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Economic analysis of the 12th man: should the fans be paid?

Luc Arrondel, Jean-Pascal Gayant and Jean-François Laslier*

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Abstract

The paper provides a theoretical sports league model to study the pricing of tickets. It considers the trade-off between maximizing match day revenues, through high prices, and maximizing the support of the most demonstrative fans, through low prices. These spectators, the “ultras” contribute to the competitive level of the club along with team-level talent, and therefore indirectly increase the number of victories, hence the revenue from TV rights. We model the role of effusive spectators and show that, in a non-cooperative Nash equilibrium, it may be optimal for the club to subsidize their presence in order to take full advantage of the 12th man effect.

key-words: Sports Economics, ticket pricing, TV rights, supporters, ultras.

1 Introduction

The aim of this paper is to study theoretically the pricing of tickets by clubs that compete in a league, taking into account the specific role of the atmosphere in the stadium. Having effusive spectators in the arena induces a home advantage and impacts the club’s sporting achievement. The next section will review what is known about this phenomenon: who are the “ultras” and what is their the role.

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In this theory paper, we are not doing an empirical estimation of the 12th man part of home field advantage but we formalize a game between the k independent clubs (Dobson and Godard 2001), using Nash equilibrium as the solution concept. We work under the assumption that each club owner desires both profit and sporting achievement, and we therefore suppose that the club maximizes a linear combination of both. This assumption is discussed and related to the literature in Section 3. The revenues are of two types: match day revenues and TV rights revenues. The only costs are the salary costs related to the remuneration of the talents. The talent in each team determines its “pure sporting level”, but the competitive level also depends on the intensity of the active spectators’ encouragement (12th man effect); this point is our contribution to the literature.

The ranking of the club within its league depends of its own competitive level compared to the competitive levels of the other clubs. A key and original feature of the analysis turns out to be that each club is facing a trade-off between maximizing match day revenues, through high prices, and maximizing the support of the most demonstrative fans, through low prices. We obtain that, in a Nash equilibrium, it may be optimal for the club to adopt a “ultra-oriented” pricing policy that makes even negative gate profit (“subsidize the fans”) in order to take full advantage of the 12th man effect and thus increase sporting achievement and the revenue from TV rights.

Our theoretical model, intended to capture the main features described in Sections 2 and 3, is presented in Section 4. The results are presented in Section 5, where we also provides some calibrations exercises in order to illustrate the previous theoretical possibilities.

2 The 12th man effect

The Covid crisis shed light on the importance of the presence of supporters for the attractiveness of sporting show. When watching matches played behind closed doors, the television viewers realised, by contrast, the decisive contribution of the most effusive spectators to the show. It became clear that team game matches are a joint production of three, rather than two, actors: the two rival teams plus the supporters. The supporters produce the “sincere soundtrack” of the emotion, which is a fundamental ingredient that generates the customer’s satisfaction, in the stadium as well as in front of the televi-

sion screen.¹ The obvious importance of the role of the spectators, and in particular of the most demonstrative ones, has resulted in the disappearance of closed-door matches during the most recent waves of the pandemic. For instance, in France, the principle of a maximum capacity (5,000 spectators) replaced the closed-door games during the 5th wave in January 2022. At the European level, the promoters of the *Super League* project for football² went even further and suggested to fully integrate the fans in the future *Major League* ecosystem: following a very strong public rejection in April 2021, the project's promoters made changes in October 2021 by (i) creating endowments to subsidize fans' travel to away games, (ii) capping the price of half of the tickets for fans, and (iii) reserving 70 per cent of the seats for the fans of the finalist teams.³

The possibility of a cap on ticket prices is a response to the discontent of a certain segment of fans, often from working-class backgrounds, who have seen ticket prices soar since the early 2000s. In the *Premier League*, in particular, the organizers have sharply increased prices in line with the growing success of the competition, which has contributed to a form of “gentrification” of the public. In a caricature, the most active and demonstrative supporters, the ultras,⁴ have been chased away by spectators with higher purchasing power but rather passive during the games. Note that these “rich and passive” spectators may easily be discouraged to come precisely by the presence of ultras and the “tyranny of the violent minority”. As a result, the atmosphere in the stadiums, now populated by wealthy, silent and demanding spectators, has deteriorated considerably. The clubs, aware of this pitfall, are therefore trying to keep a quota of relatively cheap seats, reserved for loyal season ticket holders. Thus, a form of arbitration has gradually emerged between the presence of two types of spectators in the stadiums: relatively passive spectators with a high willingness to pay on the one hand, and very demonstrative spectators with a lower willingness to pay on the

¹During some televised matches, to compensate for the absence of spectators, clubs have sometimes set up a sound system based on past recordings of background noise and spectators' reactions. The result was often mediocre, sometimes with a soundtrack that was out of sync with the real images.

²Real Madrid, FC Barcelona and Juventus Torino.

³See Wettach (2021) and the Super League web site: thesuperleague.com

⁴The term is not fully appropriate. The reality of ultras is complex (Bromberger and Lestrelin 2008) and the opposition between active and passive spectators cannot be summarized as an opposition between ultras and non-ultras. We use this denomination for convenience, while remaining fully aware of its reductive character.

other. While the former maximize match-day revenues, the latter contribute to the atmosphere in the stadium and to a possible 12th man effect that is supposed to increase the winning percentage, improve the home team's ranking and increase the share of broadcasting rights distributed at the end of the competition (Andreff 2006).

The home advantage in team sports, i.e., the phenomenon that playing in one's own stadium increases the chances of winning the game, is a well-documented phenomenon in the literature (see, for example, Schwartz and Barsky (1977), Courneya and Carron (1992), Jamieson (2010)). This advantage varies across sports but is generally stronger in football: for example, Ponzio and Scoppa (2018) estimate that, on average, the probability of winning increases by 23% for teams playing in their home arena. How then can this "home advantage" be explained?

Three factors are put forward: fan support, travel fatigue, and the habit of playing on one's home field. However, most empirical studies show that the main factor explaining this advantage is the "12th man" effect. There are two reasons for this: on the one hand, fans may encourage home players to perform better and, conversely, inhibit visiting players; on the other hand, fans may influence the referee by exerting a kind of social pressure to favor the home team ("referee bias" cf. Dohmen and Sauermann (2016)). Furthermore, long travel can tire visiting players and thus increase home advantage; for teams in the elite leagues, especially the wealthier clubs, this factor has probably diminished over time (air travel is becoming more comfortable, cf. Atkinson and Reilly (1996)). Finally, the habit of playing on one's own field, whose surface, dimensions or climatic conditions are known, may give the home team an advantage (c.f. Pollard (2002)). While empirical studies agree that the home advantage does exist, there is no consensus on the relative importance of the different factors. Moreover, the data do not always allow for differentiation between them: it is possible to measure the effect of the audience indirectly by varying attendance, travel distances or by comparing the characteristics of countries showing different degrees of advantage; for more precise analyses, it is possible to study derbies played in the same stadium or cases of temporary closures following disciplinary sanctions (due to fan violence), but these situations are rare and the sample sizes are very limited indeed. To test the home advantage hypothesis and its various assumptions, it is preferable to be able to compare the performance of teams when they play at home with and without fans, all other things being equal.

During the 2019-2020 season, the Covid-19 pandemic in Europe and else-

where forced professional football leagues to suspend championships for several months. Some leagues decided to end the season while others, such as France, went all the way through. Nevertheless, for those who resumed the competition, health regulations imposed the closed-door rule. The number of matches without spectators that could be included in a statistical analysis increased drastically. The introduction of these closed-door matches is a natural experiment to study the impact of crowd pressure on referees' decisions (referee bias), on players' behavior on the field, whether through fouls committed (yellow or red cards) or performances (number of shots and shots on target), and on the outcome of matches (number of goals or points).

Many studies have taken advantage of this closed-door environment to study the home advantage. Although these studies show a decrease in home wins when games are played behind closed doors, this does not always appear to be significant (Caiazzo et al. (2022)). On the other hand, most articles agree on the fact that a referee bias exists when the stands are full: fewer fouls are called, fewer yellow cards and fewer red cards are shown to the visiting team when the stadium doors are closed, and conversely, the home team is penalized more (c.f. Bryson et al. (2021), Endrich and Gesche (2020)). Some works also observe a decrease in the performance of home footballers (Cross and Uhrig (2023)): fewer shots on goal (both on and off target), fewer dangerous situations for the visiting team (c.f. Wunderlich et al. (2021)). In summary, the closed-door games (sometimes called "ghost games") played during the Covid pandemic, highlighted the positive role of fans on the results of their home team.

Two studies about women's football (Krumer and Smith 2022 and Szabo and Kerenyi 2023) conclude that there is no effect of the public on the performance of the home team. The explanation given by the authors is that women's football generally struggles to fill the stadiums.

While the issue of home advantage has been the subject of much recent work, the trade-off between passive spectators contributing to the maximization of match day revenues and active spectators contributing to the attractiveness of the show and the twelfth man effect has not, until now, been the subject of any modeling attempt.

For the sake of simplicity, the double question of the contribution of active spectators to the television attractiveness of the show and to the probability of victory of the host team is merged into this single 12th man effect: we consider that these two effects tend to increase the fraction of the TV rights

allocated to the team concerned.⁵.

3 League competition

The standard (neo-classical) assumption of the theory of the firm is that its objective is to maximize profit. However, it is clear that most European football clubs, especially the big ones, are not profitable (Szymanski 2015 ; Arrondel and Duhautois 2022). Economic analysis proposes possible alternatives to profit maximization: maximizing the firm's sales (W. Baumol), maximizing growth (R. Marris), satisfaction behaviour (H. A. Simons) or "utility" maximization (O. E. Williamson). All of these assumptions imply, to varying degrees, limits to the pure pursuit of profit (Sloane, 1971). For football clubs, other objectives than profitability must be considered. A football club could thus maximize a utility function' with a whole set of objectives as 'arguments' that may vary from one team to another: the number of victories for glory, stadium attendance for atmosphere, the reputation of players for ratings and popularity, competitive balance for suspense... with a constraint on the profit rate (which can be zero). All these approaches have in common the victory-profit trade-off (Fort, 2015).

Quirk and El Hodiri (1974) propose a (dynamic) model based on a profit-victory trade-off depending on the preferences of the club, constrained by the relative quality of the membership. Kesenne (1996) postulates that maximizing the club's revenue is ultimately equivalent to maximizing the utility of wins provided that the clubs reaches its break-even point.

Rascher (1997) assumes that clubs maximize a (weighted) utility function that is additively separable between wins and profits. Vrooman (1997) and Fort et al. (2009) assumes that the club maximizes a utility function that depends on the value of the membership and the performance of the team. Dietl et al. (2011) adopt the same additively separable utility function as Rascher (1997) without relatively weighting the two arguments. Finally, Madden and Robinson (2012) present the most complete specification of the club's maximization problem; they retain an additively separable utility function between profits, winning, and stadium attendance.

⁵In France, for example, the TV rights are mainly allocated according to two criteria: the team's ranking during the previous season and a measure of reputation. The double effect we are studying (ranking and attractiveness) is, schematically, in line with these criteria

To empirically settle this debate between the two logics, profit vs. wins, Garcia del Barro and Szymanski (2009) analyzed the accounts and rankings of English and Spanish clubs over the period 1994—2004. They find two very strong statistical relationships: one is between revenues and sporting performance of clubs (to increase revenues, you have to win), and another one is between performance and wage bill (to win, the team must enroll expensive players). As in any competitive environment, these relationships logically depend on the strategy of other teams: If other clubs seek to maximize their profits, the incentive to invest in players decreases, as the return on investment is smaller; and if other clubs maximize wins, all profits will go towards improving the team's ranking and therefore to signing up better players. As Garcia del Barro and Szymanski show, clubs differ with respect to the weight they seem to attach to the two objectives.

Following this literature, we use in the present paper a linear form for the club's utility function that is additively separable between wins and profit. We will not make the assumption that all clubs weight the two objectives in the same way. Diversity on this point seems to be the rule, so much that the saying goes⁶ that, in football, you have had to choose between profits and wins, glory or money, one often being incompatible with the other.

Since the profits of clubs are mainly determined by the difference between revenues and the wage bill, it is possible to identify a relationship between the expected level of profit and the position in the league table conditional on the choice of all other teams in the competition. By comparing the observed ranking of each club, the one that would allow the club to maximize its profit (for a given performance) and the one that would allow to maximize wins (for a zero profit), one can conclude: The authors show that the objective of maximizing wins best describes the behavior of the studied Spanish and English clubs: glory is preferred to money.

⁶See this comparison between the two clubs of Manchester: David Conn, "Manchester United have been owned by the Glazers for 13 years. No wonder they're struggling", *The Guardian*, 8 October 2018. In 2018, the Sheikh Mansour, in addition to the £150 million purchase of the Citizens, is said to have injected £1.3 billion into the club to achieve his ends; whereas since 2005, the Glazers have actually taken more than £1 billion from United's finances to cover interest, costs, fees and dividends. This gap in budget translated to the field, with United results collapsing compared with City's (see Arrondel and Duhautois 2022).

4 Model

4.1 The club

In a league of size $k > 1$, each club $i \in 1, \dots, k$ chooses its pricing policy and its recruitment policy. We denote by p_i the price of a ticket at club i 's stadium and by s_i the team talent level. The team talent level, which depends on the level of the recruited players, is directly determined by the remuneration of the players. The wage bill w_i of club i is therefore a function of the team talent level s_i . We will assume that the market for players is fluid and that the function $w_i = W(s_i)$ is identical for all clubs. We simply take:

$$w_i = \frac{\lambda s_i^2}{2}. \quad (1)$$

This quadratic form captures diminishing return to scale in talent with respect to wage; it should be interpreted as taking into account the well-known fact that best players are dis-proportionally expensive. See the literature on “superstar effect” (Rosen 1981, Adler 1985).

It would be quite natural to assume that the number of spectators in the stadium of team i depends on the price of the seats. To simplify the solution, we will assume that this number is fixed and equal to n_i , which is none other than the capacity of the stadium of team i . Club i 's gate revenue is thus simply $n_i p_i$ where n_i is the arena's capacity and p_i the (average) ticket price. This option is motivated by the choice to focus on the distribution of the spectators between two categories: the well-to-do and not very demonstrative spectators on the one hand, and the fans, often coming from working class backgrounds, who strongly encourage their team (the “ultras”) on the other hand. In practice, pricing policies are complex and include different tariffs, different prices for different games and different placement in the stadium for different categories of spectators, etc. At the end, the arenas are often full, which is understandable since it is generally possible to fill them with very cheap tickets. We simplify into a single dimension the complexity of the pricing policy, in order to concentrate on the feature we wish to study. Our hypothesis amounts to considering that the stadiums are always full, but that the (unique) price charged determines the breakdown between the categories of spectators. In the model, p_i for the club i is the revenue from attendance at the stadium divided by the size of the stadium, a negative p_i is possible and must be interpreted as a situation where that the charges are

not covered by the tickets sold.

The channel through which the phenomenon of encouragement passes is modeled in two steps. First we define a variable that measures the atmosphere in the home stadium, denoted a_i , which depends negatively on the price of the seats: $a_i = A_i(p_i)$. In all what follows we use the following simple form:

$$a_i = \bar{a}_i - \varepsilon_i p_i. \quad (2)$$

Here, $\varepsilon_i > 0$ is the price sensitivity of the supporters. A high value of ε_i means that, in club i the atmosphere in the arena is highly sensitive to the club's pricing policy. Then \bar{a}_i , the intercept of the linear function can be interpreted as a reference atmosphere level in the stadium, the one that would prevail if the price was nil.

Note that this linear formula does not rule out negative values for the price, a possibility that we wish to keep open for our purpose. A club may spend more money for organizing the activity of the effusive fans and the show in the stadium stands than what it gets from gross gate revenue.

As a second step, the effective competitive level of the team i , denoted c_i , depends on both the team talent level and the atmosphere in the arena. We simply take:

$$c_i = s_i + \alpha_i a_i \quad (3)$$

where $\alpha_i > 0$ measures the efficiency of the "encouragement technology" of their local team by the supporters: this can be called the 12th man effect, and it can vary from one club to the other.

The "competitive level" c_i will yield, by comparison with the competitive levels of the other teams, the achievement of team i in the league (see below the contest success function in Equation 4). Including the encouragement technology as a factor of the competitive level distinguishes the current paper from the rest of the literature.

Note there are every reason to consider that all those parameters can vary from one club to the other. Except maybe in the US, clubs are linked to different cities or regions, they have different histories, and their fans are sociologically different. Our model allows for flexibility at that level.

4.2 The league

Let us now formalize the interaction between clubs. The sporting success of a club depends on its competitive level but also on that of other clubs. We

denote this variable by $r_i = R(c_i, c_{-i})$ (as a ranking) and simply take:

$$r_i = \frac{c_i}{\sum_j c_j} = \frac{s_i + \alpha_i a_i}{\sum_j (s_j + \alpha_j a_j)}. \quad (4)$$

The sum $\sum_i r_i$ is 1 by definition so that r_i can be interpreted as a win frequency or as the relative standing of the team within the league.

The other source of revenue for the clubs is the TV rights windfall, denoted M . We will assume that the TV rights distribution rule follows the sporting success of the clubs. Therefore: $d_i = D(r_i, r_{-i})$ and, in this paper, we stick to a linear rule:⁷

$$d_i = Mr_i. \quad (5)$$

In this paper we consider that the club maximizes an objective G_i which is a linear combination of its realized profit (denoted π_i) and of the sporting achievement (r_i). Precisely, we suppose that there exists a club-specific parameter $\beta_i \geq 0$ such that:

$$G_i(p_i, s_i) = \pi_i + \beta_i r_i. \quad (6)$$

The profit of club i is:

$$\pi_i = n_i p_i + d_i - w_i \quad (7)$$

so, using the piece of notation

$$m_i = M + \beta_i, \quad (8)$$

the club's objective function can be written as:

$$G_i = n_i p_i + m_i r_i - w_i. \quad (9)$$

Following standard micro-economic theory we consider that the club maximizes its objective as a function of its two variables of action, here p_i and s_i , given the actions of the other clubs. We thereby look for a Nash equilibrium of a k -player non-cooperative game. The first order conditions for the maximization with respect to p_i and s_i , given the decisions of the other clubs are:

$$\frac{\partial G_i}{\partial p_i} = n_i + m_i \frac{\partial R}{\partial a_i} \frac{\partial A_i}{\partial p_i} = 0 \quad (10)$$

$$\frac{\partial G_i}{\partial s_i} = m_i \frac{\partial R}{\partial s_i} - \frac{\partial W}{\partial s_i} = 0 \quad (11)$$

⁷Alternative mechanisms are studied by Bergantinos and Moreno-Ternero (2020).

In our model, where the competitive level is linear with respect to the team talent level and the atmosphere, one always gets:

$$\frac{\partial R}{\partial a_i} = \alpha_i \frac{\partial R}{\partial c_i} = \alpha_i \frac{\partial R}{\partial s_i}. \quad (12)$$

The first order conditions for the club i are thus:

$$n_i = \alpha_i \varepsilon_i m_i \frac{\partial R}{\partial c_i} \quad (13)$$

$$\frac{\partial W}{\partial s_i} = m_i \frac{\partial R}{\partial c_i} \quad (14)$$

and one can remark that, for the club i , the following equation is always satisfied:

$$\varepsilon_i \alpha_i \frac{\partial W}{\partial s_i} = n_i. \quad (15)$$

With the assumption $W(s_i) = \lambda s_i^2/2$:

$$\lambda \varepsilon_i \alpha_i s_i = n_i. \quad (16)$$

Let us now solve for the entire set of teams. Recall that the ranking is proportional to the competitive level: $r_i = \frac{c_i}{\sum_j c_j}$. It follows that:

$$\frac{\partial R}{\partial c_i} = \frac{1}{\sum_j c_j} - \frac{c_i}{(\sum_j c_j)^2}. \quad (17)$$

As a consequence, writing $C = \sum_j c_j$, one gets from Equation (13):

$$n_i = \alpha_i \varepsilon_i m_i \left[\frac{1}{C} - \frac{c_i}{C^2} \right]. \quad (18)$$

It is handy to introduce the notation:

$$\omega_i = \frac{n_i}{m_i \alpha_i \varepsilon_i}; \quad \Omega = \sum_j \omega_j. \quad (19)$$

The equation (18) then can be written:

$$c_i = C - C^2 \omega_i \quad (20)$$

Summing over i the preceding equation, one obtains $C^2\Omega = (k-1)C$ and thus the remarkable relation: $C\Omega = (k-1)$, which we will write as:

$$C = \frac{k-1}{\Omega}. \quad (21)$$

Equation (20) then writes:

$$c_i = \frac{k-1}{\Omega} - \frac{(k-1)^2}{\Omega^2} \omega_i. \quad (22)$$

Now, using Equation (16) one can compute:

$$s_i = \frac{n_i}{\lambda\alpha_i\varepsilon_i} = \frac{m_i\omega_i}{\lambda} \quad (23)$$

and since $c_i = s_i + \alpha_i a_i$:

$$a_i = \frac{c_i - s_i}{\alpha_i} \quad (24)$$

which yields:

$$a_i = \frac{1}{\alpha_i} (C - (C^2 + m_i/\lambda)\omega_i) \quad (25)$$

Then, using $a_i = \bar{a}_i - \varepsilon_i p_i$, one finds the equilibrium prices from the formula: $p_i = \frac{\bar{a}_i - a_i}{\varepsilon_i}$ it comes:

$$p_i = \frac{\bar{a}_i}{\varepsilon_i} - \frac{1}{\alpha_i\varepsilon_i} \left(C - (C^2 + \frac{m_i}{\lambda})\omega_i \right) \quad (26)$$

and, after some simplifications:

$$p_i = \frac{\bar{a}_i}{\varepsilon_i} - \frac{C}{\alpha_i\varepsilon_i} + \frac{C^2\omega_i}{\alpha_i\varepsilon_i} + \frac{n_i}{\lambda\alpha_i^2\varepsilon_i^2} \quad (27)$$

from which one finally gets a well-defined value of the ticket price for team i at equilibrium, as a function of the parameters of the model:

$$p_i = \frac{\bar{a}_i}{\varepsilon_i} - \frac{(k-1)}{\alpha_i\varepsilon_i \sum_i \frac{n_i}{(M+\beta_i)\alpha_i\varepsilon_i}} + \frac{(k-1)^2 n_i}{\alpha_i^2 \varepsilon_i^2 (M+\beta_i) (\sum_i \frac{n_i}{(M+\beta_i)\alpha_i\varepsilon_i})^2} + \frac{n_i}{\lambda\alpha_i^2 \varepsilon_i^2}. \quad (28)$$

Note that having this close-form formula allows easy numerical computation of examples.

We will now check that this value, obtained from the first order condition is really a maximum. With the chosen notation, one can write:

$$R(c_i) = \frac{c_i}{C} \quad (29)$$

$$\frac{\partial R}{\partial c_i} = \frac{1}{C} - \frac{c_i}{C^2} \quad (30)$$

$$\frac{\partial^2 R}{\partial c_i^2} = \frac{-2}{C^2} + \frac{2c_i}{C^3}. \quad (31)$$

But we know that:

$$\frac{\partial \pi_i}{\partial p_i} = n_i - m_i \alpha_i \varepsilon_i \frac{\partial R}{\partial c_i} \quad (32)$$

$$\frac{\partial \pi_i}{\partial s_i} = m_i \frac{\partial R}{\partial c_i} - \lambda s_i \quad (33)$$

hence:

$$\frac{\partial^2 \pi_i}{\partial p_i^2} = m_i \alpha_i^2 \varepsilon_i^2 \frac{\partial^2 R}{\partial c_i^2} \quad (34)$$

$$\frac{\partial^2 \pi_i}{\partial p_i \partial s_i} = -m_i \alpha_i \varepsilon_i \frac{\partial^2 R}{\partial c_i^2} \quad (35)$$

$$\frac{\partial^2 \pi_i}{\partial s_i^2} = m_i \frac{\partial^2 R}{\partial c_i^2} - \lambda \quad (36)$$

Since $\frac{\partial^2 R}{\partial c_i^2} < 0$ (see the shape of the function $x \mapsto \frac{x}{x+A}$) the two second derivatives are indeed negative. It remains to check the condition on the cross derivative. Using simplified notation one finds that:

$$\left(\frac{\partial^2 \pi_i}{\partial p_i \partial s_i} \right)^2 - \frac{\partial^2 \pi_i}{\partial p_i^2} \frac{\partial^2 \pi_i}{\partial s_i^2} = m_i^2 \alpha_i^2 \varepsilon_i^2 R''^2 - m_i \alpha_i^2 \varepsilon_i^2 R'' (m_i R'' - \lambda) = \lambda m_i \alpha_i^2 \varepsilon_i^2 R'' < 0. \quad (37)$$

The value obtained for p_i at the non-cooperative equilibrium of the game is indeed the one that maximizes the objective function G_i .

Notice that this model is set to study competition between clubs that act each on their own. Collusion would obviously leave the possibility of higher aggregate profits in our model: the total manna from TV broadcast, M , is constant by assumption and there is always the possibility of cutting

down wages without changing the teams' rankings. We are not studying the case of the maximization of league profit, we are merely studying the non-cooperative Nash equilibrium.

5 Results

5.1 To pay the fans?

The optimal ticket pricing strategy is highly dependent on the size of the TV rights windfall. The most salient result is that the optimal price is likely to be zero or negative. In effect, for any team i :

$$p_i \leq 0 \iff \frac{\bar{a}_i}{\varepsilon_i} + \frac{C^2 \omega_i}{\alpha_i \varepsilon_i} + \frac{n_i}{\lambda \alpha_i^2 \varepsilon_i^2} \leq \frac{C}{\alpha_i \varepsilon_i} \quad (38)$$

which can be re-written as:

$$p_i \leq 0 \iff \alpha_i \bar{a}_i + C^2 \omega_i + \frac{n_i}{\lambda \alpha_i \varepsilon_i} \leq C. \quad (39)$$

Recall that $\lambda \varepsilon_i \alpha_i s_i = n_i$ and that $\omega_i = \frac{n_i}{(M + \beta_i) \alpha_i \varepsilon_i}$; it follows:

$$p_i \leq 0 \iff \alpha_i \bar{a}_i + s_i + \frac{n_i C^2}{(M + \beta_i) \alpha_i \varepsilon_i} \leq C. \quad (40)$$

Denote $\bar{c}_i = \alpha_i \bar{a}_i + s_i$ what can be called the competitive potential of team i , to be compared with its effective level, the previous inequality writes:

$$p_i \leq 0 \iff \bar{c}_i + \frac{n_i C^2}{(M + \beta_i) \alpha_i \varepsilon_i} \leq C \quad (41)$$

Dividing both sides by C , one gets:

$$p_i \leq 0 \iff \frac{\bar{c}_i}{C} + \frac{n_i C}{(M + \beta_i) \alpha_i \varepsilon_i} \leq 1 \quad (42)$$

The term $\frac{\bar{c}_i}{C}$ measures the relative competitive potential of team i in the league. If the sum of this competitive potential with a term that is inversely proportional to the amount of the TV manna⁸ does not exceed 1, the price

⁸The aggregate competitive level C also depends on M , but one can show that, everything else being equal, C/M decreases when M increases.

to be charged in the stadium is negative. So Equation (42) highlights the possibility of negative ticket price.

More generally, there is a complex relationship between the amount of the TV rights windfall and the price to be paid by the spectators. But whether it is to boost the attractiveness of the TV broadcast or to maximize the twelfth man effect, it may be appropriate to fill the stadium with demonstrative spectators who will create a very lively atmosphere.

The idea of free access to the stadium (or more invitations) is not new. Invitations are regularly used to fill stadiums for games where the home team is playing against a less prestigious opponent (especially if the game is being televised: it is better not to show an almost empty stadium).

The idea of negative pricing is more original. It corresponds to the intuition that a club might have an interest in paying those spectators who are the most demonstrative supporters. These demonstrative supporters or ultras create, under this modeling assumption, a part of the economic value of the television show. It is therefore economically justified to pay them like any other factor of production.

5.2 Nash equilibrium

Equilibrium simulations can be performed with different values of the parameters. In Table 1, we set up a reference equilibrium situation for which only the values of β_i differ: the teams differ only in the weight they attach to sporting success. In equilibrium, the clubs that are most focused on sporting success have the lowest ticket prices (p_i), the highest levels of atmosphere (a_i), the best sportsmen (r_i), and the highest TV rights (d_i). On the other hand, they get the lowest profits (π_i). This benchmark illustrates the spirit of this model, which uses a rich decision criterion (an objective function that combines profit and the club's sporting success) and explores the contribution of the twelfth man to satisfying this objective.

In Table 2, we vary the value of ε_i (the price sensitivity of the supporters) in addition to β_i . We find, in line with intuition, that in comparison with the benchmark situation, the clubs that are more committed to profitability are inclined to moderate their ticket prices as soon as spectators are more sensitive to price variations. Even with less emphasis on the benefits of the twelfth man effect, clubs cannot ignore the possibility that financial success is a trade-off between maximizing match day revenues and maximizing sporting success, which leads to a greater share of the TV rights windfall.

Table 1: League with 10 teams, benchmark situation. Only β varies. $M = 5$. For all i , $n_i = 1$, $\lambda_i = 1$, $\alpha_i = 1$ and $\bar{a}_i = 5$.

ε_i	s_i	w_i	β_i	m_i	r_i	d_i	a_i	p_i	π_i	G_i
1	1	0.5	14.6	19.6	0.186	0.930	1.968	3.032	3.462	6.179
1	1	0.5	14.2	19.2	0.169	0.846	1.698	3.302	3.648	6.049
1	1	0.5	13.8	18.8	0.151	0.757	1.416	3.584	3.841	5.931
1	1	0.5	13.4	18.4	0.133	0.665	1.122	3.878	4.043	5.825
1	1	0.5	13.0	18.0	0.114	0.569	0.814	4.186	4.254	5.733
1	1	0.5	12.6	17.6	0.094	0.468	0.493	4.507	4.475	5.654
1	1	0.5	12.2	17.2	0.072	0.362	0.157	4.843	4.706	5.590
1	1	0.5	11.8	16.8	0.050	0.252	-0.196	5.196	4.948	5.543
1	1	0.5	11.4	16.4	0.027	0.136	-0.565	5.565	5.201	5.512
1	1	0.5	11.0	16.0	0.003	0.015	-0.953	5.953	5.468	5.500

In Table 3, the clubs now all attach the same weight to sporting success, but they differ with respect to α_i i.e. the encouragement technology of their supporters. Unsurprisingly, the club with the most effective incentive technology is the one with the highest rank, lowest box office prices, highest G_i and lowest profit. This illustrates the role of the fans in the sporting success of the club since, *ceteris paribus*, the club that is ultimately ranked highest is the one with the best supporters.

In Table 4, we use the parameters of Table 1 (the teams differ only in the weight they attach to sporting success) but we consider a much larger TV rights windfall. While the results are qualitatively identical to those obtained in Table 1, we observe the existence of a negative box office price for the fans of the team most attached to sporting success. This illustrates equation 42 which states the condition under which it would be appropriate to “pay the fans”. As can be seen from this equation, when the TV rights windfall M becomes very high, box office receipts have a low relative importance and teams that favour sporting success will prefer to act on the lever of encouragement provided by the most demonstrative spectators, even if it means paying them.

Table 2: League with 10 teams, on top of β , ε varies too.
 $M = 5$. For all i , $n_i = 1$, $\lambda_i = 1$, $\alpha_i = 1$ and $\bar{a}_i = 5$.

ε_i	s_i	w_i	β_i	m_i	r_i	d_i	a_i	p_i	π_i	G_i
1.005	0.995	0.495	14.6	19.6	0.167	0.836	1.749	3.235	3.577	6.018
1.010	0.990	0.490	14.2	19.2	0.154	0.771	1.538	3.428	3.708	5.897
1.015	0.985	0.485	13.8	18.8	0.140	0.702	1.317	3.628	3.845	5.782
1.020	0.980	0.481	13.4	18.4	0.126	0.630	1.086	3.837	3.986	5.675
1.025	0.976	0.476	13.0	18.0	0.111	0.555	0.844	4.055	4.133	5.575
1.030	0.971	0.471	12.6	17.6	0.095	0.476	0.590	4.282	4.286	5.485
1.035	0.966	0.467	12.2	17.2	0.079	0.393	0.323	4.519	4.445	5.404
1.040	0.962	0.462	11.8	16.8	0.061	0.306	0.042	4.768	4.611	5.333
1.045	0.957	0.458	11.4	16.4	0.043	0.214	-0.254	5.028	4.784	5.273
1.050	0.952	0.454	11.0	16.0	0.024	0.118	-0.565	5.3	4.965	5.224

Table 3: Ligue with 10 teams, same β , same ε , α varies.
 $M = 5$. For all i , $n_i = 1$, $\lambda_i = 1$, $\varepsilon_i = 1$ and $\bar{a}_i = 5$.

α_i	s_i	w_i	β_i	m_i	r_i	d_i	a_i	p_i	π_i	G_i
1.05	0.952	0.454	13	18	0.139	0.696	1.251	3.749	3.992	5.803
1.04	0.962	0.462	13	18	0.131	0.655	1.125	3.875	4.068	5.771
1.03	0.971	0.471	13	18	0.123	0.613	0.993	4.007	4.148	5.742
1.02	0.980	0.481	13	18	0.114	0.570	0.856	4.144	4.233	5.714
1.01	0.999	0.490	13	18	0.105	0.526	0.714	4.286	4.322	5.689
1.00	1.000	0.500	13	18	0.096	0.481	0.566	4.434	4.416	5.667
0.90	1.010	0.510	13	18	0.087	0.436	0.411	4.589	4.514	5.647
0.98	1.020	0.521	13	18	0.078	0.389	0.250	4.750	4.618	5.630
0.97	1.031	0.531	13	18	0.068	0.341	0.082	4.918	4.728	5.615
0.96	1.042	0.543	13	18	0.059	0.293	-0.092	5.092	4.843	5.604

Table 4: League with 10 teams, M increased compared with Table 1. Only β varies. $M = 45$. For all i , $n_i = 1$, $\lambda_i = 1$, $\alpha_i = 1$ and $\bar{a}_i = 5$.

ε_i	s_i	w_i	β_i	m_i	r_i	d_i	a_i	p_i	π_i	G_i
1	1	0.5	14.6	59.6	0.128	5.203	5.739	-0.631	4.607	6.469
1	1	0.5	14.2	59.2	0.122	4.947	5.473	-0.325	4.649	6.376
1	1	0.5	13.8	58.8	0.116	4.686	5.205	-0.014	4.690	6.287
1	1	0.5	13.4	58.4	0.110	4.421	4.932	0.301	4.733	6.201
1	1	0.5	13.0	58.0	0.103	4.153	4.656	0.620	4.776	6.121
1	1	0.5	12.6	57.6	0.097	3.880	4.375	0.944	4.819	6.045
1	1	0.5	12.2	57.2	0.091	3.603	4.091	1.272	4.864	5.973
1	1	0.5	11.8	56.8	0.085	3.322	3.803	1.605	4.908	5.906
1	1	0.5	11.4	56.4	0.078	3.037	3.511	1.943	4.954	5.843
1	1	0.5	11.0	56.0	0.071	2.747	3.215	2.285	5.000	5.786

6 Conclusion

In this paper, we study the decision of a club that maximizes a mixed objective (profitability and sporting success) in terms of the trade-off between maximizing match day revenues by charging a high stadium price and maximizing the support of the most demonstrative fans to increase the number of wins and boost TV rights revenues. We show that competition between such clubs (Nash equilibrium) generates complex effects with respect to how television revenues influence the structure of equilibrium prices for stadium tickets.

Above a certain threshold of TV rights, it might be appropriate for the club to multiply the invitations or even subsidize the attendance of the most demonstrative spectators. Given that ticket sales now account for a relatively modest proportion of the budgets of most of the clubs in Europe's Big 5, decision-makers could, in the light of this article, initiate a debate on their free-seat policy.

This work could be pursued in two directions. A theoretical direction that would endogenize other sources of revenue, in particular TV rights. An empirical direction that would measure, in the real world of football leagues, the financial impact of such an hypothesis as a function of different parameters.

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