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Diversification benefits of precious metal markets: A Stochastic Spanning Approach

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Abstract

This paper investigates the diversification contribution of four main precious metals (i.e., gold, silver, platinum, and palladium) to a traditional portfolio of stocks, bonds, cash, and currencies, as well as the impact of the global financial crisis of 2008 on that contribution. We use a stochastic spanning methodology ([Arvanitis et al. 2019](#)) to test whether the traditional portfolio spans the portfolio augmented with all four precious metals in the G7 countries over three periods: before, during, and after the global financial crisis of 2008. In this study, we perform both in-sample and out-of-sample analyses of stochastic spanning tests. Our empirical results confirm the diversification benefits of precious metals for traditional assets in both spot and futures returns. These findings hold over three sub-periods of the global financial crisis of 2008. Notably, we find that precious metal futures significantly outperform precious metal spots in the aftermath of the 2008 crisis. Investors should therefore favor precious metal futures in post-crisis periods to improve their investment diversification.

Keywords: Precious metals, Stochastic spanning, Optimal portfolio, Diversification benefits, Second - order stochastic dominance

1. Introduction

Precious metals are utilized as important stores of value and monetary assets, as well as in various industrial activities. In the past, investors typically did not consider precious

metals when constructing investment portfolios. Yet precious metals have recently played an increasingly important role as potential alternatives for traditional assets (e.g., stocks, bonds, currencies) in diversified modern portfolios, especially in times of high uncertainty (Conover et al. 2009, Baur & McDermott 2010, Figuerola-Ferretti & McCrorie 2016). This trend has also increased interest in studying the potential diversification benefits of precious metals (Conover et al. 2009, Figuerola-Ferretti & McCrorie 2016). A large number of studies has assessed the diversification contributions of commodities in general (Gorton & Rouwenhorst 2006, Chong & Miffre 2010, Belousova & Dorfleitner 2012a, Narayan et al. 2013, Gao & Nardari 2018, Bessler & Wolff 2015, Wen & Nguyen 2017, Daskalaki et al. 2017a) or gold in particular (Jaffe 1989, Johnson & Soenen 1997, Ratner & Klein 2008, Agyei-Ampomah et al. 2014, Bedoui et al. 2018). Other precious metals, including silver, platinum, and palladium, which likely outperform gold in a turbulent environment (McCown & Shaw 2017, Baur & Lucey 2010), have been relatively overlooked.

The emerging financialization of precious metal markets over the last few decades has caused an intense public and scientific debate on precious metals investment and their diversification benefits. Many studies, including Jaffe (1989), Chua et al. (1990) and Hillier et al. (2006), present strong evidence that adding precious metals improves the overall performance of stock and bond portfolios. (Hillier et al. 2006), (Conover et al. 2009), (Baur & McDermott 2010) and (Jain & Biswal 2016) demonstrate that precious metals all have low correlations with stock and bond returns during turbulent periods and financial crises, which enables these metals to be a part of a well-diversified portfolio. Using M-V spanning tests, Belousova & Dorfleitner (2012a) also find that precious metals provide the highest performance improvement for traditional portfolios compared to other commodities. Several other studies, however, argue against such diversification benefits (Cao et al. 2010, Daskalaki & Skiadopoulos 2011). These studies argue that the current over-financialization of precious metal markets and the considerable rise of capital inflows into these markets may cause an increased correlation between precious metal

markets and traditional markets, reducing their diversification contribution.

Throughout these debates, the role of precious metals in portfolio diversification has constantly been changing. In particular, the more uncertain the global financial system, the more the role of precious metals is reinforced ([Białkowski et al. 2015](#), [Balcilar et al. 2015](#)). Recent publications have demonstrated that financial crisis-related uncertainty shocks significantly impact precious metal market dynamics ([Morales & Andreosso-O'Callaghan 2011](#), [Hammoudeh et al. 2011](#)). Surprisingly, however, there is limited in-depth research looking at the impact of financial crises on the diversification benefits of precious metals. As measuring the contribution of diversification under different market conditions remains challenging and requires accurate modeling of multi-asset dependency patterns, this article investigates the performance of gold, silver, platinum, and palladium over various periods of the global financial crisis of 2008.

Our research employs stochastic spanning methodology ([Arvanitis et al. 2019](#)) that focuses on the second-order stochastic dominance (SSD). This methodology allows us to effectively and accurately assess the role of using precious metals spots and futures to gain diversification benefits. This choice of methods matches the economic application of the expected utility theory and the dual theory of risk ([Yaari 1987](#)), which risk-averse investors typically use. In this study, we construct the traditional portfolio from four asset classes, including stocks, bonds, cash, and currencies. Two augmented portfolios are built by adding, in the first case, the spot returns of four precious metals and, in the second case, their future returns. Using this approach, our study sheds light on whether and how investors can benefit from diversification through investing in precious metals or their financial instruments. We conduct this research in three sub-periods: before the financial crisis of 2008 (from January 2005 to August 2007), during the crisis (from September 2007 to December 2010), and after the crisis (from January 2011 to September 2018). We also use daily data collected from the G7 countries as international evidence to clarify the diversification potential of precious metals in global financial markets.

This study includes both in-sample and out-of-sample analyses to assess the diversification benefits of precious metals for traditional assets. Under the in-sample analysis, the traditional portfolio does not span augmented portfolios of both spots and futures returns of all four metals, meaning that precious metals are good diversifiers of traditional asset classes and effective investments for enhancing diversification benefits. This finding is confirmed in an out-of-sample analysis. Notably, our out-of-sample analysis shows that futures contracts on precious metals significantly outperform precious metals spots after the 2008 global financial crisis. This finding is in line with the study of [Klein & Todorova \(2019\)](#), who find that the future contracts of precious metals have become the most widely-used instruments for risk management, particularly in more volatile periods. Finally, parametric tests of out-of-sample performance assessment reconfirm the diversification benefits of both precious metals' spots and futures.

Overall, our paper contributes to the existing literature in several ways. First, the stochastic spanning approach that we apply in this study to construct optimal portfolios and compare their performance in a non-parametric way seems to outperform the previous models. The prior works focused mainly on the issue of asset diversification employing asset pricing models ([Brewer 1981](#), [De Santis & Gerard 1997](#)), Bayesian inference ([Li et al. 2003](#)), market co-movement approach ([Christoffersen et al. 2014](#), [Reboredo 2018](#)), or M-V framework ([Huberman & Kandel 1987](#), [Belousova & Dorfleitner 2012a](#)). The advantages of stochastic spanning are discussed in more detail in section 2. Second, most of the studies mentioned above only focus on the diversification benefits of precious metals for the U.S. market ([Conover et al. 2009](#), [Figuerola-Ferretti & McCrorie 2016](#)). However, to the best of our knowledge, this study is the first research on these benefits for international portfolios (i.e., the G7 markets). Third, there is a lack of holistic research on the diversification benefits of precious metals and their derivatives. Our study provides a broad investigation into various types of these commodities by testing the diversification contributions of spot and future returns of four precious metals separately. Finally, we attempt to answer whether the diversification role of

precious metals changes between tranquil and turbulent times. Therefore, we pay attention to the global financial crisis of 2008. From this, we firmly believe that we can provide valuable recommendations to policy-makers, investors, and portfolio managers in the event of any future global financial crisis.

The rest of the article is organized as follows. Section 2 gives theoretical explanations for using stochastic spanning in our study. Section 3 describes the mathematical perspective of the stochastic spanning methodology. Section 4 reports our data and its properties. Section 5 summarizes the empirical results in both in-sample and out-of-sample analyses. Section 6 concludes and discusses the implications of our research.

2. Theoretical Framework for Assessing Diversification Benefits

According to the portfolio investment strategies, investors might hold a wide range of assets available in the financial markets. The supposed benefits of diversification (i.e., risk reduction for any expected return on an investment asset or a portfolio) might suggest that investors should strive to spend money on as many assets as possible. However, this suggestion is not entirely true. Diversification has remained limited (Bodie et al. 1992), as adding more assets could even lead to a reduction in the portfolio performance due to transaction fees or other costs. This limitation raises the question of whether introducing a new set of risky assets may improve the traditional portfolio (Kan & Zhou 2012) by bringing diversification benefits. The concept of spanning helps address the issue of limited diversification. Spanning occurs if introducing new assets or relaxing investment constraints does not improve the investment possibility set for a given class of investors (Arvanitis et al. 2019). The concept of spanning is firmly linked to the diversification benefits of new assets. Diversification benefits arise if adding new assets into the original portfolio leads to the efficient frontier shifting leftward significantly (Bekaert & Urias 1996).

Huberman & Kandel (1987) first introduced the mean-variance spanning test (hereinafter called “M-V spanning”). M-V spanning is based on the M-V framework of Markowitz (1952)

in order to assess the effect of adding more risky assets into a portfolio. This test is specifically designed to evaluate the statistical significance of improving the augmented efficient frontier over the initial frontier of benchmark assets. Spanning happens when the augmented portfolio produces an efficient frontier that coincides with the initial portfolio. However, the limitation of the M-V spanning approach is rooted in two strict assumptions. The first corresponds with the study of [Hanoch & Levy \(2013\)](#), which requires normally distributed returns of assets. The second comes from [Von Neumann et al. \(2007\)](#) assumption, which demands quadratic utility functions. If these assumptions are broken, the results become unreliable.

In order to remedy the weakness of the M-V spanning test, [Daskalaki & Skiadopoulos \(2011\)](#) applies the spanning test for non-M-V utility functions (i.e., not in the form of a quadratic function). This test uses stochastic discount factors (SDF) to analyze spanning, and the Newey and West's method ([Newey & West 1987](#)) to adjust the standard errors of coefficients. Meanwhile, [Belousova & Dorfleitner \(2012b\)](#) states that the assumption of normally distributed returns would not be a problem if conditional homoskedasticity were satisfied, driving them to propose applying GMM-based spanning test ([Ferson et al. 1993](#), [De Santis et al. 1997](#)). This test is reinforced by [Kan & Zhou \(2008\)](#), which confirms its efficacy with a regression approach.

Despite the progress in dealing with the drawbacks of the M-V approach, the methods mentioned above have not fully sorted out the issues of normally distributed returns and higher-order moment risk. To deal with these issues, [Arvanitis et al. \(2019\)](#) introduced stochastic spanning, a non-regression alternative to M-V spanning. The stochastic spanning test is conducted on a stochastic dominance (SD) platform. According to the portfolio selection theory, SD compares two investment portfolios and determines which is dominant. The framework for SD criteria was developed from the first studies of [Quirk & Saposnik \(1962\)](#), [Hadar & Russell \(1969\)](#), [Rothschild & Stiglitz \(1970\)](#), and [Bawa \(1975\)](#). Furthermore, the stochastic dominance rule is based on the expected utility paradigm and provides a uniform ordering of prospects independent of specific utility functions ([Whang 2019](#)). To define criteria

for dominance, SD frequently applies two settings. First-order stochastic dominance is suitable for both risk-averse investors and risk lovers. Second-order stochastic dominance is applied to risk averters, who are represented by concave utility functions.

Nevertheless, the main weakness of SD rules is that they can only classify two specific portfolios. It is, therefore, impossible to verify whether one portfolio dominates all others. To tackle this problem, [Post \(2003\)](#), [Scaillet & Topaloglou \(2010\)](#) and [Daskalaki et al. \(2017b\)](#) developed a statistical test of stochastic dominance efficiency (SDE) to determine the most efficient asset or portfolio out of a set of given assets or portfolios. [Arvanitis et al. \(2019\)](#) argued that the spanning test is appropriate for dealing with higher-order moment risk rather than SDE. The stochastic spanning test is justifiable since it is consistent and asymmetrically exact in the case of weakly dependent processes. Two other essential advantages of this method are that it does not assume the distribution of returns and that it considers all feasible portfolios. These advantages can help overcome the current concerns about methodologies used for testing the diversification benefits of new assets.

3. Stochastic Spanning Methodology

3.1. Definition

According to [Arvanitis et al. \(2019\)](#)

Portfolio set Λ is second-order stochastically spanned by subset $K \subset \Lambda$ if all portfolios $\lambda \in \Lambda$ are weakly second-order stochastically dominated by some portfolios $\kappa \in K$:

$$\forall \lambda \in \Lambda, \exists \kappa \in K : \forall x \in X : G(x, \kappa, \lambda; F) \leq 0$$

$$G(x, \kappa, \lambda; F) := F^2(x, \kappa) - F^2(x, \lambda)$$

In which,

$\Lambda := \{\lambda \in \mathbb{R}_+^N : \mathbf{1}_N^T \lambda = \mathbf{1}\}$ denotes the investment opportunity set which based on convex combinations of N base assets.

$X := (x_1, \dots, x_N)$ is the random vector that represents the returns of the base assets, with support bounded by $\mathcal{X}^N := [\underline{x}, \bar{x}]^N$, $-\infty < \underline{x} < \bar{x} < +\infty$.

F refers the continuous joint cumulative distribution function of X ,

$F(y, \lambda) := \int \mathbf{1}(X^T \lambda \leq y) dF(X)$ is the marginal cumulative distribution function for portfolio $\lambda \in \Lambda$. And

$$F^2(x, \lambda) := \int_{-\infty}^x F(y, \lambda) dy = \int_{-\infty}^x (x - y) dF(y, \lambda)$$

denoting the integrated cumulative distribution function.

[Arvanitis et al. \(2019\)](#) provide a measurement for deviation by spanning stochastic that is based on the scalar-valued functional of the population c.d.f F as follows:

$$\eta(F) := \sup_{\lambda \in \Lambda} \inf_{\kappa \in K} \sup_{x \in X} G(x, \kappa, \lambda; F)$$

The value of $\eta(F)$ is used to identify the likelihood of spanning. If $\eta(F) = 0$, it means no portfolio from Λ is not weakly dominated by a portfolio from K . In case $\eta(F) > 0$, stochastic spanning does not take place. [Arvanitis et al. \(2019\)](#) also suggest a transformation to estimate $\eta(F)$ by using expected utility functions as below:

$$\eta(F) = \sup_{\lambda \in \Lambda; u \in \mathcal{U}_2} \inf_{\kappa \in K} \mathbb{E}_F \left[u \left(X^T \lambda - u \left(X^T \kappa \right) \right) \right]$$

In which, \mathcal{U}_2 is a set of normalized, increasing, and concave functions that are designed from the convex combination of elementary [Russell & Seo \(1989\)](#) ramp function $r(y; x), x \in X$ as follows:

$$\mathcal{U}_2 := \left\{ u \in \mathcal{C}^0 : u(y) = \int_x^{\bar{x}} \omega(x) r(y; x) dx ; \omega \in \mathcal{W} \right\};$$

$$r(y; x) := (y - x) \mathbf{1}(y \leq x), (x, y) \in \mathcal{X}^2$$

Stochastic spanning occurs ($\eta(F) = 0$) in case no risk averters ($u \in \mathcal{U}_2$) take advantages from the enlargement $(\Lambda - K)$.

3.2. Statistical Framework and Critical Values

In theory, the c.d.f F is covert; in practice, financial analysts have data which is a discrete-time series of realized returns $(X_t)_{t=1}^T$, in which $X_t \in \mathcal{X}$. To solve this problem, [Arvanitis et al. \(2019\)](#) proposed using the empirical c.d.f to calculate the test statistic for the null hypothesis of stochastic spanning ($H_0 : \eta(F) = 0$, which represents the existence stochastic spanning) as follows:

$$\eta_T := \sqrt{T} \sup_{\lambda \in \Lambda} \inf_{\kappa \in K} \sup_{x \in X} G(x, \kappa, \lambda; F_T)$$

The null hypothesis H_0 is rejected if $\eta_T > q(\eta_\infty; 1 - \alpha)$. In empirical applications, computing the test statistic η_T is a challenging global optimization problem, that is estimated through a quasi-Monte Carlo and an embedded Linear Programming problem. The critical value corresponds with the $1 - \alpha$ quantile of the distribution regarding the relevant weak limit. However, the critical value is computed based on c.d.f F , and c.d.f F is a latent function in practice. Therefore, the basic decision rule is to reject H_0 against H_1 only if $\eta_T > q(\eta_\infty; 1 - \alpha)$ is infeasible. To make it become feasible, [Arvanitis et al. \(2019\)](#) employed a subsampling procedure and then conducted a regression to correct the estimation for the critical value. Notably, the subsampling procedure began by creating $(T - b_T + 1)$, which makes subsamples $(X_s)_{s=t}^{t+b_T-1}$, $t = 1, \dots, T - b_T + 1$ maximally overlapped. Then, test scores for each subsample are computed from the empirical joint c.d.f of $(X_s)_{s=t}^{t+b_T-1}$ given by $\eta_{b_T; T, t} = \sqrt{b_T} \eta(F_{b_T; T, t})$, from which the empirical distribution and the quantile function are presented by:

$$s_{T, b_T}(y) := \frac{1}{T - b_T + 1} \sum_{t=1}^{T - b_T + 1} \mathbf{1}(\eta_{b_T; T, t} \leq y)$$

$$q_{T, b_T}(1 - \alpha) := \inf_y \{y : s_{T, b_T}(y) \geq 1 - \alpha\}$$

The decision rule for rejecting the null hypothesis of spanning now at the given significant level is $\eta_T > q_{T, b_T}(1 - \alpha)$. This sub-sampling procedure is asymptotic exact, and reasonable if consistent assumptions about the length of the sub-sample and significance level are

always held. However, in practice, two problems may arise. Firstly, the quantile estimators $q_{T,b_T}(1 - \alpha)$ tend to be biased corresponding to the small sub-sample size b_T . Second, the choice of b_T can be quite sensitive. A regression-based method has been proposed to correct the bias (Anyfantaki & Topaloglou 2018, Chalamandaris et al. 2018, Arvanitis et al. 2019). More precisely, the quantile values $q_{T,b_T}(1 - \alpha)$ are calculated for a rational range of the sub-sample size b_T . From this calculation, OLS regression can be estimated using the following equation:

$$q_{T,b_T}(1 - \alpha) = \beta_{0;T,1-\alpha} + \beta_{1;T,1-\alpha}(b_T)^{-1} + \epsilon_{T;1-\alpha,b_T}$$

Finally, the bias-corrected quantile (at given significance level α) is estimated based on the estimators of intercept and slope of the above regression, which are the OLS predicted values in the case $b_T = T$:

$$q_T^{BC}(1 - \alpha) := \hat{\beta}_{0;T,1-\alpha} + \hat{\beta}_{1;T,1-\alpha}(T)^{-1}$$

4. Data and Properties

This section introduces the dataset used to perform stochastic spanning tests. Initial statistical analyses are given, including information about the descriptive statistics and Sharpe ratios of all considered assets and correlations between these assets over the three periods of the 2008 global financial crisis.

4.1. Data

The dataset consists of daily returns of traditional and augmented portfolios of seven countries belonging to the G7 group, which is the world's largest consumer of precious metals. All data is retrieved from Thomson Reuters DataStream. It covers three periods corresponding to three phases of the global financial crisis of 2008: pre-crisis (from January 03rd, 2005 to August 31st, 2007), crisis (from September 03rd, 2007 to December 31st, 2010), and post-crisis (from January 03rd, 2011 to September 20th, 2018), respectively. In this study, the traditional

portfolio is made up of four types of traditional assets, including shares, bonds, cash, and currencies. Notably, we use national stock market indices as proxies for stock assets ¹; 10-year government bond indices as proxies for bond assets; one-month LIBOR interest rates as proxies for cash in each country ²; and finally, exchange rates as proxies for currencies. We use each country's one-month LIBOR interest rate as the risk-free rate for its markets. As for precious metals, the study looks at two forms of these assets: spot and one-month futures.

4.2. Initial Statistical Analyses

Table 1 reports the descriptive statistics for the various asset classes, including the spot and futures return of all four precious metals and four traditional asset returns of G7 countries. The results show that the mean and standard deviation values of the spot returns of each precious metal are similar to those of their futures returns. Table 1 shows the mean is positive for all return series. Interestingly, precious metals return exhibit a higher standard deviation than traditional assets. Table 1 also summarizes the results of skewness and kurtosis. The skewness is not equal to zero and the kurtosis values are above three for most return series, indicating a non-normal distribution, which is also confirmed by a Jarque-Bera statistical test.

In addition to the descriptive statistics, the p -values of Ljung-Box (LB) statistics of the order 15 for the return series show the existence of auto-correlation in most of them. For the squared return series, however, the p -values of LB are highly significant, which reveals a long-range serial dependence. This result is confirmed by the results of the ARCH test, thus motivating our decision to use the GARCH model to filter the daily returns.

Finally, Table 1 provides the results of the traditional unit roots tests, including ADF, PP, and KPSS. These results give strong evidence against the null hypothesis of a unit root, highlighting that all the return series are stationary with a high significance level.

¹S&P500 of US, FTSE 100 of UK, S&P/TSX Composite Index of Canada, NIKKEI 225 of Japan, CAC 40 of France, DAX of Germany and FTSE MIB of Italy

²1 month US Dollar USD LIBOR interest rate, 1-month British pound sterling GBP LIBOR interest rate, Canadian dollar LIBOR rates, 1-month Japanese yen JPY LIBOR interest rate, 1-month euro LIBOR interest rate

Table 1: Descriptive Statistics and Stationary Test of Precious Metals and Traditional Assets

		Mean	Stdev	Skewness	Kurtosis	J.B.	Q(15)	Q ² (15)	ARCH(15)
Panel A: Precious metals									
Spot	Gold	0.0347	1.1306	-0.1043	4.9659	0.0000	0.0162	0.0000	0.0000
	Silver	0.0402	1.9716	-0.5029	6.1362	0.0000	0.4430	0.0000	0.0000
	Platinum	0.0085	1.3693	-0.5466	4.4916	0.0000	0.1165	0.0000	0.0000
	Palladium	0.0668	1.9143	-0.3899	3.2659	0.0000	0.4144	0.0000	0.0000
Future	Gold	0.0349	1.1518	-0.2159	5.6863	0.0000	0.0027	0.0000	0.0000
	Silver	0.0416	2.0447	-0.6362	6.2910	0.0000	0.1058	0.0000	0.0000
	Platinum	0.0090	1.4090	-0.3905	3.7328	0.0000	0.6019	0.0000	0.0000
	Palladium	0.0677	1.9335	-0.3282	3.4138	0.0000	0.0696	0.0000	0.0000
Panel B: Traditional assets of G7									
Stock	US	0.0313	1.1543	-0.1284	12.9188	0.0000	0.0000	0.0000	0.0000
	UK	0.0182	1.1196	0.0277	9.1011	0.0000	0.0000	0.0000	0.0000
	Japan	0.0308	1.4526	-0.3019	8.9664	0.0000	0.3950	0.0000	0.0000
	Canada	0.0213	1.0577	-0.5047	12.0736	0.0000	0.0000	0.0000	0.0000
	France	0.0191	1.3524	0.1477	7.5651	0.0000	0.0000	0.0000	0.0000
	Germany	0.0383	1.3144	0.1357	7.2767	0.0000	0.0552	0.0000	0.0000
	Italy	0.0016	1.5388	-0.0603	5.8304	0.0000	0.0012	0.0000	0.0000
Bond	US	0.0044	0.4620	0.1286	3.6511	0.0000	0.0032	0.0000	0.0000
	UK	0.0105	0.3985	0.1730	2.3668	0.0000	0.0027	0.0000	0.0000
	Japan	0.0067	0.1922	-0.1717	3.1283	0.0000	0.0251	0.0000	0.0000
	Canada	0.0068	0.3583	-0.0268	1.1791	0.0	0.3885	0.0000	0.0000
	France	0.0115	0.3448	-0.1659	2.6902	0.0000	0.0002	0.0000	0.0000
	Germany	0.0114	0.3515	-0.0973	2.1413	0.0000	0.0002	0.0000	0.0000
Cash	Italy	0.0081	0.4985	0.5897	16.4242	0.0000	0.0000	0.0000	0.0000
	US	0.4301	0.4935	1.1081	-0.3111	0.0000	0.0000	0.0000	0.0000
	UK	0.5160	0.5750	0.9985	-0.8541	0.0000	0.0000	0.0000	0.0000
	Japan	0.0396	0.0814	1.2103	2.5917	0.0000	0.0000	0.0000	0.0000
	Canada	0.4691	0.3636	1.0843	-0.2436	0.0000	0.0000	0.0000	0.0000
	France	0.3045	0.4348	0.9686	-0.4250	0.0000	0.0000	0.0000	0.0000
Currency	Germany	0.3045	0.4348	0.9686	-0.4250	0.0000	0.0000	0.0000	0.0000
	Italy	0.3045	0.4348	0.9686	-0.4250	0.0000	0.0000	0.0000	0.0000
	US	0.0043	0.4557	-0.2841	4.4938	0.0000	0.2807	0.0000	0.0000
	UK	0.0123	0.6193	0.8453	14.3510	0.0000	0.0000	0.0000	0.0000
	Japan	0.0047	0.6452	-0.2178	4.3681	0.0000	0.1234	0.0000	0.0000
	Canada	0.0040	0.6140	-0.0565	5.0183	0.0000	0.0319	0.0000	0.0000
Currency	France	0.0059	0.6011	-0.0914	3.2534	0.0000	0.4008	0.0000	0.0000
	Germany	0.0059	0.6011	-0.0914	3.2534	0.0000	0.4008	0.0000	0.0000
	Italy	0.0059	0.6011	-0.0914	3.2534	0.0000	0.4008	0.0000	0.0000
	France	0.0059	0.6011	-0.0914	3.2534	0.0000	0.4008	0.0000	0.0000

Note: The table shows the descriptive statistics for the daily returns series of four precious metals and four traditional assets of G7 countries over the full sample period (Jan 03, 2005, to Sep 20, 2018). J.B. refers P-value of the Jarque-Bera test for normality. Q(15) and Q2(15) denote the Ljung-Box Q-test statistics for auto-correlation in the return and squared return series, respectively, up to 15 lag. Furthermore, ARCH(15) is the (Engle 1982), LM test for ARCH effects up to 15 lags.

A Sharpe ratio (developed by William F. Sharpe) can be calculated for each asset in both the traditional and augmented portfolios by using the one-month Libor rate as a proxy for the risk-free rate. This ratio is often used to analyze the change in overall risk-return trade-off when a new asset or asset class is added to a portfolio. Generally, the greater the value of the Sharpe ratio, the more attractive the risk-adjusted return becomes. Table 2 shows the Sharpe ratio of each asset in each country, revealing that the Sharpe ratio of all four precious metals in the pre-crisis and crisis periods was positive and higher than that of stocks, bonds, and currencies in all seven countries. In contrast, this ratio of traditional markets in all G7 members was negative during the crisis and pre-crisis times, except for several stock markets in the pre-crisis period (Canada, Japan, France, Germany, and Italy). This negative ratio suggests that the incorporation of precious metals into an original portfolio of stocks, bonds, cash, and currencies could provide diversification benefits prior to and during the global financial crisis of 2008. However, it seems that the Sharpe ratio of traditional assets in the post-crisis era followed the opposite trend. More specifically, excepting palladium, the Sharpe ratio of precious metals post-crisis was negative and lower than that of traditional assets. This finding is consistent with the studies of [Cao et al. \(2010\)](#), [Delatte & Lopez \(2013\)](#), which suggest that the rapid financialization of the precious metals markets and the sharp rise in investment capital flow in these markets after the 2008 global crisis may have caused a higher correlation between precious metals and traditional assets, hence, reducing their diversification contribution.

The daily return movements of both precious metals and traditional assets over the three periods are displayed in Figure 5 in the Appendix. Almost all precious metals had considerably higher standard deviations than those of equities, bonds, cash, and currencies. The return series of silver was the most turbulent. Similar to the Sharpe ratio's movement over three periods, the behavior of precious metals returns during the crisis was not very different from that of the pre-crisis period. However, their returns in the post-crisis period demonstrated a different volatility.

Table 2: Sharpe ratios of traditional assets and precious metals in three periods of the global financial crisis

Data	USA	UK	Canada	Japan	France	Germany	Italy
A. Before global crisis, 03/01/2005 - 31/08/2007							
Stock	-0.0147	-0.0043	0.0338	0.0535	0.0393	0.0689	0.0186
Bond	-0.1448	-0.1897	-0.1229	-0.0044	-0.1220	-0.1231	-0.1289
Cash	-	-	-	-	-	-	-
Currency	-0.1170	-0.1047	-0.1007	0.0321	-0.0533	-0.0533	-0.0533
Gold	0.0247	0.0209	0.0319	0.0607	0.0383	0.0383	0.0383
Silver	0.0311	0.0290	0.0352	0.0512	0.0387	0.0387	0.0387
Platinum	0.0191	0.0153	0.0262	0.0544	0.0324	0.0324	0.0324
Palladium	0.0319	0.0297	0.0362	0.0529	0.0398	0.0398	0.0398
B. During global crisis, 03/09/2007 - 31/12/2010							
Stock	-0.0087	-0.0105	-0.0026	-0.0202	-0.0254	-0.0088	-0.0421
Bond	0.0023	-0.0125	-0.0040	0.0238	-0.0184	-0.0101	-0.0440
Cash	-	-	-	-	-	-	-
Currency	-0.0313	0.0109	-0.0226	-0.0510	-0.0191	-0.0191	-0.0191
Gold	0.0568	0.0502	0.0550	0.0637	0.0531	0.0531	0.0531
Silver	0.0512	0.0472	0.0501	0.0554	0.0490	0.0490	0.0490
Platinum	0.0223	0.0172	0.0210	0.0278	0.0195	0.0195	0.0195
Palladium	0.0482	0.0443	0.0472	0.0524	0.0461	0.0461	0.0461
C. After global crisis, 03/01/2011 - 20/09/2018							
Stock	0.0418	0.0066	0.0039	0.0316	0.0208	0.0293	0.0038
Bond	-0.0151	0.0084	-0.0101	-0.0120	0.0468	0.0426	0.0093
Cash	-	-	-	-	-	-	-
Currency	0.0039	0.0012	0.0101	0.0148	0.0145	0.0135	-0.0019
Gold	-0.0129	-0.0129	-0.0129	-0.0129	-0.0035	-0.0041	-0.0129
Silver	-0.0180	-0.0180	-0.0180	-0.0180	-0.0128	-0.0131	-0.0180
Platinum	-0.0329	-0.0329	-0.0329	-0.0329	-0.0253	-0.0258	-0.0329
Palladium	0.0110	0.0110	0.0110	0.0110	0.0165	0.0161	0.0110

Note: The table shows the Sharpe ratio of traditional assets in the G7 markets and precious metals across three phases of the 2008 financial crisis. We use the one-month Libor rate to proxy the risk-free rate.

Finally, Figure 6 reports the unconditional correlation coefficients between the individual precious metal and benchmark assets. Most precious metals have a very low correlation with traditional assets. The correlation with fixed-income markets (i.e., bonds, cash, and currencies) is negative, whereas the correlation with equity markets is slightly positive. The correlation matrix results repeatedly confirm the findings of [Cao et al. \(2010\)](#), [Delatte & Lopez \(2013\)](#), i.e., that the considerable increase in capital inflows into precious metals markets can reduce their diversification benefits because of the higher correlation between precious metals markets and stock and bond markets. As seen in Figure 6c in comparison with Figure 6a and Figure 6b, the correlation between precious metals returns and stocks, bonds, cash, and currencies in the post-crisis period was higher than during the crisis or pre-crisis. The increased correlation between assets after the crisis can be explained by the fact that when there is a significant increase in uncertainty in the global financial system, the role of precious metals becomes more essential and can be a part of a well-diversified portfolio ([Białkowski et al. 2015](#), [Balcilar et al. 2015](#)). The increased significance of precious metals results in over-investment in these markets during the crisis. Thus, the correlation between precious metals and traditional assets increased in the subsequent period.

5. Empirical Results

This section investigates whether the portfolios augmented with precious metals spot and precious metals futures are spanned by a traditional portfolio. Subsection 5.1 presents an in-sample analysis that tests the spanning hypothesis by estimating the critical value and comparing it with the test statistic value. Subsection 5.2 then tests whether the inclusion of the spot returns or future returns of four precious metals provides diversification benefits in an out-of-sample setting. The section closes by carrying out parametric tests that assess the out-of-sample performance in subsection 5.3.

5.1. In-sample Analysis

This subsection interrogates in-sample the null hypothesis that the traditional portfolio spans the augmented portfolios. In order to reduce the sensitivity to the choice of b_T in finite samples of realistic time series and cross-sectional dimensions, the bias correction proposed by [Arvanitis et al. \(2019\)](#) was applied for the quantile estimates q_{T,b_T} . Accordingly, subsample size $b_T \in [T^{0.6}, T^{0.7}, T^{0.8}, T^{0.9}]$ was chosen to obtain the subsampling distribution of the test statistic. After that, OLS regression was used on the empirical quantiles q_{T,b_T} with the significance level $\alpha = 0.05$ in order to estimate the critical value q_T^{BC} . Spanning was rejected if the test statistic η_T^* is higher than the regression estimate q_T^{BC} .

Table 3 reports the results of the test statistic η_T^* and the regression estimate q_T^{BC} . In both augmentation cases (i.e., the portfolio augmented with the spot or future of precious metals) and any period relevant to the global financial crisis of 2008, the optimal traditional portfolio cannot span any of its optimal augmented ones. These results indicate that augmenting the traditional portfolio with the spot or futures of four precious metals can improve its investment performance. Risk-averse investors might therefore be better off by performing such augmentations.

Table 3: In-sample tests

Periods	Variables	Test statistic η_T^*	Reg. Estimates q_T^{BC}	Results
Full period	Spots	0.2680	0.2435	<i>Reject Spanning</i>
	Futures	0.3210	0.2651	<i>Reject Spanning</i>
Pre-Crisis	Spots	0.3609	0.2820	<i>Reject Spanning</i>
	Futures	0.3412	0.2430	<i>Reject Spanning</i>
Crisis	Spots	0.1684	0.1567	<i>Reject Spanning</i>
	Futures	0.1743	0.1458	<i>Reject Spanning</i>
Post-Crisis	Spots	0.2645	0.1935	<i>Reject Spanning</i>
	Futures	0.2815	0.2012	<i>Reject Spanning</i>

Note: The table reports the results of the test statistic η_T^* and the regression estimate q_T^{BC} for two augmentation cases (i.e., precious metals spot and future returns) over the whole sample period (Jan 2005 to Sep 2018) and three sub-sample periods, including pre-crisis (Jan 2005 to Aug 2007), crisis (Sept 2007 to Dec 2010), and post-crisis (Jan 2011 to Sep 2018), respectively. We reject spanning if the test statistic η_T^* is higher than the regression estimate q_T^{BC} .

5.2. Out-of-sample Analysis

Next, this subsection conducts an out-of-sample setting to examine whether the augmented portfolios outperform the traditional ones.

Three optimal portfolios were built separately from various asset universes. The first portfolio includes only traditional asset classes (i.e., stocks, bonds, cash, and currencies). The second one is the traditional portfolio augmented with precious metal spots, and the last is the traditional portfolio augmented with precious metal futures. The out-of-sample analysis covers the period from 03 Jan 2005 to 20 Sep 2018. The data from the first year was used as the first training set for each day to calibrate the procedure. Thus, the out-of-sample period began from 02 Jan 2006 to 20 Sep 2018. The optimization problem was solved for the stochastic spanning test and the optimal weights were then recorded for both traditional and augmented portfolios.

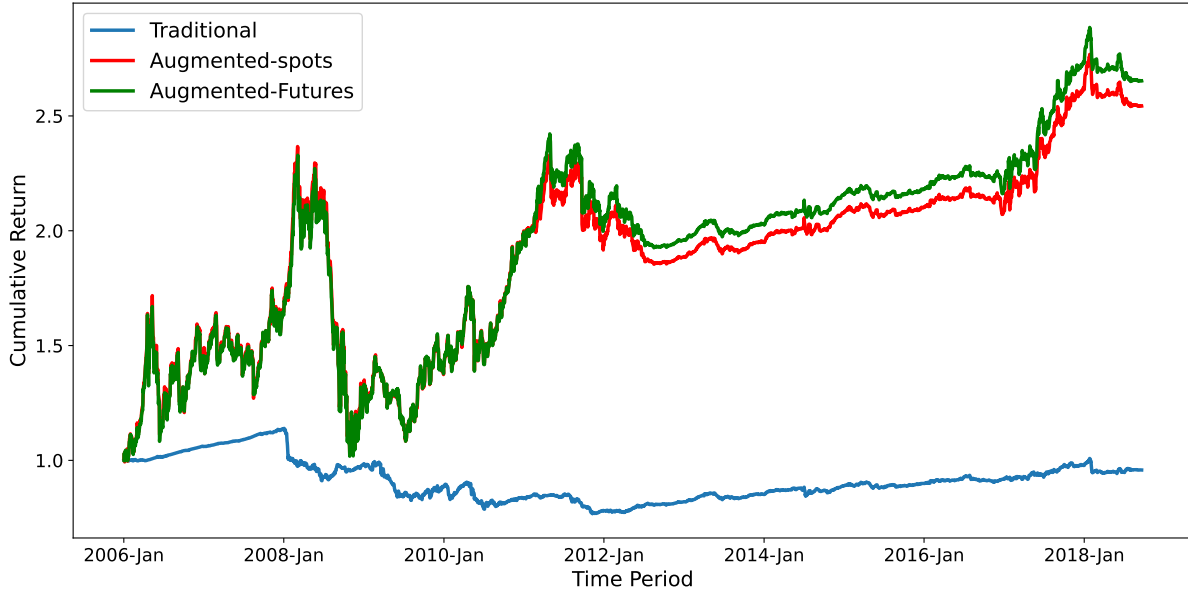


Figure 1: Cumulative performance of the traditional optimal portfolio and the first augmented optimal portfolios with spot returns and the second one with future 1-month returns of precious metals for the whole sample period from Jan 02, 2006, to Sep 20, 2018

Figures 1 to 4 exhibit the cumulative performance of the optimal traditional and augmented portfolios. The analysis was carried out for the whole sample period as well as the three sub-periods. These figures reveal that the augmented portfolios significantly outperformed the traditional portfolio in any case and period. This conclusion confirms the findings about the diversification benefits of precious metals from the in-sample analysis. Notably, figure 4 shows that, in the aftermath of the 2008 financial crisis, precious metal futures offered more diversification benefits than precious metals spots. This finding agrees with those of Kang et al. (2017), Mensi et al. (2021), which point out that the future contracts of precious metals have become the most widely used instruments for risk management in more volatile periods and suggests that investors can hold precious metal futures to improve their portfolio more than by investing in these metals spots.

5.3. Out-of-sample performance assessment

This subsection computes and compares the out-of-sample performance between the optimal augmented and the optimal traditional portfolios by using parametric tests. Several ratios suggested by Anyfantaki & Topaloglou (2018) were calculated, including the Sharpe

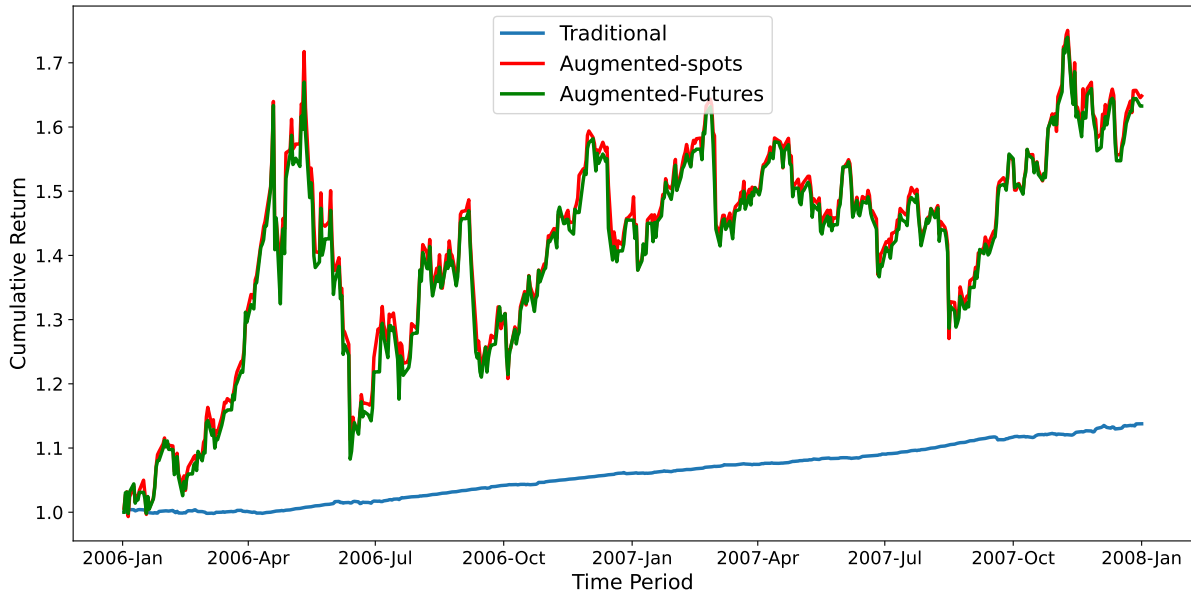


Figure 2: Cumulative performance of the traditional optimal portfolio and the first augmented optimal portfolios with spot returns and the second one with future 1-month returns of four precious metals over the pre-crisis period from Jan 02, 2006, to Jan 31, 2008

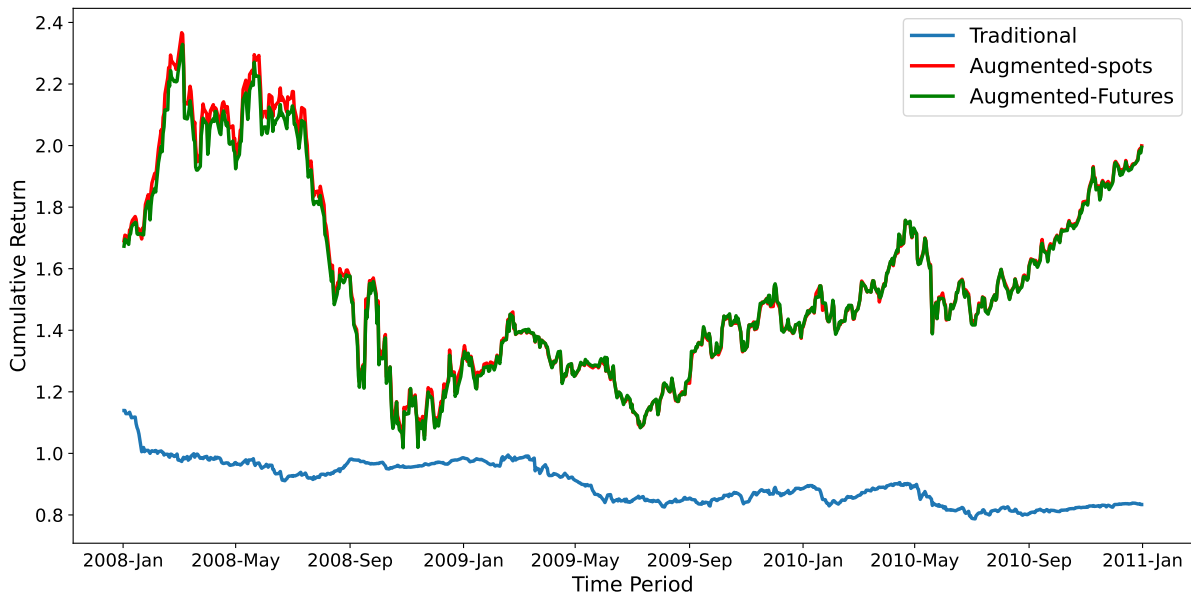


Figure 3: Cumulative performance of the traditional optimal portfolio and the first augmented optimal one with spot returns and the second one with future 1-month returns of four precious metals over the crisis period from Feb 01, 2008, to Dec 31, 2010

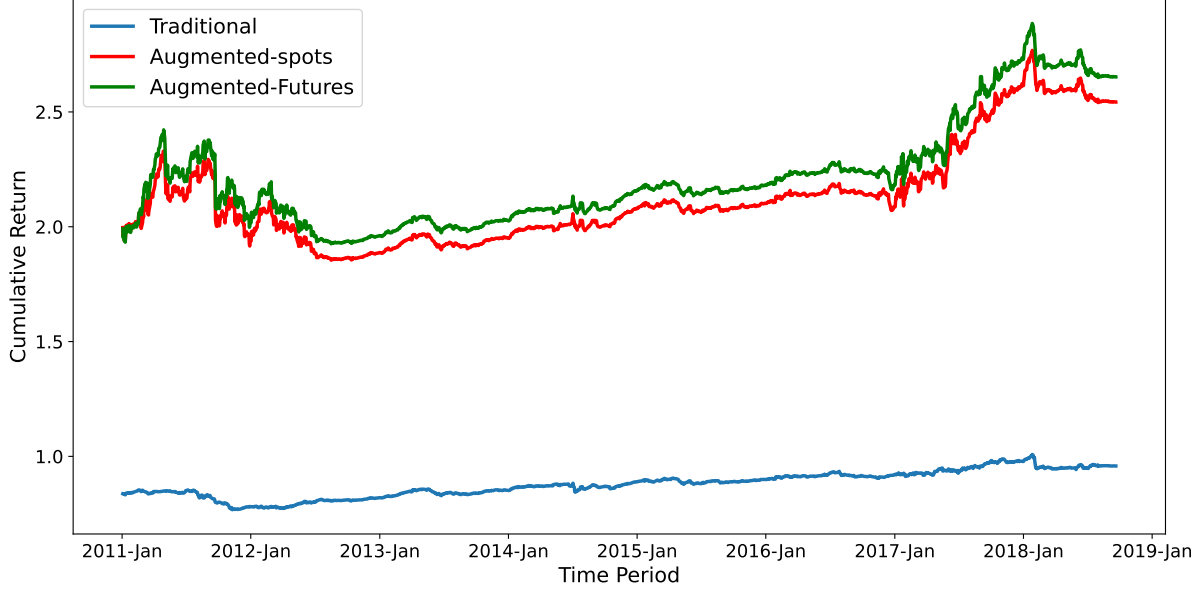


Figure 4: Cumulative performance of the traditional optimal portfolio and the first augmented optimal portfolio with spot returns and the second one with future 1-month returns of four precious metals over the post-crisis period from Jan 01, 2011, to Sep 20, 2018

ratio, the downside Sharpe ratio of [Ziembra \(2005\)](#), the upside potential and downside risk (UP) ratio of [Sortino & Van Der Meer \(1991\)](#), the opportunity cost, the portfolio turnover, and a measure of the portfolio risk-adjusted returns net of transaction costs. These ratios can cover the impact of the higher order moments and transaction costs on performance evaluation.

The downside Sharpe is considered to be a more appropriate measure of performance than the typical Sharpe ratio, given the asymmetric return distribution of precious metals ([Baur 2012](#), [Todorova 2017](#)).

The downside Sharpe ratio of [Ziembra \(2005\)](#):

$$S_P = \frac{\bar{R}_p - \bar{R}_f}{\sqrt{2}\sigma_{P-}}$$

In which, \bar{R}_p is the average back-testing period return of portfolio P and \bar{R}_f is the average risk-free rate. σ_{P-} is the downside standard deviation.

The next ratio is *the portfolio turnover (PT)*

$$PT = \frac{1}{T} \sum_{t=1}^T \sum_{i=1}^N \left(\left| \omega_{P,i,t+1} - \omega_{P,i,t} \right| \right),$$

where $\omega_{P,i,t+1}$ and $\omega_{P,i,t}$ are the optimal weights of asset i under strategy P (traditional or augmented) at time t and $t + 1$, respectively over T rebalancing points in time and across the N available assets.

From PT , [DeMiguel et al. \(2007\)](#) proposed the new ratio, named the proportional transaction cost (PTC) as a risk-adjusted returns measure. $R_{P,t+1}$ is the realized return of portfolio P at time $t + 1$. The change in the net transaction cost wealth (NW_P) is calculated as follows

$$NW_{P,t+1} = NW_{P,t} \left(1 + R_{P,t+1} \right) \left[1 - PTC \cdot \sum_{i=1}^N \left(\left| \omega_{P,i,t+1} - \omega_{P,i,t} \right| \right) \right]$$

Net transaction costs (NTC) are then determined as

$$NTC_{P,t+1} = \frac{NW_{P,t+1}}{NW_{P,t}} - 1$$

Let μ_0 , σ_0 , and μ_1 , σ_1 be the out-of-sample mean and standard deviation of daily NTC with the traditional and augmented opportunity set, respectively, from which the return loss is defined as

$$ReturnLoss = \frac{\mu_1}{\sigma_1} \sigma_0 - \mu_0$$

Finally, *the opportunity cost* θ is the return of a new asset that needs to be added to the traditional portfolio if the investor is indifferent between two different investment opportunity sets [Simaan \(1993\)](#). In other words, the opportunity cost measure is defined (functions, works, is used, is treated) as the return trade-off of choosing the traditional portfolio over the augmented portfolio and is defined as

$$\mathbb{E}\left[U\left(1 + R_0 + \theta\right)\right] = \mathbb{E}\left[U\left(1 + R_1\right)\right]$$

in which R_0 and R_1 are the returns of the optimal portfolio for the traditional and augmented asset classes, respectively. $\theta > 0$ (< 0) indicates that the investors are better (worse) off when adding new assets to their investment opportunity set.

The opportunity cost takes into account the higher order moments rather than just the first and second moment in the case of the Sharpe ratio because it covers the entire probability density function of asset returns. Thus, the opportunity cost can be used reasonably to assess investment strategies even when the asset returns do not distribute normally.

Table 4 reports the results of the parametric performance measures and the opportunity costs for the traditional and augmented portfolios in the entire period of the financial crisis, as well as the three sub-periods mentioned above. In general, the higher values of these measures correspond with a greater number of diversification benefits for investing in precious metals. The results show that the augmented portfolios' Sharpe and downside Sharpe ratios outperform those of the traditional portfolios. These results complement the conclusion of the in-sample and out-of-sample analyses, i.e., that adding precious metals into the traditional portfolio significantly improves its performance.

The traditional portfolio had a higher portfolio turnover than the augmented portfolios in all cases. This result indicates that the augmented portfolios require less frequent re-balancing and have lower cumulative transaction costs. Table 4 also shows that the return-loss measure, which accounts for transaction costs, was positive in both augmentation cases as well as in different periods.

The opportunity costs were calculated to consider higher-order moments rather than only the first and second moments. The opportunity costs were positive in all cases of augmentation, which hold for the exponential and power utility functions under the different degrees of absolute risk aversion (i.e., $ARA=2,4,6$) and relative risk aversion (i.e., $RRA=2,4,6$). These

results showed that investors are better off by adding either spot or future returns of all four precious metals into their traditional portfolio.

The parametric performance measures thus demonstrate that the investment universe of the augmented portfolios outperforms the traditional portfolio of stocks, bonds, cash, and currencies, resulting in benefits of diversity and greater investment opportunities. Notably, after the global financial crisis of 2008, precious metal futures were better than spot returns. This finding strongly reconfirms our conclusion about the diversifier role of precious metal futures stated in section 5.2, and it also explains the emerging financialization of precious metal markets over the last few decades.

Table 4: Out-of-sample performance: Parametric performance measures

	Whole period			Pre-crisis			Crisis			Post-crisis		
	Trad.	Augm. Spot	Augm. Futures	Trad.	Augm. Spot	Augm. Future	Trad.	Augm. Spot	Augm. Future	Trad.	Augm. Spot	Augm. Future
Performance Measures												
Sharpe Ratio	-0.0018	0.0295	0.0287	0.0515	0.0559	0.0535	-0.0616	0.0216	0.0221	0.0293	0.0358	0.0337
Downside Sharpe Ratio	-0.0023	0.0400	0.0389	0.0708	0.0729	0.0706	-0.0762	0.0302	0.0307	0.0384	0.0484	0.0447
Portfolio Turnover	0.2680	0.1681	0.1681	0.3202	0.0129	0.0129	0.3259	0.1320	0.1320	0.2618	0.2224	0.2224
Return Loss (vs Trad. portfolio)		0.0002	0.0002		0.0040	0.0040		0.0035	0.0034		0.0012	0.0012
Opportunity Cost												
<i>Exponential Utility</i>												
ARA = 2		0.0740	0.0731		0.1185	0.1149		0.1310	0.1350		0.0254	0.0237
ARA = 4		0.0557	0.0554		0.0921	0.0887		0.0830	0.0881		0.0227	0.0210
ARA = 6		0.0403	0.0407		0.0724	0.0692		0.0468	0.0525		0.0208	0.0190
<i>Power Utility</i>												
RRA = 2		0.0745	0.0738		0.1189	0.1152		0.1211	0.1255		0.0259	0.0239
RRA = 4		0.0517	0.0521		0.0901	0.0867		0.0624	0.0682		0.0246	0.0222
RRA = 6		0.0276	0.0294		0.0668	0.0638		0.0194	0.0259		0.0247	0.0220

Note: This table reports the various performance measures for the traditional portfolio and the portfolios augmented with precious metals spot or precious metals futures. The results for the opportunity costs are reported for different degrees of absolute risk aversion ($ARA = 2, 4, 6$) and different degrees of relative risk aversion ($RRA = 2, 4, 6$). The dataset covers the out-of-sample period from Jan 03, 2005, to Sep 20, 2018.

6. Conclusion

This chapter explores whether investors may be better off by augmenting their traditional portfolio of stocks, bonds, cash, and currencies with spots or futures of precious metals. The study focuses on the financial markets of G7 countries. It also pays special attention to the behavior of the spot and future returns of all four metals in and around the global financial crisis of 2008. This work contributes to the existing literature on behavior finance, crises, and the future markets of precious metals, showing that precious metals can offer diversification benefits for an international investment portfolio of equities, bonds, cash, and currencies. Additionally, the out-of-sample analysis compares diversification benefit potential between spot and future returns of precious metals. Precious metal futures outperformed their spots in the aftermath of the 2008 global crisis. This finding explains the increasing prevalence of precious metal futures over the last few decades. This study provides strong evidence for introducing and developing new financial innovations (e.g., futures, options, ETFs). Notably, these findings are very timely and helpful for financial market participants as the global financial markets have experienced unprecedented volatility and high demand for investment in precious metals due to the COVID-19 pandemic.

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Appendix

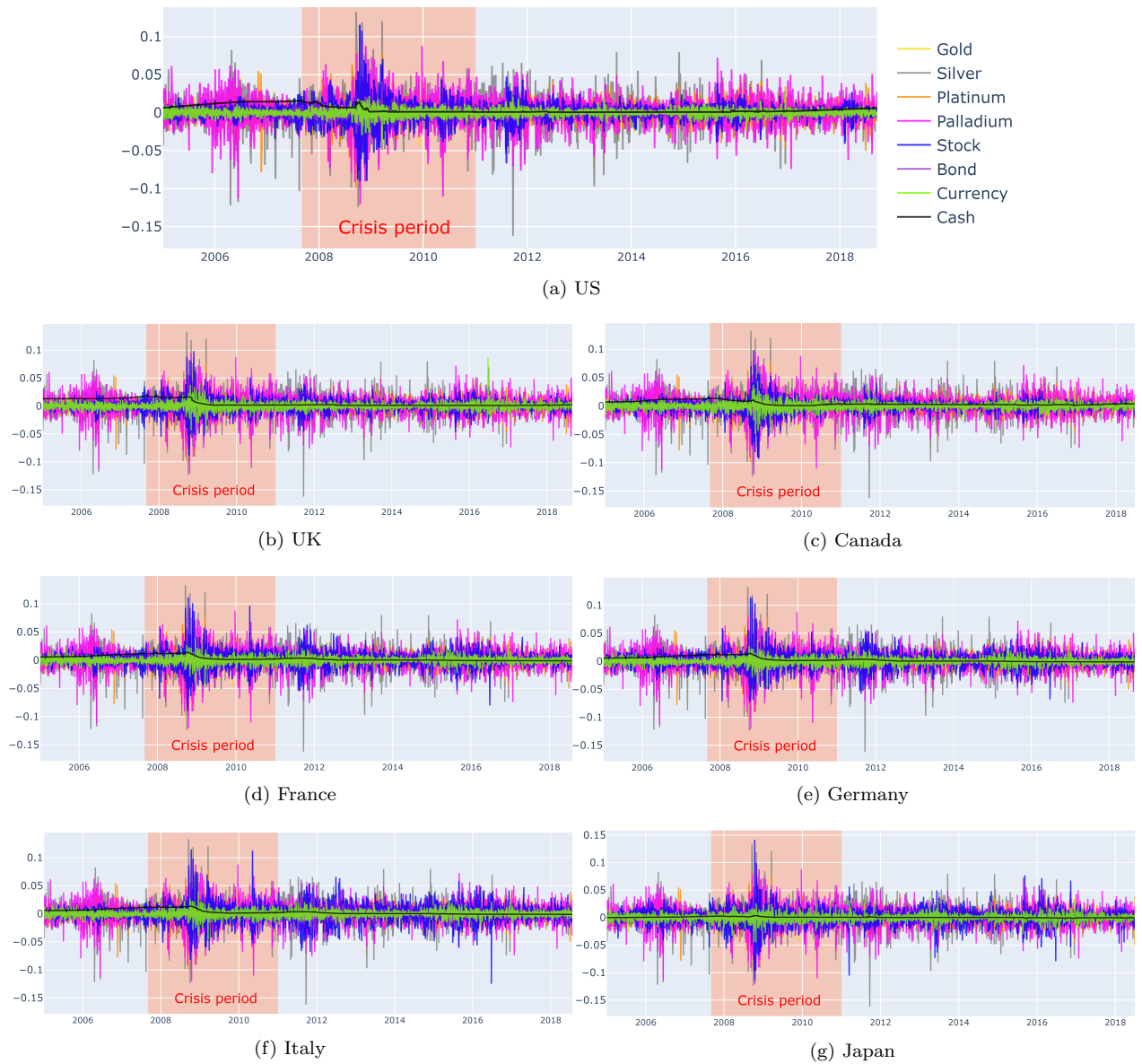
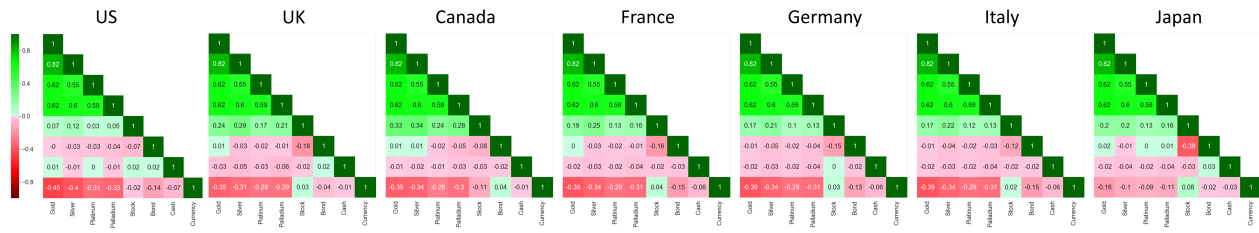
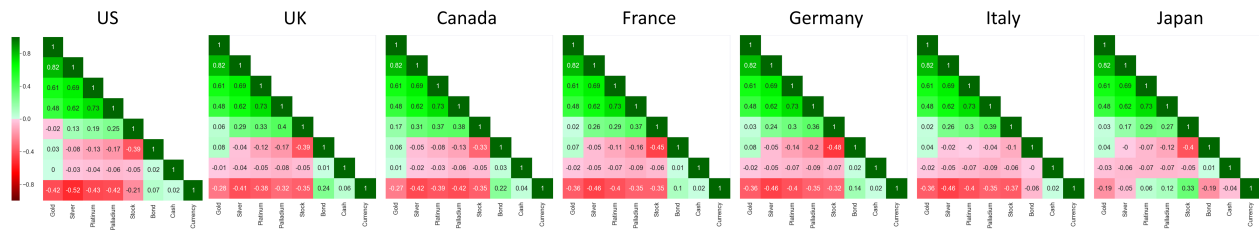


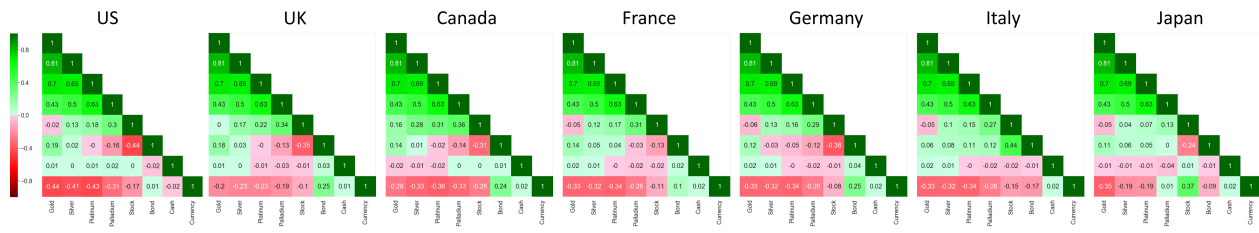
Figure 5: Precious metals and traditional assets returns of G7 markets over the three periods of the global financial crisis of 2008



(a) Before crisis



(b) During crisis



(c) After crisis

Figure 6: Correlations between precious metals and traditional assets returns of G7 countries