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ABSTRACT

This paper re-examines the incompatibility of Strong Pareto, as an axiom of sensitivity, and Strong Anonymity, as an axiom of impartiality, when comparing well-being profiles with a countably infinite number of components. We ask how far the Paretian principle can be extended without contradicting Strong Anonymity. We show that Strong Anonymity combined with four auxiliary axioms has two consequences: (i) There is sensitivity for an increase in one well-being component if and only if a co-finite set of other well-being components are at least ε (> 0) higher, and (ii) adding people to an infinite population cannot have positive social value.

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1. Introduction

Since the seminal paper of Diamond (1965), the conflict between sensitivity and impartiality in the evaluation of well-being profiles with an infinite number of components has been analyzed in many contributions, see e.g. Svensson (1980), Basu and Mitra (2003), Zame (2007) and Lauwers (2010), as well as Asheim (2010) for an overview. In particular, as shown by Van Liedekerke (1995) and Van Liedekerke and Lauwers (1997, p. 163) by means of the following two streams,

$$\mathbf{x} = (1, 1, 1, 0, 1, 0, \dots, 1, 0, \dots) \quad \text{and}$$

$$\mathbf{y} = (1, 0, 1, 0, 1, 0, \dots, 1, 0, \dots),$$

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the axioms of *Strong Pareto*, in the sense of being sensitive to an increase in any one well-being component of the profile if no other well-being component is decreased, is incompatible with the axiom of *Strong Anonymity*, in the sense of invariance to any permutation of a profile. In particular, $\mathbf{x} \succ \mathbf{y}$ if $\succsim$ satisfies Strong Pareto, while $\mathbf{x} \sim \mathbf{y}$ if $\succsim$ satisfies Strong Anonymity, as $\mathbf{x}$ is an infinite permutation of $\mathbf{y}$.

There are two main routes out of the dilemma that this impossibility poses:

- (1) Stick with Strong Pareto and weaken Strong Anonymity.
- (2) Stick with Strong Anonymity and weaken Strong Pareto.

Moreover, one can consider weakening both Strong Pareto and Strong Anonymity.

Route (1) has been extensively explored. Strong Pareto is compatible with the axiom of *Finite Anonymity*, in the sense of invariance to any *finite* permutation of a profile, at least if one is willing to give up completeness (Svensson, 1980). In fact, there is a literature on how to extend impartiality beyond Finite Anonymity. Specifically, a number of papers examine whether versions of the anonymity axiom that require invariance for specific set of permutations are compatible with Strong Pareto; see, for example, Fleurbaey and Michel (2003), Lauwers (1997a, 1998), and Sakai (2010). As showed by Lauwers (1997a), *Fixed-Step Anonymity*, which requires invariance to so-called fixed-step permutations, rearranging components of a profile within each fixed range of consecutive coordinates, is compatible with a transitive binary relation satisfying Strong Pareto. Mitra and Basu (2007) present a general analysis of anonymity axioms that are compatible with Strong Pareto from the view point of the requisite algebraic structure of a set of permutations. They show that an

anonymity axiom that is defined by a group of cyclic permutations is compatible with a transitive binary relation satisfying Strong Pareto (see also [Adachi et al., 2014](#)). However, [Lauwers \(2012\)](#) proves that a maximal group of cyclic permutations is a non-constructible object because its existence relies on the use of non-constructive mathematics like the Axiom of Choice. Therefore, it is impossible to give an explicit definition of a maximal anonymity axiom that is compatible with a transitive binary relation satisfying Strong Pareto.

In this paper we take route (2), not yet entirely explored, for avoiding the conflict between sensitivity and impartiality. Hence, we insist on Strong Anonymity as an axiom of impartiality and then ask how far the Paretian principle can be extended without contradicting Strong Anonymity.¹ Under Strong Anonymity we refer to the infinite well-being vectors as 'profiles' rather than 'streams', as we will be concerned with countably infinite well-being vectors where there might not be any natural order, like time, and where the people that experience the well-being might differ between vectors. However, we assume that each profile is presented by a particular 1-to-1 correspondence with the set of natural numbers.

In addition to the intrinsic interest in exploring route (2), there are two main reasons for making this directional choice. Firstly, there are situations where we might want to insist on Strong Anonymity. All versions of anonymity but Strong Anonymity require the existence of some underlying natural 1-to-1 correspondence between the components of the alternatives being compared. However, there might not be a natural 1-to-1 correspondence between future people in two different alternatives since the alternatives have different set of spatiotemporal positions or because the identities of future people are unobservable and cannot be assumed to remain fixed, even in comparisons with the same set of spatiotemporal positions. In fact, [Asheim et al. \(2022a, Theorem 1\)](#) show how Strong Anonymity for future people follows if future identities are unobservable. Moreover, one might argue that Strong Anonymity is needed in order to properly capture equal treatment. This position is adopted by [Zuber and Asheim \(2012\)](#) and [Asheim and Zuber \(2013\)](#); see also [Lauwers \(1997a\)](#) who discusses whether Finite Anonymity is strong enough to properly capture equal treatment.

Secondly, the axiom of Strong Anonymity has been severely criticized by many and thus has few proponents, in large part because it reduces sensitivity as in the example provided by [Van Liedekerke and Lauwers \(1997, p. 164\)](#). It is thus presumed that the usefulness of any strongly anonymous criterion is thereby severely undermined since Strong Pareto is considered a minimal requirement ([Askill, 2018](#); [Wilkinson, 2020](#)). Against this backdrop, it is pertinent to indicate that Strong Pareto might not be indispensable. In fact, the maximal sensitivity that is compatible with Strong Anonymity allows the full force of the Pareto principle on the set of streams with non-decreasing well-being. This is especially relevant in technologies that allow for development over time with non-decreasing well-being, by exhibiting positive productivity. Indeed, [Zuber and Asheim \(2012, Section 6\)](#) and [Asheim et al. \(2022b, Section 7\)](#) show how strongly anonymous criteria lead to optimal well-being streams that are both efficient and non-decreasing in economic models with positive productivity.

Strong Anonymity is clearly compatible with some sensitivity for an increase in a single well-being component, as illustrated by the *Maximin* order. Maximin is represented by the inferior of

all well-being components. It is invariant to any permutation of the profile, thus satisfying Strong Anonymity. If there is a sole well-being component j that is smaller than all others, then the goodness of the profile is determined by component j , making j a rank-positional dictator. This also means that an increase in well-being component j , keeping all other components constant, makes the profile better, provided that the other well-being components are at least ε (> 0) higher. This is an example of *rank-positional dominance*, as the sensitivity depends on the rank of the component that is increased: Maximin is sensitive to an increase in one well-being component if and only if all other well-being components are at least ε (> 0) higher. We will refer to this specific form of rank-positional dominance as *Inf-Restricted Dominance*.

The so-called *Leximin* order extends the sensitivity of Maximin by letting the second-lowest rank break the tie if the well-being of the lowest rank is the same in two profiles, and so forth. Leximin restores Strong Pareto in the setting of finite well-being profiles, while extensions of Leximin to the infinite-stream setting that satisfy Strong Pareto must necessarily contradict Strong Anonymity. For example, in the infinite-stream versions of Leximin characterized by [Asheim and Tungodden \(2004\)](#) and [Bossert et al. \(2007\)](#) Strong Pareto is maintained at the expense of weakening Strong Anonymity to Finite Anonymity. Hence, to define a version of Leximin in the infinite-profile setting that satisfies Strong Anonymity, Strong Pareto must be weakened. [Asheim and Zuber \(2013\)](#) do so by defining and characterizing an infinite-profile version of Leximin, which is called *Strongly Anonymous Leximin*. The Strongly Anonymous Leximin order compares infinite well-being profiles by applying the leximin principle to the non-decreasing rearrangements of well-being profiles that are constructed by listing those components below their limit inferiors (followed, for each profile, by the limit inferior if only a finite number of components are below the limit inferior). Notably, the Strongly Anonymous Leximin order has sensitivity for, not only an increase in the well-being of the worst-off, but an increase in any well-being component that is finitely ranked, in the sense that a co-finite set of well-being components are at least ε (> 0) higher. This stronger form of rank-positional dominance, which we will refer to as *Liminf-Restricted Dominance*, is also satisfied by the Extended Rank-Discounted Utilitarian order defined and characterized by [Zuber and Asheim \(2012\)](#).

The Maximin and Extended Rank-Discounted Utilitarian orders are representable by real-valued functions on the domain of a set of infinite well-being profiles. Their representability is in stark contrast to the impossibility of representing an order that satisfies Strong Pareto and Finite Anonymity, as demonstrated by [Basu and Mitra \(2003\)](#); see also a related result of [Dubey and Mitra \(2011\)](#). Representability of an order is an attractive property for practical purposes, in particular when applying the order to dynamic economic models. The possibility of representable orders satisfying Strong Anonymity provides an additional justification for examining to what extent sensitivity is compatible with Strong Anonymity.²

The central question posed in this paper is whether rank-positional dominance can be extended beyond Liminf-Restricted Dominance while insisting on Strong Anonymity. The answer is that Liminf-Restricted Dominance is as far as we can go under

¹ In addition to papers discussed below, [Sakai \(2016\)](#) explores the consequences of imposing Strong Anonymity. He combines Strong Anonymity and supnorm continuity with his Weak Non-Substitution axiom, which rules out sensitivity for an increase in well-being at one component.

² In the context of pursuing a representable order, another justification for weakening Strong Pareto could be provided from the viewpoint of intergenerational inequality. For example, recent work of [Feng et al. \(2022\)](#) extends [Harsanyi's \(1955\)](#) utilitarianism theorem to the setting with countably many generations having preferences over the future. They assume that the social criterion is a discounted sum of utilities. Using a weak Pareto axiom, they show that a higher discount rate is associated with more unequal utilitarian weights. Aversion to inequality thus implies less present bias.

four auxiliary axioms: *Monotonicity*, *Continuity*, *Limited Inequity*, and *Critical-Level Consistency*. We are thereby able to characterize the extent of the Paretian principle under Strong Anonymity, provided that these four axioms are imposed: *Liminf-Restricted Dominance is the maximal sensitivity axiom*.

As illustrated by Hilbert's paradox of the Grand Hotel (Hilbert, 2013, p. 730), one can augment an infinite population with a single person, or infinitely many people, without increasing the population's total size—which is already infinite. There is a relationship between, on the one hand, adding new people and moving the existing people to make room for the new ones and, on the other hand, increasing or decreasing already existing well-being components, corresponding to the alternative interpretation that everyone stays put. For example, assuming that each of infinitely many existing guests has the well-being of 0, corresponding to the well-being profile

$$\mathbf{z} = (0, 0, \dots),$$

we can interpret the infinite well-being profile $\mathbf{y} = (1, 0, 1, 0, 1, 0, \dots, 1, 0, \dots)$ as either of two different situations. One is that infinitely many new guests with the well-being of 1 are added to the odd-numbered rooms and every existing guest in room j is moved to the j th even-numbered room. The other interpretation is that the well-being of each guest in an odd-numbered room is increased to 1, leaving the well-being of the other guests in the even-numbered rooms unchanged. The sensitivity of increasing well-being below the limit inferior entailed by Liminf-Restricted Dominance means that adding people with well-being below this level has negative value as it can alternatively be interpreted as lowering already existing well-being components. Moreover, the insensitivity of increasing well-being at or above the limit inferior corresponds to zero value of adding people with well-being at or above this level. In both cases, adding people to a population that already is infinite has non-positive value. We show that this is a consequence of Strong Anonymity combined with the four auxiliary axioms. So even though Hilbert's Grand Hotel can always accommodate new guests, they are invariably unwelcome in social evaluation.

We start out in Section 2 by defining four versions of rank-positional dominance and four different complete, reflexive, and transitive binary relations that differ with respect to the rank-positional dominance axioms they satisfy. These rank-positional dominance axioms will not be used for our characterization results but will be important for the interpretation of those results. We introduce Strong Anonymity and the four auxiliary axioms in Section 3. Then, in Section 4, using the axioms presented in Section 3, we explore possibilities and impossibilities of rank-positional dominance and interpret those results in light of our four rank-positional dominance axioms. We state and prove the population-ethical result—that adding people cannot have positive social value—in Section 5. We finally discuss the merits of the Strong Anonymity axiom in light of these results in the concluding Section 6. We strengthen our results in an Appendix by weakening the Continuity axiom.

2. Rank-positional dominance

Let $\mathbb{R}$ (resp. $\mathbb{R}_+/\mathbb{R}_{++}$) denote set of all (resp. non-negative/positive) real numbers, and let $\mathbb{N}$ denote the set of all positive integers. A well-being profile with a countably infinite number of components is generically denoted by $\mathbf{x} = (x_1, x_2, \dots, x_j, \dots) \in \mathbb{R}^{\mathbb{N}}$, where $x_j \in \mathbb{R}$ is well-being component $j \in \mathbb{N}$. Throughout the paper, we restrict our attention to the set $\mathbf{X}$ of all bounded profiles, which is defined by

$$\mathbf{X} = \{\mathbf{x} = (x_1, x_2, \dots, x_j, \dots) \in \mathbb{R}^{\mathbb{N}} : \sup_{j \in \mathbb{N}} |x_j| < +\infty\}.$$

Our notation for vector dominance is as follows: For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, $\mathbf{x} \geq \mathbf{y}$ whenever $x_j \geq y_j$ for all $j \in \mathbb{N}$ and $\mathbf{x} > \mathbf{y}$ if $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$.

A permutation π of $\mathbb{N}$ is a bijection on $\mathbb{N}$. Let Π denote the set of all permutations of $\mathbb{N}$. For any $\mathbf{x} \in \mathbf{X}$ and any $\pi \in \Pi$, we write $\mathbf{x}_\pi = (x_{\pi(1)}, x_{\pi(2)}, \dots) \in \mathbf{X}$.

For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and any $N \in \{0\} \cup \mathbb{N}$, let $(\mathbf{y}_N, \mathbf{x}) \in \mathbf{X}$ be defined by

$$(\mathbf{y}_N, \mathbf{x}) = \begin{cases} \mathbf{x} & \text{if } N = 0, \\ (y_1, y_2, \dots, y_N, \mathbf{x}) & \text{if } 0 < N < +\infty. \end{cases}$$

Furthermore, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ and any $N = +\infty$, we write $\mathbf{z} = (\mathbf{y}, \mathbf{x}) = (\mathbf{y}_N, \mathbf{x})$ to mean $\mathbf{z} \in \mathbf{X}$ defined by

$$z_j = \begin{cases} y_n & \text{if } j = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_n & \text{if } j = 2n \text{ and } n \in \mathbb{N}, \end{cases}$$

that is, if $N = +\infty$, then $(\mathbf{y}_N, \mathbf{x}) = (\mathbf{y}, \mathbf{x}) = (y_1, x_1, y_2, x_2, \dots)$.³ Note that this particular form of combination of two infinite well-being profiles can be obtained from any form of combination of two profiles by applying a permutation $\pi \in \Pi$. Thus, under the axiom of Strong Anonymity, whose definition is presented in the next section, we can use this particular form of combination of two infinite profiles without loss of generality.

A binary relation $\succsim$ on $\mathbf{X}$ is a subset of $\mathbf{X} \times \mathbf{X}$. For simplicity, we write $\mathbf{x} \succsim \mathbf{y}$ instead of $(\mathbf{x}, \mathbf{y}) \in \succsim$. The asymmetric and symmetric parts of $\succsim$ are denoted by $\succ$ and $\sim$, respectively. For any $k \in \mathbb{N} \setminus \{1, 2\}$ and for any $\mathbf{x}^1, \mathbf{x}^2, \dots, \mathbf{x}^k \in \mathbf{X}$, if a binary relation $\succsim$ on $\mathbf{X}$ is transitive, we write, for simplicity, $\mathbf{x}^1 \succsim \mathbf{x}^2 \succsim \dots \succsim \mathbf{x}^k$ to mean that $\mathbf{x}^\ell \succsim \mathbf{x}^{\ell+1}$ for all $\ell \in \{1, 2, \dots, k-1\}$, so that $\mathbf{x}^\ell \succsim \mathbf{x}^{\ell'}$ holds for all $\ell, \ell' \in \{1, 2, \dots, k\}$ with $\ell \leq \ell'$.

We present four rank-positional dominance axioms which will be useful in understanding and interpreting the axiomatic analysis of possibilities and impossibilities of rank-positional dominance established in Section 4.

Axiom (Inf-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} > \mathbf{y}$ if $\inf_{j \in \mathbb{N}} x_j > y_{j'}$.

Axiom (Liminf-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} > \mathbf{y}$ if $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$.

Axiom (Sup-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} > \mathbf{y}$ if $x_{j'} > \sup_{j \in \mathbb{N}} y_j$.

Axiom (Limsup-Restricted Dominance). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$, $\mathbf{x} > \mathbf{y}$ if $x_{j'} > \limsup_{j \in \mathbb{N}} y_j$.

Since $\liminf_{j \in \mathbb{N}} x_j \geq \inf_{j \in \mathbb{N}} x_j$, it follows that Liminf-Restricted Dominance implies Inf-Restricted Dominance, as it applies to at least as many pairs of $\mathbf{x}$ and $\mathbf{y}$. For the same reason, as $\sup_{j \in \mathbb{N}} y_j \geq \limsup_{j \in \mathbb{N}} y_j$, Limsup-Restricted Dominance implies Sup-Restricted Dominance. Furthermore, whenever $\mathbf{x}$ and $\mathbf{y}$ are non-decreasing streams, $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$ holds if $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Thus, Liminf-Restricted Dominance implies unrestricted dominance defined for non-decreasing streams (in the sense that $\mathbf{x} > \mathbf{y}$ for all non-decreasing $\mathbf{x}, \mathbf{y}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$). Similarly, Limsup-Restricted Dominance implies unrestricted dominance defined for non-increasing streams.

³ These notational conventions illustrate Hilbert's paradox of the Grand Hotel (Hilbert, 2013, p. 730) by showing how one can augment an infinite population with a single individual, or infinitely many people.

Consider the following four complete preorders on $\mathbf{X}$.⁴

Maximin: $\succsim_M^+$ represented by $W_M^+(\mathbf{x}) = \inf_{j \in \mathbb{N}} x_j = \inf_{\pi \in \Pi} x_{\pi(1)}$.

Progressive Rank-Discounted (Generalized) Utilitarianism: $\succsim_R^+$ represented by

$$W_R^+(\mathbf{x}) = \inf_{\pi \in \Pi} (1 - \beta) \sum_{j=1}^{\infty} \beta^{j-1} g(x_{\pi(j)}),$$

where $0 < \beta < 1$ and g is a continuous and increasing function.

Maximax: $\succsim_M^-$ represented by $W_M^-(\mathbf{x}) = \sup_{j \in \mathbb{N}} x_j = \sup_{\pi \in \Pi} x_{\pi(1)}$.

Regressive Rank-Discounted (Generalized) Utilitarianism: $\succsim_R^-$ represented by

$$W_R^-(\mathbf{x}) = \sup_{\pi \in \Pi} (1 - \beta) \sum_{j=1}^{\infty} \beta^{j-1} g(x_{\pi(j)}),$$

where $0 < \beta < 1$ and g is a continuous and increasing function.

Maximin satisfies Inf-Restricted Dominance, but not the other rank-positional dominance axioms, and Progressive Rank-Discounted Utilitarianism satisfies Liminf-Restricted Dominance—and thus also Inf-Restricted Dominance—but not the two other rank-positional dominance axioms. Likewise, Maximax satisfies Sup-Restricted Dominance, but not the other rank-positional dominance axioms, while Regressive Rank-Discounted Utilitarianism satisfies Limsup-restricted dominance—and thus also Sup-Restricted Dominance—but not the two other rank-positional dominance axioms. Progressive Rank-Discounted Utilitarianism coincides with the Extended Rank-Discounted Utilitarian order, defined (somewhat differently) in Zuber and Asheim (2012, Definition 2). Progressive and Regressive Rank-Discounted Utilitarianism can be combined to become $\succsim_R^\alpha$ represented by $W_R^\alpha(\mathbf{x}) = \alpha W_R^+(\mathbf{x}) + (1 - \alpha) W_R^-(\mathbf{x})$, where $0 < \alpha < 1$. This is related to Sider's (1991) principle GV, but with the difference that the limit inferior and the limit superior take the place of zero well-being in Sider's setting, being the well-being level just sufficient to make a life worth living.

3. Axioms

We present five axioms for a preorder on $\mathbf{X}$. These axioms will be used to explore possibilities and impossibilities of rank-positional dominance in the next section.

Throughout this paper we will insist on impartiality in the sense of the Strong Anonymity axiom.

Axiom (Strong Anonymity). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x}_\pi = \mathbf{y}$ for some $\pi \in \Pi$, $\mathbf{x} \sim \mathbf{y}$.

Note that the four complete preorders presented in Section 2 satisfy Strong Anonymity.

As a first auxiliary axiom we will impose monotonicity in the weak sense that a weak increase in all well-being components does not make the well-being profile worse.

Axiom (Monotonicity). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$, $\mathbf{x} \succsim \mathbf{y}$.

We note that the conjunction of Liminf-Restricted Dominance and Monotonicity implies sensitivity to an increase in the limit inferior, and the conjunction of Limsup-Restricted Dominance and Monotonicity implies sensitivity to an increase in the limit supremum.

Lemma 1. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$. Let $\succsim$ be a preorder satisfying Monotonicity.

- (a) If $\succsim$ also satisfies Liminf-Restricted Dominance, then $\mathbf{x} > \mathbf{y}$ whenever $\liminf_{j \in \mathbb{N}} x_j > \liminf_{j \in \mathbb{N}} y_j$.
- (b) If $\succsim$ also satisfies Limsup-Restricted Dominance, then $\mathbf{x} > \mathbf{y}$ whenever $\limsup_{j \in \mathbb{N}} x_j > \limsup_{j \in \mathbb{N}} y_j$.

Proof. Part (a) Let $0 < \varepsilon < \liminf_{j \in \mathbb{N}} x_j - \liminf_{j \in \mathbb{N}} y_j$. By the definition of the limit inferior, there are infinitely many integers k such that $y_k < \liminf_{j \in \mathbb{N}} y_j + \varepsilon$ but only finitely many integers ℓ such that $x_\ell \leq \liminf_{j \in \mathbb{N}} y_j + \varepsilon < \liminf_{j \in \mathbb{N}} x_j$. So for infinitely many integers m , $x_m > \liminf_{j \in \mathbb{N}} y_j + \varepsilon > y_m$. Consider one of these integers, say i . Let $\mathbf{z}$ be derived from $\mathbf{x}$ by replacing x_i by y_i , where by the choice of ε and i , $\min\{x_i, \liminf_{j \in \mathbb{N}} x_j\} > y_i$. By Monotonicity, $\mathbf{z} \succsim \mathbf{y}$, and by Liminf-Restricted Dominance, $\mathbf{x} > \mathbf{z}$. Hence, by transitivity, $\mathbf{x} > \mathbf{y}$.

Part (b). The proof is similar. $\square$

In most of this paper, we also impose a continuity requirement using the supnorm topology, based on the distance function $d: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}_+$ given by, for all $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$d(\mathbf{x}, \mathbf{y}) = \sup_{i \in \mathbb{N}} |x_i - y_i|.$$

Axiom (Continuity). For any $\mathbf{x} \in \mathbf{X}$, the sets $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \succsim \mathbf{x}\}$ and $\{\mathbf{y} \in \mathbf{X} : \mathbf{x} \succsim \mathbf{y}\}$ are closed in the supnorm topology.

Note that the four complete preorders presented in Section 2 satisfy this continuity axiom. Another standard topology used in $\mathbf{X}$ is the product topology, which is larger than the supnorm topology, so that it yields a stronger continuity axiom. To the best of our knowledge, no preorder satisfying Strong Anonymity and the continuity axiom defined by the product topology have been proposed in the literature; indeed, the four complete preorders in Section 2 do not satisfy the continuity axiom defined by the product topology. Lauwers (1997b) showed that no representable social order can satisfy continuity in the product topology and even the weak finite anonymity axiom (that allows only finitely many permutations). Lauwers (1997b) actually shows that the supnorm continuity is the strongest continuity axiom compatible with equity in the context of complete social order. Thus, the continuity axiom defined by the supnorm topology is a natural choice for our analysis. On the other hand, one may argue that the supnorm topology is still a rather large topology. However, it makes it possible to prove our results in a straightforward manner. In Appendix we show how we can establish our main results also under weaker continuity properties.

To state the third auxiliary axiom, let the set $\mathcal{E}^\sim$ be defined by:

$$\mathcal{E}^\sim = \{\varepsilon \in \mathbb{R}_{++} : \text{for any } \mathbf{x}, \mathbf{y} \in \mathbf{X} \text{ with } y_{j'} < x_{j'} \leq x_{j''} < y_{j''} \text{ for some } j', j'' \in \mathbb{N}, \text{ and } x_j = y_j \text{ for all } j \in \mathbb{N} \setminus \{j', j''\}, \mathbf{x} \succsim \mathbf{y} \text{ if } y_{j'} - x_{j'} = \varepsilon (x_{j''} - y_{j''})\}.$$

Note that $\mathcal{E}^\sim = \mathbb{R}_{++}$ under the axiom of Hammond equity (Hammond, 1976). If non-leaky transfers are acceptable, then $1 \in \mathcal{E}^\sim$. Hence, $\mathcal{E}^\sim$ is also non-empty if $\succsim$ satisfies the Pigou–Dalton transfer principle in its weak version (Pigou, 1912; Dalton, 1920). Here we will be concerned with a much weaker axiom than both Hammond equity and Pigou–Dalton (but stronger than the minimal equity axiom proposed by Deschamps and Gevers, 1978): we simply ask that transfers from rich to poor are acceptable if the well-being loss of the rich is sufficiently small.⁵

⁵ To emphasize just how weak this axiom is, note that every generalized utilitarian preorder with a transformation function that is finitely non-concave satisfies Limited Inequity. So in order for $\mathcal{E}^\sim = \emptyset$, the preorder $\succsim$ has to be even less egalitarian than utilitarianism.

⁴ A preorder is a reflexive and transitive binary relation.

Axiom (Limited Inequity). $\mathcal{E}^{\approx} \neq \emptyset$.

The fourth and final auxiliary axiom relates to the effects of adding one individual (or infinitely many people) to a population. The literature on population ethics stemming from Parfit (1984) has addressed this issue, and Broome (2004) and Blackorby et al. (2005) discuss it at length. In particular, Blackorby and Donaldson (1984) and Blackorby et al. (1995) argue in favor of the existence of a critical level, such that an individual's life contributes positively to the social value of a population if and only if the well-being of the additional individual is above this critical level. In general, the critical level may depend on how well-being is distributed. The following axiom imposes some regularity in the level of the critical level for infinite populations. The axiom asserts that, if it is acceptable to add one individual at some well-being level z to a population, then it is also acceptable to add infinitely many people at this level. The following additional notation is needed: For any $z \in \mathbb{R}$ and any $\mathbf{x} \in \mathbf{X}$, $(z, \mathbf{x}) = (z, x_1, x_2, \dots) \in \mathbf{X}$, $z\mathbb{1}_{\mathbb{N}} = (z, z, \dots) \in \mathbf{X}$, and $(z\mathbb{1}_{\mathbb{N}}, \mathbf{x}) = (z, x_1, z, x_2, \dots) \in \mathbf{X}$.

Axiom (Critical-Level Consistency). For any $\mathbf{x} \in \mathbf{X}$, and any $z \in \mathbb{R}$, $\mathbf{x} \succsim (z, \mathbf{x})$ (resp. $\mathbf{x} \precsim (z, \mathbf{x})$) if and only if $\mathbf{x} \succsim (z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ (resp. $\mathbf{x} \precsim (z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$).

All four preorders of Section 2 satisfy Strong Anonymity, Monotonicity, and Continuity, as well as Critical-Level Consistency. To establish the latter claim, note that:

- $\mathbf{x} \succ_M^+(z, \mathbf{x})$ and $\mathbf{x} \succ_M^+(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ if $z < \inf_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_M^+(z, \mathbf{x}) \sim_M^+(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ otherwise,
- $\mathbf{x} \succ_R^+(z, \mathbf{x})$ and $\mathbf{x} \succ_R^+(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ if $z < \liminf_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_R^+(z, \mathbf{x}) \sim_R^+(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ otherwise,
- $\mathbf{x} \prec_M^-(z, \mathbf{x})$ and $\mathbf{x} \prec_M^-(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ if $z > \sup_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_M^-(z, \mathbf{x}) \sim_M^-(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ otherwise,
- $\mathbf{x} \prec_R^-(z, \mathbf{x})$ and $\mathbf{x} \prec_R^-(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ if $z > \limsup_{j \in \mathbb{N}} x_j$, while $\mathbf{x} \sim_R^-(z, \mathbf{x}) \sim_R^-(z\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ otherwise.

However, of the four preorders, only Maximin and Progressive Rank-Discounted Utilitarianism satisfy Limited Inequity. To see this, note that $\mathcal{E}^{\approx+}_M = \mathbb{R}_{++}$ and $1 \in \mathcal{E}^{\approx+}_R$ (under the assumptions on the rank-discount factor β and the transformation function g given by Zuber and Asheim, 2012, Proposition 6), while $\mathcal{E}^{\approx-}_M = \emptyset$ since both Maximax and Regressive Rank-Discounted Utilitarianism have sensitivity above the supremum, but no sensitivity below the limit supremum (so that well-being gains below the limit supremum are worthless and cannot compensate for a well-being loss for a maximal component).

4. Possibilities and impossibilities of rank-positional dominance

We first introduce and prove two lemmas before turning to the main results of this paper. To introduce the first of these lemmas, recall the example of two unbounded profiles used by Fleurbaey and Michel (2003, pp. 795–796) to prove that Strong Anonymity is incompatible with even the Weak Pareto axiom in the sense of being sensitive to an increase in all well-being components. A small variation of their example allows us to show the conflict between these two axioms also in our setting of bounded profiles: There exists $\pi \in \Pi$ such that

$$\mathbf{z} = \left(\frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots\right), \text{ and}$$

$$\mathbf{z}' = \left(\frac{3}{4}, \frac{2}{3}, \frac{4}{5}, \frac{1}{3}, \frac{5}{6}, \frac{1}{4}, \dots, \frac{k+2}{k+3}, \frac{1}{k+1}, \dots\right),$$

where by Strong Anonymity $\mathbf{z}$ is indifferent to $\mathbf{z}'$ even though $z_j < z'_{\pi(j)}$ for all $j \in \mathbb{N}$. Notice that $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$, where $\mathbf{z}^+$ is an increasing subsequence

$$\mathbf{z}^+ = \left(\frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \dots, \frac{k+1}{k+2}, \dots\right),$$

and $\mathbf{z}^-$ is a decreasing subsequence

$$\mathbf{z}^- = \left(\frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \dots, \frac{1}{k+2}, \dots\right).$$

Consider now the profiles $\mathbf{x}$ and $\mathbf{y}$, where

$$\mathbf{x} = \left(\frac{2}{3}, \mathbf{z}\right) = \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots\right), \text{ and}$$

$$\mathbf{y} = \left(\frac{1}{3}, \mathbf{z}\right) = \left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots\right),$$

Clearly, if $\succsim$ satisfies Monotonicity, then $\mathbf{x} \succsim \mathbf{y}$, as the only difference between the two profiles is that $x_1 = 2/3 > 1/3 = y_1$. However, the fact that $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$ with $\mathbf{z}^+$ being an increasing sequence with $x_1 = 2/3 = z_1^+$ and $\mathbf{z}^-$ being a decreasing sequence with $y_1 = 1/3 = z_1^-$ implies that there exists $\pi' \in \Pi$ such that $\mathbf{y}_{\pi'} \geq \mathbf{x}$:

$$\mathbf{y}_{\pi'} = \left(\frac{2}{3}, \frac{3}{4}, \frac{1}{3}, \frac{4}{5}, \frac{1}{3}, \frac{5}{6}, \frac{1}{4}, \dots, \frac{k+2}{k+3}, \frac{1}{k+1}, \dots\right).$$

So, if $\succsim$ satisfies also Strong Anonymity, then $\mathbf{y} \sim \mathbf{y}_{\pi'} \succsim \mathbf{x} \succsim \mathbf{y}$. Hence, by reflexivity and transitivity of $\succsim$, $\mathbf{x} \sim \mathbf{y}$, showing that $\succsim$ cannot be sensitive to an increase in a component from $1/3$ to $2/3$ when the rest of the profile equals $\mathbf{z} = (\mathbf{z}^+, \mathbf{z}^-)$. The first lemma generalizes this observation.

Lemma 2. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'} = z_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j = z_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. If $\succsim$ is a preorder satisfying Strong Anonymity and Monotonicity, then $\mathbf{x} \sim \mathbf{y}$ whenever $(z_j)_{j \in \mathbb{N} \setminus \{j'\}}$ satisfies

- there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N} \setminus \{j'\}$ such that, for all $k \in \mathbb{N}$, $x_{j'} \leq z_{f(k)} \leq z_{f(k+1)}$, and
- there exists an increasing function $g: \mathbb{N} \rightarrow \mathbb{N} \setminus \{j'\}$ such that, for all $k \in \mathbb{N}$, $y_{j'} \geq z_{g(k)} \geq z_{g(k+1)}$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j = z_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Assume that $\succsim$ is a reflexive and transitive binary relation satisfying Strong Anonymity and Monotonicity. Let $(z_j)_{j \in \mathbb{N} \setminus \{j'\}}$ satisfy (a) and (b).

Consider the permutation $\pi \in \Pi$ constructed by

- $\pi(j') = f(1)$ and $\pi(f(k)) = f(k+1)$ for all $k \in \mathbb{N}$,
- $\pi(g(1)) = j'$ and $\pi(g(k+1)) = g(k)$ for all $k \in \mathbb{N}$,

and $\pi(j) = j$ otherwise. Then $\mathbf{y}_{\pi} \geq \mathbf{x} > \mathbf{y}$, so by Strong Anonymity and Monotonicity: $\mathbf{y} \sim \mathbf{y}_{\pi} \succsim \mathbf{x} \succsim \mathbf{y}$. Hence, by reflexivity and transitivity of $\succsim$, $\mathbf{x} \sim \mathbf{y}$. $\square$

In fact, Lemma 2 can be used to show that $\mathbf{x}' \sim \mathbf{y}'$, where

$$\mathbf{x}' = (x'_1, \mathbf{z}) = \left(x'_1, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots\right), \text{ and}$$

$$\mathbf{y}' = (y'_1, \mathbf{z}) = \left(y'_1, \frac{2}{3}, \frac{1}{3}, \frac{3}{4}, \frac{1}{4}, \frac{4}{5}, \frac{1}{5}, \dots, \frac{k+1}{k+2}, \frac{1}{k+2}, \dots\right),$$

whenever $0 = \liminf_{j \in \mathbb{N}} x'_j < y'_1 < x'_1 < \limsup_{j \in \mathbb{N}} y'_j = 1$ since subsequences as specified in (a) and (b) exist. Two such subsequences do not exist if $0 = y'_1 < x'_1 \leq 1$ or $0 \leq y'_1 < x'_1 = 1$, so that Lemma 2 does not apply under these cases. However, the non-sensitivity result of Lemma 2 can be extended to cover also such circumstances if Continuity is imposed. This extension relies on the following lemma, which shows that any preorder satisfying Strong Anonymity and Continuity is invariant to adding one individual (or infinitely many people) with well-being equal to the limit inferior or the limit supremum, or indeed equal to any other cluster point.⁶

⁶ A cluster point of an infinite set of well-being levels is a point such that, for every neighborhood, there are infinitely many elements of the set with well-being levels within the neighborhood.

Lemma 3. Consider $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for $\mathbf{x}$. If $\succsim$ is a preorder satisfying Strong Anonymity and Continuity, then $\mathbf{x} \sim (z, \mathbf{x}) \sim (z \mathbb{1}_N, \mathbf{x})$.

Proof. Let $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for $\mathbf{x}$. Assume that $\succsim$ is a preorder satisfying Strong Anonymity and Continuity. We prove that $\mathbf{x} \sim (z, \mathbf{x})$ and $\mathbf{x} \sim (z \mathbb{1}_N, \mathbf{x})$.

Since z is a cluster point of $\mathbf{x}$, there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $\lim_{j \rightarrow \infty} |x_{f(j)} - z| = 0$. Hence, for every $m \in \mathbb{N}$, there exists $N(m) \in \mathbb{N}$ such that $|x_{f(j)} - z| < 1/(2m)$ for all $j > N(m)$. For each $m \in \mathbb{N}$, define the increasing function $f^m: \mathbb{N} \rightarrow \mathbb{N}$ by $f^m(j) = f(j + N(m))$ for all $j \in \mathbb{N}$. By construction, for every $m \in \mathbb{N}$ and for all $j, j' \in \mathbb{N}$,

$$|x_{f^m(j)} - x_{f^m(j')}| \leq |x_{f^m(j)} - z| + |z - x_{f^m(j')}| < \frac{1}{2m} + \frac{1}{2m} = \frac{1}{m}. \quad (1)$$

For all $m \in \mathbb{N}$, let $\tilde{\mathbf{x}}^m = (x_{f^m(k)})_{k \in \mathbb{N}}$ and we define sequences $(\hat{\mathbf{x}}^m)_{m \in \mathbb{N}}, (\tilde{\mathbf{x}}^m)_{m \in \mathbb{N}}$ in $\mathbf{X}$ as follows. For all $m \in \mathbb{N}$,

$$\hat{x}_j^m = \tilde{x}_j^m = x_j \quad \text{for all } j \in \mathbb{N} \setminus \{f^m(k) : k \in \mathbb{N}\}$$

and

$$(\hat{x}_{f^m(k)}^m)_{k \in \mathbb{N}} = (z, \tilde{\mathbf{x}}^m) \quad \text{and} \quad (\tilde{x}_{f^m(k)}^m)_{k \in \mathbb{N}} = (z \mathbb{1}_N, \tilde{\mathbf{x}}^m).$$

From (1) and the construction of $(\hat{\mathbf{x}}^m)_{m \in \mathbb{N}}$ and $(\tilde{\mathbf{x}}^m)_{m \in \mathbb{N}}$, it follows that

$$\lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |x_j - \hat{x}_j^m| = \lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |\tilde{x}_j^m - \hat{x}_{f^m(j)}^m| = 0, \quad (2a)$$

$$\lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |x_j - \tilde{x}_j^m| = \lim_{m \rightarrow +\infty} \sup_{j \in \mathbb{N}} |\tilde{x}_j^m - \tilde{x}_{f^m(j)}^m| = 0. \quad (2b)$$

Note that for each $m \in \mathbb{N}$, there exist $\pi^m, \rho^m \in \Pi$ such that

$$(z, \mathbf{x}) = \hat{\mathbf{x}}_{\pi^m}^m \quad \text{and} \quad (z \mathbb{1}_N, \mathbf{x}) = \tilde{\mathbf{x}}_{\rho^m}^m.$$

By Strong Anonymity, we obtain $(z, \mathbf{x}) \sim \hat{\mathbf{x}}^m$ and $(z \mathbb{1}_N, \mathbf{x}) \sim \tilde{\mathbf{x}}^m$ for all $m \in \mathbb{N}$. Since $\succsim$ satisfies Continuity, it follows from (2a) and (2b) that $(z, \mathbf{x}) \sim \mathbf{x}$ and $(z \mathbb{1}_N, \mathbf{x}) \sim \mathbf{x}$. $\square$

Lemma 3 can also be seen as a generalization of the observation that, under Strong Anonymity, the profiles $(1, 0, 1, 0, 1, 0, \dots)$ and $(0, 1, 0, 1, 0, 1, \dots)$ are equally good, as one profile can be obtained from the other through an infinite permutation. However, the second profile can also be obtained from the first by adding one individual with well-being equal to the limit inferior and the first profile can be obtained from the second by adding an individual with well-being equal to the limit supremum.

Using our two lemmas, we are able to prove three results regarding rank-positional dominance. The first proposition shows that any preorder satisfying Strong Anonymity, Monotonicity, and Continuity is insensitive to increasing the well-being at a particular component of the profile between the limit inferior and the limit supremum.

Proposition 1. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. The two following statements are equivalent:

- (1) For all preorders $\succsim$ satisfying Strong Anonymity, Monotonicity, and Continuity, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $\limsup_{j \in \mathbb{N}} y_j \geq x_{j'}$ and $y_{j'} \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$.

We first show that (1) implies (2). Suppose that (2) does not hold. There are two subcases:

Subcase (i): $x_{j'} > \limsup_{j \in \mathbb{N}} y_j$. Consider $\succsim_R^-$. We have that $\succsim_R^-$ is a preorder that satisfies Strong Anonymity, Monotonicity,

and Continuity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since $\succsim_R^-$ satisfies Limsup-Restricted Dominance.

Subcase (ii): $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$. Consider $\succsim_R^+$. We have that $\succsim_R^+$ is a preorder that satisfies Strong Anonymity, Monotonicity, and Continuity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since $\succsim_R^+$ satisfies Liminf-Restricted Dominance.

Next, we show that (2) implies (1). Assume that (2) holds, and assume that $\succsim$ is a preorder that satisfies Strong Anonymity, Monotonicity, and Continuity.

Both $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$ and $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ are cluster points for $\mathbf{x}$ and $\mathbf{y}$, so by repeated use of Lemma 3 we have that

$$\mathbf{x} \sim (\ell \mathbb{1}_N, \mathbf{x}) \sim (u \mathbb{1}_N, (\ell \mathbb{1}_N, \mathbf{x})) \quad \text{and} \quad \mathbf{y} \sim (\ell \mathbb{1}_N, \mathbf{y}) \sim (u \mathbb{1}_N, (\ell \mathbb{1}_N, \mathbf{y})).$$

Write $\mathbf{x}' = (u \mathbb{1}_N, (\ell \mathbb{1}_N, \mathbf{x}))$ and $\mathbf{y}' = (u \mathbb{1}_N, (\ell \mathbb{1}_N, \mathbf{y}))$. By construction, $x'_{j'} > y'_{j'}$ for some $j' \in \mathbb{N}$ and $x'_j = y'_j = z_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Furthermore, $(z_j)_{j \in \mathbb{N} \setminus \{j'\}}$ satisfies

- (a) there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N} \setminus \{j'\}$ such that, for all $k \in \mathbb{N}$, $x'_{f(k)} \leq z_{f(k)} = u$, and
- (b) there exists an increasing function $g: \mathbb{N} \rightarrow \mathbb{N} \setminus \{j'\}$ such that, for all $k \in \mathbb{N}$, $y'_{g(k)} \geq z_{g(k)} = \ell$.

By Lemma 2, $\mathbf{x}' \sim \mathbf{y}'$. Thus, $\mathbf{x} \sim \mathbf{y}$ by the transitivity of $\succsim$. $\square$

Before stating our two last results, we prove the following lemma which goes one step further by adding the Limited Inequity axiom. It shows that a binary relation that satisfies also this axiom is invariant to increasing the well-being at a particular component of the profile that is already at the limit inferior or above, in the case where the profile under consideration has a limit inferior different from its limit supremum.

Lemma 4. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Assume that $\succsim$ is a preorder satisfying Strong Anonymity, Monotonicity, Continuity, and Limited Inequity. Then $\mathbf{x} \sim \mathbf{y}$ if $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$ and $y_{j'} \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. Assume that $\succsim$ is a preorder that satisfies Strong Anonymity, Monotonicity, Continuity, and Limited Inequity. Let $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$ and $y_{j'} \geq \liminf_{j \in \mathbb{N}} x_j$.

The result follows from Proposition 1 if $\limsup_{j \in \mathbb{N}} y_j \geq x_{j'}$.

Now, suppose that $x_{j'} > \limsup_{j \in \mathbb{N}} y_j$. By Monotonicity, $\mathbf{x} \not\sim \mathbf{y}$. We need to establish that $\mathbf{y} \not\sim \mathbf{x}$. Write $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$, $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $\delta = \varepsilon(u - \ell)$, where $\varepsilon > 0$ is chosen such that $\varepsilon \in \mathcal{E}^{\succsim}$; this is possible since by Limited Inequity $\mathcal{E}^{\succsim} \neq \emptyset$.

Assume that there exists $k \in \mathbb{N} \setminus \{j'\}$ such that $u \geq x_k = y_k \geq \ell$. Define $\mathbf{y}^0$ by $y_{j'}^0 = \max\{y_{j'}, u\}$, $y_k^0 = u$ and $y_j^0 = y_j$ for $j \in \mathbb{N} \setminus \{j', k\}$. Define inductively $\mathbf{x}^n$ by $x_{j'}^n = y_{j'}^{n-1} + \delta$ and $x_k^n = \ell$ and $x_j^n = y_j^{n-1}$ for $j \in \mathbb{N} \setminus \{j', k\}$, and $\mathbf{y}^n$ by $y_k^n = u$ and $y_j^n = x_j^n$ for $j \in \mathbb{N} \setminus \{k\}$ for $n = 1, 2, 3, \dots, \bar{n}$ until $\mathbf{y}^{\bar{n}} \geq \mathbf{x}$. Then:

$$\mathbf{y} \sim \mathbf{y}^0 \succsim \mathbf{x}^1 \sim \mathbf{y}^1 \succsim \dots \succsim \mathbf{x}^{\bar{n}} \sim \mathbf{y}^{\bar{n}} \succsim \mathbf{x},$$

where $\mathbf{y} \sim \mathbf{y}^0$ and $\mathbf{x}^{\bar{n}} \sim \mathbf{y}^{\bar{n}}$ follow from Proposition 1, $\mathbf{y}^{n-1} \succsim \mathbf{x}^n$ for $n = 1, 2, 3, \dots, \bar{n}$ follow from Limited Inequity, and $\mathbf{y}^{\bar{n}} \succsim \mathbf{x}$ follows from Monotonicity.

If there does not exist $k \in \mathbb{N} \setminus \{j'\}$ such that $u \geq x_k = y_k \geq \ell$, it follows from Lemma 3 that $\mathbf{x} \sim \mathbf{y}$ if and only if $(u, \mathbf{x}) \sim (u, \mathbf{y})$ since $u = \limsup_{j \in \mathbb{N}} x_j = \limsup_{j \in \mathbb{N}} y_j$ is a cluster point for $\mathbf{x}$ and $\mathbf{y}$. Hence, we can use the argument above on the pair $((u, \mathbf{x}), (u, \mathbf{y}))$ instead of $(\mathbf{x}, \mathbf{y})$. $\square$

Lemma 4 implies that Limsup-Restricted Dominance is inconsistent with the four axioms of the lemma because such

dominance cannot be imposed on profiles where the limit inferior is strictly lower than the limit supremum. This almost establishes the result that any preorder satisfying these four axioms cannot exhibit any sensitivity beyond Liminf-Restricted Dominance. To show this result we must remove the restriction made in Lemma 4 that the profiles under consideration must have a limit inferior strictly lower than the limit supremum. The next proposition shows that it is true in general that if a preorder satisfies Strong Anonymity, Monotonicity, Continuity, and Limited Inequity, then it cannot exhibit sensitivity beyond Liminf-Restricted Dominance. Moreover, this result is tight since the converse follows from the fact that Progressive Rank-Discounted Utilitarianism satisfies these axioms as well as Liminf-Restricted Dominance.

Proposition 2. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$. The two following statements are equivalent:

- (1) For all preorders $\succsim$ satisfying Strong Anonymity, Monotonicity, Continuity, and Limited Inequity, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $y_{j'} \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $x_{j'} > y_{j'}$ for some $j' \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{j'\}$.

We first show that (1) implies (2). Suppose that (2) does not hold, implying that $\liminf_{j \in \mathbb{N}} x_j > y_{j'}$. Consider $\succsim_R^+$ with g being linear. We have that $\succsim_R^+$ is a reflexive and transitive binary relation that satisfies Strong Anonymity, Monotonicity, Continuity, and Limited Inequity. Furthermore, $\mathbf{x} \succ \mathbf{y}$ since $\succsim_R^+$ satisfies Liminf-Restricted Dominance.

Next, we show that (2) implies (1). Assume that (2) holds, and assume that $\succsim$ is a preorder that satisfies Strong Anonymity, Monotonicity, Continuity, and Limited Inequity. There are two subcases:

Subcase (i): $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_j$. The result follows from Lemma 4.

Subcase (ii): $\limsup_{j \in \mathbb{N}} y_j = \liminf_{j \in \mathbb{N}} x_j = \ell$. Since ℓ is a cluster point of $\mathbf{x}$ and $\mathbf{y}$, it follows from Lemma 3 that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \sim \mathbf{y}$. Let $(\varepsilon_n)_{n \in \mathbb{N}}$ be a sequence of positive real numbers converging to 0. By Lemma 4, $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim ((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for each $n \in \mathbb{N}$. Furthermore, $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{x}) > (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ so that $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \succ (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ by Monotonicity. By transitivity, $((\ell + \varepsilon_n) \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succ \mathbf{x}$. This is true for each $n \in \mathbb{N}$ so that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succ \mathbf{x}$ by Continuity and $\mathbf{y} \succ \mathbf{x}$ by transitivity. On the other hand, $\mathbf{x} \succsim \mathbf{y}$ by Monotonicity. $\square$

Proposition 2 shows that dominance can occur under Strong Anonymity, Monotonicity, Continuity, and Limited Inequity, when increasing the well-being at a particular component of the profile, if and only if the well-being at this component is strictly lower than the limit inferior before the increase. Any preorder satisfying these axioms is invariant to an increase in the well-being at a particular component of the profile at or above the limit inferior. Notice that Regressive Rank-Discounted Utilitarianism satisfy Limsup-Restricted Dominance and all the other axioms except for Limited Inequity. Our minimal equity requirement is essential for obtaining the result. On the other hand, Inf-Restricted Dominance and Liminf-Restricted Dominance are clearly compatible with the other axioms: both Maximin and Progressive Rank-Discounted Utilitarianism satisfy Strong Anonymity, Monotonicity, Continuity, and Limited Inequity.

Our last proposition in this section shows that we can generalize the result to the case where not only one component of the profile, but infinitely many components experience an increase in well-being above the limit inferior. To obtain this result, we need the additional axiom of Critical-Level Consistency.

Proposition 3. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} > \mathbf{y}$. The two following statements are equivalent:

- (1) For all preorders $\succsim$ satisfying Strong Anonymity, Monotonicity, Continuity, Limited Inequity, and Critical-Level Consistency, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $y_{j'} \geq \ell$ for all $j' \in \mathbb{N}$ such that $x_{j'} > y_{j'}$.

Proof. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that $\mathbf{x} > \mathbf{y}$.

We first show that (1) implies (2). Suppose that (2) does not hold, so that (i) $\liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and there exists $j' \in \mathbb{N}$ such that $\min\{x_{j'}, \liminf_{j \in \mathbb{N}} x_j\} > y_{j'}$, or (ii) $\liminf_{j \in \mathbb{N}} x_j > \liminf_{j \in \mathbb{N}} y_j$. Consider $\succsim_R^+$ with g being linear. We have that $\succsim_R^+$ is a preorder that satisfies Strong Anonymity, Monotonicity, Continuity, Limited Inequity, and Critical-Level Consistency. Furthermore, $\mathbf{x} > \mathbf{y}$ since $\succsim_R^+$ satisfies Monotonicity and Liminf-Restricted Dominance (Lemma 1(a), case (ii)).

Next, we show that (2) implies (1). Assume that (2) holds, and assume that $\succsim$ is a preorder that satisfies Strong Anonymity, Monotonicity, Continuity, Limited Inequity, and Critical-Level Consistency. Let $I_{>} = \{i \in \mathbb{N} : x_i > y_i\}$ denote the set of coordinates where utility is strictly larger in $\mathbf{x}$ than in $\mathbf{y}$. There are two subcases:

Subcase (i): $|I_{>}| < +\infty$. In this case, the result follows from repeated applications of Proposition 2 and transitivity.

Subcase (ii): $|I_{>}| = +\infty$. Since ℓ is a cluster point, by Lemma 3 we obtain $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \sim \mathbf{y}$. Let $\mathbf{z}$ and $\hat{\mathbf{z}}$ be two profiles such that:

$$z_i = \begin{cases} \ell & \text{if } i = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_i & \text{if } i = 2n \text{ and } n \in \mathbb{N} \setminus I_{>}, \\ \sup_{j \in \mathbb{N}} x_j & \text{if } i = 2n \text{ and } n \in I_{>}, \end{cases}$$

and

$$\hat{z}_i = \begin{cases} \ell & \text{if } i = 2n - 1 \text{ and } n \in \mathbb{N}, \\ x_i & \text{if } i = 2n \text{ and } n \in \mathbb{N} \setminus I_{>}, \\ \ell & \text{if } i = 2n \text{ and } n \in I_{>}. \end{cases}$$

By definition, $\mathbf{z} \geq (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \geq \hat{\mathbf{z}}$, so that by Monotonicity $\mathbf{z} \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succsim \hat{\mathbf{z}}$. Now we want to show that $\mathbf{z} \sim \hat{\mathbf{z}}$ to obtain by transitivity that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and therefore $\mathbf{y} \succsim \mathbf{x}$.

Let $J = \{i \in \mathbb{N} : \exists k \in I_{>}, i = 2k\}$. In profiles $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$, $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$, $\mathbf{z}$ and $\hat{\mathbf{z}}$, coordinates in J are those where the profiles differ. Let f be the unique increasing bijection between $\mathbb{N}$ and $\mathbb{N} \setminus J$. Define $\tilde{\mathbf{z}}$ by $\tilde{z}_n = z_{f(n)}$ for all $n \in \mathbb{N}$: profile $\tilde{\mathbf{z}}$ collects all coordinates that are the same in $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$, $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$, $\mathbf{z}$ and $\hat{\mathbf{z}}$. Let $\tilde{x} = \sup_{j \in \mathbb{N}} x_j$, by Proposition 2, $(\tilde{x}, \tilde{\mathbf{z}}) \sim (\ell, \tilde{\mathbf{z}})$. By Critical-Level Consistency, $(\tilde{x}, \tilde{\mathbf{z}}) \sim (\tilde{x} \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$ and $(\ell, \tilde{\mathbf{z}}) \sim (\ell \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$. Clearly, $\mathbf{z}$ can be obtained from $(\tilde{x} \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$ and $\hat{\mathbf{z}}$ can be obtained from $(\ell \mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{z}})$ through a permutation. Therefore, by Strong Anonymity and transitivity, $\mathbf{z} \sim \hat{\mathbf{z}}$.

We thus know that $\mathbf{y} \succsim \mathbf{x}$. But $\mathbf{x} \geq \mathbf{y}$ so that by Monotonicity $\mathbf{x} \succsim \mathbf{y}$. $\square$

An implication of Proposition 3 is that even an increase in utility at an infinite number of components may not be sufficient to guarantee social dominance. For instance, Proposition 3 applies to the following two profiles:

$$(1, 0, 1, 0, \dots, 1, 0, \dots) \quad \text{and} \quad (0, 0, 0, 0, \dots).$$

As mentioned in the introduction to this section, Fleurbaey and Michel (2003) proved that Strong Anonymity is incompatible with the Weak Pareto axiom using an example. Proposition 3 provides a larger set of cases where the Weak Pareto axiom fails when our other axioms are satisfied: all cases where there is

a strict improvement above the limit inferior without changing the limit inferior. For instance, [Proposition 3](#) also applies to the following two profiles:

$$(1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \dots, \frac{1}{k}, \dots) \text{ and } (0, 0, 0, 0, \dots).$$

Remark. Our analysis builds on the assumption that well-being is real-valued. If instead well-being is a binary variable, being equal to 0 or 1, then clearly Continuity and Limited Inequity have no bite. In this case, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} > \mathbf{y}$, the two following statements are equivalent:

- (1) For all preorders $\succsim$ satisfying Strong Anonymity, Monotonicity, and Critical-Level Consistency, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $\limsup_{j \in \mathbb{N}} y_j = 1$ and $\liminf_{j \in \mathbb{N}} x_j = 0$.

Hence, there is no sensitivity as long as both $\mathbf{x}$ and $\mathbf{y}$ have infinitely many elements at both 0 and 1. If not, $\succsim_T^\alpha$, the combined Rank-Discounted Utilitarianism defined in [Section 2](#), shows that sensitivity is consistent with Strong Anonymity, Monotonicity, and Critical-Level Consistency.

Finally, as their proofs show, our characterization results established in this section rely on that fact that the Progressive Rank-Discounted (Generalized) Utilitarian order satisfies the continuity axiom defined by the supnorm topology. An interesting question to ask is what if the stronger continuity axiom defined by the product topology is employed instead of the supnorm continuity. We leave this interesting question for future research.

5. Infinite population ethics under strong anonymity

As we have discussed before, the literature on population ethics considers the effect of adding an individual, or several people, to a population. [Parfit \(1984\)](#) has introduced the Mere Addition Principle: the addition of someone with a non-negative well-being should always be acceptable. The problem is that, under mild conditions, this principle may yield the ‘repugnant conclusion’ by which a very large population of people with lives barely worth living may be better than a large but smaller population of people with excellent lives. To avoid this conclusion, [Blackorby and Donaldson \(1984\)](#) and [Blackorby et al. \(1995\)](#) have proposed Critical-Level Utilitarianism, according to which adding only sufficiently good lives are socially acceptable.

In any case, most approaches assume that there exist levels of well-being such that adding someone with a well-being at these levels is a strict social improvement. However, this assumption is made in the setting of a finite population. In contrast, the following property is concerned with the addition of finitely or infinitely many people to a population that already has infinitely many people. It asserts that the social value of adding finitely or infinitely many lives is non-positive regardless of their well-being. Note that, in the statement below, the profile $\mathbf{x}$ can be viewed as being obtained from profile $\mathbf{y}$ by “removing” some people.

Axiom (Non-Positive Value of Additional Lives). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, if there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $x_j = y_{f(j)}$ for all $j \in \mathbb{N}$, then $\mathbf{x} \succsim \mathbf{y}$.

We obtain the surprising result that this population-ethics property is a result of the axioms that we have already imposed.

Proposition 4. Assume that $\succsim$ is a preorder satisfying Strong Anonymity, Monotonicity, Continuity, Limited Inequity, and Critical-Level Consistency. Then $\succsim$ satisfies Non-Positive Value of Additional Lives.

Proof. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ such that there exists an increasing function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that $x_j = y_{f(j)}$ for all $j \in \mathbb{N}$. Assume that $\succsim$ is a preorder satisfying Strong Anonymity, Monotonicity, Continuity, Limited Inequity, and Critical-Level Consistency.

Write $\ell(\mathbf{x}) = \liminf_{j \in \mathbb{N}} x_j$ and $\ell(\mathbf{y}) = \liminf_{j \in \mathbb{N}} y_j$, and let $z = \sup_{j \in \mathbb{N}} y_j$. By the relationship between $\mathbf{x}$ and $\mathbf{y}$, $z \geq \ell(\mathbf{x}) \geq \ell(\mathbf{y})$. Write $\tilde{\mathbf{x}} = (\ell(\mathbf{x})\mathbb{1}_{\mathbb{N}}, \mathbf{x})$ and $\tilde{\mathbf{y}} = (\ell(\mathbf{y})\mathbb{1}_{\mathbb{N}}, \mathbf{y})$. By Strong Anonymity, adding an additional individual with well-being z to $\tilde{\mathbf{x}}$ is equivalent to increasing the well-being of an existing individual from $\ell(\mathbf{x})$ to z . So by [Proposition 2](#), $(z, \tilde{\mathbf{x}}) \sim \tilde{\mathbf{x}}$. Thus, by Critical-Level Consistency, $(z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \sim \tilde{\mathbf{x}}$. By the definition of z , there exists $\pi \in \mathbb{N}$ such that $(z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \geq \tilde{\mathbf{y}}_\pi$, so by Monotonicity, $(z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \succsim \tilde{\mathbf{y}}_\pi$. By Strong Anonymity, $\tilde{\mathbf{y}}_\pi \sim \tilde{\mathbf{y}}$, while by [Lemma 3](#), $\mathbf{x} \sim \tilde{\mathbf{x}}$ and $\mathbf{y} \sim \tilde{\mathbf{y}}$. Hence, $\mathbf{x} \sim \tilde{\mathbf{x}} \sim (z\mathbb{1}_{\mathbb{N}}, \tilde{\mathbf{x}}) \succsim \tilde{\mathbf{y}}_\pi \sim \tilde{\mathbf{y}} \sim \mathbf{y}$, which by the transitivity of $\succsim$ implies that $\succsim$ satisfies Non-Positive Value of Additional Lives. $\square$

One can even be more specific about cases where adding an individual is a matter of social indifference or has negative social value. By [Lemma 3](#) and [Proposition 3](#), adding one individual—or finitely or infinitely many people—at the limit inferior or above is a matter of social indifference. Indeed, adding any number of people at the limit inferior is socially indifferent by [Lemma 3](#). Then increasing their well-being is also socially indifferent by [Proposition 3](#).

On the other hand, if a preorder $\succsim$ satisfies Liminf-Restricted Dominance, then adding at least one individual below the limit inferior has negative social value. Indeed, like before, adding any number of people at the limit inferior is socially indifferent. Then decreasing the well-being of one individual from the limit inferior to a strictly lower value has negative social value by Liminf-Restricted Dominance.⁷

6. Concluding remarks

In the view of [Van Liedekerke and Lauwers \(1997, p. 164\)](#) there are good reasons why the route taken in this paper, where we insist on Strong Anonymity and weaken Strong Pareto, has been left unexplored. In their opinion, Strong Anonymity is an unreasonable impartiality requirement. This view seems compelling if there exists a natural 1-to-1 correspondence between people in different alternatives, like in the case where the identity of people remains the same independently of the well-being they experience. For example, if the number of people in all generations is fixed, then it is not logically impossible to assume that people are the same independently of their well-being levels. So if, following an example provided by [Van Liedekerke and Lauwers \(1997, p. 164\)](#), there are 100 people in each generation, with one alternative giving 99 of them a well-being of 1 and the last one a well-being of 0, while in the other alternative 99 get a well-being of 0 and only one a well-being of 1, it might be hard to argue that the profiles are equally good, even though one profile is an infinite permutation of the other. If one agrees with this position, then the fact that Strong Anonymity together with four auxiliary axioms implies that adding people to an infinite population never has positive value can be taken as a further indication that Strong Anonymity is indeed too strong, giving more weight to the kind of arguments put forward by opponents of this axiom.

However, in comparisons where the number of people in any one generation varies between two alternatives—so that the set of spatiotemporal positions in one alternative differs from that of the other—there exists no such natural 1-to-1 correspondence. In such settings the arguments against the axiom

⁷ Hence, when Liminf-Restricted Dominance is added to the axioms of [Proposition 4](#), the Adding Goodness Principle of [Hamkins and Montero \(2000\)](#) is contradicted.

of Strong Anonymity are weakened. We have shown that it is a consequence of Strong Anonymity in combination with four auxiliary axioms that only people finitely ranked from the bottom matter. The asymmetry between the sensitivity for people finitely ranked from the bottom and the insensitivity for people finitely ranked from the top is a consequence of a weak equity axiom, Limited Inequity, stating that when making a transfer from rich to poor, the poor's change in well-being must have positive relative weight. Furthermore, we have shown that—when sensitivity is limited in this way—it follows that adding people to an infinite population cannot have positive social value, as it does not matter if the added person's well-being is at infinite rank and lead to a strictly worse profile if the added person's well-being is at finite rank. These conclusions might, at first, appear controversial. Further reflection might, however, provide insights into why it is reasonable to require insensitivity at or above the limit inferior and to conclude that an infinite population cannot be improved by adding additional people.

Appendix. Weakening continuity

In this appendix we show that the main results of this paper, i.e., Propositions 3 and 4, can be strengthened by replacing Continuity by weaker continuity axioms. To this end, we prove stronger variants of Lemma 3 and Proposition 2 with these weaker continuity axioms.

Let $d_1: \mathbf{X} \times \mathbf{X} \rightarrow \mathbb{R}_+$ be the distance function given by, for any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$,

$$d_1(\mathbf{x}, \mathbf{y}) = \min \left\{ 1, \sum_{i=1}^{\infty} |x_i - y_i| \right\}.$$

Using the distance function d_1 , we define weak continuity as follows; see Svensson (1980).

Axiom (Weak Continuity). For any $\mathbf{x} \in \mathbf{X}$, the sets $\{\mathbf{y} \in \mathbf{X} : \mathbf{y} \succsim \mathbf{x}\}$ and $\{\mathbf{y} \in \mathbf{X} : \mathbf{x} \succsim \mathbf{y}\}$ are closed in $(\mathbf{X}, d_1)$.

Clearly, Weak Continuity is implied by Continuity.

The following lemma shows that Lemma 3 can be strengthened by using Weak Continuity instead of Continuity.

Lemma 5. Consider $\mathbf{x} \in \mathbf{X}$ and z be a cluster point of $\mathbf{x}$. If $\succsim$ is a preorder satisfying Strong Anonymity and Weak Continuity, then $\mathbf{x} \sim (z, \mathbf{x}) \sim (z \mathbb{1}_N, \mathbf{x})$.

Proof. Let $\mathbf{x} \in \mathbf{X}$ and let z be a cluster point for $\mathbf{x}$. Assume that $\succsim$ is a preorder satisfying Strong Anonymity and Weak Continuity. We show that $\mathbf{x} \sim (z, \mathbf{x})$ and $\mathbf{x} \sim (z \mathbb{1}_N, \mathbf{x})$.

Since z is a cluster point of $\mathbf{x}$, there exists a subsequence of $\mathbf{x}$ that converges to z . Furthermore, it is well-known that any sequence of real numbers has a monotone subsequence and that every subsequence of a convergent sequence has the same limit. Therefore, there exists a monotone subsequence of $\mathbf{x}$ that converges to z , that is, there exists an increasing function $f: \mathbb{N} \rightarrow \mathbb{N}$ such that $(x_{f(k)})_{k \in \mathbb{N}} \in \mathbf{X}$ is monotone and converges to z . Without loss of generality, we assume that $(x_{f(k)})_{k \in \mathbb{N}}$ is non-decreasing.

Let $\tilde{\mathbf{x}} = (x_{f(k)})_{k \in \mathbb{N}}$ and, for any $n \in \mathbb{N}$, define $\tilde{\mathbf{x}}(n)$ by

$$\tilde{\mathbf{x}}(n) = (\tilde{x}_1, \dots, \tilde{x}_{n-1}, z, \tilde{x}_n, \tilde{x}_{n+1}, \dots).$$

Then, we obtain that

$$\begin{aligned} \lim_{n \rightarrow +\infty} \sum_{j=1}^{\infty} |\tilde{x}_j - \tilde{x}(n)_j| &= \lim_{n \rightarrow +\infty} \left(z - \tilde{x}_n + \lim_{N \rightarrow +\infty} \sum_{j=n+1}^N |\tilde{x}_j - \tilde{x}(n)_j| \right) \\ &= \lim_{n \rightarrow +\infty} \left(z - \tilde{x}_n + \lim_{N \rightarrow +\infty} (\tilde{x}_N - \tilde{x}_n) \right) \end{aligned}$$

$$= \lim_{n \rightarrow +\infty} 2(z - \tilde{x}_n) = 0.$$

Thus, for any $\varepsilon \in (0, 1)$, there exists an increasing sequence $\{n_t\}_{t \in \mathbb{N}}$ in $\mathbb{N}$ satisfying

$$n_t + t < n_{t+1} \text{ for each } t \in \mathbb{N}$$

and we can define the sequence $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ in $\mathbf{X}$ that satisfies

$$\sum_{j=1}^{\infty} |\tilde{x}_j^n - \tilde{x}_j^{n-1}| < \frac{\varepsilon}{2^n} \text{ for each } n \in \mathbb{N}$$

as follows:

$$\tilde{\mathbf{x}}^0 = (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, \tilde{x}_{n_1}, \tilde{x}_{n_1+1}, \dots, \tilde{x}_{n_2-1}, \tilde{x}_{n_2}, \tilde{x}_{n_2+1}, \dots),$$

$$\tilde{\mathbf{x}}^1 = (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, z, \tilde{x}_{n_1}, \dots, \tilde{x}_{n_2-2}, \tilde{x}_{n_2-1}, \tilde{x}_{n_2}, \dots),$$

$$\tilde{\mathbf{x}}^2 = (\tilde{x}_1, \dots, \tilde{x}_{n_1-1}, z, \tilde{x}_{n_1}, \dots, \tilde{x}_{n_2-2}, z, \tilde{x}_{n_2-1}, \dots),$$

and so forth. Formally, $\tilde{\mathbf{x}}^0 = \tilde{\mathbf{x}}$ and for each $n \in \mathbb{N}$, $\tilde{\mathbf{x}}^n$ is defined as follows. For each $j \in \{n_t : t \in \{1, \dots, n\}\}$,

$$\tilde{x}_j^n = z$$

and the subsequence of $\tilde{\mathbf{x}}^n$ composed of all the other components coincides with $\tilde{\mathbf{x}}$. Analogously, we define $\tilde{\mathbf{x}}^\infty$ by, for each $j \in \{n_t : t \in \mathbb{N}\}$,

$$\tilde{x}_j^\infty = z$$

and the subsequence of $\tilde{\mathbf{x}}^\infty$ composed of all the other components coincides with $\tilde{\mathbf{x}}$. By the definitions of $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ and $\tilde{\mathbf{x}}^\infty$, we obtain that

$$\sum_{j=1}^{\infty} |\tilde{x}_j^\infty - \tilde{x}_j^0| \leq \lim_{N \rightarrow +\infty} \sum_{n=1}^N \sum_{j=1}^{\infty} |\tilde{x}_j^n - \tilde{x}_j^{n-1}| < \varepsilon. \quad (\text{A.1})$$

We now define the sequence $\{\mathbf{x}^n\}_{n \in \mathbb{N} \cup \{0\}}$ using the sequence $\{\tilde{\mathbf{x}}^n\}_{n \in \mathbb{N} \cup \{0\}}$ as follows. For each $n \in \mathbb{N} \cup \{0\}$, $x_j^n = x_j$ for all $j \in \mathbb{N} \setminus \{f(k) : k \in \mathbb{N}\}$ and $(x_{f(k)}^n)_{k \in \mathbb{N}} = \tilde{\mathbf{x}}^n$. Analogously, we define $\mathbf{x}^\infty$ by $x_j^\infty = x_j$ for all $j \in \mathbb{N} \setminus \{f(k) : k \in \mathbb{N}\}$ and $(x_{f(k)}^\infty)_{k \in \mathbb{N}} = \tilde{\mathbf{x}}^\infty$. Note that $\mathbf{x}^0 = \mathbf{x}$. From (A.1) and the definitions of $\{\mathbf{x}^n\}_{n \in \mathbb{N} \cup \{0\}}$ and $\mathbf{x}^\infty$, it follows that

$$\begin{aligned} \sum_{j=1}^{\infty} |x_j^1 - x_j^0| &= \sum_{i=1}^{\infty} |\tilde{x}_i^1 - \tilde{x}_i^0| \leq \sum_{j=1}^{\infty} |\tilde{x}_j^\infty - \tilde{x}_j^0| \\ &= \sum_{j=1}^{\infty} |x_j^\infty - x_j^0| < \varepsilon. \end{aligned} \quad (\text{A.2})$$

Note that there exist permutations $\pi, \rho \in \Pi$ such that $\mathbf{x}^1 = (z, \mathbf{x})_\pi$ and $\mathbf{x}^\infty = ((z)_{\text{con}}, \mathbf{x})_\rho$. Thus, it follows from (A.2) that for any $\varepsilon \in (0, 1)$, there exist $\pi, \rho \in \Pi$ such that

$$d_1((z, \mathbf{x})_\pi, \mathbf{x}) \leq d_1((z \mathbb{1}_N, \mathbf{x})_\rho, \mathbf{x}) < \varepsilon. \quad (\text{A.3})$$

Let $m \in \mathbb{N}$. By (A.3), there exist $\pi^m, \rho^m \in \Pi$ such that

$$d_1((z, \mathbf{x})_{\pi^m}, \mathbf{x}) \leq d_1((z \mathbb{1}_N, \mathbf{x})_{\rho^m}, \mathbf{x}) < \frac{1}{m}.$$

Consider the sequences $((z, \mathbf{x})_{\pi^m})_{m \in \mathbb{N}}$ and $((z \mathbb{1}_N, \mathbf{x})_{\rho^m})_{m \in \mathbb{N}}$ in $\mathbf{X}$. By Strong Anonymity, we obtain that $(z, \mathbf{x})_{\pi^m} \sim (z, \mathbf{x})$ and $(z \mathbb{1}_N, \mathbf{x})_{\rho^m} \sim (z \mathbb{1}_N, \mathbf{x})$ for each $m \in \mathbb{N}$. Since $\succsim$ satisfies Weak Continuity and

$$\lim_{m \rightarrow +\infty} d_1((z, \mathbf{x})_{\pi^m}, \mathbf{x}) = \lim_{m \rightarrow +\infty} d_1((z \mathbb{1}_N, \mathbf{x})_{\rho^m}, \mathbf{x}) = 0,$$

we obtain $(z, \mathbf{x}) \sim \mathbf{x}$ and $(z \mathbb{1}_N, \mathbf{x}) \sim \mathbf{x}$. $\square$

Note that a variant of Lemma 4 that uses Weak Continuity instead of Continuity can be proved by using Lemma 5 instead of Lemma 3. Thus, in what follows, we will use Lemma 4 to establish a variant of Proposition 2.

To state the variant of [Proposition 2](#), we introduce another continuity axiom: *Restricted Continuity* requires an evaluation be continuous with respect to profiles that have a constant subsequence. Furthermore, it only asserts that indifference relations must be robust with respect to small changes of constant well-being levels in a very weak form by requiring social indifference must hold for all constant well-being levels in a neighborhood of a given constant well-being level.

Axiom (Restricted Continuity). For any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, any $\ell \in \mathbb{R}$, and any $\underline{\ell} < \bar{\ell}$ such that $\underline{\ell} \leq \ell \leq \bar{\ell}$, if $(\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell}) \setminus \{\ell\}$, then $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$.

The following lemma shows that Continuity implies Restricted Continuity for any preorder satisfying Monotonicity.

Lemma 6. Assume that $\succsim$ is a preorder satisfying Monotonicity. If $\succsim$ satisfies Continuity, then $\succsim$ satisfies also Restricted Continuity.

Proof. Assume that $\succsim$ is a preorder satisfying Monotonicity and Continuity. Consider any $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, any $\ell \in \mathbb{R}$, and any $\underline{\ell} < \bar{\ell}$ such that $\underline{\ell} \leq \ell \leq \bar{\ell}$. Suppose that $(\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell}) \setminus \{\ell\}$. Without loss of generality, we assume that $\bar{\ell} > \ell$. Then there exists a sequence $\{\ell^k\}_{k \in \mathbb{N}}$ in $[\underline{\ell}, \bar{\ell}]$ that converges to ℓ . Since $[\ell, \bar{\ell}] \subseteq (\underline{\ell}, \bar{\ell})$, it follows that $(\ell^k \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell^k \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for all $k \in \mathbb{N}$. Furthermore, by Monotonicity, $(\ell^k \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for all $k \in \mathbb{N}$. Since $\succsim$ is transitive, we obtain $(\ell^k \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for all $k \in \mathbb{N}$. Since the sequence $\{(\ell^k \mathbb{1}_{\mathbb{N}}, \mathbf{x})\}_{k \in \mathbb{N}}$ converges to $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$ in the sup metric, we obtain by Continuity that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$. By the same argument, we obtain $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \succsim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x})$. Thus, $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$, thereby establishing that $\succsim$ satisfies Restricted Continuity. $\square$

The following proposition is a stronger variant of [Proposition 2](#) using Weak Continuity and Restricted Continuity instead of Continuity.

Proposition 5. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $x_i > y_i$ for some $i \in \mathbb{N}$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. The two following statements are equivalent:

- (1) For all preorders $\succsim$ satisfying Strong Anonymity, Monotonicity, Weak Continuity, Restricted Continuity, and Limited Inequity, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $y_i \geq \liminf_{j \in \mathbb{N}} x_j$.

Proof. Since Continuity implies Weak Continuity and Restricted Continuity, the proof that (1) implies (2) is analogous to that of [Proposition 2](#).

We prove that (2) implies (1). Assume that $\succsim$ is a preorder on $\mathbf{X}$ that satisfies Strong Anonymity, Monotonicity, Weak Continuity, Restricted Continuity, and Limited Inequity. Let $\mathbf{x}, \mathbf{y} \in \mathbf{X}$, and assume that there exists $i \in \mathbb{N}$ such that $x_i > y_i \geq \liminf_{j \in \mathbb{N}} x_j$ and $x_j = y_j$ for all $j \in \mathbb{N} \setminus \{i\}$. There are two subcases:

Subcase (i): $\limsup_{j \in \mathbb{N}} y_j > \liminf_{j \in \mathbb{N}} x_i$. The result follows from [Lemma 4](#).

Subcase (ii): $\limsup_{j \in \mathbb{N}} y_j = \liminf_{j \in \mathbb{N}} x_i = \ell$. Since ℓ is a cluster point of $\mathbf{x}$ and $\mathbf{y}$, it follows from [Lemma 5](#) that $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim \mathbf{x}$ and $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y}) \sim \mathbf{y}$. Let $\underline{\ell}, \bar{\ell} \in \mathbb{R}$ be such that $\underline{\ell} = \ell < \bar{\ell}$. By [Lemma 4](#), we obtain $(\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell' \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ for all $\ell' \in (\underline{\ell}, \bar{\ell})$. By Restricted Continuity, $(\ell \mathbb{1}_{\mathbb{N}}, \mathbf{x}) \sim (\ell \mathbb{1}_{\mathbb{N}}, \mathbf{y})$ follows. By the transitivity of $\succsim$, we obtain $\mathbf{x} \sim \mathbf{y}$. $\square$

Using [Lemma 5](#) and [Proposition 5](#), we obtain the following stronger variants of [Propositions 3](#) and [4](#) with the weaker continuity axioms. Since their proofs are analogous to those of [Propositions 3](#) and [4](#), we state them without proof.

Proposition 6. Consider $\mathbf{x}, \mathbf{y} \in \mathbf{X}$ with $\mathbf{x} \geq \mathbf{y}$ and $\mathbf{x} \neq \mathbf{y}$. The two following statements are equivalent:

- (1) For all preorders $\succsim$ satisfying Strong Anonymity, Monotonicity, Weak Continuity, Restricted Continuity, Limited Inequity, and Critical-Level Consistency, we have that $\mathbf{x} \sim \mathbf{y}$.
- (2) $\ell = \liminf_{j \in \mathbb{N}} x_j = \liminf_{j \in \mathbb{N}} y_j$ and $y_i \geq \ell$ for all $i \in \mathbb{N}$ such that $x_i > y_i$.

Proposition 7. Assume that $\succsim$ is a preorder satisfying Strong Anonymity, Monotonicity, Weak Continuity, Restricted Continuity, Limited Inequity, and Critical-Level Consistency. Then $\succsim$ satisfies Non-Positive Value of Additional Lives.

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