

# A tale of seven cities

a new framework for the historiography of Chinese astronomy in the Period of Disunion (220–589)

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# The history of astronomy, a traditional framework

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Table of contents, Chen 2003.

Features:

- Line of **great men** and events
- Periodisation by **dynasty**
- Emphasis on 'calendar reform' (*gai li* 改曆), i.e. the adoption of an astronomical procedure text (*li* 曆) as new imperial standard for producing calendars and ephemerides.

# The history of astronomy, a traditional framework

表 8-1 回归年长度与朔望月长度

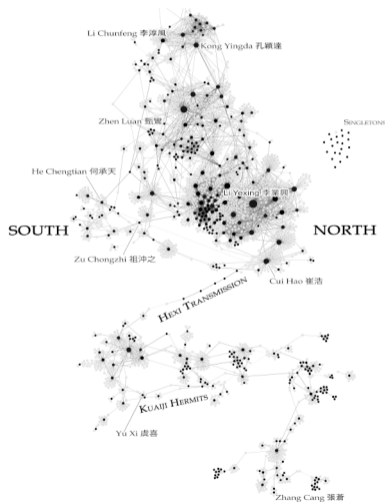
| 历法名称<br>及作者 | 年代      | 回 归 年                    |              |             | 朔 望 月                 |               |             | 调日法<br>误差(秒) | 误差(倍) |
|-------------|---------|--------------------------|--------------|-------------|-----------------------|---------------|-------------|--------------|-------|
|             |         | 据以计算回归年长度(岁)的分母与分子之比 (A) | 回归年长度(岁) (B) | 误差(秒) (C)   | 据以计算朔望月长度的分母与分子之比 (D) | 朔望月长度(日) (E)  | 误差(秒) (F)   |              |       |
| 古四分历        | 约公元前五世纪 |                          |              | 365.2500000 | 约-661                 |               | 29.5308511  | -23.1        |       |
| 太初历<br>邓平   | 公元前104  | 周天<br>562120             | 纪法<br>1539   | 0.2501624   | -664                  | 月法<br>2392    | 日法<br>81    | 0.5308642    | -24.1 |
| 四分历<br>编户等  | 公元85    | 周天<br>1461               | 日法<br>4      | 0.2500000   | -664                  | 朔日<br>27759   | 朔月<br>940   | 0.5308511    | -23.0 |
| 乾象历<br>刘徽   | 206     | 周天<br>215130             | 纪法<br>589    | 0.2461800   | -335                  | 通法<br>43026   | 日法<br>1457  | 0.5305422    | +3.7  |
| 黄初历<br>韩翊   | 220     | 斗分<br>1205               | 纪法<br>4883   | 0.2467745   | -386                  | 月法<br>356700  | 日法<br>12079 | 0.5305003    | -0.4  |
| 太和历<br>高堂隆  | 227     |                          |              | 0.2468880   | -395                  |               |             | 0.5305980    | -1.1  |
| 景初历<br>杨伟   | 237     | 周天<br>673150             | 纪法<br>1843   | 0.2468800   | -396                  | 通数<br>134630  | 日法<br>4659  | 0.5305988    | -1.1  |
| 永初历<br>王明之  | 352     | 斗分<br>1205               | 纪法<br>4883   | 0.2467745   | -387                  | 月法<br>356700  | 日法<br>12079 | 0.5305903    | 0.4   |
| 太始历<br>刘智   | 365     | 斗分<br>37                 | 度法<br>150    | 0.2466667   | -377                  | 纪日<br>1040953 | 纪月<br>35250 | 0.5305816    | +0.3  |

Table of values for the tropical year and synodic month, Chen 1995.

Features:

- Single table, works in chronological order
- Constants as ratios (1461 : 4)
- Translation into decimals for comparison (1461/4 → 365.25)
- Errors reveal:
  - sudden leaps
  - global progress

# New framework: transmission lines and social networks



Astronomers and mathematicians and their social networks (Morgan 2021).

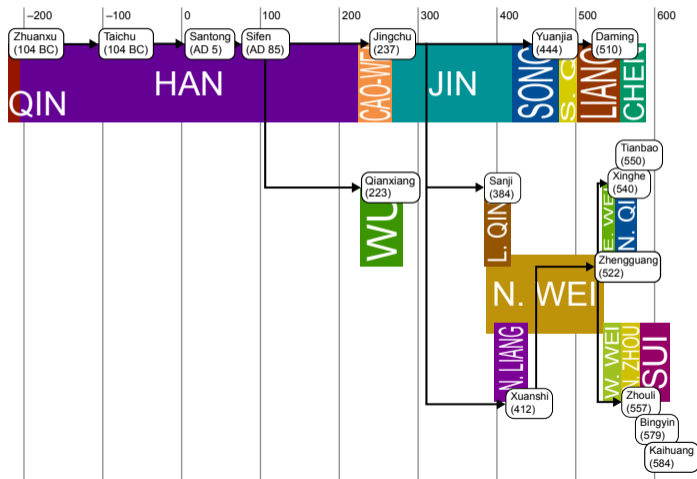
## Inspiration:

The nine constituents of a 'mathematical culture' as defined in Chemla 2009: (1) problems, (2) numbers, (3) procedures, (4) text types, (5) instruments, (6) figures, (7) proofs, (8) epistemological values, (9) types of institutions, social groups, and individual profiles.

## Findings:

- The 429 known 'mathematicians' from 221 B.C. to A.D. 670 are almost all linked by social relations, family, and transmission lines.
- Their relations formed distinct regional clusters in the Period of Disunion.
- We can trace the transmission of certain ideas, works, 'numbers', and 'procedures' between networks.

# 'Calendar reform' history and the Period of Division



## \*Note

This is obvious: after the Han, rival Chinese and barbarian courts pursued their own 'calendar reform' programmes. These courts were based in *different cities*, they were often *at war*, and it is silly to assume that their respective 'scientists' were in constant *communication and agreement*.

# A tale of seven cities (and three zones)



**DPM's current project (2022–):**  
 Reconstruction and translation of all astronomical procedure texts (*li* 曆) from the Period of Disunion in English and Python, the Python allowing dynamic worked examples and deep computational analysis.

### Framework:

Upon analysis of the mathematical texts (i.e., elements 1–8 of a 'mathematical culture' according to Chemla 2009), the only sensible solution is to organise texts *by city* (and *zone*).

**Q:** What is a *li* 曆 (calendar astronomical procedure text)?

**A:** A *li* is a set of numbers (*shu* 數) followed by a chain of procedures (*shu* 術) for calculating the civil calendar, eclipses, and the time and position of the sun, moon, planets, and their related phenomena.

**Example,** *Sifen li* 四分曆 (*Quarter-remainder Astronomy*, A.D. 85):

元法，四千五百六十。

**Origin factor:** 4560.

紀法，千五百二十。

**Era factor:** 1520. ...

推入部術曰：

Procedure to calculate [the years] entered into [the current] obscuration:

以元法除去上元，其餘以紀法除之，所得數從天紀，筭外則所入紀也。不滿紀法者，入紀年數也。Eliminate ( $\div$ ) the [years] from High Origin by the **origin factor**. Eliminate ( $\div$ ) the remainder by the **era factor**. Number from the Heaven Era by what you get, then outside the count is the Era entered. That which does not fill the **era factor** is the number of years entered into the Era. ... (preserved in *Hou Hanshu* 後漢書, 'Lüli xia' 律曆下).

Read full translations in Liu Hongtao 2003 and Cullen 2017!

## Section 2

# Of numbers and 'fraction laundering'



# Comparing resonance periods and fractions

| <i>Santong li</i> 三統曆 (A.D. 5)  |   | Modern value                  | <i>Sifen li</i> 四分曆 (A.D. 85) |   |
|---|---|-------------------------------|-------------------------------|---|
| Resonance period  | Decimal                                   |                               | Decimal                       | Resonance period  |
| $2392\text{d}/81\text{m} = 29\frac{43}{81}\text{d}/\text{m}$  | $\rightarrow 29.53086\text{ d}/\text{m}$  | $29.53059\text{ d}/\text{m}$  | $29.53085\text{ d}/\text{m}$  | $\leftarrow = 29\frac{499}{940}\text{d}/\text{m}$   |
| <p><b>RULE 章</b><br/> <math>235\text{m} = 19\text{y}</math></p> $2392\text{d}/81\text{m} \times 235\text{m}/19\text{y}$<br>$= (2392\text{d} \times 235\text{m}) / (81\text{m} \times 19\text{y})$<br>$= 562,120\text{d}/1539\text{y}$ |   |                               |                               | $1461\text{d}/4\text{y} \times 19\text{y}/235\text{m}$<br>$= (2392\text{d} \times 19\text{y}) / (4\text{y} \times 235\text{m})$<br>$= 27,789\text{d}/940\text{m}$ |
| $\leftarrow = 365\frac{385}{1539}\text{ d}/\text{y}$  | $\rightarrow 365.25016\text{ d}/\text{y}$ | $365.24217\text{ d}/\text{y}$ | $365.25000\text{ d}/\text{y}$ | $\leftarrow 1461/4 = 365\frac{1}{4}\text{ d}/\text{y}$  |

## Comparing resonance periods and fractions

**Question:** What if they were also translating?

The *Sanji li* 三紀曆 (A.D. 384) uses a mean synodic month of  $179,044 : 6063 = 29 \frac{3217}{6063}$  days, which could be a 'translation' of the *Jingchu li*'s 景初曆 (A.D. 237) value of  $134,630 : 4559 = 29 \frac{2419}{4559}$  days:

$$\text{convert: } 134,630 \times \frac{6063}{4559} \rightarrow 179,044.02$$

$$\text{round: } [179,044.02] \rightarrow 179,044$$

$$\text{MATCH! } 179,044 \div 6063 \rightarrow 29 \frac{3217}{6063}$$

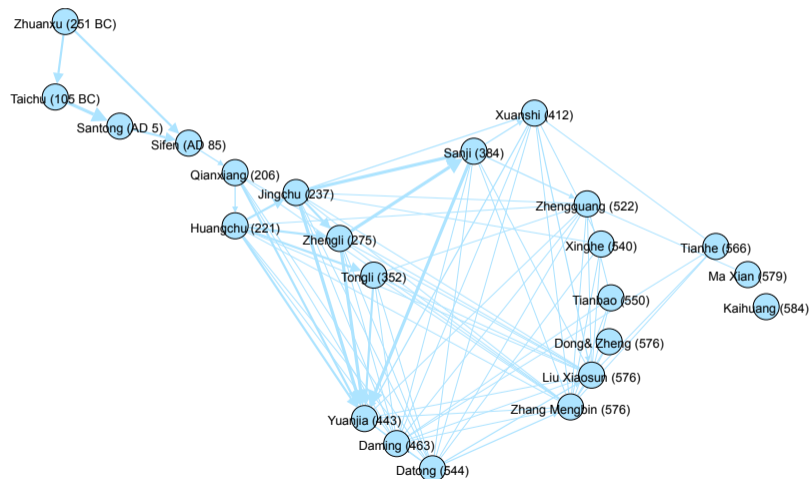
Likewise, the *Sanji li*'s mean anomalistic month of  $167,063 : 6063 = 27 \frac{3362}{6063}$  days might derive from the *Jingchu li*'s value of  $125,621 : 4559 = 27 \frac{2528}{4559}$  days:

$$\text{convert: } 125,621 \times \frac{6063}{4559} \rightarrow 167,062.98$$

$$\text{round: } [167,062.98] \rightarrow 167,063$$

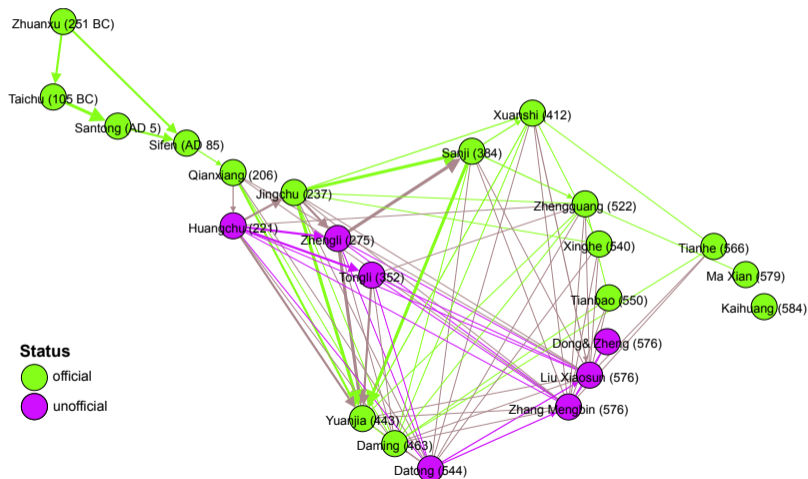
$$\text{MATCH! } 167,063 \div 6063 \rightarrow 27 \frac{3362}{6063}$$

# 'Fraction laundering'



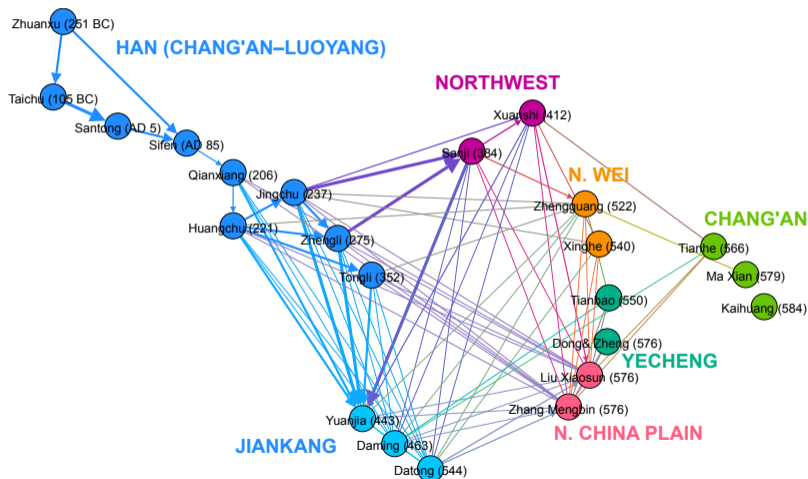
- Used Python to test every constant of every astronomy for 'fraction laundering' (translating one resonance period into the units of the other).
- The thicker the line, the more matches between astronomies.
- Observation: there is borrowing, and the direction is not always linear.

# 'Fraction laundering'



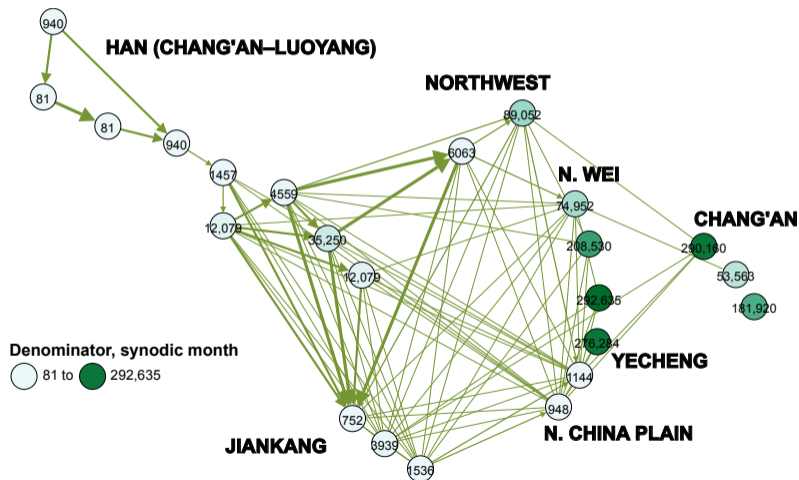
- There is no clear difference between official and unofficial astronomies in terms of weight.
- The *Jingchu li* took from the *Huangchu li* 黃初曆 rather than the *Qianxiang li* 乾象曆, which is interesting considering that the author blocked the *Huangchu li* in debate (Morgan 2017, 140–176).

# 'Fraction laundering'



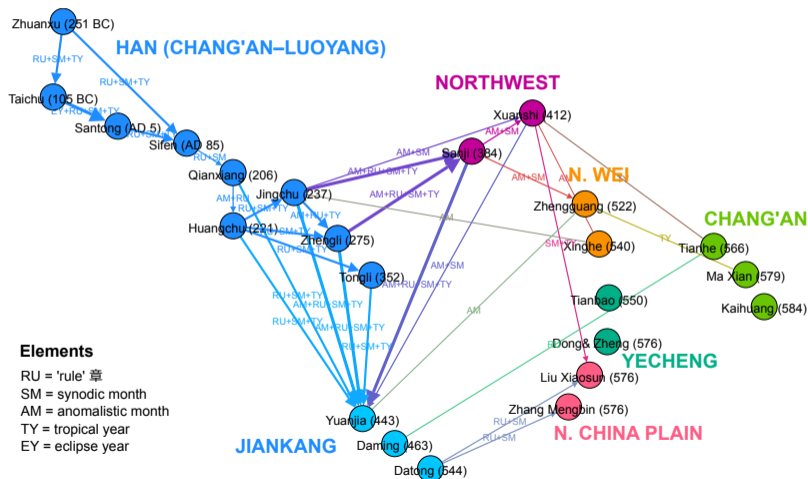
- Traces of the parallel Northwest transmission to Jiankang and N. Wei in 437–439 (Morgan 2021).
- Traces of the late 6th century integration of the South (ibid.).
- Chang'an is the most unique, borrows only from the Northwest, South and N. Wei.
- Busiest part of the diagram is Jiankang and the N. China Plain.

# 'Fraction laundering'



- Jiankang and N. China Plain prefer smaller denominators; the North prefers big ones.
- Smaller denominators make more matches, which we can interpret as noise and/or an effort to synthesise and simplify.

# 'Fraction laundering'



- Here, I have eliminated simple matches on the synodic month.
- Observation: N. China Plain has closer ties to Jiankang trad. than Chang'an (at variance with Morgan 2021).

## Section 3

# Of procedures and Python modules



```

Ouvrir 237_jingchu_li_output.txt Enregistrer
~/PycharmProjects/astrometry/working_examples
*Document 1 sans titre 237_jingchu_li_output.txt

77 #####
78 推朔術曰：
79 Proc. 2 Procedure for calculating new moon:
80 #####
81 以通數乘積月，為朔積分，
82 Mount (x) the accumulated months (prev) by the communication number (134,630) to make the new moon accumulated parts.
83
84             4,440** x 134,630 -> 597,757,200
85
86 如日法而一為積日，不盡為小餘。
87 Take for each day divisor (4,559) to make the accumulated days, and what is not exhausted makes the little
88 remainder.
89
90             597,757,200 ÷ 4,559 -> 131,115 rem 3,915
91
92 以六十去積日，餘為大餘。
93 Remove (modulo) the the accumulated days by 60, and the remainder makes the big remainder.
94
95             131,115 mod 60 -> 15
96
97 大餘命以紀，算外，所求年天正十一月朔日也。
98 Name the big remainder from era [head]. Outside the count, and that is the [sexagenary] date of new moon for the
99 celestial standard, month XI, of the year sought.
100
101             jlashen:**, name out 15 -> jlhals*
102
103 #####
104 求次月：
105 Proc. 3 To find subsequent months:
106 #####
107 加大餘二十九，小餘二千四百一十九，小餘滿日法從大餘，命如前，次月朔日也。小餘二千一百四十以上，其月大也。
108 Increase the big remainder by 29 and the little remainder by 2419. If the little remainder fills the day divisor
109 (4559), let it go with the big remainder, name as before, and that is [the sexagenary] date of the new moon of the
110 subsequent month. If the little remainder is 2140 or higher, this month is big (30 days).
111
112
113             ano 2:
114             15 d 3,915 p + 29 d 2,419 p ->
115             15 d 6,334 p + 29 d ->
116             [15 + 1] d [6,334 - 1 x 4,559] p + 29 d ->
117             16 d 1,775 p + 29 d ->
118             45 d 1,775 p
119             jlashen:**, name out 45 -> jlsls*
120             1,775 ≥ 2,140 ? Size: [s]

Texte brut  Largeur des tabulations : 8  Lig 77, Col 24  INS

```

Sample translation of the  
*Jingchu li* (a.d. 237), output:

```

549 line = f"""
550 #####
551 推朔術曰：
552 Proc. {proc} Procedure for calculating new moon:
553 #####
554 """
555 # 以通數乘積月，為朔積分，
556 ac_mo_p_CS = int(ac_months_CS * COM_NUM)
557 op_dat = af.opcount(proc, 'fetch value', ac_months_CS, np.NaN, 'impl', 'impl', False, op_
558 op_dat = af.opcount(proc, '⊗', ac_months_CS, COM_NUM, '乘', '以通數乘積月', False, op_dat)
559 chinese = """
560 以通數乘積月，為朔積分，
561 """
562 english = f"""
563 Mount (⊗) the accumulated months (prev) by the communication number ({COM_NUM:;})
564 to make the new moon accumulated parts.
565 """
566 line += af.even_text(chinese, english)
567 line += '\n\n' + ' ' * 36 + f"{ac_months_CS:;}" ⊗ "{COM_NUM:;}" -> "{ac_mo_p_CS:;}\n\n"
568 # 如日法而一為積日，不盡為小餘。
569 ac_days_CS = int(ac_mo_p_CS // DAY_DIV)
570 day_remainder_CS = int(ac_mo_p_CS % DAY_DIV)

```

Chinese

English

Python

Symbolic algebra

PROBLÈMES SORTIE CONSOLE DE DÉBOGAGE TERMINAL SQL CONSOLE

```

Proc. 30 推五星術 (0.0 seconds)
Generate extended new moon table (0.0 seconds)
Traceback (most recent call last):
File "/home/danielmorgan/PycharmProjects/astromy/Jingchu_li.py", line 4627, in <module>
>
rnd=round_push,
NameError: name 'round_push' is not defined
○ (venv) (base) danielmorgan@CDF-GSTCFG3:~/PycharmProjects/astromy$

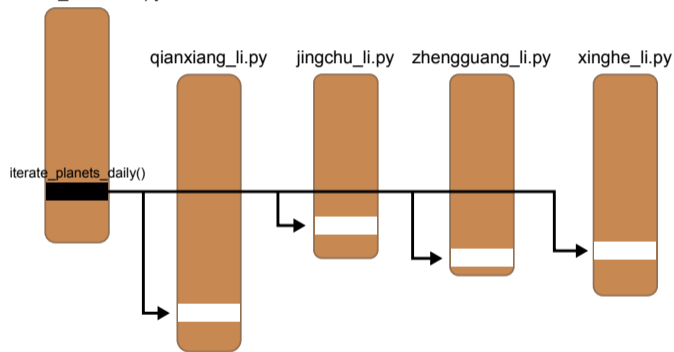
```

Bug!

## Sample translation of the *Jingchu li* (a.d. 237), code:

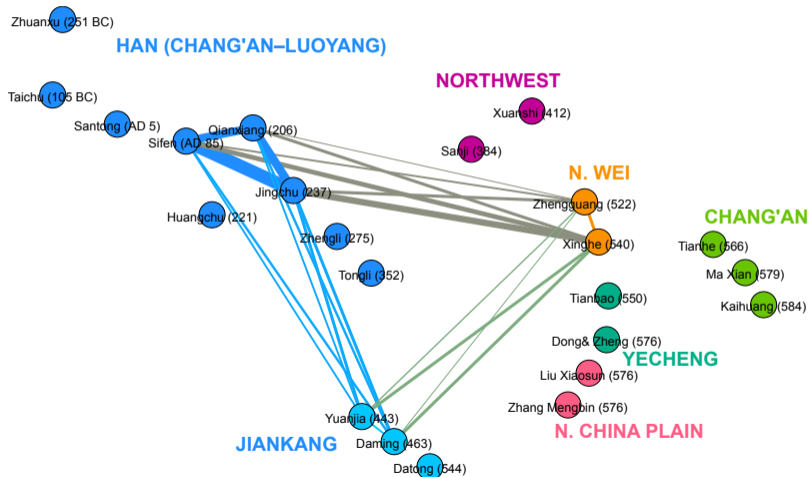
- Quadrilingual translation = complexity
- Translation into both text and practice
- Translation requires testing and debugging

common\_functions.py



- To simplify debugging, identical functions loaded from a shared module.
- Result: 'System A–System B' classification of the individual procedures.
- Result: reflection on the modularity of the sources.
- Result: data on transmission of *procedures* independent of *numbers*.

# Shared procedures (python functions)



- Only 8 extant procedure texts.
- The bigger the line, the more shared procedures (2–36).
- Nothing directly taken from the *Santong li*
- Three distinct groups:
  - Extensive borrowing in the late Han network;
  - N. Wei: heavy borrowing from the Han;
  - Jiankang: light.

# Conclusion

- My 2021 article on social networks led me to the obvious conclusion that the rival courts during the Period of Disunion (220–589) were home to distinct regional traditions in astronomy and mathematics.
- Turning to the technical literature, if we change the metric for comparing astronomical constants (*shu* 數) between incommensurable numerical structures from decimal to fractional equivalents, we find a lot of apparent borrowing between works.
- We also find patterns to that borrowing, which map on to distinct political, geographic, and social divisions.
- The same, where extant, is also true of procedures (*shu* 術).
- Therefore, the most efficient and sensible structure for a sourcebook for this period is to organise it by city and by zone/lineage.

Thank you!

# References I

- \* Chemla, Karine. 2009. “Mathématiques et culture. Une approche appuyée sur les sources chinoises les plus anciennes”. In *La mathématique: 1. Les lieux et les temps*, ed. by Claudio Bartocci and Piergiorgio Odifreddi, 105–152. Paris: Éditions du CNRS.
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- \* – . 2021. “Regional networks in Chinese mathematics and astronomy, 317–618 CE”. *East Asian Science, Technology, and Medicine* 53 (1): 1–53.