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A Note on *Arabes maxime*: A Testimony of Alfonsine Astronomers' Interest in the Toledan Tables*

Eleonora Andriani, Nick Jacobson, Samuel Gessner

DISCLAIMER: This document is a preliminary draft of the contribution, currently in its second proofs.

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The Bibliothèque nationale de France (BnF) preserves eleven manuscript volumes related to Conrad Heingarter, a Swiss physician and astrologer active in the second half of the fifteenth century.¹ One of these manuscripts, Paris, BnF lat. 7295A, contains what is perhaps the only witness of an anonymous astronomical work beginning with the expression ‘Arabes maxime’. This work has so far slipped through the net of investigation.² The aim of this contribution – besides providing introductory information about the work at issue covering folios 182^r–193^v – is to highlight some of its striking characteristics. Although the work is anonymous and its attribution to Heingarter rather doubtful, we argue that a full edition and study of this text is justified, not only for its intrinsic interest, but also in the hope that it will help refine our understanding of the history of Alfonsine astronomy. The first section of this contribution focuses on the manuscript context and the text itself, including a treatment of its sources, the time frame of its composition, and its genre. The second section comments upon

* This chapter has been written collectively. The three authors have equally contributed to the writing process and revision work. Eleonora Andriani wrote the preamble and section 1, Samuel Gessner wrote section 2a and 2b, Nick Jacobson wrote 2c and the conclusion. The edition was established collectively.

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¹ ‘The BnF collection now holds eight manuscripts related to Conrad Heingarter (lat. 7197, lat. 7273 (?), lat. 7295A (?), lat. 7314 (?), lat. 7432, lat. 7446, lat. 7447, lat. 7450)’. Matthieu Husson, ‘Mathematical Astronomy and the Production of Multiple-Texts Manuscripts in Late Medieval Europe: A Comparison of BnF lat. 7197 and BnF lat. 7432’, in *The Emergence of Multiple-Text Manuscripts*, ed. by Alessandro Bausi, Michael Friedrich and Marilena Maniaci (Berlin: De Gruyter, 2019) <<https://doi.org/10.1515/9783110645989>> [accessed: 14.04.2023], p. 249. The following manuscripts should be added to the list above: lat. 7333, lat. 11232, lat. 7305.

² An exception should be made for some marginal or rather short references in the secondary literature and catalogues. See Matthieu Husson, ‘The Art of Astrological Computations: Conrad Heingarter and the Manuscript Paris, BnF latin 7295A’, in *Exploring Written Artefacts: Objects, Methods, and Concepts*, ed. by Jörg B. Quenzer (Berlin: De Gruyter, 2021) <<https://doi.org/10.1515/9783110753301>> [accessed: 14.04.2023], p. 526; this article focuses on the manuscript 7295A and highlights its specific characteristics of a personal ‘toolbox’ for computation. See also, Fritz S. Pedersen, *The Toledan Tables: A Review of the Manuscripts and the Textual Versions with an Edition* (Copenhagen: C.A. Reitzels Forlag, 2002), p. 157; Lynn Thorndike and Pearl Kibre, *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin*, rev. ed. (London: Mediaeval Academy of America 1963), p. 124; Francis S. Benjamin Jr. and Gerald J. Toomer, *Campanus of Novara and Medieval Planetary Theory: Theorica planetarum* (Madison: University of Wisconsin Press, 1971), pp. 83–85. See also the description prepared by Laure Rioust in the BnF catalogue (June 2018); see ‘Latin 7295A’ <<https://archivesetmanuscrits.bnf.fr/ark:/12148/cc664943>> [accessed: 14.04.2023].

some of the most intriguing aspects of this work, namely its explicitly comparative approach in which the author weighs the Toledan Tables and canons against the Alfonsine procedures and the work's use of diagrams when 'demonstrating' the Toledan procedures. This section also discusses some relevant terminology and concepts related to mathematical reasoning that we have found in the text. The Appendix provides a selection of chapters excerpted from the critical edition in preparation.³

1. Manuscript context

The collection of works in Paris, BnF lat. 7295A (former shelfmarks: Colbert 979; Regius 5073. A) was produced by Conrad Heingarter at least partly during his university years in Paris (third quarter of the fifteenth century).⁴ The manuscript codex entered the Royal Library in 1732 at the same time as BnF lat. 7197, another manuscript produced by Heingarter.⁵

Heingarter seems to have written most of the folios 1^r–32^v, as well as folios 35^r–45^r, 49^r–93^r, 145^r–152^r, and 181^r–193^v, including some passages in German (ff. 64^v–92^r) and an explicit in Greek lettering (f. 45^r).⁶ He also annotated certain texts written by other copyists (e.g. ff. 99^r, 100^r, 164^r, 179^r–180^v). The manuscript consists of seventeen quires and contains several works, including the following: *Theorica planetarum Campani*, the *De armillis* by Profatius Judaeus, Ps.-Messahala's *De astrolabio* (only the part on construction), the Parisian Alfonsine Tables (augmented by the Oxford double-entry tables for latitude) accompanied by the canons of John of Saxony, and the tables of 1322 and related canons by John of Lignères.⁷

³ For the Appendix, see pp. 19–34 in the present contribution.

⁴ The watermarks identified in this manuscript are attested in Paris in the third quarter of the fifteenth century, between 1450 and 1463, which corresponds to the years when Heingarter was studying there. See Husson, 'The Art of Astrological Computations', p. 515 and the section 'présentation du contenu' in Laure Rioust's notice.

⁵ See Husson, 'Mathematical Astronomy', p. 255.

⁶ If transliterated into Latin, it would read: 'Explicit seu finis per Konradum Heingarter'. Paris, BnF lat. 7295A, f. 45^r. The explicit is bordered by three strokes drawn in red ink and is written in Greek letters. For general information on the use of Greek script in Latin manuscripts, see Walter Berschin, 'Greek Script in Latin Manuscripts', in *The Oxford Handbook of Latin Palaeography*, ed. by Frank T. Coulson and Robert G. Babcock (New York: Oxford University Press, 2020), pp. 115–17. A notable example of the practice of writing information in Latin but using Greek script is found in the manuscript Toledo, Archivo y Biblioteca Capitulares, 104 6, a Boccaccio autograph manuscript consisting of an anthology of Dante's works. Here, Giovanni Boccaccio seems to have written the following phrase in Greek script as a caption to a portrait found in the manuscript: 'Ioannes de Certaldo p[inx]it', meaning 'John of Certaldo painted it'. See Stefano Martinelli Tempesta and Marco Petoletti, 'Il ritratto di Omero e la firma greca di Boccaccio', *Italia Medioevale e Umanistica* 54 (2013), 399–409 (p. 401).

⁷ Although the last quire bears the number 18 in the bottom left margin, the quires constituting BnF lat. 7295A are only 17. The quire structure and a table of contents is available in the notice produced by Laure Rioust. For a complete overview of the works included in BnF lat. 7295A, see Husson, 'The Art of Astrological Computations', pp. 523–27.

The seventeenth and final quire is the one that contains our text, *Arabes maxime* (preceded by some shorter works), all written in the hand identified as Heingarter's. Heingarter is likely copying a preexisting work. Among the hints for this there are several inconsistencies between the text and the lettering of the diagrams. Another one is the text's reference to 'red circles' in a diagram that are not rubricated in this witness. This quire starts on folio 181^r, with a text on the colours of lunar eclipses, which Fritz S. Pedersen identified as an addition to the Toledan canons (version Cb) and defined as 'Alkharizmi'.⁸ This short excursus on eclipse colours is followed by another short text bearing the incipit 'Nota quod eclipsis lune', which mostly corresponds to a passage from the chapter on the direct, retrograde, and stationary positions of the planets from the *Theorica planetarum Gerardi*.⁹ This text ends with the explicit *Expliciunt canones secundi mobilis magistri Iohannes de Lineriis finiti Wienensi studio* indicating that the work was copied in the city of Vienne in France.¹⁰ Indeed, the previous quire (i.e. the 16th) includes the canons *Priores astrologi* (ff. 174^r–180^v) by John of Lignères. These canons were not copied by Heingarter, but include some notes in his hand, and bear traces of his careful reading. Lignères' canons end abruptly, and a remark in the lower margin, 'Hic deficiunt canones quinque, scilicet 41^{us}, 42^{us}, 43^{us}, 44^{us} et quadragesimus quintus', indicates that the canons missing from the work had not gone unnoticed to the attentive reader.

The rest of this first folio in the seventeenth quire (f. 181^r–181^v) includes a text with the incipit 'Nota secundum Alfraganum centrum ecentrici solis'. Small paragraphs starting with 'nota' and providing different kinds of definitions or clarifications show that the text is not an organic work, but rather a set of collected notes, or possibly expansions on various topics.

These short accounts defining or elucidating different subjects include the following topics: the distance of the centre of the eccentric of the Sun, the Moon, and the planets from the centre of the Earth; the semi-diameter of the epicycles of the planets; the diameter of the Sun, the Earth, and the Moon and their proportion to one another; parallaxes; planetary motion in longitude; planetary motion in latitude; lunar mean motion in latitude; the equated motion of lunar latitude; the equated argument of the Moon; the true latitude of the Moon; the apparent latitude of the Moon; the declination of the Sun; the *sinus rectus*, *sinus versus*, and the *kardaga*; the place of the stars according to longitude and latitude; a method for identifying the variation in the lengths of the daylight referring to the arcs *portio*

⁸ 'The text was popular later: part (1) is in the Alfonsine Tables, and the incipit of part (2) is quoted by Steinschneider (Études, 1881+, p. 41) from the 1518 printing of John of Saxony's rules for the Alfonsine Tables, folio 23v, where it is ascribed to John of Lignères'. See Pedersen, *The Toledan Tables*, p. 535. See Moritz Steinschneider, 'Études sur Zarkali, astronome arabe du XI^e siècle, et ses ouvrages', offprint from *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche* (ed. Boncompagni) 17 (1884), p. 781.

⁹ See *Theorica planetarum Gerardi* VI. 84–86, ed. by Francis J. Carmody (Berkeley, CA: University of California Press, Berkeley, 1942), pp. 39–40.

¹⁰ BnF lat. 7295A, f. 181^{ra}.

diei and *portio noctis*; the definition of the conditions necessary for an eclipse to take place; and various details on sexagesimal computations.¹¹ This rather heterogeneous text continues on the unruled page in a single column (f. 181^v) and is followed by a diagram, the caption of which reads ‘The figure of the solar eclipse’.¹²

This section of the manuscript with the incipit ‘Nota secundum Alfraganum’, is displayed as a single text and consists of a continuous chain of notes. Significantly, these paragraphs introduced by the word ‘nota’ suggest that the genre of this text is some sort of *catena* commentary. *Catena* commentaries are glosses that ‘break away from the immediacy of the text [...] and claim the status of independent expositions, which in turn may inspire their own glossing activity’.¹³

Opening with the incipit ‘Arabes maxime secundum motum lune’, the content of the manuscript from folio 182^{ra} until the end of the quire on folio 193^{vb} appears as a single work divided into nearly fifty unnumbered chapters of various lengths and touching upon a variety of astronomical and trigonometrical topics.¹⁴ An overview of the chapters and the topics, some of which are explored in this contribution, is displayed in the table below. The table provides the order of the chapters, the folio reference to where they begin, the incipit, and the subject area they treat. The chapter beginnings are usually marked by spaces reserved for initials.¹⁵

Chapter	Folio	Incipit	Topic
1	182 ^{ra}	<A>rabes maxime secundum motum lune tempora distinguentes	Chronology
2	182 ^{ra}	<C>um volueris scire annos, mense<s> et dies Arabum ex annis domini nostri	
3	182 ^{rb}	<S>i exordium anni vel mensis Arabici scire cupis	
4	182 ^{rb}	<C>um alicuius arcus sinum rectum sive equalem quesieris	Trigonometry and <i>primum mobile</i>
5	182 ^{vb}	<Q>ualiter invenitur per numerum portio circuli equalis	
6	183 ^{ra}	Et est sciendum quod licet inter sinum et portionem non sit proportio	
7	183 ^{ra}	<Q>uamvis in hoc opere de practica principaliter cogitemus	
8	183 ^{vb}	<E>t quoniam in premissa demonstratione	
9	183 ^{vb}	<I>tem in eadem demonstratione	
10	184 ^{ra}	<S>i aliquot gradus ecliptice cuiuscumque signi multiplicaveris	
11	184 ^{ra}	<Q>ualiter per altitudinem solis inveniantur puncta umbre	Planetary longitudes
12	184 ^{rb}	<H>actenus de motu orbis primi et pertinentibus ad ipsum tractavimus	
13	184 ^{vb}	<N>otandum quod centrum epicycli tam Veneris quam Mercurii	
14	185 ^{rb}	<S>ecundum Alfraganum luna cum fuerit in superiori parte epicycli movetur	
15	185 ^{rb}	<M>ercurius, Venus et sol in suis motibus mediis eodem modo se habent	
16	185 ^{va}	<M>ars in die movetur in epicyclo 28 minutis et percurrit eum in duobus annis	
17	185 ^{vb}	<E>t quoniam planete moventur in ecentricis ut visum est	
18	186 ^{ra}	<N>ota in tabulis tholetanis utimur annis Arabum et ponuntur ibi radices	

¹¹ Interestingly, some passages of the text beginning with ‘Nota secundum Alfraganum centrum ecentrici solis’ exhibit verbatim correspondences with certain passages included in the *Almagesti Minor*, books V–VI; see Henry Zepeda, *The First Latin Treatise on Ptolemy’s Astronomy: The Almagesti minor (c. 1200)* (Turnhout: Brepols, 2018), pp. 330 and 434.

¹² BnF lat. 7295A, f. 181^v: ‘Figura eclipsis solaris’.

¹³ Greti Dinkova-Bruun, ‘Text and Gloss’, in *The Oxford Handbook of Latin Palaeography*, ed. by Frank T. Coulson and Robert G. Babcock (New York: Oxford University Press, 2020), pp. 933–34.

¹⁴ From this point on, we will refer to this work by its incipit, that is, *Arabes maxime*.

¹⁵ Only six chapters out of 49 include initials, five of which are rubricated; additional four chapters include guide letters and only one chapter includes a heading. See the Appendix, p. 19 n. 64.

19	186 ^{ra}	<A>d inveniendum medios motus planetarum in tabulis tholetanis	
20	186 ^{rb}	<S>olem equabis per omnem motum sicut in Alphontio	
21	187 ^{rb}	<A>d habendum verum locum capitis draconis	
22	187 ^{rb}	<H>oc placuit extrahere de equatione Saturni	
23	187 ^{vb}	<V>enus et Mercurius equantur sicut tres superiores omnino hoc accepto	
24	188 ^{ra}	<C>um aliquis quinque planetarum fuerit in superiori parte epicycli	
25	188 ^{rb}	<S>i de aliquo planetarum scire volueris utrum sit stationarius	
26	188 ^{vb}	<C>um fuerit centrum planete equatum ab uno gradu in 6 signa	
27	188 ^{vb}	<O>rtum et occasum elaycum [<i>sic!</i>] planetarum sic inuenies in 3 superioribus	Planetary passions
28	189 ^{ra}	<U>trum planeta sit sub radiis vel videri possit per tabulas ad hoc constitutas	
29	189 ^{rb}	<S>i noscere desideras qua hora solis vel luna vel quilibet alius planeta	
30	189 ^{rb}	<P>ostquam de motu longitudinis planetarum dictum est	
31	189 ^{va}	<L>atitudinem lune sic inveneris	
32	189 ^{va}	<A>d cognoscendum diversitatem que accidit in latitudine 5 planetarum	Planetary latitudes
33	190 ^{rb}	<E>t si per tabulas invenire volueris quanta sit latitudo	
34	190 ^{va}	De Venere. Veneris autem et Mercurii latitudines per similes tabulas inuenies	
35	190 ^{va}	Nota quando est coniunctio vel oppositio solis et lune	
36	190 ^{va}	Cum itaque sciveris in quo mense potest fieri eclipsis solis vel lune	Syzygies
37	190 ^{va}	Certum tempus et verum locum coniunctionis et oppositionis inuenies hoc modo	
38	190 ^{vb}	Et quoniam in operatione premissa docuimus ut longitudinem	
39	191 ^{ra}	<C>um eclipsim lune absque errore prescire et sine dubio predicere volueris	
40	191 ^{va}	<Q>antum autem obscurabitur de superficie lunaris corporis	
41	191 ^{va}	<L>une dyametrum ad horam eclipsis sic inuenias	Lunar eclipses
42	191 ^{va}	<D>yametrum circuli umbre in loco transitus inuenies hoc modo	
43	191 ^{vb}	<S>i eclipsim lune in figura geometrica representare volueris	
44	192 ^{ra}	<N>unc videndum est de hiis que circa eclipsim solis considerata occurrunt	
45	192 ^{rb}	<S>i diversitatem lune invenire volueris ad horam eclipsis	
46	192 ^{vb}	<C>um scire volueris qua die et qua hora diei eclipsis solaris ventura sit	Solar eclipses
47	193 ^{rb}	<S>i scire volueris quantum de superficie solis obfuscarī debeat	
48	193 ^{rb}	<C>um solis dyametrum ad horam eclipsis invenire volueris	
49	193 ^{va}	<S>i solaris eclipsis figuram depingere volueris	

‘Arabes maxime’ (i.e. The Arabs chiefly), the initial words of this work, immediately provide the reader with a frame of reference for the main tradition underpinning it. The expression derives from the Toledan canon (canon Cb7 in the version Cb as established by Pedersen) that begins ‘Arabes vero, a lunae maxime motu ipsius temporis metientes spatia...’¹⁶ Though most of the text appears to be clearly indebted to the Cb canons, the work also relies upon other sources that could be said to belong to an ‘Arabic tradition’.¹⁷ Hence, the text is remarkable in that it establishes this interest in the Arabic authorities at the very beginning of the work, and this initial marker of interest is matched by the stress put in the text on the approach to astronomical tables that is presented as ‘Arabic’.¹⁸ As appears from the table above, the work includes forty-nine chapters, which can be distributed into several topics. These are: chronology, trigonometry and the *primum mobile*, planetary longitudes, planetary

¹⁶ See Pedersen, *The Toledan Tables*, p. 382.

¹⁷ The sources that have been identified so far, besides the canons Cb, are Euclid, al-Farghānī and Ps.-Thābit ibn Qurra. However, the ongoing work of editing the text will enable us to determine more sources. This is certainly one of the important advantages that a complete edition of this text will offer.

¹⁸ ‘Nota in tabulis tholetanis utimur annis Arabum et ponuntur ibi radices ere Arabum’. See Appendix, Ch.18, 28.

passions, planetary latitudes, syzygies, lunar eclipses, and solar eclipses. The author of the text has consciously chosen a limited set of topics, excluding some treated in the Toledan canons or other works. At times this is made explicit, such as when the author remarks that the solar and lunar motion would not be discussed in a certain passage, since they had been sufficiently treated in the *Theorica planetarum*.¹⁹

A rough temporal context for the work can be inferred from the references it includes to Toledan and Alfonsine frameworks. A few hints to define the time period are offered by the first lines of Chapters 18, 19, and 20.²⁰ Indeed, the incipit of Chapter 18 reads, ‘Note that in the Toledan Tables we use the years of the Arabs, and the radices of their epoch are put there’. The incipit of Chapter 19 reads, ‘The operation to find the mean motion of the planets in the Toledan Tables is the same as that in the Alfonsine Tables’. The incipit of Chapter 20 reads, ‘You will equate the Sun through all the motion as it is taught in Alfonso’.²¹ Some further hints from the work will be offered to narrow down the span of time in which it may have been composed (see Section 2a), although this is still tentative. Whatever the case may be, it appears that, although the time of the composition of this work was posterior to Alfonso (1270s), there was an ongoing interest in the ‘Toledan tradition’, since this work refers to the Toledan Tables quite frequently and depends largely on the Cb canons.²²

The bond that connects *Arabes maxime* to the Toledan canons is close enough to raise the question of its status as an independent work. Given its great dependency, it is possible to consider the text as an ‘Alfonsine exposition’ on the Cb canons. In his edition of the Toledan Tables and canons, Pedersen lists eleven texts that he considers as cognate to or derived from the texts of Cb.²³ He provides a short characterization of each of these texts, denoting them in a variety of ways, although his categories seem somewhat loose. The resulting classification comprises the following characterizations: ‘[a] series of excerpts similar to rules from Cb, with lengthy explanations added’;

¹⁹ ‘de motu solis et lune satis habetur in theorica planetarum’. BnF lat. 7295A, Ch.12, f. 184^{vb}.

²⁰ Note that an edition of these chapters is included in the Appendix of this contribution. See Appendix, pp. 24–34.

²¹ ‘<N>ota: in tabulis tholetanis utimur annis Arabum et ponuntur ibi radices ere Arabum’. See Appendix, Ch.18, p. 28; ‘Ad inveniendum medios motus planetarum in tabulis tholetanis eadem est operatio que in tabulis Auphontii’. See Appendix, Ch.19, p. 29; ‘<S>olem equabis per omnem motum sicut in Alphoncio docetur’. See Appendix, Ch.20, pp. 29–34.

²² For a definition of the ‘Toledan tradition’ including the works and authors that it encompasses, see Pedersen, *The Toledan Tables*, pp. 44–45.

²³ See Pedersen, *The Toledan Tables*, pp. 340–41. The texts mentioned by Pedersen are ‘Diversi astrologi’, ‘Canones Luce’ – ‘Ut annos Arabum’, ‘Kardaga est portio’, ‘Sicut dicit Hermes in libro de natura dierum’, John of Sicily’s ‘Inter cetera veritatis philosophicae documenta’, ‘Scripta Marsiliensis super canones Azarchelis’, ‘Cum in singulis scientiis’, ‘Quia ad notitiam caelestis motus secundum doctrinam magistri Arzachelis perutile est scire annos Arabum’, ‘Ad maiorem manifestationem eorum quae dicuntur de umbra supra in canonibus Arzachelis, folio 4, est istud notandum...’, ‘Circulus quilibet potest dividi in 360 partes’, and ‘De canonibus Arzachel tractaturus’. Note that Pedersen states that he mostly refrains from considering texts of the fourteenth century or later.

‘[an] expanded paraphrase of Cb’; ‘[a] literal commentary on Cb’; ‘[an] extract of texts from a literal commentary on Cb [i.e. John of Sicily]’; ‘a set of lemmatized glosses to Cb’; ‘a series of canons on chronology and trigonometry, lightly paraphrased from Cb’; ‘a comprehensive set of mixed glosses covering trigonometry (including sines of *kardagas*), planetary equations, eclipses, and risings’; and ‘trigonometrical canons [which] might be paraphrases of Cb’.

Arabes maxime seems to be one of these kinds of texts. Precisely what category best defines it, however, remains to be determined. The following paragraphs will further explore the dependency of this work upon the Toledan tradition, while also noting the subtle and sometimes sophisticated ways in which the author supplemented the mathematical reasoning of the canons through his own commentary.

2a. Comparative viewpoint on the Toledan and Alfonsine Tables

As already noted above, the incipit of our text is a true catchphrase. *Arabes maxime* seems to appeal to readers curious about the way the ‘Arabs’ understood and practiced astronomy. The text is clearly addressed to an audience used to and conversant with the Alfonsine way, but who wanted to understand the Toledan Tables as well. Indeed, several of the *Arabes maxime* chapters (19, 20, 22) make an explicit comparison between the Alfonsine approach and the one associated with the Toledan Tables.

One can imagine that this interest in comparing the Toledan Tables to the Alfonsine ones was possibly common practice from the 1270s onwards, when some astronomers were using the Alfonsine Tables (either the Castilian or the Parisian Latin version), while others continued to favour the (translated) Toledan Tables. Chapter 20 of *Arabes maxime*, opening with the sentence ‘You will equate the Sun through all the motion as it is taught in Alfonso’, contains a tantalizing hint that our text *Arabes maxime* might even refer to the Castilian version of the Alfonsine Tables before they were reformatted in Paris in the 1320s.²⁴ However, let it be said that the interpretation of the chapter is ambiguous, and no definite conclusion can be reached yet. If this view were confirmed, though, it would constitute a significant argument for an early dating of the text. In any case, to understand this hint, it is important to remember that mean motions are counted in a sidereal reference frame in the Castilian version (of the Alfonsine Tables), while in the Latin mean motions are reckoned with respect to a tropical reference frame.

The crucial sentence in this chapter translates as follows: ‘...having obtained the true place of the Sun, count it (*computa eum*) from the beginning of the movable [zodiac sign] Aries, to which certain position add the equated motion of the 8th sphere. Thus, you will have its [the Sun’s] position,

²⁴ ‘Solem equabis per omnem motum sicut in Alphontio docetur’. See Appendix, Ch.20, p. 29.

by counting most accurately from the intersection of the circles'.²⁵ Opposition is made here between measuring a position in the eighth sphere (typically from the first point of the movable Aries) on the one hand, and measuring it from the vernal point of the prime mover, defined as the rising intersection of the (moveable) ecliptic and the celestial equator, on the other.

In combination with the opening of the chapter, the straightforward reading of this passage is that while equating the Sun is done for all motions as taught by the Alfonsine astronomers, this process implies two equations: that of the argument and that of the motion of access and recess of the eighth sphere. Adding the latter equation only at the end (and not before computing the equation) agrees with the Castilian Alfonsine canons (as with the Toledan Tables) and is clearly different from the procedure for the Parisian Alfonsine Tables.²⁶

It is not entirely clear from the text, however, whether the passage provided represents an operation that is 'like it is taught in Alfonso', or whether it is given to precisely highlight one of the points of divergence of the Alfonsine from the tradition of the Toledan Tables. In general, one observes that *Arabes maxime* typically addresses the cases where the Toledan tradition is different from Alfonsine doctrine, while it does not repeat notions from the Alfonsine that are assumed (by the author of *Arabes*) to be known by the reader.

The cited passage echoes a chapter of the Toledan Cb canons on the equation of the Sun: '...and then you will get the true place of the Sun, and this is reckoned from the beginning of Aries in the eighth sphere. To this true place, add the place of the eighth sphere, and you will get the truer place'.²⁷ Yet, even in the Castilian Alfonsine canons one finds some similarity in the wording, namely in Chapter 16, which reads 'And what remains of the mean motion after its [the equation's] subtraction or addition, will be the certain position of the Sun'.²⁸ Note that the Castilian *el lugar çierto* dovetails with the Latin expression of *cui certo loco*, which one finds in both the Toledan canons Cb and in *Arabes maxime*. In all cases it refers to the position of the Sun in the eighth sphere and appears to have been a way to distinguish it from the *verus locus*, the true position with respect to the vernal point on the fixed zodiac. Both the Toledan and the Castilian canons instruct the reader to add or subtract the equation of access and recess to the already equated positions of the planets.

²⁵ 'Habitio ergo vero loco solis, computa eum a principio arietis mobilis, cui certo loco superadde motum 8ve spere equatum, et habebis locum eius certissime computando a coniunctione circulorum'. See Appendix, p. 29–30.

²⁶ With the PAT the motion of the eighth sphere must be introduced as a first step to define the position of the apogee. The apogee is needed to determine the mean anomaly and then compute the solar equation See José Chabás and Bernard R. Goldstein, *The Alfonsine Tables of Toledo*, (Dordrecht: Kluwer, 2003) pp.217-220.

²⁷ 'et tunc habebis locum solis certissime, cuius initium erit ab initio arietis 8'a sphaera. Cui certo loco addes locum 8'ae sphaerae, et habebis certiore', Pedersen, *The Toledan Tables*, Cb142, p. 436.

²⁸ 'E lo que fuere el medio curso despues del menguamiento del, o despues del añadimiento sobre el, eso sera el lugar çierto del sol'. In the chapter 'De la equaçion del Sol'. See Chabás and Goldstein, *The Alfonsine Tables of Toledo*, Ch.16, sentence 6, p. 37.

It remains uncertain whether the author of *Arabes maxime* was referring to the Castilian version of the Alfonsine doctrine or whether he was drawing from the older Toledan Cb canons. The difficulty here is to interpret the comparative perspective of the text. It is not obvious in this case whether the text describes a common or a divergent feature. One thing is clear: the text assumes that the readers were aware of two different reference frames. In the case of the Toledan and the Castilian Alfonsine traditions, planetary positions are located with respect to the eighth sphere (sidereal). In the case of the Parisian Alfonsine tradition, they are located with respect to the vernal point (tropical). The reader is further instructed to find a planetary position, *locum certissime computando*, which is the position most accurately computed from the vernal point.

2b. Invoking Euclid in clarifying the notion of *kardage*

Beyond comparing Toledan material with the Alfonsine Tables, the author of *Arabes maxime* also consistently complements the arithmetic and tabular reasoning of the canons with geometrical diagrams and commentary. This is a distinctive feature found among the various categories of commentary texts to canons.²⁹ In several places the text justifies the addition of a diagram by arguing that it clarifies the argument. The first topic treated in this way is the derivation of trigonometric values. Indeed, much space is given to the explanation and the demonstration of the sine table in the form of what is known as table of *kardage*: a way of dividing the unit circle into 24 equal arcs that goes back to Sanskrit sources and is also present in the canons of the Toledan tradition. The beginning of the passage on the *kardage* in *Arabes maxime* expresses the content of the Toledan canons numbered in Pedersen's edition as canons Cb52, Cb58 (explanation) and the canons Cb238, Cb242, and Cb244 (demonstration) treated by Pedersen as 'parallel canons' mainly derived from al-Khwārizmī.³⁰ The proof for the values of the *kardage* can be found in Chapter 7 of *Arabes maxime*.³¹ It is based on the principles also invoked in Cb244 and the following canons. These include the Pythagorean theorem with right triangles to derive the right and versed sines.³² A feature that

²⁹ The practice of adding glosses with figures to explain canons occurs of course in earlier works: for example, in Ibn al-Muthannā's commentary of al-Khwārizmī's *Ta'ālīl zīj al-Khwārizmī*. Bernard R. Goldstein, *Ibn al-Muthannā's Commentary on the Astronomical Tables of al-Khwārizmī* (New Haven: Yale University Press, 1967).

³⁰ Pedersen refers to the specific version of al-Khwārizmī's astronomical tables in its 'non-Maslama' version used by Ibn al-Muthannā whose commentary survives only in two Hebrew versions and one Latin version. Goldstein, *Ibn al-Muthannā's commentary*. Pedersen, *The Toledan Tables*, p. 51, (2.5). According to this author, Cb244 is clearly a text that was added to the end of the canons. See Pedersen, *The Toledan Tables*, p. 488.

³¹ Appendix, pp. 24–26.

³² To be exact, the *kardage* table provides the differences of the right and versed sines for six arcs 15°, 30°, 45°, 60°, 75°, 90°. The 'right sine' (*sinus rectus*) corresponds to the modern notion of sine. The 'versed sine' (*sinus versus*) corresponds to the difference of the radius and the cosine. Both are expressed with respect to the length of the radius sometimes taken, according to our text, as 150 or 60. By approximation integers are used for the sine values.

distinguishes *Arabes maxime* from the original Toledan source, however, is the invocation of propositions from Euclid’s *Elements* for many of the steps.³³

For the derivation of the sine and versed sine tables, the *Arabes maxime* goes further than the edited version of the Toledan canons does. The text has two more chapters (8 and 9) showing which geometric constructions correspond to the arithmetic operations used in Chapter 7 when the Pythagorean theorem is used.³⁴ Indeed, it repeatedly involves extracting the root of a sum or a difference of squares. The constructive steps, to join the area of two squares in one – or to construct a square equal to the difference of two squares – are justified step by step, invoking propositions from Euclid’s *Elements*.³⁵ An examination of the text shows that it contains important passages parallel to John of Gmunden’s treatise *De sinibus et arcubus* composed in 1437.³⁶ The following table contains three parallel passages excerpted from Chapters 7, 8, and 9, which reveal unmistakable similarities between the two works.³⁷

<i>Arabes maxime</i>	<i>De sinibus, chordis et arcubus</i>
[Ch. 7, p. 26, lines 50–57]	p. 92
Ex hiis que dicta sunt, manifesta est quantitas sinus tam recti quam versi cuiuslibet kardage et quarumlibet simul sumptarum, quia sinus rectus prime est sinus versus 6° et sinus rectus 2° est sinus versus 5°. Item sinus rectus duarum scilicet prime et 2° est sinus versus duarum scilicet 5° et 6°, et versus illarum est rectus istarum. Item sinus rectus 3° est sinus versus 4°, et versus 3° est rectus quarte.	Ex hiis igitur quae dicta sunt manifesta est quantitas tam sinus recti quam versi cuiuslibet cardagae et quarumlibet simul sumptarum quia sinus rectus primae cardagae est sinus versus sextae cardagae et sinus rectus secundae est sinus versus quintae. Item sinus rectus duarum cardagarum scilicet primae et secundae est sinus versus duarum ultimarum scilicet quintae et sextae et sinus versus duarum primarum

³³ This might be indicative of an arts faculty context. In medieval Latin astronomical literature, Euclid is invoked in some high-level works such as the *Almagesti minor*, which presents the material of the *Almagest modo geometrico*. Also, in treating the *kardage* and trigonometry, Richard of Wallingford’s *Quadripartitum* makes proficient use of Euclid’s *Elements* to prove the computations of sines to a great level of precision. See John D. North, *Richard of Wallingford. An Edition of his Writings with Introduction, English Translation, and Commentary*, vol.1 (Oxford: Oxford University Press, 1976), pp. 21–178. It must be kept in mind that our text is of a mathematically less advanced level than those cited examples. As we show below, the use of Euclid here is more akin to introductory mathematical texts. See note 38.

³⁴ Appendix, pp. 26–28.

³⁵ Conspicuously, though, the style of the justifications is not ‘Euclidean’.

³⁶ The sources of these passages were not indicated in detail in Busard’s 1971 edition. Hubertus. L. L. Busard, *Der Traktat De sinibus, chordis et arcubus von Johannes von Gmunden* (Vienna: Springer Verlag, 1971), edited from ÖNB Cod. 5268, ff. 44^{ra}–90^{va}. Busard notes, however, that John of Gmunden did ‘more than just use’ John of Lignères’ *Cuiuslibet arcus* canons, that is, the *Canones tabularum primi mobilis*. See also Glenn van Brummelen, *The Mathematics of the Heavens and the Earth* (Princeton: Princeton University Press, 2009), p. 218.

³⁷ The left column includes selected passages from the text of *Arabes maxime* excerpted from the Appendix. Variant readings and conjectures in these passages can be found in the edition included in the Appendix pp. 26–28. The right column provides the text of *De sinibus, chordis et arcubus* exactly as it is found in Busard’s edition. Page changes in the manuscript BnF lat. 7295A and in Busard’s edition are noted by vertical bars. As far as the diagrams are concerned, their proportions show a similarity between those supporting Gmunden’s treatise and those of *Arabes maxime*. However, the similarity does not appear in the diagrams edited by Busard, but only when examining those of the ÖNB manuscript.

<p>Similiter sinus rectus trium primarum est sinus versus trium ultimarum et sinus versus illarum est rectus istarum.</p>	<p>est sinus rectus duarum ultimarum. Item sinus rectus tertiae cardagae est sinus versus quartae et sinus versus tertiae est sinus rectus quartae. Similiter sinus rectus trium primarum cardagarum est sinus versus trium ultimarum et sinus versus trium primarum est sinus rectus trium ultimarum.</p>
<p>[Ch. 8, p. 26, lines 64–67]</p> <p><E>t quoniam in premissa demonstratione alicui docuimus ut quadrata iungerentur in unum, qualiter hoc secundum artem geometrie fieri potest, considera, si duo quadrata fuerint equalia, tunc quadretur dyiameter et habetur propositum scilicet unum quadratum equipollens duobus.</p>	<p>p. 84</p> <p>Quod unum quadratum iungitur altero tunc quando describitur unum quadratum aequale ambobus. Hoc autem secundum artem geometriae fit taliter: considera si unum quadratum alteri fuerit aequale, tunc enim quadra diametrum unius eorum et habebis propositum scilicet unum quadratum aequipollens duobus.</p>
<p>[Ch.9, 27-28, lines 77–91]</p> <p><I>tem in eadem demonstratione, quoniam diximus quadratum a quadrato esse tollendum, quod qualiter per geometriam fieri debeat, patet sic. Sit quadratum <i>hklm</i> aequale duobus quadratis suprascriptis quemadmodum ostensum est supra. <i>Abcd</i> quadratum subtrahitur ab eo hoc modo: per 3^{am} primi Euclidis ex linea <i>hm</i> abscindatur linea <i>ab</i> et lineae equalis sit <i>nm</i>. Item ex linea <i>ml</i> abscindatur alia eidem <i>nm</i> equalis et sit <i>mo</i>.</p> <p>Item per 31^{am} ex puncto <i>n</i> trahatur eque distans ipsi <i>hk</i> et sit linea <i>np</i>. Item ex puncto <i>o</i> per eandem 31^{am} ducatur eque distans ipsi <i>kl</i> et sit linea <i>oe</i>. Ad hoc per primam petitionem Euclidis trahatur <i>mk</i> dyagonaliter quae transit per punctum <i>r</i>.</p> <p>Tunc argumentatur sic: cum <i>nrom</i> quadratum ratione compositionis figure sit aequale <i>abcd</i> quadrato et ipsum totale quadratum <i>hklm</i> ex ypotesi sit aequale duobus quadratis supra descriptis, patet quod gnomo residuus erit equalis <i>efgh</i> et sic est propositum.</p>	<p>pp. 85–86</p> <p>In sequenti etiam demonstratione dicitur unum quadratum ab alio esse minuendum.</p> <p>Hoc autem per geometriam fit taliter: sit quadratum <i>h1km</i> aequale duobus quadratis suprascriptis quemadmodum ostensum est. Subtrahe ergo ab eo quadratum <i>abcd</i> hoc modo: abscinde per 3^{am} primi Euclidis ex linea <i>hm</i> lineam aequalem <i>ac</i> et sit linea <i>hn</i>. Item ex linea <i>hk</i> abscinde aliam aequalem eidem <i>hn</i> et sit linea <i>hp</i>.</p> <p>Deinde per 31^{am} primi Euclidis ex puncto <i>n</i> trahe lineam aequidistantem lineae <i>hk</i> et sit linea <i>no</i>. Similiter ex puncto <i>p</i> per eandem 31^{am} duc lineam aequidistantem lineae <i>kl</i> et sit linea <i>pq</i>. Adhuc per (<i>fol. 85^vb</i>) primam petitionem Euclidis trahe lineam <i>hl</i> diagonaliter quae transit per punctum <i>r</i>.</p> <p>Cum igitur quadratum <i>hnrp</i> ratione compositionis figurae sit aequale quadrato <i>abcd</i> et ipsum totale quadratum <i>hklm</i> ex hypothesi sit aequale duobus quadratis supradscriptis patet, quod gnomo residuus erit equalis quadrato <i>efgh</i> et hoc est propositum.</p>

One may ask oneself whether the *Arabes maxime* pre-dates or post-dates Gmunden's treatise, or whether they share a common source. The relationship between these very close passages in Gmunden's text and the *Arabes maxime* is not clear yet. We suspect that they both derive from a common source, but one could also be a reworking of the other. In any case, the quick to-and-fro we find in the *kardage* chapters between abstract arguments about given and known quantities, arithmetic

determinations, and geometricizing elaborations are characteristic of introductory texts of mathematics.³⁸

2c. Imagination and demonstration in the planetary canons

The chapters on the *kardage* are not the only places in the *Arabes maxime* that contain geometric diagrams elaborating on arithmetic canon procedures. The reader will encounter several more, for instance, accompanying the chapters on planetary motion. In these canons, the diagrams are deployed by the author as a kind of gloss on parallel material found in the Cb canons.³⁹ While basing his chapters on the Toledan corpus, however, it is worth noting that the author does not include verbatim instructions for using tables drawn from these texts.⁴⁰ Instead the contents of the chapters move beyond a mere description of the tabular steps necessary for finding true positions to include comparisons between different canon traditions, as well as illustrations of planetary models. Each illustration is accompanied by directions for composing the diagram, which effectively translate the arithmetic computations of tabular values into geometrical constructions.

In Chapter 20, for example, the reader is given a diagram of solar motion and is told to let the point *o* be the centre of the Earth, point *p* the centre of an eccentric circle, and point *a* the solar apogee or aux (Figure 1.1.). The body of the Sun is placed in two possible positions on its eccentric circle in the diagram: the first position at *b* and the second position at *c*.⁴¹ The body of the Sun in each instance is then projected onto the zodiacal circle at *d* and *m*. Given that the Sun's position is measured from the beginning of Aries at point *r* in the zodiacal circle, the reader can then easily see why the arcs *de* and *mr*, which each visually represent the solar equation, must be, respectively, subtracted or added to the Sun's mean motion to obtain the true position as viewed from the Earth at point *o*.⁴²

It is not uncommon to find worked examples in witnesses to Toledan and Alfonsine canons. In these the authors would apply the general instructions of a canon to a specific date and time in a specific place for which a planetary position is desired.⁴³ In the *Arabes maxime*, however, we find the

³⁸ For instance, the approach of the influential *Liber embadorum* by Abraham Bar Hiyya, called Savasorda (early 12th century), translated by Plato of Tivoli, is comparable. After giving numerical solutions to the problems, the *Liber* then provides a geometrical justification with recourse to lettered diagram.

³⁹ For an introductory discussion of his treatment of the Cb canons, see Pedersen, *The Toledan Tables*, 'Part 2 Cb: Canons "Quoniam cuiusque actionis"', pp. 329–42.

⁴⁰ For the chapter on the true Sun, see Appendix, pp. 29–31; compare Pedersen, *The Toledan Tables*, Part 2, pp. 435–37. For the chapter on the true Moon, see Appendix, pp. 31–34, compare Pedersen, *The Toledan Tables*, Part 2, pp. 437–39. For the chapter on the superior planets (BnF lat. 7295A, f. 187^{rb}, line 16 – f. 187^{vb}, line 2), compare Pedersen, *The Toledan Tables*, Part 2, pp. 439–41. For the chapter on the inferior planets (BnF 7295A, f. 187^{vb}, lines 3–43), compare Pedersen, *The Toledan Tables*, Part 2, pp. 440–41.

⁴¹ Appendix, p. 30, lines 130–131.

⁴² Appendix, p. 31, lines 146–155.

⁴³ In the case of the Cb version of the Toledan canons, see Pedersen, *The Toledan Tables*, Part 2, pp. 434–35 where Cremona is taken as the meridian position for a worked example for mean planetary motions. In Oxford,

author has included diagrams that belong to the *theorica planetarum* tradition. In doing so the author has adapted the convention of the worked example to the diagram provided in the chapter. He writes:

For example, when the Sun appears as it is placed at *b*, the mean motion is the arc *rqe*, but the true motion is the arc *rqd*, [and] lying between is the arc of the equation *de*. Therefore, if the aux, that is arc *rq*, should be subtracted from the mean motion, the argument will remain, namely the arc *qe*, which is smaller than 6 signs. And for this reason, the arc of the equation is subtracted from the mean motion, and the arc *rqd* remains, which is the equated motion of the Sun, and the true position of the Sun is the point *d*.⁴⁴

The order of this example follows the instructions found in the Toledan canon, but it has replaced the tabular values with their counterparts found in the diagram.⁴⁵ These steps, organized as they were diagrammatically, conform to a common technique taught in the universities by which a student would fix concepts and operations to memory by means of figures.⁴⁶ These images contained strategically labelled elements of interest, or *loci*, designed to help users recollect crucial points of comparison, connection, or distinction in their mind's eye.⁴⁷ This became a commonplace in instruction across various disciplines in the arts curriculum.⁴⁸ It would thus appear that the diagram was intended to help users to recall the procedures of the canon into the faculty of their imagination as an *aide-mémoire* for future computations.

Furthermore, in several instances the instructions for the canons of the *Arabes maxime* appeal to the reader's power of imagination – a cognitive faculty, which, according to medieval accounts of human psychology, was uniquely suited to tackling the subtleties of mathematical relations.⁴⁹ Moreover, this appears to have been, not merely for the sake of memorization, but also to clarify the canons in a way that the author believed would be demonstrative.⁵⁰ With reference to the image of

Hertford College 4, on ff. 140^r–147^v, John of Murs gives worked examples for his canons on the Alfonsine Tables throughout. See also the chapters of Chabás and Jacobson in this volume.

⁴⁴ Appendix, p. 31, lines 147–152.

⁴⁵ Compare the language of the Toledan canon (Cb141a): '... obtain [the Sun's] mean motion ... from [it] subtract the apogee of the Sun ... the remainder will be the 'argument' of the Sun ... write out the solar equation ... subtract it if the argument is less than 6 signs ... then you will get the true place of the Sun'. Pedersen, *Toledan Tables*, Part 2, pp. 435–37.

⁴⁶ See Francis Yates, *The Art of Memory* (Chicago: University of Chicago Press, 1966), pp. 70–81.

⁴⁷ Thomas Aquinas provides a detailed explication of the human capacity for recollection through geometric images in the seventh lecture of his commentary to Aristotle's *On Memory and Recollection*. See Thomas Aquinas, *Commentary on Memory and Recollection*, trans. by John Burchill, 'Thomas Aquinas, *Commentary on Aristotle, On Memory and Recollection*', in *The Medieval Craft of Memory: An Anthology of Texts and Pictures*, ed. by Mary Carruthers and Jan M. Ziolkowski (Philadelphia: University of Pennsylvania Press, 2002), pp. 181–85.

⁴⁸ As Francis Yates wrote in her seminal work on the subject of memory, Thomas Aquinas likened the strategies of the *loci* found in Cicero's *Rhetorica ad Herennium* to the stepwise memorization of mathematical instructions as described in Aristotle's base text. See Yates, *The Art of Memory*, pp. 71–72.

⁴⁹ The term *imaginari* is first introduced in the chapter on the eighth sphere (BnF lat. 7295A, f. 184rb–184va).

⁵⁰ The literature on this subject is vast, but for a few introductory articles on the subject, see Andrew G. Molland, 'Mathematics in the Thought of Albertus Magnus', in *Albertus Magnus and the Sciences: Commemorative Essays*, ed. by James A. Weisheipl (Toronto: Pontifical Institute of Mediaeval Studies, 1980),

the solar motion, the author explains why the Sun must be equated in the first place. Even though ‘the



Figure 1.1: Theory of the Sun. Illustration of the theory of the Sun found in Chapter 20 (Appendix, pp.29-31, lines 122–160) where *qetr* represents the ecliptic circle with its centre at *o*, which is the Earth. The circle *abnc* is the deferent carrying the body of the Sun with uniform angular velocity about *p*, which is drawn eccentrically to *o*. The Sun’s motion through the eccentric circle models the variation in the Sun’s apparent motion through the zodiac over the course of the year. The point *r* is the beginning of Aries, from where the position of the Sun is reckoned as projected onto the zodiac. From *t* to *a* through *o*, the line of apsides is drawn. As viewed from *o*, the Sun is slowest at the *aux*, *a*, and fastest at *t*. The mean solar anomaly is the arc from the position of the *aux*, *a*, to the position of the body of the Sun, at *b* or *c*, on the eccentric circle. The same arcs of mean anomaly appear on the ecliptic (arcs *qe* or *qr*). The true Sun is defined as the projection of the Sun at *b* or *c* onto the zodiac circle at *d* or *m* from the Earth, *o*. The arcs *de* and *mr* represent the solar equation separating the mean and true Sun. The diagram further defines two additional circles centred on *o*: one representing the external bound of the eccentric circle with a radius *oa*, and the other representing the internal bound of the eccentric circle with a radius *on*.

Sun runs through the zodiac in the same [time] in which it goes through the [whole] eccentric’, he explains that he does not mean it does so proportionally throughout the two circles. Instead, ‘on the upper half of the eccentric, which is *cab*, it moves more on the eccentric than in the zodiac’, and ‘on the lower half of the eccentric, which is *bnc*, it moves more in the zodiac than on the eccentric.’⁵¹ The diagram stands as a demonstration of this.

Here, we use the term ‘demonstration’ (*demonstratio*) just as the historical actors did themselves, but we do so advisedly. By demonstration in this context, we do not mean a rigorously deductive argument from first principles, but instead a practical exhibition or explanation of how

pp. 463–78; Edith Sylla, ‘Mathematical Physics and Imagination in the Work of the Oxford Calculators: Roger Swineshead’s *On Natural Motions*’, in *Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages: Essays in Honour of Marshall Clagett*, ed. by Edward Grant and John E. Murdoch (Cambridge: Cambridge University Press, 1987), pp. 69–103.

⁵¹ Appendix, pp. 30–31, lines 140–44.

something works.⁵² In this way, the author of the *Arabes maxime* emphasizes that diagrams can reveal the reasons for canon operations (*ratio operationis*), thereby resolving any confusion about why certain calculations should be performed to obtain a final value.⁵³ By the time of writing of the *Arabes maxime*, there was already a long tradition in the Latin West of understanding demonstrative reasoning in this way. According to Charles Burnett, a similar use of the term *demonstrare* can be discerned in Version 1 of Adelard of Bath’s twelfth-century translation of Euclid’s *Elements* from the Arabic, where ‘“demonstrare” is to demonstrate in the sense of pointing to something, to show something is true by scratching the geometrical figures in the sand or (in modern terms) on the blackboard.’⁵⁴

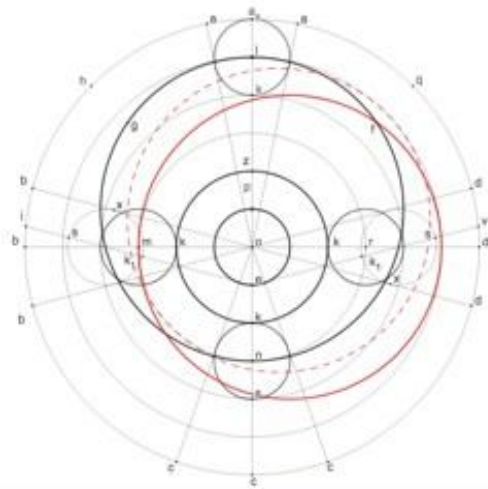


Figure 1.2: Theory of the Moon. Illustration of the diagram of the Moon in Chapter 20 (Appendix, pp. 31–34, lines 161–134), where we have superimposed the moving deferent (in red) for greater clarity. The circle *abcd* represents the line of the ecliptic in the zodiac. The tropical period of the Moon is represented by the movement of the centre of the epicycle that travels along the circumference of the deferent circle, *lmnr*, from west to east with uniform angular velocity. Meanwhile, the body of the Moon at *x* travels uniformly about the centre of its epicycle, *sk*, from east to west. These two uniform motions account for the Moon’s variable velocity in the zodiac, known as the first anomaly. The diagram also includes the second lunar anomaly, known now as evection, which Ptolemy had found to affect the position of the Moon when it was at quadratures, that is, 90° from the position of the Sun.

⁵² For the variety of medieval definitions applied to Euclidean ‘demonstration’, see Charles Burnett, ‘The Latin and Arabic Influences on the Vocabulary Concerning Demonstrative Argument in the Versions of Euclid’s *Elements* Associated with Adelard of Bath’, in *Aux origines du lexique philosophique européen: l’influence de la ‘latinitas’*, ed. by Jacqueline Hamesse (Société internationale pour l’étude de philosophie médiévale: Louvain-la-Neuve, 1997), pp. 117–35. Karine Chemla ed., *The History of Mathematical Proof in Ancient Traditions* (Cambridge: Cambridge University Press, 2012), p. 17 and pp. 41–43.

⁵³ Appendix, p. 31, lines 157–58. In the text the scribe appears to have inverted the common phrase *ratio operationis*, curiously referring to his description of the solar diagram as an *operatio rationis*.

⁵⁴ Burnett, ‘The Latin and Arabic Influences on the Vocabulary Concerning Demonstrative Argument’, p. 123.

This second anomaly manifests itself as the apparent variation in the diameter of the lunar epicycle. To model this variation, the centre of the deferent circle, p , is placed eccentrically to the Earth, which is represented by the centre point o . Thus, when the lunar epicycle is at the point of the *aux*, l , its diameter has its least apparent length from the perspective of o . When it is at the point opposite the *aux*, n , its diameter has its greatest apparent size. When the epicycle centre occupies any other points on the deferent between l and n , the argument of the Moon has to be adjusted by the equation of centre (by convention defined as the angle emo or angle ero where e is the point opposite the centre of the deferent). Finally, the centre of the deferent, p , is set in motion uniformly about o from east to west on a circle traditionally called the *circulus brevis* (though not explicitly so named in the text). One full revolution of the deferent centre around this circle represents the Moon's synodic period.

As we shall see with the case of the Moon, the author relies even more profoundly on his reader's power of imagination, describing three different places that the Moon's epicycle centre passes through. One position appears on the diagram as the point 'opposite the *aux*' (*oppositum augis*) at the base of the diagram. The other two positions, however, are indicated to be at quadratures to the line of apsides presented in the image. They are not represented as the centres of visible epicycles but rather as imagined points (*sub b vel d*), such that the author leaves their visualization to the reader's imagination. The author refers to these as the positions where the perigee occurs 'in reality' (*realiter*).⁵⁵ They are only coherent with Ptolemy's second lunar model, moreover, if the reader imagines the deferent to be in motion with its centre rotating about the Earth in the diagram itself.

The author attempts this ambitious exercise in order to help clarify the geometrical relations lying behind certain technical terms invoked in the canon procedure. He defines the equation of centre by showing the reader how to draw a line through the point opposite the centre of the lunar deferent, e , and the centre of lunar epicycle, m , thereby indicating the direction of the mean *aux* (*media aux*). The arc formed between this line and the line of the true *aux* (*vera aux*), om , constitutes the equation of centre, which is an important value in the tables. To this end, the author first lays out the figure. He writes:

For greater clarity, I shall include a figure of the Moon. Let $abcd$ be the circle of signs centred on o , which is also the centre of the sphere of the Moon whose thickness s is enclosed by the two circles sc and zk . The circle that is drawn as ac represents the exterior surface of the lunar sphere; the circle zk the interior surface. Let the deferent of the Moon be $lmnr$ centred on p . The diameter of the epicycle, sk , is, thus, aligned with the point e in every position.⁵⁶

⁵⁵ The author makes use of this adverb four times to distinguish between where a point appears in a static diagram and where it would really appear as the deferent centre, p , moves about o . See Appendix, p. 32, lines 178, 188, 190, and 196.

⁵⁶ Appendix, p. 31, lines 163–69.

After constructing the figure in this way, he goes on to say that the equation of centre will be zero when the mean centre of the epicycle is at the lunar apogee *l* and the perigee *n*, that is, when the Moon is at syzygies:

Just as *l* and *p* move to the west toward *d*, so the centre of the epicycle, which is also signified by *l*, moves to the east toward *b*, with point *e* meeting it. When the centre of the epicycle is placed at the aux of the eccentric at *l*, the Sun by [its] mean motion will, therefore, be at *a* or at *c*, which amounts to the same thing at present. [This is] because the centre of the epicycle is in mean conjunction and opposition at *l*, that is, at the aux of the eccentric. In such a position there are, in fact, no equations of centre because the centre is zero. Equations are also zero in the opposite location, namely, when the centre of the epicycle is at the place opposite the aux at point *n*, but, in reality, that happens first under *b* then under *d* since, when the Moon is distant from the Sun by 3 or 9 signs, it is at quadratures.⁵⁷

Thus, the text helps the reader to imagine the effects of the model's moving deferent centre, even though the diagram does not sensibly represent this motion. It serves as an explanation for why it is that, when the centre of the epicycle arrives 'under *b*', as the text describes it, it will be at perigee and opposite the lunar *aux* lying on the line *od*. Furthermore, it explains the reason why, when the epicycle centre proceeds to the point 'under *d*', it will have returned to perigee for a second time in its synodic cycle.

In his effort to present a diagram in motion to the reader's imagination, the author of the *Arabes maxime* was participating in what Kathleen Crowther and Peter Barker have called a 'sophisticated visual and pedagogical program' to 'train the intelligent eye'.⁵⁸ Following the analysis of David Summers, Crowther and Barker draw a distinction between 'naturalistic' illustrations, which explicitly conform to an optical experience, and 'realistic' ones, which represent real objects in nature, even if they do not do so naturalistically.⁵⁹ The *Arabes maxime* features just this sort of realism in its diagrams, which bear a strong resemblance to the figures of the Latin *theorica* and Arabo-Persian *hay'a* traditions.

Conclusion

In all of the instances we have outlined, the teaching within the *Arabes maxime* can be said to transgress certain traditional epistemic boundaries of practical knowledge to include material typically reserved for other genres, particularly that of the *theorica*. In fact, the author did so self-

⁵⁷ Appendix, pp. 31–32, lines 169–80.

⁵⁸ Kathleen M. Crowther and Peter Barker, 'Training the Intelligent Eye: Understanding Illustrations in Early Modern Astronomy Texts', *Isis* 104 (2013), 429–70 (pp. 435–36).

⁵⁹ David Summers, *The Judgment of Sense: Renaissance Naturalism and the Rise of Aesthetics* (Cambridge: Cambridge University Press, 1987), p. 3. Crowther and Barker, 'Training the Intelligent Eye', pp. 438–39.

consciously. In his chapter on the *kardage*, he even anticipates possible objections to the integration of practical and theoretical modes of reasoning within a canon text, and he justifies his approach, writing:

Although in this work we mainly deal with practical knowledge, if we nevertheless insert something pertinent to theoretical knowledge for the sake of solace, it must not be considered inappropriate. In fact, the intellect is delighted all the more by the certainty of a truth that has been attained through demonstration.⁶⁰

Of course, the *Arabes maxime* is not completely unique as a canon set that uses diagrams and geometrical arguments to reformulate and clarify tabular procedures. We find similar canons – or canon glosses, if you will – treating John of Saxony’s *Tempus est mensura motus* in Oxford, Bodleian Library, Digby 97 and John of Lignères’ *Priores astrologi* in Erfurt CA 4° 366.⁶¹ Even within the *Tempus est mensura* itself, John of Saxony occasionally explores the reason or *causa* for canon procedures by alluding to diagrams in an effort to demonstrate why certain computations are performed with the tables.⁶² If we cross the disciplinary divide to *theorica* texts, we find an author such as Campanus of Novara describing how table-makers (*tabularii*) relied on geometrical models or even ‘movable diagrams’ in the form of instruments to justify the use of their tables, and thus the line of demarcation between practical and theoretical astronomy was always porous.⁶³ Even so, the canons of the *Arabes maxime* appear to be witnesses – among a growing body of similarly identified works – to a specific sort of canon text that intentionally and systematically provided geometrical reasoning to underpin practical instructions for trigonometry and finding planetary positions.

What stands out about the *Arabes maxime*, moreover, is its close attention to the Toledan canons and the Toledan Tables at a time when Alfonsine materials had become prevalent within the medieval Latin university curriculum. It is written with the clear aim to compare the Toledan canons with the Alfonsine ones. Several questions emerge from our initial investigation of this work, including the dating of composition, the significance of the sources chosen to interpret the Toledan corpus, and the relationship between Toledan and Alfonsine material following the reception of the Castilian Alfonsine Tables in Paris in the 1320s and thereafter. This contribution serves merely as a notice of these questions, which will be the subject of ongoing research. We hope that these research areas will be further addressed and expanded upon when we release the critical edition of the entire work.

⁶⁰ Appendix, p. 24, line 1–4.

⁶¹ See *Canones in Tabulas Alphonsinas* in Oxford, Bodleian Library, Digby 97, ff. 240^r–290^v; *Canones tabularum Alfonsi* in Erfurt, Universitätsbibliothek, CA 4° 366, ff. 70^r–73^r.

⁶² See Emmanuel Poulle, *Les tables alphonsines avec les canons de Jean de Saxe* (Paris: Editions du Centre National de la Recherche Scientifique, 1984), p. 70, lines 5–9; p. 74, lines 16–29.

⁶³ See Benjamin and Toomer, *Campanus of Novara and Medieval Planetary Theory*, pp. 206–11.

APPENDIX

The aim of this Appendix is to provide a comprehensive context for the references included in the article. The Appendix includes the text of Chapters 7, 8, 9, 18, 19, and 20, excerpted from the ongoing critical edition of the work we refer to as the *Arabes maxime*. The sole witness of the work we have identified so far, namely Paris, BnF lat. 7295A, mostly lacks chapter headings and chapter initials; in the edition, the latter have been silently integrated. Both the numbering and the headings of each chapter in this Appendix are not original to the text. In fact, in the manuscript the text has no numbering and includes only the title of one chapter out of forty-nine. The headings, which in our edition consist of the *incipit* of each chapter, as well as the numbering, are therefore merely an editorial device that allows the reader to navigate the text more easily.⁶⁴

The text in the manuscript copy presents a number of corruptions. In order to overcome this, we have had to make several conjectures, which are all detailed in the apparatus and, for diagrams, in the editorial principles. Some of these conjectures are informed by the sources of the text, while others are based on a comparison of the text to other similar texts from the same period. In some cases, the editors have relied on context to arrive at plausible conjectures. It is important for readers to be aware of the potential uncertainties and limitations involved in this critical editing process that is still ongoing. Therefore, we encourage readers to approach this edition with an attentive eye, carefully considering the evidence presented in the apparatus and in the following editorial principles.

Index of the chapters

The chapters edited in this Appendix are found in Paris, BnF lat. 7295A:

Ch.7: *Quamvis in hoc opera de practica*, f. 183^{ra}–183^{vb}

Ch.8: *Et quoniam in premissa demonstratione*, f. 183^{vb}

Ch.9: *Item in eadem demonstratione*, ff. 183^{vb}–184^{ra}

Ch.18: *Nota in tabulis tholetanis utimur*, f. 186^{ra}

Ch.19: *Ad inveniendum medios motus planetarum*, f. 186^{ra}–186^{rb}

Ch.20: *Solem equabis per omnem motum*, ff. 186^{rb}–187^{ra}

Index of the diagrams

The diagrams edited in this Appendix are found in Paris, BnF lat. 7295A:

⁶⁴ Exception made for Ch.34, bearing the rubricated heading ‘De Venere’; the remaining chapters do not include any headings; see f. 190^{va}. Chapters including chapter initials, are 6 and 34, 35, 36, 37, and 38. A few chapters (i.e. 29, 33, 40, 41) includes only guide letters.

D.1: Kardage, Ch.7, f. 183^{va}

D.2: Addition of a square, Ch.8, f. 183^{vb}

D.3: Subtraction of a square, Ch.9, f. 183^{vb}

D.4: Theory of the Sun, Ch.20, f. 186^{tb}

D.5: Theory of the Moon, Ch.20, f. 187^{ra}

1. Editorial principles

1.1 The text

In preparing this Appendix, we have attempted to reproduce the text in Paris, BnF lat. 7295A as faithfully as possible, retaining its spelling in almost all instances (e.g. *Yndia*, *dyametrum*, *seccantibus*). However, we have standardised the alterations of the consonant spelling of ‘ci/ti’ (e.g. *eciam*>*etiam*, *parcium*>*partium*, *gracia*>*gratia*, *solacii*>*solatii*...), as it does not seem possible to assess the prevalence of one spelling convention over the other throughout the text. In one case, *vasis*>*basis* (Ch.8, l. 75), we have retained the original reading, and we gave the more common spelling in the *apparatus* to avoid ambiguities. Some readings referring to proper names did not appear to us to be acceptable spelling variants. In these cases (e.g. *Auphontii* for *Alphontii* in Ch.19, l. 107; *Tremonia* and *Tremonie* for *Cremona* and *Cremona* in Ch.19, ll. 117 and 121), we have presented the readings in the edition as conjectures and offered the original spelling in the *apparatus*.

To mark syntactic units, we have applied a modern system of punctuation. In a few instances, we have altered the apparent divisions of the text, and the page changes are noted in the right margin and within the text by a vertical bar.

The critical text is accompanied by an *apparatus criticus* and an *apparatus fontium*: (1) the first apparatus records the conjectures, the scribal errors, as well as the corrections made by Heingarter to the text he wrote out, and a few necessary clarifications; (2) the second apparatus includes the sources of the text. Since the edition was conducted on the basis of a single manuscript, it was deemed appropriate to avoid a redundant and unnecessary repetition of its siglum in the apparatus.

In Chapters 7 and 8 on numerous occasions it was necessary to intervene to conjecture ‘square’ in the place of ‘quarter’ (i.e. *quadratum* and *quartum* variously declined). The abbreviation for indicating both lemmas is very similar: in fact the abbreviation for ‘quarter’ normally consists of the numeral 4 (ꝛ) plus the ending indicating the case at the top right of the numeral (e.g. ꝛ^m); for ‘square’ instead the abbreviation consists of the numeral 4, the letter ‘a’ surmounting the numeral and the ending at the top right (e.g. ꝛ^a^m). Since in both chapters both the word ‘square’ and the word ‘quarter’ appear, we have decided to conjecture and show in apparatus what is reported in the text, reproducing the abbreviation as faithfully as possible for the benefit of the reader.

1.2 The diagrams

The diagrams were redrawn with respect to their proportions as they were found in the manuscript. In some aspects the diagrams in the manuscript contradict what is stated in the text. For these aspects they have generally been edited with the aim to make them consistent with the indications about them given in the text (see details below). Although the manuscript diagrams were produced using a straight edge and compasses, their finish has not been done with the greatest of care. In those cases where convergence in a point or tangency of a circle is not realized in the manuscript but can be understood from the context, the redrawn diagrams have tacitly corrected those scribal imprecisions (i.e. non convergence, non-tangency).

In general, the lettering of the diagrams was taken as a reference to correct or conjecture the letters in the text passages referring to them. Substituted letters are given in italics. Reconstructed letters (that are not present in the witness) are given in bold.

For each diagram, specific interventions were made:

- D.1: Kardage, Ch.7, f. 183^{va}. The divisions of the circle were idealised to 15° sectors.
- D.2: Addition of a square, Ch.8, f. 183^{vb}. The hatching of one of the squares is in the manuscript.
- D.3: Subtraction of a square, Ch.9, f. 183^{vb}. Where the diagram was cut in the process of a rebinding, the missing lines and vertices, and corresponding letters, were reconstituted based on the context. The reconstituted line segments are dotted, and the inferred letters are in bold.
- D.4: Theory of the Sun, Ch.20, f. 186^{rb}. In two instances, the circles confining the orb of the Sun were said to be in red (*circuli rubri*). While in the manuscript the circles are drawn in the same ink, we restore the red colour in the redrawn diagrams. The letter at the bottom of the main circle, missing from the figure, was conjectured to be **t**. It was difficult to harmonize the text and diagram. This is the case with regard to the points **q** and **a** and also the points where the solar equations are maximum. In the absence of any easy way of achieving full consistency, we opted to preserve the occurrence of these points and letters as in the manuscript.⁶⁵
- D.5: Theory of the Moon, Ch.20, f. 187^{ra}. For the Moon the text and diagram could be harmonized more easily. A substantial change was made to the diagram regarding the position of two of the epicycles **sk** when they are at mean distance. A close reading of the text shows that their centres **m** and **r** need to be placed on the lunar deferent to show the equation of centre at quadrature (line 207^{ss}). The witness puts these circles at the distance of the opposition of the apogee where the

⁶⁵ On two occasions, the ambiguity resulting from the inconsistency of the use of **q** and **a** is clarified in the *apparatus criticus* (e.g.: q] *sc.* a). Indeed, twice in the text the letter **q** is found in the place a modern reader would expect to find the letter **a**. This may not have been a cause for concern for historical actors encountering the text; points labelled as **a** and **q** might have been considered somehow interchangeable due to their visual or even theoretical sense. Our approach to these two letters was to retain their occurrence as displayed in the manuscript witness, considering it unnecessary to add conjectures to the labels on the diagram.

equation of centre would be nil. The letters *q*, *f*, *g*, and *h* were placed in accordance with what is stated in the text, halfway between the top and the quadratures. In the diagram, the letter *s* occurs four times in a script that makes it look like a capital *B*. Therefore, the letters *bk* in the text (f. 186^{rb}, line 168) were conjectured as *sk* (see also the caption to this diagram in Section 1.2).

Some diagrams were placed in the edited text where it appeared most suitable for the current edition, their original placement being noted above.

List of abbreviations

ante corr. = *ante correctionem* = ‘before the copyist corrected’

post corr. = *post correctionem* = ‘after the copyist corrected’

cf. = *confer* = ‘compare/consider’

coni. = *coniecimus* = ‘we conjectured’

deest = *deest* (or *desunt*), ‘This [word or letter]/these [words or letters] are missing’; indicates that the evidence of the manuscript is unavailable for this letter/word/passage because the manuscript is damaged or its reading is no longer legible for some other reason(s)

del. = *delevit* = ‘the copyist deleted’

fors. = *forsan* = ‘perhaps’

i. m. = *in margine* = ‘in the margin’

lect. inc. = *lectio incerta* = ‘the reading is uncertain’

litt. = *littera-ae* = ‘letter or letters’

s. l. = *supra lineam* = ‘above the line’

sc. = *scilicet* = ‘clearly’

suppl. = *supplevit* = ‘the copyist supplied’

* Variation in the word order will be pointed out, where possible, by initials.

Angular brackets (<...>) enclose words etc. that we have added.

7. Quamvis in hoc opere de practica

- <Q>uamvis in hoc opere de practica principaliter cogitemus, si tamen 183ra
gratia solatii aliquid de pertinentibus ad theoricam inserimus, non debet
inconueniens reputari, nam intellectum delectat | quam plurimum 183rb
certitudo veritatis per demonstrationem accepta.
- 5 Igitur ad demonstrandum quantitatem sinus cuiuslibet portionis pono
figuram. Sit circulus super *o* centrum descriptus *az* et *gz* dyametris
super centrum se orthogonaliter seccantibus; *ag* quarta in sex partes
sive arcus equales diuisa que partes sunt kardage sicut patet. *Dsxl*
quadratum inscribatur, *ck* exagoni inscripti signatur. Item *er*, aliud latus
10 exagoni inscripti, licet non eisdem protrahitur adhuc *ls* dyametri.
Reliquis vero lineis quemadmodum apparet in figura protractis, arguo
sic: posita certa quantitate dyametri que in tabulis kardagarum nec non
in tabulis sinus et declinationis supponitur 300 partium, nota est linea
go, cum sit medietas dyametri et consequenter *ck* et *er* linee sunt note,
15 cum sint latera exagoni qui per penultimam 4ⁱ Euclidis inscriptus est.
Item *dl* que est latus quadrati inscripti, nota erit hoc modo. *Dol* angulus
est rectus, quia cadit in quartam circuli, ergo per penultimam primi
quadratum *dl* valet quadrata *dol*. Sed illa duo quadrata sunt equalia,
ergo si coniungantur, ut eis quadratum unum, extrahatur radix et exhibit
20 quantitas *dl* sive quantitas *ds* que eadem causa est.
Itaque preter dyametros et medietates earum notas habemus 4 lineas
notas que sunt *ck dl ds er* et per consequens medietates earum sunt note
que sunt *cm dn dm en*. Tunc arguo sic: quadratum *oc* valet quadrata *cm*
et *mo*; ergo si a quadrato *oc* subtrahatur quadratum *cm* remanebit
25 quadratum *mo* a quo extrahatur radix et exit quantitas *mo*. Et ita per

7 quarta] *coni.*; 8ⁿⁱ 8 *Dsxl*] *dlxk ante corr.* 10 *ls*] *coni.*; *lf* 14 *go*] *coni.*; *to* 16 quadrati] *coni.*; 8^{si} || nota] *coni.*;
non 18 quadratum] *coni.*; 8ⁿⁱ || quadrata₂] *coni.*; quarta 19 quadratum] *coni.*; 8^m 20 *ds*] *coni.*; *df* 23 quadratum]
quadr sed 8ⁿⁱ post corr. 25 quadratum] *coni.*; quartum || *mo*₁] *ms ante corr.*

15 per penultimam 4ⁱ Euclidis] EUCLIDES, *Elementa* IV.15, 156 17 per penultimam primi] EUCLIDES, *Elementa*
I.47, 93

Item patet ex predictis quod *dn* que est sinus rectus trium kardagarum est nota, ergo eius quadratum a quadrato *do* auferatur, remanebit
45 quadratum *no*, extrahatur igitur radix et exhibit quantitas *no*; erit ergo *an* nota que est sinus versus trium kardagarum. Item ex dictis est *mn* nota; ergo et *cy* ei equalis ratione eque distantie erit nota que est sinus versus 3^e kardage. Et cum nota sit *cm*, nota erit *yn* ratione eque distantie, ergo et est *dy* que est sinus rectus 3^e kardage.

50 Ex hiis que dicta sunt, manifesta est quantitas sinus tam recti quam versi cuiuslibet kardage et quarumlibet simul sumptarum, quia sinus rectus prime est sinus versus 6^e et sinus rectus 2^e est sinus versus 5^e. Item sinus rectus duarum scilicet prime et 2^e et sinus versus duarum scilicet 5^e et 6^e et versus illarum est rectus istarum. Item sinus rectus 3^e est sinus
55 versus 4^e, et versus 3^e est rectus quarte. Similiter sinus rectus trium primarum est sinus versus trium ultimarum et sinus versus illarum est rectus istarum. |

Et si in ultimis tribus demonstrare volueris sicut iam dictum est in tribus 183vb primis, eadem habebis argumenta operando in lineis ad dexteram
60 protractis cum dyametro *go* que superius habuisti in lineis aliis, quia isti sunt illis equales. Sed in hoc solo est differentia, quod hic sinum versum concludis ubi sinum rectum conclusistis ibi et econverso. Et propterea lineam *eg* ducere oportebit loco *ac* linee que in parte alia est.

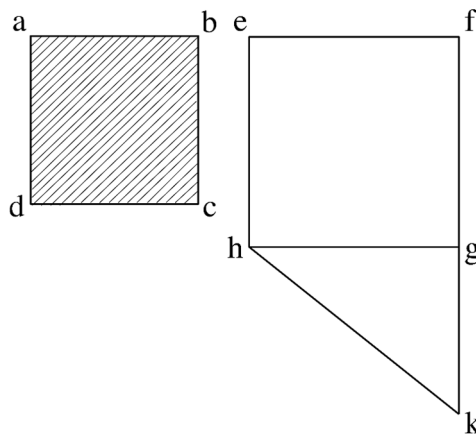
8. Et quoniam in premissa demonstratione

<E>t quoniam in premissa demonstratione alicui docuimus ut quadrata
65 iungerentur in unum, qualiter hoc secundum artem geometrie fieri potest, considera, si duo quadrata fuerint equalia, tunc quadretur dyameter et habetur propositum scilicet unum quadratum equipollens duobus.

44 quadratum] *coni.*; quartum || quadrato] *coni.*; 8^{to} 45 quadratum] *coni.*; quartum || quantitas no] no quantitas ante corr. 54 istarum] *primum scr.* Et si in ultimis tribus demonstrare volueris post istarum sed del. || est₂] *coni.*; et 64 quadrata] *coni.*; 8^a 66 quadrata] *coni.*; 8^{ta} 67 quadratum] *coni.*; quartum

Si vero inequalia, quemadmodum in presenti <figura> sunt descripta,
 70 scilicet quadratum *abcd* et quadratum *efgh* unum alteri coniungitur, hoc
 modo: per 2^{am} primi Euclidis a dato puncto *g* linee propositae scilicet *bcd*
 ducatur equalis et sic *gk*.

Item per 11^{am} eiusdem *hg* lineae ex puncto *g* trahatur perpendicularis; et
 erit *ad* presens linea eadem scilicet *gk*. Hoc facto subtendatur angulo
 75 *hgk* vasis quae est linea *hk*, quoniam *hgk* angulus est rectus. Patet <per>
 penultimam primi quod quadratum *hk* valet duo quadrata proposita.
 Quadretur ergo *hk* et habebitur propositum.



9. Item in eadem demonstratione

<I>tem in eadem demonstratione, quoniam diximus quadratum a
 quadrato esse tollendum, quod qualiter per geometriam fieri debeat,
 80 patet sic. Sit quadratum *hklm* equale duobus quadratis superscriptis
 quemadmodum ostensum est supra.

Abcd quadratum subtrahitur ab eo hoc modo: per 3^{am} primi Euclidis ex
 linea *hm* abscindatur linea *ab* et lineae equalis sit *nm*. Item ex linea *ml*
 abscindatur alia eidem *nm* equalis et sit *mo*.

85 Item per 31^{am} ex puncto *n* trahatur eque distans ipsi *hk* et sit linea *np*.
 Item ex puncto *o* per eandem 31^{am} ducatur eque distans ipsi *kl* et sit

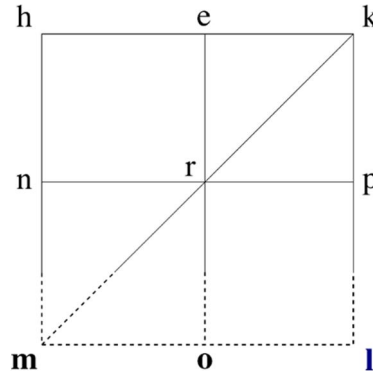
75 vasis] sc. basis

71 per 2^{am} primi Euclidis] EUCLIDES, *Elementa* I.2, 60 73 per 11^{am} eiusdem] EUCLIDES, *Elementa* I.11, 67
 82 per 3^{am} primi Euclidis] EUCLIDES, *Elementa* I.3, 61 85 per 31^{am}] EUCLIDES, *Elementa* I.31, 80 86 per eandem
 31^{am}] EUCLIDES, *Elementa* I.31, 80

linea *oe*. Ad hoc per primam petitionem Euclidis trahatur *mk* |
 dyagonaliter que transit per punctum *r*.

184ra

90 Tunc argumentatur sic: cum *nrom* quadratum ratione compositionis
 figure sit equale *abcd* quadrato et ipsum totale quadratum *hklm* ex
 ypotesi sit equale duobus quadratis supra descriptis, patet quod gnomo
 residuus erit equalis *efgh* et sic est propositum.



18. Nota in tabulis Tholetanis utimur

<N>ota: in tabulis Tholetanis utimur annis Arabum et ponuntur ibi |
 radices ere Arabum. Et sunt partes signorum sive loca in quibus planete
 fuerint in inceptione ere Arabum et <computata> sunt <ad>
 95 meridianum Tholeti cuius longitudo est 28 graduum et dimidii ab
 occidente vel medio mundi, 4 horarum et 10^e unius hore, hoc est 61
 graduum et 30 minutorum, hora enim est revolutio 15 graduum; dicitur
 autem medii mundi locus in Yndia magna, in civitate Arim, sub
 100 equinoxiali circulo. Dico autem orientem initium locorum habitabilium
 versus orientem et sic suo modo versus occidentem. Spacium autem
 terre habitabilis inter hos terminos secundum Alfraganum est tantum
 180 graduum hoc est quantum revoluitur aliquis punctus celi in 12 horis
 ab oriente in occidentem; latitudo vero Tholeti est distantia eius ab
 105 equinoxiali versus septentrionem 39 graduum et 54 minutorum.

186ra

87 oe] *coni.*; ne 89 nrom] *norom ante corr.* 93 utimur] *iti ante corr.* 103 180] *coni.*; 12 105 54] *coni.*; 4

87 per primam petitionem Euclidis] EUCLIDES, *Elementa* I. pet. 1, 58 96 Tholeti . . . 100 equinoxiali circulo] cf. *Canones Azarchelis*, Cb133, 430 100 Dico autem . . . 104 occidentem] cf. ALFARGANUS, *Differentie* 6.3, 9 104 latitudo vero . . . 105 54 minutorum] cf. *Canones Azarchelis*, Cb133, 430

19. Ad inveniendum medios motus planetarum

<A>d inveniendum medios motus planetarum in tabulis Tholetanis eadem est operatio que in tabulis Alphontii nisi quod in tabulis Tholetanis intramus cum annis a primo collectis, postea expansis, postea mensibus perfectis, postea cum horis, si volumus ultra meridiem
110 habere motum. Si autem volueris habere medios motus ad quamcumque aliam civitatem a Tholeto, considera in tabula per quot gradus longitudinis distet ista civitas a Tholeto et 15 gradibus horam constitues. Vide quot horas colligere possis et fractiones horarum non negligas igitur invenias medium cursum in tot horis cum fractionibus
115 suis, et adde eum super medium cursum quem invenisti ad Tholetum si civitas ista fuerit occidentalis. Si vero fuerit orientalis subtrahatur et habebis quod queris. Cremona distat a Tholeto versus orientem 20 gradibus, | hoc est per unam horam et tertiam partem hore. Invento 186rb igitur medio motu super Tholetum si ab eo subtraxeris quantum
120 movetur planeta in una hora et 20 minutis hore, remanebit cursus eius ad medium diem Cremone.

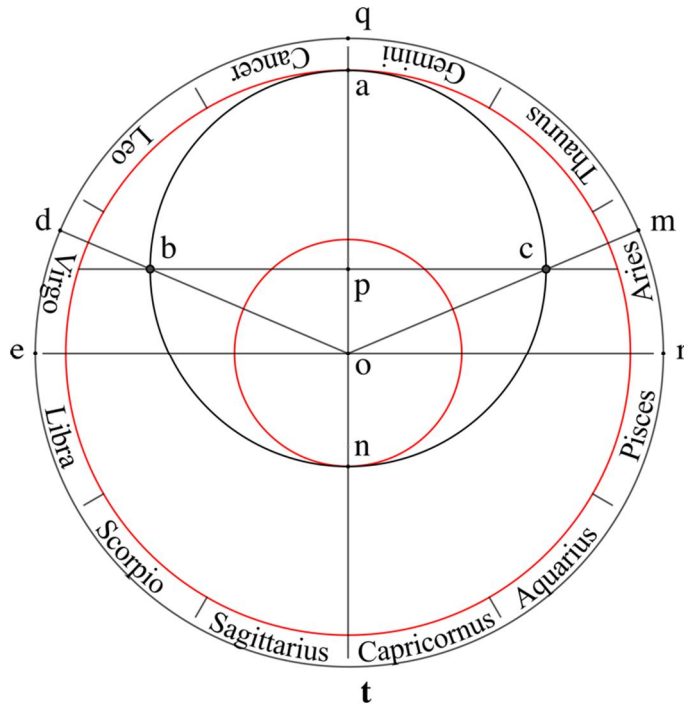
20. Solem equabis per omnem motum sicut in Alphontio

<S>olem equabis per omnem motum sicut in Alphontio docetur. Si autem partem proportionalem, sic facies: pone 60 primum in proportione, minuta argumenti secundum, differentiam totam tertium,
125 pars que queritur erit quartum. Itaque multiplica minuta argumenti per totam differentiam et quod exierit, divide per 60, quod tunc exierit erit pars quesita, quam adde prime equationi vel subtrahe secundum quod oportet. Habito ergo vero loco solis, computa eum a principio arietis

107 Alphontii] *coni.*; Auphontii 117 Cremona] *coni.*; Tremonia 121 Cremone] *coni.*; tremonie

110 Si autem . . . 113 constitues] cf. *Canones Azarchelis*, Cb140, 434 115 et adde . . . 117 queris] cf. *Canones Azarchelis*, Cb140, 434 117 Cremona . . . 118 partem hore] cf. *Canones Azarchelis*, Cb140, 434 118 Invento igitur . . . 121 Cremone] cf. *Canones Azarchelis*, Cb140, 434 127 quam adde . . . 130 circularum] cf. *Canones Azarchelis*, Cb142, 436

130 mobilis, cui certo loco superadde motum 8^{ve} spere equatum, et habebis
 locum eius certissime computando a coniunctione circulorum.



Pro maiori evidentia pono figuram: sit *o* centrum terre, *p* centrum
 ecentrici, et *a* erit punctus augis et sit *r* caput arietis, sol vero sit apud *b*
 item posterior sol sit in puncto *c* et circuli rubei comprehendant inter se
 spissitudinem spere.

135 Et dicitur exterior circulus augis scilicet puncti *q*, interior vero circulus
 sit oppositus augis, scilicet puncti *n*. Licet itaque sol in eodem percurrat
 zodiacum in quo pertransit ecentricum, non tamen est hec computatio
 in partibus circulorum proportionalibus, nisi tantum in duabus
 medietatibus; hec in puncto augis et eius opposito distinguuntur,
 140 quoniam in superiori medietate ecentrici que est *cqb* plus movetur in
 ecentrico quam in zodiaco quod patet quia medietas illa non
 correspondet medietati zodiaci proportionali ymmo parti minori. In
 inferiori vero medietate ecentrici que est *bnc* plus movetur in zodiaco

130 locum] *coni.*; locus 133 circuli rubei] *circuli non rubricati in figura* 135 q] *sc. a* 136 oppositus] *coni.*;
 oppositum 139 distinguuntur] *coni.*; *fors. distiguuntur* 140 *cqb*] *sc. cab* 143 *bnc*] *coni.*; *bvc*

quam in ecentrico. Quod patet quia medietas illa non precise
 145 correspondet medietati zodiaci ymmo arcui maiori.|
 Hinc accidit quod in ista medietate ecentrici que est ex *q* per *b* in *n* maior 186va
 est medius motus quam verus. Verbi gratia: Sole existente ut positum
 est in *b*, medius motus est arcus *rqe*, sed verus motus est arcus *rqd*,
 arcus equationis interiacens est *de*. Igitur, si a medio motu subtrahatur
 150 aux id est arcus *rq*, remanet argumentum scilicet arcus *qe* minor 6
 signis. Ideoque equationis arcus a medio motu substrahitur et remanet
 arcus *rqd* qui est motus solis equatus et locus solis verus est punctus *d*.
 Econverso accidit in alia medietate que est ex *nt* per *cm* <in> *aq* quod
 maior est verus motus quam medius. Verbi gratia: sole ut positum est
 155 posterius existente in puncto *c* medius locus solis est *r* sed verus est *m*.
 Igitur cum argumentum sit plus 6 signis arcus equationis qui est *rm*
 additur medio motui et habetur motus equatus. Et sic patet istius ratio
 operationis. Exemplum autem posuimus in locis ubi maxime est
 diversitas inter medium locum et verum, quod patet per figuram
 160 precedentem.
 Si vis equare lunam, fac totaliter sicut in Alphontio in capiendo partem
 proportionalem de diversitate dyametri: pone 60 pro primo numero,
 minuta proportionalia pro secundo, diversitatem dyametri totam pro
 tertio; multiplica igitur secundum per tertium vel e converso et divide
 165 per primum et habebis quartum.
 Pro maiori evidetia ponam figuram lune. Sit circulus signorum *abcd*
 super centrum *o* quod centrum etiam est centrum spere lune, cuius
 spissitudo *s* est inclusa duobus circulis *sc zk*. Descriptus circulus *ac*
 ostendit exteriorem superficiem spere lunaris, circulus vero *zk*
 170 interiorem superficiem.
 Deferens lune sit *lmnr* super centrum *p*; igitur *sk* dyametrum
 epicycli semper respicit punctum *e* in omni situ. Quia sicut *l* et *p*
 moventur ad occidentem versus *d*, ita centrum epicycli, quod

146 q] sc. a 153 nt] con.; post n deest 1 litt. || cm] primum scr. qsq post cm sed del. || aq] con.; qs
 155 r] con.; f 156 rm] con.; fm 157 ratio operationis] con.; rationis operatio 168 est] con.; et || zk] con.; et
 k 169 zk] et k 171 sk] con.; bk

etiam significatur per *l*, movetur ad orientem versus *b*, et ei
 175 occurrit punctum *e*. Posito ergo epicycli centro in auge ecentrici apud *l*,
 sol secundum medium cursum erit apud *a* vel apud *c*, quod in idem redit
 ad presens. Quia in coniunctione et oppositione media centrum epicycli
 est apud *l*, hoc est in auge ecentrici, in tali enim situ equationes centri
 non sunt, quia centrum nullum | est. Equationes vero minime in 186vb
 180 situ autem opposito scilicet, cum fuerit centrum epicycli in opposito
 augis in puncto *n*, quod quidem realiter contingit primo sub *b* deinde
 sub *d*. Quoniam cum luna distat a sole per 3 vel 9 signa, hoc est
 in quadraturis, quia tunc centrum epicycli est in opposito augis,
 centrum erit 6 signa et equationes eius nulle, quoniam eadem est linea
 185 ostendens medium locum et verum epicycli; equationes vero argumenti
 erunt maxime. Et hoc patet cum enim luna fuerit in puncto, ubi *oc*
 contingit epicyclum, tunc argumenti eius equatio est maxima sicut
 apparet ad sensum, scilicet *a a a* sunt minores *c c c*, qui tunc arcus
 argumenti equationis vocantur. Hec autem ostensio maioritatis
 190 equationis argumenti ex necessitate descriptionis ostensa est sub *c*, cum
 tamen, sicut dictum est, realiter contingat sub *b* vel *d*, quod est
 propter revolutionem ecentrici. In situ autem intermedio, scilicet
 cum centrum epicycli super 3 vel 9 signa ab auge, quod realiter
 contingit primo in *g* sub *h* deinde in *f* sub *q*, scilicet quando luna distat
 195 a sole 45 gradibus, ut sub *h*, vel 10 signis et 15 gradibus, ut sub *q*, tunc
 sunt equationes centri maxime.
 Equationes vero argumenti medio modo se habent, quia sunt maiores
 quam in auge, minores vero quam in opposito augis. Et hec ostensio ex
 necessitate descriptionis facta est sub *b* et *d*, licet ut dictum est, realiter
 200 sic accidat sub *h* et *q*. Posito itaque epicyclo in *m* sub *b* vel in *r* sub *d*,
 locus ipsius epicycli medius erit punctum *i*, quem ostendit linea *ei* vel
 punctum *v* quem ostendit linea *ev*; locus autem eius verus erit *b* quem
 ostendit linea *omb* vel *d* quem ostendit linea *ord*. Centrum autem eius
 medium est ex puncto *a* usque in *i* vel ultra per *c* usque in *v*. Centrum

175 epicycli] epicyclo ante corr. 194 in f] con.; infra 195 15] con.; 12 201 erit] con.; et 204 puncto] con.;
 p^t || v] con.; r

205 autem equatum sive verum est ex eodem puncto *a* usque in *b* super *m*
vel ultra per *c* usque in *d* super *r*.

Equatio centri in epicyclo, que etiam equat argumentum, est arcus *sy*.
Equatio vero centri in zodiaco est arcus *ib* vel *vd* predicto arcui
proportionalis, quia sicut se habet *sy* ad totum epicyclum, ita se habet
210 *ib* vel *vd* ad totum zodiacum. Et enim in eundem angulum cadunt, ut
patet hinc est quod harum duarum equationum centri accepta una
habetur reliqua. Item aux media epicycli est *s*, aux vera est *y* et si corpus
lune fuerit in *x*, argumentum eius medium erit arcus *sx* et hoc sub *b* vel
in alio epicyclo | *syx* et hoc erit sub *d*.

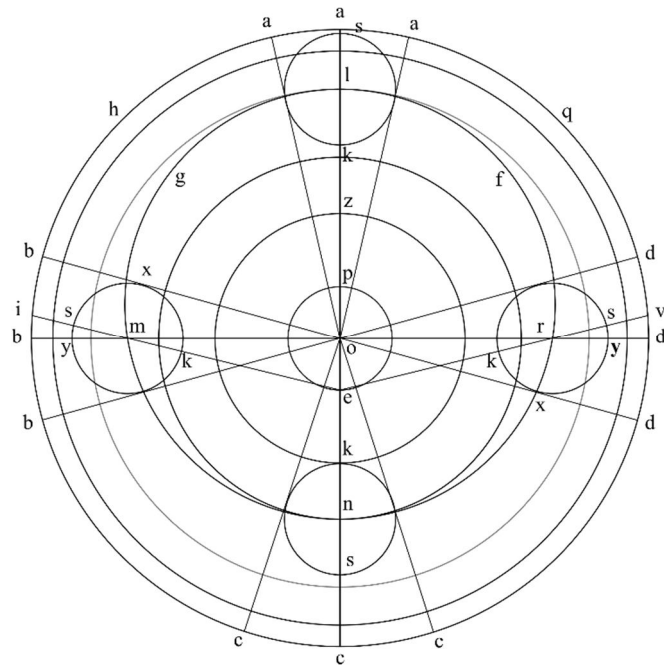
187ra

215 Argumentum autem equatum est arcus *ysx* sub *b* vel *yx* sub *d*; patet
igitur quod equatio argumenti est arcus *bb* vel arcus *dd*. Et hec equatio
equat motum lune. Hiis omnibus itaque consideratis manifestum est
quare equatio centri additur argumento si centrum est minus 6 signis ut
ys arcus sub *b*, et subtrahitur si est plus ut *ys* arcus sub *d*. Item patet
220 quare equatio argumenti zodiaci additur medio motui, cum ipsum
argumentum fuerit plus 6 signis, verbi gratia: si esset luna in puncto
contactus *pk* et subtrahitur ab eodem motu, si dictum argumentum
fuerit minus 6 signis, verbi gratia: si luna esset apud *x*. Hoc autem quod
ex diversitate dyametri additur equationi argumenti est plus vel minus
225 secundum quod epicyclus ab auge versus oppositum augis, plus vel
minus recedit. Semicirculus autem ecentrici super quem centrum
epicycli in isto recessu defertur, potest intelligi quasi per 60 partes
equales divisus, quarum partium numerus minuta proportionalia
dicuntur, sicut igitur partes pertransite se habuerint ad 60, sic de
230 diversitate dyametri additur equationi argumenti.

Habito igitur motu lune equato debes eum computare, sicut dixi tibi in
sole a capite arietis mobilis quod est in octava spera. Si autem eum
habere volueris a capite arietis fixi, scilicet ab ista intersectione
circulorum ubi caput arietis octave spera supponitur capiti arietis none
235 spera, tunc adde super motum lune equatum motum octave spera

205 b] *coni.*; k 207 in] *suppl. s. l.* 208 est] *coni.*; et 213 x] *coni.*; v 215 ysx] *coni.*; yxs 223 quod] *coni.*;
quidem 224 dyametri] *coni.*; differentiam

equatum et quod pervenerit computa a tali loco, sicut dixi. Et hec omnia patent in figura.



List of Manuscripts

Erfurt, Universitätsbibliothek, CA 4° 366

Oxford, Bodleian Library, Digby 97

Oxford, Hertford College, 4

Paris, Bibliothèque nationale de France, lat. 11232

Paris, Bibliothèque nationale de France, lat. 7197

Paris, Bibliothèque nationale de France, lat. 7273

Paris, Bibliothèque nationale de France, lat. 7295A

Paris, Bibliothèque nationale de France, lat. 7305

Paris, Bibliothèque nationale de France, lat. 7314

Paris, Bibliothèque nationale de France, lat. 7333

Paris, Bibliothèque nationale de France, lat. 7432

Paris, Bibliothèque nationale de France, lat. 7446

Paris, Bibliothèque nationale de France, lat. 7447

Paris, Bibliothèque nationale de France, lat. 7450

Toledo, Archivo y Biblioteca Capitulares (olim Biblioteca del Cabildo) 104 6

Vienna, Österreichische Nationalbibliothek, Cod. 5268

Bibliography

Almagesti minor, ed. by Henry Zepeda, *The First Latin Treatise on Ptolemy's Astronomy: The Almagesti minor (c. 1200)* (Turnhout: Brepols, 2018).

Canones Azarchelis, ed. and trans. by Fritz S. Pedersen, 'Part 2 Cb: Canons "Quoniam cuiusque actionis" ', in *The Toledan Tables. A Review of the Manuscripts and the Textual Versions with an Edition*, ed. by Fritz S. Pedersen (Copenhagen: C.A. Reitzels Forlag, 2002), pp. 382–499.

Theorica planetarum Gerardi, ed. by Francis J. Carmody (Berkeley: University of California, 1942).

Al-Farghānī, *Differentie in quibusdam collectis scientie astrorum*, ed. by Francis J. Carmody (Berkeley: 1943).

Aquinas, Thomas, *Commentary on Aristotle, On Memory and Recollection*, trans. by John Burchill, in *The Medieval Craft of Memory: An Anthology of Texts and Pictures*, ed. by Mary Carruthers and Jan M. Ziolkowski (Philadelphia: University of Pennsylvania Press, 2002), pp. 153–88.

- Berschin, Walter, 'Greek Script in Latin Manuscripts', in *The Oxford Handbook of Latin Palaeography*, ed. by Frank T. Coulson and Robert G. Babcock (New York: Oxford University Press, 2020), pp. 115–17.
- Burnett, Charles, 'The Latin and Arabic Influences on the Vocabulary Concerning Demonstrative Argument in the Versions of Euclid's *Elements* Associated with Adelard of Bath', in *Aux origines du lexique philosophique européen: L'influence de la 'latinitas'*, ed. by Jacqueline Hamesse (Louvain-la-Neuve : Société internationale pour l'étude de philosophie médiévale, 1997), pp. 117–35.
- Campanus de Novara, *Theorica planetarum*, ed. and transl. by Francis S. Benjamin Jr. and Gerald J. Toomer, *Campanus of Novara and Medieval Planetary Theory: Theorica planetarum* (Madison: University of Wisconsin Press, 1971).
- Chabás, José and Bernard R. Goldstein, *The Alfonsine Tables of Toledo* (Dordrecht: Kluwer, 2003).
- Chemla, Karine ed., *The History of Mathematical Proof in Ancient Traditions* (Cambridge: Cambridge University Press, 2012),
- Crowther, Kathleen M. and Peter Barker, 'Training the Intelligent Eye: Understanding Illustrations in Early Modern Astronomy Texts', *Isis* 104 (2013), 429–70.
- Dinkova-Bruun, Greti, 'Text and Gloss', in *The Oxford Handbook of Latin Palaeography*, ed. by Frank T. Coulson and Robert G. Babcock (New York: Oxford University Press, 2020), pp. 924–42.
- Euclides, *Elementa*, ed. by Hubertus L. L. Busard, *Campanus of Novara and Euclid's Elements* (Stuttgart: Franz Steiner, 2005).
- Goldstein, Bernard R, *Ibn al-Muthannâ's Commentary on the Astronomical Tables of al-Khwârizmî* (New Haven: Yale University Press, 1967).
- Husson, Matthieu, 'Mathematical Astronomy and the Production of Multiple-Texts Manuscripts in Late Medieval Europe: A Comparison of BnF lat. 7197 and BnF lat. 7432', in *The Emergence of Multiple-Text Manuscripts*, ed. by Alessandro Bausi, Michael Friedrich and Marilena Maniaci (Berlin: De Gruyter, 2019), pp. 247–74. <<https://doi.org/10.1515/9783110645989>> [accessed: 14.04.2023]
- , 'The Art of Astrological Computations: Conrad Heingarter and the Manuscript Paris, BnF latin 7295A', in *Exploring Written Artefacts: Objects, Methods, and Concepts*, ed. by Jörg B.

Quenzer (Berlin: De Gruyter, 2021), pp. 513–32. <<https://doi.org/10.1515/9783110753301>> [accessed: 14.04.2023]

Johannes von Gmunden, *De sinibus, chordis et arcubus*, ed. by Hubertus L. L. Busard, *Der Traktat De sinibus, chordis et arcubus von Johannes von Gmunden* (Vienna: Springer Verlag, 1971).

John D. North, *Richard of Wallingford. An edition of his writings with Introduction, English Translation, and Commentary* (Oxford: Oxford University Press, 1976).

Martinelli Tempesta, Stefano and Marco Petoletti, ‘Il ritratto di Omero e la firma greca di Boccaccio’, *Italia Medioevale e Umanistica* 54 (2013), 399–409.

Molland, Andrew G, ‘Mathematics in the Thought of Albertus Magnus’, in *Albertus Magnus and the Sciences: Commemorative Essays*, ed. by James A. Weisheipl (Toronto: Pontifical Institute of Mediaeval Studies, 1980), pp. 463–78.

Pedersen, Fritz S., *The Toledan Tables. A Review of the Manuscripts and the Textual Versions with an Edition*, 4 vols (Copenhagen: C.A. Reitzels Forlag, 2002).

Pouille, Emmanuel, *Les Tables Alphonsines avec les Canons de Jean de Saxe* (Paris: Editions du Centre National de la Recherche Scientifique, 1984).

Rioust, Laure, ‘Latin 7295A’, <<https://archivesetmanuscrits.bnf.fr/ark:/12148/cc664943>> [accessed: 14.04.2023].

Summers, David, *The Judgment of Sense: Renaissance Naturalism and the Rise of Aesthetics* (Cambridge: Cambridge University Press, 1987).

Steinschneider, Moritz, ‘Études sur Zarkali, astronome arabe du XI^e siècle, et ses ouvrages’, offprint from *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche* (ed. Boncompagni), 14, 16, 17, 18, 20 (1881–87).

Sylla, Edith, ‘Mathematical Physics and Imagination in the Work of the Oxford Calculators: Roger Swineshead’s *On Natural Motions*’, in *Mathematics and Its Applications to Science and Natural Philosophy in the Middle Ages: Essays in Honour of Marshall Clagett*, ed. by Edward Grant and John E. Murdoch (Cambridge: Cambridge University Press, 1987), pp. 69–101.

Thorndike, Lynn and Pearl Kibre, *A Catalogue of Incipits of Mediaeval Scientific Writings in Latin*, rev. ed. (London: Mediaeval Academy of America, 1963).

Van Brummelen, Glenn, *The Mathematics of the Heavens and the Earth* (Princeton: Princeton University Press, 2009).

Yates, Francis, *The Art of Memory* (Chicago: University of Chicago Press, 1966).