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Carmen Camacho, Hannes Tepper

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Carmen Camacho
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JEL Codes: O11, O21, O22.

Keywords: Reform, Economic policy, Structural change, General equilibrium



Do this or do that?

A model to prioritize reforms

Carmen Camacho*

Hannes Tepper†

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Abstract

This paper aims to fill the methodological gap in development economics that until now there exists no quantitative tool that allows to prioritize reforms in a systematic nor optimal way. Following the recent debate on the issues Randomized Control Trials (RCTs) have with establishing external validity and general equilibrium effects, this paper proposes a micro-founded Growth Diagnostics framework to consider general equilibrium effects and prioritize policy prescriptions. Contrarily to Hausmann et al. (2005), we set up two continuous-time Overlapping Generations (OLG) models to account for the different net-marginal valuations of various economic activities rigorously. We solve the household and planner problem to respectively obtain the private and social net-marginal valuations of economic activities via the corresponding Lagrange multipliers. With these in hand, we define the wedges in the net-marginal private and social valuations to set up a new planner problem (we call super policy maker problem), where the planner minimizes the sum of wedges. This final wrapping optimization problem allows to prioritize optimally economic reforms in a second-best framework, thus, to put it in the words of Rodrik (2010), to first diagnose before one prescribes the remedy.

Keywords: reform, economic policy, structural change, general equilibrium

JEL Codes: O11, O21, O22

*Paris School of Economics carmen.camacho@psemail.eu. The author thanks the grant ANR-17-EURE-0001 for its support

†Paris School of Economics Hannes.Tepper@etu.univ-paris1.fr

1 Introduction

Lionel Robbins once famously argued in 1932 that economics is the science of choice in an environment of scarcity (Robbins, 2007). The most important question then becomes what should we do first or how can we prioritize alternatives? This question is even more important in development economics where resources are even more scarce and choices even more difficult. Especially, once departing from a first-best world where most models only look on the direct effect of an economic policy and second-best interactions between economic factors are assumed away or no optimal solution for a second-best world exists. This is exactly what this paper's model will try to add to the literature: a model that allows to optimally prioritize reforms in a second-best framework.

Development economics as a whole discipline advanced a lot thanks to the invention of Randomized Control Trials (RCTs) on the micro-economic level. However, RCTs which can produce what Banerjee (2007) calls "hard evidence" have problems establishing external validity (Pritchett, 2021) and predicting what happens in the general equilibrium (Ravallion, 2020). Additionally, Deaton and Cartwright (2018) argue that their applicability and scope is limited and the estimates obtained have several shortcomings when it comes to precision and interpretation.

On the macro-economic level, Hausmann et al. (2005) (HRV hereafter) propose the growth diagnostics case study approach where the challenge lies in identifying the most binding constraint on economic growth but the empirical strategy has some shortcomings. As Aghion and Durlauf (2009) argue, the equilibrium prices in HRV used to carry out the diagnostics, do not necessarily reflect a constraint on growth. Furthermore, the framework does not allow policy prescriptions on the demand and supply side at the same time. Dixit (2006) claims that the sequential logic of HRV's decision tree misunderstands the nature of growth constraints since there can be several constraints acting simultaneously. Moreover, growth diagnostics carried out by different researchers may have low comparability. Neither do natural experiments allow to study the prioritization of reforms systematically since one can only study certain events in a retro-perspective, which implies that one would have to implement the policies first and then examine whether this was a good choice.

This paper will try to fill the methodological gap between experiments and growth diagnostics, and propose a methodology to systematically examine in a quantitative manner which reforms to prioritize. Therefore, contrarily to HRV, we will precisely define net marginal valuations and model them rigorously. Given the difference in valuations between households and policy makers, the first best will probably never be reached. This paper proposes a method to prioritize reforms in a second-best world, in which an aggregate of the valuation differences is minimized. Complementary to the argument of Banerjee (2007) that RCTs facilitate innovation because it is easy to see what works, we will argue development could be even more accelerated by seeing what does not work.

Debating global development strategies may make sense when one follows a epistemological point of view in the logic of Popper (1935). If the discipline would now fall back into debating global development strategies, one school of thought would be considered more convincing over time and its ideas may be found implemented in economic policy. Perhaps there would be what Rodrik (2010) called an "overreach of economists", illustrated on the example of the Washington Consensus where privatisation, liberalisation and deregulation finally went too far. Our model implements the main lesson from here: to consider the economy-specific context and take into account the differences in valuations of economic activities between households and policy makers.

How could a methodological framework for the prioritization of urgent economic reforms look like that does not suffer from external validity issues and accounts for what happens on a general equilibrium

level? Distortions drive a "wedge between net marginal private and social valuations of economic activities" according to (HRV p.3), where the bigger the wedge is, the bigger the distortion for economic growth results. However, notice that HRV do not properly define the net marginal valuations, leaving room for interpretation and adaptation to specific cases, depending on available information. One can think of economic activities as economic factors influencing the performance of an economy (f.ex. human capital, infrastructure, interest rates, etc.). In contrast to HRV, we model net marginal valuations explicitly. For that, we use shadow prices that we obtain as Lagrange multipliers from solving utility maximisation problems. In doing so, even though the context is a different one, there are certain parallels to parts of the literature on genuine savings (Cairns and Long, [2006](#); Dasgupta, [2009](#); Cairns, Del Campo, et al., [2019](#)) in the use of Lagrange multipliers to measure economic outcomes. Through solving a social planner's utility maximisation problem, we obtain the net marginal social (societal) valuations that are represented by the Lagrange multiplier. The solution of the household's utility maximisation problem yields the counterpart from the private side. To calculate the Lagrange multipliers, we detrend the system's optimality conditions so that we derive the Lagrange multiplier at steady state. That being so, we make sure to have obtained the net-marginal valuations of economic activities when they are at their optimum. Accordingly, this is a pre-step to get an optimal solution for the second-best problem of reform/binding constraints prioritization. For that, we go on to set up a model (we call country problem) to identify the most binding constraint on growth in the manner that HRV proposed to. The model can be underpinned with data and once calibrated, it would give a clear result where the most binding constraint can be expected. Most importantly, it solves the second-best problem in the case of prioritization of reforms and allows to prioritize constraints optimally in a second-best framework as we will explain.

The paper is structured as follows. Section 2 reviews the literature to discuss different ideas about reform strategies. Section 3 then specifies the household's utility maximisation problem to obtain the net marginal private valuations of economic activities. Section 4 develops the planner's utility maximisation problem to obtain the net marginal social valuations of economic activities. Section 5 shows how the previous obtained net marginal valuations can be used to obtain an optimization problem (country problem) from which one can extract the most binding constraint on growth. Section 6 concludes and points at challenges future works need to tackle.

2 Most binding constraints and reform strategies

In the early 2000s, following the era of what is called today the Washington Consensus, a stream of literature emerged, convinced by the fact that different countries may have different impediments to growth and thus a list of reforms applicable to all cases was less useful.

HRV developed the framework of "Growth Diagnostics" originally. Assuming that human development in developing countries has as main challenge how to increase the economic growth rate, their approach is designed to identify the most binding constraint on economic growth. How can one identify those? One first needs to go a step back and understand how distortions in economic growth can arise. HRV model them as wedges between net marginal valuations described in equation [\(1\)](#)

$$\mu_j^s(d_1, \dots, d_M) - \mu_j^p(d_1, \dots, d_M) - d_j = 0 \quad (1)$$

where d_j is the wedge in activity j , $\mu_j^s(d_1, \dots, d_M)$ are the net marginal valuations of the respective activity j by society, $\mu_j^p(d_1, \dots, d_M)$ are the net marginal valuations of the respective activity j by private agents and there is a set of wedges $d = (d_1, \dots, d_M)$. For example, one may think of it like a tax on investment that keeps private returns on capital accumulation below social returns, leading to the case that the

economy under-invests. One can therefore state that the bigger the wedge is, the bigger is the distortion for economic growth. Attention is required because the biggest wedge does not necessarily need to be the biggest constraint on economic growth. A tariff on imports may lead to a bigger wedge between social and private net marginal valuations than too few schooling, but depending on the country context, the constraint resulting from too less schooling may be more binding on growth. For that, the most binding constraint can be identified through finding the biggest Lagrange multiplier for the corresponding wedge as is shown by equation (2):

$$\frac{\partial u}{\partial d_j} = -\lambda_j + \sum_i^I \lambda_i \frac{[\mu_i^s(d, \dots) - \mu_i^p(d, \dots)]}{\partial d_j} \quad (2)$$

where λ_j is the Lagrange multiplier corresponding to the constraints associated with each of the wedges. Moreover, λ_j captures the marginal welfare benefit from reducing the distortion in market j , not considering the effect on other distorted activities. But, there may be second best complications too and this is what the summation term accounts for. That is, in other words, the interaction effects across distorted margins. If there would be only one distortion in an economy, this term would become equal to zero. Following equation (2), one needs to focus on the biggest λ_j to identify the most binding constraint on economic growth. HRV justify to only look at λ_j with the reason that in practice it may be difficult for policy makers to figure out all the second-best interactions. This may be right but focusing only on λ_j will not represent the solution to the second-best problem of how to optimally rank reforms and binding constraints. This is, because by construction, one looks only at the direct effect but not at the indirect effects: the second-best interactions. Therefore, our model will be able to quantify the interaction effects across margins. Hence, we can quantify the whole first-order derivative of (2) and solve the second-best problem in an optimal way.

2.1 Reform strategies

How does this now relate to different reform strategies? HRV give an answer, distinguishing five different approaches to reform:

1. Wholesale reform,
2. Do as much reform as you can, as best as you can,
3. Second-best reform,
4. Target the biggest distortions,
5. Focus on the most binding constraints.

Following HRV, "Wholesale reform" is a strategy that consists in eliminating all the distortions at once. Unfortunately, this strategy does not seem feasible given that it requires the knowledge of all distortions. Additionally, one would need to have the ability to remove all the distortions simultaneously. However, they dismiss this strategy due to its limited applicability but nevertheless note that this would theoretically be the best strategy. Recalling equation (2), all the Lagrange multipliers would go to zero and one has a guarantee that welfare will be improved.

"Do as much reform as you can, as best as you can" is a strategy which HRV attribute to the logic of the Washington Consensus. They call it a "laundry list approach". Their main argument against such a strategy is that in the presence of second-best interactions, one cannot be assured that more reform will lead to better results. For example, imagine a developing country that as first reform, invests into

its health system via constructing hospitals. As second reform, it starts training for surgeons. If now as a consequence massive emigration of now better qualified surgeons towards richer countries takes place, the effects of the investment in training strategy do not need to be welfare enhancing if the emigration effect outweighs the global health effect. Labonté et al. (2015) give an example of how the emigration of skilled health workers can be a serious problem in developing countries.

"Second-best reform" recognises the presence of second-best interactions that can arise. Nevertheless, HRV point towards the problem that most of these second-best interactions cannot be quantified ex-ante and are difficult to figure out, and static computable general equilibrium models are not performing well enough in this domain.

"Targeting the biggest distortion" can be understood as recalling equation (1) and focusing on the biggest d because distortions arise from the corresponding wedges. As illustrated by the schooling and import tariff example earlier, the biggest distortion on growth may not necessarily give the biggest push for economic growth. Again, HRV note that this approach would require a complete list of all distortions which may not be feasible in practice.

Therefore, HRV argue that one should identify reform priorities according to the size of the direct effect λ_j , thus focusing on the most binding constraint. This approach remains useful in practice because in case that one cannot figure out all the second-best complications and has neither a list of all distortions, this approach still delivers reasonable recommendations to policy makers. To illustrate their reasoning, HRV use a decision tree.

2.2 Diagnostic challenges

According to Dixit (2006), development economics suffers from two main problems when it comes to advice for policy makers. First, there is no consensus on which are the best measures what he calls the "cross-sectional problem". Second, the consensus shifts over time what depicts the "time series problem". This analysis follows the argument of HRV that different countries may face different challenges and thus a laundry-list approach is not useful. Inspired by the framework of HRV, Dixit (2006) rejects their decision-tree approach and develops a scheme in the form of a table. This allows for multiple causes to be identified at the same time. Each field represents a combination of a cause and an effect, together called "syndrome". Using Bayesian posterior probability, where prior probabilities are identified through econometric research to narrow down the possible causes of a syndrome, results in the identification of a problem. Finally, this allows to formulate growth policies and, in contrast to HRV, to allow several causes that act simultaneously. That is why for Dixit (2006) the sequential mode of the decision tree poses a problem.

Even if Dixit (2006) points towards a shortcoming of the work of HRV, the model this paper will develop will be based on the work of HRV. To apply Dixit (2006)'s table, one would need to estimate the prior probabilities. Dixit (2006) proposes to use historical analysis for this. He argues that his methodological approach follows principally medical diagnostic procedures. However, we will argue that one cannot deduce from historical constraints on growth on nowadays constraints on growth. As Dixit (2006) himself states development economics suffers from a cross-sectional and time-series problem. Therefore, constraints on growth from the past are linked to the consensus from the past. The "time-series problem" would not be solved. Consequently, the prior probabilities will represent the probabilities of constraints occurred in the past which can be quite different from nowadays challenges. One may think that, for example, the Covid-19 pandemic has imposed health constraints on growth as it has never happened

since World War II. If one goes even further back in time, data limitations may restrict the depth of the analysis quite heavily. Nevertheless, one needs to keep in mind the shortcoming of HRV's work Dixit (2006) points at, that one cannot identify several constraints acting simultaneously. This paper deals with these issues since it quantifies the wedges. In practice, this means that if the policy maker and the household evaluate an economic variable very differently at one moment in time for a given, we will obtain a high value for the Lagrange multiplier associated to that specific wedge.

Aghion and Durlauf (2009) points towards two major criticisms of the work of HRV. First, equilibrium prices do not need to reflect growth constraints as they illustrate with an example on a high interest rate. While HRV would consider the credit market as constrained whereas, Aghion and Bolton (1997) demonstrated that high interest rates may reflect a high degree of credit rationing. To solve this issue, this paper models the different economic activities and one gets to quantify the wedges between private and social valuations explicitly. Second, Aghion and Durlauf (2009) note that the approach à la HRV cannot lead to growth prescriptions that tackle demand and supply side problems at the same moment in time. Although this paper will not be able to eliminate this criticism completely, it will definitely smooth it. Indeed, in a first step our model will point at the biggest distortion on economic growth. In a second step, the constraints will also be quantified. If for example, the most binding constraint regards human capital, one needs to dig deeper to formulate the policy prescriptions and look on the supply and demand side.

3 A micro-founded growth diagnostics framework

The micro-founded growth diagnostics framework will consist of an overlapping generations model with economic factors. There is a continuous time stream from $t=0$ until infinity. The economy is populated by a continuum of identical households normalized to one and for simplicity no population growth exists. Following Blanchard (1985) households have an expected remaining life time that depends on the constant instantaneous probability of death ρ :

$$\frac{1}{\rho} = \int_0^x t \rho e^{-\rho t} dt. \quad (3)$$

Equation (3) is called the horizon index. If $\rho \rightarrow 0$, then the horizon of an agent becomes infinite. In this case, the problem will turn into a standard Ramsey benevolent planner problem. We will use this feature of the model together with an assumption on externalities to distinguish the planner's problem from the household's problem later on. The population size can then be calculated the following way and is normalized equal to 1

$$1 = \int_t^{-\infty} \rho e^{-\rho(t-s)} ds, \quad (4)$$

where s represents the point in time when an agent was born.

3.1 The household's problem

For one of the identical households, preferences can be represented by the following utility function:

$$\int_t^{\infty} u[c(s)] e^{(\theta+\rho)(s-t)} ds, \quad \theta \geq 0, \quad (5)$$

where $c(x)$ is consumption of the household at time s , $\theta > 0$ is the pure time discount, and $\theta + \rho$ stands for the effective discount rate. u stands for the household instant utility function. u is a positive, real

function of c , increasing and concave. Besides, $\lim_{c \rightarrow 0} u'(c) = 0$.

As in Blanchard (1985), the households will engage to sell part of their economic factors to the life insurance companies because there is no bequest motive and negative bequests are prohibited. These insurance companies face free entry, a zero-profit condition and insurance contracts are offered risklessly. Consequently, in transferring Blanchard, (1985)'s scheme to this paper's setting, we can state that households receive ρx if they do not die and pay x if they die. To put it simply, we make sure that households know where to put their savings (to the life insurance companies) and economic factors are not vanished in the next generation (since the life insurance company has them). It is important to keep in mind that ρ is not factor specific since it is only connected to the probability to die. ρ reflects the probability of death and the price of the life insurance at the same moment which should not be mistaken as a normal insurance. Note that we implicitly assume that households insure all their accumulated economic factors. Blanchard (1985) discusses alternative assumptions like actuarial bonds that guarantee the same theoretical outcome but this paper will stay with the life insurance assumption.

Households maximize their overall utility subject to two types of constraints. We will first describe the dynamics of the economic and non human production inputs, namely $\{x_1, \dots, x_n\}$. Let us denote by I_j the investment in factor j , and let us assume that the dynamics of each factor j evolves with the quantities invested in each specific factor as

$$\dot{x}_j(t) = A_j [I_1(t), \dots, I_M(t), x_j(t)] - \delta_j x_j(t), \quad j = 1, \dots, N, \quad (6)$$

where $\delta_j \geq 0$ is the broad depreciation rate of factor j . A_j is the intermediate good production function which measures the combined effect on factor j 's accumulation of the investment in each other factor and the existing stock of factor j . We assume that there is a strictly positive quantity of each economic factor at each point in time:

$$c(t) > 0, \quad x_j(t) > 0, \quad I_j(t) \geq 0 \quad \forall j = 1, \dots, M. \quad (7)$$

These constraints will ensure that the essentiality requirement of neoclassical growth functions will be met in the production of the final good later on.

Finally, the next constraint ensures that the household respects a balanced budget:

$$c(t) + \sum_{j=1}^M I_j(t) = F[x_1(t), \dots, x_M(t), L], \quad (8)$$

where $F[x_1(t), \dots, x_M(t), L]$ is final good production and L is labor.¹ We assume that $F[x_1(t), \dots, x_M(t), L] : \prod_{j=1}^n \mathbb{R}^+ \times \mathbb{R}^+ \rightarrow \mathbb{R}^+$ has constant positive diminishing returns to scale with respect to each input, and that the Inada conditions on marginal productivities are fulfilled. Note that depending on the nature of the factor, L could also be interpreted as total population, as in the case of demographic or environmental variables.

3.2 The firm's problem

There is a continuum of identical firms normalized to one. At every time $t \geq 0$, the economy produces a single final good via the previously defined function F . The firm rents production inputs from the

¹Note that for simplicity reasons we are assuming that the accumulation of each factor j does not require a specific labor allocation.

households. We will distinguish two main types of production inputs: labour L and all the other economic factors x_1, \dots, x_j . For the sake of simplicity, we assume that productivity depends linearly on the production inputs the firm rented. Following Dixit and Stiglitz (1977), we assume that production is

$$F[x_1(t), \dots, x_M(t), L] = \left[\sum_{j=1}^n x_j^{\frac{1}{\epsilon}}(t) \right]^{\epsilon} L^{\nu}, \quad (9)$$

where $\frac{1}{1-\epsilon}$ is the elasticity of substitution between the different economic factors. One can then write the firm's profit maximisation problem:

$$\Pi(t) = F[x_1(t), \dots, x_M(t), L] - \sum_{j=1}^n r_j(t)x_j(t) - \omega(t)L, \quad (10)$$

where $\sum_{j=1}^n r_j(t)x_j(t)$ is the firm's cost for the economic factors at time t and $\omega(t)L$ is the labour cost.

3.3 Optimal behaviour of the household

We can write the following present-value Hamiltonian to solve the household's problem of maximizing (5) subject to (6), (7) and (8):

$$\begin{aligned} H(c, I, x, \omega, \xi_1, \dots, \xi_M, \phi, t) = & u[c(t)] \\ & + \sum_{j=1}^M \xi_j(t) (A_j[I_1(t) \dots I_M(t), x_j(t)] - \delta_j x_j(t)) \\ & + \Phi(t) \left[c(t) + \sum_{j=1}^M I_j(t) - F[x_1(t), \dots, x_M(t), L] \right] \\ & + \sum_{j=1}^M \sigma_j(t)x_j(t) + \sum_{j=1}^M \nu_j(t)I_j(t) + \chi(t)c(t). \end{aligned} \quad (11)$$

Here, for simplicity reasons, we have defined a vector $x(t)$ of functions as $x(t) = [x_1(t), \dots, x_M(t)]$. For every $j = 1, \dots, M$, ξ_j is the co-state variable associated to x_j , σ_j ensures that the stock remain positive, and ν_j that investment is non negative. Finally, multiplier ϕ ensures that the budget is balanced, and χ that consumption is always positive.

As shown in Appendix (A.1), at every $t \geq 0$ the set of necessary optimal conditions associated to the household problem is

$$\left\{ \begin{array}{l} \dot{x}_j(t) = A_j[I_1(t) \dots I_M(t), x_j(t)] - \delta_j x_j(t), \\ \dot{\xi}_j(t) = \xi_j(t) \left[\frac{\partial A_j(t)}{\partial x_j(t)} - \delta_j \right] - \Phi \frac{\partial F[x_1(t), \dots, x_M(t), L]}{\partial x_j(t)}, \\ \Phi(t) = \sum_{k=1}^M \xi_k(t) \frac{\partial A_k[I_1(t) \dots I_M(t), x_k(t)]}{\partial x_j(t)}, \\ x_j(0), \text{ known}, \\ \lim_{t \rightarrow \infty} \sigma_j(t)x_j(t) = 0, \\ \sigma_j(t)x_j(t) = 0, \text{ and } \nu_j(t)I_j(t) = 0, \end{array} \right. \quad (12)$$

for every $j = 1, \dots, M$ and $t \geq 0$. Besides, at every time $t \geq 0$ the optimal solution must also verify that

$$\begin{cases} \Phi(t) = u'[c(t)], \\ \chi(t)c(t) = 0, \\ \phi(t) \left(c(t) + \sum_{j=1}^M I_j - F[x_1(t), \dots, x_M(t), L] \right) = 0, \end{cases} \quad (13)$$

where $\Phi(t) = \phi(t)e^{(\theta+\rho)t}$.

3.4 Equilibrium

At equilibrium, the final good, each of the intermediate goods, the financial and the labour markets need to clear. As the prize of our one single final good economy and the population are normalized to one, this is a rather simple exercise for the final good and labour market. The final good market clears under the following condition:

$$c(t) = F[x_1(t), \dots, x_M(t), L] \quad (14)$$

This equation only states that the demand for the final good resulting from the households consumption decisions in time period t , equals the amount of the final good that was produced during that period for the price $p = 1$. The continuum of firms produced the final good with the production inputs of different economic factors and labour.

Speaking of the latter, the labour market clears when the following equation is met:

$$L = 1. \quad (15)$$

As the economy is populated by a continuum of households normalized to 1 and the probability of death ρ is constant, this statement implies that all of the households are working. Thus the population is equal to the labour force.

The economic factors' market clears when each of the economic factors x_j used by the firm to produce equals all the economic factors x_j possessed by the households and the life insurance companies. The life insurance companies and the households rent the economic factors to the firms. At equilibrium, the firms rent the same amount of economic factors as the households' amount of factors that provide them with utility. Hence at equilibrium

$$\sum x_{j(firm)} = \sum x_{j(households)}, \quad \forall j = 1, \dots, M. \quad (16)$$

On the financial market and at a competitive equilibrium, the return to an economic factor x_j for the household at a given moment in time t , $r_j^{households}(t)$, is equal to the firm's cost of that economic factor, $r_j^{firm}(t)$. This is achieved once the firm has balanced its activities in such a way that the return across economic factors is equal and, thus, the firm is indifferent among the factors, that is, $r_j^{firm}(t) = r_i^{firm}(t) = r^{firm}(t)$ for all $i, j = 1, \dots, M$. Otherwise the firm would start to re-balance its activities, trying to make higher profit through changing the mix of production inputs used. The return expected by the firm is the same as the one expected by the households. Households receive interests themselves from the firm. Furthermore, recall that the life insurance companies only transfer the wealth from the deceased individuals to the individuals alive, and do not interfere in the transactions that take place between the households and the firm. We can write the following equation to capture the equilibrium condition of the financial market:

$$r_j(t)^{households} = r_j(t)^{firm}. \quad (17)$$

This model will, for simplicity, not consider a monetary market. Here, instead of money the price of the single final good serves as the numeraire, changes in prices reflect itself in changes in relative prices. That is, the prices of the economic factors (the respective interest rates) change with respect to the price of the final good.

3.5 Steady state

It is noteworthy to recall why we extract the detrended Lagrange multipliers $\bar{\xi}$ at the steady state. At steady state, we obtain the optimal Lagrange multipliers and thus our net-marginal valuations are at their optimum. Following, the information that we obtained here and feed into the country problem in Section 5 will be optimal. For that reason, our country problem in Section 5 can prioritize reforms optimally in a second-best world, that is to solve the second-best problem optimally.

In order to illustrate our model we will use in the following an example of two factors to obtain the steady state values of $\bar{\xi}_j$.² Suppose $M = j, i$ and the following functional forms:

$$\begin{cases} F(x_j, x_i, L) = x_j^{\alpha_j} x_i^{\alpha_i} L^{1-\alpha_j-\alpha_i}, \\ A_j(I_j, I_i, x_j) = I_j^{\beta_j} I_i^{\beta_i} x_j^{\gamma_j}, \\ A_i(I_j, I_i, x_i) = I_j^{\beta_j} I_i^{\beta_i} x_i^{\gamma_i}, \end{cases} \quad (18)$$

where $0 < \alpha_j, \alpha_i < 1$ and $\alpha_j + \alpha_i \leq 1$; $0 < \beta_j, \beta_i, \gamma_j, \gamma_i < 1$, with $\beta_j + \beta_i + \gamma_j \leq 1$ and $\beta_j + \beta_i + \gamma_i \leq 1$. Hence, in this setup, the specific investment I_i on factor x_i has the same effect on the targeted factor itself as in the other, competing factor, x_j if all else is equal. However, the overall effect will be different given that the pure investment function is multiplied by the stock itself. Hence, even if the "investment factor", $I_j^{\beta_j} I_i^{\beta_i}$, is identical for both x_j and x_i , its effect will be larger at time t for factor 1 if $x_j(t)^{\gamma_j} > x_i^{\gamma_i}$.

With these functional forms, the optimal dynamics of the co-state variables is described by

$$\begin{cases} \dot{\xi}_j(t) = \xi_j \left[\delta_j + \theta + \rho - \gamma_j x_j^{\gamma_j-1} I_j^{\beta_{j1}} I_i^{\beta_{j2}} \right] - \Phi \alpha_j x_j^{\alpha_j-1} x_i^{\alpha_i} L^{1-\alpha_j-\alpha_i}, \\ \dot{\xi}_i(t) = \xi_i \left[\delta_i + \theta + \rho - \gamma_i x_i^{\gamma_i-1} I_j^{\beta_{i1}} I_i^{\beta_{i2}} \right] - \Phi \alpha_i x_j^{\alpha_j} x_i^{\alpha_i-1} L^{1-\alpha_j-\alpha_i}, \\ \lim_{t \rightarrow \infty} x_j(t) \xi_j(t) = 0, \\ \lim_{t \rightarrow \infty} x_i(t) \xi_i(t) = 0. \end{cases} \quad (19)$$

The first two expressions show that ξ will contain information from all the variables in the system. This is consistent with the fact that the ξ will be used in the country problem to solve the second best problem optimally. Therefore, it is important that the ξ contain information from all the variables that exist in the environment of the agents. The wedge between ξ of the household with the ξ of the planner yields the degree to which the behaviour of the household is "non-optimal". One may call this non-optimality wedge whereas it rests conceptually close to what HRV call distortions. Minimizing the sum of non-optimality wedges will be at the heart of the country problem in section 5.

We prove in the Appendix that the household problem admits a unique non trivial steady state, which

²Our results can easily be extended to a more general case, with more than two factors and more general functional forms.

is described by the following set of four equations

$$\begin{aligned}\frac{\partial F(\bar{x}_j, \bar{x}_i)}{\partial \bar{x}_j} \left(\frac{\partial A_j}{\partial \bar{I}_j} + \frac{\partial A_i}{\partial \bar{I}_j} \frac{\frac{\partial A_j}{\partial \bar{I}_j} - \frac{\partial A_j}{\partial \bar{I}_i}}{\frac{\partial A_i}{\partial \bar{I}_i} - \frac{\partial A_i}{\partial \bar{I}_j}} \right) &= \delta_j + \theta + \rho - \frac{\partial A_j}{\partial \bar{x}_j}, \\ \frac{\partial F(\bar{x}_j, \bar{x}_i)}{\partial \bar{x}_i} \left(\frac{\partial A_i}{\partial \bar{I}_i} + \frac{\partial A_j}{\partial \bar{I}_i} \frac{\frac{\partial A_i}{\partial \bar{I}_i} - \frac{\partial A_i}{\partial \bar{I}_j}}{\frac{\partial A_j}{\partial \bar{I}_j} - \frac{\partial A_j}{\partial \bar{I}_i}} \right) &= \delta_i + \theta + \rho - \frac{\partial A_i}{\partial \bar{x}_i}\end{aligned}$$

We also prove that

$$\bar{\xi}_j \left[\frac{\partial A_j}{\partial \bar{I}_j} - \frac{\partial A_j}{\partial \bar{I}_i} \right] = \bar{\xi}_i \left[\frac{\partial A_i}{\partial \bar{I}_i} - \frac{\partial A_i}{\partial \bar{I}_j} \right],$$

so we can express $\bar{\xi}_i$ as a function of $\bar{\xi}_j$:

$$\bar{\xi}_i = \bar{\xi}_j \frac{\frac{\partial A_j}{\partial \bar{I}_j} - \frac{\partial A_j}{\partial \bar{I}_i}}{\frac{\partial A_i}{\partial \bar{I}_i} - \frac{\partial A_i}{\partial \bar{I}_j}}. \quad (20)$$

More importantly, this expression shows that our model contains the same specific feature as the model of HRV: net-marginal valuations of one specific factor are influenced by net-marginal valuations of other factors. For HRV lies here the essence of the second-best problem. As HRV argue, this allows to capture second-best interactions and we will find this feature in the same way in the social planner's problem next section.

As we show in Appendix [A.2](#), there exists a unique Steady State given by:

$$\left\{ \begin{aligned} \bar{\xi}_j &= \Phi \left[\frac{\partial \bar{A}_j}{\partial \bar{I}_j} + \frac{\partial \bar{A}_i}{\partial \bar{I}_j} \frac{\frac{\partial \bar{A}_j}{\partial \bar{I}_j} - \frac{\partial \bar{A}_j}{\partial \bar{I}_i}}{\frac{\partial \bar{A}_i}{\partial \bar{I}_i} - \frac{\partial \bar{A}_i}{\partial \bar{I}_j}} \right]^{-1}, \\ \bar{\xi}_i &= \Phi \left[\frac{\partial \bar{A}_i}{\partial \bar{I}_i} + \frac{\partial \bar{A}_j}{\partial \bar{I}_i} \frac{\frac{\partial \bar{A}_i}{\partial \bar{I}_i} - \frac{\partial \bar{A}_i}{\partial \bar{I}_j}}{\frac{\partial \bar{A}_j}{\partial \bar{I}_j} - \frac{\partial \bar{A}_j}{\partial \bar{I}_i}} \right]^{-1}, \\ \bar{x}_j &= \left(\frac{\bar{I}_j^{\beta_j} \bar{I}_i^{\beta_i}}{\delta_j} \right)^{\frac{1}{1-\gamma_j}}, \\ \bar{I}_j &= \left(\frac{\delta_j \bar{x}_j^{1-\gamma_j}}{\bar{I}_i^{\beta_i}} \right)^{\frac{1}{\beta_j}}, \end{aligned} \right. \quad (21)$$

where $\bar{A}_j = A_j(\bar{I}_j, \bar{I}_i, \bar{x}_j)$ and $\bar{A}_i = A_i(\bar{I}_j, \bar{I}_i, \bar{x}_i)$. We proof that the Mangasarian sufficient conditions hold in Appendix [\(A.1\)](#), ensuring that we find the optimal maximal solution for our problem. Having derived the private net-marginal valuations at the steady state of a detrended version of our system, we explore in the next section the planner problem from which we similarly derive the social net-marginal valuations of economic activities.

4 The social planner problem

The social planner maximizes overall welfare over an infinite time horizon and we assume $\rho = 0$, that is, the policy maker cannot die. As we will see later, this is one of the two assumptions that drive the wedge between the private and social net-marginal valuations.

$$\max_{\{c, \{I_j\}_{j \in J}\}} \int_s^\infty u[c(t)] e^{\theta(t-s)} dt, \quad (22)$$

subject to:

$$\dot{x}_j(t) = B_j[I_1(t) \dots I_M(t), x_j(t)] - \delta_j x_j(t), \quad j = 1, \dots, M, \quad (23)$$

$$c(t) + \sum_{j=1}^M I_j(t) = G[x_1(t), \dots, x_M(t), L]. \quad (24)$$

Note that the social planner values consumption as the household, and uses the same function to discount time. Besides, we assume that both the policy maker and the household observe the same depreciation rate for the production factors x_1, \dots, x_M . Hence, the only two differences between the household and the social planner problem are that households have finite lives, and they may not observe the same accumulation laws for the productive factors x_j since functions F and G are, in principle, different functions. Let us see how.

$G[x_1(t), \dots, x_M(t), L]$ fulfills the same conditions as its counterpart F in the household's problem, and is therefore a neoclassical production function. As we will further discuss in the next section, this will reflect the fact that we assume that the planner takes into account externalities and complementarities between all the production factors, which the household does/can not. We can write the following present value Hamiltonian to solve the social planner's utility maximisation problem:

$$\begin{aligned} H(c, I, x, \omega, \xi_1, \dots, \xi_M, \phi) = & u[c(t)] \\ & + \sum_{j=1}^M \xi_j(t) (B_j[I_1(t) \dots I_M(t), x_j(t)] - \delta_j x_j(t)) \\ & + \phi(t) \left(c(t) + \sum_{j=1}^M I_j(t) - G[x_1(t), \dots, x_M(t), L] \right) \\ & + \sum_{j=1}^n \sigma_j(t) x_j(t) + \sum_{j=1}^n \nu_j(t) I_j(t) + \chi(t) c(t). \end{aligned} \quad (25)$$

As it is usual, ξ_j is the co-state variable associated to x_j , σ_j ensures that the stock remain positive, and ν_j that investment is non negative for $j = 1, \dots, M$. Finally, as in the household problem, the multiplier ϕ ensures that the budget is balanced, and χ that consumption is always positive.

We show in Appendix [A.3](#) that the following set describes the necessary optimal conditions for the policy maker problem. There, we also prove that the Mangasarian second order sufficient conditions also hold. Hence, for every $j = 1, \dots, M$ the optimal solution for the central planner problem verifies that

$$\left\{ \begin{array}{l} \dot{x}_j(t) = B_j[I_1(t) \dots I_M(t), x_j(t)] - \delta_j x_j(t), \\ \dot{\xi}_j(t) = \xi_j(t) \left[\frac{\partial B_k(t)}{\partial x_j(t)} - \delta_j \right] - \Phi \frac{\partial G}{\partial x_j(t)}, \\ \Phi(t) = \sum_{k=1}^M \xi_k(t) \frac{\partial B_k(t)}{\partial x_j(t)}, \\ I_j(t) > 0, x_j(t) > 0, \\ \lim_{t \rightarrow \infty} \sigma_j(t) x_j(t) = 0, \\ \sigma_j(t) x_j(t) = 0, \quad \nu_j(t) I_j(t) = 0, \quad \forall j \\ \text{and } \chi(t) c(t) = 0, \end{array} \right. \quad (26)$$

for every $j = 1, \dots, M$ and for every $t \geq 0$, plus

$$\begin{cases} \Phi(t) = u'[c(t)], \\ \chi(t)c(t) = 0, \\ \phi(t) \left[c(t) + \sum_{j=1}^M I_j - G(x_1(t), \dots, x_M(t), L) \right] = 0, \end{cases} \quad (27)$$

for every $t \geq 0$.

As earlier in the household's problem, this detrended system allows to describe the optimal dynamics of the optimal social net-marginal valuations of activities, measured by ξ_j .

The firm problem is conceptually the same in this planner's problem as in the household's problem. The only difference consists in the fact that the firm in the planner's problem is using the final good production technology G instead of F as in the household's problem.

Hence, the planner's problem's equilibrium is the same as in the household's problem except that the good's market equilibrium is the following for every $t \geq 0$:

$$c(s, t) = G_t[x_1(t), \dots, x_M(t), L]. \quad (28)$$

The dynamic system described in (26) allows for a unique non-trivial steady state. Again, for the sake of simplicity, like for the household problem, we illustrate our results assuming that $M = 2$ and adopting the same functional forms. The steady state of the policy maker is given by

$$\begin{cases} \bar{\xi}_j = \Phi \left[\frac{\partial \bar{B}_j}{\partial I_j} + \frac{\partial \bar{B}_i}{\partial I_j} \frac{\frac{\partial \bar{B}_j}{\partial I_j} - \frac{\partial \bar{B}_j}{\partial I_i}}{\frac{\partial \bar{B}_i}{\partial I_i} - \frac{\partial \bar{B}_i}{\partial I_j}} \right]^{-1}, \\ \bar{\xi}_i = \Phi \left[\frac{\partial \bar{B}_i}{\partial I_i} + \frac{\partial \bar{B}_j}{\partial I_i} \frac{\frac{\partial \bar{B}_i}{\partial I_i} - \frac{\partial \bar{B}_i}{\partial I_j}}{\frac{\partial \bar{B}_j}{\partial I_j} - \frac{\partial \bar{B}_j}{\partial I_i}} \right]^{-1}, \\ \bar{x}_j = \left(\frac{\bar{I}_j^{\beta_j} \bar{I}_i^{\beta_i}}{\delta_j} \right)^{\frac{1}{1-\gamma_j}}, \\ \bar{I}_j = \left(\frac{\delta_j \bar{x}_j^{1-\gamma_j}}{\bar{I}_i^{\beta_i}} \right)^{\frac{1}{\beta_j}}, \end{cases} \quad (29)$$

where $\bar{B}_j = B_j[\bar{I}_1, \dots, \bar{I}_M, \bar{x}_j]$, for $j = 1, 2$.

5 Minimizing disagreement

The optimal behaviours of the household and the social planner differ. The shadow price of each variable, $\xi_j(t)$, is different for the household and the policy maker, which will obviously and surely trigger the arising wedges in the net marginal valuations of the economic activities at stake. There are two main theoretical reasons why differences in the optimal behaviours exist.

1. The household and the social planner have different life spans. Where the household is just interested in maximising utility over her life span, the social planner considers all future generations when thinking about how to maximise utility.
2. Since there may be complementarities and externalities in the final good production process, the

social planner may be able to consider the economic factors more adequately than the household. The household will, due to imperfect information, not be able to account for all the complementarities and externalities that arise. Two major examples to show that this may be a reasonable assumption are the areas of health and environment. Individual agents do often not behave optimally if one would take into account all the externalities that are caused. Concrete examples are the consumption of sugary foods or smoking for health. From an environmental point of view, households do often not know the whole impact their CO₂ emissions have.

All together, these assumptions show possible reasons why a wedge between the net marginal private and social valuations of an economic factor j can arise. In this section, we take the final step in the prioritisation of policies. First, we describe how the disagreements can be extracted from the problems we developed in the earlier section. Second, we go on to build a framework that can identify the most binding constraint in a second-best world.

5.1 Disagreement

Why can we not reach the first-best? Why is the household not adapting its behaviours? As a compromise, the policy maker chooses to identify the factor where the distance between the first-best and the second-best is the largest (here understood as the distance between the planner's and the household's), and tackles it, to reduce the gap. Note that we do not propose any additional policy to reduce the gap nor do we take its cost into consideration. We leave these important questions for future research. A step in between a first-best and a second-best world consists for HRV in obtaining the shadow prices (Lagrange multipliers) with respect to the different factors and identify the biggest wedge between the private and social net-marginal valuations. HRV call such a reform strategy "tackling the biggest distortion". Still, they argue that this is not the best reform strategy in a second-best world since we do not consider how a distortion is embedded into an economy.

In the previous sections we obtained the private and social net marginal valuations of the different economic factors, represented by the Lagrange multiplier ξ of the utility maximisation problem of the household and the social planner. Now, one can obtain the wedges between the private and social net marginal valuations by simply subtracting the household's ξ_j^P from the social planner's ξ_j^S . We define next the disagreements that policy-making has to deal with. Here, we define the disagreement on factor j as the difference in the net-marginal valuations

$$D_j(t) = \frac{\dot{\xi}_j^s(t)}{\xi_j^s(t)} - \frac{\dot{\xi}_j^p(t)}{\xi_j^p(t)}, \quad (30)$$

$\xi_j^P(t)$ is given by:

$$\xi_j^p(t) = \Phi \left[\frac{\partial A_j}{\partial I_j} + \frac{\partial A_i}{\partial I_j} \frac{\frac{\partial A_j}{\partial I_j} - \frac{\partial A_j}{\partial I_i}}{\frac{\partial A_i}{\partial I_i} - \frac{\partial A_i}{\partial I_j}} \right], \quad (31)$$

and $\xi_j^s(t)$ by:

$$\xi_j^s(t) = \Phi \left[\frac{\partial B_j}{\partial I_j} + \frac{\partial B_i}{\partial I_j} \frac{\frac{\partial B_j}{\partial I_j} - \frac{\partial B_j}{\partial I_i}}{\frac{\partial B_i}{\partial I_i} - \frac{\partial B_i}{\partial I_j}} \right], \quad (32)$$

There is one main difference between HRV's expression for the wedge [\(11\)](#) and ours. According to HRV distortions depend on all the other distortions at equilibrium, but only on distortions. Note that this would make sense in an imaginary country with high import barriers, which become more or less binding depending on the strength of the domestic manufacturing sector. In our model, all multipliers depend on the same set of fundamental variables from the household and the policymaker problems, so they

are indeed interconnected, but indirectly. Due to its micro-foundation, we are actually able to relate all disagreements to the economy's fundamentals.

The model we will lay out in this section has four main differences with Hausmann et al. (2004)'s pioneering work. First and most important, variables μ_j in the wedges defined in HRV are meant to be marginal valuations of each factor x_j . However, note that they are not properly defined leaving room for interpretation and adaptation to specific cases, probably depending on available information. Here in contrast, marginal valuations are equal to the marginal value of each factor as defined by the factor's associated multiplier in the optimal control problem of each agent. So, by definition, the variables that we use to measure disagreement are marginal valuations. Second, our model does not include a political constraint as the one of Hausmann et al. (2004). This constraint translates the trade-off between the amount of political capital and the amount of reforms.

Third, Hausmann et al. (2004)'s model is meant as a framework to rank economic reforms to obtain where intervention is most urgently needed. On the contrary, we propose here a holistic approach where we minimize an overall measure of the disagreement between the economic agents. As a result, we believe that our approach has several profound advantages over HRV. Indeed, our modelling does not necessarily pick one unique factor to tackle in full and at all times, which is the type of outcome that HRV would produce. Here, the HRV solution of picking one variable would correspond to a corner solution, which is sustained over time. As a consequence, our approach can better adapt to evolving times, situations, tastes, technological progress, etc., while minimizing an overall measure of distance between the agent's evaluations (we provide with all technical details in the following subsections).

Fourth, HRV illustrate their approach using some empirical applications which consist in calibrating an endogenous growth model's balanced growth path with an additional characterization of the interest rate. Even though Hausmann et al. (2004) illustrate their idea adding a constraint for every distortion, the framework rests conceptual and is applied in cooperation with the decision tree by the authors. On the contrary, our model is meant to be calibrated on data so that one can calculate the marginal valuations of all factors, and as a result obtain a measure of disagreement over time. As development economics as a whole has changed a lot due to the invention of RCTs, we hope to provide here with a tool that can be used as an alternative to RCTs. Where due to ethical reasons, settings with general equilibrium and problems with external validity, the use of RCTs is not possible, we hope that our model contributes to the development of methodological tools that can be used as a reliable and well funded alternative.

5.2 The most hindering disagreement

Once disagreements D_j defined, one can use them to obtain the investment functions I_1, \dots, I_M that will minimize overall disagreement. We propose a new planner optimization problem, the "super policy maker problem (SPMP)". The SPMP will take into account the optimal marginal valuations of the agents and minimize their differences. How will she do this? Through time evolving investment policies, namely investment in the different factors I_j .

Think of an economic sector which generates CO₂ emissions. Producers may derive a lot of utility from this specific sector, while the society as a whole suffers instead a lot of dis-utility. In this context, the super policy maker will take into account the difference in the marginal valuations, and try to bring together the utility of the emitters and society through a specific and optimal investment plan.

In particular, the problem of the super policy maker is

$$\text{Min}_{I_j, I_i} \int_t^\infty \left(\sum_{i=1}^M D_j(t)^2 \right) e^{\theta(t-s)} dt, \quad (33)$$

subject to (30) and

$$\dot{x}_j^p(t) = A_j [I_1(t), \dots, I_M(t), x_j(t)] - \delta_j x_j(t), \quad (34)$$

$$\dot{x}_j^s(t) = B_j [I_1(t), \dots, I_M(t), x_j(t)] - \delta_j x_j(t), \quad (35)$$

for all $j = 1, \dots, M$.

Note that the super policy maker's problem can be solved using standard optimization techniques and obtain, as announced, the trajectories for I_1, \dots, I_M that will minimize overall disagreement.

6 A two dimensional illustration

Next, we illustrate our model on an economy that has two factors: x_1 and x_2 . For simplicity we choose the household and planner problem set-ups as similar as possible. In a first step, we solve the household problem and obtain its net marginal valuation of each economic factor as measured by the associated (detrended) multipliers. In a second step, we apply the same procedure to the planner problem. Last but not least, we set up the SPMP where the super policy maker minimizes the distance between the net marginal valuations of the economic agents, here the planner and the household. Once the overall procedure is completed, we would have obtained the trajectories for investment that minimize disagreement between the economic agents. Recall that these investment functions are second best trajectories, in the sense that they are not the first best of any of the agents, but they have the valuable property of bringing them all together.

Before going any further, let us underline one of the main differences of our set-up with respect to HRV, which is at the same time our main contribution. While HRV point out at a single sector where all the policy maker's investment effort should be directed, here, the policy maker will distribute her effort among all economic factors according to an optimal scheme that evolves with time, and which will ensure that overall disagreement stays at its minimum feasible level.

6.1 Household problem

Let us assume that household's preferences can be represented with a CES utility function with parameter σ , and that the household's probability to die, ρ , is equal to zero. Then, the problem of the household becomes

$$\text{Max}_{I_1, I_2} \int_t^\infty \frac{\left(F(x_1(t), x_2(t)) - I_1(t) - I_2(t) \right)^{1-\sigma}}{1-\sigma} e^{\theta(t-s)} ds, \quad (36)$$

subject to

$$\begin{cases} \dot{x}_1(t) = I_1(t) - \delta x_1(t), \\ \dot{x}_2(t) = I_2(t) - \delta x_2(t), \\ F(x_1(t), x_2(t)) = x_1(t)^\alpha x_2(t)^{1-\alpha}. \end{cases} \quad (37)$$

We provide with the detailed solution to the problem in Appendix A.5.1. Among our results, we obtain the following relationship between the marginal valuation needs to hold at the optimal solution:

$$\lambda_1(t) = \lambda_2(t), \quad (38)$$

and this for all t . Obviously, this relationship is specific to this illustration. In a more general set-up, the household's net marginal valuations of the corresponding economic factor do not need to be equal.

We can establish several relationships that need to hold at every time t . First, we know that in final

good production, the marginal products of both factors need to be the same.

$$F_{x_1}(x_1(t), x_2(t)) = F_{x_2}(x_1(t), x_2(t)). \quad (39)$$

Plugging in the functional forms of the final good production function in (37), we have that the ratio between x_1 and x_2 needs to be constant at every time t too:

$$\frac{\alpha}{1-\alpha} = \frac{x_1(t)}{x_2(t)}. \quad (40)$$

This lets us deduct that the growth rates of both factors are necessarily equal as well, which implies in turn from (37) that the ratio between each economic factor x_1, x_2 and its corresponding investment I_1, I_2 is also equal:

$$\frac{I_1(t)}{x_1(t)} = \frac{I_2(t)}{x_2(t)} \Rightarrow I_1(t) = I_2(t) \frac{x_1(t)}{x_2(t)} = I_2(t) \frac{\alpha}{1-\alpha}. \quad (41)$$

We can read (41) as saying that the household's optimal investment in the two factors is a fixed ratio that needs to hold at every time t . As (41) shows, this ratio critically depends on the assumptions made on α since:

$$I_1(t) = \frac{\alpha}{1-\alpha} I_2(t). \quad (42)$$

Consequently, there are three broad cases. If $0 < \alpha < 1/2$, then investment into factor 1 is more important than investment into factor 2, and this, at every time t . If $\alpha = 1/2$, amounts of investment in both factors are equal at the optimum. Finally, if $1/2 < \alpha < 1$, then $I_2(t) > I_1(t)$. Hence, and very simply put, the household privileges investment in the most productive factor.

Let us focus next on the investment to stock ratio for factor 1 $J_1 = \frac{I_1}{x_1}$. Using the optimal conditions obtained above, we can describe its dynamics as:

$$\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} (J_1 - \delta) - \frac{1}{\alpha} \left[\dot{J}_1 + J_1(J_1 - \delta) \right] = \frac{1}{\sigma} \left[\alpha \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} + \sigma - \delta \right] \left[\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} - \frac{1}{\alpha} J_1 \right]. \quad (43)$$

At the steady state \bar{J}_1 , $\dot{J}_1 = 0$ so that it must be

$$\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} (\bar{J}_1 - \delta) - \frac{1}{\alpha} \bar{J}_1(\bar{J}_1 - \delta) = \frac{1}{\sigma} \left[\alpha \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} + \sigma - \delta \right] \left[\left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} - \frac{1}{\alpha} \bar{J}_1 \right]. \quad (44)$$

Equation (44) provides us with a second order polynomial for \bar{J}_1 . Let us define

$$M = \frac{1}{\sigma} \left[\alpha \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} + \sigma - \delta \right].$$

We have the following result:

Proposition 1 *Let:*

$$[M + \delta + \alpha^\alpha(1-\alpha)^{1-\alpha}]^2 > 4\alpha^\alpha(1-\alpha)^{1-\alpha}(M + \delta).$$

Then

- i) if $M + \delta < 0$, then there exists a unique positive solution for \bar{J}_1 ;*
- ii) if $M + \delta = 0$, then there is no strictly positive solution for \bar{J}_1 since $\bar{J}_1 = 0$;*
- iii) finally, if $M + \delta > 0$, then there are two positive solutions for \bar{J}_1 .*

Proposition 1 shows that if $M + \delta < 0$, then the household problem exhibits multiplicity of long-run steady states. This represents an additional source of complexity that the policy maker will need to take

care of. Indeed, what is the steady state that the household is heading towards? To somehow control this situation, the policy maker could try to lead households to a situation in which $M + \delta < 0$, that is, in which

$$\alpha^\alpha(1 - \alpha)^{1-\alpha} + \sigma < \delta(1 - \sigma).$$

Obviously, once we have obtained the steady state solution for J_1 , we also have that of J_2 since by (41) they are equal.³

6.2 Social planner problem

We set-up the social planner problem with only one difference compared to the household problem. Indeed, the policy maker knows and takes into account that there exists a spillover from sector 2 into sector 1: the specific investment in sector 2 (that equals technology in sector 2), has a direct effect on sector 1's technology. This is a typical example of a policy maker who detains more information about the relationships between the economic sectors, that are unobserved by the household. Here, we opt for a very simple formulation of this as seen in the policy maker's constraints in (46). Because of this difference in the information about technology, the household and the policy maker will value differently investment in each sector, and some disagreement about investment will emerge.

The policy maker adopts the same utility function as the representative household, that is, her preferences also take the form of a CES-function. The problem of the policy maker is then

$$\text{Max}_{I_1, I_2} \int_t^\infty \frac{\left(F(x_1(t), x_2(t)) - I_1(t) - I_2(t)\right)^{1-\sigma}}{1 - \sigma} e^{\theta(t-s)} ds, \quad (45)$$

subject to

$$\begin{cases} \dot{x}_1(t) = I_1(t)I_2(t) - \delta x_1(t), \\ \dot{x}_2(t) = I_2(t) - \delta x_2(t), \\ F(x_1(t), x_2(t)) = x_1(t)^\alpha x_2(t)^{1-\alpha}. \end{cases} \quad (46)$$

There are two solutions, either $I_1(t) = I_2(t)$ or $I_1(t) \neq I_2(t)$. If $I_1(t) = I_2(t)$, then the policy maker in our economy is having a "one size fits all" approach towards economic policy. That is, he is not distinguishing between the two sectors and is allocating the same amount of investment towards all sectors. Notice that it represents a rare case when investment into both factors is exactly equal. Given that a "one size fits all" approach runs encounter the basic intuition of our model to prioritize policies, we do not exploit this case any further.

Let ξ denote the detrended Lagrange multipliers that represent the societal net marginal valuations of the social planner. Under the assumption that $I_1(t) \neq I_2(t)$, the set of necessary optimal conditions is given by:⁴

$$\begin{cases} \dot{\xi}_1 = [\delta - \theta - I_2 F_{x_1}(x_1, x_2)] \xi_1, \\ \dot{\xi}_2 = \left[\delta - \theta - \frac{I_2}{I_1 - I_2} F_{x_2}(x_1, x_2)\right] \xi_2, \\ \lim_{t \rightarrow \infty} \xi_1(t) x_1(t) e^{-\theta t} = 0, \\ \lim_{t \rightarrow \infty} \xi_2(t) x_2(t) e^{-\theta t} = 0, \end{cases} \quad (47)$$

plus (46). With these optimal conditions, we can compute the steady state values of the different variables x_1^s , x_2^s , I_1^s and I_2^s (all details in Appendix A.5.6):

³For a full derivation and proof of the steady state turn to appendix A.5.3.

⁴See Appendix A.5.5 for a complete proof

$$\left\{ \begin{array}{l} x_2^s = C = \frac{\delta-\theta}{\delta\alpha} \left(\frac{\delta-\theta}{1-\alpha} \right)^{\frac{1-\alpha}{\alpha}} \left(1 + \frac{\delta\alpha}{1-\alpha} \right)^{\frac{\alpha-1}{\alpha}}, \\ x_1^s = \left(\frac{\delta\alpha}{\delta-\theta} \right)^{\frac{1}{1-\alpha}} C^{\frac{2-\alpha}{1-\alpha}}, \\ I_1^s = \delta \left(\frac{\delta\alpha}{\delta-\theta} \right)^{\frac{1}{1-\alpha}} C^{\frac{1}{1-\alpha}}, \\ I_2^s = \delta C. \end{array} \right. \quad (48)$$

We can compare the household and the policy maker problems at the steady state looking at the optimal ratios between the factors and investment:

	Household	Social planner
$\frac{\bar{x}_1}{\bar{x}_2}$	$\frac{\alpha}{1-\alpha}$	$\left(\frac{\delta\alpha}{\delta-\theta} \right)^{\frac{1}{1-\alpha}} C^{\frac{1}{1-\alpha}}$
$\frac{\bar{I}_1}{\bar{I}_2}$	$\frac{\alpha}{1-\alpha}$	$\left(\frac{\delta\alpha}{\delta-\theta} \right)^{\frac{1}{1-\alpha}} C^{\frac{1}{1-\alpha}}$
$\frac{\bar{J}_1}{\bar{J}_2}$	1	C^{-1}

Table 1: Ratios at Steady State

As simple as it seemed our illustration example, it turns out that it is not as simple as that. Note that the household ratios J_1 and J_2 of investment to stock are identical whereas this is not the case for the policy maker. Let us assume that both factors are equally productive, that is $\alpha = 1/2$.

	Household	Social planner
$\frac{\bar{x}_1}{\bar{x}_2}$	1	$\frac{4(\delta-\theta)^2}{(1+\delta)^2}$
$\frac{\bar{I}_1}{\bar{I}_2}$	1	$\frac{\delta}{1-\delta}$
\bar{J}_1	1	$\frac{\delta^2(1+\delta)}{4(\delta-\theta)^2}$
\bar{J}_2	1	δ
$\frac{\bar{J}_1}{\bar{J}_2}$	1	$\frac{\delta(1+\delta)}{4(\delta-\theta)^2}$

Table 2: Ratios at Steady State when $\alpha = 1/2$

These preliminary results are intuitive. Indeed, according to the second line of Table [6.2](#), long-run investment in factor 2 is larger than in factor 1 if and only $\delta < 1/2$, which is usually the case in a continuous time Ramsey type model. Besides, using the third and fourth lines in the same table, we obtain that

$$\frac{\bar{I}_1^p}{\bar{I}_2^p} - \frac{\bar{I}_1^s}{\bar{I}_2^s} = \frac{1-2\delta}{1-\delta} > 1,$$

which underlines the sizeable discrepancy between the long-term investment ratios of our two agents (even in the simplest case), showing and measuring what the role and the task of the super policy maker

is: to close the gap between these investment ratios in as much as possible.

6.3 Super policy maker problem

With the results of the previous sections in hand, the super policy maker will start by calculating the differences in net marginal valuations of both production factors. Recall that Ψ_1, Ψ_2 are the detrended Lagrange multipliers of the household problem. As such, they represent the net marginal valuation of the economic factors by the household. Similarly, ξ_1, ξ_2 are the counterpart of the planner problem. They portray how society as a whole values the corresponding economic factors x_1, x_2 . Then, as defined in (30) we compute the distance variables as

$$\begin{cases} D_1(t) = \frac{\dot{\xi}_1}{\xi_1} - \frac{\dot{\Psi}_1}{\Psi_1} = \alpha \left(\frac{x_1(t)}{x_2(t)} \right)^{\alpha-1} \left[1 - I_2(t) \right], \\ D_2(t) = \frac{\dot{\xi}_2}{\xi_2} - \frac{\dot{\Psi}_2}{\Psi_2} = (1 - \alpha) \left(\frac{x_1(t)}{x_2(t)} \right)^{\alpha} \left[\frac{F_1}{I_1(t) - I_2(t)} \right]. \end{cases} \quad (49)$$

The super policy maker's problem is now to minimize the distances between the net marginal valuations of the household and the planner. In the corner case in which one of these distances is equal to zero, the household and the planner value equally this specific economic factor, implying that the households would act optimally (regarding this specific factor).

$$\text{Min}_{I_1, I_2} \int_t^\infty \left(D_1(t)^2 + D_2(t)^2 \right) e^{\theta(t-s)} dt, \quad (50)$$

subject to

$$\begin{cases} \dot{x}_1(t) = I_1(t)I_2(t) - \delta x_1(t), \\ \dot{x}_2(t) = I_2(t) - \delta x_2(t). \end{cases} \quad (51)$$

D_1 and D_2 can be computed using the growth rates of the corresponding Lagrange multipliers from the household and social planner problem. We plug in the expressions of (49) and can then write a Hamiltonian to solve the super policy maker's problem:

$$\begin{aligned} H(x_1, x_2, I_1, I_2) = & \left[\alpha^2 \left(\frac{x_1(t)}{x_2(t)} \right)^{2(\alpha-1)} [I_1(t) - I_2(t)]^2 + (1 - \alpha)^2 \left(\frac{x_1(t)}{x_2(t)} \right)^{2\alpha} \left[\frac{I_1(t)}{I_1(t) - I_2(t)} \right]^2 \right] e^{\theta(t-s)} \\ & + \mu_1 [I_1(t)I_2(t) - \delta x_1(t)] + \mu_2 [I_2(t) - \delta x_2(t)]. \end{aligned} \quad (52)$$

In Appendix (A.5.10) we show there exists an interior optimal solution. Focusing on the long-run optimal solution of the super policy maker, we can characterize a system of equations that allows to identify I_1, I_2 :

$$\begin{cases} (1 - I_2)I_1(I_1 - I_2)^2 = \left[\frac{I_2}{\delta}(\delta - \theta) + (1 - \alpha)(1 - I_2) \right] \left[\alpha - \frac{(\delta - \theta)}{\delta} I_2(I_1 - I_2) \right], \\ (1 - I_2)^2(I_1 - I_2)^4 = \left(\frac{1 - \alpha}{\alpha} \right)^2 I_1. \end{cases} \quad (53)$$

Once we solved this system of equations numerically, we can go on and compare the different I_1, I_2 of the super policy maker, the social planner and the household problem at steady state. I_1, I_2 of the super policy maker will represent the optimal compromise. This will allow a real-world policy maker to understand, which economic factor x needs more investment. We plan to include a full numerical illustration in a more complete version of this working paper.

7 Conclusion

The aim of this paper was to construct a model that allows to prioritize economic activities where reform is the most urgently needed in a second-best framework. This will help to fill the methodological gap in development economics of how to systematically identify most urgent reforms and advice policy makers which areas to tackle first. On the one hand, the methodological shortcomings of the growth diagnostics approach do not allow for a sound empirical analysis. On the other hand, RCTs have issues establishing external validity and cannot account for general equilibrium effects. Nor are they useful to evaluate macroeconomic outcomes. Thus, a framework able to identify the most binding constraint on growth in a second-best world via a quantitative approach represents a valuable contribution to the literature on economic development.

The framework of HRV does not properly define the net marginal valuations, leaving room for interpretation and adaptation to specific cases depending on available information. On the contrary, to model the net marginal valuations rigorously, we set up two nearly identical continuous-time OLG models. Through solving the household's utility maximisation problem, one obtains the net private valuations of an economic activity j , represented by the corresponding Lagrange multiplier ξ_j^p . The social planner has a wider time horizon, and by solving his utility maximisation problem, one obtains net social valuations of the economic activity j through the Lagrange multiplier ξ_j^s . Finally, one can subtract ξ_j^p from ξ_j^s and get the wedge between private and social valuations of the economic activity, showing the biggest distortion in the economy. As HRV argued, the biggest distortion and the most binding constraint on economic growth do not need to be identical due to second-best problems. To identify the most binding constraint on economic growth in the spirit of HRV, we then set up a country problem where utility depends on the wedges in net marginal valuations that we previously obtained.

What are the real world implications of this model? This model represents a first step to allow for a quantitative growth diagnostic of a country. Together with future works to fill the gaps this paper leaves due to its scope, it can be a valuable tool for policy makers that aim at reforming their country to allow for a quicker economic development. These so-called "reformers" (Hausmann et al., 2005) have often scarce political capital and do not stay forever in office. Neither is it realistic to do all reforms at once in short time. Thus, a tool to prioritize the economic sectors to reform, that does not follow the latest trend of the economic mainstream but applies all the knowledge the economic profession has produced over the years, can speed up the debate what needs to be done in a specific country at a specific time to enhance growth.

The main topic future works with a bigger scope need to cover, is an extensive empirical analysis. This implies two parts. On the one hand, one needs to generalize this framework to allow for more than two factors. On the other hand, detailed analysis for given countries need to be undertaken to lead to valuable advice for policy makers where reform is most urgently needed.

A Appendix

A.1 First-order conditions of the household problem

The first order conditions yield:

$$\frac{\partial H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial c(t)} = 0 \Rightarrow u'(c(t))e^{-(\theta+\rho)t} - \phi(t) + \zeta(t) = 0, \quad (54)$$

$$\frac{\partial H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial I_j(t)} = 0 \Rightarrow \sum_{k=1}^M \mu_k(t) \frac{\partial A_k}{\partial I_j(t)} = \phi(t) - \eta_j(t), \quad (55)$$

$$\frac{\partial H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial x_j(t)} = -\dot{\mu}_j(t) \Rightarrow \mu_j(t) \left[\frac{\partial A_k}{\partial x_j(t)} - \delta_j \right] + \varsigma_j + \phi \frac{\partial F}{\partial x_j(t)} = -\dot{\mu}_j(t). \quad (56)$$

The transversality conditions are in this case:

$$\lim_{t \rightarrow \infty} \varsigma_j(t)x_j(t) = 0, \quad \forall j = 1, \dots, n, \quad (57)$$

plus the Kuhn-Tucker conditions which ensure that x_j , I_j and c are positive, and that the budget constraint binds:

$$\varsigma_j(t)x_j(t) = 0, \quad \forall j, \quad \eta_j(t)I_j(t) = 0, \quad \forall j \quad \text{and} \quad \zeta(t)c(t) = 0, \quad (58)$$

$$\phi(t) \left[c(t) + \sum_{j=1}^M I_j(t) - F(x_1(t), \dots, x_M(t), L) \right] = 0. \quad (59)$$

As usual, at the interior optimal solution, we have $\varsigma_j(t) = 0$, $\eta_j(t) = 0$ and $\zeta_j(t) = 0$, for all t . Like Blanchard (1985), we rule out the case of negative wealth via the assumption of only allowing non-negative wealth. We see that there is a trade-off between c and I_j and between all j . Note that since $\phi \neq 0$ we have $c(t) + \sum_{j=1}^M I_j = F(x_1(t), \dots, x_M(t), L)$. We will have to study two cases presumably at a given t : first $\eta_j = 0$ and second $\eta_j \neq 0$. We will focus on the first case because the second would imply $I_j = 0$. Moreover, we assume:

$$\lim_{I_j \rightarrow \infty} \frac{\partial A_k}{\partial I_j} \rightarrow \infty, \quad \text{then,} \quad I_j > 0, \forall j. \quad (60)$$

Since variables grow at different speeds, this problem does not have a steady state. To obtain a steady state, we need a detrended dynamic system. If we assume $\eta_j = 0$, we can build a detrended dynamic system step by step, ending up with 3 detrended dynamic equations of the detrended counterparts of x_j , I_j and μ_j . Therefore, we define the variables

$$\Phi = \phi e^{\int_0^s -\theta - \rho \, dz}, \quad (61)$$

and

$$\xi_j = \mu_j e^{\int_0^s -\theta - \rho \, dz}. \quad (62)$$

To start, let us derive a detrended expression for $\dot{\mu}_j$. We know that:

$$(\theta + \rho)\xi_j + e^{\int_0^s -\theta - \rho \, dz} \dot{\mu}_j = \dot{\xi}_j,$$

what can be transformed to:

$$(\theta + \rho)\xi_j - \dot{\xi}_j = -e^{\int_0^s -\theta - \rho \, dz} \dot{\mu}_j.$$

Then, we can write [56](#) as:

$$(\theta + \rho)\xi_j - \dot{\xi}_j = \xi_j \left[\frac{\partial A_k}{\partial x_j} - \delta_j \right] - \Phi \frac{\partial F}{\partial x_j}.$$

We check next that our first order conditions verify the Mangasarian Sufficient Conditions. In this regard, we need to proof that the Hamiltonian is jointly concave (Acemoglu, [2009](#)). We compute the second-order derivatives of H with respect to c , I_j and x_j and verify that they are indeed negative:

$$\frac{\partial^2 H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial c(t)^2} < 0 \Rightarrow u''(c(t))e^{-(\theta+\rho)t} < 0, \quad (63)$$

$$\frac{\partial^2 H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial I_j(t)^2} < 0 \Rightarrow \sum_{k=1}^M \mu_k(t) \frac{\partial^2 A_k}{\partial I_j^2} < 0, \quad (64)$$

$$\frac{\partial^2 H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial x_j(t)^2} < 0 \Rightarrow \frac{\partial^2 A_k}{\partial x_j^2} + \phi \frac{\partial^2 F}{\partial x_j^2} < 0. \quad (65)$$

Hence, the Mangasarian second order conditions are verified under the model's assumptions.

A.2 Steady state of the household problem

In the following, as mentioned in section [3.5](#), we will use an example of two factors to obtain the steady state values of $\bar{\xi}_j$. Suppose $M = 2$ and the following functional forms:

$$\begin{cases} F(x_1(t), x_2(t), L) = x_1^{\alpha_1} x_2^{\alpha_2} L^{1-\alpha_1-\alpha_2}, \\ A_1(I_1, I_2, x_1) = I_1^{\beta_1} I_2^{\beta_2} x_1^{\gamma_1}, \\ A_2(I_1, I_2, x_2) = I_1^{\beta_1} I_2^{\beta_2} x_2^{\gamma_2}. \end{cases} \quad (66)$$

At the steady state, the following is true for equation [\(6\)](#) for $j = 1, 2$:

$$A_j(\bar{I}_1, \bar{I}_2, \bar{x}_j) = \delta_j \bar{x}_j, \quad (67)$$

so that we obtain \bar{x}_j as a function of \bar{I}_1 and \bar{I}_2 .

The optimal dynamics of the co-state variables is

$$\begin{cases} \dot{\xi}_1(t) = \xi_1 \left[\delta_1 + \theta + \rho - \gamma_1 x_1^{\gamma_1-1} I_1^{\beta_{11}} I_2^{\beta_{12}} \right] - \Phi \alpha_1 x_1^{\alpha_1-1} x_2^{\alpha_2} L^{1-\alpha_1-\alpha_2}, \\ \dot{\xi}_2(t) = \xi_2 \left[\delta_2 + \theta + \rho - \gamma_2 x_2^{\gamma_2-1} I_1^{\beta_{21}} I_2^{\beta_{22}} \right] - \Phi \alpha_2 x_1^{\alpha_1} x_2^{\alpha_2-1} L^{1-\alpha_1-\alpha_2}. \end{cases} \quad (68)$$

Denoting by $\bar{\xi}_i$ the steady state of ξ_i , [\(68\)](#) implies that at the steady state

$$\begin{cases} 0 = \bar{\xi}_1 \left[\delta_1 + \theta + \rho - \gamma_1 \bar{x}_1^{\gamma_1-1} \bar{I}_1^{\beta_{11}} \bar{I}_2^{\beta_{12}} \right] - \Phi \alpha_1 \bar{x}_1^{\alpha_1-1} \bar{x}_2^{\alpha_2} L^{1-\alpha_1-\alpha_2}, \\ 0 = \bar{\xi}_2 \left[\delta_2 + \theta + \rho - \gamma_2 \bar{x}_2^{\gamma_2-1} \bar{I}_1^{\beta_{21}} \bar{I}_2^{\beta_{22}} \right] - \Phi \alpha_2 \bar{x}_1^{\alpha_1} \bar{x}_2^{\alpha_2-1} L^{1-\alpha_1-\alpha_2}, \end{cases} \quad (69)$$

as well as

$$\begin{cases} \bar{\xi}_1 \frac{\partial A_1}{\partial \bar{I}_1} + \bar{\xi}_2 \frac{\partial A_2}{\partial \bar{I}_1} = \Phi ; \quad \bar{\xi}_1 \frac{\partial A_1}{\partial \bar{I}_2} + \bar{\xi}_2 \frac{\partial A_2}{\partial \bar{I}_2} = \Phi, \\ \bar{\xi}_1 \frac{\partial A_1}{\partial \bar{I}_1} + \bar{\xi}_2 \frac{\partial A_2}{\partial \bar{I}_1} = \bar{\xi}_1 \frac{\partial A_1}{\partial \bar{I}_2} + \bar{\xi}_2 \frac{\partial A_2}{\partial \bar{I}_2}, \end{cases} \quad (70)$$

where the first line can be written as

$$\bar{\xi}_1 \left[\frac{\partial A_1}{\partial \bar{I}_1} - \frac{\partial A_1}{\partial \bar{I}_2} \right] = \bar{\xi}_1 \left[\frac{\partial A_2}{\partial \bar{I}_2} - \frac{\partial A_2}{\partial \bar{I}_1} \right],$$

and so we can express $\bar{\xi}_2$ as a function of $\bar{\xi}_1$:

$$\bar{\xi}_2 = \bar{\xi}_1 \frac{\frac{\partial A_1}{\partial \bar{I}_1} - \frac{\partial A_1}{\partial \bar{I}_2}}{\frac{\partial A_2}{\partial \bar{I}_2} - \frac{\partial A_2}{\partial \bar{I}_1}}.$$

Putting the information from the two former equations together yields:

$$\Phi = \bar{\xi}_1 \frac{\partial A_1}{\partial \bar{I}_1} + \bar{\xi}_2 \frac{\partial A_2}{\partial \bar{I}_1} = \bar{\xi}_1 \left[\frac{\partial A_1}{\partial \bar{I}_1} + \frac{\partial A_2}{\partial \bar{I}_1} \frac{\frac{\partial A_1}{\partial \bar{I}_1} - \frac{\partial A_1}{\partial \bar{I}_2}}{\frac{\partial A_2}{\partial \bar{I}_2} - \frac{\partial A_2}{\partial \bar{I}_1}} \right]. \quad (71)$$

We can use (71) in (69) to obtain that

$$0 = \bar{\xi}_1 \left[\delta_1 + \theta + \rho - \frac{\partial A_1}{\partial \bar{x}_1} \right] - \frac{\partial F_1}{\partial \bar{x}_1} \bar{\xi}_1 \bar{I}_1.$$

Hence

$$\begin{cases} 0 = \bar{\xi}_1 \left[\delta_1 + \theta + \rho - \frac{\partial A_1}{\partial \bar{x}_1} - \frac{\partial F}{\partial \bar{x}_1} \left(\frac{\partial A_1}{\partial \bar{I}_1} + \frac{\partial A_2}{\partial \bar{I}_1} \frac{\frac{\partial A_1}{\partial \bar{I}_1} - \frac{\partial A_1}{\partial \bar{I}_2}}{\frac{\partial A_2}{\partial \bar{I}_2} - \frac{\partial A_2}{\partial \bar{I}_1}} \right) \right], \\ 0 = \bar{\xi}_2 \left[\delta_2 + \theta + \rho - \frac{\partial A_2}{\partial \bar{x}_2} - \frac{\partial F}{\partial \bar{x}_2} \left(\frac{\partial A_2}{\partial \bar{I}_2} + \frac{\partial A_1}{\partial \bar{I}_2} \frac{\frac{\partial A_2}{\partial \bar{I}_2} - \frac{\partial A_2}{\partial \bar{I}_1}}{\frac{\partial A_1}{\partial \bar{I}_1} - \frac{\partial A_1}{\partial \bar{I}_2}} \right) \right]. \end{cases} \quad (72)$$

Since by (71) $\bar{\xi}_1 \neq 0$ and $\bar{\xi}_2 \neq 0$, it must be then that at the steady state

$$\begin{cases} \frac{\partial F(\bar{x}_1, \bar{x}_2)}{\partial \bar{x}_1} \left(\frac{\partial A_1}{\partial \bar{I}_1} + \frac{\partial A_2}{\partial \bar{I}_1} \frac{\frac{\partial A_1}{\partial \bar{I}_1} - \frac{\partial A_1}{\partial \bar{I}_2}}{\frac{\partial A_2}{\partial \bar{I}_2} - \frac{\partial A_2}{\partial \bar{I}_1}} \right) = \delta_1 + \theta + \rho - \frac{\partial A_1}{\partial \bar{x}_1}, \\ \frac{\partial F(\bar{x}_1, \bar{x}_2)}{\partial \bar{x}_2} \left(\frac{\partial A_2}{\partial \bar{I}_2} + \frac{\partial A_1}{\partial \bar{I}_2} \frac{\frac{\partial A_2}{\partial \bar{I}_2} - \frac{\partial A_2}{\partial \bar{I}_1}}{\frac{\partial A_1}{\partial \bar{I}_1} - \frac{\partial A_1}{\partial \bar{I}_2}} \right) = \delta_2 + \theta + \rho - \frac{\partial A_2}{\partial \bar{x}_2}. \end{cases} \quad (73)$$

Hence, we can build a four dimensional system of equations in $(\bar{I}_1, \bar{I}_2, \bar{x}_1, \bar{x}_2)$ with the four equations in (67) and (73). This set has a unique non trivial solution.

A.3 Optimal behaviour of the planner

We can write the following Hamiltonian to solve the social planner's utility maximisation problem:

$$\begin{aligned} H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi) &= u(c(t))e^{-(\theta+\rho)t} \\ &+ \sum_{j=1}^M \mu_j(t) [B_j(I_1(t) \dots I_M(t), x_j(t)) - \delta_j x_j(t)] \\ &+ \phi(t) \left[c(t) + \sum_{j=1}^M I_j(t) - G(x_1(t), \dots, x_M(t), L) \right] \\ &+ \sum_{j=1}^n \varsigma_j(t) x_j(t) + \sum_{j=1}^n \eta_j(t) I_j(t) + \zeta(t) c(t) \end{aligned} \quad (74)$$

The first order conditions yield:

$$\frac{\partial H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial c(t)} = 0 \Rightarrow u'(c(t))e^{-(\theta+\rho)t} - \phi(t) + \zeta(t) = 0, \quad (75)$$

$$\frac{\partial H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial I_j(t)} = 0 \Rightarrow \sum_{k=1}^M \mu_k(t) \frac{\partial B_k}{\partial I_j(t)} = \phi(t) - \eta_j(t), \quad (76)$$

$$\frac{\partial H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial x_j(t)} = -\dot{\mu}_j(t) \Rightarrow \mu_j(t) \left[\frac{\partial B_k}{\partial x_j(t)} - \delta_j \right] + \varsigma_j(t) + \phi(t) \frac{\partial G}{\partial x_j(t)} = -\dot{\mu}_j(t). \quad (77)$$

The transversality conditions are:

$$\lim_{t \rightarrow \infty} \varsigma_j(t) x_j(t) = 0, \quad \forall j = 1, \dots, n, \quad (78)$$

and

$$\lim_{t \rightarrow \infty} \phi(t) G(x_1(t), \dots, x_M(t), L) = 0, \quad (79)$$

plus the Kuhn-Tucker condition which ensures that x_j and w are positive:

$$\varsigma_j(t) x_j(t) = 0, \quad \forall j, \quad \eta_j(t) I_j(t) = 0, \quad \forall j \quad \text{and} \quad \zeta(t) c(t) = 0. \quad (80)$$

To fulfill the Mangasarian conditions, we need to proof that the Hamiltonian is jointly concave (Acemoglu, 2009). We therefore obtain the second-order derivatives:

$$\frac{\partial^2 H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial c(t)^2} < 0 \Rightarrow u''(c(t))e^{-\theta t} < 0, \quad (81)$$

$$\frac{\partial^2 H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial I_j(t)^2} < 0 \Rightarrow \sum_{k=1}^M \mu_k(t) \frac{\partial^2 B_k}{\partial I_j^2(t)} < 0, \quad (82)$$

$$\frac{\partial^2 H(c, I, x, \omega, \mu_1, \dots, \mu_n, \phi)}{\partial x_j(t)^2} < 0 \Rightarrow \frac{\partial^2 B_k}{\partial x_j^2(t)} + \phi \frac{\partial^2 G}{\partial x_j^2(t)} < 0. \quad (83)$$

As in the household problem, the second order Mangasarian conditions are verified under the problem's assumptions.

As usual, at the interior optimal solution, we have $\varsigma_j(t) = 0$, $\eta_j(t) = 0$ and $\zeta_j(t) = 0$, for all t . Like Blanchard (1985), we rule out the case of negative wealth via the assumption of only allowing non-negative wealth. We see that there is a trade-off between c and I_j and between all j . Note that since $\phi \neq 0$ we have $c(t) + \sum_{j=1}^M I_j = G(x_1(t), \dots, x_M(t), L)$. We will have to study two cases presumably at a given t : first $\eta_j = 0$ and second $\eta_j \neq 0$. We will focus on the first case because the second would imply $I_j = 0$. Moreover, we assume:

$$\lim_{I_j \rightarrow \infty} \frac{\partial B_k}{\partial I_j} \rightarrow \infty, \quad \text{then,} \quad I_j > 0, \forall j. \quad (84)$$

Since variables grow at different speeds, this problem does not have a steady state. To obtain a steady state, we need a detrended dynamic system. If we assume $\eta_j = 0$, we can build this detrended dynamic system step by step, ending up with 3 detrended dynamic equations of the detrended counterparts of x_j , I_j and μ_j . Therefore, we define the variables

$$\Phi = \phi e^{\int_0^s -\theta \, dz} \quad (85)$$

and

$$\xi_j = \mu_j e^{\int_0^s -\theta \, dz}. \quad (86)$$

To start, let us derive a detrended expression for $\dot{\mu}_j$. We know that:

$$\theta \xi_j + e^{\int_0^s -\theta - \rho) dz} \dot{\mu}_j = \dot{\xi}_j,$$

what can be transformed to:

$$\theta \xi_j - \dot{\xi}_j = -e^{\int_0^s -\theta - \rho) dz} \dot{\mu}_j.$$

Then, we can write (56) as:

$$\theta \xi_j - \dot{\xi}_j = \xi_j \left[\frac{\partial B_k}{\partial x_j} - \delta_j \right] - \Phi \frac{\partial G}{\partial x_j}$$

and have the following detrended system:

$$\begin{cases} \dot{\xi}_j = \xi_j \left[\frac{\partial B_k}{\partial x_j} - \delta_j \right] - \Phi \frac{\partial G}{\partial x_j}, \forall j, \\ \Phi = u'(c), \\ \Phi = \sum_{k=1}^M \xi_k \frac{\partial B_j}{\partial x_j}, \forall j \\ c(t) + \sum_{j=1}^M I_j - G(x_1(t), \dots, x_M(t), L), \\ c > 0, I_j > 0. \end{cases} \quad (87)$$

A.4 Steady state of the planner problem

The dynamic system described in (87) allows for a unique non-trivial steady state. We can characterize the steady state with the following equation

$$\bar{\xi}_j \left[\delta_j + \theta - \frac{\partial B_j}{\partial x_j} \right] - \frac{\partial G}{\partial x_j} \sum_{k=1}^M \bar{\xi}_k \frac{\partial B_k}{\partial I_j} = 0.$$

We know that at the steady state, the following is true for equation (6):

$$B_j(\bar{I}_1, \dots, \bar{I}_M, \bar{x}_j) = \delta_j \bar{x}_j,$$

so that we obtain \bar{x}_j as a function of $\bar{I}_1, \dots, \bar{I}_M$. In the following, we will use an example of two factors to obtain the steady state values of $\bar{\xi}_j$. Suppose $M = 2$ and the following functional forms:

$$\begin{cases} F(x_1(t), \dots, x_M(t), L) = \Pi_{i=1}^M x_i^{\alpha_i} L^{1-\sum \alpha_i}, \\ B_j(I_1, \dots, I_M, x_j) = \Pi_k I_k^{\beta_k} x_j^{\gamma_j}, \end{cases} \quad (88)$$

so that

$$\begin{cases} F(x_1(t), x_2(t), L) = x_1^{\alpha_1} x_2^{\alpha_2} L^{1-\alpha_1-\alpha_2}, \\ B_1(I_1, I_2, x_1) = I_1^{\beta_1} I_2^{\beta_2} x_1^{\gamma_1}, \\ B_2(I_1, I_2, x_2) = I_1^{\beta_1} I_2^{\beta_2} x_2^{\gamma_2}. \end{cases} \quad (89)$$

and

$$\begin{cases} \dot{\xi}_1 = \bar{\xi}_1 \left[\delta_1 + \theta - \gamma_1 \bar{x}_1^{\gamma_1-1} \bar{I}_1^{\beta_{11}} \bar{I}_2^{\beta_{12}} \right] - \Phi \alpha_1 \bar{x}_1^{\alpha_1-1} \bar{x}_2^{\alpha_2} L^{1-\alpha_1-\alpha_2}, \\ \dot{\xi}_2 = \bar{\xi}_2 \left[\delta_2 + \theta - \gamma_2 \bar{x}_2^{\gamma_2-1} \bar{I}_1^{\beta_{21}} \bar{I}_2^{\beta_{22}} \right] - \Phi \alpha_2 \bar{x}_1^{\alpha_1} \bar{x}_2^{\alpha_2-1} L^{1-\alpha_1-\alpha_2}, \end{cases} \quad (90)$$

as well as

$$\begin{cases} \bar{\xi}_1 \frac{\partial B_1}{\partial I_1} + \bar{\xi}_2 \frac{\partial B_2}{\partial I_1} = \Phi \quad ; \quad \bar{\xi}_1 \frac{\partial B_1}{\partial I_2} + \bar{\xi}_2 \frac{\partial B_2}{\partial I_2} = \Phi, \\ \bar{\xi}_1 \frac{\partial B_1}{\partial I_1} + \bar{\xi}_2 \frac{\partial B_2}{\partial I_1} = \bar{\xi}_1 \frac{\partial B_1}{\partial I_2} + \bar{\xi}_2 \frac{\partial B_2}{\partial I_2}, \end{cases} \quad (91)$$

where the upper part can be written as

$$\bar{\xi}_1 \left[\frac{\partial B_1}{\partial \bar{I}_1} - \frac{\partial B_1}{\partial \bar{I}_2} \right] = \bar{\xi}_1 \left[\frac{\partial B_2}{\partial \bar{I}_2} - \frac{\partial B_2}{\partial \bar{I}_1} \right],$$

and so we can express $\bar{\xi}_2$ as a function of $\bar{\xi}_1$:

$$\bar{\xi}_2 = \bar{\xi}_1 \frac{\frac{\partial B_1}{\partial \bar{I}_1} - \frac{\partial B_1}{\partial \bar{I}_2}}{\frac{\partial B_2}{\partial \bar{I}_2} - \frac{\partial B_2}{\partial \bar{I}_1}}. \quad (92)$$

Putting the information from the two former equations together yields:

$$\Phi = \bar{\xi}_1 \frac{\partial B_1}{\partial \bar{I}_1} + \bar{\xi}_2 \frac{\partial B_2}{\partial \bar{I}_1} = \bar{\xi}_1 \left[\frac{\partial B_1}{\partial \bar{I}_1} + \frac{\partial B_2}{\partial \bar{I}_1} \frac{\frac{\partial B_1}{\partial \bar{I}_1} - \frac{\partial B_1}{\partial \bar{I}_2}}{\frac{\partial B_2}{\partial \bar{I}_2} - \frac{\partial B_2}{\partial \bar{I}_1}} \right]. \quad (93)$$

We can use this to plug this into \bar{I}_1 in the following equation:

$$\dot{\bar{\xi}}_1 = \bar{\xi}_1 \left[\delta_1 + \theta - \frac{\partial B_1}{\partial \bar{x}_1} \right] - \frac{\partial G_1}{\partial \bar{x}_1} \bar{\xi}_1 \bar{I}_1,$$

to have expressions of $\dot{\bar{\xi}}$ that have the following functional form: $\dot{\bar{\xi}}_1 = \dot{\bar{\xi}}_1(\bar{\xi}_1, \bar{x}_1, \bar{x}_2, \bar{I}_1, \bar{I}_2)$ and respectively $\dot{\bar{\xi}}_2 = \dot{\bar{\xi}}_2(\bar{\xi}_2, \bar{x}_1, \bar{x}_2, \bar{I}_1, \bar{I}_2)$:

$$\begin{cases} \dot{\bar{\xi}}_1 = \bar{\xi}_1 \left[\delta_1 + \theta - \frac{\partial B_1}{\partial \bar{x}_1} - \frac{\partial G}{\partial \bar{x}_1} \left(\frac{\partial B_1}{\partial \bar{I}_1} + \frac{\partial B_2}{\partial \bar{I}_1} \frac{\frac{\partial B_1}{\partial \bar{I}_1} - \frac{\partial B_1}{\partial \bar{I}_2}}{\frac{\partial B_2}{\partial \bar{I}_2} - \frac{\partial B_2}{\partial \bar{I}_1}} \right) \right] \\ \dot{\bar{\xi}}_2 = \bar{\xi}_2 \left[\delta_2 + \theta - \frac{\partial B_2}{\partial \bar{x}_2} - \frac{\partial G}{\partial \bar{x}_2} \left(\frac{\partial B_2}{\partial \bar{I}_2} + \frac{\partial B_1}{\partial \bar{I}_2} \frac{\frac{\partial B_2}{\partial \bar{I}_2} - \frac{\partial B_2}{\partial \bar{I}_1}}{\frac{\partial B_1}{\partial \bar{I}_1} - \frac{\partial B_1}{\partial \bar{I}_2}} \right) \right]. \end{cases} \quad (94)$$

From here on, one could go on to obtain a system in \bar{I}_1 and \bar{I}_2 given that \bar{x}_1 and \bar{x}_2 could be obtained from equation (??). Due to the scope of the paper, we will renounce here to generalize on a system with more than two economic factors. We will here just rewrite the system with the information we obtained from equation (71):

$$\begin{cases} \bar{\xi}_1 = \Phi \left[\frac{\partial B_1}{\partial \bar{I}_1} + \frac{\partial B_2}{\partial \bar{I}_1} \frac{\frac{\partial B_1}{\partial \bar{I}_1} - \frac{\partial B_1}{\partial \bar{I}_2}}{\frac{\partial B_2}{\partial \bar{I}_2} - \frac{\partial B_2}{\partial \bar{I}_1}} \right] \\ \bar{\xi}_2 = \Phi \left[\frac{\partial B_2}{\partial \bar{I}_2} + \frac{\partial B_1}{\partial \bar{I}_2} \frac{\frac{\partial B_2}{\partial \bar{I}_2} - \frac{\partial B_2}{\partial \bar{I}_1}}{\frac{\partial B_1}{\partial \bar{I}_1} - \frac{\partial B_1}{\partial \bar{I}_2}} \right]. \end{cases} \quad (95)$$

A.5 Illustration

Our illustration remains simple. The economy has two factors: x_1 and x_2 . There is an externality in the accumulation of factor 1 that is only taken into account by the planner. That is, the accumulation of factor 1 depends on investment in factor 2 too. This is expressed via the first constraints in the respective problems.

A.5.1 Household problem: first-order conditions

As a first step, to solve our model we start with the household problem where we take first order conditions:

$$\frac{\partial H(x_1, x_2, I_1, I_2)}{\partial I_1} \rightarrow -(F(x_1(t), x_2(t)) - I_1(t) - I_2(t))^{-\sigma} e^{\theta(t-s)} + \lambda_1(t) = 0, \quad (96)$$

$$\frac{\partial H(x_1, x_2, I_1, I_2)}{\partial I_2} \rightarrow -(F(x_1(t), x_2(t)) - I_1(t) - I_2(t))^{-\sigma} e^{\theta(t-s)} + \lambda_2(t) = 0. \quad (97)$$

Note that variables may grow at different speeds. Therefore, we need to derive a time-stable system of equations from which we can extract the steady state values of the variables of interest x_1, x_2, I_1, I_2 . This exercise will be undertaken in the next subsection.

A.5.2 Household problem: derivation of optimality conditions

We want to derive time-stable first order conditions of the household problem. We start with the first-order conditions:

$$-\dot{\lambda}_1 = (F(x_1, x_2) - I_1 - I_2^{-\sigma} F_{x_1}(x_1, x_2)) e^{\theta(t-s)} - \delta \lambda_1, \quad (98)$$

$$-\dot{\lambda}_1 = (F(x_1, x_2) - I_1 - I_2)^{-\sigma} F_{x_1}(x_1, x_2) - \delta \lambda_1 e^{-\theta(t-s)}, \quad (99)$$

where we define Ψ with respect to λ as:

$$\Psi_1 = e^{-\theta(t-s)} \lambda_1. \quad (100)$$

From here on, we extend the expression and plug in the information we obtained earlier from taking the first-order conditions

$$\dot{\Psi}_1 = -\theta \Psi_1 + \dot{\lambda}_1 e^{-\theta(t-s)}, \quad (101)$$

$$-\theta \Psi_1 - \dot{\Psi}_1 = (F(x_1, x_2) - I_1 - I_2)^{-\sigma} F_{x_1}(x_1, x_2) - \delta \Psi_1, \quad (102)$$

$$\dot{\Psi}_1 = (\delta - \theta) \Psi_1 - (F(x_1, x_2) - I_1 - I_2)^{-\sigma} F_{x_1}(x_1, x_2). \quad (103)$$

Finally, we get two equations describing $\dot{\Psi}_1, \dot{\Psi}_2$. We add the transversality conditions. This system of six equations characterizes the optimal solution of our household problem.

$$\begin{cases} \dot{x}_1(t) = I_1(t) - \delta x_1(t), \\ \dot{x}_2(t) = I_2(t) - \delta x_2(t), \\ \dot{\Psi}_1 = \Psi_1(\delta - \theta - F_{x_1}(x_1(t), x_2(t))), \\ \dot{\Psi}_2 = \Psi_2(\delta - \theta - F_{x_2}(x_1(t), x_2(t))), \\ \lim_{t \rightarrow \infty} \Psi_1 x_1(t) = 0, \\ \lim_{t \rightarrow \infty} \Psi_2 x_2(t) = 0. \end{cases} \quad (104)$$

Equating the marginal products of both factors in the final good production yields:

$$\alpha x_1(t)^{\alpha-1} x_2(t)^{1-\alpha} = (1 - \alpha) x_1(t)^{\alpha} x_2(t)^{-\alpha}. \quad (105)$$

Moreover, we can characterize the ratio between the two derivatives of the economic factors. Similar to (40), we have that:

$$\dot{x}_1(t) = \frac{\alpha}{1 - \alpha} \dot{x}_2(t). \quad (106)$$

A.5.3 Household problem: steady state

Next, we want to obtain the steady state values of our variables of interest. We can establish the following relationships:

$$I_1 - \delta x_1 = I_2 - \delta x_2 = I_2 - \frac{1-\alpha}{\alpha} \delta x_1 \quad (107)$$

$$I_1 = I_2 - \delta x_1 \left(1 - \frac{1-\alpha}{\alpha}\right) = I_2 - \delta x_1 \left(\frac{2\alpha-1}{\alpha}\right) \quad (108)$$

$$I_1 = I_2 - \delta x_1 \left(\frac{2\alpha-1}{\alpha}\right) \quad (109)$$

$$\frac{\dot{x}_1}{x_1} = \frac{\dot{x}_2}{x_2} \quad (110)$$

$$\frac{I_1}{x_1} = \frac{I_2}{x_2} \rightarrow I_1 = I_2 \frac{x_1}{x_2} = I_2 \frac{\alpha}{1-\alpha} \quad (111)$$

$$I_1 = I_1 \frac{1-\alpha}{\alpha} - \delta x_1 \left(\frac{2\alpha-1}{\alpha}\right) \quad (112)$$

$$I_1 - \delta x_1 = \frac{\alpha}{1-\alpha} (I_2 - \delta x_2) \quad (113)$$

$$I_1 - \delta x_1 = \frac{\alpha}{1-\alpha} I_2 - \delta x_1 \quad (114)$$

Let us focus next on the dynamics of I_1 . We obtain the following expression:

$$\dot{x}_1(t) = 0 \rightarrow I_1(t) = \delta x_1(t) \quad (115)$$

$$\begin{aligned} \varphi_1 &= \left[F(x_1, x_2) - I_1 - I_2 \right]^{-\sigma} \\ -\sigma \left[F(x_1, x_2) - I_1 - I_2 \right]^{\sigma-1} &\left[F_{x_1} \dot{x}_1 + F_{x_2} \dot{x}_2 - \dot{I}_1 - \dot{I}_2 \right] \\ &= \left[F(x_1, x_2) - I_1 - I_2 \right]^{-\sigma} (\delta - \theta - F_{x_1}) \end{aligned} \quad (116)$$

$$\begin{aligned} &\alpha x_1^{\alpha-1} x_2^{1-\alpha} \dot{x}_1 + (1-\alpha) x_1^\alpha x_2^{-\alpha} \dot{x}_2 - \dot{I}_1 - \frac{1-\alpha}{\alpha} \dot{I}_1 \\ &= \frac{1}{\sigma} (F_{x_1} + \theta - \delta) (F(x_1, x_2) - I_1 - I_2) \end{aligned} \quad (117)$$

$$\begin{aligned} &\alpha \left(\frac{\alpha}{1-\alpha} \right)^{\alpha-1} (I_1 - \delta x_1) + (1-\alpha) \left(\frac{\alpha}{1-\alpha} \right)^\alpha \frac{1-\alpha}{\alpha} (I_1 - \delta x_1) - \left(1 + \frac{1-\alpha}{\alpha} \right) \dot{I}_1 \\ &= \frac{1}{\sigma} \left[\alpha \left(\frac{\alpha}{1-\alpha} \right)^{\alpha-1} + \theta - \delta \right] \left[\left(\frac{\alpha}{1-\alpha} \right)^\alpha \frac{1-\alpha}{\alpha} x_1 - x_1^\alpha x_2^{1-\alpha} - I_1 - \frac{1-\alpha}{\alpha} I_1 \right] \end{aligned} \quad (118)$$

where

$$\left(\frac{x_1}{x_2} \right)^\alpha x_2 = \left(\frac{\alpha}{1-\alpha} \right)^\alpha \frac{1-\alpha}{\alpha} x_1 \quad (119)$$

and

$$\left(\frac{\alpha}{1-\alpha} \right)^{\alpha-1} (I_1 - \delta x_1) - \frac{1}{\alpha} \dot{I}_1 = \frac{1}{\sigma} \left[\alpha \left(\frac{\alpha}{1-\alpha} \right)^{\alpha-1} + \theta - \delta \right] \left[\left(\frac{\alpha}{1-\alpha} \right)^{\alpha-1} x_1 - \frac{1}{\alpha} I_1 \right] \quad (120)$$

Following, we define the conditions under which the economy in the household problem can reach a steady state. We start with:

$$I_1 = \delta x_1, \quad (121)$$

and can write

$$\left(\frac{I_1}{x_1}\right)' = \frac{\dot{I}_1 x_1 - I_1 \dot{x}_1}{x_1^2} = \frac{\dot{I}_1}{x_1} - \frac{I_1}{x_1} \frac{\dot{x}_1}{x_1}. \quad (122)$$

We define the ratio $J_1 = \frac{I_1}{x_1}$.

$$J_1' = \frac{\dot{I}_1}{x_1} - J_1(J_1 - \delta) \quad (123)$$

J_1 reaches its steady state only if the following two conditions are fulfilled:

$$\begin{cases} J_1 = \delta \\ J_1 = \alpha \left(\frac{\alpha}{1-\alpha}\right)^{\alpha-1} \end{cases} \quad (124)$$

A.5.4 Social planner problem: first-order and transversality conditions

We obtain the first-order and transversality conditions to characterize the system of equations that yields the optimal solution to our problem:

$$\begin{cases} -(F(x_1(t), x_2(t)) - I_1(t) - I_2(t))^{-\sigma} e^{\theta(t-s)} + \lambda_1(t) I_2(t) = 0, \\ -(F(x_1(t), x_2(t)) - I_1(t) - I_2(t))^{-\sigma} e^{\theta(t-s)} + \lambda_1(t) I_1(t) + \lambda_2 = 0, \\ -\dot{\lambda}_1(t) = (F(x_1(t), x_2(t)) - I_1(t) - I_2(t))^{-\sigma} F_{x_1}(x_1(t), x_2(t)) e^{\theta(t-s)} - \delta \lambda_1(t), \\ -\dot{\lambda}_2(t) = (F(x_1(t), x_2(t)) - I_1(t) - I_2(t))^{-\sigma} F_{x_2}(x_1(t), x_2(t)) e^{\theta(t-s)} - \delta \lambda_2(t), \\ \lim_{t \rightarrow \infty} \lambda_1(t) x_1(t) = 0, \\ \lim_{t \rightarrow \infty} \lambda_2(t) x_2(t) = 0, \end{cases} \quad (125)$$

and we deduce the following relationship:

$$\lambda_1(t)(I_2(t) - I_1(t)) = \lambda_2(t), \quad (126)$$

and

$$-\dot{\lambda}_1(t) = F_{x_1}(x_1(t), x_2(t)) \lambda_1(t) I_2(t) - \delta \lambda_1(t) = \lambda_1(t)(I_2(t) F_{x_1}(x_1(t), x_2(t)) - \delta). \quad (127)$$

A.5.5 Social planner problem: derivation of the steady state

Next, we need to obtain the values of the Lagrange multipliers at steady state. We start with the derivation of the Lagrange multiplier of sector 1 at steady state ξ_1 :

$$-\dot{\lambda}_1 e^{-\theta(t-s)} = (F(x_1, x_2) - I_1 - I_2)^{-\sigma} F_{x_1}(x_1, x_2) e^{\theta(t-s)} - \delta \lambda_1 e^{-\theta(t-s)} \quad (128)$$

$$\xi_1 = \lambda_1 e^{-\sigma(t-s)} \quad (129)$$

$$\dot{\xi}_1 = \dot{\lambda}_1 e^{-\sigma(t-s)} - \theta \xi_1 \quad (130)$$

$$\dot{\xi}_1 - \theta \xi_1 = (F(x_1, x_2) - I_1 - I_2)^{-\sigma} F_{x_1}(x_1, x_2) - \delta \xi_1 \quad (131)$$

$$\dot{\xi}_1 = (\delta - \theta)\xi_1 - (F(x_1, x_2) - I_1 - I_2)^{-\sigma} F_{x_1}(x_1, x_2) \quad (132)$$

to finally obtain the detrended Lagrange multiplier ξ_1 , representing the net marginal valuations of the society for sector 1:

$$\dot{\xi}_1 = (\delta - \theta - I_2 F_{x_1}(x_1, x_2))\xi_1. \quad (133)$$

Now we turn to factor 2. We follow the same procedure as before and obtain for factor 2 the following net marginal valuations in form of the Lagrange multiplier ξ_2 :

$$-\dot{\lambda}_2 e^{-\theta(t-s)} = (F(x_1, x_2) - I_1 - I_2)^{-\sigma} F_{x_2}(x_1, x_2) e^{\theta(t-s)} - \delta \lambda_2 e^{-\theta(t-s)}, \quad (134)$$

$$\xi_2 = \lambda_2 e^{-\sigma(t-s)}, \quad (135)$$

$$\dot{\xi}_2 = \dot{\lambda}_2 e^{-\sigma(t-s)} - \theta \xi_2, \quad (136)$$

$$\dot{\xi}_2 = (\delta - \theta)\xi_2 - I_2 F_{x_2}(x_1, x_2)\xi_1. \quad (137)$$

A.5.6 Social planner problem: steady state

Now we derive the steady state. We start with the system of dynamic equations:

$$\begin{cases} \dot{\xi}_1 = 0 \rightarrow I_2 \alpha x_1^{\alpha-1} x_2^{1-\alpha} = \delta - \theta, \\ \dot{x}_1 = 0 \rightarrow I_1 = \frac{\delta x_1}{I_2} = \frac{x_1}{x_2}, \\ \dot{x}_2 = 0 \rightarrow I_2 = \delta x_2, \end{cases} \quad (138)$$

$$\delta \alpha x_1^{\alpha-1} x_2^{2-\alpha} = \delta - \theta \rightarrow x_1 = \left[\frac{\delta - \theta}{\delta \alpha} x_2^{\alpha-2} \right]^{\frac{1}{\alpha-1}} = \left(\frac{\delta \alpha}{\delta - \theta} \right)^{\frac{1}{1-\alpha}} x_2^{\frac{2-\alpha}{1-\alpha}}, \quad (139)$$

$$\begin{cases} (\delta - \theta)\xi_2 = \xi_1 I_2 F_{x_2}(x_1, x_2) = \xi_1 I_2 (1 - \alpha) x_1^\alpha x_2^{-\alpha}, \\ \xi_2 = \xi_1 (I_2 - I_1), \end{cases} \quad (140)$$

$$(\delta - \theta)\xi_2 = \frac{\xi_2}{I_2 - I_1} I_2 (1 - \alpha) \left(\frac{x_1}{x_2} \right)^\alpha, \quad (141)$$

$$(\delta - \theta) \left(\delta x_2 - \delta \frac{x_1}{x_2} \right) = \delta x_2 (1 - \alpha) \left(\frac{x_1}{x_2} \right), \quad (142)$$

$$\frac{(\delta - \theta)}{\delta(1 - \alpha)} \delta \left[x_2 - \left(\frac{\delta \alpha}{\delta - \theta} \right)^{\frac{1}{1-\alpha}} x_2^{\frac{1}{1-\alpha}} \right] = \left(\frac{\delta \alpha}{\delta - \theta} \right)^{\frac{\alpha}{1-\alpha}} x_2 \left(x_2^{\frac{\alpha}{1-\alpha}} \right), \quad (143)$$

$$\frac{\delta - \theta}{1 - \alpha} \left[1 - \left(\frac{\delta \alpha}{\delta - \theta} \right)^{\frac{1}{1-\alpha}} x_2^{\frac{\alpha}{1-\alpha}} \right] = \left[\frac{\delta \alpha}{\delta - \theta} \right]^{\frac{\alpha}{1-\alpha}} x_2^{\frac{\alpha}{1-\alpha}}, \quad (144)$$

$$x_2^{\frac{\alpha}{1-\alpha}} \left[\left(\frac{\delta \alpha}{\delta - \theta} \right)^{\frac{\alpha}{1-\alpha}} + \frac{\delta - \theta}{1 - \alpha} \left(\frac{\delta \alpha}{\delta - \theta} \right)^{\frac{1}{1-\alpha}} \right] = \frac{\delta - \theta}{1 - \alpha}, \quad (145)$$

$$x_2^{\frac{\alpha}{1-\alpha}} \left(\frac{\delta \alpha}{\delta - \theta} \right)^{\frac{\alpha}{1-\alpha}} \left[1 + \frac{\delta - \theta}{1 - \alpha} \frac{\delta \alpha}{\delta - \theta} \right] = \frac{\delta - \theta}{1 - \alpha}, \quad (146)$$

$$x_2 \frac{\delta\alpha}{\delta - \theta} \left(1 + \frac{\delta\alpha}{1 - \alpha}\right)^{\frac{1-\alpha}{\alpha}} = \left(\frac{\delta - \theta}{1 - \alpha}\right)^{\frac{1-\alpha}{\alpha}}. \quad (147)$$

With these optimal conditions, we can compute the steady state values of the different variables x_1^s , x_2^s , I_1^s and I_2^s .

$$x_2^s = \frac{\delta - \theta}{\delta\alpha} \left(\frac{\delta - \theta}{1 - \alpha}\right)^{\frac{1-\alpha}{\alpha}} \left(1 + \frac{\delta\alpha}{1 - \alpha}\right)^{\frac{\alpha-1}{\alpha}}. \quad (148)$$

A.5.7 Super policy maker problem: distances

To solve the super policy maker's problem, we first need to obtain the distances that represent the disagreements on the valuation of a respective economic factor. These distances are the differences in the growth rates of the net marginal valuations (represented by the Lagrange multipliers of the social planner and the household problem) and can be written as:

$$\begin{cases} D_1 = \frac{\dot{\xi}_1}{\xi_1} - \frac{\dot{\Psi}_1}{\Psi_1} = I_2 F_{x_1}^{PM}(x_1, x_2) + F_{x_1}^H(x_1, x_2) = -I_2 \alpha (x_1^{PM})^{\alpha-1} (x_2^{PM})^{1-\alpha} + \alpha \left(\frac{x_1^H}{x_2^H}\right)^{\alpha-1}, \\ D_2 = \frac{\dot{\xi}_2}{\xi_2} - \frac{\dot{\Psi}_2}{\Psi_2} = -\frac{I_2}{I_1 - I_2} F_{x_2}^{PM}(x_1, x_2) + F_{x_2}^H(x_1, x_2) = -\frac{I_2}{I_1 - I_2} (1 - \alpha) \left(\frac{x_1^{PM}}{x_2^{PM}}\right)^{\alpha} + (1 - \alpha) \left(\frac{x_1^H}{x_2^H}\right)^{\alpha}. \end{cases} \quad (149)$$

A.5.8 Super policy maker problem: first-order conditions

As a first step, we take first-order conditions with respect to I_1, I_2 . We have:

$$\frac{\partial H(x_1, x_2, I_1, I_2)}{\partial I_1} = (1 - \alpha)^2 \left(\frac{x_1}{x_2}\right)^{2\alpha} 2 \left(\frac{I_1}{I_1 - I_2}\right) \frac{(-I_2)e^{\theta(t-s)}}{(I_1 - I_2)^2} + \mu_1 I_2 = 0, \quad (150)$$

$$\frac{\partial H(x_1, x_2, I_1, I_2)}{\partial I_2} = -\alpha^2 \left(\frac{x_1}{x_2}\right)^{2(\alpha-1)} 2(1 - I_2)e^{\theta(t-s)} + (1 - \alpha)^2 \left(\frac{x_1}{x_2}\right)^{2\alpha} 2 \left(\frac{I_1}{I_1 - I_2}\right) \frac{-I_1 e^{\theta(t-s)}}{(I_1 - I_2)^2} + \mu_1 I_1 + \mu_2 = 0. \quad (151)$$

A.5.9 Super policy maker problem: derivation of steady state

We start to derive the steady state. Therefore, we depart from the following system of equations, consisting of the first-order conditions and the transversality conditions:

$$\begin{cases} \dot{\mu}_1(t) = \alpha^2 2(\alpha - 1)x_1(t)^{2(\alpha-1)-1} x_2(t)^{-2(\alpha-1)-1} (1 - I_2(t))^2 e^{\theta(t-s)} \\ \quad + (1 - \alpha)^2 2\alpha x_1^{2\alpha-1} x_2(t)^{-2\alpha} \left(\frac{I_1(t)}{I_1(t) - I_2(t)}\right)^2 e^{\theta(t-s)} - \delta\mu_1(t), \\ \dot{\mu}_2(t) = -\alpha^2 2(\alpha - 1)x_1(t)^{2(\alpha-1)} x_2(t)^{-2(\alpha-1)-1} (1 - I_2(t))^2 e^{\theta(t-s)} \\ \quad - (1 - \alpha)^2 2\alpha x_1(t)^{2\alpha} x_2(t)^{-2\alpha-1} \left(\frac{I_1(t)}{I_1(t) - I_2(t)}\right)^2 e^{\theta(t-s)} - \delta\mu_2(t), \\ \lim_{t \rightarrow \infty} \mu_1(t) I_1(t) = 0, \\ \lim_{t \rightarrow \infty} \mu_2(t) I_2(t) = 0. \end{cases} \quad (152)$$

We derive the optimality conditions:

$$\begin{cases} \phi_1 = \mu_1 e^{-\theta(t-s)} \\ \dot{\phi}_1 = \dot{\mu}_1 e^{-\theta(t-s)} - \theta \phi_1 \end{cases} \quad (153)$$

$$\phi_1 I_2 - (1 - \alpha)^2 \left(\frac{x_1}{x_2} \right)^{2\alpha} 2 \frac{I_1 I_2}{(I_1 - I_2)^3} = 0 \quad (154)$$

$$I_2 \left[\phi_1 - 2(1 - \alpha)^2 \left(\frac{x_1}{x_2} \right)^{2\alpha} \frac{I_1}{(I_1 - I_2)^3} \right] = 0 \quad (155)$$

We go on to obtain ϕ_2 :

$$\phi_1 I_1 + \phi_2 = 2\alpha^2 \left(\frac{x_1}{x_2} \right)^{2(\alpha-1)} (1 - I_2) + 2(1 - \alpha)^2 \left(\frac{x_1}{x_2} \right)^{2\alpha} \frac{I_1^2}{(I_1 - I_2)^3} \quad (156)$$

$$\phi_2 = 2\alpha^2 \left(\frac{x_1}{x_2} \right)^{2(\alpha-1)} (1 - I_2) \quad (157)$$

$$-\phi_1 \theta - \dot{\phi}_1 = 2\alpha^2 (\alpha - 1) x_1^{2(\alpha-1)-1} x_2^{-2(\alpha-1)} (1 - I_2)^2 + 2\alpha(1 - \alpha)^2 x_1^{2\alpha-1} x_2^{-2\alpha} \left(\frac{I_1}{I_1 - I_2} \right)^2 - \delta \phi_1 \quad (158)$$

$$\dot{\phi}_1 = \phi_1 (\delta - \theta) + (1 - I_2) \frac{\phi_2}{x_1} + \alpha \frac{I_1}{I_1 - I_2} \frac{\phi_1}{x_1} \quad (159)$$

$$\dot{\phi}_1 = \phi_1 \left(\delta - \theta - \frac{\alpha}{x_1} \frac{I_1}{I_1 - I_2} \right) + \phi_2 \frac{1 - I_2}{x_1} \quad (160)$$

and

$$-\dot{\phi}_2 - \theta \phi_2 = -2\alpha^2 (\alpha - 1) x_1^{2(\alpha-1)} x_2^{-2(\alpha-1)-1} (1 - I_2)^2 - (1 - \alpha)^2 2\alpha x_1^{2\alpha} x_2^{-2\alpha-1} \left(\frac{I_1}{I_1 - I_2} \right)^2 - \delta \phi_2 \quad (161)$$

$$\dot{\phi}_2 = \phi_2 (\delta - \theta) + (1 - \alpha) \frac{1 - I_2}{x_2} \phi_2 - \frac{I_1 (I_1 - I_2)}{x_2} \alpha \phi_1 \quad (162)$$

and we obtain $\dot{\phi}_2$

$$\dot{\phi}_2 = \phi_2 \left[\delta - \theta + (1 - \alpha) \frac{1 - I_2}{x_2} \right] - \frac{I_1 (I_1 - I_2)}{x_2} \alpha \phi_1 \quad (163)$$

We can characterise the system of equations via the following optimality conditions. Let ϕ denote the time-stable expression of μ :

$$\begin{cases} \dot{x}_1 = I_1 I_2 - \delta x_1 \\ \dot{x}_2 = I_2 - \delta x_2 \\ \dot{\phi}_1 = \phi_1 \left(\delta - \theta - \frac{\alpha}{x_1} \frac{I_1}{I_1 - I_2} \right) + \phi_2 \frac{1 - I_2}{x_1} \\ \dot{\phi}_2 = \phi_2 \left[\delta - \theta + (1 - \alpha) \frac{(1 - I_2)}{x_2} \right] - \frac{I_1 (I_1 - I_2)}{x_2} \alpha \phi_1 \\ \phi_1 = 2(1 - \alpha)^2 \left(\frac{x_1}{x_2} \right)^{2\alpha} \frac{I_1}{(I_1 - I_2)^3} \\ \phi_2 = 2\alpha^2 \left(\frac{x_1}{x_2} \right)^{2(\alpha-1)} (1 - I_2) \end{cases} \quad (164)$$

Given the simplicity of our constraints, the derivation of steady state expressions for I_1 is straightforward. Investment in x_1 depends on the ratio of x_1, x_2 whereas investment in x_2 depends only on x_2 and δ :

$$I_1 = \frac{x_1}{x_2} \text{ and } I_2 = \delta x_2. \quad (165)$$

Moreover, we can describe the ratios of the detrended Lagrange multipliers of the super policy maker problem ϕ . This a pre-step to obtain a system of equations in I_1, I_2 :

$$\begin{cases} \frac{\phi_1}{\phi_2} = \frac{\frac{1-I_2}{x_1}}{\frac{\alpha}{x_1} \frac{I_1}{I_1-I_2} - \delta + \theta}, \\ \frac{\phi_1}{\phi_2} = \frac{\delta - \theta + (1-\alpha) \frac{1-I_2}{x_2}}{\frac{I_1(I_1-I_2)}{x_2}}, \end{cases} \quad (166)$$

so that we can characterize a system of equations that allows to identify I_1, I_2 .

A.5.10 Super policy maker problem: dynamic system for numerical solution

Using (166), we can derive an expression that allows to characterize a system of equations from which we can numerically derive I_1, I_2 .

$$\frac{1 - I_2}{\alpha \frac{I_1}{I_1 - I_2} - (\delta - \theta)x_1} = \frac{x_2(\delta - \theta) + (1 - \alpha)(1 - I_2)}{I_1(I_1 - I_2)} \quad (167)$$

We need a second expression of I_1, I_2 as we have two unknowns. We can derive from the first two equations of (164):

$$\frac{\phi_1}{\phi_2} = \frac{(1 - \alpha)^2}{\alpha^2} \left(\frac{x_1}{x_2} \right) \frac{I_1}{(I_1 - I_2)^3 (1 - I_2)} \quad (168)$$

$$\begin{cases} (1 - I_2)I_1(I_1 - I_2) = \left[x_2(\delta - \theta) + (1 - \alpha)(1 - I_2) \right] \left[\alpha \frac{I_1}{I_1 - I_2} - (\delta - \theta)x_1 \right] \\ (1 - I_2)(I_1 - I_2) = \left(\frac{1 - \alpha}{\alpha} \right)^2 \left(\frac{x_1}{x_2} \right) \frac{I_1}{(I_1 - I_2)^3 (1 - I_2)} \end{cases} \quad (169)$$

$$\begin{cases} (1 - I_2)I_1(I_1 - I_2)^2 = \left[x_2(\delta - \theta) + (1 - \alpha)(1 - I_2) \right] \left[\alpha I_1 - (\delta - \theta) \frac{I_1 I_2}{\delta} x_1 (I_1 - I_2) \right] \\ (1 - I_2)^2 (I_1 - I_2)^4 = \left(\frac{1 - \alpha}{\alpha} \right)^2 I_1 \end{cases} \quad (170)$$

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