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Integrated tactical continuous production planning of two blending systems

sharing supply sources: the case of a phosphate supply chain

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Abstract. This paper deals with a problem of tactical planning of production activities. It considers the case of the world's largest phosphate production site, with several mines, several washing plants, and a drying plant, with transport of merchantable phosphate ores (MOs) by rail or pipeline. The tightening of product quality requirements by customers prevents from continuing to plan its production with the current rules. Currently, the plants are managed quite independently, source ores (SOs) are pre-assigned to the MOs for blending, and the MO production routings in the washing plants are predetermined. We propose a model that ensures flexibility in the production of MOs through the choice of dynamic blends and routings. Our approach goes beyond any pre-allocation of resources and aims at synchronising the flows of the washing plants. This specific problem of blending production and planning has not been addressed in the literature. Still, it is widely encountered in many mining industries and offers perspectives for the application of the approach proposed. Numerous tests show that this model ensures the quality constraints related to the composition of several chemical components, with a generalised use of the most economical routings while ensuring horizontal and vertical alignment of decisions.

Keywords: tactical planning, interdependent production systems, blending, alternative routings, alignment.

1 Introduction

As a result of current global developments (demographic, economic, environmental), the mining and primary processing industries are facing new challenges such as the scarcity and heterogeneity of raw materials, rising energy costs, increased customer diversity and requirements, etc. These issues have been the subject of a great deal of research based on the famous linear model of blending (Dantzig, 1982), followed by extensions of this model and the development of suitable algorithms. However, it has received little attention and application in

the specific case of the phosphate mining industry, which is the main supplier of raw materials to produce fertilisers and phosphoric acid and which have a major position in the agriculture and food industries.

These industries have a number of specific features: (i) they generally integrate several elements of the supply chain (extraction, homogenisation process, chemistry), (ii) they combine heterogeneous raw natural resources to obtain homogenous market products that meet precise and specific specifications, (iii) raw materials are supplied in push flow depending on the extraction's program, (iv) the homogenisation process often consists of several parallel treatment lines, (v) chemical plants (phosphoric acid) must be supplied continuously to avoid any defusing. For all these reasons, the traditional planning methods used in manufacturing supply chains are unsuitable.

This paper focuses on the largest phosphate rock production supply chain (SC) of OCP Group, which provides 70% of the group's total production. In 2022, OCP's revenue will be US\$ 11.5 billion, with a Morocco Gross National Product of US\$ 133 billion making OCP a worldwide leader in phosphate and its derivatives.

This SC is faced with three sets of demands linked to the type of recipient and the method of shipment: (1) internal demand to the chemical complex delivered by pipeline, (2) external demand to the port delivered by the same pipeline and (3) external demand to the port delivered by train. Each demand (Merchantable ores, **MO**) must be fulfilled in quantity, delivery date and quality requirements outlined in a quality charter (**QC**). A MO is obtained by blending different raw materials (Source Ore, **SO**) to which it is possible to apply a washing treatment that will enrich it in phosphate (three different routings coexist).

Currently, the SC focuses on a unique chemical element for piloting flow (Bone Phosphate Lime – BPL, characterising the richness of phosphate in P_2O_5) whereas the QC of each MO requires limits for other elements (Cadmium, etc.). It uses almost fixed blends to prepare the final products, although the order specifications and SO availability and chemical quality may evolve over time. The current planning organisation therefore raises a number of issues: (1)

how to ensure a finished product that satisfies QC for all components, (2) how to make better use of available SOs, and (3) how to improve the effectiveness and efficiency of SC.

To address these challenging issues, we propose a flexible and dynamic approach for piloting flow for this phosphate SC. This approach is twofold. First, we propose a new planning organisation of production in washing plants to manage the limited capacities, the parallel washing operations and the available SOs. Second, we develop an optimisation model (MILP class) to generate dynamic blends that fulfil a tactical orders book.

This paper is then the result of a collaborative research (Shani et al., 2008; van de Ven et Johnson, 2006). The aim of such research is not to consider previously identified theoretical hypotheses, but rather to explore the field to dissect the problem and identify avenues for further research and reflection. The suggested approach could be inspiring and provide valuable insights for several other mining industries adopting similar production processes.

This paper is organised as follows: Section 2 presents a literature review on planning production activities in the process industries and aligning decision. Section 3 devoted to the formulation and explanation of the mathematical model. Numerical studies and analyses are proposed in Section 4. The final section draws some managerial insights before concluding with some perspectives.

2 Literature review

As the paper deals with a processing and blending problem in the phosphate industry, we have focused the literature review on blending issues, combined with treatment/processing in the mining industry, to address the gap. We have also reviewed the concept of alignment, which deals with the coherence between decisions. In most cases, these coherence problems caused by the use of models designed to solve a particular problem are ignored by the authors of these models, implicitly assuming that the constraints induced by their solution must be taken into account by the other decision entities (which is sometimes relevant). We decided to include this issue in the analysis of the consequences of the proposed model.

2.1 *Blending production in the mining sector*

We focus on the blending problem in the mining industry in a deterministic environment, which has three specific characteristics: *i)* the inputs are extracted and not purchased. Their characteristics depend mainly on nature, which implies a relatively high instability of the characteristics of the available inputs that need to be handled through blending and processing to satisfy customer needs; this makes the problem dynamic and impacts the optimal solution more than the cost; *ii)* the blending output is a semi-raw material used later by the industry for fertilizers and phosphoric acid production; *iii)* the blending process may involve the use of alternative routings and possibly several processors that may work in series or in parallel, as we will explain later.

The articles analysed are selected based on two keywords: 'Blend/blending' and the name of a specific ore or metal such as 'iron', 'coal', 'phosphate', 'gold', *etc.*, and the analysis of the abstract. Some articles were general and did not consider the case of a specific ore/metal.

Table 1 shows the analysis grid resulting from the analysis of 15 articles dealing with blending problems in a deterministic mining environment. This analysis was carried out on the basis of several criteria. We have selected the most relevant ones, mainly five (type of ore, type of production, mathematical formulation, processing and blending process, and objectives), to compare the main characteristics of our paper with those of the analysed articles:

- **Type of ore:** the blending problem was addressed in several mining industries like iron, phosphate, copper, and coal.
- **Mathematical formulation:** Linear programming, mixed integer linear programming, non-linear programming, mixed integer non-linear programming.
- **Production's type:** Outputs made to stock or on demand.
- **Processing and/or blending:** In mining industries, the production process is composed of at most two main operations which are processing and/or blending. Different production configurations (illustrated by Figure) are possible: the first one

(configuration A) is where only the blending of SO is realised to satisfy customer requirements, and the second (configuration B) consists of blending SO and then processing the blend (washing being not the only type of treatment available) with a specific routing for enrichment. For the processing stage of this configuration, it is possible that the main one is composed of several sequential sub-processing units which justifies the case of some articles mentioned in the grid with sequential process and configuration B. The third one (configuration C) aims to treat SO and then blend the necessary inputs (enriched products and/or SO) to satisfy customer requirements. Configuration C is based on the parallel use of several ore processing lines during the same production period before blending to obtain the desired output. Other configurations using multiple processing lines at different times are possible but raise the complex problem of managing intermediate stocks combined with a scheduling problem.

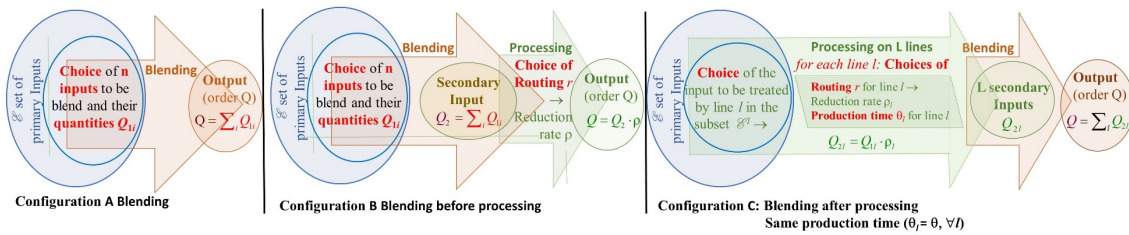


Figure 1 Caption. The different production configurations in mining industries

Figure 1 Alt text. The scheme represents three main production configurations in mining industries, configuration A concerns blending only, configuration B is related to blending raw materials before processing, and configuration C aims to blend after processing raw materials.

Recently papers were focused on phosphate blending problems and considered the case study of OCP group in Morocco (Azzamouri *et al.*, 2021; Hilali *et al.*, 2022). Those papers are kind of complementary to each other and different from ours regarding several aspects. They are adopting configuration B with a unique production system, contrary to our paper which concerns configuration C which is a bi-production system. The rest of the features are presented in the grid (Table 1).

- **Objective function:** economic (maximisation of profit or minimisation of costs), environmental, requirements targets.

Table 1 Caption. Synthetic grid for the analysis of blending literature

Table 1 Alt text. The table represents the classification of the analysed articles treating the blending concept, depending on several criteria

Numbers of articles	Author(s)	Years	Type of metal/ ore	Production's type		Processing and/or blending			Mathematical formulation				Objectives		
				For stock	On demand	Production configuration		Multi-processors	Linear programming	Non Linear Programming	Mixed integer non-linear program	Mixed integer linear program	Economic	Requirements satisfaction (volume, quality, ..)	Environmental
						Single processor	Sequencial processors								
1	Williams and Haley	1959	C		X	B		X	X				X		
2	Sherali and Puri	1993	C		X	C		X	X				X	X	
3	Chanda and Dagdelen	1995	C		X	A	X		X				X	X	
4	Shih and Frey	1995	C	X		A	X		X				X		
5	Pendharkar and Rodger	2000	C		X	A		X	X				X		
6	Kumral	2003	Fe			A	X			X			X		
7	Gholamnejad and Kasmaee	2012	Fe			A	X		X				X		
8	Liu and Li	2013	Fe		X	A	X		X				X		
9	Siemon et al	2019	Cu		X	B		X				X	X		
10	Yuan et al	2020	C	X		B		X		X			X		
11	Haonan et al	2020	-	X		A		X			X			X	
12	Azzamouri et al	2021	PO ₄ ³⁻		X	B	X					X	X		
13	Yan et al	2022	C		X	B		X	X				X		X
14	Hilali et al	2022	PO ₄ ³⁻		X	B	X					X	X		
15	Ahmad et al	2023	C		X	C		X			X		X		
16	US	2023	PO ₄ ³⁻		X	C		X				X	X		
Types of ores: PO ₄ ³⁻ -Phosphate, Fe-Iron, C-Coal, Cu-Copper															
Production configurations: A-blending, B-blending before treatment, C-blending after treatment (Figure S-Appendix)															

2.2 Alignment concept

The organisational alignment concept was defined several decades ago (Venkatraman and Camillus, 1984; Venkatraman, 1989; Tan and Tan, 2005). It is linked by some researchers with the concept of "congruence" and defined as it (Lawrence and Lorsch, 1967). Wong *et al.* (2012) have defined alignment as the 'fit' in terms of objectives, structures, and processes within and between the supply chain's various functions and production units. According to Porter (1996),

a company's ability to maintain a competitive advantage is based on its ability to acquire and deploy resources consistently and aligned with the competitive need of the organisation.

We distinguish two dimensions of alignment in a supply chain: vertical and horizontal.

- According to Kathuria *et al.* (2007), **vertical alignment** consists in developing and elaborating action plans, strategies, and decisions at all levels of the organisation's decision-making. For Bhatnagar *et al.* (1993), effective strategies leading to good performance focus on how decision-making at different organisational levels is coordinated (vertical alignment) so that operational policies are optimal for the organisation as a whole. According to Ivanov (2010), the vertical alignment of the three levels of decision is crucial, as it leads to feasible and balanced production planning and provides feedback for the audit and the adaptation of these plans to the actual performance and capacity of the processes of the supply chain. Kathuria *et al.* (2007) point out that vertical fit/alignment requires a shared understanding of the objectives of the organisation by different managers at different decision-making levels and at different units of the organisation hierarchy.
- Furthermore, **horizontal alignment** is related to the coherence of decisions taken between the different production units of the supply chain for each decision level. Kathuria *et al.* (2007) defined it as the coordination of efforts in the organisation, which is relevant in the lower levels of the strategy hierarchy. They defined it depending on the cross-functional and intra-functional integration. Cross-functional integration implies consistency of decisions between supply chain functions, such as decisions on production operations, human resources, etc., and other functions related to and complementing each other. Intra-functional integration is achieved through coherence across decision areas to achieve synergy within each function. Malone and Crowston (1994) defined horizontal

alignment as the coordination and the act of managing dependencies between entities and the joint effort of entities working together towards mutually defined objectives. Porter (1996) shed light, through his paper, on the importance of horizontal alignment within several activities of companies rather than the key (one or two) ones, in maintaining a competitive advantage over time.

3 Tactical production planning

Section 3.1 presents the general description of the SC studied. Section 3.2 details the tactical planning model we propose, which must satisfy several operational constraints, but which does not depend on the order books used, whose design must accommodate continuous internal demand and batch export demand. This is achieved with an original solution presented in section 4.

3.1 SC description and organisation

Upstream the SC, ore extraction is carried out in a push-flow mode based on an annual extraction program, leading to significant variability over time in the composition and volume of SOs, which are available for MOs production (Azzamouri *et al.*, 2018). The SO stocks built up at the mine and in the vicinity of the washing plants are significant and constitute a decoupling point in the SC (Sun *et al.*, 2008, van Donk *et al.*, 2016), implying the absence of stock management problems in the SC studied part and the possibility of local optimisation.

This SC part (see Figure) is made of *i*) five mines (extending over several hundred km²) connected to *ii*) three washing plants (with five washing lines) feeding either a drying plant or tanks in a Head Station, *iii*) the Head Station is connected, through a 183 km pipeline, with phosphoric acid production plants (whose demand is regular) and the Jorf port for exportation (of irregular demand); *iv*) MOs produced by the drying plant are delivered by trains to the Casablanca port for exportation. Two interdependent production systems coexist, the first dedicated to the order book of customers downstream of the pipeline and the second to the order

book of customers downstream of the train. These two systems (which we will refer to as the pipeline production system and the train production system) share mining resources, and the former supplies the latter with WSOs.

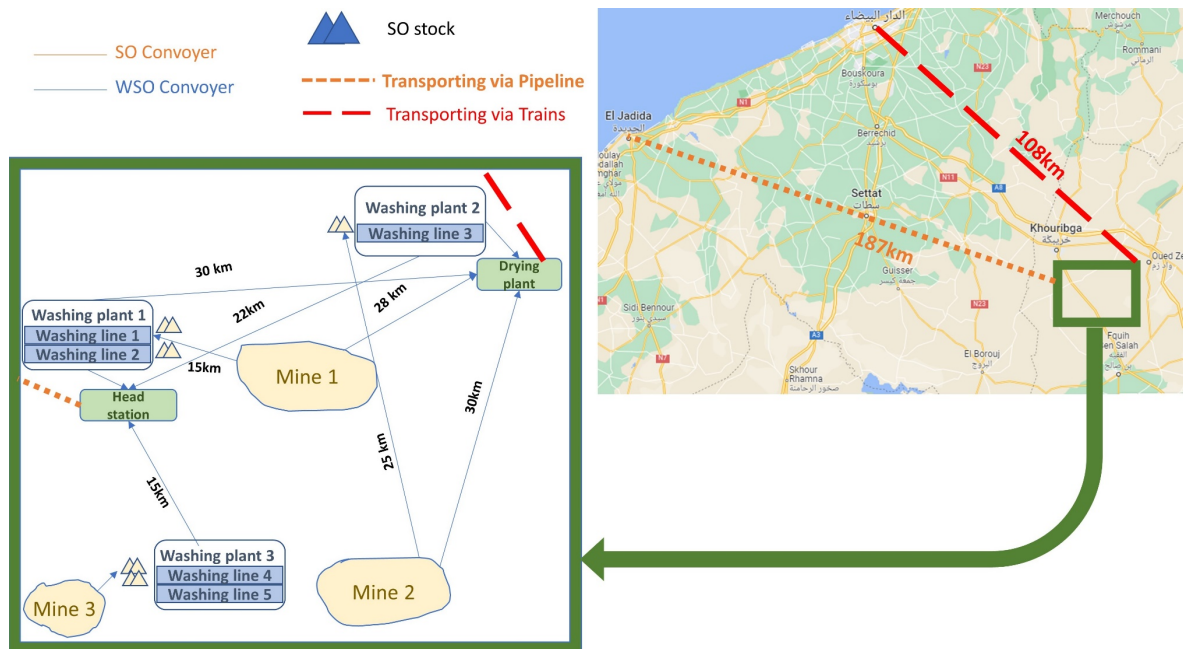


Figure 2 Caption. The studied supply chain

Figure 2 Alt text. The studied supply chain is composed of five mines, five washing lines, a drying unit, and a Head Station, linked by conveyors of several kilometres. It transforms source ores into merchantable ores for two types of customers delivered by train or pipeline.

A washing line enriches a SO to provide an intermediate product called Washed Source Ore (**WSO**) through one of three existing routings (washing, washing and flotation, washing and flotation, and grinding). Each routing enriches the phosphate component by eliminating impurities, reducing the SO's weight, and modifying its chemical composition, at an increasing cost. The more the routing is elaborated, the more the WSO is enriched but with a lower weight yield.

There are three sets of demands: *i*) irregular MOs demand for export delivered by train, produced by blending, in a drying plant, SOs with the possible addition of WSOs; *ii*) irregular MO demand for export delivered by pipeline, delivered by blending WSOs; *iii*) roughly regular demand of MO for internal demand produced by blending WSOs sent by pipeline to chemical plants. This last demand imposes a non-defusing constraint on the pipeline downstream tanks feeding the chemical plants and a non-saturation constraint on these tanks.

Currently, the production subsystem centred on the washing plants, and the one centred on the drying plant, are managed separately, using a pre-allocation of ores, with poor efficiency in terms of QC and poor effectiveness in terms of ore consumption.

As mentioned in the introduction, this SC production is a pull-flow process. The organisation of this production must be based on the strongest constraints, which are undoubtedly those of pipeline transport due to its fixed transfer rate and the constraints of not saturating the storage tanks of limited capacity located upstream and downstream of the pipeline and not defusing downstream for internal demand. The train export order book does not pose any particular problem (except that of respecting the production capacity of the drying plant), unlike the pipeline production and transport order book, which has to satisfy both continuous demand (defined by a flow rate) and discrete demand (quantities to be loaded onto a ship), that cannot be used directly. To improve the organisation, we propose a new master production schedule for the production carried out by the washing plant, based on two principles.

- Production is organised cyclically in a succession of the same set of two several alternate sequences. In the first sequence, all the washing lines produce an order for internal demand (*mono-production*), which increases the stock located downstream of the pipeline (delivery tank). In the second sequence, the washing lines are shared between an order for internal demand and an order for export demand on the other (*bi-production*), which reduces this stock. This cycle, defined to avoid saturation and depletion in the delivery tank, is arbitrarily designed using several rules presented in Section 4.1 and illustrated in Table 3.
- In mono-production as in bi-production, the lines assigned to an MO order work synchronously to produce, each of them, a WSO obtained by treating a selected SO with a selected routing. This simultaneous production, mobilising several lines over the same period with a chosen routing, implies a pre-determination of the number and quantities of inputs (SOs) used in the blending selected to produce the outputs (MOs).

Our modelling does not depend on the definition of the order book, which is considered as an input parameter to the model, whose construction logic makes it possible to reconcile continuous and discontinuous demand flows, is nevertheless a contribution of this paper.

3.2 Mathematical model

As the model is relatively technical, this paragraph describes the structure related to orders. A final order is broken down into several small orders in the case of export demands (train and pipeline) and is described by a set of orders in the case of local demands. With the production cycle described previously, each washing line produces a batch intended for a precise order (if routed by pipeline) or for an export-train final order assigned by the model. Thereby, a batch is defined by the couple (l, k) where l is the number of the line mobilised, and k is associated with an order for: *i*) the local customer (same index k since the local orders use washed ingredients only), *ii*) an export-pipeline customer (index r), or *iii*) an export-train customer (index p).

All orders routed by pipeline are constituted of a blend of WSO, whereas orders routed by train are constituted of a blend of SO and/or WSO. A local order k is obtained by blending the five mono-production or three bi-production batches produced by the lines assigned to that local MO order. An export-pipeline order r is constituted by two orders k , (assigned to r), each produced by two washing lines, which leads, consequently, to a blend of four batches. In this last case, the model will intrinsically assign, if necessary, the export-train orders to the different final commands.

Each order concerns a given MO with an associated volume and quality charter (index j). The link between an order and the merchantable ore is made through a conversion parameter $(\lambda_k, \rho_r, \pi_p)$. A parameter is used to identify the type of order to which each elementary order is attached ($A_k = \{1, 2, 3\}$).

After listing the used indexes, parameters, and variables, we first explain some restrictions with predicate definitions. These predicates will be useful for the optimisation by

reducing the number of variables. Then, we describe the mathematical model before commenting on the introduced relations.

3.2.1 Description of the used indexes, parameters, and variables.

<i>Index</i>	<i>Description</i>
t	Index of time.
i	Index of a raw ingredient/ input/ SO.
p	Index of a drying plant order performed to meet an export-train order.
r	Index of export-pipeline orders.
k	Index of an order produced in the washing plants by blending the batches of WSO produced by the washing lines assigned to that order.
j	Index of output related to an order, with $j = \lambda_k$ for the order k of the local customer, $j = \rho_r$ for the order r of the export-pipeline customers, and $j = \pi_p$ for the order p of export-train customers.
m	Index of mines.
h	Index of routings.
c	Index of chemical components.
l	Index of washing lines.

Parameters Description

General parameters

D_k	Volume of order k produced in washing plants (in m ³).
D'_p	Volume of export-train order p (in m ³).
D''_r	Volume of the export-pipeline order r (in m ³).
λ_k	Output index of local elementary orders.
ρ_r	Output index of export-pipeline orders.
ρ'_p	Output index of export-train orders.

Technical parameters

α_{ci}	Proportion of component c in the weight of input i .
η_{ih}	Weight reduction coefficient of input i by routing h . If the routing h is not used to process SO i , then $\eta_{ih} = 0$.
γ_{cih}	Distortion coefficient of component c in input i by using routing h .

$[\beta_{cj}^{\text{Min}} - \beta_{cj}^{\text{Max}}]$	Lower and upper bounds of the proportion (%) of component c in the weight of output j .
B_{kl}	Boolean is worth 1 if the elementary order k involves washing plant l .
A_k	Equal to 1 if k corresponds to the local customer, to 2 for the export-pipeline customers, and to 3 for the export-train customers.
μ_i	Mine from which the input i is extracted.
Π_{ml}	Boolean is worth 1 if the mine m feeds the washing plant l .
Π'_m	Boolean is worth 1 if the mine m feeds the drying plant.
C_k	Index of the export-pipeline order r to which the elementary order k belongs.
κ	Conversion ratio from volume (m ³) to weight (in tons) (in ton/m ³).
χ_i	Conversion ratio linked to the drying plant from entering volume of input i to exiting volume.
d_l	Hourly production rate of washing plant line l (in m ³ /h) corrected by an average percentage of availability.
\overline{M}	Maximum volume of each input to be used for each order.
\underline{M}	Minimum volume of each input to be used for each order.
<u>Time parameter</u>	
θ_k	Production time of the elementary order k .
ω_k	First period of production of the elementary order k (then the last period of production is $\omega_k + \theta_k - 1$).
π_p	Latest delivery period to start the feeding of order p in the drying plant, considering the drying time and the train transportation time and the earliest loading date on the boat.
ζ	Product transportation time by conveyor between entities (mines, washing plants, drying plant), oversized to avoid explicitly treating potential conflicts in conveyor use at a tactical level.
<u>Inventory parameters</u>	
S_i^{init}	Initial stock of input i at $t=0$ (in tons). At the tactical level, the stock of input i available in a mine is merged with the ones of that input i kept beside a washing, as transportation issues (involving short transport times) can be easily treated at the operational level.

Q_{it} Forecasted stock feeding (in tons) of input i at the beginning of period t (pushed once a day by the production program of mines).

Cost parameter

σ^E Average unitary cost of extraction of one ton of SO.

σ_h^W Treatment unitary cost of one ton of SO in a washing line (energy, water, chemicals, sludge disposal...) involved by routing h .

Variables Description

Decision variables

x_{ihlk} Boolean equals 1 if input i is processed by routing h on washing plant l to produce elementary order k forwarded via pipeline.

y_{ihlkp} Boolean equals 1 if input i is processed by routing h on washing line l to produce elementary order k to include in preparing orders p to forwarded via trains to export customers.

z_{itp} Quantity (in tons) of input i used to produce export-train order p delivered at time t .

u_{itp} Boolean equals 1 if input i is carried to the drying plant since period t for the train-export order p .

Auxiliary variables

μ_{clk} Weight of component c in the total weight of the product made (WSO) on the line l for the elementary order k .

μ'_{cp} Weight of component c in the total weight of the export-train order p of the volume D'_p .

S_{it} Level of stock of input i at the end of period t (in tons).

3.2.2 Predicates

The decision variables will not exist in the whole space defined by the combination of all the index spaces. Therefore, to reduce the size of the model, we define some predicates associated with all variables. A predicate is a logical expression ("true" or "false") that defines the domain of an index, which in turn binds the existence of a decision variable, depending on the parameters' values.

x_{ihlk} exists if four conditions of existence are present: (1) $A_k < 3$, meaning that k concerns only orders related to local customer and export pipeline ones, (2) $B_{kl} = 1$, meaning the production of order k calls for the line l , (3) $\Pi_{\mu_i l} = 1$, meaning the mine μ_i from which the SO i is extracted is linked to the washing line l by a conveyor, and (4) $\eta_{ih} > 0$, meaning that routing h can be used to treat the input i . These four conditions yield the assignment condition predicate $\Omega_1 \equiv (B_{kl} = 1)$ and the physical existence predicate $\Omega_2 \equiv (\Pi_{\mu_i l} = 1 \wedge \eta_{ih} > 0)$.

y_{ihlkp} exists if (1) $A_k = 3$ (order k deals with the production of a WSO made for a drying plant order), (2) the above predicates Ω_1 and Ω_2 are true, and (3) this WSO is produced $(\omega_k + \theta_k)$ time units prior its transportation (ζ) and use by the drying plant order p leading to the predicate $\Omega_3 \equiv (\omega_k + \theta_k - 1 + \zeta < \pi_p)$

z_{itp} exists if input i can be transported to the drying plant ($\Pi'_{\mu_i} = 1$), before the latest production period in the drying plant (considering the oversized time of conveyor transportation, ζ), yielding the predicate $\Omega_4 \equiv (\Pi'_{\mu_i} = 1 \wedge t + \zeta < \pi_p)$. u_{itp} exists if Ω_4 is true.

3.2.3 Flow characteristics

The table 2 shows the characteristics of the three types of orders (local-pipe, export-pipeline and export-train), where the production of order k aggregates the production of the lines used for this order.

Table 2 Caption. Flow characteristics

Table 2 Alt text. The table represents the flow characteristics depending on customer types, including the volumes/quantities of ingredients to consume and blend to prepare the final products according to the specificities related to the transformation process.

	Demand (m ³)	Production			Weight (ton)			Volume (m ³)	
		Duration	Start	End	Input i	Output λ_k (Before drying)	Output λ_k (After drying)	Output λ_k (Before drying)	Output λ_k (After drying)
Batch of a WSO produced by line l for order k ($x_{ihlk}=1$)		θ_k	ω_k	$\omega_k + \theta_k - 1$	$d_l \cdot \theta_k \cdot \kappa / \eta_{ih}$	$d_l \cdot \theta_k \cdot \kappa$	$d_l \cdot \theta_k \cdot \kappa \cdot \chi_i$	$d_l \cdot \theta_k$	$d_l \cdot \theta_k \cdot \chi_i$
Batch of SO i used for order p where $\sum_i u_{itp}=1$ before drying		delivered from period $\sum_i l \cdot u_{ip}$				$\sum_i z_{itp}$	$\sum_i z_{itp} \cdot \chi_i$	$\sum_i z_{itp} / \kappa$	$\sum_i z_{itp} \cdot \chi_i / \kappa$
Local customers Order k where $A_k=1$ made of WSO batches where $x_{ihlk}=1$	D_k	θ_k	ω_k	$\omega_k + \theta_k - 1$				$\sum_{l,i,h} d_l \cdot \theta_k \cdot x_{ihlk} = D_k$	
Export-pipe customers Order r made of several orders k where $A_k=2 \wedge C_k=r$	D_r''							$\sum_{k A_k=2 \wedge C_k=r} D_k = D_r''$	
Export-train customers Order p made of - orders k where $A_k=3 \wedge \sum_{ihl} y_{ihlkp}=1$ - AND/OR batches of SO i used for order p where $x_{ihlk}=1$	D'_p							relation (7)	

3.2.4 Model relations and justifications

We first present all the constraints (3.2.4.1 to 3.2.4.3); the objective-function is described in 3.2.4.4. Three production constraints exist: those dealing with the production sent by pipeline (decision variables x_{ihlk}), those dealing with the production sent by train (decision variables y_{ihlkp} and z_{itp}) and those dealing with SO Stocks (intermediate variables S_{it}).

3.2.4.1 Production constraints of the pipeline production system

$$\sum_{i,h|\Omega_2} x_{ihlk} = 1, \quad \forall k | A_k < 3, \forall l | \Omega_1 \quad (1)$$

Relation (1) forces the production elementary order k shipped by pipeline ($A_k < 3$), to use a unique allocated line ($B_{kl} = 1$), a unique input reachable by that line and a unique routing usable on that line for the input i . As presented in 3.2.1., existence of the variable x_{ihlk} is determined by the predicates Ω_1 et Ω_2 which will be inserted in all the constraints and in the objective function, involving the variable x_{ihlk} .

$$\sum_{h,l|\Omega_1 \wedge \Omega_2} x_{ihlk} \leq 1, \quad \forall i,k | A_k < 3 \quad (2)$$

Technical constraints (2) prevent feeding simultaneously two washing lines with the same input.

$$\mu_{clk} = (\kappa \cdot d_l \cdot \theta_k) \cdot \sum_{i,h|\Omega_2} \alpha_{ci} \cdot \gamma_{cih} \cdot x_{ihlk}, \quad \forall c,k,l | A_k < 3 \wedge \Omega_1 \quad (3)$$

Relation (3) calculates the weight of the chemical component c characterizing the quality of the WSO obtained from the production of line l for the elementary order k performed for the pipeline ($A_k < 3$). This production transforms SO i (represented by its chemical characteristics α_{ci}) with routing h (represented by the distortion rate γ_{cih}) for a downstream total weight equal to $(\kappa \cdot d_l \cdot \theta_k)$.

$$\beta_{c\lambda_k}^{\text{Min}} \cdot D_k \leq \sum_{l|\Omega_1} \mu_{clk} / \kappa \leq \beta_{c\lambda_k}^{\text{Max}} \cdot D_k, \quad \forall c,k | A_k = 1 \quad (4)$$

Relation (4) enforces the QC respect for local customers for each elementary order k . Recall that the Quality Charter is defined for each chemical component c and for each order j by the interval $[\beta_{cj}^{\text{Min}} - \beta_{cj}^{\text{Max}}]$, while considering the volume D_k of the elementary orders k to prepare for the local customers.

$$\beta_{cp_r}^{\text{Min}} \cdot \sum_{k|A_k=2 \wedge C_k=r} D_k \leq \sum_{k,l|A_k=2 \wedge C_k=r \wedge \Omega_1} \mu_{clk} / \kappa \leq \beta_{cp_r}^{\text{Max}} \cdot \sum_{k|A_k=2 \wedge C_k=r} D_k, \quad \forall c,r \quad (5)$$

Relation (5) enforces the QC respect for pipe-export customers for the mixing of all the elementary orders included in the pipe-export order r . Remember that an export-pipeline order is formed by combining two orders k .

An additional constraint prohibiting some combinations of routings and washing lines, in washing plants owning two lines, easy to formalise, is not presented in this paper.

3.2.4.2 Production constraints of the train production system

$$\sum_{i,h,p|\Omega_2 \wedge \Omega_3} y_{ihlp} = 1, \quad \forall l, k \mid A_k = 3 \wedge \Omega_1 \quad (6)$$

Relation (6) enforces the production of the elementary order k of an export-train order p ($A_k = 3$) by the allocated washing line to yield a WSO to include in the blending of the train order p . This is realized by respecting assignment existence condition (Ω_1), physical existence conditions (Ω_2) and time existence conditions (Ω_3). It should be noted that this relationship does not force the two WSOs produced by the two washing lines assigned to a given order k to be integrated into the same train order, which gives greater flexibility in the search for an optimal solution.

$$D'_p = \sum_{i,t|\Pi'_{it}=1 \wedge \Omega_3} \chi_i \cdot z_{itp} / \kappa + \sum_{i,h,k,l|\Omega_1 \wedge \Omega_2 \wedge \Omega_3} \chi_i \cdot d_l \cdot \theta_k \cdot y_{ihlp}, \quad \forall p \quad (7)$$

The demand D'_p of the export-train order p mixes (i) either, 0, 1 or several SOs, assuming that SO i can be conveyed to the drying plant before its use (predicate Ω_4). This corresponds to the first sum of the right side of relation (7) and (ii) either 0, 1 or several WSOs (second sum of the relation 7). During the drying process, the weight of each input i is reduced by the factor χ_i because of the existence of a screening process before the drying one.

$$\begin{cases} z_{itp} \geq \underline{M} \cdot u_{itp} \\ z_{itp} \leq \overline{M} \cdot u_{itp} \end{cases} \quad \forall i, t, p \mid \Omega_4 \quad (8)$$

Binary variable $u_{itp} = 1$, only if a positive amount z_{itp} of SO i is forwarded to the drying plant from the period t onward, to be integrated into export-train order p . The volume z_{itp} must be neither too low nor too high ($\underline{M} \cdot u_{itp} \leq z_{itp} \leq \overline{M} \cdot u_{itp}$). In addition, Relation (8) verifies $u_{itp} = 0 \Leftrightarrow z_{itp} = 0$ and enforces the z_{itp} quantity bounds to be respected if $u_{itp} = 1$.

$$\mu'_{cp} = \sum_{i,t|\Omega_4} \alpha_{ic} \cdot \chi_i \cdot z_{itp} + \sum_{i,h,k,l|\Omega_1 \wedge \Omega_2 \wedge \Omega_3} (\kappa \cdot d_l \cdot \theta_k) \cdot \alpha_{ic} \cdot \gamma_{hcl} \cdot \chi_i \cdot y_{ihlp}, \quad \forall c, p \quad (9)$$

Relation (9) calculates the weight of the chemical component c characterizing the quality of export-train orders p . This is realized while considering the contributions of raw products/ SO in the first part of relation 9, and of WSO in the second one.

$$\beta_{cp_p}^{\text{Min}}.D'_p \leq \mu'_{cp} / \kappa \leq \beta_{cp_p}^{\text{Max}}.D'_p, \quad \forall c, p \quad (10)$$

Relation (10) enforces respecting the QC for each chemical component c and for the train-export order p of D'_p volume.

3.2.4.3 Constraints on SO inventory level

$$\begin{cases} S_{i0} = S_i^{\text{init}} \\ S_{it} \geq 0 \end{cases}, \quad \forall i, t \quad (11)$$

$$S_{it} = S_{i,t-1} + Q_{it} - \sum_{p|\Omega_4} z_{ipt} - \sum_{h,k,l|A_k < 3 \wedge \Omega_1 \wedge \Omega_2 \wedge \omega_k = t} \kappa.d_l.\theta_k.x_{ihlk} / \eta_{ih} - \sum_{h,p,k,l|A_k = 3 \wedge \Omega_1 \wedge \Omega_2 \wedge \omega_k = t} \kappa.d_l.\theta_k.y_{ihlkp} / \eta_{ih}, \quad \forall i, t \quad (12)$$

Relations (11) and (12) model the SO Stock evolution. The recurrent Relation (12) is a classical stock conservation equation, where the removal of an SO from its stockpile is wholly completed at the beginning of such removal. Three removals can take place: (1) SO forwarded directly to the drying plant (z_{ipt}), (2) SO forwarded to a washing lines(plants) for a pipeline orders (x_{ihlk}) and (3) SO forwarded to a washing lines for a train orders (y_{ihlkp}).

The considered tactical horizon allows a reservation of SO depending on the need and availability. Thus, the SO are considered available upstream of washing lines, and there is not any conveying time from mines.

3.2.5 Optimisation criteria

The objective-function relates to an efficiency point of view, trying to minimise the sum of the SO extracting costs P^E (linked to train orders for first and second lines of the mathematical

formulation, and to pipeline orders for third one) and washing costs P^W (linked to pipeline orders for the first part in the relative mathematical formulation and train orders for the second one). The conveyors' and drying costs are not considered because they depend little on the chosen solution and are of low relative importance compared to other costs.

$$P^E = \sigma^E \cdot \left[\sum_{i,t|\Omega_5} z_{ipt} + \sum_{i,h,k,l,p|\Omega_1 \wedge \Omega_2 \wedge \Omega_3} \kappa \cdot d_l \cdot \theta_k \cdot y_{ihlkp} / \eta_{ih} + \sum_{i,h,k,l,p|A_k < 3 \wedge \Omega_1 \wedge \Omega_2} \kappa \cdot d_l \cdot \theta_k \cdot x_{ihlk} / \eta_{ih} \right]$$

$$P^W = \sum_h \sigma_h^W \cdot \left(\sum_{i,k,l|A_k < 3 \wedge \Omega_1 \wedge \Omega_2} \kappa \cdot d_l \cdot \theta_k \cdot x_{ihlk} / \eta_{ih} + \sum_{i,k,l,p|\Omega_1 \wedge \Omega_2 \wedge \Omega_3} \kappa \cdot d_l \cdot \theta_k \cdot y_{ihlkp} / \eta_{ih} \right)$$

$$\text{Min}(P^E + P^W) \quad (13)$$

4 Numerical experiments

Twenty tests of this model were carried out, combining randomly generated data (based on observed structures) relating to the order books of pipeline and train customers (volumes, dates, and MOs) and the availability of the SOs over the horizon under consideration, resulting from the push-flow production of SOs. This section uses the data from the first test as an illustration. Section 4.1 describes the principles retained for the design of the washing plant order book to reconcile the requirements of internal continuous demand and discrete export demand. Section 4.2 describes the other data used. Section 4.3 provides the detailed optimal solution of this test and summarises the solutions of all the tests performed. These data, generated randomly, respect the characteristics observed in the field in terms of physical production and transport systems, order books (dates, volumes, MO references) and SOs supply schedules (in volume and quality).

4.1 Design of the washing plants order book

As mentioned in section 3.1, to meet both continuous demand (internal MO) and discrete demand (export MO) to be produced by the washing plants and transported by the pipeline, we have retained an organisation that alternates between a *mono-production* where all washing

lines are allocated to the internal MO and a *bi-production* where the allocation of these lines is shared between internal and export demand. To facilitate the understanding, we will rely on the reasoning using the numerical data of the case study (close to the actual one) where the internal MO is considered as its supply is mandatory.

- At the tactical level, it is impossible to consider fluctuations in the MO chemistry demand and preventive maintenance operations and setup time in washing lines. Then the nominal production capacities are reduced in the proportion observed in the last months.
- In mono-production, the internal MO production rate is 3,240 m³/h. In bi-production, a systematic assignment of the last three lines to the internal MO production leads to a production rate of 1,980 m³/h for the internal MO and then 1,260 m³/h for the export MO with the first two lines. In both cases, these production rates consider a 10% unavailability rate (maintenance, breakdown...). This decision defines the assignments of the washing lines to the order processed for the production sent by pipeline and the WSO used by the production sent by train (parameters A_k and B_{kl}).
- The capacity of internal MO tanks downstream of the pipeline is 22,500 m³. In mono-production, these tanks, considered initially empty, are filled in $22,500/(3,240-2,500) = 30,4$ hours by a produced quantity of $30,4 \times 3,240 = 98,514$ m³ of internal MO. In bi-production, these tanks being initially full, are emptied in $22,500/(2,500-1,980) = 43.3$ hours, by a produced quantity of $43.3 \times 1,980 = 85,734$ m³. During that period, the two other washing lines may produce $43.3 \times 1,260 = 54,558$ m³, used in priority for pipeline export MOs, the residual being used (WSO) for export train MO.
- Then a possible production organisation may be based on a succession of *cycles* of approximately 73 hours made of a sequence of a 30-hour period of mono-production (gradually leading to saturation of the internal MO tanks downstream of the pipeline), followed by a sequence of 43-hour period of bi-production (gradually leading to emptying

of tanks downstream of the pipeline). With this cycle length, the time horizon used for the tactical planning is 10 cycles (730 hours), which is approximately one month.

- This type of organisation, which requires large quantities of SOs to be used simultaneously to produce MOs, in a context of relative scarcity of these SOs, not only makes it more difficult to comply with the MO quality charter but may also prevent a solution from being obtained that complies with the QC. So, the organisation chosen is based on the repetition of a 73-hour cycle in which several mono- and bi-production sequences are alternated to reduce the size of the batches to be produced and make good use of the available SOs. We have arbitrarily selected 5 sequences in a cycle comprising an alternation of 5 mono-production batches (22,680, 19,440, 19,440, 19,440 and 16,200 m³, for durations of 7, 6, 6, 6 and 5 quarters of an hour) and 5 bi-production batches (17,820, 17,820, 17,820, 15,840, 15,840 m³, for durations of 9, 9, 8, 8 and 8 quarters of an hour during which export-pipe QSLs and train QSLs can be produced). This alternation, illustrated in Table 3, uses arbitrarily chosen values that respect the constraints of non-saturation and non-de-priming. This production organisation has an impact on the data used by the planning problem, but not on the mathematical optimisation model.
- There are three additional points to consider in bi-production: *i*) each pipeline export order, whose the number and characteristics vary in time, must be split into two elementary orders (allowing a blend of 4 batches of WSO) or more, to ensure QC constraints; *ii*). a size constraint exists for the batches of WSO produced for train as an export-train order will be transported in several full trains of limited capacity (3,600 tons), where each load have to comply with the QC; to be used in the blend of a trainload, the size of WSO batches must be small enough, to allow the use of SOs in addition to WSO in blending; *iii*) a batch of WSO cannot be used immediately by the drying plant because it has to be drained and transported by conveyor belt; this implies a neutralisation period of about 42 hours at the beginning of the order book for MOs transported by train and a neutralisation period of the

same length at the end of the order book for washing plants, in order to ensure that all the batches of WSO produced are used.

- It can be added that, with a transfer rate of $4,000 \text{ m}^3/\text{h}$, the pipeline utilisation cannot exceed $3,240/4,000 = 81\%$. Pipeline transport induces a time lag between internal MO production and the feeding of delivery tanks (downstream of the pipe), which can be ignored here as we are only interested in the production problem.

Table 3 Caption. Order books for washing plants and drying plant

Table 3 Alt text. The table represents the components of the proposed sequencing program for the two first and last production cycles, including the volumes to produce, production duration and the starting production dates

WASHING PLANTS ORDER BOOK -pipeline shipment															
CYCLE 1 (from period 1 to period 73)								CYCLE 9 (periods 585 to 657)							
	Mono/Bi-production	Order k	export pipe order r	Volume D_k	Duration θ_k	Start ω_k	End $\omega_k + \theta_k - 1$		Mono/Bi-production	Volume D_k	export pipe order r	Volume D_k	Duration θ_k	Start ω_k	End $\omega_k + \theta_k - 1$
	Mono	1		22680	7	1	7		Mono	328		22680	7	585	591
	Bi-prod	2		17820	9	8	16		Bi-prod	329		17820	9	592	600
		3	1	3780	3	8	10			330	21	7560	6	592	597
		4 to 9		1260	1	11	16			331 to 333		1260	1	598	600
		10		19440	6	17	22			Mono	334		19440	6	601
	Bi-prod	11		17820	9	23	31		Bi-prod	335		17820	9	607	615
		12	1	3780	3	23	25			336	21	6300	5	607	611
		13 to 18		1260	1	26	31			337 to 340		1260	1	612	615
	Mono	19		19440	6	32	37		Mono	341		19440	6	616	621
	Bi-prod	20		17820	9	38	46		Bi-prod	342		17820	9	622	630
		21	2	3780	3	38	40			343	22	2520	2	622	623
		22 to 27		1260	1	41	46			344 to 350		1260	1	624	630
	Mono	28		19440	6	47	52		Mono	351		19440	6	631	636
	Bi-prod	29		15840	8	53	60		Bi-prod	352		15840	8	637	644
		30	2	5040	4	53	56			353	22	2520	2	637	638
		31 to 34		1260	1	57	60			354 to 359		1260	1	639	644
	Mono	35		16200	5	61	65		Mono	360		16200	5	645	649
Bi-prod	36		15840	8	66	73	Bi-prod	361		15840	8	650	657		
	37 to 44		1260	1	66	73		362 to 369		1260	1	650	657		
	Mono	45		22680	7	74	80		Mono	370		22680	7	658	664
	Bi-prod	46		17820	9	81	89		Bi-prod	371		17820	9	665	673
		47	3	3780	3	81	83			372	23	5040	4	665	668
		48 to 53		1260	1	84	89			373 to 377		1260	1	669	673
	Mono	54		19440	6	90	95		Mono	378		19440	6	674	679
	Bi-prod	55		17820	9	96	104		Bi-prod	379		17820	9	680	688
		56	3	3780	3	96	98			380	23	6300	5	680	684
		57 to 62		1260	1	99	104			381 to 384		1260	1	685	688
	Mono	63		19440	6	105	110		Mono	385		19440	6	689	694
	Bi-prod	64		17820	9	111	119		Bi-prod	386		17820	9	695	703
		65	4	5040	4	111	114			387	24	3780	3	695	697
		66 to 70		1260	1	115	115			388 to 393		1260	1	698	703
	Mono	71		19440	6	120	125		Mono	394		19440	6	704	709
	Bi-prod	72		15840	8	126	133		Bi-prod	395		15840	8	710	717
		73	4	5040	4	126	129			396	24	5040	4	710	713
		74 to 77		1260	1	130	133			397	25	2520	2	714	715
	398 & 399		1260	1	716	717									
Mono	78		16200	5	134	138	Mono	400		16200	5	718	722		
Bi-prod	79		15840	8	139	146	Bi-prod	401		15840	8	723	730		
	80 to 87		1260	1	139	146		402	25	3780	3	723	725		
Internal MO		Pipe export order				WSO for		403 to 407			1260	1	726	730	

Internal MO Pipe export order WSO for

DRYING PLANT ORDER BOOK - train shipment (3600 m ³ batches - batch drying duration of 3 hours)							
Order p	MO #	Start	End	Order p	MO #	Start	End
1	3	43	45
2	3	46	48	175	1	765	767
3	3	49	51	176	1	768	770
4	3	52	54	177	7	771	773
5	3	55	57	178	7	774	776
6	5	58	60	179	7	777	779
7	5	61	63	180	7	780	782

4.2 Other data of the planning problem

These data relate to MOs and their QCs (section 4.2.1), and to the SOs and their transformation by the routing used in the washing plants (section 4.2.2). For confidentiality reasons, the different unit costs (σ^E and σ_h^W) used in the objective function (relation 13) will

not be disclosed. The supply chain configuration defined by the conveyor links between several SC entities (Π_{ml} and Π'_m) is defined in the Mendeley file.

4.2.1 Characteristics of the MOs

Each order is characterised by a quantity, its availability date, and its MO. This last piece of information was not included in the definition of the order books (in the previous section) because the tests carried out assigned to each of the 5 order sets 2 sets of MOs randomly generated from the observed demand structures, resulting in a set of 10 order sets defined by dates, quantities and products.

For the export train order, the 8 most requested MOs were retained in these tests and are described in Table 3, together with their QCs, as well as the ones corresponding to the local and export pipeline orders.

Table 4 Caption. Merchantable Ores and their quality charts

Table 4 Alt Text. The table represents the chemical characteristics of products depending on customer types and on three chemical components (BPL, MgO and Cd).

	Merchantable Ore (output j)	Export train								Export pipeline	Local
	Chemical component c	1	2	3	4	5	6	7	8	9	10
Quality chart $[\beta_{cj}^{\text{Min}} - \beta_{cj}^{\text{Max}}]$	1 (BPL %)	[67.5-68.8]	[72-72.5]	[69.4-71]	[67.5-68.75]	69.38-71.04	70	[69.92-70.72]	72	[65-66]	>64
	2 (MgO %)	[0.3-0.65]	>2.8	[0.25-0.48]	>2.2	[0.38-0.6]	>0.38	>0.38	>0.3	[0.3-0.85]	>0.95
	3 Cd (ppm)	<19	<20	<19.5	<25	<20	<20	<22	<18	<21	<24

4.2.2 Characteristics of the SOs

The washing and drying plant are supplied by 6 mines that produce around 33 SOs in a push flow, the supply of which is considered at the beginning of each day. There is a desynchronisation between the production of a WSO prepared in a washing line and its use in an order of a drying plant, due to the time needed for draining and transport (about 30 hours).

Table 4 shows the composition of some SOs among the 33 used (more details in the Mendeley file), and Figure 2 (in the appendix) shows one of the two scenarios for the evolution of the supply of these SOs, starting from the initial stocks. Table 5 illustrates the impact of the washing

routing on the composition and weight of the first 3 SOs (see Mendeley for more information).

Table 5 Caption. Composition of available SOs

Table Alt text 5. The table represents the chemical characteristics of SOs depending on on three chemical components (BPL, MgO and Cd).

		Source Ore (input i)								
		Chemical component c	1	2	3	4	5	...	32	33
composition α_{ci}	1 (BPL %)	55	61.5	65	68	69.8	...	67	69	
	2 (MgO %)	2.3	0.9	0.9	0.8	0.54	...	4.8	4.6	
	3 Cd (ppm)	19	19	17	15	17	...	18	18.35	

Table 6 Caption. Example of impact of the washing routing on weight and composition of SOs.

Table 6 Alt text. The table represents an example of impact of washing routing on weight and composition of SOs, depending on chemical components for two different SOs.

		Source Ore $i=1$			Source Ore $i=2$		
		Routing h			Routing h		
		1	2	3	1	2	3
		Washing	Washing+flotation	Washing+flotation+grinding	Washing	Washing+flotation	Washing+flotation+grinding
Composition	1 (BPL)	1.071	1.125	1.188	1.057	1.09	1.115
distorsion of	2 (MgO)	0.57	0.558	0.5	0.7	0.67	0.6
component c	3 (Cd)	0.801	0.811	0.751	0.791	0.801	0.761
Reduction weight rate (%)		69.22	57.68	52.4	73.4	65	58.7

4.3 Results

The model was tested with 20 problem sets (detailed in the Mendely files) the solutions of which were obtained on a MacBook Air computer with an M2 chip and 24 GB of unified memory, of which 18 GB was allocated to the Parallels virtual machine running the used XPress-IVE optimisation software. which is based on AML-type modelling (Fourer, 2013). These problems involved around 2,800,000 variables and 5,000 variables; their resolution times were between 1,000 and 5,000 seconds, with gaps less than 0.002 from the theorical best solution.

The numerical results illustrate that the model perfectly meets all the constraints imposed.

For each order according to its type (local, export-train and export-pipeline). According to Table 2, the consumed SOs are: $\kappa \cdot d_l \cdot \theta_k \cdot x_{ihlk} / \eta_{ih}$ if $x_{ihlk} = 1$, used by a local order ($A_k = 1$) or an export-pipeline order ($A_k = 2$), and $\kappa \cdot d_l \cdot \theta_k \cdot y_{ihlkp} / \eta_{ih}$ if $y_{ihlkp} = 1$, or/and $z_{itp} > 0$ if $o_{itp} = 1$, used by an export train order ($A_k = 3$). Extracts of the optimal solution for the production of MO for pipeline and train customers are given in Tables 6 and (details given in the Mendeley).

Table 7 Caption. Optimal solution extract (Cycle 2 of order book & MO scenario 1 – Supply scenario 1)

Table 7 Alt text. The table represents the production recipes proposed by the model to satisfy orders to forward via pipeline for the first production cycle, including the starting production dates, volume of SO and WSO, washing line, alternative routings and product quality.

Production of Internal MO sent by pipe during cycle 1										Production of pipe export MO sent by pipe during cycle 1												
Order <i>k</i>	Start date	SO #	R#	L#	Quantity			Chemical Composition			order <i>r</i>	Order <i>k</i>	Start date	SO#	R#	L #	Quantity			Chemical Composition		
					SO	WSO	Σ	c=1 (%) BPL	c=2 (%) MgO	c=3 (ppm) Cd							SO	WSO	Σ	c=1 (%) BPL	c=2 (%) MgO	c=3 (ppm) Cd
1	1	5	1	1	3662	3150	22680	71.99	0.89	14.92	1	3	8	5	1	1	2616	1350	7560	72.98	0.60	15.48
		12	1	2	6593	5670								12	1	2	4709	2430				
		18	1	3	6501	5040								5	1	1	2616	1350				
		24	1	5	5688	4410						12	23	12	1	2	4709	2430				
		33	1	4	5127	4410								12	1	2	4709	2430				
2	8	19	1	3	7829	6480	17820	70.96	0.87	14.37	2	21	38	5	1	1	2093	1350	8820	72.98	0.60	15.48
		24	1	4	7314	5670								12	1	2	3767	2430				
		32	1	5	6931	5670						30	53	5	1	1	2093	1800				
10	17	4	1	1	3300	2700	19440	71.34	0.92	14.77	3	47	81	5	1	1	2616	1350	7560	72.98	0.60	15.48
		12	1	2	5651	4860								12	1	2	4709	2430				
		17	1	3	5808	4320						56	96	5	1	1	2616	1350				
		24	1	4	4876	3780								12	1	2	4709	2430				
		33	1	5	4395	3780								12	1	2	4709	2430				
11	23	18	1	3	8359	6480	17820	70.6	0.84	15.07	4	65	111	5	1	1	2093	1800	10080	72.98	0.60	15.48
		19	1	5	6851	5670								12	1	2	3767	3240				
		32	1	4	6931	5670						73	126	5	1	1	2093	1800				
19	32	5	1	1	3139	2700	19440	72.48	0.90	13.88	3	47	81	5	1	1	2616	1350	7560	72.98	0.60	15.48
		12	1	2	5651	4860								12	1	2	4709	2430				
		19	1	4	4567	3780								5	1	1	2616	1350				
		25	1	3	5219	4320								12	1	2	4709	2430				
		33	1	5	4395	3780								12	1	2	4709	2430				
20	38	19	1	5	6851	5670	17820	71.26	0.86	12.88	4	65	111	5	1	1	2093	1800	10080	72.98	0.60	15.48
		25	1	3	7829	6480								12	1	2	3767	3240				
		32	1	4	6931	5670								73	126	5	1	1				
28	47	5	1	1	3139	2700	19440	72.48	0.90	13.88	3	47	81	5	1	1	2616	1350	7560	72.98	0.60	15.48
		12	1	2	5651	4860								12	1	2	4709	2430				
		19	1	4	4567	3780								5	1	1	2616	1350				
		25	1	3	5219	4320								12	1	2	4709	2430				
		33	1	5	4395	3780								12	1	2	4709	2430				
29	53	19	1	4	6089	5040	15840	71.26	0.86	12.88	4	65	111	5	1	1	2093	1800	10080	72.98	0.60	15.48
		25	1	3	6959	5760								12	1	2	3767	3240				
		32	1	5	6161	5040								5	1	1	2093	1800				
35	61	5	1	1	2616	2250	16200	72.48	0.90	13.88	3	47	81	5	1	1	2093	1800	10080	72.98	0.60	15.48
		12	1	2	4709	4050								12	1	2	3767	3240				
		19	1	4	3806	3150								5	1	1	2093	1800				
		25	1	3	4349	3600								12	1	2	4709	2430				
		33	1	5	3662	3150								12	1	2	4709	2430				
36	66	19	1	4	6089	5040	15840	71.26	0.86	12.88	4	65	111	5	1	1	2093	1800	10080	72.98	0.60	15.48
		25	1	3	6959	5760								12	1	2	3767	3240				
		32	1	5	6161	5040								5	1	1	2093	1800				

Table 8 Caption. Extract of optimal solution for the drying plant focused on the orders that include WSO

Table 8 Alt text. The table represents the production recipes proposed by the model to satisfy the export train orders using WSO (k) produced during cycle 1 in washing plants, and including the volume of SO and WSO, the alternative routings, starting production periods and products quality.

	Production of WSO sent to drying plant								Production of Internal MO sent by train during the first cycle (first orders p that includes WSO produced during cycle 1)															
	Order k	Start date	SO#	R#	L#	Quantity			Order p	Start date	SO#	Order k (WSO)	Line l	Quantity			Chemical Composition							
						SO	WSO	Σ						SO (ton)	WSO (m3)	Σ (ton)	c=1 (%) (BPL)	c=2 (%) (MG O)	c=3 (ppm) (Cd)					
Cycle 1	4	11	5	1	1	523	450	1260	5	55	6	-	-	2536	-	3600	71.02	0.6	15.30					
			12	1	2	941	810																	
	5	12	12	1	2	941	810	15			-	-	182	-										
			5	1	1	523	450	12			5	2	-	810										
	6	13	5	1	1	523	450	7	58	6	-	-	2536	-	3600	71.02	0.6	15.30						
			12	1	2	941	810			13	-	-	72	-										
	7	14	12	1	2	941	810			15	-	-	182	-										
			5	1	1	523	450			12	9	2	-	810										
	8	15	12	1	2	941	810						
			5	1	1	523	450			5	5	1	-	450										
	9	16	12	1	2	941	810			5	6	1	-	450										
			5	1	1	523	450			5	16	1	-	450										
Cycle 2	13	26	5	1	2	941	810	10	77	5	24	1	-	450	3600	72	0.50	15.54						
			5	1	1	523	450			5	27	1	-	450										
	14	27	5	1	1	523	450			5	13	2	-	810										
			5	1	2	941	810			5	26	2	-	810										
	15	28	5	1	2	941	810			11	80	6	-	-					2027	-	3600	72	0.53	15.21
			5	1	1	523	450					15	-	-					19	-				
	16	29	5	1	1	523	450	5	17			2	-	810										
			5	1	2	941	810	12	8			2	-	810										
	17	30	5	1	2	941	810	12	83	13	-	-	17	-	3600	72	0.50	15.54						
			5	1	1	523	450			15	-	-	128	-										
	18	31	5	1	2	941	810			5	4	1	-	450										
			5	1	1	523	450			5	14	1	-	450										
22	41	5	1	1	523	450	5			17	1	-	450											
		5	1	2	941	810	5			22	1	-	450											
23	42	5	1	1	523	450	5			23	1	-	450											
		5	1	2	941	810	5			14	2	-	810											
24	43	5	1	1	523	450	5	16	2	-	810													
		5	1	2	941	810								
Cycle 3	25	44	5	1	1	523	450	WSO produced during cycle 1			WSO produced during cycles> 1													
			5	1	2	941	810																	
	26	45	5	1	2	941	810																	
			5	1	1	523	450																	
	27	46	5	1	1	523	450																	
			5	1	2	941	810																	

Three important observations can be made from the result of this test, which is representative of the 20 tests carried out.

- Among the 20 tests, all batches tested in a washing line use the $h=1$ routing, corresponding to a simple washing for all batches (WSO), whatever the type of order (this routing is the least expensive and consumes less water). This observation made on the optimal solution shows that an important use of the less efficient routings 2 and 3 can be avoided.
- As can be seen in Table 7 (and more extensively in the Mendeley file), a same MO order can be obtained with different SO blends (see for example orders 10, 19 and 28; orders 2, 11 and 20; orders 3, 12 and 21). This is due to the temporal and spatial interdependence of the decisions, which involves a trade-off in the use of SOs, taking into account their scarcity, which evolves over time. This observation made on the optimal solution shows that the systematic use of fixed blending recipes is not relevant.
- The WSOs produced during the first production cycle are used by orders from the drying unit in a very varied way (Table 8): an order may not include any WSOs produced during the first cycle (this is the case for the first 4 orders) or one or more WSOs from these WSOs produced during the first cycle, possibly mixed with WSOs produced late.

5 Conclusions

It is important to stress that this work is not only a comprehensive case study, but also the an original scientific approach that can and must be applied in other mining companies with similar configurations (section 5.1). This study also has interesting implications for OCP management (section 5.2).

Scientific contribution and dissemination of the approach

From a scientific point of view, the proposed model can be analysed as a joint planning problem for two logistics chains: *i)* which differ in the production processes used and the demands to be met (fairly stable continuous demand + highly irregular discrete demand for the first SC and

highly irregular demands for the second SC); *ii*) but which are linked by the sharing of primary inputs (ores extracted from 5 distant mines) and the supply, on an ancillary basis, of intermediate inputs (WSOs) produced by the first SC, to use its residual capacities, at the request of the second SC in addition to the primary inputs it can use. This type of tactical problem is specific and modelling it using mathematical programming. The problem treated in this paper represents an original assembly of primitives. The originality that can be mentioned is found upstream of the mathematical model with the construction of order books based on a cyclical organisation of the production of SC of washing plants, which makes it possible to reconcile the joint satisfaction of a regular demand and an irregular demand (this order book constituting an input data of the model).

At least a dozen other mining industries that extract and process ores of various types (See Table 9 in Appendix) produce constrained composition outputs by blending previously processed inputs in treatment lines. Many mining industries are now forced to exploit low-grade, distant deposits that require a combination of beneficiation and blending to meet quality standards, as is currently the case in the OCP supply chain studied. The increasing scarcity of phosphate ore makes this trend ineluctable.

In addition, it can be noted that the principle of alternating mono-production and bi-production should be of interest for other continuous production industries facing to a non-defusing constraint (chemical process, etc.).

5.1 Managerial impacts for OCP

The numerous tests have shown that it is possible to meet the QC requirements with minimal ore and chemical consumption, and our main contribution is to demonstrate the possibility of a more efficient management mode based on flow control, taking into account the characteristics of several chemical components and without the need for additional information, based on several advances.

The first two ones are technical.

- The change from a single component (BPL) to several components to be taken into account in the QC no longer makes it possible to manage the washing lines independently without a sharp deterioration in effectiveness (non-compliance with the QC) and efficiency (significant use of routings 2 and 3, which mobilise larger quantities of SOs and chemicals).
- The flexibility offered by replacing MO's largely fixed blend recipes by dynamic ones that take into account the entire order book and the entire ore delivery schedule, dramatically increases planning efficiency.

The other advances are of a managerial nature. Currently, the washing lines are managed independently, which is no longer possible with the scientific approach of blending, which forces a horizontal and a vertical alignment of decisions taken by productive entities.

- The replacement of parameters (predetermined 'ores' allocation and blending recipes) by decision variables in the optimisation model guarantees a *horizontal alignment* of the tactical decisions taken in these two productive subsystems at three levels: *i)* through the optimal allocation of SO between the production sent by pipeline and the production sent by train; *ii)* through the coordination of the production decisions of different production units (drying plant, washing lines) that must cooperate to ensure the satisfaction of MO demands; *iii)* through the synchronisation of the production.
- This model allows a *vertical alignment* between tactical decisions and operational decisions, in particular the combined decisions on the production of washing lines and the transport of these products by pipe, for which an operational model has been created and successfully tested (Bamoumen et al., 2020). This alignment is facilitated using the same level of detail for inputs and outputs, as well as the use of a common hourly time breakdown, which eliminates the occurrence of classical disaggregation problems. Furthermore, the operational model uses the same principle of alternating between mono and bi production. Finally, WSO production decisions made at the tactical level can be retained at the operational level, as can the sharing of WSOs between the two production systems, which

can then be managed separately. Of course, all this requires tactical planning to be updated fairly frequently, which is essential because of the hazards encountered in mining and therefore in making newly extracted SO available.

For implementing this new approach, it is now necessary to carry out some additional benchmarks to convince the OCP top management of the need for this change of perspective. It is also necessary to consider encapsulating this model in a DSS coupled with the database fed by the extraction schedule and the MOs order book.

6 Data availability statement

The data supporting this article's results are available in the Mendeley data (doi: 10.17632/5hpbyvv3rt.1). For confidentiality reasons, some data are fictitious. However, the structure of this information is close to that of the actual data.

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8 Appendix

Figure 3 Caption. Evolution of SOs supply scenario

Figure 3 Alt text. The figure represents the evolution of the different SOs supply during one month of planning

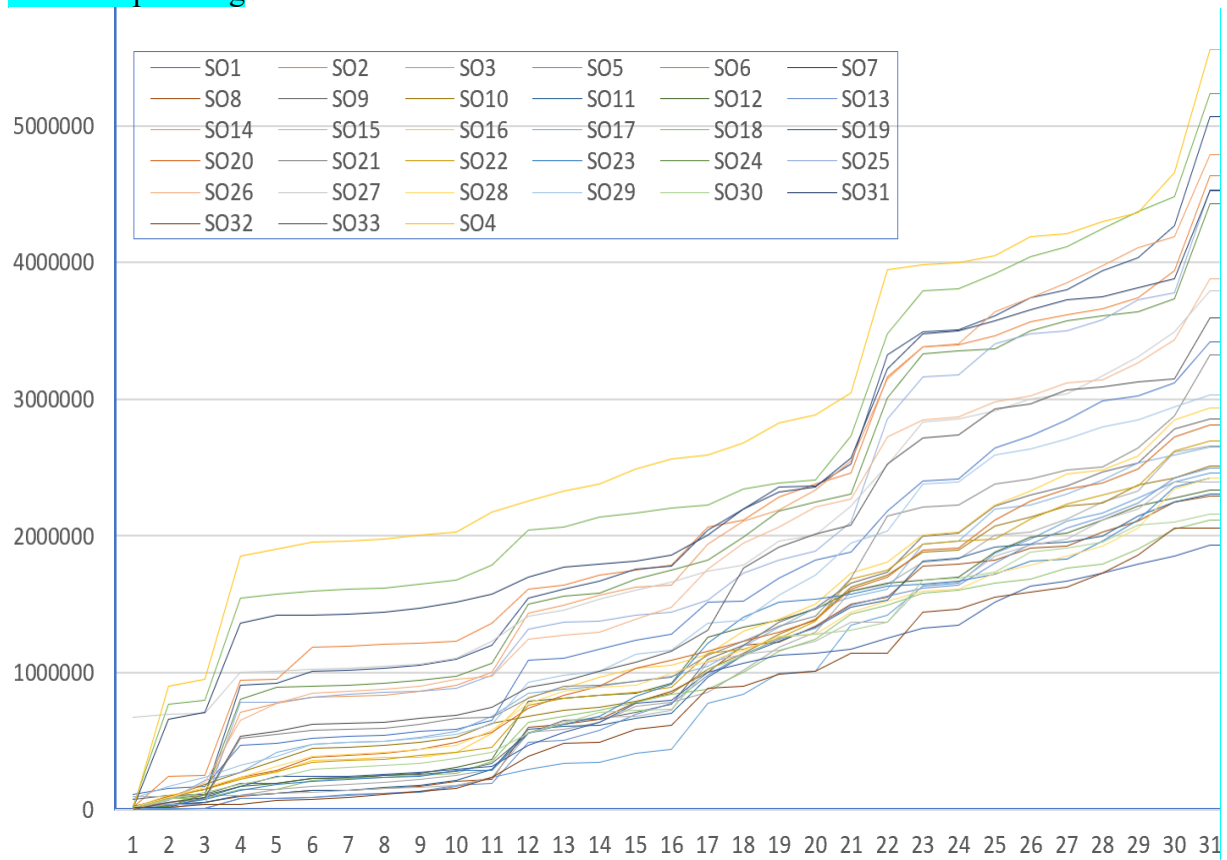


Table 9 Caption. Examples of mining companies adopting processing and blending for their production processes¹

Table 9 Alt text. The table represents the examples of 12 mining companies adopting processing and blending for their production processes depending on three main production configurations

Company name	Type of ore produced	Configuration of blending	Multiple routings	Sources
BHP Billiton	Copper, Metallurgical coal, Nickel, Potash, Iron ore	A	Multiple routings and always same: flotation, grinding, leaching concentrating	https://www.bhp.com/what-we-do/products/copper
Ma'aden	Phosphate, Gold Aluminum, and base metals, industrial minerals	NA	Alternative routings : flotation, refining grinding, leaching, concentrating	https://www.maaden.com.sa/en/business/
Nutrien	potash, phosphate	NA	Alternative routings: screening, washing, floating, dewatering, calcination. For potash, routings are always the same: crushing, grinding, desliming, flotation, drying and sizing, compaction, crystallization	https://www.nutrien.com/what-we-do/our-business/
Vale S.A.	Iron, Nickel, Manganese and ferroalloys, Copper,	A	Alternative routings depending on the quality of the ore	https://www.vale.com/w/vale-inaugurates-pilot-plant-that-allows-for-more-sustainable-mining
Glencore	Copper, Nickel Zinc & Lead, (Cobalt, Ferroalloys Other metals & minerals)	A	Alternative routings: for Nickel, grinding, flotation, drying, smelting, and refining for zinc and lead ore. For copper, crushing, grinding, flotation, leaching, and melting	https://www.glencore.com/what-we-do/metals-and-minerals/
Anglo American	Copper, Steelmaking Coal (Nickel Platinum group metals, Diamonds, Iron ore Manganese, Polyhalite)	A	Grinding, concentration using gravity separation for coal, flotation for copper,	https://www.angloamerican.com/about-us/what-we-do
Newmont Mining Corporation	Gold, silver	B	Alternative processing methods, depending on the ore, heating, heap leaching, flotation, gravity circuit	https://www.newmont.com/about-us/about-mining/the-mining-process/default.aspx
First Quantum Minerals	Iron Copper Gold zinc, pyrite	A	Alternative routings, choosing between: crushing, grinding, leaching, flotation, filtering, depending on the type of mineral and the need	https://www.first-quantum.com/English/our-operations/
Norilsk Nickel (ee)	Nickel, Palladium, Copper Platinum, Cobalt, Rhodium,	B	Mandatory and successive operations: Concentration, smelting, and refining,	https://normickel.com/business/products/
KGHM Polska Miedź	Copper, Silver and other precious metals	A	Alternative routings, depending on ore types, sifting and crushing, milling and classifying, flotation, thickening and filtration, and drying,	https://kgm.com/en/our-business/mining-and-enrichment
AngloGold Ashanti	Gold	B	Alternative routings: crushing, milling, screening, and classification processes which are common, and then they choose the cyanide leaching process or heap leach process or flotation and oxidation	https://www.aga-reports.com/11/downloads/AGA-Gold-production.pdf
Kinross Gold Corporation	Gold	NA	Alternative routings: Milling, heap leach, or milling and gravity or Milling, gravity, heap leach	https://www.kinross.com/operations/default.aspx

¹ The configurations B and C refer to the production configuration explained in **Erreur ! Source du renvoi introuvable.** (B: Blending before washing and C: Blending after washing)