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# Optimizing Portfolios for the BREXIT: An Equity-Commodity Analysis of US, European and BRICS Markets

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## Abstract

The objective of this study is to create optimal two-asset portfolios consisting of stocks from Western Europe, the United States, and the BRICS (Brazil, China, India, Russia, and South Africa), as well as sixteen commodity types during the BREXIT period. We utilized dynamic variances and covariances from the GARCH model to derive weights for the two-asset portfolios, with each portfolio consisting of one equity factor and one commodity factor. Subsequently, hedge ratios were calculated for these various assets. Our findings indicate that portfolios consisting of European stocks do not require the inclusion of commodities, whereas the other equities do.

**Keywords:** Equity markets, commodity markets, BREXIT, portfolio optimization

## 1. Introduction

In accordance with the phenomenon of financilization and the declarations of Yue-Jun Zhang et al. (2017), significant negative correlations between stocks and commodities were reported during the 2007–2008 financial crisis, various studies examined how to construct portfolios made of equities and commodities for hedging purposes. Mensi et al. (2013) used a VAR-GARCH model in order to investigate the return links and volatility spillovers between the S&P500 stock returns and price indices for energy (WTI and BRENT oil), food (wheat), gold and beverages in the periods ranging from 2000 to 2011. Their results show that there was significant volatility and return transmission between S&P500 and the various commodity markets, with shocks of volatility from the American stock market having a higher influence on gold and oil markets. Based on these results, the authors used the methodology of Kroner and Ng (1998)<sup>1</sup> in order to calculate portfolio weights and hedge ratios following Kroner and Sultan (1993)<sup>2</sup> (portfolios made of two assets). The findings suggest that the average weight for a WTI/S&P500 portfolio is 0.23, suggesting that for a \$.1 portfolio, 23 cents should be invested in WTI and 77 cents should be invested in the S&P500. For BRENT, the authors document that 43 cents should be invested in the commodity while 57 cents are to be invested in S&P500. A \$.1 portfolio of wheat and S&P500 must be 25% wheat and 75% stocks. Investors should invest 47 cents in beverages and 53 cents in S&P500. The findings also suggest that hedge ratios are overall low, suggesting a good hedging effectiveness of commodities and equities. The hedge ratios vary between 0.103 for Wheat/S&P500 and 0.203 for BRENT/S&P500. Consequently, a \$.1 long position in WTI can be hedged for 16 cents with a short position in S&P500 index. A \$.1 long position in gold can be hedged by 13 cents, a \$.1 long position in wheat can be hedged by 10 cents and a \$.1 long position in Beverages can be hedged for 11 cents. These hedges are established with a short position for the S&P500. These findings suggest that the cheapest hedge is long Wheat while the most expensive hedge is long BRENT is the most expensive hedge, with a short position of the S&P500 index.

Jebabli et al. (2014) studied shock transmission between food commodities (maize, barley, and rapeseed oil), livestock (lamb, beef, and fish), forestry products (banana, cocoa, beans, and ground nuts), energy markets (crude oil) and stock markets (MSCI world stock market index). The period of the study ranges from 1980 to 2012. Using TVP-VAR with stochastic volatility, the authors report significant evidence of volatility spillover from MSCI and crude oil markets

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<sup>1</sup> The methodology is presented in the empirical framework subsection

<sup>2</sup> The methodology is presented in the empirical framework subsection

towards commodity markets. These shocks have a short term immediate impact on commodities, which is amplified in the period of the financial crisis. Afterwards, in order to construct optimal risk-adjusted portfolios, the authors used the methodology of Kroner and Ng (1998) to calculate weights and followed Kroner and Sultan (1993) to calculate hedge ratios for two-asset portfolios. The authors constructed portfolios made of oil and food commodities and portfolios made of oil and food commodities. Their findings suggest that the hedge ratios are generally low, indicating a good hedging effectiveness. This suggests that including crude oil or stocks in a portfolio of food commodities enhances the risk adjusted performance of the resulting portfolio. Moreover, the findings show that the optimal portfolio weights for a portfolio made of crude oil and food are comparable for each kind of food commodities, except for forestry products where the investment in banana is much higher than the investment in the other components of the category. The authors also report comparing optimal weights portfolios, investors must invest more in crude oil than in equities. For some portfolios (crude oil and maize, crude oil and banana, stocks and banana, and stocks and cocoa beans), the hedge ratios are negative, suggesting that spot and futures prices may move in opposite direction in the short run. Afterwards, the authors calculated realized hedging errors in order to check the effectiveness of portfolio diversification<sup>3</sup>. The findings show that the best hedging effectiveness is (Oil, Crops), with Maize being the best risk management tool for diversification. Hedging effectiveness for portfolios made of crude oil and livestock are comparable and high, suggesting that including livestock commodities in a portfolio with crude oil improves its risk-return characteristics. For (crude oil, banana) and all the (MSCI, food portfolios), the authors recorded negative hedge effectiveness ratios. They hypothesize that this is due to higher futures return variance.

Khalifaoui et al. (2015) examined the linkages of WTI crude oil and the stock markets of the G7 countries (Canada, France, Germany, Italy, Japan, the United Kingdom and the United States) using a wavelet based MGARCH approach. More specifically, they used a bivariate GARCH-BEKK model with wavelet multi-resolution analysis to capture the multiscale volatility and mean spillover between these various assets from June 2003 to February 2012. Their findings suggest evidence of significant volatility transmission between crude oil and the stock markets. The authors also computed weights following Kroner and Ng (1998) and hedge ratios following Kroner and Sultan (2013) to construct the crude oil-stock market portfolios that allow for risk minimization without affecting the expected returns. Their findings suggest that investors

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<sup>3</sup> A methodology introduced by Ku et al. (2007), described the empirical framework subsection.

should invest considerably in crude oil (as high as 91% to 99%) when construction a portfolio with oil and one of the G7 stock markets in order to minimize the risk without lowering the expected returns. The hedge ratios documented by the authors are generally low, suggesting that construction a portfolio of a G7 stock market index and crude oil has a good hedging effectiveness. Moreover, the (S&P500, WTI) portfolio is found to have the lowest hedge ratios for both long term and short term investors.

Hasan (2017) explored the transmission of international energy price shocks (crude oil, natural gas and coal) to ten sectors (according to the Global Industry Classification Standard – GICS) of the ASX Australian stock market index (energy, materials, industrials, consumer discretionary, consumer staples, telecom and utilities). Using a MGARCH approach to model volatility, the author documented significant dynamic volatility transmission of crude oil, natural gas, coal and the different sectors of the Australian stock markets. Again, the authors computed optimal weights of two-asset portfolios constructed of each of the energy commodities with each of the stock market sectors using the methodology of Kroner and Ng (1998). The findings reveal that market participants invest highly in the GICS stocks when these are combined with coal. When the portfolio contains crude oil, the later commodity has the highest portion of investment, except when it is combined with the telecom and utility sectors. Similar to crude oil, investors holding portfolios made of GICS stocks and natural gas invest in the stocks with a 40% portion or less, especially when the commodity is combined with stocks relative to IT and health sectors.

One study that has examined the interplay between oil and food commodity prices is by Baffes and Haniotis (2017). Another relevant study is by Narayan et al. (2016), which provides a comprehensive review of the theory and empirical evidence on the relationship between oil prices and stock markets. A study by Sari and Hammoudeh (2010) investigates the shock and volatility transmission in the oil, US and Gulf equity markets. Another study that examines the dynamic relationships between stock prices and exchange rates for G-7 countries is by Tsai et al. (2017). In addition, Delatte and López-Villavicencio (2012) provide evidence of asymmetric exchange rate pass-through to consumer prices using a scanning model for the eurozone. Kanas and Ioannidis (2016) propose a new approach to modelling the dynamic co-movements between stock and bond returns. Lastly, Nguyen and Sévi (2015) perform a sector analysis of the relationship between stock returns and oil price changes in Europe. Another study that examines financial crises is by Daskalaki and Skiadopoulos (2011), who investigate whether financial

crises of equal magnitude are equal, using a wavelet analysis of contagion in the 2008 financial turmoil.

In this study, we followed the methodology adopted by Kroner and Ng's approach to calculate optimal weights and hedge ratios for portfolios consisting of one equity factor (European, American, or BRICS equities) and commodity factors (Agricultural Oils, Chemicals, Crude Oil, Electricity, Feeds, Fibers, Forestry Products, Grains, Livestock, Metals, Natural Gas, Oil Seeds, Precious Metals, Seeds, Semi-Conductors, and Softs) during the BREXIT period. We eliminated the volatility jump effect by referring to press articles to determine the crisis period.

Our findings indicate that portfolios including the European factor do not require the incorporation of commodities since European equities alone constitute optimal portfolios. This can be attributed to the fact that the European factor already encompasses equities from different countries with diverse economic characteristics. Moreover, all commodities, except for electricity, serve as good hedges for American and BRICS equities during the crisis period under study, indicating that commodities retain their reputation as hedgers and safe havens in times of crisis despite financialization.

The rest of the paper is organized as follow: Section 2 presents the empirical framework. Section 3 interprets the result. We conclude with some remarks.

## 2. Empirical Framework

In this subsection, and based on the findings of Ayadi (2021) – who documented evidence of contagion between American, European and BBRICS stock markets during the BREXIT; the works of Ayadi et al. (2021) who documented contagion between some equity and commodity markets (as well as decoupling for other assets) for various equities and commodities during four crises, we seek to make optimal portfolios which aim is to minimize the risk for investors in times of crisis. To do so, we follow Kroner and Ng (1998) and construct two-asset portfolios with the following weights:

$$w_t^{C,E} = \frac{h_t^E - h_t^{C,E}}{h_t^C - 2h_t^{C,E} + h_t^E} \quad (1)$$

$$w_t^{C,E} = \begin{cases} 0 & \text{if } w_t^{C,E} < 0 \\ w_t^{C,E} & \text{if } 0 \leq w_t^{C,E} \leq 1 \\ 1 & \text{if } w_t^{C,E} > 1 \end{cases} \quad (2)$$

Where:

- $w_t^{C,E}$  is the weight of commodities in one dollar of two assets (one of the commodity factors, one of the stock market factors) at time t
- $h_t^{C,E}$  is the conditional covariance between one commodity factors and one equity market factor
- $h_t^C$  is the conditional variance of one commodity factors
- $h_t^E$  is the conditional variance of one of the equity market factors

In this framework, the weight of the stock market factor in the considered portfolio is  $1 - w_t^{C,E}$

Afterwards, we calculate hedge ratios of these portfolios as follows:

$$\beta_t^{C,E} = \frac{h_t^{C,E}}{h_t^E} \quad (3)$$

Where:

- $\beta_t^{C,E}$  is the hedge ratio
- $h_t^{C,E}$  is the conditional covariance between one commodity factors and one equity market factor
- $h_t^E$  is the conditional variance of one of the equity market factors.

The variances and the covariances between the stock market factors and the commodity factors are obtained from the DCC-GARCH model. These dynamic variance s and covariances allowed us to obtain dynamic weights across the studied periods as well the dynamic hedge ratios.

The variances and covariances used to calculate the weights and hedge ratios are obtained from the DCC-GARCH model. The multivariate GARCH models are chosen because they have mostly been used in the literature on volatility transmission as they allow for the joint modeling of variances and covariances between different variables. Among the different specifications of the multivariate GARCH models, (i.e. CCC, DCC, VECM and BEKK), we propose to use the

DCC-GARCH of Engle (2002). This choice is motivated by the following reasons. First, according to Belgacem and Lahiani (2012), the superiority of the DCC-GARCH model over the other multivariate GARCH specifications when studying financial markets dynamics is largely documented in the literature, as it takes into account dynamic correlation between financial and economic data. Second, the conditional correlation between equity markets is shown to be time-varying. Finally, compared to the other multivariate GARCH specifications, the DCC-GARCH presents the advantage of having less parameter to estimate, which allows us to augment the model by introducing a set of macroeconomic variables<sup>4</sup>.

Engle (2002) was inspired by the *Constant Conditional Correlation* (CCC) model of Bollerslev (1990) and created the *Dynamic Conditional Correlation* (DCC-GARCH) in order to jointly model conditional variances and conditional correlations jointly. The estimation procedure contains two steps: first, the conditional variance of each variable is estimated using a univariate random ARCH process. Second, the standardised residuals of the regressors obtained from the first step are used to model correlations in an autoregressive pattern, which will end up by getting the time-varying conditional correlation matrix. The DCC process is based on the idea of decomposing conditional covariances ( $\sigma_{ij,t}$ ) as a product of conditional variances and conditional correlations ( $\rho_{ij,t}$ ).

$$\rho_{ij,t} = \frac{\sigma_{ij,t}}{\sqrt{\sigma_{ii,t}\sigma_{jj,t}}} \quad (4)$$

$$\sigma_{ij,t} = \rho_{ij,t}\sqrt{\sigma_{ii,t}\sigma_{jj,t}} \quad (5)$$

### 3. Empirical result

#### - *The portfolio weights*

This subsection focuses on the portfolio weights and the hedge ratios of the various portfolios during the BREXIT. The average commodity weights, equity weights and hedge ratios are reported in Table 1. Figure 1.a presents the commodity weights, while Figure 2.b presents the

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<sup>4</sup> Bekaert, Harvey and Ng (2005) suggested using an asymmetric GARCH model. We attempted this by comparing the DCC-GARCH to DCC-E-GARCH and DCC-GJR-GARCH. But referring to the Akaike Information Criteria (AIC), we chose the DCC-GARCH model.

hedge ratios. The weights are plotted without the last values because of their magnitude compared to the rest of the timeline.

**[Insert Table 1 about Here]**

**[Insert Table 1.a about Here]**

**[Insert Table 2.b about Here]**

By looking at the average weights across the BREXIT timeline, the findings go along with those recorded for the previous crises. Energy commodities (crude oil, natural gas and Electricity) have the lowest average weights, while forestry products have the highest average weights, followed by agricultural oils and semi-conductors. For the other commodities, the average weights vary around 4%.

Starting with crude oil, the findings show that when these commodities are included in a portfolio with European stocks during the BREXIT, an investor should allocated 0.75% of his wealth in crude oil commodities and the remaining 99.25% in European equities, in order to make an optimal one US dollar portfolio. The weights allocated to the commodities vary between 0 and 100%. These weights are practically null during all of the crisis timeline, except in the last week of the timeline. In that time, the portfolio becomes 100% crude oil commodities. Turning to the US factor, the average weight attributed to crude oil commodities when included in the same portfolio with American stocks is 0.693%. this suggests that an investor seeking to diversify a portfolio made of the S&P500 stocks during the BREXIT should make an optimal one US dollar portfolio by investing an average of 0.693% of his wealth in crude oil commodities and 99.307% in American stocks. The proportion of investor wealth allocated to crude oil range between 0% and 5.595%. The weights are 0% for most of the crisis timeline, before the occurrence of peaks between January and April 2018, followed by a drop then other peaks between October and December 2018 before dropping to 0% again in the last week of the crisis. Turning to the BRICS factor, the average weight attributed to crude oil commodities is 1.17%, making an investor construct an optimal one US Dollar portfolio by investing an average of 1.17% of his wealth in crude oil and 98.83% in BRICS equities. The portfolio weights attributed to crude oil commodities during the BREXIT are between 0% and 100%. These weights are very close to zero all along the crisis time lapse before rocketing up to 100% in the last week of the crisis.

The findings for natural gas commodities suggest that when these assets are included in a single portfolio with European stocks, the average weight they receive by investors during the BREXIT crisis time is 0.0058%. This suggests that to make an optimal one US dollar portfolio, an investor must allocate 0.0058% of his wealth to natural gas and the remaining 99.9942% to European equities. The weights attributed to natural gas commodities range between 0% and 0.565%. The weights are practically 0% all across, the crisis period; the peak is documented at the last weeks of the crisis. Turning to the US factor, the average weight allocated to natural gas commodities when included in a portfolio with US stocks in the BREXIT is 0.648%, suggesting that to make an optimal weight portfolio, an investor should allocate an average of 0.648% of his wealth to natural gas commodities and the remaining 99.352% to American stocks. The weights attributed to natural gas are between 0% and 42.563% across the crisis period. The weights are relatively high across the crisis period, with peaks around 0.9% in the first weeks of the crisis time. The weights drop to zero in January 2018 before having a very large peak of 42% in the last week of the crisis period. Regarding the BRICS factor, the average portfolio weight allocated to natural gas when included in a single portfolio with BRICS stocks during the BREXIT is 1.229%, making the optimal one US dollar portfolio made of 1.229% natural gas commodities and 98.771% BRICS stocks. The weights attributed to the commodities vary between 0% and 100% across the crisis timeline, with the highest peak of 100% recorded in the last week of the crisis. Beside the very large peak at the end of the crisis, various peaks are found across the crisis period, the highest of which is found in the earliest weeks of the crisis (around 1.5%). The peaks fluctuate for a period of time before dropping to 0% in January 2018, which precede the very large peak at the end of the crisis.

Turning to electricity, the weights allocated to these commodities when included in a single portfolio with European stocks during the BREXIT are very low, ranging between 0% and 67.4%. This peak is documented in the last week of the crisis. Accordingly, on average, an investor should allocate 0.507% of his wealth in electricity commodities and the remaining 99.493% in European stocks, in order to make an optimal one US Dollar portfolio during the BREXIT. The weights allocated to the commodities are null across most of the crisis period before having an increasing pattern starting August 2017, with a high peak around 0.0025% in December 2018, which comes before the very large peak of 100%. Regarding the US factor, the weights allocated to electricity commodities range between 0% and 51% (in the last week of the crisis), but these weights range generally between 0% 0.51%. As a result, the average weight allocated to electricity commodities when invested with American stocks to make an

optimal one US Dollar portfolio is 0.398%, thus the proportion allocated to US stocks is 99.602%. The weights are null across the crisis period before a peak of 0.5% in December 2018 which comes before the very large peak of 51% in the last week of the crisis. Regarding the BRICS factor, the average weight allocated to electricity commodities is 0.815%, thus an investor seeking to diversify a portfolio of BRICS stocks with electricity commodities should invest an average of 0.815% of his wealth in electricity commodities and the remaining 99.185% in BRICS stocks. The weights allocated to the commodities vary between 0% and 0.471%, with a very large peak of 99.68% in the last week of the crisis. The weights are null during most of the crisis period, with fluctuating peaks starting December 2017, the highest of which is in December 2018 (0.45%). These phenomena are observed before the very large peak mentioned above.

Regarding metals, the average weight attributed to these commodities during the BREXIT when included in a portfolio with European stocks is 0.751%. This suggests that to make an optimal one US Dollar portfolio, an investor should invest an average of 0.751% of his wealth in metals commodities and the remaining 99.249% in European stocks, in order to make an optimal one US Dollar portfolio. The weights allocated to the commodity are null all along the crisis period, with a large peak of 100% in the last week of the crisis. Turning to the US factor, the average weight attributed to metals during the BREXIT is 3.444% (2.712% when the 100% weight in the last week of the crisis is eliminated). This suggests that to make an optimal one US dollar portfolio, an investor is ought to allocate 3.444% of hi wealth into metals commodities and the remaining 96.556% in US stocks. The weights are generally around 0% across the crisis period, with an increasing trend starting October 2018. From this date onwards, the weights allocated to metals rocketed to reach 100% in the last week of the crisis, which is the highest value. The lowest value during the BREXIT is 0%. Regarding the BRICS factor, an investor should allocate an average of 3.532% of his or her wealth to metals and 96.468% to BRICS equities, in order to make an optimal one US Dollar portfolio during the BREXIT. The weights allocated to metals commodities vary between 0% and 100% (a peak at the last week of the crisis). These weights are generally around 2% in the early weeks of the crisis, before increasing sharply starting June 2018 to reach around 13% (before the 100% peak).

Focusing on precious metals, our results suggest that the average weight allocated to these commodities in a portfolio made of European stocks is 0.757%. That is, an investor seeking to diversify European equities should invest 0.757% of his wealth in precious metals and 99.243% in European stocks. The weights allocated to these commodities vary between 0% and 0.016%

(without the large peak of 100% in the last week of the crisis). These weights have a fluctuating trend across the crisis period with the highest values recorded in November 2017. These peaks are followed by null values of the weights that start in June 2018 before the 100% peak in the last week of the crisis. Turning to the US factor, the average weight attributed to precious metals is 4.209%, thus the optimal one US Dollar portfolio is composed as follows: 4.209% precious metals commodities and 95.791% American stocks. These weights range between 0% and 33% during the BREXIT time, before a 100% peak in the last week of the crisis. The weights are generally below 5% in most of the crisis time, with two small peaks between January and April 2018 and an upwards trend starting October 2018 onwards. The average weight attributed to precious metals commodities during the BREXIT when invested in a portfolio made of BRICS stocks is 4.435%. Consequently, the optimal one US Dollar portfolio is made of 95.565% of BRICS stocks. Except for the peak of a weight of 100% in the last week of the crisis, the weights attributed to precious metals are between 1.457% and 11.548%. The weights have values between 2% and 4% during most of the crisis period before the 11.548% in February 2018 that comes before an increasing pattern that starts in May 2018.

Focusing on agricultural oils, the average weight allocated to these commodities during the BREXIT when invested in a portfolio containing European equities is 0.763%. Consequently, the average proportion of wealth invested in European stocks is 99.237%. When the peak of 100% in the last week of the crisis is not taken into account, the weights attributed to agricultural oils vary between 0% and 0.035%, with the highest values recorded in the early weeks of the crisis and null weights during its late weeks. Investors who seek to diversify American stocks should invest an average of 8.86% of their wealth in agricultural oils and the remaining 91.14% in American equities, in order to make an optimal one US Dollar portfolio. These weights vary between 2.863% and 36.949% (when the 100% peak in the last week of the crisis is not taken into account). The weights vary between 4% and 10% during most of the crisis period, with peaks documented between February and April 2018 and an increasing trend that starts in September 2018. For the diversification of the BRICS factor, an investor should put an average of 7.559% of his or her wealth in agricultural oils and the remaining 92.441% in BRICS equities. This combination makes the optimal one US Dollar portfolio. The weights allocated to the commodity vary between 1.481% and 23.182% across the crisis period (without taking into account the 100% peak in the last week of the crisis). Again, these weights are around 5% in most of the crisis timeline, with a peak higher than 20% in February 2018 followed by an upwards trend starting April 2018.

Regarding chemicals the findings show that investors holding European equities and seeking to diversify them with chemicals commodities during the BREXIT should invest an average of 0.754% (0.002% when the 100% at the end of the crisis is eliminated) in order to make an optimal one US Dollar portfolio. The remaining 99.246% are invested in European stocks. The weights allocated to chemicals commodities vary between 0% and 100%, with no weight allocated to the commodities until April 2017. Starting that date, weights have fluctuated, showing multiple peaks across the crisis period (the highest of which is in August 2017, without counting for the 100% in the last week of the crisis period). Turning to the US factor, the results show that the average weight allocated to chemicals when they are included in a portfolio with American stocks during the BREXIT is 2.782%. This suggests that in order to make an optimal one US Dollar portfolio, an investor should put an average of 2.782% of his wealth in chemicals commodities and the remaining 97.218% in US stocks. The weights attributed to chemicals vary between 0% and 12.79% (with a 100% peak in the last week of the crisis). These weights are generally between 0% and 4%, followed by two peaks in February and April 2018. Afterwards, a drop in the weights is recorded, followed by a slight increase between September and December 2018. This evolution comes before the 100% peak at the end of the crisis period. Regarding the BRICS factor, our findings show that the average weight allocated to chemicals when they are used to diversify BRICS stocks during the BREXIT is 2.964%. This makes the composition of the optimal one US Dollar portfolio as follows: 2.964% chemicals commodities and 97.036% BRICS equities. The weights allocated to the commodities vary between 0.012% and 7.331%, followed by a 100% peak in the final week of the crisis period. These weights have a fluctuating trend across the crisis period, with the highest values recorded in February 2018 and August 2018.

Regarding feeds, the average weight allocated to these commodities in a portfolio made of European stocks is 0.757% (0.0057% without the 100% peak in the last week of the timeline). Consequently, the average optimal one US Dollar portfolio is made of 0.757% feeds and 99.243% European stocks. The weights fluctuated between 0% and 0.023% across the crisis period, with an upwards trend starting the first weeks of the crisis up to July 2017, followed by a stabilization in a 0% weight for the rest of the crisis timeline. When investors seek to diversify American stocks during the BREXIT, they should make an optimal one US dollar portfolio by investing an average of 2.334% of their wealth in feeds and the remaining 97.666% in US equities. When the 85% peak in the last week of the crisis is not taken into account, the weights vary between 0% and 25.726%. The highest weights (between 10% and 85%) are recorded in

the latest weeks of the crisis (December 2018 onwards). For the rest of the crisis time lapse, the weights did not go beyond 5%. Turning to the BRICS factor, the average weight allocated to feeds is 4.95%. That is, an investor who seeks to diversify a portfolio made of BRICS equities should introduce feeds into the portfolio at an average weight of 4.95%, while keeping the remaining 95.05% in BRICS equities. When the 100% peak in the last week of the crisis is not considered, the weights attributed to feeds vary between 1.518% and 10.377%. After fluctuations between 2% and 6%, the weights had a peak in February 2018, followed by a drop and an increasing pattern that starts from July 2017 onwards.

Regarding fibers, the findings show that the average weight allocated to these commodities when included in a portfolio with European stocks is 0.752% (0.0027% when the 99.73% in the final week of the crisis is not considered), suggesting that the optimal one US Dollar portfolio should be made of 0.752% fibers and the remaining 99.248% European stocks. The weights attributed to the commodities range between 0% and 0.019%, with the highest values recorded from April 2018 onwards. It should be mentioned that after the peak of April 2018, the weights go on a decreasing trend before the 99% peak in the final week of the crisis. Turning to the SU factor, our results show that the average weight allocated to fibers in a portfolio with American equities during the BREXIT is 3%, suggesting that to make an optimal one US Dollar portfolio, an investor should put 3% of his wealth in fibers and 97% in US stocks. The weights attributed to fibers range between 0% and 28% (61% in the last week of the crisis). These proportions of wealth are generally low, before a peak in April 2018 followed by a steep fall then an upwards trend that starts in October 2018. Our results show that if an investor wished to diversify BRICS equities with fibers during the BREXIT, he or she should invest an average of 4.94% of his or her wealth in fibers commodities and the remaining 95.06% in the BRICS stocks. The weights allocated to the commodities vary between 0.018% and 15.498% (followed by a peak of 99% in the last week of the crisis). These weights are highly fluctuating, with values that do not surpass 6% before a peak in March 2018, followed by fluctuations and values with a maximum of 14%. Afterwards, stabilization at 10% is recorded, before the 99% peak.

The estimations of portfolio weights show that when forestry products are introduced in a portfolio of European equities, the average weight allocated to the commodities is 0.753% during the BREXIT (0.002% when the 100% is not considered). Consequently, in order to make an optimal one US Dollar portfolio, an investor shall invest an average of 0.753% of his or her wealth in forestry products and the remaining 99.247% in European equities. The weights allocated to the commodities are between 0.014% (without the 100% peak). They have a

decreasing trend starting the first week of the crisis, stabilization at 0% followed by a peak in April 2018 that initiates another decreasing trend, then another stabilization at 0%. Turning to the US stocks, they are diversified by investing an average of 91.412% of investor wealth in the stocks and 8.588% in forestry products in order to make an optimal one US dollar portfolio during the BREXIT. The weights attributed to the commodities vary between 0.94% and 37.39%, before the 100% at the end of the crisis timeline. These weights have small peaks in June, November and December 2016 before stabilizing around 5%. Afterwards, peaks in February and April 2018 are recorded, followed by a steep fall and an increasing trend that starts in October 2018 onwards. For the BRICS stocks, investors can diversify them during the BREXIT by making an optimal one US Dollar portfolio that has an average of 11.472% forestry products and 88.528% BRICS equities. The weights allocated to the commodities are between 3.612% and 27.293% (without the 100% peak in the last week of the crisis timeline). These weights fluctuate around 15% until February 2017, followed by a drop to a minimum around 5% that precedes a peak in February 2018, followed by a downwards trend up to the end of the crisis timeline.

Turning to grains, our results show that when they are introduced in a portfolio with European stocks for diversification purposes during the BREXIT, investors should make an optimal one US Dollar portfolio by putting an average of 0.755% (0.0034% without the 100% at the last week of the crisis) in grains commodities and 99.245% in European equities. These weights vary between 0% and 0.014%, with the highest weights recorded from the first week of the crisis up July 2017. Afterwards, these weights drop to 0% until the end of the crisis timeline. Regarding the US factor, the results show that the average weight allocated to the commodities during the BREXIT when they are incorporated in a portfolio with American stocks is 3.428%. That is, to make an optimal one US Dollar portfolio, an investor should invest an average of 3.428% of his or her wealth in grains and the remaining 96.572% in American equities. These weights vary between 0% and 19.974% across the crisis period, with two small peaks in October and November 2016, followed by a stable pattern around 0% that goes up in a peak of 15% in January 2018 before a drop and an upwards trend starting October 2018 onwards. Regarding BRICS stocks, the results show that when diversified with grains during the BREXIT, an investor should allocate an average of 3.39% of his or her wealth in grains and 96.61% in BRICS stocks. The weights allocated to grains commodities range between 0% and 14.355% across the crisis timeline (without considering the 100% peak in the last week of the crisis). In addition to the peak documented in February 2018, the weights are generally around 2% to 4%.

Investors who seek to diversify a portfolio made of European equities with livestock commodities should invest an average of 0.569% (0.0028% when the 78% in the last week of the crisis is not considered) of their wealth in livestock commodities and the remaining 99.431% in European stocks, in order to make an optimal one US Dollar portfolio during the BREXIT. The weights allocated to livestock commodities range between 0% and 0.012%, with the highest values in the early weeks of the crisis and an upwards trend in the last weeks of the crisis timeline. Turning to the US factor, the findings show that the average weight allocated to livestock commodities when included in a portfolio with American equities is 3.242%. This suggests that an investor seeking diversification of American equities should make an optimal one US Dollar portfolio by investing only 96.758% of his or her wealth in US equities. The remaining 3.242% are invested in livestock commodities. During the BREXIT, the weights allocated to livestock commodities range between 0% and 21.254%. These weights are generally low across the crisis period, yet a peak of almost 20% is found in March 2018, followed by a drop that comes before an increasing pattern that starts in October 2018 onwards. Regarding the diversification of BRICS equities, our results show that investors should put an average of 4.25% of their wealth in livestock commodities and the remaining 95.75% in BRICS equities, in order to make an optimal one US Dollar portfolio. The weights allocated to the commodity vary between 0.099% and 14.749% across the crisis period (without considering the peak of 99.79% in the last week of the crisis). These weights are highly fluctuating with various peaks and troughs; the highest peak is documented in February 2018.

Turning to oil seeds, the findings show that during the BREXIT, the average proportion of investor wealth they receive when included in a portfolio with European stocks is 0.753% (0.0015% when the peak of 100% in the last week of the crisis is not included). This makes the optimal one US Dollar portfolio made of 0.753% oil seeds commodities and 99.9247% European stocks. These weights range between 0% and 0.0095%, with the highest values recorded between November 2016 and July 2017. For the rest of the crisis timeline, the weights allocated to oil seeds are null. Turning to the US factor, our results show that during the BREXIT, the average weight allocated to oil seeds when incorporated in a portfolio with American stocks is 1.98%. This suggests that to make an optimal one US Dollar portfolio, an investor should put an average of 1.98% of his wealth in oil seeds and the remaining 98.02% in American equities. When the 100% peak in the last week of the crisis is not included, the weights range between 0% and 16.92%. The weights are generally near 0%, with few peaks in July 2016, November 2016, February 2018 (the largest one) and March 2018. This evolution

comes before an increasing pattern that starts in October 2018. An investor holding BRICS equities and seeking to diversify his or her portfolio should invest an average of 3% of his wealth in oil seeds and the remaining 97% in BRICS stocks, in order to make an optimal one US Dollar portfolio. When the 100% peak is not taken into consideration, the weights attributed to oil seeds during the BREXIT are between 0% and 11.4%, a peak documented in February 2018. Before the peak, the weights were around 2% and after the peak, they evolve between 0% and 5%.

Turning to seeds, when they are included in a portfolio of European stocks during the BREXIT, the average proportion of investor wealth they receive is 0.752% (0.0003% when the 100% in the last week of the crisis is not considered). This suggests that the optimal one US Dollar portfolio is made of 0.752% seeds commodities and 99.248% European equities. The weights allocated to the commodities vary between 0% and 0.0034%, with the highest value recorded in the first week. The weights follow a steeply decreasing pattern until October 2016. From that date onwards, seeds receive no proportion of investor wealth for the rest of the crisis period. Turning to the US factor, the average weight allocated to seeds during the BREXIT is 2%. This suggests that investors who seek to diversify American stocks during the BREXIT are ought to invest 2% of their wealth in seeds commodities and 98% in American stocks, in order to make an optimal one US Dollar portfolio. The weights attributed to the commodities are between 0% and 18.84%, a peak documented in December 2018. This peak is preceded by an increasing pattern that starts in October 2018, which followed two peaks in February and April 2018. For the rest of the crisis timeline, the weights evolved between 0% and 4%. Market participants who wish to diversify portfolios of BRICS equities during the BREXIT with seeds commodities should invest an average of 3.34% of their wealth in seeds and the remaining 96.66% in BRICS stocks. This is the optimal combination for the one US Dollar portfolio. When the 100% peak in the last week of the crisis is not included, the weights attributed to seeds range between 0.659% and 11.427% across the crisis period. The peak is documented in March 2018. Before the peak, the weights were around 2%, they are between 2% and 4%.

Focusing on semi-conductors, the average weight allocated to these commodities when invested in a portfolio containing European stocks during the BREXIT is 0.755% (0.0032 when the peak of 100% in the last week of the crisis is not considered). This suggests that in order to make an optimal one US Dollar portfolio during the BREXIT, an investor should invest 0.755% of his wealth in semi-conductors and the remaining 99.245% in European stocks. The weights allocated to the commodities vary between 0% and 0.024%. The weights have a peak in

December 2016, drop to 0% and have another peak in October 2018 after a period of an upwards pattern. Turning to the US factor, the findings show that the average weight allocated to semi-conductors when included in a portfolio with American stocks during the BREXIT is 6.54%. Consequently, to make an optimal one US Dollar portfolio, an investor should allocated an average of 6.54% of his or her wealth in semi-conductors and 93.46% in American stocks. The weights attributed to the commodities range between 0% and 59.74% (when the 97% peak in the last week of the crisis is not considered). These weights are generally below 10% for most of the crisis period, yet the reach a peak of around 25% in April 2018 followed by a drop and an increasing trend that starts in October 2018. Turning to the BRICS factor, the results suggest that the average weight attributed to semi-conductors when they are included in the same portfolio with BRICS equities during the BREXIT is 8.978%. As a result, in order to make an optimal one US dollar portfolio, investors should put an average of 8.978% of their wealth in semi-conductors and 91.022% in BRICS equities. The weights allocated to the commodities vary between 0.24% and 41.85% (without considering the 99% peak in the last week of the crisis). These weights are generally around 5% (reaching sometimes around 10% in December 2016). Starting December 2017, the weights allocated to semi-conductors go on an increasing trend, thus the highest peak is reached in December 2018, before the large peak of 99% in the last week of the crisis timeline.

Regarding softs, the results show that when invested with European equities during the BREXIT, the average weight allocated to softs commodities is 0.756% (0.004% when the 100% peak in the last week is not considered). This means that to make an optimal one US Dollar portfolio, an investor should invest 0.756% of his wealth in softs commodities and the remaining 99.244% in European equities. The weights allocated to the commodities (without the 100% peak in the last week of the crisis) range between 0% and 0.014%, with a highly fluctuating pattern that starts with a peak at the first week of the crisis. Moreover, various peaks of different magnitudes are also documented across the crisis period. Turning to the US factor, the average weight allocated to softs commodities in a portfolio made of American stocks is 3.282. This makes the optimal one US Dollar portfolio made of 3.282% softs and 96.718% US stocks. The weights allocated to the commodities range between 0% and 16.278% across the crisis period, with values below 2% for most of the crisis period before two peaks (February and April 2018) followed by a drop and an increasing pattern that starts in October 2018. For the BRICS factor, the average weight allocated to softs is 3.342%. Consequently, an investor who seeks to diversify BRICS stocks with softs commodities should invest an average of

3.342% of his wealth in softs and 96.658% in BRICS equities. The weights allocated to the commodities (without the 100% peak in the last week of the crisis) vary between 0.243% and 10.592%, with an increasing trend starting the early weeks of the crisis onwards. A peak is documented in October 2018 and a smaller peak is found in February 2018.

The findings for the BREXIT crisis show that again, the portfolio made of the European stocks that constitute the Europe factor is already optimal, since the weight allocated to various commodities is very low. This is observed despite the fact that Europe (specifically the United Kingdom) is the origin of the crisis. Moreover, in general, the commodities receive less proportion of investor wealth than the commodities. Furthermore, the results show that when diversifying European stocks, the highest weights allocated to the commodities are observed generally in the early weeks of the crisis while when diversifying US and BRICS stocks, the peaks appear generally in a later period. Another remarkable thing is that the highest weights allocated to the commodities are generally those attributed to these assets when diversifying BRICS stocks. One possible explanation is the relationship between BRICS and European country, especially the United Kingdom, which could be impacted by the BREXIT, thus more risk is observed in BRICS stocks that needs higher diversification.

#### - *Hedge Ratios*

Now we turn our focus on the hedge ratios of the portfolios made of European stocks and the various commodities during the BREXIT. The first remarkable thing in our findings is the fact that the average hedge ratios of the portfolios made of European equities and the various commodities during the BREXIT are high in magnitude, whether they are negative or positive. This suggests that incorporating commodities in portfolios made of the equities of the European factor does not have a good hedging effectiveness. Six out of sixteen commodities had negative average hedge ratios during the BREXIT when included with European stocks, suggesting that hedging long positions in spot prices requires long positions in futures prices, since the spot and future prices go on opposite directions. These commodities are electricity (which has the lowest hedge ratio and the highest in magnitude), softs, agricultural oils, precious metals, chemicals are feeds. The lowest positive average hedge ratio is held by the portfolios including livestock (6.153). The highest one is relative to natural gas.

The portfolios incorporating European stocks and crude oil commodities exhibit an average hedge ratio of 140 during the BREXIT. They start by a decreasing trend since the first week of the crisis. Afterwards, these hedge ratios go into their highest values, which were between

November 2016 and April 2014. A smaller increase in values is documented between June 2017 and February 2018, and a smaller increase in values is documented between May and September 2018. The average hedge ratio for the portfolios incorporating European stocks and natural gas commodities during the BREXIT is 620, which is the highest hedge ratio amongst all of the commodities. The hedge ratios start negative and evolve near zero before increasing in value starting December 2017. High values are documented during 2018, the highest of which is in January and between September and October. The hedge ratios go on a sharp decreasing pattern in December 2018. When included in portfolios with European equities during the BREXIT, they generate a negatively large average hedge ratio of -2586. The highest values are found in the earliest weeks of the crisis, before dropping to large negative values in the April 2017 – December 2017 time period. Moreover, another large negative trough is documented in October 2018. The graphics show that during most of the crisis period, the hedge ratios were negative. Regarding metals, the findings suggest that when these commodities are included in portfolios with European equities during the BREXIT, the average hedge ratio is high, reaching 144. The hedge ratios are positive across the entire crisis. They evolved around values that not surpass 100 before an increase in values in July 2017. The hedge ratios then evolved around 500 – 600 before dropping gradually again to stabilize around 100 or less from March 2018 onwards. Turning to precious metals, the graphics show that the hedge ratios were mostly negative across the crisis timeline, before increasing to evolve around 0 and 20 starting June 2018. A lowest trough is documented in November 2017 – January 2018. The average hedge ratio is -37. The average hedge ratio of the portfolios made of agricultural oils and European stocks is -53 during the BREXIT period. These hedge ratios were negative until July 2017. Afterwards, they evolved in low values between 0 and 50 until the end of the crisis period. Turning to chemicals, the average hedge ratio documented during the BREXIT when they are invested with European equities is -6.99. These hedge ratios start high in the first week of the crisis, prior to a decreasing trend until April 2017. From that date onwards, the hedge ratios were mostly negative, with various troughs in different points of time (May 2017, July 2017, November 2017, October 2018 and November 2018). The hedge ratios of the portfolios including feeds and European stocks during the BREXIT have an average value of -4.712. These hedge ratios were negative until June 2017. From that date onwards, the hedge ratios become positive. The highest values are found between January and October 2018. Fibers commodities are among the ones with the lowest positive average hedge ratios amongst the commodities invested with the European stocks during the BREXIT, as it reached 11.189. These hedge ratios evolved around positive and almost null values before a large trough

between January and February 2018, after which they rose near zero but held a negative pattern until the end of the crisis period. Regarding forestry products, the average hedge ratios of these commodities when included in portfolios containing European stocks during the BREXIT is 14.165. These hedge ratios started negative before reaching high values between April and October 2017. Another peak is documented in January 2018, followed by two even higher peaks in October and December 2018. For grains, the findings show a positive average hedge ratio of the portfolios including these commodities and European equities during the BREXIT. This average hedge ratio is 50.236. This is due to the fact that the hedge ratios were mostly positive across the crisis period (from July 2017 until December 2018). The highest values are documented in the April 2018 – October 2018 time interval. In the weeks before July 2017, the hedge ratios were negative. The portfolios including livestock's and European equities during the BREXIT have the lowest positive hedge ratio, as it reached 6.153. These hedge ratios evolved very close to zero, except for a very large peak in November 2017 and a smaller trough (in magnitude) in October 2018. Focusing on oil seeds, these commodities generate an average hedge ratio of 42.357 when included in portfolios with European stocks during the BREXIT. Starting the first week of the crisis, the hedge ratios went on a decreasing trend, with the lowest trough in April 2017. Starting June 2017, the hedge ratios became positive and varied mostly between 50 and 100. A peak is reached in September 2018 before dropping since October 2018. The portfolios including seeds and European stocks generate an average hedge ratio of 25.237 during the BREXIT. These hedge ratios varied above 20 during most of crisis period, except for few weeks of negative values (from the first week until September 2016). The peaks are found in May 2017, November – December 2017 and August – October 2018. Semi-conductors exhibit low average hedge ratio during the BREXIT – when included in portfolios with European stocks; compared to the other commodities. The average hedge ratio is 6.772. Except for the period of high values between April 2017 and February 2018, the hedge ratios evolved in the negative range, in values between -50 and 0. The hedge ratios of the portfolios including softs and European stocks during the BREXIT were mostly negative and highly volatile, they the general trend is increasing. The lowest value is in the first week of the crisis, while the highest is in September 2018.

Regarding the portfolios including the US factor during the BREXIT, the findings show that when the various commodities subject of the study are included in portfolios containing the US factor, they generate relatively low hedge ratios. In fact, except for electricity, the average hedge ratios are low in magnitude, even the negative ones. This suggests that including commodities

in portfolios with the S&P500 during the BREXIT crisis period has a good hedging effectiveness. Agricultural oils, livestock's and softs have negative hedge ratios, suggesting that futures and spot prices move in opposite direction. Consequently, hedging long spot prices requires long positions in futures prices. The findings also show that the portfolios including fibers have the lowest hedge ratio, thus the best hedging effectiveness.

Starting with crude oil, the results show that when these commodities are included in portfolios with US stocks, they generate low hedge ratios. These varied between -0.742 and 2.33 across the crisis period, thus making an average hedge ratio of 0.525. The highest hedge ratio is documented in the first week of the crisis. Afterwards, these hedge ratios drop and increase again between November 2016 and April 2017. Starting May 2017, the hedge ratios evolve around 0.5, and then the values turn mostly negative from February 2018 onwards. Regarding natural gas, the hedge ratios start at their highest level at -7.73 before evolving between -5 and 0 until December 2017. Afterwards, the values increase to positive values starting January 2018, with the peak of 26.81 reached in October 2018. The average hedge ratio of the portfolios made of US stocks and natural gas during the BREXIT was 0.528. The portfolios made of electricity commodities and US stocks have the highest average hedge ratio during the BREXIT, as it reached 23.99. These hedge ratios varied between -28.596 and 80, a peak recorded in October 2016. The hedge ratios were positive and above 20 from the start of the crisis until September 2017. Afterwards, the values dropped and evolved around 0, with negative values reached from October 2018 onwards. The average hedge ratio of the portfolios made of US stocks and metals commodities during the BREXIT is 0.354. These hedge ratios varied between -0.031 and 1.112. The hedge ratios were above 0.2 during most of the crisis period. The highest values are documented between October 2017 and February 2018. The hedge ratios drop to evolve near zero from October 2018 onwards. Turning to precious metals, the average hedge ratio of the portfolios including these assets and US equities was 0.332 during the BREXIT. These hedge ratios varied between -0.136 and 1.02, with the highest values recorded between October 2016 and August 2017. Another period had smaller peaks, which is the June – October 2016 time interval. The hedge ratios for the portfolios made of agricultural oils and US stocks ranged between -1.134 and 0.151 during the BREXIT. These hedge ratios were mostly negative; thus, the average hedge ratio was -0.323. The evolution of these hedge ratios starts with steep troughs in the first week of the crisis, followed by an increasing trend, until low and positive values reached in November 2018. Regarding chemicals, the results show that the hedge ratios of the portfolios made of these commodities and American stocks ranged between -0.559 and 1.609

during the BREXIT, thus an average hedge ratio of 0.254. The peak was documented in the first week of the crisis, before dropping to evolve around values less than 0.775 during most the crisis period. Moreover, negative hedge ratios were found in the September – October 2016 period and between September 2017 and April 2018. Turning to feeds, the average hedge ratio of the portfolios including them with US equities during the BREXIT is 0.647. These hedge ratios varied between -0.208 and 1.766 during the crisis period. The highest values are found between February and September 2018. This comes after a gradual increase that started in June 2016, which followed a trend between 0 and 0.5. The hedge ratios dropped sharply from October 2018 onwards. The hedge ratios of the portfolios made of fibers and American stocks have an average of 0.219 during the BREXIT. The hedge ratios are generally between -1 and 1, except for the peak of 1.539 in May 2017 and the trough of -5.25 in February 2018. The average hedge ratios of the portfolios containing forestry products and American stocks were low during the BREXIT. These hedge ratios ranged between -0.05 and 0.765 during the crisis period. Consequently, an average of 0.288 is documented. The hedge ratios are mostly positive and above 0.2 across the crisis period. The peak is reached in February 2017. Moreover, the hedge ratios have an increasing pattern from March 2018 onwards. Turning to grains, the findings show that the average hedge ratio of the portfolios containing grains commodities and US stocks during the BREXIT is 0.333. These hedge ratios varied between -0.525 and 2.52 across the crisis timeline. The values evolved between -0.5 and 0.5 during most of the crisis period, before a sharp increase that led to the highest values in the period between April and September 2018, a date from which the hedge ratios drop again dramatically to evolve around 0.5. Focusing on livestock, the hedge ratios of the portfolio containing these assets and US equities varied between -6.979 (a trough in October 2018) and 5.8 (a peak documented in November 2017). Except for these extremes, the hedge ratios evolved near zero during all of the timeline of the BREXIT. Consequently, an average hedge ratio of 0.259 is documented during the BREXIT. Turning to oil seeds, our results suggest that when these commodities are included in portfolios with US stocks during the BREXIT, they generate hedge ratios that vary between 0.0712 and 2.216, thus an average hedge ratio of 0.832 is documented. These hedge ratios evolved between 0.5 and 1.5 during most of the crisis period, before increasing to their highest values between March and September 2018. The values then drop in October 2018 to evolve near 0.5 until the end of the crisis period. The average hedge ratio of the portfolios made of seeds and US stocks during the BREXIT is 0.414. These hedge ratios varied between -0.625 and 1.13 during the crisis period. The values started negative before rising to the positive range in October 2016. The hedge ratios had values above 0.2 during most of the crisis period, with

the highest values documented between May and September 2018. Regarding semi-conductors, the average hedge ratio of the portfolios containing these commodities and US equities during the BREXIT is 0.154. These hedge ratios varied between -0.453 and 1.151 across the crisis period. The highest values are documented between May 2017 and January 2018. During the rest of the crisis period, the values did not go above 0.4, with an evolution in the negative range from February 2018 onwards. The hedge ratios of the portfolios made of softs and US stocks were mostly negative and highly fluctuating during the BREXIT. They varied between -1.5 (a trough found in the first week of the crisis) and 1.357 (a peak documented in September 2016). Consequently, an average hedge ratio of -0.102 is documented.

Turning to the BREICS factor, our findings show that the average hedge ratios of the portfolios made of the BRICS stocks and the various commodities are low, whether they are positive or negative. Consequently, including the commodities in portfolios with the BRICS stocks during the BREXIT has a good hedging effectiveness. The portfolios containing electricity, livestock, fibers, semi-conductors and feeds have negative hedge ratios, suggesting that spot and futures prices moved in opposite direction during the BREXIT. The portfolios containing electricity have the highest average hedge ratio in terms of magnitude. Regarding the commodities with the positive hedge ratios, softs have the best hedging effectiveness as it has the lowest hedge ratio.

The average hedge ratio of the portfolios containing crude oil commodities and BRICS stocks during the BREXIT is 0.811. These hedge ratios ranged between -0.076 and 2.12 during the crisis period, with a peak in November 2016. In addition, during the BREXIT timeline, the hedge ratios have a decreasing pattern. Turning to natural gas, the findings show that the average hedge ratio of the portfolios containing these commodities and BRICS equities during the BREXIT is 0.307. These hedge ratios evolved between -7.83 and 23.915 (a peak documented in January 2018) across the crisis period. The hedge ratios were negative from the start of the crisis until the peak, after which they started having a declining trend to evolve around 5. The hedge ratios of the portfolio containing electricity commodities and BRICS equities varied between -50.85 and 36.984 during the BREXIT. These hedge ratios started with the peak in the first week of the crisis, before going into a declining trend. The lowest trough is documented in August 2017, and the hedge ratios were in the negative range from that date onwards. Consequently, an average hedge ratio of -7276 is documented. Regarding the portfolios containing metals commodities and BRICS stocks, they generated an average hedge ratio of 0.499 during the BREXIT. These hedge ratios were between -0.038 and 2.287. The

highest values were documented between July 2017 and February 2018. For the rest of the crisis period, the hedge ratios were between 0 and 0.5. The hedge ratios for the portfolios made of precious metals commodities and BRICS stocks during the BREXIT varied between -0.223 and 0.697, thus the average hedge ratio is 0.221. These hedge ratios were between -0.2 and 0.2 during most of the crisis timeline, before increasing to their highest values between December 2017 and November 2018. Turning to agricultural oils, the findings show that the average hedge ratios of the portfolios containing these commodities and BRICS stocks generated an average hedge ratio of 0.233 during the BREXIT. These hedge ratios were between -0.095 and 0.73 across the crisis period. The values were generally above 0.1, the highest of which are reached between August 2017 and February 2018. When chemicals are included in a portfolio with BRICS stocks during the BREXIT, they generate an average hedge ratio of 0.245. These hedge ratios varied between -0.469 and 0.987, with a relatively increasing pattern starting February 2017, following an evolution in the negative range. The highest value is found in August 2017. Feeds show an average hedge ratio of -0.142 when included in portfolios with BRICS stocks during the BREXIT. These hedge ratios were between -0.823 and 0.629 across the crisis period. The values were negative during most of the crisis timeline, i.e. from the troughs in the first weeks of the crisis until January 2018, before increasing and reaching the highest values during 2018. Turning to fibers, when these commodities are invested in portfolios with BRICS stocks during the BREXIT, they generate hedge ratios that vary between -15.97 (a trough documented in February 2018) and 0.98, thus an average hedge ratio of -0.464. Before the large trough mentioned above, the hedge ratios were very close to zero. After the trough, the values rose near zero but remained in the negative range. The portfolios made of forestry products and BRICS equities exhibit an average hedge ratio of 0.132 during the BREXIT. These hedge ratios varied between -0.135 and 0.469 across the crisis period. The values started negative until November 2016. Starting that date, they increased and had two large peaks (February 2017 and January 2018). Another smaller peak is found in October 2018. The hedge ratios of the portfolios including grains and BRICS equities during the BREXIT evolved around zero during most of the crisis period. This trend was held from the start of the crisis timeline until March 2018, the date from which the hedge ratios escalated until reaching the peak of 1.953 in May 2018. The lowest hedge ratio was -0.322, while the average hedge ratio during the BREXIT was 0.258. Regarding livestock's commodities, the findings show that when they are combined in portfolio with BRICS stocks during the BREXIT, they generate mostly negative hedge ratios that are also low in magnitude, except for three troughs in December 2016, October 2018 and the largest one which was in November 2017. These hedge ratios varied between -9.37 and

0.426, a peak documented in July 2017. The average hedge ratio was -0.578. Oils seeds generated hedge ratios that varied between -0.298 and 1.012, with an average of 0.282, when included in portfolios with BRICS stocks during the BREXIT. These hedge ratios evolved generally between 0 and 0.4 before increasing to reach their highest values in the time lapse between February and November 2018. From that date onwards, the hedge ratio drop again to reach values lower than 0.2. Turning to seeds, the findings show that the hedge ratios of the portfolios including seeds and BRICS stocks during the BREXIT evolve in an increasing steps-like trend, with values between -0.173 and 0.868. Consequently, the average hedge ratio is 0.225. The highest values are reached in the period that starts in May 2018 until the end of the crisis timeline, with a peak in the last week of the crisis. The hedge ratios of the portfolios containing BRICS equities and semi-conductors varied between -1 and 0.469, making an average hedge ratio of -0.248. These hedge ratios were negative until august 2017, when they increased to reach their highest values until February 2018. From that date onwards, the values dropped back to the negative range. Turning to seeds, the findings show that when these commodities are included in portfolios with BRICS stocks during the BREXIT, they generate an average hedge ratio of 0.0312. These hedge ratios vary between -1.15 and 0.838 across the crisis period. Moreover, they are highly fluctuating with various peaks (the largest one is in the first week of the crisis) and troughs (the lowest one is in August 2017).

#### **4. Conclusion**

This paper was motivated by the research of Ayadi (2021) and Ayadi et al. (2021), as well as the phenomenon of financialization in commodities, with the aim of constructing optimal portfolios consisting of various equities (USA, sixteen European countries, and the BRICS) and commodities (Agricultural Oils, Chemicals, Crude Oil, Electricity, Feeds, Fibers, Forestry Products, Grains, Livestock, Metals, Natural Gas, Oil Seeds, Precious Metals, Seeds, Semi-Conductors, and Softs) during the BREXIT. We followed the lead of previous authors, such as Mensi et al. (2013), Jebabli et al. (2014), Khalfaoui et al. (2015), and Hasan (2017), who used the Kroner and Ng (1998) methodology to calculate optimal portfolio weights and hedge ratios to test hedging effectiveness.

Ayadi (2021) provided evidence of equity market contagion from American and European stock markets to the BRICS during the BREXIT, which led to the search for a way to minimize the shock's effects. Commodities have proven to be good hedgers and safe havens for risk-

minimizing portfolios that hold equities, despite the financialization phenomenon documented by Basak and Pavlova (2016), especially during the global financial crisis, when institutional investors became interested in investing in commodities to hedge equity portfolios. As a result, commodities became an asset class in their own right.

Ayadi et al. (2021) tested for contagion from equity to commodity markets during various crises and documented evidence of shock transmission. Investors would therefore naturally seek to select the optimal combination to hedge their portfolios. Using the Kroner and Ng (1998) methodology, we calculated optimal weights and hedge ratios for various portfolios, each consisting of an equity factor and a commodity factor.

Our findings indicate that portfolios including the European factor do not need to incorporate commodities to create optimal portfolios, as European equities make optimal portfolios on their own. Moreover, all of the commodities, except for electricity, provide effective hedging for American and BRICS stocks during the crisis studied, suggesting that commodities continue to serve as effective hedges and safe havens during crises, despite financialization.

These findings will be valuable to investors and market participants in selecting the optimal combinations to create portfolios that can withstand shocks from various crises. Additionally, it helps them choose the best commodities to combine with the various equities to minimize their portfolio's risk during different crises.

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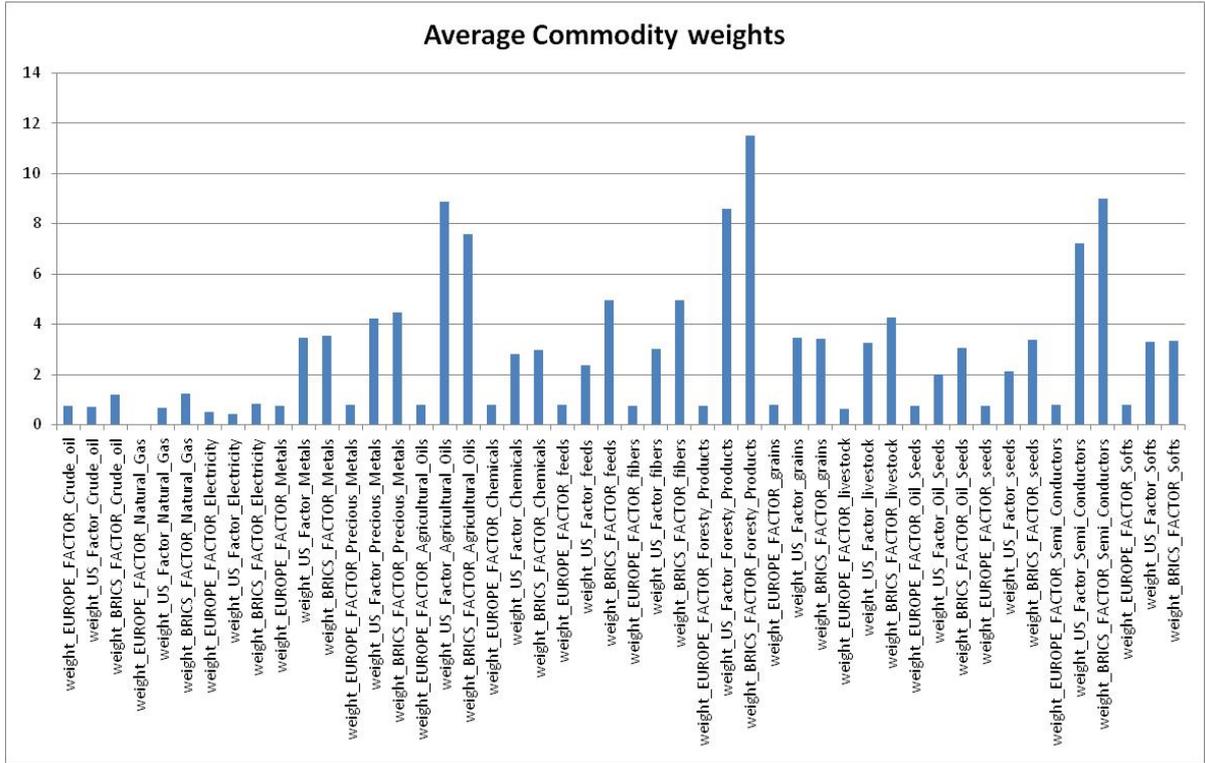
## Appendices

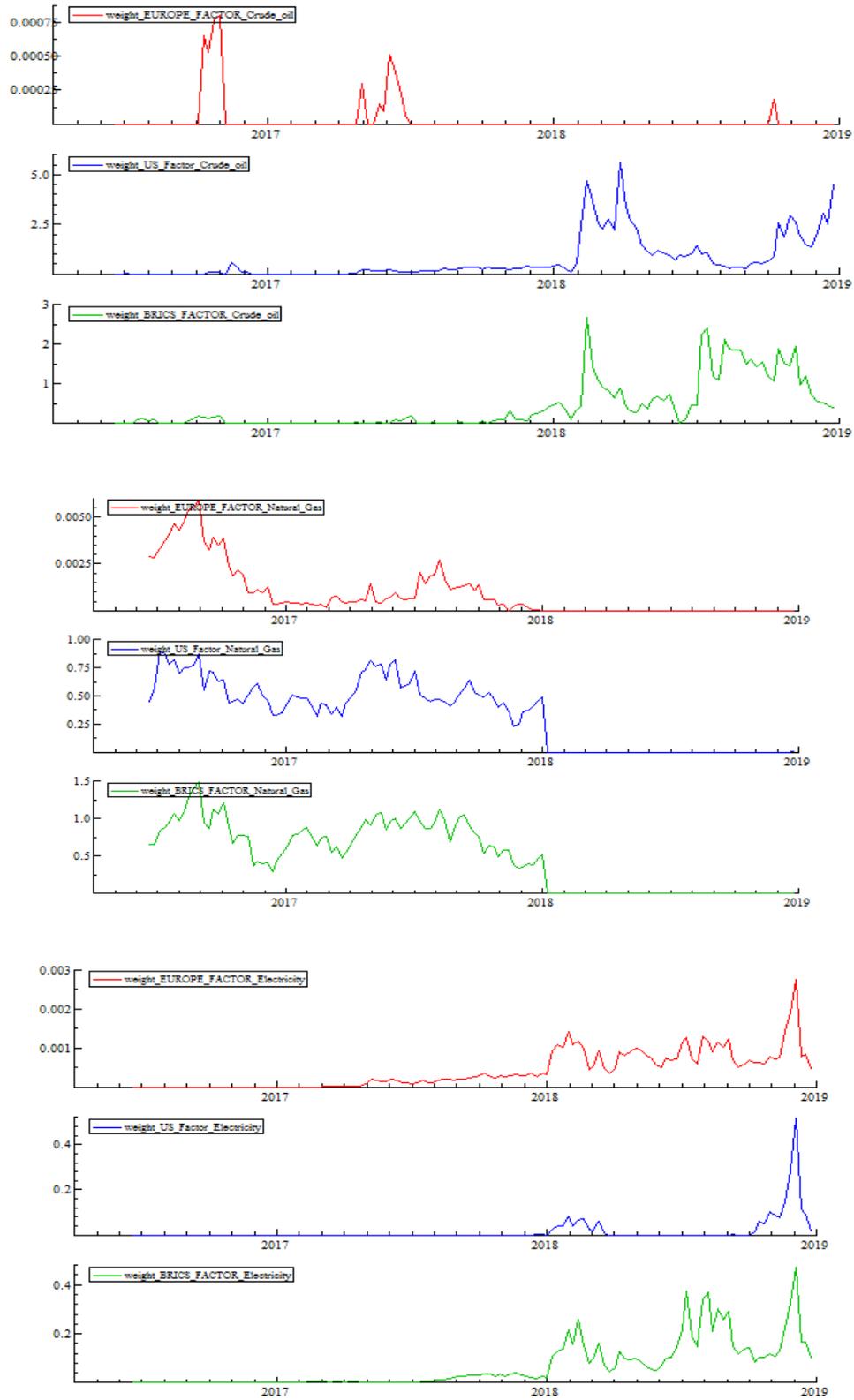
**Table 1. Weights and Hedge Ratios**

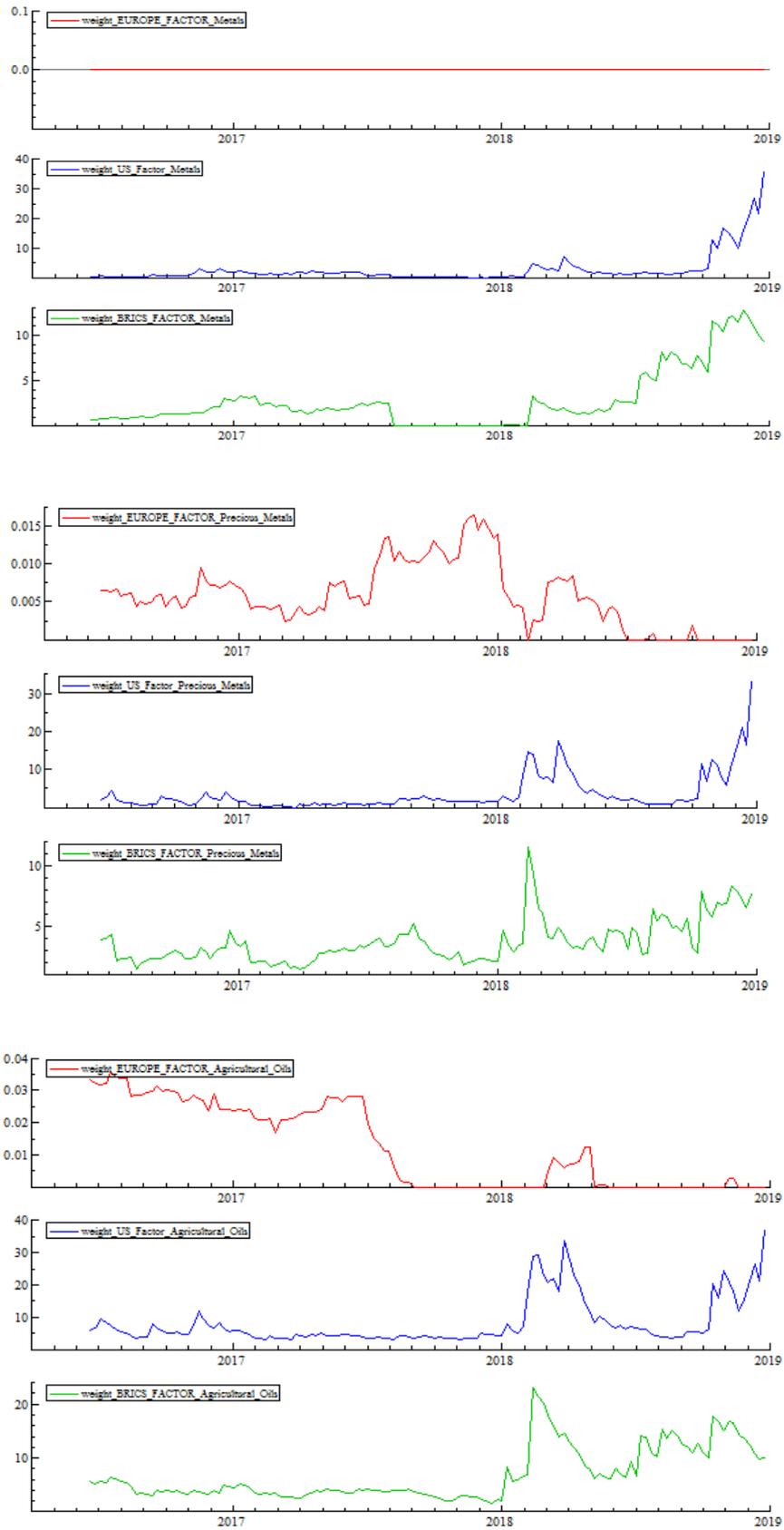
	Commodity Weights	Equity Weights	Hedge ratios
EUROPE FACTOR Crude oil	0.752	99.248	140.217
US Factor Crude oil	0.694	99.306	0.526
BRICS FACTOR Crude oil	1.170	98.830	0.811
EUROPE FACTOR Natural Gas	0.006	99.994	620.312
US Factor Natural Gas	0.649	99.351	0.528
BRICS FACTOR Natural Gas	1.229	98.771	0.308
EUROPE FACTOR Electricity	0.507	99.493	-2586.973
US Factor Electricity	0.399	99.601	23.996
BRICS FACTOR Electricity	0.816	99.184	-7.276
EUROPE FACTOR Metals	0.752	99.248	144.442
US Factor Metals	3.444	96.556	0.355
BRICS FACTOR Metals	3.533	96.467	0.499
EUROPE FACTOR Precious Metals	0.757	99.243	-37.106
US Factor Precious Metals	4.209	95.791	0.332
BRICS FACTOR Precious Metals	4.435	95.565	0.221
EUROPE FACTOR Agricultural Oils	0.764	99.236	-53.947
US Factor Agricultural Oils	8.860	91.140	-0.324
BRICS FACTOR Agricultural Oils	7.559	92.441	0.233
EUROPE FACTOR Chemicals	0.755	99.245	-6.995
US Factor Chemicals	2.782	97.218	0.254
BRICS FACTOR Chemicals	2.965	97.035	0.245
EUROPE FACTOR feeds	0.758	99.242	-4.712
US Factor feeds	2.335	97.665	0.648
BRICS FACTOR feeds	4.951	95.049	-0.143
EUROPE FACTOR fibers	0.753	99.247	11.190
US Factor fibers	3.006	96.994	0.220
BRICS FACTOR fibers	4.944	95.056	-0.465
EUROPE FACTOR Forestry Products	0.754	99.246	14.166
US Factor Forestry Products	8.589	91.411	0.288
BRICS FACTOR Forestry Products	11.473	88.527	0.132
EUROPE FACTOR grains	0.755	99.245	50.237
US Factor grains	3.429	96.571	0.333
BRICS FACTOR grains	3.400	96.600	0.259
EUROPE FACTOR livestock	0.597	99.403	6.153
US Factor livestock	3.242	96.758	-0.259
BRICS FACTOR livestock	4.259	95.741	-0.578
EUROPE FACTOR Oil Seeds	0.753	99.247	42.357
US Factor Oil Seeds	1.989	98.011	0.832
BRICS FACTOR Oil Seeds	3.040	96.960	0.282
EUROPE FACTOR seeds	0.752	99.248	58.237
US Factor seeds	2.090	97.910	0.414
BRICS FACTOR seeds	3.349	96.651	0.226
EUROPE FACTOR Semi Conductors	0.755	99.245	6.773
US Factor Semi Conductors	7.222	92.778	0.154
BRICS FACTOR Semi Conductors	8.978	91.022	-0.249
EUROPE FACTOR Softs	0.757	99.243	-65.380
US Factor Softs	3.283	96.717	-0.102
BRICS FACTOR Softs	3.342	96.658	0.031

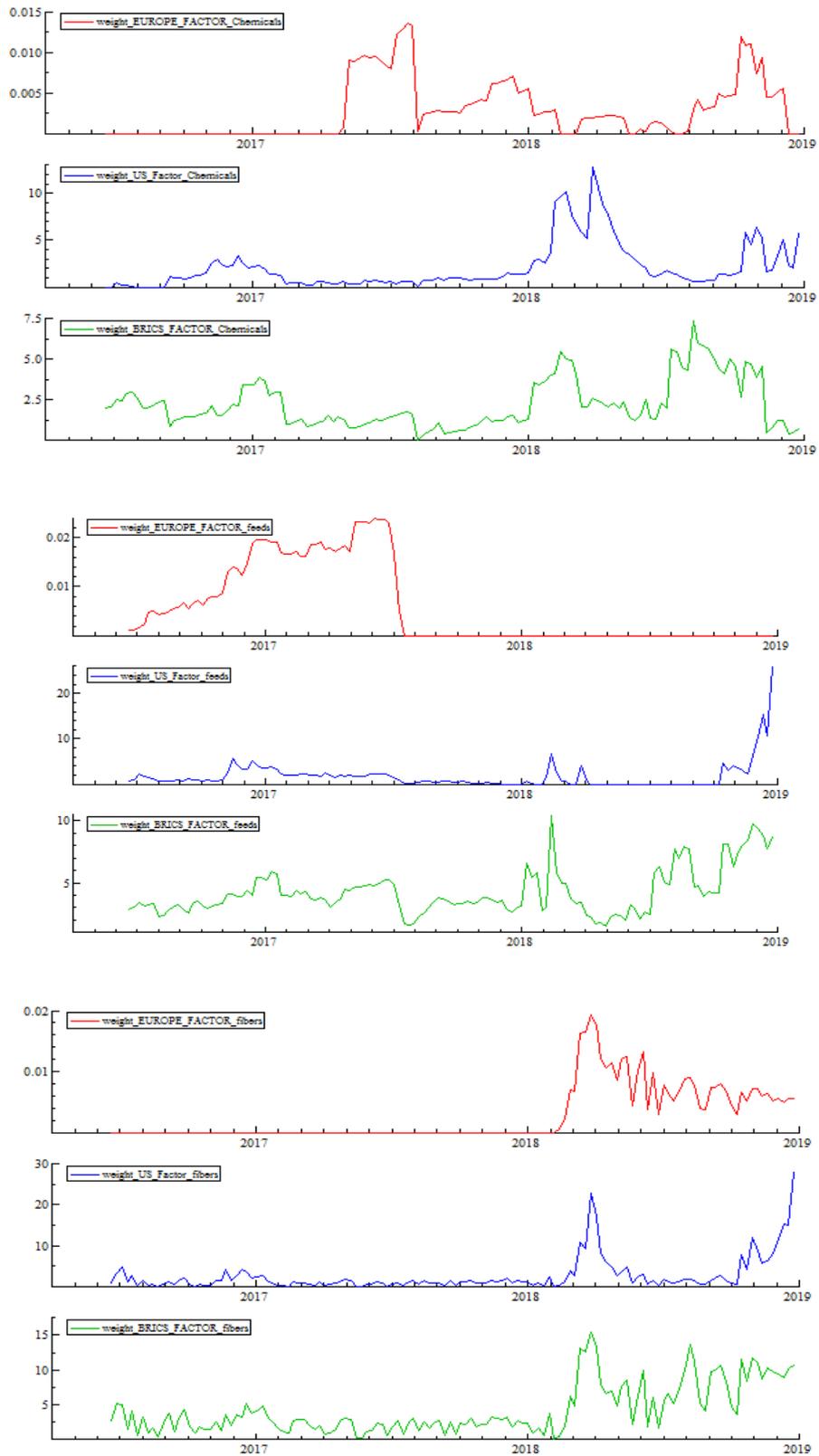
Notes: In this Table, we report the weights of each of the commodities and each of equities in the two-asset portfolios (each portfolio is made of one equity factor and one commodity factor), as well as the hedge ratios of these portfolios. The weights are calculated following Equations (1) and (2). The hedge ratios are calculated following Equation (3).

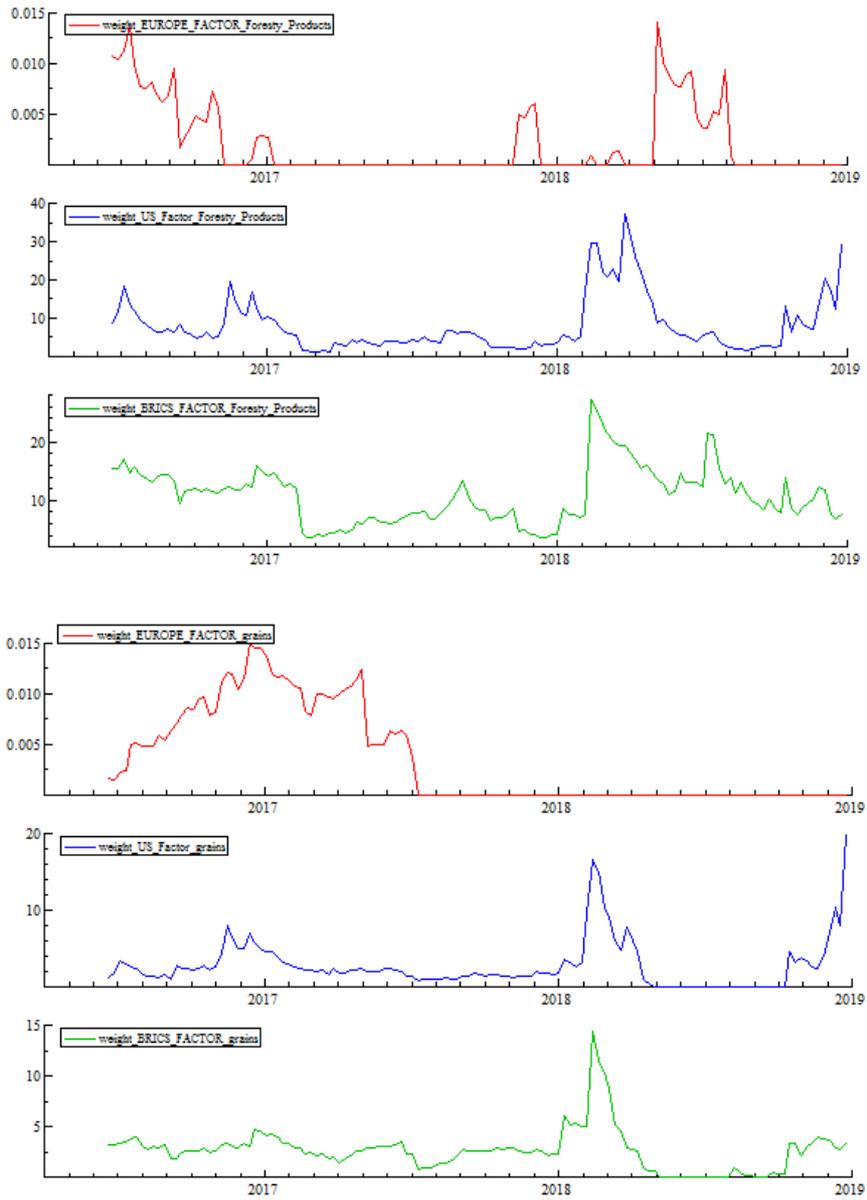
Figure 1.a: Commodity portfolio weights

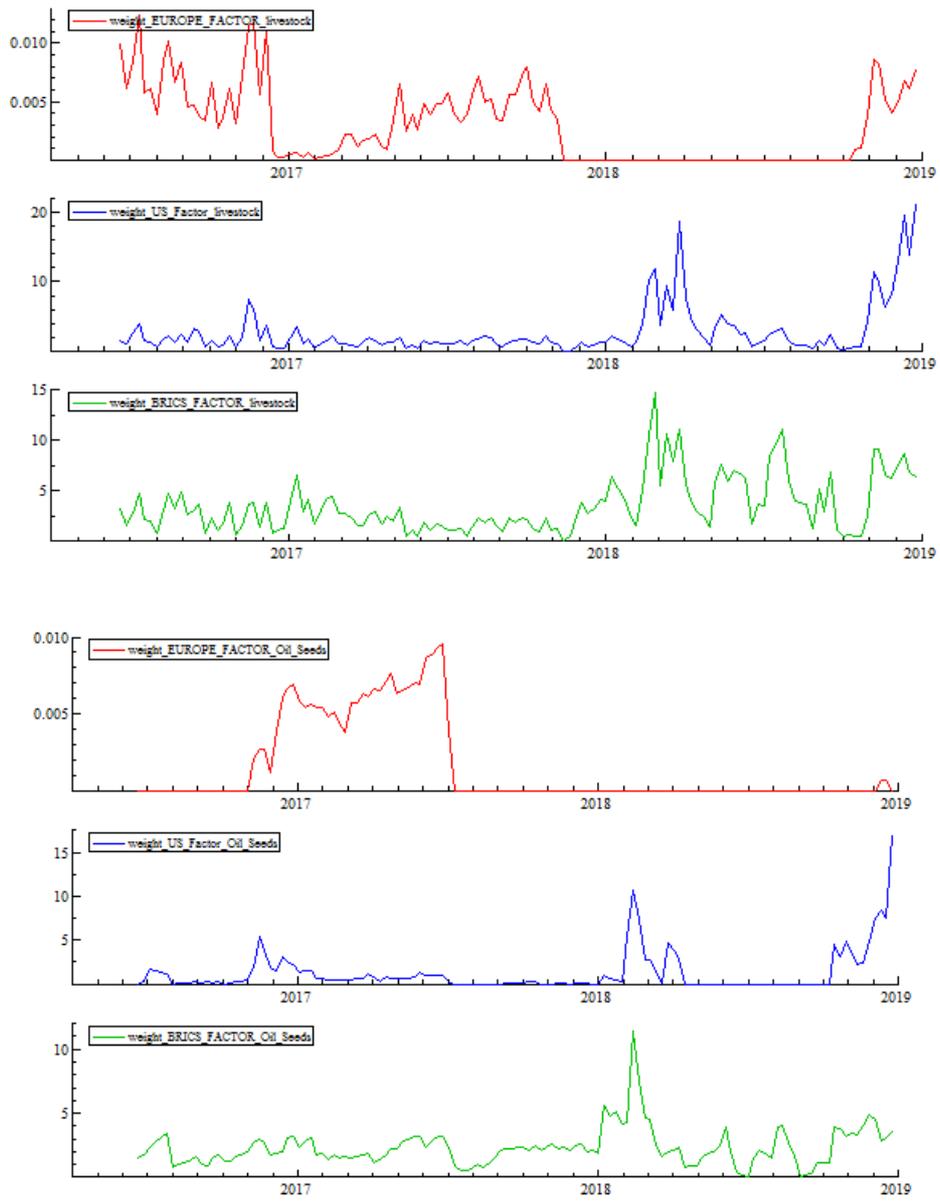












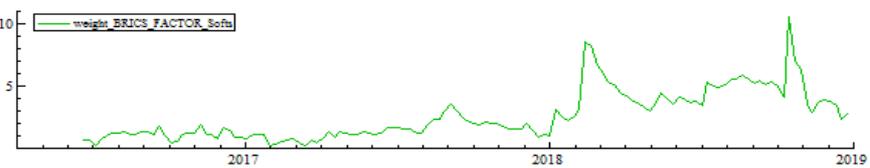
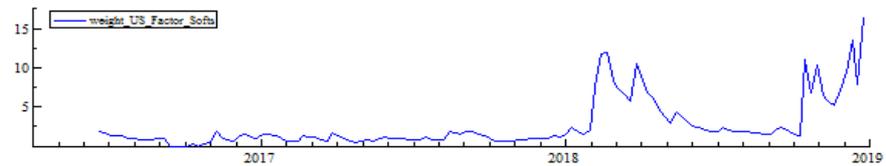
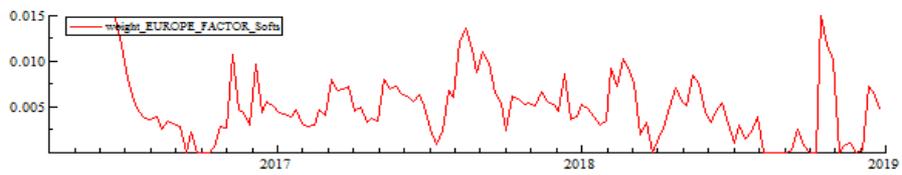
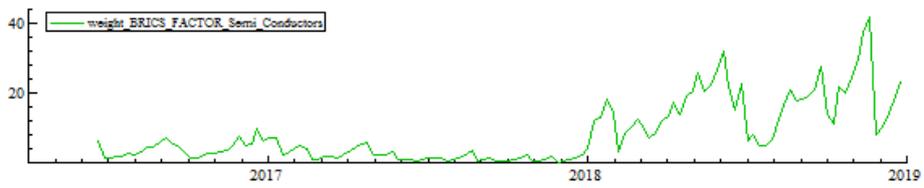
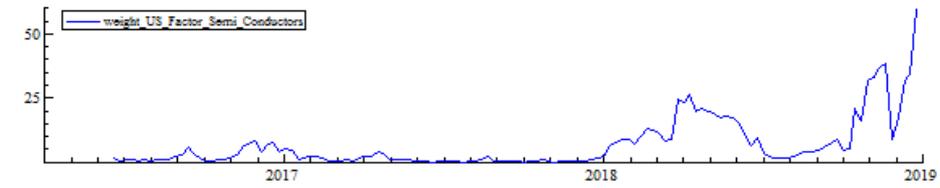
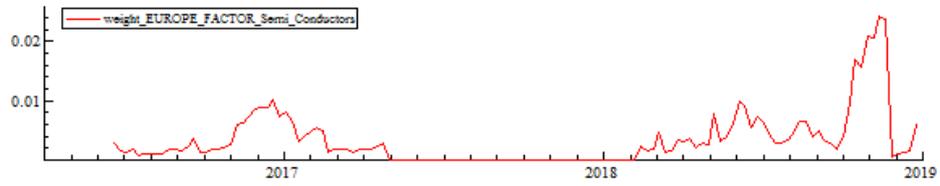
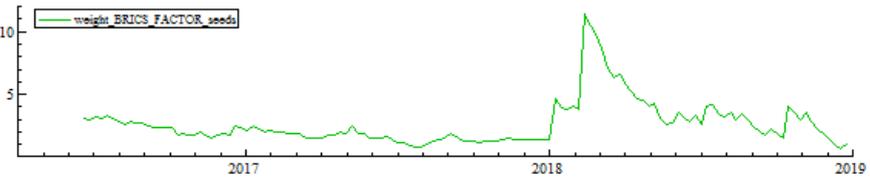
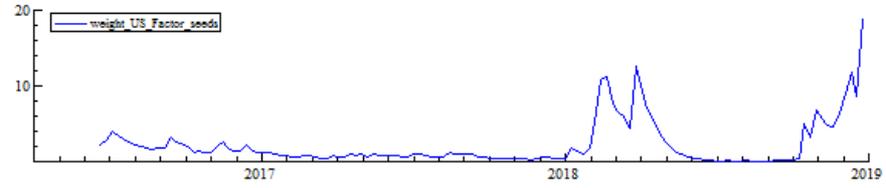
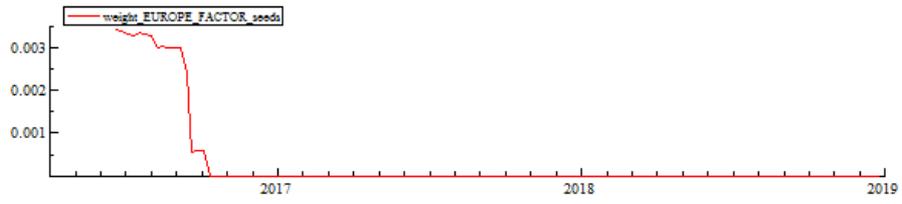


Figure 2.b: Hedge ratios

