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# The Good Cobbler, the Good Poet, and the Unseen Bride

Shahid Rahman & Walter E. Young

## 1 ARISTOTLE AND AVICENNA ON GOOD COBBLERS AND GOOD POETS

### 1.1 The Lack of Expressivity of First-Order Logic

Aristotle (*Peri Hermeneias*, chapter XI, 20b35-36 and 21a14-15, *Sophistical Refutations* 20, 177b14-15) pointed out, and rightly so, that from:

*Some x is good and a cobbler,*

one cannot, without any proviso, infer:

*Some x is a good cobbler.*

In standard classical contemporary logic, Aristotle's remark amounts to the fact that from:

$\exists x (Good(x) \wedge Cobbler(x))$

one cannot infer:

$\exists x Good-Cobbler(x).$

Notably, the latter expression is also unproductive, since it does not support any further inferences upheld by the original *Some cobbler is good (as a cobbler)*.

Clearly, the point is that:

- 1) On the one hand, in:

$\exists x (Good(x) \wedge Cobbler(x))$

“*Good*” is left totally unspecified. *Good* at what? *Good* human? *Good* runner? In fact, this logical form is compatible with *x*'s being both a *good runner* (or *good cook*, etc.) and a *cobbler* (possibly a bad cobbler).

- 2) On the other hand, the formalization:

$\exists x Good-Cobbler(x)$

does not support the inference that:

Given *some x is a good-cobbler*, it follows that *x is both a cobbler and good as a cobbler*.

In sum, the purported inference:

$\exists x (Good(x) \wedge Cobbler(x)) \vdash \exists x Good-Cobbler(x)$

is not only deductively invalid, but:

- The premise and conclusion contravene the patent meaning which is conveyed by the natural language expression, and which allows one to infer from: *some x is a good cobbler*, that *x* is both *a cobbler* and *good as a cobbler*.

Ibn Sīnā’s example,<sup>1</sup> on the other hand, explicitly uses a singular term, which accentuates both the lack of expressivity of standard contemporary first-order logic for analysing the fallacious inference by composition, and the fact that the premisses might lead to a conclusion which is **logically valid but not the targeted one**.

- (1) Imru’ al-Qays is good.  
 (2) Imru’ al-Qays is a poet.  
 -----  
 (3) Imru’ al-Qays is a good poet.

Ibn Sīnā then observes that concluding (3) from the conjunction of (1) and (2) yields a fallacious argument.

Notice, however, that a naïve first-order translation, with “*a*” standing for Imru’ al-Qays, yields a **logically valid conclusion**:

- (1) *Good(a)*  
 (2) *Poet(a)*  
 -----  
 (3) *Poet(a) ∧ Good(a)*

In fact, it may even yield the more general conclusion:

There is some *x* called “*a*” (whereby “*a*” stands for *Imru’ al-Qays*), who is a poet **and** who is good:

- (1) *Good(a)*  
 (2) *Poet(a)*  
 -----  
 (3)  $\exists x ((x = a) \wedge Good(x) \wedge Poet(x))$

There is an *x* such that { *x* is called “*a*” } & { *x* is Good (as person, or mathematician, or ...?) } & { *x* is a Poet }

Logically speaking this inference is fine. However, once again the inferred conclusion **does not express the meaning** conveyed by:

*Imru’ al-Qays is a good poet.*

Thus, in such a framework the fallacy results from attempting to infer, from (1) and (2), both

<sup>1</sup> Proposed in the logic part of his *Ishārāt* (1983, Chapter 10.1, pp. 501-502). We owe this reference to Alexander Lamprakis – cf. Rahman&Zarepour (2021, pp. 411-417).

*Good-Poet(a)*

and/or

$\exists x ((x = a) \wedge \text{Good-Poet}(x)).$

This perspective on Aristotle's example indicates that qualifying whether a step is, or is not, deductively illegitimate—at least in the context of the fallacy of composition—depends upon specifying a targeted conclusion and linking it with a set of premises that can either yield, or not yield, a deductively sound inference.<sup>2</sup>

Let us now attempt a formal analysis that renders the meaning of the natural language expression.

## 1.2 Logic in Reverse (RevLog) and the Fallacy of Composition

RevLog's leading idea is to pursue a "reverse" approach to argumentation, seeking to illuminate the principles of sound argumentation through the lens of what goes wrong in fallacious reasoning. So now that we have glimpsed what is wrong in the fallacy of composition, we should determine what the rules allowing sound inferences involving compounds actually look like. In other words, we would like to have our cake and eat it too, i.e., to render the inference rules that allow one to conclude the kind of compounds mentioned above, along with the rules that allow one to infer that *Good Cobblers* are, after all, *good as cobblers*, without falling into the fallacy of composition.

In order to do so we will return to the old Subject-Predicate structure (whereby the former expresses the bearer of attributes and the second what is attributed to the bearer), cast in the language of Per Martin-Löf's *Constructive Type Theory* (CTT), as applied to natural language by Aarne Ranta (1994). Clearly, other formal languages might offer alternative reconstructions. However, we have chosen this application of CTT because of its expressive-power, which seems very close to the grammatical and logical analyses developed in ancient and medieval logic.<sup>3</sup> After a brief discussion, we will show that this approach renders an analysis of the fallacy of composition which comes very close to the later commentary on Aristotle's treatment of the good cobbler.

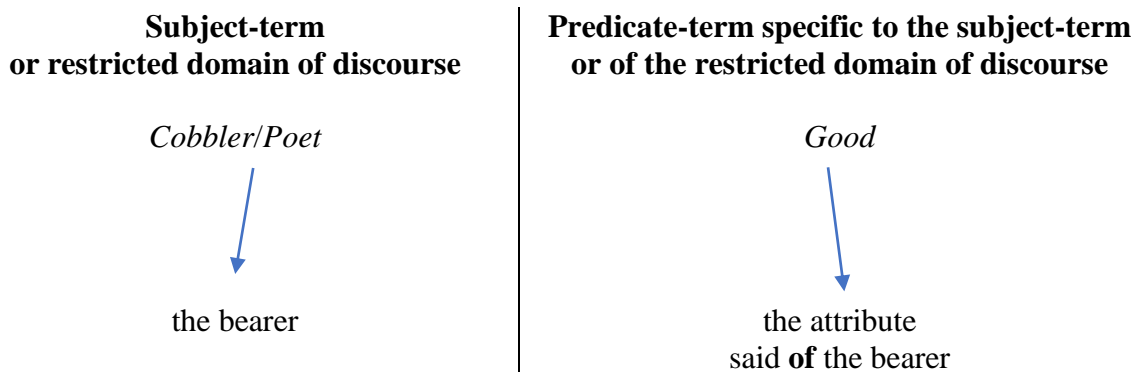
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<sup>2</sup> This seems related to Crubellier's observation (2011, 2014a,b, 2017) that Aristotle's take on the syllogism often adopts the abductive explanatory perspective rather than the deductive. In other words, Aristotle often examines syllogisms from the conclusion to the premisses, rather than from the premisses to the conclusion. From such a perspective, the syllogism is linked to the task of finding a suitable middle term. Cf. Crubellier (2011), McConaughey (2022), section 2.3.3.

<sup>3</sup> A thorough plaidoyer for the fruitfulness of this kind of analysis for Aristotelian logic, when combined with Dialogical Logic, may be found in McConaughey (2021).

### 1.2.1 Back to the Good Old Ways: Attributes and Bearers of Attributes

One way to begin analysing the examples above is to explicitly distinguish the bearers of attributes from the attributes predicated of them.



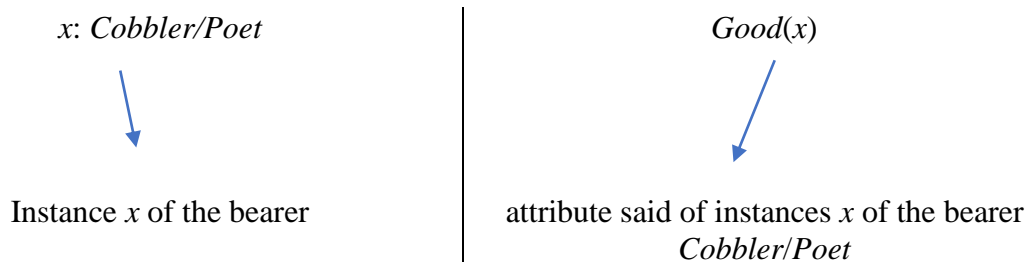
The point here is that **the attribute is restricted** to instances of some specific bearer: **it is one of the things that you can say of some specific bearer**. Thus, it is apparent that:

*Good Cobbler/Poet*

means

*Good as a cobbler/poet.*

In other words, those  $x$  of whom it is said that they are good, are instances of the bearer being a cobbler/poet. Thus,  $Good(x)$  assumes that  $x$  is an instance/token of *Cobbler/Poet*, i.e.,  $x: Cobbler/Poet$ .



Notice that  $x: Cobbler$  also admits the following readings (with, of course, similar readings applying to  $x: Poet$ ):

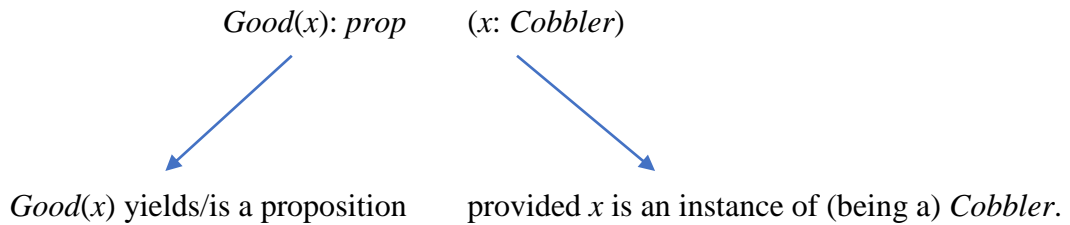
- $x$  **is** a token of the *Cobbler*;
- $x$  **is** an element of the set *Cobbler*;
- $x$  **verifies** that the set/type *Cobbler* is not empty;
- $x$  **is** some instance of the Subject-Term *Cobbler*.

In other words, the logical form of  $Good(x)$  is that of a propositional function defined over the restricted domain of *Cobblers*, *Poets*, *Cooks*, and so on. Thus,  $Good(x)$  yields a proposition if one substitutes for  $x$  instances of a specific bearer to which this attribute of *Good* is said to apply:

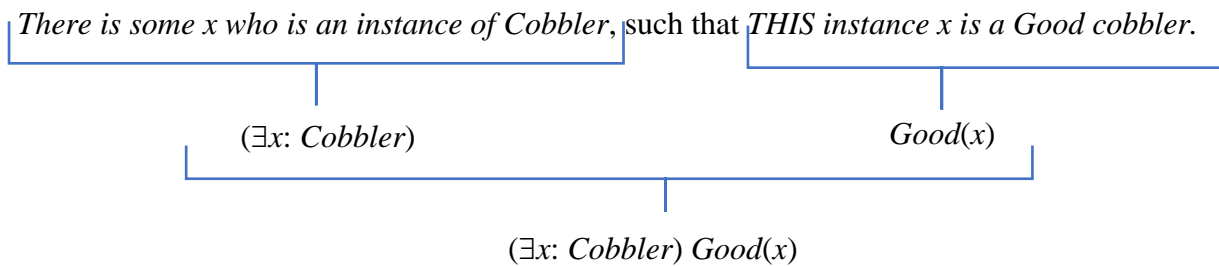
*Good Cobbler*       $Good(x)$  yields a proposition if  $x$  is an instance of (being a) *Cobbler*

*Good Poet*            *Good(x)* yields a proposition if  $x$  is an instance of (being a) *Poet*  
 ...  
*Good B*                *Good(x)* yields a proposition if  $x$  is an instance of (being a) *B*

Formally, we express this with the following well-formation rule for the propositional function *Good(x)*:



Then if we quantify existentially over the domain *Cobbler*, we obtain the following:



Something similar, of course, applies to  $(\exists x: Poet) Good(x)$ .

Notice again that here we do not have the ambiguity of *Good* mentioned above. *Good* is **dependent** upon the set of *Cobblers/Poets*; *Cobbler* and *Good* are **not** two attributes such that the second can be dependent upon another domain, such as *Good* cooks (or some other subdomain of human being). In short, the composition is **not achieved by the conjunction** of *Cobbler(x)* and *Good(x)*, but by expressing *Good* as a compound attribute said of good cobblers/poets.

So far so good, but we would also like to have inference rules which avoid the fallacy of composition and which render both:

1) the synthetic procedure by way of which we obtain, from the premises that  $a$  is a cobbler and that  $a$  is a good cobbler, the fact that at least one  $x$  enjoys the compound attribute of being a *good cobbler*,

and

2) the analytic procedure, by way of which we decompose  $(\exists x: Cobbler) Good(x)$ , into the facts that there is indeed an instance  $x$  of *Cobbler*, and that one of those instances enjoys the attribute of being *good as a cobbler*.

## 1.2.2 Synthetic and Analytic Procedures of Composition

Let us start with the rules that prescribe how to constitute a meaningful expression out of the predicate *Being good* (as a cobbler) and the subject or domain of *Cobblers*. These *well-formation rules* comprise well-formation procedures for the constitution of a proposition which are at the same time syntactic and semantic. According to this approach, an expression is well-formed or meaningful iff it satisfies both semantic and syntactic conditions for the constitution of a proposition (independently of the truth or falsity of the resulting proposition). In other words, and applying the traditional terminology to our main example: The predicate *Being Good (cobbler)* yields a well-formed proposition, such as *Some Cobblers are Good (cobblers)*, iff this proposition has instances of *Cobbler* as its subject.

### Well-Formation Rules

From the above discussion we have learned that  $(\exists x: \text{Cobbler}) \text{Good}(x)$  is well-formed if *Cobbler* is a set/domain of discourse and *Good(x)* is a propositional function defined over that set/domain. Thus:

Formation Rule	Subject-Predicate Reading	Natural-Language (Contemporary) Gloss
<i>Cobbler: set</i>	→	Instances of <i>Cobbler</i> provide the subject of propositions about cobblers.
<i>Good(x): prop</i> $(x: \text{Cobbler})$	→	In order to build a proposition out of the predicate <i>Being Good</i> (as a cobbler), the subject of this proposition must be constituted by instances of <i>Cobbler</i> .
-----		
$(\exists x: \text{Cobbler}) \text{Good}(x): \text{prop}$	→	The proposition <i>Some cobbler is good (as a cobbler)</i> predicates <i>being Good (as a cobbler)</i> of at least one of the cobblers.
		<i>Some cobbler is good (as a cobbler)</i> is a (well-formed) proposition.

Provided this formation rule—which establishes that the existential has been built by combining a domain and a predicate over this domain, such that the predicate is *restricted* to this domain (in our example, that *Good* is said of good cobblers)—we can say:

*a* is a *Cobbler*  
 this *a* is *Good (as a cobbler)*

-----  
 There is at least one instance of *Cobbler* who is a good cobbler

The most delicate step in the inference is the second premise. We need a procedure that combines the first premise with the second one in such a way that it **expresses semantic composition but not conjunction** (which would lead to the fallacy).

The solution appears to be a simple one. We need a *method* that transforms instances of *being a cobbler* into instances of *being good*, in such a way that the composition at stake in the compound is understood as expressing the semantic dependence of the propositional function *Good(x)* upon the set *Cobbler*—i.e., the scope of the meaning of being *Good* is restricted to the set of cobblers. (Linguists

would point out that the predicate *Good* **modifies** the predicate *Cobbler*.) Such a method is a function—that is, a function that takes instances of *being a cobbler* and yields instances of *being good* (at *being a cobbler*):

$a: \text{Cobbler}$

$b(a): \text{Good}(a)$ .

Now, given both premises, we can synthesize them into the compound:

$(\exists x: \text{Cobbler}) \text{Good}(x)$

**Remark:**

Notice that if we wish to render both *Cobbler* and *Good* as attributes of, say, the species of *Human Beings*, but at the same time stress the fact that *Good* is dependent upon *Cobbler* (i.e. that *Good* modifies *Cobbler*), the structure requires the embedding of two existentials expressing that *Cobbler* is said of *Human Beings*, and *Good* is said of those human beings who are cobblers. This notation is a bit more complex, but in fact amounts to an embedding of two existentials:

$(\exists y: (\exists x: \text{Human Being}) \text{Cobbler}(x)) \text{Good}(\text{this-}y)$

There is an instance  $y$  of { being one of those humans  $x$  who are cobblers }, such that { this instance  $y$  is good (as a cobbler) }.

In order to include the anaphoric “*this-y*” within the formalization we need the notation  $(\exists y: \{ x: \text{Human Being} \mid \text{Cobbler}(x) \} \text{Good}(\text{left}(y)))$ , which explicitly expresses that the instance of those of which it is said that they are good as cobblers, is of a human – the left component Human-Cobbler –, who is of course a cobbler.

For the sake of simplicity, we will deploy the simpler structure for now.

Notice that, since  $(\exists x: \text{Human Being}) \text{Cobbler}(x)$  can be rendered as the set of *all those humans who are cobblers*—i.e., the set  $\{ x: \text{Human Being} \mid \text{Cobbler}(x) \}$ —we have:

$(\exists y: \{ x: \text{Human Being} \mid \text{Cobbler}(x) \} \text{Good}(\text{left}(y)))$

There is an instance  $y$  of { those human beings  $x$  who are cobblers }, such that { this  $y$  is good (as a cobbler) }

This yields the searched inference rule, called the “**introduction rule**,” which renders the conditions which must be fulfilled in order to assert  $(\exists x: \text{Cobbler}) \text{Good}(x)$ :

**Synthesis**

**Introduction Rule**

**Natural-Language Gloss**

$a: \text{Cobbler}$



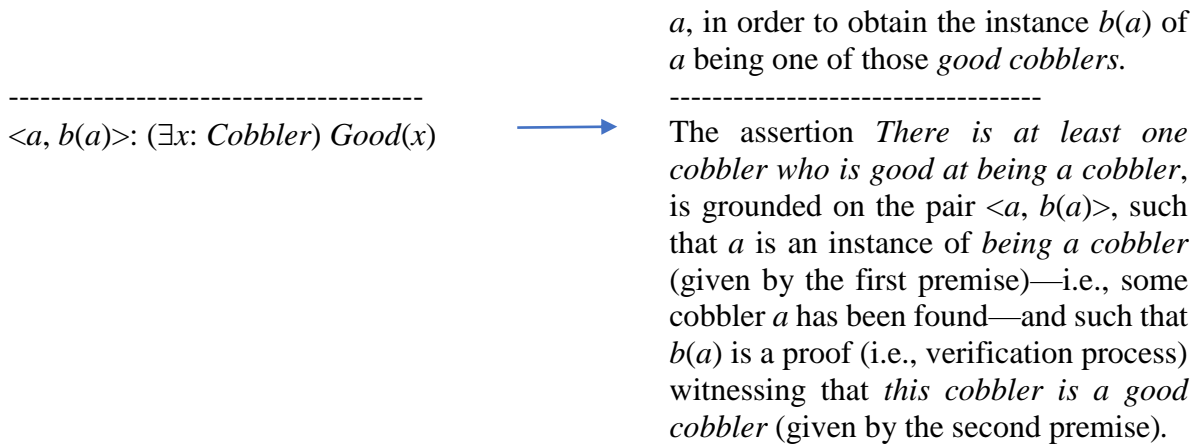
$a$  is a *Cobbler*

$b(a): \text{Good}(a)$



There is a way to associate instances of *being a cobbler* with instances of *being a good cobbler*, by means of the function  $b(x)$ ; and this function can be applied to





Or, in a horizontal notation, with the turnstile “ $\vdash$ ” standing for the inference bar:

$$a: \text{Cobbler}, b(a): \text{Good}(a) \vdash \langle a, b(a) \rangle: (\exists x: \text{Cobbler}) \text{Good}(x)$$

### Analysis

Let us now assume that we have an instance  $c$  of the compound of something *being a good cobbler*—i.e., let us assume  $c$  witnesses  $(\exists x: \text{Cobbler}) \text{Good}(x)$ . Notice that if  $c$  witnesses  $(\exists x: \text{Cobbler}) \text{Good}(x)$ , then  $c$  is an element of the set of *all those cobblers who are Good*.<sup>4</sup> Thus, if  $c$  is an unanalysed compound  $c$ , and we assert that this  $c$  is some instance of the set of *all those cobblers who are good (as cobblers)*, i.e.,

$$c: (\exists x: \text{Cobbler}) \text{Good}(x)$$

then we can infer—by recalling that instances, such as  $c$ , of *being a good cobbler*, are built by the compounds prescribed by the synthesis rule—that:

1. The analysis of the left component of  $c$  should yield some instance, say  $a$ , of *being a cobbler*
2. The analysis of the right component of  $c$  should yield an instance  $b(a)$ , witnessing that cobbler  $a$  is a *good cobbler*.

That is, from:

*There is an instance  $c$  of being a good cobbler,*

we can infer:

1. *Some instance of cobbler can be found,*
2. *There is a way to associate the found cobbler with an instance of being a good cobbler.*

Thus, the analysis procedure yields two decomposition rules: one for the left component of  $c$  and one for the right. These rules are called “**elimination rules.**” They prescribe what inferences we

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<sup>4</sup> Recall that in our remark on notation we indicated that an existential, such as  $(\exists x: \text{Cobbler}) \text{Good}(x)$ , can be rendered as the set  $\{ x: \text{Cobbler} \mid \text{Good}(x) \}$  of all those cobblers who are good as cobblers.

commit to when asserting that there is an instance  $c$  witnessing the existential *At least one cobbler is good as a cobbler*.

In other words,

- **The first elimination rule** allows us to obtain, from the compound  $c$ , its left component—i.e., an instance, let us call it  $c_1$ , of *being a cobbler*.

	→	<b>Natural Language Gloss</b>
$c: (\exists x: \text{Cobbler}) \text{Good}(x)$	→	$c$ is one of these cobblers who are good as cobblers
-----		-----
$\text{left}(c): \text{Cobbler}$		the <i>left component</i> of the instance $c$ of one of those cobblers who are good, is a cobbler
 Whereby:		
$a: \text{Cobbler}$	→	$a$ is a cobbler
$b(a): \text{Good}(a)$	→	$b(a)$ yields an instance of cobbler $a$ being a good cobbler
-----		-----
$\text{left}(c)=\text{left}\langle a, b(a)\rangle=a: \text{Cobbler}$	→	the <i>left component</i> of the instance $c$ of one of these cobblers who are good is cobbler $a$

- **The second elimination rule** allows us to obtain, from the compound  $c$ , its *right* component—i.e., an instance,  $b(a)$ , witnessing the left component to be *a good cobbler*.

	→	<b>Natural Language Gloss</b>
$c: (\exists x: \text{Cobbler}) \text{Good}(x)$	→	$c$ is one of these cobblers who are good as cobblers
-----		-----
$\text{right}(c): \text{Good}(\text{left}(c))$	→	the <i>right component</i> of the instance $c$ (of one of those cobblers who are good) witnesses that its left component (the chosen cobbler) is good (as a cobbler).
 Whereby:		
$a: \text{Cobbler}$	→	$a$ is a cobbler
$b(a): \text{Good}(a)$	→	$b(a)$ yields an instance of $a$ being a good cobbler
-----		-----
$\text{right}(c)=\text{right}(a, b(a))=b(a): \text{Good}(a)$	→	the <i>right component</i> of the instance $c$ (of one of those cobblers who are good) namely $b(a)$ , witnesses that its left component, i.e. $a$ , is good (as a cobbler).

### 1.2.3 The Fallacy of Composition Revisited

The upshot of the preceding considerations is the following rendering of the fallacy of composition:

$$\exists x (Good(x) \wedge Cobbler(x)) \not\equiv (\exists x: Cobbler) Good(x)$$

But in fact this is not well-formed, since the first half quantifies implicitly over some universal domain, while the second half has a restricted domain.

Alternately, we may restrict the first half to the domain of humans, in the premise, and restrict *Good* to *Cobbler*, in the conclusion, as follows:

$$(\exists x: Human\ Being) (Good(x) \wedge Cobbler(x)) \not\equiv (\exists x: Cobbler) Good(x)$$

This rendering, as pointed out to me by Zoe McConaughey, coincides with Catherine Dalimier’s (2007, p. 343, n. 6) comment on the cobbler passage in her French translation of the *Peri Hermeneias*. In brief, she notes that the fallacy lies in the fact that, when saying: “he is good and he is a cobbler,” “good” is predicated of the man (just as cobbler is predicated of that man); but when saying: “he is a good cobbler,” “good” is predicated of the cobbler.<sup>5</sup>

### 1.3 The Fallacy of Composition as an Illegitimate Argumentative Move

In a dialogical setting, fallacious deductive inferences by composition amount to bringing forward illegitimate argumentative moves while justifying an assertion involving *compound attributes*. Recall that, as mentioned above, in an argumentative setting justification proceeds from conclusion to premisses. More precisely, if, when asked to justify their assertion:

*Some x is a good cobbler,*

a respondent **X** responds:

*a is a cobbler and a is Good,*

then these will be deemed *illegitimate responses* to the request of justifying the composition at work in the assertion. In fact, they are illegitimate because the questioner can now retort by showing that these responses are compatible with cobbler *a* being good, say, as a musician, but still bad as a cobbler.

Assertion	Request	Illegitimate Responses	The Antagonist’s Rejoinder
<b>X</b> ! <i>Some x is a good cobbler</i>	$Y ?_L$ <hr/> $Y ?_R$	$X ! Cobbler(a)$ (X responds: <i>a is a cobbler</i> ) <hr/> $X ! Good(a)$ (X responds: <i>a is good</i> )	$Y ! Cobbler\ a\ is\ a\ good\ musician,\ but\ still\ he\ is\ a\ bad\ cobbler$

<sup>5</sup> Dalimier also provides the reference to the *Sophistical Refutations* 20, 177b14-15.

(X claims: <i>Some x is a good cobbler</i> )	(Questioner Y asks X to justify the left and/or right component of <i>good cobbler</i> )		
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Note that canonical rules prescribing the legitimate responses corresponding to our discussion above can also be declined into rules of *composition or synthesis* and rules of *decomposition or analysis*. The synthesis rules prescribe which moves lead to the justification of X’s assertion *Some x is a good cobbler*. Moreover, if successful, X’s responses should lead to constructing the compound that justifies their initial claim.

SYNTHESIS			
Assertion	Request	Canonical Responses	Final Synthetic Outcome <sup>6</sup>
X ! (∃x: <i>Cobbler</i> ) <i>Good</i> (x) <sup>7</sup>	Y ? <sub>L</sub>  Y ? <sub>R</sub>	X <i>a</i> : <i>Cobbler</i> (X responds: <i>a is a cobbler</i> )  X <i>b</i> ( <i>a</i> ): <i>Good</i> ( <i>a</i> ) (X responds: <i>b</i> ( <i>a</i> ) witnesses cobbler <i>a being a good cobbler</i> )	X < <i>a</i> , <i>b</i> ( <i>a</i> )>: (∃x: <i>Cobbler</i> ) <i>Good</i> (x)  (X justifies the initial claim by means of the compound < <i>a</i> , <i>b</i> ( <i>a</i> )>—constituted by <i>a</i> , which verifies that a cobbler, namely <i>a</i> , has been found—and the verification process <i>b</i> ( <i>a</i> ), witnessing that this <i>a</i> is indeed a <i>good cobbler</i> . The initial justified claim now becomes an assertion.)
(X claims: <i>Some x is a good cobbler</i> )	(Questioner Y asks X to justify the left and/or right component of <i>good cobbler</i> )		

The analysis rules, on the other hand, again prescribe how to decompose a non-articulated compound *c*, which is claimed to witness both the fact that there is at least one cobbler, and the fact that this cobbler is good as a cobbler. Moreover, the questioner can continue their interrogation, requiring X to bring forward concrete instances of each component, by asking questions such as: *Who is the cobbler that you claim gives witness to there being at least one (cobbler)? How do you verify that this cobbler is a good one?*

ANALYSIS (I)		
Assertion	Request	Canonical Responses
X <i>c</i> : (∃x: <i>Cobbler</i> ) <i>Good</i> (x)	Y ? <sub>L</sub>  Y ? <sub>R</sub>	X <i>left</i> ( <i>c</i> ): <i>Cobbler</i> (X responds: the left component of <i>c</i> witnesses that there is indeed a cobbler)

<sup>6</sup> The final synthetic outcome is the result of an argumentative process we call “recapitulation.” The term “recapitulation” was inspired by Michel Crubellier, who pointed out that this was in fact one of the earliest senses of the term *sylogismos*, and represents one of most important phases in ancient Greek dialectic (cf. Kapp [1942, pp. 14–16 & 71], Rahman, et al. [2018, section 7.7.2]). The recapitulation process, when achieved, indicates that X has a winning strategy for justifying the initial assertion (which corresponds to the proof-theoretical notion of having a demonstration).

<sup>7</sup> Note that the exclamation sign will disappear when the instance of the proposition is made explicit in the responses. The point is that, with X ! *A*, the respondent X claims that there is some instance verifying his claim *A*, and, with X *a*: *A*, the respondent X brings forward instance *a*. The initial statement has an exclamation sign because it has no explicit proof object. This must be constructed during the interaction.

(X claims: the compound <i>c</i> justifies <i>Some x is a good cobbler</i> )	(Questioner Y asks X to justify the left and/or right component of <i>good cobbler</i> )	<b>X right(c): Good(left(c))</b> (X responds: the right component of <i>c</i> witnesses that the left component is a good cobbler)
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ANALYSIS (II)		
Assertion	Request	Identifying the Components
<b>X left(c): Cobbler</b> (X claims: the left component of <i>c</i> witnesses that there is indeed a cobbler)	<b>Y ? left(c)</b> (Y asks: who is this left component of <i>c</i> ?)	<b>X left(c)=a: Cobbler</b> (X responds: the left component of <i>c</i> is cobbler <i>a</i> )
<b>X right(c): Good(left(c))</b> (X claims: the right component of <i>c</i> witnesses that the left component is a good cobbler)	<b>Y ? right(c)</b> (Y asks: how do you verify that the left component is a good cobbler?)	<b>X right(c)=b(a): Good(a)</b> (X responds: the right component of <i>c</i> is the verifier <i>b(a)</i> , which witnesses that cobbler <i>a</i> is a good cobbler)

The point of discussing these composition-governing inferential and argumentative rules is to show that such rules are illegitimate (and fallacious) precisely when they do not correspond to the right way of composing the meaning of the dependent attributes prescribed by the well-formation rules. Rules for the well-formation of the existential within a dialogical framework are quite straightforward. In short, the questioner will require the respondent to make the components of *Some x is a Good-Cobbler* explicit. The respondent will then make explicit how being good is dependent upon being a cobbler.

WELL - FORMATION			
Assertion	Request	Responses	Final Formal Outcome
<b>X Some x is a Good-Cobbler: prop</b>	<b>Y ?FL</b>	<b>X Cobbler: set</b> (X responds: <i>Cobbler</i> is a set)	} <b>X (∃x: Cobbler) Good(x): prop</b>
(X claims: <i>Some x is a Good-Cobbler</i> is a well-formed proposition)	<b>Y ?FR</b> (Questioner Y asks respondent X to make explicit each of the existential's components)	<b>X Good(x): prop (x: Cobbler)</b> (X responds: <i>Good(x)</i> is a proposition provided <i>x</i> is some element of the set of cobblers)	
			(X concludes that the existential <i>There is at least one Cobbler who is Good as Cobbler</i> is a well-formed proposition)

Now, through the example of “breaking” (*kasr*), a move developed by medieval Muslim jurist-dialecticians for objecting to incorrectly composed compound causes, we will examine what illegitimate moves can be triggered by reasoning with compounds outside the realm of syllogism or logically valid inferences.

## 2 KASR, OR HOW TO “BREAK” A COMPOUND OCCASIONING FACTOR<sup>8</sup>

<sup>8</sup> This section is a succinct presentation of certain parts of Rahman&Young (2022), adding no further results. The aim, rather, is to provide an example of the analysis of an illegitimate move which involves composition but which does not reduce to the syllogistic form of a fallacy of composition.

## 2.1 Composition in the Argumentation Framework of *Qiyās* (Correlational Inference)

In argumentative frameworks that do not focus (primarily, or at all) on logical inference, illegitimate moves involving composition nevertheless assume the kind of well-formation rules described in the previous section. We cannot expect, however, that in argumentative frameworks with a broader scope than logical validity illicit reasoning involving compound attributes reduces to illicitly inferring in precisely the manner described by Aristotle’s discussion of the fallacy of composition.

The framework of our case study is Islamic juridical dialectic (*jadāl / munāẓara*), which shaped arguments targeting the *legal validity* (or lack thereof) of a norm or action, through the justifications of a proponent, the *Respondent* (R), and the critiques of an opponent, the *Questioner* (Q). An important genus of arguments employed by mainstream Sunni jurists in such a framework is called *qiyās*, or *correlational inference*, and takes the form of an inference by parallel reasoning.<sup>9</sup> More precisely, the aim of *qiyās* is to provide a rational ground for the application of a juridical ruling (*ḥukm*)—including such qualifications as *obligatory* (*wājib*, *fard*, *lāzim*), *forbidden* (*ḥarām*, *maḥẓūr*), *permissible* (*mubāḥ*), *valid* (*ṣaḥīḥ*), and *invalid* (*fāsid*)—to a given case, called the branch-case (*farʿ*), which has not been directly and unequivocally pronounced upon in the primary juridical sources (i.e., the Qurʾān, Sunna [Prophet’s example], and *Ijmāʿ* [consensus]).

The more rigorous, archetypal *qiyās* method—known as *qiyās al-ʿilla*, or “correlational inference of the occasioning factor”—begins with an attempt to determine that property (*wasf*) or set of properties in the root-case (*aṣl*) which constitutes the *ʿilla*, or *occasioning factor* (or *legal cause*, *ratio legis*, etc.), giving rise to its ruling. If it is epistemically certain, or at least probable, that this property occasions the ruling, and it is shared by the branch-case, then we may infer that it is equally productive of that ruling in the branch-case. If, on the other hand, the *ʿilla* cannot be determined, then other (but less epistemically satisfying) forms of *qiyās* may be attempted.<sup>10</sup>

With such focus on correctly determining the *ʿilla* in *qiyās* arguments, Muslim jurists elaborated complex discussions around *ʿilla*-oriented justifications and critiques. This included considerations of the “compound occasioning factor” (*ʿilla murakkaba*), which in effect demonstrated different ways in which arguments involving composition can trigger illegitimate moves when deciding whether or not a given ruling applies to a contended case. Particularly relevant is the dialectical objection called *kasr*, or “breaking,” a move in which Q, by taking apart and disposing of its components, aims to destroy that *compound of properties* which R claimed to be the occasioning factor (*ʿilla*) for a specific juridical ruling.

Generally speaking, *kasr* is when Q objects that one (or some) of the properties constituting R’s claimed compound *ʿilla* exists in another case—the “problem-case of breaking,” or *kasr*-case (*masʿalat al-kasr*)—but with a different and incompatible juridical ruling. As such, *kasr* divides into types and subtypes, for some of which several forms of rejoinders were also considered. Rahman&Young (2022b) studied the two main types, and in the present paper we will make reference to what we identified in that study as Type 1a, along with a rejoinder, as illustrated by al-Shīrāzī.<sup>11</sup> Here, the objection consists in Q’s bringing a parallel compound *ʿilla* that challenges the causal efficiency of R’s claimed compound *ʿilla*. As we will discuss below, Q aims to bring forward a compound counterexample which, though different from R’s, seems (at least at first sight) to preserve

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<sup>9</sup> *Qiyās* is often translated as *analogy*, though, strictly speaking, most conceptions of analogy correspond only to the weakest type of *qiyās*, “correlational inference of resemblance” (*qiyās al-shabah*), deemed to be of the most limited epistemic value.

<sup>10</sup> See the types of *qiyās* detailed in Hassan (1986), Hallaq (1997), and Young (2017).

<sup>11</sup> See *al-Maʿūna*, 246; *Sharḥ al-Lumaʿ*, 893, 898-9.

the same kind of (legal) meaning (*ma'nā*). In other words, it is something more precise than, e.g., merely substituting *good cobbler* with *good runner*, as in the cases above.

## 2.2 *Kasr* and the Unseen Bride

In this example, bear in mind that the occasioning factor (or legal cause, *ratio legis*, etc.) of R's *qiyās* is a "compound occasioning factor" (*'illa murakkaba*) consisting of a pair of properties we have labelled *A* and *B*:

<i>A Disputation on Sale of What the Buyer Has Not Seen</i>	
Questioner (Q)	Respondent (R)
<p><b>1. [Q's Demand for R's Ruling]</b></p> <p>[Though it does not appear in al-Shīrāzī's abbreviated examples, his prescribed procedure dictates that Q would first ask for R's ruling—i.e., solution—to the problem-question (<i>mas'ala</i>) at hand.]</p> <p><b>3. [Q's Demand for R's Proof]</b></p> <p>[Again, though absent from the example, al-Shīrāzī's procedure dictates that Q would now ask for R's proof (<i>dalil</i>).]</p>	<p><b>2. R's Ruling</b></p> <ul style="list-style-type: none"> <li>• "Sale of what the buyer has not seen..." <ul style="list-style-type: none"> <li>○ [= the contended branch-case (<i>far'</i>)]</li> </ul> </li> <li>• "... is not valid." <ul style="list-style-type: none"> <li>○ [= the claimed ruling (<i>hukm</i>)]</li> </ul> </li> </ul> <p><b>4. R's Proof by <i>Qiyās al-'Illa</i></b></p> <ul style="list-style-type: none"> <li>• "Because it [i.e., what the buyer has not seen] is [A] an object of sale (<i>mabī'</i>); and it is [B] unknown of attribute to a contractor at the time of contract..." <ul style="list-style-type: none"> <li>○ [= the claimed compound legal cause (<i>'illa</i>)]</li> </ul> </li> <li>• "...like when the seller merely says 'I sell you a garment.'" <ul style="list-style-type: none"> <li>○ [= the authoritative root-case (<i>aṣl</i>), invalid by consensus (<i>ijmā'</i>)]<sup>12</sup></li> </ul> </li> </ul> <p>[R's "indicant" (<i>dalīl</i>) is thus a "correlational inference of the occasioning factor" (<i>qiyās al-'illa</i>). He argues: the branch-case (<i>far'</i>) has the same legal cause (<i>'illa</i>) as the root-case (<i>aṣl</i>)—"what the buyer has not seen" and some unspecified "garment" are both objects of sale unknown of attribute to the buyer at the time of contract. Therefore, both should have the same ruling (<i>hukm</i>)—i.e., that same legal cause should occasion the same ruling ("not valid") in the branch-case as it does in the root-case (where it is invalid by consensus).]</p> <p><b>5. Q's Objection by <i>Kasr</i></b></p>

<sup>12</sup> See *MF*, s.v. [البيع، المبيع وأحكامه وأحواله، حضور المبيع وغيابه، غياب المبيع]، 9.23.

<ul style="list-style-type: none"> <li>• “This is broken (<i>yankasiru</i>) by the [case of the unseen] bride...” <ul style="list-style-type: none"> <li>○ [= the “problem-case of breaking” (<i>mas’alat al-kasr</i>)]</li> </ul> </li> <li>• “...because she is [A*] an object of marriage (<i>mankūḥa</i>) who is [B] unknown of attribute to a contractor at the time of contract...” <ul style="list-style-type: none"> <li>○ [= the parallel compound legal cause (<i>‘illa</i>), with A* substituted for A]</li> </ul> </li> <li>• “...but [the contract] is valid.” <ul style="list-style-type: none"> <li>○ [= the opposite of R’s claimed ruling (<i>ḥukm</i>)]</li> </ul> </li> </ul> <p>[Q’s objection is Type 1a <i>kasr</i>. He attempts to nullify R’s compound legal cause (<i>‘illa</i>) by replacing one of its properties (<i>A</i>, <i>mabī’</i>: <i>object of sale</i>) with a supposed parallel in meaning (<i>A*</i>, <i>mankūḥa</i>: <i>object of marriage</i>)<sup>13</sup> and showing that the property of primary concern (<i>B</i>: <i>unknown of attribute to a contractor at the time of contract</i>) is thus co-present with the opposite of R’s claimed ruling (<i>ḥukm</i>), in a seemingly parallel case (the <i>mas’alat al-kasr</i>: marriage of an unseen bride).</p>	<p><b>6. R’s Counter-Objection by <i>Farq</i></b></p> <ul style="list-style-type: none"> <li>• “Marriage is not like sale with respect to this ruling, because lack of knowledge (<i>jahāla</i>) has an efficiency in the category of sale which it does not have in marriage.” <ul style="list-style-type: none"> <li>○ [= the counter-objection by disqualifying difference (<i>farq</i>)]</li> </ul> </li> <li>• “If he marries an unseen bride and then sees her, he has no option [to rescind] (<i>khiyār</i>); but if he buys some commodity he hasn’t seen and then sees it, he has the option [to rescind].” <ul style="list-style-type: none"> <li>○ [= demonstration of the disqualifying difference (<i>farq</i>)]</li> </ul> </li> <li>• “Thus it is possible that being unseen does not prevent validity of marriage, while it prevents validity of sale.” <ul style="list-style-type: none"> <li>○ [= the conclusion of R’s rejoinder]</li> </ul> </li> </ul> <p>[R counter-objects, nullifying Q’s “problem-case of breaking” (<i>mas’alat al-kasr</i>) with a charge of disqualifying difference (<i>farq</i>), and then demonstrating that difference: marriage differs from sale with regard to property <i>B</i> because one has the option to rescind with an object of sale having property <i>B</i>, but not so with an object of marriage having property <i>B</i>.]</p>
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In the above sequence, first notice that Q’s objection by *kasr* (move #5) is based on the idea that both objects of sale and objects of marriage are objects of contractual transaction—i.e., they are in the same meaning (*ma’nā*). Thus, according to Q, they should have the same ruling when they share property *B* (“unknown of attribute to a contractor at the time of contract”), but—and this inconsistency

<sup>13</sup> Although the replacing property, (*A\**) *mankūḥa*, is not made explicit in the *Ma’ūna*’s abbreviated text, it is in the *Sharḥ al-Luma’* (893, 898-9).



is at the heart of the *kasr* critique—in fact they do not have the same ruling. R’s counter-objection, however, specifically targeting Q’s *kasr*-case of the unseen bride, demonstrates that his substituting object of marriage (*mankūḥa*) for object of sale (*mabīʿ*) in fact contributes a *kasr*-case marred by a disqualifying difference (*farq*)—i.e., they are not in the same meaning. A commodity whose attributes are unknown to the buyer at the time of a sales contract is relevantly different from a bride whose attributes are unknown to the groom at the time of a marriage contract, as proven by the buyer’s right to rescind and the groom’s lack thereof. Thus the ruling of proscription applies only to selling the unseen object of sale, and not to marrying the unseen bride.

In other words, the efficiency of property *B* (“unknown of attribute to a contractor at the time of contract”) in occasioning a ruling of proscription is not evident outside of apt substitutions of similar sets of objects—i.e., objects in the same *maʿnā*—such as should allow property *B* to play the same causal role in Q’s opposing *kasr*-case as in R’s original, authoritative root-case. Q’s substitution should preserve the *meaning dependences* of the original case even as it disproves R’s juridical ruling’s dependence upon this parallel compound. However, R’s counter-objection by disqualifying difference (*farq*) shows that it does not preserve these meaning dependences. This constitutes a novel approach to illicit moves concerning composition as we have encountered them in previous sections (i.e., the cobbler and the poet). Not only does Type 1a *Kasr* and its unseen bride example occur in a context of determining (legal) causation, but it concerns primarily the *illegitimate substitution* of one of a compound cause’s relevant components.

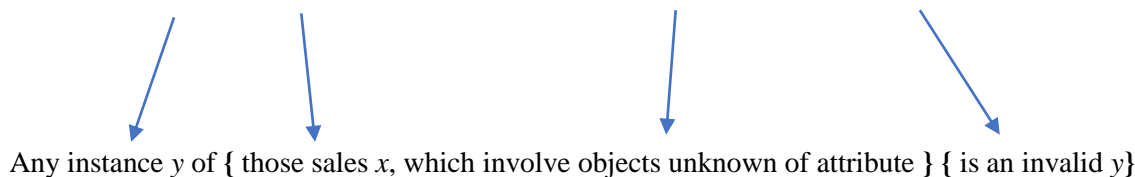
Note the logical structure displays a *double meaning dependence*, namely:

1. In the context of R’s root-case, the ruling *not valid* is linked to a cause composed of two properties: *object of sale* and *attributes unknown to contractor at time of contract* (i.e., *attributes unknown to buyer at time of buying*), and not to any other compound cause.
2. In the context of R’s root-case, the property *attributes unknown to contractor at time of contract* refers only to objects of sale, and not to objects of any other contract (such as marriage).

Thus, we may begin our formalization as follows:

Selling objects of sale, unknown of attribute, is not valid.

$(\forall y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \} \textit{Invalid}(y)$



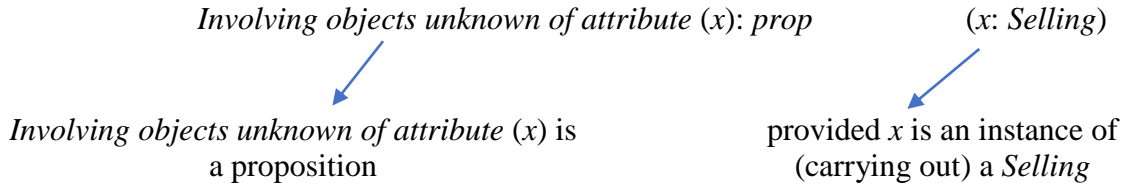
In fact, we can be more precise and indicate that what is invalid is actually the *selling of something* unknown of attribute—that is, what is (contractually) invalid is the *left component* of *y*. Indeed, if:

$\{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute} (x) \}$

is the set of all those sales of objects unknown of attribute, and *y* is an element of this set:

$y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute } (x) \}$

then this already assumes the meaning dependence at work:



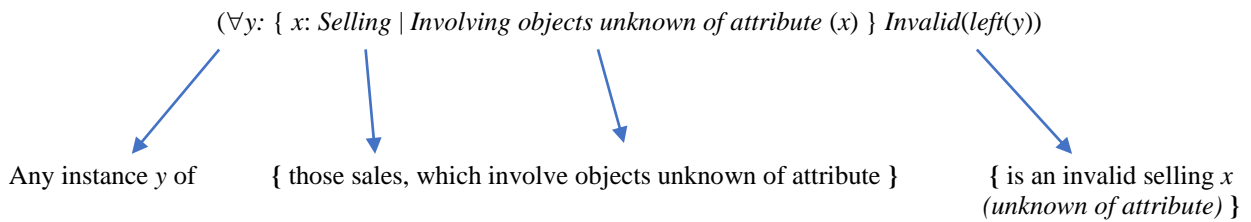
With such being the case, then the left component of  $y$  is the **selling  $x$**  (which is of those sales involving objects unknown of attribute),<sup>14</sup> that is:

$\textit{left}(y)=x: \textit{selling}$ ,

and the right component of  $y$  verifies that this selling  $x$  **involves an object unknown of attribute**:

$\textit{right}(y)=z(x): \textit{Involving objects unknown of attribute } (x)$ .

This leads to the formulation:



The illegitimate use of *kasr* is thus due to the fact that the following substitution does not hold:

$(\forall y: \{ x: \textit{Marrying} \mid \textit{Involving objects unknown of attribute } (x) \} \textit{Invalid}(\textit{left}(y)))$

And why does this not hold? It does not hold due to a disqualifying difference (*farq*), whereby a telling norm holds true for objects of sale but not for objects of marriage—for commodities, but not brides:

### Disqualifying difference (*farq*)

$(\forall y: \{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute } (x) \} \textit{Rescinding-allowed}(\textit{left}(y)))$

$(\forall y: \{ x: \textit{Marrying} \mid \textit{Involving objects unknown of attribute } (x) \} \textit{Rescinding-not-allowed}(\textit{left}(y)))$

## 3 Logic in Reverse (RevLog) and Illegitimate Moves

As mentioned above, the leading idea of “Logic in Reverse” is to study the principles of sound argumentation at work in different traditions, by focussing on what constitutes the criteria for qualifying a move as admissible or not admissible, in the context of certain precise reasoning patterns.

<sup>14</sup> Notice that the set  $\{ x: \textit{Selling} \mid \textit{Involving objects unknown of attribute } (x) \}$  is constituted by all those sales which have been carried out while the object is unknown of attribute. Thus, for every  $x$  in this set, one can say that it is a sale involving objects unknown of attribute.

In the case of the fallacy of composition, from the inferential point of view, the inference rules should yield the *meaning composition* at work. This also holds if the rules for reasoning with compounds are cast in a dialogical framework, whereby the rules for question and answer are built on top of the rules of syntactic and semantic well-formation. The latter establish that the compounds involved assume the *meaning dependence* of one component upon the other.

Within Islamic dialectic, the same governs the criteria of admissible moves with respect to objections against claims involving compound causes. Moves aimed at testing the legal causal efficiency of a compound of attributes cannot simply proceed by replacing that compound with another in which one of the components has been switched out for an arbitrary other. An admissible Type 1a *Kasr* objection must be based on a counterexample (*kasr*-case) that preserves the relevant legal meaning of the original compound.

The compounds of our Islamic case study—namely, marriages involving unseen brides and sales involving unseen commodities—can both trigger inferential fallacies of the kind described by Aristotle’s good cobbler and Avicenna’s good poet. However, the contribution of Type 1a *Kasr* lies not only in its offering a case wherein we find a complex embedding of components, such that the legal decision is also dependent upon this compound. The discussions around *kasr* also—or perhaps even primarily—concern the development of a reasoning pattern based on comparing alternative compounds in relation to the objectives set by the argumentative framework at work (in our case study, determining the compound’s efficiency vis-à-vis the legal invalidity claimed by R for the contended case).

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