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**Unequal inequality aversion within and among  
countries and generations**

Marc FLEURBAEY, Stéphane ZUBER

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# Unequal inequality aversion within and among countries and generations

Marc Fleurbaey\*

Stéphane Zuber†

## Abstract

Suppose that, for whatever reason, it is decided that inequalities within countries are more offensive than inequalities between countries, and that inequalities between populations living together are more offensive than inequalities between generations living in different times. Can a social welfare function express that preference? We show that it is actually difficult to incorporate such a localist preference into a social welfare function, except in a limited way (i.e., from a situation of specific similarity between countries). We also show that in order to obtain such preferences, the relative size of inequality aversion within and between countries may be counter-intuitive in some relevant cases, in the sense that a greater inequality aversion may happen to be required across countries than within countries. This research highlights new social welfare functions that aggregate the outcomes of evaluations over pairs of agents.

**Keywords:** inequality aversion, transfer principle, within-country preference.

**JEL:** D04, D63, D64, D78.

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## 1 Introduction

It has been suggested that there are reasons why one may prefer transfers within countries (where individuals have strong links and share political values) to transfers between countries. For instance, Schelling (1995) famously suggested that distance in time and distance in space might justify certain degrees of receding priority: “we may prefer beneficiaries who are closer in time, in geographical distance, in culture, surely in kinship” (p. 395). In the philosophical literature, the debate about global justice and cosmopolitanism has been the occasion to hear voices justifying some lower priority for transfers across borders in defining social goals (Rawls, 2001; Nagel, 2005). Certainly, this is controversial, and very strong cosmopolitan arguments have been raised (Beitz, 2005; Brock, 2009) against such views which appear to echo extremist political discourse emanating from chauvinistic movements.

Pragmatic reasons may obviously justify showing greater concern for inequality at home than farther away. For instance, there may be greater costs in transfers across countries, and less control of how the distribution is targeted. But we are not interested in this issue as it does not bear on the social objective, only on implementation constraints. There may also be reasons to worry about relative income (or consumption) comparisons, which matter more for people’s well-being at the local level than across countries or generations. Sen (1983) famously argued that a concern for absolute poverty in capabilities and functionings can justify a concern for relative poverty in economic resources. This may justify caring more about the poor in rich countries than the equally poor in less affluent countries. This would imply prioritizing within-country transfers originating in rich countries, and between-country transfers originating in poor countries. The latter is counter-intuitive, but not so relevant to practical policy.

Regarding intergenerational inequality, authors worried about the strong demands put on earlier generations by utilitarian social welfare functions. Arrow (1999) and Dasgupta (2019) have concluded that it was hard to avoid putting some bias in favor of the present generation and against future generations, and even that an agent-relative approach could be used. That is, the current generation could use a different social objective than future generations when they will make their own decisions. This may raise issue with intertemporal consistency of planning, but we will not examine agent-relative approaches here and will instead focus on unbiased social welfare functions.<sup>1</sup>

Retaining an unbiased (not agent-relative) approach, Anthoff and Emmerling (2019) and Berger and Emmerling (1995) discuss and use nested social welfare functions with different degrees of

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<sup>1</sup>We use the term “unbiased” rather than “impartial” social welfare functions because we want to allow for social welfare functions that do not satisfy the Anonymity principle. The social welfare functions are however unbiased in the sense that they do not systematically favor a group of persons, for instance people in a given country or in a given time period.

inequality aversion for different partitions of the population, and suggest that natural orderings of the coefficients of inequality aversion can adequately reflect preferences for equality of unequal degrees. It is worth noting that, unlike in philosophy, the economic literature on such approaches is thin, probably due to the low appeal of considerations which may superficially appear similar to nationalist political discourse.

In this paper, we completely abstract from this normative debate and simply examine if a preference for localism in space (country) or time (generation) can be *coherently* embedded in a global social welfare function. In other words, suppose that, for whatever reason, it is decided that inequalities within countries are more offensive than inequalities between countries, and that inequalities between populations living together are more offensive than inequalities between generations living in different times. Can a social welfare function express that preference? To reiterate a key point, let us be very clear that we are not interested in a biased social welfare function that would simply stipulate that the citizens of, say, the United States are always more important than others (“America First”), or that present generations are always more important just because they live now rather than in the future. We are looking for social welfare functions that give preference to inequality reduction happening within countries rather than across countries, and within generations rather than across generations. So a country may give priority to their citizens, and another country to their own, but it is not the case that both countries A and B would like to prioritize the citizens of country A.

In this note, we show that it is actually difficult to incorporate such a localist preference into a social welfare function, except in a limited way (from a situation of specific similarity between countries, and sometimes for transfers below some upper bounds). We also show that, in order to obtain such preferences, the relative size of inequality aversion within and between countries may be counter-intuitive in some relevant cases, in the sense that a greater inequality aversion may happen to be required across countries than within countries. An interesting byproduct of this research is that it highlights social welfare functions that aggregate the outcomes of evaluations over pairs of agents, a rarely noticed feature of the Gini social welfare function (whereas it is well-known for the Gini coefficient of inequality). Such social welfare functions offer interesting possibilities and may deserve further attention.

## 2 Two-level inequalities

Consider first a static setting where there are two levels: countries, and individuals within countries.<sup>2</sup> Countries are labelled by  $z \in Z$ , and individuals are labelled  $i \in I_z$  in each country  $z$ . We

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<sup>2</sup>Formally, this first framework could also be interpreted as generations (time periods) instead of countries, and individuals within generations, where there is a finite number of time periods.

	Country A	Country B
Status quo	(10, 2)	(9, 2)
Transfer within country	(9, 3)	(9, 2)
Transfer between countries	(9, 2)	(9, 3)

  

	Country A	Country B
Status quo	(9, 2)	(10, 2)
Transfer within country	(9, 2)	(9, 3)
Transfer between countries	(9, 3)	(9, 2)

Table 1: Counter-example for General within-country preference

denote  $I = \cup_{z \in Z} I_z$  the set of all individuals. Let  $n = |I|$  the total population,  $n_z = |I_z|$  be the population of country  $z$ . For each individual  $i \in I$ ,  $x_i$  is the income of individual  $i$ .  $\bar{x}_z$  is the average income in country  $z$ . Let  $x_z$  denote the income distribution in country  $z$ :  $x_z = (x_i)_{i \in I_z}$ , and  $x$  the world distribution  $x = (x_z)_{z \in Z}$ .

## 2.1 A first impossibility

A natural requirement to represent the fact that we prefer within-country transfers to between-country transfers is the following, which says that a rich donor making a transfer to a poor recipient produces more good for society if the recipient is in the same country, other things equal:

**General within-country preference:** Let  $z, z' \in Z$ ,  $z \neq z'$ . If  $(i, j, k) \in I_z \times I_z \times I_{z'}$  are such that  $x_i > x_j = x_k$ , a progressive transfer from  $i$  to  $j$  is better than a progressive transfer from  $i$  to  $k$ .

This requirement is impossible to satisfy. This is easily seen in a setting with two countries and two individuals per country. Suppose the status quo is  $((10, 2), (9, 2))$  (the first pair is the distribution in the first country; the second pair in the second country). We should then prefer  $((9, 3), (9, 2))$  to  $((9, 2), (9, 3))$ . But if the status quo is  $((9, 2), (10, 2))$ , we should prefer  $((9, 2), (9, 3))$  to  $((9, 3), (9, 2))$ . We thus have a cycle: social preferences cannot be consistently defined. Table 1 summarizes the example.

## 2.2 A more general impossibility

There is a fundamental difficulty in making boundaries play a role in defining social priorities, within the framework of a social welfare approach. Consider a general class of weaker requirements that would apply within-country preference only when the distributions of income in the donor and the receiver countries are related in a specific way. Let  $R$  denote a relation between distributions,

and  $(x_z, x_{z'}) \in R$  mean that these two distributions are related according to  $R$ . For instance, it could mean that the population size is the same in  $z$  and  $z'$  and that  $x_{z'}$  is proportional to  $x_z$ .

**$R$ -restricted within-country preference:** Let  $z, z' \in Z$ ,  $z \neq z'$ , be such that  $(x_z, x_{z'}) \in R$ . If  $(i, j, k) \in I_z \times I_z \times I_{z'}$  are such that  $x_i > x_j = x_k$ , a progressive transfer from  $i$  to  $j$  is better than a progressive transfer from  $i$  to  $k$ .

Here is why such a weaker requirement is hard to satisfy unless  $R$  is very restrictive. Consider a twice continuously differentiable monotonic social welfare function  $W$  defined over the set of world distributions. Consider a transfer  $t$  from an individual  $i$  to an individual  $j$ . A Taylor expansion gives that the change in the value of  $W$  induced by the transfer  $t$  from  $i$  to  $j$  is:

$$\Delta_{i,j,t} = \left( \frac{\partial W}{\partial x_j} - \frac{\partial W}{\partial x_i} \right) t + \frac{1}{2} \left( \frac{\partial^2 W}{\partial x_i^2} + \frac{\partial^2 W}{\partial x_j^2} - 2 \frac{\partial^2 W}{\partial x_i \partial x_j} \right) t^2 + O(t^2).$$

A similar expression is obtained for a change in welfare resulting from a transfer  $t$  from  $i$  to another individual  $k$  (it suffices to replace  $\partial x_j$  by  $\partial x_k$ ). Thus, to know whether it is preferable to make the transfer  $t$  to  $j$  rather than making it to  $k$ , we need to look at the difference:

$$\Delta_{i,j,t} - \Delta_{i,k,t} = \left( \frac{\partial W}{\partial x_j} - \frac{\partial W}{\partial x_k} \right) t + \frac{1}{2} \left( \frac{\partial^2 W}{\partial x_j^2} - \frac{\partial^2 W}{\partial x_k^2} \right) t^2 - \left( \frac{\partial^2 W}{\partial x_i \partial x_j} - \frac{\partial^2 W}{\partial x_i \partial x_k} \right) t^2 + O(t^2).$$

For a small transfer  $t$ , and when  $\frac{\partial W}{\partial x_j} \neq \frac{\partial W}{\partial x_k}$ , the first term dominates the decomposition and a transfer to  $j$  is preferred to a transfer to  $k$  if and only

$$\frac{\partial W}{\partial x_j} > \frac{\partial W}{\partial x_k}.$$

The important fact is that this condition does not involve the origin of the transfer, namely, the country of the donor  $i$ . Thus, if the donor  $i$  is from  $z$ , a within-country transfer is preferred in this case, whereas if the donor is from  $z'$ , the opposite should occur. Now, suppose that  $R$  is a symmetric relation (such as proportionality and same population size). Then, one has  $(x_z, x_{z'}) \in R$  if and only if  $(x_{z'}, x_z) \in R$ , and within-country preference should apply both when the donor is from  $z$  and from  $z'$ . This would require  $\frac{\partial W}{\partial x_j} > \frac{\partial W}{\partial x_k}$  and  $\frac{\partial W}{\partial x_j} < \frac{\partial W}{\partial x_k}$  as the same time, which is impossible.

To formulate a general impossibility result formally, the following definition is needed:

**Definition 1** *Let  $R$  be a relation between allocations and  $W$  be a differentiable social welfare function representing social judgements about allocations. We say that  $R$  is minimally symmetric and*

rich with respect to  $W$  if there exist a conceivable world with at least two countries  $z, z' \in Z$ , and a possible allocation  $x$  such that  $(x_z, x_{z'}) \in R$ ,  $(x_{z'}, x_z) \in R$ ,  $x_i = x_{i'} > x_j = x_k$ , for some individuals  $(i, j) \in I_z$ ,  $(i', k) \in I_{z'}$ , and  $\partial W/\partial x_j \neq \partial W/\partial x_k$ .

We obtain the next proposition, which summarizes the above argument:<sup>3</sup>

**Proposition 1** *Let  $W$  be a social welfare function and  $R$  be a relation. If  $W$  is twice continuously differentiable and  $R$  is minimally symmetric and rich with respect to  $W$ , then  $W$  cannot satisfy  $R$ -restricted within-country preference.*

Proposition 1 is fairly general as relation  $R$  does not need to contain many allocations (we just need two country allocations related symmetrically), and we only need differentiability of  $W$  at one point (which means that this result carries over to rank-weighted welfare functions in the spirit of the Gini social welfare function).

This section implies that in order to satisfy  $R$ -restricted within-country preference, one should specify  $R$  in such a way that either  $R$  is asymmetric (i.e., bears on countries in certain unequal relative positions), or  $\frac{\partial W}{\partial x_j} = \frac{\partial W}{\partial x_k}$  whenever  $(x_z, x_{z'}) \in R$  and  $x_j = x_k$ . In the remainder of this paper we explore an array of possibilities along these two veins.

### 2.3 Social welfare functions

In order to explore possibility results, one needs to define social orderings which give a role to country boundaries without abandoning a basic form of unbiasedness, according to which no individual within a country, or country in the world, is given a special position only due to their label  $i$  or  $z$ .

Here we will focus on three classes of ordering represented by the following social welfare functions (SWF):

$$W^* = \left( \sum_{z \in Z} \frac{n_z}{n} \left( \frac{1}{n_z} \sum_{i \in I_z} x_i^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} \right)^{\frac{1}{1-\beta}}$$

$$W^{**} = \sum_{i, j \in I} \left( \frac{x_i^{1-\gamma_{ij}} + x_j^{1-\gamma_{ij}}}{2} \right)^{\frac{1}{1-\gamma_{ij}}}$$

$$W^{***} = \sum_{i, j \in I} \gamma_{ij} \left( \frac{x_i^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}}$$

where  $\gamma_{ij} = \alpha$  if  $i, j \in I_z$  for some  $z \in Z$ , and  $\beta$  otherwise.

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<sup>3</sup>We warmly thank a referee for suggesting this line of reasoning.



Function  $W^*$  has been proposed by Anthoff and Emmerling (2019)<sup>4</sup> in order to disentangle within-country and between-country inequality aversion. Functions  $W^{**}, W^{***}$  are new, as far as we know, and embody the intuitive idea that either the degree of inequality aversion between any pair of individuals can depend on their belonging to the same country or not ( $W^{**}$ ), or that the weight of the inequality between any pair of individuals can also depend on the same ( $W^{***}$ ).

The  $W^{**}, W^{***}$  SWFs are actually Atkinson-type variants of the Gini SWF. Indeed, the Gini SWF can be written as

$$W_{\text{Gini}} = \frac{1}{2n^2} \sum_{i,j \in I} \frac{x_i + x_j}{2} (1 - \text{Gini}\{x_i, x_j\})$$

where  $\text{Gini}\{x_i, x_j\} = \frac{1}{4} |x_i - x_j| / \frac{x_i + x_j}{2}$ , which applies to  $x_i, x_j$  the general formula of the Gini coefficient:

$$\text{Gini} = \frac{1}{2n^2 \bar{x}} \sum_{i,j \in I} |x_i - x_j|.$$

With equal parameters  $\gamma_{ij} = \gamma$ , the functions  $W^{**}, W^{***}$  would both be proportional to

$$\sum_{i,j \in I} \frac{x_i + x_j}{2} (1 - \text{Atk}_\gamma \{x_i, x_j\})$$

where  $\text{Atk}_\gamma \{x_i, x_j\}$  is the Atkinson inequality index with inequality aversion coefficient  $\gamma$ :

$$\text{Atk}_\gamma = 1 - \left( \frac{1}{n} \sum_{i \in I} \left( \frac{x_i}{\bar{x}} \right)^{1-\gamma} \right)^{\frac{1}{1-\gamma}}.$$

Remark that there are also generalized Gini inequality measures that have been proposed for the case of multiple goods (Gajdos and Weymark, 2005). However, the framework here is different as we do not necessarily have the same number of individuals per country (while there is the same number of goods for each individual in the multiple goods framework). The dyadic social welfare functions<sup>5</sup> that we consider here have not been studied before in the literature as far as we know. It may be that such functions have interesting properties that have nothing to do with localism, but we leave it to future research. For the present paper, they are convenient to consider because they permit to isolate inequalities between pairs and to put arbitrary weight on those inequalities. This offers a lot of flexibility, especially when we consider three levels of group in Section 3.

<sup>4</sup>Blackorby, Donaldson and Auersperg (1981) used a similar nesting of equally-distributed equivalent values in the study of the decomposition of inequality measurement between subgroups. Maasoumi (1986) also looked at similar generalized entropy measured for inequality with multiple goods where we can disentangle the inequality coming from each good in special cases.

<sup>5</sup>I.e., social welfare functions that evaluate social welfare over pairs before aggregating.

## 2.4 A paradoxical possibility for identical countries

Consider a version of  $R$ -restricted within-country preferences, involving the relation  $R_0$  which holds only when the two countries have identical distributions (including population size).

**$R_0$ -restricted within-country preference:** Let  $z, z' \in Z$ ,  $z \neq z'$ , have the same size and same income distribution. If  $(i, j, k) \in I_z \times I_z \times I_{z'}$  are such that  $x_i > x_j = x_k$ , a progressive transfer from  $i$  to  $j$  is better than a transfer from  $i$  to  $k$ .

Intuitively, one should think that within-country preference will be satisfied if  $\alpha > \beta$ , i.e., if inequality aversion is stronger within countries than across countries. In fact, the opposite is true, as stated in this proposition.

**Proposition 2**  $W^*$  and  $W^{***}$  satisfy  $R_0$ -restricted within-country preference if and only if  $\alpha < \beta$ , whereas  $W^{**}$  does not satisfy it for any configuration of parameters.

**Proof.** See the appendix ■

In order to get some intuition for the result about  $W^*$  and  $W^{***}$ , consider that the two countries are assumed to have identical distributions initially. Therefore a transfer across countries generates inequality between the two countries. A strong aversion to inequality between countries therefore is key to downgrade transfers between equal countries. This result also suggests that  $W^{**}$  is not a promising SWF, and this will be confirmed in other results of this section.

There is a formal similarity between the result for  $W^*$  and the literature on multidimensional inequality. Consider the simple case of two-individual countries, depicted in Table 2. The transfer between countries produces a situation in which country B enjoys the highest values for both the high-income and low-income positions and country A the lowest, whereas the transfer within A produces a more mixed situation. If individual positions were goods and countries were individuals, moving from the latter configuration to the former would be a “correlation-increasing transfer”, giving an individual the best component of the two initial individual bundles in each dimension. It can be deduced from (Tsui, 1999, Th. 2 and Eq. (3)) that a separable function like  $W^*$  decreases under such transfers provided the individual function (i.e.,  $\left(\frac{\sum_{i \in I_z} x_i^{1-\alpha}}{n_z}\right)^{\frac{1-\beta}{1-\alpha}}$  in the case of  $W^*$ ) is strictly subadditive (for the case  $\beta < 1$ ), which is equivalent to its cross-derivatives being negative. One checks that this indeed means that  $\alpha < \beta$ .

## 2.5 Unequal countries

Those who advocate national preference actually have in mind countries with unequal distributions. Is it possible, in spite of the impossibility stated at the beginning, to satisfy such a preference

	Country A	Country B
Status quo	(10, 2)	(10, 2)
Transfer within country	(9, 3)	(10, 2)
Transfer between countries	(9, 2)	(10, 3)

Table 2: Correlation-increasing majorization in this model

between countries at unequal levels of income? Here is one tentative formulation, which restricts national preference to the case in which the recipient country has the same distribution as the other, but scaled down by a single factor. In this case, the recipient in the poorer country has a higher rank in the distribution, if the two potential recipients must have the same income initially. Let us introduce the relation  $R_1$  such that  $(z, z') \in R_1$  when they have the same population size and the distribution in  $z'$  is a proportional reduction of the distribution in  $z$ .

**$R_1$ -restricted within-country preference :** Let  $z, z' \in Z$ ,  $z \neq z'$ , have the same size and different income distributions that are proportional to one another,  $z'$  being strictly poorer by a factor  $\lambda < 1$ . If  $(i, j, k) \in I_z \times I_z \times I_{z'}$  are such that  $x_i > x_j = x_k$ , a progressive transfer of an amount  $\delta \leq (1 - \lambda)x_i$  from  $i$  to  $j$  is better than a transfer from  $i$  to  $k$ .

The restriction on the amount transferred is meant to keep the donor richer than her  $\lambda$ -poorer counterpart in the poorer country. With this condition, more intuitive results are obtained.

**Proposition 3**  $W^*$  and  $W^{***}$  satisfy  $R_1$ -restricted within-country preference if and only if  $\alpha > \beta$ , whereas  $W^{**}$  does not satisfy it for any configuration of parameters.

**Proof.** See the appendix. ■

The proof also shows that if the recipient country is at least as rich as the donor country ( $\lambda \geq 1$ ), a preference for a within-country transfer obtains only if  $\alpha < \beta$ , in the case of  $W^*$  and  $W^{***}$ .

In a nutshell, in order to implement national preference in transfers, international inequalities take precedence when considering transfers to another country that is not poorer than the country of the donor, and it is only when the donor is in the richer country that internal inequalities should matter more. An American donating to a poor Indian should rather give to an equally poor American if national inequalities loom large in the social welfare function, whereas an Indian donating to a poor American should rather give to an equally poor Indian if, instead, international inequalities loom large. This is rather intuitive, again. An Indian giving to a poor American is worsening international inequalities, whereas if national inequalities mattered a lot, the poor

American, living among a richer population, would deserve greater priority than an equally poor Indian living among less rich fellow citizens.

### 3 Three-level inequalities

If we want to apply these concepts to an inter-temporal framework, we need to introduce a third layer of differentiation, namely generations. Let us introduce generations in addition to countries and individuals. Let  $I_{gz}$  denote the population of generation  $g \in G$  in country  $z \in Z$ . Populations sizes are denoted  $n_{gz}, n_g, n_z$  for country-generation (i.e., people belonging to the same country and same generation), generation, and country, respectively.

For the hierarchical approach with  $W^*$ , the question is then to decide in which order we want to aggregate welfare. There are two options: treating generations (of all countries) as the proximate group above the current population of the country, or treating the country (with all its generations) as the proximate group. In the first case, we first aggregate across countries at a given period in time and then across generations. In the second case, we first consider each country and aggregate across generations, and then we aggregate across countries.

There are therefore two approaches:

- Generation approach:

$$W^* = \left( \frac{1}{n} \sum_{g \in G} n_g \left( \frac{1}{n_g} \sum_{z \in Z} n_{gz} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{1-\beta}{1-\alpha}} \right)^{\frac{1-\gamma}{1-\beta}} \right)^{\frac{1}{1-\gamma}}$$

- Country approach:

$$W^* = \left( \frac{1}{n} \sum_{z \in Z} n_z \left( \frac{1}{n_z} \sum_{g \in G} n_{gz} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{1-\beta}{1-\alpha}} \right)^{\frac{1-\gamma}{1-\beta}} \right)^{\frac{1}{1-\gamma}}$$

In both approaches,  $\alpha$  measures inequality aversion within country-generations, while  $\beta, \gamma$  measure inequality aversion across countries and generations, respectively, for the generations approach, whereas they measure inequality over generations and countries, respectively, for the country approach.

In this section we focus on marginal transfers and first-order analysis for simplicity.

Consider the most relevant case of a transfer going to a poorer country-generation that belongs to a poorer country, but also to a richer generation (living in the future). Then, in order to favor

transfers within the country-generation over such an international-intergenerational transfer, one must have  $\beta < \alpha, \gamma$  with the generation approach, and  $\alpha > \beta > \gamma$  with the country approach. The proof, and a general analysis of all possible inequality patterns across donor and recipient country-generations, is provided in the appendix. Intuitively, since, with the generation approach,  $\beta$  represents inequality aversion over countries (within every generation), such inequality aversion must be low in order to avoid prioritizing international transfers to poor countries. In the country approach,  $\beta$  represents inequality aversion over generations (within every country), and it must be sufficiently high in order to avoid transfers toward richer generations, but still lower than inequality aversion among individuals within a country-generation.

The weighted bilateral approach  $W^{***}$  can still be defined by the formula

$$W^{***} = \sum_{i,j \in I} \gamma_{ij} \left( \frac{x_i^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}},$$

where the weights  $\gamma_{ij}$  can depend on whether  $i, j$  belong to the same country or the same generation. Let us start with the simple case in which the parameters are defined as follows:  $\gamma_{ij} = \alpha$  if they belong to the same country and same generation,  $\gamma_{ij} = \beta$  for different country but same generation, and these become  $\alpha', \beta'$  when they belong to different generations. Then, under some assumptions detailed in the appendix, an international transfer toward a poorer country in a richer generation will be considered less appealing than an internal transfer within a country-generation if  $\alpha' > \beta' > \alpha, \beta$ . This confirms the paradoxical pattern in which distant pairs, in time in this case, much have greater weight than close pairs. A detailed analysis is provided in the appendix.

It is important to note that the  $W^{***}$  approach provides more parametric flexibility than nested social welfare functions. Keeping its general formula unchanged, the parameters  $\gamma_{ij}$  can be a decreasing function of the distance between  $i$  and  $j$  in time and space. When this decreasing profile of parameters is adopted, the correlation between parameters  $\gamma_{ij}$  and  $x_i$  becomes an essential determinant of the marginal social value of  $x_j$ . Indeed, one computes (see the appendix for further details)

$$\frac{\partial W^{***}}{\partial x_j} \propto \sum_{i \in I} \gamma_{ij} \left( \left( \frac{x_i}{x_j} \right)^{1-\gamma} + 1 \right)^{\frac{\gamma}{1-\gamma}}.$$

Thus, one obtains that a recipient  $j$  who is on average closer to rich people (i.e., has a positive correlation between  $\gamma_{ij}$ , which increases with proximity to  $i$ , and  $x_i$ ) should receive priority over one who is on average less close to them. If donors belong to rich groups, then this implies favoring proximate transfers.

Importantly, this has nothing to do with proximity with the donor per se, it only derives from

the fact that  $j$  is close to rich people. If the donor belonged to a poor population, distant transfers would be favored because greater priority is given to recipients who are close to rich groups. In particular, if one is interested in investments for future generations and growth renders them richer than the donor, then one sees that investments for the future will be favored if the parameters  $\gamma_{ij}$  decrease with time distance, which is the opposite of the intuitive idea that giving less importance to the future should undermine such investments. It is due to the fact that  $\gamma_{ij}$  decreasing with time does not mean that the future counts less, but that time distance to rich people makes a potential recipient in the present less of a social priority than a potential recipient in the future who is closer to the rich future generations.

In conclusion, with  $W^{***}$ , for transfers within country-generations to be favored generally, in a context of growth, it is necessary that weights increase with time distance, rather than the other way around.

## 4 Conclusion

It is far from easy to incorporate a preference for transfers that are local rather than distant, in space and time, in a social welfare function that is otherwise not agent-relative, i.e., that does not simply stipulate that some individuals are more important than others, in a “My Country First” fashion.

This is perhaps the occasion to come back to the normative debate. One key argument of the advocates of localist preferences is that social justice requires a framework of mutual cooperation and solidarity and that such a framework is stronger at the local (esp. the national) level than at the global level. Cosmopolitan critics retort that this argument puts the cart before the horse, since the normative debate should be about principles guiding the design of institutions rather than be contingent on the existing institutional framework. According to them, the right question is not: Can there be global justice without a global state? Instead, it should be: Does global justice require specific global institutions?

Our analysis does not shed light on this debate, but it shows that a global evaluation criterion, in the form of a social welfare function, cannot easily condone a localist preference. However, it is very easy, for a fully unbiased and not localist social welfare function, to prefer implementing local transfers when costs are attached to distant transfers. Just as parents are, on average, better equipped to take care of their own children than their neighbors’, solidarity mechanisms are smoother at the local level and have lower transaction costs. This observation is very similar to the premise of the advocates of localism. But economists immediately recognize that it has to do with feasibility constraints, not with the definition of the normative goals for society. Local solidarity

can be justified in practice without relying on localist normative principles.

However, in the case of intergenerational transfers, transaction costs for transfers across distant generations (in the form of uncertainty about transmission through intermediate generations) are generally more than compensated by the returns to investment. Such returns, for typical utilitarian social welfare functions, demand huge savings efforts from earlier generations. But egalitarian social welfare functions (e.g., of the maximin type) are, on the contrary, criticized for killing growth. Obviously, with substantial but finite inequality aversion, one can obtain intermediate and attractive conclusions about the desired level of savings effort.

All in all, we are skeptical about the need to introduce localist preferences in social welfare functions. Standard social welfare criteria are perfectly able to yield palatable conclusions about local solidarity and savings efforts demanded on populations. In contrast, localist social preferences appear not only hard to justify. They are, to start with, hardly consistent.

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## References

- Anthoff D., J. Emmerling 2019, “Inequality and the social cost of carbon,” *Journal of the Association of Environmental and Resource Economists* 6: 29–59.
- Arrow K.J. 1999, “Discounting, morality, and gaming,” in P.R. Portney and J.P. Weyant (eds.), *Discounting and Intergenerational Equity*, New York: Resources for the Future.
- Beitz C.R. 2005, “Cosmopolitanism and global justice,” *Journal of Ethics* 9: 11–27.
- Berger L., J. Emmerling 2020, “Welfare as equity equivalents,” *Journal of Economic Surveys* 34: 727–752.

- Blackorby C., D. Donaldson, M. Auersperg 1981, “A new procedure for the measurement of inequality within and among population subgroups,” *Canadian Journal of Economics* 14: 665–685.
- Brock G. 2009, *Global Justice: A Cosmopolitan Account*, Oxford University Press.
- Dasgupta P. 2019, *Time and The Generations. Population Ethics for A Diminishing Planet*, New York: Columbia University Press.
- Gajdos T., J. Weymark 2005, “Multidimensional generalized Gini indices,” *Economic Theory* 26(3): 471–496.
- Maasoumi E. 1986, “The Measurement and Decomposition of Multi-Dimensional Inequality,” *Econometrica* 54(4): 991–997.
- Nagel T. 2005, “The problem of global justice,” *Philosophy and Public Affairs* 33: 113–147.
- Rawls J. 2001, *The Law of Peoples*, Cambridge, Mass.: Harvard University Press.
- Tsui K.-Y. 1999, “Multidimensional Inequality and Multidimensional Generalized Entropy Measures: An Axiomatic Derivation,” *Social Choice and Welfare*, 16 (1): 145–157.
- Schelling, T. 1995, “Intergenerational Discounting,” *Energy Policy* 23: 395–401.
- Sen A.K. 1983, “Poor, relatively speaking,” *Oxford Economic Papers* 35: 153–169.

## Appendix: Proofs and detailed results

As in the formulation of within-country preference requirements, the key notations involve  $z, z' \in Z$ ,  $z \neq z'$ , and  $(i, j, k) \in I_z \times I_z \times I_{z'}$ , such that  $x_i > x_j = x_k$ .

### $R_0$ -restricted within-country preference

*Case 1:  $W^*$ .*

Let  $W^*(i \rightarrow j)$  denote the value of  $W^*$  when  $i$  gives  $\delta$  to  $j$ , and  $W^*(i \rightarrow k)$  when  $i$  gives to  $k$ . When  $\beta < 1$ , one has  $W^*(i \rightarrow j) > W^*(i \rightarrow k)$  if and only if (after eliminating redundant terms and taking account of the fact that the distribution in  $z'$  is the same as in  $z$ , so that one can write



everything in terms of  $z$ 's population):<sup>6</sup>

$$\begin{aligned} & \left( (x_i - \delta)^{1-\alpha} + (x_j + \delta)^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} + \left( x_i^{1-\alpha} + x_j^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} > \\ & \left( (x_i - \delta)^{1-\alpha} + x_j^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} + \left( x_i^{1-\alpha} + (x_j + \delta)^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}}, \end{aligned}$$

or, rearranging the terms, if and only if  $f(x_i - \delta) > f(x_i)$ , where:

$$f(y) = \left( y^{1-\alpha} + (x_j + \delta)^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} - \left( y^{1-\alpha} + x_j^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}}.$$

When  $\beta > 1$ , a necessary and sufficient condition is  $f(x_i - \delta) < f(x_i)$ .

A necessary and sufficient condition for  $W^*(i \rightarrow j) > W^*(i \rightarrow k)$  is therefore that  $f$  be decreasing when  $\beta < 1$  (resp. increasing when  $\beta > 1$ ). The derivative of  $f$  with respect to  $y$  is equal to

$$(1 - \beta) \left[ \left( y^{1-\alpha} + (x_j + \delta)^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{\alpha-\beta}{1-\alpha}} - \left( y^{1-\alpha} + x_j^{1-\alpha} + \sum_{h \in I_z \setminus \{i,j\}} x_h^{1-\alpha} \right)^{\frac{\alpha-\beta}{1-\alpha}} \right] x_i^{-\alpha}.$$

Subcase 1:  $\beta < 1$ . Then  $W^*(i \rightarrow j) > W^*(i \rightarrow k)$  if and only if  $f$  is decreasing hence  $\alpha < \beta$ .

Subcase 2:  $\beta > 1$ . Then  $W^*(i \rightarrow j) > W^*(i \rightarrow k)$  if and only if  $f$  is increasing hence  $\alpha < \beta$ .

*Case 2:  $W^{**}$ .*

It suffices to look at marginal transfers: if the properties does not work for marginal transfers, then it does not work in general, which is what we want to prove. Consider the decomposition (for a transfer  $t$  from  $i$  to  $j$ ):

$$dW = \left( \frac{\partial W}{\partial x_j} - \frac{\partial W}{\partial x_i} \right) t + \frac{1}{2} \left( \frac{\partial^2 W}{\partial x_i^2} + \frac{\partial^2 W}{\partial x_j^2} - 2 \frac{\partial^2 W}{\partial x_i \partial x_j} \right) t^2 + O(t^2).$$

When  $z, z'$  have identical distributions, necessarily  $\frac{\partial W}{\partial x_j} = \frac{\partial W}{\partial x_k}$  and  $\frac{\partial^2 W}{\partial x_j^2} = \frac{\partial^2 W}{\partial x_k^2}$  since there is reason to distinguish  $j$  from  $k$  for an unbiased SWF, so that it is better to do a small transfer from  $i$  to  $j$

<sup>6</sup>The inequality is in the other direction when  $\beta > 1$ .

rather than from  $i$  to  $k$  if and only if:

$$\frac{\partial^2 W}{\partial x_i \partial x_j} < \frac{\partial^2 W}{\partial x_i \partial x_k}.$$

For  $W^{**}$ , one has

$$\frac{\partial W^{**}}{\partial x_i} = \sum_{j \in I_{z_i}} \left( \frac{x_i^{1-\alpha} + x_j^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} \frac{x_i^{-\alpha}}{2} + \sum_{j \notin I_{z_i}} \left( \frac{x_i^{1-\beta} + x_j^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} \frac{x_i^{-\beta}}{2}$$

Thus, if  $i, j$  are in the same country,

$$\frac{\partial^2 W^{**}}{\partial x_i \partial x_j} = \alpha \frac{x_i^{-\alpha} x_j^{-\alpha}}{4} \left( \frac{x_i^{1-\alpha} + x_j^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}-1}$$

whereas for different countries,

$$\frac{\partial^2 W^{**}}{\partial x_i \partial x_k} = \beta \frac{x_i^{-\beta} x_k^{-\beta}}{4} \left( \frac{x_i^{1-\beta} + x_k^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}-1}$$

These expressions are not monotonic in  $\alpha$  and  $\beta$ . One computes

$$\begin{aligned} & \frac{\partial^3 W^{**}}{\partial \alpha \partial x_i \partial x_j} \propto 1 - \alpha^2 \ln(x_i x_j) \\ & + \alpha \left( \frac{x_i^{1-\alpha} + x_j^{1-\alpha}}{2} \right)^{\frac{2\alpha-1}{1-\alpha}} \left[ \frac{1}{(1-\alpha)^2} \ln \left( \frac{x_i^{1-\alpha} + x_j^{1-\alpha}}{2} \right) - \frac{2\alpha-1}{2(1-\alpha)} \left( x_i^{1-\alpha} \ln x_i + x_j^{1-\alpha} \ln x_j \right) \right], \end{aligned}$$

where the right-hand side expression is close to 1 when  $\alpha$  is small and is dominated by  $-\alpha^2 \ln(x_i x_j)$  when  $\alpha$  is large. Thus, typically (in particular for  $x_i x_j > 1$ ), the above expressions  $\frac{\partial^2 W^{**}}{\partial x_i \partial x_j}$  and  $\frac{\partial^2 W^{**}}{\partial x_i \partial x_k}$  have an inverted U shape with respect to  $\alpha$  and  $\beta$ , respectively. Moreover, the greater the gap between  $x_i$  and  $x_j = x_k$ , the smaller the value of the parameter  $\alpha$  or  $\beta$  at the peak. Therefore, for a small gap between  $x_i$  and  $x_j = x_k$ , these expressions are increasing in their parameter, leading to within-country preference being satisfied only for  $\alpha < \beta$ , whereas for a large gap, within-country preference will be satisfied for  $\alpha > \beta$ .

*Case 3:  $W^{***}$ .* Similarly as before, let  $W^{***}(i \rightarrow j)$  denote the value of  $W^{***}$  when  $i$  gives  $\delta$  to

$j$ , and  $W^{***}(i \rightarrow k)$  when  $i$  gives to  $k$ . After elimination of identical terms, one computes

$$W^{***}(i \rightarrow j) - W^{***}(i \rightarrow k) = 2(\alpha - \beta) \left[ \left( \left( \frac{(x_i - \delta)^{1-\gamma} + (x_j + \delta)^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} - \left( \frac{(x_i - \delta)^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} \right) - \left( \left( \frac{x_i^{1-\gamma} + (x_j + \delta)^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} - \left( \frac{x_i^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} \right) \right]. \quad (1)$$

The expression

$$\left( \frac{y^{1-\gamma} + (x_j + \delta)^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} - \left( \frac{y^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}}$$

is increasing in  $y$  because its derivative with respect to  $y$  is equal to

$$\left[ \left( \frac{y^{1-\gamma} + (x_j + \delta)^{1-\gamma}}{2} \right)^{\frac{\gamma}{1-\gamma}} - \left( \frac{y^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{\gamma}{1-\gamma}} \right] x_i^{-\gamma},$$

which is positive, and therefore the term in brackets in (1) is negative, implying that  $W^{***}(i \rightarrow j) - W^{***}(i \rightarrow k) > 0$  if and only if  $\alpha < \beta$ .

### **$R_1$ -restricted within-country preference**

Consider a transfer toward a country with a distribution that is  $\lambda$ -rescaled compared to the distribution of the donor country. The case  $\lambda = 1$  is covered by within-country preference, thus we focus here on  $\lambda \neq 1$ . The axiom  $R_1$ -restricted within-country preference itself focuses on  $\lambda < 1$ , but it is worth examining the more general case.

*Case 1:  $W^*$ .*

Assume that  $\beta < 1$ .<sup>7</sup> One has  $W^*(i \rightarrow j) > W^*(i \rightarrow k)$  if and only if (after eliminating redundant terms and taking account of the fact that the distribution in  $z'$  is a proportional reduction of the distribution in  $z$ , so that one can write everything in terms of  $z$ 's population, letting  $k_z$  denote an individual in  $z$  whose income is such that  $x_k = x_j = \lambda x_{k_z}$ ):

---

<sup>7</sup>As for the case of  $R_0$ -restricted within-country preference, the whole reasoning is easily adapted for the case  $\beta > 1$  by reverting the inequalities and monotonies when necessary.

$$\begin{aligned}
& \left( (x_i - \delta)^{1-\alpha} + (x_j + \delta)^{1-\alpha} + \left(\frac{x_j}{\lambda}\right)^{1-\alpha} + \sum_{h \in I_z \setminus \{i, j, k_z\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} \\
& + \left( (\lambda x_i)^{1-\alpha} + (\lambda x_j)^{1-\alpha} + x_j^{1-\alpha} + \sum_{h \in I_z \setminus \{i, j, k_z\}} (\lambda x_h)^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} > \\
& \left( (x_i - \delta)^{1-\alpha} + x_j^{1-\alpha} + \left(\frac{x_j}{\lambda}\right)^{1-\alpha} + \sum_{h \in I_z \setminus \{i, j, k_z\}} x_h^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} \\
& + \left( (\lambda x_i)^{1-\alpha} + (\lambda x_j)^{1-\alpha} + (x_j + \delta)^{1-\alpha} + \sum_{h \in I_z \setminus \{i, j, k_z\}} (\lambda x_h)^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}}. \tag{2}
\end{aligned}$$

Write  $A = (x_i - \delta)^{1-\alpha} + \left(\frac{x_j}{\lambda}\right)^{1-\alpha} + \sum_{h \in I_z \setminus \{i, j, k_z\}} x_h^{1-\alpha}$ ,  $B = (\lambda x_i)^{1-\alpha} + (\lambda x_j)^{1-\alpha} + \sum_{h \in I_z \setminus \{i, j, k_z\}} (\lambda x_h)^{1-\alpha}$ . This can be written:

$$\begin{aligned}
& \left( A + (x_j + \delta)^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} - \left( A + (x_j)^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} > \left( B + (x_j + \delta)^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} - \left( B + (x_j)^{1-\alpha} \right)^{\frac{1-\beta}{1-\alpha}} \\
\iff & (1 - \beta) \int_{x_j}^{x_j + \delta} (A + z^{1-\alpha})^{\frac{\alpha-\beta}{1-\alpha}} dz > (1 - \beta) \int_{x_j}^{x_j + \delta} (B + z^{1-\alpha})^{\frac{\alpha-\beta}{1-\alpha}} dz.
\end{aligned}$$

When  $x_i - \delta \geq \lambda x_i$  (which implies  $\lambda < 1$ ), and given that  $x_h > \lambda x_h$  and  $\frac{x_h}{\lambda} > x_h$ , we necessarily have that  $(A + z^{1-\alpha})^{\frac{1-\beta}{1-\alpha}} > (B + z^{1-\alpha})^{\frac{1-\beta}{1-\alpha}}$ . So, when  $\beta < 1$  a necessary and sufficient condition for the above inequality to hold is that  $\alpha > \beta$ .

When  $\lambda > 1$ , and  $x_i - \delta \leq \lambda x_i$  we obtain the same inequality but  $(A + z^{1-\alpha})^{\frac{1-\beta}{1-\alpha}} < (B + z^{1-\alpha})^{\frac{1-\beta}{1-\alpha}}$ . So the necessary and sufficient condition for  $W^*(i \rightarrow j) > W^*(i \rightarrow k)$  is that  $\alpha < \beta$ .

*Case 2:  $W^{**}$ .*

As before, in order to have a counter-example, it suffices to look at marginal transfers. One has, taking account of the fact that  $x_j = x_{j'}$  and that to every  $k \in I_{z_j}$ , there is  $k' \in I_{z_{j'}}$  such that  $x_{k'} = \lambda x_k$ :

$$\begin{aligned}
\frac{\partial W^{**}}{\partial x_j} &= \sum_{k \in I_{z_j}} \left( \frac{x_j^{1-\alpha} + x_k^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} \frac{x_j^{-\alpha}}{2} + \sum_{k \in I_{z_j}} \left( \frac{x_j^{1-\beta} + (\lambda x_k)^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} \frac{x_j^{-\beta}}{2} + \sum_{k \notin I_{z_j} \cup I_{z_{j'}}} \left( \frac{x_j^{1-\beta} + x_k^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} \frac{x_j^{-\beta}}{2} \\
\frac{\partial W^{**}}{\partial x_{j'}} &= \sum_{k \in I_{z_j}} \left( \frac{x_j^{1-\alpha} + (\lambda x_k)^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} \frac{x_j^{-\alpha}}{2} + \sum_{k \in I_{z_j}} \left( \frac{x_j^{1-\beta} + x_k^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} \frac{x_j^{-\beta}}{2} + \sum_{k \notin I_{z_j} \cup I_{z_{j'}}} \left( \frac{x_j^{1-\beta} + x_k^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} \frac{x_j^{-\beta}}{2}
\end{aligned}$$

so that

$$\begin{aligned}
\frac{\partial W^{**}}{\partial x_j} - \frac{\partial W^{**}}{\partial x_{j'}} &= \frac{x_j^{-\alpha}}{2} \sum_{k \in I_{z_j}} \left( \left( \frac{x_j^{1-\alpha} + x_k^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} - \left( \frac{x_j^{1-\alpha} + (\lambda x_k)^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} \right) \\
&\quad - \frac{x_j^{-\beta}}{2} \sum_{k \in I_{z_j}} \left( \left( \frac{x_j^{1-\beta} + x_k^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} - \left( \frac{x_j^{1-\beta} + (\lambda x_k)^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} \right) \\
&= \left( \frac{x_j^{-\alpha}}{2} - \frac{x_j^{-\beta}}{2} \right) \sum_{k \in I_{z_j}} \left( \left( \frac{x_j^{1-\alpha} + x_k^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} - \left( \frac{x_j^{1-\alpha} + (\lambda x_k)^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} \right) \\
&\quad + \frac{x_j^{-\beta}}{2} \left( \left( \frac{x_j^{1-\alpha} + x_k^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} - \left( \frac{x_j^{1-\alpha} + (\lambda x_k)^{1-\alpha}}{2} \right)^{\frac{\alpha}{1-\alpha}} - \left( \frac{x_j^{1-\beta} + x_k^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} + \left( \frac{x_j^{1-\beta} + (\lambda x_k)^{1-\beta}}{2} \right)^{\frac{\beta}{1-\beta}} \right)
\end{aligned}$$

Take for instance the case  $\lambda < 1$ . The first row is negative if  $\alpha > \beta$  and  $x_j > 1$ , but positive if one of these inequalities is reversed, and similar dependence on the level of  $x$  occurs for the second row. This shows once again that with  $W^{**}$ , it is hard to obtain any systematic result.

*Case 3:  $W^{***}$ .*

One has  $W^{***}(i \rightarrow j) > W^{***}(i \rightarrow k)$  if and only if (after eliminating redundant terms):

$$\begin{aligned}
&\alpha \sum_{h \in I_z \setminus \{i, j\}} \left( \frac{(x_j + \delta)^{1-\gamma} + x_h^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} + \alpha \left( \frac{(x_j + \delta)^{1-\gamma} + (x_i - \delta)^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} \\
&\quad + \beta \sum_{h \in I_{z'} \setminus \{k\}} \left( \frac{(x_j + \delta)^{1-\gamma} + x_h^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} + \beta \left( \frac{(x_j + \delta)^{1-\gamma} + x_k^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} > \\
&\beta \sum_{h \in I_z \setminus \{i, j\}} \left( \frac{(x_k + \delta)^{1-\gamma} + x_h^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} + \beta \left( \frac{(x_k + \delta)^{1-\gamma} + (x_i - \delta)^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} \\
&\quad + \beta \left( \frac{(x_k + \delta)^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} + \alpha \sum_{h \in I_{z'} \setminus \{k\}} \left( \frac{(x_k + \delta)^{1-\gamma} + x_h^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}},
\end{aligned}$$

i.e., if and only if

$$(\alpha - \beta) \left[ \sum_{h \in I_z \setminus \{i, j\}} \left( \frac{(x_j + \delta)^{1-\gamma} + x_h^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} + \left( \frac{(x_j + \delta)^{1-\gamma} + (x_i - \delta)^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} \right]$$

$$- \sum_{h \in I_{z'} \setminus \{k\}} \left[ \left( \frac{(x_j + \delta)^{1-\gamma} + x_h^{1-\gamma}}{2} \right)^{\frac{1}{1-\gamma}} \right] > 0.$$

The term in brackets is positive when for all  $h \in I_z \setminus \{j\}$ ,  $h' \in z'$  such that  $x_{h'} = \lambda x_h$ , one still has  $x_h + t_h \geq x_{h'}$  after  $h$  is submitted to transfer  $t_h$ , with a strict inequality for at least one  $h$ . This is satisfied when  $x_i - \delta \geq \lambda x_i$ . Under this condition,  $\alpha > \beta$  is necessary and sufficient to satisfy the desired property.

When  $\lambda > 1$ , the term in brackets is negative and the conclusion is reversed, requiring  $\alpha < \beta$ .

### Three-level inequalities

*Case 1:  $W^*$ , generation approach*

$$\begin{aligned} \frac{\partial W^*}{\partial x_j} &= \frac{1}{n} \left( \sum_{g \in G} \frac{n_g}{n} \left( \sum_{z \in Z} \frac{n_{gz}}{n_g} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{1-\beta}{1-\alpha}} \right)^{\frac{1-\gamma}{1-\beta}} \right)^{\frac{\gamma}{1-\gamma}} \\ &\quad \left( \sum_{z \in Z} \frac{n_{gz}}{n_g} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{1-\beta}{1-\alpha}} \right)^{\frac{\beta-\gamma}{1-\beta}} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{\alpha-\beta}{1-\alpha}} x_j^{-\alpha}. \end{aligned}$$

Consider a transfer either to  $j$  or to  $k$ , and the latter belongs to a different country or generation.

*Subcase 1:  $k$  belongs to the same generation but to a country that is  $\lambda$ -rescaled compared to the donor's country. Then*

$$\frac{\partial W^*}{\partial x_k} = \lambda^{\alpha-\beta} \frac{\partial W^*}{\partial x_j}.$$

*Subcase 2:  $k$  belongs to a generation and a country that is  $\lambda$ -rescaled. Then*

$$\frac{\partial W^*}{\partial x_k} = \lambda^{\alpha-\gamma} \frac{\partial W^*}{\partial x_j}.$$

*Subcase 3:  $k$  belongs to a generation that is  $\lambda$ -rescaled and a country that is  $\lambda'$ rescaled. Then*

$$\frac{\partial W^*}{\partial x_k} = \lambda^{\beta-\gamma} \lambda'^{\alpha-\beta} \frac{\partial W^*}{\partial x_j}.$$

Thus, within-country-generation preference will be obtained when the term in  $\lambda$  is less than one. The conditions induced on the parameters  $\alpha, \beta, \gamma$  are listed in Table 2.

*Case 2:  $W^*$ , country approach*

Generation	Country-generation	Parameters
$\lambda > 1$	$\lambda' > 1$	$\alpha < \beta < \gamma$
$\lambda > 1$	$\lambda' < 1$	$\beta < \alpha, \gamma$
$\lambda < 1$	$\lambda' > 1$	$\beta > \alpha, \gamma$
$\lambda < 1$	$\lambda' < 1$	$\alpha > \beta > \gamma$
$\lambda = 1$	$\lambda' > 1$	$\alpha < \beta$
$\lambda = 1$	$\lambda' < 1$	$\alpha > \beta$
$\lambda > 1$	$\lambda' = \lambda$	$\alpha < \gamma$
$\lambda < 1$	$\lambda' = \lambda$	$\alpha > \gamma$

Table 3: How to obtain within-country-generation preference

$$\frac{\partial W^*}{\partial x_j} = \frac{1}{n} \left( \frac{1}{n} \sum_{z \in Z} n_z \left( \frac{1}{n_z} \sum_{g \in G} n_{gz} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{1-\beta}{1-\alpha}} \right)^{\frac{1-\gamma}{1-\beta}} \right)^{\frac{\gamma}{1-\gamma}}$$

$$\left( \sum_{g \in G} \frac{n_{gz}}{n_z} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{1-\beta}{1-\alpha}} \right)^{\frac{\beta-\gamma}{1-\beta}} \left( \frac{\sum_{i \in I_{gz}} x_i^{1-\alpha}}{n_{gz}} \right)^{\frac{\alpha-\beta}{1-\alpha}} x_j^{-\alpha}.$$

Consider a transfer either to  $j$  or to  $k$ , and the latter belongs to a different country or generation.

*Subcase 1:*  $k$  belongs to the same country but to a generation that is  $\lambda$ -rescaled compared to the donor's generation. Then

$$\frac{\partial W^*}{\partial x_k} = \lambda^{\alpha-\beta} \frac{\partial W^*}{\partial x_j}.$$

*Subcase 2:*  $k$  belongs to a generation and a country that is  $\lambda$ -rescaled. Then

$$\frac{\partial W^*}{\partial x_k} = \lambda^{\alpha-\gamma} \frac{\partial W^*}{\partial x_j}.$$

*Subcase 3:*  $k$  belongs to a country that is  $\lambda$ -rescaled and a generation that is  $\lambda'$ rescaled. Then

$$\frac{\partial W^*}{\partial x_k} = \lambda^{\beta-\gamma} \lambda'^{\alpha-\beta} \frac{\partial W^*}{\partial x_j}.$$

Again, within-country-generation preference will obtain when the term in  $\lambda$  is less than one. The conditions induced on the parameters  $\alpha, \beta, \gamma$  are the same as in Table 3, replacing the first column "Generation" with "Country".

Consider the most relevant case of a transfer going to a poorer country-generation that belongs to a poorer country, but also to a richer generation. Then, in order to favor transfers within the country-generation over such an international-intergenerational transfer, one would need to have

$\beta < \alpha, \gamma$  with the generation approach, and  $\alpha > \beta > \gamma$  with the country approach.

*Case 3:  $W^{***}$ .*

*Subcase 1: Weights  $\gamma_{ij} = \alpha, \beta, \alpha', \beta'$ .*

Individual  $i$  can transfer either to  $j$  in the same country-generation  $(z, t)$  or to  $k$  in country-generation  $(z', t')$ . One has  $x_i > x_j = x_k$ . Let  $I_{zt}$  be the generic symbol denoting the population in any country-generation  $(z, t)$ , and use the symbols  $\bar{z}, \bar{t}$  to denote the population that is not in  $z$  or  $z'$ , and not in  $t$  or  $t'$ , respectively.

To save notation, we will simply drop the term  $\left(\frac{x_h^{1-\gamma} + x_j^{1-\gamma}}{2}\right)^{\frac{\gamma}{1-\gamma}} = \left(\frac{x_h^{1-\gamma} + x_k^{1-\gamma}}{2}\right)^{\frac{\gamma}{1-\gamma}}$  which appears within each summation of the type  $\sum_{h \in I_{..}}$ . One computes

$$\begin{aligned} \frac{\frac{\partial W^{***}}{\partial x_j}}{\frac{x_j^{-\gamma}}{2}} &= \alpha \sum_{h \in I_{zt}} + \beta \left( \sum_{h \in I_{z't}} + \sum_{h \in I_{\bar{z}t}} \right) + \alpha' \left( \sum_{h \in I_{z't'}} + \sum_{h \in I_{\bar{z}t'}} \right) \\ &+ \beta' \left( \sum_{h \in I_{z't'}} + \sum_{h \in I_{\bar{z}t'}} + \sum_{h \in I_{z'\bar{t}}} + \sum_{h \in I_{\bar{z}\bar{t}}} \right). \end{aligned}$$

Likewise, one has:

$$\begin{aligned} \frac{\frac{\partial W^{***}}{\partial x_k}}{\frac{x_k^{-\gamma}}{2}} &= \alpha \sum_{h \in I_{z't'}} + \beta \left( \sum_{h \in I_{z't}} + \sum_{h \in I_{\bar{z}t}} \right) + \alpha' \left( \sum_{h \in I_{z't'}} + \sum_{h \in I_{\bar{z}t'}} \right) \\ &+ \beta' \left( \sum_{h \in I_{z't}} + \sum_{h \in I_{\bar{z}t}} + \sum_{h \in I_{z'\bar{t}}} + \sum_{h \in I_{\bar{z}\bar{t}}} \right). \end{aligned}$$

Subtracting, one obtains

$$\begin{aligned} \frac{\frac{\partial W^{***}}{\partial x_j} - \frac{\partial W^{***}}{\partial x_k}}{\frac{x_j^{-\gamma}}{2}} &= \alpha \left[ \sum_{h \in I_{zt}} - \sum_{h \in I_{z't'}} \right] + \beta \left[ \sum_{h \in I_{z't}} + \sum_{h \in I_{\bar{z}t}} - \sum_{h \in I_{z't'}} - \sum_{h \in I_{\bar{z}t'}} \right] \\ &+ \alpha' \left[ \sum_{h \in I_{z't'}} + \sum_{h \in I_{\bar{z}t'}} - \sum_{h \in I_{z't}} - \sum_{h \in I_{\bar{z}t}} \right] + \beta' \left[ \sum_{h \in I_{z't'}} + \sum_{h \in I_{\bar{z}t'}} + \sum_{h \in I_{z'\bar{t}}} - \sum_{h \in I_{z't}} - \sum_{h \in I_{\bar{z}t}} - \sum_{h \in I_{z'\bar{t}}} \right] \\ &= (\alpha - \beta') \left[ \sum_{h \in I_{zt}} - \sum_{h \in I_{z't'}} \right] + (\beta - \alpha') \left[ \sum_{h \in I_{z't}} - \sum_{h \in I_{z't'}} \right] \end{aligned}$$



$$+ (\beta - \beta') \left[ \sum_{h \in I_{z\bar{t}}} - \sum_{h \in I_{z't'}} \right] + (\alpha' - \beta') \left[ \sum_{h \in I_{z\bar{t}}} - \sum_{h \in I_{z'\bar{t}}} \right].$$

Depending on whether 1) country  $z$  is uniformly<sup>8</sup> richer than country  $z'$ , 2) generation  $t$  is uniformly poorer than generation  $t'$ , 3) country-generation  $(z, t)$  is uniformly richer than  $(z', t')$ , and 4)  $(z, t)$  is uniformly richer than  $(z', t)$ , one obtains sufficient conditions for  $\frac{\partial W^{***}}{\partial x_j} > \frac{\partial W^{***}}{\partial x_{j'}}$ , thus ensuring that the local transfer is socially preferred. This is laid out in Table 3. It is assumed that all country-generations have the same population size, in order to avoid mechanical effects due to demographics. Note that when  $z$  is richer than  $z'$  and  $t$  is poorer than  $t'$ , necessarily  $(z, t)$  is richer than  $(z', t)$ , and the only unknown is whether  $(z, t)$  is richer than  $(z', t')$ . Therefore, under particular assumptions for (1) and (2), only one of (3) and (4) remains to free.

	$t$ richer than $t'$		$t$ poorer than $t'$	
$z$ richer than $z'$	$(z, t)$ richer than $(z', t)$	$\alpha > \beta', \alpha' > \beta > \beta'$	$(z, t)$ richer than $(z', t')$	$\alpha, \alpha' > \beta' > \beta$
	$(z, t)$ poorer than $(z', t)$	$\alpha > \beta', \beta > \alpha' > \beta'$	$(z, t)$ poorer than $(z', t')$	$\alpha' > \beta' > \alpha, \beta$
$z$ poorer than $z'$	$(z, t)$ richer than $(z', t)$	$\alpha, \beta > \beta' > \alpha'$	$(z, t')$ richer than $(z', t)$	$\alpha < \beta', \beta' > \alpha' > \beta$
	$(z, t)$ poorer than $(z', t)$	$\beta > \beta' > \alpha, \alpha'$	$(z, t')$ poorer than $(z', t)$	$\alpha < \beta', \beta' > \beta > \alpha'$

Table 4: Sufficient conditions for preference for local transfer under  $W^{***}$

To illustrate these computations, let us focus on the case in which country  $z$  is richer than country  $z'$  and generation  $t$  is poorer than generation  $t'$ . Taking account of the fact that  $\left(\frac{x_k^{1-\gamma} + x_j^{1-\gamma}}{2}\right)^{\frac{\gamma}{1-\gamma}}$  is increasing in  $x_k$  (as  $\gamma > 0$ ), this implies that

$$\sum_{h \in I_{z't}} - \sum_{h \in I_{z't'}} < 0; \quad \sum_{h \in I_{z\bar{t}}} - \sum_{h \in I_{z't'}} < 0; \quad \sum_{h \in I_{z\bar{t}}} - \sum_{h \in I_{z'\bar{t}}} > 0.$$

The sign of  $\sum_{h \in I_{z\bar{t}}} - \sum_{h \in I_{z't'}}$  is, however, indeterminate. Two cases must be distinguished.

*Subsubcase A:*  $(z, t)$  is richer than  $(z', t')$ , i.e.,  $\sum_{h \in I_{z\bar{t}}} - \sum_{h \in I_{z't'}}$   $> 0$ . Then a sufficient condition for  $\frac{\partial W^{***}}{\partial x_j} > \frac{\partial W^{***}}{\partial x_{j'}}$  is

$$\alpha - \beta' > 0; \beta - \alpha' < 0; \beta - \beta' < 0; \alpha' - \beta' > 0$$

which obtains if  $\alpha, \alpha' > \beta' > \beta$ .

*Subsubcase B:*  $(z, t)$  is poorer than  $(z', t')$ , i.e.,  $\sum_{h \in I_{z\bar{t}}} - \sum_{h \in I_{z't'}}$   $< 0$ . Then a sufficient condition for  $\frac{\partial W^{***}}{\partial x_j} > \frac{\partial W^{***}}{\partial x_{j'}}$  is

$$\alpha - \beta' < 0; \beta - \alpha' < 0; \beta - \beta' < 0; \alpha' - \beta' > 0$$

which obtains if  $\alpha' > \beta' > \alpha, \beta$ .

<sup>8</sup>A population is uniformly richer if there is a bijection from it to the other population such that every member is richer than their correspondent in the other population.

In sum, the inequality  $\alpha' > \beta' > \beta$  is always part of the relevant sufficient condition, showing that the greatest weight must go to pairs made of different generations in the same country and the smallest weight to contemporaneous people from different countries. Whether “local” pairs (same country-generation) have a high or low weight  $\alpha$  depends on whether the donor is in a richer or poorer country-generation than the recipient.

In this sense, Table 3 bears similarities with the two-level analysis, where the case of a transfer to a poorer country yielded the intuitive ranking of parameters, local pairs having a greater weight, whereas the opposite occurs in the case of a transfer to a richer country. Indeed, here, when  $(z, t)$  is richer than  $(z', t')$ , the parameter  $\alpha$  is ranked high, whereas it ranks low in the opposite case.

*Subcase 2: General weights  $\gamma_{ij}$ .*

Again, consider a transfer to either  $j$  or  $k$ , with  $x_j = x_k$ . One has

$$\begin{aligned} \frac{\partial W^{***}}{\partial x_j} - \frac{\partial W^{***}}{\partial x_k} &= \frac{x_j^{-\gamma}}{2} \sum_{i \in I} (\gamma_{ij} - \gamma_{ik}) \left( \frac{x_i^{1-\gamma} + x_j^{1-\gamma}}{2} \right)^{\frac{\gamma}{1-\gamma}} \\ &= 2^{\frac{1}{\gamma-1}} \sum_{i \in I} (\gamma_{ij} - \gamma_{ik}) \left( \left( \frac{x_i}{x_j} \right)^{1-\gamma} + 1 \right)^{\frac{\gamma}{1-\gamma}} \end{aligned}$$

Thus, for a given value of  $x_j$ , this expression is positive if, on average,  $\gamma_{ij} > \gamma_{ik}$  when  $x_i$  is high and  $\gamma_{ij} < \gamma_{ik}$  when  $x_i$  is low. Assume that weights decrease with distance in time and space. Then,  $\gamma_{ij} > \gamma_{ik}$  when  $j$  is closer (in time or space) to  $i$  than  $k$  is. The above expression is positive, therefore, if  $j$  is closer to rich people in  $I$  than  $k$ . If the donor belongs to a rich group (country, generation), this favors proximate transfers. If the donor belongs to a poor group, the opposite obtains and distant transfers are favored.

This result is a mere generalization of the results obtained with two-level inequalities, where it was assumed that the donor belonged to a richer country and  $j$  to the same country, thus being closer, on average, to rich populations than  $k$ .