Models of urban morphogenesis to link urban form and function

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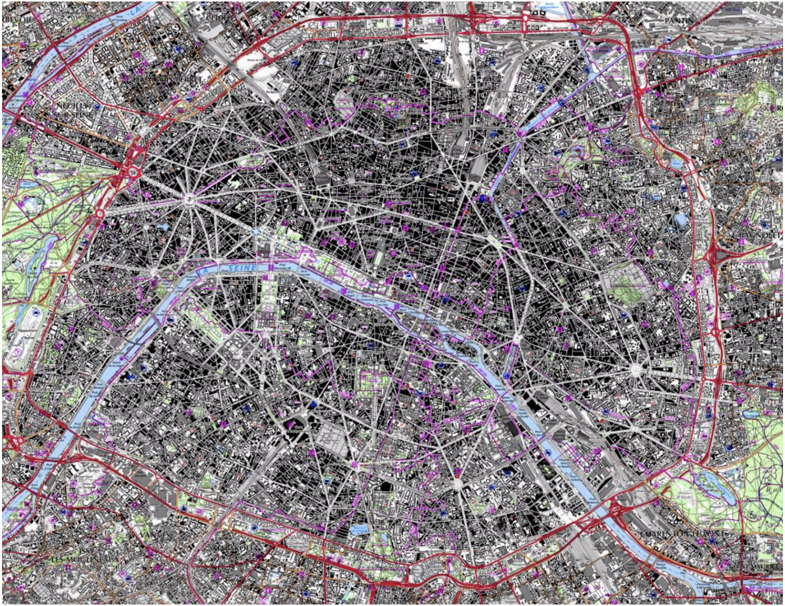
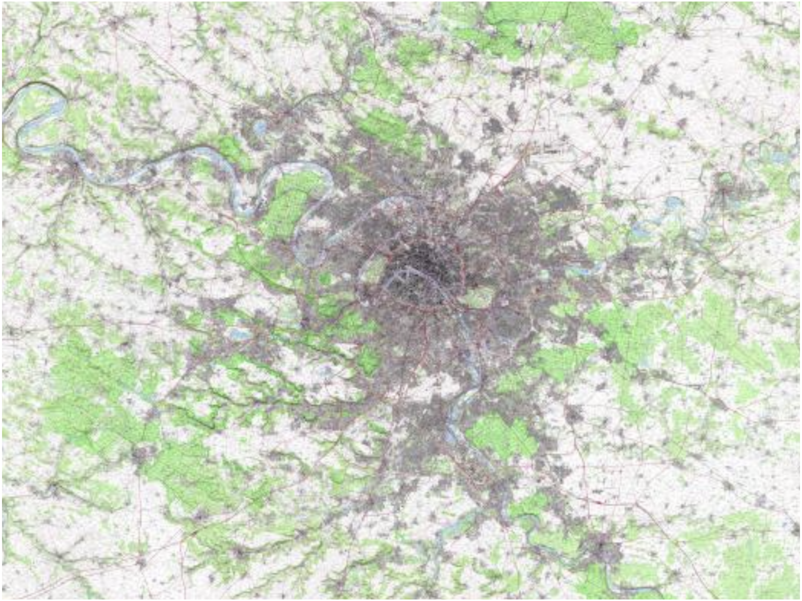
TQG Debates 2019
3.1: Fractals and Multi-fractals
November 15th 2019

Complex processes of Urban Morphogenesis



Complex processes of Urban Morphogenesis



What is Morphogenesis ?

Morphogenesis (*Oxford dictionary*)

1. *Biology* : The origin and development of morphological characteristics
2. *Geology* : The formation of landforms or other structures.

History of the notion

→ Started significantly with embryology around 1930 [Abercrombie, 1977]

→ Turing's 1952 paper [Turing, 1952], linked to the development of Cybernetics

→ first use in 1871, large peak in usage between 1907-1909, increase until 1990, decrease until today. *Scientific fashion ?*

What is Morphogenesis ? Examples

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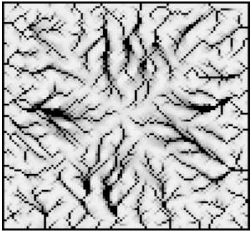
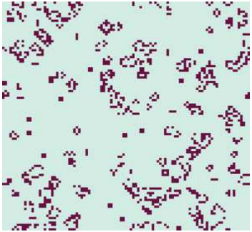
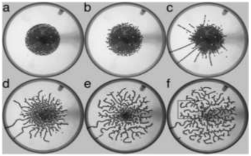
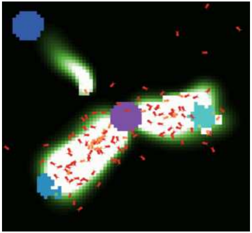
Non Functional			
Functional			

Sources (in order by column). Ants, Erosion, Game of Life: NetLogo Library; Arbotron [Jun and Hübler, 2005]; Industrial design [Aage et al., 2017]; Swarm chemistry [Sayama, 200

Defining Morphogenesis

Proposition of an interdisciplinary definition

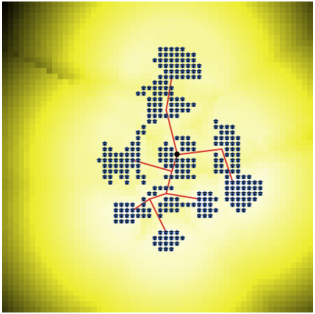
Meta-epistemological framework of imbricated notions:
Self-organization $\supsetneq$ Morphogenesis $\supsetneq$ Autopoiesis $\supsetneq$ Life

Properties:

- Architecture links form and function
- Emergence strength [Bedau, 2002] increases with notion depth, as bifurcations [Thom, 1974]

Definition of Morphogenesis : *Emergence of the form and the function in a strongly coupled manner, producing an emergent architecture [Doursat et al., 2012]*

Which models for Urban Morphogenesis ?



Example: a basic hybrid model based on elementary processes for density and network [Raimbault et al., 2014]

→*At the crossroad between Urban Simulation and Artificial Life, few models try to integrate and explain the link between Urban Form and Function*

→*Importance of parcimonious, stylized models: modeling as a tool to understand processes*

Research Objective : Explore simple models to capture morphogenesis based on abstract representation of urban processes; test their ability to reproduce existing urban systems.

A simple Reaction-diffusion model

$\rightarrow$ Crucial role of the interplay between concentration forces and dispersion forces [Fujita and Thisse, 1996] in keeping Urban Systems at the border of chaos

$\rightarrow$ Potentiality of aggregation mechanisms (such as Simon model) to produce power laws [Dodds et al., 2017]

$\rightarrow$ Link with Reaction-diffusion approaches in Morphogenesis [Turing, 1952]

$\rightarrow$ Extension of a DLA-type model introduced by [Batty, 1991], with simple abstract processes of population aggregation and diffusion

Raimbault, J. (2018). Calibration of a density-based model of urban morphogenesis. PloS one, 13(9), e0203516.

Model Formalization

$\rightarrow$ Grid world with cell populations $\left(P_i\left(t\right)\right)_{1\leq i\leq N^2}$.

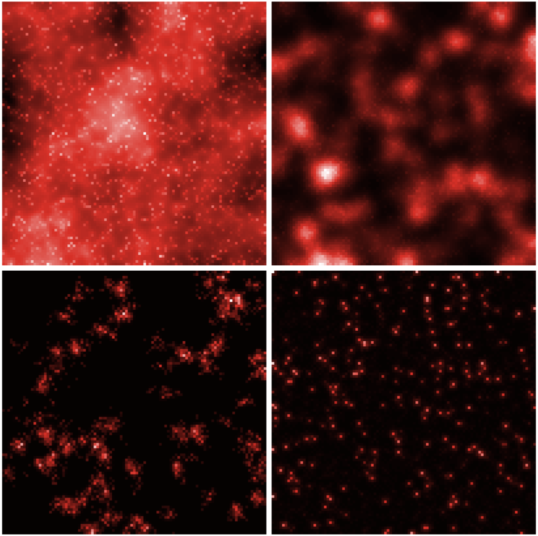
$\rightarrow$ At each time step:

1. Population growth with exogenous rate N_G, attributed independently to a cell following a preferential attachment of strength α
2. Population is diffused n_d times with strength β

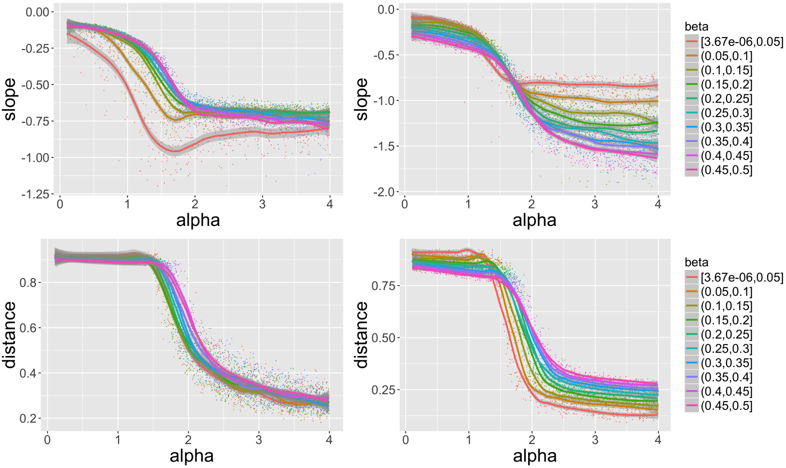
$\rightarrow$ Stopping criterion: fixed maximal population P_m.

$\rightarrow$ Output measured by morphological indicators: Moran index, average distance, rank-size hierarchy, entropy.

Generating Population Distributions



Examples of generated territorial shapes



Phase transitions of indicators unveiled by exploration of the parameter space (80000 parameter points, 10 repetitions each)

Path-dependence and frozen accidents

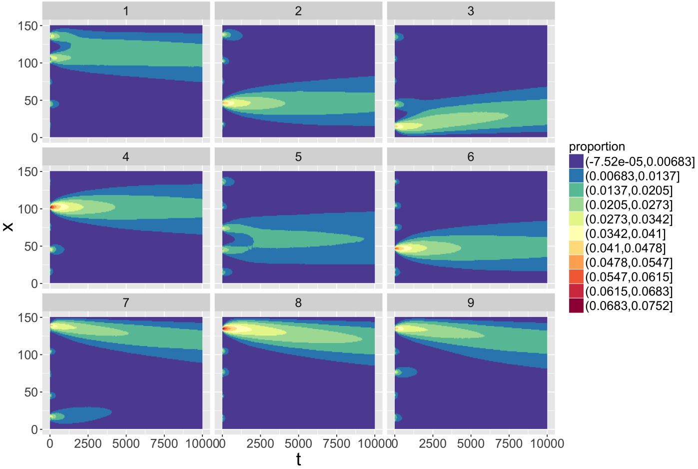
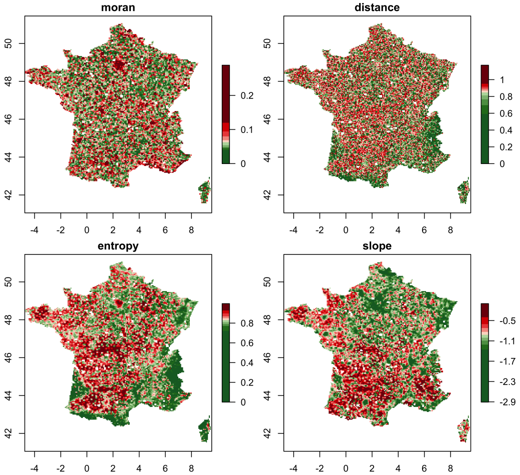
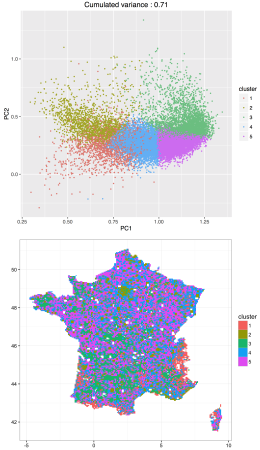


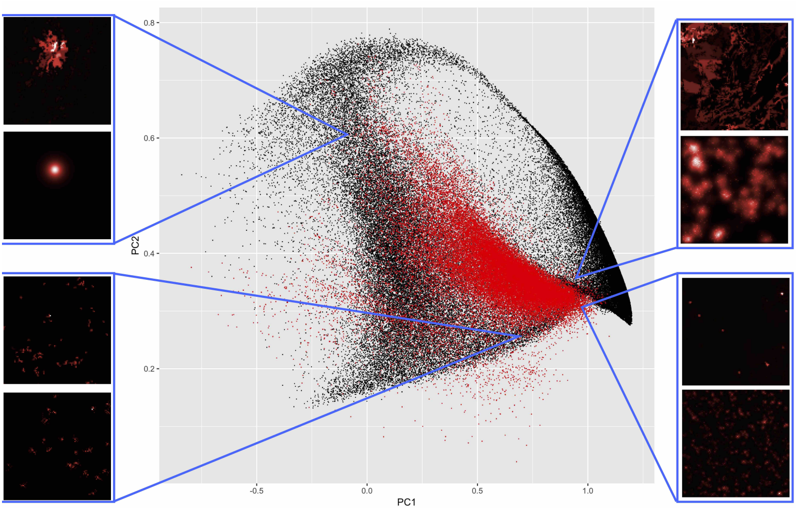
Illustration of path-dependence in a simplified one-dimensional version of the model: cell trajectories in time for 9 independent repetitions from the same initial configuration.





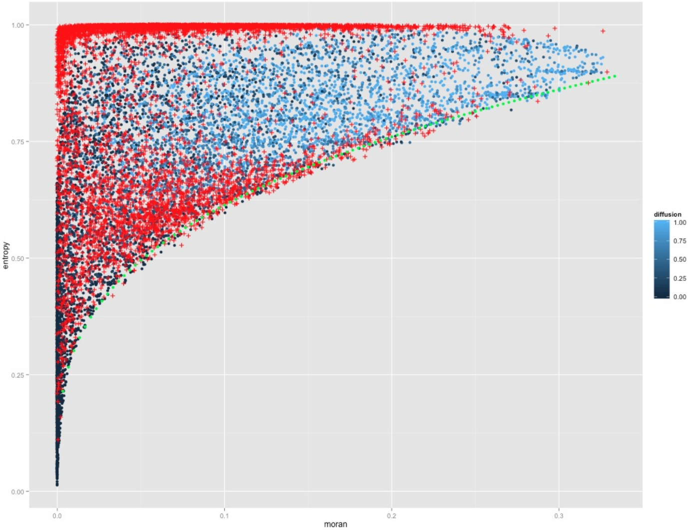
Computation of morphological indicators on population density data for Europe (shown here on France), morphological classification.

Model Calibration



Brute force calibration by exploring the parameter space. Reproduction of most existing configuration in the morphological sense (here in principal plan).

Model Targeted Exploration



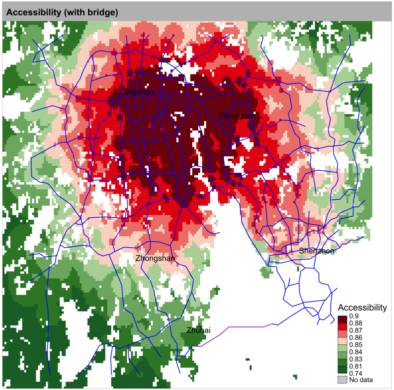
Potentialities of targeted model explorations: here feasible space using Pattern Space Exploration algorithm [Chérel et al., 2015].

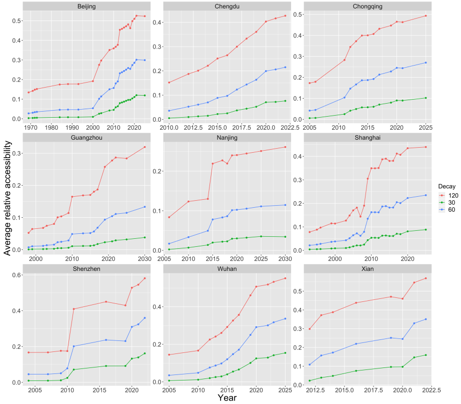
Including more complex processes ?

Which ontology to include more complex functional properties ?

$\rightarrow$ Territorial systems as the strong coupling between territories and (potential and realized) networks [Dupuy, 1987].

$\rightarrow$ Networks convey functional notions of centralities and accessibility, among others; have furthermore proper topological properties.





Accessibility as part of complex processes of co-evolution between transportation networks and territories.

Raimbault, J. (2019). Evolving accessibility landscapes: mutations of transportation networks in China. In Aveline-Dubach, N., ed. *Pathways of sustainable urban development across China - the cases of Hangzhou, Datong and Zhuhai*, pp 89-108. Imago. ISBN:978-88-94384-71-0

A Morphogenesis Model of co-evolution

$\rightarrow$ Coupled grid population distribution and vector transportation network, following the core of [Raimbault et al., 2014]

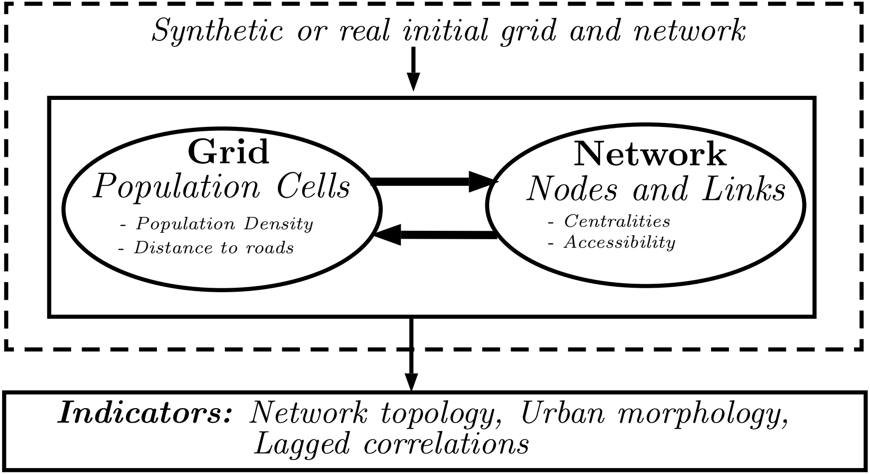
$\rightarrow$ Local morphological and functional variables determine a patch-value, driving new population attribution through preferential attachment ; combined to population diffusion (reaction-diffusion processes studied before)

$\rightarrow$ Network growth is also driven by morphological, functional and local network measures, following diverse heuristics corresponding to different processes (multi-modeling)

Local variables and network properties induce feedback on both, thus a strong coupling capturing the ***co-evolution***

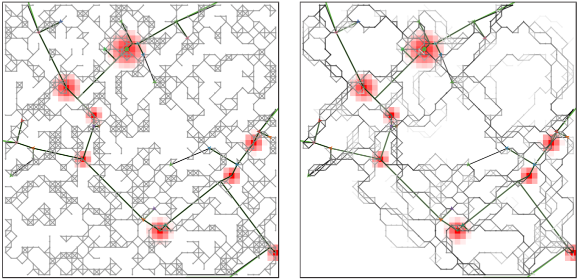
Raimbault, J. (2019). An urban morphogenesis model capturing interactions between networks and territories. In The Mathematics of Urban Morphology (pp. 383-409). Birkhäuser, Cham.

Raimbault, J. (2018). Multi-modeling the morphogenesis of transportation networks. In Artificial Life Conference Proceedings (pp. 382-383).


Synthetic or real initial grid and network
Grid
Population Cells
- Population Density
- Distance to roads
Network
Nodes and Links
- Centralities
- Accessibility
Indicators: Network topology, Urban morphology, Lagged correlations

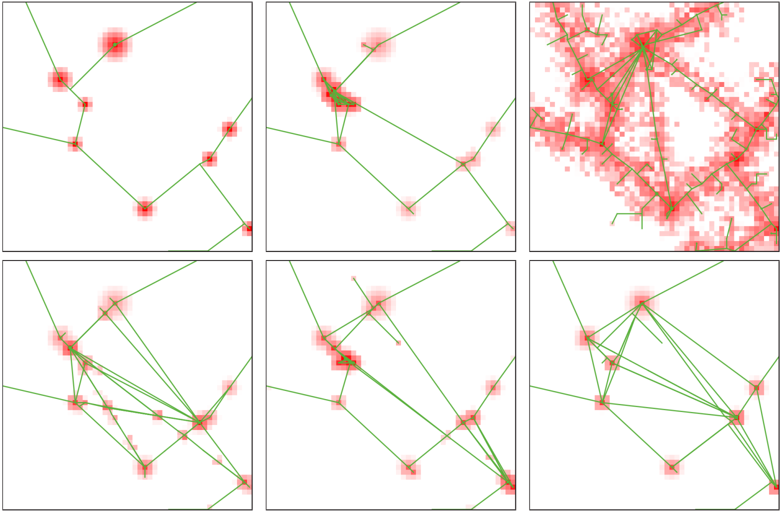
At fixed time steps :

1. Add new nodes preferentially to new population and connect them
2. Variable heuristic for new links, among: nothing, random, gravity-based deterministic breakdown, gravity-based random breakdown (from [Schmitt, 2014]), cost-benefits (from [Louf et al., 2013]), biological network generation (based on [Tero et al., 2010])



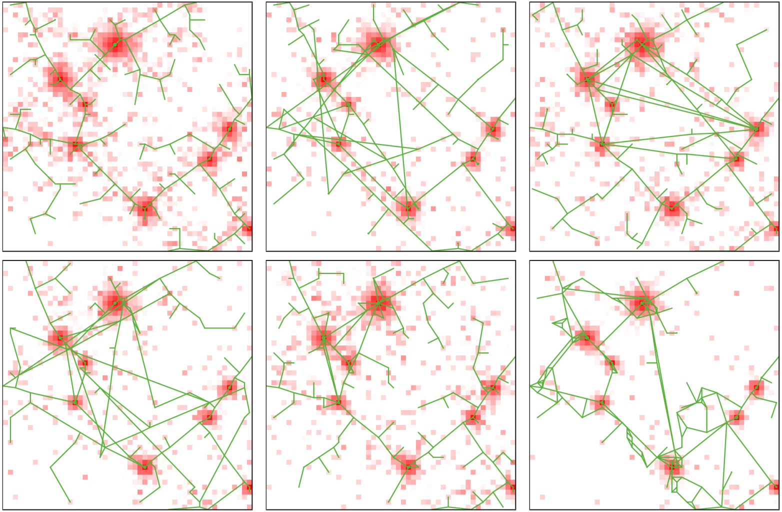
Intermediate stage for biological network generation

Generated Urban Shapes: Urban Form



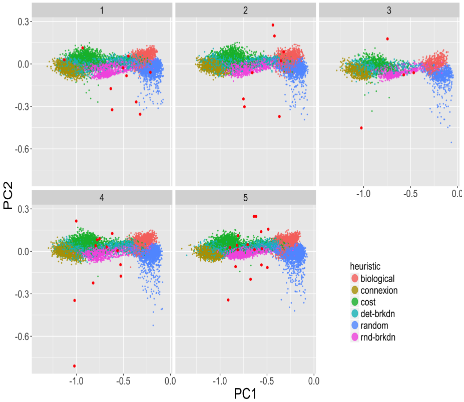
In order: setup; accessibility driven; road distance driven; betweenness driven; closeness driven; population driven.

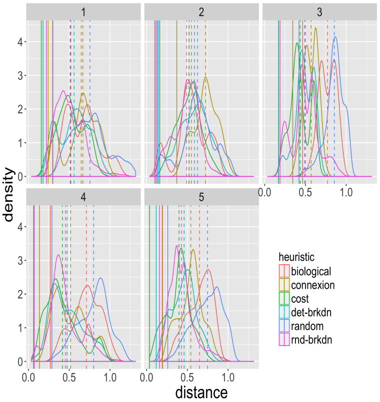
Generated Urban Shapes: Network



In order: connection; random; deterministic breakdown; random breakdown; cost-driven; biological.

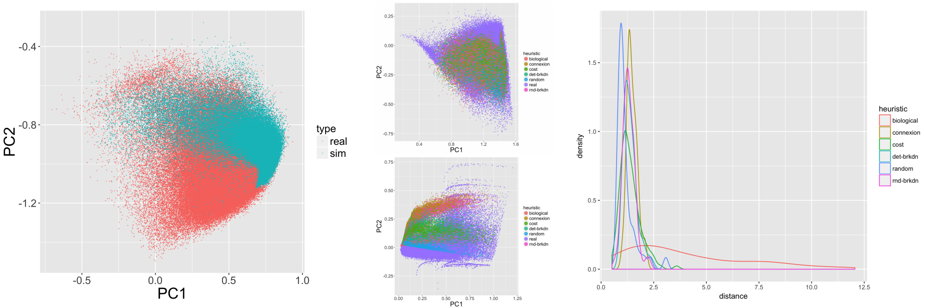
Comparison of feasible space for network indicators with fixed density





(Left) Feasible spaces by morphological class and network heuristic; (Right) Distribution of distances to topologies of real networks

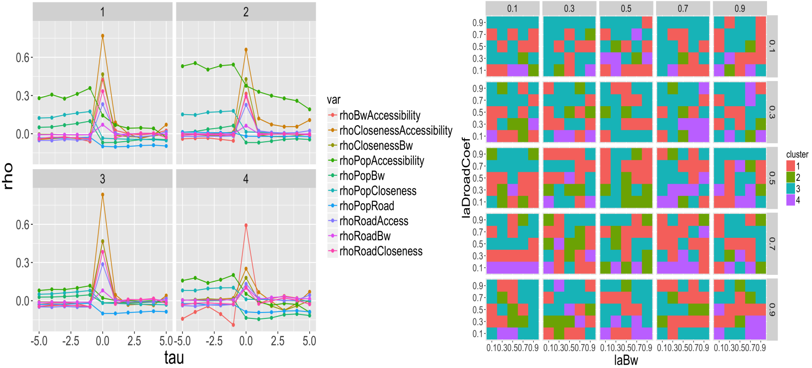
Calibration (model explored with OpenMole [Reuillon et al., 2013], $\sim 10^6$ model runs) at the first order on morphological and topological objectives, and on correlations matrices.



(Left) Full indicator space; (Middle) Morphological and Topology, by network heuristic; (Right) Distance distribution for cumulated distance for indicators and correlations.

Unsupervised learning on lagged correlations between local variables unveils a diversity of causality regimes

→ Link between *co-evolution regime* and morphogenetic properties of the urban system



(Left) Lagged correlation profiles of cluster centers; (Right) Distribution of regimes across parameter space

Implications

$\rightarrow$ This rather simple model reproduces most of existing urban forms in Europe for both population distribution and road network : which intrinsic dimension to the urban system and its morphological aspect ?
$\rightarrow$ Ability to reproduce static correlations and a variety of dynamical lagged correlation regimes suggests that the model captures some of the processes of co-evolution

Developments

$\rightarrow$ Towards a dynamical calibration ? Need of dynamical data
$\rightarrow$ Investigate the link between spatial non-stationarity and non-ergodicity through simulation by the model
$\rightarrow$ Compare network generation in a “fair” way (correcting for additional parameters, open question for models of simulation)

Link with fractals

Morphogenesis and fractals already linked in the biological literature: for example [Nelson and Manchester, 1988] with network morphogenesis, [Matsuyama and Matsushita, 1993] with a DLA model for bacteria self-organization

Also links in Urban Science: DLA model [Batty, 1991], fractal models of urban growth [Frankhauser, 2008]

Open questions:

→ Formal link between fractal properties and the dynamics of form and function [Batty, 1999]

→ Relating fractal indicators of urban form with other dimensions

→ Link between multi-fractal properties [Salat et al., 2017] and multi-scalar models of urban systems [Raimbault, 2019a]

More realistic models?

→ Introducing more concrete ontologies, economic processes [Bonin and Hubert, 2014], qualitative differentiation [Bonin and Hubert, 2012] governance processes [Le Néchet and Raimbault, 2015]

→ Possible bridges with Land-use change models/Land-use Transport models [Wegener and Fürst, 2004], with systems of cities models [Pumain and Reuillon, 2017]

More data-driven models?

→ Work in progress: calibration of the reaction-diffusion model on world urban areas with the Global Human Settlements Layer database

→ Link with sustainability indicators: GHG emissions, economics, etc. [Raimbault, 2019b]

→ Study models on hybrid synthetic data [Raimbault et al., 2019]: systematic conclusions for policies

Conclusion

$\rightarrow$ A novel model of urban morphogenesis at the mesoscopic scale systematically explored: **need for more coupling and comparison of models.**

$\rightarrow$ At the macro scale of the system of cities? **Need for multi-scale models.**

$\rightarrow$ With more refined urban characteristics and other dimensions ? **Need for more interdisciplinarity.**

- Code, data and results available at `https://github.com/JusteRaimbault/CityNetwork`

- Acknowledgments: Thanks to the *European Grid Infrastructure* and its *National Grid Initiatives* (*France-Grilles* in particular) to give the technical support and the infrastructure.

Reserve Slides

Morphogenesis Overview

[Bourgine and Lesne, 2010] : interdisciplinary workshop on morphogenesis

→*To what extent the notion is indeed transdisciplinary, i.e. are there common definitions across disciplines ? What are the concepts shared or the divergence ?*

- **Biology**
  - External phenotype morphogenesis (ant colony) [Minter et al., 2012]
  - Symbiosis of species [Chapman and Margulis, 1998]
  - Botany [Lord, 1981]
- **Social Sciences** : Archeology [Renfrew, 1978]
- **Epistemology** : [Gilbert, 2003]
- **Artificial Intelligence** : From self-assembly to Morphogenetic Engineering [Doursat et al., 2013]. Synthetic Biology ?
- **Geomorphology** : dunes formation [Douady and Hersen, 2011]
- **Physics** : Arbotrons playing Tetris ?
- etc. . .

- **Morphogenesis and Self-Organisation** : when does a system exhibit an architecture ? Insights from Morphogenetic Engineering [Doursat et al., 2013]. Architecture : the relation between the form and the function ?

- **Scales, Units and Boundaries** From local interactions to global information flow (Holland's *signal and boundaries* [Holland, 2012]: morphogenesis as the development of Complex Adaptive Systems ?)

- **Symmetry and Bifurcations** : on quantitative becoming qualitative. René Thom's *theory of catastrophes* [Thom, 1974]

- **Life and Death** : link with autopoiesis and cognition [Bourgine and Stewart, 2004] ; co-evolution of subsystems as an alternative definition ? In psychology, attractors of the mind.

A system is viewed as its internal state X_w, where $w \in W$ is a control parameter.

Catastrophe set $K \subset W$ is where the system endures phase transition.

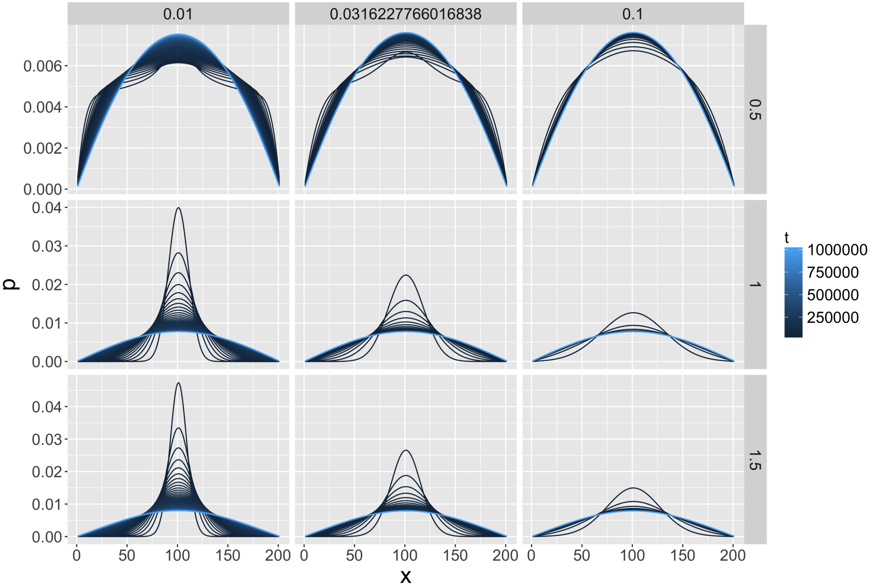
Thom classified possible topologies for K depending on the dimension of W.

[Makse et al., 1998] correlated growth;
[Murcio et al., 2015] multi-scale migration and percolation;
[Bonin and Hubert, 2012] qualitative differentiation of urban function;
[Achibet et al., 2014] procedural model at the micro-scale

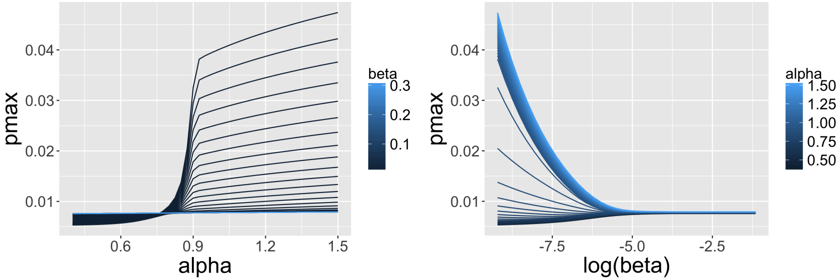
The one-dimensional model verifies the PDE :

$$\delta t \cdot \frac{\partial p}{\partial t} = \frac{N_G \cdot p^{\alpha}}{P_{\alpha} t} + \frac{\alpha\beta\left(\alpha - 1\right)\delta x^2}{2} \cdot \frac{N_G \cdot p^{\alpha-2}}{P_{\alpha}\left(t\right)} \cdot \left(\frac{\partial p}{\partial x}\right)^2 + \frac{\beta\delta x^2}{2} \cdot \frac{\partial^2 p}{\partial x^2} \cdot \left[1 + \alpha\frac{N_G p^{\alpha-1}}{P_{\alpha t}}\right] \quad (1)$$

Stationary behavior of 1D model



Stationary behavior of 1D model



1. Rank-size slope γ, given by $\ln\left(P_{\tilde{i}}/P_0\right) \sim k + \gamma \cdot \ln\left(\tilde{i}/i_0\right)$ where $\tilde{i}$ are the indexes of the distribution sorted in decreasing order.
2. Entropy of the distribution:

$$\mathscr{E} = \sum_{i1}^{M} \frac{P_i}{P} \cdot \ln \frac{P_i}{P} \tag{2}$$

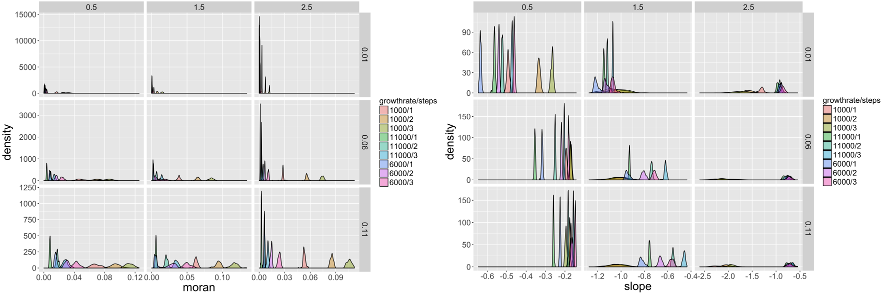
   $\mathscr{E} = 0$ means that all the population is in one cell whereas $\mathscr{E} = 0$ means that the population is uniformly distributed.
3. Spatial-autocorrelation given by Moran index, with simple spatial weights given by $w_{ij} = 1/d_{ij}$

$$I = M \cdot \frac{\sum_{ij/} w_{ij}\left(P_i - \bar{P}\right) \cdot \left(P_j - \bar{P}\right)}{\sum_{ij/} w_{ij} \sum_i \left(P_i - \bar{P}\right)^2}$$

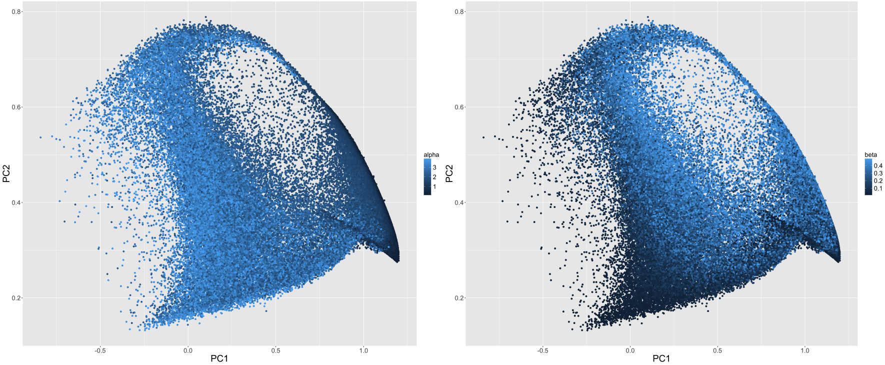
4. Mean distance between individuals

$$\bar{d} = \frac{1}{d_M} \cdot \sum_{i<j} \frac{P_i P_j}{P^2} \cdot d_{ij}$$

Large number of repetitions show good convergence properties



Model behavior



Empirical indicators computation

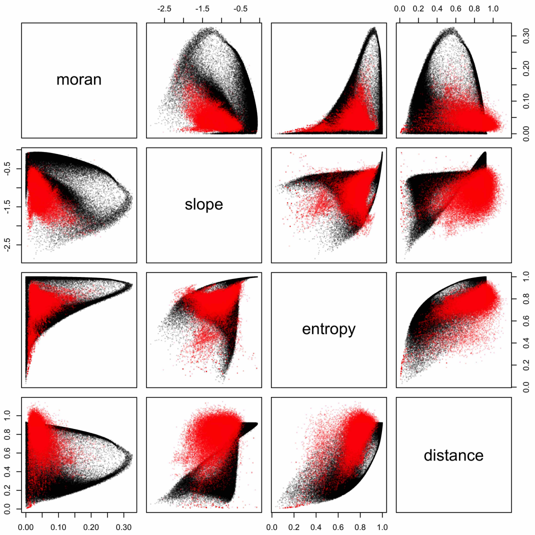
→ Eurostat population density raster (100m, simplified at 500m resolution)

→ Overlapping (10km offset) squares of 50km side : equivalent to smoothing, removes window shape effect. Not very sensitive to window size (tested with 30km and 100km)

→ Indicators computed using Fast Fourier Transform Convolution

→ Classification using repeated k-means ; number of clusters taken at transition in clustering coefficient.

Model calibration: all indicators



Defining co-evolution

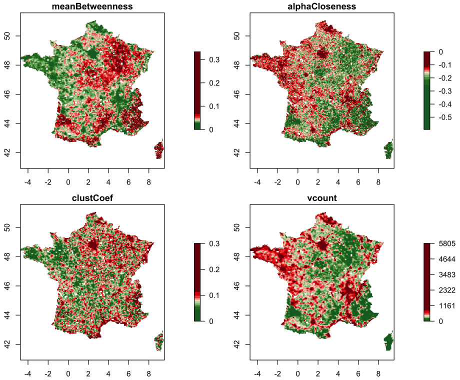
No clear definition of co-evolution in the literature : [Bretagnolle, 2009] distinguishes "reciprocal adaptation" where a sense of causality can clearly be identified, from co-evolutive regimes

Identification of multiple causality regimes in a simple strongly coupled growth model $\rightarrow$ to be put in perspective with a theoretical definition of co-evolution based on the conjunction of Morphogenesis and the Evolutive Urban Theory, given in [Raimbault, 2018]

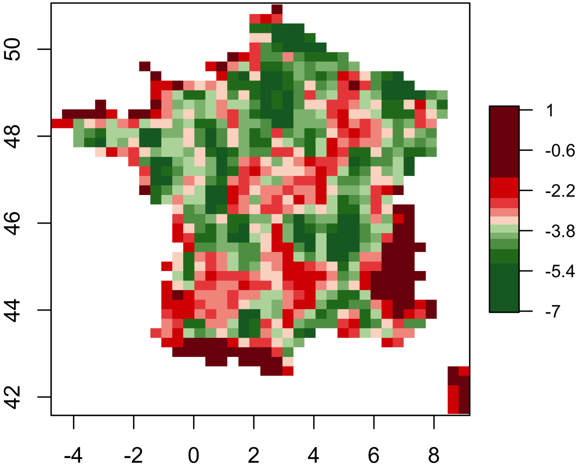
Modeling Co-evolution

[Baptiste, 2010] system dynamics with evolving capacities
[Wu et al., 2017] population diffusion and network growth
[Blumenfeld-Lieberthal and Portugali, 2010] and [Schmitt, 2014]: random potential breakdown for network growth.
[Barthelemy and Flammini, 2009] geometrical network growth model making network topology co-evolve with vertex density

Empirical Data : network indicators



Empirical Data : correlations



Network Indicators

Network Topology measured by:

- Betweenness and Closeness centralities: average and hierarchy
- Accessibility (weighted closeness)
- Efficiency (network pace relative to euclidian distance)
- Mean path length, diameter

Model specification

Patch utility given by $U_i = \sum_k w_k \cdot \tilde{x}_k$ with $\tilde{x}_k$ normalized local variables among population, betweenness and closeness centrality, distance to roads, accessibility ; aggregation done with probability $\left(U_i / \sum_k U_k\right)^\alpha$; diffusion among neighbors n_d times with strength β

Network Generation:
Adding a fixed number n_N of new nodes: for patches such that $d_r < d_0$, probability to receive a node is

$$p = P/P_{max} \cdot \left(d_M - d\right)/d_M \cdot \exp\left(-\left(\left(d_r - d_0\right)/\sigma_r\right)^2\right)$$

Nodes connected the shortest way to existing network.

General model parameters :

- Patch utility weights w_k
- General network generation parameters: growth time steps t_N, maximal additional links

1. Gravity potential given by

$$V_{ij}\left(d\right) = \left[\left(1-k_h\right) + k_h \cdot \left(\frac{P_i P_j}{P^2}\right)^{\gamma}\right] \cdot \exp\left(-\frac{d}{r_g\left(1\ d/d_0\right)}\right)$$

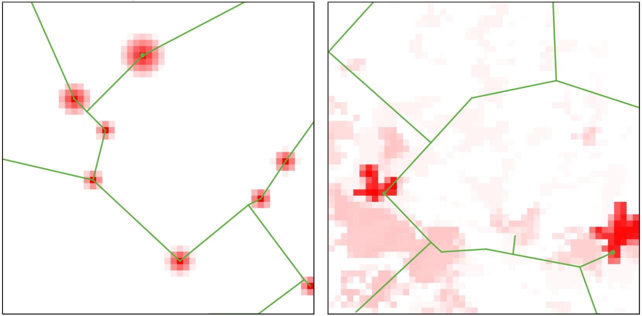
2. $k \cdot N_L$ links are selected with lowest $V_{ij} d_N / V_{ij} d_{ij}$, among which N_L links with highest (lest costly) are realized
3. Network is planarized

Adding new links with biological heuristic:

1. Create network of potential new links, with existing network and randomly sampled diagonal lattice
2. Iterate for k increasing ($k \in \{1, 2, 4\}$ in practice) :
   - Using population distribution, iterate $k \cdot n_b$ times the slime mould model to compute new link capacities
   - Delete links with capacity under θ_d
   - Keep the largest connected component
3. Planarize and simplify final network

Synthetic setup: rank-sized monocentric cities, simple connection with bord nodes to avoid bord effects

Real setup: Population density raster at 500m resolution (European Union, from Eurostat)

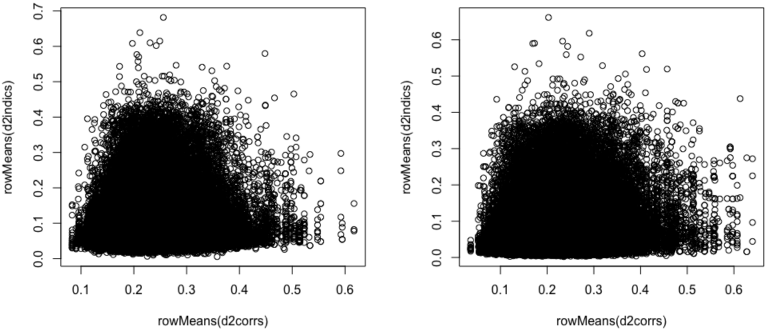


Stopping conditions: fixed final time; fixed total population; fixed network size.

Calibration Method

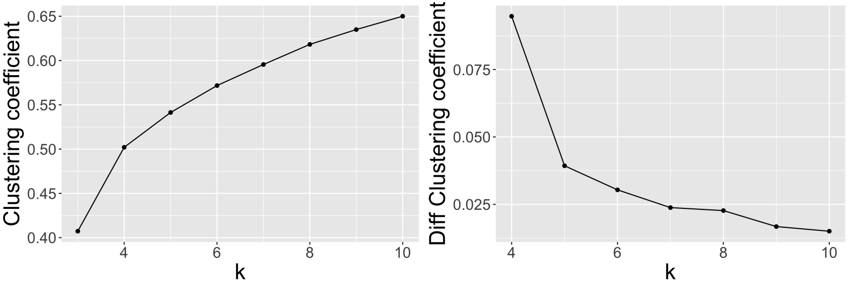
- Brute force exploration of a LHS sampling, 10 repetitions of the model for each parameter point.
- For each simulated point, closest in indicator space (euclidian distance for normalized indicators) among real points are selected.
- Among these, point with lowest distance to correlation matrix are taken.

Calibration: optimal points



Pareto plots of distance to indicators and distance to correlation matrices, for a given simulated configuration and all real points.

Causality regimes: clustering



Clustering coefficient (left) and its derivative (right) as a function of number of clusters

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