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Shahid Rahman, Flavia Marcacci

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# Dialogical Logic



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**Shahid Rahman**

[shahid.rahman@univ-lille.fr](mailto:shahid.rahman@univ-lille.fr)

Université de Lille  
UMR-CNRS 8163: STL, France

**Flavia Marcacci**

[flaviamarcacci@gmail.com](mailto:flaviamarcacci@gmail.com)

Pontificia Università Lateranense  
Stato della Città del Vaticano

## Introduction

*Dialogical logic* is a dialogue-based framework for meaning, knowledge and logic rooted in a research tradition on argumentation that goes back to dialectics in Greek Antiquity, when semantic, epistemic and ethical issues were approached through debates in which opposing parties discussed a thesis through questions and answers

Contemporary "Dialogic logic" was conceived by Paul Lorenzen by 1958, the year of his talk *Logik und Agon* held in Rome at the XII<sup>th</sup> World Congress of Philosophy and was further developed by Kuno Lorenz. Initially Dialogical Logic aimed at overcoming some limitations of Lorenzen's (1955) own *Operative Logik*.<sup>1</sup>

The work of Lorenzen and Lorenz and the *Constructivist School of Erlangen*, took later on the shape of an overall pragmatist and game-theoretical *Dialogical Framework* to philosophy of language, logic and sciences. This framework was the outcome of the intertwining of constructive mathematics and logic,<sup>2</sup> a dialectical reading of emergence of philosophy of language and logic in the Ancient Greek tradition,<sup>3</sup> and Wittgenstein's notion of language games,<sup>4</sup> with a special accent on the ethical undergirding of conceptual thinking.<sup>5</sup> Lorenz added three important methodological perspectives to the project, namely (i) the reconstruction of ancient theories of meaning and knowledge including not only Plato but also Indian traditions, (ii) a semiotic, Peircean, understanding of language games (iii) the explicit use of mathematical game theory in logic.<sup>6</sup>

Dialogical logic studies dialogues; but it also takes the form of dialogues. In a dialogue, two parties (players) argue on a thesis (a certain statement that is the subject of the whole argument) and follow certain fixed rules in their argument.

The player who states the thesis is the Proponent, called "**P**", and his interlocutor, the player who challenges the thesis, is the Opponent, called "**O**". In challenging the Proponent's thesis, the Opponent is requiring of the Proponent that he defends his statement by a sequent moves, shaped by the adopted rules for the development of a dialogue, involving declarative utterances, called *statements*, and interrogative utterances called *requests* (in the original setting of Lorenzen and Lorenz, some requests, may adopt the form of a statement, namely, when stating the antecedent of an implication (or

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<sup>1</sup> see Lorenz (2001), Schroeder-Heister (2008).

<sup>2</sup> see Lorenzen & Lorenz (1978).

<sup>3</sup> See Ebbinghaus (1964, 2016), Lorenz & Mittelstrass (1966, 1967), Lorenz (2011).

<sup>4</sup> see Lorenz (1970), Kamlah & Lorenzen (1972), Lorenz (2009, 2011).

<sup>5</sup> see Lorenzen (1969), Lorenzen & Schwemmer (1975); Lorenz (2021, chapter 2).

<sup>6</sup> see Lorenz (1961, 1998, 2009, 2010, 2014, 2021).

negation), in order to challenge a consequent).

In fact, the rules that shape a dialogue are divided into two kinds of rules: *particle rules* and *structural rules*. Whereas the first determine *local meaning* of an expression the second determines its *global meaning*. More precisely, the rules for local meaning explain the meaning of an expression independently of the rules setting the development of a dialogue (play) and independently of who of the interlocutors is the one setting the thesis – i.e. these rules prescribe how to challenge a statement and to defend it but it does not fix who the challenger and who the defender is. Norms for *winning-strategies*, the dialogical analogue to inference rules, select those plays shaped by the structural rules in which both players make optimal moves. In fact, validity, as discussed below, is defined via the notion of winning strategy.

Thus, there is a play perspective on plays and a strategic perspective on plays, whereby the latter presupposes the former. The very point of just playing, can be seen as mechanism for checking if the thesis brought forward, does or not enjoy the property of dialogue definiteness: it does not matter who wins, so long as one does. The strategic perspective then, chooses among the plays on a dialogue definite thesis, those where the moves are optimal with regard to justifying the thesis.

**Nota Bene:** Notice that since dialogue definiteness is defined on **individual** plays, the play perspective on plays does not preclude the coexistence of two (or more) plays on the same thesis but where different sequences of moves lead to different outcomes. This allows shaping plays where *revisions* take place by choosing one rather than another sequence of moves (for example by the explicit collaboration of the interlocutors), and/or plays aiming at conciliation, whereby the different sequences merge into a new play on an expanded, modified, new thesis.

The local meaning of an expression sets the core of its interactive meaning, which we call its *local dialogical meaning explanation* is *invariant*<sup>7</sup>.

in relation to the players,  
to the development of a play (including winning and losing it),  
to the constitution of winning strategy.

Global meaning set the meaning of an expression in the context of some specific form of developing a dialogue given by rules called *structural rules* (since they generate at the strategy level the known structural rules of Gentzen systems). Accordingly, we say that the structural rules provide *global dialogical meaning explanations*. Global Meaning explanations are only *partially* invariant in relation to the players, since, as we will discuss below, there is a rule which allows one player, namely **O**, to state elementary statements, that cannot be challenged. Still, *global dialogical meaning explanations are invariant*

in relation to the players, when they state non-elementary statements,  
to the development of a play (including winning and losing it),  
to the constitution of winning strategy.

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<sup>7</sup> The expression *invariant core* has been borrowed from *Linear Logic* and *Ludics*. Ludics proposes a kind of theory of meaning as interaction focused only on what we just called local dialogical meaning explanations. Indeed, within Ludics, structural rules are reduced to local rules by introducing unary connectives into the object language such as an operator for repetition ranks that allows to convert an intuitionistic negation into a classical one – cf. Lecomte & Quatrini (2011a,b). Ludics proposes a dynamic interaction process involving only the rules preserving the invariant meaning core, whereby structural and strategic rules are constituted. The invariant core amounts to the bare interaction of questions and answers or *expectations* and *intentions*, to use Lecomte & Quatrini's (2011a) words.

Central to the dialogical perspective is that *preserving local meaning*, plays the role that *admissibility* had in Lorenzen's earlier *Operative Logik*. Indeed, as pointed out by Schröder-Heister (2008), admissibility was Lorenzen's proof-theoretical venue to meaning and what distinguished his approach from formalism: admissibility allows, to identify a core of meaning invariant in relation to a set of rules. Admissibility continues to be one of the main tenets of contemporary proof-theoretical approaches as the one of Substructural Logics, Linear Logic and Ludics, after Prawitz's developed attributions of admissibility into a system of normalization procedures.<sup>8</sup>

After the dialogical turn, the identification of the invariant core of meaning came down to identifying the local meaning at work in both structural rules and strategic rules. Moreover, attributions of admissibility to an inference are now replaced by attributions of *dialogue-definiteness*: an expression enjoys the property of dialogue-definiteness, if it **can** be stated as thesis in an individual finite Zero-Sum-play, that ends with loss or win and is shaped by structural rules that preserve local dialogical meaning explanations. Actually dialogue-definiteness is the condition for being counted as a proposition – see Lorenz (2001, p. 258). However, win or loss amounts neither to truth-value definiteness nor to proof-theoretical definiteness. Indeed according to the novel approach, truth-value indefinite propositions such as the Law of Excluded Middle, can give place to a decidable play on the thesis stating that Law, where **P** loses the game, but this of course does not mean that it false or that there is a proof for its refutability – i.e. that **P** has a winning strategy for its negation.

Furthermore, in the dialogical setting, the justification of inference rules, amounts to show that they preserve local meaning, that is that they preserve a core of dialogical meaning explanation invariant in relation to players, structural and strategic rules.<sup>9</sup>

- In short, from the Dialogician's perspective, the root of inference rules as expressed by rules for a winning strategy, is the player independent local dialogical meaning explanations put at work by structural rules that preserve them. Dialogical rules are not a dialogical interpretation of inference rules but they justify them.
- The former point is crucial for any form of dialogical pluralism since choosing between different patterns of reasoning assumes an invariant core in relation to which the norms for determining both the development of a play and of a winning strategy are to be constituted.

Actually, as pointed out by Lion (2021, pp. 125-159; 265-310), the invariant core itself is the result of a prior dynamic semiotic process whereby this core meaning has been fixed – though the process can later on be launched again. According to this view, the choices inherent to dialogical pluralism, start before the discussion on structural rules come to the fore.

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<sup>8</sup> See Lecomte & Quatrini (2011a,b).

<sup>9</sup> This is why tonk-like operators can be dismissed in the dialogical framework – see Rahman & Redmond (2016). Moreover, on one hand Crubellier et al. (2019) – further developed by McConaughey (2021), interpret Aristotelian proofs by *ecthesis*, as laying down the local meaning of the quantifiers given the *Dictum de Omni*, on the other we know since the work of Ebbinghaus (1964), that Aristotelian reductions to the perfect syllogism, inspired Lorenzen for the invention of admissibility, who by the way was the first on proposing a dialogical reconstruction of Aristotle's proof by *ecthesis*. Thus, according to Lorenzen's interpretation, reductions to perfect syllogism, if successful, show that the reduced rules preserve the local meaning layed down by the dialectical rules for quantifiers as encoded by the rules for *ecthesis*.

Let us now put this into work in the context of the logical constants for first-order logic. This represents indeed quite of a restricted context but it has the advantage of providing a simple language game for the study of the different levels of meaning shaping the Dialogical Framework.

### Particle Rules and Local Dialogical Meaning Explanations

The dialogical meaning explanations for the standard first order logical constants, in the original setting of Lorenzen and Lorenz, are the following: <sup>10</sup>

|  | Request  | Answer   |
|--|--|--|
| $X ! A \vee B$<br>(player <b>X</b> states a disjunction)                   | $Y ?_{\vee}$<br>(Questioner asks <b>X</b> to choose which side of the disjunction s(h)e is willing to defend)                                | $X ! A$<br>-----OR-----<br>$X ! B$<br>( <b>X</b> chooses the sides to be defended)           |
| $X ! A \wedge B$<br>(player <b>X</b> states a conjunction)                 | $Y ?_{L\wedge}$<br>-----OR-----<br>$Y ?_{R\wedge}$<br>(Questioner chooses to ask either for the Left or the Right)                           | $X ! A$<br>-----<br>$X ! B$<br>( <b>X</b> defends the side requested by <b>Y</b> )           |
| $X ! A \supset B$<br>(player <b>X</b> states an implication)               | $Y ! A$<br>( <b>Y</b> : I grant you the antecedent. Are you able to state the consequent given this grant of mine?)                          | $X ! B$<br>( <b>X</b> : yes, indeed)   |
| $X ! \neg A$<br>(player <b>X</b> states a negation)                        | $Y ! A$<br>( <b>Y</b> : On the contrary, <i>A</i> is the case)   | ———<br>(No defence is available. <b>X</b> might try a counterattack on <b>Y</b> ! <i>A</i> ) |
| $X ! \exists x A(x)$<br>(player <b>X</b> states an existential quantifier) | $Y ?_{\exists}$<br>( <b>Y</b> : Who/Which is the <i>x</i> you said to be an <i>A</i> ?)  | $X ! A(a/x)$<br>( <b>X</b> : <i>a</i> is one of those that is an <i>A</i> )                  |
| $X ! \forall x A(x)$<br>(player <b>X</b> states a universal quantifier)    | $Y ?_{\forall a}$<br>( <b>Y</b> : Since you stated that every <i>x</i> is an <i>A</i> , this should also hold for <i>a</i> . <u>Right</u> ?) | $X ! A(a/x)$<br>( <b>X</b> : Indeed, <i>a</i> is an <i>A</i> )                               |

### Structural Rules and Global Dialogical Meaning Explanations

As already mentioned, structural rules set the local dialogical meaning explanations in the context of the development of a play. This renders their global dialogical meaning explanations. Several development rules are possible. Each determine what pattern of reasoning counts as meaningful and which not, e.g., the choice between the rules **SR1i** and **SR1c**. below, determines if the reasoning is shaped by either an intuitionistic or classical theory of meaning. The present rules are *basic* in the sense that different logics and meaning approaches might require extending them by either adding new rules or adding further conditions to the existing rules. Some of these extensions will be mentioned in the commentaries of the appendix.

<sup>10</sup> see Rückert (2011), Clerbout (2014a,b), Clerbout & McConaughey (2022).

**SR0 (starting rule):** A play starts with a player stating a proposition called the thesis; that player becomes the Proponent (**P**) and the move is labelled move 0.<sup>11</sup>

The other player—the Opponent (**O**)—chooses a repetition rank determining how many times she is allowed to challenge or defend any move in a play. It is usually enough for **O** to choose a repetition rank of 1 ( $m := 1$ ), it is move 1. **P** then chooses a repetition rank: 2 is usually enough ( $n := 2$ ); it is move 2.<sup>12</sup>

**SR1i (intuitionistic game-playing rule):** After the repetition ranks have been chosen, each move is a challenge or a defence in reaction to a previous move, in accordance with the rules for **local** dialogical meaning explanations. Each player can challenge the same previous move at most  $n$  times, where  $n$  is the player's repetition rank, or defend against the adversary's *last unanswered* challenge.<sup>13</sup>

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<sup>11</sup> This labelling has the effect that whereas even numbers indicate moves by **P**, odd numbers indicate moves by **O**.

<sup>12</sup> This rule should assure that plays are finite (though there might be an infinite number of them). Indeed, one important feature of Lorenzen-Lorenz conception of proposition and statement is its finiteness: for an expression to count as a proposition  $A$  there must exist an individual play about the statement  $!A$ , in a play which must reach a final position after a **finite number** of moves according to definite particle and structural rules – cf. Lorenz (2001, p. 258). If language-games are to be conceived as mediators of meaning carried out by social interaction, these language-games must be games that we **can actually perform**. This is what Lorenz's notion of **dialogue-definiteness** is about. Technically speaking, finiteness is implemented by the explicit introduction of a repetition rank which bounds **the number of times a move can be challenged and/or defended** - for discussion on this point See Rahman et al. (2018, pp. 278-282). As pointed out by Lorenz (2001, p. 260) the bounds fixed by **the repetition rank is the play-level source of the strategy level rule for contraction**. Blass (1992) related the repetition rank with *Linear Logic*. Clerbout (2014a,b,c) delved into the metalogical consequences of the repetition rank and showed how it is related to the semi undecidability of first-order logic. One important general philosophical result of Clerbout work is that though every play must be finite, it does not prevent the existence of an infinite number of them. This also answers to a usual misunderstanding of Lorenzen-Lorenz dialogues: **most of the them, are in principle, zero-sum plays, but winning or losing a play does not preclude strategic Turing-Church-semi undecidability of first-order logic: we cannot in general fix a uniform repetition rank for every thesis stating a first order proposition**. Here again, we must recall the distinction between play perspective on plays and strategic perspective on plays : the **zero-sum** feature of plays is part of the mechanism for checking dialogue definiteness, but it **does not in general transmit to the strategy level**. Clerbout also showed that repetition rank  $m := n$  for **O** and  $n := n+1$  for **P**, are the optimal choices for developing a winning strategy for intuitionistic and classical logic.

An additional interesting proof-theoretical feature of arguments with restricted contraction or explicit repetition rank, is that they can show the degree of dependence of a proof upon an axiom. Indeed, assume that for some proof the axiom of choice is used twice, it makes perfect sense to ask if it one can develop a proof with only one use of the axiom of choice – this is a paraphrase of Mares&Paoli (2014, p. 452f). So, it can be very useful, to make the repetition rank at use explicit – to mention just another example, recall that in order to prove the validity of the double negation of third excluded in a dialogical setting for constructive logic, the Proponent needs to challenge twice Opponent's statement  $\neg(A \vee \neg A)$ : if the repetition rank were one, then Proponent would not be able to win the thesis  $\neg\neg(A \vee \neg A)$ .

<sup>13</sup> **Choosing between SR1i and SR1c: Dialogical Pluralism.** Perspicuity and explanatory power won by a dialogical setting, become a salient issue when it aims at motivating the choice of some structural rules over others, such as the choice between **SR1i and SR1c**. Indeed, Lorenzen and Lorenz dialogical logic offered a *substructural* response (to use the terminology launched by Peter Schröder-Heister and Kosta Došen (1993) years after the birth of dialogical logic) to W. v. Quine's (1986, second edition, p. 81) dictum, that when non-classical logics speak of a *deviant* connective they change the subject of conversation. The point of Quine can be put as positing the question on how to establish a meaningful dialogue on the standards of inferential rationality associated to an expression if those standards differ. The answer of Lorenzen and Lorenz, was to distinguish *local meaning* and *global meaning*. Thus, classical and intuitionistic connectives share the same local meaning, but differ only on one rule on global, namely whereas in classical logic the global ruling allows the Proponent to, so to say, *redo* his response to a challenge (or ignore the last challenge launched by the Opponent and answer to a previous one) such kind of backwards moves, are not allowed when in an intuitionistic setting – which, as mentioned above allows **P** to bring forward two theses and generates the rejection of weakening to the right at the proof-theoretical level. Thus, if there is some common ground between classical and intuitionist approaches to meaning, this common ground, is in a dialogical setting the *local dialogical meaning explanations*, given by player independent rules. By 1996 this stance on plural approaches to meaning and knowledge was generalized and developed into the project of *Dialogical Pluralism* launched by S. Rahman, H. Rückert and L. Keiff – see Keiff (2007), Rückert (2011).



**SR1c (classical game-playing Rule):** After repetition ranks have been chosen, each move is a challenge or a defence in reaction to a previous move, in accordance with the particle rules shown previously. Each player can challenge *the same previous move, or defend against the same previous challenge, at most  $n$  times*, where  $n$  is that player's repetition rank.

**SR2 (Formal Rule and Socratic Rule).**<sup>14</sup>

**SR2.1 (Formal Rule):** **P** may not state an elementary proposition unless **O** stated it first. Elementary propositions cannot be challenged.

This rule shapes *formal plays or purely assertoric plays*, whereby no thesis can state an elementary proposition.

Modal and temporal logic require some additional conditions for **P** overtaking an elementary proposition stated by **O**.

**SR2.2 (Socratic Rule):** **P** may not state an elementary proposition unless **O** stated it first. Elementary propositions stated by **P** can be challenged, but not those of **O**.

This rule shapes *material plays or material assertoric plays*. In the case that the thesis states an elementary proposition such as  $!A$ , **O** can launch the challenge  $?A$ , iff she (**O**) did not state the same elementary proposition before. The elementary propositions of material plays have content.

This requires a special rule for each of the elementary propositions involved in a play, which stipulate that **O** can state some primitives when requested.

For example, in the case of statements involving natural numbers, the Socratic Rule

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The rationale behind Dialogical Pluralism is that if deductive rationality is declined as dialogical interaction and the latter is governed by the general properties of the inference-relation, disagreeing with such standards of rationality amounts to disagreeing with the structural rules that shape inference. One new brand of Dialogical Pluralism, called the *Built-in-Opponent Pluralism* (BIO-Pluralism) has been recently developed by Dutilh Novaes & French (2018) and Dutilh Novaes (2020, pp. 78-84). Whereas the Saarbrücken-Lille-Pluralism is led by the idea that different logics are the result of different conceptual architectures which lay down what **counts as a meaningful expression** by setting the rules that shape the development of plays of giving and asking for reasons, BIO-Pluralism focuses on how cognition is gathered by **classifying the psychological and sociological responsive mechanisms at work** in the use of different logics.

<sup>14</sup> **Formal and Material Plays.** These rules constitute one of the main features of the dialogical framework. In fact, while working out a new concept of judgmental content, Per Martin Löf (2015, 2017a,b, 2019, 2020) highlighted the contribution of the dialogical framework to the notion of *assertoric* knowledge and to Göran Sundholm's (1997, 2012, 2019) germane concept of *epistemic assumption*, which turns out to have an essential role in an explanation of validity of inference that does not fall into the very well-known justification-circle. The interaction of statements, governed by these rules constitutes the root of what at the strategic level yields assertoric knowledge. The *Socratic Rule*, which owes its name to Marion & Rückert (2016), who indicate some crucial texts in Plato's and Aristotle's *Dialectics* as its historical source – see too Crubellier et al. (2019) and McConaughy (2022), extends Lorenzen & Lorenz's (1978) *Formal Rule* to material dialogues. The main idea is that allows the Proponent to state some specific statements on the sole ground that the Opponent stated them before. Moreover, the rationale behind is that if the thesis is to be challenged under the toughest conditions, the primitive (or declared as primitive) components of the Opponent's challenge shall be considered as holding. If such a stipulation is left out, the Opponent's challenge could be brought to a breakdown **without the thesis having undergone the** strongest possible scrutiny. Usually, the statements governed by this rule are elementary statements and definitions – see Clerbout & McConaughy (2022). These rules are absent in Jaakko Hintikka's *Game Theoretical Semantics* (GTS) and also in the further developments of it such as the ones by van Benthem and collaborators – cf. Hintikka (1973, pp.77-82); to this matter see too the further developments of GTS-approach published in the collection *Texts in Logic and Games*, launched by Johan van Benthem in 2007.

The Formal (Socratic) Rule has been the source of quite an amount of misunderstandings. It has been claimed, for examples, that it does not preserve the player independent meaning explanation of implication, since, if **O** states an implication such that the antecedent is an elementary proposition and **O** did not yet assert that antecedent, then **P** cannot challenge it -see e.g. Tulenheimo (2011). However, this objection misconstrues the prescriptions at work in local dialogical meaning explanations. In the case of implication, the local meaning rules prescribe that a challenge requires the challenger **being able** to state the antecedent. The Formal (Socratic) Rule restricts the space of possibilities for that challenge: **P can** challenge the implication if he **can** state the antecedent, however this does not contravene its local meaning explanation. Notice that for example, the repetition rank, might also restrict the number of challenges on a conjunction, but this does not defeat the dialogical meaning explanation of conjunction which prescribes that a conjunction can be challenged if the challenger can request for the left and the right component of the conjunction.

stipulates that **P** can always ask **O** to state that 0 is a natural number. Once **O** responded to such a request **P** can overtake it. This is combined with a rule for local meaning establishing that for any statement by **X** of the form “the successor of  $n$  is a natural number”, the questioner can request **X** to further state that  $n$  is also a natural number, In the case of empirical elementary propositions some verification process determined by epistemological context at work will determine which elementary propositions can be stated by **O** without challenge.

**SR3 (winning rule):**<sup>15</sup> The play ends when it is a player’s turn to make a move but (s)he has no available move left. That player loses and the other player wins.

In the dialogical approach validity is defined via the notion of *winning strategy*, where winning strategy for **X** means that for any choice of moves by **Y**, **X** has at least one possible move at his disposal such that (s)he (**X**) wins:

**Validity (definition):** A proposition is valid in a certain dialogical system iff **P** has a formal winning strategy for this proposition set as thesis of the dialogue.

Thus,

- $\alpha$  is intuitionistically valid if there is a winning strategy for **P** for plays governed by structural rules shaped by **SR1i**.
- $\alpha$  is classically valid if there is a winning strategy for **P** for plays governed by structural rules shaped by **SR1c**.

### Strategies

A *strategy* for player **X** in a dialogical game is a complete conditional plan of action. It is conditional in the sense that the strategy informs how **X** plays depending on the moves of the adversary. It is complete in the sense that it must inform how **X** plays for every possible choice of move by the adversary.

There are several ways to define a winning strategy within a dialogical framework – see e.g. Krabbe (1985), Clerbout (2014a,b,c), Rahman et al. (2018), Lion (2023a,b). For the sake of a simple presentation we will yield a variation of Felscher (1985) which can be also found in Redmond & Fontaine’s (2011) dialogical tables with branches, and in Piecha (2015) – Rahman et al. (2018, pp. 90-109) describe a method for finding a winning strategy out of a succession of plays.

### Winning strategy

- A player **X** has a winning strategy if for every move made by the other player **Y**, player **X** can make another move, such that each resulting play is eventually won by **X**.

<sup>15</sup> **Winning a play by **P** does not amount to the validity of the thesis.** The notion of a winning a play is not enough to render the notion of inference or of logical validity. In the following example, the thesis is of course not valid, but **P** wins. However, **P** wins because **O** made the wrong choice. In fact, **O** loses the play since when she defends the disjunction she chooses to state precisely what she asking Proponent to state, and the chosen repetition rank does not allow her to challenge twice the same move.

| Opponent |                  | Proponent                       |   |
|----------|------------------|---------------------------------|---|
|          |                  | $! A \vee B \supset B \wedge A$ | 0 |
| 1        | $m := 1$         | $n := 2$                        | 2 |
| 3        | $! A \vee B$     | $! B \wedge A$                  | 4 |
| 5        | $?_{L^{\wedge}}$ | $! B$                           | 8 |
| 3        | $B$              | $?_{\vee}$                      | 6 |

There is obviously another play, where **O** wins, namely, asking for the left side of the conjunction. Dually a valid thesis can be lost because **P** this time, makes the wrong choice; e.g. **P** can lose a play with the thesis  $! A \wedge B \supset B$ , if he chooses repetition rank 1 and asks **O** to state the left side of the conjunction. Another telling example has been mentioned by Piecha (2015), namely the case of the thesis  $\neg A \vee (A \supset A)$ , which can be lost by **P** when playing with **SR1i**, though it is intuitionistically valid, namely when **P** chooses to answer the challenge on the disjunction with the left side. All these considerations show that we need to introduce the notion of winning strategy.



As already mentioned, in dialogical logic validity is defined in relation to winning strategies for the proponent **P**.

- A proposition is valid if **P** has a winning strategy for a thesis stating this proposition
- A *winning strategy for P* for a thesis  $A$  is a tree  $T$  the branches of which are plays won by **P** for that thesis, and where the nodes are those moves, such that
  1.  $T$  has the move  $\mathbf{P} A$  as root node (depth of the tree 0),<sup>16</sup>
  2. if the node is an **O**-move (i.e. if the number of the node is odd), then it has exactly one successor node (which is a **P**-move),<sup>17</sup>
  3. if the node is a **P**-move (i.e. if the number of a node is even), then it has as many successor nodes as there are possible moves for **O** at this position.

Branches are introduced by **O**'s choices such as when she challenges a conjunction or when she defends a disjunction.

### Finite winning strategies

Winning strategies for quantifier-free theses are always finite trees, whereas winning strategies for first-order theses can, in general, be trees of countably infinitely many finite branches (each branch is a play).

For example, if **P** states some universal quantifier, then each choice of the adversary triggers a different play and will yield a tree with infinite number of branches: one for each possible option. Since temporal (and modal) operators can also quantify over an infinite number of histories or moments (or modal contexts), they can also trigger infinite winning strategies. Infinite winning strategies for **P** can be avoided by introducing some provisos grounded on the following rationale:

- Because of the Formal (Socratic) Rule, when **O** has the chance to choose at some move  $n$  on how to challenge a universal or defend an existential, her optimal move is to always choose an  $a$  which is new with regard to the development of the play at that move  $n$ .
- On the contrary **P**, who will do his best to force **O** to state the elementary proposition she asked **P** for, when he challenges a universal of **O** or defends an existential, will copy **O**'s own choices (when **she** challenged an universal or an universal (if **O** did already make such a choice)).

These take us to the following restrictions:

1. If the depth of a node  $n$  is even such that **P** stated a universal at  $n$ , and if among the possible choices for **O** she can choose an  $a$  which is new in the branch, then this move counts as the only immediate successor node of  $n$ .
2. If the depth of a node  $n$  is odd such that **O** stated an existential at  $n$ , and if among the possible choices for **O** she can choose an  $a$  which is new in the branch, then this move counts as the only immediate successor node of  $m$ , i.e. the node where **P** launched the attack on  $n$ .<sup>[1]</sup>
3. If it is **P** who has the choice, then only one of the plays triggered by the choice will be kept.

In a nutshell, the two new main innovations of dialogical logic are

- (i) the introduction of a level of meaning explanations invariant in relation to players, plays and strategies;
- (ii) the introduction of a level of plays upon which a mechanism for checking dialogue definiteness is built and winning strategies are generated.

Currently, the philosophical development of the dialogical framework experiences a thriving interest especially in the field of argumentation theory, history and philosophy of logic and mathematics, computer science, applied linguistics, legal reasoning, and

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<sup>16</sup> Recall that the depth of a node in the tree is the length of the path from the root to the node.

<sup>17</sup> In this case **P** can either challenge that move or answer (if the **O** move is a challenge). If there are many options for **P** only one will be retained. Indeed if there is a winning strategy for **P** it is built out of one of **P**'s choices.

artificial intelligence.<sup>18</sup>

Let us first briefly mention some of the most important research programs that either emerged from the dialogical framework or share some of its main tenets, before we present the main elements of intuitionistic and classical dialogical logic. Then we will present the main elements of a new development that arose by combining the perspectives of the Dialogical Framework with the ones of Per Martin L f's Constructive Type Theory.

## 2 Main Research Programs Related to the Dialogical Framework

1. **Dialogical Logic and Constructivist Approaches to Meaning and Knowledge.** This is the main tradition of dialogical logic and in the next section we will focus on new developments in that direction. Felscher (1985) re-writes winning strategies using the semantic tableaux of Beth (1955), and his formalizations are strongly present in nowadays literature on the dialogical framework or some sequent-calculus variant of it – see Ferm ller (2003), S rensen and Urzyczyn (2007); Alama et al. (2011); Uckelman et al. (2014); Researchers at the U. Saarland and Lille, have made dialogical logic a framework for expressing various logics other than the initial intuitionistic logic and this launched *Dialogical Pluralism*. The rationale behind Dialogical Pluralism is that if deductive rationality is declined as dialogical interaction and the latter is governed by the general properties of the inference-relation, disagreeing with such standards of rationality amounts to disagreeing with the structural rules that shape inference. – see Rahman & R ckert (2001), Rahman and Keiff (2005), Keiff (2011), R ckert (2011). The most recent version of Dialogical Pluralism is the one of Dutilh Novaes (2015; 2020). We will come back to it in our discussion on Argumentation theory.

Ranta (1988) launched a dialogical interpretation of Martin L f's Constructive Type Theory (CTT), whereby winning strategies are identified with proof-objects. This interpretation opened the way to develop is Type Theoretical Grammar for natural languages based on language games – see Ranta (1994), Ginzburg (2012) –. By these times Coquand (1995) applied dialogical logic for the foundations of mathematics and logic – see too Sterling (2021) –.

More recently, while working out a new concept of judgmental content, Martin L f (2015, 2017a,b, 2019, 2020) highlighted the contribution of the dialogical framework to the notion of *assertoric* knowledge, which turns out to have an essential role in an explanation of validity of inference that does not fall into the very well-known justification-circle. This produced different research projects linking CTT and the dialogical framework such as Klev's (2022, 2023) dialogical rules for assertoric knowledge within CTT and Rahman et al. (2018) *Immanent Reasoning* that shall be discussed in the last section of our paper.

2. **Game Theoretical Semantics.** This approach developed by Hintikka (1968) shares the game-theoretical tenets of dialogical logic for logical constants, but switches to standard truth-functional model theory at the level of elementary statements. This approach gave rise to large and successful programme on dynamic epistemic logics launched by van Benthem in Amsterdam – see the collection *Texts in Logic and Games*. Lorenz's *semantization of pragmatics*, just mentioned, targets these kind of game theoretical approaches.

A different direction of GTS, closer to the constructivist tenet of the dialogical framework is Hintikka & Sandu's Independence Friendly Logic (IF-Logic), which deals with games with incomplete information where third-excluded is not generally valid. It is well known that cases of the so-called Henkin branching quantifiers such as  $\forall x \forall y \exists z \exists u A(x, y, z, u)$  present an important limitation of first-order logic. In Hintikka's IF-notation, this is written:  $(\forall x)(\forall y)(\exists z \ / \forall y)(\exists u \ / \forall x) \exists u A(x, y, z, u)$ . The dialogical way to deal with it is to define the meaning of  $(\exists z \ / \forall y)A$  by requiring that when **X** defends a challenge on  $\exists$  and has therefore to choose a value for  $z$ , s(h)e that player *does not know* the choice for  $y$  made by his **Y** – see Hintikka (1996), Hintikka & Sandu (1997).

3. **Argumentation theory and Dialogical Logic.** This framework initiated by Barth and Krabbe (1982; see also Gethmann 1979) links dialogical logic with informal logic, which originated in the work of Perelman and Olbrechts-Tyteca (1958) and Toulmin's (1958) argumentation theory. It also includes the work of Walton (1984), Johnson (1999), Woods' argumentation theory – see Woods (1989) and Woods et al. (2000) –, and critical thinking – see Vaidya 2013, among others).

Here we must also mention the work of Dutilh Novaes (2015; 2020); Dutilh Novaes and French (2018); French (2021), who combine informal logic with the formal setting as formalized by S rensen and Urzyczyn (2007). Indeed, Dutilh Novaes approach, called the *Built-In-Opponent*

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<sup>18</sup> see Clerbout & McConaughey (2022).

(BIO), amounts to a dialogical interpretation of inferential steps carried out in an already pre-determined inferential system. Thus, according to the BIO approach, the dialogical perspective has as main purpose to lead to convincing and/or persuading interlocutors concerning the reasoning behind the inferential steps employed in a deduction carried out by means of a sequent calculus. This tenet also grounds her *Dialogical Pluralism*, which focus on a dialogical interpretation – in the sense just mentioned - of the structural rules at work in Gentzen-style systems. More generally, in order to analyse the convincing power of an argument a *three-tiered model of epistemic exchange* is deployed, consisting in (i) identifying possible sources for epistemic exchange, (ii) choosing among those possible sources, and (iii) engaging with the content. This approach that amounts to assume a given inferential system and add a dialogical interpretative layer on it, is of course very different to Lorenzen and Lorenz proposal who aimed at providing dialogical foundations of meaning and logic. According to Lorenz’s characterization such kind approaches fall under the category of *pragmatization of semantics*.

Another project towards linking the dialogical framework with informal argumentation is the one of Prakken (2005) who deploys the dialogical framework in order to shape non-monotonic reasoning particularly in the context of legal reasoning. Another important strand in the literature closer to the work of Krabbe than to work of Lorenzen and Lorenz is Eemeren & Grootendorst (2004) pragma-dialectics whereby consensus is seen as the main goal of an argumentative process.

4. **Ludics and Linear Logic.** This tradition provides an overall theory of proof-theoretical meaning based on interactive computation. Blass (1992) proposed a dialogical semantics (in the sense of tradition 1) for Girard’s (2001) linear logic, to which *Ludics* is often associated - see Lecomte & Quatrini (2011a,b).

Ludics proposes a kind of theory of meaning as interaction focused only on what we called above the invariant meaning core at work within local dialogical meaning explanations. Within Ludics, structural rules are reduced to local rules by introducing special operators ; e.g. in order to introduce contraction an operator for repetition ranks is explicitly introduced in the object language as a unary connective, which allows turn an intuitionistic negation into a classical negation – see Lecomte & Quatrini (2011a,b). Moreover, Ludics proposes a dynamic interaction process, involving only those rules that preserve the invariant meaning core, whereby structural and strategic rules are constituted. The invariant core amounts to the bare interaction of questions and answers or *expectations* and *intentions*, to use Lecomte & Quatrini’s (2011a) words.

5. **Dialogical Perspectives in the History of Logic.** Research in the history of logic with particular accent on the dialectical genealogy of Logic is experiencing nowadays a huge revival that includes studies not only in the ancient Greek and Medieval (Arabic, Byzantine, Hebrew and Latin traditions), but also in Indian, Chinese and Japanese traditions. The literature on the subject is immense and is not possible in this paper to even attempt to provide a short overview. Notice that, very much to our regret, with few exceptions, scholars working in contemporary Argumentation theory do not know or neglect the work of their colleagues in the vast historical sources on the subject or have quite of a superficial understanding of it. So, allow us to mention just a few that have been directly influenced by the work of Lorenzen and Lorenz and Game Theoretical Approaches. Before doing so, let us stress one methodological, hermeneutical, point that might elucidate the rationale behind the use of the Dialogical Framework for reconstructing historical patterns for logical reasoning. According to the dialogical stance, meaning is dialectical interaction including that of logical constants. Thus, in order to explain the meaning of some expression within a dialectical context we need bring to the fore the rules for request and answer involving such an expression. To put it bluntly, dialogical meaning explanations, serve as a bridge between informal and formal analyses for the study of logical constants and the underlying inference theory. Most of the handful of scholars mentioned below, explicitly or implicitly adopted this methodological principle. In ancient Greek philosophy, let us mention the work a Dialectical reading of Aristotelian Syllogism of McConaughey (2021), who on one hand develops further the work, already mentioned, of Ebbinghaus (1964), Lorenz & Mittlestrass (1966, 1967), and on the other she deploys the formal means of recent studies on dialogical logic and syllogism – see Castelnérac & Marion (2009), Crubellier (2011), Marion & Rückert (2016), Crubellier et al. (2019). Hintikka’s (2004) includes reprints of papers discussing Aristotle’s dialectics under the background the GTS. The GTS reading of traditional logic and Keffer’s (2001) dialogical reconstruction of *Obligationes* underlies Dutilh-Novaes (2007) and Uckelman’s (2013) work on Medieval debate theory and Meaning. One landmark in this field is the work of Yrjönsuuri (2001) on Obligations and Insolubles. In relation to the Arabic tradition, so far we know, Miller (1984, repr. 2020) was the first to explicitly deploy Lorenzen and Lorenz Dialogues for the analysis of Arabic Debate Theory. The most impressive work in this direction is the one of Young (2017, 2021), who influenced also Iqbal’s (2022) reconstruction of the reception of Arabic Juridical Debate Theory in Indonesia. Notice that the texts in Arabic Disputation theory represent most probably the richest source of manuscripts on the subject in Medieval Times – by large yet unexplored and neglected.<sup>19</sup> Let us

<sup>19</sup> Dutilh Novaes (2020), in her book on the *Dialogical Roots of Deduction*, astonishingly, not only does she

finish by mentioning Gorisse’s (2017, 2018) dialogical reconstruction of Jaina dialectics – see too Clerbout et al. (2011).

### 3 Immanent Reasoning: A Venue in History and Philosophy of Logic.

#### 3.1 Enriching the Expressivity of the Dialogical Framework

Dialogues are games of giving and asking for reasons; yet, in the standard dialogical framework, the reasons for each statement are left implicit

The *Immanent Reasoning* (IR) framework imports Martin Löf’s 1984 Constructive Type Theory (CTT) form of a judgement in order to make these reasons explicit.<sup>20</sup> Fully developed, statements thus have the following form:

$$\mathbf{X} ! a : B$$

where  $B$  is a proposition and  $a$  its local reason, i.e. the particular, circumstantial reason that entitles one to state  $B$ , which is read as  $\mathbf{X}$  states that  $a$  provides evidence for  $B$ . In this fashion, the reasons interlocutors have for making a statement are specified at the object-language level. As expected, IR distinguishes local reasons and strategic reasons. Local reasons are brought forth in particular plays, whereas strategic reasons are a recapitulation of all the relevant plays for constituting a winning strategy:

Moreover, IR allows to enrich the standard dialogical framework within an interactive theory of meaning that makes use of the expressivity of the fully interpreted languages at work in CTT. This expansion of the dialogical framework does not only allow to integrate the CTT analyses on anaphoric constructions such as the famous sentences, branching quantifiers, and generalized quantifiers (such as Most, At-Most, At-Least, and so on),<sup>21</sup> but is also provided a venue for studies in the history and philosophy of logic that brings to the light the rationale behind ancient and medieval logical analyses,<sup>22</sup> the dialogical stance on contemporary and historical legal reasoning.<sup>23</sup>

Indeed, the framework allows to introduce explicitly into the object language the domain of discourse over which the scope of quantifiers have been defined. So, in this framework the sentences *All Cobblers are Good* (*cobblers*), and *Some Students are Good* (*students*) – to use an old Aristotelian example, can be analyzed as having the following logical form:

$$(\forall x \varepsilon \text{ Cobblers}) \text{ Good}(x)^{24}$$

$$(\exists x \varepsilon \text{ Cobblers}) \text{ Good}(x)$$

That admit the glosses

All (elements) of the (set/domain of) cobblers are good

Some of the (elements of the set/domain of) cobblers are good

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almost not acknowledge the existence of Islamicate Disputation theory – letting by side her rather thin overview and interpretation of Arabic Logic, but also seems to neglect the studies of scholars in the history of the dialogical perspective on Logic in general, when she develops her own theory of dialogues and argumentation.

<sup>20</sup> Cf. Rahman et al. (2018).

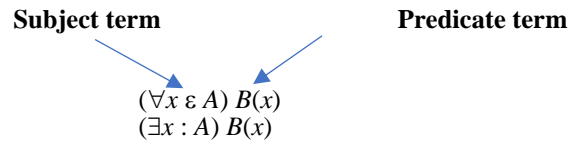
<sup>21</sup> See Sundholm (1986, 1989), Ranta (1994)

<sup>22</sup> See Crubellier et al. (2019), Iqbal (2022).

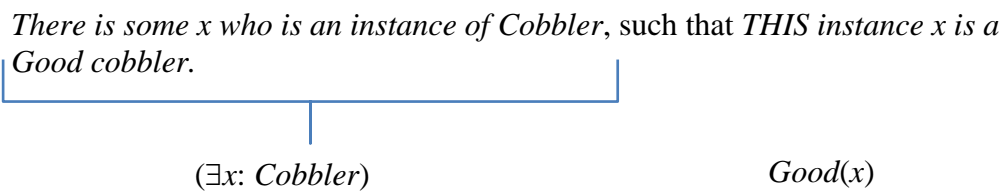
<sup>23</sup> See Rahman et al. (2021).

<sup>24</sup> Whereby “ $\varepsilon$ ” stands for type-token relation or element of a set. The most recent notation in CTT is “ $\cdot$ ”, however, we use “ $\varepsilon$ ” which was used by Lorenz and Mittlestrass (1966, 1967) for the instantiation-relation within their dialogical analysis of Plato’s theory of meaning.

Notice that this analysis renders the idea of traditional logic that predication amounts to stating that the Predicate-Term applies to any/some instance of the Subject Term:



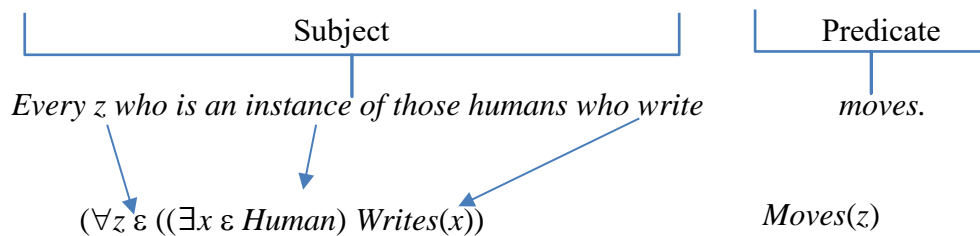
More precisely:



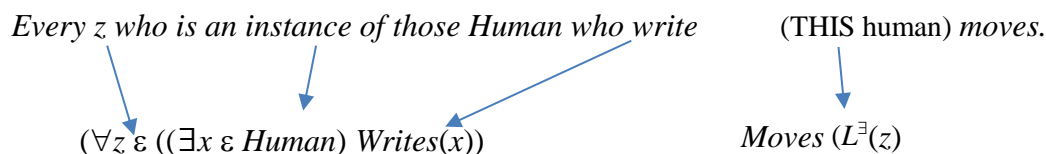
The framework allows, to add complexity to the subject, such as in Avicenna’s famous example

*Every human who writes, moves (while writing)*

Letting the temporal component by side, we obtain:<sup>25</sup>



If we add operators such as  $L^\exists(x)$ ”, “ $R^\exists(x)$ ”, which renders the left component of the complex subject we can be more precise and indicate the specific instance of man who constitutes the head of the anaphoric construction:



### 3.2 Immanent Reasoning Revisited

Ansten Klev's (2022, 2023) dialogue rules for Constructive Type Theory, develops further Martin-Löf’s rules shaping the dialogical perspective, and have as main purpose to serve as formalized meaning explanations for the forms of assertoric content employed

<sup>25</sup> In fact Avicenna added spans of time – See Rahman & Zarepour (2021).

in type theory. Thus, his dialogue rules for CTT, at least at this initial stage, do not aim at proving an assertion but only justifying an inference on the basis of them. So, Klev's use of the Dialogical Framework is not independent of the inferential system of CTT.

Klev's approach really constitutes a progress for linking CTT and dialogical logic, particularly since it avoids the proliferation of notation, used in Rahman et al; (2018). Moreover, Martin L f's remarks on the contribution of dialogical logic to assertoric logic and valid inference is an important point is an important point towards motivating such an interactive stance.

However, it is only depicts one side of the the dialogical contribution. The dialogical framework aims at furnishing a purely interactive notion of meaning invariant to introduction and elimination rules and to the underlying structural rules. Justifying an inference rule eventually comes down to showing that it emerges from the interaction local meaning explanations of the expressions involved. Moreover instead of the Type-checking decidable mechanism assumed by the CTT framework, the dialogical framework is endowed with the decidable notion of dialogue-definiteness.

In other words, the two main aims of dialogical logic are

- (i) to introduce a level of meaning explanations invariant in relation to players, plays and strategies;
- (ii) to introduce a level of plays upon which a mechanism for checking dialogue definiteness is built and winning strategies are generated.

Let us finish our paper with a glimpse on some of the rules for local meaning. Recall that in fact they are the standards ones plus the explicit occurrence of reasons . Reasons occur at the left of the epsilon ;

|   | Request  | Answer   | Request  | Answer   |
|---|--|--|--|--|
| $X ! d \varepsilon A \vee B$<br><br>(X claims to have a reason $d$ for the disjunction) | $Y ?_{\vee} d$<br><br>(Y asks for which side of the disjunction $d$ is a reason for) | $X ! d \Rightarrow L^{\vee}(a) \varepsilon A \underline{\vee} B$<br>-----OR-----<br>$X ! d \Rightarrow R^{\vee}(b) \varepsilon A \underline{\vee} B$<br><br>(Answerer states the $d$ stands for some reason backing the left or the right side of the disjunction) |  |  |
| $X ! d \varepsilon A \vee B$  | $Y ?_{\vee}$   | $X ! L^{\vee}(a) \varepsilon A \underline{\vee} B$<br>-----OR-----<br>$X ! R^{\vee}(b) \varepsilon A \underline{\vee} B$   | $Y ?_{L^{\vee}(a)}$<br>-----OR-----<br>$Y ?_{R^{\vee}(b)}$<br><br>(Questioner, requests Y to specify the reason for the chosen side) | $X ! a \varepsilon A$<br>---OR---<br>$X ! b \varepsilon B$ |

### Implication, Universal Quantifier

In the case of implication, the rules prescribe the challenger to provide a local reason for the antecedent (the left side of the implication) and the defender to provide a local reason for the consequent. The notation  $(L^{\supset}Y)R^{\supset X} \varepsilon A \supset B$  indicates that the defence of the implication requires to defend the consequent after the challenger Y commits to the



antecedent by stating  $a \in A$ , and the defender X commits to  $b \in B$ .<sup>26</sup>

| Assertion  | Request       | Answer   |
|--|---------------|--|
| $X ! d \in A \supset B$                            | $Y ? \supset$ | $X ! (LY^\supset)RX^\supset \in A \supset B$                     |
| $X ! d \in (\forall x \in A)B$                     | $Y ? \forall$ | $X ! (LY^\forall)RX^\forall \in (\forall x \in A)B$              |
|  | $Y ?_d$       | $X ! d \Rightarrow (LY^\supset)RX^\supset \in A \supset B$       |
|  | $Y ?_d$       | $X ! d \Rightarrow LY^\forall)RX^\forall \in (\forall x \in A)B$ |
| $X ! (LY^\supset)RX^\supset \in A \supset B$       | $Y ! a \in A$ | $b \in B$  |
| $X ! LY^\forall)RX^\forall \in (\forall x \in A)B$ | $Y ! a \in A$ | $b \in B(a/x)$   |

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