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# Pollution in a globalized world: are debt transfers among countries a solution?* 

Marion Davin ${ }^{\dagger}$, Mouez Fodha ${ }^{\ddagger}$ and Thomas Seegmuller ${ }^{\S}$

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#### Abstract

We analyze the effects of a debt relief, i.e. a decrease of public debt of a low-income country financed by a high-income country, on environmental quality. People in the highincome country are the most patient. Under perfect mobility of assets, the debt relief increases the overall capital stock, and environmental quality when public abatements are sufficiently efficient. Welfare in both countries can also improve. In contrast, under a weak mobility of assets, capital does no more increase in the richest country, but environmental quality can still improve. This comes from a crowding-out effect of debt in the high-income country, which does no more take place when the mobility of assets is significant.


JEL classification: F43, H23, Q56.
Keywords: Global pollution; Overlapping generations; Public debt; Capital market integration.

## 1 Introduction

The trend in $\mathrm{CO}_{2}$ emissions predicts a global temperature increase of more than $3^{\circ} \mathrm{C}$ by the end of the century with severe consequences. In addition, climate change is accompanied by other global environmental problems such as plastic waste, pesticides and persistent organic pollutants (POPs), which can cause both locally and globally damaging effects. Loss of agricultural yield, loss of biodiversity, sea level rise, climate migration, extreme weather events, increased health

[^0]problems... are all phenomena that will increase and the brunt of the economic damages will be probably borne by poorest countries (Schelling, 1992; Mendelsohn et al., 2006). To achieve a $1.5^{\circ} \mathrm{C}$ consistent pathway, to limit the discharge of plastic waste into the oceans or to stop the accumulation of POPs in the atmosphere, the international community calls for immediate global response.

Environmental objectives are a particular concern for low-income countries in which these challenges are compounded by high levels of external economic vulnerability and public debt. They recurrently use public debt to absorb the impact of external macroeconomic shocks and, recently, of more frequent natural disasters. In turn, higher levels of public debt associated with weak macroeconomic situation increase fiscal pressure and constrain the capacity of countries to address vulnerabilities and to mitigate pollution. ${ }^{1}$ According to IMF classification, a large part of low-income countries presents a high risk of experiencing a debt crisis, which underlines the importance to compose with countries debt profiles to address environmental challenges. ${ }^{2}$

When we examine the diversity of instruments used to provide climate finance, debt transfers, in the form of debt reliefs granted by developed countries to developing ones, currently represent only an insignificant proportion of financing strategies (see the Table 1 built by Fenton et al., 2014). Yet, debt transfers are justified if they enhance the environmental quality and economic variables (consumption, savings, capital...) simultaneously.

Insert Table 1

Does a debt transfer in favor of low-income countries a relevant macroeconomic policy to promote pollution reduction? This question is of interest when international environmental policies have not been implemented to fight pollution effectively. Therefore, classical economic policies must regain legitimacy. This article contributes to this question by using a theoretical approach combining debt transfers and global pollution issues. We aim at analyzing the consequences of a debt transfer from high-income to low-income countries on pollution, GDP and well-being. We only study second-best policies, without seeking to achieve optimality.

We develop a two-country overlapping generations (OLG) model with global pollution externalities. Production deteriorates the environmental quality, harming the welfare of future generations, but public abatement which linearly increases with income improves it. Total-factor productivity determines the country profile: we define the low-income (resp. the high-income) country as the one with the lower (resp. the higher) level of productivity. According to the empirical evidence, people of the richest country are the more patient and hence have the highest saving rate. Countries are also characterized by different level of debts. Issuing public debt

[^1]and tax on agents' income are used to finance public abatement and debt services. The overall debt remains constant, even after international debt transfers have taken place. This implies a constant global debt supply and rules out issues related to unsustainable global debt.

We study the environmental and economic consequences of a debt relief in favor of the lowincome country, funded by the high-income country. Under perfect mobility of assets, the debt transfer implies a redistribution of income net of tax between the countries and modifies the debt burden and public abatement through a general equilibrium effect. When the interest rate is low, which is supported by several recent empirical studies (Ahn, 2003; Blanchard, 2019; Geerolf, 2018; Rachel and Summers, 2019), issuing more debt in the high-income country reduces taxation and increases savings. In this context, debt relief raises capital because this country has the highest saving rate. Environmental quality increases because public abatements per unit of income are assumed to be sufficiently efficient. In this context, a debt relief can therefore be beneficial for both capital accumulation and the environment, but also welfare, when the interest rate is low. Hence, our paper contributes to the literature that aims to rethink or redefine policies under low interest rates (Blanchard, 2019; Rachel and Summers, 2019).

We then analyze the robustness of these results when the mobility of assets is imperfect. To that extent, we consider the polar case without mobility. In this case, savings of each country should finance its own public debt. Since a debt relief means that public debt increases in the high-income country and decreases in the low-income country, there is a crowding-out effect on capital in the former country, whereas we observe exactly the opposite effect in the last one. Environmental quality still improves if the effect of public abatement net of pollution raises sufficiently in the low-income country, which happens if debt in this country is high enough. Therefore, the main difference between perfect mobility of assets and weak mobility concerns the real side of the economy. The question is whether debt emission in country with high income can be financed or not by over-saving of the other country. If it is mostly not the case, capital does not raise in the high-income country because of a crowding-out effect.

Our paper contributes to two strands of the literature, namely the interplay between public debt and environmental quality, and the links between economic development and the environment in an international context. Connecting these two strands, we provide new intuitions about the effects of international transfers on welfare and the environment.

Debt financing has been introduced in dynamic models with environmental concerns (Bovenberg and Heijdra, 1998; Heijdra et al., 2006; Fernández et al., 2010). Debt policy only makes possible to redistribute welfare gains from future to existing generations: there is no debt financing for environmental protection. Fodha and Seegmuller (2012, 2014) and Fodha et al. (2018) analyze debt financing schemes for public and private mitigation. They show that efficient environmental fiscal reforms may be designed. This literature, however, considers closed economies, which is a limitation as capital markets are interconnected and globalized, as climate and some global pollution issue.

Our work complements prior contributions that deal with global environmental issues in
international frameworks. Firstly, John and Pecchenino (1997) focus on the effects of cooperation between two countries in mitigating global pollution. Their approach considers neither capital nor savings arbitrage and they only assume lump-sum transfers between countries. They conclude that the developing countries must be compensated for their environmental expenditures. In our article, we focus on financial markets and debt reduction. The new thing is that the transfer we highlight goes through the financial market and has effects through the debt service reduction. The second is the work of Bednar-Friedl et al. (2010). Countries differ in their levels of public debt per capita. They find that if the country which reduces its emissions permits is a net creditor to the world economy, the domestic welfare costs are smaller. While in this work mitigation efforts are exogenous, we differ by studying in details the effect of debt transfers. Third, MullerFurstenberger and Schumacher (2017) propose a dynamic model where all agents contribute to a global externality, but only those in a specific region suffer from it. They show that if agents suffering from the externality are low-income, they may be stuck in an environmental poverty trap, but capital market integration helps to escape from the trap. We differ from this article by considering public expenditure for the environment and by examining the influence of public debt on saving and capital accumulation.

The literature has also questioned the characteristics of international transfers to protect the environment to see whether donor countries should prioritize technology transfers oriented towards mitigation or adaptation (Barañano and San Martin, 2015; Sakamoto et al., 2017). A related literature considers the interplay between environmental policy and development aid transfers in low-income countries and conclude that these transfers are in general uneffective (Bretschger and Suphaphiphat, 2014; Eyckmans et al., 2016). In our article, we depart from this literature as we limit aid transfers to untied debt cancellation.

Finally, in our paper, debt relief is undertaken without any counterpart (i.e. debt relief in return for improved environmental protection). Our model does not take into account debt for nature swaps (Deacon and Murphy, 1997; Cassimon et al., 2011, 2014). The literature on economic development underlines the inefficiencies of this type of aid as soon as donor country monitoring is not put in place. The risk of inefficiency of this policy is all the higher as the recipient country faces problems of quality governance, or extreme poverty. These issues are discussed in depth in Azam and Laffont (2003). They show how the optimal aid contract is complicated when institutional details of the real world are taken into account.

The rest of this paper is organized as follows. Section 2 presents the two-country OLG model with environmental externalities. Section 3 defines the configuration of perfect mobility of assets. Section 4 studies the robustness under imperfect mobility. Section 5 provides concluding remarks. Technical details are relegated to an Appendix.

## 2 The model

The world consists of two competitive economies indexed by $i \in\{D, F\}$. Within each country, a new generation of two-period lived agents is born at each period of time. Therefore, two generations are alive in each period $t$ : the workers and the retired people. In each country, the population size is constant and normalized to one. There is no mobility of labor between countries, whereas we will first assume that there is perfect mobility of the assets and a unique final good, as in Persson (1985). Perfect capital mobility can be seen as an approximation of countries with highly integrated financial markets and few capital controls. In a second step, we will extend our analysis to a world with imperfect mobility of assets.

### 2.1 Global environmental quality

Global environmental quality evolves as the opposite of the global stock of pollution, representing the stock of greenhouse gases in the atmosphere or the stock of plastics in the oceans. We measure this aggregate environmental quality by an index, which deteriorates with emission flow from production in each country and is enhanced by public abatement measures. Abatement could be reforestation, plastic waste management, anti-pollution devices (filters, particulate filter), insulation of buildings, energy transition, waste collection and sorting or carbon capture and sequestration technologies. Following John and Pecchenino (1994), Ono (2002) or Mariani et al. (2010), we interpret $E$ as a measure of the amenity value of the environmental quality.

Global environmental quality $E_{t}$ evolves according to:

$$
\begin{equation*}
E_{t+1}=(1-m) E_{t}+\phi_{D} g_{D t}+\phi_{F} g_{F t}-\theta_{D} y_{D t}-\theta_{F} y_{F t} \tag{1}
\end{equation*}
$$

where $y_{i t}$ and $g_{i t}$ represent respectively production and public abatement of country $i$, and $E_{0}>0$. The pollution flow resulting from production is given by the emission factor $\theta_{i}>0$ and efficiency of abatement is given by factor $\phi_{i}>0$. We keep these parameters different between the two countries.

In the absence of human activity, the quality of the environment has an autonomous level of zero; the parameter $m \in(0,1)$ measures the speed of reversion of environmental quality to this level. ${ }^{3}$ For some current environmental issues, natural "cleaning up" of the environment is not possible. This last point is all the more important as the environment is already highly polluted.

[^2]The equation is equivalent to:

$$
E_{t+1}=(1-m) E_{t}+m \widetilde{E}+\phi_{D} g_{D t}+\phi_{F} g_{F t}-\theta_{D} y_{D t}-\theta_{F} y_{F t}
$$

By taking $\widetilde{E}=0$, we find (1).

A high level of pollution compromises a favourable evolution of the environment, as shown by the case of plastics in the oceans and the loss of marine biodiversity it induces, or the consequences of climate change on biodiversity; nature cannot restore the natural state, unless huge investments are undertaken (Müller-Fürstenberger and Schumacher, 2017).

### 2.2 Firms

In each country, firms use capital and labor for the production of a unique final good, which is the numéraire. The technology used is Cobb-Douglas. As there is no labor mobility and labor input equals one in each country, the production function writes $y_{i t}=A_{i} k_{i t}^{\alpha}$, where $k_{i t}$ denotes capital, $A_{i}>0$ the country-specific productivity and $\alpha \in(0,1)$ the capital share in income. Profit maximization gives:

$$
\begin{align*}
w_{i t} & =(1-\alpha) A_{i} k_{i t}^{\alpha}  \tag{2}\\
R_{i t} & =\alpha A_{i} k_{i t}^{\alpha-1} \tag{3}
\end{align*}
$$

with $w_{i t}$ the wage and $R_{i t}$ the return of capital in country $i .{ }^{4}$

### 2.3 Households

A generation born at period $t$ derives utility from consumption when young $c_{i t}$ and old $d_{i t+1}$, and from environmental quality at both periods. Accordingly, the lifetime utility is given by:

$$
\begin{equation*}
\ln c_{i t}+\delta_{i} \ln E_{t}+\beta_{i}\left(\ln d_{i t+1}+\delta_{i} \ln E_{t+1}\right) \tag{4}
\end{equation*}
$$

where $\beta_{i} \in(0,1)$ denotes the discount factor in country $i$, and $\delta_{i} \geqslant 0$ the vulnerability to pollution stock. We assume that these preference parameters are country specific, which means that $\beta_{D} \neq \beta_{F}$ and $\delta_{D} \neq \delta_{F}$. On the one hand, heterogeneity between the discount factors $\beta_{i}$ of the different countries is supported by empirical studies (Lawrance, 1991; Tanaka et al., 2010; Wang et al., 2016). On the other hand, $\delta_{i}$ aggregate both sensitivity and vulnerability to climate change which are also different among high-income and low-income countries (Schelling, 1992; Mendelsohn et al., 2006). Environmental quality when young $E_{t}$ and old $E_{t+1}$ is taken as given by agents. It acts as a pure externality in the utility function.

When young, each agent inelastically supplies one unit of labor and receives real wage $w_{i t}$. A lump-sum tax $\tau_{i t}$ is levied on this income. Net income, $w_{i t}-\tau_{i t}$, is shared between consumption $c_{i t}$ and savings $s_{i t}$. Consumption when old $d_{i t+1}$ is entirely financed by the remunerated savings.

[^3]Therefore, the two budget constraints faced by an agent born at period $t$ write:

$$
\begin{align*}
c_{i t}+s_{i t} & =w_{i t}-\tau_{i t}  \tag{5}\\
d_{i t+1} & =R_{i t+1} s_{i t} \tag{6}
\end{align*}
$$

A young agent maximizes her utility (4) taking into account the two budget constraints (5) and (6). We obtain $d_{i t+1}=\beta_{i} R_{i t+1} c_{i t}$ and deduce that:

$$
\begin{equation*}
s_{i t}=\frac{\beta_{i}}{1+\beta_{i}}\left(w_{i t}-\tau_{i t}\right), \quad c_{i t}=\frac{1}{1+\beta_{i}}\left(w_{i t}-\tau_{i t}\right), \quad d_{i t+1}=\frac{\beta_{i}}{1+\beta_{i}} R_{i t+1}\left(w_{i t}-\tau_{i t}\right) \tag{7}
\end{equation*}
$$

### 2.4 Government

The government engages in public abatement $g_{i t}$ to directly reduce pollution. The government collects lump sum tax $\tau_{i t}$ on workers and uses bonds as debt instrument $b_{i t}$. Its expenditures include repayment of debt and interest payments, as well as public spendings which correspond here to public abatement. In addition to the engagement in abatement, the government can also use debt to postpone the financing of expenditures and modify current taxes to increase income.

In each country, the government faces the following budget constraint:

$$
\begin{equation*}
b_{i t+1}=R_{i t} b_{i t}-\tau_{i t}+g_{i t} \tag{8}
\end{equation*}
$$

with the initial public debt level $b_{i 0} \geqslant 0$ given. In this paper, we consider public debt as being a target and a policy instrument, we thus assume a constant level of debt $b_{i t}=b_{i}>0$. Concerning public expenditures on environmental protection, review of data from Eurostat and IMF shows a disparity between countries and relative stability over time when expenditure is evaluated in terms of GDP. ${ }^{5}$ We thus assume that the ratio of public spending over GDP is constant and country specific, i.e. $g_{i t}=g_{i} y_{i t}$, with $g_{i}>0$.

Introducing these different ingredients in the government budget (8), we obtain:

$$
\begin{equation*}
\tau_{i t}=\left(R_{i t}-1\right) b_{i}+g_{i} y_{i t} \tag{9}
\end{equation*}
$$

Taxation is endogenous and varies to satisfy the government budget constraint at each period of time.

We aim at studying the effect of a debt relief, i.e. a reallocation of debt between the two countries, in the long run. We will in particular analyze the effect of a debt relief through an increase of $b_{D}$ which allows to decrease $b_{F}$. Such a policy implies a redistribution of income net of tax between the two countries, as the tax paid in each country depends on $b_{i}$. However, we will see that the effects of a debt transfer differ from a standard redistribution of income because it goes

[^4]through different channels. Indeed, while a debt transfer directly modifies taxes, it also modifies the debt burden, i.e. the interest rate, and public abatement through a general equilibrium effect.

Considering debt issues and its interplay with the global environment in an integrated world, the degree of capital market integration is important. We start by studying the effect of a debt relief on environmental quality and productive capital of each country when the mobility of assets is perfect. Then, we will analyze the robustness of the result when the mobility of assets becomes imperfect.

## 3 Perfect mobility of assets

As international capital mobility is assumed to be perfect, foreign and domestic assets yield the same rate of return. Market clearing also requires world savings equal to world investment:

$$
\begin{align*}
R_{D t} & =R_{F t}  \tag{10}\\
s_{D t}+s_{F t} & =k_{D t+1}+b_{D t+1}+k_{F t+1}+b_{F t+1} \tag{11}
\end{align*}
$$

Given the public abatements an the production technologies in each country, the law of motion for environmental quality (1) is given by:

$$
\begin{equation*}
E_{t+1}=(1-m) E_{t}+\left(\phi_{D} g_{D}-\theta_{D}\right) A_{D} k_{D t}^{\alpha}+\left(\phi_{F} g_{F}-\theta_{F}\right) A_{F} k_{F t}^{\alpha} \tag{12}
\end{equation*}
$$

and $E_{t}>0$. Using (3), equation (10) means that:

$$
\begin{equation*}
k_{F t}=k_{D t}\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}} \tag{13}
\end{equation*}
$$

We consider a domestic economy richer than the foreign one. Numerous studies emphasize that the major part of the difference in incomes between high-income and low-income countries is due to differences in total factor productivity. We can refer to the survey by Caselli (2005) and the study by Hsieh and Klenow (2010). Moreover, in line with empirical evidences (Tanaka et al., 2010; Wang et al., 2016), wealthy population or population in wealthier countries are characterized by preferences with lower discount rate and thus tend to be more patient. We thus assume a domestic economy more patient than the foreign one.

Assumption $1 A_{D}>A_{F}$ and $\beta_{D}>\beta_{F}$.
Despite this assumption on the productivities between countries, we do not impose any ranking on the respective debt levels, i.e. $b_{D}$ higher or smaller than $b_{F}$.

Substituting (2), (3) and (9) into (7), we obtain savings in both countries:

$$
\begin{equation*}
s_{i t}=\frac{\beta_{i}}{1+\beta_{i}}\left[\left(1-\alpha-g_{i}\right) A_{i} k_{i t}^{\alpha}-\left(\alpha A_{i} k_{i t}^{\alpha-1}-1\right) b_{i}\right] \tag{14}
\end{equation*}
$$

Using the condition of perfect mobility of capital (13), pollution evolves according to:

$$
\begin{equation*}
E_{t+1}=(1-m) E_{t}+\left[\phi_{D} g_{D}-\theta_{D}+\left(\phi_{F} g_{F}-\theta_{F}\right)\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}\right] A_{D} k_{D t}^{\alpha} \tag{15}
\end{equation*}
$$

We ensure that environmental quality is positive for each level of $m$ and at a steady state. Moreover, public spending over GDP $g_{i}$ should be low enough to ensure positive savings whatever the level of debt. Accordingly, we assume:

Assumption $2 \phi_{D} g_{D}-\theta_{D}+\left(\phi_{F} g_{F}-\theta_{F}\right)\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}>0$ and $g_{i}<1-\alpha$ for $i=D, F$.
The first condition implies that efficiencies of abatements per unit of income should be sufficiently high with respect to emission rates. In particular, the benefit resulting from one unit of abatement has to be larger than the environmental damage resulting from one unit of production at least in one country. This seems a reasonable condition because we consider the environmental quality that is perceived by agents. In this way, governments invest in abatement activities that especially target environmental issues while pollution is simply a side-effect of the aggregate production process. Hence, the marginal benefit of abatement can be assumed to be relatively higher than the marginal damage of production. The second condition means that public spending over GDP should be lower than the labor share of income. In other word, the fiscal burden has to be lower than labor income to keep the wage net of taxes positive.

Using (13) and (14), the equilibrium on the asset market (11) rewrites:

$$
\begin{align*}
& k_{D t+1}\left[1+\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}\right]=\left[\frac{\beta_{D}}{1+\beta_{D}}\left(1-\alpha-g_{D}\right)+\frac{\beta_{F}}{1+\beta_{F}}\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}\left(1-\alpha-g_{F}\right)\right] A_{D} k_{D t}^{\alpha} \\
& -\left(\frac{b_{D}}{1+\beta_{D}}+\frac{b_{F}}{1+\beta_{F}}\right)-\left(\frac{\beta_{D}}{1+\beta_{D}} b_{D}+\frac{\beta_{F}}{1+\beta_{F}} b_{F}\right) \alpha A_{D} k_{D t}^{\alpha-1} \equiv G\left(k_{D t}\right) \tag{16}
\end{align*}
$$

Given $E_{0}>0$ and $k_{D 0}>0$, equations (15) and (16) drives the dynamics of the economy. In fact, (16) determines the sequence of $\left(k_{D t}\right)_{t \geqslant 0}$ and given it, (15) determines the sequence of $\left(E_{t}\right)_{t \geqslant 0}$.

We note that $G\left(k_{D t}\right)$ is a strictly increasing and concave function with $G(0)<0$ and $\lim _{k_{D t} \rightarrow+\infty} G\left(k_{D t}\right) / k_{D t}<1+\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}$. Therefore, one can easily show that for $b_{D}$ and $b_{F}$ not too high, the dynamics of $k_{D t}$ is characterized by two steady states $k_{l}>0$ and $k_{h}\left(>k_{l}\right)$, where $k_{l}$ is unstable $\left(G^{\prime}\left(k_{l}\right)>1+\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}\right)$ and $k_{h}$ is stable $\left(G^{\prime}\left(k_{h}\right)<1+\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}\right)$. See also Figure 1.

Using (15), we easily see that $E_{j}$, the level of environmental quality associated to $k_{j}$ for $j=\{h, l\}$, is given by:

$$
\begin{equation*}
E_{j}=\frac{1}{m}\left[\phi_{D} g_{D}-\theta_{D}+\left(\phi_{F} g_{F}-\theta_{F}\right)\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}\right] A_{D} k_{j}^{\alpha} \tag{17}
\end{equation*}
$$

which is positive under Assumption 2. Since $d E_{t+1} / d E_{t}=1-m \in(0,1)$ and both $k_{D t}$ and $E_{t}$
are predetermined variables, the steady state $\left(k_{l}, E_{l}\right)$ is an unstable saddle, whereas $\left(k_{h}, E_{h}\right)$ is stable.


Figure 1 - Dynamics of capital

We now focus on the effect of debt relief which means an increase of debt $b_{D}$ and a decrease of $b_{F}$ such that $b_{D}+b_{F}$ does not change, i.e. $d b_{D}=-d b_{F}=d b>0$, on the stable steady state $\left(k_{h}, E_{h}\right)$. We examine whether debt relief increases capital. It is of particular interest because, according to (17), it also improves environmental quality. We deduce the following proposition:

Proposition 1 Under Assumptions 1 and 2, a debt relief implies:

1. An increase of $k_{h}$ and $E_{h}$ if $R_{h}<1$ (low interest rate);
2. A decrease of $k_{h}$ and $E_{h}$ if $R_{h}>1$ (high interest rate).
with $R_{h} \equiv \alpha A_{D} k_{h}^{\alpha-1}$.
Proof. See Appendix 6.1.
Since the economies share a common asset market, the debt transfer has no direct crowdingout effect on capital: the larger level of public debt in country $D$ is exactly compensated by the lower level in country F. The effects entailed by the policy therefore go through the variation of taxes. As the domestic country is more patient $\left(\beta_{D}>\beta_{F}\right)$, the debt transfer entails an increase in capital stock only when there is a low interest rate ( $R_{h}<1$ ). In that case, issuing more debt reduces the fiscal pressure. The transfer of debt is associated to a transfer of income favorable for the patient economy. Of course, we have the opposite result when countries are characterized by a high interest rate ( $R_{h}>1$ ).

Because the efficiencies of public abatements per unit of income is sufficiently high with respect to emissions rates, environmental quality evolves in the same direction than capital. Public abatements therefore plays a crucial role in our analysis. If they were not sufficiently
efficient and if they did not respond to income, environmental quality and GDP per capita could not improve together.

Since debt relief is efficient to increase environmental quality and capital when there is overaccumulation, we now examine the conditions to have such a configuration. Let us define $\widetilde{k}_{D}$ such that $G^{\prime}\left(\widetilde{k}_{D}\right)=1+\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}$. Of course, $\widetilde{k}_{D}<k_{h}$ and $R_{h}=\alpha A_{D} k_{h}^{\alpha-1}<1$ if $\alpha A_{D} \widetilde{k}_{D}^{\alpha-1}<1$. Since $G^{\prime}\left(\widetilde{k}_{D}\right)$ is decreasing in $\widetilde{k}_{D}$ and using (16), this last condition is satisfied if:

$$
\begin{align*}
& \frac{\beta_{D}}{1+\beta_{D}}\left(1-\alpha-g_{D}\right)+\frac{\beta_{F}}{1+\beta_{F}}\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}\left(1-\alpha-g_{F}\right) \\
& +(1-\alpha)\left(\frac{\beta_{D}}{1+\beta_{D}} b_{D}+\frac{\beta_{F}}{1+\beta_{F}} b_{F}\right)\left(\alpha A_{D}\right)^{-\frac{1}{1-\alpha}}>1+\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}} \tag{18}
\end{align*}
$$

We deduce the following corollary:
Corollary 1 Under Assumptions 1 and 2, and inequality (18), a debt relief implies an increase of $k_{h}$ and $E_{h}$.

By inspection of inequality (18), a debt relief improves the environmental and real sides of the economy if $A_{D}$ is sufficiently low (given the ratio $\left.A_{F} / A_{D}\right)$, the saving rates $\beta_{i} /\left(1+\beta_{i}\right)$ and the debt levels $b_{i}$ are high enough.

It is also important to note that having low interest rate has a significant empirical support in macroeconomics (Ahn, 2003; Blanchard, 2019; Geerolf, 2018; Rachel and Summers, 2019), and can be explained by over-saving and secular stagnation, for instance. This means that Proposition 1 and Corollary 1 establish the relevance of debt relief to increase capital and environmental quality under some realistic conditions. When the cost of the debt is low ( $R_{h}<1$ ), the increase in debt in country D does not translate into a higher tax burden. Having such properties is interesting as this country is characterized by a higher saving rate than country F. In this way, environmental engagement increases following the policy. This also means that our paper contributes to this literature that suggests to rethink about fiscal policies when the interest rates are low (Blanchard, 2019; Rachel and Summers, 2019).

Considering that Corollary 1 holds, we then examine the effect of the debt transfer on the agent's welfare in both economies $i=D, F$. Using agent's consumption choices (7), its indirect utility $V_{i h}$ evaluated at the stable steady state $\left(k_{h}, E_{h}\right)$ is given by:

$$
\begin{equation*}
V_{i h}=\left(1+\beta_{i}\right) \ln \left(w_{i h}-\tau_{i h}\right)+\beta_{i} \ln \left(R_{h}\right)+\left(1+\beta_{i}\right) \delta_{i} \ln \left(E_{h}\right)+\beta_{i} \ln \beta_{i}-\left(1+\beta_{i}\right) \ln \left(1+\beta_{i}\right) \tag{19}
\end{equation*}
$$

Following a debt relief, $E_{h}$ increases. Therefore, we have to investigate whether the utility associated to consumptions raises too or not. This is shown in the following corollary:

Corollary 2 Under Assumptions 1 and 2, inequality (18) and $\alpha \geqslant 1 / 3$ and $R_{h} \geqslant 1 / 3$, a debt relief implies an increase of the welfare evaluated at the steady state $\left(k_{h}, E_{h}\right)$ in country $D$, while the effect on the welfare in country $F$ is positive only if preferences for environmental quality $\delta_{F}$ and/or debt level $b_{F}$ is high enough.

## Proof. See Appendix 6.2.

When interest rate is low enough ( $R_{h}<1$ ), capital goes up in each country and the environment improves following the debt relief. Such capital improvement leads to opposite effects on the well being, by improving wage and reducing the return of saving. At the same time, lower interest rate reduces the cost of debt. For country D, as long as the share of capital and interest rate satisfy $\alpha \geqslant 1 / 3$ and $R_{h} \geqslant 1 / 3$, which seems to be empirically relevant assumptions, the positive effects on income outweigh the negative effect associated to the lower return of saving. Accordingly, the welfare increases in country D because both the utility of consumptions and environmental quality raise. To observe an improvement of welfare in country F additional conditions are required. Indeed, the effect of the policy on the utility for consumptions is not clear-cut. This is because the constraint to keep the aggregate level of debt constant implies that taxation may go up in the foreign economy. The reduction of the debt burden has to be sufficiently important to avoid such increase and/or the weight associated to environmental quality has to be sufficiently high to observe a raise in welfare effects in the foreign economy.

In a context of low interest rate and perfectly integrated capital market, a debt transfer from the patient to the impatient country is a way to increase capital stock and hence environmental engagement. This may increase the welfare in both country. Note that even in a context with $R_{h}<1$, keeping $b_{D}+b_{F}$ constant is important as it allows to avoid crowding-out effects. We can also underline that a simple transfer of income would not provide such welfare benefits. Indeed, if the domestic patient country transfers revenue to the foreign impatient country, the aggregate capital stock would go down, unless to mix such a transfer to conditions aiming at modifying behavior in the recipient country. We do not explore this kind of policy in our analysis. ${ }^{6}$

As a part of the problem of pollution in low income countries could be related to their low efficiency in abatement activities, it is also possible to consider that the fall in debt cost entailed by the debt transfer could be used to improve this efficiency. This would be beneficial for global environment but could question the welfare benefit of the transfer for the foreign country. Indeed, in that scenario, the foreign country would not benefit of a lower fiscal burden and this could make the transfer costly for the real side of the economy.

## 4 Imperfect capital mobility

Despite the increased importance of international capital flows, some countries are characterized by imperfect capital mobility, implying that foreign and domestic assets do not necessarily yield the same rate of return. External debt, defined as $k_{t+1}^{i}+b_{t+1}^{i}-s_{t}^{i}$, depends on the interest rate differential between countries. Market clearing always requires world savings equal to world investment.

To understand the difference between the effect of debt relief under perfect and imperfect

[^5]mobility, we consider the limit case where the two countries are closed but share a common environment. We can use (7) to define the equilibrium on the asset market in each country $i=D, F$ as follows:
\[

$$
\begin{equation*}
k_{i t+1}+b_{i}=\frac{\beta_{i}}{1+\beta_{i}}\left[(1-\alpha) A_{i} k_{i t}^{\alpha}-\tau_{i t}\right] \tag{20}
\end{equation*}
$$

\]

Using (3) and (9), we obtain:

$$
\begin{equation*}
k_{i t+1}=\frac{\beta_{i}}{1+\beta_{i}}\left(1-\alpha-g_{i}\right) A_{i} k_{i t}^{\alpha}-\frac{b_{i} \beta_{i}}{1+\beta_{i}} \alpha A_{i} k_{i t}^{\alpha-1}-\frac{b_{i}}{1+\beta_{i}} \equiv G_{i}\left(k_{i t}\right) \tag{21}
\end{equation*}
$$

Given the levels of capital in each country, the global level of environmental quality is still defined by the law of motion (12).

We note that $G_{i}\left(k_{i t}\right)$ is a strictly increasing and concave function with $G_{i}(0)<0$ and $\lim _{k_{i t} \rightarrow+\infty} G_{i}\left(k_{i t}\right) / k_{i t}<1$. We deduce that when $b_{i}$ is not too high, in each economy, there are two steady states, the unstable one $k_{i 1}>0$, with $G_{i}^{\prime}\left(k_{i 1}\right)>1$, and the stable one $k_{i 2}\left(>k_{i 1}\right)$, with $G_{i}^{\prime}\left(k_{i 2}\right) \in(0,1)$. As in the previous section, we focus on this last one to study the effect of the policy on the equilibrium of the global economy. We easily see that an increase of debt $b_{i}$ decreases $k_{i 2}$. Indeed, differentiating $k_{i 2}=G_{i}\left(k_{i 2}\right)$, we deduce that:

$$
\begin{equation*}
\frac{d k_{i 2}}{d b_{i}}=-\frac{1+\beta_{i} \alpha A_{i} k_{i 2}^{\alpha-1}}{\left(1-G_{i}^{\prime}\left(k_{i 2}\right)\right)\left(1+\beta_{i}\right)} \tag{22}
\end{equation*}
$$

has the opposite sign of $d b_{i}$ because $G_{i}^{\prime}\left(k_{i 2}\right) \in(0,1)$. Here, countries do not share a common capital market, as a result, the debt transfer affects capital stock through a direct crowding-out effect and an indirect income effect (whose sign depends on the level of the interest rate). ${ }^{7}$ The first effect is more important. Therefore, a debt relief which corresponds to an increase of debt $b_{D}$ and a decrease of $b_{F}$, always implies a decrease of $k_{D 2}$ and an increase of $k_{F 2}$.

Let $E_{2}$ be the stationary level of environmental quality associated to $k_{D 2}$ and $k_{F 2}$. It is given by:

$$
\begin{equation*}
E_{2}=\frac{1}{m}\left[\left(\phi_{D} g_{D}-\theta_{D}\right) A_{D} k_{D 2}^{\alpha}+\left(\phi_{F} g_{F}-\theta_{F}\right) A_{F} k_{F 2}^{\alpha}\right] \tag{23}
\end{equation*}
$$

Since $m \in(0,1)$, the steady state $\left(k_{D 2}, k_{F 2}, E_{2}\right)$ is the only one stable. To ensure that environmental quality and the levels of capital are positive, Assumption 2 is replaced by:

Assumption $3 \phi_{i} g_{i}-\theta_{i}>0$ and $g_{i}<1-\alpha$ for $i=D, F$.
Note that this assumption implies Assumption 2. As in the previous section, we focus on the effect of debt relief which means an increase of debt $b_{D}$ and a decrease of $b_{F}$ such that $b_{D}+b_{F}$ does not change, i.e. $d b_{D}=-d b_{F}=d b>0$, on the stable steady state $\left(k_{D 2}, k_{F 2}, E_{2}\right)$. Differentiating

[^6]equation (23), we get:
\[

$$
\begin{align*}
\frac{d E_{2}}{d b}= & -\frac{\left(\phi_{D} g_{D}-\theta_{D}\right) \alpha A_{D} k_{D 2}^{\alpha-1}}{m} \frac{1+\beta_{D} \alpha A_{D} k_{D 2}^{\alpha-1}}{\left(1-G_{D}^{\prime}\left(k_{D 2}\right)\right)\left(1+\beta_{D}\right)} \\
& +\frac{\left(\phi_{F} g_{F}-\theta_{F}\right) \alpha A_{F} k_{F 2}^{\alpha-1}}{m} \frac{1+\beta_{F} \alpha A_{F} k_{F 2}^{\alpha-1}}{\left(1-G_{F}^{\prime}\left(k_{F 2}\right)\right)\left(1+\beta_{F}\right)} \tag{24}
\end{align*}
$$
\]

We can rationalize that for a given level of $k$, we have $G_{D}(k)>G_{F}(k)$ because $\beta_{D}>\beta_{F}$ and the fiscal pressure (measured by $b_{i}$ and $g_{i}$ ) in country $D$ is not higher than in country $F$. This implies that $k_{F 2}$ is lower than $k_{D 2}$. However, this is not enough to give a clear cut sign of $d E_{2} / d b$. A positive sign of this expression is however obtained if $G_{F}^{\prime}\left(k_{F 2}\right)$ is sufficiently close to 1 .

Proposition 2 Under Assumptions 1 and 3, and $\alpha(2-\alpha)<1$, there exists a stationary level $\widehat{k}_{F 2}$ such that $G_{F}^{\prime}\left(\widehat{k}_{F 2}\right)=1$ for a given level of $b_{F}=\widehat{b}_{F}$. For a slightly lower level of debt, we have $k_{F 2}>\widehat{k}_{F 2}$ but close to $\widehat{k}_{F 2}$. In this case, a debt relief implies a decrease of $k_{D 2}$ and an increase of $k_{F 2}$ and $E_{2}$.

Proof. See Appendix 6.3.
We deduce from this proposition that the two countries cannot benefit of the effect of the policy in terms of capital. Indeed, since the economies are closed, debt has a crowding-out effect on capital. When debt increases, less savings is devoted to finance capital investment, whereas when debt decreases, we have of course the opposite effect on capital. This explains that $k_{D}$ decreases and $k_{F}$ increases. When capital was perfectly mobile, this would not occur because the larger level of public debt in country D was financed by the excess of savings of country F.

As regards environmental quality, it improves if green public spendings net of pollution increases. Because public spendings over GDP is constant, green public spendings net of pollution linearly raises with the country's income. As a result, public environmental engagement increases in economy F while effort in country D becomes lower. An improvement of environmental quality thus requires a sufficiently high difference between incomes variation in countries F and D . Then, Proposition 2 implies that the effect of the marginal increase of income in country F dominates the effect of the marginal decrease of income in country D. We note that it does not require a low interest rate for all countries, but a sufficiently high level of debt in country F.

Then, we can examine the effects of the transfer on the welfare when economies do not share a common capital market. Based on the indirect utility (19), it is easy to see that the welfare effects are not clear-cut. According to Proposition 2, labor income increases in country F and decreases in country D while global environmental quality improves. At the same time, the transfer entails a direct effect on taxes which is negative in country D and positive in country F if the interest rate is lower than one. Reversely, the transfer increases taxes in country D and reduces it in country F if the interest rate is higher than one. Therefore, even if it is not clear-cut, an increase of welfare associated to the consumptions over the life-cycle is more likely to occur in country F than in country D. In this last one, an increase of welfare would require a sufficiently high
preference for the environmental quality. Hence, the implementation of such a policy based on debt relief could depend on the acceptability of country D which could experience a decrease of income and welfare.

Our analysis allows to assess that debt relief can improve environmental quality, whatever the degree of integration between countries. Nevertheless, a difference emerges concerning the real side of the economy. This difference is important because it can condition the acceptability of the policy. Let us consider the relevant case with low interest rate $\left(R_{h}<1\right)$. When capital mobility is sufficiency high, the results presented in Proposition 1 prevail and both countries can benefit from the debt transfer. But when capital mobility is low enough, country D can experience a decrease of income, as presented in Proposition 2. The difference observed between the two cases comes from the fact that when capital mobility is not perfect, country D experiences a crowdingout effect, which is not compensated by the crowding-in effect in country F. More precisely, an important question is to know how the increase of debt in country D is financed. If there is perfect mobility, the increase of $b_{D}$ is financed with the over-saving of country F. Since the global amount of public debt is constant, there is no crowding-out effect. In contrast, as we have seen, if there is no mobility, an increase of $b_{D}$ is not financed by external funds, which creates a crowding-out effect on capital of country D. More generally, a lower mobility of asset reduces this possibility of external funding of the increase of public debt $b_{D}$ and reinforces the crowding-out effect of debt on capital in country D.

## 5 Concluding remarks

We examine in this paper the interplay between public debt and global environment. In a context of low interest rate, redefine debt policies and then examine its effects for environmental engagement is important. Indeed, low interest rate goes with the absence of fiscal costs but, at the same time, public debt can be costly as it can reduce capital accumulation. In an international context, the existence of this cost crucially depends on how the overall debt level evolves and how capital market are integrated. In this paper, we question the effect of debt transfer in favor of low-income countries, the overall debt level remaining unchanged. We study the effects on both GDP and the environment when capital mobility is perfect or not.

When capital is perfectly mobile between countries, such a policy is not accompanied by negative crowding-out effect and it makes possible to increase GDP in both countries. As public spending for the environment goes up with country's income and is sufficiently efficient, the environmental quality increases as well. As the high-income economy is more patient, this scenario can emerge when the interest rate is sufficiency low. In that case, the debt transfer relaxes the fiscal pressure in high income country and global capital goes up. At the same time, the policy reduces the debt burden for the recipient country and hence it can increase the welfare of both economies. If the mobility of capital is imperfect, the increase in debt in high income countries goes with a fall in capital accumulation. Environmental quality can still improve, meaning that
the main difference with perfect mobility mainly concerns the real side of the economy. The acceptability of the implementation of a debt relief could therefore depend on the degree of international openness. However, whatever the mobility of assets, debt relief could be welfare improving when environmental preferences are high enough.

## 6 Appendix

### 6.1 Proof of Proposition 1

Differentiating equation (16) at the stable steady state $k_{h}$, we get:

$$
\begin{equation*}
\frac{d k_{h}}{d b}\left[1+\left(\frac{A_{F}}{A_{D}}\right)^{\frac{1}{1-\alpha}}-G^{\prime}\left(k_{h}\right)\right]=\frac{1+\beta_{F} \alpha A_{D} k_{h}^{\alpha-1}}{1+\beta_{F}}-\frac{1+\beta_{D} \alpha A_{D} k_{h}^{\alpha-1}}{1+\beta_{D}} \tag{25}
\end{equation*}
$$

Note that $\frac{1+\beta_{i} \alpha A_{D} k_{h}^{\alpha-1}}{1+\beta_{i}}$ is increasing in $\beta_{i}$ if and only if $R_{h} \equiv \alpha A_{D} k_{h}^{\alpha-1}>1$ and decreasing if and only if $R_{h}<1$. We deduce that:

- $d k_{h} / d b<0$ if $R_{h}>1$;
- $d k_{h} / d b>0$ if $R_{h}<1$;

Using equation (17) and Assumption 2, when debt relief increases (decreases) capital, it also improves (reduces) environmental quality.

### 6.2 Proof of Corollary 2

Let:

$$
\begin{equation*}
\widetilde{V}_{i h}=\left(1+\beta_{i}\right) \ln \left(w_{i h}-\tau_{i h}\right)+\beta_{i} \ln \left(R_{h}\right) \tag{26}
\end{equation*}
$$

The welfare in a country $i$ increases for all values of $\delta_{i}$ if following an increase of $b_{D}$ and a decrease of $b_{F}$ such that $d b_{D}=-d b_{F}=d b>0, \widetilde{V}_{i h}$ raises. Using (2), (3) and (9), equation (26) writes:

$$
\begin{equation*}
\widetilde{V}_{i h}=\left(1+\beta_{i}\right) \ln \left[\left(1-\alpha-g_{i}\right) A_{i} k_{i}^{\alpha}+\left(1-\alpha A_{i} k_{i}^{\alpha-1}\right) b_{i}\right]+\beta_{i} \ln \alpha A_{i} k_{i}^{\alpha-1} \tag{27}
\end{equation*}
$$

Differentiating this equation with respect to debt for country D and F , we get:

$$
\begin{align*}
\frac{d \widetilde{V}_{D h}}{d b}= & \frac{\left(1+\beta_{D}\right)\left(1-R_{h}\right)}{\left(1-\alpha-g_{D}\right) A_{D} k_{D}^{\alpha}+\left(1-R_{h}\right) b_{D}}  \tag{28}\\
& +\frac{\left[\left(1-\alpha-g_{D}\right) R_{h}\left(1-\beta_{D}(1-2 \alpha) / \alpha\right)+(1-\alpha)\left(b_{D} / k_{D}\right)\left(R_{h}-\beta_{D}\left(1-2 R_{h}\right)\right)\right] d k_{D} / d b}{\left(1-\alpha-g_{D}\right) A_{D} k_{D}^{\alpha}+\left(1-R_{h}\right) b_{D}} \\
\frac{d \widetilde{V}_{F h}}{d b}= & -\frac{\left(1+\beta_{F}\right)\left(1-R_{h}\right)}{\left(1-\alpha-g_{F}\right) A_{F} k_{F}^{\alpha}+\left(1-R_{h}\right) b_{F}}  \tag{29}\\
& +\frac{\left[\left(1-\alpha-g_{F}\right) R_{h}\left(1-\beta_{F}(1-2 \alpha) / \alpha\right)+(1-\alpha)\left(b_{F} / k_{F}\right)\left(R_{h}-\beta_{F}\left(1-2 R_{h}\right)\right)\right] d k_{F} / d b}{\left(1-\alpha-g_{F}\right) A_{F} k_{F}^{\alpha}+\left(1-R_{h}\right) b_{F}}
\end{align*}
$$

where $R_{h}, k_{F}$ and $k_{D}$ are the values of $R_{i t}, k_{F t}$ and $k_{D t}$ evaluated at the steady state $\left(k_{h}, E_{h}\right)$. Under Corollary 1, we have $d k_{D} / d b>0$ and $d k_{F} / d b>0$. Moreover, $\alpha \geqslant 1 / 3$ implies $\beta_{i}<1 \leqslant$ $\alpha /(1-2 \alpha)$ and $R_{h} \geqslant 1 / 3$ implies $\beta_{i}<1 \leqslant R_{h} /\left(1-2 R_{h}\right)$.

We deduce that $d \widetilde{V}_{D h} / d b>0$ which means that the welfare increases for country $D$, whereas the sign of $d \widetilde{V}_{F h} / d b$ is not clear-cut. It is positive if debt level in country $\mathrm{F}, b_{F}$, is sufficiently high. Otherwise, since $d E_{h} / d b>0$, the welfare increases in country F if the weight in the utility associated to environmental quality $\delta_{F}$ is high enough.

### 6.3 Proof of Proposition 2

Let $\widehat{k}_{F 2}$ be a steady state of $k_{F t+1}=G_{F}\left(k_{F t}\right)$ such that $G_{F}\left(k_{F t}\right)$ is tangent to the $45^{0}$ degree line in the plane $\left(k_{F t}, k_{F t+1}\right)$. Such an equilibrium exists if we could find $\widehat{k}_{F 2}$ and $\widehat{b}_{F}$ solving $G_{F}\left(\widehat{k}_{F 2}\right)=\widehat{k}_{F 2}$ and $G_{F}^{\prime}\left(\widehat{k}_{F 2}\right)=1$. Using (21), we get:

$$
\begin{align*}
& \widehat{k}_{F 2}=\frac{\beta_{F}}{1+\beta_{F}}\left(1-\alpha-g_{F}\right) A_{F} \widehat{k}_{F 2}^{\alpha}-\frac{\widehat{b}_{F} \beta_{F}}{1+\beta_{F}} \alpha A_{F} \widehat{k}_{F 2}^{\alpha-1}-\frac{\widehat{b}_{F}}{1+\beta_{F}}  \tag{30}\\
& \alpha \frac{\beta_{F}}{1+\beta_{F}}\left(1-\alpha-g_{F}\right) A_{F} \widehat{k}_{F 2}^{\alpha-1}+(1-\alpha) \frac{\widehat{b}_{F} \beta_{F}}{1+\beta_{F}} \alpha A_{F} \widehat{k}_{F 2}^{\alpha-2}=1 \tag{31}
\end{align*}
$$

Multiplying the second equation by $\widehat{k}_{F 2}$ and substituting in the first one, we get:

$$
\begin{equation*}
A_{F} \widehat{k}_{F 2}^{\alpha-1}=\frac{1+\beta_{F}}{\beta_{F}\left(1-\alpha-g_{F}\right)}\left(\frac{1-\alpha}{1+\beta_{F}} \frac{\widehat{b}_{F}}{\widehat{k}_{F 2}}+2-\alpha\right) \tag{32}
\end{equation*}
$$

whereas equation (31) may rewrite:

$$
\begin{equation*}
A_{F} \widehat{k}_{F 2}^{\alpha-1}\left[\alpha \frac{\beta_{F}}{1+\beta_{F}}\left(1-\alpha-g_{F}\right)+(1-\alpha) \frac{\beta_{F}}{1+\beta_{F}} \alpha \frac{\widehat{b}_{F}}{\widehat{k}_{F 2}}\right]=1 \tag{33}
\end{equation*}
$$

Combining these last two equations, we get:

$$
\begin{equation*}
\frac{1+\beta_{F}}{\beta_{F}\left(1-\alpha-g_{F}\right)}\left(\frac{1-\alpha}{1+\beta_{F}} \frac{\widehat{b}_{F}}{\widehat{k}_{F 2}}+2-\alpha\right)\left[\alpha \frac{\beta_{F}}{1+\beta_{F}}\left(1-\alpha-g_{F}\right)+(1-\alpha) \frac{\beta_{F}}{1+\beta_{F}} \alpha \frac{\widehat{b}_{F}}{\widehat{k}_{F 2}}\right]=1 \tag{34}
\end{equation*}
$$

The left-hand side of this equation is a decreasing function of $\widehat{k}_{F 2} / \widehat{b}_{F}$, which tends to $+\infty$ when $\widehat{k}_{F 2} / \widehat{b}_{F}$ tends to 0 and tends to $\alpha(2-\alpha)$ when $\widehat{k}_{F 2} / \widehat{b}_{F}$ tends to $+\infty$. Therefore, $\alpha(2-\alpha)<1$ ensures the existence and uniqueness of a solution $\widehat{k}_{F 2} / \widehat{b}_{F}$. Given such a solution, equation (33) gives a unique $\widehat{k}_{F 2}$. The existence and uniqueness of $\widehat{b}_{F}$ of course follows.

Taking into account the concavity of $G_{F}\left(\widehat{k}_{F 2}\right)$, for $b_{F}$ slightly smaller than $\widehat{b}_{F}$, there exists a steady state $k_{F 2}$ higher than and close to $\widehat{k}_{F 2}$ with $G_{F}^{\prime}\left(k_{F 2}\right)$ smaller but close to 1 . In this case, equation (24) allows to deduce that $d E_{2} / d b>0$, whereas we already know from (22) that $d k_{D 2} / d b<0$ and $d k_{F 2} / d b>0$.

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| Financial instruments | Amount (million, US\$) | Share |
| :---: | :---: | :---: |
| Grants and related instruments | 14379.1 | $45.2 \%$ |
| Unknown | 614.4 | $1.9 \%$ |
| Multiple | 419.3 | $1.3 \%$ |
| Loans, guarantees and insurance | 14840.2 | $46.7 \%$ |
| Debt relief | 82.5 | $\mathbf{0 . 3} \%$ |
| Capital contribution | 1457.4 | $4.6 \%$ |
| Total | 31792.9 | $100 \%$ |

Table 1 - Instrument used to fulfill climate finance commitments.
From "Debt relief and financing climate change action", A. Fenton, H. Wright, S. Afionis, J. Paavola and S. Huq, Nature Climate Change (4), 2014. Based on data collected by the World Resource Institute http://go. nature. com. inshs. bib. cnrs. fr/rMhVxK


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[^1]:    ${ }^{1}$ Collard et al. (2015) show empirically the extent to which debt levels can be a limit for financing future investments. They propose a measure of maximum sustainable government debt for advanced economies that strongly varies across countries.
    ${ }^{2}$ See the debt sustainability framework of IMF and World Bank for public and external debt sustainability analysis in low-income countries.

[^2]:    ${ }^{3}$ We could however also interpret $m$ as pollution absorption, and consider equation (1) as a linear approximation of a more complicated relationship between production, public abatement, and the quality of the environment. For instance, let $P_{t}$ be the stock of pollution, such that $P_{t+1}=(1-m) P_{t}-\phi_{D} g_{D t}-\phi_{F} g_{F t}+\theta_{D} y_{D t}+\theta_{F} y_{F t}$. Then assume that the environmental quality index is defined by $E_{t}=\widetilde{E}-P_{t}$, with $\widetilde{E} \geqslant 0$ is the long-term natural value of $E$. We obtain:

    $$
    \widetilde{E}-E_{t+1}=(1-m)\left(\widetilde{E}-E_{t}\right)-\phi_{D} g_{D t}-\phi_{F} g_{F t}+\theta_{D} y_{D t}+\theta_{F} y_{F t}
    $$

[^3]:    ${ }^{4}$ For simplification and taking into account the duration of a period, we assume full depreciation of capital after one period of use.

[^4]:    ${ }^{5}$ See also Ercolano and Romano (2018) for a study on spending for environmental protection in Europe.

[^5]:    ${ }^{6}$ The acceptability of a transfer of income from foreign (less advanced) to domestic (advanced) economy is excluded.

[^6]:    ${ }^{7}$ Indeed, equation (20) also writes $k_{i t+1}+b_{i}=\frac{\beta_{i}}{1+\beta_{i}}\left[\left(1-\alpha-g_{i}\right) A_{i} k_{i t}^{\alpha}-\left(R_{i t}-1\right) b_{i}\right]$. We easily see that income increases or not with public debt if the interest rate is smaller or higher than one.

