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## ► To cite this version:

| Shahid Rahman. Dialogues and Defeasible Reasoning. 2023. halshs-04152642

**HAL Id: halshs-04152642**

**<https://shs.hal.science/halshs-04152642>**

Preprint submitted on 5 Jul 2023

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# Dialogues and Defeasible Reasoning

Shahid Rahman

(draft 29 April 2011)

**Abstract:** The standard logical approaches to model argumentation forms that involve defeasibility stress either the point on the non monotonic property of the turn-style or introduce a specific conditional (or a combination of the two). In a game-theoretic approach this means that either it is a property of winning strategies or it is about the rules that define a new logical constant (particle rules). In the present paper we will make use of an alternative to the approaches mentioned about available to the dialogical approach. The dialogical approach, as developed in the appendix, distinguishes between

- the rules defining the constants (*local meaning*),
- the rules that determine how to play (*global meaning*) and
- the rules that delineate the ways to win (*strategic level*) if winning is possible
- the notion of winning at play level from winning at the strategic level

In this context, defeasible reasoning is about introducing some restrictions on the rules on how to play (global meaning), it consists neither in introducing a new conditional (at the local level) nor in changing the winning-strategies. According to our view, defeasibility principle involves semantic features at the global level. More precisely, according to the dialogical approach, defeasibility amounts to the task of verifying that the proponent's thesis is compatible with the last concession(s) introduced in the play. Notice that the theory of meaning underlying dialogical logic provides a uniform semantics that though it is neither model-theoretic non-proof-theoretic can capture the features of both of those approaches within one frame.

Moreover, this approach allows to distinguish John Pollock's *justification of a belief relative* to the actual defeasible reasoning at a given stage from *warrant* which assumes the set of all possible inferences that can be drawn by an idealized player at a given stage. In the dialogical framework, while *warrant* corresponds to the strategic level (triggered by the extensive form of the play at a given stage) *justification* corresponds to *the play level*. Precisely the dialogical level where the semantics is generated and where Pollock's notion of *multiple assignments* find its natural place.

It should be also pointed out that the dialogical framework provides to the development defeasible reasoning a level beyond the one of play and strategy: the level of *cycles* linked to the notion of dialogue-definiteness. Dialogical definiteness is the dialogical way to deal with Church's theorem on the non-decidability of first-order logic.

The analysis of defeasible reasoning within the dialogical framework should help to build a link between the approaches of default-logic and the argument-based approach initiated by John Pollock [1987], which defines notions like argument, counterargument, attack and defeat, and defines consequence notions in terms of the interaction of arguments for and against certain conclusions.

## Introduction

The term “non-monotonic reasoning” covers a family of formal frameworks devised to capture and represent *defeasible inference*, i.e., that kind of inference in which reasoners draw conclusions tentatively, reserving the right to retract them in the light of further information. Such inferences are called “non-monotonic” because the set of conclusions warranted on the basis of a given knowledge base, given as a set of premises, does not increase (in fact, it can shrink) with the size of the knowledge base itself. This is in contrast to standard logical

frameworks (e.g., classical first-order) logic, whose inferences, being deductively valid, can never be “undone” by new information.<sup>1</sup>

The most known of the examples is build around the poor *Tweety* that is a bird but can not fly.

**Ex. 1:** Typically, it is assumed that birds fly. Thus, if a given animal (namely *Tweety*) is known to be a bird, and nothing else is known, it can be assumed to be able to fly. The default assumption must however be retracted if it is later learned that the considered bird is a penguin.

Another famous example from the literature is the so-called *Nixon diamond*.

**Ex. 2:** Suppose our knowledge base contains (defeasible) information to the effect that a given individual, Nixon, is both a Quaker and a Republican. Quakers, by and large, are pacifists, whereas Republicans, by and large, are not. The question is what defeasible conclusions are warranted on the basis of this body of knowledge, and in particular whether we should infer that Nixon is a pacifist or that he is not pacifist. If there no reason to prefer either conclusion (“Nixon is a pacifist;” “Nixon is not a pacifist”) to the other one, one kind of reasoner (usually called *credulous*, will definitely commit to one or the other. A different kind of reasoner, the *sceptical* reasoner, recognizes that this is a conflict not between hard facts and defeasible inferences, but between two different defeasible inferences. Since the two possible inferences in some sense “cancel out,” the sceptical reasoner will refrain from drawing either one.

A tricky example stems from Hans Rott<sup>2</sup>:

**Ex. 3:** Imagine you are walking along a beach. It is a beautiful night and you are hungry. But you know that at the end of the beach there are two restaurants, one of the them run by Anil, the other by Zorba. Now, you are still far away from the restaurants but you happen to perceive a shimmering light coming from that direction. This shimmering is enough to lead you to the belief that *either Anil’s or Zorba’s restaurant is open* (Thus you assert that under such conditions you are willing to accept the conditional: *If Anil’s restaurant is not open, then Zorba’s is.* ( $\sim p \rightarrow q$ )). However, awhile approaching to the restaurants, you realize *that Anil’s restaurant is open and Zorba’s is closed*. Consequently, it seems sensible to assume that you will withdraw the previous conditional (though perhaps you would not withdraw it if you understand the conditional not as indicative but as subjunctive: *If Anil’s restaurant had not open, then Zorba’s would*). If we disallow the use of explosion the thesis will be no longer defensible in relation to the new information. Another way to analyse this failure is to involve possible conclusions of the thesis. Indeed, given the premise  $p \vee q$  it is not only possible to conclude  $\sim p \rightarrow q$  but also to infer  $q$  from that conclusion (i.e.,  $\sim p \rightarrow q$ ), but the new premise  $\sim q$  is apparently incompatible with the latter inference ( $q$ ).

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<sup>1</sup> Cf ; G. Aldo Antonelli Non-monotonic Logic. Entry at the Stanford Encyclpaedia of Philosophy 2010.

<sup>2</sup> The example is in fact a slight variation of one of Hans Rott ([1989]).

Interesting is the fact that while rejecting a logical justification of empirical induction already David Hume point out that conclusions drawn by logical reasoning (what he calls: *conclusions formed by reason* to be contrasted by conclusions stemming *from experience*) remains unchanged if new information is added.<sup>3</sup>

Actually this remark of Hume could be used to launch an attack on non-monotonic reasoning: If we give up monotony, don't we blur the distinction between inductive and deductive reasoning? The discussion is complex and involves the further question if non-monotonic reasoning is logic after all. We will not address this question here. We will instead start with exploring the possibilities to provide a analysis in a framework for logic (namely, the dialogical framework) where it's underlying semantics is linked to inferential forms of argumentation.

The analysis of defeasible reasoning within the dialogical framework should help to build a link between the approaches of default-logic and the argument-based approach initiated by John Pollock [1987], which defines notions like argument, counterargument, attack and defeat, and defines consequence notions in terms of the interaction of arguments for and against certain conclusions.<sup>4</sup>

It should be mentioned that J. Ph. Hoepelman and A.J.M. Hoof already initiated by 1988 the study of the use of a failure-operator in what they call *dialogical tableaux systems*. However, it is not clear if their failure operator also triggers a switch in relation to who of both players has to play under the restriction on atomic formulae imposed by the formal rule – this feature that is characteristic of the dialogical approach is not discussed. Be that as it may these tableaux systems, that they call dialogical, are in fact closer to Beth-tableaux than to the dialogical approach, since they are essentially player dependent: there is a particular rule for **P** and one different rule for **O**, both for the operator and the default conditional. There is something deeply unsatisfactory with this approach, the asymmetry is so strong that the negation of such a conditional on **O**- side, does not really carry over to the other side after **P**'s challenge on it – and vice versa. In other words, the connective on one of the player's side is not even the dual of the other: **P** plays with one connective and **O** with another.

Gildas Nazokou developed in his thesis [2010] a dialogical approach to non-monotonic reasoning argumentation in African traditional law. The work of Nzokou provides important insights on the dialogical view on non-monotonic reasoning and motivated the further developments of the present paper. However, he uses belief-revision operators instead of defeasibility. Moreover, the difference between global and particle rules is not that neat (though it might perhaps be reformulated in that way).

## 1. Defeasibility and the semantics of dialogical logic

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<sup>3</sup> Now it seems evident, that, if this conclusion were formed by reason, it would be as perfect as first, and upon one instance, as after so long a course of experience. Hume [1772], An Enquiry concerning Human Understanding. Oxford : Oxford, Philosophical Texts: chapter 4, paragraph 2, p. 116, 1999.

<sup>4</sup> In fact, John Pollock [1987] developments were based on his earlier work in epistemology, e.g. [1974], and the computer scientist Ronald Loui [1987] The first artificial intelligence paper on argumentation systems was [Loui, 1987]. The argumentation-approach to non-monotonic reasoning has become popular in legal reasoning [Loui *et al.*, 1993, Prakken, 1993, Sartor, 1994, Gordon, 1995, Loui & Norman, 1995, Prakken & Sartor, 1996, Freeman & Farley, 1996, Prakken & Sartor, 1997a, Prakken, 1997, Gordon & Karacapilidis, 1997]. For further developments and an overview see Prakken/Vreeswijk [2001].

The standard logical approaches to model argumentation forms that involve defeasibility stress either the point on the non monotonic property of the turn-style or introduce a specific conditional (or a combination of the two). In a game-theoretic approach this means that either it is a property of winning strategies or it is about the rules that define a new logical constant (particle rules). In the present paper we will make use of an alternative to the approaches mentioned about available to the dialogical approach. The dialogical approach, as developed in the appendix, distinguishes between

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- the rules that determine how to play (*global meaning*) and
- the rules that delineate the ways to win (*strategic level*) if winning is possible
- the notion of winning at the play level from the notion of winning at the strategic level.

In this context, defeasible reasoning is about introducing some restrictions on the rules on how to play (global meaning), it consists neither in introducing a new conditional (at the local level) nor in changing the winning-strategies. According to our view, defeasibility principle involves semantic features at the global level. More precisely, according to the dialogical approach, defeasibility amounts to the task of verifying that the proponent's thesis is compatible with the last concession(s) introduced in the play. Notice that the theory of meaning underlying dialogical logic provides a uniform semantics that though it is neither model-theoretic nor proof-theoretic can capture the features of both of those approaches within one frame.

Moreover, this approach allows to distinguish Pollock's *justification of a belief relative* to the actual defeasible reasoning at a given stage from *warrant* which assumes the set of all possible inferences that can be drawn by an idealized player at a given stage.<sup>5</sup> In the dialogical framework, while *warrant* corresponds to the strategic level (triggered by the extensive form of the play at a given stage) and justification corresponds to *the play level*. Precisely the dialogical level where the semantics is generated and where (as discussed in chapter 4 below) Pollock's notion of *multiple assignments* finds its natural place.

It should be also pointed out that the dialogical framework provides to the development of defeasible reasoning a level beyond the one of play and strategy: the level of *cycles* linked to the notion of *dialogue-definiteness*. Dialogical definiteness is the dialogical way to deal with Church's theorem on the non-decidability of first order logic. .

## 1.1 Defeasibility and the Semantics of dialogical logic I: Particle rules

As already mentioned we will not introduce a particle rule for conditional, however we will make use of two operators, namely the *defendability* operator  $\mathcal{V}$  and the *attackability* operator

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<sup>5</sup> Pollock [2008]. *Reasoning: Studies of Human Inference and its Foundations*, ed. Jonathan Adler and Lance Rips, Cambridge University Press. 451-470

$\mathcal{F}$ . These operators have been developed by Rahman and Rückert [2001] in a different context.<sup>6</sup>

### The operator $\mathcal{V}$

In uttering the formula  $\mathcal{V}_{\mathcal{P}i} \phi$  the argumentation partner **X** is committed to the claim that, given concession(s)  $\mathcal{P}i$  he (**X**) can defend successfully  $\phi$  in a play where he does not play under the restriction of the formal rule. The other argumentation partner **Y** challenges  $\mathcal{V}_{\mathcal{P}i} \phi$  by claiming that under such circumstances  $\phi$  can not be defended. Thus, the challenger of **Y** compels **X** (who stated  $\mathcal{V}_{\mathcal{P}i} \phi$  in the so-called *upper dialogue*) to open a *subdialogue* with concession(s)  $\mathcal{P}i$  where he (**X**) utters  $\phi$  and **Y** challenges it. Now, because of the scope of challenge, *the challenger must play formally*. Graphically:

X: $\mathcal{V}_{\mathcal{P}i} \phi$	<b>Challenge</b>	<b>Defence</b>
	Y: ? $\mathcal{V}_{\mathcal{P}i} \phi$	
	<i>Subdialogue</i>	<i>Subdialogue</i>
	(The challenger must play formally in the subdialogue)	X: $\phi \wedge \mathcal{P}i$ . (The defender chooses the subdialogue)

Notice that the upper dialogue and their subdialogues are sections of just one dialogical play where one of the argumentation partners wins or loses.

Notice also that the particle rules of the operator  $\mathcal{V}$  allow a change in the right to introduce atomic formulae, that is, the Proponent is in this version of dialogical logic the argumentation partner who stated the thesis which motivated the whole dialogue play, not the argumentation partner who plays formally.<sup>7</sup> Thus the formal structural rule has to be reformulated. We will do this later; for our present purposes we will introduce a graphic mark that signalises which of the argumentation partners has to argue formally – let us call this restriction *formal restriction*. We will do this by shading the column of the argumentation partner who plays under the formal restriction. By means of this device both cases of arguing with the operator **V** (with and without changing the formal restriction) can be distinguished.

In order to keep track of different parts of a dialogue play for defeasible argumentation we will distinguish:

**Upper-section  $d_1$  of a play:** that identifies the first -sections of a play where **P** defended a thesis given the initial concession(s)  $\mathcal{P}i$ .

**Subdialogue  $d_{i,n}$  of section  $i$ .** The subdialogue has been triggered by the defending a  $\mathcal{V}$ (or  $\mathcal{F}$ )-operator at section  $i$ .

The initial section (called also the initial dialogue) will be the section 1. It is assumed that, if the play continues after section 1, P wins at this section – otherwise the play stops there.

<sup>6</sup> Rahman introduced these operators in his Habilitationsschrift 1997 and were later on thorough studied by Rahman/Rückert ([2001]) in order to develop a dialogical semantics for connexive logic

<sup>7</sup> From the point of view of modeltheoretic semantics this operator corresponds to satisfiability.

<b>d<sub>i</sub></b>	<b>Y</b>	<b>X</b>
	... $? \mathcal{V}_{\mathcal{P}i}$	$\mathcal{V}_{\mathcal{P}i} \phi$
<b>d<sub>i,m</sub></b>	<b>Y</b>	<b>X</b>
	...	$\phi \wedge \mathcal{P}i$

Case 2:

<b>d<sub>i</sub></b>	<b>X</b>	<b>Y</b>
	... $? \mathcal{V}_{\mathcal{P}i}$	$\mathcal{V}_{\mathcal{P}i} \phi$
<b>d<sub>i,m</sub></b>	<b>X</b>	<b>Y</b>
	...	$\phi \wedge \mathcal{P}i$

### The operator $\mathcal{F}$

The condition free-operator  $\mathcal{F}$  is the dual of  $\mathcal{V}$ . In fact  $\mathcal{V}\phi =: \mathcal{F}\sim\phi$ . If we add conditions  $\mathcal{P}i$  the duality is the following  $\mathcal{V}(\phi \wedge \mathcal{P}i) =: \mathcal{F}(\mathcal{P}i \rightarrow \sim\phi)$ : Thus, in uttering the formula  $\mathcal{F}_{\mathcal{P}i} \phi$  the argumentation partner **X** claims that given concession(s)  $\mathcal{P}i$   $\phi$  can be successfully challenged in a play where he (**X**) does not play under the formal rule. The other argumentation partner **Y** challenges  $\mathcal{F}_{\mathcal{P}i} \phi$  by asserting that there is no condition under which  $\phi$  can be challenged successfully even if he (**X**) concedes  $\mathcal{P}i$ . Thus, the challenge of **Y** compels **X** to open a *subdialogue* where he (**X**) utters  $\sim(\mathcal{P}i \rightarrow \phi)$  and **Y** attacks it conceding  $\mathcal{P}i$ . Again, *the challenger must play formally*:

<b>X: <math>\mathcal{F}_{\mathcal{P}i} \phi</math></b>	<b>Challenge</b>	<b>Defence</b>
	<b>Y: <math>? \mathcal{F}_{\mathcal{P}i}</math></b>	
	<i>Subdialogue</i>	<i>Subdialogue</i>
	<b>Y: <math>\mathcal{P}i \rightarrow \phi</math></b> $\phi$ <b>(Y must defend <math>\phi</math>).</b>  <b>Y must play formally in the subdialogue</b>	<b>X: <math>\sim(\mathcal{P}i \rightarrow \phi)</math></b>  <b>X: <math>\mathcal{P}i</math></b> <b>(X challenges the conditional)</b>  <b>The defender chooses the subdialogue</b>

Again two cases (with and without changing the formal restriction) should be distinguished here:

Case 1:

<b>d<sub>i</sub></b>	<b>Y</b>	<b>X</b>
	$? \mathcal{F}_{\mathcal{P}i}$	$\mathcal{F}_{\mathcal{P}i} \phi$
<b>d<sub>i,m</sub></b>	<b>Y</b>	<b>X</b>
	.	$\sim(\mathcal{P}i \rightarrow \phi)$

$\mathcal{P}i \rightarrow \varphi$ $\varphi$	$\mathcal{P}i$
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Case 2:

<b>d<sub>i</sub></b>	<b>Y</b>	<b>X</b>
...	$\mathcal{F}_{\mathcal{P}i} \varphi$	
<b>d<sub>i,m</sub></b>	<b>Y</b>	<b>X</b>
$\mathcal{P}i \rightarrow \varphi$ $\varphi$	$\sim(\mathcal{P}i \rightarrow \varphi)$ $\mathcal{P}i$	

## 1.2 Defeasibility and the Semantics of Dialogical logic II: Structural Rules

### 1.2.1 Sections of a play:

- Each section represents a part of the play where the Proponent defends the main thesis with **additional** concession(s) in relation to the concession(s) of initial section that starts the whole dialogue play.

- The whole play is unfolded in the following way

- 1 The dialogue starts with a main thesis and a concession, the play continues until the Proponent wins.
- 2 A second section of the dialogue starts with the same main thesis but with the addition of new concession(s), where the Proponent might lose.
- 3 In general, when a player loses on one section, he might start with a different one. In particular, if it is the opponent; he might start a new section (bigger than 1) by uttering:

**O:**  $\mathcal{P}_i^\alpha?$  (to be read: can you show that the thesis  $\alpha$  is compatible with the additional concession  $\mathcal{P}_i?$ )

If it is the Proponent, he might start a new section by uttering:

**P:**  $\mathcal{P}_j^\alpha!$  (to be read: I can show that the thesis  $\alpha$  is (still) compatible with the additional concession  $\mathcal{P}_j$ )

- It the proponent wins the whole play, we say that the thesis in the context of the concessions of the first section is *undefeated*. Otherwise it is *defeated*.
- Sometimes we will call a conclusion with his supportive concession(s) an *inference*.



The operators introduced above will be implemented by structural rules that, for short, establish that if **P** utters an atomic formula; this formula must be compatible with the newest concession(s).

### 1.2.2 S-DF-I: Structural rule for defeasibility I: The formal rule

#### *Changes of the formal restriction:*

At the start of a dialogue **P** plays under the formal restriction. Changes of the formal restriction are regulated solely by the particle rules of  $\mathcal{F}$  and  $\mathcal{V}$ .

#### *Utterance of atomic formulae by the argumentation partner X who plays without the formal restriction:*

The argumentation partner **X** who does not play under the formal restriction in a determinate section/subdialogue may utter an atomic formula in this section/subdialogue whenever needed.

#### *Utterance of atomic formulae by the argumentation partner who plays under the formal restriction:*

The argumentation partner who plays under the formal restriction in a determinate section/subdialogue may utter in this section only atomic formulae which his argumentation partner has already stated in this section/subdialogue.

- Atomic formulae can in general not be challenged. Now; if the thesis is an atomic formula or (as we will see below when we defined a more general kind of plays) if it is a conclusion of the thesis, then they can be challenged by means of a compatibility requirement of the form described above. These kind of challenges trigger a transition to another section of the play where the atomic formula at stake is defended in the context of possibly new concessions.

### 1.2.3 S-DF-II: Structural rule for defeasibility II:

The argumentation partner **X** in a determinate section/subdialogue may attack (in accordance with the particle and other structural rules) any (complex) formula stated by **Y** in this section/subdialogue.

### 1.2.4 S-FD-III: Structural rule for defeasibility III:

#### III.1 Acceptability of Concessions:

At any section different from the initial one, before defending the main thesis, the Proponent has the right to check the acceptability of the new concessions in relation to pre-established criteria. The criteria establish one order of importance in relation to the competing conclusions of new incoming concessions.

It is important to point out that these criteria might be determined by an epistemic and/or socio-cultural background or they might be formal, such as **specificity** (e.g.: *Tweety is a penguin* is more specific than *Tweety is a bird*, and *Tweety is genetically modified penguin* is more specific than the other two)

The right procedure of checking must be determined in relation to these criteria. As pointed out by Prakken/Vreeswijk [2001]:

*Argumentation systems vary in their ground for the evaluation of arguments. In artificial intelligence the specificity principle, which prefers arguments based on the most specific defaults, is by many regarded as very important, but several researchers, e.g. Vreeswijk [1989], Pollock [1995] and Prakken & Sartor [1996], have argued that specificity is not a general principle of common-sense reasoning but just one of the many standards that might or might not be used. Moreover, some have claimed that general, domain-independent principles of defeat do not exist or are very weak, and that information from the semantics of the domain will be the most important way of deciding among competing arguments [Konolige, 1988, Vreeswijk, 1989]. For these reasons several argumentation systems are parametrised by user-provided criteria. Some, e.g. Prakken & Sartor, even argue that the evaluation criteria are debatable, just as the rest of the domain theory is, and that argumentation systems should therefore allow for defeasible arguments.<sup>8</sup>*

In general we will assume the criteria are user-provided. However we will develop our system in consideration of the criterion of specificity.

### III.1a Acceptability of new concessions

#### Specificity:

If the criteria is specificity, and Y added a new concessions  $\mathcal{P}_j$ . X may ask Y to show that *at least one* of the new concessions entails *at least one* of the former ones  $\mathcal{P}_i$  – assuming that they are different.

Accordingly, X might require Y to show that  $\mathcal{P}_j \rightarrow \mathcal{P}_i$  is the case (where “ $\rightarrow$ ” is the material implication). X expresses this requirement by uttering  $\mathcal{F}(\mathcal{P}_j \rightarrow \mathcal{P}_i)$ . X must now defend formally the conditional that results from his own choice of which of the concessions of  $\mathcal{P}_j$  will build the head of the conditional and which of the concessions of  $\mathcal{P}_i$  will tail of the conditional.

- **Remark:** such checking subdialogues are not to overlap with the ones triggered by checking the compatibility of a new concession. In fact, the latter assume already that the new concession has been accepted

Take the old concessions to be  $\mathcal{P}_i = \{Tweety \text{ is a bird } (Bt), \text{ All birds can Fly } (\forall x(Bx \rightarrow Fx))\}$ , and the new concessions to be  $\mathcal{P}_j = \{Tweety \text{ is a penguin } (Pt), \text{ All penguins are birds } (\forall x(Px \rightarrow Bx), \text{ No penguin can fly } (\forall x(Px \rightarrow \neg Fx))\}$ . Thus, after X challenge  $\mathcal{F}(\mathcal{P}_j \rightarrow \mathcal{P}_i)$ , Y will run a subdialogue showing that  $((Pt \wedge \forall x(Px \rightarrow Bx)) \rightarrow Bt)$  is indeed the case

### III.1.b Acceptability of new concessions

#### Explosion:

Some complications might arise if the concessions are inconsistent. If the new concession is contradictory it will entail the second one by explosion.

If the initial concessions are contradictory then one can adopt three strategies, namely:

- a) reject those concessions as fit to trigger defeasible reasoning,

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<sup>8</sup> Prakken/Vreeswijk [2001], chapter 3

- b) accept the explosion and stop the argument there,
- c) adopt some strategy to extract a consistent part (if there is any) of the concession at stake.

A possible one is the following:

The player X playing under the formal rule must utter all of his possible defences and all of those atomic formulae that his antagonist Y uttered to challenge formulae of the thesis. If X did not fulfil these conditions Y might require this fulfilment only after the the corresponding standard play finished.

E.g.: If the pending defensive move involves the utterance of formula  $\alpha$ . Y might state the following interrogative utterance:  $?\alpha$

Accordingly explosion will not work since the conclusion will never be uttered.

### III.1c *Unsolved conflicts*

Assume that we run two different dialogues in relation to one pre-agreed criterion C, such as specificity, for two conflicting inferences, one starting with the first inference of the Nixon diamond-example (mentioned in the introduction) and one starting with the second inference of this diamond. In both dialogues the proponent might reject each of the new concessions. In such a case, we say that the conflict is “unsolved”.

This adds an important feature to our analysis, namely the possibility to compare two different arguments.

We will discuss this point in chapter 4. Until then we will assume that all conflicts are not unsolvable in the sense specified above.

## 2 Play level, Types of Ranks and Strategic Level

One of the main features of the dialogical approach to logic is the distinction between play level and strategy level. Roughly the play level indicates on how to play, but not how to develop a winning strategy. The play level is the level in which you play against a real player with all his skills and limitations. This level seems to be crucial in the study of defeasible reasoning. As already mentioned in the introduction to chapter 1 this seems to correspond to Pollock’s [2008] notion of *justification* of an inference relative to a given stage. Certainly, this does not exclude a strategic level – Pollock’s notion of *ideal warrant* - that defines an appropriate notion of defeasible validity, as we will discuss below.

Accordingly we will introduce a typical dialogical device: ranks. We will make use of two kind of ranks, namely, **S** ranks (section-ranks) and **R**-ranks (repetition ranks). The **S**-rank fixes the number of sections that may define a whole play. Now, in general we assume that this rank is bigger than three and is imposed from outside. In this context this seems to be reasonable. External limits such as time or deadlines usually determine the length of the concatenation of arguments and counterarguments - external ranks for non-monotonic reasoning have been used for the first time by Nzokou [2010].

An important related issue is the free choice of the concessions: We can assume that the choice of concessions is a choice of the players during the play (let us call them *internal*

*choices of concessions*) but limited by the choice within some background finite source (such as in law) or internal and infinite or that there is a dynamic external procedure that provides the concessions (*external choices of concessions*). The background source of concessions could be thought as a kind of oracle that either provides at different stages of the play different concessions or allows to choose some of them.<sup>9</sup> This source is not to be thought as a set of concessions from which the inferences will be drawn but rather as a dynamic system that provides the set of concessions from which inferences will be drawn *if these concessions are acceptable*.

One reasonable idea is to combine external choices of **S**-ranks with a limited internal source. A strategy that I think captures a good deal of defeasible argumentation forms. Indeed, assume that the **S**-rank is  $m$ , and within the range of this scope, and in relation to the (acceptable) concessions provided by the source the proponent always wins. Then; we can distinguish at  $m$  the play level, that is the level at which the proponent won his thesis in relation to the actual moves of the opponent; and the strategic level. The level at which each of the possible relevant moves of the opponent have been considered at  $m$ . Pollock [2008], calls this the *diachronically fixed ideal warrant* (here fixed at level  $m$ ).

Now, as rightly stressed by Pollock [2008] because of Church's theorem on the undecidability of first-order logic, the ideal warrant might never be reached. Here we will briefly introduce an additional notion of rank that allows to distinguish a further level. In dialogical logic we speak of repetition ranks ( **R**-ranks) in the following way:

- After the move that sets the thesis players **O** and **P** each choose a natural number  $n_i$  and  $n_j$  respectively (termed their repetition ranks). Thereafter the players move alternately, each move being a request or an answer.
- In the course of the dialogue, **O** (**P**) may attack or defend any single (token of an) utterance at most  $n_i$  (or  $n_j$ ) times.
- This notion of **R**-rank assumes that the attacks –and defences, are of the same kind. Recall that in our context there might be three kinds of challenge: (i) those determined by the involved particles, (ii) the compatibility-challenges  $\mathcal{P}_t^{\mathcal{A}^?}$ ,  $\mathcal{P}_t^{\mathcal{A}^!}$ , (iii) the acceptability challenge. Thus, if the **R**-rank is 1 one formula might be challenged once with each of the kinds of challenge just mentioned. ,

The **R**-rank, determine the level of *cycles* of plays that can be played at given **S**-rank. Take the **S**-rank to be two, if the concessions involves quantification the number of plays at **S**-rank 2 might be infinite. However at this level we could further distinguish the number of cycles of plays to be played at the fixed **S**-rank 2, where each cycle of plays is characterized by the number  $n_i$  (or  $n_j$ ) of times that a given formula can be challenged or defended at a play the **S**-rank of which is 2.

For a detailed discussion on this issue see chapter 4

- In the following examples and chapters, because of reasons of simplicity, we will not take into consideration the dynamics aspects of the oracle but the result of it. In fact we will assume that with each new **S**-rank a possible new set of concessions will result.

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<sup>9</sup> This is linked to the Hintikka's uses of the oracle in the context of enquiry games (cf. Hintikka/Halonen/Mutanen 1988, 47-90).

The **R**-ranks is the way Church's undecidability of first-order logic manifest itself in the context of dialogical logic. Indeed as pointed out by Tero Tulenheimo, for some non-valid formulae – namely those that only have infinite countermodels - no single cycle of plays suffices for creating a counter-model: one has to look at the totality of cycles, one cycle of plays for every rank chosen by the proponent. More importantly, Tulenheimo remarked that there is no finite **R**-rank which uniformly (the same **R**-rank in any dialogue) suffices to prove the validity of a formula (in classical logic). However, if the thesis is valid there is a finite **R**-rank with the help of which a winning strategy can be built – though it might be different for each such formula.<sup>10</sup> The point is, that this kind of ranks will always produce finite cycles that are *dialogue definite* (one can always establish within the scope of a such a cycle who won). For short, though a winning strategy might require a number infinite of cycles, meaning is based on finite plays.

### Example 1:

Let us display here a very well known example:

We assume that the **S**- and the **R**-rank is 2:

$\mathcal{P}1$ : Pigs do not fly ( $\forall x (Px \rightarrow \sim Fx)$ ). Babe is a pig. ( $Pb$ )

*Thesis*: Babe does not fly

$\mathcal{P}2$ :. Genetically modified pigs can fly ( $\forall x (Px \wedge Gx \rightarrow Fx)$ ). Babe is a genetically modified pig ( $Pb \wedge Gb$ ).

$d_1$	O	P
$\mathcal{P}1.1$ :	$\forall x (Px \rightarrow \sim Fx)$	$\sim Fb$ 0
$\mathcal{P}1.2$ :	$Pb$	
1	$Fb$ 0	
3	$Pb \rightarrow \sim Fb$	$\mathcal{P}1.1$ ?- $\forall x/b$ 2
5	$\sim Fb$	3 $Pb$ 4
7	—	5 $Fb$ 6
7	? $\mathcal{P}2^{\sim Fb}$ 0	$\mathcal{U}(\sim Fb \wedge \mathcal{P}2)$ 8
9	? $\mathcal{V}$ 8	
$d_{1.1}$	O	P
11	? $\wedge 2$ 10	$\sim Fb \wedge \mathcal{P}2$ 10
13	? $\wedge 1$ $\mathcal{P}.2.2$ : 12	$\mathcal{P}.2.1$ : $\forall x (Px \wedge Gx \rightarrow Fx)$ 12
15	? $\wedge 2$ $\mathcal{P}.2.2$ : 12	$\mathcal{P}.2.2$ : $Pb \wedge Gb$ 12
17	?- $\forall x/b$ $\mathcal{P}.2.1$ :	$Pb$ 14
12		$Gb$ 16
19	?- $Pb \wedge Gb$ 20	$Pb \wedge Gb \rightarrow Fb$ 18
21	? $\wedge 1$ 12	$Fb$ 20
23	?- $Fb$ 22	$\sim Fb$ 22
		—

<sup>10</sup> Tero Tulenheimo [2011], personal communication in the context of a discussion of a paper in preparation on the metalogic of dialogical logic by Nicolas Clerbout.

	The proponent loses and thus the inference has been defeated
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Notice the following:

- 1) If, instead of immediately running the subdialogue, the proponent decides to counterattack in the upper dialogue the concession of both concessions, this will not change the end of the dialogue. Indeed, as established by the structural rules, the subdialogue must be developed. This is certainly one consequence that checking compatibility of the thesis (of the Proponent) with each of the concessions (of the Opponent) is the very name of the game.
- 2) If, instead of defending immediately the conditional with move 20 (of the subdialogue), the proponent decides to counterattack 19. This will not change the result. Indeed, the proponent has already conceded the correspondent atomic formulae with moves 14 and 16.
- 3) The Proponent, could try to test the acceptability of the second concession challenging the Opponent to show that the second implies the first. However, this will not lead to a win of the proponent either. Indeed, the opponent will trivially win a dialogue on the relevant conditional:  $Pb \wedge Gb \rightarrow Pb$

### 3 Defeasibility and the Challenge of Intermediate Steps: Towards an Argument-based approach of Dialogues for Defeasible Reasoning

Let us delve into Roth's example mentioned in the introduction , that requires a more general approach to compatibility.

*P1.: Either Anil's or Zorba's restaurant is open ( $p \vee q$ )*

*P2. Anil's restaurant is open ( $p$ ), Zorbas restaurant is not open ( $\sim q$ )*

**Thesis:** *If Anil's restaurant is not open, then Zorba's is . ( $\sim p \rightarrow q$ )*

Interesting is that, if we allow explosion, the thesis is compatible with *P2*. Let us assume that we do not allow explosion using the acceptability device c) mentioned at III.1b, namely

The player X playing under the formal rule must utter all of his possible defences and all of those atomic formulae that his antagonist Y uttered to challenge formulae of the thesis. If X did not fulfil these conditions Y might require this fulfilment only after the the corresponding standard play finished.

In this case, the proponent will loose since, when required to utter *q* (see move 17 below) he will loose. Notice that even with this device, the proponent wins the thesis in the context of

$\mathcal{P}1$ . Indeed, the proponent, can win since the opponent will utter both of the disjuncts in the same play. Certainly the use of this device takes us closer to relevant logic.

$d_1$	O		P	
$\mathcal{P}1$ :	$p \vee q$		$\sim p \rightarrow q$	0
1	$\sim p$	0	$q$	6
3	$p$		$\mathcal{P}1$ $? \vee$	2
	—		1 $p$	4
5	$q$			
7	$? \mathcal{P}2 \sim p \rightarrow q$	0	$\mathcal{V} \sim p \rightarrow q \wedge \mathcal{P}2$	8
9	$? \mathcal{V}$	8		
$d_{1,1}$	O		P	
			$\sim p \rightarrow q \wedge \mathcal{P}2$	10
11	$? \wedge 2$	10	$\mathcal{P}.2.1: p$	12
			$\mathcal{P}.2.2: \sim q$	12
13	$? \wedge 1$	10	$\sim p \rightarrow q$	14
15	$\sim p$	14	$q$	18
	—		$p$	16
17	$? \neg q$	14		
19	$q$	$\mathcal{P}.2.2: 12$	—	
			The proponent loses and thus the inference has been defeated	

- Notice that from the standard point of view the play finished at move 16. However the opponent will use his right to require the proponent to fulfil his commitment to the defence of the conditional uttered at 14.
- Notice too that the first concession can be inferred from second. In other words the proponent will lose if he utters  $\mathcal{H}(\mathcal{P}_2 \rightarrow \mathcal{P}_1)$ .

Another way to analyse this argument is to involve possible conclusions of the thesis. Indeed, given the concession  $p \vee q$  it is not only possible to conclude  $\sim p \rightarrow q$  but also to infer  $q$  from that conclusion (i.e.,  $\sim p \rightarrow q$ ), but the new concession  $\sim q$  is apparently incompatible with the latter inference ( $q$ ).

- An important upshot of this way to see things is that *steps* leading to a thesis can be challenged and defeated by new information. Such kind of challenges build the core of the argumentative approach to defeasible reasoning of John Pollock and Praaken above.

This might motivate the following extension

- 1 The dialogue starts with a main thesis and a concession, the play continues until the Proponent wins.
- 2 A second section of the dialogue starts with the same main thesis but with the addition of new concession(s), where the Proponent might lose.

- 3 In general, when a player loses on one section, he might start with a different one. In particular, if it is the opponent; he might start a new section (bigger than 1) by uttering:

- O:**  $\mathcal{P}_i^\alpha?$  (to be read: can you show that the thesis  $\alpha$  is compatible with the additional concession  $\mathcal{P}_i?$ ) *and/or*
- O:**  $\mathcal{P}_i^\beta?$  (to be read: can you show that the defensive move  $\beta$  involved in the defence of  $\alpha$  is compatible with the additional concession  $\mathcal{P}_i?$ )

If it is the Proponent, he might start a new section by uttering:

- P:**  $\mathcal{P}_j^\alpha!$  (to be read: I can show that the thesis  $\alpha$  is (still) compatible with the additional concession  $\mathcal{P}_j$ ) *and/or*
- P:**  $\mathcal{P}_j^\beta!$  (to be read: I can show that the defensive move  $\beta$  is compatible with the additional concession  $\mathcal{P}_j$ ). This second possibility assumes that the opponent challenged the compatibility of the defensive move in an upper section of the play.

This, form might shorten some plays at the level of the subdialogue. Indeed let us see how it works in relation to Rott's example:

$d_1$	O		P	
$\mathcal{P}1:$	$p \vee q$		$\sim p \rightarrow q$	0
1	$\sim p$	0	$q$	6
3	$p$		$\mathcal{P}1$ $?\vee$	2
	—		1 $p$	4
5	$q$			
7	$?\mathcal{P}2^q$	0	$\forall q \wedge \mathcal{P}2)$	8
9	$?\mathcal{V}$	8		
$d_{1,1}$	O		P	
			$q \wedge \mathcal{P}2$	10
11	$?\wedge 2$	10	$\mathcal{P}.2.1: p$	12
			$\mathcal{P}.2.2: \sim q$	12
13	$?\wedge 1$	10	$q$	14
15	$q$	12	—	
			The proponent loses	

Actually, we hope that this incipient work can be developed further, in order to capture some of the features of the argument-based approach to defeasible reasoning. A further motivation towards this development is linked to the Nixon's diamond problem where sets of arguments are to be taken into consideration.



## 4 Nixon Diamond, Defeasibility and Super-Criteria in the Dialogical Framework

Let us briefly recall once more Nixon's diamond problem.

Suppose our knowledge base contains (defeasible) information to the effect that a given individual, Nixon, is both a Quaker ( $Q_n$ ) and a Republican ( $R_n$ ). Quakers, by and large, are pacifists ( $\forall x(Qx \rightarrow Px)$ ), whereas Republicans, by and large, are not ( $\forall x(Rx \rightarrow \neg Px)$ ).

These allows the following two inferences with contradictory concessions:

- 1) Infer  $P_n$  from  $\forall x(Qx \rightarrow Px)$  and  $Q_n$
- 2) Infer  $\neg P_n$  from  $\forall x(Rx \rightarrow \neg Px)$  and  $R_n$

As mentioned above, if there no reason to prefer either conclusion ("Nixon is a pacifist;" "Nixon is not a pacifist") to the other one, one kind of reasoner (usually called *credulous*, will definitely commit to one or the other at random (or both alternatively) and further explore its consequences. A different kind of reasoner, the *sceptical* reasoner, will refrain from drawing either one. The sceptical approach is often defended by saying that since in an unresolvable conflict no argument is stronger than the other, neither of them can be accepted as justified, while the credulous approach has sometimes been defended by saying that the practical circumstances often require a person to act, whether or not this person has conclusive reasons to decide which act to perform.<sup>11</sup>

Notice neither of the concessions is more specific than the other. Thus, if we start with one the second could be rejected. But this works too if we start the other way cycle. If there were a third concession that defeats one of the conclusions and this concession satisfy some agreed criterion; then a solution is possible since the non defeated will be re-established. For example take that the criterion is specificity and we have a third concession that states that "Quakers who live in Chicago are not pacifists" and "Nixon lives in Chicago" then it is apparent that the inference 2) has been re-established.

But what happens if this is not the case. Well, we can stick to our criterion and hold that since there is no reason to prefer one of them over the other we refrain from both.

Reiter [1980] and Dung [1995] have proposed a solution with help of the notion of possible extensions:

- Given a set  $\Gamma$  of defaults, an extension for  $\Gamma$  represents a set of inferences that can be *reasonably* and *consistently* drawn using defaults from  $\Gamma$

The idea is, roughly, then to endorse a conclusion  $\phi$  if and only if  $\phi$  is contained in *every* extension of the theory. Since this will not be possible in the case of the Nixon diamond nothing can be concluded.

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<sup>11</sup> Prakken and Vreeswijk give a more fine-grained picture of these antagonist positions. Cf. Prakken and Vreeswijk [2001], chapter 4.3.

Pollock [2008] produced some counterexamples and Makinson [1994] observed that sceptical consequences based on the notion of extension fail to be cautiously monotonic.

One first approach would be to follow assign arbitrary one criterion  $C_i$  that yields that one of both inferences, say inference 1, is defeated and proceed further on. If the further development shows that according to this criterion  $C_i$  the inference 1 is defeated after all, then start again with the second inference. The point is that this approach considers each inference individually and not a whole that interacts. The latter motivates Pollock's notion of *multiple assignments*. Pollock's idea is to introduce the idea of partial multiple assignments – that have some echoes of van Fraassen's supervaluations, where an assignment is a partial function defined over the set {defeated, undefeated} to steps of an argument: Instead of studying all the possible inferences, the idea is to study the possible assignments to some steps in the inference. The interesting point is that this super-assignment technique can be performed at the play level. Accordingly,

Given a set of arguments  $X$  and a relation of defeat on  $X$  an argument is *justified* iff it receives the assignment 'defeated' in all status assignments to  $X$ .

We will provide only the first steps towards implementing these ideas by designing two kinds of dialogues, where the first is closer to the credulous and the second closer to the sceptical approach:

Assume that we run two different dialogues in relation to one pre-agreed criterion  $C$ , such as specificity, for two conflicting inferences, one starting with the first inference of the Nixon diamond and one starting with the second inference of this diamond. In both dialogues the proponent might reject each of the new concessions, thus, as pointed out in III.1c (chapter 1.2.4) the conflict is *unsolved*.

Super-Criteria:

- 1) If, according to one pre-agreed criterion the conflict between two inferences is unsolved, then let the proponent choose a criterion for the acceptability of a new concession in order to test the thesis or to some defensive steps of the thesis. If, according to the criterion, the proponent wins then the first inference is said to be undefeated. Otherwise it has been defeated.
- 2) If, according to one pre-agreed criterion the conflict between two inferences is unsolved, then let the opponent choose a criterion for the acceptability of a new concession in order to test the thesis or to some defensive steps of the thesis. If, according to the criterion, the opponent wins then we say that conflict has been defeated. Otherwise it is undefeated.

We assume here the approach that allows to challenge defensive steps developed in the preceding chapter.

## 5 Extensive Games and Winning Strategies

## **5.1 Extensive Games and Winning Strategies**

## **5.2 Winning Strategies**

## **5.3 For and Against Cautious Monotony**

### **Conclusions and the way ahead:**

From the dialogical point of view, defeasible reasoning is about a global rule that establishes the compatibility of a thesis or steps defending the thesis in the context of some concessions. Its semantics is situated at the play-cycle level and this provides its dynamic features. Moreover, according to our view the approach allows to combine the “logical” with the “argumentative” approaches

There is quite a big deal work to do, let us enumerate some pressing ones:

- 1) To develop further the approach in order to incorporate the interaction of different arguments but also between arguments and subarguments
- 2) To compare the advantages and disadvantages of the dialogical approach to multiple assignments with the one of Pollock
- 3) To study the case of self-defeating arguments: that is those arguments where a step of an argument conflict with the conclusion of the same argument. In particular if an how to exempt self-defeating steps from multiple-assignments.
- 4) To re-consider the approach in the context of a limited choice of possible inferences.
- 5) To study the fruitfulness of the present approach in view of some known and new fields of applications such as Nzokou’s challenging proposals mentioned above.

## Appendix: Dialogical Logic

### 1. Introduction:

Dialogical logic (DL) searches both to recover the philosophical and technical link between argumentation and logic (logic as Agon) via the development of a *pragmatist semantics*. This semantics provides the basis for the notion of *formal strategy* by the means of which inference is understood dynamically, i.e., as a kind of a rational interaction of agents.

Developed by Paul Lorenzen and Kuno Lorenz DL was the result of a solution to some of the problems that arouse in Lorenzen's *Operative Logik* (1955).<sup>12</sup> As pointed out by Peter Schroeder-Heister, the insights of Operative logic had lasting consequences in the literature on proof-theory and still deserve attention nowadays.<sup>13</sup> Moreover, the notion of *harmony* formulated by the antirealists and particularly by Dag Prawitz has been influenced by Lorenzen's notions of *admissibility*, *eliminability* and *inversion*. However, on my view, the dialogical tradition is rather a rupture than a continuation of the operative project and it might be confusing to start by linking conceptually both projects together.

Historically speaking, dialogical logic, was suggested at the end of the 1950s by Paul Lorenzen and then worked out by Kuno Lorenz.<sup>14</sup> Inspired by Wittgenstein's *meaning as use* the basic idea of the dialogical approach to logic is that the meaning of the logical constants is given by the norms or rules for their use. This feature of its underlying semantics quite often motivated the dialogical approach to be understood as a *pragmatist semantics*.<sup>15</sup>

The point is that those rules that fix meaning may be of more than one type, and that they determine the kind of reconstruction of an argumentative and/or linguistic practice that a certain kind of language games called dialogues provide. As mentioned above the dialogical approach to logic is not a logic but a semantic rule-based framework where different logics could developed, combined or compared using the same logical and semantic "terminology".

In a dialogue two parties argue about a thesis respecting certain fixed rules. The player that states the thesis is called Proponent (**P**), his rival, who puts into question the thesis is called Opponent (**O**). In its original form, dialogues were designed in such a way that each of the plays end after a finite number of moves with one player winning, while the other loses. Actions or moves in a dialogue are often understood as *utterances*<sup>16</sup> or as speech-acts<sup>17</sup>. The point is that the rules of the dialogue do not operate on expressions or sentences isolated from the act of uttering them.<sup>18</sup> The rules are divided into particle rules or rules for logical constants (*Partikelregeln*) and structural rules (*Rahmenregeln*). The structural rules determine the general course of a dialogue game, whereas the particle rules regulate those moves (or

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<sup>12</sup> Cf. Lorenz 2001.

<sup>13</sup> Schröder-Heister 2008.

<sup>14</sup> The main original papers are collected in Lorenzen/Lorenz 1978. A detailed account of recent developments can be found in Felscher 1985, Keiff 2004, Keiff 2007, Rahman 2009, Rahman/Keiff 2004, Rahman/Clerbout/Keiff 2009, Keiff 2009, Fiutek/Rückert/Rahman 2010, Rahman/Tulenheimo 2006, Rückert 2001, Rückert 2007. For a textbook presentation (in French), see Fontaine/Redmond 2008.

<sup>15</sup> Quite often it has said that dialogical logics has a *pragmatic* approach to meaning. I concede that the terminology might be misleading and induce one to think that the theory of meaning involved in dialogic is not semantics at all. Helge Rückert proposes the more appropriate formulation *pragmatistische Semantik (pragmatist semantics)*.

<sup>16</sup> Cf. Rahman/Rückert 2001, 111 and Rückert 2001, chapter 1.2.

<sup>17</sup> Cf. Keiff 2007.

<sup>18</sup> Tulenheimo 2010.

utterances) that are requests (to the moves of a rival) and those moves that are answers (to the requests).

Crucial for the dialogical approach, and that distinguishes it from all other approaches are the following points (that will motivate some discussion further on)

1. The distinction between local (rules for logical constants) and global meaning (included in the structural rules) – it is not the difference between introduction-elimination rules and structural rules.
2. The player independence of local meaning (recall that in a tableaux system, for example, the meaning of the logical constants is dependent on the sides: True-rules and False-Rules)
3. The distinction between the play level (local winning or winning of a play) and the strategic level (global winning; or existence of a winning strategy) – the notion of play does not correspond neither to winning in a model nor to a branch in branch proof\_tree)
4. the notion of formal play and strategy (this does not correspond to true in any model, but true in a play where the proponent does not know about the justification of the atomic formulae)

## 2. Dialogical Logic and Meaning

### 2.1 Local Meaning

#### Particle rules:

In dialogical logic, the particle rules are said to state the *local semantics*: what is at stake is only the request and the answer corresponding to the utterance of a given logical constant, rather than the whole context where the logical constant is embedded.

- The standard terminology makes use of the terms *challenge* or *attack* and *defence*.<sup>19</sup> However let me point out that at the local level (the level of the particle rules) this terminology should be devoid of strategic underpinning.
- *Declarative utterances* involve the use of formulae, *interrogative utterances* do not involve the use of formulae

The following table displays the particle rules, where X and Y stand for any of the players **O** or **P**:

$\vee, \wedge, \rightarrow, \neg, \forall, \exists$	Challenge	Defence
<b>X:</b> $\alpha \vee \beta$	<b>Y:</b> $?\neg\vee$	<b>X:</b> $\alpha$ or <b>X:</b> $\beta$ ( <b>X</b> chooses)

<sup>19</sup> See Keiff 2007, Rahman/Clerbout/Keiff 2009. Tero Tulenheimo pointed out that this might lead the reader to think that already at the local level there are strategic features and that this contravenes a crucial feature of the dialogical framework. Indeed, Laurent Keiff [2007] introduced the terminology *requests* and *answers*. However the dialogical vocabulary has been established with the former choice and it would be perhaps confusing to change it once more.

$X: \alpha \wedge \beta$	$Y: ? \wedge 1$ or $Y: ? \wedge 2$ ( $Y$ chooses)	$X: \alpha$ <i>respectively</i> $X: \beta$
$X: \alpha \rightarrow \beta$	$Y: \alpha$ ( $Y$ challenges by uttering $\alpha$ and requesting $B$ )	$X: B$
$X: \neg \alpha$	$Y: \alpha$	— (no defence available)
$X: \forall x \alpha$	$Y: ? - \forall x/k$ ( $Y$ chooses)	$X: \alpha [x/k]$
$X: \exists x \alpha$	$Y: ? \exists$	$X: \alpha [x/k]$ ( $X$ chooses)

In the diagram,  $A[x/k]$  stands for the result of substituting the constant  $k$  for every occurrence of the variable  $x$  in the formula  $A$ .

One interesting way to look at the local meaning is as rendering an abstract view (on the semantics of the logical constant) that distinguishes between the following types of actions:

- a) Choice of declarative utterances (=: disjunction and conjunction).
- b) Choice of interrogative utterances involving individual constants (=: quantifiers).
- c) Switch of the roles of defender and challenger (=: conditional and negation). As we will discuss later on we might draw a distinction between the switches involved in the local meaning of negation and the conditional).

Let us briefly mention two crucial issues

- **Player independence:** The particle rules are symmetric in the sense that they are player independent – that is why they are formulated with the help of variables for players. Compare with the rules of tableaux or sequent calculus that are asymmetric: one set of rules for the *true*(left)-side other set of rules for the *false*(right)-side. The symmetry of the particle rules provides, the means to get rid of tonk-like-operators.
- **Sub-formula property:** If the local meaning of a particle # occurring in  $\phi$  involves declarative utterances, these utterances must be constituted by sub-formulae of  $\phi$ .<sup>20</sup>

## 2.2 Global Meaning

**Structural rules:**

(SR 0) (*starting rule*):

The initial formula is uttered by **P** (if possible). It provides the topic of the argumentation. Moves are alternately uttered by **P** and **O**. Each move that follows the initial formula is either a request or an answer.

**Comment:** The proviso *if possible* relates to the utterance of atomic formulae. See formal rule (SR 2) below.

(SR 1) (*no delaying tactics rule*):

<sup>20</sup> This has been pointed out by Laurent Keiff and by Helge Rückert in several communications.

Both **P** and **O** may only make moves that change the situation.

**Comments:** This rule should assure that plays are finite (though there might be an infinite number of them). There are several formulations of it with different advantages and disadvantages. The original formulation of Lorenz made use of ranks; other devices introduced explicit restrictions on repetitions. Ranks seem to be more compatible with the general aim of the dialogical approach of distinguishing between the play level and the strategic level. Other non-repetition rules seem to presuppose the strategic level. One disadvantage of the use of ranks is that they make metalogical proofs quite complicated. Let us describe here the rule that implements the use of ranks.

- After the move that sets the thesis players **O** and **P** each choose a natural number  $n$  and  $m$  respectively (termed their *repetition ranks*). Thereafter the players move alternately, each move being a request or an answer.
- In the course of the dialogue, **O** (**P**) may attack or defend any single (token of an) utterance at most  $n$  (or  $m$ ) times.

(SR 2) (*formal rule*):<sup>21</sup>

**P** may not utter an atomic formula unless **O** uttered it first. Atomic formulae can not be challenged.

The dialogical framework is flexible enough to define the so-called *material dialogues*, that assume that atomic formulae have a fixed truth-value:

(SR \*2) (*rule for material dialogues*):

Only atomic formulae standing for true propositions may be uttered. Atomic formulae standing for false propositions can not be uttered.

(SR 3) (*winning rule*):

X wins iff it is Y's turn but he cannot move (either challenge or defend).

### Global meaning

These rules determine the meaning of a formula where a particle occurs as a main operator in every possible play.

(SR 4i) (*intuitionist rule*):<sup>22</sup>

In any move, each player may challenge a (complex) formula uttered by his partner or he may defend himself against the last challenge that has not yet been defended.

or

(SR 4c) (*classical rule*):

In any move, each player may challenge a (complex) formula uttered by his partner or he may defend himself against any challenge (including those challenges that have already been defended once).

<sup>21</sup> See a discussion of this rule below in the commentaries about the dialogical notion of validity

<sup>22</sup> In the standard literature on dialogues, there is an asymmetric version of the intuitionist rule, called E-rule since Felscher [1985]. For a discussion of this see appendices 1 and 2.

- Notice that the dialogical framework offers a fine-grained answer to the question: Are intuitionist and classical negation the same negations? Namely: The particle rules are the same but it is the global meaning that changes.

In the dialogical approach validity is defined via the notion of *winning strategy*, where winning strategy for X means that for any choice of moves by Y, X has at least one possible move at his disposal such that he (X) wins:

*Validity (definition):*

A formula is valid in a certain dialogical system iff **P** has a formal winning strategy for this formula.

Thus,

- A is classically valid if there is a winning strategy for **P** in the formal dialogue Dc(A).
- A is intuitionistically valid if there is a winning strategy for **P** in the formal dialogue Dint(A).

### Examples:

In the following examples, the outer columns indicate the numerical label of the move, the inner columns state the number of a move targeted by an attack. Expressions are not listed following the order of the moves, but writing the defence on the same line as the corresponding attack, thus showing when a round is closed. Recall, from the particle rules, that the sign “—” signalises that there is no defence against the attack on a negation.

For the sake of a simpler notation we will not record in the dialogue the rank choices but assume the uniform rank **O**: n=1 **P**: m=2:

#### Ex. 1: Classical and intuitionistic rules

In the following dialogue played with classical structural rules **P**’ move 4 answers **O**’s challenge in move 1, since **P**, according to the classical rule, is allowed to defend (once more) himself from the challenge in move 1. **P** states his defence in move 4 though, actually **O** did not repeat his challenge – we signalise this fact by inscribing the not repeated challenge between square brackets.

<b>O</b>			<b>P</b>		
				$p \vee \neg p$	0
1	$?_{\vee}$	0		$\neg p$	2
3	$p$	2		—	
[1]	[ $?_{\vee}$ ]	[0]		$p$	4

Classical rules. **P** wins.

In the dialogue displayed below about the same thesis as before, **O** wins according to the intuitionistic structural rules because, after the challenger’s last attack in move 3, the intuitionist structural rule forbids **P** to defend himself (once more) from the challenge in move 1.

<b>O</b>			<b>P</b>		
				$p \vee \neg p$	0
1	$?_{\vee}$	0		$\neg p$	2
3	$p$	2		—	

Intuitionist rules. **O** wins.



## Comments on the dialogical notion of validity:

Helge Rückert (2011) pointed out, and rightly so, that the formal rule triggers a novel notion of validity.<sup>23</sup> Validity, is not being understood as being true in every model, but as *having a winning strategy independently of any model* or more generally independently of any *material* grounding claim (such as truth or justification). The copy-cat strategy implicit in the formal rule is not copy cat of groundings but copy-cat of declarative utterances involving atomic formulae. Moreover one should add that there is the notion of *formal play*, that does not seem to correspond to nothing in model-theoretic approach: a formal play is not playing in a model.

In fact, one could see the formal rule as process the first stage of which starts with what Laurent Keiff called *contentious* dialogues.<sup>24</sup> Contentious dialogues are dialogues where a player X utters one or more atomic formulae that are dependent upon a given ground and X is not prepared to put this ground into question – one can think of it as a claim of having some kind of ground (or a claim of truth) for it. Moreover the antagonist is willing to concede this ground for the sake of the argument.<sup>25</sup> Now, if we would like to avoid to have the result that an atomic formula is true by the only reason that the player X stated it – that is, if we want to find a way out of contentious dialogues, then there are two possible ways:

- either we accept some principle of grounding external to the dialogue itself (and thus external to the interaction of the players)  
or
- we look for a player principle of grounding that is internal to the dialogue and dependent on the interaction of the players.

The first way leads to material dialogues the second to formal ones

If we are willing to accept something like *material truth*, then we can think that the grounds upon which the atomic formulae depend are facts of the world and a grounded atomic formulae is a way to say that it is true. However, something more general might be thought too, such as true in virtue of some player independent ground. This is the basis on which the rule for material dialogues has been formulated

Rückert pointed out that the formal rule establishes a kind of game where one of the players must play without knowing what the antagonist's justifications of the atomic formulae are. Thus, according to this view, the passage to formal dialogues relates to the switch to some kind of games with incomplete information. Now, if the ultimate grounds of a dialogical thesis are atomic formulae and if this is implemented by the use of a formal rule, then the dialogues are in this sense necessarily asymmetric. Indeed, if both contenders were restricted by the formal rule no atomic formula can ever be uttered. Thus, we implement the formal rule by designing one player, called the *proponent*, whose utterances of atomic formulae are, at least, at the start of the dialogue restricted by this rule.

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<sup>23</sup> Talk at the workshop Proofs and Dialogues, Tübingen, Wilhelm-Schickard Institut für Informatik, 25-27; February 2011/.

<sup>24</sup> Cf. Clerbout/Keiff/Rahman 2009 and in Keiff/Rahman 2010.

<sup>25</sup> Cf. Keiff/Rahman 2010 (156-157), where this is linked to some specific passages of Plato's *Gorgias* (472b-c).

Apparently, the formal rule introduces an asymmetry in relation to the commitments of **O** and **P** particularly so in the case of the utterance of the conditional. Indeed, if **O** utters a conditional, then **P**'s commits him to a series of moves that must at the end be based on atomic moves of **O**. If it is **O** that challenges a conditional no such commitment will be triggered. But it would be a mistake to draw from this fact the conclusion that the local meaning of the conditional is not symmetric. The very point of player independence is that it is a property of the meaning of the logical particles not of the dialogue as a whole where **P** is committed to the validity of the thesis. More precisely the asymmetry of the winning strategy is triggered by the semantic asymmetry of the formal rule. It is the possibility to isolate meaning (local and global) from validity commitments that allows dialogicians to speak of the symmetry of the logical constants and this prevents tonk-like operators from being introduced in the dialogical framework.

### 2.3 Play level and Strategic Level

As mentioned above in the dialogical approach validity is defined via the notion of *winning strategy*. A systematic description of the winning strategies available for **P** in the context of the possible choices of **O** can be obtained from the following considerations<sup>26</sup>:

If **P** is to win against any choice of **O**, we will have to consider two main different situations, namely

- the dialogical situations in which **O** has uttered a complex formula, and
- those in which **P** has uttered a complex formula.

We call these main situations the **O**-cases and the **P**-cases, respectively. In both of these situations another distinction has to be examined:

- P** wins by *choosing* between two possible challenges in the **O**-cases or between two possible defences in the **P**-cases, iff he can win with *at least one* of his choices.
- When **O** can *choose* between two possible defences in the **O**-cases or between two possible challenges in the **P**-cases, **P** wins iff he can win *irrespective* of **O**'s choices.

The description of the available strategies will yield a version of the semantic tableaux of Beth that become popular after the landmark work on semantic-trees by Raymond Smullyan (1968), where **O** stands for **T** (left-side) and **P** for **F** (right-side) and where situations of type **ii** (and not of type **i**) will lead to a branching-rule.

(P)-Chooses	(O)-Chooses
$(P)\alpha \vee \beta$ <hr style="border-top: 1px dashed black;"/> $\langle O? \rangle (P)\alpha$ $\langle O? \rangle (P)\beta$ <i>The expressions of the form <math>\langle X... \rangle</math> constitute interrogative utterances</i> $(O)\alpha \wedge \beta$	$(P)\alpha \wedge \beta$ <hr style="border-top: 1px dashed black;"/> $\langle O? \wedge 1 \rangle (P)\alpha \mid \langle P? \wedge 2 \rangle (P)\beta$ <i>The expressions of the form <math>\langle X... \rangle</math> constitute interrogative utterances</i> $(O)\alpha \vee \beta$

<sup>26</sup> Lorenzen 1978, 217-220. The relation with natural deduction has been recently worked out in Rahman/Clerbout/Keiff 2009, 301-336.

$\begin{array}{c} \text{-----} \\ \langle \mathbf{P} ? \wedge 1 \rangle (\mathbf{O}) \alpha \\ \langle \mathbf{P} ? \wedge 2 \rangle (\mathbf{O}) \beta \\ \\ (\mathbf{P}) \alpha \rightarrow \beta \\ \text{-----} \\ (\mathbf{P}) \alpha \\ (\mathbf{O}) \beta \\ \\ \text{No choice} \\ (\mathbf{P}) \neg \alpha \\ \text{-----} \\ (\mathbf{O}) \alpha \end{array}$	$\begin{array}{c} \text{-----} \\ \langle \mathbf{P} ? \rangle (\mathbf{O}) \alpha \mid \langle \mathbf{P} ? \rangle (\mathbf{O}) \beta \\ \\ (\mathbf{O}) \alpha \rightarrow \beta \\ \text{-----} \\ (\mathbf{P}) \alpha \\ (\mathbf{O}) ? \dots \mid (\mathbf{O}) \beta \\ \text{(Opponent has the choice between} \\ \text{counterattacking or defending)} \\ \text{No choice} \\ (\mathbf{O}) \neg \alpha \\ \text{-----} \\ (\mathbf{P}) \alpha \end{array}$
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However, tableaux are not dialogues. The main point is that dialogues are built up bottom up, from local semantics to global semantics and from global semantics to validity. This establishes the priority of the play level over the winning-strategy-level. The levels are to be thought as defining an order. From the dialogical point of view, to set the meaning of the logical constants via validity is like trying to define the (meaning) moves of the king in the game of chess by the strategic rules of how to win a play. Neither semantic tableaux nor sequent calculus give priority to the play level. The point is not really that sequent calculus or tableaux do not have a play level, if with this we mean that one could not find the steps leading to the proof though there is one. What distinguishes the dialogical approach from other approaches is that in the other approaches – if there is something like a play level – the play level is ignored: the logical constants are defined via the rules that define validity.<sup>27</sup> The dialogical approach takes the play level as the level where meaning is set and on the basis of which validity rules should result. Within the dialogical approach, the more basic step of meaning at the play level is the setting of player-independent particle-rules (i.e. symmetric rules): the difference between **O** (**T**)-rules and the **P** (**F**)-rules results from the asymmetry introduced by the formal rule at the strategic level. These considerations lead us to tonk. One can build tableaux-rules for tonk and tonk-like operators but, from the dialogical point of view, they have no semantic underpinning.

<sup>27</sup> The point that other systems have also a play level has been stressed by Luca Tranchini in the workshop Workshop Amsterdam/Lille: *Dialogues and Games: Historical Roots and Contemporary Models*, 8-9 February 2010, Lille.

## Literature

- Anderson, A. / Belnap, N. D. (1975). *Entailment the logic of Relevance and Necessity*. Princeton: Princeton U. Press, 1975, vol. 1.
- Blass A. (1992). A game semantics for linear logic. *Annals of Pure and Applied Logic*, 56: 183–220, 1002.
- Clerbout, N. / Keiff, L. / Rahman, S.: Dialogues and Natural Deduction. In G. Primiero and S. Rahman (ed.) *Acts of Knowledge, History, Philosophy, Logic*. G. Primiero (ed.), London : College Publications. chapter 4.
- R. Dyckhoff / N. Francez (2007). A note on harmony. Typescript, August 2007.
- Dunn, M. (1976). Intuitive Semantics for First-Degree-Entailments and Coupled-Trees. *Philosophical Studies* 1976, 29, 149-168.
- Felscher, W.(1985). Dialogues as a foundation for intuitionistic logic. In D. Gabbay, D. and F. Guenther, (editors), *Handbook of Philosophical Logic*, Volume 3, Dordrecht: Kluwer, 341–372, 1985.
- Fontaine, M. / Redmond, J. (2008). *Logique Dialogique: une introduction, Volume 1: Méthode de Dialogue: Règles et Exercices*. London : College Publications, London, 2008.
- Fiutek, V. / Rückert, H. / Rahman, S. (2010) *A Dialogical Semantics for Bonanno's System of Belief Revision*. To appear in *Constructions* . P. Bour et alii (ed.), London : College Publications, 2010.
- Fontaine, M. /Redmond, J.
- Genot, E. (2009a). The Game of Enquiry: The interrogative Approach to Enquiry and Belief. *Synthese*, 2009, vol. 171/2, 271-289.
- Genot, E. (2009b) Extensive Questions: From Research Agendas to Interrogative Games. *Logic and its applications* (LNAI 5378), 2009, 131-145.
- Genot, E. (2009c). The Game of Enquiry. PHD-Thesis, 2009, University of Lille.
- Genot, E. J. / Rahman, S. (2010), Interrogative Dialogues. A (Dialogically Inspired) Game-Theoretic Semantics for Hintikka's Interrogative Tableaux, Paper presented at the 3rd World Congress on Universal Logic, Lisbon, April 2010.
- Gentzen, G. (1969). *The Collected Papers of Gerhard Gentzen*, tr. M. Szabo. Amsterdam: North-Holland 1969.
- Gentzen, G. 'Investigations concerning logical deduction', trans. in Gentzen 1969, 68-131.
- Hintikka, J. (1999). *Inquiry as Inquiry: A Logic of Scientific Discovery*. Dordrecht: Kluwer.
- Hintikka, J., Halonen, I., and Mutanen, A. (1999). Interrogative Logic as a General Theory of Reasoning. In Hintikka 1999, pages 47–90.
- Hintikka, J., / Sandu, G., 1989. Informational Independence as a Semantical Phenomenon. In *Logic, Methodology and Philosophy of Science*, vol. 8, J. E. Fenstad, I. T. Frolov, and R. Hilpinen (editors.), Amsterdam: Elsevier, 571–589, 1989.
- Hoepelman, J. Ph. / Hoof, A.J.M., 1988. The Success of Failure. In *Proceedings of COLING, Budapest*, Budapest 1988, 250-55.
- Keiff, L. (2004). Heuristique formelle et logiques modales non normales. *Philosophia Scientiae*, vol. 8-2, 39-59, 2004.
- Keiff, L. (2004). Introduction à la dialogique modale et hybride. *Philosophia Scientiae*, vol. 8-2, 89-105, 2004.
- Keiff, L. (2007). Approches dynamiques à l'argumentation formelle. PHD thesis, Lille : Université de Lille, 2007.
- Keiff, L. (2009). Dialogical Logic, Entry in the Stanford Encyclopaedia of Philosophy, 2009, <http://plato.stanford.edu/entries/logic-dialogical/>
- Keiff, L. / Rahman, S. (2010). La Dialectique entre logique et rhétorique. *Revue de Métaphysique et Morale*, Avril-June 2010, vol. 2, 149-178.
- Lorenz, K. Basic objectives of dialogue logic in historical perspective. *Synthese*, 2001, vol. 127: 255–263.
- Lorenzen, P. *Einführung in die operative Logik und Mathematik*. Berlin, Göttingen, Heidelberg: Springer, 1955.
- Lorenzen P. / Lorenz K (1978). *Dialogische Logik*. Darmstadt: Wissenschaftliche Buchgesellschaft, 1978.
- Prawitz, D. (1965). *Natural Deduction*. Stockholm: Almqvist & Wiksell 1965.
- Prawitz, D (1979). Proofs and the meaning and completeness of the logical constants. In J. Hintikka, I. Niiniluoto, and E. Saarinen (editors), *Essays on Mathematical and Philosophical Logic*, Dordrecht: Reidel, 1979, 25–40.
- Priest, G. (2001). *An Introduction to Non-Classical Logic*. Cambridge: Cambridge U. Press, 2001.
- Priest, G. (2010). Realism, Antirealism and Consistency. In G. Primiero and S. Rahman (editors), *(Anti)Realism. The Realism-Antirealism Debate in the Age of Alternative Logics*. Forthcoming, 2010.
- Rahman, S (1993). *Über Dialogue, Protologische Kategorien und andere Seltenheiten*. Frankfurt/Paris/ N. York: P. Lang, 1993.
- Rahman, S. (2009). A non normal logic for a wonderful world and more. In J. van Benthem et alia *The Age of Alternative Logics*, chez Dordrecht: Kluwer-Springer, 311-334, 2009.
- Rahman, S. / Clerbout, N. / Keiff, L. (2009). Dialogues and Natural Deduction. *Acts of Knowledge, History, Philosophy, Logic*. G. Primiero (ed.), London : College Publications, 301-336.

- Rahman, S. / Keiff, L. (2004). On how to be a dialogician. In D. Vanderveken (ed.), *Logic, Thought and Action*, Dordrecht: Kluwer, 2004, 359-408.
- Rahman, S. / Rückert, H. (2001). New Perspectives in Dialogical Logic. S.Rahman/H.Rückert *Synthese*, 127, 2001.
- Rahman, S. / Tulenheimo, T. (2009). From Games to Dialogues and Back: Towards a General Frame for Validity. In O. Majer, A-V. Pietarinen and T. Tulenheimo (editors), *Games: Unifying Logic, Language and Philosophy*, Part III, Dordrecht, Springer, 2009.
- Read, S. (2000). Harmony and autonomy in classical logic. *Journal of Philosophical Logic*, 2000, 29, 123–154.
- Read, S. (2005). The unity of the fact. *Philosophy*, 2005, 80, 317–42.
- Read, S. (2008). Harmony and modality. In C. D'egremont, L. Kieff, and H. Rückert (editors), *Dialogues, Logics and Other Strange Things: Essays in Honour of Shahid Rahman*, London: College Publications, 2008, 285–303.
- Read, S. (2010). General Elimination Harmony and the Meaning of the Logical Constants. Typescript, March 2010.
- Rott, H.: Nonmonotonic Conditional Logic for Belief Revision. Part I: Semantics and Logic. In Proceedings of the Workshop on Nonmonotonic Reasoning, 1989, GMD Arbeitspapier 443, 45-51.
- Rückert, H. (2001). Why Dialogical Logic? In H. Wansing (ed.), *Essays on Non-Classical Logic*, N. Jersey, London ...: World Scientific, 2001, 165-185.
- Rückert, H. (2007). Dialogues as a dynamic framework for logic. PHD-Thesis, Leyden, 2007. Online in: [https://openaccess.leidenuniv.nl/dspace/bitstream/1887/12099/1/R%C3%BCckert\\_PhD\\_Dialogues\\_neu.pdf](https://openaccess.leidenuniv.nl/dspace/bitstream/1887/12099/1/R%C3%BCckert_PhD_Dialogues_neu.pdf).
- Rückert, H. (2011). The Conception of Validity in Dialogical Logic. Talk at the workshop Proofs and Dialogues, Tübingen, organized by the Wilhelm-Schickard Institut für Informatik (Universität Tübingen), 25-27 February 2011.
- Schröder-Heister (2008). P. Lorenzen's operative justification of intuitionistic logic. In M. van Atten, P. Boldini, M. Bourdeau, G. Heinzmann (eds.), *One Hundred Years of Intuitionism (1907-2007)*, Basel: Birkhäuser 2008.
- Sundholm, B. G. (1983a). Constructions, proofs and the meaning of the logical constants. *Journal of Philosophical Logic*, Vol. 12, 151-72.
- Sundholm, B. G. (1983b). Systems of deduction', chapter I:2 in: Gabbay, D., and F. Guenther, *Handbook of Philosophical Logic*, Vol. I, Reidel, Dordrecht.
- Sundholm, B. G. (2010). Proofs as Acts and Proofs as Objects: Some Questions for Dag Prawitz. *Theoria*, forthcoming.
- Tulenheimo, T. (2009). Independence Friendly Logic. Entry in the Stanford Encyclopaedia of Philosophy, 2009: <http://plato.stanford.edu/entries/logic-if/>
- Tulenheimo, T. (2010). On the dialogical approach to semantics. Talk at the Workshop Amsterdam/Lille: *Dialogues and Games: Historical Roots and Contemporary Models*, 8-9 February 2010, Lille. Online in <http://www.tulenheimo.webs.com/talks.html>
- Wansing, H. (2001). Negation. In L. Goble (editors.), *The Blackwell Guide to Philosophical Logic*, ch. 19. Oxford: Blackwell, 2001.