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The concrete numbers of the “primitive” societies: an historiographical approach

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Abstract – From the 19th century onwards, some mathematicians and philosophers (G. Peacock, L. Conant, L. Lévy-Bruhl, L. Brunschvicq et al.), analyzed the numerical systems in use in the so-called primitive societies as having a more “concrete” (or less “abstract”) character than those developed in more “advanced” societies. This article aims to better understand—and compare—what this opposition between “abstract numbers” and “concrete numbers” actually meant for these authors, as well as the characteristics of the “primitive mentality”—which they believed—this opposition would highlight.

Résumé – Dès le XIX^e siècle, certains mathématiciens et philosophes (G. Peacock, L. Conant, L. Lévy-Bruhl, L. Brunschvicq et al.), ont analysé les systèmes de numération en usage dans les sociétés dites “primitives” comme ayant un caractère plus “concret” (ou moins “abstrait”) que ceux développés dans les sociétés plus “civilisées”. Cet article vise à mieux comprendre – et à comparer – ce que cette opposition entre “nombres abstraits” et “nombres concrets” signifiait pour ces auteurs, ainsi que les caractéristiques de la “mentalité primitive” que cette opposition entendait mettre en évidence.

MSC: 01A55, 00A30, 01A11, 01A12, 01A07, 01A85

Keywords: primitive societies, mentality, abstract numbers, concrete numbers, numeration, abstraction

Mots-clés : sociétés primitives, mentalité, numération, nombres abstraits, nombres concrets, abstraction

Introduction

From the 19th century onwards, some mathematicians and philosophers have analyzed the numerical systems in use in the so-called primitive societies—documented either by missionaries, explorers, or ethnologists—, as having a more “concrete” (or less “abstract”) character than those developed in more “advanced” societies. This paper aims to grasp—and compare—what this opposition between “abstract numbers” and “concrete numbers” meant for some of the major 19th and early 20th century authors, and the characteristics of the “primitive mentality” that this opposition was supposed to highlight. This will uncover dated views about numeration developed in “primitive” societies, to be understood in the colonial context of the time, but sometimes still prevalent to this day.

To do this, we will first outline an outstanding contribution by Cambridge algebraist Reverend George Peacock (1791–1858). This contribution is of particular importance since Peacock could be one of the first mathematicians who analyzed the numerals elaborated by the so-called barbarous-primitive peoples, in an introductory part of a long text devoted to Arithmetic (1845).

A half century later, American mathematician Levi Leonard Conant (1857–1916) published the first book devoted to numerical systems developed in “primitive” societies, and the issue of the origins of the concept of number in particular. This study, based on the analysis of ethnographic sources, mainly originating from 19th century anthropology, has been significantly influenced by the evolutionist theories of the time, and particularly by anthropologist Edward Tylor’s work on *Primitive Cultures* (1871).

The article then turns to philosophical approaches to numerations in use among “primitive” peoples. At the beginning of the 20th century, the theory developed by French philosopher and sociologist Lucien Lévy-Bruhl (1857–1939)—holder of the Chair of history of modern philosophy at Sorbonne University, and cofounder (with Marcel Mauss and Paul Rivet) of the Ethnological Institute (*Institut d’ethnologie*) of the University of Paris—significantly influenced the discussions on the forms of thought, reasoning, or logic, at work in “inferior” societies. This theory of primitive/prelogical/mystical mentality has first been developed in the book *Les fonctions mentales dans les sociétés inférieures* (1910), thereafter translated into English under the title *How Natives Think* (authorized translation, 1926). We will particularly focus/concentrate on a chapter wholly devoted to numerations developed in these inferior societies; such numerical systems described as “concrete numerations”, through which—according to this author—prelogical thinking expresses itself. Our last author will be philosopher Léon Brunschvicg (1869–1944), who succeeded Lévy-Bruhl (in 1927) to the Chair of history of modern philosophy at Sorbonne University, and was a member of the French “Académie des sciences morales et politiques” (from 1932 onward). Brunschvicg’s work has inspired many philosophers and historians of the 20th century and thereafter. In particular, his book *Les étapes de la philosophie mathématique* (1912) has had a considerable impact in the field of history and philosophy of mathematics, frequently quoted—to this day—in many works published in this field. It is in the first chapter of this book that Brunschvicg reflects on the issue of ontogenesis of numbers, making a detour (given the lack of historical sources to tackle this issue) through the numeration elaborated in “barbarous” tribes.

1. George Peacock and the numerals of barbarous peoples

In 1845, the Cambridge algebraist Rev. George Peacock (1791–1858) published an “impressive history of Arithmetic in the *Encyclopaedia Metropolitana*”. As pointed out by historian of mathematics Marie-José Durand Richard (2011)¹, this long text by Peacock (written two decades earlier before its publication) “gathered together his own early research on languages and arithmetical notations, given at the very first meetings of the Cambridge Philosophical Society”². Peacock first raises the issue of the elaboration of the “idea of numbers” in children’s mind. He analyzes this elaboration as an abstraction process related to language, through the formation of words for counting:

¹ This article by historian of mathematics Marie-José Durand Richard is devoted to Peacock’s (history of) Arithmetic. More precisely, this paper investigates “why this (History of) Arithmetic was conceived and how it reinforced Peacock’s general view of algebra as a symbolizing process”.

² Cf. (Durand-Richard, 2011 : 282, note 70).

Abstraction is the creature of language, and without the aid of language, he [the child] will never separate the idea of any number from the qualities of the objects with which it is associated. He will have a distinct idea of four cows, as distinguished from five cows; but it by no means follows, that the idea of the number four, as connected with four cows, will be perfectly identical in his mind with the idea of the number four as connected with four horses; as they would in both cases be blended with his ideas of the individual qualities of the objects themselves: but if his idea of the number four be *registered* in the memory by a specific word, independent of the qualities of the objects with which it was in the first instance associated, he will become accustomed, after a more enlarged experience, to pronounce the word without reference to such associations, though they must necessarily spring up in one form or other in the mind, upon a farther analysis of the idea, of which the word is the general symbol (1845: 369).

Therefore, as mentioned by Durand-Richard, “because words precede signs in the development of arithmetic, Peacock was going to examine, through words, the traces of arithmetical experiences” (Durand-Richard, 2011: 264), as it would have occurred through the elaboration of the various systems of numerals (*i.e.* number words) worldwide. From the beginning of his article, Peacock immediately differentiates between “numbers into abstract and concrete”, though, according to him, “abstraction exists merely in the word by which any number is designated, or in the equivalent symbol by which it is represented in different arithmetical systems” (1845: 369). To him, we might suppose the “process for the formation of abstract numbers” (*i.e.* disconnection of numerals from the “qualities of the [counted] object”)

to be completely effected by attaching names to the series of natural numbers, beginning from unity; but if such names were perfectly arbitrary and independent of each other, our progress in numeration would be extremely limited, as the memory would be overwhelmed with a multitude disconnected words (1845: 370).

By contrast, the large number of different systems of numerals that have been developed in the various societies worldwide, including the numerals in use in the more “primitive/rude” ones, would have been created, in Peacock’s view, through “regular principles”:

we might readily credit the narrations of the travellers who have limited the powers of numeration of some savage tribes to five, or to ten; but it will be found, upon an examination of the numerical words of different languages, that they have been formed upon regular principles, subordinate to those methods of numeration which have been suggested by nature herself, and which may suppose to have been more or less practised amongst all primitive people [...] (1845: 370).

Peacock illustrates this point by analyzing/comparing many different—quinary, denary or vicensary—vernacular numerical systems developed in different “primitive” societies, considering that the languages of “the most primitive and barbarous people” might be less altered than others (Durand-Richard, 2011: 265). For example, regarding the quinary numerical

systems of the Melanesians from the Islands of New Caledonia, Tanna and Malekula (both the latter New Hebrides Islands, actual Vanuatu) respectively, he explains:

In the first of these systems, six, seven, eight, and nine, are expressed by five one, five two, five three, and four; in the second, by more one, more two, more three, more four; in the third, by the combination of the word *goo*, of which we do not know the meaning, with one, two, three, four; in the second, ten is expressed by the repetition of the term for five [...] (1845: 385).

Table 1: Numerical systems of the Melanesians from the Islands of New Caledonia, Tanna and Malekula (Peacock 1945: 385)

New Caledonia	Tanna	Mallicollo [Malekula]
1. Pātai	1. Rettec	1. thkai
2. Pā-ròo	2. Carroo	2. Ery
3. Par-ghen	3. Kāhār	3. Erey
4. Par-bai	4. Kafā	4. Ebàts
5. Pā-nim	5. Karirrom	5. Erihm
6. Pānim-gha	6. Ma-riddee	6. Tsukài
7. Pānim-roo	7. Ma-carroo	7. Goory
8. Pānim-ghen	8. Ma-kāhār	8. Goorey
9. Pānim-bai	9. Ma-kafā	9. Goodbàts
10. Pārooneek	10. Karirrom - karirrom	10. Seneām

The examination of more or less sophisticated systems of numerals leads Peacock to the conclusion that the “regular principles”—at stake in the elaboration of these systems—are derived from “practical methods of numeration” (1845: 385). In his view, these “practical methods” would have been developed involving the body (fingers, hands, feet in particular), but also different artifacts such as kernels, pebbles, etc., as confirmed by the analysis of some numerical systems developed in South American indigenous societies in particular.

The practical methods of counting of American tribes, however, are not in all cases restricted to the fingers and toes, and their numeration is not necessarily confined to twenty, the radix of their scale, when destitute of the aid of names, whether arbitrary or not, for higher numbers, or when they cannot call in the assistance of other persons. The Guaranies make heaps of maize, each consisting of twenty grains, two, three, four, &c. of which are used to denote 40, 60, 80, &c. the excess above any one of this series of articulate numbers being reckoned in the ordinary way [...] (1845: 390).

Thanks to the various systems of numerals analyzed in the first part of his history of arithmetic, Peacock argues that, although the quinary and vicensary scales has often been used for counting in many “rude tribes” (1945: 383), the decimal scale seems to be the more common and universal one. “The very general adoption of the decimal system of notation”, would find its origin, according to this author, in the “very natural practice of numbering by fingers on the two hands” (1845: 370)³.

³ As I have pointed out elsewhere (Vandendriessche & Petit, 2017), such a corporeal origin of numbering systems had already been suggested by the German naturalist and geographer Alexander von Humboldt (1769-1859) in an article published in German in 1826, thereafter translated into French in 1851 under the title “Des systèmes de

In fact, Peacock was going to assert that all languages kept the marks of the original decimal scale, and that its universal presence arose from its natural origin: the organization of the human body, with its symmetry and the ten fingers, was considered as the first “natural abacus” [370 § 4]. That first instrument for counting could be praised as universal, because of its “natural” origin, which makes it existing before human thinking itself. Establishing this natural character of the decimal scale was a very strong argument for Peacock in asserting that arithmetical language was universally established (Durand-Richard, 2011: 265).

As mentioned above, Peacock introduces a distinction between abstract and concrete numbers. However, in his view, this distinction does not have a fundamental importance in the development of arithmetic:

In arithmetics we consider both kinds of numbers, though the operations are in all cases the same as if the numbers were perfectly abstract; the association of qualities being merely of use in directing us to the particular operations or reductions to be performed, and in assisting us in the proper interpretation of the result [...] (1845: 369)⁴.

Although Peacock acknowledges that sophisticated numerical systems have been found in different primitive/barbarous societies, he notices, however, that there could be significant differences from one of these systems to another regarding their abstraction level. Some South American tribes, in particular, would have developed extremely limited system of numerals, while using very concrete terms to designate the numbers.

chiffres en usage chez différents peuples, et de l’origine de la valeur de position des chiffres indiens”. In this paper, Humboldt argues that a “common corporeal organization (four extremities, each of which divided into five parts)” provides natural “stopping points” for counting, probably at the origin of the groups of 5, 10, and 20, recurrent in numbering systems, and often named in reference to hands and feet (1851, 378). According to Humboldt (1851, 382), the groups of units resulting from these “stopping points” are not only perceptible in various vernacular languages, but also sometimes in “numerative hieroglyphics” (that of the Aztecs in particular). He also argues that in addition to the “reciprocal influence” existing between writing and language (perceptible in the writing of numbers in particular), the instruments of “palpable arithmetic” (such as the Chinese abacus and the quipu of Peru) “offered to the eye the first ways of designating graphically groups of different orders” (offert “à l’œil les premières manières de désigner graphiquement des groupes de différents ordres”) (1851, 387). They would thus have made it possible to write and read “numbers with great regularity”; the “impressions left by these operations” (*impressions que laissaient ces opérations*) would have influenced “the beginnings of the writing of numerals/numbers” (“les commencements de la graphique numérique”) (1851, 389). It is noteworthy that although both Peacock and Humboldt wrote their contribution to the history of arithmetic at the same period in the 1820s, they do not mention their respective works.

⁴ This extract is part of a footnote, in which Peacock further indicates that some “authors have defined abstract or discrete numbers to be those which have no denomination annexed to them; considering all others as contract or concrete”. This was indeed a definition commonly used over the 18th and 19th century (*cf.* articles by Wang, Feirrer & Schubring, and Chambris & Visnovska in this volume). However, according to Peacock, upon this definition, “a difficulty arose about the class to which fractional numbers were to be referred; the unit of the numerator being limited in value by the denominator, and consequently being in this respect different from abstract whole numbers. The solution of this difficulty is to be found in the meaning of the word *denomination*, which in our definition would be confined to designate a quality of the subject which the subject is supposed to denote” (1845: 369).

In the languages of these rude tribes, **abstract terms** are almost entirely unknown, and their expressions from mere poverty, in many cases assume a highly figurative form, being obliged to refer to natural objects and the most common relations of life, to express ideas which do not otherwise come within the compass of their languages; thus in the Caribbean language, the fingers are termed the children of the hand, and the toes the children of the feet; and the phrase for ten, *Chon oucabo raim*, all the children of the hands (1845: 390).

Before moving on to the next section, let's keep in mind that, to Peacock, regular (arithmetic) principles underlie the elaboration of numerals in every language. In this domain, abstraction (of numbers) is related to language through the process of elaborating numerals, that occurs even in the less civilized societies. Peacock notices, however, that some “barbarous” tribes have developed poor number systems using “figurative” terminology for numbers.

The decades that have followed Peacock's analysis of numerals (of the so-called primitive societies) have seen the development of the fledgling discipline of anthropology, through the dominant evolutionist theory—first elaborated by the British anthropological school, and developed by Edward Tylor (1832–1917) in his book *Primitive Culture: Researches Into the Development of Mythology, Philosophy, Religion, Languages, Art and Customs* (1871) in particular. The latter theory defends the thesis that human cultures evolve in a unilineal manner through three stages of development: savagery, barbarism, and civilization. Basically, the lowest stage (savagery) is defined as “that in which man subsists on wild plants and animals, neither tilling the soil nor domesticating creatures for his food. [Tylor considers] that men reach the barbarian stage when they take to agriculture, and pass from the barbaric to the civilized stage by acquiring the art of writing” (Powell, 1885: 175). To Tylor, the stage of savagery

corresponds in a considerable degree to that of modern savage tribes, who, in spite of their difference and distance, have in common certain elements of civilization, which seem remains of an early state of the human race at large. [Therefore, the] European “may find among the Greenlanders or Maoris many a trait for reconstructing the picture of his own primitive ancestors (Tylor, 1871: 21)⁵.

In the chapter “The Art of counting” of the book *Primitive Culture*, Tylor seeks to illustrate the evolutionist theory by bringing to light how numeration has been “developed by rational invention” from a “low stage up to that in which we ourselves possess it” (Tylor, 1871 : 22). In

⁵ This paradigm—as well as the unilineal development of cultures—has been criticized since the early 20th century. In particular, in his book *The Mind of Primitive Man* (1911), anthropologist Franz Boas argues that while “it is certainly true that analogues can be found between the types of culture represented by primitive people and those conditions which prevailed among the ancestors of the present civilized peoples at the dawn of history, and that these analogues are supported by the evidence furnished by survivals, the evidence of archaeology does not support the complete generalization. The theory of parallel development, if it is to have any significance, would require that among all branches of mankind the steps of invention should have followed, at least approximately, in the same order, and that no important gaps should be found. The facts, so far as known at the present time, are entirely contrary to this view” (1911: 179). Some decades later, Lévi-Strauss (1952) will argue that indigenous societies (*sociétés sans écriture*) also have History, and, therefore, cannot be considered as comparable to the earliest human societies.

particular, although Tylor does not explicitly introduce both categories “abstract” and “concrete” in relation to numbers, he argues that, in all cases, the names given to numerals in the primitive societies originate from the vernacular terms which designate the body units (finger, hand, foot, man, etc.), as involved in gestural numerations analyzed as the first stage of the elaboration of a numerical system⁶.

The evolutionist theory in its link to “the art of counting” has had a significant impact on the history of mathematics, and that of numbers specifically. Indeed, the elaboration of a system of numbers has often been presented (and sometimes still is, *cf.* Dantzig 1930, Struik 1948, Ifrah *et al.* 2000, Boyer 2011, Keller 2016...) in contrasting the (supposed) extreme concreteness of the (early) number concept in indigenous societies with the (so-called) abstract concept number elaborated in more advanced societies, through a unilineal development (*cf.* Ferrara & Valerio in this volume). As we will see in the next section, it is through this paradigm, and significantly influenced by evolutionary theory of his time, and by Tylor’s *Primitive Culture* in particular, that American mathematician Levi Leonard Conant analyzed the origin and the development of the “concept of number”.

2. Levi Leonard Conant and the “Number Concept”

Levi L. Conant received a PhD in mathematics from Syracuse University, New York State, in 1893. He began his career as a professor of mathematics at the Dakota School of Mines, then at Clark University, Massachusetts, before being recruited in the same state by the Worcester Polytechnic Institute. His most significant work is undoubtedly the book *The Number Concept: Its Origin and Development* (1896). To my knowledge, it is the first book fully devoted to a comparative study of the numerical systems in use in the so-called primitive societies, with a special focus on “North American tribes”, and drawn from the ethnographic sources available at the end of the 19th century.

More precisely, Conant borrows the latter ethnographic sources from studies in both anthropology/ethnology and linguistics (Alfred C. Haddon, Franz Boas, Daniel Brinton, John Murdoch, Friedrich Müller, etc.). Several times, Conant also relies on the ethnographic documentation presented by Peacock in his *Arithmetic* (1845 [1826]), without, however, commenting on Peacock’s analysis.

Contents of the book :

- 1- Counting
- 2- Number System Limits
- 3- Origin of Number Words
- 4- Origin of Number Words (continued)
- 5- Miscellaneous Number Bases
- 6- The Quinary System

⁶ Surprisingly, Tylor does not refer to Peacock’s *History of Arithmetic*, and refers only once to A. Humboldt’s article ([1826] 1851); *cf.* (Tylor, 1871: 260).

7- The Vigesimal System

It is at the end of Chapter 3 that Conant introduces an opposition between “abstract numbers” and “concrete numbers”. Therefore, we will focus our analysis on the first half of Conant’s book in order to highlight the intellectual path that led him to this opposition. The purpose of the first chapter (Counting) is essentially to show the preponderant and universal role of fingers (and possibly other artifacts) in the numerical practices of lower societies. The use of such artifacts would just become a convenience when societies climb the ladder of development.

The place occupied, in the intellectual development of man, by finger counting and by the many other artificial methods of reckoning,—pebbles, shells, knots, the abacus, etc.,—seems to be this: The abstract processes of addition, subtraction, multiplication, division, and even counting itself, present to the mind a certain degree of difficulty. To assist in overcoming that difficulty, these artificial aids are called in; and, among savages of a low degree of development, like the Australians, they make counting possible. A little higher in the intellectual scale, among the American Indians, for example, they are employed merely as an artificial aid to what could be done by mental effort alone. Finally, among semi-civilized and civilized peoples, the same processes are retained, and form a part of the daily life of almost every person who has to do with counting, reckoning, or keeping tally in any manner whatever. They are no longer necessary, but they are so convenient and so useful that civilization can never dispense with them (Conant 1896: 19).

The second chapter (Number System Limits) intends to show that many primitive societies would have developed very limited numerical systems, involving a small number of numerals (1, 2, 3 or 4), as it generally occurs in Australian Aboriginal societies, as well as in Latin American societies such as the Indians from the Chaco (1896: 22). Drawing on the ethnographic literature at his disposal, and in line with Tylor’s analysis of numeration (*cf.* *The art of counting, Primitive Culture*, 1871), he argues that societies at the bottom of the scale (leading to civilization) would have a low apprehension of numbers, resulting in a low ability to grasp the “abstract idea of number”, without explaining here though what “abstract” precisely means to him in this context:

At first thought we might, then, state it as a **general law** that those races which are lowest in the scale of civilization, have the feeblest number sense also; or in other words, the least possible power of grasping the abstract idea of number (1896: 30).

Nevertheless, according to Conant, “there are many and important exceptions” to this “general law” (1896: 31). To illustrate this point, he indicates that some “savage tribes” actually use numbers as large as 100, 1000, or even 1 million. Trades would have contributed to these exceptional numerical abilities. Like Tylor, he mentions the case of Tongans who have developed numerals to count up to 100,000; as well as the Thembu, Fingu, Mpondo, and a dozen other South African societies who are able to count up to 1 million. He also notices that

when the numeration system is reduced to a few numerals, the “savage”⁷ can generally count far beyond these numbers, with the help of his fingers in particular (1896: 31).

Conant, however, qualifies the exceptional numerical skills in use in some primitive tribes, asserting that even though the “savages” can sometimes count up to 10, 20 or 100, they would have a “real mental conception” of numbers only for the smallest numbers. To him, this point is confirmed by the lack of precision (“absence of exactness”) in particular, that the “savage” generally demonstrates in the use of (large) numbers (1896: 32).

The third chapter of Conant’s book deals with the issue of the “origin of number words” or numerals. According to Conant, although the primary meaning of numerals used in the more advanced societies is most often entirely lost, it is sometime possible to “detect these meanings” in “savage languages”. Conant explains that “number words” are

the one, two, three, etc., of English; the eins, zwei, drei, etc., of German; words which must at some time, in some prehistoric language, have had definite meanings entirely apart from those which they now convey to our minds. In savage languages it is sometimes possible to detect these meanings, and thus to obtain possession of the clue that leads to the development, in the barbarian’s rude mind, of a count scale—a number system. But in languages like those of modern Europe, the pedigree claimed by numerals is so long that, in the successive changes through which they have passed, all trace of their origin seems to have been lost (1896: 39).

Similarly to Peacock’s approach, Conant further argues that a “general method” underlies the elaboration of every numerical system worldwide, through the use of a limited number of numerals, in order to not saturate one’s memory with a multitude of different number words.

The formation of numeral words shows at a glance the general method in which any number scale has been built up. The primitive savage counts on his fingers until he has reached the end of one, or more probably of both, hands. Then, if he wishes to proceed farther, some mark is made, a pebble is laid aside, a knot tied, or some similar device employed to signify that all the counters at his disposal have been used. Then the count begins anew, and to avoid multiplication of words, as well as to assist the memory, the terms already used are again resorted to; and the name by which the first halting-place was designated is repeated with each new numeral. [...] (1896 : 40).

To Conant, this “general method” is a universal phenomenon, “presenting us with nothing but local variations, which are, relatively speaking, entirely unimportant” (1896: 41). Like Peacock, Conant shows that different principles underlie the formation of numerals through additive, multiplicative, and subtractive methods (1896: 43–46). He further asserts that the formation of numerals is in all cases achieved by following the same “invariable” law, stating that

⁷ Conant constantly refers to the (abstract) entity that he calls “the savage”, thus taking a high level of generality, while hypothesizing the existence of invariant features related to the “savage” mind at work in the “primitive” societies.

[...] all numeral greater than the base⁸ of a system are expressed by compound words, except such as are necessary to establish some new order of unit, as hundred or thousand (1896: 42).

If the etymological meaning of numerals is—according to Conant—usually difficult to establish in our societies (and sometimes also in less advanced societies), he gives different examples in which the terms used for numerals (and particularly the “bases”) refer to the vernacular terms used to designate fingers, hands, feet, or even a man (1896: 47), *cf.* ethnographic examples of the numerals of Zuñi Indians and the Eskimos from Point Barrow, table below.

Like Peacock and Tylor before him, Conant hypothesizes a continuity between the use of the body parts for counting (and of the hands in particular) and the development of numerals. Unlike Peacock and Tylor, however, he does not think that this continuity is a universal phenomenon, but simply frequent (1896: 54).

Table 2: Numeral of the Zuñi Indians and of the Point Barrow “Eskimos”

Numerals of the Zuñi Indians (documented by F. H. Cushing, 1892) (Conant, 1896: 48)	Numerals of the Eskimos from Point Barrow (documented by J. Murdoch, 1890)
1. <i>töpinte</i> = taken to start with. 2. <i>kwilli</i> = put down together with. 3. <i>ha'i</i> = the equally dividing finger. 4. <i>awite</i> = all the fingers all but done with. 5. <i>opte</i> = the notched off. 6. <i>topal'ik'ya</i> = another brought to add to the done with. 7. <i>kwilli'k'ya</i> = two brought to and held up with the rest. 8. <i>hailik'ye</i> = three brought to and held up with the rest. 9. <i>tenalik'ya</i> = all but all are held up with the rest. 10. <i>astem'thila</i> = all the fingers. 11. <i>iistem'thla topaya'thl'tona</i> = all the fingers and another over above held. The process of formation indicated in 11 is used in the succeeding numerals up to 19. 20. <i>kwillik'yenastem'thlan</i> = two times all the fingers. 100. <i>assiastem'thlak'ya</i> = the fingers all the fingers. 1000. <i>assiastem'thlanak'yeniistem'thla</i> = the fingers all the fingers times all the fingers.	1 = atauzik; 2= madro; 3 = pinasuu; 4 = sisaman. 5 = tudlemut. (pure number word) 6. <i>atauyimin akbinigiii [tudlimu(t)]</i> = 5 and 1 on the next. 7. <i>madronin akbinigin</i> = twice on the next. 8. <i>pinasunin akbinigin</i> = three times on the next. 9. <i>kodlinotaila</i> = that which has not its 10. 10. <i>kodlin</i> = the upper part <i>i.e.</i> the fingers. 14. <i>akimixotaityuna</i> = I have not 15. 15. <i>akimia</i> . [This seems to be a real numeral word.] 20. <i>inyuina</i> = a man come to an end. 25. <i>inyuina tudlimunin akbinidigin</i> = a man come to an end and 5 on the next. 30. <i>inyuina kodlinin akbinidigin</i> = a man come to an end and 10 on the next. 35. <i>inyuina akimiamin aipalin</i> = a man come to an end accompanied by 1 fifteen times. 40. <i>madro inyuina</i> = 2 men come to an end.

As mentioned above, at the end of the third chapter (Origin of Number Words), Conant introduces an opposition between “abstract numbers” and “concrete numbers”, the latter being analyzed as characteristic of the numerations developed by primitive peoples. The main argument is that these numerations—whose numerals are generally related to digital or body practices—are anchored in a concrete reality, the only one accessible to the intelligence of the “savages”.

⁸ In this context, the term “base” refers to the (few) “root words” from which the other number words are elaborated through “cyclic patterns” (Salzmann, 1950) involving “arithmetical” operations between the “base(s)” and the first numerals 1, 2, 3... (*cf.* Table 1).

In the sense in which the word is defined by mathematicians, *number* is a pure, abstract concept. But a moment's reflection will show that, as it originates among savage races, number is, and from the limitations of their intellect must be, entirely concrete (1896: 72).

In the following passage, Conant clarifies how, according to him, the numbers in use in these primitive societies have an exclusively “concrete” aspect. His argument stems from the fact that the “mental image” of the number would be a “material image”, often related to a part of the body in particular; and that of the hand for the number 5 specifically.

An abstract conception is something quite foreign to the essentially primitive mind, as missionaries and explorers have found to their chagrin. The savage can form no mental concept of what civilized man means by such a word as “soul”; nor would his idea of the abstract number 5 be much clearer. When he says five, he uses, in many cases at least, the same word that serves him when he wishes to say hand; and his mental concept when he says *five* is of a hand (Conant, 1896: 72).

In general, the understanding of numbers in these societies is analyzed by Conant as a process of relating a “material image” to a collection of objects. He asserts:

The concrete idea of a closed fist or an open hand with outstretched fingers, is what is uppermost in his mind. He knows no more and cares no more about the pure number 5 than he does about the law of the conservation of energy. He sees in his mental picture only the real, material image, and his only comprehension of the number is, “these objects are as many as the fingers on my hand.” (1896: 72).

As we will see in Section 4, this principle of one-to-one correspondence between parts of the body and objects to be counted will be, for philosopher Léon Brunschvicg, one of the key steps leading to the elaboration of the “concept of number”.

As mentioned above, Conant conceives the emergence of the “concept of number” through a slow evolution that would occur in conjunction with the cultural development of each human society, passing progressively from the lower stage savagery to civilization (according to the evolutionist theory). He further asserts that “in general, the growth of the number sense keeps pace with the growth of the intelligence in other respects” (1896: 33). Through this process, numbers would gradually dissociate themselves from their material representation. Such dissociation being generally concomitant to a transformation of the numerals as well as the oblivion of their primary meaning.

Then, in the lapse of the long interval of centuries which intervene between lowest barbarism and highest civilization, the abstract and the concrete become slowly dissociated, the one from the other. First the actual hand picture fades away, and the number is recognized without the original assistance furnished by the derivation of the word. But the number is still for a long time a certain number of objects, and not an independent concept. It is only when the savage ceases to be wholly an animal, and becomes a thinking human being, that **number in the abstract can come** within the grasp

of his mind. It is at this point that mere reckoning ceases, and arithmetic begins (Conant 1896: 72).

In this view, the development of numerical abilities and the abstract conception of numbers is thus explicitly linked to the evolution of human societies as well as the development of intellectual abilities of the people who constitute them. Therefore, due to a (hypothetical) lesser capacity for abstraction, number systems would have emerged with difficulty in “lower” societies, while remaining anchored for a long time in the concrete:

By the slow, and often painful, process incident to the extension and development of any mental conception in a mind wholly unused to abstractions, the savage gropes his way onward in his counting from 1, or more probably from 2, to the various higher numbers required to form his scale. The perception of unity offers no difficulty to his mind, though he is conscious at first of the object itself rather than of any idea of number associated with it. The concept of duality, also, is grasped with perfect readiness. This concept is, in its simplest form, presented to the mind as soon as the individual distinguishes himself from another person, though the idea is still essentially concrete (Conant 1896: 72).

In summary, we have seen that Conant’s approach is profoundly influenced by the evolutionist theories of the time. Since it was believed that, at the lowest stage of evolution, “savage” peoples do not use abstract concepts, they would have only elaborate numerical systems anchored in the concrete. As we will see in the following, Conant’s analysis of “savage tribes” numerical systems has been a source for further reflection on the topic; carried out in the field of philosophy in particular, in France, in the first decades of the 20th century.

3. Lucien Lévy-Bruhl and the theory of prelogical mentality

Rooted in Auguste Comte’s positive philosophy, Lévy-Bruhl’s scientific project was to carry out, through a positive and sociological approach, and at odds with evolutionary theories, a comparative analysis of ethnological facts—as they are described in the major 19th century anthropological works (Tylor, 1871; Boas, 1890; Haddon 1890, Frazer 1911-1915, K. von den Steinen 1894, *et al.*). Lévy-Bruhl’s goal has been to bring to light the logic of the “primitive mentality” (*i.e.* ways of “feeling, thinking, and acting” of primitive peoples) and “establish how it is organized” (Keck, 2010: 15).

In his book *Les fonctions mentales dans les sociétés inférieures*, published in 1910 (translated into English by Lilian A. Clare, under the title *How Natives Think*, 1926), Lévy-Bruhl sought a “general law” governing the links between the “collective representations”⁹ at work in these

⁹ Lévy-Bruhl begins his 1910 book by defining these “collective representations” as follows: “The representations which are termed collective, defined as a whole without entering into detail, may be recognized by the following signs. They are common to the members of a given social group; they are transmitted from one generation to another; they impress themselves upon its individual members, and awaken in them sentiments of respect, fear, adoration, and so on, according to the circumstances of the case. Their existence does not depend upon the individual; not that they imply a collective unity distinct from the individuals composing the social group, but because they present themselves in aspects which cannot be accounted for by considering individuals merely as such” (Lévy-Bruhl, 1966: 1). Although Lévy-Bruhl does not mention it explicitly, these considerations on

“lower” societies; collective representations “by virtue of which objects, whatever they may be, living beings, inanimate objects, or articles manufactured by man, always present themselves charged with mystic properties” (1966 [1925]: 60). According to Lévy-Bruhl, the latter properties imply, in turn, “a ‘participation’ between persons or objects which form part of a collective representation” (1966: 61). This led him to call “the law of participation” the principle which is “peculiar to the primitive mentality”, and governs the connection of these representations.

Lévy-Bruhl illustrates the “law of participation” through the example, borrowed from ethnologist Karl Von den Steinen (1855-1929), of the Amazonian Bororo who claim that “Bororo are Arara (red parakeets)”:

“The Bororos,” says Von den Steinen, who would not believe it, but finally had to give in to their explicit affirmations, “give one rigidly to understand that they are araras *at the present time*, just as if a caterpillar declared itself to be a butterfly.” It is not a name they give themselves, nor a relationship that they claim. What they desire to express by it is actual identity (1966: 62)¹⁰.

As explained by Keck (2010: 58), in Lévy-Bruhl’s view, the Bororo’s statement (asserting that they are araras through an “essential identity”) does not refrain from contradiction (*i.e.* does not follow the principle of non-contradiction), since it states that “A is non-A” (a man cannot be a bird). Therefore, it must be governed by a different principle of logic, what he calls “participation”, according to which “A participates with non-A” (men participate with the spirit of the bird with whom they perceive a relation of kinship). Lévy-Bruhl introduces the term “prelogical” to designate such phenomenon. To him, this mentality can thus be described as “mystical”, as well as “prelogical”: “mystical” if we look at the content of the collective representations, and “prelogical”¹¹ if we look at the relationships between beings and/or objects within the collective representations (Chimisso 2008: 66; Vandendriessche [upcoming](#)).

collective representations seems to have been borrowed from Durkheim’s article “Représentations individuelles et représentations collectives” (1898). The objective of this paper is to show that the collective representations go far beyond the mental life of each individual of the society, and cannot be understood by studying only the mental life of individuals,—in the same way that individual representations cannot be fully understood through physiological causes related to the brain functions (Durkheim, 1898). Therefore, according to Durkheim, these collective representations (shared by individuals) have their own laws that needs to be studied for themselves, and this could not be achieved only by an “internal analysis” (Durkheim, 1898: 302).

¹⁰ Original text: Les Bororô, dit M. von den Steinen, [...], “les Bororô donnent froidement à entendre qu’ils sont actuellement des araras, exactement comme si une chenille disait qu’elle est un papillon”. Ce n’est pas un nom qu’ils se donnent, ce n’est pas une parenté qu’ils proclament. Ce qu’ils veulent faire entendre, c’est une identité essentielle (Lévy-Bruhl, 1910: 77-78).

¹¹ According to Frédéric Keck (2010: 13-14), the use of the term “prelogical” to characterize the primitive mentality gave rise “to the greatest misunderstandings”. The latter term “was losing the strength of Lévy-Bruhl’s anti-evolutionist stance, looking at primitive societies, not as a kind of childhood mistake, but characterized by a social operational mode of thought [...]”. Using the term “prelogical”, Lévy-Bruhl wanted to describe “another logic” than the “logic of signs”: this “other logic” is still present in every human mind, but its expression became less visible, so that a detour through primitive societies allows us to bring it to light. So, according to Keck, under the appearance of a classic evolutionism, “it is a surprisingly relativist project that emerged to pluralize the logical activity itself by renewing it to its social genesis”. See Section 4, Brunschvicq’s interpretation of the concept of prelogism.

To Lévy-Bruhl, all of these characteristic features of the primitives' collective representations thus entail—for those peoples—a lesser ability to abstract and generalize, and conduct a deductive reasoning (1922: 505–506), since their way of thinking is not governed by the same logical principles as ours. This would be particularly perceptible, according to Lévy-Bruhl, in the development of the “concrete” numerical systems, as it would have occurred within various primitive societies.

In a chapter entitled “Prelogical Mentality in Relation to Numeration” (1910: 202–257, 1966: 158–198), Lévy-Bruhl relates the nature of mathematics in primitive societies to the prelogical mentality that, in his view, governs them. He intends to show that the “various methods of counting and calculating, of forming and using the names for numbers”, enable us to see “actually at work, the mentality of primitives where it differs specifically from logical thought” (1966: 158). His argument stems from the fact that the prelogical mentality does not have abstract concepts, therefore it does not allow the primitives to abstract the number of counted objects, by detaching it from “what it is the number of”: this leads Lévy-Bruhl to describe as “concrete” the numerations developed/elaborated in primitive societies.

It is usually admitted as a natural fact, requiring no examination, that numeration starts with the unit, and that the different numbers are formed by successive additions of units to each of the preceding numbers. This is, in fact, the most simple process, and the one which imposes itself upon logical thought when it becomes conscious of its functioning. [...] Prelogical mentality, however, which has **no abstract concepts** at command, does not proceed thus. It does not distinctly separate the number from the objects numbered. That which it expresses by speech is not really numbers, but “number-totals”, the units of which it has not previously regarded singly. To be able to imagine the arithmetical series of whole numbers, in their regular order, starting from the unit, it must have separated the number from that which the number totals, and this is precisely what it does not do. It imagines, on the contrary, collections of entities or objects which are familiar to it both by nature and by number, the latter being felt and perceived, though not conceived in the abstract (1966: 169)¹².

Lévy-Bruhl illustrates the concept of “number-totals” (or “number-sets”, *ensemble-nombre*) with different examples borrowed from the anthropological literature. In particular, he borrows some ethnographic facts from the book *The Melanesian languages* (1885) by the Anglican Pastor and anthropologist Robert Henry Codrington (1830–1922): the latter reported that in Fiji

¹² Original text: On admet en général sans examen, et comme une chose naturelle, que la numération part de l'unité, et que les différents nombres se forment par l'addition successive de l'unité à chaque nombre précédent. C'est là en effet le procédé le plus simple, celui qui s'impose à la pensée logique quand elle prend conscience de son opération. Mais la mentalité prélogique, qui ne dispose point de concepts abstraits, ne procède pas ainsi. Pour elle, le nombre ne se sépare pas nettement des objets nombrés. Ce qu'elle exprime dans le langage, ce ne sont pas les nombres proprement dits, ce sont des « ensembles-nombres », dont elle n'a pas isolé préalablement les unités. Pour se représenter la série arithmétique des nombres entiers, dans leur succession régulière, à partir de l'unité, il faudrait qu'elle eût détaché le nombre de ce dont il est le nombre. C'est précisément ce qu'elle ne fait pas. Elle se représente au contraire des collections d'êtres ou d'objets, qui lui sont familières à la fois par leur nature et par leur nombre, celui-ci étant senti et perçu, mais non abstraitement conçu (1910 : 219–220).

and the Solomon Islands (South Pacific), there are/were collective nouns (analyzed as “number-totals” by Lévy-Bruhl), “signifying tens of things very arbitrarily chosen: neither the number nor the name of the thing being expressed”. Thus, “in Fiji *bola* is a hundred canoes, *koro* a hundred coconuts, and *salavo* a thousand coconuts, etc.” (1966: 169).

Fig. 1. Tsimshian numeration system. Table extracted from (Lévy-Bruhl, 1966: 171).

	Counting in general.	Flat objects.	Round objects.	Human beings.	Long objects.	Canoes.	Measures.
1	gyak	gak	g'erel	k'al	k'awutskan	k'amaet	k'al
2	t'epqat	t'epqat	goupel	t'epqadal	gaopskan	g'alpēelk	gulbel
3	guant	guant	gutle	gulal	galtskan	galtskantk	guleont
4	tqalpq	tqalpq	tqalpq	tqalpqdal	tqaapskan	tqalpsqk	tqalpqalont
5	kctofic	kctōnc	kctoñc	kcenecal	k'etōentskan	kctoōnsk	kctonsilont
6	k'alt	k'alt	k'alt	k'alidal	k'aoltskan	k'altk	k'alidelont
7	t'epqalt	t'epqalt	t'epqalt	t'epqaldal	t'epqaltskan	t'epqaltk	t'epqaldeont
8	guandalt	yuktalt	yuktalt	yuktleadal	ek'tlaedskan	yuktaltk	yuktalidelont
9	kctemac	kctemac	kctemac	kctemacal	kctemaetskan	kctemack	kcteasmilont
10	gy'ap	gy'ap	kpēel	kp'al	kpēetskan	gy'apsk	kpēont

Similarly, Lévy-Bruhl borrows from anthropologist Franz Boas (1858–1942) the example of the numerical system of the Tsimshian (from North America) who use/used seven different series of numbers to count different classes of objects. Lévy-Bruhl notes that

the first class, that of the words which are used for counting in general, is almost identical with the second, with the exception of a slight difference in 1 and 8. It is therefore allowable to suppose that the first class is not formed at the same time as the others, or independently of them, but, on the contrary, that there were special numerals for certain categories of objects before there were any for simple counting (1966: 172).

As further evidence that the primitive mentality does not separate the numbers/numerals from the enumerated objects, Lévy-Bruhl gives the example of numerical systems with “classifiers”. Classifiers have been used/introduced in many languages (not only those of oral tradition societies; such classifiers are found in Chinese and Japanese in particular, Crump 1992). They are either prefixes or suffixes, attached to numerals, in order to specify a particular class of objects to which the counted objects belong. Lévy-Bruhl gives the example of the numerical system of the Heiltsuk¹³, documented by Franz Boas, the first ethnographer—to my

¹³ The Heiltsuk's numerical system with classifiers had already been mentioned by Conant as Lévy-Bruhl indicates, pointing out that in “his very serviceable book, Conant has collected a good many facts of this kind, of which I shall quote a few only” (1966: 170 ; 1910: 223). However, Conant analyzes these numerical systems as

knowledge—who collected and examined such numerical systems (*Sixth Report on the North-Western Tribes of Canada*, 1890).

Table 3. Classifier numerical system of the Heiltsuk. Adapted from (Lévy-Bruhl, 1966: 172).

Object counted	One	two	Three
Animated being	menok	maalok	yutuk
Round object	menskam	masem	yutqsem
Long object	ments'ak	mats'ak	yututs'ak
Flat object	menaqa	matlqsa	yutqsa
Day	op'enequils	matlp'enequils	yutqp'enequils
Fathom	op'enkh	matlp'enkh	yutqp'enkh
United	—	matloutl	yutoutl
Group	nemtsmots'ut	matltsmots'utl	yutqtsmots'utl
Full cup	menqtlala	matl'aqtlala	yutqtlala
Empty cup	menqtlala	matl'aqtlala	yutqtlala
Full box	menskamala	masemala	yutqsemala
Empty box	menskam	masem	yutqsem
Loaded canoe	mentsake	mats'ake	yututs'ake
Canoe and its crew.	ments'akis	mats'akla	yututs'akla
All on the shore	—	maalis	—
All in the house	—	maalitl	—

Lévy-Bruhl mentions that similar “classifier numerical systems” have been recorded by Boas in neighboring societies in British Columbia, while emphasizing that the number of series of number-names can be in some cases almost “unlimited”:

Of the Kwakiutl, Boas says: “Besides the class-suffixes for animated beings, round, flat, long objects, days, fathoms, the numerals may take any of the noun suffixes [...] The number of classes is unlimited. They are simply compounds of numerals and noun-suffixes” (1966: 173)¹⁴

Lévy-Bruhl comments on this “unusual copiousness”, which can be readily understood—according to him—, “when we look back upon the general character of these languages, which are but slightly abstract, and pre-eminently ‘pictorial’. It is hardly surprising that the numerals do not stand alone” (1966: 173). Our author further asserts that these classifying number systems are related to a particular form of abstraction—which the prelogical mentality implies—and which leads to a weak propensity to search for generality.

[...] these facts are traceable to the general trend of primitive mentality, for as its **abstractions are always specializing rather than generalizing**, it does, at a certain degree of development, form numerals; but **they are not abstract numbers such as we use**. They are invariably the number-names of certain classes or persons or things, and these classes most frequently depend upon the conformation, attitude, position and

part of the discussion regarding the origin of the numerals (*cf.* section 2), and, unlike Lévy-Bruhl, does not describe them as “concrete numeration”.

¹⁴ Original text : Chez les Kwakiutl, outre les suffixes de classe pour les êtres animés, pour les objets ronds, longs, plats, pour les jours, pour les brasses... les nombres peuvent encore prendre tous les suffixes de noms. Le nombre des classes est illimité. Ce sont simplement des composés des nombres et des suffixes des noms (1910: 224).

movement of the objects. Now we have already seen how much importance these primitive languages accord to everything that expresses the contour and the relative position and movement of objects in space. This is carried to such a point that it would frequently be possible to superpose the detail of that which the words signify, upon the delineation which translates it into a reality for the eyes, and the gesture-language which expresses it by movement (1966: 174–175)¹⁵

To Lévy-Bruhl, the mentality of the primitive's mind implies that “number is undifferentiated” from two standpoints: on the one hand, as we have just seen, in its practical use “number still more or less adheres to the objects counted” (*cf.* number-totals, and classifier numerical systems). On the other hand, according to this author, “in the collective representations the number and its name still participate so closely in the mystic qualities of the ensembles represented that they are indeed mystic realities themselves, rather than arithmetical units” (1966: 182)¹⁶. Lévy-Bruhl further asserts that apart from simple counting (number of hours of work, number of fishes caught, for instance), every time the primitive man

imagines a number *qua* number, he necessarily pictures it with the mystic property and value appertaining to that number, and to it alone, by virtue of participations which are equally mystic. The number and its name are indifferently the vehicle of these participations.

Thus each number has its own individual physiognomy, a kind of mystic atmosphere, a “field of action” peculiar to itself. Every number, therefore, is imagined—we might also say, felt—especially for itself, and without comparison with the others. From this standpoint numbers do not constitute a homogeneous series, and they are accordingly quite unsuited to the simplest logical or mathematical operations. The mystic personality contained in each makes them unable to be added, subtracted, multiplied or divided. The only processes they admit of are themselves mystic processes and not, like arithmetical operations, subject to the law of contradiction (1966: 182)¹⁷.

¹⁵ Original text: Ces faits se ramènent, selon nous, à la disposition générale de la mentalité des sociétés inférieures. Comme ses abstractions sont toujours plutôt particularisantes que généralisantes, elle forme bien, à un certain degré de son développement, des noms de nombre ; mais ce ne sont pas des noms de nombre *in abstracto*, comme ceux dont nous usons. Ce sont toujours les noms de nombre de certaines classes d'êtres ou d'objets. Ces classes dépendent le plus souvent de la conformation, de l'attitude, de la situation, du mouvement des objets. Or nous avons vu plus haut quelle importance les langues de ces sociétés attachent à tout ce qui exprime les contours, les déplacements et les situations relatives des objets dans l'espace : à tel point que souvent une sorte de superposition serait possible entre le détail de ce qui est rendu par la phrase, le dessin traduisant la même réalité pour les yeux, et la phrase du langage par gestes qui l'exprimerait au moyen de mouvements (1910 : 227).

¹⁶ Original text: [...] le nombre et son nom participent encore si étroitement aux propriétés mystiques des ensembles représentés, qu'ils sont bien plutôt des réalités mystiques eux-mêmes, que des unités arithmétiques » (1910: 236-237).

¹⁷ Original text : Toutes les fois qu'il se représente un nombre *comme nombre*, il se le représente nécessairement avec une vertu et une valeur mystiques qui appartiennent à ce nombre-là, et à lui seul, en vertu de participations également mystiques. Le nombre et son nom sont, indistinctement, le véhicule de ces participations. Chaque nombre a ainsi sa physionomie individuelle propre, une sorte d'atmosphère mystique, de « champ de force » qui lui est particulier. Chaque nombre est donc représenté, — ou pourrait dire aussi, senti, — spécialement pour lui-même et sans comparaison avec les autres. De ce point de vue, les nombres ne constituent pas une série homogène, et, par suite, ils sont tout à fait impropres aux opérations logiques ou mathématiques les plus simples. L'individualité mystique de chacun d'eux fait qu'ils ne s'additionnent ni ne se soustraient, ne se multiplient ni ne

Lévy-Bruhl points out, however, that this concerns small numbers; since—according to him—numbers known in lower societies rarely exceed a decade. He further notices that “peoples who have risen to an abstract conception of number, the value and mystic power of numbers may indeed be preserved for a very long time, when it is a case of those which formed part of the very earliest collective representations”¹⁸ (1966: 182). “Even among the most advanced nations”, Lévy-Bruhl explains, “there are traces of this mystic character still discernible both in religion and metaphysics” (1966: 184).

Lévy-Bruhl illustrates the “mystical power” of numbers through several examples borrowed from the ethnographic literature: he mentions, for instance, the omnipresence of the number 4 in the great Navajo epic, with their “four bear gods, four porcupines, four squirrels, four long-bodied goddesses, four holy young men, four lightning birds, etc.”¹⁹ (1966: 185).

To give a concrete example of the non-respect of the principle of non-contradiction in primitive numerical practices, Lévy-Bruhl refers to the 1878 book *La religion védique, d’après les hymnes du rig-veda* by Abel Bergaigne, and more precisely to the chapter “L’arithmétique mythologique” (Mythological Arithmetic). In this extract, Bergaigne asserts the “extraordinary facts” that different numbers can be nonetheless equal to one another.

The simultaneous and impartial use of three and seven [...] proves but one thing: their complete equivalence [...] The various numbers we have found used for one another, because they all express, in different systems of division, the sum of the parts of the universe, have for the same reason been capable of being used, by a kind of pleonasm, side by side with each other. In fact, this has frequently been done. In this way, three is the same as seven, or as nine²⁰ (1966: 195).

Lévy-Bruhl interprets this mystical equivalence between numbers as an “absurdity to logical thought” (*scandale de la pensée logique*), even though it seems quite natural for the “prelogical mentality”. Since it is above all preoccupied with mystic participations, this mentality

does not regard these numbers in an **abstract relation to other numbers**, or with respect to the arithmetical law in which they originate. The primitive mind considers

se divisent. Les seules opérations qu'ils comportent sont des opérations mystiques elles-mêmes, et non soumises, comme les opérations arithmétiques, au principe de contradiction (1910: 237).

¹⁸ Original text: Dans les sociétés qui se sont élevées à la conception abstraite du nombre, la valeur et la vertu mystiques des nombres peuvent bien se conserver très longtemps, quand il s’agit justement de ceux qui entraînent dans les représentations collectives les plus anciennes (1910: 237).

¹⁹ « [...] les dieux sont tous au nombre de quatre, et tous se rangent aux points cardinaux, peints de la couleur propre à chacun de ces points. Il y a quatre dieux ours, quatre porcs-épics, quatre écureuils, quatre déesses au long corps, quatre jeunes saints, quatre oiseaux de l’éclair, etc. Le héros a quatre jours et quatre nuits pour raconter son histoire ; quatre jours sont employés à sa purification, etc. » (1910: 241).

²⁰ Original text: L’emploi simultané et indifférent de trois et de sept [...] ne prouve qu’une chose: leur parfaite équivalence [...] Les divers nombres que nous avons vus employés les uns pour les autres, parce qu’ils expriment tous, dans différents systèmes de division, la somme des parties de l’univers, ont pu, pour la même raison, être employés, par une sorte de pléonasm, les uns à côté des autres. Et, en effet, ils l’ont été souvent. De la sorte, trois est la même chose que sept, ou que neuf (Bergaigne, 1878: 148).

each as a reality grasped by itself, and not needing for its definition to be regarded as a functioning of other numbers. Thus, every number has an inviolate individuality which allows it to correspond exactly with another number, itself equally inviolate²¹ (1966: 195–196).

As seen in this section, in Lévy-Bruhl's views, and as a consequence of the prelogical mentality at work in the “lower” societies, the numerals they use are not genuine numbers, but only concrete “number-totals”, deeply steeped in mystical atmosphere. And thus, unsuitable for the development of arithmetic skills. Although Lévy-Bruhl's 1910 book had the merit of seeking to deepen the issue of rationality and abstraction as expressed in primitive peoples' mode of thinking, his analysis of numerical practices in their links to prelogical and mystical mentality has undoubtedly contributed to nourishing a demeaning view of mathematical practices developed by these peoples. Moreover, as I have mentioned elsewhere (Vandendriessche, upcoming), this theory of a “prelogical mentality” has significantly influenced scholarly discussions/studies in the field of history and philosophy of science, in the first half of the 20th century, when comparing—between “civilizations”—the emergence/development of “science”.

In particular, Lévy-Bruhl's influence is perceptible in the work of philosopher Léon Brunschvicg (1869–1944)—his successor at Sorbonne University (chair in the History of Modern Philosophy)—through his analysis of the numerical systems developed in the “primitive” societies, while attempting to grasp the ontogenesis process of numbers.

4. Léon Brunschvicg and the primitive numerations

In the continuity of Lévy-Bruhl's work, and that of both Conant and Tylor, philosopher Léon Brunschvicg devoted a chapter of his book *Les étapes de la philosophie mathématique* (1912) to the issue of the ontogenesis of numbers (1912, Chap 1: L'ethnographie et les premières opérations numériques, p. 7–25). Unlike Lévy-Bruhl, Brunschvicg takes an evolutionist perspective²², intending to grasp the emergence of the concept of number, while attempting to go back to the “early signs which give evidence of the awakening in mankind of scientific thinking”. A difficult task, given the lack of historical sources, whose Rhind papyrus would be the most important. To overcome this difficulty, Brunschvicg asserts that the study on “the primitive era of our civilization” could be substituted with research carried out on the inferior societies.

²¹ [Cette mentalité] préoccupée avant tout de participation mystique, ne considère pas ces nombres dans leur relation abstraite avec les autres nombres, ni dans la loi arithmétique qui les engendre. Chacun d'eux est, à ses yeux, une réalité qui est saisie en elle-même, et qui n'a pas besoin, pour se définir, d'être considérée en fonction des autres nombres. Chaque nombre a ainsi une individualité irréductible, qui lui permet de correspondre exactement à un autre nombre, individualité non moins irréductible (1910 : 255).

²² As noticed by Chimisso (2008: 71), Brunschvicg's philosophy takes the “history of science” as its “object” of study, and therefore can be analyzed as “a reflection on the history of science”. It is in that perspective that he wrote his book *Les étapes de la philosophie mathématique*. The use of the term “étapes” (stages) reflects Brunschvicg's methodology, conceiving mathematics' development as a unilinear phenomenon, and in an evolutionist perspective. It can also be noticed the ethnocentrism perspective taken by this Brunschvicg's book, concentrating mostly on European traditions in mathematics. See also (Vandendriessche, upcoming 2021).

Ethnography, exercising a form of vicarious function, helps us fill in a large part of the gaps of prehistory, and, through a hypothesis that is certainly unverifiable, but at least very probable, it should help to outline the natural course of human evolution²³ (Brunschvicg, 1912: 3–4)—my translation.

The analysis of ethnographic data extracted from previous studies on numbers (Tylor 1871-French translation 1920, Conant 1896, and Lévy-Bruhl 1910, in particular) allows Brunshvicg to argue that the increasing complexity of numerical systems occurs through three stages of development. The first stage is called the “stage of enumeration”, and corresponds to the cognitive process of relating a collection of objects to a list of names.

Fig. 2. “Numerical names” given to children from Adelaide District, Australia. Table extracted from (Brunshvicg, 1912: 9); adapted from *Journals of expeditions of discovery into Central Australia and Overland from Adelaide to King George’s Sound (1840-1841)*, by Edward John Eyre, Vol. II, London, 1845, p. 324.

Le 1 ^{er}	enfant serait appelé	ou	<i>Kertameru</i> (garçon)	ou	<i>Kertanya</i> (fille)
Le 2 ^e	—	—	ou <i>Warritya</i>	—	ou <i>Warriarto</i> —
Le 3 ^e	—	—	ou <i>Kudnulya</i>	—	ou <i>Kudnarto</i> —
Le 4 ^e	—	—	ou <i>Monaitya</i>	—	ou <i>Monarto</i> —
Le 5 ^e	—	—	ou <i>Milaitya</i>	—	ou <i>Milarto</i> —
Le 6 ^e	—	—	ou <i>Marrulya</i>	—	ou <i>Marruarto</i> —
Le 7 ^e	—	—	ou <i>Wangulya</i>	—	ou <i>Wangwarto</i> —
Le 8 ^e	—	—	ou <i>Ngarlaitya</i>	—	ou <i>Ngarlarto</i> —
Le 9 ^e	—	—	ou <i>Pouarna</i> ² ».		

Brunshvicg bases his argument/demonstration on an example already mentioned in Tylor’s book (French translation, Vol. 1, 1920: 202) showing how, in some Australian aboriginal societies, children are named in terms of their birth order. For each brotherhood, the same ordered series of names is used, adding the suffixes *ru* and *ya* to distinguish boys and girls respectively (*cf.* Fig. 2). These names can be analyzed as “numerical names” (*noms numériques*), each of them “becoming like numbers” (*comme un numéro*) (1912: 9).

Given that these people do not have “cardinal numbers” beyond three, Brunshvicg asserts that the “Australian father of nine children will know if his family is complete without, however, having the representation of the number nine, neither the abstract idea of the cardinal number or even ordinal number” (1912: 9). Therefore, it does not mean that this man counts up to nine, but instead, he pushes to the end the “qualitative distinction of terms” (*distinction qualitative des termes*), forming what we might perceive as an orderly series. By means of an enumeration, the primitive mind would simply match a collection of objects with a list of names/terms/words (1912: 9). At this very first stage, numerical thinking would be “contained in things, rather than being present in the human mind”.

The second stage of development of numbers consists, in Brunshvicg’s view, in the elaboration of a “concept of order” related to any collection (ordinal numbers). This would happen by

²³ Original text: [Notre seule ressource est de tourner la difficulté, de substituer] aux recherches sur l’ère primitive de nos civilisations, les observations que, de nos jours, on fait directement sur les sociétés inférieures. L’ethnographie, exerçant une sorte de fonction vicariante, permet de combler en une large mesure les lacunes de la préhistoire, et, par une hypothèse qui est assurément invérifiable, mais qui du moins a pour elle la vraisemblance, de rétablir dans ses grandes lignes le cours naturel de l’évolution humaine (Brunshvicg 1912: 3-4).

generalizing “the idea of order, [already] subtended by the qualitative distinction of the terms of a [given] series, so that, instead of being applied to the same group of individuals, this distinction becomes a point of reference for the ordination of arbitrary objects” (1912: 9).

Brunschvicg supports this point by describing a numerical system in use in the Murray Islands of the Torres Strait, South Pacific. This system—described a few decades earlier by Missionary Archibald E. Hunt (1899: 13) (and later on mentioned by Lévy-Bruhl, 1910: 209)—enables one to count up to 31, using body-counting practices through a principle of correspondence between body parts and a series of names, equivalent to “concrete” numeration (*équivalent concret de la numération*, Brunshvicq, 1912: 10), allowing the serialization of a group of objects.

The counting, commenced at the little finger of the left hand, thence counting the digits, wrist, elbow, armpit, shoulder, hollow above the clavicle, thorax, and thence in reverse order down the right arm, ending with little finger of right hand. This gives twenty-one²⁴ (Hunt, 1899: 13; Brunshvicq, 1912: 10²⁵).

Quoting anthropologist Alfred C. Haddon (1890), who has observed such body-counting practices in the same area/region in 1888, Brunshvicg asserts that these sorts of counting routines allow a man, in a matter of trade, to “remember how far along his person a former number of articles extended, and by beginning again on the left little finger he could recover the actual number” (Haddon, 1890: 305; Brunshvicq, 1912: 10²⁶).

Brunshvicg then raises the question of how the “proper concept of number” would have emerged through counting practices. To him, this corresponds to the third and final stage of the elaboration of numbers. This would occur through the “intuition of sets” that would allow the transformation of the “ordinal series” into a “cardinal collection”, and thus the emergence of the “abstract idea of cardinal number”. Our author thus explains:

By going through, for each problem to be solved, the different fingers of the hands and feet, or the different parts of the body which are the terms of the reference series, we no longer need to repeat this preliminary work. The memory fixes the result, it remembers the rank of each of the terms as the result of a previous calculation, as being given in advance for a future calculation. It is then possible, by naming this term, to evoke in a single image, in a simultaneous collection, what had first had to be given by a series of successive movements; in the enunciation of the word that will mean hand will be implied the memory of all the fingers that have been touched or raised beforehand. The word is

²⁴ Actually, although neither Brunshvicq nor Lévy-Bruhl mention it, Hunt indicates that once 31 is reached the “toes are then resorted to, and these give ten more” (Hunt, 1899: 13).

²⁵ Original translation by Brunshvicq: “on commence par le petit doigt de la main gauche, puis on passe par les doigts, le poignet, le coude, l’aisselle, l’épaule, le creux au-dessus de la clavicle, le thorax, ensuite dans l’ordre inverse le long du bras droit, pour finir par le petit doigt de la main droite” (1912: 10).

²⁶ Original translation by Brunshvicq: Ainsi, « dans une question d’affaires, [...], un homme se rappellera jusqu’à quel point de sa personne un nombre d’objets était allé, et en recommençant par son petit doigt gauche, il retrouvera le nombre cherché » (1912: 10).

thus enriched with a cardinal numerical meaning that ends up, in the languages of civilized peoples, by erasing the original representation²⁷ (1910: 12)—my translation.

Once again, we see here the fundamental/central role assigned by the author to the body, and the hands in particular, in the process of “transforming the ordinal series into a cardinal collection”; a process fostered by the double role played by the fingers of the hands, being simultaneously “instruments for counting” and “objects to be counted” (1912: 16–17).

Although the methods of calculation or counting systems in use among primitives are “phenomena embedded in the human mind” (*phénomènes dont les esprits humains sont le siege*, 1912: 4), these phenomena do not result, according to Brunshvicg, from conscious reflections (of the same human mind). Primitive peoples are engaged in exchange, computing operations, etc. without having any idea of equality rules, laws of addition or multiplication, that give to their practice a “character of truth” (1912: 4). At the same time, and like Peacock before him (however without referring to this author though), Brunshvicg points out the ingenuity of the various numerical systems revealed by ethnographical sources; numerical systems based on various processes, involving combinations of additions, multiplications, duplications, or even divisions by two (*deminution*) and exponents/powers (*élévation aux puissances*) (1912: 21). Brunshvicg further underlines that this diversity of numerical systems attests

the intensity and fruitfulness of intellectual activities. Here, primitives are inventors: going forward into numerical ideas, expanding the circle of their rudimentary processes. However, sure of their conjectures, they do what inventors do, that is to say, as best as they can. They use the most diverse means, without any concern for this scholastic aesthetic, generating the elegance of simplicity and uniformity (Brunshvicg, 1912: 21)—my translation.

To Brunshvicg, it is also a “lack of methodical coordination and regularity” that characterize the first operations on numbers, as they have been carried out in primitive societies. Thus, it is the latter phenomenon that makes the primitive mentality appear as “prelogical” (1922: 22). Although Brunshvicg explicitly borrowed this term/concept from Lévy-Bruhl (1910), he reinterprets it. We have seen above that, for Lévy-Bruhl, primitive numeration/arithmetic systems are governed by the “law of participation”, and therefore do not refrain from contradiction (a given number can be perceived as identical to a different number, say 3 and 4, in certain circumstances through mystical representations). In Brunshvicg’s reinterpretation, in the context of the elaboration of numerical systems, “prelogical” refers to a lack of regularity in the “logical discourse” (1912: 22–23)

²⁷ Original text : [...] : à force de parcourir, pour chaque problème à résoudre, les différents doigts des mains et des pieds, ou les différentes parties du corps qui sont les termes de la série de référence, on n’a plus la peine de refaire ce travail préliminaire. La mémoire en fixe le résultat, elle retient le rang de chacun des termes comme exprimant le résultat d’un calcul antérieur, comme étant donné à l’avance pour le calcul futur. Il est possible alors, en nommant ce terme, d’évoquer en une seule image, en une collection simultanée, ce qu’il avait fallu se trouver d’abord par une série de mouvements successifs ; dans l’énonciation du mot qui signifiera main, se trouvera impliqué le souvenir de tous les doigts qui ont été touchés ou levés au préalable. Le mot s’enrichit ainsi d’un sens numérique cardinal qui finit, dans les langues des peuples civilisés, par effacer la représentation originaire (1910 : 12).

In fact, if we limit the consideration of prelogism to the computation practices, we see that prelogism preludes to the rules of logical discourse rather than to the rules of logical thought. Prelogism would not be prior to logic if, by logic, we understand the rational [...]. And, indeed, the field of computation is also the domain of individual practices, in which the primitive shows an intelligence similar to ours²⁸ (1912: 22-23)—my translation.

Furthermore, Brunshvicg disagrees with Lévy-Bruhl regarding the mystical aspects of numbers “that would be mystical realities rather than arithmetic units” (1910: 236–237). In the context of numeracy, according to Brunshvicg, the primitives would not have considered

these singular forms of solidarity between beings and things which contrast so strongly with causality in space and time; on the contrary, what strikes the observer is the accuracy of the processes involved, which appear so rudimentary, from the point of view of our theories. On the other hand, mystical analogies play only a secondary role here. They are developed afterwards, in civilizations that are of a relatively higher type, already similar to those of the historical period. Numerical systems can lead to mystical analogies; however, they are not made through them (1912: 22–23)—my translation.

Brunshvicg thus builds on Lévy-Bruhl’s approach of prelogical numeration as part of his project of describing/studying the emergence of the (early) concept of numbers. By doing so, as noticed by Chimisso (2008: 71), Brunshvicg interprets the Lévy-Bruhl concept of “prelogical mentality” (at least the one at work in the elaboration of numbers) as the mentality of the earliest human societies or human beginnings²⁹.

In conclusion

We have seen that the same idea of an abstraction process related to the elaboration of numbers—a process consisting in detaching/distancing numbers from what is counted—can be found in the four authors’ theses discussed in the present article. However, these authors have indeed different views regarding how this process does—or does not—occur among “primitive” peoples in particular. To Peacock, this abstraction process is primarily linked to language, through the formation of number words or numerals. These numerals are always elaborated

²⁸ Original text: M. Lévy-Bruhl a fortement insisté sur la signification du prélogique, qui n’est nullement antilogique ou alogique. De fait, quand on limite la considération du prélogique à ce qui concerne le calcul, on voit que le prélogique prélude bien plutôt aux règles du discours logique qu’aux règles de la pensée logique. Le prélogique ne serait nullement antérieur au logique si, par logique, on entendait le rationnel, comme fait Galois dans le passage que nous venons de citer. Et en effet, le domaine du calcul est aussi le domaine de la pratique individuelle où le primitif fait preuve d’une intelligence analogue à la nôtre (Brunshvicg, 1912: 22-23)

²⁹ Two decades after the publication of *Les étapes de la philosophie mathématique* though, Brunshvicg published a paper entitled “Nouvelles études sur l’âme primitive” (New studies on primitive soul) (1932)²⁹, whose a significant part of which is devoted to the introduction of Lévy-Bruhl’s work. In this paper, Brunshvicg explicitly approved Lévy-Bruhl’s position of rejecting “two sociological dogmas: (1) the assumption that the mind and its categories are the same in all times and places; and (2) the interpretation of different ways of thinking simply as stages of a universal mental evolution” (Chimisso 2008: 71, Vandendriessche upcoming-2021).

through “regular arithmetic principles” even in the less civilized societies. Conant has a different point of view. To him, this abstraction process does not occur in the case of numerical systems developed in societies that are at the lower stage of their evolution. According to him, this is due to the fact that—at the “savagery” stage—numbers are still anchored in concrete reality, in relation to the body parts in particular. Numbers are then “concrete” in the sense that they are no more than mental images of material objects. Lévy-Bruhl also defends the idea that “primitive” peoples do not have the concept of “abstract numbers”. However, to him, this is due to the fact that the “units” that compose each number have not been isolated by the primitive mind. In his view, these numbers are therefore not genuine numbers but “number-totals”, unsuitable for the development of an arithmetic. Finally, to Brunshvicg, the concept of “abstract numbers” coincides with that of “cardinal numbers”. The detour through “primitive” societies allows him to make hypotheses on the ontogenesis of numbers. Three principal evolution stages (1-enumeration, 2-ordinality, and 3-cardinality) would lead to the abstraction of numbers (and thus to our concept of number).

As seen in this article, studies devoted to numerical systems of the so-called primitive societies have contributed to larger debates on the nature of the primitive mind/mentality. Numbers developed by Indigenous peoples being studied with the goal of supporting evolutionist or prelogical/primitive mentality theories, while tracing a line between those whose mentality is governed by abstraction and logic, and the (inferior) others. As we have seen throughout this paper, these anthropological/historical approaches to “otherness” is steeped in a profound racism.

In the second half of the 20th century, an epistemological turning point occurred, particularly through the work of anthropologist Claude Lévi-Strauss (1962), regarding how anthropology addresses the issue of “logical forms” or “rationality” as expressed in various non-Western societies (Vandendriessche & Petit, 2017). And it is, indeed, in taking more account of anthropological work that some mathematicians were at the origin of the development of the interdisciplinary research field called Ethnomathematics, aiming to study the cultural variations in the uses of mathematical ideas and knowledge. As mentioned by Stephen Chrisomalis (2009: 502), “ethnomathematics provides a useful antidote to the sometimes aggressive Eurocentrism of earlier decades, and has brought contemporary anthropological insights to the study of mathematics”.

This is particularly true regarding the research on numbers as they are/were developed in Indigenous societies worldwide. Indeed, ethnomathematical studies have deconstructed both the idea of the universality of decimal numerical systems and the analysis of classifiers as a marker of concreteness in the elaboration of numbers. This already appears in the work by American mathematician Marcia Ascher (1935–2013), one of the founders of ethnomathematics. In her 1991 book, regarding the “often-repeated idea that numerals involving cycles based on ten are somehow more logical because of human fingers”, she explains that the Yupi of California “are said to believe that their cycles based on eight are most appropriate for *exactly* the same reason. The Yuki, however, are referring to the interfinger spaces”! (Ascher, 1991: 9) As for the classifier number systems, Ascher (1991: 14–15) strongly disagrees with historians who have asserted that people using classifiers for counting “do not

understand that two human fishermen and two long things bananas both contain the same concept of two [while concluding] that such people are incapable of abstraction”. To her, it is clear that numeral classifiers “combined into or coupled with number words do not interfere with the concept of number. It would be no less abstract and no less understanding of seven hundred if one had to say seven hundred human traffic fatalities rather than seven hundred traffic fatalities [...] the implications of this misunderstanding were not so mathematically crucial and pervasive as well as persistent. There is ample evidence that it is time to move ahead”.

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