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# Dealing with redundancies among criteria in multicriteria decision making through independent component analysis

Guilherme Dean Pelegrina<sup>a,b,\*</sup>, Leonardo Tomazeli Duarte<sup>a</sup>, Michel Grabisch<sup>b,c</sup>, João Marcos Travassos Romano<sup>d</sup>

<sup>a</sup>*School of Applied Sciences, University of Campinas, 1300 Pedro Zaccaria Street, CEP 13484-350, Limeira, Brazil*

<sup>b</sup>*Centre d'Économie de la Sorbonne, Université Paris I - Panthéon-Sorbonne, 106-112 Boulevard de l'Hôpital, 75647 Paris Cedex 13, France*

<sup>c</sup>*Paris School of Economics, Université Paris I – Panthéon-Sorbonne, 106-112 Boulevard de l'Hôpital, 75647 Paris Cedex 13, France*

<sup>d</sup>*School of Electrical and Computer Engineering, University of Campinas, 400 Albert Einstein Avenue, CEP 13083-852, Campinas, Brazil*

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## Abstract

Redundancies among criteria are frequently observed in multicriteria decision making problems. This aspect may introduce bias in the achieved ranking, since alternatives that have a good performance only in redundant criteria will be favored in comparison with other ones. Aiming at dealing with this inconvenience, several sophisticated approaches were developed in the literature. Most of them require subjective information provided by the decision maker in order to define the set of parameters. In this paper, we consider one of the simplest and most used method, called weighted arithmetic mean, and propose some approaches to automatically adjust the weights in order to deal with redundant criteria. For this purpose, we formulate the problem as a blind source separation one and apply independent component analysis techniques to extract the information used to perform the adjustments. Numerical experiments in both synthetic and real data attest our proposal. Moreover, we extend the proposed approaches to deal with group multicriteria decision making problems.

*Keywords:* multicriteria decision making, redundant criteria, independent component analysis, weighted arithmetic mean

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\*Corresponding author. Phone number: +55 19 981280518

*Email addresses:* [guidean@unicamp.br](mailto:guidean@unicamp.br) (Guilherme Dean Pelegrina), [leonardo.duarte@fca.unicamp.br](mailto:leonardo.duarte@fca.unicamp.br) (Leonardo Tomazeli Duarte), [michel.grabisch@univ-paris1.fr](mailto:michel.grabisch@univ-paris1.fr) (Michel Grabisch), [romano@dmoe.fee.unicamp.br](mailto:romano@dmoe.fee.unicamp.br) (João Marcos Travassos Romano)

## 1. Introduction

Several situations have been formulated as multicriteria decision making (MCDM) problems (Figueira et al., 2016). For example, recent works include applications in green supply chain management (Uygun & Dede, 2016; Sari, 2017), failure mode and effect analysis (Liu et al., 2019), supplier selection (Ghorabae et al., 2016; Wan et al., 2017; Fei et al., 2019; Chai & Ngai, 2020; Liang et al., 2020), project investments selection (Jia & Liu, 2019; Ye et al., 2020) and hospital management and evaluation (Wu et al., 2018; Fei et al., 2020). A typical MCDM problem consists in ranking a set of objects (or alternatives) by taking into account their associated consequences. These consequences are generally measured in terms of decision criteria, which are expected to satisfy some properties (Keeney & Raiffa, 1976). For instance, one expects that the set of criteria is (i) complete, in the sense that it is adequate and sufficient to describe the goals expected by the decision maker, (ii) operational, since criteria should have a clear meaning and be understood by all actors involved in the problem, (iii) as small as possible, in order to be easily managed by the decision maker, and (iv) non-redundant.

Property (iv) is desirable since one should associate each criterion to a different latent factor describing the alternatives. If this is not achieved, one may have two or more criteria that measure the same latent factor and, therefore, the ranking may be biased towards alternatives with good evaluations only with respect to this latent factor. Redundancy is easily verified when we take the correlation between pairs of criteria. For instance, if two criteria are positively correlated, one may say that there is a latent factor influencing both of them.

Although we expect to satisfy property (iv), we often deal with MDCM problems with correlations (or more generally, dependencies) in the decision data. Aiming at dealing with such situations, several methods have been proposed in the literature. Roughly speaking, they differ one from the other in the mechanism used to aggregate the criteria evaluations. Examples are the Choquet integral (Grabisch, 1996) and the multilinear model (Owen, 1972), which are defined on a set of  $2^m$  capacity coefficients (Choquet, 1954), where  $m$  is the number of criteria. Since in these aggregation functions the parameters are associated with all possible coalitions of criteria, one may model interactions between them (Marichal, 2000; Yager, 2018) and, therefore, deal with correlations in the decision data. Several applications in the literature adopted the Choquet integral (Zhang et al., 2016; Meng et al., 2021; Siskos & Burgherr, 2021). Moreover, it has also been combined with other MCDM methods in order to exploits criteria interactions, such as the QUALIFLEX method (Liang et al., 2020), fuzzy logic (Chen et al., 2020; Büyüközkan et al., 2021) and both Robust Ordinal Re-

gression (ROR) and the Stochastic Multicriteria Acceptability Analysis (SMAA) (Angilella et al., 2018). With respect to the multilinear model, a recent work dealt with dependent criteria by means of an unsupervised approach based on the Sobol's index (Pelegrina et al., 2020). The price to be paid in these aggregation functions is that the set of parameters increases exponentially with the number of criteria, which may be an inconvenience in real scenarios.

In the literature, one may also find some works that exploit redundancies among criteria by means of an extended version of TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method (Hwang & Yoon, 1981), called TOPSIS-M (Antuchevičienė et al., 2010; Chang et al., 2010; Vega et al., 2014; Wang & Wang, 2014). In the classical TOPSIS method, the overall evaluation (frequently called closeness measure) of each alternative is calculated based on the Euclidean distances from this alternative and the positive and the negative ideal alternatives. Therefore, alternatives which are closer to the positive ideal one will achieve a better position in the ranking. The difference in TOPSIS-M is that the overall evaluations are calculated based on the Mahalanobis distance (Mahalanobis, 1936; De Maesschalck et al., 2000) and, therefore, the covariance matrix of the decision data is used in this procedure. The advantage of this method in comparison with TOPSIS is that one tries to mitigate redundancies with a simple modification in the distance calculation. However, as highlighted in (Pelegrina et al., 2019a), the second order information may not be sufficient to deal with criteria that are statistically dependent. A recent work (Aduba, 2021) also used TOPSIS in the proposed method. The author exploited the Pearson correlation coefficients extracted from the decision data and proposed an approach that combines the TOPSIS and the fuzzy analytical hierarchical process. Moreover, another existing approach is the system of pairwise comparisons used in the Analytic Network Process (ANP), proposed by Saaty (1996). For instance, Dağdeviren & Yüksel (2010) and Wan et al. (2017) consider intercriteria relations by applying fuzzy ANP models and Ziemba (2019) applied the ANP in wind energy management.

Although the aforementioned techniques have their advantages in dealing with redundancies in MCDM problems, one may note that either they use a set of parameters that increases exponentially with the number of criteria (the case of the Choquet integral and the multilinear model) or they adopt a sophisticated procedure to derive the ranking of alternatives (e.g., TOPSIS-based methods and ANP). In this paper, we aim at dealing with redundancies among criteria by using one of the simplest and largely used aggregation function: the Weighted Arithmetic Mean (WAM). In summary, the WAM aggregates the criteria evaluations according to a set of  $m$  weighs, which compensate scaling disparities (if the criteria are not normalized) and indicate the relative importance

given to each criterion. This relative importance is generally defined by means of a subjective or an objective weighting method (Zardari et al., 2015; Alemi-Ardakani et al., 2016). In the subjective case, the decision maker provides the sufficient information (preferences with respect to the set of criteria) to define all weights. On the other hand, objective methods extract relevant information from the decision data in order to derive the WAM parameters. Remark that, even if one has differences in the practical meanings of different criteria, one may have redundancies between them. For example, in performance measurement systems frequently used in industrial environments (Berrah et al., 2008), some redundant indicators such as the equipment availability and the bottleneck productivity may be measured in different scales (in % and in number of items manufactured per hour, respectively).

Since the decision maker may not be aware of the redundancies among criteria (or the degree of such a redundancy), a subjective weighting method (also called elicitation) may lead to unfair ranking of alternatives. Therefore, the combination of both subjective and objective methods may improve the results (Ma et al., 1999). For instance, the decision maker may provide his/her opinion about the criteria weights and an objective method may be applied to adjust these weights in order to take into account information extracted from the decision data (correlations, for example). For this purpose and inspired by the work conducted by Pelegrina et al. (2019a), we formulate the decision problem as a blind source separation (BSS) (Comon & Jutten, 2010) one and investigate if the information obtained by Independent Component Analysis (ICA) (Hyvärinen et al., 2001) techniques is enough to perform the adjustments on the set of weights. More precisely, we attempt to use this information to penalize (resp. favor) the weights associated with positively (resp. negatively) correlated criteria and, therefore, overcome biased results provided by redundancies in the decision data. It is worth mentioning that ICA is a classical technique used in signal separation and has been adopted in industrial applications such as in process monitoring (Pelegrina et al., 2016; Wang et al., 2016; Li et al., 2017), where attributes of interest are continuously observed in order to detect abnormal patterns (which may indicate machine failures). The novelty of this paper consists in bringing such a technique to MCDM problems, which is, as far as we know, hardly exploited in the literature. Moreover, we aim at contributing to the development of data-driven methods that address redundant criteria in the decision data. For instance, in the aforementioned performance measurement systems example (Berrah et al., 2008), the application of our proposal can mitigate the effect of redundant indicators (e.g, equipment availability and the bottleneck productivity) when evaluating industrial performances.

In order to attest our proposal, we conduct experiments in different decision scenarios and compare our results with the ones obtained by adopting existing objective methods, such as the entropy (Hwang & Yoon, 1981) and CRITIC (CRiteria Importance Through Intercriteria Correlation) (Diakoulaki et al., 1995). For instance, CRITIC has also been used in the literature to exploit correlations in the decision data (Song et al., 2021).

The rest of this paper is organized as follows. In Section 2, we discuss the addressed MCDM problem and some issues about redundancies in the decision data. Section 3 describes the theoretical aspects of BSS problems and a technique to solve it. In Section 4, we present our ICA-based proposal to deal with MCDM problems with redundancies in the decision data. The experiments conducted in this paper and the obtained results are described in Section 5. Finally, we present our conclusions and future perspectives in Section 6.

## 2. Problem statement

Consider a problem of ranking a set of  $n$  alternatives whose consequences are measured through  $m$  decision criteria. Moreover, let us define  $\mathbf{U} = (u_{j,i})_{m \times n}$  as the matrix whose element  $u_{j,i}$  ( $j = 1, \dots, m$  and  $i = 1, \dots, n$ ) represents the evaluation of alternative  $i$  with respect to the criterion  $j$ . The problem consists in aggregating the criteria evaluations through an aggregation function  $F(\mathbf{u}_i) = F(u_{1,i}, \dots, u_{m,i})$ , where  $\mathbf{u}_i$  is the  $i$ -th column of  $\mathbf{U}$ . Based on these overall values, one may rank the available alternatives.

One of the simplest and most used aggregation function is the weighted arithmetic mean, defined by

$$F(\mathbf{u}_i) = \mathbf{w}^T \mathbf{u}_i = \sum_{j=1}^m w_j u_{j,i}, \quad (1)$$

where the weights  $\mathbf{w} = [w_1, \dots, w_m]^T$  ( $w_j \geq 0$ ,  $\forall j = 1, \dots, m$ , and  $\sum_j^m w_j = 1$ ) represent the relative importance given to each criterion in the decision problem<sup>1</sup>. If we consider, for example, the illustrative situation described in Table 1 (adapted from (Grabisch, 1996)), which comprises the problem of ranking a set of students based on their grades in calculus (C), physics (P) and literature (L),  $w_1$  will represent the relative importance given to the first discipline (calculus, in this case) in the aggregation of all grades. For example, if we consider  $\mathbf{w} = [w_1, w_2, w_3] = [0.4, 0.4, 0.2]$  (more

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<sup>1</sup>The weights  $\mathbf{w}$  may also represent scaling factors. However, in this paper, we consider that the criteria evaluations are normalized and, therefore, it is safe to say that the weights represent the relative importance. For a detailed discussion on criteria weights interpretation, see (Choo et al., 1999).

importance is given to the scientific disciplines), the obtained overall evaluations  $\mathbf{r} = [r_1, r_2, r_3]$ , where  $r_i = F(\mathbf{u}_i)$ , and the ranking positions are presented in Table 1. In this case, Student 1 achieves the first position in the ranking.

Table 1: Illustrative example.

Students	Grades			For $\mathbf{w}$		For $\hat{\mathbf{w}}$	
	C	P	L	$r_i$	Pos.	$r_i$	Pos.
Student 1	18	16	10	15.6	1	14.9	2
Student 2	10	12	18	12.4	3	13.1	3
Student 3	15	15	16	15.2	2	15.3	1

As mentioned in the last section, an important aspect that we should be aware of is the presence of redundancies in the decision data. In the illustrative example, one may consider that both calculus and physics measure the same latent factor (e.g. scientific skills). Therefore, if we do not consider this redundancy, the obtained result may be biased towards students with good grades in the two correlated disciplines but a weak performance in literature. This is the case of Student 1, which have the better grades in calculus and physics and the worst one in literature.

In such situations, it may be interesting to penalize redundant criteria in order to compensate the bias introduced by positive correlations. For instance, in the illustrative example, if one decreases both  $w_1$  and  $w_2$  and increases  $w_3$ , one penalizes the importance given to both calculus and physics. For example, by assuming  $\hat{\mathbf{w}} = [0.35, 0.35, 0.3]$ , one obtains the results presented in Table 1. In this case, Student 3, which has the more balanced grades, achieves the first position.

Based on the aforementioned discussions, instead of subjectively redefining the weights in order to take into account the redundancies in the decision data, we would like to adopt an automatic system to provide it. In that respect, we propose an approach based on independent component analysis. The next section presents the blind source separation problem, whose formulation will be used in our proposal.

### 3. Blind source separation: ICA and whitening

A typical problem in signal processing is the blind source separation (Comon & Jutten, 2010). Consider a set of sources  $\mathbf{s}(t) = [s_1(t), \dots, s_m(t)]$  that were mixed through an instantaneous linear

mixing process represented by<sup>2</sup>

$$\mathbf{q}(t) = \mathbf{A}\mathbf{s}(t) + \mathbf{g}(t), \quad (2)$$

where the vector  $\mathbf{q}(t) = [q_1(t), \dots, q_m(t)]$  represents the set of mixed signals,  $\mathbf{A} = (a_{j,j'})_{m \times m}$  is the mixing matrix and  $\mathbf{g}(t) = [g_1(t), \dots, g_m(t)]$  represents the Additive White Gaussian Noise (AWGN). The goal in BSS consists in retrieving  $\mathbf{s}(t)$  based on the mixed signals  $\mathbf{q}(t)$  without the knowledge on the mixing matrix parameters. Therefore, in order to estimate the sources, one assumes some properties about them. For example, one may assume that at most one source is Gaussian and that they are independent and identically distributed. In such a scenario, one may formulate an optimization problem whose aim is to obtain the parameters of a separation matrix  $\mathbf{B}$  that leads to a set of estimates  $\hat{\mathbf{s}}(t) = [\hat{s}_1(t), \dots, \hat{s}_m(t)]$ , given by

$$\hat{\mathbf{s}}(t) = \mathbf{B}\mathbf{q}(t), \quad (3)$$

as independent (or non-Gaussian) as possible. This technique is known as Independent Component Analysis (ICA) (Hyvärinen et al., 2001). Note that, ideally,  $\mathbf{B} = \mathbf{A}^{-1}$ . Moreover, although one may achieve a good separation process, one may face scaling and/or permutation ambiguities (Comon & Jutten, 2010). These ambiguities are inherent to ICA techniques, since retrieving sources multiplied by a constant factor (scaling ambiguity) or in different order (permutation ambiguity) does not interfere on the measures associated with statistical dependence or non-Gaussianity.

Several algorithms can be used in ICA, such as the FastICA (Hyvärinen et al., 2001) and JADE (Joint Approximate Diagonalization of Eigenmatrices) (Cardoso & Souloumiac, 1993). In order to simplify the application of these algorithms, one generally conducts a preprocessing step known as *whitening*, whose goal is to transform the mixtures  $\mathbf{q}(t)$  into a set of uncorrelated signals  $\mathbf{z}(t)$  with unit variance, given by

$$\mathbf{z}(t) = \mathbf{P}\mathbf{q}(t), \quad (4)$$

where  $\mathbf{P}$  is the whitening matrix. An interesting aspect in whitening is that it can be interpreted as a transformation that rotates and stretches the data. Moreover,  $\mathbf{P}$  can be easily found through the Principal Component Analysis (PCA) (Jolliffe, 2002; Shlens, 2014), a technique largely used in data compression and visualization. The application of PCA leads to a matrix  $\bar{\mathbf{P}}$  such that the

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<sup>2</sup>Usually, BSS deals with scenarios in which the number of mixed signals is greater or equal to the number of signal sources. In this paper, as will be discussed in the next section, we assume that the number of sources is equal to the number of mixed signals.



signals

$$\bar{\mathbf{z}}(t) = \bar{\mathbf{P}}\mathbf{q}(t) \quad (5)$$

are uncorrelated, i.e., the covariance matrix of  $\bar{\mathbf{z}}$ , represented by  $\mathbf{C}_{\bar{\mathbf{z}}} = \mathbb{E} [\bar{\mathbf{z}}\bar{\mathbf{z}}^T]$ , is diagonal.

It is known that the solution of PCA is quite simple (see (Shlens, 2014) for further details). Since the covariance matrix of  $\mathbf{q}(t)$  is a symmetric matrix, it can be decomposed into its eigenvectors  $\mathbf{E}$  (in each column) and eigenvalues  $\mathbf{\Lambda}$  (in the diagonal), i.e.,  $\mathbf{C}_{\mathbf{q}} = \mathbf{E}\mathbf{\Lambda}\mathbf{E}^T$ . By setting  $\bar{\mathbf{P}}$  as the transpose of  $\mathbf{E}$ , the diagonalization of  $\mathbf{C}_{\bar{\mathbf{z}}}$  can be obtained as follows<sup>3</sup>:

$$\mathbf{C}_{\bar{\mathbf{z}}} = \bar{\mathbf{P}}\mathbf{q}\mathbf{q}^T\bar{\mathbf{P}}^T = \bar{\mathbf{P}}\mathbf{C}_{\mathbf{q}}\bar{\mathbf{P}}^T = \bar{\mathbf{P}}\mathbf{E}\mathbf{\Lambda}\mathbf{E}^T\bar{\mathbf{P}}^T \quad (6)$$

and, by setting  $\bar{\mathbf{P}} = \mathbf{E}^T$  and remembering that  $\mathbf{E}$  is an orthogonal matrix, i.e.,  $\mathbf{E}^T = \mathbf{E}^{-1}$ ,

$$\mathbf{C}_{\bar{\mathbf{z}}} = \mathbf{E}^T\mathbf{E}\mathbf{\Lambda}\mathbf{E}^T\mathbf{E} = \mathbf{I}\mathbf{\Lambda}\mathbf{I} = \mathbf{\Lambda}, \quad (7)$$

where  $\mathbf{I}$  is the identity matrix.

Although  $\mathbf{C}_{\bar{\mathbf{z}}}$  is diagonal, whitening also requires unit variance. A transformation frequently used in the literature (Hyvärinen et al., 2001) consist in multiplying  $\bar{\mathbf{P}}$  by  $\mathbf{E}\mathbf{\Lambda}^{-1/2}$ , which leads to the whitening matrix

$$\mathbf{P} = \mathbf{E}\mathbf{\Lambda}^{-1/2}\mathbf{E}^T. \quad (8)$$

One may verify that the transformed data

$$\mathbf{z}(t) = \mathbf{E}\mathbf{\Lambda}^{-1/2}\mathbf{E}^T\mathbf{q}(t) \quad (9)$$

is white, since

$$\mathbf{C}_{\mathbf{z}} = \mathbf{E}\mathbf{\Lambda}^{-1/2}\mathbf{E}^T\mathbf{C}_{\mathbf{q}}\mathbf{E}\mathbf{\Lambda}^{-1/2}\mathbf{E}^T = \mathbf{E}\mathbf{I}\mathbf{E}^T = \mathbf{I}. \quad (10)$$

Let us recall that whitening uses second-order statistics (covariance matrix, in this case) to decorrelate the data. However, a decorrelation procedure does not lead, necessarily, to independence (Papoulis & Pillai, 2002). Therefore, based on the whitened data  $\mathbf{z}(t)$ , ICA is completed by finding a rotation matrix  $\mathbf{V}$  such that the estimates  $\hat{\mathbf{s}}(t) = \mathbf{B}\mathbf{q}(t) = \mathbf{V}\mathbf{P}\mathbf{q}(t)$  are as independent (or non-Gaussian) as possible. In our experiments, we adopted the JADE algorithm, which diagonalizes a set of cumulant matrices of  $\mathbf{z}(t)$ . Since this cumulant matrices carry higher-order statistics, the diagonalization provided by JADE algorithm will lead to independent data (Cardoso & Souloumiac, 1993). Figure 1 illustrates the BSS problem and both whitening and ICA techniques.

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<sup>3</sup>We assume that  $\mathbf{q}$  has zero mean. Otherwise, we need to centralize it by subtracting its mean, i.e.  $\mathbf{q} \leftarrow \mathbf{q} - \mathbb{E}[\mathbf{q}]$ .

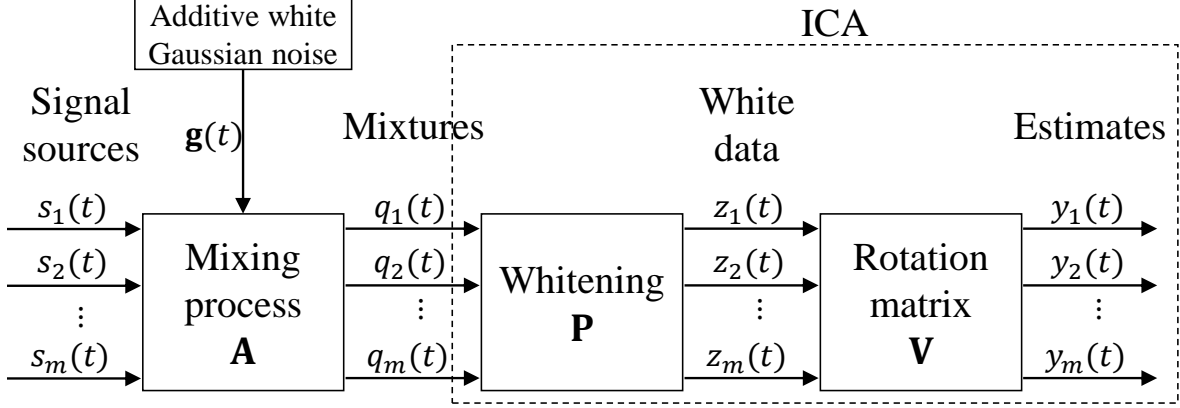


Figure 1: Whitening and ICA technique in BSS problem.

## 4. Methodology

In this section, we present the proposed approach to adjust the criteria weights and, therefore, to deal with redundancies in the decision data. We also extend our proposal to group MCDM problems

### 4.1. The proposed ICA-based approach

As mentioned in Section 1, we formulate the MCDM problem as a BSS one and, by solving it, we extract the relevant information used in the weights adjustments. For this purpose, we assume that the redundancies that we observe in the decision data  $\mathbf{U}$  come from a mixture of latent variables<sup>4</sup>  $\mathbf{L}$ , i.e.,  $\mathbf{U} = \mathbf{A}\mathbf{L}$ . Since we assume that there is no redundancies among  $\mathbf{L}$  (recall that the rows in  $\mathbf{L}$  are as independent as possible), these variables satisfy the non-redundancy property mentioned in Section 1. Therefore, we consider that  $\mathbf{L}$  is the “correct” decision matrix that should be used in WAM to derive that ranking of alternatives. In this scenario, the vector  $\mathbf{r}$ , which contains the overall scores for all alternatives, can be obtained by

$$\mathbf{r} = \mathbf{w}^T \mathbf{L} = \mathbf{w}^T \mathbf{A}^{-1} \mathbf{U} = \hat{\mathbf{w}}^T \mathbf{U}, \quad (11)$$

where  $\mathbf{w}$  is the initial weights vector provided by the decision maker (through an elicitation method, for example) and  $\hat{\mathbf{w}} = (\mathbf{w}^T \mathbf{A}^{-1})^T$  is the adjusted weights vector which carries the information used to mitigate the bias introduced by correlations in the decision data. Therefore, in order to calculate

<sup>4</sup>By using the BSS formulation described in Equations (2) and (3),  $\mathbf{L}$  and  $\mathbf{U}$  represent the set of sources  $\mathbf{s}(t)$  and the set of mixtures  $\mathbf{q}(t)$ , respectively.

$\hat{\mathbf{w}}$ , we need to estimate the mixing matrix  $\mathbf{A}$ . In this paper, we deal with such a task by means of a BSS technique.

A first remark in Equation (11) is that, since  $\mathbf{w}$  is applied on  $\mathbf{L}$ , the number of elements in  $\mathbf{w}$  must be equal to the number of latent variables. This is ensured by the BSS formulation presented in Section 3 in which the number of sources is equal to the number of mixtures. The rationale behind such a formulation is that each criterion is predominantly associated to a single latent factor and different criteria are predominantly associated to different latent factors. Moreover, as mentioned in the previous section and discussed in (Pelegrina et al., 2019a), there are possible ambiguities in the latent variable estimation. For instance, in the case of a permutation ambiguity, the  $j$ -th latent variable may not be associated with the  $j$ -th observed criteria in Equation (11). As a consequence, the weight  $w_j$  is not associated with criterion  $j$ , which makes the elicitation process difficult for the decision maker.

In order to overcome the aforementioned inconvenience and ensure a correct adjustment on the weights vector, one performs adjustments on the estimated mixing matrix  $\hat{\mathbf{A}}$  and, consequently, on the estimated latent variables  $\hat{\mathbf{L}}$ . For this purpose, we assume that (i) the collected evaluations in  $\mathbf{U}$  are in similar range (or normalized) and (ii) the diagonal elements in  $\hat{\mathbf{A}}$  are positive and greater (in absolute value) than the off-diagonal elements in the same row. Although hypothesis (ii) may be strong in BSS, we consider that it is feasible in MCDM problems since we expect that each latent variable has a positive majority influence in each observed criterion. Therefore, given  $\hat{\mathbf{A}}$  (resp.  $\hat{\mathbf{L}}$ ), we permute its columns (resp. rows) in order to obtain a mixing matrix whose diagonal elements are (in absolute value) greater than the off-diagonal ones in the same row. This procedure will ensure that the  $j$ -th latent variable (and, consequently, the weight  $w_j$ ) is directly associated with the  $j$ -th observed criteria. Therefore, in the elicitation method, the decision maker may define the weights  $w_1, \dots, w_m$  according to his preference on the latent variables, which are associated with the observed criteria, without taking into account redundancies among them.

With respect to the scaling ambiguity, we normalize  $\hat{\mathbf{A}}$  in such a way that the elements in the diagonal are equal to 1. Therefore, for each column  $j$  in  $\hat{\mathbf{A}}$ , we divide all the elements by  $\hat{a}_{j,j}$ . Let us illustrate these procedures in the following matrix:

$$\hat{\mathbf{A}} = \begin{bmatrix} -0.22 & 0.28 \\ -0.29 & -0.06 \end{bmatrix}.$$

In order to ensure our assumptions, we achieve

$$\begin{bmatrix} -0.22 & 0.28 \\ -0.29 & -0.06 \end{bmatrix} \rightarrow \begin{bmatrix} 0.28 & -0.22 \\ -0.06 & -0.29 \end{bmatrix} \rightarrow \begin{bmatrix} 0.28 & 0.22 \\ -0.06 & 0.29 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 0.76 \\ -0.21 & 1 \end{bmatrix}.$$

For more details about the aforementioned procedure used to mitigate scaling and/or permutation ambiguities, see (Pelegrina et al., 2019a). Once  $\hat{\mathbf{A}}$  is obtained, we calculate the weights vector  $\hat{\mathbf{w}}$  and normalized it in order to guarantee that  $\sum_i^m \hat{w}_i = 1$ .

In this paper, other than the ICA technique, we also verify if the information obtained in whitening step is enough to deal with the redundancies. Therefore, in Equation (11), we consider two scenarios: (i)  $\hat{\mathbf{w}} = (\mathbf{w}^T \hat{\mathbf{A}}^{-1})^T$ , where  $\hat{\mathbf{A}}^{-1} = \mathbf{B} = \mathbf{V}\mathbf{P}$ , and (ii)  $\hat{\mathbf{w}} = (\mathbf{w}^T \mathbf{P})^T$ . Figure 2 illustrates the proposed approach. All the steps are presented in Algorithm 1. It is worth remarking that the adjustments conducted to mitigate the aforementioned ambiguities should also be performed on  $\mathbf{P}^{-1}$  if one only considers the whitening step.

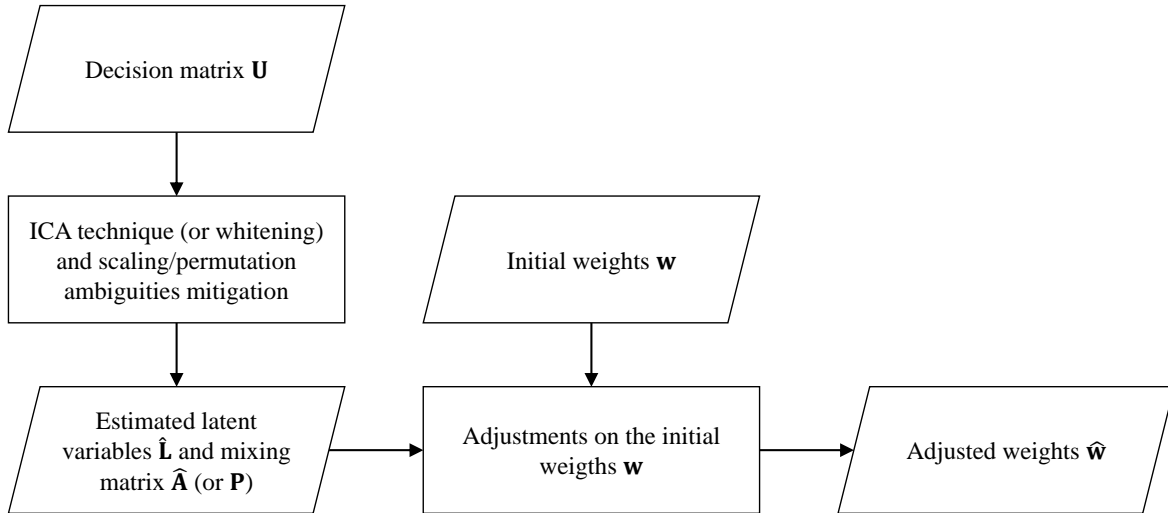


Figure 2: The proposed ICA-based approach.

#### 4.2. An extension to group MCDM

This section extends our proposal to deal with correlations among both criteria and decision makers in group MCDM problems. We address the situation in which each decision maker  $d_k$ ,  $k = 1, \dots, l$ , provides his/her own criteria evaluations  $u_{j,i}^k$  for all alternatives  $i$ ,  $i = 1, \dots, n$  and all criteria  $j$ ,  $j = 1, \dots, m$ . In other words, each decision maker  $d_k$  provides his/her own decision

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**Algorithm 1** *ICA-based approach.*


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**Input:** Decision data  $\mathbf{U}$  and initial weights vector  $\mathbf{w}$ .

**Output:** Adjusted weights vector  $\hat{\mathbf{w}}$  and overall scores  $\mathbf{r}$ .

1: **Mixing matrix estimation.** Based on the decision data  $\mathbf{U}$ , apply an ICA technique (or whitening) in order to estimate the mixing matrix  $\mathbf{A}$ .

2: **Ambiguities mitigation.** Perform adjustments in both estimated mixing matrix and latent criteria in order to mitigate permutation and/or scaling ambiguities and determine  $\hat{\mathbf{A}}$  (or  $\mathbf{P}$ ) and  $\hat{\mathbf{L}}$ .

3: **Adjusted weights calculation.** Compute the adjusted weights vector  $\hat{\mathbf{w}} = (\mathbf{w}^T \hat{\mathbf{A}}^{-1})^T$  (or  $\hat{\mathbf{w}} = (\mathbf{w}^T \mathbf{P})^T$ ).

4: **Overall scores and ranking determination.** Based on  $\hat{\mathbf{w}}$ , calculate the overall scores by computing  $\mathbf{r} = \hat{\mathbf{w}}^T \mathbf{U}$  and determine the ranking of alternatives.

---

matrix

$$\mathbf{U}^k = \begin{bmatrix} u_{1,1}^k & u_{1,2}^k & \cdots & u_{1,n}^k \\ u_{2,1}^k & u_{2,2}^k & \cdots & u_{2,n}^k \\ \vdots & \vdots & \ddots & \vdots \\ u_{m,1}^k & u_{m,2}^k & \cdots & u_{m,n}^k \end{bmatrix} \quad (12)$$

Given all  $\mathbf{U}^k$ ,  $k = 1, \dots, l$ , one aggregates the collected information according to predefined weights for criteria and decision makers. Mathematically, the overall evaluation of alternative  $i$  is calculated by

$$r_i = \sum_{k=1}^l w_k^d \left( \sum_{j=1}^m w_j u_{j,i}^k \right) = \sum_{k=1}^l \sum_{j=1}^m \omega_{j,k} u_{j,i}^k, \quad (13)$$

where  $w_k^d$  ( $w_k^d \geq 0$ ,  $\forall k = 1, \dots, l$ , and  $\sum_{k=1}^l w_k^d = 1$ ) is the weighting factor associated with  $d_k$  and  $\omega_{j,k} = w_k^d w_j$ .

Our aim here is to perform adjustments on the weights  $w_j$  and  $w_k^d$  in order to overcome bias introduced by correlations between criteria and between decision makers, respectively. Although some group decision problems in the literature are solved by means of a consensus reaching process (Dong et al., 2016), i.e., by finding a solution with a mutual agreement among the actors, there may be real scenarios in which there is a group of decision makers that, intentionally, combine their opinions in order to favor (or penalize) specific alternatives. One may cite as examples the contests (e.g., sport and music), where a committee put scores on the performance of every participant, then these scores are averaged. If a subset of the committee members combine their scores, the achieved

ranking may be biased towards the interests of such individuals. In this paper, we consider that this unfair result should be avoided. Therefore, by adjusting both criteria weights, which mitigates the effect of redundant criteria, and decision makers weights, which deals with ill-intentioned actors, we may achieve a fairer result.

Aiming at extracting the statistics used in our proposals, we assume that, for each criterion, the evaluations provided by different decision makers may be concatenated into a single vector. Similarly, we assume that, for each decision maker, the evaluations with respect to different criteria may also be concatenated into a single vector. By doing these procedures for all criteria and decision makers, one obtains the “extended criteria” matrix  $\mathbf{U}^{\mathcal{C}}$ , defined by

$$\mathbf{U}^{\mathcal{C}} = \left[ \mathbf{U}^1, \dots, \mathbf{U}^l \right], \quad (14)$$

and the “extended decision makers” matrix  $\hat{\mathbf{U}}^{\mathcal{D}}$ , defined by

$$\hat{\mathbf{U}}^{\mathcal{D}} = \begin{bmatrix} \text{reshapeR} \left( \hat{\mathbf{U}}^1 \right) \\ \vdots \\ \text{reshapeR} \left( \hat{\mathbf{U}}^l \right) \end{bmatrix}, \quad (15)$$

where the operator  $\text{reshapeR} \left( \hat{\mathbf{U}}^k \right)$  converts the matrix  $\hat{\mathbf{U}}^k$  into a row vector, i.e.,

$$\text{reshapeR} \left( \hat{\mathbf{U}}^k \right) = \left[ u_{1,1}^k, u_{1,2}^k, \dots, u_{1,n}^k, u_{2,1}^k, u_{2,2}^k, \dots, u_{2,n}^k, \dots, u_{m,1}^k, u_{m,2}^k, \dots, u_{m,n}^k \right]. \quad (16)$$

Let us illustrate these assumptions with a simple example composed by 3 alternatives, 4 criteria and 3 decision makers, as presented in Figure 3.

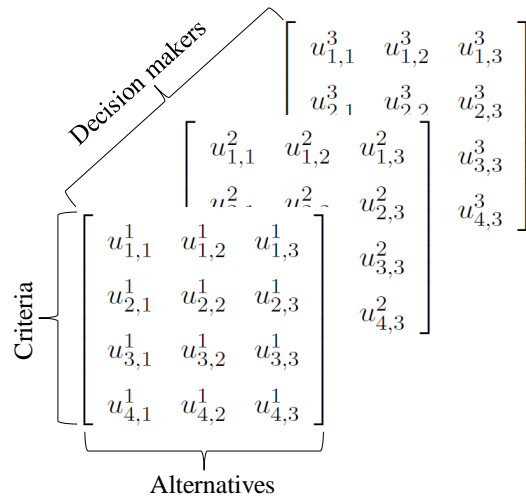


Figure 3: Illustrative example of a dataset in group MCDM.

By concatenating these data, one obtains the following matrices:

$$\mathbf{U}^{\mathcal{C}} = \begin{bmatrix} u_{1,1}^1 & u_{1,2}^1 & u_{1,3}^1 & u_{1,1}^2 & u_{1,2}^2 & u_{1,3}^2 & u_{1,1}^3 & u_{1,2}^3 & u_{1,3}^3 \\ u_{2,1}^1 & u_{2,2}^1 & u_{2,3}^1 & u_{2,1}^2 & u_{2,2}^2 & u_{2,3}^2 & u_{2,1}^3 & u_{2,2}^3 & u_{2,3}^3 \\ u_{3,1}^1 & u_{3,2}^1 & u_{3,3}^1 & u_{3,1}^2 & u_{3,2}^2 & u_{3,3}^2 & u_{3,1}^3 & u_{3,2}^3 & u_{3,3}^3 \\ u_{4,1}^1 & u_{4,2}^1 & u_{4,3}^1 & u_{4,1}^2 & u_{4,2}^2 & u_{4,3}^2 & u_{4,1}^3 & u_{4,2}^3 & u_{4,3}^3 \end{bmatrix} \quad (17)$$

and

$$\mathbf{U}^{\mathcal{D}} = \begin{bmatrix} u_{1,1}^1 & u_{1,2}^1 & u_{1,3}^1 & u_{2,1}^1 & u_{2,2}^1 & u_{2,3}^1 & u_{3,1}^1 & u_{3,2}^1 & u_{3,3}^1 & u_{4,1}^1 & u_{4,2}^1 & u_{4,3}^1 \\ u_{1,1}^2 & u_{1,2}^2 & u_{1,3}^2 & u_{2,1}^2 & u_{2,2}^2 & u_{2,3}^2 & u_{3,1}^2 & u_{3,2}^2 & u_{3,3}^2 & u_{4,1}^2 & u_{4,2}^2 & u_{4,3}^2 \\ u_{1,1}^3 & u_{1,2}^3 & u_{1,3}^3 & u_{2,1}^3 & u_{2,2}^3 & u_{2,3}^3 & u_{3,1}^3 & u_{3,2}^3 & u_{3,3}^3 & u_{4,1}^3 & u_{4,2}^3 & u_{4,3}^3 \end{bmatrix}. \quad (18)$$

For both  $\mathbf{U}^{\mathcal{C}}$  and  $\mathbf{U}^{\mathcal{D}}$ , we can apply the ICA-based approach described in the previous section (see Algorithm 1) and adjust the weights associated with criteria and decision makers, respectively.

## 5. Numerical experiments

In order to verify the application of our proposal to deal with redundancies in the decision data, we conduct numerical experiments on several decision scenarios. We also compare our proposal with existing objective weighting methods. They are described in the sequel.

### 5.1. Objective weighting methods used as benchmarks

As mentioned in Section 1, we compare our results with the ones obtained by adopting two existing objective methods: entropy and CRITIC. As the name suggests, the former is based on a concept in information theory called Shannon entropy (Shannon, 1948). In summary, it represents a measure of the information (or uncertainty) contained in a discrete probability distribution. In the context of MCDM, the reasoning is to assign greater relative weights to criteria that carry greater information. In other words, one assigns high relative weights to the criteria whose evaluations lead to a high entropy value. By using such a method, after normalizing the decision data, the weights can be defined by (for further details, see (Hwang & Yoon, 1981)):

$$u'_{j,i} = \frac{u_{j,i}}{\sum_{i=1}^n u_{j,i}}, \quad j = 1, \dots, m, \quad i = 1, \dots, n, \quad (19)$$

$$H_j = -\frac{\sum_{i=1}^n u'_{j,i} \ln(u'_{j,i})}{\ln(n)}, \quad j = 1, \dots, m, \quad (20)$$

$$\bar{w}_j = \frac{1 - H_j}{\sum_{j=1}^m (1 - H_j)}, \quad j = 1, \dots, m. \quad (21)$$

One may note that the entropy method does not consider intercriteria information in the weights definition. However, the second objective weighting method that we consider as a benchmark, called CRITIC, exploits the linear correlation coefficient between pairs of criteria. It starts by calculating the following:

$$\gamma_j = \sigma_j \sum_{j'=1}^m (1 - \rho_{j,j'}), \quad j = 1, \dots, m, \quad (22)$$

where  $\sigma_j$  is the standard deviation of the criterion evaluations  $j$ , and  $\rho_{j,j'}$ , defined by

$$\rho_{j,j'} = \frac{n \left( \sum_{i=1}^n u_{j,i} u_{j',i} \right) - \left( \sum_{i=1}^n u_{j,i} \right) \left( \sum_{i=1}^n u_{j',i} \right)}{\sqrt{\left( n \sum_{i=1}^n u_{j,i}^2 - \left( \sum_{i=1}^n u_{j,i} \right)^2 \right) \left( n \sum_{i=1}^n u_{j',i}^2 - \left( \sum_{i=1}^n u_{j',i} \right)^2 \right)}}, \quad \forall j, j' = 1, \dots, m, \quad (23)$$

is the linear correlation coefficient between the criteria evaluations  $j, j'$ . Given all  $\gamma_j, j = 1, \dots, m$ , the weights are defined by

$$\bar{w}_j = \frac{\gamma_j}{\sum_{j=1}^m \gamma_j}, \quad j = 1, \dots, m. \quad (24)$$

Note that, for both entropy and CRITIC methods, if the decision maker has predefined weights  $w_j, j = 1, \dots, m$ , that he/she wants to include in the elicitation process, the weights can be modified by

$$\hat{w}_j = \frac{w_j \bar{w}_j}{\sum_{j=1}^m w_j \bar{w}_j}, \quad j = 1, \dots, m. \quad (25)$$

## 5.2. Experiment in a synthetic dataset

The first experiment consists in comparing the considered approaches on a synthetic dataset composed by 1000 alternatives and 3 decision criteria. The criteria evaluations, illustrated in Figure 4, were generated according to a uniform distribution on  $[0, 1]$  and with a positive correlation<sup>5</sup> ( $\rho_{2,3} = 0.7$ ) between criteria 2 and 3.

If one assumes  $\mathbf{w} = [1/3, 1/3, 1/3]$ , the obtained rankings are presented in Table 2. One may note that, since the entropy method does not consider the correlation between pairs of criteria in the weights estimation, it leads to very similar values for all criteria. On the other hand, the application of both CRITIC and the proposed ICA-based approaches penalizes criteria 2 and 3. This penalization was expected since both criteria are positively correlated. However, if we compare the results provided by the application of ICA, whitening and CRITIC, we note that the last two methods led to a similar penalization for both redundant criteria. A possible explanation for this

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<sup>5</sup>In this paper, all correlation coefficients are referred to as the Pearson's correlation coefficient (Chen & Popovich, 2002).



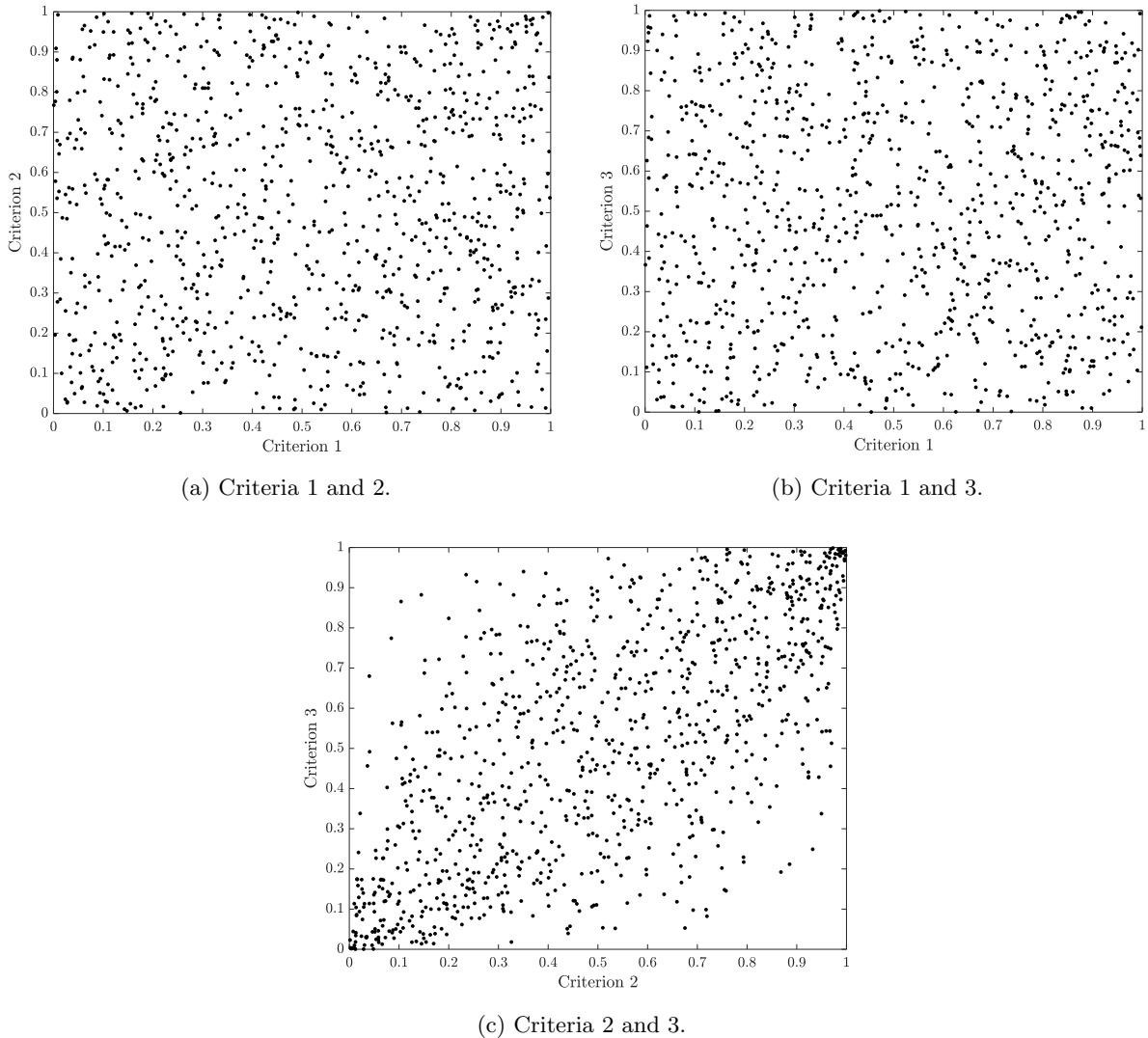


Figure 4: Scatter plot of pairs of criteria - Synthetic data.

difference is that, since ICA exploits higher-order statistics, this technique needs more data to achieve stable results. We further investigate this result in the next experiment.

Remark that the adjusted weights presented in Table 2 assumed an equal relative importance between all criteria (i.e.,  $\mathbf{w} = [1/3, 1/3, 1/3]$ ). However, the decision maker may consider, for example, that criterion 3 is more important than the criteria 1 and 2. In this case, he/she may assume  $\mathbf{w} = [1/4, 1/4, 2/4]$ , which leads to the adjusted weights presented in Table 3. One may note that all methods led to  $\hat{w}_3$  higher than the weights associated to criteria 1 and 2. However, CRITIC and the proposed approaches led to  $\hat{w}_3$  lower than the value achieved by the entropy method. This is explained by the penalization imposed to both criteria 2 and 3. Although the decision maker's

Table 2: Weights adjusted by the proposed approaches and other objective methods (for  $\mathbf{w} = [1/3, 1/3, 1/3]$ ).

Adjusted weights	Method			
	Entropy	CRITIC	ICA	Whitening
$\hat{w}_1$	0.34	0.42	0.39	0.39
$\hat{w}_2$	0.33	0.29	0.32	0.30
$\hat{w}_3$	0.33	0.29	0.29	0.31

opinion was responsible to the increase of  $\hat{w}_3$ , due to the redundancy between criteria 2 and 3, the automatic methods could reduce this value and increase  $\hat{w}_1$  (the independent one). This situation highlights the importance of combining subjective (which satisfies the decision maker’s opinion) and objective (which mitigate possible bias) weighting methods when dealing with redundant criteria.

Table 3: Weights adjusted by the proposed approaches and other objective methods (for  $\mathbf{w} = [1/4, 1/4, 2/4]$ ).

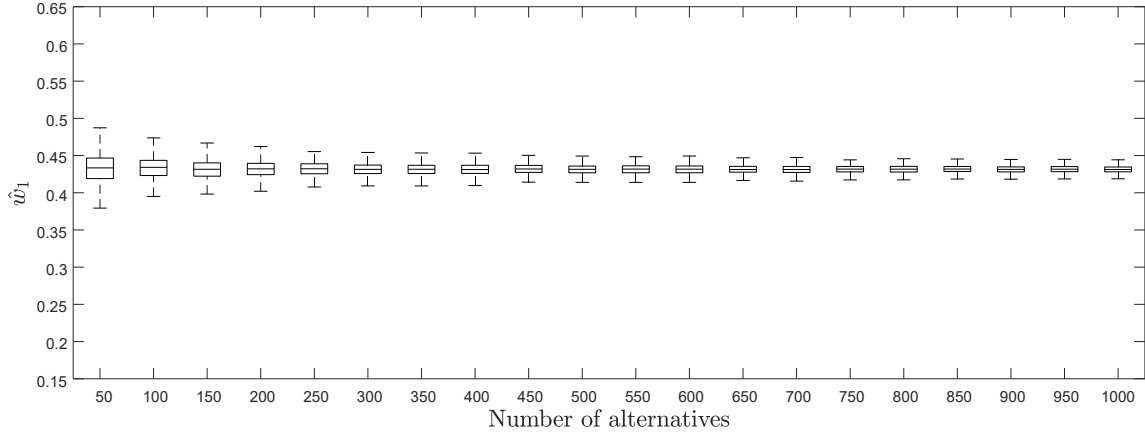
Adjusted weights	Method			
	Entropy	CRITIC	ICA	Whitening
$\hat{w}_1$	0.25	0.33	0.31	0.30
$\hat{w}_2$	0.25	0.22	0.25	0.23
$\hat{w}_3$	0.50	0.45	0.44	0.47

### 5.3. Experiments varying the number of alternatives

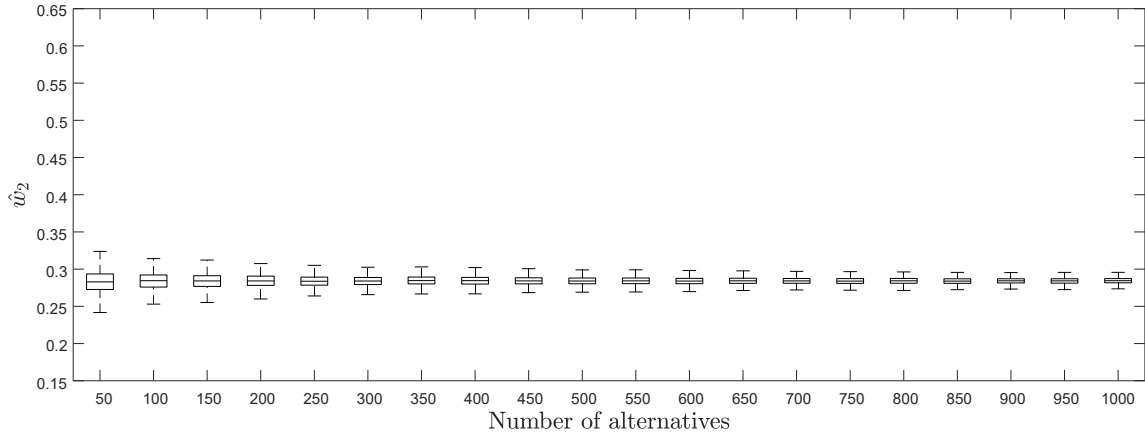
In the previous experiment, we conducted an experiment based on a single decision scenario. In this section, aiming at further investigating the stability in the weights adjustments, we performed an experiment varying the number of alternatives from 50 to 1000 (the other parameters were the same as in the last experiment). Based on 1000 simulations, Figures 5, 6 and 7 present boxplots of the obtained weights<sup>6</sup>.

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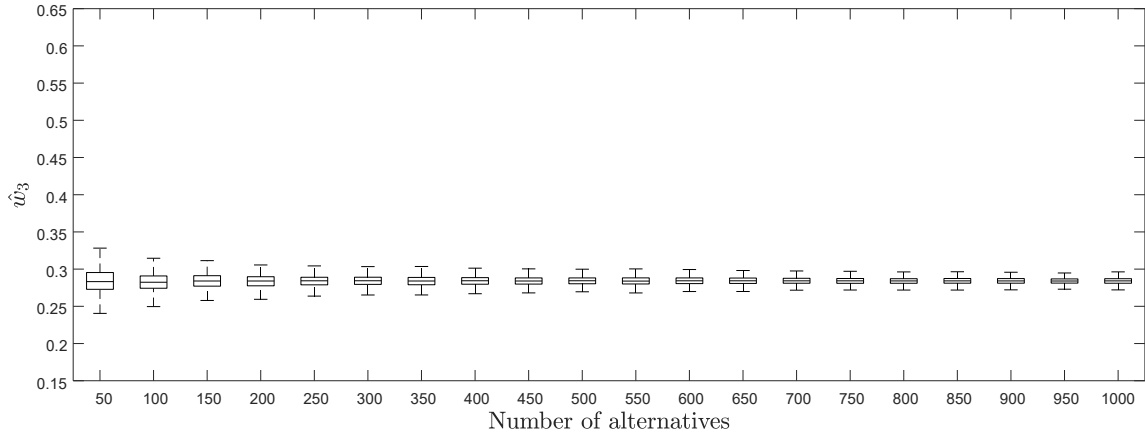
<sup>6</sup>In each box, the central mark, the top edge and the bottom edge represent the median ( $Q_2$ ), the 75th percentile ( $Q_3$ ) and the 25th percentile ( $Q_1$ ), respectively. Moreover, the whiskers extend until the extremes points that lies in the range  $[Q_3 - 1.5(Q_3 - Q_1), Q_1 + 1.5(Q_3 - Q_1)]$ , which covers 99,3% of the points, approximately, if the data are normally distributed. The points outside this range are considered as outliers and were removed from the boxplot.



(a) Weight for criterion 1.

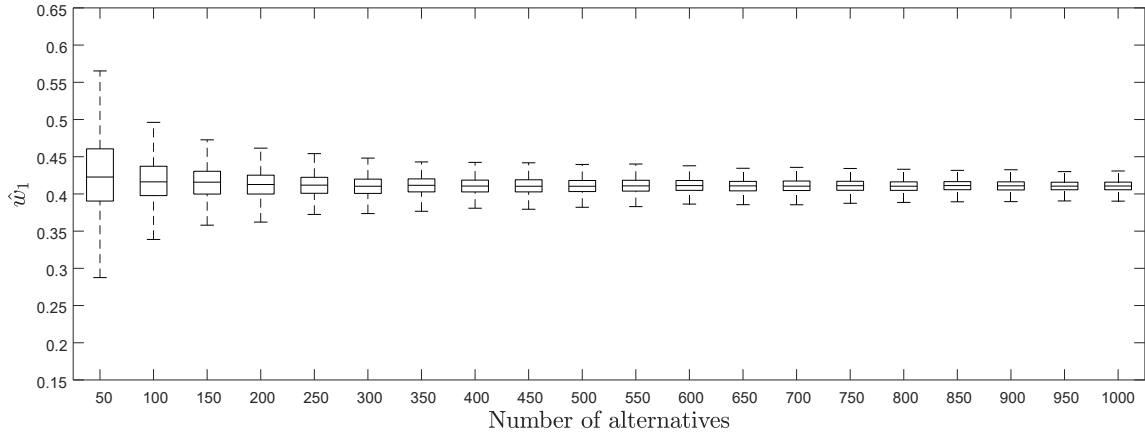


(b) Weight for criterion 2.

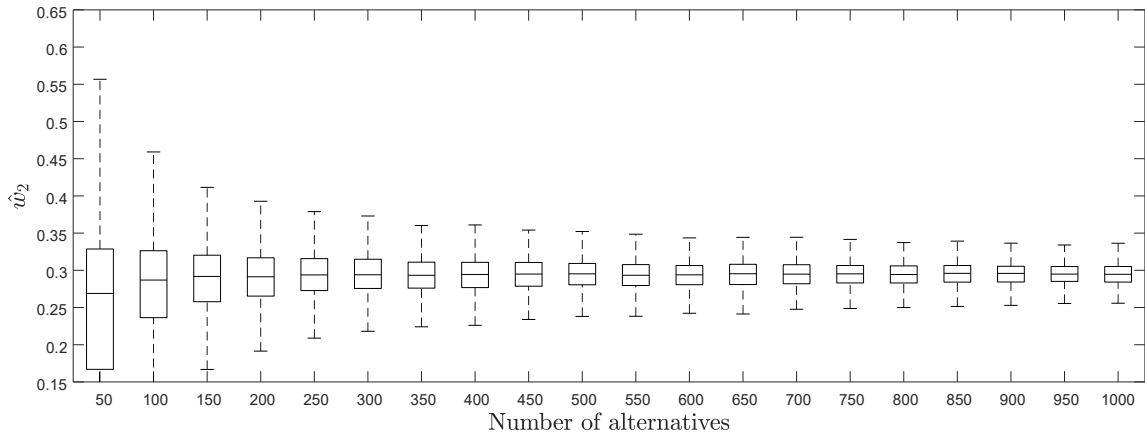


(c) Weight for criterion 3.

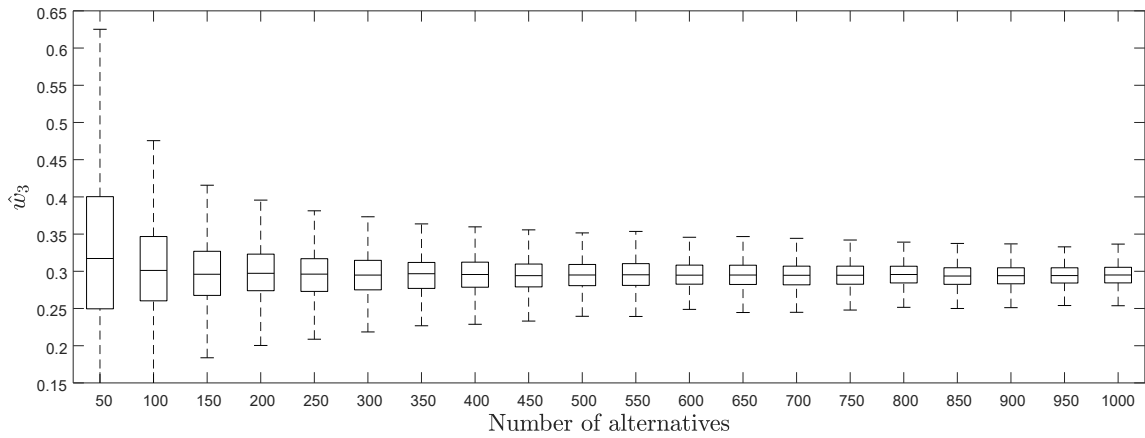
Figure 5: Adjusted weights by using CRITIC method.



(a) Weight for criterion 1.

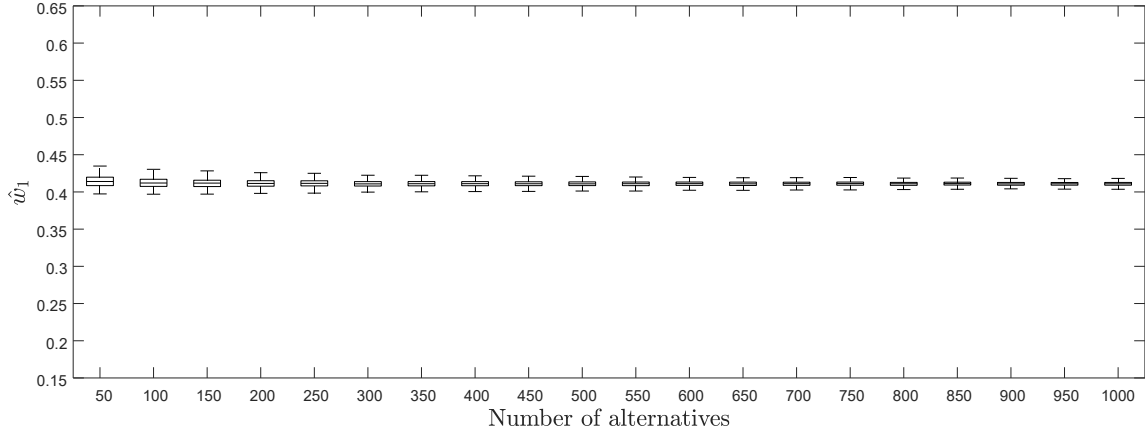


(b) Weight for criterion 2.

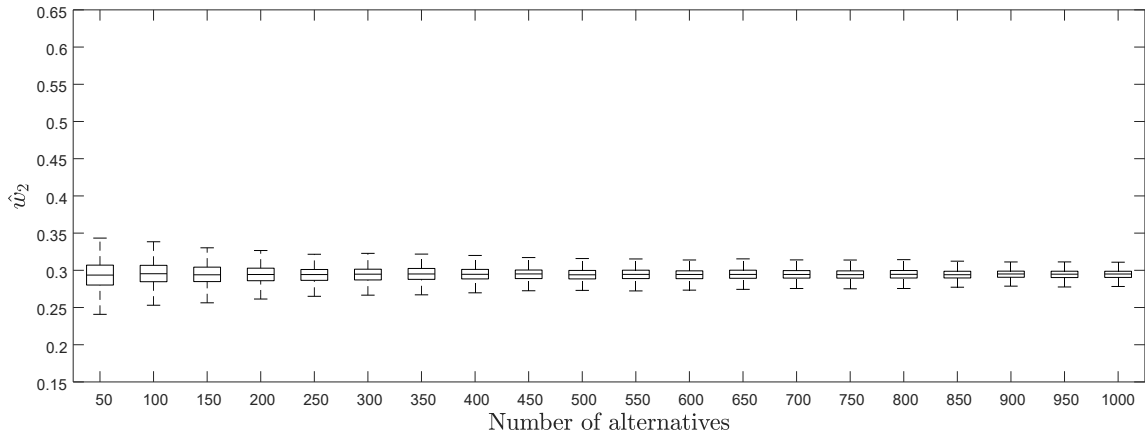


(c) Weight for criterion 3.

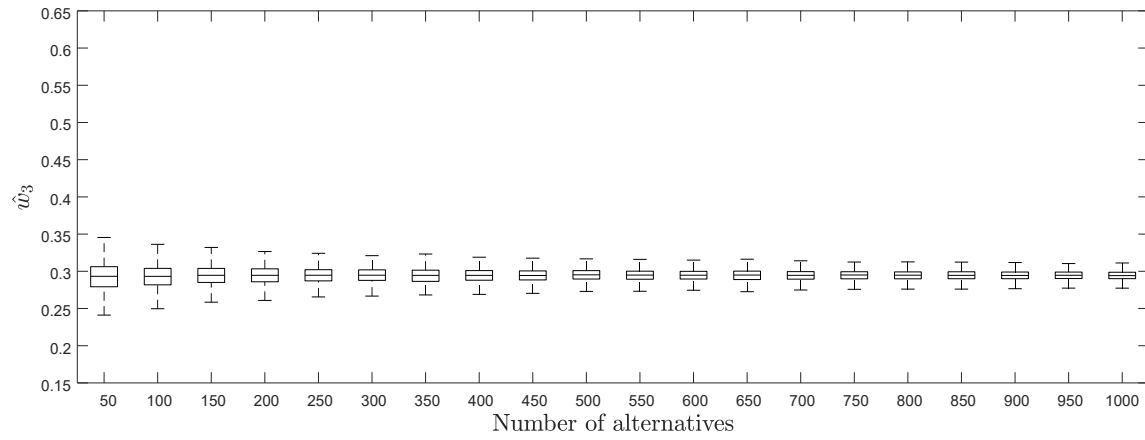
Figure 6: Adjusted weights by using ICA.



(a) Weight for criterion 1.



(b) Weight for criterion 2.



(c) Weight for criterion 3.

Figure 7: Adjusted weights by using whitening.

We note that the stability of the results provided by the application of ICA increases when the number of alternatives also increases. With respect to both CRITIC and whitening, the weights are stable even with a small dataset. This suggests that the use of the latter methods, even if only a decorrelation procedure is taken into account, is enough to mitigate the redundancies. If one considers a stable scenario, one achieves  $\hat{\mathbf{w}}_{CRITIC} = [0.43, 0.29, 0.28]$ ,  $\hat{\mathbf{w}}_{ICA} = [0.41, 0.29, 0.30]$  and  $\hat{\mathbf{w}}_{White} = [0.41, 0.30, 0.29]$ , i.e., very similar penalization for criteria 2 and 3.

Besides the stability in the weights adjustments, we also verified the averaged time execution of each considered approach<sup>7</sup>. There were few variations even with different number of alternatives, especially for CRITIC and whitening. For 100, 500 and 1000 alternatives, the time (in seconds) spent for CRITIC was  $9.42 \times 10^{-5}$ ,  $1.01 \times 10^{-4}$  and  $1.44 \times 10^{-4}$ , respectively. For ICA, we spent  $2.00 \times 10^{-3}$ ,  $2.00 \times 10^{-3}$  and  $2.08 \times 10^{-3}$  and, for whitening,  $4.23 \times 10^{-4}$ ,  $4.69 \times 10^{-4}$  and  $6.23 \times 10^{-4}$ . As expected, CRITIC is the least time expensive since there are only a few computations. On the other hand, ICA is the most time expensive due to the steps in estimating the mixing matrix. However, the time spent by all methods are very acceptable in practical MCDM problems.

#### 5.4. Experiments varying the degree of correlation

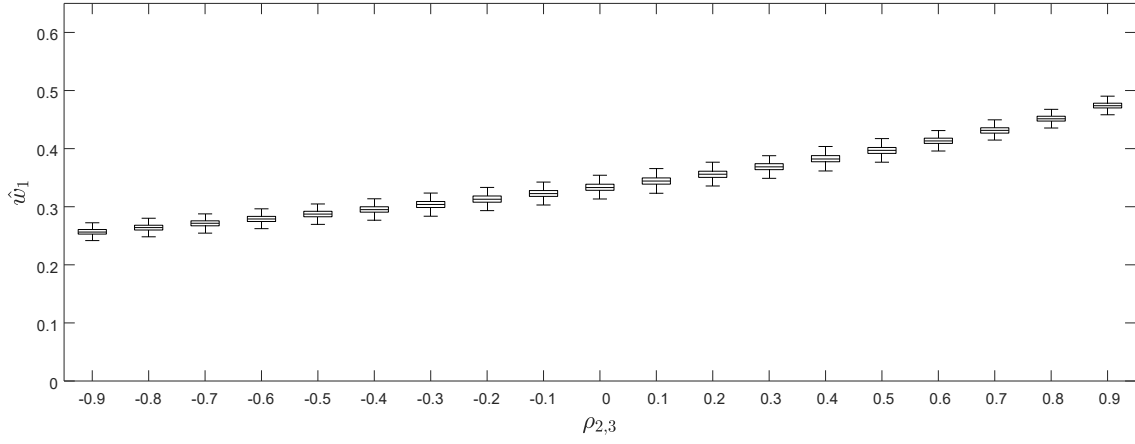
Instead of tackling scenarios with different numbers of alternatives, in this experiment, we vary the degree of correlation between criteria 2 and 3. For this purpose, we generate synthetic data with 500 alternatives and  $\rho_{2,3}$  in the range  $[-0.9, 0.9]$  (the other parameters were kept the same). Based on 1000 simulations, Figures 8, 9 and 10 present boxplots of the obtained weights.

As expected, the higher the correlation, the higher the adjustments on the criteria weights. However, if we analyze the scenarios in which  $\rho_{2,3} < 0$ , the values of both  $\hat{w}_2$  and  $\hat{w}_3$  (resp.  $\hat{w}_1$ ) increase (resp. decreases) faster in our proposal in comparison with the CRITIC method. Therefore, our proposal favored negatively correlated criteria more in comparison with CRITIC method. For instance, in the extreme case where  $\rho_{2,3} \approx -0.9$ , the weights associated with criteria 2 and 3 for both ICA and whitening are close to 0.42 and for CRITIC is close to 0.37. Depending on the addressed situation, this may raise a problem since our proposal associates a small weight to the independent criteria ( $\hat{w}_1 \approx 0.16$ , in this case), which decreases its impact on the overall evaluations.

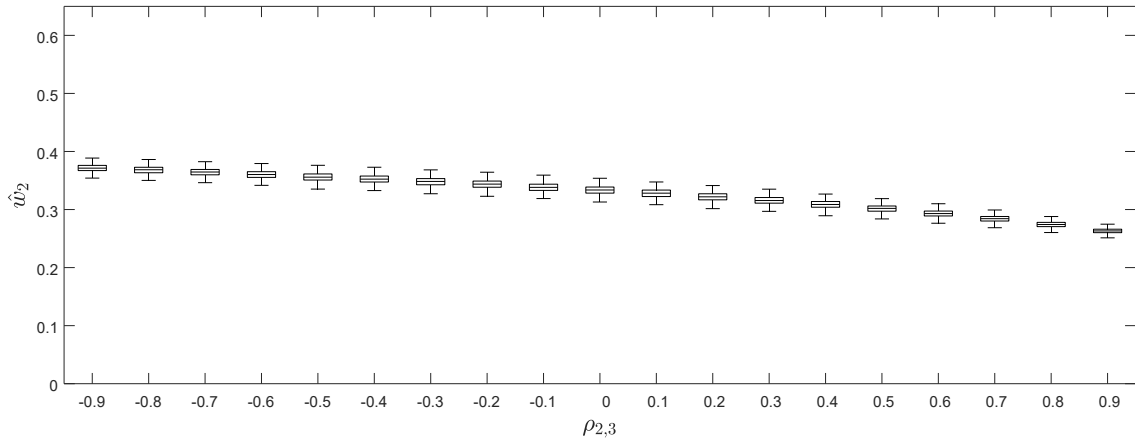
One may also remark on the obtained results that the weights adjusted by the ICA technique are not stable, even with 500 alternatives, in scenarios with a strong correlation between criteria 2 and 3 ( $\rho_{2,3} \approx -0.9$  and  $\rho_{2,3} \approx 0.9$ ). The explanation for such a result is that, since a strong correlation

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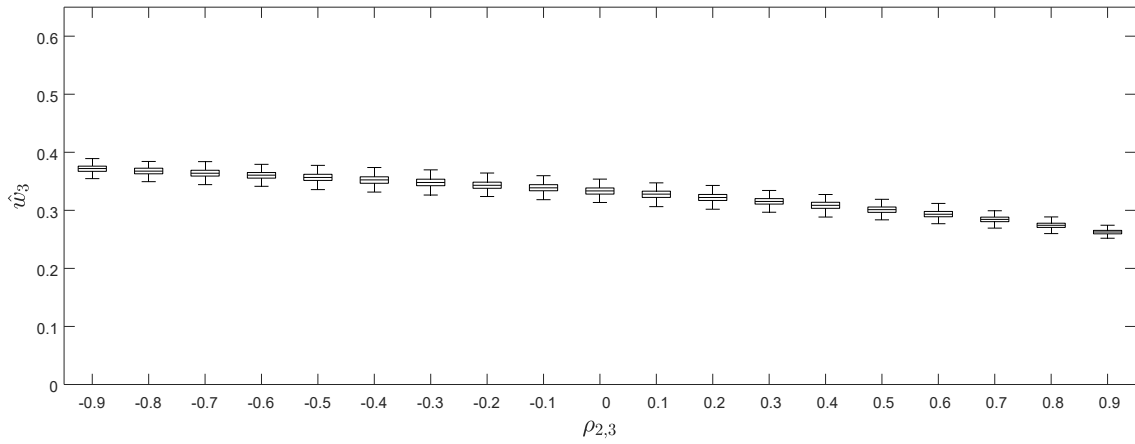
<sup>7</sup>Computing device: Intel Core i7-8565U, CPU 1.80 GHz, 8.00 GB RAM, software MATLAB 2015a.



(a) Weight for criterion 1.

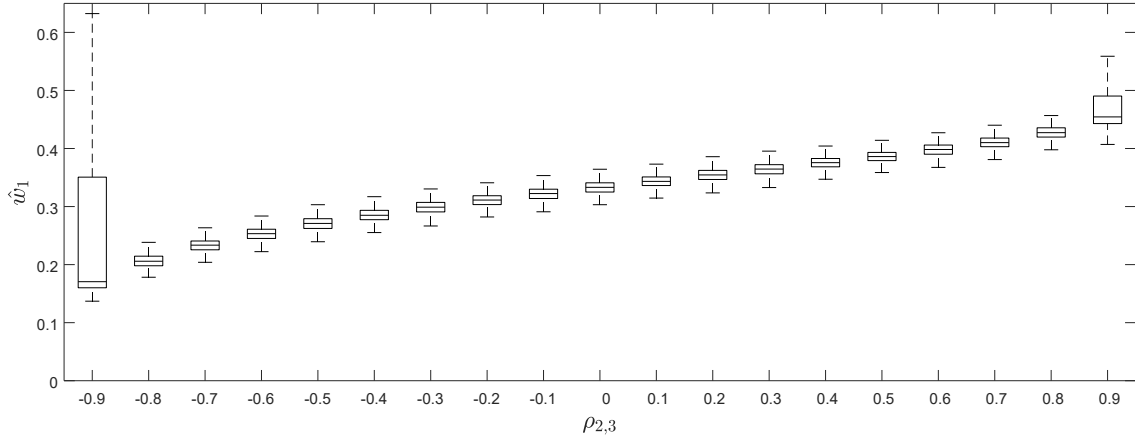


(b) Weight for criterion 2.

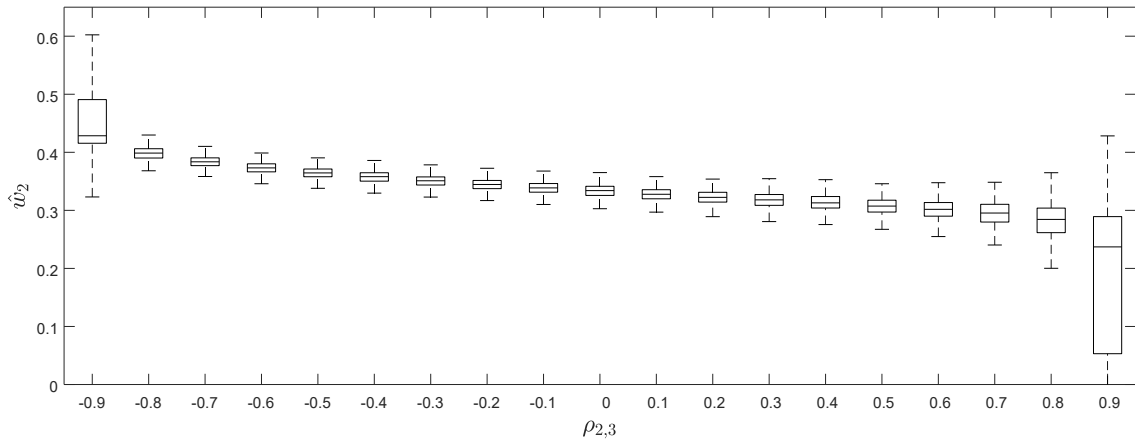


(c) Weight for criterion 3.

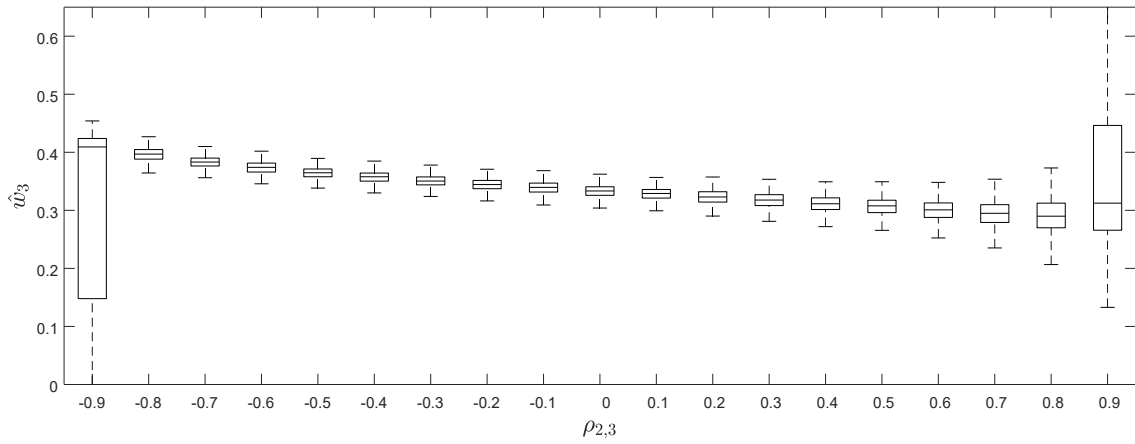
Figure 8: Adjusted weights by using CRITIC method.



(a) Weight for criterion 1.



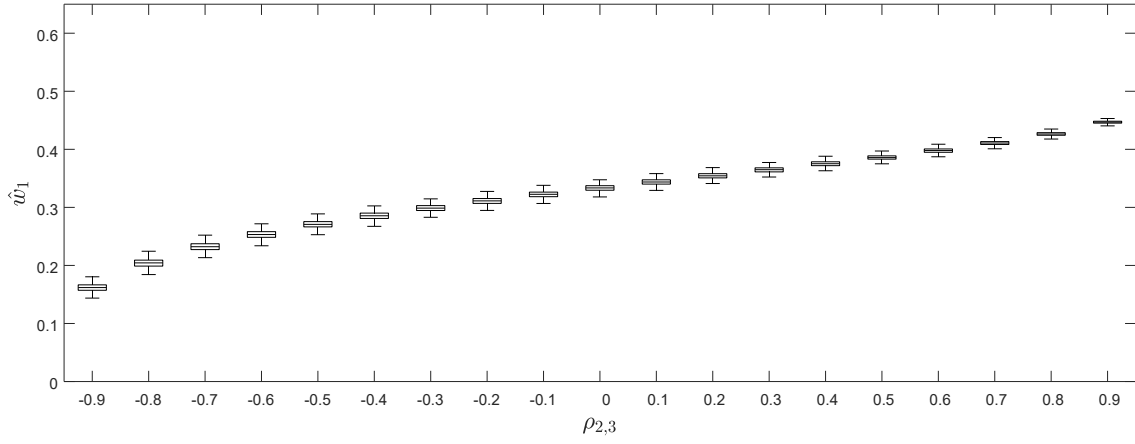
(b) Weight for criterion 2.



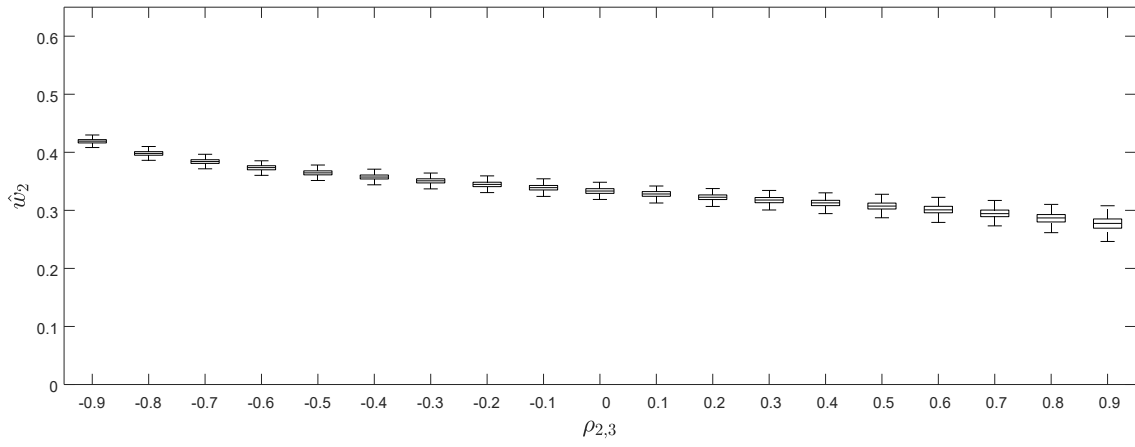
(c) Weight for criterion 3.

Figure 9: Adjusted weights by using ICA.

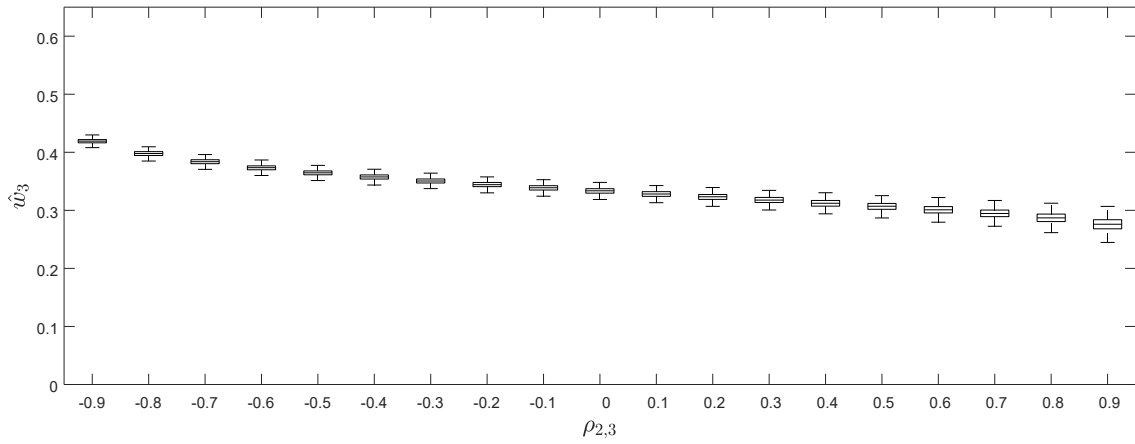




(a) Weight for criterion 1.



(b) Weight for criterion 2.



(c) Weight for criterion 3.

Figure 10: Adjusted weights by using whitening.

indicates that two criteria are practically the same (or the opposite, in a negative correlation), the assumed hypothesis (ii) (see Section 4.1) is violated (a single latent factor will have a major influence in two observed criteria). As a consequence, the procedure used to mitigate the permutation and scaling ambiguities does not work and, therefore, the adjustment of the weights fails. In such scenarios, the use of whitening is more suitable to avoid undesirable results.

We also verify the averaged execution time when varying the degree of correlation. The time spent for CRITIC and whitening remained practically unchanged regardless the degree of correlation. For  $\rho_{2,3} \approx -0.8, 0$  and  $0.8$ , CRITIC spent  $1.16 \times 10^{-4}$ ,  $1.15 \times 10^{-4}$  and  $1.16 \times 10^{-4}$  seconds and whitening spent  $4.97 \times 10^{-4}$ ,  $4.96 \times 10^{-4}$  and  $5.01 \times 10^{-4}$  seconds, respectively. On the other hand, ICA consumed more time with the increase in the degree of correlation. For instance, for  $\rho_{2,3} \approx -0.8, 0$  and  $0.8$ , ICA spent  $2.09 \times 10^{-3}$ ,  $1.86 \times 10^{-3}$  and  $2.04 \times 10^{-3}$  seconds, respectively. Besides the difference in the execution time, all methods are fast enough to be used in practical MCDM problems.

### 5.5. Application on real data

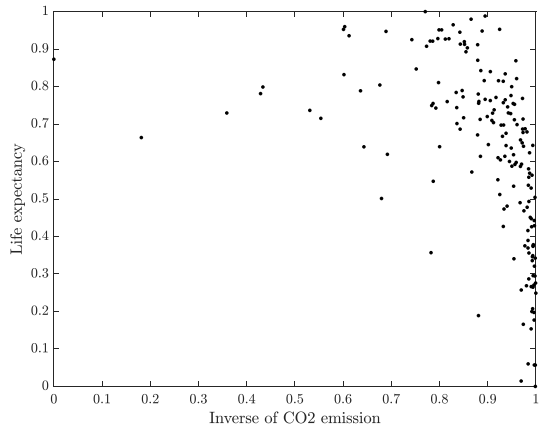
In order to attest our proposals on a real scenario, we consider a problem of ranking<sup>8</sup>  $n = 176$  countries according to  $m = 4$  indicators: the inverse of  $CO_2$  emissions (criterion 1, in 1/metric tons per capita), life expectancy at birth (criterion 2, in years), forest area (criterion 3, in % of land area) and gross national income (GNI) per capita (criterion 4, in current US\$). These data were collected from the World Bank at <http://www.worldbank.org> and refer to 2016 (we normalized the data in the interval  $[0, 1]$ ). The scatter plots of pairs of criteria and the associated correlation coefficients are presented in Figure 11 and Table 4, respectively.

Table 4: Pearson’s correlation coefficients.

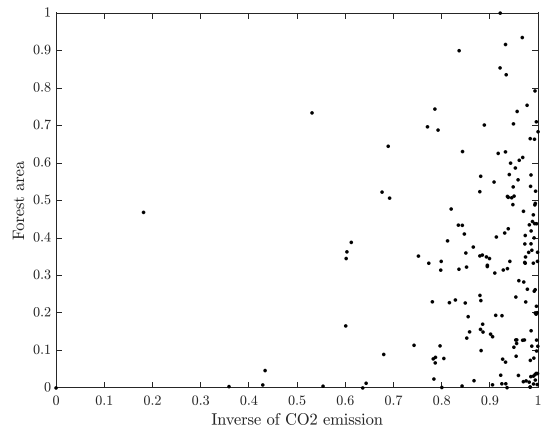
	Inverse of $CO_2$ emission	Life expectancy	Forest area	GNI
Inverse of $CO_2$ emission	1	-0.46	0.12	-0.60
Life expectancy		1	0.01	0.64
Forest area			1	-0.03
GNI				1

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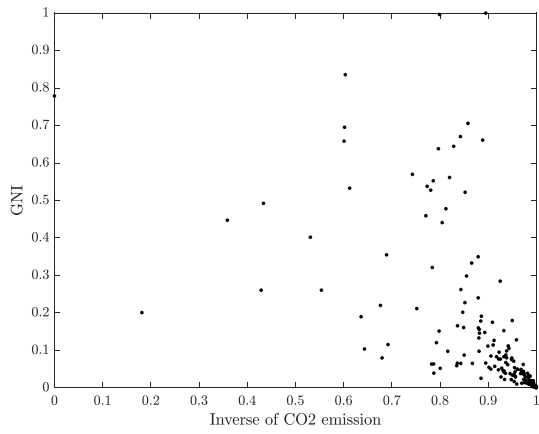
<sup>8</sup>In this paper, we consider a scenario with 4 criteria only to attest our proposals in a real dataset. For a complete analysis, more criteria can be used to rank countries.



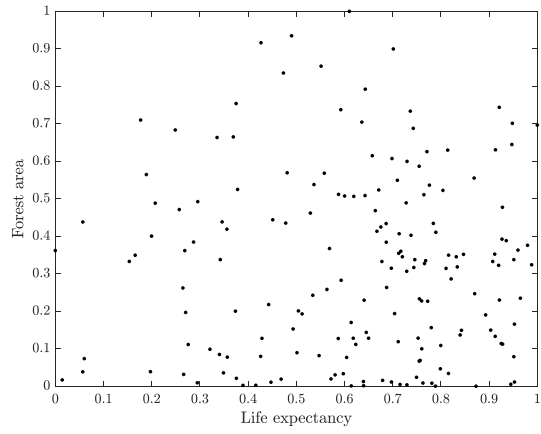
(a) Criteria 1 and 2.



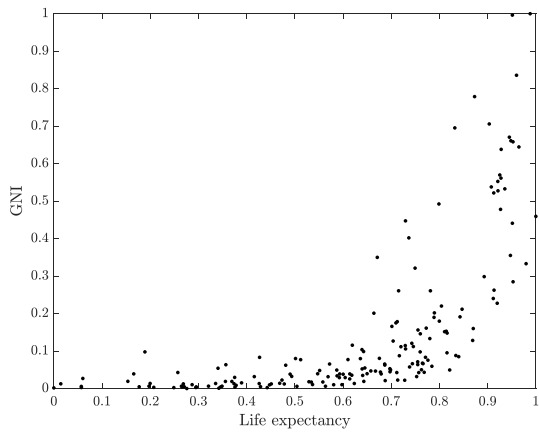
(b) Criteria 1 and 3.



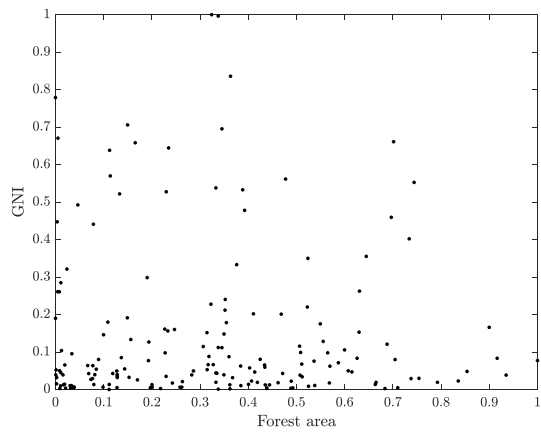
(c) Criteria 1 and 4.



(d) Criteria 2 and 3.



(e) Criteria 2 and 4.



(f) Criteria 3 and 4.

Figure 11: Scatter plot of pairs of criteria - Real data.

Although the inverse of  $CO_2$  emission reflects an environmental aspect, this indicator is negatively correlated with the GNI (an economic indicator). Moreover, one may note that both life expectancy and GNI are positively correlated. Since the former may be affected by several factors, such as social and economic ones, this indicator may be dependent on the GNI. Therefore, there are latent factors (e.g., environmental, social, land use and economic) that may be associated with more than one indicator. Since one may not be aware of the degree of such redundancies, the method used to aggregate the collected information should take this characteristic into account.

By assuming that  $\mathbf{w} = [1/4, 1/4, 1/4, 1/4]$ , the application of the considered approaches lead to the weights presented in Table 5. It is interesting to remark that the CRITIC method provided few adjustments on the weights, even if there are redundancies on the decision data. For instance, both ICA and whitening favored the weight associated with the inverse of  $CO_2$  emission. This is explained by the negative correlation of this criterion with respect to both life expectancy and GNI. Moreover, since the life expectancy is more positively correlated with GNI than negatively correlated with the inverse of  $CO_2$  emission, the whitening method penalized the associated weight ( $\hat{w}_2$  lower than 0.25). These results suggest that whitening could capture more intercriteria relations in comparison with CRITIC.

Another interesting result was obtained for  $\hat{w}_4$ . Both CRITIC and whitening provided a practically insignificant adjustment on the weight associated with GNI since the positive correlation is practically canceled by the negative ones.

Table 5: Adjusted weights - Real data.

Adjusted weights	Method		
	CRITIC	ICA	Whitening
$\hat{w}_1$	0.23	0.34	0.35
$\hat{w}_2$	0.26	0.25	0.20
$\hat{w}_3$	0.27	0.22	0.21
$\hat{w}_4$	0.24	0.19	0.24

### 5.6. Experiment on a group MCDM problem

In the previous experiments, we tackled MCDM problems with a single decision maker. In contrast, in this section we deal with correlations in the decision data in group MCDM problems.

In order to attest our proposal on a simple example, we conducted an experiment on a synthetic dataset made with 50 alternatives, 4 criteria and 3 decision makers. All data were randomly generated according to a uniform distribution in the range  $[0, 1]$ . Figures 12 and 13 present scatter plots of pairs of criteria and pairs of decision makers evaluations, respectively. In these figures, we also highlight the origin of each point in the concatenated vector. One may note that criteria 2 and 3 and decision makers 1 and 2 are correlated ( $\rho_{2,3} \approx 0.61$  and  $\rho_{1,2}^d \approx 0.84$ , respectively).

Consider that, initially, we define  $\mathbf{w} = [1/4, 1/4, 1/4, 1/4]$  and  $\mathbf{w}^d = [1/3, 1/3, 1/3]$ . With the application of the considered approaches on both  $\hat{\mathbf{U}}^C$  (to adjust the criteria weights  $w_i$ ) and  $\hat{\mathbf{U}}^D$  (to adjust the decision makers weights  $w_k^d$ ), one obtains the results described in Table 6. Similarly as in the previous experiments, all approaches provide a penalization of correlated criteria and correlated decision makers.

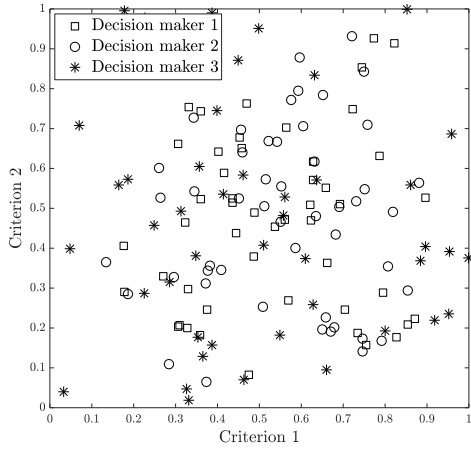
Table 6: Criteria and decision makers weights adjusted by the considered approaches.

Methods	Adjusted weights						
	$\hat{w}_1$	$\hat{w}_2$	$\hat{w}_3$	$\hat{w}_4$	$\hat{w}_1^d$	$\hat{w}_2^d$	$\hat{w}_3^d$
CRITIC	0.26	0.23	0.22	0.29	0.23	0.23	0.54
ICA	0.29	0.17	0.36	0.18	0.32	0.26	0.42
Whitening	0.29	0.21	0.20	0.30	0.29	0.28	0.43

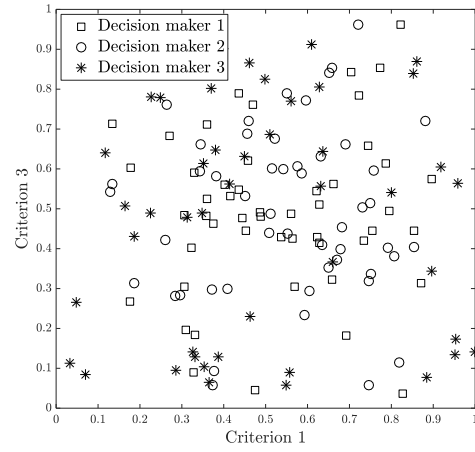
It is important to mention that the adjustments on the criteria and/or decision makers weights are associated with the degrees of redundancy between these entities. Therefore, if the decision makers opinions are uncorrelated, our approach will not modify the decision makers weights. Moreover, in a scenario in which all individuals share the same opinion (e.g., when we achieve consensus), the redundancies (or correlations) between different decision makers will be practically the same. In this case, the same penalization will be applied on all decision makers and, after normalizing the weights, they will remain the same as they were initially defined.

## 6. Conclusions

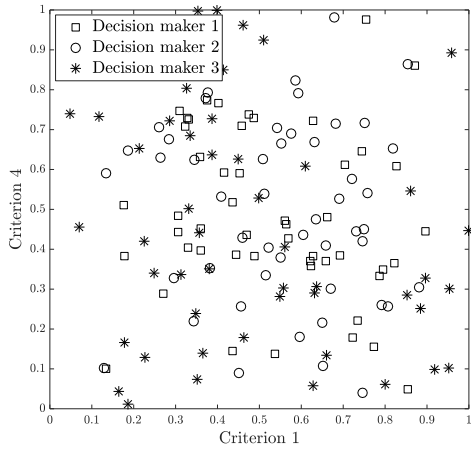
Ranking alternatives based on a set of criteria is a typical problem in multicriteria decision making. In such situations, it is desirable that the considered criteria do not contain redundancies among them, i.e., that there is no correlation in the decision data. However, these redundancies are



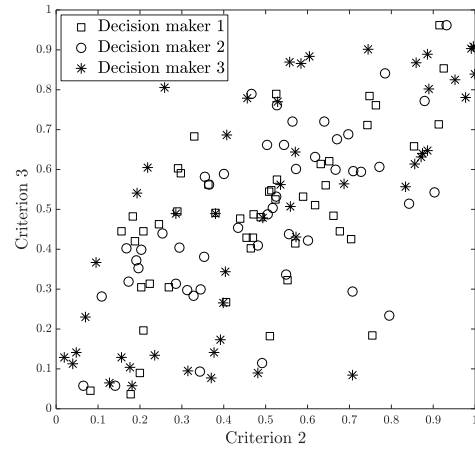
(a) Criteria 1 and 2.



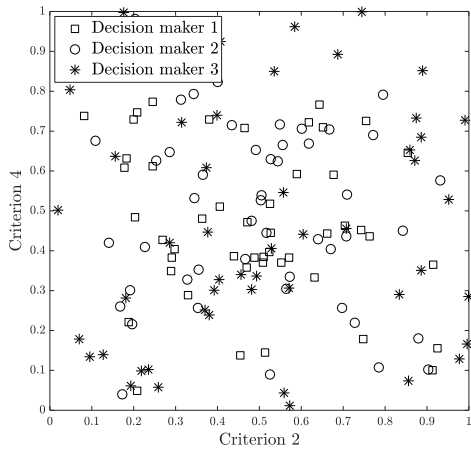
(b) Criteria 1 and 3.



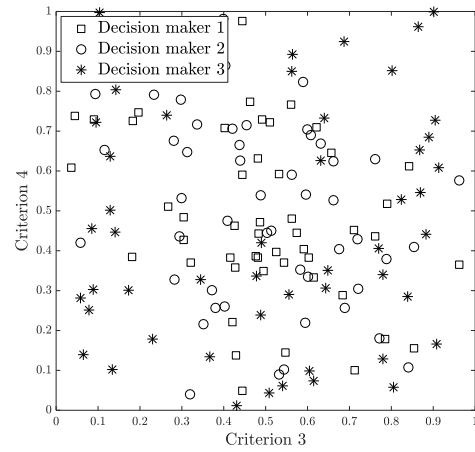
(c) Criteria 1 and 4.



(d) Criteria 2 and 3.

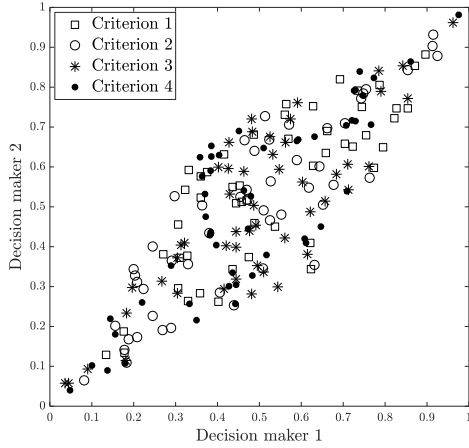


(e) Criteria 2 and 4.

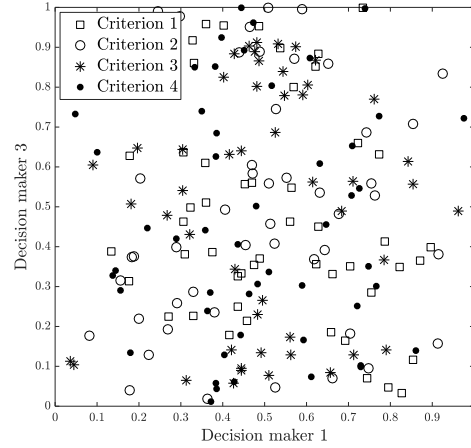


(f) Criteria 3 and 4.

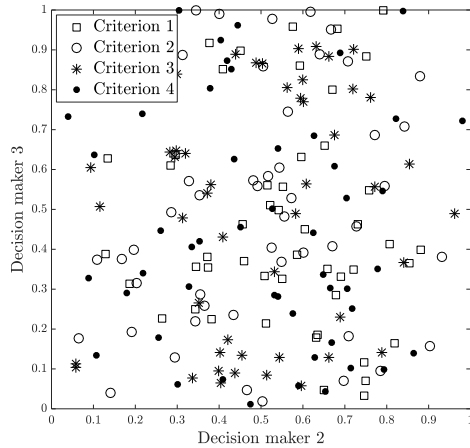
Figure 12: Scatter plots of (concatenated) criteria.



(a) Decision makers 1 and 2.



(b) Decision makers 1 and 3.



(c) Decision makers 2 and 3.

Figure 13: Scatter plots of (concatenated) decision makers evaluations.

often observed in real decision problems. Therefore, the development of techniques that take this characteristic into account is of importance in the research community.

In this paper, we proposed an approach that is able to deal with redundancies among criteria through the weighted arithmetic mean with adjusted weights. For this purpose, we applied both ICA and whitening techniques to extract the information used to adjust the criteria weights. In situations in which two criteria are redundant, they are penalized and the weight associated with an independent criterion increases. If the criteria are negatively correlated, our proposals favor them by increasing the associated weights. This is the case of the inverse of  $CO_2$  emission in the experiment with a real dataset. On the other hand, in situations with no redundancies among criteria, the whitening matrix is diagonal and, therefore, no adjustments will be performed. Moreover,

positive and negative correlations may cancel each other, therefore, our proposal will not lead to a considerable adjustment on the associated weights.

Other than ICA and whitening, we also considered in the experiments both entropy and CRITIC methods. Since the entropy method does not consider information between pairs of criteria in the weights definition, it does not mitigate the effect of redundant criteria. On the other hand, CRITIC could deal with such redundancies. However, by comparing both CRITIC and whitening, the obtained results are similar for low to medium degrees of correlation but our approach penalizes the associated criteria more than the CRITIC method when this relation is strong (or favors, in the case of a strong negative correlation). This can be explained by how these methods use the second-order statistics in the weights adjustments. CRITIC method performs the adjustment on a criterion weight based on the linear correlation coefficients between this criterion and the other ones (see Equations (22) and (24)). On the other hand, since whitening consists in a decorrelation procedure followed by a power normalization, all intercriteria information are exploited simultaneously. Therefore, we consider that whitening captures more information in comparison with CRITIC to adjust the criteria weights.

By comparing both whitening and ICA, we note that the former led to better results, specially in terms of the stability in the adjusted weights, in scenarios with small number of data. If we have a large number of data, they achieve a similar performance.

Besides the application in MCDM problems with a single decision maker, we also extended our proposal to the group MCDM. In this scenario, the weights associated with decision makers can be adjusted in order to penalize the ill-intentioned ones who have combined their opinions. Therefore, we provided adjustments in both criteria and decision makers weights by applying our proposal twice.

Future works may address decision data in which there exist other kind of relation (nonlinear, for example) among criteria. Moreover, we also encourage novel studies to exploit the case where the number of latent variables is different from the number of observed criteria. This scenario may be tricky for the decision maker, since he/she will need to either define the initial weights by assuming an additional information about the latent variables or, in the absence of such an information, set the same value for all weights.



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