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# **Sustainable Financial Risk Modelling Fitting the SDGs: Some Reflections**

by

**Christian Walter**

## **ABSTRACT**

This article argues that any ecological finance theory devised to fit the SDGs needs a paradigm shift in financial risk modelling. Any ecological finance theory has to take over from neoclassical finance theory by integrating the characteristics of “nature” into the modelling carried out. If nature is properly described by fractals, hence ecological finance theory has to include fractal properties of nature into financial risk modelling. This paper lays the ground for the “fractalisation” of financial risk modelling to meet ecology and calls upon to build new kinds of models. It argues that the main problem with unsustainable neoclassical modelling of financial risk comes from its probabilistic ingredients: continuous stochastic processes and the unquestioned use of mathematical expectation. It approaches this problem from a philosophical point of view, presenting Leibniz’s principle of continuity and Quetelet’s theory of averages. Then it turns to a cultural approach by defining two risk cultures in finance, Brownian finance and non-Brownian finance. Next, the notion of fractal is introduced as a tool adapted to ecology and then a fractalisation of financial risk modelling is presented as a way to move from neoclassical financial theory to ecological finance theory. The conclusion proposes a new agenda for future researches.

**KEYWORDS:** fractal; ecology; ecological finance theory; scaling laws; power laws; Brownian finance; principle of continuity; financial risk modelling; Value-at-Risk; stochastic processes; Lévy processes

**JEL classification:** G00

## 1. Introduction

In the 2030 Agenda for Sustainable Development, a part of the Sustainable Development Goals (SDGs), it is strongly asserted that “we are determined to protect the planet from degradation”. In particular, “we are determined to take the bold and transformative steps which are urgently needed to shift the world onto a sustainable and resilient path”. A quite striking example of an unsustainable and non-resilient path is given by finance and financialisation of the world economy (Epstein, 2005) due to the financial approach of the neoclassical theory. Thus, among the bold and transformative measures that are urgently needed to put the world on a sustainable and resilient path is the urgent need to rebuild finance on an ecological basis, to re-embed financial systems within ecological constraints and to develop an “ecological finance theory” (Lagoarde-Segot and Martinez, 2020). This paper is a piece for contributing to this stake in order to interlock financial systems with the objectives of the 2030 Agenda.

An ecological finance theory will have to take over from neoclassical financial theory, by integrating the characteristics of “nature” into the modelling carried out. If neoclassical finance theory superimposes financial logics on to the analysis of the environment, ecological finance theory calls upon us to turn this order upside down and to build new kinds of models. Among the models to be developed, one of them is particularly important and sensitive, the financial risk modelling. This article argues that ecological finance theory devised to fit the SDGs needs a paradigm shift in financial risk modelling.

What are the possible avenues of research for new and sustainable financial risk modelling? If one considers that sustainability needs to take account the “nature of nature” and that to imitate the nature can “save the planet” (Dicks, 2019; Dicks and Block, 2019; Kennedy et al., 2015), one has to take account of a very specific “signature” of nature, its fractal geometry (Mandelbrot, 1982). It seems that “fractality” meets the aims and constraints of sustainability (*Triple Pundit*, 2011). Assuming that nature is properly described by fractals, hence sustainable financial risk modelling has to include fractal properties of nature. The need of building risk models with fractal methods follows. Consequently, financial risk modelling will be sustainable if it fits to the basic principles of biomimicry with fractals: nature as model, nature as measure, and nature as mentor (Benyus, 1997). To put it in a nutshell, if the relevant geometry of nature is fractal, then financial risk modelling will have to be anchored on fractal representations in order to be in line with nature and become sustainable. Consequently, one step which is urgently needed to shift the world onto a sustainable and resilient path is the fractalisation of financial risk modelling.

This paper lays the ground for the fractalisation of financial risk modelling to meet ecology. Section 2 describes the influence of any financial risk modelling in the real economy, due to the quantification conventions acting in technical tools and regulations, illustrating this with the example of the Li’s copula, the “formula that killed Wall Street”. In an Anthropocene market society, it is extremely important to understand this phenomenon that a risk model provides a strong performative influence, as this performative influence extends to the modelling of natural risks. This is the “financial ontology” of catastrophe (Kob, 2020). Section 3 enters this problem with a philosophical angle that of the Leibnizian continuous principle combined with the Quetelet’s theory of averages as a pervasive mental model for neoclassical financial risk modelling. Section 4 echoes Keynes’ suggestion of two geometries in economics and introduces two risk geometries as two risk cultures in finance; I name Brownian finance and non-Brownian finance, as two ways of thinking risk. Section 5 presents the notion of fractals and scaling laws as tools for discontinuously devised structures adapted to nature and human geography (the “law of 80-20”). Conclusion proposes a new agenda for future researches.

## 2. Neoclassical financial risk modelling: “we were wrong”.

It is a well-documented fact today that one of the central problems in the 2008 financial crisis lays in a specific pricing equation, a mathematical formula that priced credit default swaps (CDS), the financial products supposed to provide financial protection against default risks: the mathematical copula devised by David Li and known as “Li’s copula”. This formula was faulty, not in the sense that the risks were miscalculated, but in the sense that they were mismodelled. Now I elaborate on this.

## 2.1. The formula that killed Wall Street

There have been many debates about this equation, which a famous article by Felix Salmon called a “Recipe for disaster: The Formula That Killed Wall Street” (Embrechts, 2009; MacKenzie and Spears, 2014; Salmon, 2009). The devil, it is said, is in the details. Playing with the words, I would say that, in this case, the detail was in the D-tails, D for “distribution” tails. The tails of a probability distribution describe the behaviour of a random variable in the zone that is far from its central value. Distribution tails can be “thin” (indicating a very low probability) or “fat” or “heavy” (indicating the opposite: a very high probability). Li thought that the risks of simultaneous credit default could be modelled using a mathematical coupling function called a “copula”. But he chose to use a probability of joint default based on Gaussian distribution (a Gaussian copula formula). The cognitive framework of the Gaussian distribution underestimates the probability of rare events, and creates the illusion that the risk is under control. The Gaussian copula formula encouraged excessive risk-taking because it gave precisely that cognitive illusion (Walter, 2008).

In Walter (2013, 2020), I analysed in detail a movie that is crucial to grasping the epistemological issues of financial risk modelling, the Jeffrey Chandor’s movie, *Margin Call* (2011). According to the philosopher Stanley Cavell (2005), cinema can be for us a means of ethical teaching which does not use learned reasoning but only the power of images that touch us. The American philosopher Cora Diamond gave a reason for this: images can often better convey ethical thinking than reasoned reflections. Images touch on an infra-rational level. That is what we are doing now, considering a particular scene from the movie *Margin Call*.

This movie tells the story of the fall of a merchant bank. The faulty risk modelling problem is very well presented with great epistemological accuracy in one central dialogue: a conversation between the risk management officer (Sarah Robertson) and the head of fixed income securities (Jared Cohen). This dialog perfectly describes the epistemological and ethical issues carried by the probabilistic assumptions. The scene happens late at night, just after they have realized there is a problem with the mathematical risk assessment formula. They are beginning to assess the consequences of the wrong risk formula, in terms of potential losses for the bank. It is such an almost-perfect illustration of the difference between risk calculation and risk modelling that it should be part of every course aiming to teach financial ethics. I now quote the dialog. Robertson and Cohen discuss about Li’s formula:

- (Sarah Robertson) It’s legit... the kid killed it. The formula’s worthless.
- (Jared Cohen) What does that mean?
- (Sarah Robertson) It’s broken.
- (Jared Cohen) There are 8 trillion dollars of paper around the world relying on that equation!
- (Sarah Robertson) Well, we were wrong.

This dialogue illustrates that a mathematical equation, here Li’s formula, led to financial disaster because it was ill-conceived (“the formula’s worthless”). A simple, but ill-conceived, equation led to financial disaster.

In this sense, the financial crisis can’t be simplified in a “casino capitalism” approach, an expression which is a conflation of a particularly unhelpful kind (Cassidy, 2009) because it fails to help to understand the role of risk modelling issue. Similarly, the financial crisis can’t be reduced to a massive misjudgement about the consequences of financial innovation (Engelen *et al.*, 2012) because, if there was a massive misjudgement, it concerns, first, the risk modelling issue. In neoclassical finance, something has not worked, which is not only linked to the avidity or greed of financial actors, but to the mathematical structure of financial risk modelling, the practices and techniques of a given risk culture carrying financialized tools. It is therefore of primary importance to investigate what, in the neoclassical modelling of financial risk, was inherently dangerous for the sustainability of the economy. For any ecological financial theory will have to start by getting rid of these dangerous elements and replace them with models adapted to the characteristics of nature.

In the next section, I adopt a Foucauldian approach (from the French philosopher and historian Michel Foucault) with the use of two important notions, that of “discourse” and “dispositive” (see for example Larroche, 2019). I address the issue of the financialisation of economy through socio-technical devices (“dispositive”) and risk modelling theories (“discourse”) that legitimizes them.

## 2.2. The performativity of financial risk modelling: dispositive and discourse

The idea that mathematical or financial models have an influence on reality leads to a very rich current of thought on the sociology of financial models, in particular the social studies of finance approach and the performativistic framework. MacKenzie (2006) and MacKenzie *et al.* (2007) are a good introduction to it.

### 2.2.1. Socio-technical instruments: the financialised tools

An important aspect of the performativity of mathematical financial risk modelling is the socio-technical dimension of the mathematical models. Financial instruments derived from mathematical models and financialised evaluation play an important role in the financialisation of the economy (Chiapello, 2015, 2020; Chiapello and Gilbert, 2019).

To better understand how a particular risk culture is created based on a probabilistic hypothesis, it is interesting to note a detail of the dialogue between Sarah Robertson and Jared Cohen in *Margin Call*: Robertson's remark "we were wrong". The use of the word "we" denotes a form of socially elaborated and shared knowledge with a practical aim, which helps to construct a culture of models common to a financial group, the culture of "how model works" (Svetlova, 2018). The culture of models is an "epistemic culture" (Knorr Cetina, 1999) specific to each group of financial practitioners, but diffuses a general way of thinking about technical objects, especially for financial models that are overloaded with probabilistic techniques. This culture of models is based on calculation and quantification conventions (Desrosières, 2008; Mennicken and Espeland, 2019). Quantification conventions ensure the same risk culture for financial practitioners. For example, probabilistic techniques have had an enormous influence on risk assessment and the formation of quantification conventions (Chiapello and Walter, 2016). Much work has been carried out on the basis of these methodological premises and we take the liberty of referring the reader to the references indicated so as not to lengthen the text excessively.

### 2.2.2. Discourse of financialisation: the financial Logos

I will now present the discourse of neoclassical risk modelling I have called the "financial *Logos*". In Walter (2016), I proposed to consider risk models as a "speech act" in the Austin (1955)'s sense (see for example Brisset, 2018). For Austin, any statement must be considered an action, which he calls an "act of speaking", a "speech act". There are three dimensions to this act. A first dimension is that of the speech itself, the content of what is said. This is the dimension called "locutionary". The classic example is that of baptism. When the officiant pronounces the phrase "I baptize you", the words of this phrase represent the locutionary dimension of the speech. The second dimension expresses the desired effect of the speech, which is done in saying so. This effect is generally achieved if certain conventions are observed. In the case of baptism, the officiant must truly be a priest or pastor. Then, social conventions ensure that the baptism will have been performed. But it can happen that unexpected effects, either unforeseen or unwanted by those who play the social convention of baptism, may occur. For example, if someone in the assembly is unbelieving and suddenly becomes unhappy that a new human being is baptized, his anger is an unintended and unwanted effect of the baptismal act of speech. Obviously, this is not what was desired by the participants in the baptism. Austin calls these unintended effects of the speech act (in the sense that they appear when there was no reason to believe that they would appear) the "perlocutionary" dimension of the act of speech.

To sum up, when we talk, three distinct things happen. There is what is said (the content of the discourse), what is done in saying so (the social conventions that ensure the performative effect of the discourse), and what is done by the way of saying it (the unexpected consequences of performativity). These three dimensions – locutionary, illocutionary, and perlocutionary – characterize a speech act.

The proposal I made to analyse the 2008 financial crisis is to consider the neoclassical risk modelling as a speech act. The three dimensions of this speech act (the *Logos* of financial risk modelling) are then as follows. The locutionary of the financial *Logos* is the mathematical content of financial risk modelling. Here, it is the mathematical writing of the probable that constitutes the "speech" on risk. The discourse on financial risk is taking shape and becoming performative because international financial regulations and globally accepted quantification conventions impose it on players in the financial world.

Financial regulations and financial techniques represent the illocutionary dimension of the financial *Logos*. Finally, the 2008 crisis is an example of the unintended effect of the massive use of neoclassical risk modelling. This is the perlocutionary dimension of the financial *Logos* (Walter, 2020).

My proposal makes it possible to combine the mathematical aspects of financial risk modelling with the social and conventional aspects of this writing of the probable. It allows us to understand how a reality imagined in mathematical finance research laboratories takes shape in the real economy. It allows us to understand how probabilistic hypotheses have an effect on decision-making processes and risk assessment. It allows us to include unintended effects, in this case the consequence of the poor risk assessment of CDS, in the probabilistic speech act. It is in this sense that the probabilistic assumptions of neoclassical finance have made it possible to build a reality common to all financial practitioners.

### 2.3. Epistemic ethics and sustainable finance

This way of thinking the socio-technical influence of mathematical models raises ethical issues on epistemic responsibility in finance. This is because, as MacKenzie said at the end of *An Engine not a Camera*, “the notion of performativity prompts the most important question of all: what sort of a world do we want to see performed?” (2006, p. 275). This sentence echoes “what should be” from the ecological finance theory (Lagoarde-Segot and Martinez, 2020) and raises the question of epistemic ethics, i.e. taking into account the impact of the choice of model on the economy (De Bruin, 2015; Lamy, 2020). More precisely, the framing of decision-making by probabilistic assumptions exhibits a link between sustainable finance and financial risk modelling issues. It is in this perspective that it is interesting to read again the work of the French physicist Pierre Duhem in the early 20th century.

In *The Aim and Structure of Physical Theory* (1906), Duhem questioned the role of models in science. He puts forwards the following statement on the relationship between models and accidents: “We shall remind industrialists, who have no care for the accuracy of a formula provided it is convenient, that the simple but false equation sooner or later becomes, by an unexpected act of revenge of logic, the undertaking which fails, the dike which bursts, the bridge which crashes; *it is financial ruin when it is not the sinister reaper of human lives*” (our italics). One can see how this Duhem’s reflection is relevant to answer the ethical question raised by MacKenzie (2006) and to address the issue of action raised by Lagoarde-Segot and Martinez (2020).

I am applying Duhem’s reflection to financial risk modelling. Choosing a financial risk model means making an ethical choice. Li’s formula was convenient, but not relevant as regards the accuracy of the probabilistic assumptions in the case of risk modelling of neoclassical finance. Neoclassical financial risk modelling created the *hubris* and the decoupling between finance and economy, and gave rise to the 2008 crisis (Walter, 2008). The short-termism of finance is strongly linked to the probabilistic assumptions embedded in the technical objects of everyday financial practice.

## 3. Philosophy behind neoclassical finance

A very important aspect of the probabilistic assumptions of neoclassical finance theory is the continuity of the stochastic processes used in the construction of financial models. Neoclassical view of financial risk was based on continuous stochastic processes. One consequence of the use of continuous stochastic processes is the importance given to the average in neoclassical finance. Continuity and average represent two intellectual foundations of neoclassical financial theory.

The shift from continuous risk modelling to a discontinuous view with a possible abandonment of the use of the average as the benchmark, are important issues for the implementation of any sustainable finance and ecological finance theory. Let us elaborate on this point.

### 3.1. The principle of continuity and the theory of averages

Once again, I am relying here on previous work, with the aim of using these achievements so as not to lengthen the text excessively. I argued that one of the key issues of this problem can be illuminated by reference to a familiar debate in philosophy over the principle of continuity (Walter, 2019). Although this philosophical debate may seem to be a scholastic preoccupation within a tight circle of specialists in philosophy of science, far from the financial stakes of risk modelling and with no impact on concrete

financial practices, I argue, on the contrary, that the divergent positions about the mindset behind the risk modelling implicate entirely different views of what it is important to capture and how to model it.

The principle of continuity is a principle from natural philosophy postulating that in nature, things change gradually rather than suddenly. The most compact expression is found in the Leibniz' words *Natura non facit saltus* (meaning "nature does not make a leap"). This principle was the source of differential and integral calculus as performed by Leibniz, then Newton. Note the ambiguous status of this principle, which can be understood either as mathematical or metaphysical. It provided the foundations for the ideas of Carl von Linné on classification of species, and later Charles Darwin on the theory of evolution. It was then taken up by Alfred Marshall in his *Principles of Economics* (1890): "If the book has any special character of its own, that may perhaps be said to lie in the prominence which it gives to [...] the Principle of Continuity."

In the 20th century, both physics and genetics abrogated the principle of continuity. Quantum mechanics postulated discrete energy levels, and genetics took discontinuities into account. Yet economics, from which neoclassical financial theory originated, remained outside of these important intellectual transformations. Despite much evidence of economic phenomena that could not be explained by continuity, this principle remained in force until the end of the 20th century. Everything was going on as if, for mysterious reasons that had nothing to do with science, the principle of continuity was considered "natural" for finance and in any case preferable to discontinuous approaches.

The same reasoning can be held for the principle of average. Quetelet is credited with developing a theory of the average man in the 19th century. This theory had an extraordinary influence on statistics in the 19th century, and then in the 20th century. The index numbers, from which market indices are derived, are a consequence of this theory. The average has been used extensively in financial modelling, particularly in portfolio management. It has been used to define portfolio benchmarks. It has justified the use of passive index management. Since the 1980s, it has been known that the performance of a portfolio is poorly explained by its average, but the average has continued to be used as a management benchmark. Even if it was established that performance was concentrated on a small number of securities or days, the average remained the unassailable benchmark for measuring risk-adjusted performance. To repeat the same sentence as previously, everything was going on as if, for mysterious reasons that had nothing to do with science, the principle of average was considered "natural" for finance and in any case preferable to other approaches.

### **3.2. From philosophy to financial techniques**

The principle of continuity – change is continuous – permeated all neoclassical economic models, which was the source of (unsustainable) finance theory. The financial risk modelling is an application of this continuity principle. The formulae developed by Fisher Black, Myron Scholes and Robert Merton in 1973 and later the fundamental theorem of asset pricing of Harrison, Kreps and Pliska in 1979 and 1981, assume continuous price change. The principle of continuity was the cognitive representation that governed academics' intuition in the mathematical financial risk modelling, in their work and in their teaching of finance.

The principle of continuity was at the heart of the probabilistic assumptions in financial risk modelling. It contained methods of reasoning for financial practitioners derived from risk models based on the continuity assumption. For example, as MacKenzie and Spears (2014, p. 401) put it: "it is the strategy of Black-Scholes modelling writ large: find a perfect hedge, a *continuously-adjusted portfolio* of more basic securities that will have the same payoff as the derivative, whatever happens to the price of the underlying asset" (emphasis added). To say it differently, the principle of continuity was at the core of a large number of financial techniques. With a mental representation built on continuity, financial risk disappears since if prices change gradually and steadily, their future path is predictable and hedging can be found with financial derivatives, which are all based on the principle of continuity. The principle of continuity can be seen as the principle of the riskless economy. And an economy in which risk is thought to have disappeared is likely to become unsustainable. A riskless economy leads to a risk-free economy.

The academic trajectory of the theory of average in finance was no less than that of the principle of continuity. This theory was the basis of all portfolio management techniques. The formulae for obtaining

the mean-variance optimal of Harry Markowitz in 1959 and later the Capital Asset Pricing Model of William Sharpe in 1964 are based on the relevance of the averages. As with the principle of continuity, the theory of average was the cognitive representation that governed academics' intuition in the mathematical portfolio management, in their work and in their teaching of finance.

Some researchers considered that neoclassical economic theory had constructed a mathematical utopia of the market (Chen, 2017; Lawson, 1997, 2003]. I argue that the math choice was due to philosophical preferences. Hence, to shed a light to the Lawson (1997, p. 3) statement "contemporary academic economics is not in a healthy state" and to answer the question asked by Chen (2017, p. 17) "what's wrong with economic math", I would say: the presupposition of the principle of continuity coupled with the theory of average, the whole forming the Brownian representation of finance and financial risk. This allows me to introduce an important new notion, that of the cultural existence of two kinds of finance which I call "Brownian finance" and "non-Brownian" finance. These two kinds of finance define two "risk cultures" (Roeschmann, 2014), in the sense given in *Risk Culture's Critical Role in ERM*: "an organization's 'risk culture' is the way in which its management and personnel collectively perceive and respond to risk".

Just before turning to this presentation, I would like to point out that, as early as 1966, Wiener pointed out that "here some recent work of Mandelbrot is much to the point. He has shown the intimate way in which the commodity market is both theoretically and practically subject to random fluctuations arriving from the very contemplation of its own irregularities is something much wilder and much deeper than has been supposed, and that the *usual continuous approximations to the dynamics of the market must be applied with much more caution* than has usually been the case, or not at all" (Wiener, 1966, p. 92, emphasis added). Wiener's criticism shows that the issue of continuity and the relevance of the average in the presence of strong irregularities were raised at an early stage. Very early on, comments challenged Marshall's attempt to impose continuity as a basis for economic modelling. What is interesting about Wiener's comment is that he does so from knowledge of Mandelbrot's work. Indeed, Mandelbrot was the first to assert that continuity and average were dangerous for financial risk modelling, and all his later work was a development of how to take discontinuities into account. Before turning to Mandelbrot's conception of fractal-based financial risk modelling, let us look at how the two kinds of finance position each other and define two different risk cultures.

#### **4. Two risk cultures: the smooth and the rough**

Let us start with a remark by Keynes, to which we will propose an answer. In *The General Theory*, Keynes stated: "The classical theorists resemble Euclidean geometers in a non-Euclidean world who, discovering that in experience straight lines apparently parallel often meet, rebuke the lines for not keeping straight – as the only remedy for the unfortunate collisions which are occurring. Yet, in truth, there is no remedy except to throw over the axiom of parallels and to work a non-Euclidean geometry. Something similar is required today in economics." I would now like to respond to Keynes's call and to do "something similar" in finance, a change of geometry for financial risk modelling leading to a new risk culture. This is also an answer to the 2008 call of IIF Final Report on Market Best Practices for Financial Institutions and Financial Products: "Cultivation of a consistent 'risk culture' throughout firms is the most important element in risk management" (Green and Jennings-Mares, 2009; Levy et al., 2010).

##### **4.1. Brownian finance and non-Brownian finance**

It is well known that Euclidian geometry is well adapted to a flat world. But, as space-time is not flat, space-time geometry is not Euclidian. A non-Euclidian geometry is geometry of curved space-time. This type of geometry has become essential for incorporating the effects of gravity when measuring radio wave frequencies: for example, the atomic clocks of the satellites in the Global Positioning System (GPS) need adjustments due to the Earth's gravitational field. In the same vein, financial risk modelling based on the principle of continuity is devised to manage risk in a flat world. In a flat world, risk has only one dimension, the volatility. Risk assessment, risk management and risk control are designed to tame and to mitigate volatility. But in the real world, there is another dimension to the risk, the intensity of jumps, creating discontinuities and roughness on the paths of the economic variables. For taming the jumps, risk modelling needs to "throw over the axiom" of neoclassical finance: the "flatness" of risk.

To incorporate the effects of non-flat world, it is necessary to use curved risk geometry, i.e. a new geometry including roughness at the heart of the risk models. This new risk geometry defines a new finance. Just as Euclidian geometry and non-Euclidian geometry both exist, I define “Brownian finance” and “non-Brownian finance” which both exist and I suggest that non-Brownian finance is just as important for modelling durable finance as non-Euclidian geometry is for modelling durable travel. Now I'm clarifying these notions.

By “Brownian finance”, I mean a framework to risk modelling in which the dynamics of price variations uses a motion (stochastic process) devised in 1827 by the Scottish botanist, Robert Brown named for this reason as “Brownian motion”. Although in an implicit form and not quoted as such, the Brownian motion was introduced into finance by Louis Bachelier in his 1900 thesis on *Theory of speculation* (David and Etheridge, 2006). Brownian motion then appeared in physics with Einstein in 1905 and was then mathematically described by Norbert Wiener in 1923. The Brownian motion became the basic component of all equations of financial dynamics starting in the 1950s. With the Brownian motion, financial engineering was able to develop sophisticated calculation techniques to evaluate the price of derivative products and resolve the question of contingent claims (“options”) raised by Condorcet in 1782 (Walter, 2013, p. 233). The solution to Condorcet’s problem was the nobelized work of Black, Scholes and Merton in 1973. The Fourier heat equation was used to find the price of an option. Brownian finance is fundamentally smooth finance. There is no roughness in the paths, no fractures in markets and no financial crises. Brownian finance paints a picture of a regular, reassuring risk. Risk definitely exists, but is not too unpredictable because it is smooth. Brownian finance is dangerous for the real-life economy because the illusion of continuity can give birth to financial *hubris*: an excessive confidence in smooth risk modelling. The risk culture resulting from the use of Brownian finance does not allow us to imagine financial accidents. Brownian finance is unsustainable for this reason.

By “non-Brownian finance”, I mean the opposite. Non-Brownian finance describes a framework in which the roughness dominates, in which stock market paths are made of jumps at all times and scales. Non-Brownian finance incorporates jumps creating the “depth” of space, i.e. the depth of the risk. In non-Brownian finance, risk is perceived as “rough”, like the curved space of non-Euclidian geometry. In non-Brownian finance, risk has “depth” and “relief”, preventing the belief that it can be controlled simply by calculating statistical indicators, such as volatility. When the jump occurs, linear forecasts are found wanting. The stochastic processes of non-Brownian finance differ from Brownian motion as non-Euclidian geometry differs from Euclidian geometry. Non-Brownian stochastic processes paint a very different picture of risk, with much more surprises, as the path of the asset “jumps” all the time. The first stochastic processes that gave rise to non-Brownian finance were conceptualised by French mathematician Paul Lévy in 1924 and are named for this reason “Lévy processes”. Even if the time length between two points on a Lévy path decreases towards zero, the discontinuities remain. Lévy processes have discontinuous paths. A more intuitive way to express this is to say that the paths “jump” all the time, whatever the scale of analysis. Roughness is the essence of non-Brownian paths. The culture of risk stemming from the use of non-Brownian finance encourages caution at all times, and leads us to consider the financial fragility of the systems to ensure their long-term survival.

Benoît Mandelbrot introduced Lévy processes in finance in the 1960s. In a nutshell, in the 20th century, the two scientists who gave birth to the two views of finance (flat and non-flat), the two risk cultures in finance, were Bachelier and Mandelbrot.

To perceive visually and in a direct intuitive way the difference between Brownian motion and Lévy processes, just consider the charts of the paths. Although both show irregular moves up and down, there is a clear visual distinction between the paths followed by the two processes. Although both trajectories have irregularities in upward and downward motion, unpredictable up and down, the big difference between Brownian motion and Lévy processes lies in the visual aspect of the trajectory. It appears smooth in a Brownian motion, and rough in a Lévy process. Moves look “smoother” in Brownian motion than in a Lévy process, which have “jumps” or “discontinuities” almost everywhere at every scale. Following Mandelbrot’s vocabulary, the difference between these two kinds of finance, between these two risk cultures, is the difference between “smooth” and “rough”.

Playing with words, I say that that Brownian finance is a “Brown finance” i.e. a “brown finance” as opposed to “green finance”. Hence, I argue that a key condition for the making of a sustainable finance

(“green finance”) is to change the risk culture, i.e. to remove all traces of continuity (“brown finance”) from the today financial system. Then it will be a matter of building non-Brownian finance. To say it differently, the global conceptual framework of finance needs to be thoroughly reviewed in order to “detoxify finance” from the Brownian representation of financial risk modelling. The risk culture must be emptied of the Brownian representation of risk.

Coming back to Keynes’ statement (“the classical theorist (...) rebuke the lines for not keeping straight”), it is possible to show that a similar phenomenon occurred in financial risk modelling with Brownian finance. For forty years, academic research into financial risk modelling deliberately ignored the jumps, posing as an axiom a continuity assumption supposed sufficient to understand the nature of financial risk (De Bruin and Walter, 2017). Without any validated tests, Brownian motion has become the paradigmatic stochastic process used in neoclassical financial risk modelling and built a risk culture that ignored the real risks. For this reason, the persistence of Brownian motion in financial risk modelling can be considered as an epistemological puzzle (Walter, 2019). This is an intriguing fact to be explained by sociologists.

## **4.2. The risk culture of Brownian finance and its pitfalls**

A good example of the need for detoxifying finance is made of the prudential regulation rules. It is known that multilateral institutions have acknowledged the need for a profound reform of the global financial system with emergence of Principles for Responsible Banking (PRB), Principles for Sustainable Insurance (PSI) and Principles for Responsible Investment (PRI). Although not mentioned in the previous list, responsible regulation is at least as important as responsible banking or responsible investment, sustainable regulation is at least as important as sustainable insurance. But the prudential regulations established after the 2008 crisis have had unexpected effects, just as dangerous as the ones they sought to address. The risk culture of Brownian finance is indeed not adapted to protect against financial accidents, and may even, through the regulation it carries, cause them. Black swans and extreme values can thus be understood as effects of the regulation of Brownian finance.

### **4.2.1. Brownian regulation and financial Black swans**

Official reports have shown that international regulation has pro-cyclical effects and creates potentially more dangerous financial situations than those that prevailed before the regulations were put in place. In recent work, it has been shown that neoclassical financial regulation has the potential to create the crashes it seeks to avoid and that there is therefore a “regulation risk” (Le Courtois *et al.*, 2020). This is because neoclassical regulation uses the Brownian risk culture to assess the capital requirement. This is not trivial and the reasons for this are explained now.

The objectives of international prudential regulation are to protect financial institutions from failures that could cause a damaging systemic risk to the economy. To this end, international prudential regulation has established rules for calculating the minimum capital that financial institutions must have available to face market shocks. These rules have been formulated by the Basel Committee for the banks and by the European regulatory authorities for the insurance companies: respectively Basle III and Solvency II. The problem was that the rules for calculating the capital requirement, e.g. Solvency Capital Requirement (SCR), had to use probabilistic assumptions to extrapolate the 10-day SCR from the 1-day SCR. The prudential rule says that the capital requirement for a 10-day horizon has to be calculated as the capital requirement for a 1-day horizon multiplied by the square root of 10. This is the “square-root-of-time-rule” for calculating minimum capital. This rule comes from the Brownian representation of financial risk modelling. The reasons for this extrapolation rule come from the properties of Brownian motion and will be explained below. To say it differently, Brownian risk culture permeates international prudential regulation. The connection between mathematical assumptions (the square-root-of-time-rule) and the regulatory framework can be thought as the effects of the quantification convention that structures the world of neoclassical finance.

One of the consequences of the use of Brownian finance is the inability to deal with the problem of extreme values and the resulting intellectual cleavage in the appreciation of complex situations. A very good example of this inability to properly understand what is at stake is given by Alan Greenspan’s editorial (16/03/08) in the *Financial Times* about the 2008 financial crisis: “we will never be able to

anticipate all discontinuities in financial markets”. Greenspan cannot imagine discontinuities being incorporated into a probabilistic description of the market. For Greenspan, (financial) nature does not make a leap and discontinuities are just unthinkable. But then how to consider the discontinuities? The answer is given by Nassim Taleb (2007): discontinuities are unpredictable “black swans”. Le Courtois *et al.* (2020) have shown that financial regulation of Brownian finance could create market crashes. This is because the risk culture of Brownian finance prevents us from understanding the influence of continuity on the synchronisation of financial risk management practices.

We said earlier that the neoclassical representation of financial risk could be seen as the locutionary act of the financial *Logos* of neoclassical finance. Regulations that ensure the use of neoclassical probabilistic assumptions for risk measurement can be considered as the illocutionary act of financial *Logos*. Similarly, the production of unintended effects of regulation is akin to the perlocutionary act of financial *Logos*. Thus my proposal is to interpret financial black swans as unexpected effects of regulation in Brownian finance (Walter, 2020).

#### **4.2.2. Extreme values and irrational exuberance**

Compared to the previous analysis which sees financial black swans as a consequence of Brownian regulation, Greenspan and Taleb’s analysis sees them as unpredictable events that just fell out of the sky. Discontinuities are outliers. Discontinuities are assimilated to disordered movements of markets subject to irrational exuberance. In fact, one of the cognitive consequences of the Brownian-based risk culture is the truncation of financial time series into “normal” periods (continuous “rational” market) and periods of “insanity” where markets are deemed “irrational” (discontinuous “irrational” market). This dichotomy leaves the practitioners unable to explain the transition from one period to another. In Brownian finance, the only way to add jumps is to combine a continuous stochastic process with a stochastic jump process. Brownian finance needs the use of extreme value theory to be able to tackle the jump problem of financial risk. This is why, starting in the 1990s, extreme value theory (EVT) was rediscovered in financial risk modelling. For some two decades, the only way to incorporate discontinuities into financial risk modelling was to use EVT. But it is also clear how the integration of extreme value theory does not solve the problem of discontinuities at all scales of observation. Only large discontinuities are taken into account, and it is assumed that small discontinuities can be assimilated to a continuous path.

The risk culture of Brownian finance prevents itself from conceiving that small jumps are present on small scales. The assimilation of small jumps to quasi-continuity makes it impossible to measure the fragility of the market. Small jumps make it possible to evaluate the erratic structure of a stock market path, and give an indication of its irregularity. Considering small market jumps as “micro-crashes” makes it possible to try to anticipate the occurrence of large jumps, which are, those, dangerous. Small jumps are like weak signals in the structure of financial risk. The risk culture of Brownian finance ignores small jumps because it considers them to be unimportant. In doing so, it must add extreme values to account for discontinuities.

To say it differently, the hinge of my alternative can be summarized as follows. There are two fundamentally different risk cultures for financial risk modelling. The first is “Leibnizian”: it takes the continuity as a cornerstone for financial risk modelling. In this philosophical framework, following Bachelier’s (1900) thesis, risk is modelled by continuous diffusion processes and jumps (market crashes, financial accidents etc.) are added to this main process as part of another stochastic jump process. This is the “Bachelier legacy” which leads to a “diffusion + jumps” conceptual framework, a “smooth” risk culture. On the contrary, “anti-Leibnizian” position holds that discontinuity exists at all scales, even very small scales. The presence of discontinuities at all scales, even micro-scales (micro-crashes), allows grasping the profound nature of financial risk. According to this view, financial risk modelling is described by discontinuous pure jump processes, as for instance Lévy processes. This is the “Mandelbrot legacy”, a “rough” risk culture. Mandelbrot’s pivotal move of the 1960s consisted in conceptualizing the discontinuity of price changes as tenet problem for financial risk modelling.

Now is the time to turn to the work of Benoît Mandelbrot.

## 5. A possible solution for the sustainable modelling of financial risk: fractal geometry

In a very influential book published in 1982 that had considerable audience, *The fractal geometry of nature*, Benoît Mandelbrot introduced the idea that one of the main characteristics of “nature” was that it possessed so-called “scaling laws” (or “power laws”), a property related to the fractal structure of nature. The word “nature” designates both physical geography and human geography. Mandelbrot's idea was that many natural patterns could be described mathematically as fractal.

### 5.1. The “law of 80-20” and fractals in the real world

There are so many references to fractals today that it is out of the scope of this paper to give even a glimpse of them here. Only the main characteristics are presented here with the aim of introducing the usefulness of fractal geometry for the building of ecological finance theory.

#### 5.1.1. Pareto distributions, scaling laws and scale invariance

A convenient way to get into fractal geometry is to start with what everyone knows and experiences every day: the adage that “very few have much and many have very little”. This concentration of a phenomenon on its extremes can be found everywhere in the geosciences and in the real economy. The Pareto distribution of income (the so-called “law of 80-20”) is a canonical example of this. The “law of 80-20” states that for many phenomena, about 80% of the total (e.g. total portfolio gain, liquid mass of water on planet Earth) comes from about 20% of the population (e.g. very few stocks in the portfolio, very few oceans or large lakes on planet Earth). There are so many natural or economic phenomena that follow this law that it is impossible to give even an initial reference list here. What all these phenomena have in common is that very few oceans or large lakes concentrate most of the water on Earth, very few titles concentrate most of the total gain, but also very few scientific articles concentrate most of the quotations, very few words are found very often in speeches (the so-called Zipf's law), very few books are sold in very large numbers, and so forth (Newman, 2005; Redner, 1998; Zipf, 1949). The Lorenz curve represents this phenomenon on a graph whose coordinates are the percentage of the population concerned and the percentage of the total obtained.

Let us express this phenomenon of concentration by describing it with the cumulative distribution function of the population under consideration. To understand it intuitively, one has to ask oneself the question: how many values are greater than or equal to a given value? Using probabilities, the question is  $\Pr(X > x)$  for  $x > a$  where  $a$  is a given threshold.  $\Pr(X > x) = 1 - F(x)$  is the opposite of the cumulative distribution function (also called survival function or tail function). In the case of a phenomenon with a high concentration of the total on the largest values of the distribution beyond a given threshold, we find that  $\Pr(X > x) = (a/x)^\alpha$  for  $x > a$ . We find a power law. The name of this function comes from the fact that one of the quantities (here the cumulative empirical frequency) varies as the power of another (here the values exceeding the threshold). This is the Pareto type I distribution. A very large number of situations are possible, especially with other power laws, but we don't intend to make the presentation more complicated here. Turning now to the empirical data and empirical frequencies, a rank-frequency diagram allows us to visually capture a characteristic property of Pareto's power laws. The straight line of the log-log plot has the slope  $-\alpha$ . In fact, when logarithms are taken instead of raw values, if  $Y = aX^{-\alpha}$ , then  $\ln Y = -\alpha \ln X + k$ . We deduce from this that the “signature” of a power law is linear relationship between the two quantities in log-log diagram.

The parameter  $\alpha$  is the exponent of Pareto's law, the slope of the log-log line, and indicates the degree of concentration of the distribution. The value of the parameter  $\alpha$  determines the existence of the moments of the distribution. The variance (central moment of order 2) exists if and only if  $\alpha > 2$  and the mathematical expectation exists if and only if  $\alpha > 1$ . If  $\alpha < 2$ , the variance does not exist (it is infinite). If  $\alpha < 1$ , the mathematical expectation does not exist (it is infinite). It can be seen from this elementary example that, in the case of high concentration distributions, the variance, or mathematical expectation, may not exist. Averages can always be calculated, but since the theoretical values of moments are infinite, the averages will be unstable or diverge. And they will be sensitive to the larger values of the distributions. Thus, Pareto's distributions invalidate the simple use of Quetelet-style averages in risk modelling.

Power laws have the property to be “scale invariant”. That means that scaling the quantity by a constant  $c$  multiplies the power-law relation by  $c^{-\alpha}$ . If the scale invariance property is validated, objects do not change if scales of length are multiplied by a common factor. It represents a kind of universality. Like Russian dolls, the shape of the object repeats itself at all sizes. It is said that there is a self-similarity of the phenomenon being studied. So there is a relationship between the law of power and self-similarity. Here again, in practice, there are several forms of self-similarity, but we do not want to complicate the presentation in the context of this introductory article. This power law property / self-similarity seems to be the signature of “how nature works” (Bak, 1997), like a “law of nature”, a “life law” (West, 2018; West *et al.*, 2004). Power laws appear widely in many fields from physics to geosciences. They are found everywhere from molecules to trees and forests. This is the reason why applications of fractal modelling in the geosciences are huge (Gao and Zong-Guo, 1996; Ghanbarian and Hunt, 2017; Puente *et al.*, 2017).

Some criticisms and controversies about the universality of fractal geometry addressed the issue of limited scaling ranges of data, which could invalidate the global fractal approach ad infinitum (Avnir *et al.* 1998; Shenker, 1994). The answer to these criticisms was to say that the existence of a scaling range defines the validity of fractal approach for properly describing a given phenomenon (Mandelbrot *et al.*, 1998).

### 5.1.2. Irregularity as universal pattern

Let us now move from a Paretian representation of unequal distributions of static phenomena (oceans, book sales, etc.) to the analysis of a dynamic phenomenon with the same concerns. Let us consider the values, no longer of record sales or the liquid volume of lakes, but of variations in a quantity that we follow over time (rainfall, river level, wind speed etc.). We will find on these variations the same characteristics as on the static samples: concentration of the total variation on the strongest variations. If we then consider the shape of the trajectory examined, we can guess that the more the variations are concentrated on a few very large values, the more *irregular* the trajectory appears. It can be seen that the irregularity is closely related to the concentration property, and that the greater the concentration, the greater the irregularity will be, i.e. (in the previous example) the parameter  $\alpha$  is small. In the absence of variance, and even a mean, the irregularity will be extremely large. At certain moments, the path will give the impression of “jumping” strongly, and large discontinuities will appear. There is therefore a relationship between concentration, scaling laws, discontinuity and irregularity.

It is the origin of the word “fractal”. The word “fractal” is a neologism created by Mandelbrot in 1974 from the Latin root “*fractus*”, which means broken, irregular. In a nutshell, Mandelbrot’s idea was that fractal geometry describes well the irregular face of “nature”. Using fractal geometry, Mandelbrot argued, the irregular natural objects, once considered unmeasurable, could now be approached in rigorous and vigorous quantitative fashion: “clouds are not spheres, mountains are not cones, and lightning does not travel in a straight line”. Hence, “the complexity of nature’s shapes differs in kind, not merely degree, from that of the shapes of ordinary geometry, the geometry of fractal shapes” (Mandelbrot, 1967, 1982). A fractal pattern, or “fractal”, is in first approximation a curve, a surface, a volume of irregular or fragmented shape that is created by following deterministic or stochastic rules involving internal homothety. A fractal is created from an initial shape that is fragmented into small pieces. Infinite repetition (iteration) is one of the essential aspects of fractal geometry. More generally, a fractal designates objects whose structure is invariant by change of scale. Fractality is another way to understand scale invariance.

The infinite repetition of the same pattern is a central feature of fractals and is related to the notion of self-similarity. By zooming in or out on the object under examination, its structure is invariant. The whole looks like the part, which itself looks like a smaller part. The pattern repeats itself ad infinitum. A self-similar object is an object which is exactly or approximately similar to a part of itself (i.e., the whole has the same shape as one or more of the parts).

It seems that approximate fractal shapes are easily observed in nature. Fractals occur in an extraordinarily diverse range of phenomena. The description of nature by fractal geometry provides information in several scientific fields such as geology (study of relief, coasts and rivers, rock structures, avalanches), meteorology (clouds, vortexes, pack ice, rogue waves, turbulence), volcanology (prediction

of volcanic eruptions, earthquakes), astronomy (structures of the universe, craters on the Moon, distribution of galaxies), human geography (urban structure, changing demographics), and economics (prediction of stock market crashes). In all these fields, scaling laws appear as the property of extremely irregular or fractured objects, irregularity visible at different scales.

## 5.2. Fractalisation of financial risk modelling

To summarize, if the relevant description of nature is fractal, then financial risk modelling will have to be anchored on fractal representations in order to be in line with nature and become sustainable. As Mandelbrot argued, “even when the present fractal models become superseded, fractal tools are bound to remain central to finance”. He added: “the reasons are that the main feature of price records is roughness and that the proper language of the theory of roughness in nature and culture is fractal geometry” (Mandelbrot, 2005). The fractalisation of financial risk modelling becomes the new frontier for ecological finance theory. If price changes have fractal properties, then it is a matter of matching these fractal properties to the fractal properties of nature. In this sense, financial fluctuations will be aligned with changes in the natural quantities of the economic, geographic or human environment.

Let us recall again that it is precisely because of the misalignment between the fractal properties of nature and the non-fractal descriptions of financial risk that the disconnection of finance from the economy was made possible. This disconnection came about because we wanted to cover the rough with the smooth. At the risk of insisting even more, let us say that financial *hubris* stems from the desire to hide the roughness with smoothness.

### 5.2.1. How to fractalise financial risk

How to fractalise the financial risk modelling? Anew, the question is so vast that it would be impossible here to even summarize the work that has been undertaken on this subject. To paint a general picture, let us say that, in order to model financial risk as fractal, we must quantify the price changes with fractal properties. The simplest way to consider price changes is to use the distribution of changes  $\Delta X$  over a given time scale  $\Delta t$ . *If price fluctuations can be described by fractal properties, then we need to search for fractal properties on the distributions of price changes  $\Delta X$  at different scales  $\Delta t$ .* The statistical structure of price change  $\Delta X$  has to be described at different scales (small and large  $\Delta t$ ). To be accurate, let us say that  $\Delta X(t)$  is the log-difference of prices  $S$ , i.e.  $\ln S(t) - \ln S(t-1)$  for normalized time intervals. To say it differently,  $X(t)$  is the continuous log-return of the stock price  $S$  between time 0 and time  $t$ , i.e.  $X(t) = \ln S(t) - \ln S(0)$ . In the standard model of stock market variations (neoclassical finance theory),  $X(t)$  follows a Brownian motion with trend  $\mu$  and standard deviation  $\sigma$ , which is the “volatility” of price fluctuations. The standard model of stock market variations is  $X(t) = \mu t + \sigma W(t)$  where  $W(t)$  is a Wiener process i.e.  $W(0) = 0$  and  $\mathbf{E}(W_1) = 1$ . In the standard model of stock market variations, the “square-root-of-time” rule describes a scaling law on volatility. This scaling law allows annualizing the volatility from monthly measures of empirical volatility.

More generally, the issue arises of moving from one time horizon  $t$  to another time horizon  $at$  i.e. a time horizon which is a multiple of  $t$ . The mathematical formulation of this problem is as follows. Let  $\lambda(t)$  be a parameter of the distribution of  $X(t)$ . The objective is to determine the parameter  $\lambda(at)$  of the distribution of  $X(at)$ . Among the parameters useful to qualify the risk of a distribution are the moments of order  $k$ , the  $k$ -th moments. The moments give an idea of the shape of the distribution (flat, peaked, symmetric, asymmetric etc.). In particular, if  $m_k(X, t)$  is a central moment of order  $k$  of the random variable  $X(t)$ , the presence of a scaling law on  $X(t)$  implies the existence of a scaling law on moments. Hence the issue is to obtain the central moments  $m_k(X, at)$  at a scale  $at$  knowing the central moments  $m_k(X, t)$  at scale  $t$ . This is given by using the convolution product of the distributions of random variables  $\Delta X$ . And so we see that the problem of the scaling laws of financial risk is closely linked to the issue of scale invariance of risk: is the shape of the distribution the same on different time scales? If the answer is “yes”, hence, there is a fractal property of financial risk (Walter, 2009).

In Brownian finance, as the distribution used is Gaussian, there is a fractal property of volatility: the shape of financial risk remains Gaussian at every scales. This is the fractal property of Brownian motion. It means that, implicitly but clearly, in neoclassical finance, risk is assumed fractal because the volatility is supposed to follow the celebrated “square-root-of-time-rule” (Danielsson and Zigrand,

2006). This represents the time-scaling of risk in neoclassical finance. The “square-root-of-time-rule” in finance states that the annual volatility is given by the monthly volatility multiplied by the square root of the duration measured in months, i.e. 12. More generally, in neoclassical finance, the risk is scaled by the square root of the time horizon. This relation describes a scale invariance property of the Brownian motion used in neoclassical risk modelling. The annual risk has the same shape as the monthly risk: a Gaussian distribution. It can therefore be seen that the fractality of risk is not in itself sufficient to align finance with nature, since the equations of neoclassical finance contain a fractal property. Some fractals are relevant, some are not. The fractalisation of risk modelling has to deal with the determination of relevant fractals.

The presence of discontinuities at every scale in financial time series gives a light on this puzzle. Fractals built with a Gaussian base are not rough enough. It is necessary to capture the roughness of stock market paths to avoid financial *hubris*. Examples of rough fractals with discontinuities can be found in nature. It is then a matter of transposing the fractal characteristics of nature to the fractal characteristics of financial risk. A risk fractality adjustment can be made by measuring the fractality of the real phenomenon being investigated in the economy. For example, in the case of the financial valuation of a “green project”, in order to take into account the fractal nature of the project's risks for its funding, the fractal characteristics of the environment will have to be measured, and then a new measure, a “green metric”, will have to be defined for the valuation of the project. In this case, it will not be a question of modifying the discount rate or the numbers of the future values and future risks of the green project, but of modifying the calculation technique itself, independently of the parameters chosen to feed the valuation models. A green metric is not a neo-classical metric greened by green parameters, but a metric created with entirely new fundamental principles based on the fractal characteristics of nature. We will return to the issue of green metrics below.

To give a first and elementary idea of what the fractalisation of a risk can be, let us consider the relation of neoclassical finance on the central moment of order 2, the variance. The variance is – according to the definition of the Brownian motion – linear in time, we have:

$$m_2(X, at) = a \times m_2(X, t)$$

From which comes the “square-root-of-time-rule” on volatility which is:

$$\sigma(X, at) = \sqrt{a} \times \sigma(X, t)$$

This is the reason why:

$$\text{volatility}(X, \text{one year}) = \sqrt{12} \times \text{volatility}(X, \text{one month})$$

Considering the quantity called “Value-at-Risk” (VaR), we will have in the same way:

$$\text{VaR}(X, 10 \text{ days}) = \sqrt{10} \times \text{VaR}(X, 1 \text{ day})$$

These are exactly the capital requirement calculations recommended by the Basel III and Solvency II regulations. We see how neoclassical finance unconsciously uses fractals. But not rough fractals: smooth fractals.

If now the fractal characteristic of financial risk is not the one defined by the fractal properties of Brownian motion, we can consider that there will be another scaling-law on volatility, which will be:

$$\sigma(X, at) = a^H \times \sigma(X, t)$$

$H$  represents the fractality exponent.  $H$  will be obtained by the “natural” properties of the object (for example properties given in geophysical records) whose financial risk is to be modelled. The “square-root-of-time-rule” would be in this case a “time-power- $H$ -rule”. In the simplest case,  $H$  is a constant. But we can alternatively have  $H(t)$  which would mean that the exponent of fractality varies with time:

$$\sigma(X, at) = a^{H(t)} \times \sigma(X, t)$$

The scaling property is not the same as previous. As we can see in this elementary overview, introducing fractality in a financial risk modelling is like transforming some important constants of neoclassical finance into variables that introduce roughness where previously there was smooth.

In most cases, and in particular in the basic example given above, the simple fractals corresponding to the first Mandelbrot models are not sufficient to account for the complexity of the phenomenon to be modelled. Let us give a brief overview of Mandelbrot’s models. Mandelbrot developed his fractal models in two strands of papers. A good introduction to the usefulness of his models for finance is given in *Fractals and Scaling in Finance* (Mandelbrot, 1997). Let us just quickly say that he Mandelbrot new

way of modelling price changes was based on concentration, discontinuities, scaling laws and time-change processes. Mandelbrot designed several models of fractals for finance, and presented them in several overlapping layers of articles or chapters, often intertwined with each other, with back and forth between models through time. So the financial fractal landscape is quite complicated to explore (in a sense, it is itself fractal).

To put it in a nutshell, there were two distinct problems to deal with: that of heavy distribution tails (high probability of large risks) and that of long-range dependence (long memory of stock market movements). Mandelbrot's early models dealt with the two problems separately. The 1962 model resolves the issue of heavy distribution tails but within a framework of independent market movements, so the question of long dependence remains. The 1967 model deals with the question of long dependence but in a Brownian framework, so the question of heavy non-Gaussian tails remains. Finally, in a third model in 1972, Mandelbrot tried to combine the two problems, that of distribution tails and that of long dependence, by assuming that market time did not follow the time of the calendar clock. Market time was like an intrinsic time, a time different from physical time. Using multifractal time, Mandelbrot showed that it was possible to model financial risk with a Brownian motion transformed by a time-change. Instead of  $X(t) = B(t)$  where  $B(t)$  is a standard Brownian motion, we have  $X(t) = B(T(t))$  where  $T(t)$  is the trading time. We then introduce a subordinated stochastic process. This third model represents the transition from unifractal models to multifractal models. Finally, these ideas were synthesized in a series of three articles published in 2001, which represented Mandelbrot's comeback to the financial risk modelling arena. A summarized mathematical presentation of these three models can be found in Walter (2015) and a historical overview of the "Mandelbrot's programme" can be found in Walter (2017).

### **5.2.2. Green metrics and green premium puzzles**

At this point, it is important to emphasise something non-trivial in order to fully understand how ecological finance theory is different from green finance. The neoclassic way to evaluate a green project is to change the parameters of the evaluation by adding green attributes. This is a financial strategy that nudges the market by "greening" neoclassical metrics. This way of doing things has been noticed in Lagoarde-Segot and Martinez (2020, n. 14), where it is said that recent research on "sustainable finance" superimposes a neoclassical financial risk modelling to the analysis of sustainability concerns. This kind of sustainable finance analyses new ecological concerns through the lens of technical tools based on Brownian finance, rather than questioning the foundations of neoclassical risk modelling to respond to the sustainability challenge. It is important here to clearly understand that the fractal way of risk modelling is not a neoclassical risk modelling "tinkering" with "green parameters", but a new epistemological framework to design new tools and new regulations built for new ecological stakes. The "green bonds premium puzzle" (Bachelet *et al.*, 2019) can be seen as a trace of this confusion between neoclassical greened metric and fractal metric.

Coming back to geosciences and giving just one example, it is interesting to observe that multifractals help to model geophysical records (Ghanbarian and Hunt, 2017). We can therefore see how the fractalisation of financial risk can take place. From the geophysical records, we capture the multifractal characteristics of the geophysical phenomenon and we can then model the financial risk in harmony with these characteristics.

## **6. Conclusions and future research**

Sustainable financial risk modelling has to take account of a specific "signature" of nature, its fractal geometry. Fractals and scaling laws seem tools for discontinuously devised structures adapted to nature and human geography (the "law of 80-20"). Assuming that nature is properly described by fractal geometry, hence sustainable financial risk modelling has to include fractal properties in the mathematical structure of models. I started from this statement to establish a new perspective on the 2008 crisis and unsustainable finance, by proposing to consider that this unsustainability was due to the lack of consideration of fractality in financial risk modelling, creating a faulty risk culture. I have defined the main problem of neoclassical financial risk modelling as being due to the widespread use of the continuity principle and the theory of averages, i.e. two non-fractal representations of nature. I have

enlightened this puzzle from a philosophical perspective, exploring the presuppositions at work in finance from Leibniz's principle of continuity and Quetelet's theory of averages, and making them ubiquitous mental models. Next, moving to a cultural approach to these issues, I followed Keynes' point of view on the two geometries in economics, specifying it in terms of two geometries of financial risk. I have defined two risk cultures as two types of financial risk geometries: Brownian finance and non-Brownian finance. Finally, starting from a simplified presentation of the fractals, I tried to lay the ground for the fractalisation of financial risk modelling with the objective of contributing to future development of Ecological Finance Theory by providing a first insight in the usefulness of fractals for SDGs.

It is not possible in the context of this paper to present in detail how we get from the fractal geometry to sustainable financial risk modelling. I simply wish to set out an agenda for future research by discussing what the main steps of this trip might be. My proposition is as follows:

1) To verify how good is fractal representation to model important features of physical and human geographies in the sense of SDGs.

2) If a relevant financial risk modelling should take these fractal characteristics into account, to seek to construct "green metrics" in the fractal sense to lay the groundwork for ecological finance theory as an alternative of neoclassical finance.

3) To apply these new metrics to portfolio management and risk measurement techniques, by following the tracks indicated in Walter (2013).

4) To introduce financial risk modelling issues and metrics into a philosophical reflection on epistemic ethics because financial models and financial risk modelling contribute to shape the real world. This last issue is important keeping in mind that ecological finance theory aims to move from "what is" to "what should be".

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