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**Insurance Supervision under Climate Change:
A Pioneers Detection Method**

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JEL Codes: G22, G28, Q54.

Keywords: Climate change, Insurance market stability, Opinion pooling.



INSURANCE SUPERVISION UNDER CLIMATE CHANGE: A PIONEERS DETECTION METHOD *

Eric Vansteenbergh[†]

Abstract

I present the Pioneers Detection Method, a supervisory tool I developed to enhance resilience in insurance markets facing the challenges posed by climate change. Based on a theoretical model of the insurance industry, I consider a scenario in which independent experts determine premiums according to their individual risk assessments. Due to the segmented nature of the private insurance market, accurately estimating the tail parameter of loss distribution is difficult, especially given the rarity of extreme events. My method leverages temporal directional change and convergence to integrate expert opinions, giving greater emphasis to those who effectively identify trend shifts after climatic tipping points. A series of simulations reveals that the Pioneers Detection Method outperforms traditional pooling methods within a Bayesian framework. Furthermore, this approach appears to be notably effective in improving welfare in an insurance market with a limited number of private entities.

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[†]Paris School of Economics - EHESS; Banque de France - ACPR: eric.vansteenbergh@acpr.banque-france.fr

1 INTRODUCTION

There have recently been many press articles raising blunt questions about the insurability of our planet and academic articles warning of imminent climate tipping point (Ditlevsen and Ditlevsen, 2023).¹ Climate change, characterized by long-term shifts in temperatures and weather patterns, leads to both direct (e.g., heatwaves, droughts) and indirect (e.g., desert expansion, wildfires, storms) extreme events with increasing frequency and intensity (Stott et al., 2016). Although a well-established phenomenon², there is no consensus on whether climate change will be continuous or whether there will be unexpected shocks that will pose significant challenges for financial systemic risks, which fall within the mandate of financial supervisors (Svartzman et al., 2021). One such challenge is determining the insurability of risks in a constantly changing environment and one of the supervisory tools for this challenge is to leverage insurance company expertise by pooling their opinions.³ In the foundational work by Stone (1961), opinion pooling is presented as the method of merging experts' subjective probability distributions to form a collective judgment. However, traditional pooling methods were not specifically devised for extreme events and a loss distribution tail parameter that can never be exactly estimated. As a result, they encounter two primary hurdles: potential inconsistencies in expert opinions over time due to model shortcomings, and slow decision-making processes in accepting or rejecting insurability hypotheses. In this paper, an insurance market is modeled to design, validate, and estimate the welfare benefits of introducing a novel opinion pooling strategy specifically designed for this context: a Pioneers Detection Method.

I start by designing an insurance market model derived from Raviv (1979). I depart from it by introducing beliefs heterogeneity among insurance experts and study its equilibrium to determine the optimal insurance contracts when reinsurance is not available. When this heterogeneity arises from information limited to small private samples, I study remedies that an insurance supervisor can implement to limit potential welfare loss. Climate change impacts both the frequencies and magnitudes of events; in the model, yearly aggregated insurance losses for an asset class follow a Pareto distribution with an unobservable tail index.⁴ This

¹“What to do when the US becomes uninsurable” Financial Times 12th June 2023.

²Early works on climate change include those by Fourier (1824), Foote (1857), Arrhenius (1896), Callendar (1938). Recent contributions are reviewed in Hegerl et al. (2007) and (Change, IPCC Climate and others, 2014).

³For insurability criteria, see Berliner (1985).

⁴The aggregate or magnitude component of losses can be understood as yearly aggregates of losses faced by an insurance company for a given asset class. The paper assumes that insurance companies cannot completely discriminate policyholders, so premium pricing is equivalent to modeling the magnitude components.

paper models non-cooperative private insurance experts with proprietary information sets and modeling expertise.⁵ I model a tipping point where reinsurers, exogenous to the market, decide to withdraw.⁶ After this distinct and unanticipated shock, there is a situation of heterogeneous beliefs among insurance experts as they update their risk models to determine whether, and under what conditions, they provide insurance coverage for the affected asset class.⁷ An insurance supervisor, mandated with ensuring financial stability, is also included in the model. The supervisor decides if there is a need to intervene in the insurance market. The goal of this intervention would be to limit welfare losses by influencing the terms under which insurance is provided for specific asset classes. To do so and for the supervisor to estimate the risk after a tipping point, the context calls for a new insurance supervision tool.

In this paper, I introduce an innovative supervisory tool, namely the Pioneers Detection Method (PDM). It is particularly designed for the constantly changing climate, where the absence of solid, empirical data challenges expertise evaluations grounded on past estimations. The primary objective of this method is to assign substantial weightage to dependable pioneers—experts who have consistently underscored the intensifying gravity of climate-centric risks. These experts employ transparent models and datasets, and their viewpoints receive tacit validation from their professional peers over time.⁸ The opinion pooling weights can be understood as unspoken inter-temporal votes shared among experts. The method’s core strength lies in its dependency on directional shifts to configure weights, rather than a primary emphasis on reducing disparities between opinions.

When the insurability of an asset class becomes suddenly uncertain⁹, and furthermore if reinsurance is not available, two problematic scenarios can arise from a financial stability

⁵The European Court of Justice established in 1987 (Judgement of 27.1.198) that EU competition law is fully applicable to the insurance sector. Some exemptions were allowed by the Insurance BER” (Regulation (EEC) No. 3932/1992) as Collaboration between insurance undertakings [...] in the compilation of information (which may also involve some statistical calculations) allowing the calculation of the average cost of covering a specified risk [...] makes it possible to improve the knowledge of risks [...]. This can in turn facilitate market entry and thus benefit consumers.” The “Insurance BER” was stopped on April 1, 2017. Automatic exemptions were discarded to avoid misapplications, considering that guidelines were sufficient (Stancke, 2017).

⁶This concept refers to a critical threshold in a complex system, where minor perturbations can result in substantial, often irreversible, changes in the system’s overall behavior or state (Gladwell (2006) and Lenton et al. (2008)).

⁷Belief heterogeneity between insurance buyers and sellers is not necessary in this model to reach our policy recommendations. I discuss the implications of this additional heterogeneity. A class can be understood here as either a region or a business line.

⁸Examples include Meadows et al. (1972) and Jean-Marc Jancovici (Jancovici, 2004), whose diagnostic strategies are built on transparent data and have paved the way for predictions and guidelines that are progressively being embraced by the mainstream.

⁹Uncertainty in the sense of Knight (1921), which is known to result in higher premiums than under unambiguous risks (Hogarth and Kunreuther, 1989).

perspective. The first concern is that some insurance companies might underestimate the impacts of the change and be exposed to ruin. This paper primarily focuses on the second scenario, where non-cooperative insurance companies increase their premiums, potentially close to the coverage limit.¹⁰ To determine whether the reduction in insurance supply is justified by the increase in risks or the inability of insurance experts to accurately estimate risks, the supervisor can decide to collect expertise using ad-hoc on-site inspections of insurance companies and pool expertise. Although there is a rich literature on stress tests ([Battiston et al., 2017](#)) and their use in learning about the insurance market and guiding supervisory actions, little research has been conducted on recommending ad-hoc inspections for pooling opinions for the same purpose.¹¹ This paper aims to fill this gap by demonstrating how supervisors can use their regular supervision or ad-hoc inspection activities to reinforce their knowledge and understanding of climate change’s disruptive potential for the insurance markets. An additional application would be to apply the method on heterogeneous bottom-up stress test projections.¹²

More broadly, this paper contributes to the continuing discourse on insurance supervision in the context of climate change. In countries like France, debates exist on whether housing insurance state guarantees for clay soil risks should be scrapped and how a supervisor can act once the market is fully private. Currently, 40% of the state natural catastrophe scheme (CatNat) is used to cover losses for houses exposed to the risk of Withdrawal-Swelling of Clay Soils (WSCS) under the effect of drought.¹³ The French Court of Auditors is advising the exclusion of WSCS from the CatNat regime, given that the unpredictable nature of the risk is no longer being addressed due to the effects of climate change ([Cour des comptes, 2022](#)). If WSCS was removed from the CatNat regime and if external reinsurers decided to discontinue the coverage for certain areas, the supervisor could use the PDM to learn as fast as possible from insurance companies internal expertise. An alternative approach would be to use historical data from the scheme and build his own granular model. When

¹⁰[Winter \(1994\)](#) acknowledges that supply will always be positive at some price but mentions premiums reaching 90% of the coverage limit for firms removing asbestos.

¹¹The ECB Banking Supervision launched a 2022 climate risk stress test using macroeconomic scenarios from the NGFS. The closest strategy to inspection is NAIC’s Climate Risk Disclosure Survey, a voluntary risk management tool through which state regulators can request annual, non-confidential disclosure of insurers’ assessment and management of their climate-related risks. Source: Moody’s report on Insurance Conference 2022.

¹²In a bottom-up stress test exercise, each insurance company projects his exposure based on a stressed scenario common to all companies. The **ACPR** launched its second **climate stress test** covering the **insurance sector** on the 2023-07-06. The author thanks Jeffrey Czajkowski for suggesting this application for the PDM.

¹³[Fleureau et al. \(1993\)](#) characterizes the phenomenon, and [Babaeian et al. \(2019\)](#) describes the modeling challenges compounded by limitations of remote sensing of soil moisture.

granular data are not readily available, and the supervisor’s capacity is limited in modeling the risk for each asset class, the novel methodology enhances supervision. It aggregates expertise at the insurance group level, rather than initiating procedures to obtain granular underwriting data for evaluating climate-related financial risks post-shock. While the United States Department of the Treasury’s Federal Insurance Office (FIO) has considered collecting granular data from property and casualty insurers regarding homeowners’ insurance, it has faced resistance.¹⁴ A policy recommendation from this paper is to allow the use of this new tool to help supervisors determine if an asset class is insurable. Upon determining the insurability of an asset class, the supervisor can then advise the regulator on whether to utilize public or private insurance mechanisms, such as syndication or reinsurance, for coverage.

Literature The main contribution of this paper is on the leadership and pioneership literature. The etymology and historical application of the terms ‘leadership’ and ‘pioneership’ have origins rooted deeply in military contexts. According to the Oxford English Dictionary (2015), a ‘leader’ was initially defined as an individual who ‘conducts, precedes as a guide’, while a ‘pioneer’ was described as ‘a member of an infantry group [...] ahead of an army [...] to clear terrain in readiness for the main body of troops’. In financial contexts, leadership connotes certain interactions, with some investors being influenced by the actions of their counterparts. This effect is so significant that it’s considered a ‘first-order effect’ (Devenow and Welch, 1996). Such interactions might result in ‘information cascade’, ‘when it is optimal for an individual, having observed the actions of those ahead of him, to follow the behavior of the preceding individual without regard to his own information’ (Bikhchandani, Hirshleifer and Welch, 1992). Hirshleifer and Hong Teoh (2003) categorises information cascade as a subset of herding, which is recognized as behavioural uniformity stemming from individual interactions. Notably, in financial literature, the concept of pioneership is largely overlooked. It emerges ‘where individuals act similarly due to a parallel, independent influence of a shared external factor’ (Hirshleifer and Hong Teoh, 2003). In relation to climate change, a distinction emerges between leaders and pioneers. Specifically, a ‘leader state has the explicit ambition to lead others, while a pioneer state’s priority is to develop its own pioneering activities without paying (much) attention to attracting followers.’ (Lieberink and Wurzel, 2017). In this paper, the supervisor assumes a leadership role, whether on a temporary or permanent basis, guiding insurers to address climate change challenges. Conversely, a pioneer is delineated as an expert who temporarily serves as an indicator (predictor) for

¹⁴The FIO issued a request for comment on a [proposed data call](#) on October 18, 2022.

his peers in the subsequent time step. The primary motive driving pioneership is the intent to assess risks judiciously and calibrate premiums to prevent financial downturns for the insurance entity. With our innovative Pioneers Detection Method (PDM), multiple experts might qualify as pioneers during a particular timeframe, albeit with varying magnitudes. It is worth noting that in our definition, pioneership lacks inertia; an expert’s status as a pioneer at a given moment doesn’t guarantee their continued pioneership in future periods. Our approach significantly diverges from conventional financial literature in two pivotal ways. First, experts, in our context, do not leverage information from fellow experts’ premiums. As experts are in competition, their primary focus is on pioneership to bolster their insurance firm’s fiscal health, rather than leadership or mimicking another expert. Only the insurance supervisor is equipped to gather information and discern the pioneership intensity of each expert. Second, unlike traditional financial paradigms, our approach acknowledges that the supervisor cannot ever truly discern the risk’s actual realization. Hence, pioneership can only be discerned when an expert’s actions seem to be emulated by peers over successive periods. Implicit in this is the belief that experts’ precision will refine over time. An alternative perspective on pioneership intensity involves weighting experts to expedite convergence towards a more precise risk assessment.

This paper contributes to the literature on opinion pooling and combination forecasting, building upon previous reviews by [Genest and Zidek \(1986\)](#), [Clemen \(1989\)](#), [Timmermann \(2006\)](#), and [Wang et al. \(2022\)](#). This literature concludes that in a Gaussian context it is almost impossible to beat a mean method potentially excluding outliers. In a context with fat tails, extreme events do occur but are rare, hence learning is slow and I design a new method that relies on inter-temporal opinion changes to weigh experts that are the most likely to learn faster about extreme events.

This paper adds to the rich literature and ongoing debate surrounding the impacts of climate change on insurance losses. There are mixed conclusions on the effects of climate change on loss trends (especially after normalizing with the local gross domestic products), as discussed by [Mills \(2005\)](#), [Pielke Jr et al. \(2008\)](#), [Stern \(2008\)](#), [Kousky \(2014\)](#), [Hoeppe \(2016\)](#), [Hsiang \(2016\)](#), and [Botzen, Deschenes and Sanders \(2020\)](#). I use publicly available data to model the evolution of losses over time in [Appendix A](#). I find that the combined influences of climate and macroeconomic changes lead to increases in both average expectations and tail fatness of losses. The modeling addresses heavy-tailed distributions, drawing on the Extreme Value Theory (EVT) literature and its applications in insurance, as seen in the seminal works of [Frechet \(1927\)](#), [Fisher and Tippet \(1928\)](#), [Gnedenko \(1943\)](#), [de Haan \(1970\)](#), [Balkema and](#)

De Haan (1974). EVT has been applied to various aspects of climate change, including the analysis of extreme weather events (Coles et al., 2001; Palutikof, Subak and Agnew, 1997) and the estimation of extreme financial losses (McNeil, Frey and Embrechts, 2015). The primary challenge, which remains unresolved, is that loss tails must be estimated with limited observations. Consequently, after an unforeseen tipping point, expert model uncertainty increases. This paper adds to the literature on modeling loss tails under uncertainty, building upon the works of Danielsson et al. (2001), Scarrott and MacDonald (2012), and Raftery et al. (2017), by exploring the combination of EVT and expert opinion.

This paper builds upon the literature on insurability and the challenges climate change poses to insurability. Climate change is testing the insurability of various business lines and geographic regions, as demonstrated by Kunreuther (1996), Jaffee and Russell (1997), Mills (2007), Charpentier (2008), Kousky and Cooke (2012), and Surminski (2014). Both pandemic and climate change risks are difficult to mutualize in a cross-sectional manner due to their complex nature and widespread impacts. While pandemics have an episodic nature, insuring them intertemporally presents challenges due to their potential for widespread impact and external moral hazard Richter and Wilson (2020). Nevertheless capital markets (Cat bonds) and provisions can be efficiently used for pandemics with government promoting intertemporal behavior Hartwig, Niehaus and Qiu (2020), a model for private pandemic insurance is suggested by Gründl et al. (2021). Insurance companies cannot adopt the same strategies against climate change, as it is expected to be an irreversible trend with potential tipping points. This paper demonstrates that, although preventive actions against climate change are primarily the responsibility of regulators, a supervisor operating further down the line for insurance market stability can utilize this paper to direct his prudential activities to avoid disturbances caused by uncertainty. My approach differs from the work of Jaffee and Russell (1997) as I do not focus solely on catastrophes. Jaffee and Russell (1997) consider that catastrophes are actuarially insurable as they are infrequent, local and uncorrelated.¹⁵ In my approach, I consider yearly cumulative losses each insurance company faces and how climate change fattens the subjectively estimated tail of this distribution up to uninsurability.

Section 2 presents a model of the insurance market under climate change, where a risk-averse insurance buyer faces potential loss to an asset, and insurance companies operate in a competitive market with heterogeneous beliefs. The model also considers the effects of climate change on the tail parameter over time, the absence of reinsurance, and the role of an insurance supervisor who can influence the market by estimating and acting on the tail

¹⁵Jaffee and Russell (1997) considers that uninsurability is mainly due to the lack of (tax) incentives for insurance companies to accumulate a pool of liquid assets to meet catastrophe losses.

parameter. Section 3 introduces the PDM, a novel approach to identify experts (pioneers) who deviate from the majority but toward whom other experts' opinions converge over time, especially in the context of rare events impacted by a changing climate. The method relies on four main steps, including identifying trends, distance reduction, orientation change for convergence, and proportion of convergence attributed to a pioneer, and it also discusses alternative approaches. Section 4 details the testing and validation of the PDM against alternative opinion pooling methods in various scenarios, including insurance supervision under climate change and demonstrates that the PDM outperforms traditional measures in producing precise estimates earlier. Section 5 suggests that the decision to use the PDM in insurance markets depends on the market configuration and the evolution of the tail parameter. Through mathematical models and simulations, it demonstrates that in configurations with more than five insurance companies, the benefit of using the PDM depends on the cost of collecting information. However, in configurations with fewer than five insurance companies, it is always more beneficial to use the PDM to improve welfare, regardless of the cost function. Section 6 explores the complexities of policy recommendations for dealing with heterogeneous beliefs between insurance buyers and insurance companies in the context of climate change. Finally, Section 7 summarizes the contributions to the fields of opinion pooling, climate change impacts, insurability, and insurance supervision, highlighting the introduction of the PDM as a practical tool to assess insurability and enhance market supervision amid climate change challenges; it emphasizes the method's speed and advantage in extreme events.

2 A MODEL OF INSURANCE MARKET UNDER CLIMATE CHANGE

A risk-averse insurance buyer (IB) with an initial aggregated wealth w and a von Neumann-Morgenstern utility function denoted by U , with $U'(\cdot) > 0$ and $U''(\cdot) < 0$, faces a risk of (yearly) aggregated loss of x^t to his asset, where x^t is a random variable following a Pareto distribution $\mathcal{P}(x^t, \alpha^t)$ with an unknown tail parameter α^t and a threshold normalized at unity.¹⁶ In this model, the IB is assumed to have one asset class facing a single risk, meaning that there is no consideration of background risks, thus neither moral hazard nor adverse

¹⁶Hogg and Klugman (1983) presents applications to long tailed skewed loss distributions, Hogg and Klugman (1984) presents loss distributions modeled with a Pareto on grouped data, Embrechts, Klüppelberg and Mikosch (2013). Balkema and De Haan (1974) introduces the class of functions for which exceedances over a large threshold converge to the Generalized Pareto Distribution (GPD).

selection problems.¹⁷

An insurance contract for an insurance company (IC) i is defined as a pair $(I(x^t), \Pi_i(I(x^t)))$, with an indemnity schedule $I(x^t)$ and an insurance premium $\Pi_i(I(x^t))$. Climate change, combined with macroeconomic factors such as inflation, impacts the tail parameter over time t with potentially disturbing tipping points, but the realization of α^t is never observable.¹⁸ Each IC is considered an expert who estimates the tail parameter and calibrates its insurance contract based on public information and its private claim histories.¹⁹ I consider m risk-neutral ICs that operate in a perfectly competitive insurance market without access to reinsurance. These ICs have homogeneous market shares for the asset class but potentially heterogeneous beliefs denoted by $\hat{\alpha}_i^t$. Each IC has its own private cost policy, denoted as γ_i .²⁰ I make the usual assumption that $\gamma_i'(\cdot) \geq 0$ to reflect the monitoring and auditing efforts made during the claims processing. There is no modeling of strategic behaviors of IC, and market shares are considered as fixed and exogenous.²¹ Hence, each IC offers the insurance premium (1).

$$\Pi_i\left(\frac{1}{m}x^t\right) = \mathbb{E}_{\hat{\alpha}_i^t}\left[I\left(\frac{1}{m}x^t\right) + \gamma_i\left(I\left(\frac{1}{m}x^t\right)\right)\right] \quad (1)$$

The IB is offered an aggregated premium $\Pi = \frac{1}{m} \sum_{i=1}^m \Pi_i$ for its asset, and I assume there exists a synthetic tail parameter belief $\bar{\alpha}^t$ such that $\Pi(x^t) = \mathbb{E}_{\bar{\alpha}^t}[I(x^t) + \gamma(I(x^t))]$ where γ , as a sum of increasing functions, is increasing. The optimal insurance contracts are solution of the simplified program (2). There are two sources of beliefs heterogeneity in this model. The first source of belief heterogeneity comes from the heterogeneous claim histories that will drive the posterior IC belief $\hat{\alpha}_i^t$, but will not impact how program (2) is solved compared with the literature. The second source of heterogeneity comes from the IB subjective probability $\hat{\alpha}_b^t$ which can differ from the IC belief $\bar{\alpha}^t$ and is the main point of attention on how program (2) is solved.

¹⁷Schlesinger (2013) provides a survey on demand for insurance for single risk.

¹⁸In this paper, both climate and economic effects on risk are referred to as “climate change” for simplicity.

¹⁹Experts can be Bayesian or Frequentist; both cases are considered for the simulations. Brunnermeier, Simsek and Xiong (2014) emphasize distortion in updating as a key source of heterogeneous beliefs. In this model, different information is the unique source of heterogeneity. Furthermore, heterogeneity in prior beliefs is not studied.

²⁰I follow the notation of Brunnermeier, Simsek and Xiong (2014) for beliefs heterogeneity: $\mathbb{E}_{\hat{\alpha}_i^t}(x) = \int x \mathcal{P}(x, \hat{\alpha}_i^t) dx$.

²¹An extension is to use the supervisory data to estimate the demand sensitivity to premiums to make the market shares endogenous to the model, resulting in heterogeneous private information and incentives to invest in modeling each class’s idiosyncratic risk. This would allow introducing situations where an insurer with low exposure to an asset class could offer loss-making premiums to grow its share, introducing a ruin gamble not present in this model.

$$\begin{cases} \max_{I(x^t), \Pi} W(I(x^t), \Pi) := \mathbb{E}_{\hat{\alpha}_b^t} [U(w - x^t + I(x^t) - \Pi)] \\ \text{subject to } \Pi(x^t) = \mathbb{E}_{\hat{\alpha}^t} [I(x^t) + \gamma [I(x^t)]] \\ \text{and } 0 \leq I(x^t) \leq x^t \end{cases} \quad (2)$$

Raviv (1979) extends Arrow (1963) and demonstrates that if there are no belief heterogeneity between the IB and the IC, an optimal contract is full insurance above a straight deductible. Gollier (2013) shows that optimism on the side of the IB in the sense of a monotone likelihood ratio explains coinsurance above the optimal deductible. Chi (2019) shows that full insurance over a straight deductible is always optimal if and only if the IC is more optimistic about the conditional loss given non-zero loss than the IB in the sense of monotone hazard rate order (MHR). I follow the MHR hypothesis for this paper, which allows for simple simulations. Additional configurations are discussed in sections 6.3 and Appendix B. Without loss of generality, in the MHR case, I normalize this straight deductible to unity, which can later be multiplied by the order of magnitude of the asset class under consideration. This simplifies the insurance contract to a one-dimension parameter problem where the indemnity follows a Pareto with a varying tail parameter $\hat{\alpha}_i^t$ and a unity threshold.²²

An insurance supervisor, denoted as S and mandated for ensuring financial stability, completes the model.²³ S is a social planner whose program aims to maximize the welfare of the IB and IC, which is equivalent to maximizing the IB welfare by solving the program (2).²⁴ Following Brunnermeier, Simsek and Xiong (2014), S is aware of the presence of belief heterogeneity among IC but is unaware of the objective belief. Therefore, S has to form his own tail parameter estimate, $\hat{\alpha}_S^t$. S has two main options: 1) to exploit the estimates $\hat{\alpha}_i^t$ at a fixed cost c_S and pool their expertise $\hat{\alpha}_{pool}^t$, with c_S as a fixed cost for dedicating a team to this task based on available data) or 2) to audit IC (e.g. with on-site ad-hoc inspections) to estimate $\hat{\alpha}_{audit}^t$ after collecting detailed claim history at a cost $c_S + \lambda \#x$, with a term linear in the size of the information to collect $\#x$.²⁵ I make two assumptions about how S

²²The tail parameter is sufficient to characterize both the magnitude of the expected indemnity and the tail of the distribution: $\mathbb{E}^i [I(x^t)] = \frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1}$.

²³Real-life examples will be the International Association of Insurance Supervisors (IAIS)'s members: European Insurance and Occupational Pensions Authority (EIOPA), France's Autorité de Contrôle Prudentiel et de Résolution (ACPR), Germany's Bundesanstalt für Finanzdienstleistungsaufsicht (BAFIN), UK's Prudential Regulation Authority (PRA), etc.

²⁴This is a "benevolent supervisor assumption" which is equivalent to consider that S dislike leaving a rent to the IC or considering S maximizes IB and IC surplus when the IC is risk-neutral in a competitive insurance market.

²⁵As part of his supervision activities, S already collects internal model reporting from IC and thus has access to their individual estimates $\hat{\alpha}_i^t$.

modifies the program (2). First, I assume that the functioning costs of S are embedded in the costs of the IC. As ICs pay fees to the supervisor, these fees are passed on to the IB as part of the IC cost function. Secondly, I assume that S announcement is fully trusted by an IC for their offered contracts.²⁶ Equation (3) formalizes the two impacts of possible S interventions. Traditional opinion pooling methods have not been optimally designed for the EVT context. In an EVT context, extreme events do occur but with low probability, hence ICs will have heterogeneous learning speeds of the tail parameter, depending on whether their small samples contain extreme realizations. I design a Pioneers Detection Method for this context in Section 3.

$$\begin{aligned} \text{Intervention after pooling: } & \mathbb{E}_{\hat{\alpha}_{pool}^t} \left[U \left(w - x^t + I(x^t) - \Pi \left[x^t \right] + c_S \right) \right] \\ \text{Intervention after audit: } & \mathbb{E}_{\hat{\alpha}_{audit}^t} \left[U \left(w - x^t + I(x^t) - \Pi \left[x^t \right] + c_S + \lambda \#x \right) \right] \end{aligned} \quad (3)$$

3 A PIONEERS DETECTION METHOD

The design of a new method for weighting expert estimates is informed by two primary assumptions of the model. First, estimations can never be compared to a realization of the tail parameter for weight calculation based on historical performance or forecast errors (Genest and Zidek, 1986; Stock and Watson, 2004). As a result, the tool must rely on cross-sectional and temporal comparisons of expert model outcomes. Second, the distributions exhibit fat tails, indicating that extreme events occur infrequently but do happen. Experts learn from their claims; hence, experts already exposed to an extreme event at time t will have faster learning rates compared with experts not yet exposed to tail events.²⁷ This is a significant departure from the traditional literature on expert combination, where only a minority exhibits a bias (Min and Zellner, 1993). In a climate change environment, a subset of experts might accurately internalize the change before the majority converges towards this initial subset. The Pioneers Detection Method (PDM) aims to identify this subset of pioneers as early as possible to increase the learning of tail parameters.

Pioneers are experts who deviate from the majority opinion, but other experts' opinions converge toward them over time, even though the experts do not cooperate.²⁸ This can also

²⁶For completeness, once calibrated, S should integrate in the simulation how his announcement, $\hat{\alpha}_S^t$, can be integrated by an expert depending on his evaluation of his credibility and alignment with his objectives.

²⁷Equation (40) provides an expression of their posterior when experts' estimations are Bayesian.

²⁸Liefferink and Wurzel (2017) introduce a clear distinction between pioneers as being 'ahead of the troops or the pack' and leaders which have 'the explicit aim of leading others, and, if necessary, to push others in

be thought of as implicit inter-temporal voting among experts to identify pioneers.

In section [Appendix C](#), I introduce alternative novel approaches in this context and will test them against the PDM. In [Appendix C.1.](#), I study the convergence properties of the PDM.

The main benefit of the PDM is that the supervisor recognizes the convergence as soon as it begins and does not wait for the differences in estimates to narrow; a convergence in direction is enough to trigger a shift in weights. This PDM could also be envisaged for other contexts where non-cooperating experts are learning on rare events. I introduce four steps for the PDM.

Initialization without Pioneer situations

At time $t = 0$, no past observation exists, hence the weights w_i^0 have to be initialized and can be taken as a linear combination, i.e. $w_i^0 = \frac{1}{m}$.

There can be situations where no Pioneer is identified. A simple example is a case where there are only two experts and at time t both are diverging from one another. In this case, the weights from the period $t - 1$ can be replicated.

There can be some exotic situation where since the initialization period no Pioneer can be detected, hence the linear weights w_i^0 are still applied all along. This situation is not so much a weakness of the PDM, but rather a characterization of a situation where no implicit consensus emerges among experts. Taking a linear combination of diverging opinions might not be optimal and S would need to investigate to understand why a persistent divergence can exist before building his own estimate.

Step 1: identify trends

First, I smooth the time series to identify underlying trends in pioneers and followers. This can be achieved using moving averages or filtering techniques, depending on the specific phenomenon being studied.

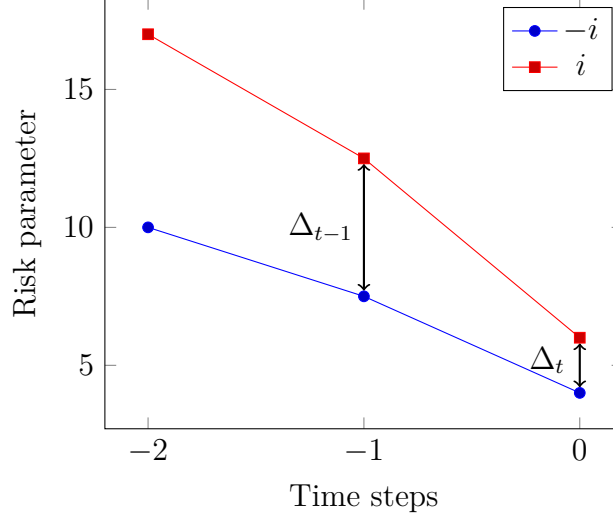
Step 2: distance reduction condition

Second, I determine if the distance between an expert's estimate and the average of the other experts' estimates has decreased between $t - 1$ and t . This is represented by a dummy variable, $\delta_{\text{distance}}^t$, which serves as a necessary but not sufficient condition for an expert to

a follower position'.

be considered a pioneer.

FIGURE 1
Distance reduction dummy to identify Pioneers



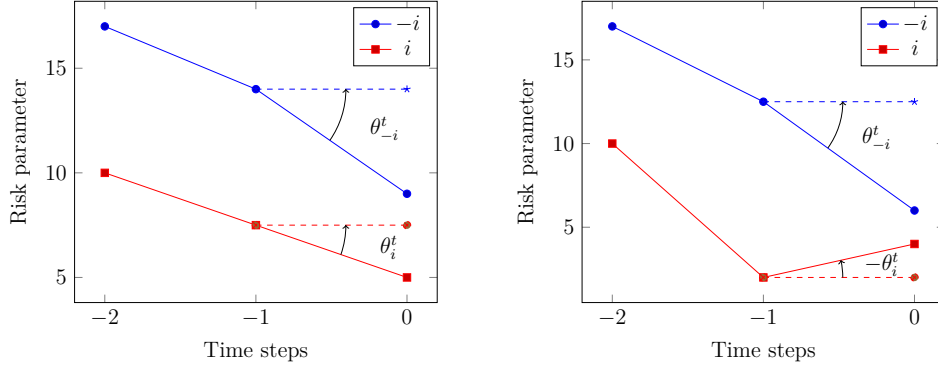
Notes: i represents the expert of interest and $-i$ the average estimate of his competitors (i excluded). $\delta_{\text{distance}}^t = \mathbb{1}_{\Delta_t < \Delta_{t-1}}$

Step 3: orientation change for convergence condition

Third, I determine whether the orientation of the segments between an expert's estimate and the average of the other experts' estimates has decreased between $t - 1$ and t . This is represented by a dummy variable, $\delta_{\text{orientation}}^t$. If the second step results in a positive dummy, this step helps determine whether the significant convergence of trends is due to the other experts agreeing with the expert (who is then considered a pioneer) or if the expert is the one converging toward the average of the other experts' judgments (and is thus a follower).²⁹ This distinction is made by attributing the change in direction primarily to either the expert's own estimate or the average of the estimates of his peers. If the change in direction is mainly due to the average of the expert's peers, then the expert is considered a pioneer.

²⁹ *Agreeing* here means: Their estimates change direction over time towards the expert's estimate, and both time series converge (or are cointegrated).

FIGURE 2
Orientation change dummy to identify Pioneers



Notes: i represents the expert of interest and $-i$ the average estimate of his competitors (i excluded). The measure is $\delta_{\text{orientation}}^t = \mathbb{1}_{\theta_{-i}^t > \theta_i^t}$. In both panels, i can be considered a pioneer. The right panel is interesting because i is not consistent, an aspect that is taken care of both in the smoothing of the time series and in the second characteristic of the measure.

Step 4: proportion of the convergence attributed to a Pioneer

Fourth, I calculate the proportion of the orientation decrease attributable to the average of the other experts' estimates, $-i$, with respect to the considered expert's estimate, i . The pioneer score for each expert, indexed by i , is then derived from a combination of the four steps, as shown in Equation (4) and is used as a weighing for his opinion in the final estimate of S : $\hat{\alpha}_S^t = \sum_i w_i^t \hat{\alpha}_i^t$.

$$w_i^t = \delta_{\text{distance}}^t \times \delta_{\text{orientation}}^t \times \frac{|\theta_{-i}^t|}{|\theta_{-i}^t| + |\theta_i^t|} \quad (4)$$

where $-i$ is the mean of all expert estimates, i excluded.

4 TESTING AND VALIDATING THE PIONEERS DETECTION METHOD

I test and validate the PDM against alternative opinion pooling methods, using the Root Mean Square Error (RMSE) as a criterion to evaluate the precision of the estimate. I will use time series settings and evaluate the capacity of the PDM to produce a precise estimate earlier than traditional opinion pooling methods. I demonstrate the importance of this early capacity in the context of insurance supervision under climate change. In a severe stress

scenario where all reinsurance companies are foreign and out of supervision and regulation scope of S , after the estimated tail of the yearly aggregate losses of an asset class jumped due to climate change, reinsurance companies can decide to leave this market. Insurance experts are left alone to decide at which cost they can offer insurance coverage and experts can have heterogeneous beliefs. I model heterogeneous beliefs with Bayesian experts estimating the tail parameter based on their private samples.³⁰ I evaluate the capacity of the PDM to help S leverage on a limited count of insurance expert estimates and a limited time series history (two to five years) to decide on optimal supervision and regulation actions.

I start with the seminal data set on combination forecasts as a preliminary check. I find that the PDM produces coherent results and outperforms traditional measures. This first part is not for drawing a general conclusion; it serves as a sanity check to ensure that the PDM, if applied in a non-EVT context, would still perform. For the main validation, I model an insurance market with Bayesian experts who have potential heterogeneous beliefs, learning a fat tail parameter from private samples taken from the same loss EVT distribution. Across simulations, to validate the relevance of the PDM in different market settings, I vary the tail parameter and the number of experts to confirm that the PDM performs best, as early as possible, in any configuration. I cannot determine *ex ante* which experts will be identified as pioneers and to which extent, which will determine their weight over time, hence I will ultimately have to use Monte Carlo simulations with a known fat tail distribution for each simulation. I use a Pareto distribution as I argued in footnote 16 that it covers generic situation of aggregate yearly losses with fat tails.

4.1 Comparison the Pioneers Detection Method with Existing Combination Methods

I compare the PDM with existing combination forecasting methods. Following the work of [Stock and Watson \(2004\)](#), which relies on the dataset from [Bessler and Brandt \(1981\)](#), all methods are evaluated using their RMSE relative to the autoregressive model of order one calibrated at each period on all past periods.³¹ As with the original paper, I find that combination forecasting improve upon autoregressive forecasts (Table 1). The PDM performs

³⁰It is common practice that insurance experts leverage also on public information and consulting companies or model provider can reduce the heterogeneity. While reliance on common model might raise other supervision concerns, I limit my modeling to estimates based on private information.

³¹An autoregressive model of order one, AR(1), for a stationary time series α^t is the simplest form $\alpha^t = \phi\alpha^{t-1} + \nu^t$ with ϕ a constant that can be calibrated with ordinary least squares method and ν^t an error term. If a more complex model cannot beat the AR(1) wrt RMSE, then this complex model may be discarded in favor of the parsimonious AR(1) ([Box et al., 2015](#)).

best, although with only three forecasters, there are some situations where no expert can be identified as a pioneer (e.g., when all three experts are diverging from one another). This is a limitation in the application of this method for combining forecasts. Note that the three novel methods introduced—Granger Causality, Lagged Correlation, and Pioneers—do not rely on past performance and do not include the actual true time series to be estimated.

4.2 Validation of the Pioneers Detection Method with Fragmented Insurance Market Shares

I now want to evaluate the capacity of the PDM to “learn as fast as possible” a loss distribution after a unique and unexpected change. For this test, I will use configurations with limited count of insurance experts $m \in [2, 20]$ and tail parameter after the change around the problematic range $\alpha \in [0.5, 3]$.³²

I evaluate the PDM on a fragmented insurance market through the use of Monte Carlo simulations. In a hypothetical steady state, IB and IC would agree on an estimation of α , α^- , although there can be heterogeneous beliefs among IC. This is equivalent to modeling that each IC would have his own estimates $\hat{\alpha}_i^-$, but the representative offered insurance contract in the steady states follows α^- . IB are not explicitly modeled in the simulations but are assumed to solve the program (2) for the optimal insurance contract. This optimal contract form is important for the simulations. If beliefs’ heterogeneity does not align with my main assumption in programme (2), I discuss in [Appendix B](#) a workaround where IB have a reservation premium. I start the simulations with Bayesian experts who have uninformative priors and allow for two time periods, $t \in [0, 1]$, as burn-in to account for beliefs’ heterogeneity. At $t = 2$ a climate change enters a unique and unexpected tipping point. The unknown tail parameter jump to a new value following the climate tipping point and I simulate that after this tipping point the climate has stabilized to an unobservable state, hence α remains stable. In this controlled environment, I can calculate the RMSE for each expert estimation against the true α , after time $t = 2$. I benchmark the performance of the PDM against existing opinion pooling techniques as well as other novel approaches introduced within this study.

As IC experts can be either Bayesian or Frequentist, I test two extreme configurations, a full Bayesian and a full Frequentist.

³²For a Pareto loss distribution, as soon as the tail parameter is below 2 the variance is unbounded and as soon as the tail parameter is below 1 the mean is unbounded.

4.2.1 A full Bayesian configuration

I model Bayesian insurance experts who learn losses generated from a Pareto type I distribution with a tail parameter close to unity, meaning a fat tail environment. Each expert’s observations are independent from one another both in cross-section and over time. In such an environment, experts will observe extreme values over time, but might have to wait for some time before observing an extreme event and being able to refine their calibration of the tail parameter. The PDM starts with at least one set of past data points, and it weights pioneers as soon as a change of direction to their estimates occurs. The supervisor needs at least one past period of estimations to form his own subjective opinion. Once the estimation can be formed, the PDM outperforms the other opinion pooling methods (Table 2). The performance of the new method is significant in early stages, especially at the first period where an estimate can be formed ($t = 2$). The performance is also found to be more stable over time, using mean, median, and the standard deviation over the first ten periods (Table 3). An interesting challenger is to take the minimum tail parameter at each step.³³ With heterogeneous beliefs, this can lead to taking the most conservative premium, but has the benefit of being a stable approach (Table 3, standard deviation). If subjective (non-Bayesian) experts judgments are added to the tail parameter estimate in a crisis, then taking the minimum estimate would exacerbate the *panic* or *run* on the insurance coverage provision.

I test the capacity of the new method to identify linear relationships between time series (Section Appendix D). As expected from the literature, in a Gaussian context, the linear opinion pooling performs best, and the new PDM does not improve performance, but its performance converges in the long term with the linear method (Table 4). This performance is robust to scaling and non-linear transformations such as logarithmic (Table 4), the method is not scale but ordinally invariant.

Table 5 reports robustness checks for the new supervision tool where the tail parameter α and the number of Bayesian experts are varied. The PDM minimizes the RMSE for all configurations.

4.2.2 Frequentist experts

I follow the same approach as section 4.2.1 except that I now consider Frequentist insurance experts. When I assume that all experts are Frequentist, Table 6 reports robustness checks

³³The author thanks Arthur Charpentier for suggesting this method in an EVT context as part of his discussion at the ACPR 2023 research seminar.

for the new supervision tool where the tail parameter α and the number of Frequentist experts are varied. The PDM minimizes the RMSE for configurations with the least expert counts.

I therefore recommend using the PDM in a Bayesian context, but this conclusion is more nuanced when some experts can be considered as Frequentist. I discuss the implication of Frequentist expert section [Appendix E](#).

5 POLICY RECOMMENDATIONS

Introducing the PDM into the domain of insurance supervision presents a potential advancement in risk estimation in a climate change context, albeit with accompanying budget considerations. Building upon the academic paradigms delineated by [Mossin \(1968\)](#) and intertwining utility considerations, this section aims to formulate policy recommendations that not only address the pragmatic deployment of PDM but also engage with broader concerns of market stability and confidence. These policy suggestions coalesce around thorough cost-benefit analysis and a calculated, phased rollout to ensure the financial and operational feasibility of implementing PDM.

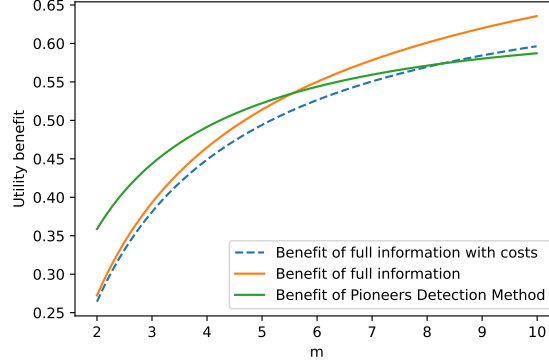
The recommendation to use the PDM depends on the insurance market configuration and the evolution of the tail parameter. I use a logarithmic utility as in [Mossin \(1968\)](#) and apply the Delta method to estimate the gain or loss in utility at the 95% lower confidence interval bound.³⁴ The loss of utility is proportional to the variance of the estimate of α .³⁵ I normalize the variance with the Bayesian estimate variance in the case of a market with a unique expert, then I can express the variance for the fully informed S as $\frac{1}{mt}$ using expression [\(16\)](#).

As there is no closed form for the variance of the estimate outcome with the PDM, I apply Monte Carlo simulations with a tail parameter $\alpha = 1.5$. I vary the number of ICs, m , and find a strong linear relationship between m and the ratio of the estimate standard deviations, $\frac{\sigma_{\hat{\alpha}_{\text{tool}}}}{\sigma_{\hat{\alpha}_{\text{full}}}} = a + bm$. [Figure 3](#) illustrates the utility improvement versus the cost S has to

³⁴The logic is the same as the mean-variance principle for the premium applied by the IC, following [Young \(2014\)](#), $\frac{\hat{\alpha}^t}{\hat{\alpha}^t - 1} + \beta \frac{(\hat{\alpha}^t)^2}{t} \frac{1}{(\hat{\alpha}^t - 1)^4}$, $\hat{\alpha}^t > 1$, for which the variance coefficient would need to be determined in an experiment. [Cao et al. \(2022\)](#) demonstrate that in a Stackelberg competition, the ambiguity aversion determines this coefficient. For consistency, I use the confidence interval, where the representative contract offered by the IC assures they are profit-making 95% of the time.

³⁵It is expressed by applying a Taylor expansion to $Var\left(\log\left[c - a\frac{\alpha}{\alpha-1}\right]\right) \simeq \left(\frac{a}{(\mu_\alpha-1)[c(\mu_\alpha-1)-a\mu_\alpha]}\right)^2 Var(\alpha)$ around the mean μ_α of the tail parameter with c and a being constants.

FIGURE 3
Cost versus utility benefit of S actions



Source: author's computation.

spend. In Figure 3, I display the normalized benefit of full information without the linear cost and find that S would never find it beneficial to spend the effort to collect the full information set from IC using on-site inspection when there are fewer than five ICs. When S incurs a linear cost to collect granular information, this threshold shifts to the right, I illustrate this with a dashed line.

With my model and setup, in a configuration with more than five ICs, whether it is welfare-improving for S to collect the full set of information to reduce the uncertainty in the premium estimation depends on the cost to collect granular information. In a configuration with less than five ICs, it is always more beneficial for S to use the PDM, regardless of the cost function, to improve the welfare.

6 DISCUSSION

6.1 On the existence and persistence of pioneers in a competitive insurance market

In Section 2, I introduced a stylized model of a competitive insurance market. Here, I seek to provide insight into why pioneers emerge within such a market, drawing inspiration from an end-of-year examination I conducted in 2021 with master 2 students from University Paris 1 and observations for the French insurance market.

Consider a Gaussian context: even with a limited sample size (fewer than ten observations), a Bayesian expert's first and second moment estimations will be unbiased. In

such a scenario, a supervisory tool wouldn't outperform a linear combination that excludes outliers. One can infer this intuitively. Given a Gaussian distribution, each observation is equally likely to lie to the right or the left of the true first moment. This resembles the law of large numbers, where the average of means is itself Gaussian-distributed. However, when dealing with extreme values, the law of large numbers isn't applicable when the tail index implies an unbounded mean or variance. A sample of small size might not effectively represent the distribution's tail. In such contexts, a supervisor relying solely on a linear combination of tail indices might be misguided.

The informal findings of my end-of-year examination of my master 2 students open the possibility for me to study further pioneership in a rigorous experimental design, not just to validate the PDM but also to test underlying assumptions. Firstly, I intend in future work to examine the very concept of a pioneer. A primary test would involve determining if an expert can differentiate between a Gaussian and an Extreme Value (EV) context. Subsequently, within an EV setting, it would be intriguing to see if human experts adhere to Bayesian principles or whether they incline towards excessive reactions, setting *exuberant* premiums. If such overreactions are identified, it could challenge the PDM's current framework, potentially necessitating modifications—perhaps introducing inertia in pioneer identification to prevent overshooting.

Turning our attention to the French insurance market overseen by the ACPR, AXA SA stands out as the predominant player. This insurer has even carved out a dedicated entity, AXA Climate, which has ventured into consultancy. Evidently, size heterogeneity and market share play pivotal roles, enabling certain players to establish specialized teams focused on discerning climate change implications for the insurance sector. While I refrain from commenting on the commitment or caliber of any specific French firms in climate change modeling, size disparities naturally pave the way for the birth of pioneers. Since market shares are fluid, it's conceivable that a pioneering insurer might be better poised to make informed decisions regarding which asset classes to insure or exclude ahead of competitors. Such dynamics could skew the insurance market. While these facets aren't encapsulated in my current model, they could be fertile ground for future experiments, especially in exploring how markets might evolve over time under the influence of EV risks. In this paper's design of the PDM, the weights are a ratio of angles of changes between the previous period and the current observation's period, in other words there is no inertia in our main specification of the PDM. This can be updated based on equation (4) where the weight for each expert becomes $\sum_{k=0}^l \theta^k w_i t - k$, now the optimal lag order (l) selection becomes

an additional research question that cannot rely on the traditional time series approach [Lütkepohl \(2005\)](#) as the supervisor has no access to the realization of the tail parameter for calibration.

6.2 On the role of leader of a supervisor

This paper has delineated policy recommendations, highlighting configurations in which a supervisor can judiciously employ the PDM for meticulous risk estimation to inform decisions pertaining to intervention. Equation (3), alongside computations considering welfare change (either gains or losses), encompasses supervisory interventions as a leader that either directly or indirectly disclose the supervisor’s own estimations of the tail parameter, thereby acting as an indicative benchmark of the most astute available estimate at the time of disclosure. This evokes a pivotal inquiry into the incentives for private insurance companies to amplify their expertise when confronting the prospect that their discernment will subsequently be utilized by a supervisor, synthesized through a PDM, and disseminated amongst all insurance entities.

This aforementioned trade-off is not rigorously operationalized in the present work and somewhat parallels earlier discourses concerning the potential veracity of the *Efficient Market Hypothesis* (EMH). The hypothesis postulates that financial markets are efficient in reflecting all available information in asset prices. This brings forth the Grossman-Stiglitz Paradox, which presents a conundrum: if every piece of information is already embodied in prices, the motivation for any investor to incur costs to acquire information becomes obfuscated ([Grossman and Stiglitz, 1980](#)). Analogously, in the context of insurance, if supervisors utilize and disclose expert risk estimations via interventions, it could ostensibly dampen insurance companies’ incentive to enhance their own expertise. Essentially, why would insurance companies bear the costs of accruing and refining expertise if supervisory interventions eventually render such proprietary insights into public knowledge?

An elaborate exploration into this domain, potentially integrating the underpinnings of the Grossman-Stiglitz Paradox, may further illuminate the dynamics between supervisory interventions, informational asymmetry, and the consequent incentives (or lack thereof) for expertise development within insurance entities. Establishing a harmonious balance where supervisory interventions and the development of proprietary expertise by insurance companies coexist beneficially necessitates a thorough understanding of these interlinked paradigms.

6.3 Climate change, welfare and supervision actions under heterogeneous beliefs between IB and IC

The main purpose of this paper is to study possible policy recommendations to deal with the heterogeneous beliefs of insurance experts when the objective loss is unknown. There is additional heterogeneity in beliefs between insurance buyers and sellers, which can impact both the supply and demand of insurance coverage (Hogarth and Kunreuther, 1989). In this context, S has several possible actions to first ensure the stability of the insurance market and second improve the welfare. When S believes an IC is too optimistic ($\hat{\alpha}_i^t > \hat{\alpha}_S^t$) and subject to ruin, the traditional approach is for S to ask the regulator to modify the IC solvency constraint, either temporarily or permanently. In a model à la Kleindorfer and Klein (2003), S defines a constraint such that $Pr(x^t > x^*) \leq \rho$, where x^* represents the yearly aggregate losses above which the IC becomes insolvent. If there is a disagreement on the estimate of the tail parameter, S can compensate by setting a lower hence more restrictive value of ρ . In a climate change context, IC can be more pessimistic than IB about the loss; hence, the straight deductible result from the MHR hypothesis might not hold, and its form might be undefined (Marshall, 1992; Ghossoub, 2017; Chi, 2019). When S believes that an IC is pessimistic and decides to reduce insurance coverage, the actions are not as straightforward as in the latter case. In an increasingly risky environment due to climate change, S would be reluctant to alleviate solvency constraints to increase IB coverage at the expense of IC resilience, either for credibility concerns or long term incentives to IC and IB to prepare for the increase in risk. In the meantime, S can impose restrictions on the insurance contracts. There are two possible dilemmas: one is the risk of distorting IB incentives, causing them to relocate their portfolio to less exposed asset classes (Oh, Sen and Tenekedjieva, 2022), I study the second, where S can breach the IC participation constraint. I adapt Averch and Johnson (1962) where S can decide to regulate the rate-of-return of IC and show Appendix F that such a regulation cannot meet both the IC participation constraint and the aim to reduce an IC pessimist bias. Finally, I develop a tractable model à la Lee (2017) to study the effect of an increase in the right tail risk and uncertainty, because, when ICs are more pessimistic than IB, the straight deductible result doesn't hold. The IB is subject to a loss x , which used to occur with a probability $p \in (0, 1)$. Now with climate change, it can take two values each with probability of occurring $\frac{1}{2}$, $p_1 = p$ and $p_2 = p + \delta$, with $\delta \in (p, 1 - p)$.³⁶ The IB is optimistic because he doesn't internalize the increase in risk due to climate change

³⁶This is the second main departure from the original model, it is not a mean-preserving spread (Stiglitz and Rothschild, 1970), but a risk skewed to the right.

and considers that the probability remains p . I assume that ICs are biased and pessimistic, meaning they internalize the increased risk with climate change and overestimate this risk, considering that the probability p_2 is B , where $1 > B > \frac{1}{2}$. In this set up, I implicitly consider that the risk is still insurable and I consider for simplicity that the cost is linear with a factor λ , then the premium for a risk-neutral insurer in perfect competition is set to cover the expected loss and administrative costs and profits. In this simplified model, x is not a random variable, which also simplifies the expression (5) of the premium where I define $\Phi = (1 + \lambda)(p + B\delta)$. The expected utility W of the optimistic IB depends on his wealth w , straight deductible D , and premium Π , with the implicit assumption that the loss $x > D$ and follows (6).

$$\Pi(D) = \Phi(x - D) \quad (5)$$

$$W(D, \Phi) := pU(w - \Pi(D) - D) + (1 - p)U(w - \Pi(D)) \quad (6)$$

Proposition 1. *The effect of an increase in risk perception bias is ambiguous on the realized welfare of the IB and on the profit of the IC.*

Proposition 1 is demonstrated in [Appendix G](#). The risk perception bias in this model is denoted B and when doing the welfare derivation, the sign lies in the concavity of the IB utility, and hence, in the disutility of insurance coverage reduction. This is against S advising on a regulation of the premium loading as in Equation (32). The proposition put forward suggests that even if S can disentangle risk perception bias from uncertainty, its impact on the welfare is of an ambiguous sign when it tries to lower the risk perception bias with announcements.

In this case, an alternative policy recommendations is to limit supervisors to considering that IB have reservation premium and how the PDM can be used in this context, section [Appendix B](#).

6.4 Absence of reinsurance in the model

A central hypothesis in this model is the absence of any form of reinsurance, neither from outside reinsurers nor between ICs. This simplifying assumption can be understood as a situation where after a tipping point for an asset class in a country supervised by S , foreign reinsurance companies (not under direct and unique S supervision) decide to exit the market

or provide deterrent reinsurance premium during the January renewal.³⁷

A real-life hypothetical example is the housing insurance context in France, where an estimated half of the houses are exposed to WSCS due to the effects of drought, which is intensifying under climate change. If the legislation was modified where these risks would be taken out of state guarantee (Cour des comptes, 2022) and would be left to private (re)insurance, reinsurers might withdraw from some areas and IC might be left alone to determine the premium to offer to IB for this asset class.

As a simplification, neither dynamic entry nor exit of reinsurers or ICs is allowed before or after the tipping point. This is an interesting avenue for extension of the model where S can influence entry or exit of (re)insurers depending on his own evaluation of the risk-benefit assessment.

According to Plantin (2006), insurers rarely trade risks with each other. In his model, reinsurers emerges as informed capital providers that closely monitors IC to mitigate moral hazard problem because they have granular (and soft) claim information, and more risk-management skills than outside financiers. Once reinsurers exit a given market, S can decide to take over this monitoring role and start collecting granular claim information or use opinion pooling method to synthesize IC expertise.

7 CONCLUSION

This paper contributes to the literature on opinion pooling, climate change impacts on losses, insurability, and insurance supervision in the context of climate change. By introducing a Pioneers Detection Method that pools expertise, the paper offers a practical approach to assess the insurability of asset classes after a tipping point. It tests the policy recommendation that insurance supervisors should focus on on-site inspections to collect granular information or should rely on the pooling of expertise. In the event of a sudden shock or panic in the insurance market for a given asset class, a supervisor must determine as quickly as possible whether the asset class is insurable to fulfill his stability mandate. The new tool relies on readily available model parameters to identify experts who are quick at understanding the implications of climate change on aggregate tail losses. I demonstrate that, compared to traditional opinion pooling methods, the Pioneers Detection Method is the fastest and has

³⁷This is a strong modeling assumption with limited historical precedent such as the liability insurance crisis of the mid-1980s, due to underestimation of long-tail liabilities, legal and macroeconomic (interest rates) changes (Berger, Cummins and Tennyson, 1992). As an anecdotal recent evidence post COVID-19, on 8th December 2020, the reinsurer Munich Re announced it would stop covering big event cancellation. The author thanks Jeffrey Czajkowski for suggesting adding this consideration to the paper.

an advantage when there are limited observations available. The choice to collect granular information via on-site inspection will depend on the supervisor’s capacity and his estimate of the cost-benefit. I demonstrate that, as long as there is a limited number of independent insurance companies covering an asset class, relying on the Pioneers Detection Method is always preferred.

In terms of policy implications, after building their own estimates, supervisors can reduce uncertainty and help avoid crises and equilibrium shifts in insurance supply by making forward-looking announcements and influencing expert opinions. A supervisor can also use the Pioneers Detection Method to monitor the insurability of asset classes and advise a regulator to design appropriate public or private insurance schemes. Additionally, it promotes transparency and collaboration within the insurance sector, contributing to a more resilient market in the face of climate change.

Further research on the behavior of insurance experts should follow, especially to test if they exhibit an optimism or pessimism bias due to the lack of extreme observations in their private datasets, or because of modeling choices. I recommend setting up experiments to determine how actuaries would behave in this context, particularly to test whether their approaches are Bayesian or Frequentist. The paper also highlights the need for further research in the area of insurance supervision and climate change, exploring topics such as the endogeneity of market shares, demand sensitivity to premiums, and ruin gambles in the context of a changing climate.

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TABLE 1
New supervision tool compared with combination methods

	RMSE	Constant	Econometric	ARIMA	Expert
Expert	1.35				
Econometric	1.22				
Method C out-sample	1.00	-22.21	0.93	0.22	0.31
AR	1.00				
Method A out-sample	0.79	0	-1.11	4.63	-2.62
Method B out-sample	0.69	0	-1.01	4.63	0.28
ARIMA	0.64				
Median	0.64				
Minimum MSE adaptive	0.61		0.32	0.41	0.27
Discounted MSFE	0.57		0.19	0.46	0.35
Minimum MSE adaptive, expert excluded	0.57		0.41	0.59	
Correlation	0.56		0.30	0.35	0.36
Two forecast Simple average	0.55		0.50	0.50	
GC	0.55		0.50	0.50	0.00
Minimum Variance, expert excluded	0.55		0.48	0.52	
Three forecast Simple average	0.52		0.33	0.33	0.33
Minimum Variance	0.48		0.53	0.25	0.22
Method B	0.47	0	0.54	0.25	0.22
Method A	0.47	0	0.53	0.25	0.22
Method C	0.44	15.19	0.10	0.40	0.18
Pioneers	0.42		0.70	0.17	0.12

NOTE.—The data set is taken from [Bessler and Brandt \(1981\)](#). The reported weights are the average when they are varying over time for the method. Simple averages, Minimum MSE adaptive and Minimum Variance are implemented as in [Bessler and Brandt \(1981\)](#). Methods A, B and C are from [Granger and Ramanathan \(1984\)](#), weights were computed on past performances for the out-sample. Discounted MSFE is the first method in [Bates and Granger \(1969\)](#). Principal Component forecast combination was not added as this is mainly to tackle situations with a large number of forecast. I add three novel methods, Granger Causality (GC), (lagged) Correlation and the Pioneers Detection Method.

TABLE 2

New supervision tool validation - full Bayesian							
	Pioneers	Linear	Median	Minimum	Pioneers distance-weighted	Correlation	GC
2	1.00	6.57	2.46	1.13	1.83	792.00	8.18
3	1.00	5.94	2.00	1.27	1.68	315.40	6.93
4	1.00	2.84	1.68	1.52	1.22	87.38	3.75
5	1.00	2.10	1.52	1.69	1.05	391.41	2.83
6	1.00	1.72	1.39	1.76	0.95	62.55	2.42
7	1.00	1.48	1.29	1.80	0.89	46.78	2.17
8	1.00	1.31	1.20	1.81	0.84	51.38	1.97
9	1.00	1.18	1.13	1.79	0.81	53.14	1.82

NOTE.—The tail parameter is taken from a Pareto type one distribution with $\alpha = 1.5$. Five non-cooperative bayesian experts are modeled with independent observations from the loss distribution, heterogeneous beliefs are allowed. 10^5 Monte Carlo simulations are run. The new Pioneers Detection Method outperform established opinion pooling methods (linear, median and minimum) as well as alternative methods introduced in this paper: pioneer with weight based on distance rather than angle, lagged correlation and Granger Causality.

TABLE 3

New supervision tool validation - full Bayesian							
	Pioneers	Linear	Median	Minimum	Pioneers distance-weighted	Correlation	GC
mean	1.00	3.62	1.74	1.51	1.29	172.00	4.59
median	1.00	1.92	1.46	1.72	1.00	85.12	2.64
std	1.00	10.34	3.16	0.71	2.50	125.00	12.34

NOTE.—The tail parameter α is fixed at 1.5 and a Monte Carlo simulation is run with 10^5 runs. Five non-cooperative Bayesian experts are modeled with independent observations from the loss distribution. The mean, median and standard deviation of the Root Mean Square Errors are reported over the first 10 estimation period.

TABLE 4
New supervision tool validation, robustness to transformation - full Bayesian

(a) New supervision tool validation,
robustness to scaling

	Pioneers	Linear	Median
2	1.00	2.62	1.34
3	1.00	2.33	1.22
4	1.00	1.48	1.14
5	1.00	1.31	1.12
6	1.00	1.24	1.11
7	1.00	1.20	1.10
8	1.00	1.18	1.10
9	1.00	1.17	1.09

(b) New supervision tool validation,
robustness to non-linear transformation

	Pioneers	Linear	Median
2	1.00	1.68	1.49
3	1.00	1.16	1.05
4	1.00	0.74	0.80
5	1.00	0.60	0.73
6	1.00	0.59	0.72
7	1.00	0.61	0.72
8	1.00	0.63	0.74
9	1.00	0.65	0.74

(c) New supervision tool validation,
relevance in a Gaussian context

	Pioneers	Linear	Median
2	1.00	0.58	0.70
3	1.00	0.74	0.88
4	1.00	0.65	0.77
5	1.00	0.73	0.87
6	1.00	0.80	0.96
7	1.00	0.87	1.04
8	1.00	0.91	1.09
9	1.00	0.95	1.14

NOTE.—The tail parameter is taken from a Pareto type one distribution with $\alpha = 1.5$. Five non-cooperative Bayesian experts are modeled with independent observations from the loss distribution. 10^5 Monte Carlo simulations are run. In Table 4a, estimates have been scaled by 100. In table 4b, the estimates have been transformed with a logarithm $\log(c + \alpha)$ with $c = 2$ a constant to avoid issues when $\hat{\alpha}$ are close to 0. In table 4c, the loss samples are taken from a standard normal law.

TABLE 5
New supervision tool robustness checks - full Bayesian

α	experts	Linear RMSE	Median RMSE	Pioneers RMSE
0.5	2	3.48	3.48	1.00
0.5	5	3.53	1.51	1.00
0.5	20	3.08	1.21	1.00
1.1	2	3.29	3.29	1.00
1.1	5	3.51	1.54	1.00
1.1	20	3.25	1.26	1.00
2.0	2	3.19	3.19	1.00
2.0	5	3.70	1.56	1.00
2.0	20	3.73	1.30	1.00
3.0	2	3.03	3.03	1.00
3.0	5	3.57	1.41	1.00
3.0	20	4.52	1.34	1.00

NOTE.—The tail parameter α and the number of Bayesian experts are varied. The last three columns report the average Root Mean Square Errors for the competing opinion pooling tools over the three initial estimation periods.

TABLE 6
New supervision tool robustness checks - full Frequentist

α	experts	Linear RMSE	Median RMSE	Pioneers RMSE
1.1	2	2.32	2.32	1.00
1.1	5	1.48	0.85	1.00
1.1	20	0.99	0.21	1.00
2.0	2	2.60	2.60	1.00
2.0	5	1.62	0.73	1.00
2.0	20	0.91	0.11	1.00
3.0	2	2.88	2.88	1.00
3.0	5	2.01	0.72	1.00
3.0	20	1.23	0.06	1.00

NOTE.—The tail parameter α and the number of Frequentist experts are varied. The last three columns report the average Root Mean Square Errors for the competing opinion pooling tools over the three initial estimation periods.

TABLE 7
New supervision tool validation

	x	y	z
Pioneers	0.60	0.22	0.19
Correlation	0.46	0.48	0.06
GC	0.62	0.38	0.00

NOTE.—The time series are random sample of length 10. $b = d = e = .9$ and $a = c = .1$. 10^5 Monte Carlo simulations are run and the average of each weights are reported. The weights per method are expected to be ranked such that on average $w_x > w_y > w_z$.

TABLE 8
New supervision tool validation

	x	y	z
Pioneers	0.31	0.21	0.48
Correlation	1.28	0.09	-0.37
GC	0.50	0.44	0.06

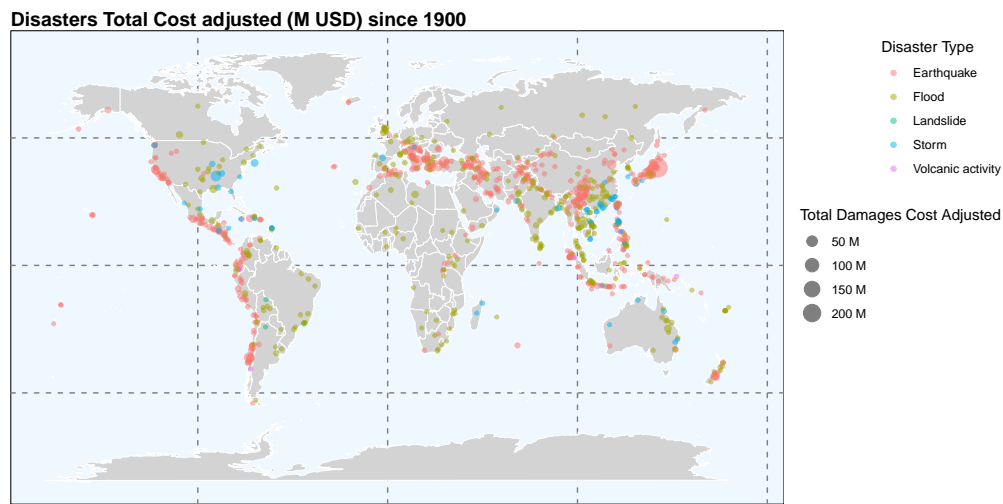
NOTE.—The time series are random sample of length 10. $b = d = e = .1$ and $a = c = .9$. 10^5 Monte Carlo simulations are run and the average of each weights are reported. The weights per method are expected to be ranked such that on average $w_x > w_y > w_z$.

I APPENDIX

Appendix A Climate change impact on losses distribution

I play the role of an insurance expert using a public data set, the Emergency Events Database (EM-DAT) ³⁸. Figure A.1 displays the information with total damage adjusted cost for each disaster from 1990 to 2021.

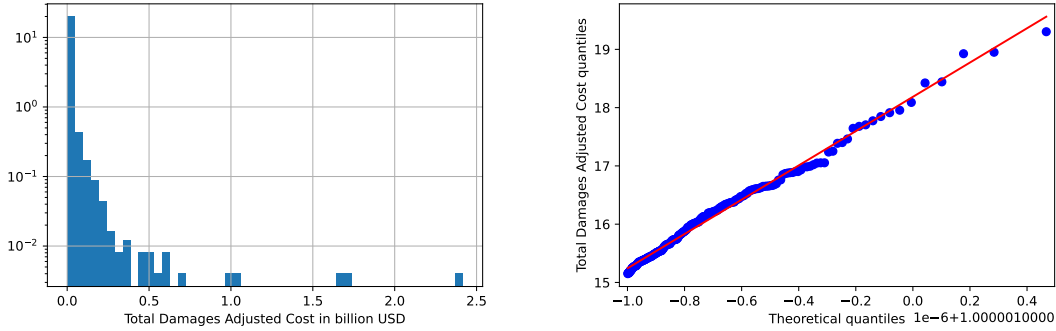
FIGURE A.1
Disaster Total Cost Adjusted (M USD) - map



Source: EM-DAT, CRED / UCLouvain, Brussels, Belgium – www.emdat.be (D. Guha-Sapir). Total damages in adjusted billion USD.

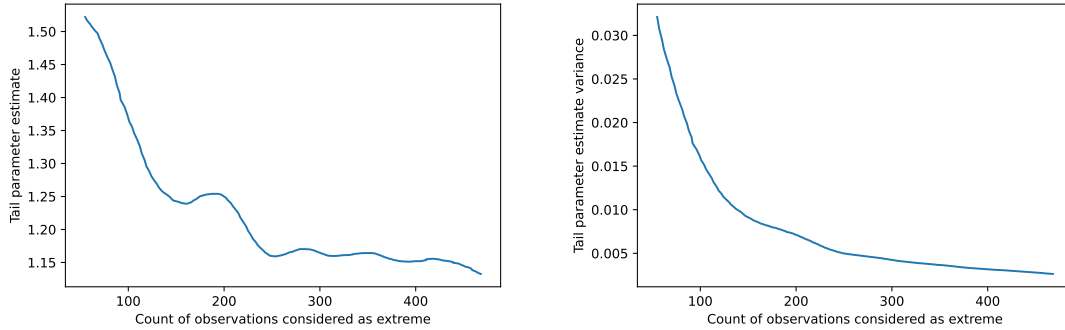
³⁸EM-DAT: The Emergency Events Database - Université catholique de Louvain (UCL) - CRED, D. Guha-Sapir - www.emdat.be, Brussels, Belgium.

FIGURE A.2
Disaster Total Cost Adjusted (M USD) - histogram and Q-Q plot



Source: EM-DAT, CRED / UCLouvain, Brussels, Belgium – www.emdat.be (D. Guha-Sapir). Total damages in adjusted billion USD. Author's computation. The quantile-quantile plot is done against a Pareto type 1 distribution.

FIGURE A.3
Heavy-tail shape parameter: the observation count trade-off



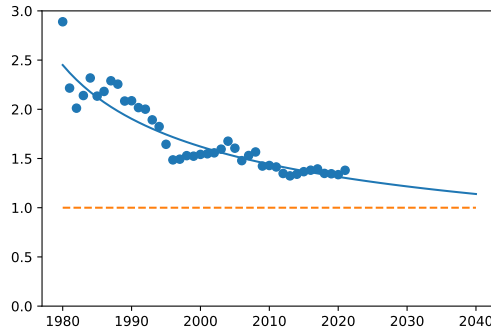
Source: Author's computation on EM-DAT.

To simulate a learning from an evolving environment, I apply a 80-year rolling-window estimation exercise without the possibility to change the model calibration methods as the window was rolled. Hence, I design a model and calibration approaches that I apply to each 41 year from 1980 to 2021 on the natural disaster losses. In the left panel of Figure A.2, the distribution of total damaged adjusted costs displays a heavy-tail property, which leads us to use EVT to model the distributions. Hence, it is current practice for insurance experts to test extreme events quantiles against parametric heavy tail distributions quantiles. The right panel of Figure A.2 displays a quantile-quantile plot of the 95-th percentile of damages against a Pareto type I distribution. This can be accepted or rejected based on expert experience with more complex distributions. There is no one-size-fit-all approach in EVT

to define a threshold above which values can be considered as part of a tail ³⁹, I face the trade-off each expert is facing when dealing with fat tailed loss distributions. Figure A.3 illustrates for a simple Pareto type I law the trade off between limiting the fit on "relatively" few observations and the increase variance in the parameter estimates as the observations left in the sample decrease. As the observations count kept to fit a Pareto distribution increases, the value of the parameter estimated decreases toward one, which is not an indication of increased risk but rather of including observations that do not belong to the tail. As the observations increase, the parameter estimate variance improves. Hence, each expert faces a trade-off between limiting the sample to extremes and improving his estimations variance with more observations.

I report Figure A.4 the time series of estimated tail parameters (α^t). I also project Figure A.4 how the Pareto exponent estimates of climate change damages decrease over time for the upcoming 20 years with a linear fit post Box-Cox transformation. In Table 1, two linear regressions—pre (1) and post (2) Box-Cox transformation—of the Pareto coefficient over time are applied. A significant downward trend indicates that a threshold of 1 could be reached within the next century. Therefore, my model and this data set leads me to conclude on an increase in the insurance losses over time attributed to climate change up to becoming uninsurable for the considered natural catastrophes (unbounded expectation).

FIGURE A.4
Pareto estimates over time



Source: Author's experiment and computation.

³⁹Rolski et al. (1999) demonstrated the difficulty to verify the heavy-tail property statistically

TABLE 1
Pareto exponent evaluation

	(1)	(2)
Intercept	2.269*** (0.052)	0.386*** (0.006)
t	-0.027*** (0.002)	-0.005*** (0.000)
Observations	42	42
R^2	0.794	0.888
Adjusted R^2	0.789	0.885
Residual Std. Error	0.172(df = 40)	0.020(df = 40)
F Statistic	154.184*** (df = 1.0; 40.0)	318.005*** (df = 1.0; 40.0)

Note:

*p<0.1; **p<0.05; ***p<0.01

NOTE.—The first column is estimated over α and the second after a Box-Cox transformation.

Appendix B Insurability testing and optimal contract when the IB has a reserve premium

An asset becomes uninsurable when the optimal premium exceeds what the IB can afford for coverage. I model a simple situation where the IB has a reservation premium ⁴⁰ Π^* that for simplicity is constant ⁴¹ over the observation period. S tests the insurability hypothesis. For this test, the IC cost function is simplified, leaving out $(H_S^t : \Pi^t = \mathbb{E}_{\pi_i^t}[I(x^t)] = \frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1} \leq \Pi^*)$, and becomes $(H_S^t : \alpha^t > \alpha^*, \text{ if } \alpha^* > 1)$.

When the insurability condition is met, then the probability for an IC i to actually make a profit is given by a Pareto cdf

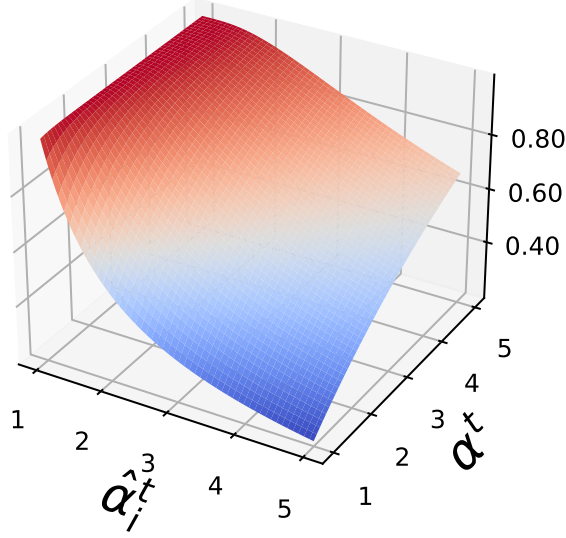
$$z(\hat{\alpha}_i^t, \alpha) := P\left(\text{loss} \leq \frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1}\right) = 1 - \left(\frac{\hat{\alpha}_i^t}{\hat{\alpha}_i^t - 1}\right)^{-\alpha}$$

$$\frac{\partial z}{\partial \alpha} = (1 - z) \ln\left(\frac{\hat{\alpha}_i^t - 1}{\hat{\alpha}_i^t}\right) > 0, \forall \alpha > 1, \hat{\alpha}_i^t > 1$$

⁴⁰The reservation premium is dependent on the IB outside option and ability to pay premiums based on their income and asset value.

⁴¹This inelasticity can be interpreted as buyers having limited capacity to estimate climate change impacts on losses and the reservation premium representing their ability to pay premiums based on their income and asset value. In practice this reservation premium would need to be corrected at least with inflation and wage changes.

FIGURE A.5
IC profit probability as a function of his estimate and the tail parameter



Source: author's computation.

$$\frac{\partial z}{\partial \hat{\alpha}_i^t} = -\frac{\alpha}{(\hat{\alpha}_i^t)^2} \left(\frac{\hat{\alpha}_i^t - 1}{\hat{\alpha}_i^t} \right)^{\alpha-1} < 0, \forall \alpha > 1, \hat{\alpha}_i^t > 1$$

z is increasing in α and decreasing in $\hat{\alpha}_i^t$, illustrated Figure A.5. When $\hat{\alpha}_i^t$ decreases the profit likelihood of IC increases as the IB is willing to pay more, hence an expert would want to set his offered premium based on the IB reserve α^* . As α increases, the likelihood of losses decreases so the expected profit of insurance companies increases.

B.1. Case of interset: reserve premium above the true insurable risk

When $1 < \alpha^* < \alpha^t$, this means that the IB reserve premium is above the true cost of the risk exposure and the asset is certainly insurable. When the expert's estimate respects $\alpha^* \leq \hat{\alpha}_i^t$, then the expert advises offering protection, and the insurance company is likely to make a profit. When at time t the expert's estimation is such that $\hat{\alpha}_i^t < \alpha^*$, then she advises her IC to exit the market, which is a missed opportunity for her company. Hence, the situation of interest for S is when $\hat{\alpha}_i^1 < \alpha^* < \alpha$, meaning that, after the first year, expert i wants to exit the insurance market and, hence, stop learning. B.2. provides the stopping condition for S based on the risk estimate and time spent. As there is no closed form formula for the PDM, I adopt a welfare consideration framework.

B.2. Stopping condition for S depending on the tail parameter and the time spent

The probability that at time t , $\hat{\alpha}_i^t > \alpha^*$ for an expert i is given by the cumulative distribution function (cdf) of a Gamma distribution, this is the probability that expert i exit the insurance market:

$$P\left(\hat{\alpha}_i^t = \frac{t}{\sum_t \ln x_i^t} \leq \alpha^*\right) = \frac{1}{\Gamma(t)} \gamma\left(t, \alpha^* \sum_t \ln x_i^t\right) \quad (7)$$

Taking the expectation

$$E\left[P\left(\hat{\alpha}_i^t \leq \alpha^*\right)\right] = \frac{1}{\Gamma(t)} \gamma\left(t, \frac{t\alpha^*}{\alpha}\right) \quad (8)$$

If at time T , $\hat{\alpha}_i^T < \alpha^*$ and α^* is outside of the acceptable confidence region (e.g. 95%) of the posterior density $p_i(\alpha^t|x)$, then there won't be any time t such that $t > T$ and α^* will move back inside the same confidence region criteria of the posterior density. So when this is the case, the only rational decision for S is to advise insurance experts to stop providing coverage. Now, looking at the claim level. Let's imagine that $\exists t, \hat{\alpha}_i^t = \frac{t}{\sum_t \ln x_i^t} < \alpha^*$, then it is worth covering the asset class if there is some non-zero probability that $\hat{\alpha}_i^{t+1} = \frac{t}{\sum_t \ln x_i^{t+1}} \geq \alpha^*$, this condition is equivalent to

$$x_i^{t+1} \leq \exp\left[\frac{t+1}{\alpha^*} - \sum_t \ln x_i^t\right] = \tilde{x} \quad (9)$$

As with only one expert, S has only as much information as the expert, then both their believes are

$$P\left(x_i^{t+1} \leq \tilde{x} \mid \hat{\alpha}_i^t \leq \alpha^*\right) = 1 - (\tilde{x})^{-\hat{\alpha}_i^t} \leq 1 - (\tilde{x})^{-\alpha^*} \quad (10)$$

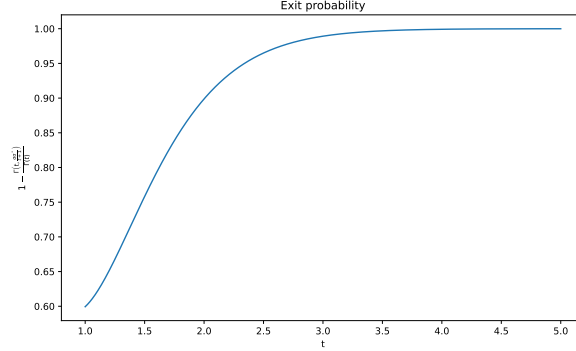
As there is a lower bound on x , the process stops whenever

$$\exp\left[\frac{t+1}{\alpha^*} - \sum_t \ln x_i^t\right] \leq 1 \quad (11)$$

that is whenever $\sum_t \ln x_i^t \geq \frac{t+1}{\alpha^*}$. As $\sum_t \ln x_i^t$ follows an inverted Gamma distribution and taking realist tail parameters, the process is likely to stop after 3 iterations, illustrated Figure [A.6](#).

$$P\left(\sum_t \ln x_i^t > \frac{t+1}{\alpha^*}\right) = 1 - \frac{\Gamma\left(t, \frac{\alpha\alpha^*}{t+1}\right)}{\Gamma(t)} \quad (12)$$

FIGURE A.6
Stopping probability when there is a unique IC



Source: Author's computation, $\alpha = 1.5$ and $\alpha^* = 1.6$.

In a multiple IC configuration, when at least one expert estimates the risk to be higher than what policyholders are ready to pay ($\exists i, \hat{\alpha}_i^t < \alpha^*$), the decision-maker S might want to intervene if he has enough evidence to believe that the willingness to pay leaves enough room for an insurance market to exist; that is, even after the tipping point, the true risk is insurable without any public intervention ($\alpha < \alpha^*$). This writes

$$P\left(\alpha^* \leq \hat{\alpha}_S^t \mid \hat{\alpha}_i^t \leq \alpha^*\right) \quad (13)$$

S can form $\hat{\alpha}_S^t$ with the Pioneers Detection Method introduced in this paper, but for which there is no closed form formula. Hence I test this method in a welfare consideration framework with simulations Section 4.

Appendix C Discussion of the PDM and alternative novel approaches

C.1. Pioneers Detection Method convergence properties

I show that the new supervision has some desired convergence properties. m represents the number of experts and t the observation periods which are also the observations count.

Property 1. When Bayesian experts' claims are independent and identically distributed (iid), as $m \rightarrow \infty$, the PDM converges to the mean of the experts' estimations.

Writing S 's subjective opinion o_t , it is constructed with the PDM assigning a vector w_t to the experts judgment \hat{A}_t , $o_t = \hat{A}_t w_t$. As each expert posterior of α follows a Gamma distribution with all the same shape t and a rate parameter that tend to $\frac{1}{\alpha}$ as $t \rightarrow \infty$, then as demonstrated in Mathai (1982), an omniscient expert opinion will follow a Gamma distribution with mt as a shape parameter, hence the mean of the experts' opinions. As $m \rightarrow$

∞ , the Bayes omniscient expert opinion will converge to the true value of α , [de Zea Bermudez and Turkman \(2003\)](#) find this Bayesian approach robust and having lower variance than Maximum Likelihood Estimation. Then as $m \rightarrow \infty$, it becomes impossible for any expert to lead the omniscient expert, which is also the average of all other experts, hence the PDM applies a weight of 1 to the omniscient expert and 0 on all experts, which is equivalent to assigning a weight $\frac{1}{m}$ to each expert's opinion.

Property 2. When Bayesian experts' claims are iid, as $t \rightarrow \infty$, the PDM converges to the mean of the experts' estimations.

For an expert, his posterior's rate parameter is an exponential autoregressive process as defined in [Gaver and Lewis \(1980\)](#), with a unit root: $r_i^{t+1} = r_i^t + \epsilon_i^t$ with $\epsilon_i \sim \text{Exp}(\alpha), \forall i$. As such, $\text{cov}(r_i^{t+1}, r_i^t) = \frac{1}{\alpha^2}t$ hence as $t \rightarrow \infty$ the covariance between two experts estimates cannot be distinguished as leading any other and all experts will be treated equivalently by the PDM, hence $w_i^t \xrightarrow[t \rightarrow \infty]{} \frac{1}{m}$.

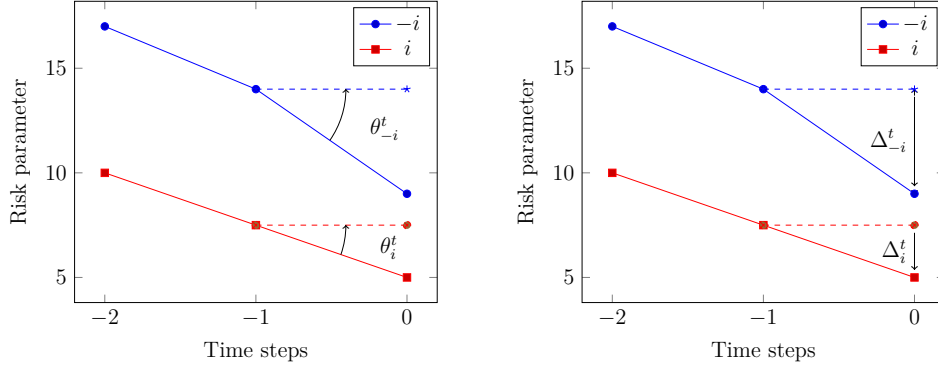
C.2. Weighing convergence with distances or angles

In the last two steps of the PDM, the weights can be defined with angles or distances as illustrated in the right panel Figure [A.7](#). Angles are the preferred approach because they allow the supervisor to take into account the speed of convergence between time series. If θ is the angle between vector \vec{u} and \vec{v} , it can be computed as

$$\begin{aligned} \theta &= \cos^{-1} \left(\frac{u_x v_x + u_y v_y}{\sqrt{u_x^2 + u_y^2} \sqrt{v_x^2 + v_y^2}} \right) \\ &= \cos^{-1} \left(\frac{s^2 + u_y v_y}{\sqrt{s^2 + u_y^2} \sqrt{s^2 + v_y^2}} \right) \end{aligned} \tag{14}$$

the distance relevant to the measure are taken from the y-axis u_y and v_y and the weight is computed as $\frac{|v_y|}{|u_y| + |v_y|}$ which do not include the x-axis $u_x = v_x = s$ the step between two observation points. This weighting with distance is found to be non-robust, as shown in Table [2](#).

FIGURE A.7
Alternative to the orientation change dummy to identify Pioneers



Notes: i represents the expert of interest and $-i$ the average estimate of his competitors (i excluded). The weight in the left panel is $\frac{|\theta_{-i}^t|}{|\theta_{-i}^t| + |\theta_i^t|}$. The weight in the right panel is $\frac{|\Delta_{-i}^t|}{|\Delta_{-i}^t| + |\Delta_i^t|}$.

Alternative inter-temporal pioneers detection methods could also be implemented with more traditional time series methods. I list the five candidate methods and demonstrate how they boil down to implementing the Granger Causality, lagged-correlation and probabilistic combinations methods as alternatives to the PDM.

C.3. Granger Causality

The core principle of the PDM entails assigning indirect votes based on expert estimates. While experts are non-cooperative and do not exert influence on one another, this approach exhibits certain similarities with the identification of Granger Causality, which tests whether a given time series is beneficial in predicting another. It examines whether an expert's opinion change appears to precede a similar opinion change from his competitors. I implement the test introduced by [Granger \(1969\)](#). [Toda and Yamamoto \(1995\)](#) take into account potential integration and cointegration between time series. [Hasbrouck \(1995\)](#) also introduces cointegration to determine the information share of each random variable. In the context of climate change and small time series (maximum lag of 2), the integration or cointegration orders cannot be tested robustly. Therefore, I assume that the time series are locally stationary and apply a Granger Causality test with a lag of one.

C.4. Lagged Correlation

An alternative approach involves measuring correlations between lagged estimates from each expert and the estimates from his competitors. This can be measured using the Pearson

coefficient (Pearson, 1895), as in Sakurai, Papadimitriou and Faloutsos (2005) and Forbes and Rigobon (2002) for financial applications.

C.5. Probabilistic combinations

When estimates are not points but probabilistic distributions, on top of the above methods, two additional methods are considered. The first is the Bayesian Model Averaging (BMA, Draper (1995)) which presents three challenges (Wang et al., 2022), a blocking one for my approach is to elicit a prior which comes back to doing a combination choice. Quantile combinations (Vincent, 1912) present the advantage to keep location-scale family after the transformation. I apply the vincentization as a candidate in a probabilistic set up, results are available from the author.

C.6. Multivariate Linear Regressions

A related approach to the Granger causality test involves using multivariate linear regressions of each expert’s estimates on the average estimate from his competitors, as in Yi et al. (2000). If significant, the coefficients can be considered as voting weights for each expert. This approach with limited history returns to searching for correlation and Granger causality between time series.

C.7. Information Transfer

I also explore the information transfer literature and measures (Schreiber, 2000). The idea is to measure whether the time series act as if there were transfer entropy, that is, information transport from one series to another. For financial applications with non-Gaussian random variables, this approach requires discretizing continuous time series with bins. Dimpfl and Peter (2014) limit their analysis to three bins and divide the return data along the 5% and 95% quantiles, as they “assume that extreme (tail) events are more informative than the median observation.”

The information transfer method tests whether an expert’s opinion change appears to be informative to his competitors. With non-cooperative experts, no information is exchanged, and therefore, if an apparent information transfer is detected, it is as if one expert learnt from the Data Generating Process (DGP) before his competitors, and then the DGP informs the competitors, which is similar to the expert directly informing his competitors if the expert learns faster. Barnett, Barrett and Seth (2009) demonstrate that this method is similar to Granger causality if the random variables are Gaussian. When experts are Bayesian, the

posteriors are the random variables of interest and can be approximated as Gaussian, hence I implement the Granger causality method.

Appendix D Pioneers Detection Method and linearly related time series

I test the PDM capacity to identify linearly related time series. I define three time series as the experts x , y and z in the spirit of (Granger and Newbold, 1974):

$$\begin{aligned}x_t &= x_{t-1} + \epsilon_t \\y_t &= ay_{t-1} + bx_{t-1} + \nu_t \\z_t &= cz_{t-1} + dy_{t-1} + ex_{t-1} + \xi_t\end{aligned}\tag{15}$$

with ϵ , ν and ξ white noises, I set $x_0 = y_0 = z_0 = 0$. The results are sensitive to the coefficients, I set auto-regressive coefficients significantly lower than cross linear relationships so the cross-relationships can be captured by the method. x is the main pioneer of the group as the innovations x faces are passed on to other time series as x has a unit root. On the contrary, y has an auto-regressive coefficient below unity and his innovations are transient. The main aim of this test is to confirm that all methods can identify the pioneership of x and measure to which extent the effect of y on z can be identified. Table 7 reports the average weight each of the three novel Pioneers Detection Methods assigns to each time series. Table 8 reports the detection capacity when the coefficient has a lesser dampening effect on noise.

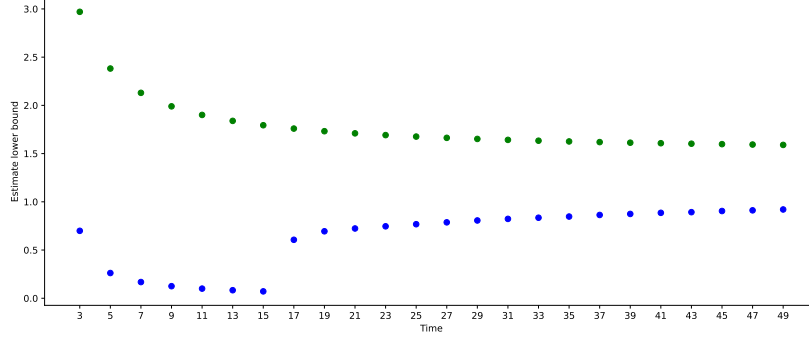
Appendix E Bayesian versus Frequentist experts

In this paper, I have mostly considered Bayesian insurance experts. This allows me to run simulations and work with an explicit formula for the precision of estimates based on the size of the sample. I discuss the implications of having insurance experts who might not behave in a Bayesian way but can use Frequentist methods. I demonstrate that the results would converge, but as I work with small samples the conclusions might differ based on the expert approaches. This is following the literature on distortions in updating approach (Brunnermeier, Simsek and Xiong, 2014).

E.1. Bayesian experts

Following Arnold and Press (1989), each Bayesian insurance expert's natural conjugate prior family for the Pareto exponent is a Gamma distribution of shape s_i^t and rate r_i^t , detailed in Appendix H. The posterior of the tail parameter of each expert is unimodal and roughly

FIGURE A.8
Bayesian versus Frequentist lower confidence interval bound



Source: author's computation. In green the Bayesian lower bound evolution with the observations count and in blue the Frequentist. With $\alpha = 1.5$ and 10^5 Monte Carl runs.

symmetric, as demonstrated by [Le Cam \(1953\)](#), and can be approximated by a normal distribution.

$$\pi(\alpha^t | \cup_t x_i^t) \sim \mathcal{N}\left(\tilde{\alpha}_i^t, \frac{[\tilde{\alpha}_i^t]^2}{s-1}\right) \quad (16)$$

where $\tilde{\alpha}_i^t$ is the mode of the Gamma posterior.

E.2. Frequentist experts

Frequentist experts will use maximum likelihood estimation. [Zajdenweber \(1996\)](#) demonstrates that above a unit threshold, the tail parameter can be estimated ⁴² with t claims as

$$\frac{1}{\hat{\alpha}_i} = \frac{1}{t} \sum_{k=1}^t \log x_i^k \quad (17)$$

As x_i^1, \dots, x_i^t are independent and identically Pareto distributed, then $\ln(x_i^k)$ are independent and identically (exponentially) distributed, as demonstrated in ([Rytgaard, 1990](#)),

$$\sqrt{t}(\hat{\alpha}_i - \alpha) \rightarrow \mathcal{N}(0, \alpha^2) \quad (18)$$

Therefore, asymptotically the Frequentist and Bayesian methods estimate follow the same normal distribution. Nevertheless, this is not true with few yearly observations as in the insurance market model, therefore I compare the bias that each approach can introduce in the estimation of the tail parameter.

⁴²This is called the Hill estimator, ([Hill, 1975](#)).

E.3. Comparison of Bayesian and Frequentist expert approaches

To compare Bayesian with Frequentist approaches, I run Monte Carlo simulations with the same tail parameter at 1.5 and observation counts from a minimum of 3 to 50. I use the asymptotic distribution of the estimate to compute the 95% confidence interval lower bound, $\hat{\alpha} - \Phi(.025)\frac{\hat{\alpha}^2}{t}$ where $\Phi(.)$ is the percentage point function of the normal distribution. Figure A.8, the frequentist lower bound evolution is non linear. When estimating the tail parameter, the lowest observation is taken as the Pareto threshold and when the sample size is small, this initially limits the capacity to improve the tail estimate. As the sample size grows, the threshold is more likely to be close to the actual Pareto threshold, and the likelihood of observing tail events increases; hence, the tail parameter estimation improves with the sample size. Depending on the IC approach—Bayesian or Frequentist—the IC in the model can be either optimistic or pessimistic. S can elicit the IC method to determine the most likely sign of the bias to evaluate the opportunity to intervene.

Appendix F Rate-of-return regulation for IC

I follow assumptions 2, 3 and 9 of Laffont and Tirole (1993), S can observe $\gamma_i(I(x^t))$, but needs to invest to observe $I(x^t)$, hence the profit of the IC. The IC can refuse to provide insurance cover for x^t if its profit is strictly negative. The IC profit is written as:

$$z(\Pi(x^t), I(x^t)) = \Pi(x^t) - I(x^t) - \gamma_i(I(x^t)) \quad (19)$$

I consider a regulator that can put some constraint on the rate of return of the IC, r . I consider that the acquisition of IB is at a cost c_1 of premium and normalize this cost to one and the program becomes for the IC:

$$\begin{cases} \max_{\Pi(I), I} \Pi(I) - I - \gamma_i(I) \\ \text{subject to } \Pi(I) - \gamma_i(I) - r\Pi(I) \leq 0 \end{cases} \quad (20)$$

The Lagrange function writes:

$$\Lambda = \Pi(I) - I - \gamma_i(I) - \lambda [\Pi(I) - \gamma_i(I) - r\Pi(I)] \quad (21)$$

The Kuhn-Tucker conditions are:

$$\Lambda_{\Pi} = 1 - \lambda(1 - r) \leq 0 \quad (22)$$

$$\Lambda_I = \Pi'(I) - 1 - \gamma'_i(I) - \lambda[\Pi'(I) - r\Pi'(I) - \gamma'_i(I)] \leq 0 \quad (23)$$

$$\Lambda_\lambda = \Pi(I) - \gamma_i(I) - r\Pi(I) \leq 0 \quad (24)$$

We end up with a constraint on the cost function

$$\gamma'_i(I) \geq \frac{1-r}{r} \quad (25)$$

Hence, a condition on the premium becomes $\Pi[I(x)] \leq \left(I(x) + \frac{1-r}{r}I(x)\right) = \frac{1}{r}I(x)$. This is incompatible with the IC participation constraint as long as $r < 1$, and this regulatory approach cannot alleviate the pessimistic bias of the IC embedded in a premium.

Appendix G Imperfect information and uncertain probability following [Lee \(2017\)](#)

The IC risk perception bias is denoted by B , and the uncertainty by δ . S can try to reduce B with an announcement. However, in this section, we demonstrate through Proposition 1 that the effect of a change in B on the sign of change in welfare is ambiguous.

The first-order condition to find the optimal deductible that maximizes the IB's expected utility yields Equation (26) where subscript 1 denotes the state with the loss and 2 without the loss. For an interior solution, it has to be better for the IB to insure, Equation (27) and therefore restriction on Φ applies Equation (28).

$$\frac{\partial W}{\partial D} = -p(1 - \Phi)U'_1 + (1 - p)\Phi U'_2 = 0 \quad (26)$$

$$\begin{aligned} [w - \Pi(D) - D] - [w - x] &= x - \Pi(D) - D \\ &= x - D - \Phi(x - D) = (x - D)(1 - \Phi) > 0 \end{aligned} \quad (27)$$

$$\Phi < 1 \quad (28)$$

The second-order condition for the maximization problem is

$$\frac{\partial^2 W}{\partial D^2} = A = p(1 - \Phi)^2 U''_1 + (1 - p)\Phi^2 U''_2 < 0 \quad (29)$$

I totally differentiate the first-order Equation (30), which must be null, to rearrange and determine the sensitivity of the optimal deductible to the parameter Φ , as shown in Equation (31).

$$\begin{aligned} \frac{\partial^2 W}{\partial D \partial \Phi} &= pU'_1 + (1-p)U'_2 - p(1-\Phi) \left[-(x-D) + (\Phi-1) \frac{\partial D}{\partial \Phi} \right] U''_1 + (1-p)\Phi \left[-(x-D) + \Phi \frac{\partial D}{\partial \Phi} \right] U''_2 \\ &\quad (30) \\ \frac{\partial D^*}{\partial \Phi} &= -\frac{1}{A} [pU'_1 + (1-p)U'_2 - (1-p)(x-D)\Phi U''_2 + p(1-\Phi)(x-D)U''_1] \quad (31) \end{aligned}$$

Contrary to Lee (2017), the sign in equation 31 is ambiguous as the sign of $p(1-\Phi)(x-D)U''_1$ is negative and opposite to the other right hand terms, S cannot impose $\Phi > 1$ which is equivalent to saying that the premium loading λ must be above a minimum threshold such that Equation 32 is respected. The extent of actual risk and IC bias B has an indeterminate effect on the utility-maximizing choice of deductible and insurance coverage.

$$\lambda > \frac{1}{p + B\delta} - 1 \quad (32)$$

To conclude the demonstration, I study the impact of a change in bias B on the IB realized utility Equation (33) and find that the sign is ambiguous Equation (34). Indeed, for an interior solution respecting Equation 28, the sign of $\frac{\partial D^*}{\partial \Phi}$ is ambiguous. Under this circumstances, an increase in risk uncertainty is taken into account by the IC and the effect on the IB welfare is ambiguous and the need for regulation is not clear.

$$\bar{W} = (p + \frac{\delta}{2})U_1 + (1 - p - \frac{\delta}{2})U_2 \quad (33)$$

$$\begin{aligned} \frac{\partial \bar{W}}{\partial B} &= -(1+\lambda)\delta \left[(p + \frac{\delta}{2})(x-D)U'_1 \right. \\ &\quad \left. + (1-p - \frac{\delta}{2})(x-D)U'_2 \right. \\ &\quad \left. + \frac{\partial D^*}{\partial \Phi} \left([1 - (1+\lambda)\delta\Phi] U'_1 - (1+\lambda)\delta\Phi U'_2 \right) \right] \quad (34) \end{aligned}$$

In conclusion, $\frac{\partial \bar{W}}{\partial B} > 0$ is intuitive, but it cannot be guaranteed. No strong policy recommendation can be formulated when ICs are more pessimistic than IB.

As an additional result, the IC profit, as a function of its own bias, can be expressed

as $Z(B) = \Pi[D^*(B)] - (p_{\frac{\delta}{2}}(x - D^*(B)))$. The effect of the bias on the IC profit has an ambiguous sign, as shown in Equation 35.

$$Z(1) - Z(B) = \left(\Phi - p - \frac{\delta}{2} \right) [D^*(B) - D^*(1)] \quad (35)$$

Appendix H Bayesian prior and posterior distribution of the Pareto exponent

In this paper, I consider that the insurance deductible (the Pareto threshold) is known ⁴³. Following Arnold and Press (1989), each Bayesian insurance expert's natural conjugate prior family for the tail parameter is a Gamma distribution. Each expert i receives n_i^t observations (claims) per year t . i updates his posterior shape parameter as $s_i^t = s_i^{t-1} + n_i^t$ and rate parameter as $r_i^t = r_i^{t-1} + \sum_{k=1}^{n_i^t} \ln(x_k^t)$. If I consider yearly claim aggregates, then $\forall i, t$, $n_i^t = 1$, so the evolution of the shape parameter is deterministic, $s_t = t$, and the evolution of the rate parameter is a random variable following an inverted gamma distribution $r_i^t \sim \text{invGamma}(t, \alpha)$.

I follow Meyers (1996), the prior for the α of the Pareto distribution follows a Gamma distribution:

$$\pi_A(\alpha) = \frac{r^s}{\Gamma(s)} \alpha^{s-1} e^{-r\alpha} \quad (36)$$

The model pdf is denoted $f_{X|A}(x|\alpha)$, with x a sample from X and α our parameter from the parameter space A . For a Pareto distribution, this pdf is:

$$f_{X|A}(x|\alpha) = \frac{\alpha^n}{\prod_{i=1}^n x_i^{\alpha+1}} \quad (37)$$

The posterior distribution is the conditional probability distribution of the parameters given the observed data. According to Bayes' theorem it is:

$$\pi_{A|X}(\alpha|x) = \frac{f_{X|A}(x|\alpha) \pi(\alpha)}{\int f_{X|A}(x|\alpha) \pi(\alpha) d\alpha} \quad (38)$$

which yields a Gamma distribution with shape $n + s$ and rate $r + \sum_i \ln(x_i)$:

$$\pi_{A|X}(\alpha|x) = \frac{\alpha^{n+s-1} \exp[-\alpha(\sum_i \ln(x_i) + r)]}{(n+s-1)! \left(\frac{1}{\sum_i \ln(x_i) + r} \right)^{n+s}} \quad (39)$$

⁴³Arnold and Press (1989) demonstrate that if this threshold is unknown, a modified Lwin prior should be implemented.

Furthermore, Equation 16 can be developed for each Bayesian expert because the mode can be expressed using the Pareto-distributed loss realizations.

$$\pi_i(\alpha^t | \cup_t x_i^t) \sim \mathcal{N} \left(\frac{t-1}{\sum_{k=1}^t \ln x_i^k}, \frac{t-1}{\left(\sum_{k=1}^t \ln x_i^k\right)^2} \right) \quad (40)$$

Following Meyers (1996), the Bayesian posterior will follow a normal distribution:

$$\pi(\alpha | \hat{\alpha}) \sim \mathcal{N} \left(\hat{\alpha}, \frac{\hat{\alpha}^2}{t} \right) \quad (41)$$