



# Outside the Logic of Necessity: Deontic Puzzles and 'Breaking' Compound Causal Properties in Islamic Legal Theory and Dialectic

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**Outside the Logic of Necessity:  
Deontic Puzzles and ‘Breaking’ Compound Causal Properties in Islamic Legal Theory  
and Dialectic.**

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**Abstract:** This study examines an Islamic dialectical objection called *kasr*, or “breaking,” as treated by the dialecticians and legal theorists Abū Ishāq al-Shīrāzī (d. 1083 CE) and Abū al-Walīd al-Bājī (d. 1081 CE). *Kasr* has both legitimate and illegitimate applications, and we confront the fallacious modes denounced by our theorists with deontic paradoxes and puzzles falling under the rubric of *logical extrapolation fallacy*. Our primary claim is that, whereas logical extrapolation produces fallacies or paradoxes by unsafely applying inference rules of standard alethic and/or logical necessity to the deontic realm, the fallacies generated by illegitimate modes of *kasr* in Islamic legal theory (wherein logical rules are expressed dialectically) constitute a genuine source for reflecting on what patterns of reasoning should be endorsed for determining causality in legal—and, perhaps, natural—epistemological contexts. Ultimately, this is the first step towards a larger study of *kasr*, which will compare it with other dialectical objections and treat relevant critical discourses from later Muslim legal theorists and dialecticians.

**Keywords:** Islamic juristic dialectic (*jadal* / *munāẓara*); correlational inference (*qiyās*); breaking (*kasr*); logical extrapolation fallacy; compound causes; dialogical logic.

## **1 Introduction**

This study will examine an important group of illegitimate moves involving causal properties as identified by Medieval Muslim jurists in the intertwined domains of legal theory (*uṣūl al-fiqh*) and dialectic (*jadal*). More precisely, we will focus on the discourse surrounding the dialectical objection called *kasr*, or “breaking,” which deliberates certain proper and improper paths to challenging and defending the causal components of a correlational argument (*qiyās*) in which the *ratio legis* (*illa*) of the root-case’s ruling (*ḥukm*) is a compound of two or

more properties. The developmental history of this dialectical objection is complicated; long and heated controversies centred on which modes of *kasr* (and responses to *kasr*) were fallacious and which were not. There were even those who rejected *kasr* at the outset, some with arguments paralleling classical and medieval Latin claims that one cannot refute an argument whose premises have a meaning *in sensu composito* by the blunt separation of its parts.

In the present study, we will restrict our analyses to the relevant discussions of two 11<sup>th</sup> century CE theorists: the renowned Shāfi‘ī jurist Abū Ishāq al-Shīrāzī (d. 1083 CE) and his equally prominent, one-time pupil, the Mālikī jurist Abū al-Walīd al-Bājī (d. 1081 CE), both of whom elaborated two main pathways to “breaking” an opponent’s compound ‘*illa*, either by (1) replacing one of its properties with something similar, or (2) removing one of its non-efficient properties. The rules and responses prescribed in the sources we examined for each type make it clear that they correspond to two primary conjunctive modes for an ‘*illa*’s valid composition: either (type 1) *meaning dependence* of one property (or more) upon the other (with all of them together being claimed to occasion the ruling), or (type 2) *independent conjunctive composition*, in which one or more properties might be efficient, but not the remainders (and from which it follows that if *A*, *B*, ..., occasion the ruling, then either *A* occasions the ruling or *B* occasions it, or ...).

Moreover, we will briefly confront the additional, but fallacious, modes of *kasr* denounced by al-Shīrāzī and al-Bājī with forms of certain deontic paradoxes and puzzles we may group under the rubric *logical extrapolation fallacy*. These play a dominant role in contemporary deontic logic, although they have roots in the medieval Latin tradition (or, arguably, are already present in Megarian and Stoic logic). More generally, fallacies of *logical extrapolation* should include the illegitimate reduction of some pattern of argumentation to a pre-existing logical system (be it modal or otherwise).

Our primary claim is that, whereas logical extrapolation produces fallacies or paradoxes by unsafely applying inference rules of standard alethic and/or logical necessity to the deontic realm, the fallacies generated by invalid modes of *kasr* in Islamic legal theory (wherein both logical rules and semantic rules for reasoning with deontic modalities are expressed dialectically) constitute a genuine source for reflecting on what patterns of reasoning should be

endorsed for determining causality in Law—and, perhaps, more generally, also for establishing causality in certain natural (as opposed to normative) epistemological contexts.

## 2 The Logical Extrapolation Fallacy

Contemporary approaches to Standard Deontic Logic (SDL) have followed G. H. von Wright's idea of applying the developed framework of modal logic to the deontic realm.<sup>1</sup> According to this approach—further extended with the model theoretical semantics of Jaakko Hintikka and Saul Kripke<sup>2</sup>—obligation was conceived as a special reading of the necessity operator of basic modal propositional logic. According to this view deontic reasoning can be reduced to a special kind of modal logic.

Possible-world interpretations of deontic necessity struggled, however, with a wealth of philosophical and logical puzzles that threatened the framework right from the very start—already by the sixties, and at the propositional level.<sup>3</sup> Of course, deontic necessity is still a kind of necessity. But standard possible-world semantics—without deep modifications—is not the instrument required, vis-à-vis the prescriptive dimension on performances of actions, to grasp the meaning of those norms governed by the assumption of liberty—or, more precisely: of legal or ethical *liability*.

Paul McNamara and Risto Hilpinen provide, in their contribution to the *Handbook on Deontic Logic and Normative Systems*, a systematic, historical overview of such problematic, reductionist projects. One of the puzzles they discuss is the following reconstruction of Stephen Langton's (1150-1228 CE) paradox.<sup>4</sup> It results from reducing (Aristotelian) modal necessity to

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<sup>1</sup> See von Wright, Georg Henrik, *An Essay in Modal Logic*, Studies in Logic and the Foundations of Mathematics (Amsterdam: North-Holland Publishing Company, 1951); von Wright, Georg Henrik, *Norm and Action: A Logical Enquiry* (New York: Humanities Press, 1963).

<sup>2</sup> Hintikka, Jaakko, 'Modality as Referential Multiplicity', *Ajatus*, 20 (1957), 49–64.; Hintikka, Jaakko, *Knowledge and Belief: An Introduction to the Logic of the Two Notions*, 2<sup>nd</sup> ed., Texts in Philosophy, 1 (London: College Publications, 1962 [2005]); Hintikka, Jaakko, 'Language-Games for Quantifiers', in *Studies in Logical Theory*, ed. by Nicholas Rescher, American Philosophical Quarterly Monograph Series, 2 (Oxford: Basil Blackwell, 1968), pp. 46–72.; Kripke, Saul, 'Semantical Considerations on Modal Logic', *Acta Philosophica Fennica*, 16 (1963), 83–94.

<sup>3</sup> For recent overviews of these challenges see Hilpinen, Risto, and Paul McNamara, 'Deontic Logic: A Historical Survey and Introduction', in *Handbook of Deontic Logic and Normative Systems*, ed. by Dov Gabbay, John Horty, Xavier Parent, Ron van der Meyden, and Leandert van der Torre (London: College Publications, 2013), pp. 3–136; Navarro, Pablo E. and Jorge L. Rodriguez, *Deontic Logic and Legal Systems* (Cambridge: Cambridge University Press, 2014).

<sup>4</sup> Hilpinen and McNamara, 'Deontic Logic', p. 8.

deontic necessity, and constitutes an excellent example for our reflection on the genealogy of fallacious arguments within the deontic realm.<sup>5</sup>

*Necessarily, if this man visits his sick father, then the father is sick.*

But it does not follow that

*If this man ought to visit his sick father, then his father ought to be sick.*

These and similar counterexamples are rooted in the following form of reductionism for which Leibniz has been praised as much as blamed:

$N(A \supset B)$  (whereby “N” stands for necessity)

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$(OA \supset OB)$  (inferred via modus ponens from  $N(A \supset B)$ , followed by introduction of implication and substituting N with “O”, obligatory)

Notice that this example of the sick father is not specific to the modal necessity operator but extends to inferences governed by logical necessity as well. This also holds for many other well-known deontic puzzles, including: A. Ross’s *Burning-Letter* paradox;<sup>6</sup> R. M. Chisholm’s puzzle on *Conditional Obligations*;<sup>7</sup> and A. N. Prior’s *The Good Samaritan* paradox,<sup>8</sup> in which latter there follows from: *It is obligatory that Jones help Smith who is being mugged*, that: *It is obligatory that Smith is being mugged* (assuming the premise is rendered as a conjunction).

More generally, the problem may be seen as an illicit extrapolation from the modal logic of necessity to the set of rules governing deontic reasoning in the moral and/or legal realms. This signifies a special kind of fallacious argumentation: identifying illegitimate moves which consist in extrapolating a set of rules for logical reasoning to a pattern of reasoning within some

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<sup>5</sup> Our rendering of the example is based on p. 77 of Knuuttila, Simo and Oli Hallamaa, ‘Roger Roseth and Medieval Deontic Logic’, *Logique et Analyse*, 38 (1995), 75-87.

<sup>6</sup> Ross, Alf, ‘Imperatives and Logic,’ *Theoria*, 7 (1941), 53–71.

<sup>7</sup> Chisholm, Roderick M., ‘Supererogation and Offence: A Conceptual Scheme for Ethics’, *Ratio (Misc.)*, 5.1 (1963), 1–15.

<sup>8</sup> Prior, Arthur N., ‘Escapism: The Logical Basis of Ethics’, in *Essays in Moral Philosophy*, ed. by Abraham Irving Melden (Seattle: University of Washington Press, 1958), pp. 135–146.

specific field of knowledge, and drawing a paradoxical or puzzling consequence that patently contravenes the conceptual architecture of that field. As mentioned, we will call this the *fallacy of logical extrapolation*, though it is not new. Ample Megarian and Stoic debates on the fruitfulness of material implication for temporal and modal contexts long ago involved discussions of such puzzles. The garden of logical extrapolation fallacies is quite rich.

This is particularly important in the context of Islamic Dialectical theory, wherein rules for legal reasoning were not constituted by importing logical rules from somewhere else and subjoining them to rules of dialectic, but the dialectical framework itself originated dialectical rules for legal-logical reasoning. In the general dialectical theory of Shams al-Dīn al-Samarqandī (d. 1322 CE), the *ādāb al-baḥṭh wa-l-munāẓara*, which streamlined and universalized a predecessor juristic dialectic, logical rules were (as in the predecessor dialectic) consciously and rigorously formulated as dialogical rules.<sup>9</sup> This explains why we will not, in principle, find fallacies of extrapolation therein. However, there is no dearth of cases for the identification and study of illegitimate moves in legal and deontic reasoning more broadly.

### 3 Islamic Deontic Imperatives: Outside Logical Extrapolation

#### 3.1 Ibn Ḥazm's Deontic Imperatives

In previous joint studies by S. Rahman, W. E. Young, and F. Zidani,<sup>10</sup> the authors argue that Ibn Ḥazm of Córdoba's (994-1064 CE) *Facilitating the Understanding of the Rules of Logic and Introduction Thereto, with Common Expressions and Juristic Examples*, wherein, among other things, he thoroughly investigates deontic notions and their modal counterparts, assures him a place among the fathers of the logic of norms. Moreover, in these studies we show that, in the context of what we called *Islamic heteronomous imperatives*, “puzzles” of extrapolation are not puzzles at all. The point is that this approach does not require one to block

<sup>9</sup> See Miller, Larry Benjamin, *Islamic Disputation Theory: The Uses & Rules of Argument in Medieval Islam* (Cham: Springer, 2020), pp. 103-123.

<sup>10</sup> Most of this current article's sections 3.1 and 3.2 has been reproduced from the following: Rahman, Shahid, Johan-Georg Granström, and Ali Farjami, 'Legal Reasoning and Some Logic After All. The Lessons of the Elders', in *Natural Arguments. A Tribute to John Woods*, ed. by Dov Gabbay, Lorenzo Magnani, Woosuk Park, and Ahti-Veikko Pietarinen (London: College Publications, 2019), pp. 743-780; Rahman, Shahid, Walter Edward Young, and Farid Zidani, 'It Ought to be Forbidden! Islamic Heteronomous Imperatives and the Dialectical Forge', in *Lógica Conocimiento y Abducción. Homenaje a Angel Nepomuceno*, ed. by Cristina Barés, Francisco J. Salguero, and Fernando Soler, Cuadernos de Lógica, *Epistemología y Lenguaje*, 15 (London: College Publications, 2021), pp. 97-114; Rahman, Shahid, Farid Zidani, and Walter Edward Young, 'Ibn Ḥazm on Heteronomous Imperatives. A Landmark in the History of the Logical Analysis of Legal Norms', in *Agency, Norms, Inquiry, and Artifacts: Essays in Honor of Risto Hilpinen*, ed. by Paul McNamara, Andrew Jones, and Mark Brown, Synthese Library 454, Studies in Epistemology, Logic, Methodology, and Philosophy of Science (Cham: Springer, 2022), pp. 139-171.

the use of any logical rules as standard solutions do, but allows logical inference rules to cohabit with deontic rules—if, that is, sufficient care is given to the meaning constitution of deontic assertions generated by the logic that governs the normative realm. Insights from the Islamic tradition that facilitate our reconstruction include the following:

- Prescriptions are understood as prescriptions to **do** rather than prescriptions that take us from one state of affairs to another: *Tun Sollen* rather than *Sein Sollen*.
- Actualizations (performances) of prescriptions are subjects of predication. In other words, performances of actions are bearers of qualifications such as *law-abiding* (rewarded) or *law-breaking* (sanctioned).<sup>11</sup> The distribution of reward and sanction yields the classification of deontic modalities into *obligatory*, *forbidden*, *recommended permissible*, *reprehended permissible*, and *evenly permissible*.
- Norms presuppose freedom of choice or moral and legal liability. In other words, each type of prescription (obligatory, forbidden, etc.) assumes as hypothesis that the corresponding type of action can be deliberately carried out or not carried out.

### 3.2 Ibn Hazm's Deontic Imperatives

In the general realm of human actions, Muslim jurists identified five deontic qualifications.<sup>12</sup> Ibn Hazm defines them as follows:<sup>13</sup>

- 1 **wājib, farḍ, lāzim**. Obligatory action is the one which: If we do it we are rewarded. If we do not do it we are sanctioned.
- 2 **ḥarām, maḥẓūr**. Forbidden action is the one which: If we do it we are sanctioned. If we do not do it we are rewarded.
- 3 **mubāḥ mustaḥabb**. Recommended permissible action is the one which: If we do it we are rewarded. If we do not do it we are neither sanctioned nor rewarded.

<sup>11</sup> By way of clarification, “sanction” in this study is meant only in its sense of “penalty,” and “sanctioned” as “penalized.”

<sup>12</sup> Contractual qualifications (valid, invalid, null and void) were also developed, but will not be discussed here.

<sup>13</sup> Ibn Hazm, *Al-Iḥkām fī Uṣūl al-Aḥkām*, 8 vols., ed. by Aḥmad Muḥammad Shākir (Cairo: Maktabat al-Khānjī, 1926), vol. 3, p. 77; Ibn Hazm, *Kitāb al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-alfāz al-‘Āmmiyya wa-l-Amthila al-Fiqhiyya*, ed. by Iḥsān ‘Abbās (Beirut: Dār Maktabat al-Ḥayāh, 1959), p.86; Ibn Hazm, *Kitāb al-Taqrīb li-Ḥadd al-Manṭiq wa-l-Mudkhal ilayhi bi-l-alfāz al-‘Āmmiyya wa-l-Amthila al-Fiqhiyya*, ed. by Aḥmad b. Farīd b. Aḥmad al-Mazīdī (Beirut: Manshūrāt Muḥammad ‘Alī Bayḍūn, Dār al-Kutub al-‘Ilmiyya, 2003), pp. 83-4.

- 4     **mubāḥ makrūh**. Reprehended permissible action is the one which: If we do not do it we are rewarded. If we do it we are neither sanctioned nor rewarded.
- 5     **mubāḥ mustawin**. Evenly permissible action is the one which: If we do it we are neither sanctioned nor rewarded. If we do not do it we are neither sanctioned nor rewarded.

In the aforementioned studies we proposed a logical analysis based on A. Ranta's *Type Theoretical Grammar*<sup>14</sup> and Per Martin-Löf's Constructive Type Theory (CTT).<sup>15</sup> This fully interpreted framework allows a logical analysis of imperatives that makes explicit the conceptual links at work here. More precisely it allows one to express for each deontic qualification both (1) the dependence of rewarding or sanctioning on the presupposition that there was a real choice, and (2) the fact that these performances actualize the type of action specific to the imperative at stake (obligatory, forbidden, etc.) Thus, if we are describing an *obligatory action*, we need to express the following:

**Obligatory action:**

- (i) *If the individual **g** made the choice to perform an action of type A (i.e., if there is a performance by **g** that actualizes the **left side** of the disjunction) then some (legal) procedure **b**, determined by the Law, attributes a form of **reward** specific to this type of performance.*
- (ii) *If the individual **g** made the choice to omit performing an action of type A (i.e., if there is a performance by **g** that actualizes the **right side** of the disjunction) then some (legal) procedure **b**, determined by the Law, attributes a form of **sanction** specific to this type of omission.*

### 3.3 Rendering the Conceptual Analysis Explicit

Before presenting the notation that makes our analysis explicit, let us provide first the main keys for reading the CTT formal terminology employed here and further on in the study:

- If “A” stands for a type of action, and “y” stands for some not-yet-actualized performance of an action of type A, then we will write  $y: A$ . Similarly, the “x” in “ $x: A \vee \sim A$ ”, stands for the performance of a type of action of either A or  $\sim A$ . Thus, more loosely, we can say that

<sup>14</sup> Ranta, Aarne, *Type-Theoretical Grammar* (Oxford: Clarendon Press, 1994).

<sup>15</sup> Martin-Löf, Per, *Intuitionistic Type Theory. Notes by Giovanni Sambin of a Series of Lectures Given in Padua, June 1980* (Naples: Bibliopolis, 1984).



“ $x$ ” in “ $x: A \vee \sim A$ ” stands for the choice of carrying out  $A$  or intentionally refraining from carrying out  $A$ .

### Non-actualized Type of Action $A$

Some arbitrary performance  $y$



$y: A$

### Actualized Type of Action $A$

Performance  $a$



$a: A$

Type of Action

### Non-actualized Type $x: A \vee \sim A$

Some arbitrary performance  $x$   
standing for one of the sides of the  
disjunction.



$x: A \vee \sim A$

### Actualized Type $a: A \vee \sim A$

Performance  $a$  actualizes the  
disjunction by actualizing the *left*  
side of it—i.e., the left of the  
disjunction has been chosen.



$a: A \vee \sim A$

Type of Action

Choosing

- If “ $y$ ” stands for some not-yet-actualized performance of an action of type  $A$ , then “ $R(y)$ ” stands for attributing the predicate *Reward* to the performance  $y$  of the type of action  $A$ . Thus, “ $R(y)$ ” is an expression of the type *prop* (i.e., of the type proposition), whereby  $y$  is an element of the set  $A$  of performances of action. In a more standard terminology, “ $R(y)$ ” stands for a propositional function over the set  $A$  of performances. Something similar holds for the propositional function “ $S(z)$  defined over the set of omissions  $\sim A$ ”.

**Performance**

**Type of Action**



$R(y): \text{prop}(\text{osition})$  provided  $y: A$ —in short  $R(y): \text{prop } (y: A)$

**Predicate**

**Hypothetical judgment or  
normative prescription**

$$R(y) \text{ true } (y: A)$$

Any arbitrary performance  $y$  of  $A$  makes  
it *true that this performance is*  
*rewarded*

$$d(y): R(y) \text{ (} y: A \text{)}$$

(legal) procedure  $d(y)$  attributes *Reward*  
to any arbitrary performance  $y$  of  $A$

**Categorical judgment or  
result of carrying out a prescription**

$$R(a) \text{ true}$$

It is *true* that the performance  $a$  is  
*rewarded*

$$d(a): R(a)$$

(legal) procedure  $d(a)$  attributes  
*Reward* to performance  $a$  of  $A$

- If “ $x$ ” indicates the choice between carrying out  $A$  or intentionally refraining from carrying out  $A$ , then “ $\mathbf{left}^\vee(y) =_{\{H\}} x$ ” indicates that the result of this choice is to perform  $A$  (i.e., it is identical to a performance  $y$  of  $A$ ); and something similar holds for “ $\mathbf{right}^\vee(z)$ ”.

Accordingly, the first component of the conjunction:

$$(\forall y : A) \mathbf{left}^\vee(y) =_{\{H\}} x \supset R(y) \text{ (} x : A \vee \sim A \text{)}$$

reads:

*Assuming that, given the choice  $x$  of performing or not performing an action of type  $A$ , performing it has been chosen (i.e., if the left side of the disjunction has been chosen to be performed), then, for any performance  $y$  of the type of action  $A$  that is identical to the choice  $x$ , there follows reward (for performing this action).*

A similar reading applies to the second component:

$$(\forall z : \sim A) \mathbf{right}^\vee(z) =_{\{H\}} x \supset S(z) \text{ (} x : A \vee \sim A \text{)}.^{16}$$

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<sup>16</sup> The notation for propositional identity, namely “ $x =_{\{D\}} y$ ”, standing for “ $x$  is identical to  $y$  within the set  $D$ ,” is closer to what is employed in first-order logic.

Thus, if we pull all this together and employ the abbreviation  $\{H\}$  for the hypothesis  $x: A \vee \sim A$ , expressing the fact that the agent can choose to either actualize (perform)  $A$  or actualize an omission of performing  $A$ , we obtain:

$$b(x) : [ (\forall y : A) \textbf{left}^\vee(y)=\{H\}x \supset R(y) ] \wedge [ (\forall z : \sim A) \textbf{right}^\vee(z)=\{H\}x \supset S(z) ] (x : A \vee \sim A).$$

- Given  $x: A \vee \sim A$ , “ $b(x)$ ” stands for a procedure (expressed by a function) that distributes **reward** and **sanction** depending upon the actual choice made (by the agent  $g$ ) between performing  $A$  or  $\sim A$  and specific to the type of action chosen to be performed. In short, the function relates performances of actions to the kind of sanction or reward specified by the Law. In our context, the procedure is a legal one carried out by the competent legal authority (in this world, or the next, or both).

The hypothetical can be glossed as follows:

**Obligatory action:**

*Given the choice  $x$  between performing  $A$  or intentionally refraining from doing so:*

*If performing an action of type  $A$  has been chosen (i.e., if there is a performance  $y$  that actualizes the **left side** of the disjunction)—viz.  $\textbf{left}(y)=x$ —then (legal) procedure  $\mathbf{b}$  attributes a form of **reward** specific to this type of performance.*

*If intentionally refraining from performing  $A$  has been chosen (i.e., if there is a performance  $z$  that actualizes the **right side** of the disjunction)—viz.  $\textbf{right}(z)=x$ —then (legal) procedure  $\mathbf{b}$  attributes a form of **sanction** specific to this type of omission.*

The above yields:

**wājib, farḍ, lāzim:** Doing  $A_1$  is rewarded. Intentionally refraining from doing  $A_1$  is sanctioned.

$$b_1(x) : [ (\forall y : A_1) \textbf{left}^\vee(y)=\{H\}x \supset R_1(y) ] \wedge [ (\forall z : \sim A_1) \textbf{right}^\vee(z)=\{H\}x \supset S_1(z) ] (x : A_1 \vee \sim A_1).$$

As mentioned, this approach provides a general framework that prevents the kind of puzzles discussed above. It only works, however, if the framework is embedded in a general theory of the meaning of legal norms that also accounts for reasoning involving deontic modalities. It is important to observe that in Islamic jurisprudence there is no theory of deontic syllogisms as such. Rather, within the dialectical theory and practice of *uṣūl al-fiqh*, the distribution and transmission of the deontic modalities described above is governed by argumentation rules on how to justify a claimed legal norm, how to object to such justifications,

and how to respond to such objections. In what follows, we will focus on arguments concerning compound premises expressing the causal factors for legal rulings. Inquiring into the causal factors of rulings was regarded as a critical epistemic endeavour, and the rules for challenging arguments in support of a compound causal factor's occasioning of a legal norm were among the more controversial and subtle within Islamic juristic dialectic.

## 4 Beyond Fallacies of Logical Extrapolation

It is interesting to conjecture as to why extrapolation fallacies involving modal necessity drew the attention of premodern scholars in the Latin tradition but not (at least not explicitly) in the Arabic tradition. From a historical point of view, one possible answer—though perhaps problematic in over-generalizing—might be that Aristotle's modal logic, or at least Aristotle's modal notions as discussed in the *Peri hermeneias* and commented upon by Latin scholars as early as Boethius, took conceptual priority over deontic notions in the Latin tradition, whereas a dominant practice of dialectical legal reasoning in Islamic thought inversed this order of priorities for the Arabic tradition. If true, a possible effect of this might be the fact that, in the further maturation of dialectical theory following the success of al-Samarqandī's *ādāb al-baḥth*, discourse on valid and invalid reasoning with regard to both legal and natural causality appears to have blurred older distinctions and embraced a general causality pattern.<sup>17</sup>

Whatever the reason, one might be justified in asking whether fallacies bounded by deontic or legal reasoning can be identified within Islamic disputation theory. The answer is a resounding “Yes.”<sup>18</sup> This is particularly so in relation to deontic and natural causal necessity in the context of that set of patterns for parallel reasoning called *qiyās*, the basics of which we will now quickly review.

### 4.1 *Qiyās* Basics

The many debates and elaborations on *qiyās*, which might be translated “correlational inference” (more often, if less accurately, “analogy”),<sup>19</sup> together constitute one of the finest

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<sup>17</sup> See Young, Walter Edward, ‘Concomitance to Causation: Arguing *Dawarān* in the Proto-*Ādāb al-Baḥth*’, in *Philosophy and Jurisprudence in the Islamic World*, ed. by Peter Adamson (Berlin: De Gruyter, 2019), pp. 205–281; and Young, Walter Edward, ‘Islamic Legal Theoretical and Dialectical Approaches to Fallacies of Correlation and Causation (7th–8th/13th–14th centuries)’, in *Islamic Legal Theory: Intellectual History and Uṣūl al-Fiqh*, ed. by Robert Gleave and Murteza Bedir (Leiden: Brill, forthcoming).

<sup>18</sup> See sources in previous note.

<sup>19</sup> See Young, Walter Edward, *The Dialectical Forge: Juridical Disputation and the Evolution of Islamic Law* (Cham: Springer, 2017), p. 10.

outcomes of the argumentative approach to legal reasoning within Islamic Law. A particularly lucid example is the systematization of the respected Shāfi‘ī theoretician Abū Ishāq al-Shīrāzī (1003-1083 CE), upon which the following is based.<sup>20</sup>

Among the majority of Sunni jurists, *qiyās* consists in a set of methods with varying epistemic grades, the highest of which belongs to that mode, called *qiyās al-‘illa*, in which the rule-occasioning factor or cause (‘*illa*) may be identified, located, and proven efficacious. The aim of this archetypal form of *qiyās* is to provide a rational ground for the application of a juridical ruling (*ḥukm*) to a given case, called the branch-case (*far‘*), which has not been directly and unequivocally pronounced upon in the primary juridical sources (i.e., the Qur’ān, Sunna [Prophet’s example], and *Ijmā‘* [consensus]). The method starts by attempting to determine the property (*wasf*) or set of properties in the root-case which constitutes the causal or occasioning factor or factors, or *ratio legis* (‘*illa*) giving rise to its ruling. If it is “probable” (*ẓannī*) that this property occasions the ruling, and it is shared by the branch-case, then we may infer that it is equally productive of that ruling in the branch-case.<sup>21</sup>

A cardinal feature of al-Shīrāzī’s take on *qiyās al-‘illa* is his particular notion of efficiency (*ta’thīr*), which tests whether the property *P* purported to be efficient in occasioning the ruling at stake is indeed so. For al-Shīrāzī, *ta’thīr* consists of two complementary procedures:

co-presence (*tard*): whenever the property is present, the ruling is also present

and

co-absence (‘*aks*): whenever the property is absent, the ruling is also absent.

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<sup>20</sup> A landmark on the subject of *qiyās* is Hasan, Ahmad, *Analogical Reasoning in Islamic Jurisprudence: a Study of the Juridical Principle of Qiyās* (Islamabad: Islamic Research Institute, 1986). Young provides a summary of al-Shīrāzī’s systematization of *qiyās* (*Dialectical Forge*, pp. 110-128), and on this basis Rahman, Iqbal, and Soufi develop a logical analysis—see Rahman, Shahid, Muhammad Iqbal, and Youcef Soufi, *Inferences by Parallel Reasoning in Islamic Jurisprudence: Al-Shīrāzī’s Insights into the Dialectical Constitution of Meaning and Knowledge* (Cham: Springer, 2019). See also Iqbal, Muhammad, *Arsyad al-Banjari’s Insights on Parallel Reasoning and Dialectic in Law: The Development of Islamic Argumentation Theory in the 18th Century in Southeast Asia* (Cham: Springer, 2022).

<sup>21</sup> Though admitting a range of degrees, the “probability” indicated by the term *ẓannī* does not refer to the statistical view conceived by the time of Leibniz. See Hallaq, Wael B., *A History of Islamic Legal Theories: An Introduction to Sunnī Uṣūl al-Fiqh* (Cambridge: Cambridge University Press, 1997), p. 39. In fact, we might suggest *ẓann* probability is a kind of *qualitative* expression for comparing degrees of acceptability, such as: “there are more indications in favour of accepting that the property *P* occasions the ruling than not”.

While co-presence examines whether ruling  $\mathcal{K}$  follows from verifying the presence of property  $\mathcal{P}$ , co-absence examines whether exemption from ruling  $\mathcal{K}$  follows from verifying the absence of  $\mathcal{P}$ .<sup>22</sup>

## Remarks

1. It is important to bear in mind that *qiyās* procedure involves two primary dialectical steps: one is the epistemological and hermeneutical task of identifying the properties claimed to occasion the ruling, the other—that is, the logical step—assumes that the first has been achieved. The first involves sub-arguments and counter-arguments that are not formal, but contentual or material; the second involves dialectical rules for logical reasoning. Contesting a legal argument might involve contesting a logical mistake, but this concerns fallacies more in line with those in the peripatetic syllogistic tradition, which constitute a group of their own known as *mughālaṭāt*. In the context of legal reasoning, the most relevant form of objection concerns the first step whereby a semantic link between the property and the ruling is claimed. The sub-arguments and counterarguments concerning this step are not formal and are therefore always prone to further revision. Agreement can be achieved at some point, but the whole process can also start over again.<sup>23</sup>

2. In section 5 we will distinguish between *occasioning factors* or *properties*, and *occasioning procedure*: the efficient causation that transforms instances of factors or properties into concrete applications of the ruling. For the time being, however, we will leave this finer distinction aside.

## 4.2 *Kasr* or: “How to Break Apart a Compound Occasioning Factor”

An important set of recognized, fallacious moves in Islamic legal argumentation theory relates to challenging claims that a property (*waṣf*) or composition of properties constitutes the efficient *‘illa*, i.e., the occasioning (causal) factor upon which the ruling (*ḥukm*) is grounded.

<sup>22</sup> See the preface in Rahman, Iqbal, and Soufi, *Inferences by Parallel Reasoning*. NB: this test of a property’s causal efficiency is elsewhere and more commonly called “co-presence and co-absence” (*al-ṭard wa-l-‘aks*) or “concomitance” (*dawarān*), and listed among the “modes of causal justification” (*masālik al-ta’līl*). See Young, ‘Concomitance to Causation,’ and Hasan, *Analogical Reasoning*, pp. 315-330. As for “efficiency” (*ta’thīr*), such as al-Ghazālī deemed it to be a direct designation of the cause (*‘illa*) by either univocal source-text (*naṣṣ*) or consensus (*ijmā’*), while others held different notions (see Hasan, *Analogical Reasoning*, pp. 272-3, 284).

<sup>23</sup> See Rahman, Iqbal, and Soufi, *Inferences by Parallel Reasoning*, pp. 46-47.

Among other things in this context, what may be subject to fallacy is the form an *objection* takes in challenging a claim that some property (or properties) constitutes the ‘*illa*’ for a debated case—or the *response* to that objection. Fallacious objections to claimed legal or natural occasioning factors, however, appear not to have been included among the peripatetic tradition’s *mughālaṭāt* (usually delimited by *σόφισμα*). Rather, we find them described in legal-theoretical and dialectical works as “invalid” (*fāsid*, *ghayr ṣaḥīḥ*) objections and responses, while objections and responses that contribute to legitimate refutations and defences are deemed “valid” (*ṣaḥīḥ*).

Of course, in Islamic legal-theoretical contexts, as in Roman Law, “valid” bears the sense of “legally sound and effective”, as in “a valid contract”, while also admitting the deontic reading “legally allowed”. In Islamic juristic dialectic, however, “valid” (*ṣaḥīḥ*) and “validity” (*ṣiḥḥa*) are also applied to arguments—and, by extension, to moves that produce valid arguments—with validity understood as conformity with some set of dialectical rules establishing both the dialectical meaning of the expression involved and the procedural debate-protocol to be followed.<sup>24</sup> On the other hand, “invalid” (*fāsid*) and “invalidity” (*fasād*) seems also to relate to the notion of a winning strategy: that is, Q’s objection is invalid if, no matter what Q does after some specific move, R wins without further intervention.<sup>25</sup>

Among the most virulent challenges to an ‘*illa*’-claim is the charge of *naqḍ* (inconsistency), whereby Q aims to “destroy” R’s claimed ‘*illa*’ by bringing a case in which it is found without R’s claimed ruling (*ḥukm*)—thus failing to exhibit the requisite co-presence (*ṭard*). Notably, a legitimate or valid *naqḍ* brings a parallel case more general than R’s, wherein the property claimed to be the occasioning factor is present but R’s desired ruling is not, whereas

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<sup>24</sup> See Rahman, Shahid, Zoe McConaughy, Ansten Klev, and Nicolas Clerbout, *Immanent Reasoning or Equality in Action: A Plaidoyer for the Play Level* (Cham: Springer, 2018), pp. 57-105, and pp. 279-282 for *refutation*.

<sup>25</sup> The use of “valid” and “invalid” in argumentative contexts of course relates to logical terminology, but an important caveat must be observed. As mentioned, the notion of *invalidity*—and, by extension, *invalid move*—includes both semantically unsound justifications and objections and the notion of *logically* invalid, whereas its counterpart, *validity*, does not refer to logically valid arguments (at least not in its most common usage). As we shall see below, however, in Islamic legal reasoning an *invalid move* may also refer to a move by Q whereby R wins, no matter what Q attempts to do afterwards (since Q’s move contravened the meaning and/or procedural rules established for the thesis involved), while a *valid move* does not automatically amount to a winning strategy. It refers rather to a legitimate move that can produce the win of one *play* of one of the contenders, but does not prevent finding another play with a different outcome. Put more simply: “validity” applies to winning just one play, not all relevant possible plays (see the previous reference to Rahman, McConaughy, Klev, and Clerbout, *Immanent Reasoning*). Islamic debates—operating primarily in the domain of probability as opposed to certainty—thus also operated primarily at the play level.

an illegitimate or invalid *naqd* may bring a parallel case that in fact constitutes a particularization or further specification of R's root-case.<sup>26</sup>

Here we will focus on a special form of *naqd* called *kasr*, aimed at destroying a *compound of properties* claimed to be the occasioning factor by “breaking” apart and disposing of its components. In short, Q objects that one (or some) of the purportedly efficient properties of R's ‘*illa* exists in another case—the *kasr*-case, or “problem-case of breaking” (*mas’alat al-kasr*)—but with a different ruling (*ḥukm*).

Formulating the move is delicate and requires some fine distinctions, beyond the pair *sensus composito* and *sensus diviso*. It is therefore not surprising that *kasr* triggered long and heated debates over whether it should be admissible at all. The notions of *kasr* discussed here are those elaborated by al-Shīrāzī and al-Bājī, who, well aware of the pitfalls involved, developed two main pathways to “breaking” an opponent's compound ‘*illa*:

- (1) by substitution (*ibdāl*): producing a parallel *kasr*-case, similar to R's root- and branch-case, but wherein one of the properties in R's compound ‘*illa* is replaced by another in the same *ma'nā* (meaning, intension), and that parallel compound is present despite the absence of R's desired ruling (thus, the unaltered property is proven non-efficient in occasioning the ruling);
- (2) by removal (*isqāṭ*): producing a parallel *kasr*-case similar to R's root- and branch-case, but wherein one of the non-efficient properties in R's compound ‘*illa* is omitted, while the remainder is present despite the absence of R's desired ruling (thus, the remaining property [or properties] is proven non-efficient in occasioning the ruling).

Each type is further subdivided into subtypes that produce valid refutations of R's ‘*illa* and subtypes that do not. As evident in the above descriptions, a key consideration is that the rules of challenge and defence governing the building of a valid *kasr*-case must be *meaning preserving* in relation to both the form of composition of properties which R claims to constitute the ‘*illa* and to the conceptual dependence between that compound and the ruling.

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<sup>26</sup> See Young, *Dialectical Forge*, pp. 169-173.



This of course assumes that the framework is thoroughly dialectical. Indeed, the conceptual links are made explicit by what contemporary dialogicians call *dialogical meaning explanations*. That is, the conceptual links expressed by connectives or other terms, are given by rules of challenge and defence. In the sources, these meaning explanations manifest in the context of debates on the legitimacy of a concrete refutation attempt. It is here that the precise mode of composition assumed by each way of constructing a *kasr*-case is determined.

In the *Sharḥ al-Luma*,<sup>27</sup> al-Shīrāzī appears to be led by this meaning-preservation principle or caveat when dividing approaches to building a *kasr*-case into the aforementioned types and subtypes. Our texts provide sufficient elements to render a precise formulation of the rules governing valid and invalid moves for each type, though the terse examples, aimed at an audience proficient in Islamic substantive law and its points of derivation, are often difficult to follow.<sup>28</sup> In the following presentation we have attempted, with an eye to intrinsic coherence, to reconstruct arguments regarding the classification of types; but this should still be considered work in progress.

## **Type 1: *Kasr* by Substitution (*ibdāl*) in Dependent Conjunctive Compounds.**

### **Type 1a: Valid *kasr* by *ibdāl***

- **Valid objection (*i ʿtirāḍ*):** Given a compound of properties *AB*, claimed by *R* to constitute the *ʿilla*, a valid *kasr*-refutation by *ibdāl* is one in which *Q* substitutes *A\** for *A*, with *A\** being in the same *maʿnā* as *A*, in a *kasr*-case of the same *maʿnā*, and shows the *naqḍ* (inconsistency) of this parallel compound *A\*B*.
- **Valid response (*jawāb*):** Although *R* might acknowledge that, in principle, the above conditions for a valid refutation have been accomplished by *Q*, *R* can nevertheless rejoin if he can produce an argument showing that *A\** is *not* in the same *maʿnā* as *A*, and thus *Q*'s *kasr*-case involves a significant deviation from

<sup>27</sup> al-Shīrāzī, Abū Ishāq, *Sharḥ al-Luma*, ed. by ʿAbd al-Majīd Turkī (Beirut: Dār al-Gharb al-Islāmī, 1988), pp. 892-909, §§1035-1051.

<sup>28</sup> The original sources for our study also included al-Shīrāzī, Abū Ishāq, *Al-Maʿūna fī l-Jadal*, ed. by ʿAlī b. ʿAbd al-ʿAzīz al-ʿUmayrīnī (Al-Ṣafāh, Kuwait: Manshūrāt Markaz al-Makhtūṭāt wa-al-Turāth, 1987); al-Bājī, Abū al-Walīd, *Kitāb al-Minhāj fī Tartīb al-Ḥijāj* (*L'art de la Polémique: Édition Critique d'al-Minhāj de Bāḡī*), ed. by ʿAbd al-Majīd Turkī (Paris: Maisonneuve et Larose, 1978); al-Bājī, Abū al-Walīd, *Iḥkām al-Fuṣūl fī Ahkām al-Uṣūl*, 2 vols., ed. by ʿAbd al-Majīd Turkī (Beirut: Dār al-Gharb al-Islāmī, 1995). Since al-Bājī's classification mostly follows that of his teacher, al-Shīrāzī, and the discussion of *kasr* in the latter's *Sharḥ al-Luma* is more thorough than the one in his *Maʿūna*, we will base our presentation mostly on al-Shīrāzī's notion of *kasr* as presented in the *Sharḥ al-Luma*, sometimes completing the presentation of his *Maʿūna* with further details found therein. We will also add some of al-Bājī's remarks should they help clarify the subtypes.

the *ma'nā* of R's root- and branch-cases. R's response thus takes the form of a (counter-) objection by “disqualifying difference” (*farq*), invalidating Q's *kasr*-case as a proof of R's inconsistency. Importantly, that disqualifying difference also shows that substituting *A\** for *A* triggers a change in the meaning of *B*—the *ma'nā* of *B* in the context of the new *A* should not be expected to contribute to occasioning a ruling as did the *ma'nā* of *B* in the context of the old *A*.

### **Type 1b: Invalid *kasr* by *ibdāl***

- Given a compound of properties *AB*, claimed by R to constitute the '*illa*', an invalid *kasr*-refutation by *ibdāl* is one in which Q substitutes *A\** for *A*, but *A\** (and thus the *kasr*-case itself) is *not* in the same *ma'nā*. (Q's new compound *A\*B* may even prove irrelevant for occasioning the ruling of his own *kasr*-case.) Q has therefore failed to show the *naqd* of R's '*illa AB*', and R is not bound to do anything beyond pointing out that Q's *kasr* is invalid.
- The point here is that the new compound in an invalid (type 1b) *kasr*-case does not preserve the meaning dependence with its ruling in the way that R's original compound had done.

In fact, it is the rule for R's response that signals how his compound is to be understood in the first place. In the next section, we will provide further details of logical analyses of the form of composition *AB* assumed in type 1 and type 2 *kasr* objections. But for the moment let us say that intrinsic to type 1 *kasr* objections and responses are compounds wherein the components are claimed to occasion the ruling *together*, not in isolation. Note that, on the topic of compound '*illas*', W. B. Hallaq observes:<sup>29</sup>

The ratio may also consist of more than one attribute, all of which must be considered as 'causing' a normative rule to arise from them. For instance, the ratio of the theft penalty encompasses five attributes: (1) the taking away of something by stealth; (2) the stolen object must be of a minimum value...; (3) the object must in no way be the property of the thief; (4) it must be taken out of custody (*hirz*); and (5) the thief must have full legal capacity. All of these attributes must obtain for an act to qualify as theft (*sariqa*) punishable

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<sup>29</sup> Hallaq, Wael, *Sharī'a: Theory, Practice, Transformations* (Cambridge: Cambridge University Press, 2009), p. 102.

by cutting off the hand. Each attribute is necessary; no single one by itself suffices to produce the ratio legis.

More generally, and although not plainly stated in our sources in these terms, we may infer from the examples that, in a successful *kasr*-case, the meaning of property *B* must also be dependent upon the meaning of *Q*'s *A*\* in such a way that *Q*'s substitution does not alter substantially the meaning of *B* from what it enjoyed when paired with *R*'s original *A*. In short, *B* is a propositional function defined over *A* or its meaning-preserving substitutes. Rejecting a refutation based on *kasr* by *ibdāl* amounts to showing that *Q*'s argument contravenes (partially or totally) the meaning-preservation caveat regarding the dependences inherent to *R*'s compound '*illa*.

### Examples: Type 1

**Type 1a:** Al-Shīrāzī provides the following example of a valid attempt at *kasr* and its valid response.<sup>30</sup> The '*illa* of *R*'s *qiyās* is a compound '*illa* ('*illa murakkaba*) consisting of properties *A* and *B*:

<b>contended branch-case (<i>far</i>):</b>	Sale of what the buyer has not seen...
<b>claimed ruling (<i>ḥukm</i>):</b>	is not allowed...
<b>claimed legal cause ('<i>illa</i>):</b>	because it is [ <i>A</i> ] an object of sale ( <i>mabī'</i> ), and it is [ <i>B</i> ] unknown of attribute ( <i>majhūl al-ṣifa</i> ) to the contractor at the time of contract...
<b>endoxon root-case (<i>aṣl</i>):</b>	like when the seller merely says "I sell you a garment" (which we know, by juristic consensus [ <i>ijmā'</i> ], is not allowed).

- *Q* attempts to invalidate *R*'s '*illa AB* by replacing one of its properties (*A*, *mabī'*: *object of sale*) with another (*A*\*, *mankūḥa*: *object of marriage*)<sup>31</sup> and showing that *B* is thus co-present with the opposite of *R*'s ruling (i.e., it is valid, rather than not allowed) in the seemingly parallel *kasr*-case of marriage.

<sup>30</sup> Al-Shīrāzī, *Sharḥ al-Luma'*, pp. 893, 898-9.

<sup>31</sup> Although this is not made explicit in the example in the *Ma'ūna*, it is in the example in the *Sharḥ al-Luma'* (al-Shīrāzī, *Sharḥ al-Luma'*, pp. 893, 898-9), where (*A*) *mabī'* is replaced by (*A*\*) *mankūḥa*.

So, the one disagreeing with him [i.e., Q] says: ‘This is broken (*inkasara*) by [the fact that] if he marries a woman whom he has not seen, then she will be [[A\*] an object of marriage (*mankūḥa*) and] [B] unknown of attribute (*majhūl al-ṣifa*) to the contractor at the time of contract—yet it [i.e., the contract] is valid.’<sup>32</sup>

- R responds by invalidating Q’s *kasr*-case through an objection of disqualifying difference (*farq*): marriage is not the same as sale with regard to property *B* because the buyer has the option to rescind (*khiyār*) in the case of a sale exhibiting property *B*, but the groom does not have that option in the case of a marriage exhibiting property *B*.

Importantly, Q’s *kasr*-case of marriage shares property *B* with the contended case. It is a specific form of ignorance (*jahāla*)—namely, “unknown of attribute” (*majhūl al-ṣifa*)—which, as Hallaq explains, “presumes existence [of the object of contract] but involves lack of reasonable knowledge of the thing’s characteristics”.<sup>33</sup> However, “object of sale” (A) and “object of marriage” (A\*) take different paths in relation to the contractor’s subsequent recourse to option (*khiyār*). Thus, the meaning of *B*, “unknown of attribute”, changes if A\*, “object of marriage”, is substituted for A, “object of sale”; and Q’s objection of *kasr* is thus nullified.

**Type 1b:** In this example, we begin with the same *qiyās al-‘illa*, but Q’s objection is invalid because his substitution and *kasr*-case are clearly not in the *ma’nā* of R’s original property and the contended case.

- Q attempts to invalidate R’s *‘illa* by bringing the *kasr*-case of bequeathing (i.e., in a will) something which is [A\*] an object of bequest (*mūṣā bihi*) and [B] unknown of attribute (*majhūl al-ṣifa*) to the contracting party [i.e., the legatee] when the contract was made, but which is nevertheless allowed, in contrast to the contended case of sale.

Such is like when the one drawing indication [R] says: ‘It is [A] an object of sale which is [B] unknown of attribute to the one contracting at the time of contract, so it resembles when he says: “I sell you a garment.”’ So he [Q] says to him:

<sup>32</sup> Al-Shīrāzī, *Ma’ūna*, p. 246, §127.

<sup>33</sup> Hallaq, *Sharī‘a*, p. 244.

‘This is broken (*yankasiru*) by [A\*] the object of bequest (*mūṣā bihi*), for it is [B] unknown of attribute to the one contracting at the time of contract, but the bequeathing of it is valid.’<sup>34</sup>

Thus, Q has replaced [A] with [A\*], then attempted to show the *naqḍ* of this parallel compound [A\*B] in the *kasr*-case.

- However, Q’s replacing of A (*mabīʿ*, object of sale) with A\* (*mūṣā bihi*, object of bequest) in the *kasr*-case is invalid. Why? Because A and A\* are not in the same *maʿnā*. As al-Shīrāzī explains in the *Sharḥ al-Lumaʿ*:

Bequest (*waṣīyya*) is not a parallel case (*naẓīr*) for sale in terms of lack of knowledge (*jahāla*), nor is it in [sale’s] quality / meaning (*maʿnā*). Don’t you see that no [kind of] lack of knowledge (*jahālāt*) prevents the validity of the bequest? For this reason, if he were to say: ‘I bequeath to you a garment, or something, or wealth / property,’<sup>35</sup> it would be permitted. But the like of this is not valid in sale.

Thus, Q attempts to replace A “object of sale” with A\* “object of bequest”, despite the fact that a legatee’s ignorance of the nature or characteristics of an object of bequest has no efficiency whatever with regard to the bequest contract’s validity. According to our understanding, the case is deceptive precisely because, though sharing that property of ignorance (*B*) which is deemed efficient in R’s root- and branch-case, its irrelevance in the context of bequest puts Q’s substitution and *kasr*-case outside the *maʿnā* of R’s original property and cases. The legatee’s ignorance of what has been bequeathed to him simply does no work in occasioning that contract’s validity or lack thereof, whereas the buyer’s ignorance of what has been sold to him does.

### Conclusions regarding Type 1.

Our texts indicate that the compounds targeted by type 1 *kasr* involve meaning dependences of the unchanged property (or properties) upon the property replaced. The relation is a conceptual link, whereby the second property [B] in fact constitutes a specification of the first [A]. That is, what is unknown of attribute to the buyer at the time of sale is a specific kind

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<sup>34</sup> Al-Shīrāzī, *Sharḥ al-Lumaʿ*, second half of §1041.

<sup>35</sup> Reading [بخير] instead of [بخير].

of object of sale—*almost* in the way that a bride unknown of attribute to the groom at the time of marriage is a specific kind of object of marriage (but not quite, since the groom has no option to rescind), and *certainly not* in the way of a legatee’s ignorance of the object of bequest (which is in fact a normal state of bequest, not a specific kind).

## **Type 2: *Kasr* by Removal (*isqāṭ*) in Independent Conjunctive Compounds.**

In contrast to type 1, a valid type 2 *kasr* works by omitting a non-efficient property of R’s compound ‘*illa* and proving the remainder inconsistent via *naqd*. Although at this stage of research we have uncovered no explicit positive statements to this effect, we may also conjecture, by sheer logical analysis of the examples discussed in the sources, that type 2 *kasr* was understood (again in contrast to type 1) to target compounds wherein *each of the components was thought (by R) to contribute independently to the occasioning of the ruling*. However our jurists may in fact have conceived of type 2 *kasr*, we will carry this assumption forward into our analyses in order to explore the full potential of this objection and its responses without invalidating it from the start.

### **Type 2a: Valid *kasr* by *isqāṭ*.**

- **Valid objection (*i tirād*):** Given a compound of properties *ABC*, which R claims to be the ‘*illa*, a valid *kasr* by *isqāṭ* is when Q (1) brings a parallel *kasr*-case in which one of R’s properties (e.g., *A*) is omitted but the remainder (e.g., *BC*) are present; (2) justifies this *kasr*-case by showing that *A* lacks efficiency (a subsidiary objection of ‘*adam al-ta’thīr*), and can thus be removed; and (3) shows that this *kasr*-case constitutes a *naqd*-case for what remains of R’s ‘*illa* (*BC*)—i.e., that *BC* is present therein without R’s *ḥukm*. Note that Q thus contests R’s components in two moves, first denying efficiency to one, and then destroying what remains via *naqd*.
- **Valid response (*jawāb*):** R therefore has recourse to either (1) contesting Q’s objection that *A* lacks efficiency (i.e., his subsidiary move of ‘*adam al-ta’thīr*), or (2) contesting Q’s *kasr*-case in that it does not demonstrate the *naqd* of *BC*.

### **Type 2b: Invalid *kasr* by *isqāṭ*.**

- Given a compound of properties *ABC*, which R claims to be the ‘*illa*, an invalid *kasr* by *isqāṭ* is when Q’s *kasr*-case omits *A* despite there being clear evidence that *A* is in fact causally efficient. Having thus wrongly removed the truly causal

portion of R's compound *'illa*, the *kasr*-case's subsequent *naqd* of what remains (*BC*) does no real harm, having failed to show inconsistency for the causally efficient *A*.

In sum, it may help to think of type 2 *kasr* by *isqāṭ* as Q accusing R of “stuffing” his *'illa* with extra, non-efficient properties. Thus, in subtype 2a, Q rightly jettisons the non-efficient stuffing and asserts *naqd* of the efficient remainder, while in subtype 2b, Q wrongly jettisons the efficient component and asserts *naqd* only of the non-efficient stuffing. In the following, we will only examine R's response to 2a as discussed by al-Bājī,<sup>36</sup> since it yields a complete illustration of this subtype, whereas 2b represents a special case in which Q gets it wrong from the start.

### Example: Type 2

#### Type 2a: valid *kasr* and valid response

- R asserts that intent (*niyya*) is not obligatory for the minor ritual ablution (*wuḍū'*) because *wuḍū'* is [*A*] a means (*sabab*) of reaching prayer, and [*B*] not a substitute act (*badal*); therefore, *niyya* is not obligatory, as in the root-case of removing filthiness (*najāsa*), which also has both properties *AB*.<sup>37</sup> In al-Bājī's words:

As for when [*Q*] omits a non-efficient property, it is a valid *kasr*. This is like when the Ḥanafī [*R*], for the [issue] of intent (*niyya*) in minor ritual ablution (*wuḍū'*), draws indication that [intent] is not obligatory, because [ritual ablution] is [*A*] a means of access / mediate cause (*sabab*)<sup>38</sup> by means of which one reaches prayer, which is [*B*] not in the way of a substitution (*badal*),<sup>39</sup> so intent is not obligatory for it—like [in the case of] removal of impure filthiness (*izālat al-najāsa*).

Thus, R's *qiyās* has the following components:

<sup>36</sup> Al-Bājī, *Minhāj*, §436.

<sup>37</sup> The relevant substantive legal category here is called *ibdāl*, or substitution, and is when an original, obligatory act of ritual worship, the *aṣl*, is replaced by another kind of “substitute” act, the *badal*, either because one was not able to do the *aṣl*, or one has neglected / invalidated the *aṣl* and needs to make up for it. In this example, a condition (*sharṭ*) of valid prayer is valid *wuḍū'*: the minor ritual ablution with water (constituting the original act, or *aṣl*); but should one find oneself in, e.g., a waterless waste, one may instead perform *tayammum*: ritual ablution with sand or clean earth (constituting the substitute act, or *badal*). *Tayammum* is a *badal* for the originally obliged *aṣl* of *wuḍū'*.

<sup>38</sup> On the *sabab*, see Hasan, *Analogical Reasoning*, pp. 369 ff.

<sup>39</sup> On substitution (*ibdāl*) in general, see Wizārat al-Awqāf wa-l-Shu'ūn al-Islāmiyya, *Al-Mawsū'a al-Fiqhiyya*, 45 Vols. (Kuwait: Wizārat al-Awqāf wa-l-Shu'ūn al-Islāmiyya, 1986-2000), vol. 1, pp. 140 ff., s.v. [إبدال].

<b>contended branch-case (<i>far</i>):</b>	Intent ( <i>niyya</i> ) in the minor ritual ablution ( <i>wuḍū'</i> )...
<b>claimed ruling (<i>ḥukm</i>):</b>	is not obligatory...
<b>claimed legal cause (<i>'illa</i>):</b>	because <i>wuḍū'</i> is [A] a means ( <i>sabab</i> ) of reaching prayer, and [B] not a substitute act ( <i>badal</i> )...
<b>endoxon root-case (<i>aṣl</i>):</b>	like removal of impure filthiness ( <i>izālat al-najāsa</i> ).

- Now, in objecting via type 2a *kasr*, Q “breaks” R’s *'illa* in three steps:

First, Q brings the *kasr*-case of *tayammum*, the ritual ablution with sand or earth, which contains *A* (being a *sabab*) though not *B* (not being a *badal*)—in fact, *tayammum* is the very *badal* for *wuḍū'*—but for which *niyya* is obligatory (the opposite of R’s ruling). Property *B* is thus omitted in Q’s *kasr*-case (*tayammum* is *A* but not *B*).

Second, Q justifies this omission by claiming that *B* cannot be an efficient property vis-à-vis non-obligation of *niyya* in the contended case, since when it comes to *niyya* whatever holds for the *badal* always holds for the *aṣl* and vice versa—it is never the case that one requires *niyya* and the other doesn’t (as is the consequence of R’s claim). Q thus attempts to show that *B*, not being a *badal*, has a lack of efficiency (*'adam al-ta'thīr*) in occasioning a ruling (*ḥukm*) of non-obligation of *niyya*.

Third, once *B* is shown to be non-efficient “stuffing”, Q’s *kasr*-case automatically becomes a *naqd*-case for the remaining property *A*. In the case of *tayammum*, we find property *A* (being a *sabab*), but not R’s ruling (non-obligation of *niyya*). R’s *'illa* is present despite the absence of R’s *ḥukm*—the very definition of *naqd*. In al-Bājī’s words:

So the Mālikī [Q] says to him: ‘This is broken (*yankasiru*) by ablution with dry earth or sand (*tayammum*), for it is [A] a means of access / mediate cause (*sabab*) by means



of which one reaches prayer, yet it requires intent.<sup>40</sup> This is a valid *kasr*, because the property by way of which [R] guarded against [inclusion of] ablution with dry earth or sand (*tayammum*) [that is, *B*: it is not a substitution (*badal*)] is not efficient in the falling away of intent (*niyya*).<sup>41</sup> Don't you see that original [acts] (*uṣūl*)<sup>42</sup> and substitutions (*abdāl*)<sup>43</sup> are one with regard to the subject of intent in God's Law?

- R, however, successfully escapes this objection with a valid response. In short, he revives the possibility that property *B* (not being a *badal*) can have efficiency, by destroying Q's claim of *aṣl-badal* correspondence vis-à-vis *niyya* with counter-examples wherein an *aṣl* does not require *niyya* but its *badal* does. With the possibility of *B*'s efficiency thus restored, R's *qiyās* evades Q's type 2a *kasr* by *isqāṭ*.

## Conclusions regarding Type 2.

Our texts on this type of *kasr* suggest that R implicitly assumes that the compound *AB*, in order to occasion the ruling, must be constituted by a conjunctive antecedent.

Now, if we link an antecedent of the form  $A \wedge B$  to R's ruling with some form of implication, and the components of the antecedent do not constitute a chain of dependences as in type 1 *kasr*, then it logically follows that these components distribute **disjunctively** in relation to R's ruling.

This logical fact explains why Q's challenge in type 2a requires both denying efficiency to one property and destroying the other: challenging a conjunction that distributes over an implication requires challenging all of the resultants of the disjunctive distribution. What the dialectical setting adds to this purely logical analysis is the fact that Q's challenges are not of the same form (one claims lack of

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<sup>40</sup> Note that *tayammum* is itself the substitution done in place of *wuḍū'* when there is no water, and so Q has omitted property [B] "not being in the way of a substitution."

<sup>41</sup> That is, R likely formulated his compound *'illa* with property B (not in the way of substitution) in order to exclude the endoxon case of *tayammum*, which has the opposite (intent is obligatory) of the desired *ḥukm*, but that property B has no efficiency in the negation of intent in the case at hand. So Q's omission of property B in his *kasr*-case of *tayammum* (which is in the way of substitution) is valid, and exposes the inconsistency (*naqḍ*) of the other property A: that is, property A is present in the *kasr*-case of *tayammum*, but without R's *ḥukm*. Thus, it is a valid *kasr*.

<sup>42</sup> In this case, minor ritual ablution (*wuḍū'*).

<sup>43</sup> In this case, ablution with dry earth or sand (*tayammum*).

efficiency, the other builds a *naqd*-case). This likewise explains why R can respond by countering one of these different challenges.

Moreover, it is the disjunctive distribution of the conjunction over the implication that explains why, in type 2b, the move that contests the “redundant” components in the antecedent does not destroy the main claim, given that the component known to be efficient has been omitted from the charge of inconsistency.

Thus, logical analysis suggests that we may think of type 2 *kasr*’s target *qiyās* as being implicitly understood by R to ground such assertions as:

Performing actions of the types containing *A, B* ... is forbidden... iff performing types containing *A* is forbidden or performing types containing *B* is forbidden.

In fact, if we do not assume such a logical analysis, then type 2 *kasr* is wrong from the start. Certainly, we cannot read its target *qiyās* as being of the sort aimed at by type 1 *kasr*, as made plain through considering once more the example of *wuḍū’*. R claims it does not require *niyya* because it is a *sabab* and not a *badal*; but if we read that according to the (meaning dependence) manner of type 1 *kasr*’s target *qiyās* composition, then the attempted *kasr*-case of *tayammum* would be precluded from the outset.

## 5 Towards a Dialogical Meaning Explanation of *Kasr*:

Due to space constraints we will focus on developing a logical analysis that will constitute the building blocks for *kasr*’s dialogical framework. In this study, however, we will neither be able to describe all the required dialogical steps nor to build the actual dialogue.

### 5.1 Functional Analysis of the Occasioning Factor

Rahman and Iqbal have proposed a CTT analysis of the conceptual link between the ruling and the properties constituting the occasioning factor.<sup>44</sup> The idea is that a ruling is conceptually dependent upon the properties that occasion that ruling. Certainly, the *Forbidden*

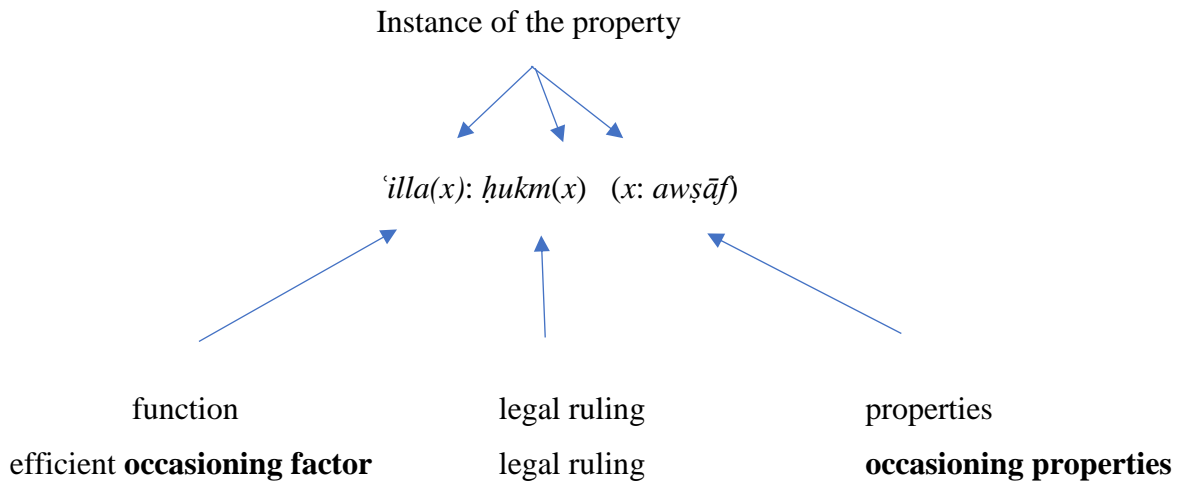
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<sup>44</sup> Cf. Rahman, Shahid and Muhammad Iqbal, ‘Unfolding Parallel Reasoning in Abu Ishāq al-Shīrāzī’s System of Co-Relational Inferences of the Occasioning Factor’, *Arabic Sciences and Philosophy*, 28 (2018), 67–132, which is further developed in Rahman, Iqbal, and Soufi, *Inferences by Parallel Reasoning*, pp. 31–40, and in Iqbal, *Arsyad al-Banjari’s Insights*.

occasioned by intoxicating drinks is not the same as the *Forbidden* occasioned by murder. The legal system attends to this distinction by the quality and degree of sanctions that penalize such acts.

In the context of a CTT framework, functions are the means to express such dependences; and a function is a method or procedure for relating concepts such that one is dependent upon the other. In legal contexts, executing this function is a legal procedure by means of which performances of a type of action occasion the ruling's application to such performances. In concrete cases, it is the legal authority who carries out the procedure.

The upshot of such an analysis is that it allows one to distinguish between the property constituting the *occasioning factor*, and the *actual legal procedure that occasions* the ruling. In other words, according to this view, it is useful to distinguish between the property (Ar. *wasf*, pl. *awṣāf*) and the properly efficient cause / occasioning factor (*'illa*), that is, the procedure that transforms instances of occasioning (causal) factors into applications of the ruling to these instances. If we recall our notation in section 3 we have:



- Notice that the CTT framework allows the introduction of potentially infinite types, each of them intensionally defined—they are not extensional sets. This not only allows one to distinguish between different types of action but between different types of properties as qualities; legal contexts require certain multi-sorted domains. Thus, this multi-sorted language allows one to introduce performances of actions and instances of qualities into the object language. With the term *awṣāf* we may refer to any of these. The distinction between types of actions and types of quality is to

be determined in context. And there might be certain arguments supporting the reduction, in legal contexts, of all types to types of actions, and in natural contexts, of all types to types of events.

If the above is granted, we might take a step further and delve into the conceptual links constituting compound occasioning factors.

## 5.2 Compound Occasioning Factors within *Kasr*-Cases

As discussed in the previous section the *kasr* method of refutation targets compound occasioning factors: the first type targeting compounds wherein the meaning of one property is dependent upon another, the second targeting meaning independent components.

### 5.2.1 Dependent Composition

The compound *AB* at work in type 1 *kasr*-cases by substitution may be understood as *B* constituting a *propositional function* over *A*, in the manner described in section 3; that is, the meaning constitution of *B* is based on *A*. (Recall Aristotle’s point on the role of “Good” in “Good Cobbler”).<sup>45</sup> Notice that in this form of *kasr* Q’s objection is based on the idea that both objects of sale and objects of marriage are objects of contractual transaction—i.e., they are in the same meaning (*ma’ nā*). Thus, according to Q, they should have the same ruling when they share the property “involving objects unknown of attribute to a contractor at the time of contract,” but—and this inconsistency is at the heart of the *kasr* critique—in fact they do not have the same ruling. R’s counter-objection, however, specifically targeting Q’s *kasr*-case of the unseen bride, demonstrates that his substituting object of marriage (*mankūḥa*) for object of sale (*mabī’*) in fact contributes a *kasr*-case marred by a disqualifying difference (*farq*)—i.e., they are not in the same meaning. A commodity whose attributes are unknown to the buyer at the time of a sales contract is relevantly different from a bride whose attributes are unknown to the groom at the time of a marriage contract, as proven by the buyer’s right to rescind and the groom’s lack thereof. Thus, the ruling of proscription applies only to selling the unseen object of sale, and not to marrying the unseen bride.

In other words, the efficiency of the property “involving objects unknown of attribute to a contractor at the time of contract” in occasioning a ruling of proscription is not evident

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<sup>45</sup> See *De Int.* 9, 20b35-36 and 21a14-15; *Soph. El.* 20, 177b14-15.

outside of apt substitutions of similar sets of objects. That is, objects in the same *ma 'nā* should allow the property to play the same causal role in Q's opposing *kasr*-case as in R's original, authoritative root-case. Q's substitution should preserve the *meaning dependences* of the original case even as it aims to disprove R's juridical ruling's dependence upon this parallel compound. However, R's counter-objection by disqualifying difference (*farq*) shows that it does not preserve these meaning dependences. This constitutes a novel approach to illicit moves concerning composition. Not only does Type 1a *Kasr* and its unseen bride example occur in a context of determining (legal) causation, but it concerns primarily the *illegitimate substitution* of one of a compound cause's relevant components.

Note the logical structure displays a *double meaning dependence*, namely:

1. In the context of R's root-case, the ruling *not valid* is linked to a cause composed of two properties: *object of sale* and *attributes unknown to contractor at time of contract* (i.e., *attributes unknown to buyer at time of buying*), and not to any other compound cause.
2. In the context of R's root-case, the property *attributes unknown to contractor at time of contract* refers only to objects of sale, and not to objects of any other contract (such as marriage).

Thus, we may begin our formalization as follows:

Selling objects of sale, unknown of attribute, is not valid.

$(\forall y: \{ x: \text{Selling} \mid \text{Involving objects unknown of attribute } (x) \} \text{ Invalid}(y))$



Any instance  $y$  of  $\{ \text{those sales } x, \text{ which involve objects unknown of attribute } \} \{ \}$  is an invalid  $y$

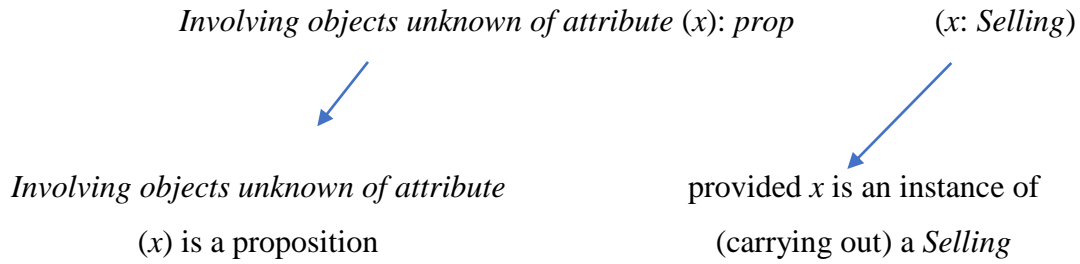
In fact, we can be more precise and indicate that what is invalid is actually the *selling of something* unknown of attribute—that is, what is (contractually) invalid is the *left component* of  $y$ . Indeed, if:

$$\{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \}$$

is the set of all those sales of objects unknown of attribute, and  $y$  is an element of this set:

$$y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \}$$

then this already assumes the meaning dependence at work:



With such being the case, then the left component of  $y$  is the ***selling  $x$***  (which is of those sales involving objects unknown of attribute),<sup>46</sup> that is:

$$\textit{left}(y)=x: \textit{selling},$$

and the right component of  $y$  verifies that this selling  $x$  ***involves an object unknown of attribute***:

$$\textit{right}(y)=z(x): \textit{Involving objects unknown of attribute} (x).$$

This leads to the formulation:

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<sup>46</sup> Notice that the set  $\{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \}$  is constituted by all those sales which have been carried out while the object is unknown of attribute. Thus, for every  $x$  in this set, one can say that it is a sale involving objects unknown of attribute.

$$(\forall y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute } (x) \} \textit{Invalid}(\textit{left}(y)))$$


Any instance  $y$  of  $\{ \textit{those sales, which involve objects unknown of attribute } \}$   $\{ \textit{is an invalid selling } x \textit{ (unknown of attribute) } \}$

The illegitimate use of *kasr* is thus due to the fact that the following substitution does not hold:

$$(\forall y: \{ x: \textit{Marrying} \mid \textit{Involving objects unknown of attribute } (x) \} \textit{Invalid}(\textit{left}(y)))$$

And why does this not hold? It does not hold due to a disqualifying difference (*farq*), whereby a telling norm holds true for objects of sale but not for objects of marriage—for commodities, but not brides:

Disqualifying difference (*farq*)

$$(\forall y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute } (x) \} \textit{Rescinding-allowed}(\textit{left}(y)))$$

$$(\forall y: \{ x: \textit{Marrying} \mid \textit{Involving objects unknown of attribute } (x) \} \textit{Rescinding-not-allowed}(\textit{left}(y)))$$

This should clarify why type 1 *kasr* can only be carried out by substitution: what is asserted is that the ruling ***forbidden*** applies only to those *Sales*  $y$  *involving objects unknown of attribute*. Substitution should preserve the meaning dependences of original case and also the dependence of the juridical ruling upon this compound.

### 5.2.2 Independent Composition

The approach to the compound *AB* which is at work in type 2 *kasr* cases by *removal* may be understood as omitting from the conjunction the non-efficient component and building an inconsistency-proving *naqd*-case for the remainder. As mentioned above, logically speaking

such a move is only possible if the meaning of the component that builds the *naqd*-case is not dependent upon the one removed.<sup>47</sup> Let us study the logical structure behind this, step by step:

$$A \wedge B: \text{prop}$$

What makes this proposition true is in fact a pair (or however many components constitute the conjunction) such that the first element of the pair verifies *A*, the second verifies *B*, the third verifies *C*, and so on:

$$(x_1, x_2): A \wedge B$$

(Whereby  $x_1$  stands for a function that takes elements of the domain *D* and asserts of them that they are *A*, and something similar holds for  $x_2$ .)

Recall that in our *wuḍū*’ example the two members in the conjunction are the properties *being a sabab (intermediate means) for reaching prayer* **and** *not being a badal* (i.e., a substitute for a ritual component missed or contextually unobtainable).

$$(x_1, x_2): \textit{Sabab} \wedge \textit{Not-Badal}$$

If we then relate the ruling to the compound, we might be tempted to express it as:

$$\textit{'illa}(x,y): \textit{ḥukm} [(x, y): (A \wedge B)]$$

However, this does not express the fact that the second property might be superfluous “stuffing”—as in fact it is in the example under consideration. Thus, what we need is to make apparent the logical consequence of an analysis that contemplates cases wherein one of the components is not efficient. As mentioned above, this amounts to a disjunctive distribution of the conjunction over the (claimed) occasioned ruling:

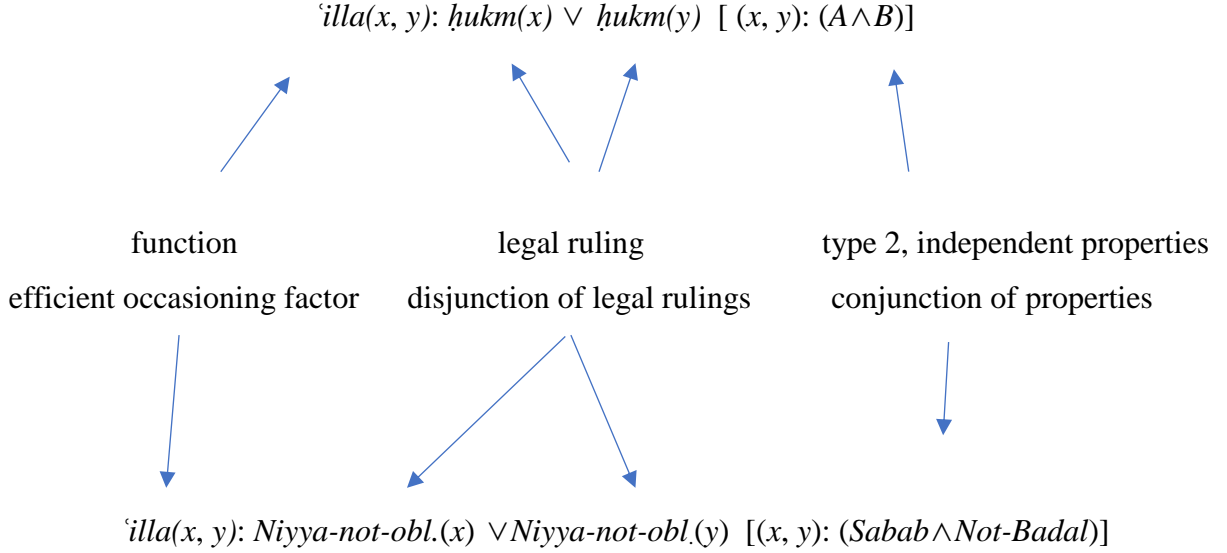
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<sup>47</sup> Each component expresses a property belonging to some domain of objects (or actions)—for the sake of simplicity we leave the domain tacit. In the context of CTT grammar, such a process is called *sugaring*: it is the reverse of formalization—see Ranta, *Type-Theoretical Grammar*, pp. 7-11. Sugaring procedures are those transformations which, step by step, produce a natural language expression. In the case of ambiguity in a natural language expression, one traces bottom-up the meaning-constitution sugaring process that yielded this expression.



$$'illa(x,y): \text{ḥukm}(fst(x,y)) \vee \text{ḥukm}(snd(x,y)) \quad [(x,y): (A \wedge B)]$$

And since  $fst(x,y)=x: A$ , and  $snd(x,y)=y: B$ , we may indulge in the following shortcut:



This explains why, if an unskilful Q removes a property known to be efficient, the refutation aborts: verification of one of the components is sufficient. Moreover, it explains why, in order to refute R's claim, Q needs to contest the efficiency of one of the components and to build a *naqd*-case for the other. In the *wuḍū'* example, it amounts to contesting the efficiency of:

$$'illa(wuḍū'₂): \text{Niyya-not-obligatory}(wuḍū'₂) \quad (wuḍū'₂: \text{Not-Badal})$$

and building a *naqd*-case against:

$$'illa(wuḍū'₁): \text{Niyya-not-obligatory}(wuḍū'₁) \quad (wuḍū'₁: \text{Sabab})$$

by replacing *wuḍū'* with *tayammum*. Given:

$$(wuḍū'₁, wuḍū'₂): (Sabab \wedge \text{Not-Badal})$$

Whereby:                       $wuḍū'₁: \text{Sabab}$

### 5.2.3 Conclusion regarding Dependent and Independent Types

The above CTT analysis reveals that the two different compositional modes of compound factors occasioning a juridical ruling targeted by a *kasr* refutation decline into two different ways of linking the members of such compounds by means of what, in natural language, we call conjunction. In short, this conjunction either (1) might assume that components display some form of conceptual dependence (as in type 1's target *qiyās*), or (2) might assume that they do not (as in type 2's target *qiyās*).

Note that the dialectic rules established for building a *kasr*-case for both forms of composition require basically the same form of verifiers: namely, a verifier for each of the components (though in the first type [at least], one of the verifiers is a function having as its argument the other verifiers of the compound). This strongly indicates that the two types of compound may be included under a single category we might call “conjunctive composition.”

Here we may appeal to a famous insight of Avicenna's: namely, that an implication can be reduced to a universally quantified proposition if its components *share content*.

All conditional and disjunctive propositions, and in particular the conditional in which the antecedent and the consequent share one part, can be reduced to categorical propositions.<sup>48</sup>

Accordingly, implication can be generalized as distinguishing, within the category of implications, those for which the content of the antecedent does not occur in the consequent (propositional implications) from those for which it does (universal quantifiers with restricted domains). In CTT and its grammar, the category including both forms of implication (but not disjunction) is known as *Π-Type*, also called the *cartesian product of a family of sets*.<sup>49</sup>

<sup>48</sup> Avicenna, *Al-Shifā' : al-Manṭiq : 4. al-Qiyās*, ed. by Sa'īd Zāyid (Cairo: al-Maṭba'at al-Amīriyya, 1964), p. 256, lines 11–15. Reference (and translation) is from Street, Tony, ‘Arabic Logic’, in *Handbook of the History and Philosophy of Logic*, Vol. I, ed. by John Woods and Dov Gabbay (Amsterdam: Elsevier, 2004), p. 533.

<sup>49</sup> See Ranta, *Type-Theoretical Grammar*, pp. 45–46. Notice that  $(\forall x: A)B$  and  $A \supset B$  express the same proposition. However, if  $B$  and  $A$  “share a part”, and, more precisely, if  $B$  is dependent upon  $A$ , we have  $(\forall x: A)B(x)$ . So Avicenna is right: every implication can be subsumed under the general universal  $(\forall x: A)B[x]$ , where the square brackets indicate that “ $x$ ”, the “part” of  $A$ , might occur or not occur in  $B$ .

This  **$\Pi$ -Type** is the dual, i.e. the logical counterpart, of a category we might call *conjunctive* and which includes conjunctions wherein some of the members *share content with the other* (existential quantifiers with restricted domains or subset separation: those elements in  $A$  that are  $B$ ) and those that do not (propositional conjunctions). In CTT, the general category that includes both forms of conjunction is called  **$\Sigma$ -Type** or *disjoint union of a family of sets*  $\Sigma$ -Type.<sup>50</sup> The upshot of all this is that whereas our analysis of type 1 amounts to *subset separation* (those elements in  $A$  that are  $B$ ), type 2 corresponds to *propositional conjunction*. Both types of conjunctive composition are at work in the two types of *kasr* we have examined.

Notice that in the case where components of a  $\Sigma$ -Type are dependent, the elimination rule holds, but each eliminated component will encode the information of the dependence. More precisely, from the compound *all those  $A$  that are  $B$* ,

$$(y, d(y)): \{y: A \mid d(y): B(y)\})$$

the function  $\text{fst}(y, d(y)) = y : A$  renders “those  $A$  that witness  $B$  *within* the compound,” and  $\text{snd}(y, d(y)) = d(y): B(y)$  renders “those  $B$ s that are  $A$ s *within* that same compound.”

And if we again recall our example of the object of sale,  $\text{fst}(y, d(y)) = y : A$  renders “a sale, involving an object unknown of attribute to the buyer,” and  $\text{snd}(y, d(y)) = d(y): B(y)$  attributes the property “involving objects unknown of attribute” to instances of sales.

Note that, in fact, interdiction in the sale example has, as its scope,  $\text{fst}(y, d(y)) = y : A$ , i.e., “those acts of selling involving objects unknown of attribute to the buyer.” Thus, if *selling a garment* is one specific sale (of an object of unknown of attribute), then  $\text{fst}(y, d(y)) = y : A$  can be read as the anaphoric construction:

“If selling a garment is selling one of those objects unknown of attribute to the buyer, then ‘this’ selling (of a garment) constitutes a law-breaking selling.”

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<sup>50</sup> Ranta, *Type-Theoretical Grammar*, pp. 43-45.

### 5.3 Dialogical Meaning Explanations

Rahman and Iqbal, prompted by Young's *Dialectical Forge*, proposed certain first steps towards a dialogical reconstruction of al-Shīrāzī's notion of parallel reasoning, within the dialogical framework called *Immanent Reasoning*.<sup>51</sup> This latter is a framework that integrates the fully interpreted language of CTT into Lorenzen & Lorenz's dialogical logic,<sup>52</sup> in order to achieve a framework which is sensitive both to content and to the perspective of the meaning play level over the strategic and logical validity level.

Being that we cannot here reproduce all the rules for parallel reasoning developed in Rahman, Iqbal, & Soufi,<sup>53</sup> we will restrict ourselves to the case of *kasr*, which was only very schematically and briefly discussed by the authors in 20 lines. Moreover, we will only mention the more crucial steps in the context of the examples discussed in our sources above.

#### 5.3.1 Dialogical Meaning Explanation of *Kasr* Type 1

Let us recall al-Shīrāzī's example, where the 'illa of R's *qiyās* is a compound 'illa ('illa *murakkaba*) consisting of properties A: being an object of sale (*mabī'*) and B: being unknown of attribute (*majhūl al-ṣifa*) to the contractor at the time of contract, while the claimed ruling (*ḥukm*) is "not allowed." Q attempts to invalidate R's 'illa by replacing one of its properties (namely, A) with another ( $A^*$ , *mankūḥa*: *object of marriage*) and showing that the parallel compound  $A^*B$  is thus co-present with the opposite of R's ruling (i.e., it is valid) in the seemingly parallel *kasr*-case of marriage. However, Q's case is based on a substitution that changes the meaning, since the substitute carries with it the additional meaning of *having the option to rescind*. The point, dialogically speaking, is the following:

Interlocutor **X** observes that if we delve into the meaning of *B*, *involving objects unknown of attribute*, it is apparent that it encompasses more than one kind of "being unknown of attribute" in contexts of contractual transactions, including those with the option to rescind (*khiyār*) and those without. Both are different specifications of *involving objects unknown of attribute*.

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<sup>51</sup> See Rahman and Iqbal, 'Unfolding Parallel Reasoning,' with further development in Rahman, Iqbal, and Soufi, *Inferences by Parallel Reasoning*; and Iqbal, *Arsyad al-Banjari's Insights*. As for immanent reasoning, see Rahman, McConaughey, Klev, Clerbout, *Immanent Reasoning*.

<sup>52</sup> Lorenzen, Paul and Kuno Lorenz, *Dialogische Logik* (Darmstadt: Wissenschaftliche Buchgesellschaft, 1978).

<sup>53</sup> Rahman, Iqbal, and Soufi, *Inferences by Parallel Reasoning*.

So, let  $T$  stand here for the general set of *acts of transaction* (which include acts of sale and marriage);  $B$ , as before, for *involving objects unknown of attribute*;  $R_1$  for the property *involving objects unknown of attribute with option to rescind*, as said of acts of Transaction; and  $R_2$  for the property *involving objects unknown of attribute without option to rescind*. Thus, the original proposition and the substitute do not share the same meaning, i.e., they *separate* two sets within the set of transactions  $T$ , namely:

- the set of those transactions with option to rescind (including acts of selling) involving objects unknown of attribute;
- the set of those transactions without this option to rescind (including acts of marriage) involving objects unknown of attribute.

Accordingly, **X** concludes that  $R_1$  and  $R_2$  yield different rulings. Whereas the former is legally not allowed, the latter is:

**X**      *'illa(fst(x)): ~allowed(fst(x))*     $(x: (\exists z: \{y: T \mid d(y): R_1(y)\}) B(z) )$

**X**      *'illa(fst(x)): allowed(fst(x))*     $(x: \{ \exists z: \{y: T \mid d(y): R_2(y)\} \mid B(z) \})$

Note that at this point of the debate **Y** might either concede or demand evidence for the latter assertions; and providing such evidence might commit **X** to finding cases in the sources, and thus providing rival occasioning factors.

### 5.3.2 Dialogical Meaning Explanation of *Kasr* Type 2

In relation to our paradigmatic example of removal, let us recall that an original, obligatory act of ritual worship, the *aṣl* of *wuḍūʿ*, claimed not to require intent (*niyya*), was confronted by the case of a “substitute” act, the *badal* of *tayammum*, which in fact *does* require intent. This proved that whether something is a *badal* or not has no efficiency in relation to requiring intent.

#### Valid *Kasr* Type 2

As noted, it follows from the logical analysis of claims constituted by a compound antecedent, said to occasion a ruling, that it is the disjunctive distribution of that ruling over the

claimed occasioning factor that makes challenges by *removal* even possible. The explicit claim has the form:

$$\mathbf{X} \quad \textit{‘illa}(x,y): \textit{hukm}(\textit{fst}(x,y)) \vee \textit{hukm}(\textit{snd}(x,y)) \quad [(x, y): (A \wedge B)]$$

This shapes the dialogical interaction. Q must (1) attempt to show that property *B* (not being a *badal*) is not efficient—despite being paired with *A* (being a *sabab*)—and that it can therefore be removed, **and** then (2) proceed to build an inconsistency-proving case against *A*. The disjunctive distribution requires that both kinds of challenges be carried out. R can escape with a valid response, supposing R can restore the removed disjunct by reviving the possibility that property *B* (not being a *badal*) can have efficiency.

## 6 Overall Conclusions and Work Ahead

The current study is only a first step towards a larger investigation of *kasr*, which will encompass both a comparison between *kasr* and other dialectical objections and analyses of critical discourses from later Muslim legal theorists and dialecticians. In fact, such work should contribute to large-scale research on the notion of illegitimate moves in Islamic dialectics. Here we will advance, very briefly, some remarks—in fact, working hypotheses—that might motivate or lead such research.

Fallacious—that is, illegitimate—arguments in a specific field of knowledge are arguments that contravene procedural or meaning-setting rules that prescribe what should count as a sound argument in relation to the conceptual framework of that field. From this point of view, the so-called deontic paradoxes or puzzles that we have referred to as extrapolation fallacies find a solution—not by adjusting *ad hoc* some inference rule, but by observing the reasoning patterns for linking concepts established in a specific field.

The setting of what is correct and not correct is implemented by a notion that seems to prefigure what we nowadays in proof-theory call “admissibility.” The Aristotelian theory of the syllogism formulates a general form of this notion. And one way to interpret the Medieval focus on classifying fallacies relates to the task of identifying one or more admissibility criteria for shaping reasoning patterns. In this light, the Islamic tradition seems to have distinguished

admissibility criteria at work in argumentation theory from criteria closer to the perspective on syllogistic fallacies.

The key point, so we conjecture, is that, whereas syllogism-based criteria of admissibility focus on the strategic point of view (namely, that what is admissible is what constitutes optimal moves from the point of view of logical validity), dialectic-based based criteria of admissibility focus on the play-level. In other words, what is admissible is what allows one to carry out dialectical interactions according to rules that establish the meaning relevant to the concepts at work in a debate. An inadmissible move is what contravenes the rules on how to debate, not on how to win the debate in regard to any possible move of the contender. Note that this leaves room for valid moves, that is admissible moves, which are not always successful.

Islamic texts wisely include discussions on both dialectical illegitimate moves (with a focus on the play level) and syllogistic-based illegitimate moves (with a focus on the strategy level of logic). Both points of view are complementary: plays are what strategies are made of. What we should not do is impose a purportedly universal, abstract, strategic admissibility criteria on debates wherein concrete interlocutors dispute content constituted and agreed upon during the interaction. And this, we dare to suggest, is what the dialectical stance in Islamic argumentation theory is all about.

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