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François Conne, Jean Brun

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# CONTENT AND PROCESS: THE CASE OF TEACHING WRITTEN CALCULATION AT PRIMARY SCHOOL.

F. CONNE, J. BRUN. FPSE, University of Geneva.

This paper discusses reseach still in progress supported by the Swiss National Fund for Scientific Research, no 11-25448-88, entitled : L'étude des algorithmes de calculs dans la transmission et la constitution des connaissances numériques. , conducted by J. Brun, F. Conne, and R. Schubauer, of University of Geneva, and J. Retschitzki, of University of Fribourg.

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## SUMMARY

By this presentation, we want to study the case of teaching written calculation, and try to answer to the two questions inscribed in this topic area:

- How does the content affect the process of teaching?
- How do pedagogical and curricular knowledge relate to each other?

We shall limit ourselves to the classical teaching of elementary arithmetics. Our first task will be to define what can be considered as a CONTENT of this teaching. Shall we take into account the aims of curriculum, or what teaching books content (in the french-speaking part of Switzerland, there is an official methodological book for teaching maths, imposed to primary teachers), or, which is different, the mathematical knowledge of teachers? That discussion leads to answer at the two questions mentioned above. We shall treat the following main points:

1. Though written calculation is a traditional object of teaching, we must consider it more as a didactical object than as a mathematical one.
2. Trying to make it modern, new pedagogy? Perhaps, but also new epistemological point of view.
3. Twenty years after, the point of situation: mathematical or pedagogical resistances?

This presentation will be based on the analyse of teaching handbooks. Our research, still in progress, will confront this analysis with two others, concerning more precisely pupils productions, and teachers representations.

## INTRODUCTION

In this paper, we will examine the two questions presented by the organizers in this topic area:

- How does content affect the process of teaching?
- How do pedagogical and curriculum knowledge relate to each other?

We propose to treat this by descriptions and comparisons of teaching handbooks in Suisse Romande (french-speaking states of Switzerland), more

precisely with the chapters concerning the teaching of written algorithm of division.

But before we start our analysis, let us make some general comment. First of all, concerning the two questions above: does content really affect the process of teaching?

The answer is yes, with certain evidence from elementary mathematics. But where does this evidence come from? If we consider the result of the teaching process as building up a certain knowledge in the pupil's mind, we shall immediately ask the following question: does that knowledge have to be built up directly in its achieved form? We mean, of course, achieved in the point of view of mathematics. This simple question shows that we have to manage three orders of building. Let's say, in vague terms: first is the structural organization and order of mathematics (one aspect of content), second the order of knowledge development of the pupil, third is the order of teaching organization itself. These three orders are not the same, and are not naturally synchronized, but give an indication of how the content and the teaching process are linked.

Another more specific argument, gives us further indications. It concerns the teaching of written algorithms at the primary school level, and will permit us to approach our subject. Upon examination of the algorithms of calculation, we can affirm that they are particularly adapted to the traditional way of teaching elementary mathematics, which, we must say, is still current in most of our schools. Some of the most traditional methods even trace their progression on the algorithm's rules and consequently on the skills that it allows to the calculator. But there is another way of tracing the progress of the teaching on the algorithm: it is to propose activities for the pupils which build up the algorithm itself. It is the solution chosen by modern teaching handbooks in Suisse Romande which shall be examined more closely now.

### FRAME OF OUR RESEARCH.

At the end of the sixties, the teaching of mathematics at the primary school level was reformed in Suisse Romande. Regarding this teaching, it was decided that there would be the same curriculum in all the different "cantons" of this part of the country. Teachers and pupils would receive the same handbooks, the teachers receiving a methodological edition, and the pupils receiving an exercise edition. So, for each grade of primary school, there are two handbooks. After a couple of years, this curriculum was evaluated, and the handbooks were reedited, with some corrections according to the results of these evaluations.

Despite the very precise instructions about content, curriculum, and teaching methodology given to teachers, for many teachers there is no general vision about elementary numerical school activities. For example, teachers of 5th grade who are in charge of teaching the written division algorithm, do not know if the pupils in the 7th or 8th grade are still solicited to make such calculations. This result concerns all the grades of school and is not necessarily reserved to the link between primary and secondary school. But when we consider the handbooks themselves, despite the cyclical character of the curriculum, we can understand this lack for those who are satisfied with a single reading of the text. Without serious study, the handbook looks like a collection of meal recipes, put together with descriptions of content and didactical ingredients for these meals. Furthermore, in order to understand the organization of the content, one would have to enter into the mind of the author of the text, and forget one's own role of a "teacher-preparing-his-lesson-for-tomorrow".

We are concerned with studying the teaching of mathematics in a non-prescriptive perspective. One of the goals of this kind of research is building general principles for teachers to promote thinking and perceiving in a global manner the elementary numerical activities at primary school in such a way that it takes into consideration the three orders mentioned in the introduction. It appears that these principles must be based on the distinction of two major aspects of mathematical knowledge, the general distinction of actions and relations. One key for understanding these two aspects and their relations lies in the symbolic systems used as tools of action, and at the same time, to support the relations. Because these tools are didactical tools too, they are also key for studying the classroom activities, the school handbooks, and more generally the planning and progression of teaching. Our research studies and puts into evidence the role of the diagram of an algorithm as one of these symbolic supporting elements, and aims to treat the question of the relations between actions and relations in the teaching of mathematics. It is the reason why we have entitled it: The study of algorithms of calculation in transmitting and establishing of numerical knowledge. ( L'étude des algorithmes de calculs dans la transmission et la constitution des connaissances numériques. ) Here we present the results of one aspect of this research, still in progress, the analysis of handbooks.

We shall now have a rapid and quite superficial look at the school's grade organization in Suisse Romande, and more precisely, about the distribution of elementary numerical activities during these grades. The primary school mathematics program concerns the first six grades of school, which are in their turn divided into three pairs of grades. The average ages required for beginning

each of these grades range from seven to twelve. As far as numbers and numerical knowledge are concerned, the mathematical program presents some basic activities with quantity and numbers, as well as the four arithmetic operations. Calculation is taught for each of these operations in both written and mental forms. Regarding this subject, the aims of the program are traditional. Despite the coherence and continuity of this program, one notices a difference between the first four grades and the last two. During the four first grades of school, if a subject is presented in a constructive way, going back if necessary to the beginning of concrete manipulations, then as subjects become more complex, in the fifth and sixth grade, their presentation is done in a more analytic way. The teaching of written division as it takes place between fourth and fifth grade, serves as a turning point in the whole curriculum which is one of the reasons we decided to study this subject.

### MATHEMATICAL CONTENT OR DIDACTICAL CONTENT?

For the purposes of our study, one must consider calculation not only as a mathematical object, but also as a didactical one. Our interest in content is an interest in mathematics. Some of our questions are for example: Which type of mathematics works at school? How the mathematical knowledge both of pupils and teachers operates? We investigate the field where the knowledge of each one interrelates. This means that for us the matter is not only mathematical but also didactical, for the content is an object of the didactical exchange between teachers and pupils. We examine the handbooks from this point of view, since they place conditions for this exchange, and indicate how mathematics will then intervene.

Mathematically speaking, the algorithms of calculation are quite poor and anecdotal objects. They are of historical interest, and perhaps also of interest for the science of algorithms. However, they have much more importance from a cultural perspective, the reason being the role they have in teaching mathematics, since the role they play in everyday life is tiny nowadays. We can say that in our culture, the algorithms of calculation are emblematic objects in elementary mathematical education. But we must be more precise. Concerning the algorithm of division, it is clear that it is no longer an important technique to learn at school. Consequently the exercises proposed in the handbooks are rather simple, since, for example, they do not present more than five digits dividends and two digits divisors, or simple three digits divisors. These limits have didactical consequences

which weakens the aim of teaching division. Why should one teach or even learn such a sophisticated procedure such as the division algorithm for such simple calculations, where more simple procedures, like trial and error or chains of subtraction, appear as efficient?

Given that a matter taught at school is a didactical object, we will examine how it is incorporated in the teaching content, and the subsequent function it assumes in that system. This function is quite complex as is the system of teaching. In the following sections of this paper we will illustrate the three important aspects already pointed out in the introduction, by the study of the teaching of division. Let us formulate these aspects in a slightly different way.

1. A didactical object is structured, as is the content's organization which is set in a way that enlarges the meaning of the notions taught for pupils.

2. A didactical object is implied in an epistemological perspective. Content's organization respects the epistemological statements adopted in the curriculum.

3. A didactical object adaptes to the school and classroom work style. Content's organization is made suitable for teaching practices.

The mathematics developed by the authors in their handbooks are not free of these crossed constraints which must be taken into account. So we shall consider the content of the handbooks as responses, if not solutions, to the problems which are presented to them.

### HOW THE HANDBOOKS PLAN THE ENLARGEMENT OF MEANING OF DIVISION.

The handbooks present a structured matter for teaching. The subject is not the written division calculation but more widely the notion of division itself, and the construction of the algorithm is an occasion to work some general notions, not only the calculation of division, but also other ones which are related to it, such as approximative evaluations of calculation results, decimal numbers, proportionality, and so on. In order to analyse these links, and the way in which they are treated in the handbooks, we look at the situations referred to in the books, and the way these references evolve during the progress of the teaching. The reference to sharing discrete units introduces the calculation of division. At first, division is calculated by a chain of subtraction. The calculation method is shortened and made more sophisticated. Observations, conclusions, and rules are abstracted from calculating experiences, which in turn introduce the meaning of division by decimal divisors, and then the calculation of division among decimal numbers. Let us describe this more precisely.

At first the algorithm of division is referred to a situation of sharing. But we must be more precise and add the fact that this sharing is conceived as a distribution of discrete units. Why is this precision so important? Because it makes us realize that this reference focuses on the meaning of an additive treatment of sharing, namely distribution, but not on other aspects of sharing experiences like, for example, dividing real objects by cutting them. We notice first that the pupil's attention is focused on the procedural aspect, secondly that it concerns number, and counting procedures, and cumulative relations of repeating distribution, but does not involve the relation of proportion between the whole and the obtained parts. This can be understood as the result of the desire to simplify things at the beginning. But that limitation has further consequences that force the authors of methodological books to seek a way of enlarging its meaning, in order to cover all the aspects involved in division. In our case, it is clear that the reference to a sharing situation does not provide a meaning for division like "  $6 : 3.4$  ", namely the case of a decimal divisor. But this is not the only case where the presentation proves too limited. The numerical property on which the algorithm of division is built is the distributivity of multiplication on addition, in other words the combination of additive and multiplicative relations. In order to achieve its objectives, the methodology can not be satisfied by additive references, but needs, somewhere during its progression, to refer to multiplication as well. Here we point out a didactical problem set by the mathematical structure of the algorithm of division. The solution found by the authors of our handbooks lies in two main references.

The first one is to introduce the multiplicative relations by the reference to situations of exchange of units of different value (studied before in the curriculum). This reference could be a reference to the concept of value, and equivalence of values, but here again only one procedural aspect is retained: the aspect of sharing a collection of chips of different values, and exchanging high valued chips for lower valued ones when necessary (in order to achieve the sharing). Here again it is a way of dealing with the situation by an additive procedure involving discrete quantities. This way makes the link between a sharing-by-distribution situation and an exchanging-chips situation very clear. Yet the didactical difficulty is not to show and train pupils to share by distributing and exchanging chips of different values, but rather to make them consider the situation of dividing numbers as a situation of sharing and exchanging units of different value. As the sharing situation reference was made to globally introduce the meaning of division, the exchanging situation reference is made to provide a meaning to some sub-operation of the algorithm itself. Here we point out an

inversion of the direction of reference: first, the starting point is the sharing situation and it introduces the calculation, second the starting point is the calculation which is examined as an exchanging situation. In the next section of this paper we shall come back to this point, and show how it relates to the epistemological choice of the authors of the handbook.

The second reference which introduces multiplicative relations is made to the relations of proportionality. Here the proportionality between the three terms in division (dividend, divisor, quotient) is studied. Then it is extended to the relations between dividend, divisor quotient and remainder, which become more complicated. The pupils are asked to calculate, then to compare some of their calculations in order to find some rules between their data and and give verbal expressions of them. This reference is quite abstract. The calculation experiences provide an occasion to point out multiplicative properties, and it is hoped that they will, in return, help pupils to check their calculations. This presents didactical difficulties. First of all, the pupils are asked to make observations and inferences while they are calculating. Second the situation of comparison itself is quite artificial, it's aim is not very clear. The didactical solution suggested by the handbooks is to inscribe data and results in tables which will support the comparisons and verbalisations of the pupils. The same tables are also used in the treatment of "concrete situations". But what is the true purpose of these exercises? In fact it is a study of calculation, but in the handbooks, for both teachers and pupils, the emphasis is put on abstract properties of the operation rather than on tools of calculation. The reason for this lies in the fact that the distinction between mathematical operations and calculations is never clearly made. One will not find them mentioned in the handbooks, even in the notices which are especially addressed to the teachers. After that, the pupils are required to use these rules in some calculation exercises. Paradoxically, they are then encouraged to calculate mentally, and not with the written algorithm. Linking together mental and written calculations is another didactical difficulty encountered. The solution provided by our handbooks is to articulate both mental and written calculation in the situation of estimating and approximating result of difficult division. But upon solving the algorithm, mental sub-calculations have already occurred to calculate in written division, for example the estimation of the quotient digits. These sub-calculations are never referred to in the handbooks, as if the authors deliberately chose to avoid them. Why? We assume that the authors do not want to suggest activities to the pupils which are too specifically dedicated to calculation. Another didactical difficulty concerning the relationship of proportionality is studied here in a very abstract way, without the support of

“concrete situations” references: it is not the proportionality of some concrete “quantities” (amount, measurements,...) which are studied here, but rather the proportionality among the data involved in the calculations.

Those who study the handbooks of Suisse Romande will be very astonished by how much they insist on proposing such treatment of proportionality. The function of such exercises is crucial to this method because it permits the teaching process to introduce important mathematical notions. We have already observed the calculating notion of estimating the quotient’s size (when it is an integer). After which, both rules and meaning of division among decimal numbers are based on that reference to proportionality. Consequently the notion of estimating the quotient’s size is enlarged to the notion of “approximative-decimal-quotient of integers”.

So, if we look back at the progression of the references made in this teaching, we can make the following observations. The starting point is the reference in sharing situations which introduces the calculating situation. Then, starting from the calculating situation, a second reference is made to the sharing-exchanging situation, which permits calculation to be shortened. Then from a situation of comparison of calculations, relations of proportionality among division data are derived, which will in turn provide significance to division among decimals, and in particular division by a decimal divisor. For each one of these references, an explicit procedure of operation is shown to pupils who are then required to perform them (sharing by distribution and exchange, doing chains of subtraction, dividing, setting tables for comparison of division data and results, estimating quotients, and so on). We can see here how the handbooks plan to make these references meaningful for pupils.

We have described backward and forward motions in the use of references during the progression of teaching as it is planned in the handbooks. It is clear that they do not exactly describe what will be the actual teaching taking place in the real classrooms. Furthermore, we can be sure that the references made by teachers will be wider and more complex than in the handbooks. It is clear that the largest part of the instructional activity is at the discretion of the teachers, even if they receive very detailed instructions on how to proceed. Yet the solutions given in the handbooks remain significant because they show not only the major references made, but especially how the control of these references is carried out in order to achieve the desired goals of the teaching. We can be sure that the teachers will develop very similar procedures of control. In our case, we can qualify this control as: “procedural control”, because the meaning of each reference is always reduced to some narrow task that the pupil has to perform. Nothing in that

method assures that the mathematical underlying concepts will really be involved (in a controlled manner). The handbooks propose their own solutions to didactical problems, which can be taken as a prototype of the teaching process. Of course the teachers will not follow these solutions exactly. But above all, they will infer from their reading of the handbooks the degree to which a conceptual understanding of the calculation is required for the pupils. We cannot expect them to be more rigorous than the handbooks.

### EPISTEMOLOGICAL CONCEPTIONS OF HANDBOOKS.

The analysis we have made in the last section describes how the teaching of division is planned in the mathematical handbooks of Suisse Romande. But it also gives some indications as to why such planning is done. In particular, we have mentioned the general problem of distinguishing between the general notion of mathematical operations and the calculation of these operations as a mathematical tool. This comes from general epistemological concepts which are realised through the organization of teaching. In fact this point goes back to the reform of teaching modern mathematics.

Traditional skill-oriented teaching of written division follows the enlargement of the field of application of the algorithm. The modernised one, in Suisse Romande, tries to follow a formal building of the algorithm itself. What was new in this idea? Looking back two hundred years, we find the famous arithmetic handbook of Condorcet: Moyens d'apprendre à compter sûrement et avec facilité., in which very precise descriptions of algorithms are given. Our modern authors seem to have been inspired if not directly by Condorcet's version, than by a derived one. So let us compare these versions of the presentation of division. We begin with Condorcet's text, which is written like an ex-cathedra (academic if you prefer) lecture. In the next section we will point out that in comparison, the version in the actual handbooks is an adaptation of Condorcet's version to another style of teaching and classwork, requiring more participation from the pupils, and thus more "modern". In these handbooks the lessons are organised as activities for team-work, and whole-class-discussions. But before this there is an important epistemological distinction to note.

The text of Condorcet is admirably clear. The stages of its presentation are exactly the ones which our handbooks propose, the difference lying in the way these stages are realised. Let us summarize them. The general idea is to present the algorithm of division as a structured and shortened procedure of reiterated

subtraction. Condorcet presents six different stages in order to finally reach a concise version of the algorithm.

1. The pragmatic situation of reference for dividing involves sharing of objects by distribution among a group of people. There are two possible cases which can occur, depending on which is given, the number of parts or the value of one part. But numerically speaking, they can both be solved by a succession of subtraction.

2. That succession of subtraction can be shortened. Instead of subtracting again and again the divisor from the dividend, one can subtract at the same time, according to an estimation of the quotient, the divisor times a certain multiple of ten (hundred, thousand, a.s.o...). This will determine the digits of the quotient.

3. This determination can be simplified as one observes that it is unnecessary to take into consideration, each time, all the digits of the dividend.

4. The successive determination of the digits of the quotient is made by a trial and error estimation within the table of the first ten multiples of the divisor.

5. Returning to the reference of sharing, he rediscusses the meaning of the remainder in terms of fractions, according to the two situations as the amount of fractioned objects, or as a fraction of one part.

6. Verification of result procedures.

Such a presentation proceeds like a demonstration of a tool. As Condorcet builds it up for the reader, he shows him how it functions. It is important to note that it provides an outstanding objectivity to the tool itself. Here the actual handbooks differ widely from Condorcet's ex-cathedra presentation, because they request pupils to take an active part in the construction of the algorithm, making their relation to it much more "subjective". Here we encounter another didactical problem which we will call an "objectivation" problem, the question being to transform procedures of pupils into objective tools. Let us further examine this concept.

As we have already seen, the general concept of our handbooks consists of setting up the algorithm of division. The idea is the same as introducing division as a succession of subtraction, via reference to distribution of objects among a group of people. Then it continues in the same way to make that calculation shorter and more direct, in order to reach the final concise algorithm, in other words the form of algorithm which is given in the curriculum. Yet once again the difference lies in the fact that the modern method is not satisfied by only demonstrating its concepts to pupils, but want to make them also do the entire construction of algorithm, by themselves, and through team activities. It is quite an ambitious aim, which clearly arise from the idea that knowledge must be

constructed. That statement implies that the pupils develop and work with well self-appropriated tools, that they must not be solely spectators, but actors of the construction of their algorithm as well. Paradoxically that statement interferes with another one. Modern teaching dissociates itself from traditional teaching, and proposes to place less emphasis on skills of calculation. What solution is found for such a contradiction? In our handbooks it consists of at least three closely related didactical aspects, the third one being the main one.

1. In that teaching, the emphasis is not placed on the degree of skillfulness the pupils accomplish in their learning, but on the confidence they have in the effectiveness of their procedures, and the capacity they have in reflecting upon them. This method encounters an important didactical difficulty. In the next section, we shall describe how accommodations have been made in order to bypass it. We have also noticed that in this case it is rather difficult to convince the pupil to shorten already well adapted procedures.

2. That teaching substitutes concrete manipulation skills for numerical abstract skills. For example let us compare the way these methods introduce the order of division from “left to right”, in other words from the higher range of dividend digits to the lower ones. We have seen, in point 2 of our description, that Condorcet has introduced it as a simplification of an overly long succession of subtraction, and how he related it to the estimation of the quotient. In our handbooks a specific situation is introduced for this purpose: the reference to sharing by distribution and exchange of a quantity of units of different values. The pupils are required to experiment with such sharings, and to establish that it is more convenient to proceed from higher valued units to lower valued ones. By specific exercises, and notations, the pupils are encouraged to transfer these observations to the calculation of division itself. In general we can qualify that didactical procedure in the following terms: for each aspect of the algorithm, a model is created in order to provide a “concrete” and simple experiment for the pupils. It is assumed that these substitutions of reality, and transfer of meaning can be required from the pupils. The underlying statement of this lies in an epistemological conception of mathematics, which is the third point we want to develop here.

3. Modern mathematical reform is based on criticism of traditional teaching. One such very important criticism consists of the consideration of the narrowness of traditional mathematics (which one must consider as synonymous to traditionally taught mathematics). Modernists insist on affirmations like the following: “Mathematics is not only, and not even fundamentally, a question of numbers, measures, and calculations.”; “Mathematical properties can be observed

in much wider fields of reality, than is traditionally presented.”; “One can do mathematics, and particularly algebra, almost with everything.”; “Mathematics is a language for reality.”; and so on. Our purpose is not to discuss these statements, but to analyse how they changed the didactics of mathematics itself. Since the emphasis was put on properties of numerical operations, more than on the way these properties intervene in specific mathematical tools, it stimulated didactical imagination, because according to that concept, many tricky models could be suitable to make the pupils pick out the mathematical properties underlying the mathematical situations. The claim was that a clear identification and a thorough understanding of these properties would be sufficient to provide pupils with a firm knowledge of mathematics. We now see how this third point is the turning point of the two previous ones. Pupils are intended to be mathematicians more than calculators.

The final consequences of what we want to relate to here is how our modern teaching books solve the ambiguity between an arithmetical operation and its calculations. We have already observed it. Very simply, this teaching does not deal with calculation as one of its content-notions. Of course, it remains one of the content-skills of this teaching. That concept forces some conceptual contorsions into the methodology, particularly when euclidean division (division with remainder) is presented to the teachers, in a formalised way, as an “operation from pairs of integers to pairs of integers” (which is not exact mathematically speaking), and when that operation is distinguished with the operation of “exact division”, which is “an operation from pairs of integers to a single integer, not defined everywhere”. But there are other less formal examples of these confusions: procedures of estimation of quotients are introduced, after that, they are related, in a quite abstruse way, to approximation of non integer quotients by division. In the previous section, we have already seen in detail how the study of proportionality among the factors of the division considered as an operation was substituted for the study of the division considered as a calculation. The epistemological statement of our handbooks blocks the consideration of calculation (as an objective entity), and consequently the consideration of algorithms as tools by which one can deal with calculations of decimals or rational numbers like calculations of integers. The same problems are encountered with subtraction in the second grade handbooks (F. Conne 1981), and then, at the sixth grade of primary school, with relative numbers and their calculation (rules of signs, for example).

#### THE FIFTEEN YEARS AFTER ADAPTATIONS.

As we related above, the teaching handbooks of mathematics in Suisse Romande have been evaluated and corrected before being reedited. To compare the changes between the two editions is enlightening. What has been changed for the teaching of division? In fact while they have a very different presentation, they are very similar on a mathematical point of view. They present the same organization of notions, the same progression in the teaching. The main difference consist of modifications of some of the situations of reference, and in changes in the style of classroom activities. In the first edition the general pattern for these activities was the type of problems we call in French: “problèmes-à-tiroir” (drawer-problems) in which a given situation provides a succession of joint sub-problems. It generally makes these activities long and heavy, difficult for the teachers to grasp in a single concept, and quite boring for every one concerned. Some of these activities, for example, must be developed in classrooms over a period of many lessons, which can last for a couple of days. In addition to that, these activities are very closely planned. Though these long lasting and wide ranging activities are designed for team work in the classroom, with the pupils working altogether at the construction of their knowledge, it rapidly became evident that such a teaching was much too rigid. Thus, the principal adaptations of the new edition of the handbooks has resulted in reserving the long lasting problems for less-scholastic activities called: “mathematical workshops”. The other activities have then been shortened and cut into a collection of different more particular activities, all of them relating to some specific aspect of division. We see that the same movement, which operated in the reform of mathematical teaching, has been pursued. As a matter of fact, as we have pointed out in the previous section, the modern teaching substitutes a succession of “concrete” procedural models for the traditional explanation of algorithm as a specific mathematical tool. The authors of the second edition have illustrated the succession of “explaining” models of the first edition by a genuine collection of corresponding situations. According to that the traditional word problems, which had previously almost disappeared of the handbooks, became much more numerous in the second edition. The evolution of the terminology used in the handbooks clearly reveals this. In the first grade handbook, the term “play” was used, each “play” presented a particular model of the notion to be taught; then for the other grades, “play” was changed for the term “activity”, an “activity” regrouping a succession of related “plays”; and in turn, for the second edition of the 5th and 6th grade handbooks, the term “activity” was replaced by the term of “theme”, a “theme”

regrouping the collection of situations related to the previous models (i.e. in the first edition) developed in the corresponding “activities”.

The result of these adaptations seems to simplify the presentation of mathematical notions; yet we do not think it is efficient in helping the teachers. As a matter of fact, these modifications make the didactical and the epistemological principles in accordance to which the handbooks were drafted less obvious. In comparison to the first edition of the handbooks, the corrected version of the second edition is more attractive and perhaps more suitable for immediate use, but it is also less accessible for didactical thinking.

To conclude this section, let us say that this description would not be complete if we had not mentioned that the new presentation of mathematical activities is more suitable for individualised teaching, which is the mode nowadays in Switzerland.

## CONCLUSION.

Our description gives a view of the complexity of teaching and schooling, even for such a well defined matter as traditional algorithm of calculation. During our presentation we suggested general principles to deal with it. Let us repeat two of them.

Our first conclusion concerns the analysis of the mathematical handbooks of Suisse Romande. We can now characterise this teaching of calculation as the following. It is based on the didactical principle of substitution of “concrete” procedures for the algorithm, by providing procedural models of general mathematical properties. Substitution of procedures for algorithm..., what a nice tautology! It does not escape from the “good forms” of the traditional didactic.

Before ending, we want to look back at two last questions: What can be defined as content of teaching? How shall we take into account altogether the aims of curriculum, the content of handbooks and the knowledge they induce for teachers and pupils? Our second conclusion will give a preliminary answer. During our analysis we have continually emphasized considering the content as related to the solutions of didactical problems. There is something upstream of content. But perhaps is it precisely that, which lies upstream of the obvious content, that is the real content of teaching.

We claim that it is at this “underlying” level, that the different didactical analysis of pupils productions, classrooms events, content of handbooks, teachers conceptions, and so on, can be related. Our research tends to collect factual improvements to establish that statement.