



HAL
open science

Evaluating Climate Policies by the Pareto Principle: Efficiency When Future Identities Are Unobservable

Geir B. Asheim, Kohei Kamaga, Stéphane Zuber

► **To cite this version:**

Geir B. Asheim, Kohei Kamaga, Stéphane Zuber. Evaluating Climate Policies by the Pareto Principle: Efficiency When Future Identities Are Unobservable. 2022. halshs-03760333

HAL Id: halshs-03760333

<https://shs.hal.science/halshs-03760333>

Preprint submitted on 25 Aug 2022

HAL is a multi-disciplinary open access archive for the deposit and dissemination of scientific research documents, whether they are published or not. The documents may come from teaching and research institutions in France or abroad, or from public or private research centers.

L'archive ouverte pluridisciplinaire **HAL**, est destinée au dépôt et à la diffusion de documents scientifiques de niveau recherche, publiés ou non, émanant des établissements d'enseignement et de recherche français ou étrangers, des laboratoires publics ou privés.

**Evaluating Climate Policies by
the Pareto Principle:
Efficiency When Future
Identities Are Unobservable**

Geir B. Asheim, Kohei Kamaga, Stéphane Zuber

Impressum:

CESifo Working Papers

ISSN 2364-1428 (electronic version)

Publisher and distributor: Munich Society for the Promotion of Economic Research - CESifo GmbH

The international platform of Ludwigs-Maximilians University's Center for Economic Studies and the ifo Institute

Poschingerstr. 5, 81679 Munich, Germany

Telephone +49 (0)89 2180-2740, Telefax +49 (0)89 2180-17845, email office@cesifo.de

Editor: Clemens Fuest

<https://www.cesifo.org/en/wp>

An electronic version of the paper may be downloaded

- from the SSRN website: www.SSRN.com
- from the RePEc website: www.RePEc.org
- from the CESifo website: <https://www.cesifo.org/en/wp>

Evaluating Climate Policies by the Pareto Principle: Efficiency When Future Identities Are Unobservable

Abstract

Climate change is an externality since those who emit greenhouse gases do not pay the long-term negative consequences of their emissions. In view of the resulting inefficiency, it has been claimed that climate policies can be evaluated by the Pareto principle. However, climate policies lead to different identities of future people, implying that the Pareto principle is not applicable. Assuming that there are infinitely many future people whose identities are not observable, we specify conditions under which their spatiotemporal positions do not matter. This implies that the Suppes-Sen principle whereby ranked streams are compared plays an important role and justifies that following dominance relation: A state a is said to *dominate* another state b if a Pareto dominates b for existing people and Suppes-Sen dominates b for future people, with at least one of the two being strict. We illustrate the consequences of this dominance definition for policy choice.

JEL-Codes: D610, D630, D710, Q540.

Keywords: climate change, efficiency, intergenerational equity, population ethics, infinite streams.

Geir B. Asheim
Department of Economics
University of Oslo / Norway
g.b.asheim@econ.uio.no

Kohei Kamaga
Department of Economics
Sophia University / Tokyo / Japan
kohei.kamaga@sophia.ac.jp

Stéphane Zuber
Paris School of Economics – CNRS
Paris / France
stephane.zuber@univ-paris1.fr

14 February 2022

We thank Walter Bossert, Maya Eden, Lars Ehlers, Adam Jonsson, Justin Leroux, Frikk Nesje, Paolo Piacquadio, Marcus Pivato, Sébastien Pouget, and seminar participants at Australian National University, Concordia University, ETH, Hitotsubashi University, Université de Montpellier, Université de Paris 1 (CES), Sophia University, University of Oslo, Yale University, the FAERE conference 2021, and an International Seminar on Social Choice for comments. Asheim thanks IMéRA (Institut d'études avancées – Aix Marseille Université) for providing an environment where this research was started and CIREQ (Centre Interuniversitaire de Recherche en Économie Quantitative) for facilitating his visits to Montreal. Kamaga thanks University of Oslo, CES (Centre d'Économie de la Sorbonne) at Université Paris 1 and CIREQ in Montreal for their hospitality and acknowledges financial support from JSPS KAKENHI Grant No. 20K01565. Zuber acknowledges support by the Agence nationale de la recherche through the Fair-ClimPop project (ANR-16-CE03-0001-01) and the Investissements d'Avenir program (PGSE-ANR-17-EURE-01). The paper is part of the research activities at the Centre for the Study of Equality, Social Organization and Performance (ESOP) at the Department of Economics at the University of Oslo.

1 Introduction

Among others, Duncan Foley and John Broome (Foley, 2009; Broome and Foley, 2016; Broome, 2016, 2018) have pointed to the importance of the Pareto principle in the context of climate policies. Climate change is an externality, since those who emit greenhouse gases do not pay the long-term negative consequences of their emissions. Stern (2007) has even characterized climate change as “the greatest market failure the world has ever seen.” Correcting for a market failure by internalizing externalities usually improves efficiency and provides an opportunity for realizing a Pareto improvement whereby some parties gain without other parties losing.

A Pareto improvement is in principle possible in the climate change setting by reducing emissions and investments now, improving the situation both for the present and the future. One can argue that Nordhaus (2008) seeks to find the potential for efficiency gains when using observed behavior to motivate discounting of poor and future people through Negishi weights and time discounting in the DICE and RICE models.

However, climate policies require pervasive action that will profoundly change present and future behavior. In particular, climate policies will influence fertility, leading to a different number of people in each future generation. For example, population measures might serve as a means to reducing greenhouse gas emissions. Moreover, reproductive choices will be affected, implying that different people will exist in the future; this is what philosophers call the “non-identity effect” (Parfit, 1984). The concept of Pareto improvement applies only to same people choices, where there is a natural 1-to-1 correspondence between people in the two alternatives, comparing the vector of wellbeings in two alternatives individual-by-individual. In contrast, as a result of fertility choices and population measures, we are faced with different people–different number choices in the case of climate policies, so that no natural 1-to-1 correspondence exists (Broome, 2018).

While the ratio of people that have ever lived to the ones living now is low (estimated to 15:1 by Kaneda and Haub, 2021), there might potentially be many people living in future.¹ This observation might justify the modeling choice often made in the literature on sustainability and intergenerational equity, namely that the future will consist of infinitely many people, independently of the policy choices that are made. Moreover, one might argue that the world is actually infinite; Askill (2018, Section 1.1) and Pivato (2021, Section 1) refer to such evidence.

Assuming that the number of people is countably infinite independently of policy choice also leads to same number choices, since the cardinality of the resulting populations remains unchanged. Hence, in a formal sense there exist 1-to-1 correspondences between people in any two alternatives. For example, people might be counted in the order of time of birth and compared according to this sequence. However, with different fertility choices in different alternatives, the person that experiences wellbeing at component j in one alternative will not coincide with the

¹With 500 million years left of the earth as acceptable habitat for humans, population stable at 1.1 billion with an average length of life equal to 70 years, the ratio of people who will potentially live in the future to people living now is about 1 million to 1.

person that experiences wellbeing at component j in another alternative. Therefore, comparison of wellbeings according to the 1-to-1 correspondence that follows the temporal sequence of birth does not represent a proper application of the Pareto principle.

An underlying problem is that, when policy chooses an observable outcome, the set of future identities that this choice will bring into life is unobservable. In contrast to the comparison of the wellbeings for present people, where identities are fixed, we do not know whether a particular future individual that will be alive if one outcome is realized would also be alive if a different outcome were implemented. In our analysis we assume that, for a given pair of outcomes, we cannot observe the sets of future people that will be alive when these outcomes are realized. We refer to this assumption as *Unobservability of Future Identities*. This assumption means the concepts of \mathcal{A} -domination and \mathcal{P} -domination, proposed by Golosov, Jones and Tertilt (2007) when population is endogenous and finite, cannot be generalized to our situation with an infinite population, as these concepts require that future identities are observable.²

The central question posed in the present paper is *how to define and justify a concept of dominance, and thus a concept of efficiency, which can be used for policy evaluation when the Unobservability of Future Identities limits the applicability of the Pareto principle*. We build on a suggestion by Broome (2018, pp. 13-14) and argue that a generalized Suppes-Sen principle whereby ranked streams are compared plays an important role under such circumstances. When generalizing the finite population Suppes-Sen principle (Suppes, 1966; Sen, 1970) to the case with an infinite population, we utilize the Pareto principle on derived rank-ordered streams. These streams are wellbeing profiles where wellbeing is rank-ordered starting with the worst-off and completed with wellbeings equal to the smallest cluster point if there is only a finite number of wellbeing components that can be rank-ordered in this manner.³ We propose a dominance definition where a state a is said to *dominate* another state b if a Pareto dominates b for existing people and generalized Suppes-Sen dominates b for future people, with at least one of the two being strict. We show as our main result how this Dominance Definition can be justified.

We assume that, for any policy choice, the spatiotemporal distribution of wellbeings is observable, but not the identities of the individuals that will be alive at these spatiotemporal positions. Examples of sets of spatiotemporal positions are structures where a population is constant through time and the n (≥ 1) people per period inhabit a spatial dimension. Other examples involve spatiotemporal structures where population grows over time. Such spatiotemporal structures are essential when determining an efficient set of wellbeing profiles subject to a feasibility constraint,

²The concept of \mathcal{A} -dominance compares only people that are alive, while the concept of \mathcal{P} -dominance compares all potential people, who are assigned utility even if not alive. Golosov, Jones and Tertilt (2007) assume that the n th child of a parent has a fixed identity that depends solely on being the n th child of this parent. However, the identity of the n th child will depend on time of birth, which in turn might be influenced by the number of siblings.

³A *cluster point* of an infinite set of wellbeings is a point such that, for every neighborhood, there are infinitely many components of the set with wellbeings within the neighborhood. This generalization of the Suppes-Sen principle differs from the one given in Asheim, Buchholz and Tungodden (2001), based on the conjunction of the axioms of Strong Pareto and Finite Anonymity (to be defined later in the introduction).

since the feasible per capita wellbeing might depend on per-period population size in the former examples and on population growth in the latter examples.

Furthermore, we assume that there is a countably infinite set of potential future identities. When an outcome is realized, an infinite subset of identities are selected and turned into future alive people that inhabit a set of spatiotemporal positions. We do not impose restrictions on the selection of future identities and their distribution on the spatiotemporal positions. This *Richness* assumption does not mean that there exists a technology that can implement any such selection; rather, it signifies that we do not rule out any selection of future identities from consideration.

In line with our emphasis on the Pareto principle, in the sense of comparing wellbeings identity-by-identity, we assume that people are the bearers of value, not the spatiotemporal positions. Related philosophical literature—see, e.g. Broome (1991) and Vallentyne and Kagan (1997) as well as more recent contributions (Askell, 2018; Wilkinson, 2020; Jonsson, 2021)—provides arguments for both people and spatiotemporal positions being the bearers of value. A normative theory that seeks to treat people in an equal manner lends support to the view (called *Qualitativeness* by Askell, 2018) that spatiotemporal positions should be bearers of value, as the identities of the people inhabiting the different positions should not matter if they are treated equally. However, concepts of dominance and efficiency based on the Pareto principle do not entail that people are necessarily treated in an equal manner.

Even if we do not impose as a primitive condition that spatiotemporal positions are bearers of value, we show how it follows from our assumptions of Richness and Unobservability of Future Identities that future people must be treated equally under the following condition:

- *Pareto Monotonicity*, deeming one observable outcome at least as good as another if there exists a set of identities of future people that are possible in each of the two outcomes, and where all existing and future individuals are at least as well off in the former outcome as in the latter.

Furthermore, the derived equal treatment of future people is in the form of *Strong Anonymity*, implying that it does not matter in what order their wellbeings are listed. The result is a Welfareism Theorem where a consequentialist evaluation based on the stream of wellbeings arises from informational constraints rather than normative judgement.

It is well-known (Van Liedekerke, 1995; Van Liedekerke and Lauwers, 1997, p. 163) that equal treatment in the sense of Strong Anonymity is incompatible with sensitivity in the sense of *Strong Pareto*, which requires that an outcome be deemed better than another if it increases the wellbeing of some component without decreasing the wellbeing of any other component. For example, the stream $(0, 1, 0, 1, 0, \dots, 0, 1, \dots)$ can be transformed to the stream $(1, 1, 0, 1, 0, \dots, 0, 1, \dots)$ by reordering an infinite number of components, implying by Strong Anonymity that the two are equally good. However, by applying Strong Pareto component-by-component, the latter is strictly better than the former. The resulting decreased sensitivity of imposing Strong Anonymity

is often deemed unacceptable for various reasons (Van Liedekerke and Lauwers, 1997; Askill, 2018; Wilkinson, 2020). However, as illustrated by the Maximin order and the Extended Rank-Discounted Utilitarian order (Zuber and Asheim, 2012), Strong Anonymity can be combined with some sensitivity for individual wellbeing. Indeed, Strong Anonymity is compatible with sensitivity for an increase in the wellbeing of any component that is finitely ranked, in the sense that there is a cofinite set of people that have wellbeings that are at least ε (> 0) higher.⁴

To explore a sensitivity concept that compatible with Pareto Monotonicity under the assumptions of Richness and Unobservability of Future Identities, we examine the unanimous agreement of—i.e., the intersection of—all preorders that satisfy the axiom of Pareto Monotonicity; see Donaldson and Weymark (1997), Sen (1973), and Suzumura (1983, p. 69) for an approach that considers the intersection of evaluation relations. In this approach domination is defined as the asymmetric part of the preorder⁵ on outcomes obtained by taking the intersection of all preorders satisfying Pareto Monotonicity. In our setting, under the assumptions of Richness and Unobservability of Future Identities, any such preorder satisfies Strong Anonymity, leading to an enlarged symmetric part. Therefore, compared to the case where Strong Pareto could have been imposed, the concept of dominance will be changed. In particular, we will show how the Strong Anonymity removes sensitivity for wellbeing at infinite rank while strengthening sensitivity for wellbeing at finite rank by allowing for comparisons of rank-ordered streams. These results are obtained if we impose three auxiliary axioms: *Continuity* as a robustness axiom, *Limited Inequity*, requiring that the poor’s change in wellbeing must have positive relative weight when making a transfer from a rich to a poor, and *Critical-Level Consistency*, imposing that if it is good to add one individual, then it is good to add infinitely many with the same wellbeing.

Thus, we associate dominance with the asymmetric part of the preorder obtained by the taking the intersection of all preorders satisfying Pareto Monotonicity, Continuity, Limited Inequity, Critical-Level Consistency under the assumptions of Richness and Unobservability of Future Identities and show the following Justification Theorem: *A profile of wellbeings is preferred to another profile of wellbeings according to the dominance preorder thus obtained if and only if the former Pareto dominates the latter for existing people and generalized Suppes-Sen dominates the latter for future people, with at least one of the two being strict.*

Our framework treats existing (past and present) people asymmetrically from future people, based on the assumption that the former have fixed identities, while the latter have variable and unobservable identities. This provides an interesting perspective on Parfit’s (1984, p. 361) ‘depletion’ example (see also Jonsson, 2021), where depletion of a resource, as opposed to conservation, leads to higher wellbeing for present people, but lower wellbeing for future people. Furthermore, the identities of present people remain the same, while the identities of future people are entirely

⁴In Appendix A we refer to such sensitivity as *Liminf-Restricted Dominance* and show how this is a maximal sensitivity axiom under the auxiliary axioms of Continuity, Limited Inequity, and Critical-Level Consistency.

⁵A preorder is a reflexive and transitive binary relation.

different under depletion than under conservation. In the context of this example one might deem depletion advantageous, as it is advantageous to some people—namely the present—while hurting no one. Our analysis provides one answer that permits weight to be given to future people also under such circumstances, so that conservation is not a dominated choice.

In Section 2 we define dominance: a state is said to dominate another state if the former Pareto dominates the latter for existing people and Suppes-Sen dominates it for future people, with at least one of the two being strict. We also illustrate the consequences of this definition in various settings. Then, in Section 3, we establish a Welfarism Theorem which implies Strong Anonymity for future people by showing that spatiotemporal positions and future identities do not matter under Pareto Monotonicity when future identities are unobservable. In Section 4 we justify the Dominance Definition on the basis of the Welfarism Theorem by adding the auxiliary axioms. In the subsequent Section 5 we relate to relevant literature, while in the final Section 6 we discuss our results. Finally, in Appendix A we provide results on maximal sensitivity under Strong Anonymity, used to establish the Justification Theorem, while in Appendix B we show how the Continuity axiom invoked to obtain our results can be weakened.

2 Defining dominance

For any real vectors \mathbf{x} and \mathbf{y} of countably infinite dimension, we write $\mathbf{x} \geq \mathbf{y}$ whenever $x_j \geq y_j$ for all components j , and $\mathbf{x} > \mathbf{y}$ if $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$. Furthermore, we follow the usual convention of saying that one vector \mathbf{x} *dominates* another vector \mathbf{y} if $\mathbf{x} > \mathbf{y}$. So when we in this section *define* dominance, we are actually concerned with the determination of the vectors \mathbf{x} and \mathbf{y} that should be compared in our setting with an infinite number of future people whose identities are unobservable. When determining such vectors, derived rank-ordered streams will play an essential role, so that domination is determined by the Suppes-Sen principle. However, not all infinite vectors can be rank-ordered starting with the worst-off; $(1, 0, \dots, 0, \dots)$ is an example.

Therefore, for any countably infinite subset Q of the set \mathbb{Q} of rational numbers, and any vector $\{x_q\}_{q \in Q}$ where, for all $q \in Q$, x_q takes on a value in the set \mathbb{R} of real numbers, derive the rank-ordered stream $\mathbf{x}_{[]} = (x_{[1]}, x_{[2]}, \dots, x_{[r]}, \dots)$ as follows. Let Π_Q denote the set of all bijective functions from the set \mathbb{N} of natural numbers to Q , so that $s_r(\{x_q\}_{q \in Q})$ denotes the infimum of the sum of any selection of $r \in \mathbb{N}$ components from $\{x_q\}_{q \in Q}$:

$$s_r(\{x_q\}_{q \in Q}) = \inf_{\pi \in \Pi_Q} \sum_{j=1}^r x_{\pi(j)}. \quad (1)$$

Set $s_0(\{x_i\}_{i \in Q}) = 0$ and define $x_{[r]}$ for all $r \in \mathbb{N}$ by

$$x_{[r]} = s_r(\{x_q\}_{q \in Q}) - s_{r-1}(\{x_q\}_{q \in Q}). \quad (2)$$

Let $\ell(\{x_q\}_{q \in Q})$ denote the smallest cluster point or, equivalently, the limit inferior of $\{x_q\}_{q \in Q}$, and let $L(\{x_q\}_{q \in Q}) = \{q \in Q : x_q < \ell(\{x_q\}_{q \in Q})\}$ denote the set of components of $\{x_q\}_{q \in Q}$

that are smaller than the limit inferior. The construction of $\mathbf{x}_{[\cdot]}$ depends on whether or not there are infinitely many components smaller than the limit inferior. If $L(\{x_q\}_{q \in Q})$ is infinite, then $\mathbf{x}_{[\cdot]}$ is the rank-ordered infinite subsequence consisting of the elements in $L(\{x_q\}_{q \in Q})$. If $L(\{x_q\}_{q \in Q})$ is finite, then $\mathbf{x}_{[\cdot]}$ is the rank-ordered finite subsequence consisting of the elements in $L(\{x_q\}_{q \in Q})$ followed by a constant stream where wellbeing equals $\ell(\{x_q\}_{q \in Q})$. In the case of $\mathbf{x} = (1, 0, \dots, 0, \dots)$, $\ell(\mathbf{x}) = 0$ and $L(\mathbf{x}) = \emptyset$ so that $\mathbf{x}_{[\cdot]} = (0, 0, \dots, 0, \dots)$.

The derived rank-ordered streams allow us to define a *generalized Suppes-Sen principle* whereby, for any $\{x_q\}_{q \in Q}$ and $\{y_q\}_{q \in Q}$, $\{x_q\}_{q \in Q}$ is better than $\{y_q\}_{q \in Q}$ if $\mathbf{x}_{[\cdot]} > \mathbf{y}_{[\cdot]}$ and at least as good as $\{y_q\}_{q \in Q}$ if $\mathbf{x}_{[\cdot]} = \mathbf{y}_{[\cdot]}$.

Let A be the *set of social states*, where each $a \in A$ is a complete description of all relevant aspects of the social state, including information of the individuals alive. Hence, each social state determines the wellbeings of existing and future individuals as well as the identities of the individuals that will live in the future. We assume that each state has a finite number of existing individuals and a countably infinite number of future individuals, each experiencing a real-numbered wellbeing level. The wellbeings that individuals experience are at least interpersonally level comparable.

Let $E = \{-1, -2, \dots, -e\}$ be the finite set of the (past and present) existing individuals, with fixed identities, where the number of existing individuals, $e \in \mathbb{N}$, is fixed and finite. Including also past people is a standard approach in population ethics where the notion of total population is associated not only with the people that are alive at any one time, but also with people that will ever live and those that have ever lived (Blackorby, Bossert and Donaldson, 2005a). For the purpose of our Dominance Definition of this section, there is no need to know past people's wellbeing if their wellbeing is the same across the compared alternatives. Also, for the Justification Theorem of Section 4, we can associate their identities with their index $i \in E$ given that their identities can be assumed to be fixed and not influenced by the choice of alternatives.

Let \mathbb{N} be the set of *all potential future identities*. Let F^* generically denote an infinite subset of \mathbb{N} ; formally, $F^* \subseteq \mathbb{N}$ and $|F^*| = +\infty$. This will be used to denote a set of the future individuals alive in a framework where their identities are specified. Let \mathcal{N} be the collection of all infinite subsets F^* of \mathbb{N} ; formally, $\mathcal{N} = \{F^* \subseteq \mathbb{N} : |F^*| = +\infty\}$.

Let \mathbb{P} be a countably infinite set of *all potential future spatiotemporal positions*. An element p of \mathbb{P} determines the point in time and space at which some future identity might live. Specifying how alive individuals are positioned through time and space is important when determining feasibility. For example, a given constant wellbeing level which is feasible if population growth is small might not be feasible if population growth is higher. In this example, the spatial dimension captures the number of people alive at any point in time. Let P^* generically denote an infinite subset of \mathbb{P} ; formally, $P^* \subseteq \mathbb{P}$ and $|P^*| = +\infty$. This will be used to denote the set of future spatiotemporal positions that will be inhabited. Let \mathcal{P} be the collection of all infinite subsets P^* of \mathbb{P} ; formally, $\mathcal{P} = \{P^* \subseteq \mathbb{P} : |P^*| = +\infty\}$.

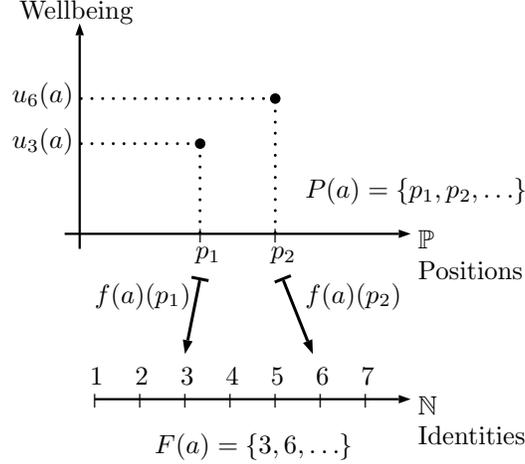


Figure 1: Inhabited positions and alive identities in social state a

For any subset $P^* \in \mathcal{P}$, let \mathcal{F}_{P^*} be the collection of all injective functions $f^*: P^* \rightarrow \mathbb{N}$. Each $f^* \in \mathcal{F}_{P^*}$ selects the identities of the future individuals that will be alive and orders them in a 1-to-1 correspondence with the set P^* of inhabited positions.⁶ Let $P: A \rightarrow \mathcal{P}$ be a function that, for any state $a \in A$, determines an infinite set $P(a) \in \mathcal{P}$ of inhabited positions. Let $f: A \rightarrow \bigcup_{a \in A} \mathcal{F}_{P(a)}$ be a function that for any state $a \in A$ determines an injective function $f(a) \in \mathcal{F}_{P(a)}$, and write $\mathcal{F} = \{f(a) : a \in A\}$. Hence, for social state $a \in A$, $f(a): P(a) \rightarrow \mathbb{N}$ selects the identities of the future individuals alive in a and orders them in a 1-to-1 correspondence with the set $P(a)$ of inhabited positions; that is, for each $p \in P(a)$, $f(a)(p)$ is the future individual alive at position p in state a . This means that, for each $a \in A$, the range $f(a)(P(a)) \in \mathcal{N}$ is the set of future individuals alive in a , which we denote by $F(a)$ instead of writing $f(a)(P(a))$.

For each existing individual $i \in E$, let $u_i: A \rightarrow \mathbb{R}$ be i 's utility function, which determines i 's wellbeing as a function of the state. For each potential future identity $i \in \mathbb{N}$, let A_i denote the set of social states in which the individual with identity i is alive, that is, $A_i = \{a \in A : i \in F(a)\}$. Likewise, for each $F^* \in \mathcal{N}$, let A_{F^*} denote the set of states $\{a \in A : F(a) = F^*\}$ for which F^* is the set of alive future individuals. For each $i \in \mathbb{N}$, $u_i: A_i \rightarrow \mathbb{R}$ is a utility function of the potential future individual with identity i ; it determines i 's wellbeing in states where i is alive. Throughout, we assume that there exists a single fixed profile $u = (u_1, u_2, \dots)$ of utility functions of all potential future identities. Furthermore, we assume that, for each $a \in A$, $\sup_{i \in F(a)} |u_i(a)| < +\infty$. Therefore, a vector of wellbeings of future alive individuals is bounded.

Figure 1 illustrates key elements of our model, namely, future spatiotemporal positions, future alive identities, and their wellbeings. In state a of the figure, future identities 3, 6, and so on, are alive at positions p_1, p_2 , and so on, so that these alive identities are related to inhabited positions

⁶In contrast to de la Croix and Doepke (2021) we assume that each identity i lives only once and is not a 'soul' who might experience multiple incarnations over time.

as $f(a)(p_1) = 3$ and $f(a)(p_2) = 6$. Also, the future alive identities 3 and 6, who live at p_1 and p_2 , attain the wellbeing levels $u_3(a)$ and $u_6(a)$ in state a .

For any $a \in A$, write $\mathbf{x}_E(a) = (u_{-1}(a), u_{-2}(a), \dots, u_{-e}(a))$ for the wellbeings of existing people. For any $a \in A$ and any $p \in P(a)$, write $x_p(a) = u_{f(a)(p)}(a)$ for the wellbeing of the individual at position p in state a , so that $\mathbf{x}_{[]}(a) = (x_{[1]}(a), x_{[2]}(a), \dots, x_{[r]}(a), \dots)$ derived from $\{x_p(a)\}_{p \in P(a)}$ through (1) and (2) denotes the rank-ordered stream of future wellbeings.

With this notation in place we can state our Dominance Definition.

Definition (Dominance). A social state $a \in A$ *dominates* another social state $b \in A$ if $(\mathbf{x}_E(a), \mathbf{x}_{[]}(a)) > (\mathbf{x}_E(b), \mathbf{x}_{[]}(b))$.

As a first illustration, the Dominance Definition is applied to Parfit's (1984, p. 361) 'depletion' example, where depletion of a resource, as opposed to conservation, leads to higher wellbeing for present people, but lower wellbeing for future people. We assume that the identities of present people remain the same, while the identities of future people are unobservable. Let c correspond to a state of world in which conservation is chosen, and let d be a state of the world in which depletion is chosen. If distribution is egalitarian within the subset of future people, then $x_i(c) < x_i(d)$ for all $i \in E$ and $x_{[r]}(c) > x_{[r]}(d)$ for all $r \in \mathbb{N}$. This means that, by the Dominance Definition, neither does conservation dominate depletion nor the opposite. Hence, the choice between depletion and conservation cannot be made on the basis of a notion of dominance alone.

As a second illustration of the same kind, consider a 'climate change' example, where a corresponds to a state of world where abatement is undertaken, while b represents the state of the world with business as usual. Furthermore, in comparison with state b , state a does not only involve greater abatement efforts by present people, but also smaller investments, so that the wellbeings of current people remain unchanged. However, future people strictly prefer a to b , as the better climate resulting from present abatement efforts more than compensates for smaller inherited stocks of reproducible capital. We assume that the identities of present people remain the same, while the identities of future people are unobservable. If distribution is egalitarian within the subset of future people, then $x_i(a) = x_i(b)$ for all $i \in E$ and $x_{[r]}(a) > x_{[r]}(b)$ for all $r \in \mathbb{N}$. This means that, by the Dominance Definition, abatement dominates business as usual.

We can also consider examples in terms of growth models used to illustrate problems of intergenerational distribution. The simplest of these is the Ramsey model, where output in each period can be split into current consumption and next period's capital stock, and where output in the next period depends on next period's capital stock and population size.

If population size (and thus, labor) is constant, then a stream of decreasing wellbeings when people are listed according to time, like

$$\text{State } a: \quad \left(\underbrace{2}_E, \underbrace{\frac{3}{2}, \frac{4}{3}, \dots, \frac{p+2}{p+1}, \dots}_{P^* = \mathbb{N} \text{ (time)}} \right),$$

can be realized if the investment in next period's capital stock is not sufficient to maintain the productive capacity. Consider also a state that, with the same present investment, has a constant future wellbeing level of $4/3$:

$$\text{State } b: \quad \left(\underbrace{2}_E, \underbrace{\frac{4}{3}, \frac{4}{3}, \dots, \frac{4}{3}, \dots}_{P^*=\mathbb{N} \text{ (time)}} \right).$$

Finally, consider a state with constant future wellbeing level of 1, but where present consumption is increased to 3:

$$\text{State } c: \quad \left(\underbrace{3}_E, \underbrace{1, 1, \dots, 1, \dots}_{P^*=\mathbb{N} \text{ (time)}} \right).$$

In this example, $E = \{-1\}$ and

$$\begin{aligned} (x_{-1}(a), \mathbf{x}_{[\cdot]}(a)) &= (2, 1, 1, \dots, 1, \dots), \\ (x_{-1}(b), \mathbf{x}_{[\cdot]}(b)) &= (2, \frac{4}{3}, \frac{4}{3}, \dots, \frac{4}{3}, \dots), \\ (x_{-1}(c), \mathbf{x}_{[\cdot]}(c)) &= (3, 1, 1, \dots, 1, \dots), \end{aligned}$$

since all future people are infinitely ranked under state a . It follows from the Dominance Definition that state a is dominated by state b , since b is identical for the present individual while better than a for future people according to the generalized Suppes-Sen principle. Also, state a is dominated by state c , since c is better for the present individual while being equally good as a for future people according to the generalized Suppes-Sen principle. Finally, states b and c are not comparable according to dominance because there is a trade-off between present and future wellbeing.

If, on the the other hand, the investment in next period's capital stock exceeds what is needed to maintain the productive capacity, then a stream of increasing wellbeings, like

$$\text{State } a: \quad \left(\underbrace{2}_E, \underbrace{\frac{5}{2}, \frac{8}{3}, \dots, \frac{3p+2}{p+1}, \dots}_{P^*=\mathbb{N} \text{ (time)}} \right),$$

can be realized. Consider also a state that, with the same present investment, has a constant future wellbeing level of $8/3$:

$$\text{State } b: \quad \left(\underbrace{2}_E, \underbrace{\frac{8}{3}, \frac{8}{3}, \dots, \frac{8}{3}, \dots}_{P^*=\mathbb{N} \text{ (time)}} \right).$$

Finally, consider a state with a constant future wellbeing level of 3, but where present consumption

is decreased to 1:

$$\text{State } c: \quad \left(\underbrace{1}_E, \underbrace{3, 3, \dots, 3, \dots}_{P^* = \mathbb{N} \text{ (time)}} \right).$$

In this example, all future people are finitely ranked also in state a , implying that:

$$\begin{aligned} (x_{-1}(a), \mathbf{x}_{[\cdot]}(a)) &= \left(2, \frac{5}{2}, \frac{8}{3}, \dots, \frac{3p+2}{p+1}, \dots \right), \\ (x_{-1}(b), \mathbf{x}_{[\cdot]}(b)) &= \left(2, \frac{8}{3}, \frac{8}{3}, \dots, \frac{8}{3}, \dots \right), \\ (x_{-1}(c), \mathbf{x}_{[\cdot]}(c)) &= (1, 3, 3, \dots, 3, \dots). \end{aligned}$$

In this case, it follow from the Dominance Definition that, of the three state, no two states dominate each other. We have that b is identical to a for the present individual while not being comparable to a for future people according to the generalized Suppes-Sen principle. Moreover, c is worse than a and b for the present individual while being better than a and b for future people according to the generalized Suppes-Sen principle.

If we consider population growth in the Ramsey model, we might compare constant per capita wellbeing with population growth,

$$\text{State } a: \quad \left(\underbrace{2}_E, \underbrace{(2, 2), (2, 2, 2, 2), \dots, (2, \dots, 2)}_{\substack{2^{t+1} \text{ times} \\ P^* \subseteq \mathbb{N}^2 \text{ (time} \times \text{space)}}} \right),$$

with higher constant per capita wellbeing in the case where population is constant,

$$\text{State } b: \quad \left(\underbrace{3}_E, \underbrace{3, 3, \dots, 3, \dots}_{P^* \subseteq \mathbb{N}^2 \text{ (time} \times \text{space)}} \right),$$

since investments then do not have to compensate for the dilution effect of population growth. In this example,

$$\begin{aligned} (x_{-1}(a), \mathbf{x}_{[\cdot]}(a)) &= (2, 2, 2, \dots, 2, \dots), \\ (x_{-1}(b), \mathbf{x}_{[\cdot]}(b)) &= (3, 3, 3, \dots, 3, \dots). \end{aligned}$$

Clearly, state a is dominated by state b , since both the present individual as well as all future people are better off under state b . The last example indicates that high rates of population growth might be inefficient under the proposed Dominance Definition if the feasibility constraint trades off population growth against per capita wellbeing.

The Dominance Definition will be justified in two steps. First, in the subsequent section, we establish a Welfarism Theorem showing why identities and spatiotemporal positions do not matter when future identities are unobservable. Then, in Section 4, we use the Welfarism Theorem, combined with auxiliary results obtained in Appendix A, to establish a theorem that justifies the Dominance Definition.

3 Establishing welfarism

The purpose of this section is to use the framework of Section 2, where future identities are explicitly modeled, to establish a Welfarism Theorem that specifies conditions under which spatiotemporal positions and future identities do not matter when future identities are unobservable. Thus, we derive as a result the requirement that future people be treated in an impartial manner.

The framework we employ for establishing the Welfarism Theorem assumes that there exists a single and fixed profile of individuals' utility functions. Thus, our Welfarism Theorem is an infinite and variable population variant of the single-profile welfarism theorems established by Blackorby, Donaldson and Weymark (1990) and Blackorby, Bossert and Donaldson (2006) in a finite and fixed population framework, and it is an infinite population analogue of the results of Blackorby, Bossert and Donaldson (1999, 2005a) who employed a finite and variable population framework assuming a multi-profile setting.⁷

Throughout, we make the following assumption on the set A of social states.

Richness. For any set of positions $P^* \in \mathcal{P}$, any selection of identities $f^* \in \mathcal{F}_{P^*}$, and any wellbeings $\{u_i^*\}_{i \in E \cup f^*(P^*)}$ for existing and future individuals $E \cup f^*(P^*)$ satisfying $\sup_{i \in f^*(P^*)} |u_i^*| < +\infty$, there exists $a \in A$ such that $P(a) = P^*$, $f(a) = f^*$, and $u_i(a) = u_i^*$ for all $i \in E \cup f^*(P^*)$.

This assumption means that any spatiotemporal profile of wellbeings and any selection of future identities will be considered in our analysis. This is analogous to the assumption that any consumption bundle in the n -dimensional orthant is considered in regular consumer theory. Hence, the assumption of Richness is not a claim about feasibility. For example, the assumption of Richness entails that we do not exclude any wellbeing profile among past people from consideration. However, given that the wellbeings of past people cannot be changed, some states are counterfactual in the sense that the wellbeings of past people in these states differ from their factual wellbeings. Likewise, the assumption of Richness means that we do not rule out any selection of future identities from consideration. This does not imply nor require that there exists a technology that allows us to choose any such selection.

We assume that the set of inhabited positions and the wellbeings of the people at these positions are observable, but not necessarily their identities. Thus, social states a and b are observationally distinguishable even if we cannot observe the identities of the future individuals alive in a and b , if (i) $P(a) \neq P(b)$ so that the sets of inhabited positions are different, or (ii) $P(a) = P(b) = P^*$ but $u_i(a) \neq u_i(b)$ for some $i \in E$ or $u_{f(a)(p)}(a) \neq u_{f(b)(p)}(b)$ for some $p \in P^*$ so that the sets of inhabited positions coincide but the profiles of wellbeings are different. In contrast, consider a and b with $P(a) = P(b) = P^*$, $u_i(a) = u_i(b)$ for all $i \in E$, and $u_{f(a)(p)}(a) = u_{f(b)(p)}(b)$

⁷Other related but different versions of multi-profile welfarism theorem were presented by d'Aspremont and Gevers (1977), Blackorby, Bossert and Donaldson (2005a, 2005b) and Hammond (1979) in a finite and fixed population setting and also by d'Aspremont (2007) and Kamaga (2016) in the context of intergenerational wellbeing evaluation that relies on the use of the order of generations.

for all $p \in P^*$. Then, whether a and b are observationally distinguishable depends on whether the identities of the future individuals alive in a and b are observable or not. These identities are selected by $f(a)$ and $f(b)$.

We formalize such an information structure by introducing an information function that generates a partition of set A of social states according to the observational indistinguishability. Let $\{\mathcal{F}(a)\}_{a \in A}$ be a partition of \mathcal{F} that satisfies that (i) for each $a \in A$, $f(a) \in \mathcal{F}(a)$ and (ii) for any $a, b \in A$, $\mathcal{F}(a) = \mathcal{F}(b)$ or $\mathcal{F}(a) \cap \mathcal{F}(b) = \emptyset$. Given a partition $\{\mathcal{F}(a)\}_{a \in A}$, each element $\mathcal{F}(a)$ is interpreted as the set of functions in \mathcal{F} (for instance, $f(a)$ and $f(b)$) where the underlying social states (for instance, a and b) are not distinguished by these functions. Since, for any $a \in A$, the set of inhabited positions $P(a)$ is observable, we have that, for any $a \in A$, $\mathcal{F}(a) \subseteq \mathcal{F}_{P(a)}$.

Given a partition $\{\mathcal{F}(a)\}_{a \in A}$ of \mathcal{F} , define the *information function* associated with $\{\mathcal{F}(a)\}_{a \in A}$ by the mapping $\mathcal{I}: A \rightarrow 2^A \setminus \{\emptyset\}$ such that, for each $a \in A$,

$$\mathcal{I}(a) = \{b \in A : f(b) \in \mathcal{F}(a), u_i(b) = u_i(a) \text{ for } i \in E, \text{ and } u_{f(b)(p)}(b) = u_{f(a)(p)}(a) \text{ for } p \in P(a)\}.$$

That is, the information function \mathcal{I} associates to each social state $a \in A$ the set $\mathcal{I}(a)$ of social states which are observationally indistinguishable due to the sameness of their observable infinite wellbeing profiles and their observable sets of inhabited spatiotemporal positions and the unobservability of the identities of people alive in the underlying social states.

Given a partition $\{\mathcal{F}(a)\}_{a \in A}$ of \mathcal{F} , the associated information function \mathcal{I} induces the partition $\{\mathcal{I}(a)\}_{a \in A}$ of the set A of social states since it satisfies the following properties: (i) $a \in \mathcal{I}(a)$ for all $a \in A$ and (ii) for all $a, b \in A$, $\mathcal{I}(a) \cap \mathcal{I}(b) \neq \emptyset$ implies $\mathcal{I}(b) = \mathcal{I}(a)$. The first property is straightforward from the definition of \mathcal{I} . The second property is verified as follows. Let $a, b \in A$ and suppose that $\mathcal{I}(a) \cap \mathcal{I}(b) \neq \emptyset$. Let $c \in \mathcal{I}(a) \cap \mathcal{I}(b)$. Then, $\mathcal{F}(a) = \mathcal{F}(b) = \mathcal{F}(c)$ follows and we obtain $\mathcal{I}(a) = \{d \in A : f(d) \in \mathcal{F}(c), u_i(d) = u_i(c) \text{ for } i \in E, \text{ and } u_{f(d)(p)}(d) = u_{f(c)(p)}(c) \text{ for } p \in P(c)\} = \mathcal{I}(b)$. The term ‘observable outcomes’ already used in the previous sections refers to elements of the information partition $\{\mathcal{I}(a)\}_{a \in A}$.

The structure of the induced partition $\{\mathcal{I}(a)\}_{a \in A}$ of the set A of social states depends on assumptions on observability of future identities that are determined by the structure of the underlying partition $\{\mathcal{F}(a)\}_{a \in A}$ of \mathcal{F} . Among all possible assumptions, the two polar cases are worth noting. One is the assumption of perfect observability of future identities defined as follows.

Perfect Observability of Future Identities. $\mathcal{F}(a) = \{f(a)\}$ for all $a \in A$.

The polar opposite assumption is unobservability of future identities defined as follows.

Unobservability of Future Identities. $\mathcal{F}(a) = \mathcal{F}_{P(a)}$ for all $a \in A$.

So, for any $a \in A$, $\mathcal{I}(a) = \{a\}$ under the assumption of Perfect Observability of Future Identities, while $\mathcal{I}(a) = \{b \in A : P(b) = P(a), u_i(b) = u_i(a) \text{ for } i \in E, \text{ and } u_{f(b)(p)}(b) = u_{f(a)(p)}(a) \text{ for } p \in$

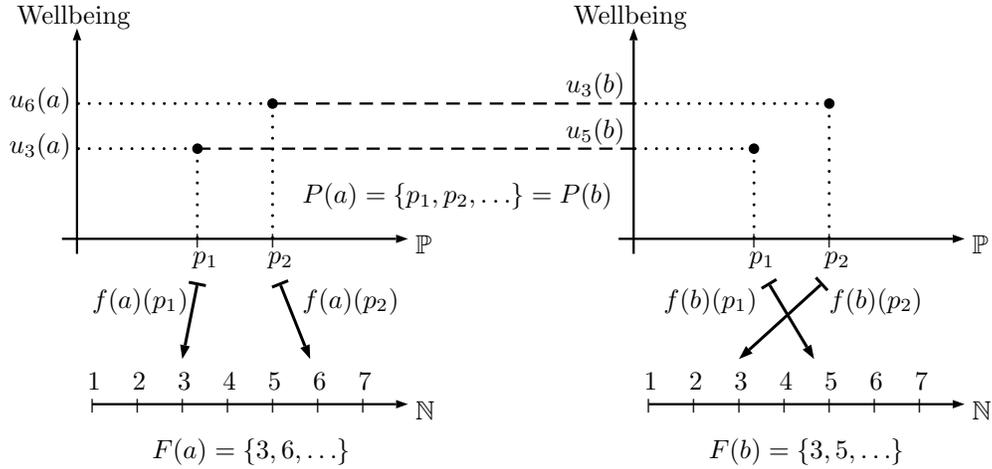


Figure 2: Indistinguishable states a and b under Unobservability of Future Identities

$P(a)$ under the assumption of Unobservability of Future Identities. Figure 2 illustrates two indistinguishable social states, a and b . As illustrated, the sets of inhabited positions in a and b coincide, i.e., $P(a) = P(b) = \{p_1, p_2, \dots\}$. Furthermore, while the identities of the future individuals who live at these positions are different, e.g., $f(a)(p_1) = 3 \neq 5 = f(b)(p_1)$ and $f(a)(p_2) = 6 \neq 3 = f(b)(p_2)$, the alive individual at each of these positions has the same wellbeing level in states a and b . Thus, states a and b are observationally indistinguishable under Unobservability of Future Identities, so that $a, b \in \mathcal{I}(a) = \mathcal{I}(b)$ holds, since, for each position in $P(a) = P(b)$, the alive individual at this position has the same wellbeing level in states a and b .

Under the assumption of Unobservability of Future Identities the level of wellbeing at an inhabited position p depends on the set P^* of inhabited positions, as well as the allocation of resources among these inhabited positions. This allows for the possibility that the individual at inhabited position p has preferences about time and space and exhibits altruism towards individuals that inhabit other positions. However, the assumption of Unobservability of Future Identities rules out that the wellbeing of the individual at position p depends on the identity of this individual, as wellbeing is assumed to be observable while identity is assumed to be unobservable. We acknowledge that this implication of our framework might be problematic. For example, Broome (2018, pp. 12–14) considers it to be a serious problem that the wellbeing derived by the individual living at a given position cannot depend on the “innate nature” of this individual. Indeed, on this basis he argues against the use of the Suppes-Sen principle for defining dominance under such circumstances. However, a *No-Difference View* (Parfit, 1984), rejecting that non-identity makes a moral difference, might also, in practical use, rule out that the wellbeing at a particular position depends on the identity of the individual that lives at this position.

Let \succsim^* be a reflexive and transitive binary relation, thus a *preorder*, defined on the elements of the information partition $\{\mathcal{I}(a)\}_{a \in A}$, expressing a social preference over these elements. Note

that under Perfect Observability of Future Identities we obtain that \succsim^* is a preorder on A . The following axiom imposed on \succsim^* is a person-affecting principle in a setting where identities might be unobservable and allowed to vary also between outcomes with the same set of positions. In particular, the term ‘Pareto’ is used for same people comparisons.

Axiom (Pareto Monotonicity). For any $a, b \in A$, if there exist $c \in \mathcal{I}(a)$, $d \in \mathcal{I}(b)$, and $F^* \in \mathcal{N}$, with $c, d \in A_{F^*}$ and $u_i(c) \geq u_i(d)$ for all $i \in E \cup F^*$, then $\mathcal{I}(a) \succsim^* \mathcal{I}(b)$.

The axiom of Pareto Monotonicity means that $\mathcal{I}(a) \succsim^* \mathcal{I}(b)$ if there is a possible state in $\mathcal{I}(a)$ that Pareto dominates or is Pareto indifferent to a possible state in $\mathcal{I}(b)$. In other words, the outcome $\mathcal{I}(a)$ is deemed to be at least as good as the outcome $\mathcal{I}(b)$ under the preorder \succsim^* if there exists a set of identities of future people that is possible in *each* of the two outcomes, and where all existing and future individuals are at least as well off in the former outcome as in the latter.⁸ Note that under Perfect Observability of Future Identities the axiom simply means that $a \succsim^* b$ if a Pareto dominates or is Pareto indifferent to b .

The Welfarism Theorem of this section shows how, under the axiom of Pareto Monotonicity, the preorder \succsim^* defined on $\{\mathcal{I}(a)\}_{a \in A}$ induces to a binary relation \succsim defined on wellbeing profiles in $\mathbb{R}^e \times \mathbf{X}$, where \mathbf{X} denotes the set of all bounded streams:

$$\mathbf{X} = \left\{ \mathbf{x} = (x_1, x_2, \dots, x_j, \dots) \in \mathbb{R}^{\mathbb{N}} : \sup_{j \in \mathbb{N}} |x_j| < +\infty \right\}.$$

Define the correspondence $A : \mathbb{R}^e \times \mathbf{X} \rightarrow 2^A$ by, for any $(\mathbf{x}_E, \mathbf{x}) \in \mathbb{R}^e \times \mathbf{X}$,

$$A(\mathbf{x}_E, \mathbf{x}) = \left\{ a \in A : \begin{array}{l} \text{for all } i \in E, u_i(a) = x_i \text{ and} \\ \text{there exists } \pi \in \Pi_{P(a)} \text{ such that, for all } j \in \mathbb{N}, u_{f(a)(\pi(j))} = x_j \end{array} \right\}.$$

Hence, $A(\mathbf{x}_E, \mathbf{x})$ is the set of states under which the wellbeings of existing people equal \mathbf{x}_E and the wellbeings of future people equal the stream \mathbf{x} under some order of listing the inhabited positions. Under the condition of Richness, $A(\mathbf{x}_E, \mathbf{x})$ is non-empty for all $(\mathbf{x}_E, \mathbf{x}) \in \mathbb{R}^e \times \mathbf{X}$.

Define the binary relation \succsim on $\mathbb{R}^e \times \mathbf{X}$ by

$$(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{y}_E, \mathbf{y}) \text{ if and only if } \mathcal{I}(a) \succsim^* \mathcal{I}(b) \text{ for all } a \in A(\mathbf{x}_E, \mathbf{x}) \text{ and } b \in A(\mathbf{y}_E, \mathbf{y}). \quad (3)$$

The induced binary relation \succsim inherits transitivity from \succsim^* , but need not be reflexive unless further assumptions are imposed on A , \mathcal{I} , and \succsim^* .

For any $\mathbf{x} \in \mathbf{X}$ and any $\pi \in \Pi_{\mathbb{N}}$, write $\mathbf{x}_\pi = (x_{\pi(1)}, x_{\pi(2)}, \dots) \in \mathbf{X}$. To examine the implication of Pareto monotonicity of \succsim^* for the induced relation \succsim , consider the following two axioms on \succsim .

⁸It might be that c being the true state if outcome $\mathcal{I}(a)$ is chosen rules out that d is the true state when outcome $\mathcal{I}(b)$ is chosen. In particular, Pareto Monotonicity is consistent with requiring that a particular identity i if brought to life must live at a particular position p , as long as the association between i and p is not known.

Axiom (Monotonicity). For any $(\mathbf{x}_E, \mathbf{x}), (\mathbf{y}_E, \mathbf{y}) \in \mathbb{R}^e \times \mathbf{X}$ with $(\mathbf{x}_E, \mathbf{x}) \geq (\mathbf{y}_E, \mathbf{y})$, $(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{y}_E, \mathbf{y})$.

Axiom (Strong Anonymity). For any $\mathbf{x}_E \in \mathbb{R}^e$ and any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x}_\pi = \mathbf{y}$ for some $\pi \in \Pi_{\mathbb{N}}$, $(\mathbf{x}_E, \mathbf{x}) \sim (\mathbf{x}_E, \mathbf{y})$.

Each of these two axioms on \succsim implies that \succsim is reflexive. Since \succsim is transitive, the conjunction of them yields the following axiom on \succsim .

Axiom (Strong Suppes-Sen Monotonicity). For any $(\mathbf{x}_E, \mathbf{x}), (\mathbf{y}_E, \mathbf{y}) \in \mathbb{R}^e \times \mathbf{X}$ with $(\mathbf{x}_E, \mathbf{x}_\pi) \geq (\mathbf{y}_E, \mathbf{y})$ for some $\pi \in \Pi_{\mathbb{N}}$, $(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{y}_E, \mathbf{y})$.

Note that the permutations involved in the axioms of Strong Anonymity and Strong Suppes-Sen Monotonicity concern streams of future wellbeings. The prefix ‘strong’ indicates that $\Pi_{\mathbb{N}}$ contains all bijective functions from \mathbb{N} to \mathbb{N} , also those that reorder an infinite number of components.

These three axioms imposed on \succsim consider different people comparison in a welfarist framework. However, they correspond to the axiom of Pareto Monotonicity imposed on \succsim^* , and which considers same people comparisons in a non-welfarist framework. This is established through the following result which shows that we obtain Monotonicity (and thereby reflexivity) and Strong Suppes-Sen Monotonicity (and thereby Strong Anonymity) of \succsim under Richness and Unobservability of Future Identities, provided that the axiom of Pareto Monotonicity is imposed on \succsim^* .

Theorem (Welfarism). *Assume that the set A of social states satisfies Richness and the information partition $\{\mathcal{I}(a)\}_{a \in A}$ satisfies Unobservability of Future Identities. Then the following statements are equivalent, where \succsim denotes the induced binary relation on $\mathbb{R}^e \times \mathbf{X}$ defined by (3).*

- (i) *The preorder \succsim^* satisfies Pareto Monotonicity.*
- (ii) *The binary relation \succsim satisfies Monotonicity.*
- (iii) *The binary relation \succsim satisfies Strong Suppes-Sen Monotonicity.*

Furthermore, each of these statements implies:

- (iv) *For any $(\mathbf{x}_E, \mathbf{x}), (\mathbf{y}_E, \mathbf{y}) \in \mathbb{R}^e \times \mathbf{X}$ and any $a, b \in A$ with $a \in A(\mathbf{x}_E, \mathbf{x})$ and $b \in A(\mathbf{y}_E, \mathbf{y})$, $(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{y}_E, \mathbf{y})$ if and only if $\mathcal{I}(a) \succsim^* \mathcal{I}(b)$.*

Proof. Assume that the set A of social states satisfies Richness and the information partition $\{\mathcal{I}(a)\}_{a \in A}$ satisfies Unobservability of Future Identities. Let \succsim be the induced binary relation on $\mathbb{R}^e \times \mathbf{X}$ defined by (3).

First, we show that (i) implies (iv). Assume that \succsim^* satisfies Pareto Monotonicity. Consider any $(\mathbf{x}_E, \mathbf{x}), (\mathbf{y}_E, \mathbf{y}) \in \mathbb{R}^e \times \mathbf{X}$ satisfying that there exist $a \in A(\mathbf{x}_E, \mathbf{x})$ and $b \in A(\mathbf{y}_E, \mathbf{y})$ such that $\mathcal{I}(a) \succsim^* \mathcal{I}(b)$. We must show that $\mathcal{I}(c) \succsim^* \mathcal{I}(d)$ for all $c \in A(\mathbf{x}_E, \mathbf{x})$ and $d \in A(\mathbf{y}_E, \mathbf{y})$. By

transitivity of \succsim^* , it is sufficient to show that $\mathcal{I}(a) \sim^* \mathcal{I}(c)$ for all $a, c \in A(\mathbf{x}_E, \mathbf{x})$ (and likewise that $\mathcal{I}(b) \sim^* \mathcal{I}(d)$ for all $b, d \in A(\mathbf{y}_E, \mathbf{y})$). Hence, assume $a, c \in A(\mathbf{x}_E, \mathbf{x})$. Under Richness, there exist $a', c' \in A(\mathbf{x}_E, \mathbf{x})$ with:

$$\begin{aligned} F(a') &= F(c') = F^* \text{ and } u_i(a') = u_i(c') \text{ for all } i \in F^*, \\ P(a) &= P(a') \text{ and } u_{f(a)(p)}(a) = u_{f(a')(p)}(a') \text{ for all } p \in P(a), \\ P(c') &= P(c) \text{ and } u_{f(c')(p)}(c') = u_{f(c)(p)}(c) \text{ for all } p \in P(c), \end{aligned}$$

Since $u_i(a) = u_i(a') = u_i(c') = u_i(c)$ for all $i \in E$, it follows by Pareto Monotonicity that $\mathcal{I}(a') \sim^* \mathcal{I}(c')$ and by Unobservability of Future Identities that $\mathcal{I}(a) = \mathcal{I}(a')$ and $\mathcal{I}(c') = \mathcal{I}(c)$. By reflexivity and transitivity of \succsim^* , $\mathcal{I}(a) \sim^* \mathcal{I}(c)$. This establishes the if part of (iv). The only-if part of (iv) follows from (3).

Next, we show that (i) implies (iii). Assume that \succsim^* satisfies Pareto Monotonicity. Then, by the first part, statement (iv) holds. Consider any $\mathbf{x}_E \in \mathbb{R}^e$ and any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_\pi \geq y$ for some $\pi \in \Pi_{\mathbb{N}}$. It follows from the definition of the correspondence $A : \mathbb{R}^e \times \mathbf{X} \rightarrow 2^A$ that, under Richness, there exist $a \in A(\mathbf{x}_E, \mathbf{x})$ and $b \in A(\mathbf{x}_E, \mathbf{y})$ with $F(a) = F(b) = F^*$ and $u_i(a) \geq u_i(b)$ for all $i \in E \cup F^*$. By Pareto Monotonicity, $\mathcal{I}(a) \succsim^* \mathcal{I}(b)$. Hence, by (iv), $(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{x}_E, \mathbf{y})$.

Then, we show that (iii) implies (ii). This is straightforward since $\Pi_{\mathbb{N}}$ contains the identity permutation.

Finally, we show that (ii) implies (i). Assume that statement (ii) holds and consider any $a, b \in A$ for which there exist $c \in \mathcal{I}(a)$, $d \in \mathcal{I}(b)$, and $F^* \in \mathcal{N}$, with $c, d \in A_{F^*}$ and $u_i(c) \geq u_i(d)$ for all $i \in E \cup F^*$. Then, even under Unobservability of Future Identities, $u_i(a) = u_i(c) \geq u_i(d) = u_i(b)$ for all $i \in E$. Also, since $c \in \mathcal{I}(a)$ and $d \in \mathcal{I}(b)$, there exist bijective functions $\pi : P(a) \rightarrow \mathbb{N}$ and $\rho : P(b) \rightarrow \mathbb{N}$ such that $u_{f(a)(\pi(j))} \geq u_{f(b)(\rho(j))}$. This implies that $a \in A(\mathbf{x}_E, \mathbf{x})$ and $b \in A(\mathbf{x}_E, \mathbf{y})$, where, for all $i \in E$, $x_i = u_i(a) \geq u_i(b) = y_i$ and, for all $j \in \mathbb{N}$, $x_j = u_{f(a)(\pi(j))} \geq u_{f(b)(\rho(j))} = y_j$. It follows by (ii) and (3) that $\mathcal{I}(a) \succsim^* \mathcal{I}(b)$, thereby establishing (i). \square

To verify that the conditions of the theorem are logically consistent, we provide a concrete example of a pair of preorders defined on $\{\mathcal{I}(a)\}_{a \in A}$ and $\mathbb{R}^e \times \mathbf{X}$, respectively, that satisfy these conditions. For this purpose, write, for any $z \in \mathbb{R}$, $z^+ = \max\{z, 0\}$ and $z^- = \min\{z, 0\}$.

Example. Let, for each $\varepsilon \in (0, 1]$, \succsim^ε on $\mathbb{R}^e \times \mathbf{X}$ be defined by $(\mathbf{x}_E, \mathbf{x}) \succsim^\varepsilon (\mathbf{y}_E, \mathbf{y})$ if and only if

$$\varepsilon \left(\sum_{i=-1}^{-e} (x_i - y_i)^+ + \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^j (x_j - y_j)^+ \right) + \sum_{i=-1}^{-e} (x_i - y_i)^- + \sum_{j=1}^{\infty} \left(\frac{1}{2}\right)^j (x_j - y_j)^- \geq 0.$$

Let $\mathbf{x}_{[\]}$ and $\mathbf{y}_{[\]}$ be the rank-ordered streams of future wellbeing derived from \mathbf{x} and \mathbf{y} through (1) and (2), and let $\succsim_{[\]}^\varepsilon$ on $\mathbb{R}^e \times \mathbf{X}$ be defined by $(\mathbf{x}_E, \mathbf{x}) \succsim_{[\]}^\varepsilon (\mathbf{y}_E, \mathbf{y})$ if and only if $(\mathbf{x}_E, \mathbf{x}_{[\]}) \succsim^\varepsilon (\mathbf{y}_E, \mathbf{y}_{[\]})$. For $\varepsilon = 1$, $\succsim_{[\]}^\varepsilon$ is equally weighted utilitarian for existing people and rank-weighted utilitarian

for future people, while in the limit, when $\varepsilon \rightarrow 0$, $\succsim_{[\cdot]}^\varepsilon$ combines the Pareto principle for existing people and the generalized Suppes-Sen principle for future people. It can be verified that $\succsim_{[\cdot]}^\varepsilon$ is a preorder satisfying Monotonicity and Strong Anonymity, hence, by transitivity, also Strong Suppes-Sen Monotonicity. Let $\succsim_{[\cdot]}^{\varepsilon*}$ on $\{\mathcal{I}(a)\}_{a \in A}$ be defined by $\mathcal{I}(a) \succsim_{[\cdot]}^{\varepsilon*} \mathcal{I}(b)$ if and only if there exist $c \in \mathcal{I}(a)$ and $d \in \mathcal{I}(b)$ such that $(\mathbf{x}_E(c), \mathbf{x}_{[\cdot]}(c)) \succsim^\varepsilon (\mathbf{x}_E(d), \mathbf{x}_{[\cdot]}(d))$. It can be verified that $\succsim_{[\cdot]}^{\varepsilon*}$ is a preorder satisfying Pareto Monotonicity under Richness and Unobservability of Future Identities.

The Welfarism Theorem means that, under Richness and Unobservability of Future Identities, the evaluation of observable outcomes according to a preorder \succsim^* satisfying Pareto Monotonicity is represented by a binary relation \succsim for wellbeing profiles that satisfies Strong Suppes-Sen Monotonicity; thus, \succsim is not only transitive but reflexive, i.e., a preorder. Strong Suppes-Sen Monotonicity implies that the preorder \succsim must satisfy Strong Anonymity, in the sense that any reorder of the stream of future wellbeings leads to a stream that is equally good. The underlying issue is that there is no natural 1-to-1 correspondence between future people in two different alternatives since the identities of future people are unobservable and cannot be assumed to remain fixed, even in comparisons with the same set of spatiotemporal positions.⁹

As we discuss in the subsequent section and analyze formally in Appendix A, Strong Anonymity limits the kind of sensitivity that can be imposed on the wellbeings of future people. This in turn has a bearing on how to define and justify the concept of dominance under such circumstances.

4 Justifying the Dominance Definition

In the usual setting, with a finite number of individuals with fixed identities, dominance coincides with *Pareto dominance* and derives from the axiom of Strong Pareto—the axiom which requires a preorder to be sensitive to an increase of the wellbeing of any one individual if the wellbeing of no other individual is decreased. A wellbeing profile Pareto dominates another if and only if it is better according to the axiom of Strong Pareto.

As mentioned in the introduction, the axiom of Strong Pareto cannot be imposed on a preorder for infinite streams that satisfies Strong Anonymity (Van Liedekerke, 1995; Van Liedekerke and Lauwers, 1997, p. 163). This impossibility result is an illustration of the conflict between sensitivity and impartiality in the evaluation of infinite wellbeing streams which has been analyzed in many contributions, including Diamond (1965), Svensson (1980), Basu and Mitra (2003), Zame (2007), and Lauwers (2010), as well as Asheim (2010) for an overview.

⁹The proof of the Welfarism Theorem shows, under the assumptions of Richness and Unobservability of Future Identities, that condition (iv) is equivalent to the *Pareto Indifference* of \succsim^* (in the sense that, for any $a, b \in A$, if there exist $c \in \mathcal{I}(a)$, $d \in \mathcal{I}(b)$, and $F^* \in \mathcal{N}$, with $c, d \in A_{F^*}$ and $u_i(c) = u_i(d)$ for all $i \in E \cup F^*$, then $\mathcal{I}(a) \sim^* \mathcal{I}(b)$) as well as the Strong Anonymity of \succsim . This version of the Welfarism Theorem can be seen to be an infinite and variable population variant of the single-profile anonymous welfarism established by Blackorby, Bossert and Donaldson (2006).

There are two main routes out of the dilemma that this impossibility poses:

- (1) Stick with Strong Pareto and weaken Strong Anonymity.
- (2) Stick with Strong Anonymity and weaken Strong Pareto.

Moreover, one can consider weakening both Strong Pareto and Strong Anonymity.

Route (1) has been extensively explored. Strong Pareto *is* compatible with the axiom of Finite Anonymity, in the sense of invariance to any *finite* reordering of a stream (Svensson, 1980). In fact, there is a literature on how to extend impartiality beyond Finite Anonymity, while remaining compatible with Strong Pareto; see, for example, Lauwers (1997, 1998, 2012), Fleurbaey and Michel (2003), Mitra and Basu (2007), Sakai (2010), and Adachi, Cato and Kamaga (2014). However, even though the axiom of Strong Anonymity has been severely criticized by many (primarily because it reduces sensitivity as in the example provided by Van Liedekerke and Lauwers, 1997, p. 164) and thus has few proponents, all versions of anonymity but Strong Anonymity require the existence of some underlying natural 1-to-1 correspondence between the components of the alternatives being compared.

In view of the Welfarism Theorem, we here follow route (2), not yet entirely explored, for avoiding the conflict between sensitivity and impartiality.¹⁰ Since the axiom of Strong Pareto cannot be imposed, we instead adapt an approach to dominance proposed by Donaldson and Weymark (1997) and Suzumura (1983, p. 69; 1999). When applying this approach in the usual setting, with a finite number of individuals with fixed identities, Pareto dominance can be characterized as the asymmetric part of the intersection of all preorders satisfying Pareto Monotonicity in the sense that one wellbeing profile is deemed at least as good as another if each wellbeing component of the former profile is at least as high as the corresponding wellbeing component of the latter. We show how this approach generalizes to our setting with an infinite number of individuals with unobservable identities and use it to investigate how far sensitivity can be extended without contradicting Strong Anonymity. By applying this approach we are able to prove a theorem that justifies our Dominance Definition, based on auxiliary results obtained in Appendix A.

Consider the collection of induced binary relations \succsim on $\mathbb{R}^e \times \mathbf{X}$ under the assumptions that the set A of social states satisfies Richness, the information partition $\{\mathcal{I}(a)\}_{a \in A}$ satisfies Unobservability of Future Identities, and the preorder \succsim^* on $\{\mathcal{I}(a)\}_{a \in A}$ which induces \succsim satisfies the axiom of Pareto Monotonicity. By the Welfarism Theorem, any element \succsim of this collection satisfies Strong Suppes-Sen Monotonicity, leading to an enlarged symmetric part that is incompatible with Strong Pareto. Therefore, compared to the case where Strong Pareto could have been imposed, dominance will be changed when this concept is derived from the approach to dominance

¹⁰Also Sakai (2016) explores the consequences of imposing Strong Anonymity. However, he combines Strong Anonymity and supnorm continuity with his Weak Non-Substitution axiom, which rules out sensitivity for the wellbeing at any one component.

proposed by Donaldson and Weymark (1997) and Suzumura (1983, p. 69; 1999), adapted to our setting. The question is what kind of dominance concept this approach leads to.

The concept of dominance will be particularly tractable if we let the collection of induced binary relations \succsim on $\mathbb{R}^e \times \mathbf{X}$ satisfy three auxiliary axioms. The first of those is the following robustness axiom, ensuring that small wellbeing changes do not reverse the evaluation.

Axiom (Continuity). For any $(\mathbf{x}_E, \mathbf{x}) \in \mathbb{R}^e \times \mathbf{X}$, the sets $\{(\mathbf{y}_E, \mathbf{y}) \in \mathbb{R}^e \times \mathbf{X} : (\mathbf{y}_E, \mathbf{y}) \succsim (\mathbf{x}_E, \mathbf{x})\}$ and $\{(\mathbf{y}_E, \mathbf{y}) \in \mathbb{R}^e \times \mathbf{X} : (\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{y}_E, \mathbf{y})\}$ are closed in the supnorm topology.

To state the next auxiliary axiom, let the set \mathcal{E}^{\succsim} be defined by:

$$\mathcal{E}^{\succsim} = \{\varepsilon \in \mathbb{R}_{++} : \text{for any } \mathbf{x}_E \in \mathbb{R}^e \text{ and any } \mathbf{x}, \mathbf{y} \in \mathbf{X} \text{ with } y_{j'} < x_{j'} \leq x_{j''} < y_{j''} \text{ for some } j', \\ j'' \in \mathbb{N}, \text{ and } x_j = y_j \text{ for all } j \in \mathbb{N} \setminus \{j', j''\}, (\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{x}_E, \mathbf{y}) \text{ if } y_{j''} - x_{j''} = \varepsilon (x_{j'} - y_{j'})\}.$$

Note that $\mathcal{E}^{\succsim} = \mathbb{R}_{++}$ under the axiom of *Hammond equity* (Hammond, 1976). If non-leaky transfers are acceptable, then $1 \in \mathcal{E}^{\succsim}$. Hence, \mathcal{E}^{\succsim} is also non-empty if \succsim satisfies the *Pigou-Dalton transfer principle* in its weak version (Pigou, 1912; Dalton, 1920). Here we will be concerned with a much weaker axiom than both Hammond equity and Pigou-Dalton (but stronger than the minimal equity axiom proposed by Deschamps and Gevers, 1978): we simply ask that transfers from rich to poor are acceptable if the wellbeing loss of the rich is sufficiently small.¹¹

Axiom (Limited Inequity). $\mathcal{E}^{\succsim} \neq \emptyset$.

The third and final auxiliary axiom relates to the effects of adding one individual (or infinitely many people) to a population. The literature on population ethics stemming from Parfit (1984) has addressed this issue, and Broome (2004) and Blackorby, Bossert and Donaldson (2005a) discuss it at length. In particular, Blackorby and Donaldson (1984) and Blackorby, Bossert and Donaldson (1995) argue in favor of the existence of a critical level, such that an individual's life contributes positively to the social value of a population if only and if the wellbeing of the additional individual is above this critical level. In general, the critical level may depend on how wellbeing is distributed. The following axiom imposes some regularity in the level of the critical level for infinite populations. The axiom asserts that, if it is acceptable to add one individual at some wellbeing level z to a population, then it is also acceptable to add infinitely many people at this level. The following additional notation is needed: For any $z \in \mathbb{R}$ and any $\mathbf{x} \in \mathbf{X}$, $(z, \mathbf{x}) = (z, x_1, x_2, \dots) \in \mathbf{X}$, $z\mathbb{1}_{\mathbb{N}} = (z, z, \dots) \in \mathbf{X}$, and $(z\mathbb{1}_{\mathbb{N}}, \mathbf{x}) = (z, x_1, z, x_2, \dots) \in \mathbf{X}$.

Axiom (Critical-Level Consistency). For any $(\mathbf{x}_E, \mathbf{x}) \in \mathbb{R}^e \times \mathbf{X}$ and any $z \in \mathbb{R}$, $(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{x}_E, (z, \mathbf{x}))$ (resp. $(\mathbf{x}_E, \mathbf{x}) \precsim (\mathbf{x}_E, (z, \mathbf{x}))$) if and only if $(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{x}_E, (z\mathbb{1}_{\mathbb{N}}, \mathbf{x}))$ (resp. $(\mathbf{x}_E, \mathbf{x}) \precsim (\mathbf{x}_E, (z\mathbb{1}_{\mathbb{N}}, \mathbf{x}))$).

¹¹To emphasize just how weak this axiom is, note that every generalized utilitarian preorder with a transformation function that is finitely non-concave satisfies Limited Inequity. So in order for $\mathcal{E}^{\succsim} = \emptyset$, the preorder \succsim has to be even less egalitarian than utilitarianism.

Consider the collection \mathcal{D} of binary relations \succsim on $\mathbb{R}^e \times \mathbf{X}$ satisfying:

- \succsim is induced by (3) from some preorder \succsim^* on the elements of the information partition $\{\mathcal{I}(a)\}_{a \in A}$, where A satisfies the assumption of Richness, $\{\mathcal{I}(a)\}_{a \in A}$ satisfies the assumption of Unobservability of Future Identities, and \succsim^* satisfies the axiom of Pareto Monotonicity.
- \succsim satisfies Continuity, Limited Inequity, and Critical-Level Consistency.

By the Welfarism Theorem, any derived binary relation \succsim on $\mathbb{R}^e \times \mathbf{X}$ in \mathcal{D} is a preorder and satisfies Strong Suppes-Sen Monotonicity, Continuity, Limited Inequity, and Critical-Level Consistency, so that Lemma 3 and Proposition 3 of Appendix A apply for any element of this set.

Note that $\mathcal{D} \neq \emptyset$ since, for any $\varepsilon \in (0, 1]$, $\succsim_{[\cdot]}^\varepsilon$ is an element of \mathcal{D} , where $\succsim_{[\cdot]}^\varepsilon$ is the preorder on $\mathbb{R}^e \times \mathbf{X}$ defined in the Example. To verify this, let $\varepsilon \in (0, 1]$. The preorder $\succsim_{[\cdot]}^\varepsilon$ clearly satisfies Monotonicity and Strong Anonymity, hence, by transitivity, also Strong Suppes-Sen Monotonicity. Moreover, it is evident that $\succsim_{[\cdot]}^\varepsilon$ satisfies Continuity. To establish Limited Inequity of $\succsim_{[\cdot]}^\varepsilon$, consider any $\mathbf{x}_E \in \mathbb{R}^e$ and any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $y_{j'} < x_{j'} \leq x_{j''} < y_{j''}$ for some $j', j'' \in \mathbb{N}$, and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j', j''\}$, where $y_{j''} - x_{j''} = \varepsilon(x_{j'} - y_{j'})$. If $x_{j''} \geq \lim_{r \rightarrow \infty} x_{[r]}$, then $\mathbf{x}_{[\cdot]} \geq \mathbf{y}_{[\cdot]}$ and, thus, $(\mathbf{x}_E, \mathbf{x}) \succsim_{[\cdot]}^\varepsilon (\mathbf{x}_E, \mathbf{y})$. If $x_{j''} < \lim_{r \rightarrow \infty} x_{[r]}$, then there exists $r' \in \mathbb{N}$ such that $x_{j'} \leq x_{[r']} \leq x_{j''}$. Furthermore, in this case:

$$\sum_{r=1}^{\infty} \left(\frac{1}{2}\right)^{r-r'} (x_{[r]} - y_{[r]})^+ \geq x_{j'} - y_{j'} > 0 > \sum_{r=1}^{\infty} \left(\frac{1}{2}\right)^{r-r'} (x_{[r]} - y_{[r]})^- \geq x_{j''} - y_{j''}.$$

Since, by assumption, $\varepsilon(x_{j'} - y_{j'}) + x_{j''} - y_{j''} = 0$, it follows from the definition of $\succsim_{[\cdot]}^\varepsilon$ that $(\mathbf{x}_E, \mathbf{x}) \succsim_{[\cdot]}^\varepsilon (\mathbf{x}_E, \mathbf{y})$ also in this case. Hence, $\varepsilon \in \mathcal{E}^{\succsim_{[\cdot]}^\varepsilon}$, implying that $\mathcal{E}^{\succsim_{[\cdot]}^\varepsilon} \neq \emptyset$. To establish Critical-Level Consistency of $\succsim_{[\cdot]}^\varepsilon$, note that, for any $(\mathbf{x}_E, \mathbf{x}) \in \mathbb{R}^e \times \mathbf{X}$ and any $z \in \mathbb{R}$, $(\mathbf{x}_E, \mathbf{x}) \sim_{[\cdot]}^\varepsilon (\mathbf{x}_E, (z, \mathbf{x}))$ and $(\mathbf{x}_E, \mathbf{x}) \sim_{[\cdot]}^\varepsilon (\mathbf{x}_E, (z \mathbb{1}_{\mathbb{N}}, \mathbf{x}))$ if $z \geq \lim_{r \rightarrow \infty} x_{[r]}$ and $(\mathbf{x}_E, \mathbf{x}) \succ_{[\cdot]}^\varepsilon (\mathbf{x}_E, (z, \mathbf{x}))$ and $(\mathbf{x}_E, \mathbf{x}) \succ_{[\cdot]}^\varepsilon (\mathbf{x}_E, (z \mathbb{1}_{\mathbb{N}}, \mathbf{x}))$ if $z < \lim_{r \rightarrow \infty} x_{[r]}$.

We are now in the position of stating a theorem that justifies our Dominance Definition in a similar way as the concept of Pareto dominance is characterized in the usual setting with a finite set of individuals with fixed identities. For this purpose, define the *Dominance Preorder* $\succsim^{\mathcal{D}}$ by $(\mathbf{x}_E, \mathbf{x}) \succsim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y})$ if and only if $(\mathbf{x}_E, \mathbf{x}) \succsim (\mathbf{y}_E, \mathbf{y})$ for all \succsim on $\mathbb{R}^e \times \mathbf{X}$ in \mathcal{D} . Note that $\succsim^{\mathcal{D}}$ embodies the unanimous evaluation of all preorders in \mathcal{D} . Let $\sim^{\mathcal{D}}$ and $\succ^{\mathcal{D}}$ be the symmetric and asymmetric parts of $\succsim^{\mathcal{D}}$. The following theorem justifies our Dominance Definition by showing that it is derived as the strict preference of the unanimous evaluation of all preorders in \mathcal{D} .

Theorem (Justification of the Dominance Definition). *If the set A of social states satisfies Richness and the information partition $\{\mathcal{I}(c)\}_{c \in A}$ satisfies Unobservability of Future Identities, then the following statements are equivalent.*

- (i) $(\mathbf{x}_E, \mathbf{x}) \succ^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y})$.
- (ii) $(\mathbf{x}_E(a), \mathbf{x}_{[\cdot]}(a)) > (\mathbf{x}_E(b), \mathbf{x}_{[\cdot]}(b))$ for any $a \in A(\mathbf{x}_E, \mathbf{x})$ and $b \in A(\mathbf{y}_E, \mathbf{y})$.

Proof. Consider any $a, b \in A$, and assume that the set A of social states satisfies Richness and the information partition $\{\mathcal{I}(c)\}_{c \in A}$ satisfies Unobservability of Future Identities.

Consider any $(\mathbf{x}_E, \mathbf{x}), (\mathbf{y}_E, \mathbf{y}) \in \mathbb{R}^e \times \mathbf{X}$ with $a \in A(\mathbf{x}_E, \mathbf{x})$ and $b \in A(\mathbf{y}_E, \mathbf{y})$. We have that $\mathbf{x}_E = \mathbf{x}_E(a)$ and $\mathbf{y}_E = \mathbf{x}_E(b)$. Write \mathbf{x}' and \mathbf{y}' for the streams in \mathbf{X} defined by $x'_r = x_{[r]}(a)$ and $y'_r = x_{[r]}(b)$ for all $r \in \mathbb{N}$. Also, write $\ell = \lim_{j \rightarrow \infty} x'_j$ and $\mu = \lim_{j \rightarrow \infty} y'_j$. By the construction of $(\mathbf{x}_E(a), \mathbf{x}_{[\cdot]}(a))$ and $(\mathbf{x}_E(b), \mathbf{x}_{[\cdot]}(b))$ and the definition of $A(\mathbf{x}_E, \mathbf{x})$ and $A(\mathbf{y}_E, \mathbf{y})$, there exist $\pi \in \Pi_{\mathbb{N}}$ and $\rho \in \Pi_{\mathbb{N}}$ such that $\mathbf{x}_\pi \geq (\mathbf{x}_E, (\ell \mathbb{1}_M, \mathbf{x}'))$ and $\mathbf{y}_\rho \geq (\mathbf{y}_E, (\mu \mathbb{1}_N, \mathbf{y}'))$, where $M = \emptyset$, $M = \{1, 2, \dots, m\}$ for some $m \in \mathbb{N}$, or $M = \mathbb{N}$, and likewise for N . Writing $\mathbf{x}'' = (\ell \mathbb{1}_M, \mathbf{x}')$, we have that $\liminf_{j \rightarrow \infty} x_{\pi(j)} = \ell$ and $x''_{j'} = \ell$ for all $j' \in \mathbb{N}$ such that $x_{\pi(j')} > x''_{j'}$. Likewise, writing $\mathbf{y}'' = (\mu \mathbb{1}_N, \mathbf{y}')$, we have that $\liminf_{j \rightarrow \infty} y_{\rho(j)} = \mu$ and $y''_{j'} = \mu$ for all $j' \in \mathbb{N}$ such that $y_{\rho(j')} > y''_{j'}$.

By Strong Anonymity of any $\succsim \in \mathcal{D}$, it follows that $(\mathbf{x}_E, \mathbf{x}) \sim^{\mathcal{D}} (\mathbf{x}_E, \mathbf{x}_\pi)$ and $(\mathbf{y}_E, \mathbf{y}) \sim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y}_\rho)$. Furthermore, since any $\succsim \in \mathcal{D}$ satisfies Strong Suppes-Sen Monotonicity, Continuity, Limited Inequity, and Critical-Level Consistency, Proposition 3 of Appendix A implies that $(\mathbf{x}_E, \mathbf{x}_\pi) \sim^{\mathcal{D}} (\mathbf{x}_E, \mathbf{x}'')$ and $(\mathbf{y}_E, \mathbf{y}_\rho) \sim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y}'')$. Finally, by Strong Anonymity and Continuity of any $\succsim \in \mathcal{D}$, Lemma 3 of Appendix A implies that $(\mathbf{x}_E, \mathbf{x}'') \sim^{\mathcal{D}} (\mathbf{x}_E, \mathbf{x}')$ and $(\mathbf{y}_E, \mathbf{y}'') \sim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y}')$. Thus, since $\succsim^{\mathcal{D}}$ is transitive, it holds that $(\mathbf{x}_E, \mathbf{x}) \succsim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y})$ if and only if $(\mathbf{x}_E, \mathbf{x}') \succsim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y}')$.

(ii) *implies (i).* Assume that (ii) holds: $(\mathbf{x}_E(a), \mathbf{x}_{[\cdot]}(a)) > (\mathbf{x}_E(b), \mathbf{x}_{[\cdot]}(b))$. We have that $(\mathbf{x}_E, \mathbf{x}') \succsim (\mathbf{y}_E, \mathbf{y}')$ for all $\succsim \in \mathcal{D}$, since $(\mathbf{x}_E, \mathbf{x}') = (\mathbf{x}_E(a), \mathbf{x}_{[\cdot]}(a)) > (\mathbf{x}_E(b), \mathbf{x}_{[\cdot]}(b)) = (\mathbf{y}_E, \mathbf{y}')$ and any $\succsim \in \mathcal{D}$ satisfies Monotonicity. Furthermore, it is not the case that $(\mathbf{y}_E, \mathbf{y}') \succsim (\mathbf{x}_E, \mathbf{x}')$ for all $\succsim \in \mathcal{D}$, since $(\mathbf{y}_E, \mathbf{y}') \not\prec_{[\cdot]}^{\varepsilon} (\mathbf{x}_E, \mathbf{x}')$ for any element in the subcollection $\{\prec_{[\cdot]}^{\varepsilon}\}_{\varepsilon \in (0,1]} \subseteq \mathcal{D}$. This implies that $(\mathbf{x}_E, \mathbf{x}') \succ^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y}')$. Hence, $(\mathbf{x}_E, \mathbf{x}) \succ^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y})$, showing that (i) holds.

(i) *implies (ii).* Suppose that (ii) does not hold: $(\mathbf{x}_E(a), \mathbf{x}_{[\cdot]}(a)) \not\geq (\mathbf{x}_E(b), \mathbf{x}_{[\cdot]}(b))$.

First subcase: $(\mathbf{x}_E(a), \mathbf{x}_{[\cdot]}(a)) = (\mathbf{x}_E(b), \mathbf{x}_{[\cdot]}(b))$. Then $(\mathbf{x}_E, \mathbf{x}') = (\mathbf{y}_E, \mathbf{y}')$ and $(\mathbf{x}_E, \mathbf{x}') \sim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y}')$ since $\succsim^{\mathcal{D}}$ is reflexive. Hence, $(\mathbf{x}_E, \mathbf{x}) \sim^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y})$, showing that (i) does not hold.

Second subcase: $(\mathbf{x}_E(a), \mathbf{x}_{[\cdot]}(a)) \not\geq (\mathbf{x}_E(b), \mathbf{x}_{[\cdot]}(b))$. Then $(\mathbf{x}_E, \mathbf{x}')$ and $(\mathbf{y}_E, \mathbf{y}')$ satisfy that there exists $i \in E$ such that $x_i < y_i$ or $r \in \mathbb{N}$ such that $x'_r < y'_r$. In either case, there exists $\varepsilon' > 0$ such that $(\mathbf{x}_E, \mathbf{x}') \not\prec_{[\cdot]}^{\varepsilon'} (\mathbf{y}_E, \mathbf{y}')$. Since $\prec_{[\cdot]}^{\varepsilon'} \in \mathcal{D}$, this implies that $(\mathbf{x}_E, \mathbf{x}') \not\prec^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y}')$. Hence, $(\mathbf{x}_E, \mathbf{x}) \not\prec^{\mathcal{D}} (\mathbf{y}_E, \mathbf{y})$, showing again that (i) does not hold. \square

5 Related literature

Our paper differs from most of the related literature in two ways:

- We do not assume that there is a natural 1-to-1 correspondence (or counterpart relation) between the wellbeings in any two states of the world.
- We do not assume equal treatment but derive such impartiality as a result.

Literature that presupposes a natural 1-to-1 correspondence can be divided in two classes. One considers 1-to-1 correspondences that derive from spatiotemporal positions. A recent example is

Jonsson (2021) who argues that the relevant 1-to-1 correspondence should be based on time of birth, in comparisons where one individual is born in each time period, thereby lending support to the very common procedure of aggregating the wellbeing of each generation before considering streams of generational wellbeing. If a natural 1-to-1 correspondence derives from spatiotemporal positions, then evaluation can be made according to spatiotemporal utilitarian averages or sums, based on to an expansionist view of infinite aggregation (Vallentyne and Kagan, 1997; Wilkinson, 2020). Within this class one can distinguish between literature which insists on completeness and numerical representation, and another which allows for incompleteness. The former subclass includes the recent contributions by Khan and Stinchcombe (2018) and Pivato (2020, 2021). This approach implies that there is no sensitivity for any finite set of spatiotemporal positions, thus contradicting the Pareto principle. The latter subclass, which includes evaluation based on catching-up and overtaking criteria, as well as other kinds of limiting procedures (see, e.g., Kamaga and Kojima, 2010; Jonsson and Voorneveld, 2018), restores the Pareto principle at the cost of introducing incomparability.

The other class considers 1-to-1 correspondences based on people. These 1-to-1 correspondences are preserved independently how people are counted, provided that people are counted in the same manner in each of the compared alternatives. Such invariance, which are referred to as *Isomorphism Invariance* (Lauwers and Vallentyne, 2004), *Strong Relative Anonymity* (Asheim, d’Aspremont and Banerjee, 2010), and the *Permutation Principle* (Askell, 2018), leads to even more incomparability (Wilkinson, 2020), but does not in itself contradict the Pareto principle. Isomorphism invariance rules out evaluation based on spatiotemporal utilitarian averages. Furthermore, generalized utilitarian (prioritarian) criteria that satisfy Isomorphism Invariance must be based on unconditional convergence; conditional convergence does not suffice. We discuss these issues in more detail in a companion paper (Asheim, Kamaga and Zuber, 2022).

All of these contributions assume the axiom of *Finite Anonymity*, implying that it does not matter in what order a *finite* subset of wellbeings are listed. Lauwers (1997) discusses whether Finite Anonymity is strong enough to properly capture equal treatment. Consequently, later contributions consider strengthenings of Finite Anonymity that do not contradict Strong Pareto. Zuber and Asheim (2012) and Asheim and Zuber (2013) go further and impose equal treatment in the form of Strong Anonymity, showing how this strong form of equal treatment is compatible with completeness and even numerical representation, while maintaining some sensitivity for single components. Also contributions that yield welfarism results, as Blackorby, Bossert and Donaldson (1999) in the multi-profile setting and Blackorby, Bossert and Donaldson (2006) in the single-profile setting, impose axioms of anonymity. Here, in contrast, we do not impose equal treatment in the form of an anonymity axiom as a primitive condition. Rather, equal treatment (in the form of Strong Anonymity) of future people and welfarism arise from considering evaluation that respects informational constraints, where these informational constraints are explicitly modeled.

6 Concluding remarks

We have presented a framework and analysis where future people are anonymous in a literal sense: we do not know who they are. Furthermore, when the axiom of Pareto Monotonicity is imposed under the assumptions of Richness and Unobservability of Future Identities, there is no limit to this literal anonymity, leading to Strong Anonymity whereby there is invariance also for permutations that reorder an infinite number of components. The kind of literal anonymity that arises from the assumption that we cannot observe the identities of future people is different from the procedural anonymity that stems from the normative principle that everybody should be treated equally. Indeed, in the present paper we have studied dominance that can be used to determine what outcomes are efficient without imposing such equal treatment as a normative principle. We are led to deviate from the Pareto principle toward a generalized Suppes-Sen principle precisely because we do not have the information needed in order to make comparisons identity-by-identity, and moreover, identities of future people are in any case likely to have a small overlap when comparing one outcome to another (Parfit, 1984, p. 361).

Since future people are strongly anonymous, the axiom of Pareto Monotonicity leads to a Dominance Definition that combines the Pareto principle for existing people with a generalized Suppes-Sen principle for future people, provided that three auxiliary axioms are invoked. The Dominance Definition implies that only people finitely ranked from the bottom matter. The asymmetry between the sensitivity for people finitely ranked from the bottom and the insensitivity for people finitely ranked from the top is a consequence of a weak equity axiom, Limited Inequity, stating that when making transfers from rich to poor, the poor's change in wellbeing must have positive relative weight. Also, the Dominance Definition entails that adding future people to an infinite population cannot have positive social value, as it does not matter if the added person's wellbeing is at infinite rank and lead to a strictly dominated outcome if the added person's wellbeing is a finite rank.¹² When interpreting this result it is important to realize that it is obtained in a setting where it cannot be observed if, in fact, additional future people are added to a population, as population size remains infinite and future identities are unobservable.

The result that future people are strongly anonymous imposes much indifference between wellbeing profiles for existing and future people. This changes the concept of dominance, as it removes sensitivity for wellbeing at infinite rank while strengthening sensitivity for wellbeing at finite rank by allowing for comparisons of rank-ordered stream. The fact that sensitivity is retained for people that are finitely ranked suggests that our Dominance Definition might be useful in economic environments where technology exhibits net positive productivity, thereby allowing for development over time with non-decreasing wellbeing. Indeed, as shown by Zuber and Asheim (2012, Section 6), there are wellbeing streams in economic growth models that are non-decreasing and, moreover, can be seen to be optimal under strongly anonymous criteria that go beyond

¹²See also Proposition 4 of Appendix A.5.

mere efficiency. Also, our Dominance Definition has a consequence in terms of sustainability for future people, in the sense that streams with decreasing wellbeing are inefficient if wellbeing in the near future can be reinvested into higher wellbeing in the distant future. Finally, our Dominance Definition has implications for population policy, in the sense that it does not permit people's level of wellbeing to be traded off for population growth, as long as the eventual total population remains infinite. In particular, a high rate of population growth combined with a low level of wellbeing cannot be efficient if lower growth leads to higher wellbeing, even if the low wellbeing level leads to acceptable lives. In this manner, the Dominance Definition suggests how the *Repugnant Conclusion* (Parfit, 1984, Chapter 17) might be easier to escape in the infinite population setting.

Applied to the evaluation of climate policies, the main implication is that one should prefer policies that increase the wellbeing of future people that are finitely ranked and avoid future lives with low wellbeing. In contrast, increasing the wellbeing of future people that infinitely ranked or adding future lives with high wellbeing will not have social value.

Since existing people are assumed to have fixed identities, our analysis does not imply that present people are treated equally among themselves, or in comparison with future people. Any vector of positive relative welfare weights on existing people as well as the finitely rank-ordered wellbeings of future people is consistent with the Dominance Definition. In particular, the definition permits that some present people are given higher priority than others—as exemplified by Negishi weights—and it allows for more weight on present wellbeing relative to future wellbeing—through time discounting. It does, however, require that future people with unobservable identities be treated equally among themselves, a requirement that follows from the axioms and the Richness assumption on which the analysis rests.

Thus, through our Justification Theorem we provide a new normative foundation for a kind of quasi-hyperbolic discounting (Frederick, Loewenstein and O'Donoghue, 2002) where future wellbeing is discounted relative to present wellbeing, but where only the derived rank-ordered stream of future wellbeings matters, not the timing of these future wellbeings. Therefore, as time moves forward and a future individual is born and thereby turned into an existing individual, the weight on this individual might increase—in particular if their lifetime wellbeing is high—leading to an interesting time-inconsistency problem. This time inconsistency can be avoided by imposing that existing and future people must be treated in an equal manner. However, whereas the present paper argues that equal treatment of future people derives from the Pareto principle under informational constraints, imposing equal treatment of existing and future people appears to require a normative foundation that goes beyond the notions of dominance and efficiency.

A Maximal sensitivity under Strong Anonymity

In Appendices A and B we consider infinite streams of future wellbeings: $\mathbf{x} = (x_1, x_2, \dots, x_j, \dots) \in \mathbb{R}^{\mathbb{N}}$, where $x_j \in \mathbb{R}$ is the wellbeing of component $j \in \mathbb{N}$. As before, we restrict our attention to the set \mathbf{X} of all bounded streams. To simplify notation, we here keep the wellbeings of existing people fixed, without being explicitly given. Hence, we will consider a preorder \succsim on \mathbf{X} without, as in the main text, consider the wellbeing profile for present people. As before, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \geq \mathbf{y}$ whenever $x_j \geq y_j$ for all $j \in \mathbb{N}$, and $\mathbf{x} > \mathbf{y}$ if $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.

For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and any $N \in \{0\} \cup \mathbb{N}$, let $(\mathbf{y}_N, \mathbf{x}) \in \mathbf{X}$ be defined by

$$(\mathbf{y}_N, \mathbf{x}) = \begin{cases} \mathbf{x} & \text{if } N = 0, \\ (y_1, y_2, \dots, y_N, \mathbf{x}) & \text{if } 0 < N < +\infty. \end{cases}$$

Furthermore, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and any $N = +\infty$, we write $\mathbf{z} = (\mathbf{y}, \mathbf{x}) = (\mathbf{y}_N, \mathbf{x})$ to mean $\mathbf{z} \in \mathbf{X}$ defined by

$$z_j = \begin{cases} y_n & \text{if } j = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_n & \text{if } j = 2n \text{ and } n \in \mathbb{N}, \end{cases}$$

that is, if $N = +\infty$, then $(\mathbf{y}_N, \mathbf{x}) = (\mathbf{y}, \mathbf{x}) = (y_1, x_1, y_2, x_2, \dots)$.¹³

A.1 Strong Anonymity does not rule out sensitivity

The *Maximin* order illustrates that Strong Anonymity is compatible with some sensitivity for the wellbeing at a single component of the stream. Maximin is represented by the inferior of wellbeing taken over all components. It is invariant to any reordering of the stream, thus satisfying Strong Anonymity. If there is a sole component indexed by the natural number j' with wellbeing that is at least ε (> 0) smaller than all other components, then the goodness of a stream is determined by component j' , making this component a rank-positional dictator. This also means that an increase in the wellbeing at component j' , keeping the wellbeing of all other components constant, makes the stream better. This is an example of *rank-positional dominance*, as the sensitivity depends on the rank position of the component whose wellbeing is increased: Maximin is sensitive to an increase in the wellbeing at a single component if and only if the wellbeing at this component is at least ε (> 0) smaller than the wellbeing at all other components. We will refer to this specific form of rank-positional dominance as **Inf-Restricted Dominance**.

The *Leximin* order constitutes a way of extending the sensitivity of Maximin. Leximin restores Strong Pareto in the setting of finite wellbeing streams, while extensions of Leximin to the infinite-stream setting that insist on satisfy Strong Pareto must necessarily contradict Strong Anonymity.

¹³These notational conventions illustrate Hilbert's paradox of the Grand Hotel (Hilbert, 2013, p. 730), one can augment an infinite population with a single individual, or infinitely many people, without increasing the population's total size—which is already infinite.

For example, in the infinite-stream versions of Leximin characterized by Asheim and Tungodden (2004) and Bossert, Sprumont and Suzumura (2007), Strong Pareto is maintained at the expense of weakening Strong Anonymity to Finite Anonymity. Hence, to define a version of Leximin in the infinite-stream setting that satisfies Strong Anonymity, Strong Pareto must be weakened. Asheim and Zuber (2013) do so by defining and characterizing an infinite-stream version of Leximin that has sensitivity for an increase in, not only the wellbeing of the worst-off, but the wellbeing at any component that is finitely ranked, in the sense that there is a cofinite set of people that have wellbeings that are at least ε (> 0) higher. This stronger form of rank-positional dominance, which we will refer to as **Liminf-Restricted Dominance**, is also satisfied by the Extended Rank-Discounted Utilitarian order defined and characterized by Zuber and Asheim (2012).

In this appendix we ask whether rank-positional dominance can be extended beyond Liminf-Restricted Dominance while insisting on Strong Anonymity. The answer is that Liminf-Restricted Dominance is as far as we can go under seemingly innocent auxiliary axioms.

A.2 Rank-positional dominance

Consider the following four rank-positional dominance axioms:

Axiom (Inf-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} \succ \mathbf{y}$ if $\inf_{j \in \mathbb{N}} x_j > y_{j'}$.

Axiom (Liminf-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} \succ \mathbf{y}$ if $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$.

Axiom (Sup-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} \succ \mathbf{y}$ if $x_{j'} > \sup_{j \in \mathbb{N}} y_j$.

Axiom (Limsup-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} \succ \mathbf{y}$ if $x_{j'} > \limsup_{j \in \mathbb{N}} y_j$.

Since $\liminf_{j \in \mathbb{N}} x_j \geq \inf_{j \in \mathbb{N}} x_j$, it follows that Liminf-Restricted Dominance implies Inf-Restricted Dominance, as it applies to at least as many pairs of \mathbf{x} and \mathbf{y} . For the same reason, as $\sup_{j \in \mathbb{N}} x_j \geq \limsup_{j \in \mathbb{N}} x_j$, we have that Limsup-Restricted Dominance implies Sup-Restricted Dominance. Furthermore, whenever \mathbf{x} and \mathbf{y} are non-decreasing streams, $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$ holds if $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Thus, Liminf-Restricted Dominance implies unrestricted dominance defined for non-decreasing streams (in the sense that $\mathbf{x} \succ \mathbf{y}$ for all non-decreasing \mathbf{x}, \mathbf{y} with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$). Similarly, Limsup-Restricted Dominance implies unrestricted dominance defined for non-increasing streams.

Consider the following four total preorders on \mathbf{X} .

Maximin: \succsim_M^+ represented by $W_M^+(\mathbf{x}) = \inf_{j \in \mathbb{N}} x_j = \inf_{\pi \in \Pi_{\mathbb{N}}} x_{\pi(1)}$.

Progressive Rank-Discounted (Generalized) Utilitarianism: \succsim_R^+ represented by

$$W_R^+(\mathbf{x}) = \inf_{\pi \in \Pi_{\mathbb{N}}} (1 - \beta) \sum_{j=1}^{\infty} \beta^{j-1} g(x_{\pi(j)}),$$

where $0 < \beta < 1$ and g is a continuous and increasing function.

Maximax: \succsim_M^- represented by $W_M^-(\mathbf{x}) = \sup_{j \in \mathbb{N}} x_j = \sup_{\pi \in \Pi_{\mathbb{N}}} x_{\pi(1)}$.

Regressive Rank-Discounted (Generalized) Utilitarianism: \succsim_R^- represented by

$$W_R^-(\mathbf{x}) = \sup_{\pi \in \Pi_{\mathbb{N}}} (1 - \beta) \sum_{j=1}^{\infty} \beta^{j-1} g(x_{\pi(j)}),$$

where $0 < \beta < 1$ and g is a continuous and increasing function.

Maximin satisfies Inf-Restricted Dominance, but not the other rank-positional dominance axioms, and Progressive Rank-Discounted Utilitarianism satisfies Liminf-Restricted Dominance—and thus also Inf-Restricted Dominance—but not the two other rank-positional dominance axioms. Likewise, Maximax satisfies Sup-Restricted Dominance, but not the other rank-positional dominance axioms, while Regressive Rank-Discounted Utilitarianism satisfies Limsup-restricted dominance—and thus also Sup-Restricted Dominance—but not the two other rank-positional dominance axioms. Progressive Rank-Discounted Utilitarianism coincides with the Extended Rank-Discounted Utilitarian order, defined (somewhat differently) in Zuber and Asheim (2012, Definition 2). Progressive and Regressive Rank-Discounted Utilitarianism can be combined to become \succsim_R^α represented by $W_R^\alpha(\mathbf{x}) = \alpha W_R^+(\mathbf{x}) + (1 - \alpha) W_R^-(\mathbf{x})$, where $0 < \alpha < 1$. This is related to Sider’s (1991) principle GV, but with the difference that the limit inferior and the limit superior take the place of zero wellbeing in Sider’s setting, being the wellbeing level just sufficient to make a life worth living.

A.3 Axioms

We insist on the axioms of Monotonicity and Strong Anonymity since these are implied by Pareto Monotonicity under the assumptions of the Welfarism Theorem. They are here restated when the wellbeings $\mathbf{x}_E \in \mathbb{R}^e$ of existing people are kept fixed, without being explicitly given.

Axiom (Monotonicity). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$, $\mathbf{x} \succsim \mathbf{y}$.

Axiom (Strong Anonymity). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x}_\pi = \mathbf{y}$ for some $\pi \in \Pi_{\mathbb{N}}$, $\mathbf{x} \sim \mathbf{y}$.

We note that the conjunction of Liminf-Restricted Dominance and Monotonicity implies sensitivity to an increase in the limit inferior, and the conjunction of Limsup-Restricted Dominance and Monotonicity implies sensitivity to an increase in the limit supremum.

Lemma 1. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$. Let \succsim be a preorder satisfying Monotonicity.

- (a) If \succsim also satisfies *Liminf-Restricted Dominance*, then $\mathbf{x} \succ \mathbf{y}$ whenever $\liminf_j x_j > \liminf_j y_j$,
- (b) If \succsim also satisfies *Limsup-Restricted Dominance*, then $\mathbf{x} \succ \mathbf{y}$ whenever $\limsup_j x_j > \limsup_j y_j$.

Proof. Part (a). Let $0 < \varepsilon < \liminf_j x_j - \liminf_j y_j$. By the definition of the limit inferior, there are infinitely many integers k such that $y_k < \liminf_j y_j + \varepsilon$ but only finitely many integers ℓ such that $x_\ell \leq \liminf_j y_j + \varepsilon < \liminf_j x_j$. So for infinitely many integers m , $x_m > \liminf_j y_j + \varepsilon > y_m$. Consider one of these integers, say i . Let \mathbf{z} be derived from \mathbf{x} by replacing x_i by y_i , where by the choice of ε and i , $\min\{x_i, \liminf_j x_j\} > y_i$. By Monotonicity, $\mathbf{z} \succsim \mathbf{y}$, and by Liminf-Restricted Dominance, $\mathbf{x} \succ \mathbf{z}$. Hence, by transitivity, $\mathbf{x} \succ \mathbf{y}$.

Part (b). The proof is similar. \square

We also impose the three auxiliary axioms of Section 4:

Axiom (Continuity). For any $\mathbf{x} \in \mathbf{X}$, the sets $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \succsim \mathbf{x}\}$ and $\{\mathbf{y} \in \mathbf{X} : \mathbf{x} \succsim \mathbf{y}\}$ are closed in the supnorm topology.

The supnorm topology is based on the distance function $d: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}_+$ which is defined by, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$d(\mathbf{x}, \mathbf{y}) = \sup_{j \in \mathbb{N}} |x_j - y_j|.$$

The supnorm topology is a rather large topology, but it makes it possible to prove our results in a straightforward manner. In Appendix B we show how we can establish our main results also under weaker continuity properties.

Axiom (Limited Inequity). $\mathcal{E}^{\succsim} \neq \emptyset$.

As we in the appendices keep the wellbeing of existing people fixed, without being explicitly given, \mathcal{E}^{\succsim} is here defined as:

$$\begin{aligned} \mathcal{E}^{\succsim} = \{ \varepsilon \in \mathbb{R}_{++} : \text{for any } \mathbf{x}, \mathbf{y} \in \mathbf{X} \text{ with } y_{j'} < x_{j'} \leq x_{j''} < y_{j''} \text{ for some } j', j'' \in \mathbb{N}, \\ \text{and } x_j = y_j \text{ for all } j \in \mathbb{N} \setminus \{j', j''\}, \mathbf{x} \succsim \mathbf{y} \text{ if } y_{j''} - x_{j''} = \varepsilon (x_{j'} - y_{j'}) \}. \end{aligned}$$

Axiom (Critical-Level Consistency). For any $\mathbf{x} \in \mathbf{X}$, and any $z \in \mathbb{R}$, $\mathbf{x} \succsim (z, \mathbf{x})$ (resp. $\mathbf{x} \precsim (z, \mathbf{x})$) if and only if $\mathbf{x} \succsim (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ (resp. $\mathbf{x} \precsim (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$).

All four preorders of subsection A.2 satisfy Monotonicity, Strong Anonymity, and Continuity, as well as Critical-Level Consistency. To establish the latter claim, note that:

- $\mathbf{x} \succ_M^+ (z, \mathbf{x})$ and $\mathbf{x} \succ_M^+ (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ if $z < \inf_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_M^+ (z, \mathbf{x}) \sim_M^+ (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ otherwise,
- $\mathbf{x} \succ_R^+ (z, \mathbf{x})$ and $\mathbf{x} \succ_R^+ (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ if $z < \liminf_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_R^+ (z, \mathbf{x}) \sim_R^+ (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ otherwise,
- $\mathbf{x} \prec_M^- (z, \mathbf{x})$ and $\mathbf{x} \prec_M^- (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ if $z > \sup_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_M^- (z, \mathbf{x}) \sim_M^- (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ otherwise,

- $\mathbf{x} \prec_R^-(z, \mathbf{x})$ and $\mathbf{x} \prec_R^-(z \mathbb{1}_\mathbb{N}, \mathbf{x})$ if $z > \limsup_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_R^-(z, \mathbf{x}) \sim_R^-(z \mathbb{1}_\mathbb{N}, \mathbf{x})$ otherwise.

However, of the four preorders, only Maximin and Progressive Rank-Discounted Utilitarianism satisfy Limited Inequity. To see this, note that that $\mathcal{E}^{\succ_M^+} = \mathbb{R}$ and $1 \in \mathcal{E}^{\succ_R^+}$ (under an assumption given by Zuber and Asheim, 2012, Proposition 6), while $\mathcal{E}^{\succ_M^-} = \mathcal{E}^{\succ_R^-} = \emptyset$ since both Maximax and Regressive Rank-Discounted Utilitarianism have sensitivity above the supremum, but no sensitivity below the limit supremum (so that wellbeing gains below the limit supremum are worthless and cannot compensate for a wellbeing loss for a maximal component).

A.4 Possibilities and impossibilities of rank-positional dominance

We first introduce and prove two lemmas before turning to the main results of this appendix. To introduce the first of these lemmas, recall the example of two unbounded streams used by Fleurbaey and Michel (2003, pp. 795–796) to prove that Strong Anonymity is incompatible with even the Weak Pareto axiom. A small variation of their example allows us to show the conflict between these two axioms also in our setting of bounded streams: There exists $\pi \in \Pi_\mathbb{N}$ such that

$$\mathbf{z} = \left(\frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots \right), \text{ and}$$

$$\mathbf{z}_\pi = \left(\frac{3}{4}, \frac{2}{3}, \frac{4}{5}, \frac{1}{3}, \frac{5}{6}, \frac{1}{4}, \dots, \frac{k+2}{k+3}, \frac{1}{k+1}, \dots \right),$$

where by Strong Anonymity \mathbf{z} is indifferent to \mathbf{z}_π even though $z_j < z_{\pi(j)}$ for all $j \in \mathbb{N}$. Notice that $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$, where \mathbf{z}^+ is an increasing subsequence

$$\mathbf{z}^+ = \left(\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots, \frac{k+1}{k+2}, \dots \right),$$

and \mathbf{z}^- is an decreasing subsequence

$$\mathbf{z}^- = \left(\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots, \frac{1}{k+2}, \dots \right).$$

Consider now the streams \mathbf{x} and \mathbf{y} , where

$$\mathbf{x} = \left(\frac{2}{3}, \mathbf{z} \right) = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots \right), \text{ and}$$

$$\mathbf{y} = \left(\frac{1}{3}, \mathbf{z} \right) = \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots \right),$$

Clearly, if \succsim satisfies Monotonicity, then $\mathbf{x} \succsim \mathbf{y}$, as the only difference between the two streams is that $x_1 = 2/3 > 1/3 = y_1$. However, the fact that $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$ with \mathbf{z}^+ being an increasing sequence with $x_1 = 2/3 = z_1^+$ and \mathbf{z}^- being an decreasing sequence with $y_1 = 1/3 = z_1^-$ implies that there exists $\pi'_\mathbb{N} \in \Pi$ such that $\mathbf{y}_{\pi'} \geq \mathbf{x}$:

$$\mathbf{y}_{\pi'} = \left(\frac{2}{3}, \frac{3}{4}, \frac{1}{3}, \frac{4}{5}, \frac{1}{3}, \frac{5}{6}, \frac{1}{4}, \dots, \frac{k+2}{k+3}, \frac{1}{k+1}, \dots \right).$$

So, if \succsim satisfies also Strong Anonymity, then $\mathbf{y} \sim \mathbf{y}_\pi \succsim \mathbf{x} \succsim \mathbf{y}$. Hence, by reflexivity and transitivity of \succsim , $\mathbf{x} \sim \mathbf{y}$, showing that \succsim cannot be sensitive to an increase of a component from 1/3 to 2/3 when the rest of the stream equals $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$. The first lemma generalizes this observation.

Lemma 2. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'} = z_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j = z_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. If \succsim is preorder satisfying Strong Anonymity and Monotonicity, then $\mathbf{x} \sim \mathbf{y}$ whenever $(z_j)_{j \in \mathbb{N} \setminus \{j'\}}$ satisfies*

- (a) *there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N} \setminus \{j'\}$ such that, for all $k \in \mathbb{N}$, $x_{j'} \leq z_{f(k)} \leq z_{f(k+1)}$, and*
- (b) *there exists an increasing function $g : \mathbb{N} \rightarrow \mathbb{N} \setminus \{j'\}$ such that, for all $k \in \mathbb{N}$, $y_{j'} \geq z_{g(k)} \geq z_{g(k+1)}$.*

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j = z_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Assume that \succsim is a reflexive and transitive binary relation satisfying Strong Anonymity and Continuity. Let $(z_j)_{j \in \mathbb{N} \setminus \{j'\}}$ satisfy (a) and (b).

Consider the permutation $\pi \in \Pi_{\mathbb{N}}$ constructed by

- $\pi(j') = f(1)$ and $\pi(f(k)) = f(k+1)$ for all $k \in \mathbb{N}$,
- $\pi(g(1)) = j'$ and $\pi(g(k+1)) = g(k)$ for all $k \in \mathbb{N}$,

and $\pi(j) = j$ otherwise. Then $\mathbf{y}_\pi \geq \mathbf{x} > \mathbf{y}$, so by Strong Anonymity and Monotonicity: $\mathbf{y} \sim \mathbf{y}_\pi \succsim \mathbf{x} \succsim \mathbf{y}$. Hence, by reflexivity and transitivity of \succsim , $\mathbf{x} \sim \mathbf{y}$. \square

In fact, Lemma 2 can be used to show that $\mathbf{x}' \sim \mathbf{y}'$, where

$$\begin{aligned} \mathbf{x}' &= (x'_1, \mathbf{z}) = \left(x'_1, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots \right), \text{ and} \\ \mathbf{y}' &= (y'_1, \mathbf{z}) = \left(y'_1, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots \right), \end{aligned}$$

whenever $0 = \liminf_{j \in \mathbb{N}} x'_j < y'_1 < x'_1 < \limsup_{j \in \mathbb{N}} y'_j = 1$ since subsequences as specified in (a) and (b) exist. Two such subsequences do not exist if $0 = y'_1 < x'_1 \leq 1$ or $0 \leq y'_1 < x'_1 = 1$, so that Lemma 2 does not apply under these cases. However, the non-sensitivity result of Lemma 2 can be extended to cover also such circumstances if Continuity is imposed. This extension relies on the following lemma, showing that any preorder satisfying Strong Anonymity and Continuity is invariant to adding one individual (or infinitely many people) with wellbeing equal to the limit inferior or the limit supremum, or indeed equal to any other cluster point.

Lemma 3. *Consider $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for \mathbf{x} . If \succsim is a preorder satisfying Strong Anonymity and Continuity, then $\mathbf{x} \sim (z, \mathbf{x}) \sim (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$.*

Proof. Let $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for \mathbf{x} . Assume that \succsim is a preorder satisfying Strong Anonymity and Continuity. We prove that $\mathbf{x} \sim (z, \mathbf{x})$ and $\mathbf{x} \sim (z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$.

Since z is a cluster point of \mathbf{x} , there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $\lim_{j \rightarrow \infty} |x_{f(j)} - z| = 0$. Hence, for every $m \in \mathbb{N}$, there exists $N(m) \in \mathbb{N}$ such that $|x_{f(j)} - z| < 1/(2m)$ for all $j > N(m)$. For each $m \in \mathbb{N}$, define the increasing function $f^m: \mathbb{N} \rightarrow \mathbb{N}$ by $f^m(j) = f(j + N(m))$ for all $j \in \mathbb{N}$. By construction, for every $m \in \mathbb{N}$ and for all $j, j' \in \mathbb{N}$,

$$|x_{f^m(j)} - x_{f^m(j')}| \leq |x_{f^m(j)} - z| + |z - x_{f^m(j')}| < \frac{1}{2m} + \frac{1}{2m} = \frac{1}{m}. \quad (\text{A.1})$$

For all $m \in \mathbb{N}$, let $\tilde{\mathbf{x}}^m = (x_{f^m(k)})_{k \in \mathbb{N}}$ and we define sequences $(\hat{\mathbf{x}}^m)_{m \in \mathbb{N}}$, $(\check{\mathbf{x}}^m)_{m \in \mathbb{N}}$ in \mathbf{X} as follows. For all $m \in \mathbb{N}$,

$$\hat{x}_j^m = \check{x}_j^m = x_j \quad \text{for all } j \in \mathbb{N} \setminus \{f^m(k) : k \in \mathbb{N}\}$$

and

$$(\hat{x}_{f^m(k)}^m)_{k \in \mathbb{N}} = (z, \tilde{\mathbf{x}}^m) \quad \text{and} \quad (\check{x}_{f^m(k)}^m)_{k \in \mathbb{N}} = (z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}^m).$$

From (A.1) and the construction of $(\hat{\mathbf{x}}^m)_{m \in \mathbb{N}}$ and $(\check{\mathbf{x}}^m)_{m \in \mathbb{N}}$, it follows that

$$\lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |x_j - \hat{x}_j^m| = \lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |\tilde{x}_j^m - \hat{x}_{f^m(j)}^m| = 0, \quad (\text{A.2a})$$

$$\lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |x_j - \check{x}_j^m| = \lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |\tilde{x}_j^m - \check{x}_{f^m(j)}^m| = 0. \quad (\text{A.2b})$$

Note that for each $m \in \mathbb{N}$, there exist $\pi^m, \rho^m \in \Pi$ such that

$$(z, \mathbf{x}) = \hat{\mathbf{x}}_{\pi^m}^m \quad \text{and} \quad (z\mathbb{1}_{\mathbb{N}}, \mathbf{x}) = \check{\mathbf{x}}_{\rho^m}^m.$$

By Strong Anonymity, we obtain $(z, \mathbf{x}) \sim \hat{\mathbf{x}}^m$ and $(z\mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \check{\mathbf{x}}^m$ for all $m \in \mathbb{N}$. Since \succsim satisfies Continuity, it follows from (A.2a) and (A.2b) that $(z, \mathbf{x}) \sim \mathbf{x}$ and $(z\mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$. \square

Lemma 3 can also be seen as a generalization of the observation that, under Strong Anonymity, the streams $(1, 0, 1, 0, 1, 0, \dots)$ and $(0, 1, 0, 1, 0, 1, \dots)$ are equally good, as one stream can be obtained from the other through an infinite permutation. However, the second stream can also be obtained from the first by adding one individual with wellbeing equal to the limit inferior and the first stream can be obtained from the second by adding an individual with wellbeing equal to the limit supremum.

Using our two lemmas, we are able to prove three results regarding rank-positional dominance. The first proposition shows that any reflexive and transitive binary relation satisfying Monotonicity, Strong Anonymity, and Continuity is insensitive to increasing the wellbeing at a particular component of the stream between the limit inferior and the limit supremum.

Proposition 1. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. The two following statements are equivalent:

- (1) For all preorders \succsim satisfying Monotonicity, Strong Anonymity, and Continuity, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $\limsup_{j \in \mathbb{N}} y_j \geq x_{j'}$ and $y_{j'} \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$.

We first that (1) implies (2). Suppose that (2) does not hold. Then there are two subcases:

Subcase (i): $x_{j'} > \limsup_{j \in \mathbb{N}} y_j$. Consider \succsim_R^- . We have that \succsim_R^- is a preorder that satisfies Monotonicity, Strong Anonymity, and Continuity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^- satisfies Limsup-Restricted Dominance.

Subcase (ii): $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$. Consider \succsim_R^+ . We have that \succsim_R^+ is a preorder that satisfies Monotonicity, Strong Anonymity, and Continuity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^+ satisfies Liminf-Restricted Dominance.

Next, we show that (2) implies (1). Assume that (2) holds, and assume that \succsim is a preorder that satisfies Monotonicity, Strong Anonymity, and Continuity.

Both $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$ and $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ are cluster points for \mathbf{x} and \mathbf{y} , so by repeated use of Lemma 3 we have that

$$\mathbf{x} \sim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (u \mathbb{1}_{\mathbb{N}}, (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})) \text{ and } \mathbf{y} \sim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \sim (u \mathbb{1}_{\mathbb{N}}, (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})).$$

Write $\mathbf{x}' = (u \mathbb{1}_{\mathbb{N}}, (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}))$ and $\mathbf{y}' = (u \mathbb{1}_{\mathbb{N}}, (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}))$. By construction, $x'_{j''} > y'_{j''}$ for some $j'' \in \mathbb{N}$ and $x'_j = y'_j = z_j$ for all $j \in \mathbb{N} \setminus \{j''\}$. Furthermore, $(z_j)_{j \in \mathbb{N} \setminus \{j''\}}$ satisfies

- (a) there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N} \setminus \{j''\}$ such that, for all $k \in \mathbb{N}$, $x'_{j''} \leq z_{f(k)} = u$, and
- (b) there exists an increasing function $g : \mathbb{N} \rightarrow \mathbb{N} \setminus \{j''\}$ such that, for all $k \in \mathbb{N}$, $y'_{j''} \geq z_{g(k)} = \ell$.

By Lemma 2, $\mathbf{x} \sim \mathbf{x}' \sim \mathbf{y}' \sim \mathbf{y}$. Thus, $\mathbf{x} \sim \mathbf{y}$ by the transitivity of \succsim . \square

Before stating our two last results, we prove the following lemma which goes one step further by adding the Limited Inequity axiom. It shows that a binary relation that satisfies also this axiom is invariant to increasing the wellbeing at a particular component of the stream that is already at the limit inferior or above, in the case where the stream under consideration has a limit inferior different from its limit supremum.

Lemma 4. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Assume that \succsim is a preorder \succsim satisfying Monotonicity, Strong Anonymity, Continuity, and Limited Inequity. Then $\mathbf{x} \sim \mathbf{y}$ if $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$ and $y_{j'} \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Assume that \succsim is a preorder that satisfies Monotonicity, Strong Anonymity, Continuity, and Limited Inequity. Let $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$ and $y_{j'} \geq \liminf_{j \in \mathbb{N}} x_j$.

The result follows from Proposition 1 if $\limsup_j y_j \geq x_{j'}$.

Now, suppose that $x_{j'} > \limsup_{j \in \mathbb{N}} y_j$. By Monotonicity, $\mathbf{x} \succsim \mathbf{y}$. We need to establish that $\mathbf{y} \succsim \mathbf{x}$. Write $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$, $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $\delta = \varepsilon(u - \ell)$, where $\varepsilon > 0$ is chosen such that $\varepsilon \in \mathcal{E}^{\succsim}$; this is possible since by Limited Inequity $\mathcal{E}^{\succsim} \neq \emptyset$.

Assume that there exists $k \in \mathbb{N} \setminus \{j'\}$ such that $u \geq x_k = y_k \geq \ell$. Define \mathbf{y}^0 by $y_{j'}^0 = \max\{y_{j'}, u\}$, $y_k^0 = u$ and $y_j^0 = y_j$ for $j \in \mathbb{N} \setminus \{j', k\}$. Define inductively \mathbf{x}^n by $x_{j'}^n = y_{j'}^{n-1} + \delta$ and $x_k^n = \ell$ and $x_j^n = y_j^{n-1}$ for $j \in \mathbb{N} \setminus \{j', k\}$, and \mathbf{y}^n by $y_k^n = u$ and $y_j^n = x_j^n$ for $j \in \mathbb{N} \setminus \{k\}$ for $n = 1, 2, 3, \dots, \bar{n}$ until $\mathbf{y}^{\bar{n}} \geq \mathbf{x}$. Then:

$$\mathbf{y} \sim \mathbf{y}^0 \succsim \mathbf{x}^1 \sim \mathbf{y}^1 \succsim \dots \succsim \mathbf{x}^{\bar{n}} \sim \mathbf{y}^{\bar{n}} \succsim \mathbf{x},$$

where $\mathbf{y} \sim \mathbf{y}^0$ and $\mathbf{x}^{\bar{n}} \sim \mathbf{y}^{\bar{n}}$ follow from Proposition 1, $\mathbf{y}^{n-1} \succsim \mathbf{x}^n$ for $n = 1, 2, 3, \dots, \bar{n}$ follow from Limited Inequity, and $\mathbf{y}^{\bar{n}} \succsim \mathbf{x}$ follows from Monotonicity.

If there does not exist $k \in \mathbb{N} \setminus \{j'\}$ such that $u \geq x_k = y_k \geq \ell$, it follows from Lemma 3 that $\mathbf{x} \sim \mathbf{y}$ if and only if $(u, \mathbf{x}) \sim (u, \mathbf{y})$ since $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$ is a cluster point for \mathbf{x} and \mathbf{y} . Hence, we can use the argument above on the pair $((u, \mathbf{x}), (u, \mathbf{y}))$ instead of (\mathbf{x}, \mathbf{y}) . \square

Lemma 4 implies that Limsup-Restricted Dominance is inconsistent with the four axioms of the lemma because such dominance cannot be imposed on streams where the limit inferior is strictly lower than the limit supremum. This almost establishes the result that any preorder satisfying these four axioms cannot exhibit any sensitivity beyond Liminf-Restricted Dominance. To show this result we must remove the restriction made in Lemma 4 that the streams under consideration must have a limit inferior strictly lower than the limit supremum. The next proposition shows that it is true in general that if a preorder satisfies Monotonicity, Strong Anonymity, Continuity, and Limited Inequity, then it cannot exhibit sensitivity beyond Liminf-Restricted Dominance. Moreover, this result is tight since the converse follows from the fact that Progressive Rank-Discounted Utilitarianism satisfies these axioms as well as Liminf-Restricted Dominance.

Proposition 2. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. The two following statements are equivalent:

- (1) For all preorders \succsim satisfying Monotonicity, Strong Anonymity, Continuity, and Limited Inequity, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $y_i \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$.

We first show that (1) implies (2). Suppose that (2) does not hold, implying that $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$. Consider \succsim_R^+ with u being linear. We have that \succsim_R^+ is a reflexive and transitive binary relation that satisfies Monotonicity, Strong Anonymity, Continuity, and Limited Inequity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^+ satisfies Liminf-Restricted Dominance.

Next, we show that (2) implies (1). Assume that (2) holds, and assume that \succsim is a preorder that satisfies Monotonicity, Strong Anonymity, Continuity, and Limited Inequity. There are two subcases:

Subcase (i): $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$. The result follows from Lemma 4.

Subcase (ii): $\limsup_{j \in \mathbb{N}} y_j = \liminf_{j \in \mathbb{N}} x_j = \ell$. Since ℓ is a cluster point of \mathbf{x} and \mathbf{y} , it follows from Lemma 3 that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \sim \mathbf{y}$. Let $(\varepsilon_n)_{n \in \mathbb{N}}$ be a sequence of positive real numbers converging to 0. By Lemma 4, $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim ((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for each $n \in \mathbb{N}$. Furthermore, $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{x}) > (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ so that $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \succ (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ by Monotonicity. By transitivity, $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succ \mathbf{x}$. This is true for each $n \in \mathbb{N}$ so that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succ \mathbf{x}$ by Continuity and $\mathbf{y} \succ \mathbf{x}$ by transitivity. On the other hand, $\mathbf{x} \succ \mathbf{y}$ by Monotonicity. \square

Proposition 2 shows that dominance can occur under Monotonicity, Strong Anonymity, Continuity, and Limited Inequity, when increasing the wellbeing at a particular component of the stream, if and only if the wellbeing at this component is strictly lower than the limit inferior before the increase. Any preorder satisfying these axioms is invariant to an increase in the wellbeing at a particular component of the stream at or above the limit inferior. Notice that Regressive Rank-Discounted Utilitarianism satisfy Limsup-Restricted Dominance and all the other axioms except for Limited Inequity. Our minimal equity requirement is essential for obtaining the result. On the other hand, Inf-Restricted Dominance and Liminf-Restricted Dominance are clearly compatible with the other axioms: both Maximin and Progressive Rank-Discounted Utilitarianism satisfy Monotonicity, Strong Anonymity, Continuity, and Limited Inequity.

Our last proposition in this section shows that we can generalize the result to the case where not only one component of the stream, but infinitely many components experience an increase in wellbeing above the limit inferior. To obtain this result, we need the additional axiom of Critical-Level Consistency.

Proposition 3. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} > \mathbf{y}$. The two following statements are equivalent:*

- (1) *For all preorders \succsim satisfying Monotonicity, Strong Anonymity, Continuity, Limited Inequity, and Critical-Level Consistency, we have that $\mathbf{x} \sim \mathbf{y}$.*
- (2) *$\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $y_{j'} \geq \ell$ for all $j' \in \mathbb{N}$ such that $x_{j'} > y_{j'}$.*

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $\mathbf{x} > \mathbf{y}$.

We first show that (1) implies (2). Suppose that (2) does not hold, so that (i) $\liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and there exists $j' \in \mathbb{N}$ such that $\min\{x_{j'}, \liminf_{j \in \mathbb{N}} x_j\} > y_{j'}$, or (ii) $\liminf_{j \in \mathbb{N}} x_j > \liminf_{j \in \mathbb{N}} y_j$. Consider \succsim_R^+ with u being linear. We have that \succsim_R^+ is a preorder that satisfies

Monotonicity, Strong Anonymity, Continuity, Limited Inequity, and Critical-Level Consistency. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since \succsim_R^+ satisfies Monotonicity and Liminf-Restricted Dominance (Lemma 1(a), case (ii)).

Next, we show that (2) implies (1). Assume that (2) holds, and assume that \succsim is a preorder that satisfies Monotonicity, Strong Anonymity Continuity, Limited Inequity, and Critical-Level Consistency. Denote $I_{>} = \{i \in \mathbb{N} : x_i > y_i\}$ the set of coordinates where utility is strictly larger in \mathbf{x} than in \mathbf{y} . There are two subcases:

Subcase (i): $|I_{>}| < +\infty$. In this case, the result follows from repeated applications of Proposition 2 and transitivity.

Subcase (ii): $|I_{>}| = +\infty$. Since ℓ is a cluster point, by Lemma 3 we obtain $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \sim \mathbf{y}$. Let \mathbf{z} and $\hat{\mathbf{z}}$ be two streams such that:

$$z_i = \begin{cases} \ell & \text{if } i = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_i & \text{if } i = 2n \text{ and } n \in \mathbb{N} \setminus I_{>}, \\ \sup_{j \in \mathbb{N}} x_j & \text{if } i = 2n \text{ and } n \in I_{>}, \end{cases}$$

and

$$\hat{z}_i = \begin{cases} \ell & \text{if } i = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_i & \text{if } i = 2n \text{ and } n \in \mathbb{N} \setminus I_{>}, \\ \ell & \text{if } i = 2n \text{ and } n \in I_{>}. \end{cases}$$

By definition, $\mathbf{z} \geq (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \geq \hat{\mathbf{z}}$, so that by Monotonicity $\mathbf{z} \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succsim \hat{\mathbf{z}}$. Now we want to show that $\mathbf{z} \sim \hat{\mathbf{z}}$ to obtain by transitivity that $(\ell \mathbb{1}_{\mathbb{N}} \mathbf{y}) \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and therefore $\mathbf{y} \succsim \mathbf{x}$.

Let $J = \{i \in \mathbb{N} : \exists k \in I_{>}, i = 2k\}$. In streams $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$, $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$, \mathbf{z} and $\hat{\mathbf{z}}$, coordinates in J are those where the streams differ. Let f be the unique increasing bijection between \mathbb{N} and $\mathbb{N} \setminus J$. Define $\tilde{\mathbf{z}}$ by $\tilde{z}_n = z_{f(n)}$ for all $n \in \mathbb{N}$: stream $\tilde{\mathbf{z}}$ collects all coordinates that are the same in $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$, $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$, \mathbf{z} and $\hat{\mathbf{z}}$. Let $\bar{x} = \sup_{j \in \mathbb{N}} x_j$, by Proposition 2, $(\bar{x}, \tilde{\mathbf{z}}) \sim (\ell, \tilde{\mathbf{z}})$. By Critical-Level Consistency, $(\bar{x}, \tilde{\mathbf{z}}) \sim (\bar{x} \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$ and $(\ell, \tilde{\mathbf{z}}) \sim (\ell \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$. Clearly, \mathbf{z} can be obtained from $(\bar{x} \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$ and $\hat{\mathbf{z}}$ can be obtained from $(\ell \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$ through a permutation. Therefore, by Strong Anonymity and transitivity, $\mathbf{z} \sim \hat{\mathbf{z}}$.

We thus know that $\mathbf{y} \succsim \mathbf{x}$. But $\mathbf{x} \geq \mathbf{y}$ so that by Monotonicity $\mathbf{x} \succsim \mathbf{y}$. □

An implication of Proposition 3 is that even an increase in utility at an infinite number of components may not be sufficient to guarantee social dominance. For instance, Proposition 3 implies that the following two streams are equivalent:

$$(1, 0, 1, 0, \dots, 1, 0, \dots) \quad \text{and} \quad (0, 0, 0, 0, \dots).$$

As mentioned in the introduction to this section, Fleurbaey and Michel (2003) proved that Strong Anonymity is incompatible with the Weak Pareto axiom using an example. Proposition 3 provides a larger set of cases where the Weak Pareto axiom fails when our other axioms are satisfied: all cases where there is a strict improvement above the limit inferior without changing the limit inferior. For instance, the following two streams are equally good:

$$(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{k}, \dots) \quad \text{and} \quad (0, 0, 0, 0, \dots).$$

Remark. Our analysis builds on the assumption that wellbeing is real-valued. If instead wellbeing is a binary variable, being equal to 0 or 1, then clearly Continuity and Limited Inequity have no bite. In this case, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} > \mathbf{y}$, the two following statements are equivalent:

- (1) For all preorders \succsim satisfying Monotonicity, Strong Anonymity, and Critical-Level Consistency, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $\limsup_{j \in \mathbb{N}} y_j = 1$ and $\liminf_{j \in \mathbb{N}} x_j = 0$.

Hence, there is no sensitivity as long as both \mathbf{x} and \mathbf{y} have infinitely many elements at both 0 and 1. If not, \succsim_T^α , the combined Rank-Discounted Utilitarianism defined in subsection A.2, shows that sensitivity is consistent with Monotonicity, Strong Anonymity, and Critical-Level Consistency.

A.5 Infinite population ethics under strong anonymity

As we have discussed before, the literature on population ethics considers the effect of adding an individual, or several people, to a population. Parfit (1984) has introduced the Mere Addition Principle: the addition of someone with a non-negative wellbeing should always be acceptable. The problem is that, under mild conditions, this principle may yield the ‘repugnant conclusion’ by which a very large population of people with lives barely worth living may be better than a large but smaller population of people with excellent lives. To avoid this conclusion, Blackorby and Donaldson (1984) and Blackorby, Bossert and Donaldson (1995) have proposed Critical-Level Utilitarianism, according to which adding only sufficiently good lives are socially acceptable.

In any case, most approaches assume that there exist levels of wellbeing such that adding someone with a wellbeing at these levels is a strict social improvement. However, this assumption is made in the setting of a finite population. In contrast, the following property is concerned with the addition of finitely or infinitely many people to a population that already has infinitely many people. It asserts that the social value of adding finitely or infinitely many lives is non-positive regardless of their wellbeing. Note that, in the statement below, the stream \mathbf{x} can be viewed as being obtained from stream \mathbf{y} by “removing” some people.

Axiom (Non-Positive Value of Additional Lives). *For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $x_j = y_{f(j)}$ for all $j \in \mathbb{N}$, then $\mathbf{x} \succsim \mathbf{y}$.*

We obtain the surprising result that this population-ethics property is a result of the axioms that we have already imposed.

Proposition 4. *Assume that \succsim is a preorder satisfying Monotonicity, Strong Anonymity, Continuity, Limited Inequity, and Critical-Level Consistency. Then \succsim satisfies Non-Positive Value of Additional Lives.*

Proof. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ such that there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $x_j = y_{f(j)}$ for all $j \in \mathbb{N}$. Assume that \succsim is a preorder satisfying Monotonicity, Strong Anonymity, Continuity, Limited Inequity, and Critical-Level Consistency.

Write $\ell(\mathbf{x}) = \liminf_{j \in \mathbb{N}} x_j$ and $\ell(\mathbf{y}) = \liminf_{j \in \mathbb{N}} y_j$, and let $z = \sup_{j \in \mathbb{N}} y_j$. By the relationship between \mathbf{x} and \mathbf{y} , $z \geq \ell(\mathbf{x}) \geq \ell(\mathbf{y})$. Write $\tilde{\mathbf{x}} = (\ell(\mathbf{x})\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and $\tilde{\mathbf{y}} = (\ell(\mathbf{y})\mathbb{1}_{\mathbb{N}}, \mathbf{y})$. By Strong Anonymity, adding an additional individual with wellbeing z to $\tilde{\mathbf{x}}$ is equivalent to increasing the wellbeing of an existing individual from $\ell(\mathbf{x})$ to z . So by Proposition 2, $(z, \tilde{\mathbf{x}}) \sim \tilde{\mathbf{x}}$. Thus, by Critical-Level Consistency, $(z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \sim \tilde{\mathbf{x}}$. By the definition of z , there exists $\pi \in \Pi$ such that $(z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \geq \tilde{\mathbf{y}}_{\pi}$, so by Monotonicity, $(z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \succsim \tilde{\mathbf{y}}_{\pi}$. By Strong Anonymity, $\tilde{\mathbf{y}}_{\pi} \sim \tilde{\mathbf{y}}$, while by Lemma 3, $\mathbf{x} \sim \tilde{\mathbf{x}}$ and $\mathbf{y} \sim \tilde{\mathbf{y}}$. Hence,

$$\mathbf{x} \sim \tilde{\mathbf{x}} \sim (z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \succsim \tilde{\mathbf{y}}_{\pi} \sim \tilde{\mathbf{y}} \sim \mathbf{y},$$

which by the transitivity of \succsim implies that \succsim satisfies Non-Positive Value of Additional Lives. \square

One can even be more specific about cases where adding an individual is a matter of social indifference or has negative social value. By Lemma 3 and Proposition 3, adding one individual—or finitely or infinitely many people—at the limit inferior or above is a matter of social indifference. Indeed, adding any number of people at the limit inferior is socially indifferent by Lemma 3. Then increasing their wellbeing is also socially indifferent by Proposition 3.

On the other hand, if a preorder \succsim satisfies Liminf-Restricted Dominance, then adding at least one individual below the limit inferior has negative social value. Indeed, like before, adding any number of people at the limit inferior is socially indifferent. Then decreasing the wellbeing of one individual from the limit inferior to a strictly lower value has negative social value by Liminf-Restricted Dominance.¹⁴

¹⁴Hence, when Liminf-Restricted Dominance is added to the axioms of Proposition 4, the *Adding Goodness Principle* of Hamkins and Montero (2000) is contradicted.

B Weakening continuity.

In this appendix we show that the main results of Appendix A, i.e., Propositions 3 and 4, can be strengthened by replacing Continuity by weaker continuity axioms. To this end, we prove stronger variants of Lemma 3 and Proposition 2 with these weaker continuity axioms.

Let $d_1: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}_+$ be the distance function given by, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$d_1(\mathbf{x}, \mathbf{y}) = \min \left\{ 1, \sum_{i=1}^{\infty} |x_i - y_i| \right\}.$$

Using the distance function d_1 , we define weak continuity as follows; see Svensson (1980).

Axiom (Weak Continuity). For any $\mathbf{x} \in \mathbf{X}$, the sets $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \succsim \mathbf{x}\}$ and $\{\mathbf{y} \in \mathbf{X} : \mathbf{x} \succsim \mathbf{y}\}$ are closed in (\mathbf{X}, d_1) .

Clearly, Weak Continuity is implied by Continuity.

The following lemma shows that Lemma 3 can be strengthened by using Weak Continuity instead of Continuity.

Lemma 5. Consider $\mathbf{x} \in \mathbf{X}$ and z be a cluster point of \mathbf{x} . If \succsim is a preorder satisfying Strong Anonymity and Weak Continuity, then $\mathbf{x} \sim (z, \mathbf{x}) \sim (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$.

Proof. Let $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for \mathbf{x} . Assume that \succsim is a preorder satisfying Strong Anonymity and Weak Continuity. We show that $\mathbf{x} \sim (z, \mathbf{x})$ and $\mathbf{x} \sim (z \mathbb{1}_{\mathbb{N}}, \mathbf{x})$.

Since z is a cluster point of \mathbf{x} , there exists a subsequence of \mathbf{x} that converges to z . Furthermore, it is well-known that any sequence of real numbers has a monotone subsequence and that every subsequence of a convergent sequence has the same limit. Therefore, there exists a monotone subsequence of \mathbf{x} that converges to z , that is, there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $(x_{f(k)})_{k \in \mathbb{N}} \in \mathbf{X}$ is monotone and converges to z . Without loss of generality, we assume that $(x_{f(k)})_{k \in \mathbb{N}}$ is non-decreasing.

Let $\tilde{\mathbf{x}} = (x_{f(k)})_{k \in \mathbb{N}}$ and, for any $n \in \mathbb{N}$, define $\tilde{\mathbf{x}}(n)$ by

$$\tilde{\mathbf{x}}(n) = (\tilde{x}_1, \dots, \tilde{x}_{n-1}, z, \tilde{x}_n, \tilde{x}_{n+1}, \dots).$$

Then, we obtain that

$$\begin{aligned} \lim_{n \rightarrow +\infty} \sum_{j=1}^{\infty} |\tilde{x}_j - \tilde{x}(n)_j| &= \lim_{n \rightarrow +\infty} \left(z - \tilde{x}_n + \lim_{N \rightarrow +\infty} \sum_{j=n+1}^N |\tilde{x}_j - \tilde{x}(n)_j| \right) \\ &= \lim_{n \rightarrow +\infty} \left(z - \tilde{x}_n + \lim_{N \rightarrow +\infty} (\tilde{x}_N - \tilde{x}_n) \right) = \lim_{n \rightarrow +\infty} 2(z - \tilde{x}_n) = 0. \end{aligned}$$

Thus, for any $\varepsilon \in (0, 1)$, there exists an increasing sequence $\{n_t\}_{t \in \mathbb{N}}$ in \mathbb{N} satisfying

$$n_t + t < n_{t+1} \quad \text{for each } t \in \mathbb{N}$$

and we can define the sequence $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ in \mathbf{X} that satisfies

$$\sum_{j=1}^{\infty} |\tilde{x}_j^n - \tilde{x}_j^{n-1}| < \frac{\varepsilon}{2^n} \quad \text{for each } n \in \mathbb{N}$$

as follows:

$$\begin{aligned} \tilde{\mathbf{x}}^0 &= (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, \tilde{x}_{n_1}, \tilde{x}_{n_1+1}, \dots, \tilde{x}_{n_2-1}, \tilde{x}_{n_2}, \tilde{x}_{n_2+1}, \dots), \\ \tilde{\mathbf{x}}^1 &= (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, z, \tilde{x}_{n_1}, \dots, \tilde{x}_{n_2-2}, \tilde{x}_{n_2-1}, \tilde{x}_{n_2}, \dots), \\ \tilde{\mathbf{x}}^2 &= (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, z, \tilde{x}_{n_1}, \dots, \tilde{x}_{n_2-2}, z, \tilde{x}_{n_2-1}, \dots), \end{aligned}$$

and so forth. Formally, $\tilde{\mathbf{x}}^0 = \tilde{\mathbf{x}}$ and for each $n \in \mathbb{N}$, $\tilde{\mathbf{x}}^n$ is defined as follows. For each $j \in \{n_t : t \in \{1, \dots, n\}\}$,

$$\tilde{x}_j^n = z$$

and the subsequence of $\tilde{\mathbf{x}}^n$ composed of all the other components coincides with $\tilde{\mathbf{x}}$. Analogously, we define $\tilde{\mathbf{x}}^\infty$ by, for each $j \in \{n_t : t \in \mathbb{N}\}$,

$$\tilde{x}_j^\infty = z$$

and the subsequence of $\tilde{\mathbf{x}}^\infty$ composed of all the other components coincides with $\tilde{\mathbf{x}}$. By the definitions of $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ and $\tilde{\mathbf{x}}^\infty$, we obtain that

$$\sum_{j=1}^{\infty} |\tilde{x}_j^\infty - \tilde{x}_j^0| \leq \lim_{N \rightarrow +\infty} \sum_{n=1}^N \sum_{j=1}^{\infty} |\tilde{x}_j^n - \tilde{x}_j^{n-1}| < \varepsilon. \quad (\text{B.1})$$

We now define the sequence $\{\mathbf{x}^n\}_{n \in \mathbb{N} \cup \{0\}}$ using the sequence $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ as follows. For each $n \in \mathbb{N} \cup \{0\}$, $x_i^n = x_j$ for all $j \in \mathbb{N} \setminus \{f(k) : k \in \mathbb{N}\}$ and $(x_{f(k)}^n)_{k \in \mathbb{N}} = \tilde{\mathbf{x}}^n$. Analogously, we define \mathbf{x}^∞ by $x_j^\infty = x_i$ for all $j \in \mathbb{N} \setminus \{f(k) : k \in \mathbb{N}\}$ and $(x_{f(k)}^\infty)_{k \in \mathbb{N}} = \tilde{\mathbf{x}}^\infty$. Note that $\mathbf{x}^0 = \mathbf{x}$. From (B.1) and the definitions of $\{\mathbf{x}^n\}_{n \in \mathbb{N} \cup \{0\}}$ and \mathbf{x}^∞ , it follows that

$$\sum_{j=1}^{\infty} |x_j^1 - x_j^0| = \sum_{i=1}^{\infty} |\tilde{x}_i^1 - \tilde{x}_i^0| \leq \sum_{j=1}^{\infty} |\tilde{x}_j^\infty - \tilde{x}_j^0| = \sum_{j=1}^{\infty} |x_j^\infty - x_j^0| < \varepsilon. \quad (\text{B.2})$$

Note that there exist permutations $\pi, \rho \in \Pi$ such that $\mathbf{x}^1 = (z, \mathbf{x})_\pi$ and $\mathbf{x}^\infty = ((z)_{\text{con}}, \mathbf{x})_\rho$. Thus,

it follows from (B.2) that for any $\varepsilon \in (0, 1)$, there exist $\pi, \rho \in \Pi$ such that

$$d_1((z, \mathbf{x})_\pi, \mathbf{x}) \leq d_1((z \mathbb{1}_\mathbb{N}, \mathbf{x})_\rho, \mathbf{x}) < \varepsilon. \quad (\text{B.3})$$

Let $m \in \mathbb{N}$. By (B.3), there exist $\pi^m, \rho^m \in \Pi$ such that

$$d_1((z, \mathbf{x})_{\pi^m}, \mathbf{x}) \leq d_1((z \mathbb{1}_\mathbb{N}, \mathbf{x})_{\rho^m}, \mathbf{x}) < \frac{1}{m}.$$

Consider the sequences $((z, \mathbf{x})_{\pi^m})_{m \in \mathbb{N}}$ and $((z \mathbb{1}_\mathbb{N}, \mathbf{x})_{\rho^m})_{m \in \mathbb{N}}$ in \mathbf{X} . By Strong Anonymity, we obtain that $(z, \mathbf{x})_{\pi^m} \sim (z, \mathbf{x})$ and $(z \mathbb{1}_\mathbb{N}, \mathbf{x})_{\rho^m} \sim (z \mathbb{1}_\mathbb{N}, \mathbf{x})$ for each $m \in \mathbb{N}$. Since \succsim satisfies Weak Continuity and

$$\lim_{m \rightarrow +\infty} d_1((z, \mathbf{x})_{\pi^m}, \mathbf{x}) = \lim_{m \rightarrow +\infty} d_1((z \mathbb{1}_\mathbb{N}, \mathbf{x})_{\rho^m}, \mathbf{x}) = 0,$$

we obtain $(z, \mathbf{x}) \sim \mathbf{x}$ and $(z \mathbb{1}_\mathbb{N}, \mathbf{x}) \sim \mathbf{x}$. □

Note that a variant of Lemma 4 that uses Weak Continuity instead of Continuity can be proved by using Lemma 5 instead of Lemma 3. Thus, in what follows, we will use Lemma 4 to establish a variant of Proposition 2.

To state the variant of Proposition 2, we introduce another continuity axiom: *Restricted Continuity* requires an evaluation be continuous with respect to streams that have a constant subsequence. Furthermore, it only asserts that indifference relations must be robust with respect to small changes of constant wellbeing levels in a very weak form by requiring social indifference must hold for all constant wellbeing levels in a neighborhood of a given constant wellbeing level.

Axiom (Restricted Continuity). *For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, any $\ell \in \mathbb{R}$, and any $\underline{\ell} < \bar{\ell}$ such that $\underline{\ell} \leq \ell \leq \bar{\ell}$, if $(\ell' \mathbb{1}_\mathbb{N}, \mathbf{x}) \sim (\ell' \mathbb{1}_\mathbb{N}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell}) \setminus \{\ell\}$, then $(\ell \mathbb{1}_\mathbb{N}, \mathbf{x}) \sim (\ell \mathbb{1}_\mathbb{N}, \mathbf{y})$.*

The following lemma shows that Continuity implies Restricted Continuity for any preorder satisfying Monotonicity.

Lemma 6. *Assume that \succsim is a preorder satisfying Monotonicity. If \succsim satisfies Continuity, then \succsim satisfies also Restricted Continuity.*

Proof. Assume that \succsim is a preorder satisfying Monotonicity and Continuity. Consider any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, any $\ell \in \mathbb{R}$, and any $\underline{\ell} < \bar{\ell}$ such that $\underline{\ell} \leq \ell \leq \bar{\ell}$. Suppose that $(\ell' \mathbb{1}_\mathbb{N}, \mathbf{x}) \sim (\ell' \mathbb{1}_\mathbb{N}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell}) \setminus \{\ell\}$. Without loss of generality, we assume that $\bar{\ell} > \ell$. Then there exists a sequence $\{\ell^k\}_{k \in \mathbb{N}}$ in $(\underline{\ell}, \bar{\ell})$ that converges to ℓ . Since $[\ell, \bar{\ell}] \subseteq (\underline{\ell}, \bar{\ell})$, it follows that $(\ell^k \mathbb{1}_\mathbb{N}, \mathbf{x}) \sim (\ell^k \mathbb{1}_\mathbb{N}, \mathbf{y})$ for all $k \in \mathbb{N}$. Furthermore, by Monotonicity, $(\ell^k \mathbb{1}_\mathbb{N}, \mathbf{y}) \succsim (\ell \mathbb{1}_\mathbb{N}, \mathbf{y})$ for all $k \in \mathbb{N}$. Since \succsim is transitive, we obtain $(\ell^k \mathbb{1}_\mathbb{N}, \mathbf{x}) \succsim (\ell \mathbb{1}_\mathbb{N}, \mathbf{y})$ for all $k \in \mathbb{N}$. Since the sequence $\{(\ell^k \mathbb{1}_\mathbb{N}, \mathbf{x})\}_{k \in \mathbb{N}}$ converges to $(\ell \mathbb{1}_\mathbb{N}, \mathbf{x})$ in the sup metric, we obtain by Continuity that $(\ell \mathbb{1}_\mathbb{N}, \mathbf{x}) \succsim (\ell \mathbb{1}_\mathbb{N}, \mathbf{y})$. By the same argument, we obtain $(\ell \mathbb{1}_\mathbb{N}, \mathbf{y}) \succsim (\ell \mathbb{1}_\mathbb{N}, \mathbf{x})$. Thus, $(\ell \mathbb{1}_\mathbb{N}, \mathbf{x}) \sim (\ell \mathbb{1}_\mathbb{N}, \mathbf{y})$, thereby establishing that \succsim satisfies Restricted Continuity. □

The following proposition is a stronger variant of Proposition 2 using Weak Continuity and Restricted Continuity instead of Continuity.

Proposition 5. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. The two following statements are equivalent:*

(1) *For all preorders \succsim satisfying Monotonicity, Strong Anonymity, Weak Continuity, Restricted Continuity, and Limited Inequity, we have that $\mathbf{x} \sim \mathbf{y}$.*

(2) *$y_i \geq \liminf_{j \in \mathbb{N}} x_j$.*

Proof. Since Continuity implies Weak Continuity and Restricted Continuity, the proof that (1) implies (2) is analogous to that of Proposition 2.

We prove that (2) implies (1). Assume that \succsim is a preorder on \mathbf{X} that satisfies Monotonicity, Strong Anonymity, Weak Continuity, Restricted Continuity, and Limited Inequity. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that there exists $i \in \mathbb{N}$ such that $x_i > y_i \geq \liminf_{j \in \mathbb{N}} x_j$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. There are two subcases:

Subcase (i): $\limsup_{j \in \mathbb{N}} y_j > \liminf_{i \in \mathbb{N}} x_i$. The result follows from Lemma 4.

Subcase (ii): $\limsup_{j \in \mathbb{N}} y_j = \liminf_{i \in \mathbb{N}} x_i = \ell$. Since ℓ is a cluster point of \mathbf{x} and \mathbf{y} , it follows from Lemma 5 that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \sim \mathbf{y}$. Let $\underline{\ell}, \bar{\ell} \in \mathbb{R}$ be such that $\underline{\ell} = \ell < \bar{\ell}$. By Lemma 4, we obtain $(\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell})$. By Restricted Continuity, $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ follows. By the transitivity of \succsim , we obtain $\mathbf{x} \sim \mathbf{y}$. \square

Using Lemma 5 and Proposition 5, we obtain the following stronger variants of Propositions 3 and 4 with the weaker continuity axioms. Since their proofs are analogous to those of Propositions 3 and 4, we state them without proof.

Proposition 6. *Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$. The two following statements are equivalent:*

(1) *For all preorders \succsim satisfying Monotonicity, Strong Anonymity, Weak Continuity, Restricted Continuity, Limited Inequity, and Critical-Level Consistency, we have that $\mathbf{x} \sim \mathbf{y}$.*

(2) *$\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $y_i \geq \ell$ for all $i \in \mathbb{N}$ such that $x_i > y_i$.*

Proposition 7. *Assume that \succsim is a preorder satisfying Monotonicity, Strong Anonymity, Weak Continuity, Restricted Continuity, Limited Inequity, and Critical-Level Consistency. Then \succsim satisfies Non-Positive Value of Additional Lives.*

References

- Tsuyoshi Adachi, Susumu Cato, Kohei Kamaga, Extended anonymity and Paretian relations on infinite utility streams, *Mathematical Social Sciences* 72 (2014) 24–32
- Geir B. Asheim, Intergenerational equity, *Annual Review of Economics* 2 (2010) 197–222
- Geir B. Asheim, Claude d’Aspremont, Kuntal Banerjee, Generalized time-invariant overtaking, *Journal of Mathematical Economics* 46 (2010) 519–533
- Geir B. Asheim, Wolfgang Buchholz, Bertil Tungodden, Justifying sustainability, *Journal of Environmental Economics and Management* 41 (2001) 252–268
- Geir B. Asheim, Kohei Kamaga, Stéphane Zuber, Infinite population utilitarian criteria, CESifo Working Paper No. 9576 (2022)
- Geir B. Asheim, Bertil Tungodden, Resolving distributional conflicts between generations, *Economic Theory* 24 (2004) 221–230
- Geir B. Asheim, Stéphane Zuber, A complete and strongly anonymous leximin relation on infinite streams, *Social Choice and Welfare* 41 (2013) 819–834
- Amanda Askill, Pareto Principles in Infinite Ethics. Ph.D. dissertation, New York University (2018)
- Claude d’Aspremont, Formal welfarism and intergenerational equity. In: John Roemer, Kotaro Suzumura (eds.), *Intergenerational Equity and Sustainability*. Palgrave, London (2007)
- Claude d’Aspremont, Luis Gevers, Equity and the informational basis of collective choice, *Review of Economic Studies* 44 (1977), 199–209
- Kaushik Basu, Tapan Mitra, Aggregating infinite utility streams with intergenerational equity: the impossibility of being Paretian, *Econometrica* 71 (2003) 1557–1563
- Charles Blackorby, Walter Bossert, David Donaldson, Intertemporal population ethics: critical-level utilitarian principles, *Econometrica* 63 (1995) 1303–1320
- Charles Blackorby, Walter Bossert, David Donaldson, Information invariance in variable-population social-choice problems, *International Economic Review* 40 (1999), 403–422
- Charles Blackorby, Walter Bossert, David Donaldson, *Population Issues in Social Choice Theory, Welfare Economics, and Ethics*. Cambridge University Press, Cambridge, UK (2005a)
- Charles Blackorby, Walter Bossert, David Donaldson, Multi-profile welfarism: A generalization, *Social Choice and Welfare* 24 (2005b), 253–267
- Charles Blackorby, Walter Bossert, David Donaldson, Anonymous single-profile welfarism, *Social Choice and Welfare* 27 (2006), 279–287
- Charles Blackorby, David Donaldson, Social criteria for evaluating population change, *Journal of Public Economics* 25 (1984) 13–33
- Charles Blackorby, David Donaldson, John A. Weymark, A welfarist proof of Arrow’s theorem, *Recherches Economiques de Louvain* 56 (1990), 259–286
- Walter Bossert, Yves Sprumont, Kotaro Suzumura, Ordering infinite utility streams, *Journal of Economic Theory* 135 (2007) 579–589

- John Broome, *Weighing Goods: Equality, Uncertainty and Time*, Oxford University Press, Oxford, UK (1991)
- John Broome, *Weighing Lives*, Oxford University Press, Oxford, UK (2004)
- John Broome, Do not ask for morality. In: Adrian Walsh, S ade Hormio, Duncan Purves (eds.), *Ethical Underpinnings of Climate Economics*. Routledge, London (2016)
- John Broome, Efficiency and future generations, *Economics & Philosophy* 34 (2018) 221–241
- John Broome, Duncan Foley, A world climate bank. In: Inigo Gonz alez-Ricoy and Axel Gosseries (eds.), *Institutions for Future Generations*, Oxford University Press, Oxford (2016)
- David de la Croix, Matthias Doepke, A soul’s view of the optimal population problem, *Mathematical Social Sciences* (2021) 98–108
- Richard Deschamps, Louis Gevers, Leximin and utilitarian rules: A joint characterization, *Journal of Economic Theory* 17 (1978) 143–163
- Peter A. Diamond, The evaluation of infinite utility streams, *Econometrica* 33 (1965) 170–177
- Hugh Dalton, The measurement of the inequality of incomes. *Economic Journal* 30 (1920) 348–361
- David Donaldson, John A. Weymark, A quasiordering is the intersection of orderings, *Journal of Economic Theory* 78 (1997) 382–387
- Marc Fleurbaey, Philippe Michel, Intertemporal equity and the extension of the Ramsey criterion, *Journal of Mathematical Economics* 39 (2003) 777–802
- Shane Frederick, George Loewenstein, Ted O’Donoghue, Time discounting and time preference: A critical review, *Journal of Economic Literature* 40 (2002) 351–401
- Duncan Foley, The economic fundamentals of global warming. In: Jonathan M. Harris, Neva R. Goodwin (eds.), *Twenty-First Century Macroeconomics: Responding to the Climate Challenge*. Edward Elgar, Cheltenham (2009)
- Mikhail Golosov, Larry E. Jones, Mich ele Tertilt, Efficiency with endogenous population growth, *Econometrica* 75 (2007) 1039–1071
- Joel David Hamkins, Barbara Gail Montero, Utilitarianism in infinite worlds, *Utilitas* 12 (2000) 91–96
- Peter J. Hammond, Equity, Arrow’s conditions, and Rawls’ difference principle, *Econometrica* 44 (1976) 793–804
- Peter J. Hammond, Equity in two person situations: Some consequence, *Econometrica* 47 (1979) 1127–1135
- David Hilbert,  ber das Unendliche. In: William Ewald, Wilfried Sieg (eds.), *David Hilbert’s Lectures on the Foundations of Arithmetics and Logic 1917–1933*. Springer Verlag, Heidelberg (2013)
- Adam Jonsson, Infinite utility: Counterparts and ultimate locations, mimeo, Lule  University of Technology (2021)
- Adam Jonsson, Mark Voorneveld, The limit of discounted utilitarianism, *Theoretical Economics* 13 (2018) 19–37
- Kohei Kamaga, Infinite-horizon social evaluation with variable population size, *Social Choice and Welfare* 47 (2016) 207–232

- Kohei Kamaga, Takashi Kojima, On the leximin and utilitarian overtaking criteria with extended anonymity, *Social Choice and Welfare* 35 (2010) 377–392
- Toshiko Kaneda, Carl Haub, How many people have ever lived on earth? Population Reference Bureau, <https://www.prb.org/articles/how-many-people-have-ever-lived-on-earth/> (2021)
- Urmee Khan, Maxwell B. Stinchcombe, Planning for the long run: Programming with patient, Pareto responsive preferences, *Journal of Economic Theory* 176 (2018) 444–478
- Luc Lauwers, Infinite utility: insisting on strong monotonicity, *Australasian Journal of Philosophy* 75 (1997) 222–233
- Luc Lauwers, Intertemporal objective functions: Strong Pareto versus anonymity, *Mathematical Social Sciences* 35 (1998) 37–55
- Luc Lauwers, Ordering infinite utility streams comes at the cost of a non-Ramsey set, *Journal of Mathematical Economics* 46 (2010) 32–37
- Luc Lauwers, Intergenerational equity, efficiency, and constructibility, *Economic Theory* 49 (2012) 227–242
- Luc Lauwers, Peter Vallentyne, Infinite utilitarianism: More is always better, *Economics and Philosophy* 20 (2004) 307–330
- Tapan Mitra, Kaushik Basu, On the existence of Paretian social welfare quasi-orderings for infinite utility streams with extended anonymity. In: John Roemer, Kotaro Suzumura (eds.), *Intergenerational Equity and Sustainability*. Palgrave, London (2007)
- Tim Mulgan, Transcending the infinite utility debate. *Australasian Journal of Philosophy* 80 (2002) 164–177
- William Nordhaus, *A Question of Balance: Weighing Options on Global Warming Policies*. Yale University Press, New Haven & London (2008)
- Derek Parfit, *Reasons and Persons*. Oxford University Press, Oxford (1984)
- Arthur C. Pigou, *Wealth and Welfare*. Macmillan, London (1912)
- Marcus Pivato, A characterization of Cesàro average utility, mimeo, CY Cergy Paris Université (2020)
- Marcus Pivato, Population ethics in an infinite universe, mimeo, CY Cergy Paris Université (2021)
- Toyotaka Sakai, A characterization and an impossibility of finite length anonymity for infinite generations, *Journal of Mathematical Economics* 46 (2010) 877–883
- Toyotaka Sakai, Limit representations of intergenerational equity, *Social Choice and Welfare* 47 (2016) 481–500
- Amartya K. Sen, *Collective Choice and Social Welfare*, Oliver and Boyd, Edinburgh (1970)
- Amartya K. Sen, *On Economic Inequality*, Oxford University Press, Oxford (1973), expanded edition (1997)
- Theodore Sider, Might theory X be a theory of diminishing marginal value? *Analysis* 51 (1991) 265–271
- Nicholas Stern, *The Stern Review on the Economics of Climate Change*. HM Treasury, London (2007)

- Patrick Suppes, Some formal models of grading principles, *Synthese* 6 (1966) 284–306
- Kotaro Suzumura, *Rational Choice, Collective Decisions, and Social Welfare*. Cambridge University Press, Cambridge, UK (1983)
- Kotaro Suzumura, Paretian welfare judgements and Bergsonian social choice, *Economic Journal* 109 (1999) 204–220
- Lars-Gunnar Svensson, Equity among generations, *Econometrica* 48 (1980) 1251–1256
- Peter Vallentyne, Shelly Kagan, Infinite value and finitely additive value theory, *Journal of Philosophy* 94 (1997) 5–26
- Luc Van Liedekerke, Should utilitarians be cautious about an infinite future? *Australasian Journal of Philosophy* 73 (1995) 405–407
- Luc Van Liedekerke, Luc Lauwers, Sacrificing the patrol: Utilitarianism, future generations and infinity, *Economics and Philosophy* 13 (1997) 159–174
- Hayden Wilkinson, Infinite aggregation: Expanded addition, *Philosophical Studies* 178 (2020) 1917–1949
- William R. Zame, Can intergenerational equity be operationalized? *Theoretical Economics* 2 (2007) 187–202
- Stéphane Zuber, Geir B. Asheim, Justifying social discounting: The rank-discounted utilitarian approach, *Journal of Economic Theory* 147 (2012) 1572–1601