efficient allocation $\bar{a} = (0)$ can be implemented as a *truthful resale-proof* equilibrium in the bidding game modified by the possibility of resale by means of the following (monotonic) truthful schedules:

(5.2)
$$\widetilde{T}_i(a) = \max \left\{ \frac{1}{2} \sum_{i=0,1} \Pi_j(0) + \frac{1}{2} \left(\Pi_i(a) - \Pi_{-i}(a) \right) - \widetilde{\Pi}_i; 0 \right\}.$$

Resale has, of course, some consequences on the distribution of equilibrium payoffs, as explained in the next proposition.

PROPOSITION 4. There exists a unique truthful resale-proof equilibrium. Profits in this equilibrium are given by the following:

(5.3)
$$\overline{\Pi}_0^{rp} = \frac{1}{2} \Big(\Pi_0(0) + \Pi_1(0) + \Pi_0(1) - \Pi_1(1) \Big),$$

$$\overline{\Pi}_1^{rp} = \Pi_1(0),$$

$$\overline{\Pi}_a^{rp} = \frac{1}{2} \Big(\Pi_0(0) + \Pi_1(1) - \Pi_0(1) - \Pi_1(0) \Big).$$

A (resp. F_0 , F_1) always earns more (resp. less, the same) profits in the unique truthful resale-proof equilibrium than in the unique truthful equilibrium without resale.

With resale, A can obtain a greater share of the industry profit; thus, that of F_0 decreases while F_1 's share remains unchanged. Intuitively, A can now threaten each downstream firm to sell the rights to its rival as a base for future resale; this is a very unattractive outcome that forces the downstream firm that values the most those rights, i.e., F_0 , to pay a lot for exclusivity.

It is straightforward to check that the payoff vector $(\overline{\Pi}_0^{rp}, \overline{\Pi}_1^{rp}, \overline{\Pi}_a^{rp})$ belongs to Σ . These results come as no surprise. The unique truthful resale-proof equilibrium implements the constrained-efficient allocation, gives to the dominated firm its reservation payoff and, by construction, satisfies all incentive constraints that must hold in equilibrium, as all equilibrium payoffs that belong to Σ must do.

Our results suggest that the sole role of banning resale is to change the distribution of profits between the dominant firm F_0 and the producer of rights A. This distribution is of course neutral for both the allocation of rights that arises at equilibrium and consumer welfare. Hence, the only justification that can be found for such a policy is that it might have an indirect impact on investment. Suppose indeed that F_0 's promotional effort is endogenized. With resale, F_0 's profits are lower, and we may expect lower incentives to

exert such effort. In turn, F_0 and F_1 may become more symmetric, and a nonexclusive distribution of rights may become more attractive. In other words, banning resale might have a role in preventing exclusivity; however, this seems to be a very indirect role.

6. Package Bidding

We now extend our previous approach to analyze settings in which firms in the downstream market can acquire several sets of premium contents from the upstream seller. To fix ideas, we suppose that two sets of rights are available. Exclusivity can be given to either firm. Much as before, F_0 is the dominant player in both markets, as it is able to improve the quality of contents more than what F_1 would do if given these rights. Viewers are still located on the [0,1] segment, but they now choose their service provider by comparing prices and the overall quality of the contents, taking into account the possibility that one of these downstream firms owns exclusivity for both sets of rights. For simplicity, we assume that these rights have a symmetric impact on profits and surpluses across markets. Under those conditions, the model is rather similar to that explained in our previous setup.

The only difference is that we hereafter suppose that a joint distribution of either set of rights is no longer possible. There are two reasons for this assumption. First, it simplifies the analysis by limiting the number of configurations under scrutiny. However, this is only a minor restriction since the possibility remains that each downstream firm enjoys exclusively one set of rights, giving rise to a rather balanced market structure. Second, most real-world auctions for the broadcasting of premium sporting events involve exclusive distribution only. The main concern expressed by some competition authorities is to prevent the dominant firm from obtaining exclusivity over all sets of rights to maintain a long-term competitive balance in the market.

We now present the model and the structure of the equilibria with such $package\ bidding.^{24}$ We then analyze the consequences of a ban on package bidding on the equilibrium allocations.

6.1. Preliminaries

Formally, we denote by $b=(a^1,a^2)$ any arbitrary allocation of rights, where a^k (for k=1,2) belongs to $\mathcal{A}_*=\mathcal{A}/\{c\}$. In other words, in each market, A can give exclusivity to either firm or refuse to distribute its rights. The set of possible allocations is thus the cross-product \mathcal{A}^2_* . Thanks to our assumption of additivity, the value-enhancing parameter that applies to firm F_i 's services can thus be written as $\beta_i=\sum_{k=1,2}\alpha_i^k$, where the superscript k indices the set of rights with, as in our baseline model, $\alpha_0^k \in \{0, \alpha + \Delta\}$ and $\alpha_1^k \in \{0, \alpha\}$, depending on how rights are allocated across firms. From there, we can derive the expressions of F_i 's profits for a given allocation of rights b and the induced value-enhancing parameters, as $\pi(\beta_i - \beta_{-i})$. To illustrate, had F_0 acquired the bundle of exclusivity rights, i.e., b=(0,0), $\beta_0=2(\alpha+\Delta)$ and $\beta_1=0$, so that $\Pi_0(0,0)=\pi(2(\alpha+\Delta))$ and $\Pi_1(0,0)=\pi(-2(\alpha+\Delta))$. Similar formulas follow for other configurations of rights.

The next Lemma shows that allocating both sets of rights to F_0 maximizes the overall profits of the industry, as expected.

²⁴Milgrom (2007) offers an exhaustive review of the literature on package auctions.

LEMMA 3. Suppose that F_0 is dominant in both markets. Then, $b^* = (0,0)$ is the constrained-efficient allocation, 25 or

(6.1)
$$\sum_{i=0,1} \Pi_i(0,0) = \arg \max_{b \in \mathcal{A}^2_*} \sum_{i=0,1} \Pi_i(b).$$

An immediate consequence of Lemma 3 is that (0,0) is also market-by-market constrainedefficient; i.e., F_0 should be given the second set of rights if it already owns the first one.
In other words, rights are strict complements for the dominant firm; this is a property
that again holds thanks to the strict convexity of the $\pi(\cdot)$ function. Formally, condition
(6.1) indeed implies the following:

(6.2)
$$\sum_{i=0,1} \Pi_i(0,0) = \arg\max_{a \in \mathcal{A}_*} \sum_{i=0,1} \Pi_i(a,0) = \arg\max_{a \in \mathcal{A}_*} \sum_{i=0,1} \Pi_i(0,a).$$

This property is key to understanding that when F_0 is a dominant firm for both sets of rights, selling those rights as a bundle or separately in different auctions has no impact on the constrained efficiency of the equilibrium allocation. Of course, whether rights are sold as a bundle or separately has an impact on bids and payoffs. We consider those two scenarios in the following.

6.2. Package Bidding Equilibria

Suppose first that both sets of rights are simultaneously on sale by means of the same tender procedure. Downstream firms are unrestricted in their bidding strategies on those packages. Formally, a bidding schedule T_i is thus any arbitrary nonnegative mapping on \mathcal{A}^2_* . Such mapping determines F_i 's bid $T_i(b)$ to A for each feasible package $b \in \mathcal{A}^2_*$. Still relying on our definition of monotonicity, it is straightforward to recast our analysis of Proposition 2. We again denote by $(\overline{T}_0, \overline{T}_1, \overline{b})$ any arbitrary equilibrium pair of bidding schedules and the ensuing allocation of rights.

Proposition 5.

- 1. All monotonic equilibria are constrained-efficient, i.e., $\bar{b} = (0,0)$.
- 2. The set Σ of profit levels $(\overline{\Pi}_0, \overline{\Pi}_1, \overline{\Pi}_a)$ that can be achieved in any monotonic equilibrium is defined by the following conditions:

(6.3)
$$\Pi_0(1,1) \leq \overline{\Pi}_0 \leq \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1),$$

$$\overline{\Pi}_1 = \Pi_1(0,0),$$

$$\Pi_0(0,0) - \Pi_0(1,1) \geq \overline{\Pi}_a \geq \Pi_1(1,1) - \Pi_1(0,0).$$

Proposition 5 echoes, in a multimarket context, our earlier findings for Proposition 2. There still exists a whole range of equilibrium profits corresponding to different distributions of the maximal industry profit between A and the dominant firm F_0 . Indeed, A obtains a greater share of this profit when it can threaten F_0 to sell exclusivity rights

 $^{^{25}}$ In the context of the Hotelling model used in Section 3 being extended to the case of two sets of rights, it turns out that (0,0) is better for welfare than (1,0) or (0,1). However, dynamic considerations such as maintaining a competitive balance in the downstream market may lead a competition authority to prefer a more balanced allocation of rights.

in both markets to F_1 at price $\overline{T}_1(1,1) \ge \Pi_1(1,1) - \Pi_1(0,0)$. Offering such an attractive option in its bidding schedule remains, of course, a weakly dominated strategy for F_1 .

To eliminate those weakly dominated strategies and focus on the distribution of profits that is the most favorable to F_0 , we again consider truthful bidding schedules of the following form:

(6.4)
$$T_i(b) = \max\{\Pi_i(b) - \Pi_i; 0\}.$$

Those bidding schedules are of course monotonic, and any truthful equilibrium is thus constrained-efficient. Echoing our earlier findings in Proposition 3, such an equilibrium determines a unique distribution of profits that again corresponds to an extremal point of the set described in Proposition 5 giving to F_0 the highest possible profit.

PROPOSITION 6. There exists a unique equilibrium with truthful schedules with package bidding. Equilibrium profits are given by the following:

(6.5)
$$\overline{\Pi}_0^b = \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1) > 0,$$

$$\overline{\Pi}_1^b = \Pi_1(0,0) > 0,$$

$$\overline{\Pi}_a^b = \Pi_1(1,1) - \Pi_1(0,0) > 0.$$

6.3. Split Auctions: Pairwise-Proof Quasi-Equilibrium

Consider now a scenario where each set of rights is sold separately by means of a different auction procedure. A final allocation \bar{b} is thus determined market by market. To tackle the analysis of such a complex web of bilateral relationships and provide some predictive insights, we first follow an approach that has been extensively used in the literature on vertical relationships. Following Crémer and Riordan (1987), Hart and Tirole (1990), Horn and Wolinsky (1988) and O'Brien and Shaffer (1992), works in the field have often made the simplifying assumption that, in each bilateral contract (or bargain) it is part of, a given party takes as given the outcome of other bilateral contracts (or bargains) in which it participates. The benefits of this approach are of course tractability.²⁶

There are a number of necessary requirements that any such outcome should satisfy. Mimicking our earlier analysis, downstream firms should of course play to an equilibrium in each bidding market taken separately.

DEFINITION 2. A pairwise-proof quasi-equilibrium is an array $(\overline{T}_0^k, \overline{T}_1^k, \overline{a}^k)_{k=1,2}$ that satisfies the following necessary conditions.

1. Profit maximization for A in market k: A chooses an allocation in market k within the best-response correspondence $\overline{\mathcal{A}}(\overline{T}_0, \overline{T}_1)$ defined as follows:

(6.6)
$$\overline{\mathcal{A}}(T_0^k, T_1^k) = \arg\max_{a^k \in \mathcal{A}_*} \sum_{i=0,1} T_i^k(a) \quad \forall (T_0^k, T_1^k).$$

²⁶To back up this approach, Hart and Tirole (1990) and McAfee and Schwartz (1994) have shown that such an outcome might be sustained as an equilibrium in some structured environments provided that out-of-equilibrium beliefs following unexpected offers are passive. A simple, often found and more practical motivation is that each firm may have delegated to a different department the responsibility to bid in a particular market, thereby avoiding a joint design of bidding schedules in both markets. The tractability of this approach can be useful for empirical purposes, as shown in Dubois and Sætre (2020).

2. Profit maximization for F_i in market k (for k = 1, 2): \overline{T}_i^k satisfies the following:

(6.7)
$$\Pi_{i}(\overline{a}^{k}, \overline{a}^{-k}) - \overline{T}_{i}^{k}(\overline{a}^{k}) \in \arg \max_{\substack{T_{i}^{k} \geq 0 \\ \widetilde{a}^{k} \in \overline{\mathcal{A}}(T_{i}^{k}, \overline{T}_{-i}^{k})}} \Pi_{i}(\widetilde{a}^{k}, \overline{a}^{-k}) - T_{i}^{k}(\widetilde{a}^{k}), \quad i = 0, 1;$$

Our goal here is to exhibit a plausible outcome of our bidding game and study its properties. To this end, we describe a pairwise-proof allocation such that dominant firm F_0 obtains exclusivity in each market. By construction, the corresponding allocation $\overline{a}^k = (0)$ inherits all the properties that were highlighted in Proposition 2 for a market where only one set of rights is sold. In particular, we already know that, by insisting on strategies that are not weakly dominated, the outcome $\overline{a}^k = (0)$ in market k can be implemented with truthful schedules of the following form:

(6.8)
$$\overline{T}_i^k(a^k, 0) = \max\{\Pi_i(a^k, 0) - \overline{\Pi}_i^k; 0\}.$$

It should be stressed that because of the extra benefit of owning two sets of rights, the fees $\overline{\Pi}_i^k$ are no longer equilibrium profits as in the single-right scenario. The next proposition describes profits in such pairwise-proof equilibrium and the corresponding values of $\overline{\Pi}_i^k$.

Proposition 7.

1. The allocation $\bar{b} = (0,0)$ is a pairwise-proof allocation sustained with truthful schedules of the form (6.8) with

(6.9)
$$\overline{\Pi}_0^k = \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(0,1) \text{ and } \overline{\Pi}_1^k = \Pi_1(0,1), \quad k = 1, 2.$$

2. The profits in this pairwise-proof allocation are given by the following:

(6.10)
$$\overline{\Pi}_0^u = \Pi_0(0,0) + 2(\Pi_1(0,0) - \Pi_1(0,1)) > \overline{\Pi}_0^k,$$

$$\overline{\Pi}_1^u = \Pi_1(0,0),$$

$$\overline{\Pi}_a^u = 2(\Pi_1(0,1) - \Pi_1(0,0)) < \overline{\Pi}_a.$$

As in the case of the truthful equilibrium with package bidding, the pairwise-proof allocation also gives all exclusivity rights to F_0 . However, the distribution of profits is more favorable to this dominant firm. The intuition is again straightforward. Recall that F_0 finds it even more attractive to acquire a second set of rights if it already owns a first set of rights. In a pairwise-proof allocation, all players anticipate that F_0 already has exclusivity in market -k when bidding for exclusivity rights in market k. As a result, k can only threaten k0 to sell the rights in market k1 to k1 in case k2 bid in this market is not attractive enough. With package bidding, k2 can instead use the more efficient threat of giving exclusivity to both sets or rights to k3 in case k4 bid to obtain such joint exclusivity is not high enough. k4 thus forces k5 to pay more to obtain all exclusivity rights. At the same time, k5 remains dominated, and the worst scenario in each market is, for that firm, that k6 is exclusive. From Proposition 1 (especially condition (4.6)), k5

²⁷At a rough level, the argument bears some similarity with the literature on repeated games in multimarket contexts (Bernheim and Whinston, 1990; Spagnolo, 1999). In the literature, it is shown that collusion may be easier to enforce under the threat of simultaneous deviations in several markets.

makes a null bid for such an outcome whether it arises through package bidding or as a pairwise-proof allocation. Henceforth, splitting auctions has for the sole consequence of redistributing profit from A to F_0 in comparison with package bidding, with again no impact on F_1 .

MULTILATERAL DEVIATIONS. In the specific context of vertical manufacturer-retailer relationships or in broader contexts of contracting with externalities, McAfee and Schwartz (1994) and Rey and Vergé (2004) have criticized the concept of pairwise-proofness and showed that such a pairwise-proof allocation might unfortunately fail to be a perfect Bayesian equilibrium of the extensive form games under scrutiny. While those authors have focused on the offer-game scenario where the party (manufacturer) at the nexus of all contracts makes secret offers to his or her agents (retailers), Segal and Whinston (2003) have also studied the bidding game scenario discussed in this paper. In our context, we might also wonder whether a pairwise-proof allocation is still immune to multilateral deviations in which one party simultaneously deviates by jointly modifying the bidding schedules in both markets. Of course, the same nonexistence issue could also a priori arise in our bidding environment. Before addressing this issue, we state the following definition.

DEFINITION 3. A pairwise-proof quasi-equilibrium $(\overline{T}_0^k, \overline{T}_1^k, \overline{a}^k)_{k=1,2}$ is an equilibrium if it is robust to multilateral deviations, i.e.,

$$(6.11) \quad \Pi_{i}(\overline{a}^{1}, \overline{a}^{2}) - \sum_{k=1,2} \overline{T}_{i}^{k}(\overline{a}^{k}) = \max_{\substack{T_{i}^{k} \geq 0, k=1,2 \\ \widetilde{a}^{k} \in \overline{\mathcal{A}}(T_{i}^{k}, \overline{T}_{-i}^{k}), k=1,2}} \Pi_{i}(\widetilde{a}^{0}, \widetilde{a}^{1}) - \sum_{k=1,2} T_{i}^{k}(\widetilde{a}^{k}), \quad i = 0, 1.$$

The robustness requirement (6.11) is of course satisfied by the equilibrium bidding schedules under package bidding. By definition, such schedules prevent any deviation from the nonequilibrium distribution of rights in both markets. Viewed through this lens, condition (6.11) imposes a similar condition when the overall bidding schedule $\sum_{k=1,2} \overline{T}_i^k(\overline{a}^k)$ is additive across markets, as requested with pairwise-proof allocations. In our context, this robustness requirement is in fact quite demanding, as shown below.

PROPOSITION 8. The pairwise-proof quasi-equilibrium sustained with truthful schedules (6.8)-(6.9) is not immune to multilateral deviations.

With a pairwise-proof allocation, we already know that F_0 ends up paying too little to obtain exclusivity in both markets, which is the most attractive allocation from its point of view. More precisely and thanks to symmetry, its bid to obtain exclusivity in each market is as follows:

(6.12)
$$\overline{T}_0^k(0) = \max\{0; \Pi_0(0,0) - \overline{\Pi}_0^k\} = \Pi_1(0,1) - \Pi_1(0,0), \quad k = 1, 2.$$

This bid is the *per-market* Vickrey-Clarke-Groves payment that is needed to prevent F_1 from acquiring exclusivity in only one market at the time. Given that the pairwise-proof allocation implements a constrained-efficient allocation, F_0 pays only $2(\Pi_1(0,1)-\Pi_1(0,0))$ to obtain exclusivity in both markets.

In any package auction, F_0 would instead pay at minima the truthful payment:

(6.13)
$$\overline{T}_0^b(0,0) = \Pi_1(1,1) - \Pi_1(0,0), \quad k = 1,2$$

to prevent A from selling both rights to F_1 . Clearly, the convexity of F_1 's profit function implies the following:

$$\Pi_1(1,1) - \Pi_1(0,0) > 2(\Pi_1(0,1) - \Pi_1(0,0)).^{28}$$

Henceforth, F_0 pays too little to obtain exclusivity in both markets in a pairwise-proof allocation. From there, it follows that F_0 certainly has no incentives to enter into multi-lateral deviations that would also implement $\bar{b} = (0, 0)$.

Instead, taking advantage of such a low bid from F_0 , F_1 can bid more aggressively in both markets to obtain bundle (1,1) rather than the putative pairwise-proof allocation (0,0). To see how it can be so, observe that by paying $T_1(1) = \overline{T}_0^k(0) + \varepsilon$ in each market, F_1 can obtain exclusivity in both markets and obtain a net profit worth the following:

$$\Pi_1(1,1) - 2(\Pi_1(0,1) - \Pi_1(0,0)) - 2\varepsilon > \Pi_1(0,0),$$

where the inequality holds from the fact that F_1 's profit is convex and the fact that ε is small enough. Hence, the pairwise-proof allocation is not immune to multilateral deviations by the dominated firm F_1 .

With a pairwise-proof quasi-equilibrium, the final allocation of rights remains unchanged in comparison with package bidding. Consumer surplus also comes unchanged by banning package bidding. Again, such a restriction only redistributes profits from the seller to the dominant firm. As such, and by an argument that mirrors the one made in Section 5, this policy could only have a positive indirect effect in boosting investment by the dominant firm. However, such a presumption is based on the wrong premise that multilateral deviations are not attractive to dominated firms. In other words, making policy recommendations on the predictions of pairwise-proofness seems a rather flawed approach.

Pairwise-proofness is a flawed equilibrium concept; thus, we return to the mere definition of bidding schedules in a multimarket context and impose additional constraints on them. These constraints capture the fact that auctions for each set of rights are run separately. Instead of allowing arbitrary bidding schedules contingent on global allocations, as with the scenario of package bidding, we might view the requirement of *split bidding in each market* as an additivity constraint of the following kind:

(6.14)
$$T_i(a^1, a^2) = \sum_{k=1,2} T_i^k(a^k) \text{ for } i = 1, 2.$$

The benefit of such additivity restrictions on bidding schedules is that pairwise-proofness is obtained for free. Multilateral deviations add no further constraints on putative equilibrium allocations. The cost of such a restriction is instead that a downstream firm's overall bidding strategy might no longer reflect the payoff complementarity that appears when it owns both sets of rights, such as what arises with a truthful schedule. In particular, one might expect that F_0 would bid less than its incremental value to obtain

²⁸Observe that this condition rewrites as $\pi(2\alpha) + \pi(-2\alpha - 2\Delta) > 2\pi(-\Delta)$, which holds since $\pi(\cdot)$ is convex.

exclusivity on only any single set of rights. Under such circumstances, intuition might suggest that imposing the additivity constraint (6.14) on feasible strategies could *a priori* lessen competition between downstream firms.²⁹

Of course, the additive bidding schedule (6.14) is monotonic when each component $T_i^k(a^k)$ is itself monotonic. A monotonic equilibrium in additive schedules thus implements the constrained-efficient allocation $\bar{b} = (0,0)$. The next proposition characterizes equilibrium profits. Surprisingly, and in contrast with the intuition just sketched, the additivity constraint does not restrict the set of equilibrium outcomes in comparison with the scenario where package bidding is possible.

PROPOSITION 9. Any profit levels $(\overline{\Pi}_0^s, \overline{\Pi}_1^s, \overline{\Pi}_a^s)$ in Σ that can be achieved with monotonic equilibria when package bidding is allowed can also be achieved with additive bidding schedules of the form (6.14) when auctions are split.

An immediate corollary of our findings is that the extreme payoffs obtained in (6.5) by means of truthful schedules can still be achieved with split auctions and additive bidding schedules. Let us see how this can be so with this particular allocation. By construction, the truthful equilibrium schedules (6.4) are not additive. Indeed, F_0 benefits from some complementarity in obtaining exclusivity in both markets and, in a truthful equilibrium, is ready to pay the large price $\overline{T}_0^b(0,0)$ given by (6.13). As discussed earlier, this bid is higher than what F_0 pays in a pairwise-proof allocation, although the same allocation (0,0) remains implemented. When restricted to additive bid schedules, F_0 can still prevent A from selling both sets of rights to F_1 with a bid $\overline{T}_0(0)$ that would replicate what the truthful payment does, namely,

$$2\overline{T}_0(0) = \overline{T}_0^b(0,0) = \Pi_1(1,1) - \Pi_1(0,0).$$

This bid $\overline{T}_0(0)$ is now much larger than in the pairwise-proof allocation, since

$$\overline{T}_0(0) = \frac{1}{2}(\Pi_1(1,1) - \Pi_1(0,0)) > \Pi_1(0,1) - \Pi_1(0,0) = \overline{T}_0^k(0),$$

where the inequality again follows from the convexity of F_1 's profit. Multilateral deviations by F_1 are thus still prevented. The flip side of the large bid $\overline{T}_0(0)$ is that one may wonder if F_0 could not simply prefer to acquire exclusivity in only one market rather than duplicating those bids in each market. This is not the case since

$$\overline{T}_0(0) \le \Pi_0(0,0) - \Pi_0(0,1);$$

which is an inequality that now follows from the convexity of F_0 's profit. Therefore, a restriction to split auctions does not change the distribution of profits and surplus.

²⁹In the literature on package auctions, which is mainly inspired by applications to the telecom industry, this phenomenon is referred to as the risk of *demand exposure*. A bidder who cannot express his or her willingness to pay for a bundle reduces his or her bid on each component because he or she is exposed to the risk that a local competitor, who is interested in only one item, prevents him or her from acquiring both items. In such contexts, not only is competition lessened, but there is also a risk that the final allocation will be inefficient. See Kagel and Levin (2005) and Milgrom (2007).

7. Conclusion

Motivated by the prevalence of exclusivity in broadcasting rights in pay-TV markets, we have developed a model of competition in which firms compete both upstream for the acquisition of broadcasting rights and downstream to attract viewers. Profit functions in the downstream market exhibit a fundamental convexity property. This property implies that giving exclusive broadcasting rights to one firm, namely, a dominant player, maximizes the industry profit. We characterize all equilibria when firms can freely bid for any possible allocation of the broadcasting rights and show that all monotonic equilibria implement this profit-maximizing allocation. Light-handed regulation such as banning resale or limiting the lot size under tenders in the case of multiple rights does not change this outcome. At best, such regulation might modify the distribution of profits that is achieved. As a result, and if anything, the dominant firm's incentives to further invest in enhancing the quality of services might be modified (and possibly diminished); however, this appears to be a very indirect way of promoting downstream competition.

Therefore, our analysis suggests that if exclusivity is to be avoided because it fails to maximize welfare, more heavy-handed tools may prove useful. As an example, the European Commission required the UK Premier League to ensure that at least one package of broadcasting rights to the English soccer championship would go to an operator other than the dominant one, thereby forcing de facto a competitive outcome. Perhaps unexpectedly, such a 'no single buyer rule' is now almost always implemented by major European soccer leagues, even though competitive and regulatory landscapes differ. One may wonder, however, whether systematically forcing a nonexclusive outcome is warranted. We believe that this issue should be resolved on a case-by-case basis.

Specifically, with regard to soccer broadcasting rights, banning package bidding is a remedy that forces soccer fans to subscribe to several pay-TV offers to have full access to the televised games of a championship, which is an outcome that is ruled out by assumption in our modeling. Whether profit functions exhibit convexity when some viewers from the downstream market may buy simultaneously from several pay-TV firms remains an open question.

We have also remained silent on another issue that is salient in pay-TV markets and sports events broadcasting, namely, whether sports leagues that coordinate the selling of broadcasting rights on behalf of teams form a cartel and thus harm competition. Again, competition authorities differ in their assessments. Our model could be extended to take into account both the joint and individual selling of broadcasting rights and analyze how this impacts the vertical agreements that emerge between sellers of rights and pay-TV firms. This investigation is left for future research.

APPENDIX

PROOF OF LEMMA 1. The marginal viewer who is indifferent between buying from F_0 or from F_1 is located at $x_0(p_0, p_1, \alpha_0, \alpha_1)$ which is defined by the indifference condition $v + \alpha_0 - p_0 - tx = v + \alpha_1 - p_1 - t(1 - x_0)$, or $x_0 = 1/2 + (1/2t)(\alpha_0 - \alpha_1 + p_1 - p_0)$. From this indifference condition, and from the fact that viewers are uniformly distributed over the unit segment, we easily deduce

³⁰See Commission Decision of 22/3/2006 Case COMP/38.173 – Joint selling of the media rights to the FA Premier League, available at https://eur-lex.europa.eu/legal-content/EN/ALL/?uri=CELEX% 3A52008XC0112%2803%29.

the following expression for the demand faced by F_0 :

$$D_0(p_0, p_1, \alpha_0, \alpha_1) = \Pr(x \le x_0(p_0, p_1, \alpha_0, \alpha_1)) = \frac{1}{2} + \frac{1}{2t}(\alpha_0 - \alpha_1 + p_1 - p_0).$$

Assuming (for simplicity) that v is large enough that the market is always fully covered for the equilibrium prices that we will derive below, the demand for F_1 is then simply $1-D_0(p_0, p_1, \alpha_0, \alpha_1)$.

Turning now to the equilibrium prices $(p_0^*(\alpha_0, \alpha_1), p_1^*(\alpha_0, \alpha_1))$ chosen at the last stage of the game, F_0 's best response is given by $p_0 = \arg\max_{\widetilde{p}_0} \widetilde{p}_0 D_0(\widetilde{p}_0, p_1^*(\alpha_0, \alpha_1), \alpha_0, \alpha_1)$. This yields the following first-order condition: $2p_0 = t + p_1^*(\alpha_0, \alpha_1) + \alpha_1 - \alpha_0$. Similarly, F_1 's best response satisfies $p_1 = \arg\max_{\widetilde{p}_1} \widetilde{p}_1(1 - D_0(p_0^*(\alpha_0, \alpha_1), \widetilde{p}_1, \alpha_0, \alpha_1))$, which yields the corresponding first-order condition: $2p_1 = t + p_0^*(\alpha_0, \alpha_1) + \alpha_0 - \alpha_1$. The Nash equilibrium in prices $(p_0^*(\alpha_0, \alpha_1), p_1^*(\alpha_0, \alpha_1))$ is thus given by $p_i^*(\alpha_i, \alpha_{-i}) = P^*(\alpha_i - \alpha_{-i})$, i = 0, 1 where $P^*(x) = t + \frac{x}{3}$. At equilibrium, F_0 's market share is thus given by $D_0(p_0^*(\alpha_0, \alpha_1), p_1^*(\alpha_1, \alpha_0), \alpha_0, \alpha_1) = \frac{1}{2} + \frac{1}{6t}(\alpha_0 - \alpha_1)$. The expressions of the profits respectively made by F_0 and F_1 in (3.1) follow. Assuming that the market is covered (which requires v large enough), total welfare is equal to the following (omitting some notations to streamline the exposition):

$$\widetilde{SW}(\alpha_0, \alpha_1) = \int_0^{D_0(p_0^*, p_1^*, \alpha_0, \alpha_1)} (v + \alpha_0 - tx) dx + \int_{D_0(p_0^*, p_1^*, \alpha_0, \alpha_1)}^1 (v + \alpha_1 - t(1 - x)) dx,$$

$$= v + \alpha_1 + \frac{1}{2} \left(1 + \frac{1}{3t} (\alpha_0 - \alpha_1) \right) (\alpha_0 - \alpha_1)$$

$$- \frac{t}{2} \left(D_0^2(p_0^*, p_1^*, \alpha_0, \alpha_1) + (1 - D_0(p_0^*, p_1^*, \alpha_0, \alpha_1))^2 \right),$$

$$= v + \alpha_1 + \frac{1}{2} \left(1 + \frac{1}{3t} (\alpha_0 - \alpha_1) \right) (\alpha_0 - \alpha_1)$$

$$- \frac{t}{2} \left(\left(\frac{1}{2} + \frac{1}{6t} (\alpha_0 - \alpha_1) \right)^2 + \left(\frac{1}{2} - \frac{1}{6t} (\alpha_0 - \alpha_1) \right)^2 \right),$$

which ultimately simplifies as follows:

(A.1)
$$\widetilde{SW}(\alpha_0, \alpha_1) = v - \frac{t}{4} + \frac{1}{2}(\alpha_0 + \alpha_1) + \frac{5}{36t}(\alpha_0 - \alpha_1)^2.$$

Using (3.1) and (A.1), we can express consumer surplus as follows:

$$\begin{split} \widetilde{CS}(\alpha_0, \alpha_1) &= \widetilde{SW}(\alpha_0, \alpha_1) - \sum_{i=0,1} \widetilde{\Pi}_i(\alpha_i, \alpha_{-i}), \\ &= v - \frac{5t}{4} + \frac{1}{2}(\alpha_0 + \alpha_1) + \frac{1}{36t}(\alpha_0 - \alpha_1)^2. \end{split}$$

PROOF OF LEMMA 2. From (3.1) we have the following:

$$\sum_{i=0,1} \Pi_i(0) = \frac{t}{2} \left(1 + \frac{1}{3t} (\alpha + \Delta) \right)^2 + \frac{t}{2} \left(1 - \frac{1}{3t} (\alpha + \Delta) \right)^2$$

$$\geq \sum_{i=0,1} \Pi_i(1) = \frac{t}{2} \left(1 + \frac{1}{3t} \alpha \right)^2 + \frac{t}{2} \left(1 - \frac{1}{3t} \alpha \right)^2$$

$$\geq 2\Pi(c) = t$$

where the string of inequalities follows from Jensen inequality and $\Delta > 0$.

We define social welfare in terms of the allocation a as $SW(a) = \widetilde{SW}(\alpha_0, \alpha_1)$ for an allocation

a inducing extra-valuations α_0 and α_1 . A similar definition is used for consumer surplus: $CS(a) = \widetilde{CS}(\alpha_0, \alpha_1)$. Then, notice that

$$SW(c) \ge SW(\{0\}) \ge SW(\{1\})$$

amounts to the following:

$$\alpha \geq \max \Big\{ \frac{1}{2}\alpha + \frac{5}{36t}\alpha^2, \frac{1}{2}(\alpha + \Delta) + \frac{5}{36t}(\alpha + \Delta)^2 \Big\},$$

or

$$\alpha \ge \frac{1}{2}(\alpha + \Delta) + \frac{5}{36t}(\alpha + \Delta)^2,$$

which amounts to (3.3).

PROOF OF PROPOSITION 1. Because $\overline{T}_i(a) \geq 0$ for all $a \in \mathcal{A}$, we have the following:

$$(A.2) \overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) = \max_{a \in \mathcal{A}} \overline{T}_i(a) + \overline{T}_{-i}(a) \ge \max_{a \in \mathcal{A}} \overline{T}_{-i}(a).$$

The first equality writes as (4.4).

CLAIM 1. (4.6) holds.

PROOF OF CLAIM 1. Suppose to the contrary that $\overline{T}_i(a_{-i}) > 0$. Consider the new bidding schedule as follows:

(A.3)
$$\tilde{T}_i(a) = \max\{\overline{T}_i(a) - \overline{T}_i(a_{-i}); 0\}.$$

By construction, we have the following:

$$\tilde{T}_i(a_{-i}) = 0.$$

Since $\overline{T}_i(a) \geq 0$ for all $a \in \mathcal{A}$, we also have the following:

$$\tilde{T}_i(a) \leq \max{\{\overline{T}_i(a);0\}} = \overline{T}_i(a).$$

Moreover, the following string of inequality holds:

$$\begin{aligned} \max_{a \in \mathcal{A}} \, \tilde{T}_i(a) + \overline{T}_{-i}(a) &= \max_{a \in \mathcal{A}} \left\{ \max \{ \overline{T}_i(a) - \overline{T}_i(a_{-i}) + \overline{T}_{-i}(a); \overline{T}_{-i}(a) \} \right\}, \\ &= \max \left\{ \max_{a \in \mathcal{A}} \overline{T}_i(a) - \overline{T}_i(a_{-i}) + \overline{T}_{-i}(a); \max_{a \in \mathcal{A}} \overline{T}_{-i}(a) \right\}, \\ &= \max \left\{ \overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) - \overline{T}_i(a_{-i}); \overline{T}_{-i}(a_{-i}) \right\}, \\ &= \overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) - \overline{T}_i(a_{-i}), \\ &< \tilde{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}), \end{aligned}$$

where the last equality follows from the definition of \overline{a} and the last inequality from the definition of $\tilde{T}_i(a)$ given in (A.3). From this, we deduce that \tilde{T}_i implements \overline{a} as follows:

$$\overline{a} \in \arg\max_{a \in \mathcal{A}} \tilde{T}_i(a) + \overline{T}_{-i}(a)$$

and does so at a weakly lower cost for F_i than \overline{T}_i ; which ends the proof.

Claim 2. (4.5) holds.

PROOF OF CLAIM 2. By definition, we have $\overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) = \max_{a \in \mathcal{A}} \overline{T}_i(a) + \overline{T}_{-i}(a)$ and $\overline{T}_{-i}(a_{-i}) = \max_{a \in \mathcal{A}} \overline{T}_{-i}(a)$. Suppose to the contrary that

$$\overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) > \overline{T}_{-i}(a_{-i}).$$

There thus exists $\varepsilon > 0$ small enough such that $\overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) > \varepsilon + \overline{T}_{-i}(a_{-i})$. Consider the new bidding schedule as follows:

$$\tilde{T}_i(a) = \max{\{\overline{T}_i(a) - \varepsilon; 0\}}.$$

Proceeding as in the Proof of Claim 1, we obtain the following:

$$\begin{split} \max_{a \in \mathcal{A}} \, \tilde{T}_i(a) + \overline{T}_{-i}(a) &= \max_{a \in \mathcal{A}} \left\{ \max\{\overline{T}_i(a) - \varepsilon + \overline{T}_{-i}(a); \overline{T}_{-i}(a)\} \right\}, \\ &= \max\left\{ \max_{a \in \mathcal{A}} \overline{T}_i(a) - \varepsilon + \overline{T}_{-i}(a); \max_{a \in \mathcal{A}} \overline{T}_{-i}(a) \right\}, \\ &= \max\left\{ \overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) - \varepsilon; \overline{T}_{-i}(a_{-i}) \right\}, \\ &= \overline{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}) - \varepsilon, \\ &\leq \tilde{T}_i(\overline{a}) + \overline{T}_{-i}(\overline{a}). \end{split}$$

From this, we deduce that \tilde{T}_i again implements \bar{a} , or

$$\overline{a} \in \arg\max_{a \in A} \tilde{T}_i(a) + \overline{T}_{-i}(a).$$

and does so at a weakly lower cost for F_i than \overline{T}_i ; which ends the proof.

Claims 1 and 2 taken together allow to prove Proposition 1. \Box

PROOF OF PROPOSITION 2.

ITEM 1. First observe that a monotonic equilibrium is necessarily such that $a_{-0} = (1)$ and $a_{-1} = (0)$. From (4.6), it immediately follows that

$$\overline{T}_0(1) = \overline{T}_1(0) = 0.$$

From (4.5), we write the following:

$$(A.5) \overline{T}_0(\overline{a}) + \overline{T}_1(\overline{a}) = \overline{T}_1(1) = \overline{T}_0(0).$$

From (4.3), any equilibrium allocation must satisfy

(A.6)
$$\sum_{i=0,1} \Pi_i(\overline{a}) + \sum_{i=0,1} \overline{T}_i(\overline{a}) \ge \sum_{i=0,1} \Pi_i(0) + \sum_{i=0,1} \overline{T}_i(0).$$

Inserting (A.4) and (A.5) into (A.6) and simplifying yields the following:

$$\sum_{i=0,1} \Pi_i(\overline{a}) \ge \sum_{i=0,1} \Pi_i(0).$$

Since (0) is constrained-efficient, we necessarily have $\bar{a} = (0)$, which ends the proof.

ITEM 2. We now turn to the characterization of equilibrium payments in monotonic equilibria. We start with the following lemma.

LEMMA A.4. The set of equilibrium bids in monotonic equilibria is defined by the following conditions:

(A.7)
$$\overline{T}_0(0) \in [\Pi_1(1) - \Pi_1(0), \Pi_0(0) - \Pi_0(1)],$$

$$(A.8) \overline{T}_0(0) = \overline{T}_1(1) \ge 0,$$

$$\overline{T}_0(1) = \overline{T}_1(0) = 0,$$

(A.10)
$$\overline{T}_0(c) \in [0, \overline{T}_0(0) + \Pi_1(0) - \Pi_1(c)],$$

(A.11)
$$\overline{T}_1(c) \in [0, \Pi_0(0) - \Pi_0(c)],$$

with

$$(A.12) \overline{T}_0(c) + \overline{T}_1(c) \le \overline{T}_0(0).$$

PROOF OF LEMMA A.4. Necessity. Since in any equilibrium $\bar{a} = (0)$, F_1 should not want to deviate to induce the alternative allocation a = (1) by offering a payment $T_1(1) \geq \overline{T}_0(0)$. The corresponding incentive constraint becomes as follows:

(A.13)
$$\Pi_1(0) \ge \max_{T_1(1) \text{ s.t. } T_1(1) \ge \overline{T}_0(0)} \Pi_1(1) - T_1(1) = \Pi_1(1) - \overline{T}_0(0)$$

which gives the lower bound in (A.7).

Additionally, F_1 should not want to deviate from $\overline{a} = a_{-1} = (0)$ to induce an outcome with joint distribution, *i.e.*, a = (c), by offering a payment $T_1(c)$ such that $\overline{T}_0(c) + T_1(c) \ge \overline{T}_0(0)$. The corresponding incentive constraint becomes as follows:

(A.14)
$$\Pi_{1}(0) \geq \max_{T_{1}(c) \text{ s.t. } \overline{T}_{0}(c) + T_{1}(c) \geq \overline{T}_{0}(0)} \Pi_{1}(c) - T_{1}(c) = \Pi_{1}(c) + \overline{T}_{0}(c) - \overline{T}_{0}(0).^{31}$$

From which, we obtain the upper bound in (A.10).

Second, turning to F_0 's incentives to abide to the equilibrium strategy rather than letting a = (1) emerges, which is simply obtained by not paying since $a_{-0} = (1)$, it must be that

(A.15)
$$\Pi_0(0) - \overline{T}_0(0) \ge \Pi_0(1).$$

which gives the upper bound in (A.7).

Additionally, F_0 should not be willing to induce a = (c) either, which requires the following:

$$(A.16) \qquad \Pi_0(0) - \overline{T}_0(0) \ge \max_{T_0(c) \text{ s.t. } T_0(c) + \overline{T}_1(c) \ge \overline{T}_0(0)} \Pi_0(c) - T_0(c) = \Pi_0(c) + \overline{T}_1(c) - \overline{T}_0(0)$$

where the constraint in the maximand of the right-hand side follows from (A.4). Simplifying yields (A.11).

Putting together (A.13) and (A.15) yields (A.7).

Using (4.4) yields the following:

$$\overline{T}_0(c) + \overline{T}_1(c) < \overline{T}_0(0) + \overline{T}_1(0).$$

Taking into account that $a_{-1} = (0)$ (and thus $\overline{T}_1(0) = 0$) yields (A.12). The right-hand side inequality follows from bid being non-negative.

Using (4.5), the fact that $\overline{a} = a_{-1} = (0)$ and thus $\overline{T}_1(0) = 0$ (from (4.6)) implies (A.8). Last, (A.9) follows from (4.6).

Sufficiency. Any pair of bidding schedules $(\overline{T}_0, \overline{T}_1)$ that satisfy conditions (A.7) to (A.12) also satisfy, by construction, all the incentive constraints for F_0 , F_1 and A that must hold at an equilibrium.

Gathering everything and noting that the overall profit of the industry must be $\Pi_0(0) + \Pi_1(0)$ yields (4.8).

PROOF OF PROPOSITION 3. First, constrained efficiency follows from Proposition 2. Second, we remind that monotonicity implies $a_{-0}=(1)$ and $a_{-1}=(0)$, and we note that truthful schedules of the form $T_i(a)=\max\{\Pi_i(a)-\overline{\Pi}_i,0\}$ satisfy those monotonicity conditions. Following Laussel and Le Breton (2001), we define the value of any arbitrary coalition $S\subseteq N=\{F_0,F_1,A\}$ as $W(S)=\max_{a\in\mathcal{A}}\sum_{i\in S}\Pi_i(a)$. The cooperative game with transferable utility defined through these values is strongly subadditive if, for all $S\subseteq N$, $T\subseteq N$ such that $S\cup T=N$, $W(N)\leq W(T)+W(S)-W(S\cap T)$; this is a condition that can be readily verified since we have the following:

$$W(\{01\}) = \Pi_0(0) + \Pi_1(0), \quad W(\{0\}) = \Pi_0(0), \quad W(\{1\}) = \Pi_1(1), \quad W(\emptyset) = 0$$

and thus

$$W({01}) < W({0}) + W({1}) \Leftrightarrow \Pi_1(0) < \Pi_1(1).$$

Following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair $(\overline{\Pi}_0, \overline{\Pi}_1)$ must lie on the Pareto frontier of the set defined by the three following constraints

$$(A.17) \overline{\Pi}_0 \le W(\{01\}) - W(\{1\}) = \Pi_0(0) + \Pi_1(0) - \Pi_1(1),$$

(A.18)
$$\overline{\Pi}_1 \le W(\{01\}) - W(\{0\}) = \Pi_1(0),$$

(A.19)
$$\overline{\Pi}_0 + \overline{\Pi}_1 \le W(\{01\}) = \Pi_0(0) + \Pi_1(0).$$

Because of strong sub-additivity, the following inequality holds:

$$W({01}) - W({1}) + W({01}) - W({0}) < W({01}).$$

It implies that (A.19) necessarily holds when (A.17) and (A.18) do (Bernheim and Whinston, 1986a, Corollary 1) and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\overline{\Pi}_0, \overline{\Pi}_1) = (\Pi_0(0) + \Pi_1(0) - \Pi_1(1), \Pi_1(0)).$$

This finally gives us the expressions of profits for the whole industry in (4.9).

PROOF OF PROPOSITION 4. We adapt the methodology developed in Proposition 3 to a scenario with renegotiation. For any arbitrary coalition $S \subseteq N = \{F_0, F_1, A\}$, we may define the coalitional payoff with renegotiation as $W^{rp}(S) = \max_{a \in \mathcal{A}} \sum_{i \in S} \tilde{\Pi}_i(a)$. By Definition (5.1), we have the following:

$$W^{rp}(\{01\}) = \Pi_0(0) + \Pi_1(0), \quad W^{rp}(\{0\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} \Big(\Pi_0(0) - \Pi_1(0) \Big),$$

$$W^{rp}(\{1\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) + \frac{1}{2} \Big(\Pi_1(1) - \Pi_0(1) \Big), \quad W^{rp}(\emptyset) = 0.$$

The cooperative game so-defined is again strongly subadditive since

$$W^{rp}(\{01\}) < W^{rp}(\{0\}) + W^{rp}(\{1\}) \Leftrightarrow \Pi_1(0) - \Pi_0(0) < \Pi_1(1) - \Pi_0(1).$$

Again following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair $(\overline{\Pi}_0^{rp}, \overline{\Pi}_1^{rp})$ lies on the Pareto frontier of the set defined by the three following constraints:

(A.20)
$$\overline{\Pi}_0^{rp} \le W^{rp}(\{01\}) - W^{rp}(\{1\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) - \frac{1}{2} \Big(\Pi_1(1) - \Pi_0(1) \Big),$$

(A.21)
$$\overline{\Pi}_1 \le W^{rp}(\{01\}) - W^{rp}(\{0\}) = \frac{1}{2} \sum_{j=0,1} \Pi_j(0) - \frac{1}{2} \Big(\Pi_0(0) - \Pi_1(0) \Big),$$

(A.22)
$$\overline{\Pi}_0^{rp} + \overline{\Pi}_1^{rp} \le W^{rp}(\{01\}) = \Pi_0(0) + \Pi_1(0).$$

Strong sub-additivity implies that (A.22) necessarily holds when (A.20) and (A.21) (Bernheim and Whinston, 1986a, Corollary 1) do and that the Pareto frontier of that set is reduced to the extremal point as follows:

$$(\overline{\Pi}_0^{rp}, \overline{\Pi}_1^{rp}) = \left(\frac{1}{2} \sum_{i=0,1} \Pi_i(0) - \frac{1}{2} \left(\Pi_1(1) - \Pi_0(1)\right), \Pi_1(0)\right).$$

Observe that F_1 's profits with or without resale are the same (see (4.9) and (5.3)). Then A's profit in the unique truthful resale-proof equilibrium is greater than that in the unique truthful equilibrium without resale since

$$\frac{1}{2}\Big(\Pi_0(0)+\Pi_1(1)-\Pi_1(0)-\Pi_0(1)\Big)>\Pi_1(1)-\Pi_1(0).$$

This concludes the proof.

PROOF OF LEMMA 3. The proof is straightforward and left to the reader. It immediately follows from the fact that $\pi(\cdot)$ is increasing and convex as the Proof of Lemma 2.

PROOF OF PROPOSITION 5.

ITEM 1. First observe that a monotonic equilibrium is necessarily such that $b_{-0} = (1,1)$ and $b_{-1} = (0,0)$ where $b_{-i} = \arg\max_{b \in \mathcal{A}^2_*} \overline{T}_{-i}(b)$. From (4.6), it follows that

(A.23)
$$\overline{T}_0(1,1) = \overline{T}_1(0,0) = 0.$$

From (4.5), we write the following:

$$\overline{T}_0(\overline{b}) + \overline{T}_1(\overline{b}) = \overline{T}_1(1,1) = \overline{T}_0(0,0).$$

From (4.3), any equilibrium allocation must satisfy the following:

(A.25)
$$\sum_{i=0,1} \Pi_i(\bar{b}) + \sum_{i=0,1} \overline{T}_i(\bar{b}) \ge \sum_{i=0,1} \Pi_i(0,0) + \sum_{i=0,1} \overline{T}_i(0,0).$$

Inserting (A.23) and (A.24) into (A.25) and simplifying yields the following:

$$\sum_{i=0,1} \Pi_i(\bar{b}) \ge \sum_{i=0,1} \Pi_i(0,0).$$

Since (0,0) is constrained-efficient, we necessarily have $\overline{b} = (0,0)$; which ends the proof.

ITEM 2. We now turn to the characterization of equilibrium payments in monotonic equilibria.

Lemma A.5. The whole set of bids in monotonic equilibria is defined by the following inequalities:

(A.26)
$$\overline{T}_0(0,0) = \overline{T}_1(1,1) \in [\Pi_1(1,1) - \Pi_1(0,0), \Pi_0(0,0) - \Pi_0(1,1)],$$

(A.27)
$$\overline{T}_0(1,1) = \overline{T}_1(0,0) = 0,$$

(A.28)
$$\overline{T}_0(0,1) \in \left[0, \overline{T}_0(0,0) + \Pi_1(0,0) - \Pi_1(0,1)\right],$$

$$\overline{T}_0(1,0) \in \left[0, \overline{T}_0(0,0) + \Pi_1(0,0) - \Pi_1(1,0)\right],$$

(A.29)
$$\overline{T}_1(0,1) \in [0, \Pi_0(0,0) - \Pi_0(0,1)],$$

$$\overline{T}_1(1,0) \in [0, \Pi_0(0,0) - \Pi_0(1,0)],$$

(A.30)
$$\max\{\overline{T}_0(0,1) + \overline{T}_1(0,1), \overline{T}_0(1,0) + \overline{T}_1(1,0)\} \le \overline{T}_0(0,0) = \overline{T}_1(1,1).$$

Proof of Lemma A.5.

Necessity. Since in any equilibrium $\bar{b} = (0,0)$, the first equality in (A.26) follows from (4.5) and (4.6).

Also F_1 should not want to deviate to induce the alternative allocation a = (1, 1). The corresponding incentive constraint becomes as follows:

$$(A.31) \qquad \Pi_1(0,0) \ge \max_{T_1(1,1) \text{ s.t. } T_1(1,1) \ge \overline{T}_0(0,0)} \Pi_1(1,1) - T_1(1,1) = \Pi_1(1,1) - \overline{T}_0(0,0).$$

Similarly, F_1 should not induce a deviation towards a = (0, 1), which requires the following:

$$\begin{array}{ll} (\mathrm{A}.32) & \Pi_1(0,0) \geq \\ & \max_{T_1(0,1) \text{ s.t. } T_0(0,1) + T_1(0,1) \geq \overline{T}_0(0,0)} \Pi_1(0,1) - T_1(0,1) = \Pi_1(0,1) + T_0(0,1) - \overline{T}_0(0,0); \end{array}$$

thus, the first condition in (A.28) follows.

 F_1 should also not induce a deviation towards a = (1,0), which requires the following:

(A.33)
$$\Pi_1(0,0) \ge \max_{T_1(1,0) \text{ s.t. } T_0(1,0) + T_1(1,0) \ge \overline{T}_0(0,0)} \Pi_1(0,1) - T_1(0,1) \equiv \Pi_1(0,1) + T_0(0,1) - \overline{T}_0(0,0);$$

thus, the second condition in (A.28) follows.

Let us now turn to F_0 's incentives to abide to the equilibrium strategy rather than to induce a = (1, 1). Such deviation is simply obtained with a null bid since $a_{-0} = (1, 1)$. F_0 's incentive constraint is thus written as follows:

(A.34)
$$\Pi_0(0) - \overline{T}_0(0,0) \ge \Pi_0(1,1).$$

Putting together (A.31) and (A.34) yields the second condition in (A.26).

Also, F_0 should not induce a deviation towards a = (0, 1). Since, A is indifferent between $\overline{a} = (0, 0)$ and a = (1, 1) (from the first equality in (A.26)), this requires avoiding that A switches

to (1,1) in case F_0 no longer offers $\overline{T}_0(0,0)$. We write this incentive constraint as follows:

$$\begin{array}{ll} (\mathrm{A.35}) & \Pi_1(0,0) - \overline{T}_0(0,0) \geq \\ & \max_{T_0(0,1) \text{ s.t. } T_0(0,1) + \overline{T}_1(0,1) \geq \overline{T}_1(1,1)} \Pi_0(0,1) - T_0(0,1) = \Pi_0(0,1) + \overline{T}_1(0,1) - \overline{T}_1(1,1). \end{array}$$

Again taking into account the first equality in (A.26), (A.35) simplifies as follows:

$$\Pi_1(0,0) - \overline{T}_0(0,0) \ge \Pi_0(0,1) + \overline{T}_1(0,1) - \overline{T}_0(0,0).$$

Therefore, the upper bound for $\overline{T}_1(0,1)$ in (A.29) immediately follows.

Finally, F_0 should also not induce a deviation towards a = (1,0). Replicating lines above, it must be that

$$\begin{array}{ll} (\mathrm{A}.36) & \Pi_0(0,0) - \overline{T}_0(0,0) \geq \\ & \max_{T_0(1,0) \text{ s.t. } T_0(1,0) + \overline{T}_1(1,0) \geq \overline{T}_1(1,1)} \Pi_0(1,0) - T_0(1,0) = \Pi_0(1,0) + \overline{T}_1(1,0) - \overline{T}_1(1,1). \end{array}$$

Again taking into account the first equality in (A.26), (A.36) simplifies as follows:

$$\Pi_0(0,0) - \overline{T}_0(0,0) \ge \Pi_0(1,0) + \overline{T}_1(1,0) - \overline{T}_0(0,0).$$

Therefore, the upper bound for $\overline{T}_1(1,0)$ in (A.29) immediately follows.

Sufficiency. Any pair of bidding schedules $(\overline{T}_0, \overline{T}_1)$ that satisfy conditions (A.26) also satisfy F_0 , F_1 and A's incentive constraints from the above analysis, which defines an equilibrium. \square

The expression of profits immediately follows from the existing bounds on bids characterized in Lemma A.5.

PROOF OF PROPOSITION 6. Truthful strategies are monotonic; thus, any putative equilibrium with truthful bidding schedules implements a constrained-efficient outcome. We again adapt the methodology already used in Propositions 3 and 4 to a scenario with multiple rights. For any arbitrary coalition $S \subseteq N = \{F_0, F_1, A\}$, we may define the coalitional payoff as $W(S) = \max_{b \in \mathcal{A}_s^2} \sum_{i \in S} \tilde{\Pi}_i(a)$. We immediately check that

$$W(\{01\}) = \Pi_0(0,0) + \Pi_1(0,0), \quad W(\{0\}) = \Pi_0(0,0),$$

 $W(\{1\}) = \Pi_1(1,1), \quad W^{rp}(\emptyset) = 0.$

The cooperative game with the so-defined coalitional payoffs is again strongly sub-additive since

$$W(\{01\}) < W(\{0\}) + W(\{1\}) \Leftrightarrow \Pi_1(0,0) < \Pi_1(1,1).$$

Again following Bernheim and Whinston (1986a, Theorem 2), any equilibrium pair $(\overline{\Pi}_0^{rp}, \overline{\Pi}_1^{rp})$ lies on the Pareto frontier of the set defined by the following three constraints

(A.37)
$$\overline{\Pi}_0 \le W(\{01\}) - W(\{1\}) = \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1),$$

(A.38)
$$\overline{\Pi}_1 \le W(\{01\}) - W(\{0\}) = \Pi_1(0,0),$$

(A.39)
$$\overline{\Pi}_0 + \overline{\Pi}_1 \le W(\{01\}) = \Pi_0(0,0) + \Pi_1(0,0).$$

Strong sub-additivity implies that (A.39) necessarily holds when (A.37) and (A.38) (Bernheim and Whinston, 1986a, Corollary 1) do and that the Pareto frontier of that set is reduced to the

extremal point as follows:

$$(\overline{\Pi}_0^b, \overline{\Pi}_1^b) = (\Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1), \Pi_1(0,0)).$$

Gathering everything, we finally obtain the expressions of equilibrium profits $\overline{\Pi}_1^b$ and $\overline{\Pi}_a^b$ in the unique truthful equilibrium which are given in (6.5).

PROOF OF PROPOSITION 7.

ITEM 1. Players conjecture that $\bar{a}^{-k} = (0)$ in market -k. Then, (6.2) ensures that $\bar{a}^k = (0)$ is the constrained-efficient allocation when restricted on market k. Mimicking our earlier findings from Proposition 2, any pairwise-proof allocation in market k implements $\bar{a}^k = (0)$. We can then apply our earlier findings from Proposition 3 to show that the truthful strategies (6.8) together with the expressions (6.9) form an equilibrium in market k. Note that we use symmetry to simplify the expressions of profits.

ITEM 2. When it offers the truthful schedules determined in (6.8)-(6.9), F_0 obtains at the pairwise-proof allocation, a payoff worth the following:

$$\overline{\Pi}_0^u = \Pi_0(0,0) - \overline{T}_0^1(0,0) - \overline{T}_0^2(0,0)$$

or

$$\overline{\Pi}_0^u = \Pi_0(0,0) - 2\left(\Pi_0(0,0) - (\Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(0,1))\right),\,$$

where again the above expression uses symmetry. Simplifying yields the expression of $\overline{\Pi}_0^u$ in (6.10). We check the following:

$$\overline{\Pi}_1^u > \overline{\Pi}_1^b$$

or

$$\Pi_0(0,0) + 2(\Pi_1(0,0) - \Pi_1(0,1)) > \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1),$$

which amounts to the following:

$$\Pi_1(0,0) + \Pi_1(1,1) > 2\Pi_1(0,1) \Leftrightarrow \pi(-2\alpha - 2\Delta) + \pi(2\alpha) > 2\pi(-\Delta),$$

which is a property that holds thanks to the convexity of $\pi(\cdot)$.

We turn now to F_1 . F_1 obtains at the pairwise-proof allocation a payoff worth the following:

$$\Pi_1(0,0) - \overline{T}_1^1(0,0) - \overline{T}_1^2(0,0) = \Pi_1(0,0) - 2(\Pi_1(0,0) - \Pi_1(0,0))$$

where again the above expression uses symmetry. Simplifying yields the expression of $\overline{\Pi}_1^u$ in (6.10).

Notice that

$$\overline{\Pi}_0^u + \overline{\Pi}_1^u + \overline{\Pi}_a^u = \overline{\Pi}_0^b + \overline{\Pi}_1^b + \overline{\Pi}_a^b = \Pi_0(0,0) + \Pi_1(0,0);$$

thus, we obtain the expression of $\overline{\Pi}_a^u$ in (6.10).

Finally, since $\overline{\Pi}_1^u = \overline{\Pi}_1^b$, we obtain the following:

$$\overline{\Pi}_a^u < \overline{\Pi}_1^b$$
.

PROOF OF PROPOSITION 8. First, observe that the best multilateral deviation for F_0 should induce (0,0). In each market k, we would have $a_{-0}^k = (1)$. Thus, inducing (0,0) with a pair of

bidding schedules giving total payoff $\sum_{k=1,2} T_0^k(a^k)$ to A, requires to give at least $\sum_{k=1,2} T_0^k(0)$ such that

$$\sum_{k=1,2} T_0^k(0) + \sum_{k=1,2} \max\{\Pi_1(0,0) - \overline{\Pi}_1^k; 0\} = \sum_{k=1,2} \max\{\Pi_1(0,1) - \overline{\Pi}_1^k; 0\}.$$

Taking into account the expression of $\overline{\Pi}_1^k$ given in (6.9) and simplifying yields the following:

$$\sum_{k=1,2} T_0^k(0) = 2(\Pi_1(0,1) - \Pi_1(0,0)).$$

The maximal payoff that a multilateral deviation could yield to F_0 is thus

$$\Pi_0(0,0) - 2(\Pi_1(0,1) - \Pi_1(0,0)) = \overline{\Pi}_0^u$$

a payoff which is also obtained at the *pairwise-proof quasi-equilibrium*. This shows that multilateral deviations are not attractive for F_0 .

We now turn now to F_1 's benefit of a multilateral deviation. The best multilateral deviation for F_1 should induce (1,1). In each market k, we would have $a_{-1}^k = (0)$. Thus, inducing (1,1) with a pair of bidding schedules giving total payoff $\sum_{k=1,2} T_1^k(a^k)$ to A requires to give at least $\sum_{k=1,2} T_1^k(1)$ such that

$$(A.40) \quad \sum_{k=1,2} T_1^k(1) + \sum_{k=1,2} \max\{\Pi_0(1,1) - \overline{\Pi}_0^k; 0\} = \sum_{k=1,2} \overline{T}_0^k(0) = \sum_{k=1,2} \max\{\Pi_0(0,0) - \overline{\Pi}_0^k; 0\}.$$

First, observe that

$$\Pi_0(1,1) < \overline{\Pi}_0^k$$

since the following string of inequalities holds

$$\Pi_0(1,1) + \Pi_1(0,1) < \Pi_0(1,1) + \Pi_1(1,1) < \Pi_0(0,0) + \Pi_1(0,0).$$

Hence, the right-hand side of (A.40) can be reduced to the following:

(A.41)
$$\sum_{k=1,2} T_1^k(1).$$

Second, observe that

$$\Pi_0(0,0) > \overline{\Pi}_0^k$$

since

$$\Pi_0(0,0) > \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(0,1).$$

Hence, the left-hand side of (A.40) can be reduced to the following:

(A.42)
$$2(\Pi_1(0,1) - \Pi_1(0,0)).$$

Inserting (A.41) and (A.42) into (A.40) we obtain the following:

(A.43)
$$\sum_{k=1,2} T_1^k(1) = 2(\Pi_1(0,1) - \Pi_1(0,0)).$$

Therefore, the maximal gain for a multilateral deviation towards (1,1) by F_1 is as follows:

$$\Pi_1(1,1) - 2(\Pi_1(0,1) - \Pi_1(0,0)).$$

 F_1 gains from a multilateral deviation since

$$\Pi_1(1,1) - 2(\Pi_1(0,1) - \Pi_1(0,0)) > \overline{\Pi}_1 = \Pi_1(0,0)$$

amounts to

$$\pi(2\alpha) + \pi(-2\alpha - 2\Delta) > 2\pi(-\Delta),$$

which holds thanks to the convexity of $\pi(\cdot)$. This shows that a multilateral deviation is attractive for F_1 .

Proof of Proposition 9.

BIDS. Inserting the additivity requirement (6.14) into constraints (A.26) to (A.30) gives a characterization of the whole set of equilibrium transfers on split markets as those satisfying the following set of linear constraints:

(A.44)
$$2\overline{T}_0(0) = 2\overline{T}_1(1) \in [\Pi_1(1,1) - \Pi_1(0,0), \Pi_0(0,0) - \Pi_0(1,1)],$$

$$\overline{T}_0(1) = \overline{T}_1(0) = 0,$$

(A.46)
$$\overline{T}_0(0) \ge \Pi_1(0,1) - \Pi_1(0,0) \ge 0,$$

(A.47)
$$0 \le \overline{T}_1(1) \le \Pi_0(0,0) - \Pi_0(0,1).$$

The set of possible values for $\overline{T}_0(0) = \overline{T}_1(1)$ satisfying all those constraints is non-empty whenever

(A.48)
$$\max\{2(\Pi_1(0,1) - \Pi_1(0,0)); \Pi_1(1,1) - \Pi_1(0,0)\} \le \min\{2(\Pi_0(0,0) - \Pi_0(0,1)); \Pi_0(0,0) - \Pi_0(1,1)\}.$$

Expressing profits in terms of the $\pi(\cdot)$ function, we find for the left-hand side of (A.48):

$$(A.49) \max\{2(\Pi_1(0,1) - \Pi_1(0,0)); \Pi_1(1,1) - \Pi_1(0,0)\} = \max\{2(\pi(-\Delta) - \pi(-2\alpha - 2\Delta)); \pi(2\alpha) - \pi(-2\alpha - 2\Delta)\}$$
$$= \pi(2\alpha) - \pi(-2\alpha - 2\Delta).$$

Since, $\pi(\cdot)$ being convex, we have the following:

$$2\pi(-\Delta) < \pi(-2\alpha - 2\Delta) + \pi(2\alpha).$$

The right-hand side of (A.48) becomes as follows:

$$(A.50) \min\{2(\Pi_0(0,0) - \Pi_0(0,1)); \Pi_0(0,0) - \Pi_0(1,1)\} = \min\{2(\pi(2\alpha + 2\Delta) - \pi(\Delta)); \pi(2\alpha + 2\Delta) - \pi(-2\alpha)\},$$

$$= \pi(2\alpha + 2\Delta) - \pi(-2\alpha).$$

Since $\pi(\cdot)$ is convex, we have the following:

$$2\pi(\Delta) < \pi(2\alpha + 2\Delta) + \pi(-2\alpha).$$

Gathering (A.49) and (A.50) and inserting into (A.48), the set of possible values for $\overline{T}_0(0) = \overline{T}_1(1)$ is non-empty whenever

$$\pi(2\alpha) - \pi(-2\alpha - 2\Delta) \le \pi(2\alpha + 2\Delta) - \pi(-2\alpha),$$

but again, this property follows from $\pi(\cdot)$ being convex.

Finally, observe that Conditions (A.44) to (A.47) are necessary but also sufficient for an

equilibrium with additive bidding schedules. Sufficiency follows from the existence of bids satisfying those constraints.

PROFITS. From (A.44), we deduce the following characterization of F_0 , F_1 and A's profits as follows:

$$\Pi_0(1,1) \leq \overline{\Pi}_0^s \leq \Pi_0(0,0) + \Pi_1(0,0) - \Pi_1(1,1),$$

$$\overline{\Pi}_1^s = \Pi_1(0,0),$$

$$\Pi_0(0,0) + \Pi_1(0,0) - \overline{\Pi}_0^s = \overline{\Pi}_a^s \geq \Pi_1(1,1) - \Pi_1(0,0).$$

This ends the proof.

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