

How to misunderstand a diagram: The eccentric/equant model in a 17th century Jesuit Chinese text

Diagram Diversity in the Light of Digital Humanities: Types and Ambiguous Cases
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Plan of this talk

- Jesuit astronomers in late Ming dynasty China, and the compilation of the *Chong zhen li shu* 崇禎曆書 ‘Documents on calendrical astronomy [prepared during] the Chongzhen reign (1628-1644)’.
- Solar tables and the equation of centre.
- The solar models of Tycho and Kepler.
- The explanation of the equation of centre table in the *Chong zhen li shu*, and its connection with Kepler’s innovations.
- Recalculating the equation table: correct and incorrect methods.
- The authorship of the table, and the origin of the incorrect method.

Acknowledgements

- The research reported here is part of a continuing project on Jesuit astronomy in early Qing dynasty China, jointly with Catherine Jami (CNRS).
- The following article contains important substantive discussions of the solar theory in the *Chong zhen li shu*:
 - Chu Longfei 褚龙飞 and Shi Yunli 石云里 (2012). "Chong zhen li shu xi lie li fa zhong de tai yang yun dong li lun 《崇祯历书》系列历法中的太阳运动理论 (The Theory of Solar Motions in the Chongzhen Lishu Series)." *Ziran kexueshi yanjiu* 自然科学史研究 **31** (4): 410-427.
- The earliest use of the tangent formula used here to recalculate the Keplerian equant/eccentric solar equation of centre known to me is:
 - Duke, Dennis (2008). "An Interesting Property of the Equant." *DIO: The International Journal of Scientific History* **15**: 24-37.
- I am grateful to Jim Evans (Puget Sound) for helpful email discussions of points connected with this paper.

Jesuits in China in the late Ming dynasty (1368-1644)

- Matteo Ricci (1552-1610) the first Jesuit missionary to establish residence in Beijing (1601). He translates parts of *Elements of Geometry* into Chinese with Xu Guangqi 徐光啓 (1562-1633), a Catholic convert who later achieves high office.
- In 1629 Xu proposes to the Chongzhen 崇禎 emperor (reigned 1628-1644) that a 'Calendar Office' *Li ju* 曆局 should be set up, in which Jesuits expert in astronomy should lead work to prepare a reform of the calendar.
- First Jesuit appointed was Johann(es) Schreck, also Terrenz or Terrentius, (Deng Yuhan 鄧玉函, 1576-1630), assisted by Giacomo Rho (Luo Yagu 羅雅谷, 1593-1638), and Adam Schall von Bell (Tang Ruowang 湯若望, 1591-1666).

The formation of the *Chong zhen li shu* 崇禎曆書 ‘Documents on calendrical astronomy [prepared during] the Chongzhen reign (1628-1644)’ (CZLS)

- A series of texts are prepared and submitted to the throne when ready, beginning in 1630. Schreck dies in May of that year.
- First set of tables of the sun are submitted on 25 October 1630; second set on 28 February 1631. Both sets have Giacomo Rho named as principal author, followed by list of assistants, European and Chinese.
- Position of the sun as seen from the earth at any instant is found by using starting point tabulated for the given year, then calculating mean motion along ecliptic to the given day, finally correcting to true apparent motion by adding or subtracting ‘additive or subtractive differences for the sun’s orbit’ *ri chan jia jian cha* 日躔加減差 tabulated against sun’s distance from perigee.
- In European practice, the mean motion is measured from the apogee, and called the ‘mean anomaly’ or ‘mean centrum’, and the additions or subtractions are called the ‘equation of centre’.

The basis of Jesuit astronomy in China

The content of the *Chong zhen li shu* is based on the astronomical system of Tycho Brahe (1546-1601).

Tycho's cosmology was geocentric, but broadly kinematically equivalent to that of Copernicus.

Tycho's prolonged and systematic observations, carried out by himself and his assistants with the large and well-made instruments of his Uraniborg observatory, provided the best data available at the time.

The use of Tycho's system therefore provided good predictive power, without the doctrinal (and physical) problems of a moving and rotating earth.

Some tables in the *Chong zhen li shu* are directly taken from those found in Brahe, Tycho (1610). *Tychonis Brahe Dani, Astronomiae instauratae progymnasmata*.



Start of solar equation of centre table from *Chong zhen li shu*

The translation of the red framed section is shown.

Values of the equation in degrees minutes and seconds are given in vertical alignment in the original. The first 6 signs counting top to bottom show additive equations for 0° in each sign, hence at anomalies of 0°, 30°, 60° and so on. The next six signs counting bottom to top show subtractive equations for 30° in each sign (their zero points are at the end of the table).

The next column to the left in the original shows values for 0° 10' in the signs 1 to 6, and 29° 50' in the signs 7 to 12, then in the next column for 0° 20' and 29° 40', and so on.

In this table, the angle (*yin shu* 引數 'guide number') against which the equation is tabulated at 10 second intervals is measured from perigee, not from apogee, and hence differs by 180° from the angle on which European tables are based.

Values for 0° 0' in sign; Additive			
D	M	S	Sign
0	0	0	1, 12
1	2	33	2, 11
1	47	40	3, 10
2	3	10	4, 9
1	45	46	5, 8
1	0	40	6, 7
Values for 30° 0' in sign; subtractive			

引數	加	度	分	秒
十五	〇	〇	〇	〇
十四	〇	〇	〇	〇
十三	〇	〇	〇	〇
十二	〇	〇	〇	〇
十一	〇	〇	〇	〇
十	〇	〇	〇	〇
九	〇	〇	〇	〇
八	〇	〇	〇	〇
七	〇	〇	〇	〇
六	〇	〇	〇	〇
五	〇	〇	〇	〇
四	〇	〇	〇	〇
三	〇	〇	〇	〇
二	〇	〇	〇	〇
一	〇	〇	〇	〇
〇	〇	〇	〇	〇
減	〇	〇	〇	〇
一十	〇	〇	〇	〇
二十	〇	〇	〇	〇
三十	〇	〇	〇	〇
四十	〇	〇	〇	〇
五十	〇	〇	〇	〇

Tycho's
(1610)
table of
solar
equation of
centre in
Progymn.
1, 60-61

60

TYCHONIS BRAHE LIB. I.

TABVLA PROSTHAPHAERESIVM SOLARIVM

PRÆSVPPONENS ECCENTRICITATEM 3584. aut p. 2. / 9.

0 Subtr: Diff: A.				1 Subtr: Diff: A.				2 Subtr: Diff: A.				G.	
G.	P.	/.	//.	P.	/.	//.	P.	/.	//.	P.	/.	//.	G.
0	0	0	0	2	5	0	59	44	1	49	1	44	30
1	0	2	5	2	5	1	1	33	1	48	1	45	29
2	0	4	10	2	4	1	3	21	1	46	1	46	28
3	0	6	14	2	4	1	5	8	1	47	1	47	27
4	0	8	18	2	4	1	6	54	1	46	1	48	26
5	0	10	22	2	4	1	8	38	1	44	1	49	25
6	0	12	26	2	4	1	10	21	1	43	1	50	24
7	0	14	30	2	4	1	12	3	1	42	1	51	23
8	0	16	33	2	3	1	13	44	1	41	1	52	22
9	0	18	36	2	3	1	15	24	1	39	1	53	21
10	0	20	39	2	3	1	17	3	1	37	1	54	20
11	0	22	41	2	2	1	18	40	1	37	1	55	19
12	0	24	43	2	2	1	20	16	1	36	1	55	18
13	0	26	45	2	2	1	21	51	1	35	1	56	17
14	0	28	47	2	2	1	23	24	1	33	1	57	16
15	0	30	48	2	1	1	24	56	1	32	1	57	15
16	0	32	49	2	0	1	26	27	1	29	1	58	14
17	0	34	49	1	59	1	27	56	1	27	1	59	13
18	0	36	48	1	59	1	29	23	1	26	1	59	12
19	0	38	47	1	58	1	30	49	1	24	2	0	11
20	0	40	45	1	58	1	32	13	1	23	2	0	10
21	0	42	43	1	57	1	33	36	1	22	2	0	9
22	0	44	40	1	57	1	34	58	1	20	2	1	8
23	0	46	36	1	56	1	36	18	1	18	2	1	7
24	0	48	31	1	55	1	37	36	1	16	2	2	6
25	0	50	25	1	54	1	38	52	1	14	2	2	5
26	0	52	18	1	53	1	40	6	1	13	2	2	4
27	0	54	11	1	53	1	41	18	1	11	2	2	3
28	0	56	3	1	52	1	42	29	1	9	2	2	2
29	0	57	0	1	51	1	43	0	1	8	2	3	1
30	0	59	44	1	50	1	44	46	1	8	2	3	0

G. 11 Adde Diff: S. 10 Adde Diff: S. 9 Adde Diff: S. G.

DE STELLA NOVA ANNI 1572.

61

NOSTRIS TEMPORIBVS ACCOMMODATA.

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3 Subtr: Diff: A.S.				4 Subtr: Diff: S.				5 Subtr: Diff: S.								
G.	P.	/.	//.	P.	/.	//.	P.	/.	//.	P.	/.	//.	P.	/.	//.	G.
0	2	3	9	0	4	1	48	36	1	3	1	3	33	1	53	30
1	2	3	13	0	2	1	47	33	1	5	1	1	40	1	55	29
2	2	3	15	0	2	1	46	28	1	5	0	59	45	1	57	28
3	2	3	13	0	2	1	45	21	1	7	0	57	48	1	59	27
4	2	3	9	0	4	1	44	12	1	9	0	55	49	2	1	26
5	2	3	3	0	8	1	43	1	1	14	0	53	48	2	1	25
6	2	2	55	0	10	1	41	47	1	16	0	51	47	2	2	24
7	2	2	45	0	12	1	40	31	1	17	0	49	45	2	2	23
8	2	2	33	0	14	1	39	14	1	19	0	47	43	2	3	22
9	2	2	19	0	14	1	37	55	1	19	0	45	40	2	3	21
10	2	2	2	0	17	1	36	35	1	20	0	43	36	2	4	20
11	2	1	43	0	19	1	35	13	1	24	0	41	31	2	5	19
12	2	1	22	0	21	1	33	49	1	26	0	39	25	2	7	18
13	2	0	58	0	26	1	32	23	1	29	0	37	18	2	8	17
14	2	0	32	0	28	1	30	54	1	31	0	35	10	2	9	16
15	2	0	4	0	28	1	29	23	1	31	0	33	1	2	9	15
16	1	59	34	0	30	1	27	50	1	33	0	30	52	2	10	14
17	1	59	2	0	32	1	26	16	1	34	0	28	42	2	10	13
18	1	58	27	0	35	1	24	40	1	36	0	26	32	2	11	12
19	1	57	50	0	37	1	23	3	1	37	0	24	21	2	11	11
20	1	57	18	0	39	1	21	25	1	38	0	22	10	2	11	10
21	1	56	30	0	41	1	19	46	1	39	0	19	59	2	11	9
22	1	55	46	0	44	1	18	5	1	41	0	17	47	2	12	8
23	1	55	0	0	46	1	16	22	1	43	0	15	35	2	12	7
24	1	54	12	0	48	1	14	36	1	46	0	13	22	2	13	6
25	1	53	21	0	51	1	12	48	1	48	0	11	9	2	13	5
26	1	52	28	0	53	1	10	59	1	49	0	8	56	2	14	4
27	1	51	33	0	55	1	9	9	1	50	0	6	42	2	14	3
28	1	50	36	0	57	1	7	18	1	51	0	4	28	2	14	2
29	1	49	0	0	59	1	5	0	1	52	0	2	14	2	14	1
30	1	48	36	1	1	1	3	33	1	53	0	0	0	2	14	0

30	1	48	36	1	1	1	3	33	1	53	0	0	0	2	14	0
G.	8 Adde			Diff: S.A.			7 Adde			Diff: A.			6 Adde			G.

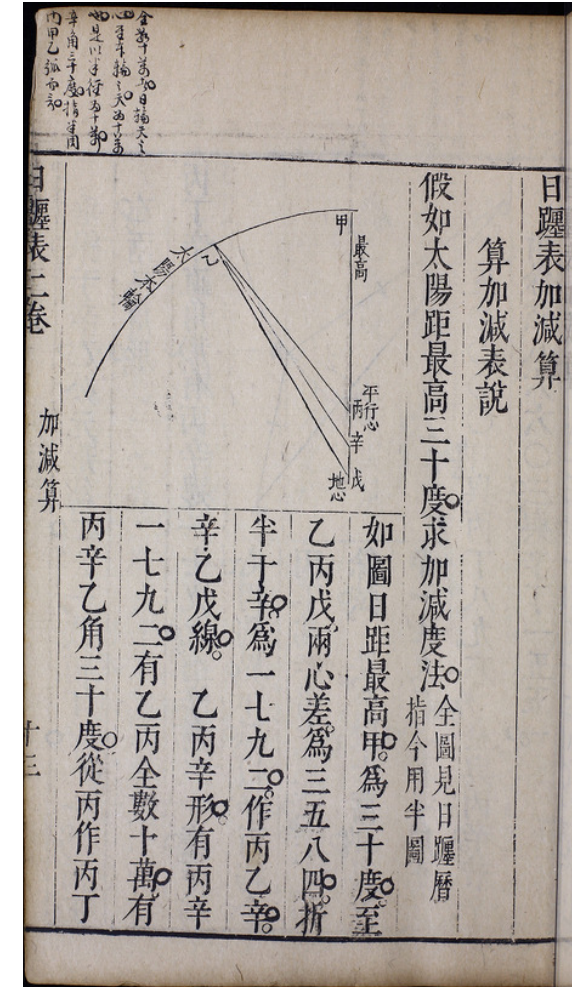
Chong zhen li shu and *Progymnasmata* values

<i>Chong zhen li shu</i> Values for 0° 0' in sign; Additive			
D	M	S	Sign
0	0	0	1, 12
1	2	33	2, 11
1	47	40	3, 10
2	3	10	4, 9
1	45	46	5, 8
1	0	40	6, 7
Values for 30° 0' in sign; subtractive			

<i>Progymnasmata</i> Values for 0° 0' in sign; Additive			
D	M	S	Sign
0	0	0	1, 12
1	3	33	2, 11
1	48	36	3, 10
2	3	9	4, 9
1	44	46	5, 8
0	59	44	6, 7
Values for 30° 0' in sign; subtractive			

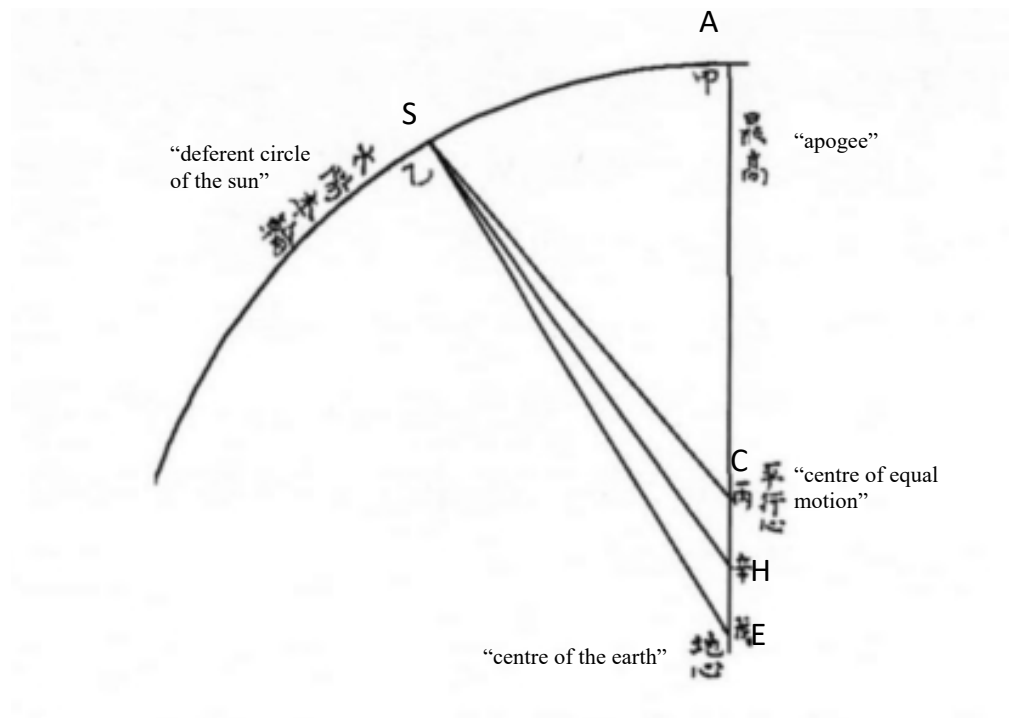
How was the *CZLS* equation table calculated?

- Some *CZLS* texts have no explanation of how the equation table is calculated.
- Other editions (Bodleian Library, Oxford, and Kyujanggak, Seoul) have further material after the table.
- Headings on right: ‘Calculation of additive and subtractive [differences] in solar tables’; ‘Explanation of calculating the table of additive and subtractive [differences]’.

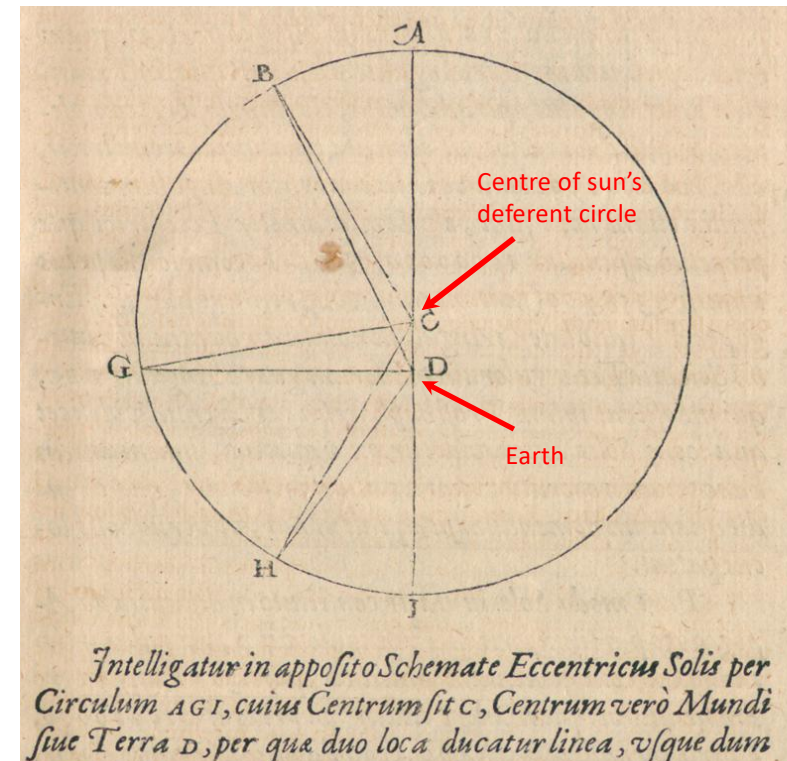


Where does the CZLS diagram come from?

CZLS solar motion diagram has an equant and an eccentric earth



Tycho *Progymnasmata* solar motion diagram has eccentric earth, no equant.



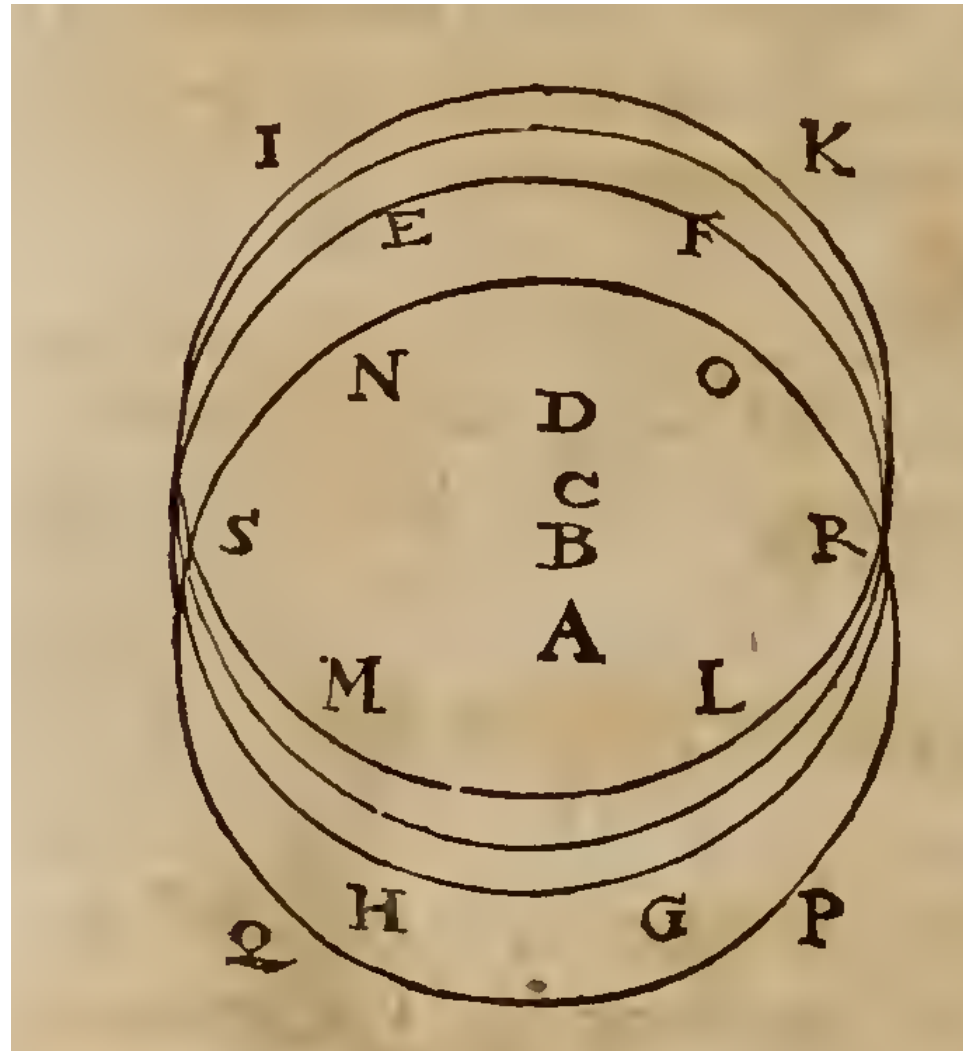
Kepler 1596 *Mysterium
Cosmographicum*, ch XXII:
a hypothesis ...

A: Sun, at centre of cosmos

B: Centre of deferent circle of
planet, round which planet
moves; sun is 'eccentric' with
respect to this point.

D: Equant, from which radius
vector to planet rotates at
constant angular speed.

*... haec incertalis sub Astronomo
iudice pendeat.* “ ... let this
remain undecided under [the
decision of] the astronomer judge



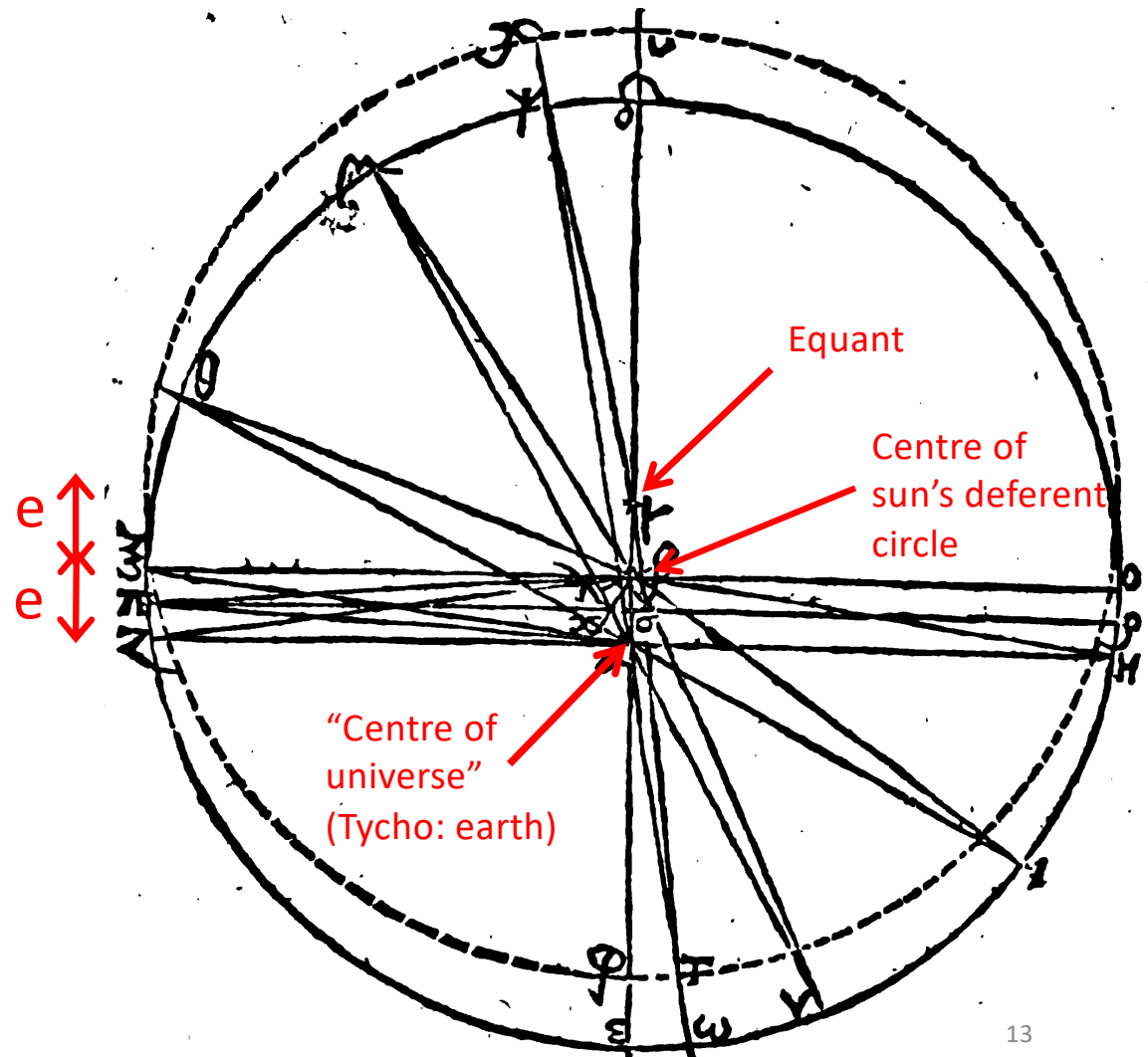
Kepler 1609

Astronomia Nova, ch XXXII

Kepler shows that the old eccentric model for the sun cannot be made consistent with Tycho's accurate observations. He thus applies to the sun the equant/eccentric model for the outer planets used by Ptolemy in the *Almagest* (2nd century CE).

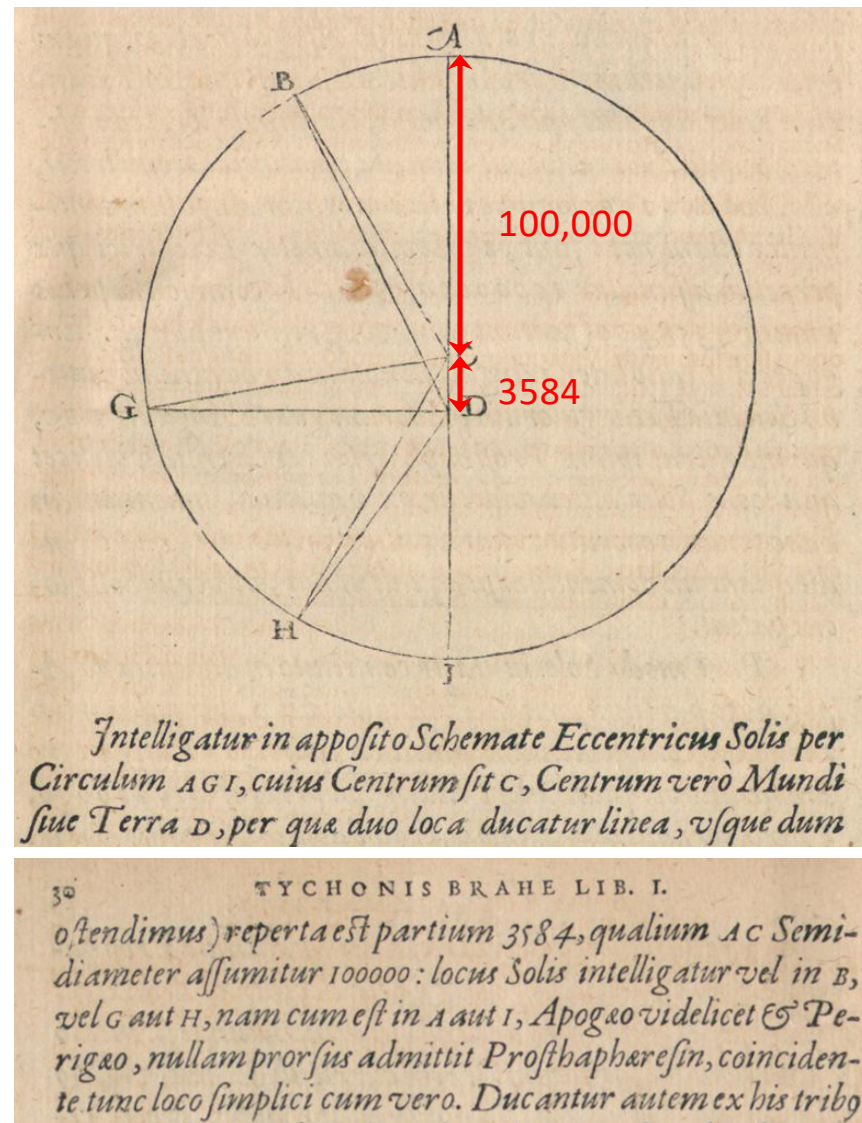
.... Cum ergo in Mysterio meo
Cosmographico ... dici non posset an etiam
Sol vel Terra puncto aequatorio & ejus
eccentricitas bisectione utatur ... Hoc inquam
jam demonstrato ...

‘...While in my *Mysterium Cosmographicum*
I could not say whether the sun [Tycho] or
earth [Copernicus] should make use of an
equant point and the bisection of the
eccentricity ... I say this has now been
proved ...



‘Bisecting the eccentricity’: 1

Kepler notes (1609
Astronomia Nova ch XXIII)
that in Tycho’s system, for a
solar deferent circle of radius
100,000 units, the earth is
‘off-centre’ (eccentric) by
3584 units – as stated in
Tycho *Progymnasmata* on the
page following the diagram
showing eccentric earth in
relation to deferent.

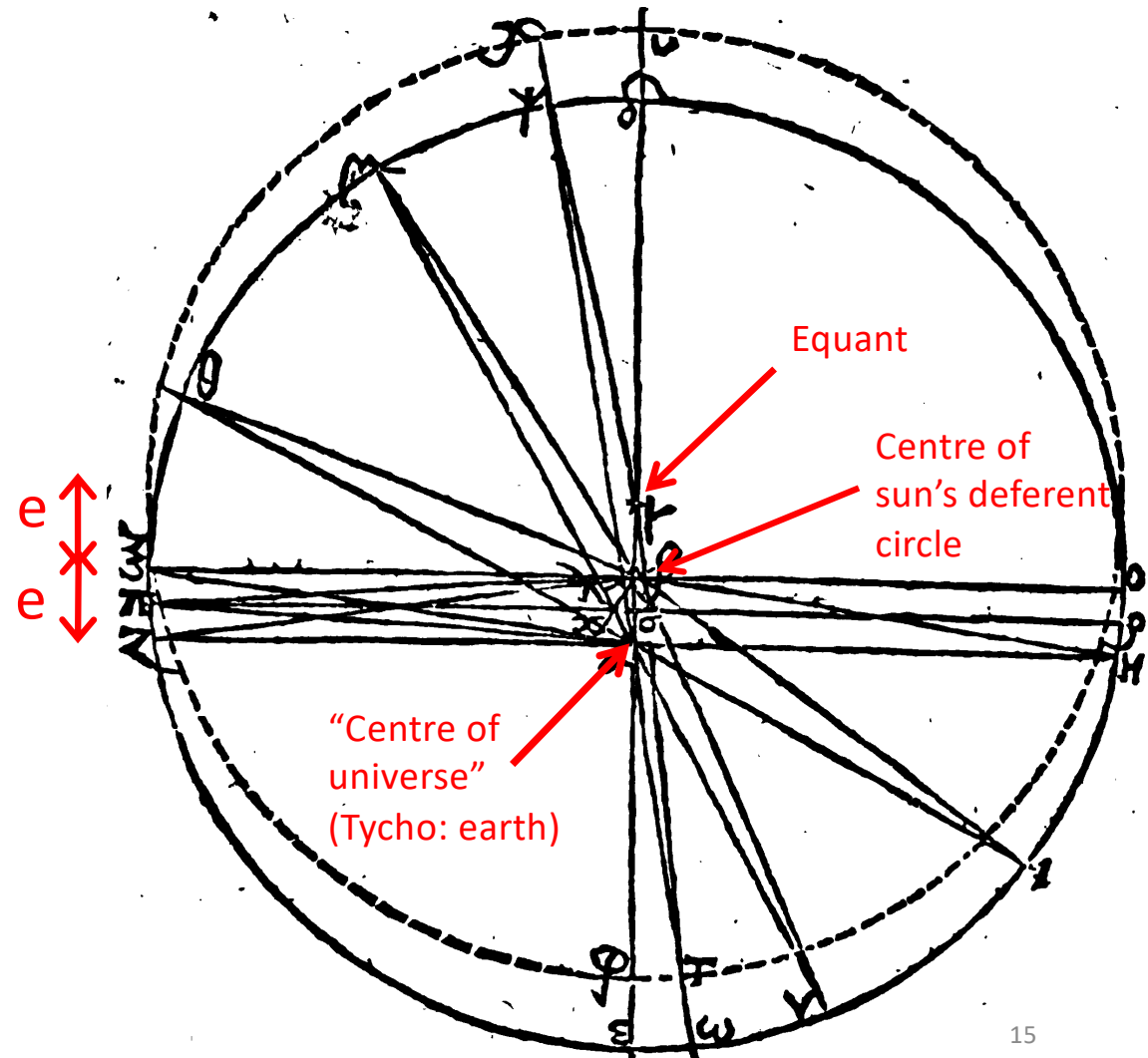


‘Bisecting the eccentricity’: 2

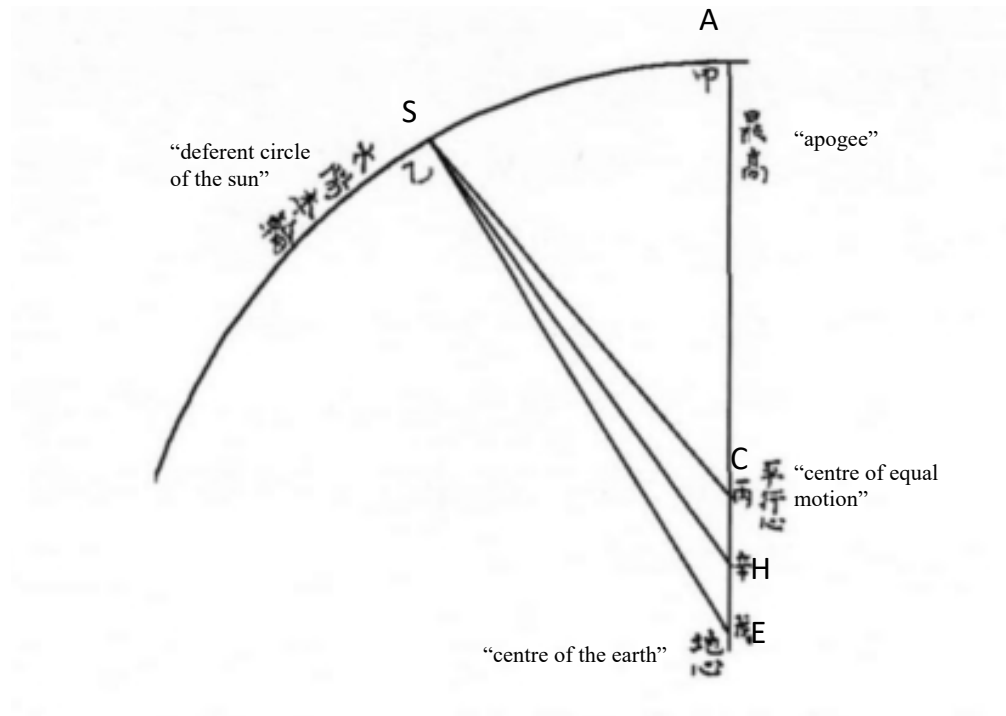
Kepler states (ch XXIII) that in fact Tycho’s eccentricity of 3584 for a 100,000 radius should be split into two equal parts, each of 1792 units, representing the distance of the earth (in Tycho’s system) and of the solar equant from the centre of the solar deferent circle.

So in the diagram, $e = 1792$.

(In fact Kepler prefers the rounded value of 1800 for his own purposes.)



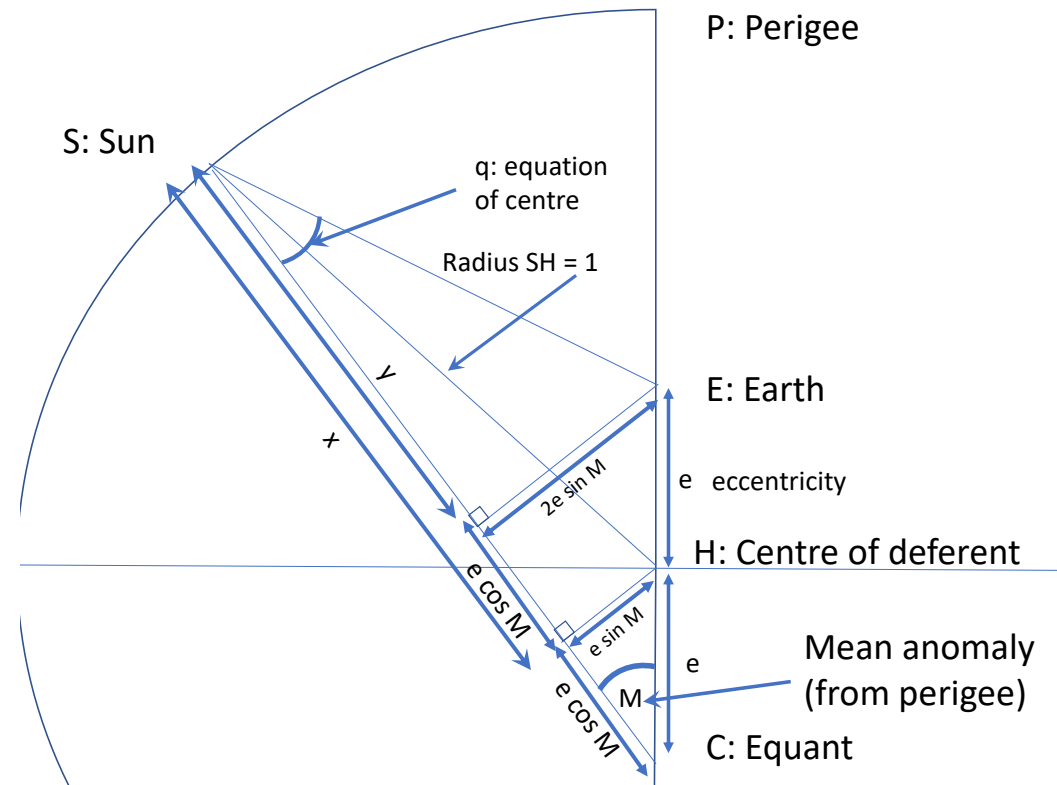
Eccentricity in the CZLS ‘explanation’ diagram



- The text accompanying the diagram states:
- 兩心差。為三五八四。折半于辛。為一七九二。 ‘The separation of the two centres [C and E] is 3584. Bisect it at H, and that makes 1792.’
- This is clearly Kepler’s bisection of Tycho’s eccentricity.

Recalculating q, the equation of centre

- For convenience, the diagram shows a unit radius deferent (rather than 100,000), and $e = 0.01792$ (rather than 1792). Mean anomaly is measured from perigee as in *CZLS*, not apogee.
- Now $\tan q = (2e \sin M)/y$
- And $y = x - e \cos M$
- While $1^2 = x^2 + (e \sin M)^2$
- Hence $x = \sqrt{1 - (e \sin M)^2}$
- Thus:
- $q = \arctan \left(\frac{2e \sin M}{\sqrt{1 - e^2 \sin^2 M} - e \cos M} \right)$

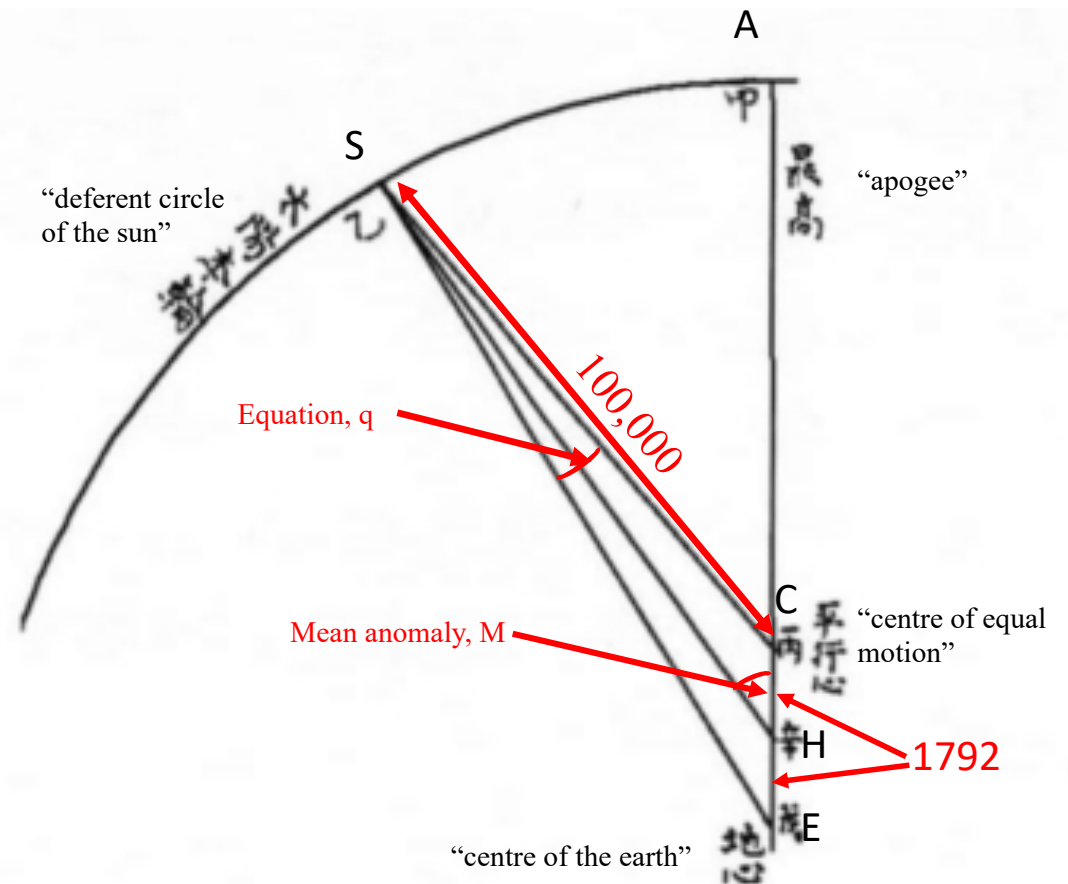


- Calculating the equation of centre using the formula with $e = 0.01792$ and unit radius gives agreement with *CZLS* tabulated values within 1 or 2 seconds. Changing to 0.01800 (corresponding to Kepler's preferred value of 1800 for a 100,000 radius) increases errors to a range of 16 to 34 seconds.
- Evidently the table has been correctly calculated using the diagram with equant and eccentric, and Kepler's bisection of Tycho's eccentricity from 3584 to 1792.

Eccentricity used in table				CZLS table of solar equation of centre				Calculated equation using $e = 0.01792$				Difference, seconds
0.01792				D	M	S	Additive	D	M	S		
House	Mean anomaly from perigee	Mean anomaly from apogee										
1	0	180		0	0	0	Additive	0	0	0		
2	30	210		1	2	33	Additive	1	2	34		1
3	60	240		1	47	40	Additive	1	47	39		-1
4	90	270		2	3	10	Additive	2	3	11		1
5	120	300		1	45	46	Additive	1	45	44		-2
6	150	330		1	0	40	Additive	1	0	40		0
7	180	360		0	0	0	Subtractive	0	0	0		0
8	210	30		1	0	40	Subtractive	-1	0	-40		0
9	240	60		1	45	46	Subtractive	-1	-45	-44		-2
10	270	90		2	3	10	Subtractive	-2	-3	-11		1
11	300	120		1	47	40	Subtractive	-1	-47	-39		-1
12	330	150		1	2	33	Subtractive	-1	-2	-34		1
13	360	180		0	0	0	Subtractive	0	0	0		0

The mistaken interpretation

- In the ‘Explanation of how to calculate the equation table’, the bisected eccentricity is correctly given as 1792. But instead of the deferent radius HS being given as 100,000, this value is assigned to CS, the (varying) length of the radius vector from the equant to the sun. Further, the mean anomaly is not measured at the equant C, but at H, the centre of the deferent.
- Both errors show a misunderstanding of Kepler’s eccentric/equant model of the sun’s motion under Tycho’s geocentric scheme.



The incorrect results

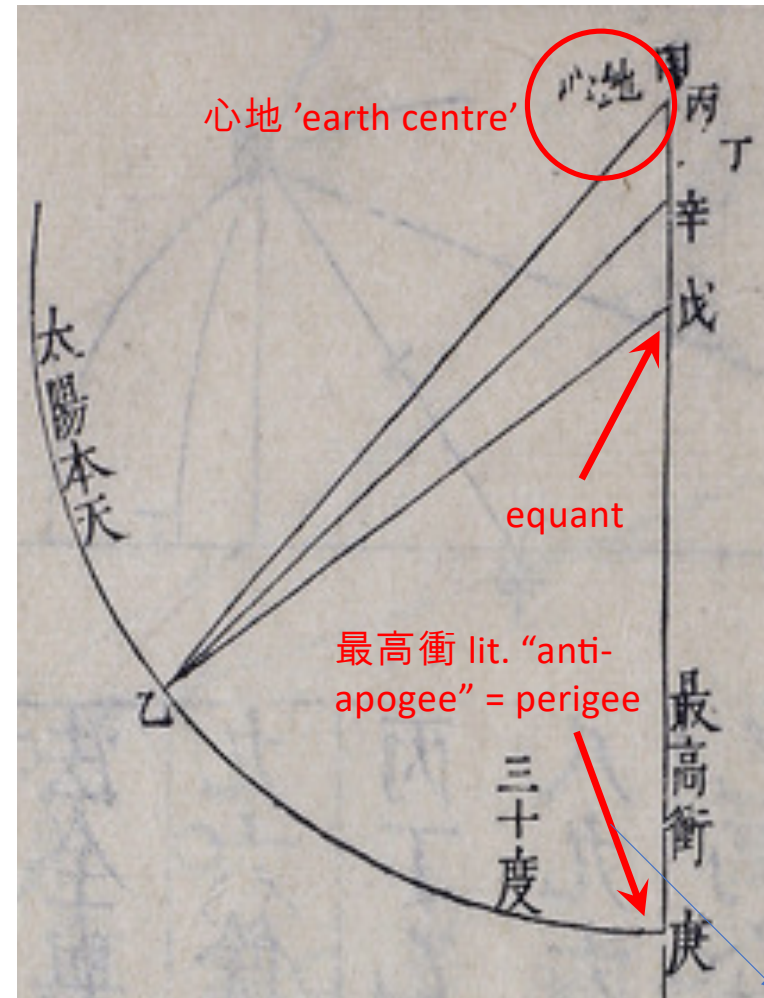
- The ‘explanation’ text calculates two values of the equation on this basis, one for an angle past the apogee of 30° (210° past perigee), and one for an angle before the perigee of 30° (hence 330° past perigee, 150° past apogee).
- The results (both said to be *ru biao* 如表 ‘as tabulated’) are given as:

Position of sun	Result in ‘explanation’	Result in <i>CZLS</i> table	Recalculated result using stated eccentricity
30° past apogee	$1^\circ 00' 29''$	$1^\circ 00' 40''$	$1^\circ 00' 40''$
30° before perigee	$1^\circ 20' 30''$	$1^\circ 02' 33''$	$1^\circ 02' 34''$

- The second ‘explanation’ result is the result of an arithmetical error, without which it would have been $1^\circ 02' 30''$
- The results given in the ‘explanation’ are not, as claimed, identical to those in the *CZLS* equation table.

Creating an incorrect diagram

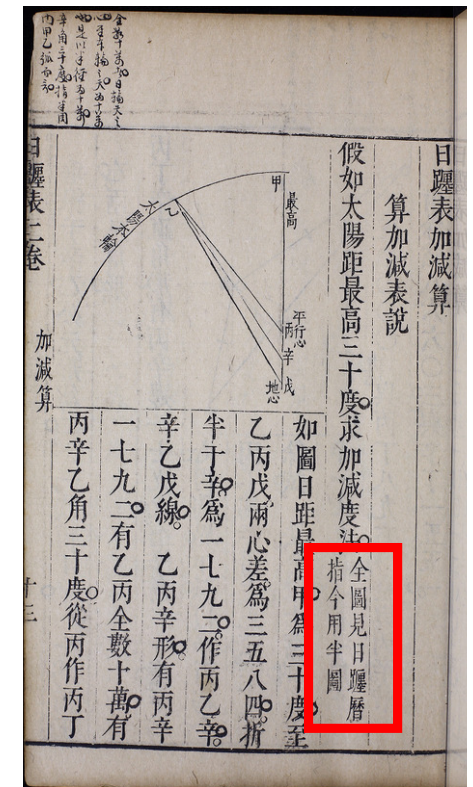
- Evidently the *CZLS* table was calculated by somebody who understood Kepler's eccentric/equant model correctly, and used Kepler's 1792 bisection of Tycho's 3584 eccentricity to calculate the tabulated values of equation of centre.
- The person who wrote the explanation that follows the diagram cannot have understood Kepler's model. What is more, for their second calculation they added an additional diagram in which the earth is shown as further away from the perigee than the centre of the deferent!



An unfulfilled promise ...

- At the start of the explanation, we are told: “For the whole diagram, see the *Ri chan li zhi* ‘Instructions on calculating the sun’s path’. Here we use only the half diagram.”
- But no such diagram is found in the relevant section of the *Chong zhen li shu*.

全圖見日躔曆
指今用半圖



曰。以日月食推地徑與日輪本天徑之比例。歌白泥定地半徑與日天半徑之比例。若一與一千一百四十三。如上前圖。甲戊丁爲太陽本圈。甲爲最高。乙爲其心。丙爲地心。乙丙爲兩心之差。日在戊。甲戊爲日距最高度之弧。乙戊爲本圈之半徑。今欲求自地相離之線。曰。戊乙爲直泉三角。形。有乙戊半徑全數。又有兩心之差乙丙。三五八。又有甲乙戊角之餘角爲戊乙丙。而求丙戊邊其法如增圖。全數乙丙與乙丙邊外。若戊乙丙角餘角之正弦。丁丙。

But the Ptolemaic eccentric/equant model for Saturn appears in the 五緯曆指 *Wu wei li zhi* , dealing with planetary theory:



A plan unfulfilled

- It seems that there may at some stage have been a plan to introduce the Keplerian eccentric/equant model for solar motion into the part of the *Chong zhen li shu* that explains solar theory (*Ri chan li zhi*).
- But this was not done – although the solar equation table was calculated on that basis, and at least one correct diagram with eccentric and equant may have been prepared for use and kept in working files.
- Could it have been Shreck's death that led to the abandonment of the plan to revise the *Ri chan li zhi*?

Who did what, when?

- It is likely that the actual calculation of the table was carried out by, or under the close supervision of Schreck, whose mathematical and astronomical skills were acknowledged while he was still in Europe. He is known to have been in regular contact with Kepler and they were both in Prague in 1607, during Kepler's period as Astronomer at the court of Rudolph II (1600-1612). Schreck did not leave for China until 1618, and he may well have possessed a copy of Kepler's *Astronomia Nova* (1609) and probably Tycho's *Progymnasmata* (1610).
- In 1623 Schreck wrote a letter to the Jesuit mathematicians of Ingolstadt, asking for help with astronomical questions. This was passed to Kepler, who wrote comments on it in 1627. It is not known whether this was ever sent to China, and is unlikely to have reached Schreck during his lifetime. Kepler's *Tabulae Rudolphinae* (1627), to which he refers in his comments, did not reach China until 1646.
- Schreck died in May 1630, before the two parts of the solar tables were ready for presentation in late 1630 and in early 1631 (when the second part of the tables, with the equation table, was presented). The tables thus could not be presented under his name, and Giacomo Rho's name appears instead.

Signs of Kepler's work known to early 17th C Jesuits

- See Bernard, Henri (1937). "L'encyclopédie astronomique du Père Schall (part 1)." *Monumenta Serica* 3 (1): 35-77, p. 67 n.113 who notes MS annotations referring to Kepler's work (from content, clearly *Astronomia Nova* (1609)) in copies of the following two early 17th C texts in the Beitang library:
 - Mulerius, Nicolaus (1611). *Tabulae Frisicae lunae-solares quadruplices e fontibus Cl. Ptolemaei, Regis Alfonsi, Nic. Copernici, et Tychonis Brahe ... Quibus accessere solis tabulae totidem, hypotheses Tychonis illustratae: kalendarium Rom. vetus, cum methodo Paschali emendatâ*. Alcmariae, Excudebat Iacobus Meesterusven. Apud Wilhelmum Ianssonium. (This is in Rho's copy, Verhaeren 1444)
 - Magini, Giovanni Antonio (1614). *Supplementum ephemeridum, ac tabularum secundorum mobilium, Io. Antonii Magini ... in quo habentur ratio, & methodus perfacilis promptissime supputandi verum motum Solis, Lunae, & Martis ex nouis tabulis secundum Tychonicas obseruationes, nunc primum accuratè constructis*. Venice, Apud Hæredem Damiani Zenarii. (Rho had a copy of this: Verhaeren 2152. Is this it?)

113) Dans l'exemplaire des **Tabulae frisicae*, de MULLER, 1611, marqué "Alex. Rhau d. s. j.", p. 24, 28-29, "Ex Origan, Kepler, Copernico, Magino, Mario, Mestlino"; p. 138 "pequinense meridianum facimus orientaliorem h. l. 25' 6. 25' ". — **Astronomia danica*, de LONGOMONTANUS, contient beaucoup de citations manuscrites: v. g. p. 220 "in Kepler fol. 54", p. 310 (pro anno 1620),... ; au titre "ejus meridianus occidentior francoforti 12' et alexandrini orientiori h. l. 35' ". — **Supplementum ephemeridum Magini*, Venise, 1614: p. 58 "Haec tabella cum Keplero f. 162 fere respondet"; *Tabulae...*, p. 158: "Keplero f. 101". — Comme on le voit, les livres de Kepler ont été certainement connus et employés par Terrentius.

Who wrote the incorrect explanation?

- At some later stage before the end of the Ming dynasty, someone thought that it was appropriate to add an explanation of how the tables were calculated. It is possible that the first diagram had been prepared by Schreck for use with an explanation that he never wrote. It was however not properly understood, and a second incorrect diagram was added in addition to the incorrect explanation of the calculation procedure.
- The error in the explanation was not detected, and remained in place in new versions of the *CZLS* text produced under different titles by Schall during the early Qing (from 1644).