

## Navigating the sea of histories of mathematics Agathe Keller

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# Navigating the sea of histories of mathematics

### **Agathe Keller**

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### Navigating the sea of histories of mathematics

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#### ABSTRACT

This essay argues against a history of mathematics that celebrates a unique Greek 'outlier' in world history, while raising the question of the new type of histories of mathematics that we should write today. Taking examples from the history of mathematical sources in Sanskrit and histories of mathematics written in South Asia, this essay deconstructs some assumptions behind narratives of Greek and European exceptionalism. In particular, it challenges the notion that ancient Greece possessed a unique democratic culture that fostered scientific debate and that only Greece possessed brash authors able to challenge common sense. The essay provides a reflection on the political histories that European exceptionalism – in particular regarding mathematics – has directly or indirectly shaped.

**KEYWORDS** 

Far-right; Hindu nationalism; white supremacy; śulbasūtra; Āryabhaṭa; Brahmagupta; city-states; mathematical proofs

He looked into the water and saw that it was made up of a thousand thousand thousand and one different currents, each one a different colour, weaving in and out of one another like a liquid tapestry of breathtaking complexity; and Iff explained that these were the Streams of Story [...] And because the stories were held here in fluid form, they retained the ability to change, to become new versions of themselves, to join up with other stories and so become yet other stories. (Rushdie 1990, 72)

#### 1. Introduction

On the 22nd of July 2011, Anders Behring Breivik killed 77 people in and around Oslo. In *2083: A European Declaration of Independence*, the manifesto he published online to explain his act, he proclaims a war to defend Europe against its degradation by 'cultural marxists' and its overtaking by Islam. Copying blog posts by Fjordmann, a far-right Norwegian blogger with an interest in history of science, Breivik celebrates science as European and Christian, encouraging his readers to explore the writings of Edward Grant, David C. Lindberg, Toby E. Huff, Victor J. Katz, James C. Evans and John North.<sup>1</sup> Of

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<sup>&</sup>lt;sup>1</sup>In his manifesto, Breivik writes: 'If you are looking for books about the history of science, I recommend everything written by Edward Grant. *The Beginnings of Western Science* by David C. Lindberg is very good, though slightly more politically correct than Grant when it comes to science in the Islamic world. *The Rise of Early Modern Science: Islam, China and the West* by Toby E. Huff is excellent and highly recommended. These books are easy to read for an educated, mainstream audience. For books that are excellent, yet more specialised and slightly more difficult, I can recommend Victor J. Katz for the history of mathematics and *The History and Practice of Ancient Astronomy* by James Evans for the history of pre-

course, none of our colleagues wrote to fuel far-right declarations let alone mass murders, nor do such appropriations of their scholarship reflect on the quality of their works that we continue to read and use. Indeed, the goal of some of the scholars Fjordmann refers to was precisely to promote the knowledge of scientific sources beyond those of classical Greece and Europe (Lindberg on science in Islam, Evans on Babylonian astronomy, etc.): they could have just as easily been labelled as 'cultural Marxists' or 'Islamists' by Breijvik. But while spectacular, this use of history of science by the far-right white nationalists - and by religious fundamentalists who are sometimes their allies - is not an isolated case. It is now a global phenomenon.<sup>2</sup> What consequences should we draw from these facts? Of course, we cannot be held responsible for all mis-uses and mis-representations of our research. Nonetheless, I believe that far-right and religious fundamentalists' exploitation of our works invite us all, as historians of science, to consider what people might read into our publications. How should we deal with the political claims that our research can be subject to? What responsibilities do we have as professional historians of science for statements on history of science made in the public sphere, especially when they invoke our work? I do not profess to offer any definitive answer to these questions, but Reviel Netz's essay brought me back to them. Let me be clear: I do not think that Reviel Netz is complacent in any way to far-right appropriations of his work. Yet his essay raises historiographic questions that need critical examination. To be more direct: Brejvik's use of works by historians of science reminds us that, unknowingly, Eurocentrism, forms of white supremacy and sexism have shaped much of past publications in the history of science. This statement might strike some as lacking in nuance. I hope to convince my readers that if there is any truth in it, we need urgently to critically deconstruct past histories that contain such assumptions and simultaneously wonder whether documents of the past allow us to construct new histories of science and new histories of mathematics.

Netz argues that Archimedes radically shaped the world we live in: his work enabled the scientific revolution and the subsequent industrial revolution. Archimedes, according to Netz, initiated an improbable mode of studying conics and introduced mathematics into the study of physical phenomena that enabled mathematicians to elaborate an objective way of thinking of the world, a way of abstracting from it, that was especially fruitful.

Reviel Netz likes provocation. His essay swims upstream against standard trends in history of mathematics. As he notes himself, his essay invokes the importance of a unique hero, Archimedes, while academic history of science has largely set aside eulogies of great men. But of course, there is a greater provocation in Reviel Netz's statement, which brings us back to the uses the far right can make of history: are Greek and European exceptionalism still the only narrative that our sources testify to? Is this the only story that can be told?

telescopic astronomy up to and including Kepler. Evans' book is extremely well researched and detailed, almost too much so on European and Middle Eastern astronomy, but contains virtually nothing on Chinese or Mayan astronomy. For a more global perspective, *Cosmos: An Illustrated History of Astronomy and Cosmology* by John North is good and not too difficult to read (Fjordman's tips),' 2083: A European Declaration of Independence (772). (Accessed online 18/08/22).

<sup>&</sup>lt;sup>2</sup>For the uses of history of science by Hindu nationalists see section 3.4. For an other instance in which history and philosophy of science was used in an argument fuelled by the far-right, see, in France, (Gougenheim 2008) who claimed that Islam was not fit for science, and that the role of scientific texts written in Arabic had been overplayed in the way historians wrote the history of the European renaissance. The publication of this book raised many academic disclaimers, concerning science notably (Bellosta 2009).

Netz argues for this exceptionalism, if I understand him correctly, for reasons that are in part different from those developed in the history of mathematics in the past, by using new approaches to history. In this reply, nonetheless, I argue against a history which celebrates a unique Greek 'outlier', and more largely against a kind of global history of mathematics written 'from above'. I will argue that some world-histories take approaches that in fact look down at the world from a very partial point of view. But more than deconstructive criticism, my goal is to point a way forward to the kind of histories of mathematics we could write today.

In what follows I will first analyse some of the statements in Netz's essay, reflecting critically on how they might be understood if we take a world-wide perspective in history of mathematics. In the second part, I will reflect on the political histories that European exceptionalism has directly or indirectly produced. My counter-examples will be taken from the history of mathematical sources in Sanskrit and histories of mathematics written in South Asia. Reflecting on how to write new histories of mathematics, Section 1 will deal with how Netz writes contextual global histories of mathematics, using a specific case study as a counter-example to some of his claims, that of the oldest south Asian mathematical texts the *Aphorisms on Ropes (śulbasūtra*). In section 2, turning directly to the politics of histories of science I will explore how the colonial and post-colonial histories in South Asia were shaped by histories of European exceptionalism. Finally, in the conclusion I will point to the kind of collective histories of mathematics we should write in the future.

#### 2. Looking at it from above: contextual global histories

Part of what makes Reviel Netz's essay thought-provoking is his effort, as always, to take a bird's eye view on the problem he wants to tackle. Indeed, his reflections on the different ways that mathematics and its history have contributed to shaping today's world invite us, refreshingly, to step back from what might be our too great involvement in the intricate details of case studies. There is certainly a need for a history of mathematics with such a perspective. But the kind of point of view adopted by Netz requires a distance from which the reasoning takes place – and this distance is a part of the problem.

#### 2.1. The historian's bird-eye's view

Indeed, such a distance might provide a point of view that is too general, and thus provoke assessments that lack nuance. Take, for example, one of Netz's arguments in favour of Greek exceptionalism that has to do with the social structure of classical Greek society: that democratic city states were the exceptional setting in which such exceptional mathematics was developed. Because this concerns the social and political environment in which mathematical texts are produced, it also raises questions about how we might write contextual histories of mathematics.

#### 2.1.1. Mathematics and the state

Netz argues that states were important for the development of mathematics. Building on a contrast between what he calls 'simple societies' with 'state societies', he affirms that the

first would have no specialized mathematical practitioners, while the second would have state-funded ones, such as school teachers and priests.<sup>3</sup> The Sanskrit *Aphorisms on Ropes* (*śulbasūtra*) offer a counter example to these too simple assumptions. The *Aphorisms on Ropes* are the oldest mathematical texts that have been preserved from South Asia. They are a complex set of layered texts, the most ancient strata dated anywhere from 800 to 600 BCE and the most recent to 300 CE or even later.<sup>4</sup> The *Aphorisms on Ropes* are a set of different works belonging to different schools of ritual. Each has an official author, although some might argue that Baudhāyana, Āpastamba, etc. are rather mythical figures – school founders at best. The mathematics of the *Aphorisms on Ropes* include practical algorithms to construct real ritual altars together with general mathematical reflections: from different versions of the Pythagorean theorem to exploring how geometrical figures are related to numerical relations, especially how rectangles, squares and trapeziums relate to Pythagorean triplets.<sup>5</sup>

So, were the *Aphorisms on Ropes* composed in a state-society or a simple-society, to take Netz's subdivisions? Well, neither. It is difficult to sketch the political structures in which these texts were composed, since the information we gather from Vedic literature does not correspond to what we know of the archeology and history of their period of composition. Standardly, before the Mauryan empire (ca. 320-ca.185 BCE) historians question whether any state existed in South Asia.<sup>6</sup> If we were to accept the axiom that 'simple societies' do not have professional mathematicians, then how should we treat the societies that produced the earliest *Aphorisms on Ropes*? Are they to be seen as 'outliers'? The risk with general historical rules is to make of true historical cases a list of exceptions to them.

But there is more. Following a long trail of scholars, Netz argues that Greek society and its political organization into democratic city-states would have enabled the fostering of the 'Greek exception'.<sup>7</sup> Democracy – as public deliberation of equal citizens according to written laws – would have fuelled the development of reason. City-states could subsidize intellectual activities, but not sufficiently. Consequently, intellectuals – and notably mathematicians – were forced into trying to be remarkable to be funded.<sup>8</sup> But it seems that what Netz and others see as the unique conjunction – of on the one hand the city-state and on the other its democratic institutions – was not so rare. The analysis of archeological excavations dating from a range of prehistoric and antique sites show that both the Indus Valley, the Megasites of Ukraine and Georgia or Meso-America had different versions of such social structure: an autonomous city with no apparent ruler and large halls where deliberations of a great number of citizens could take

<sup>&</sup>lt;sup>3</sup>For a critique of evolutionary historian's categories of 'simple societies', including a nice quotation of Ursula K. Le Guin, see (Graeber and Wengrow 2021, 297).

<sup>&</sup>lt;sup>4</sup>(Hayashi 2001, 761). Debates on the dates of Vedic sources (initiated by (Bronkhorst 2007, Part III)), might further change such a chronology making the *Aphorisms on Ropes* less antique then usually accepted.

<sup>&</sup>lt;sup>5</sup>(Hayashi 2002, 362) implicitly criticizes (Lloyd 1990, 104) argument that the *Aphorism on Ropes (sulbasūtras*) were essentially practical mathematics. The *sulbasūtras* also contain algorithms describing how to transform with strings and pegs given geometrical figures into another figure of same area or how to construct ritual altars of sophisticated shapes with a fixed number of square and rectangular bricks. Numerical methods to approximate the square-root of two were also developed in these texts.

<sup>&</sup>lt;sup>6</sup>(Bronkhorst 2016, 2).

<sup>&</sup>lt;sup>7</sup>Notably (Lloyd 1990, 8) – inspired by (Vernant 1962, 1965)—and (Ober 2015).

<sup>&</sup>lt;sup>8</sup>Netz's (and Ober's) version of this argument resonates with the structure of research in today's world, it maybe gives to this reasoning a new relevance; although if there were any universal truth in it, this would imply state policies to structurally keep scientific research underfunded to push for innovations ...

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place.<sup>9</sup> In the case of the prehistoric Indus Valley excavations, we lack texts that could help us document elements of the political and administrative organizations, such as how deliberations were carried out, and who could participate in them. However, these sites clearly show the absence of a king's palace or of wealthy neighbourhoods for a ruling class.<sup>10</sup> On the contrary, they testify to large egalitarian neighbourhoods, cemeteries and important large public buildings.<sup>11</sup> By the time we have texts, the situation is of course more complicated. Vedic texts, as it is well known, install a caste society for which a crude depiction would be that priests (brahmins, skt. brahman) are on the top counselling warrior kings (ksatriya) who rule on a cast of merchants and farmers (vaiśya) and low-casts (śudra). This is, of course, a vision of society promoted by Brahmins in the texts they composed. Bronkhorst (2016) argues however that it is not before the sixth or seventh century of the common era that they actually were able to enforce such a position on the whole of society. Buddhists and Jains, whose religions were born in the fifth century BCE as a protest against such a vision not only of religion but also of society, composed texts that evoke large debating assemblies (Bronkhorst 2016, 182). Vedic, Brahmanical, Buddhist and Jain texts all testify to a culture of debate. These debates might have taken place within a certain hierarchy - organized by a king or a religious superior - but they nonetheless could include a great number of people considered as equal during the discussion.<sup>12</sup> Little is known about the modes of deliberation that existed within them, although occasionally texts might provide some rules of how deliberations should take place.<sup>13</sup> Jain logical schools developed procedures to create consensus among multiple divergent views which inspire logicians today (Clerbout, Gorisse, and Rahman 2011). This shows that modes of arriving at a consensus were an important part of the culture related to these religious schools of thought and maybe were related to how their infrastructures/institutions worked. Arriving at an agreement is the basis of much of the philosophical school debates documented in Sanskrit sources. Indeed, from the third century BCE, different philosophical schools developed epistemological reflections, discriminating between modes of argumentation, their persuasiveness and ability to derive reliable knowledge. The role of inference (anumāna) in relation to traditional knowledge (āgama) and direct perception (pratyaksa) structured much of the philosophical conversations across schools. The Indus cities and settlements, followed by schools of philosophy in early South Asia are just one example. Many other modes of democratic deliberation among city-states can be testified for in other places of the world in antiquity. But different modes of democratic deliberation and critical rational debates also existed in other venues, such as courts of law, monasteries, and in

<sup>&</sup>lt;sup>9</sup>(Graeber and Wengrow 2021; chapter 8: 283–333) For a treatment of South Asia, but also more generally a history of assumptions on states and rulers by archeologists see (Green 2021).

<sup>&</sup>lt;sup>10</sup>Graeber and Wengrow argue that non-egalitarian societies leave archeological traces that egalitarian societies do not. The presence of the first is thus easier to prove than the presence of the second. (Graeber and Wengrow 2021, 326, 337), see also for a similar argument (Green 2021, 161; 182).

<sup>&</sup>lt;sup>11</sup>The elaborate craftsmanship associated with this society, shows that everything from beads, bangles and seals to city planning was made by collaborating bodies of experts who exchanged know-how enabling constant technological innovation. Archeologists thus believe that negotiations and discussions within different corporate groups on all different matters took place in the various public halls all the cities display.

<sup>&</sup>lt;sup>12</sup>Eric Gurevitch has added pointedly that kings could indeed be challenged during these debates, as illustrated famously in the *Milindapañha*, in which the king Milinda is challenged by the sage Nagasena.

<sup>&</sup>lt;sup>13</sup>(Muhlenberg and Paine 1996, 35–6) mentioned by (Graeber and Wengrow 2021, 595) refer to *Mahavagga* 1.28 and *Kullavagga* 4.9-14 concerning Buddhist monastic rules and the votes to be carried to out in case of an unresolved dispute.

a more abstract 'republic of letters.' In other words, city-states and democratic debates can take, and have indeed taken, many shapes and forms that still need very much to be documented. If the ability for debates and critical thinking seems an obvious necessity to make any form of knowledge thrive, these social conditions were certainly not only specific to ancient Greece.<sup>14</sup>

Despite the difficulties of assessing their context of elaboration and composition, or maybe because of this difficulty, the Aphorisms on Ropes are one of the rare sets of mathematical texts in Sanskrit for which different interpretations of their context have been given. Were their mathematics traces of the mathematical knowledge linked to ritual of a larger Indo-European tribal culture with roots that go beyond historical times? (Van der Waerden 1983, 32-34) Or would they rather be, as (Chattopadhyaya 1986) argues with a Marxist leaning, instantiating the knowledge of the construction workers of the prehistorical Indus cities replete with bricks? Such a knowledge would have been later appropriated by Brahmins as their own. In what kind of society did the schools of ritual to which the Aphorisms on Ropes (sulbasūtra) belong thrive? Another view of the social setting of the Aphorisms on Ropes, one anchored in a celebration of the intellectual works in religious settings, insists on the importance of the families and village-based teaching institutions in which these texts were studied and practiced. A certain number of historians, more or less explicitly, trace a continuity in between the Indus village settlements, the Vedic households in which brahmins of the same family lineage transmitted and studied vedic texts (*sākhas*), the Budhdist congregations (sangha), the Hindu monasteries devoted to learning (māthas) also anchored in family lineages and often instantiated in specific villages (grāma, south Indian illam). Religious congregations would have thus created an enduring infrastructure in which the development of ideas was made possible by changing political circumstances outside of the state.<sup>15</sup> These institutions that embodied specific schools of thought would have notably kept alive and transformed mathematical and astronomical ideas.<sup>16</sup>

Were the *Aphorisms on Ropes* the mathematical outcome of tribal societies, antique cities with or without states, or autonomous religious communities? It is hard to tell. But in any case, the existence of these mathematical texts fundamentally troubles the broad-strokes account of historical change that Netz sketches. This cursory introduction to the *Aphorisms on Ropes*, highlights how the distance Netz created in assessing both Greek society and the rest of the world homogenizes complex societies and does away with their particular histories. Netz himself observes that favourable social and economic conditions are necessary but not sufficient for the making of good mathematics. He suggests that these conditions made possible the key singular and exceptional elements of Greek mathematics: the systematic proofs of the second generation of Greek practitioners of mathematics and the encouragement of surprising, counterintuitive and

<sup>&</sup>lt;sup>14</sup>In a private conversation, Eric Gurevitch has pointed out to me that the structure of this argument might lead one to precisely the opposite conclusion: That it was the deeply inegalitarian nature of Greek society (Finley 1959) and the absolute prince of early modern Italy (Biagioli 1993) that provided the elites praised in the paper with the appropriated resources and leisure time to produce their intellectual achievements. This is not to endorse this historiographic claim, but to show the tendentious nature of the original point.

<sup>&</sup>lt;sup>15</sup>For a study that attempts to relate the different layers of Vedic literature with the social structure that stabilized themby a study of their modes of transmission see (Houben 2016).

<sup>&</sup>lt;sup>16</sup>(Plofker 2009, 179–181, 217–218), (Divakaran 2018, 87; 258) Jain and Buddhist monasteries are often included in these descriptions.

complex results. In contrast to this Greek situation, Netz argues that 'other civilizations' do not have mathematics detached from the social context that brought them into being, and so never succeeded in becoming exceptional.

The way the social context of the Aphorisms on Ropes has been studied shows us how contextual histories of mathematics are more often than not steeped in their politics. Such histories further raise the question of how much the social contexts of the production of Aphorisms on Ropes might impact their mathematical content. Let us note first then, that as texts which have many layers of redaction that span across several centuries, they obviously do not testify to a single context of production. But let us look at some of their common features. All the Aphorisms on Ropes share a certain number of procedures, notably those to construct squares and rectangles with pegs and ropes, followed by different versions of the Pythagorean theorem stated as a property of the diagonal of squares and rectangles. These rules are never given for specific cases, they are given for generic figures. In most cases, rules to construct a given figure are never given alone: a first procedure is always given with a second, alternative one. Take for instance the Aphorisms on Ropes considered to be authored by Baudhāyana. He gives two rules to construct rectangles, one resting tacitly on the construction of a circle, the other on a Pythagorean triplet (15, 12, 13). He also gives two rules to construct a square, first by means of a circle, then by means of the Pythagorean triplet (12, 9, 15). How should these constructions be understood? Are they really about constructing rectangles and squares with pegs and ropes? Are they not also a way of underlining that these figures are connected *in a general way* to the numerical relations that underline them? Or that there could be two ways of considering orthogonality, by considering that it is not only structurally based on a Pythagorean triplet but also on a circle? What appears as a procedure to construct generic figures, can also be read then as containing general mathematical characterizations. But there is more. In later mathematical literature in Sanskrit, giving alternatives to a given procedure is sometimes considered as a way of proving a procedure. The fact that these construction procedures come in pairs, might also have been understood then by those who worked with these texts as a way of asserting that the procedures given were correct. But what then is the link of these rules with the social context in which they were produced and used? It is difficult to tell. Despite the assertion of scholars who perennially read them through a religious lens, much of the rules given in these texts are not about the rituals they are attached to. The ritual context, the knowledge of brick constructions they display, all appear to be mere elements of a larger mathematical body of knowledge.

The mathematics of the *Aphorisms* owes something to the multiple social contexts in which those who composed them were imbedded and to the rituals they describe. Such is the case for all human productions. But it makes no sense to qualify their contents only by this context, especially because they openly present themselves as belonging to no specific time and place. Furthermore, they were copied and recited for millennia after their initial production. Greek mathematicians of antiquity were not the only ones to thrive in places where debates could be harsh and competitive, nor were they unique in considering that mathematical knowledge was technical but also had a form of universality, even if what was understood as 'universality' by any of our actors might have been very different from what we understand by this word today.

#### 2.1.2. A-historical concepts

So, how many 'significant mathematical authors' did other ancient cultures, preceding the Greeks, usually have? In an important sense, the answer is *zero*. (Netz 2022, 305) If you run the game of civilization a hundred times, how many times do you get Archytas? Euclid? Archimedes? A somewhat absurd question, of course: but if we raise the question of Archimedes' place in world history, this, effectively, is what we ask. (Netz 2022, 311) Empirically, Archimedes occurred, in the span of two thousand years, once. I think this suggests that he, too, was rare. (Netz 2022, 313)

The precise social context of the various documents dealing with mathematics and more generally the great diversity of existing mathematical sources of antiquity are addressed but only marginally in Reviel Netz's essay. While he carefully analyzes the different generations of Greek mathematical readers of Archimedes, and provides a specific biographic backgrounds for them, when he extends his analyses to 'the rest of the world', he simply attributes an identity harnessed to their language or their religion to these authors and the mathematics they practice. Sometimes, these identities are extended to a geographical area, like 'the Mediterranean', but this is a 'cultural' geography which defines a space that is separated from what is constructed as an 'other', even if this other could also be geographically part of this area. The world seems thus made of 'civilizations' or 'cultures', in which Greece and later Western Europe appear as exceptional outliers.

Brett Bowden, inspired by Quentin Skinner, has described how the term 'civilization' is both descriptive and condemning: it includes in its history the idea of civilizing the un-civilized, before being included in a vision of the world made of different, classifiable civilizations. In other words, it contains the memory of the imperial agenda in which it was first developed. Karuna Mantena, prolonging the study of the intellectual justifications of colonial agendas, has studied how the term 'culture' and the idea of traditional societies were used by the comparative historian and jurist Henry Maine (1822–1888) or the political economist James Stuart Mill (1806–1873) as part of their justification of the British colonial state in India.<sup>17</sup> European exceptionalism was central to the argument used to subject what was deemed lesser rational people. Homogenising groups of people by their use of language, their religion or the place they live in and attributing to them characteristics that would be outside of time was a way of denying to such people a history and an intellectual life worth studying.<sup>18</sup> And indeed, in Netz's essay, it is striking that outside of Greece and Europe, everything is chronologically vague. For instance, when Netz evokes China or India, is he referring to a time-frame similar to that of the Greek antique texts he is speaking of, or of all texts written in China and India before a time they would clearly have encountered European scientific texts? This distance, which allows imprecise statements, which homogenizes people and their intellectual productions, is also used to quantify the production of mathematical texts.

#### 2.2. Quantifying

Part of Netz's argument for Greek exceptionalism finds its acme in quantification, and specifically in the quantifying of what he calls 'significant mathematical authors'. For

<sup>&</sup>lt;sup>17</sup>(Bowden 2009, 8–9), (Mantena 2010, Chapter 2).

<sup>&</sup>lt;sup>18</sup>Netz is provocative here because if he seems to follow the Sartonian reification of civilizations, he doesn't manifest any Sartorian inclination to demonstrate that every part of the world participated in the making of modern science and modern mathematics.

Netz, important Greek mathematical texts were an unlikely contingency in 'the civilisational system' they belonged to. This part of his argument is innovative and surprising: what we are used to seeing as something central to human history, and in particular to the history of mathematics, Netz insists was highly improbable, and purely contingent. It very well may never have occurred. Such a historical approach departs radically from all the forms of nineteenth and early twentieth century histories made of necessary causes and outcomes. The contingency of Archimedes's existence, and the uncommonness not only of such a personality but also of his posterity, together make him extremely precious. His precarious rarity and great value are then used to justify claims to Greek exceptionalism. This rarity is anchored in a dubious mode of quantification that Netz engages in, when he tries to count the great men of different civilizations. But there are many unwarranted assumptions laying behind the label 'significant mathematical authors'.

What lies behind this label? Is it the significant texts that Netz, as an observer, considers significant? Is it the making of the category of 'author' and consequently of 'significant author' by different actors in history? The count made by Netz raises problems if we consider it as referring to significant mathematical texts: can we really count significant mathematical texts in several languages dating from 400 BCE to 100 BCE? Such a count would suppose that all the mathematical literature of the world has been equally edited, translated, studied, assessed and known to the one making the count. We all know that this is not the case.<sup>19</sup> If we concentrate on the notion of 'significant author', we might ask: are the criteria used to establish who is a significant mathematical author meaningful and fair for authors of mathematical texts all over the world? Hinging an argument for exceptionalism on a pregiven notion of significant authors, obviously distorts the conclusion which is already included at the outset, within the original assumptions. As such, any quantification of significant authors across cultures should not be treated as an objective measure of comparison enabling definitive judgements.

There are many ways in which we can think of Archimedes's place in world history. Part of the task of the historian of mathematics might be here to find the right scale of analysis and explanation that balances attributing agency to lone heroes and to anonymous collectives.

#### 2.3. Heroes in history

The named author, I suggested, is related to the contents. Indeed, the results found by Greek mathematicians – starting right with the earliest attested – tend to be strikingly counter-intuitive, or at the very least require very complicated arguments. (...) Other civilizations produce a mathematics that is often hard, but this, generally speaking, has to do with the complexity of calculations and algorithms and there is definitely no deliberate effort to seek counter-intuitive results. (...)

In China, indeed, as in India, there arises eventually a tradition of named mathematical authorship, which, however, is constituted mostly through the vehicle of commentary on the canonical mathematical works. While authors in this commentary tradition do

<sup>&</sup>lt;sup>19</sup>Concerning sources from South Asia, Sanskrit texts have been overwhelmingly more studied than those in other languages. Strictly mathematical texts have been over studied when compared to those devoted to astral sciences. For recent relevant studies of astral texts somewhat contemporary to the timeframe given by Netz see (Geslani et al. 2017).

produce innovative mathematical contributions, the emphasis is subtly different, on presenting the author as a supreme teacher of a knowledge external and preceding him.<sup>20</sup> (Netz 2022, 306)

Let us come back to the idea of 'significant mathematical author', and more generally to Netz's emphasis on authorship. Netz believes that in part Greek exceptionalism was linked to the rise of authorship seen as a break from what preceded, and a unique Greek reality. One of the arguments held by Netz is that during the second period of Greek mathematics, most practitioners shared the idea that a good mathematical author had to be subtly surprising.

In drawing this contrast between Greek authors and scholarly mathematical texts from other parts of the world, Netz conflates several elements. First, he seems to consider that all texts should or would be meant to accomplish the same things. Second, he seems to consider that there is only one unique and correct conception of authorship. Finally, he seems to neglect the fact that different conceptions of originality could have been upheld by all sorts of different actors. For Netz, it is only the convergence of these three elements that gives rise to what is for him the one and only good brand of mathematics.

The Aphorisms on Ropes (sulbasūtras) can also be used here as a convenient counter example. We have seen that they are presented as texts related to ritual doctrinal schools, although they have named authors. Each Aphorisms on Ropes is made of different layers of texts. They share common parts with other Aphorisms on Ropes but also give voice to debates and differences with other doctrinal schools. Consequently, individually or considered together, the Aphorisms on Ropes should not be understood as embodying a single time and place. They each have their own singularity, their different ways of conceiving measures and mathematics, of constructing a square or a given altar. They are not uniquely about rituals, neither solely about the pure mathematics of a 'significant' author, nor about religious figures. Rather, they are somewhere in between all of this. The Aphorisms on Ropes thus serves as a reminder that ancient mathematical texts should not be assessed with the too simplistic framework of the original author opposed to the servile ritual school followers. Even if we set aside the Baudhāynas and Āpastambas who might not have been - according to us - true authors of the Aphorisms on Ropes that actors attributed to them, there are many singular and striking self-claimed authors of other mathematical texts in Sanskrit. I will mention a few here who, although they arrive later than Archimedes, are just as witty, wry, and surprising as Greek and European authors. Take Āryabhața (b. 476), who believes that the earth turns on itself and not the sky around it. This is a provocation, since he contradicts the cosmology of Hindu religious texts. The Aryabhatiya contains one of the oldest known sine tables, and developed witty algorithms to solve linear indeterminate problems, etc. He affirms that he is re-casting the 'knowledge of Kusumapura' - a prestigious centre of learning in Eastern India. He nonetheless proclaims that he is the author of his treatise, the Aryabhațīya (499). Bhāskara (629), in commenting on this passage, compares this endorsement to the cry of a hero yielding a towering sword before running to fight in a battle: this was indeed a world of fierce debates. The *Āryabhatīya* is also a literary feat playing with forms of orality. It contains onomatopoetic verses encoding numbers, verses like little libraries

<sup>&</sup>lt;sup>20</sup>Such affirmations had already been debated more than 20 years ago, see (Netz 1998) and (Chemla 1999).

of many procedures obtained by omitting some of their words, and rules with double meanings (Keller 2016). It is thus a text tailored to make an effect on those who would hear or read it. We, as observers, might call this effect 'surprise', even if it was not named as such by the author or his commentators. Nonetheless, Āryabhața quite clearly claims to be the author of these effects. Despite the difficulties of the text, there was no break in the transmission of the  $\bar{A}ryabhat\bar{i}ya$ , which was commented upon until the end of the nineteenth century. Next, let us consider Brahmagupta, a wellknown ironical and harsh critic of Āryabhața. Here is somebody with hero material, who wrote an astronomical encyclopaedia and curricula (Brāhmasphutasiddhānta-628) and a practical manual of astronomy (Khandakhādyaka-665) which was translated and adapted into Arabic and maybe Chinese (Pingree 1975; Gupta 1989). In these two treatises, he develops a number of recursive numerical procedures to make approximations of all sorts of astronomical parameters (fixed point algorithms, that he names *asakrt*- i.e. 'without end'), also developing a complex space geometry along the way that we have reasons to believe greatly influenced the numerical techniques of astronomers writing in Arabic and Persian (Pingree 1976; Brummelen 2014). He is known to have studied the property of cyclic quadrilaterals, indeterminate analysis of the second degree, and more broadly algebraic rules to work on surds and zero, etc. attracting the attention of nineteenth-century European mathematicians. His work is unique but his treatises have largely been neglected by scholarship outside of South Asia. There is no translation and study of the totality of his encyclopaedia in a European language, despite the fact that his works were known to European scholars from the beginning of the nineteenth century.

So there were 'remarkable authors of mathematical texts' outside of Europe–although they are often less well known by historians of science than their European and Greek counterparts. These authors sought to write remarkable texts, and they did so without fear of criticizing their predecessors, and using at times literary effects. They were not the slaves to tradition that Netz makes them out to be. But should we replace the history of one culture's hero with yet another hero?

By and large, the history of mathematics in South Asia has been written as a competition involving great men.

Dhruv Raina (2010) has shown how from the eighteenth century onwards, European orientalists and mathematicians set out to look for canonical texts in South Asia and encouraged their Indian interlocutors to provide them with such texts. It is striking that one of the first histories of mathematics and astronomy written in Sanskrit was thus a bibliographical list of great mathematicians (Dvivedin 1892). The legacy of the great heroes in nationalist historiographies can also be seen in the strange subdivision of Amulya Kumar Bag's manual on the history of mathematics in India (Bag 1979): its first chapter ('scholars of mathematics') provides a time-frame with the biographies of great men on the one side, while the following chapters develop a thematic history of arithmetic, geometry, algebra, etc. as if the articulation of both was impossible.

Histories of mathematics dealing with South Asia have often – with a few important exceptions – been concerned with showing that Sanskrit mathematical texts should be included in a world history of mathematics – whether through the canonical authors who composed them or the outstanding early mathematical results they contain.

Another version of this rather ahistorical and technical approach to the history of mathematics and astronomy is embodied in the notion of doctrinal school. Thus David Pingree distinguishes different doctrinal schools of astronomy - to which he gives a generic Sanskrit name paksa - according to their parameters (Pingree 1981). The history of astronomical schools does not always coincide well with the history of great men, since some great men like Brahmagupta wrote multiple treatises that seem to belong to conflicting schools (if we are to accept the criteria of parameters as definitional of a school.) What then was meant by these doctrinal schools, how important they were to those who belonged to them, all of this needs more elaboration. The same tension between the different focusses of history-writing can be seen in the existing histories of the 'Kerala school', which is told both as a lineage of great men and that of a mathematical doctrinal school. Their mathematical achievements might be attributed indistinctly to all authors, or to the founder of the school. Thus, the methods and reasoning of Nīlakantha (fl. ca 1444–1545) and Śańkara (fl. ca. 1540) quoting Mādhava (fl.ca. 1400) as they geometrically derive the numerical procedures by which they approximate  $\pi$  or sines with non finite series are often ascribed to the whole school or to Mādhava.

We see then how heroic histories, civilizational histories raise real difficulties for those who want to write a history of mathematics in South Asia. We might then attempt to reformulate the question of the kind of history of mathematics that we might want to write in the light of these reflections. How can we write a history with proper scaling, one that would do justice to all the stories that are to be told and that would not erase either the role of collectives in the making of the history of mathematics or of individual agency in different forms?

#### 2.4. Scaling

Netz's essay serves as a warning that we need to tread delicately. We must not do away with the nuances that micro studies and close readings afford, but we cannot remain limited to the narrow scholarly worlds often created by such studies. How can we balance detailed micro analysis with big-picture studies?

Scholars of South Asia have long interrogated the position of the historian. In the 1990s, historians of colonial India programmatically proclaimed the necessity of taking the point of view of the subaltern, against a history that takes the point of views of the powerful and the winners.<sup>21</sup> We could also reformulate this as the necessity of taking a point of view from within the world, and not towering above it.<sup>22</sup> This leads to the larger question of how to construct a situated knowledge without doing away with the critical detachment of the scientist, historian or observer we also embody (Haraway 1988).<sup>23</sup> One convenient way of writing a history which deals with different points of view, is to concentrate on actors who navigate in between several worlds. In the history of science, there has been in recent years a great emphasis on 'brokers',

<sup>&</sup>lt;sup>21</sup>It is often said that writing subaltern histories is made impossible for lack of subaltern sources, but concerning colonial India at least, it is especially that sources in regional languages have for a long time been too neglected.

<sup>&</sup>lt;sup>22</sup>The position of the scientist looking 'objectively' at the world 'from above' has also been denounced as a way of attempting to make domination a-political; it is also now seen as an attitude towards what it studies that has led to today's ecological disaster. (Bonneuil and Fressoz 2013, 14). This does not mean doing away with domination or conflict in the writing of history of course.

<sup>&</sup>lt;sup>23</sup>G. Catren has pointed out to me this classic as a way of thinking of the issues of a scientific situated knowledge.

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looking at the networks of those who enabled the transmission and travelling of knowledge with its many transformations and misunderstandings, looking at networks of exchanges and focusing on those remarkable agents of changes that are go-betweens (Schaffer et al. 2009). Multiplying points of view in the history of South Asia is also a question of multiplying the sources and their languages- which means not restricting ourselves to Sanskrit sources but reading also texts in regional languages. There has been in recent years a renewed interest in the mathematical and astral sources written in Odissi, Bengali, Kannada, Malayalam and Tamil, among others, which promises to deeply change how we perceive the history of mathematics in South Asia. Eric Gurevitch remarks that studying vernacular scholarly literature in medieval South Asia is also a way of studying how actors themselves sought to break away from a scholarly culture of 'universal knowledge that could travel' (written in Sanskrit) to create what they considered a more local perspective.<sup>24</sup> Gurevitch further argues that studies of individual disciplines might make us overlook issues and sources that initiate dialogs with other disciplines or break out of previous disciplinary boundaries. In other words, he invites us to look at the multiple scales that the actors of our sources were aiming at.

Netz, in this essay, explores new methods to contextualize history of mathematics, using quantification but also more broadly attempts at modelling the structure of societies and their outcomes. These processes might be fruitful, but as we have seen, they would benefit from integrating the multiple reflections of colleagues of the twentieth and the twenty-first centuries on the problems raised by Eurocentrism in the history of science. In the final part of this paper, I turn to the consequences of the politics of Netz's narrative, as it addresses the larger question of how different histories of mathematics can or cannot dialogue with one another.

#### 3. Politics

I have argued that the global point of view is striking for what it focusses on, what it sees ... and all that remains in the shadow. Contrary to what is assumed, at a distance not everything comes to view- it all depends on the point of view you take, and what your eyes are trained to see.

#### 3.1. How should we compare?

While we do not usually phrase this in such probabilistic terms, we are now used to comparative claims which, whether explicitly or not, are statements of probabilistic correlations. (Netz 2022, 311)

Euclid was satisfied by compiling his Elements, Ptolemy was satisfied by completing Hipparchus' project and, as we will note immediately below, everyone was satisfied by merely extending this ancient project, keeping to the same terms, through many centuries to come, all the way down to (very much including) Copernicus. Why not, after all? The patrimony was extremely impressive (and impressive, further, by being unparalleled by any other civilization). (Netz 2022, 313)

<sup>&</sup>lt;sup>24</sup>(Gurevitch 2022, 7). For a study in the history of science in South Asia dealing both with sources in Kannada and Sanskrit and discussing the scale in which to write history of science see (Gurevitch 2022, 3–15). I read his PhD after writing a first version of this section: his reflections are more erudite and on a larger scope than mine, and his PhD work is a perfect example of the kind of history he suggests to carry out.

Early historians of science like Sarton (and Joseph Needham) were driven by the desire to demonstrate the ancient and medieval or early modern contributions of Eastern civilizations. But once the narrative of the rise of Western science was set in place, other counter narratives were implied, with their distinctive vocabulary of stagnation, decline and dark ages. After all, once one begins to extol the virtues of past Golden Age, one is left with the inevitable question 'What went wrong?'. (Elshakry 2010, 107)

The first problem, the most obvious with the narrator of Netz's essay, is that he is obviously prejudiced. He has ideas of what constitutes good and bad mathematics, important and less important mathematical texts, and he expects us, his readers, to go along with him. Presentism here fuels also something of a Panglossian approach to the value of the texts: Netz only sees the 'good' in Greek mathematical texts, and what is good is what was deemed useful by future users of these texts. Future usages of Greek mathematical texts then reversely appear as the best possible interpretation of these texts. The retrospective good, useful and true interpretation of Archimedes is thus used to construct not only the exceptionalism of Greek mathematics, but also of European civilization. Many of the questions raised by R. Netz are oriented by this framework in such a way that Archimedes and what he calls 'Greek' mathematics are the only possible positive answers to the questions he raises. The way that this framework distorts all questions and comparisons can be found in the devil of many details. Let us come back to the time-range adopted by R. Netz to count 'significant mathematical authors'. Throughout the text, the time-range actually changes according to whether he wants to assess the uniqueness of Greek authors in antiquity or the uniqueness of Archimedes in world history. Accordingly, he limits his time-range to a couple of hundred years in antiquity, and at others to a span of 2000 years. But obviously such variations change significantly who can be considered as a relevant author (either by actors or observers), or what text can be included in the history of mathematics: whether for instance, to come back to my heroes, Brahmagupta or Āryabhata would be allowed in the competition or not. No fair comparison is possible in such a framework. It is flawed from the beginning because it takes one 'outlier' and measures all 'others' in relation to this given 'outlier'. But if from the outset, the outlier is 'unparalleled', it is by definition incomparable. All the rest then measure a failure.

The problems with the framework on which Netz's reasoning is based become more obvious when we look at the contents of what makes 'good' mathematics. Netz repeats that Greek exceptionalism is rooted in its unique focus on a geometry detached from real life, while those of 'others' would never have attained that level of abstraction.<sup>25</sup> But whether they were detached from real life or not, would that mean we should dismiss the trigonometrical circle of sine derivations, the elaborate space and spherical geometry of mathematical astronomy, the volume of conch shells, the volume of a universe thought to be an infinite set of piled pyramids in a Jain cosmos, the geometry of tantric drawings, etc. that can be found in Sanskrit mathematical and astral texts?<sup>26</sup>

<sup>&</sup>lt;sup>25</sup> The motivation of mathematical statements, in most early civilizations, is usually transparently pre-mathematical: the heaps of grain and the fields and the eclipses. The beginning and end of one's work are questions that can be understood by the layperson. The figures studied are "normal", those whose names are available from ordinary language: one looks for the volume of a cube or a sphere. Authors such as Archimedes, however, find the volume of, say, the paraboloid of revolution (which is the object that you get when you find a curve by cutting a cone in particular way, and then taking that resultant curve and rotating it around its axis). A layperson no longer knows what is even discussed.' (Netz 2022, 307).

<sup>&</sup>lt;sup>26</sup>Among many studies, see for instance (Hayashi 1996; Morice-Singh 2017; Chiodo 2021).

More strikingly, Netz's history seems to do away with a history of algebra, arithmetic, numerical procedures considered as secondary ...

Netz also argues that one of the incredible strengths of Archimedes lies in his 'willingness to engage with radically simplified and idealized versions of reality' (Netz 2022, 327). Bhāskara I in the seventh century argued, I think, precisely for something similar, thinking of mathematics, and in particular geometry, as a tool to describe reality, not the truth of reality itself. He writes:

A means is merely a tool to fulfil a goal. Therefore, every computation (*prakriyā*) by which the true motion of planets is established is untrue (*asatya*). It is not truth (*satya*) that is explained [by those desirous to know the highest reality (*paramārtha*) insofar as their means are untrue]. Indeed, it is just as physicians practice [procedures] such as incisions on things such as lotus stalks. Barbers practice activities such as shaving on things such as pots, those who know the sacrificial treatises practice such rituals as sacrifices on [<such things as> dried sacrificial bricks], and grammarians adduce words with such things as word roots, affixes, derivatives, augments, phonemes, elisions and transpositions. It is in that manner that in this instance too astronomers adduce the true motions of planets with practices such as mean motions, slow and fast corrections, their circumferences, sines, arcs, bases, uprights and hypotenuses. Therefore, there is no objection to be made considering that the means are untrue, when the purpose is explaining in the best way possible a truth.<sup>27</sup>

In other words, for Bhāskara, mathematical models in astronomy do not describe the truth of the motion of planets but are rather just a method, an artificial expedient, to approach this truth, to explain it. He uses the same reasoning when explaining why astronomers should be familiar with the armillary sphere. While doing so, the words he uses for the mathematics of the sphere (circles, perpendiculars, axis etc.), as can be seen in the quotation of note 27, could also be different material elements of the armillary sphere (rings, plumb lines, rods, etc.). This is his way of inviting astronomers to work on a simplified version – precisely a mathematical model – of the cosmos.<sup>28</sup>

<sup>&</sup>lt;sup>27</sup>BAB.3.17 (Shukla 1976, 217), attention to this passage was given to me by Sho Hirose; it is translated in (Lu 2015, 3), [] indicate additions to the text made by the editor (Shukla), <> and () those of the translator (me).

kevalam tüpeyasādhakā upāyāh asmād iyam sarvā prakriyāsatyā, yayā grahāņām sphuṭagatih sādhyate [evam ca paramārthajijñāsubhih asatyopāye]na satyam pratipadyate | tathā hi bhiṣajo hy utpalanālādiṣu vadhādīny abhyasyante | nāpitāh piṭharādiṣu muṇḍanādīni, yajñaśāstravidah [śuṣkeṣṭikayā] yajñādīni, śābdikāh prakrti-pratyaya-vikāra-āgamavarņa-lopa-vyatyayādibhih śabdān pratijānate evam atrāpi madhyama-mandocca-sīghrocca-tatparidhi-jyā-kāṣṭhabhujā-koṭi-karṇādi-vyavahāreṇa sāmvatsarā grahāṇām sphuṭagatim pratijānate tasmād upāyeṣv asatyeṣu satyapratipādanapareṣu na codyam asti

A similar sentence is given in BAB.4.introduction (Shukla 1976, 240): evam paramārthajijīnāsavo hi asatyapūrvakam satyam pratipadyante | tad yathā bhisajo hi utpalanālādisu sirāvedhanādīni pratipadyante, yajīnāsāstravidah suskestyā yajīnādīni [pratipadyante], vaiyākaranāh prakrtipratyayalopāgamavarņavikārādibhih sādhušabdam pratipadyante, evam atrāpi sāmvatsarāh vrttasalākāsūtrāvalambakādibhih kṣetragaņitavišeṣaih pāramārthikam golam pratipadyante | tasmād dinmātrapradarsanam evaitat ārabhyate, asakyatvād asēṣapradarsanasya | ko hi citrayan nimeṣonmeṣādy api citrayati |In that manner, those who desire to know the ultimate reality, will explain indeed truth through untruth. For instance, doctors explain things such as the piercing of body vessels on things such as a lotus stalk; those who know sacrifice treatises [explain] rituals such as sacrifices through acts such as an empty rite (i.e. play acting); grammarians explain correct words through such things as word roots, affixes, elision, augment, phonemes, and derivatives. In the same way, here also, astronomers explain the Sphere which is the ultimate reality (*pāramārthika*) through specific geometry such as circles, rods axis (*salākā*), threads/lines (*sūtra*), plumb lines/perpendiculars. Therefore, only showing a mere direction will be undertaken, for it is impossible to show everything. Indeed who can paint in a momentary glance <movements> such as the fluttering of eyes?

<sup>&</sup>lt;sup>28</sup>It is possible to argue that this way of seeing things was enabled, partly, by something Archemedian that would have come down from the astronomical and astrological legacy of Hellenistic astronomy in early medieval Sanskrit scholarly sources. Johannes Bronkhorst (2016, 276–302) could have argued for this, but he excludes Bhāskara from such an

#### 3.2. The persistence of ignorance

There is a virtuous cycle, bringing together proof and surprise. The urge to make surprising claims persuasive makes it all the more urgent to find a method of proof-for, without proof, how would anyone believe them? But, once proof is found, what was once surprising becomes, instead, a given (in a process well known already to Aristotle). (Netz 2022, 309)

I have argued that many of the exceptional characteristics that Netz attributes to Greek authors – such as belonging to a critical democratic culture, having a conception of authorship that competitively pushed towards innovation, having an abstract geometry or even maybe the use of a toy world to reflect on the cosmos – could as convincingly be applied to all sorts of other scholarly cultures. But what is surprising is that much of what I have enumerated above, and more of what I have not enumerated, has been well known for a very long time. Take the idea of proofs. I have touched on this before, but Sanskrit mathematical and astronomical texts (treatises and commentaries) are replete with many different types of reasonings, and notably all sorts of proofs. The diversity of modes of justification of mathematical results used in texts that were not written in Greek, Latin, Arabic or Persian – some of which were systematic – has been known and partly documented since the beginning of the nineteenth century (Chemla 2014). How then is it possible to dismiss the considerable literature documenting these ecologies of reasoning?

There is something a bit harrowing in the persistence of this ignorance, that many have tried in their own way to break, with their flaws, and true to their time. Louis Karpinsky (1878–1956), Bardtel Leendert Van der Waerden (1903–1996), or Adolf Pavlovitch Youschevitch (1906–1993) and others seem to have published in vain for a great number of members of our community. Historians and philosophers have tried to understand these phenomena of persisting ignorance. Achille Mbembe (2021) suggests to call this form of Eurocentrism 'late Eurocentrism' – characterized as a conservative reaction in a world that now knows that Europe is neither alone nor especially exceptional intellectually. Or should we understand this enduring ignorance as a form of blindness due to the persistence of structures of domination in global histories?

#### 3.3. Colonial responses

In the nineteenth and early-twentieth century, Indian scholars and scientists read and sometimes interacted with European and American authors who wrote histories of science and mathematics. Partly in reaction to discourses claiming that science was modernity and that modernity was European, or even Greek, a now well-documented nationalist historiography was elaborated. Some Indian scholars were shocked that the British could consider that Euclid was Greek when he was so evidently an Arabic Muslim author.<sup>29</sup> A number of studies have shown how emerging scientists tried in many different ways to reconcile what they knew of their rich scholarly traditions

influence. He further understands Bhāskara's notion of mathematical objects differently from me: he suggests that for Bhāskara mathematical objects would be generalizations made by observing real objects.

<sup>&</sup>lt;sup>29</sup>Private conversation with Irfan S. Habib. The conceptions of science and its history of such actors is described notably in (Habib 2004).

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with the new sciences that were taught to them, that they practiced, and which were allegedly of foreign origin (Nandy 1995; Raina and Habib 2004). In the history of mathematics, the trope that most of the interesting mathematics that had been done in India was numerical and algebraic and not geometric was taken for granted. Thus, the first manual on the history of 'Hindu' mathematics published in English (Datta and Singh 1935-1938) focuses on numbers and algebra. The parts on geometry and trigonometry would only be published separately more than 50 years later. One might be struck by the way Datta & Singh make comparisons. They aim to compare the achievements found in mathematical texts in Sanskrit to those found in European texts. Their construction of a more or less homogeneous 'Hindu mathematics' enables such a comparison, made by apposition, showing what 'Hindu' authors might have achieved before a famous European author. This sometimes extends to supposing a secret influence of Indian achievements on European ones. Like the proofs of Sanskrit mathematical texts that piqued the curiosity of many European mathematicians in the nineteenth century, the mathematical achievements of the 'Kerala school' were known since the mid nineteenth century. Vengeful histories of influence and of posthumous greatness have been written that proclaim the possibility that the mathematical results developed by the 'school of Mādhava' were transmitted to Leibniz and Newton by travelling Jesuits (e.g. Joseph 2009). Most histories of mathematics treating with South Asia are written to prove that something interesting in relation to what is well-known and documented for Europe has also been carried out in India.

Netz insists that each history of mathematics should be written on its own terms. The problem then – as raised by those who already, over a century ago, started to write a history of mathematics from sources in Sanskrit – is indeed how to integrate this 'history on its own terms' in a wider history of mathematics; that is, how to allow such a history to enter a global conversation. Or, on the contrary, should historians show how each history is exceptional and without a need for a dialogue?

#### 3.4. Post-colonial histories

On seeing this kind of work actually being performed by the little children, the doctors, professors and other 'big-guns' of mathematics are wonder struck and exclaim: – 'Is this mathematics or magic'? And we invariably answer and say: 'It is both. It is magic until you understand it; and it is mathematics thereafter'; and then we proceed to substantiate and prove the correctness of this reply of ours! (Tirtha 1965, xvii–xviii)

From the end of the nineteenth century in South Asia, non-academic histories of science were written that replied in a radical way to discourses of European exceptionalism. Aimed at showing that scientific modernity did not necessarily have to be 'western', one branch of these alternative histories constructed new scientific heroes. To the necessary separation of science and religion these new histories opposed a 'holistic' science *enabled* by religion. One such example is embodied in the figure of Bhāratī Kṛṣṇa Tīrtha (1884–1965), a religious nationalist figure of the early-twentieth century. Trained in modern sciences, he wrote at the end of his life a book published posthumously under the title *Vedic Mathematics*. It has been a long-standing bestseller in India. This text did not deal with the geometry of Vedic altars as

historical Vedic mathematical texts like the Aphorisms of the Ropes do but dealt with numerical and algebraic procedures said to be the product of religious revelation. In other words, beyond the question of whether Vedic Mathematics is a historical text that belongs to South Asia's Vedic antiquity - a sometimes debated question - it contains mathematics revealed to a seer, exactly like the most sacred texts of Hinduism, the Vedas. Furthermore, it implicitly endorsed the assumption that the past mathematical feat of Sanskrit texts was mainly arithmetical and algebraical. Nonetheless, Bhāratī Krsna Tīrtha is explicit that to be mastered, all the mathematical results he gives should be understood rationally with the help of proofs. Vedic Mathematics became popular in the 1970s and 1980s in Transcendent Meditation (TM) circles not only in India but also in the United States, England and Australia. These circles promoted the idea that such mathematics were made possible through the 'magic' of meditation on an obscure and lost recension of the sacred Vedas- i.e. through a specific Hindu religious practice. The subtext of the numerous ads for Vedic Mathematics is that by meditating and/or practicing Vedic Mathematics your brain can be transformed into some kind of super-computer. The religious haze around Vedic mathematics is one of the many kinds of 'religion' based sciences promoted by the Hindu nationalists in power today in India. But such movements do not belong to South Asia alone.

The blogs of the far-right, the Fjordman's and others, explain that only Christian countries (and specifically not Muslim ones) could make scientific and technological advancements.<sup>30</sup> Islamists, Hindu and Jewish bloggers of all sorts seem to reply that such scientific facts can be read back into their scriptures.<sup>31</sup> At times such statements might be a way of insinuating that religious traditions also arrived at exceptional scientific revelations by religiously based means. More broadly, the multiplication of such blog posts shows a very religious public trying to redefine the relation of science and religion. In other words, writing and reflecting on how rational inquiry has been enmeshed with religious beliefs and practices is a hot topic in non-academic circles. In this perspective, we need to be aware that the books and essays we publish are used (and of course at times distorted) by a public reading us from every corner of the world, seeking answers about their own histories. This brings us back to our tasks as academic historians (and philosophers) of mathematics.

#### 4. Conclusion. History of science storytelling

The chronicler who narrates events without distinguishing between major and minor ones acts in accord with the following truth: nothing that has ever happened should be regarded as lost to history. Of course only a redeemed mankind is granted the fullness of its past-which is to say, only for a redeemed mankind has its past become citable in all its moments. (Benjamin 1996, 45)

<sup>&</sup>lt;sup>30</sup>At times, fundamentalists of different religions can have common goals, and thus can show a certain familiarity with one another. Thus, Breivik quoted a Hindu Nationalist, Konrad Elst in his manifesto, precisely to criticize Islam. Koenrad Elst himself recognizes critically this in the following blog post: https://bharatabharati.in/2011/09/10/when-andersbreivik-quotes-koenraad-elst-on-islam-koenraad-elst/ (accessed 21/09/22)

<sup>&</sup>lt;sup>31</sup>See for instance, https://www.chabad.org/library/article\_cdo/aid/71692/jewish/How-Scientific-is-Torah.htm (accessed 21/09/22)

Histories of exceptionalism contribute to bringing about a deeply divided world, a world for which we are told that some are more worthy than others. In other words, such histories are political and have political consequences that we must take into account.

Can fictions help us make better history? Maybe we should consider history writing as an effort not unlike Rushdie's sea of stories, to take together different threads that we can look at from different perspectives and braid into different stories.

Reviel Netz, in a part of his essay, argues for history of science fictions as a way of escaping presentist histories. His fiction of a history without Archimedes fails because to be able to imagine a world without Archimedes he would need to know so much more about the mathematical worlds that have existed outside of Archimedes, their physics and mathematics. Maybe history of science fictions could be a way to imagine other histories, less to prove that the presence of one person changed the world, than to explore other possible worlds, one in which we could give voice to those that we have heard too little, to explore lost and abandoned theories and authors that are part of humanity's common legacy.

The problems raised by Netz's essay help us reflect on the histories of mathematics we might want to write. His essay invites us to re-open the way we think of the social and political context in which mathematical reflections and mathematical texts were produced, in particular in antiquity. Exploring the relation of mathematics with state and stateless societies, democratic and non-democratic forums, and the like, we might also want to document our historical actors' conceptions of authorship, of the style of text they should author, whether or not they think of mathematics in terms of universality, generality or for situated, local and specific or specialized purposes. Netz's essay also invites us to look closer at the characteristics of the different collectives that practiced mathematics in the past, beyond religious, language or geographical boundaries, and to raise questions about how we may integrate these histories of mathematics beyond case studies.

I hope to have convinced my readers that it is crucial to find ways of initiating conversations between different situated history of mathematics. In the present context of resurgent white supremacy, insurgent nationalisms and religious fundamentalisms, Reviel Netz's essay challenges us to radically rethink what we do as historians of mathematics. His essay convinces us that we have to re-think what are our basic tools of narration. It obliges us to confront the dark past of our profession's history. It requests from us also, urgently, to read each other's works, and discuss.

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