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# Regulation risk

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## Abstract

Market risk regulations adopted in response to recent crises aim to reduce financial risks. Nevertheless, a large number of practitioners feel that, if these rules seem to succeed in lowering volatility, they appear to rigidify the financial structure of the economic system and tend to increase the probability of large jumps: prudential rules seem to produce an unexpected effect, the swap between volatility risk and jump risk. The aimed reduction of volatility is accompanied by an increase in the intensity of jumps. The new regulations seem create a new risk. This paper discusses this idea in three ways. First, we introduce a conventionalist framework to put some light on this unexpected effect. Second, we precisely define volatility risk and the intensity of jumps to document the risk swap effect by analysing a daily time series of the S&P500. Third, we propose a model which allows one to appreciate a practical consequence of this swap on the risk measures. We conclude by challenging the main objective of regulation: we argue that concentrating on reducing the sole volatility can create a new type of risk which increases the potential losses, that we term regulation risk.

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## Introduction

The purpose of this paper is to explore a possible inverse relationship between volatility and erraticity as it appears in recent years, and, following [41], to suggest an unexpected influence of regulation for explaining this inverse relationship.

In the literature, “regulatory risk” has multiple definitions. A 2014 research report of the joint Risk Management Sections of Casualty Actuarial Society, Canadian Institute of Actuaries and Society of Actuaries on “Regulatory Risk and North American Insurance Organizations”[24] proposes a survey of many instances, which are related to a multitude of bodies that influence their framework. Among others, regulatory risk can refer to the exposure to financial loss arising from the probability that regulatory agencies will make changes in the current rules (or will impose new rules) that will negatively affect the already-taken trading positions. It can be also the inability to predict a regulatory outcome. More generally, the research report emphasizes the fact that the notion of regulatory risk refers to main two strands of thought: on the one hand, *the potential and actual challenges faced by insurers and regulators under a supervisory regime arising from changes to products and regulations*, and on the other hand, *the unintended results of regulations that put at risk the ability of policyholders, shareholders or regulators to achieve their legal or fiduciary objective*. Our aim in this paper is to explore, evidence and document some facts related the second meaning above of regulatory risk. In order to delineate precisely our object of study, we shall use below the term “regulation risk” to refer to the possibility that prudential regulations may have, in some circumstances, an *unexpected effect* : the amplification of market risk.

In the post-financial crisis environment, the US and EU are each pursuing the modernization of their regulatory frameworks [9]. Hence, the regulatory landscape is inherently more complex, and the recent regulations aiming at reducing the risk to all financial institutions lead to a fundamental redesign of the conceptual frameworks. The paths taken by the US and EU are each impacted by these conceptual frameworks. Although different, they share a same vision of the regulation building: market risk regulations are now *model based*. This new model-based approach which specifies the characteristics of Basle III or Solvency II rules, is at the origin of what we term *regulation risk* because of an *unexpected effect* of the mathematical models in the financial context: a negative spillover causing a new type or risk.

Tail risks appear at two levels in relation with prudential regulations: at the level of the models themselves and at the level of the probabilistic tools (VaR,

TCE, ...) used to measure risks. While it is now known that the use of inaccurate or inappropriate models (for instance, the Samuelson lognormality assumption in the Black-Scholes model that fails to take into account extreme risks has often been considered one of the causes of the 1987 crash) can lead to tail risk, it is less known that the use of apparently model-free tools, such as the historical Value-at-Risk, can lead to extreme risks as well. This article discusses this latter feature by examining the risks associated with risk measures.

Indeed, the negative spillover of risk modelling in regulatory design is the topic of this paper: we argue that flaws in understanding the nature of financial uncertainty may perversely lead to the amplification of market risk. Saying that, we echo the feeling of a large number of practitioners: more precisely, if the new rules achieve to reduce volatility, they tend to rigidify the financial structure of the economic system and, consequently, to increase the probability of large moves, i.e. the probability of large jumps. These rules seem to swap volatility for the intensity of jumps, in the sense that the aimed for reduction of volatility is accompanied by an unwanted increase in the intensity of jumps. Volatility risk is apparently exchanged for jump risk. This is the unexpected effect. In this situation, the new rules of regulators seem create a new risk, which is precisely the one we call regulation risk.

The remaining of this paper is organized as follows. Section 1 introduces a conventionalist framework to emphasize the importance of the dichotomy continuity / discontinuity for the quantification of financial uncertainty and its implication for the policymaking about risk supervision. We build on the Comprehensive Actuarial Risk Evaluation (CARE) 2010 report to explain how the fundamental market risk methodologies are embedded in the regulatory process. Section 2 presents an empirical study of the S&P 500 based on a modelling with stable motion. We find that, for the recent period, a reduction in volatility has come along with an increase in the local intensity of the jumps. By analysing how these evolutions echo on measures of risk as the VaR and TCE, we document the conjecture of [13] that the choice of the VaR by the regulator could lead to very large increases of the market risk and provoke crashes. Finally, section 3 makes explicit how this effect is translated into a regulation risk which, combined with a model risk, produces a market risk.

Let us emphasize that this article does not try to develop new financial or mathematical models. All those used here are well known. Our contribution lies in: 1) the introduction of the notion of regulation risk by using a conventionalist approach of quantification; 2) an empirical analysis showing the simultaneity of the reduction in volatility and in the increase in the local intensity of the jumps;

3) the quantification of a negative spillover illustrated with a simple example on the calculation of the capital needs using the VaR and TCE measures with stable motions.

## 1 A conventionalist approach to risk metrology

Building on the approach taken in several works in sociology (for example [11, 15]), this article rests on the idea that a connection can be established between quantification conventions and regulation framework. When one decides to use a mathematical model to quantify risk, an extremely large number of choices must be made. These choices are not what we refer to here in the concept of the "quantification convention". A quantification convention is more like a meta-convention: its name covers a configuration or a coherent set of operations both cognitive and normative, including selection of the items to take into account, relevant judgment criteria, choices of mathematical schemas, etc.

The Comprehensive Actuarial Risk Evaluation (CARE) 2010 report [25] emphasizes that the use of convenient mathematical models to quantify risk is similar to looking for your lost keys under the nearest lamp post. To go beyond the metaphor, we consider here the lamp post as a meta-convention of quantification and we understand the warning of the CARE report in the light of the conventionalist approach of models. The interest of this approach is that it allows the risk managers and the regulators to be equipped with a valuable toolkit coming from the social sciences, and also permits to address the issue of the "nearest lamp post" and to avoid the negative spillover due to a flawed regulation.

Every quantification convention has an epistemic, a pragmatic and a political dimension. The *epistemic dimension* is built on a set of assumptions regarding uncertainty; the *pragmatic dimension* makes certain actions possible, such as trading, arbitraging and managing risk; the *political dimension* of each convention authorizes specific actors – not necessarily the same actors for each convention. Each arrival of a new convention enables the growth of certain practices and re-configurations of some professions. Part of the epistemic dimension is the selection of relevant predictive factors drawn from today's world that can be used to construct a decision, *i.e.* selection of what is true. Financial data, representations of financial theory and conceptualizations of the regulator joint to co-construct a financial "reality". In this sense, statistics must be conceived as simultaneously conventional and real [16].

We now present the two main meta-conventions of quantification of uncer-

tainty and the resulting risk metrology.

## **1.1 The two quantification conventions of uncertainty**

Risk management and financial regulation seek to grasp a future which, by definition, is uncertain. It therefore requires assumptions concerning what “shape” of uncertainty the future will exhibit, which is in practice an assumption regarding the stochastic dynamic of financial and economic variables.

There are two fundamentally different ways of viewing uncertainty in finance, each of them being at the roots of a quantification convention. One assumes the principle of continuity, the other does not. According to the first view, following Bachelier (1900)’s legacy, price movements are modelled by continuous diffusion processes, as for instance Brownian motion. According to the other view, following Mandelbrot (1963)’s legacy, price movements are modelled by discontinuous processes, as for instance Lévy processes [34]. The nature of the risk that we highlight in this paper can essentially be described in one statement: some prudential rules based on the first quantification convention picture the risk with only a single dimension, as if the regulator’s belief was embedded in a Brownian (continuous) representation of stock market fluctuations. In doing so, these rules leave aside many factors which are nevertheless crucial to ensure a proper taming of risk in a broader context.

To keep the discussion at an elementary level, we will focus here on the fact that, in a discontinuous paradigm, price changes are due to two factors: the instantaneous variance (or an equivalent scale parameter if the variance is infinite), and the local intensity of jumps (see Section 2.1 for precise definitions). A crucial point is that, by restricting the risk measurement to the sole “volatility”, the regulator aggregates these two factors. This is a consequence of the belief that prices essentially evolve in a continuous fashion. Under such an assumption, the variance may be identified to volatility and it provides all the relevant information needed to assess risk. But, as we shall quantify, ignoring jumps, or at least the fact that they produce variations non-reducible to variance, has damaging consequences on both the measurement of risk and its perception. Therefore, volatility (understood as variance) must be supplemented by another factor. We meet again here the “nearest lamp post” syndrome (quantification convention) emphasized in the CARE report.

## 1.2 Risk metrology

We now define the consequences of the choice of a quantification convention for the risk metrology. Firstly, we recall that volatility appears to be sufficient in the continuity representation. Secondly, we address the issue of the jumps risks.

### 1.2.1 The first convention: continuity and volatility risk

In physics, the so-called principle of continuity states that change in nature is continuous rather than discrete. Leibniz and Newton, inventors of differential calculus, stated the maxim: “*Natura non facit saltus*” (nature does not make jumps). This same principle underpinned not only the works of Linnaeus on the classification of species and later Charles Darwin’s theory of evolution (1859). It is also a crucial assumption in Alfred Marshall’s 1890 *Principles of Economics*, allowing the use of differential calculus in economics and the subsequent development of neoclassical economic theory. Modern financial theory grew out of neoclassical economics and naturally assumes the same principle of continuity. One of the great success stories of modern financial theory is the valuation of derivatives. Examples include the formulas of Fisher Black, Myron Scholes, and Robert Merton (1973) for valuing options, and the subsequent fundamental theorem of asset pricing that emerged from the work of Michael Harrison, Daniel Kreps, and Stanley Pliska between 1979 and 1981. These success stories rest on the principle of continuity.

Early in the 20th century, physics and genetics abrogated the principle of continuity in favour of discontinuity: it is now widely recognized that nature does make jumps. Quantum mechanics postulated discrete energy levels while genetics took discontinuities into account. But economics, including modern financial theory, stood back from this intellectual revolution. The early attempt by Mandelbrot in 1962 to take explicit into account discontinuities at all scales in stock market prices led to huge controversies in the profession [46]. But, by the 1980’s, despite the repeated financial crises following the 1987 stock market crash and despite rather overwhelming empirical counter evidence, continuity maintained popularity and the academic consensus reaffirmed the principle of continuity. Many popular financial techniques, such as portfolio insurance or the calculation of capital requirements in the insurance industry assume that (financial) nature does not make jumps and therefore promote continuity. Most statistical descriptions of stochastic evolutions in finance assume continuity. This is the *epistemic dimension* of the quantification convention.

It follows that Brownian representation became the standard model, part and parcel of finance curricula across the globe. It is the point of reference of most top journals in the field of finance; it is the dominant view in the financial industry itself. This is the *pragmatic dimension* of the quantification convention. And it underlies almost all prudential regulation worldwide: for instance, the so-called square-root-of-time-rule underlying the regulatory requirements (Basel III and Solvency II) for calculating minimum capital is a very narrow subset of time scaling rules of risk, and comes directly from the hypothesis that returns are independent and stationary normal, *i.e.* a Brownian framework. In other words, the principle of continuity was adopted not only in purely academic circles, but also in policy making. This is the *political dimension* of the quantification convention.

But if volatility perfectly describes risk in a Brownian universe, that is to say with a continuous representation of financial fluctuations, this is no longer the case as soon as we consider another representation of financial uncertainty where discontinuities of all sizes exist.

### **1.2.2 The second convention: discontinuity and jump risk**

There exist large sudden variations in the markets. This is a known and documented phenomenon. The first response of regulator to the presence of these large variations was to prescribe the use of internal models. A look at the professional practices for the implementation of internal models reveals the “come-back” of a well-established statistical theory, the so-called extreme value theory. This theory focuses on the most important variations of a random phenomenon, whether continuous or discontinuous. Without distinguishing between the two representations of uncertainty (continuous or discontinuous), prudential regulations attempted to deal with large variations in isolating them, that is to say by splitting the markets into two regimes, in effect adding large variations to smooth and small variations. The dissociation between these two market regimes is an intellectual consequence of the adoption of a paradigm of continuity. On the contrary, [29] shows that, with a discontinuous paradigm, it is not necessary to split market movements into two regimes: this paradigm allows one to unify in the same mindset quiet periods and periods of strong movements.

The first convention of quantification (continuous representation of stock market fluctuations) is dangerous for this very precise reason: discontinuities occur out of the blue, and only at large scales. Adopting this convention explains the words of Alan Greenspan, former chairman of the Federal Reserve of the United States: ‘We will never be able to anticipate all discontinuities in financial mar-



kets. Discontinuities are, of necessity, a surprise' (*Financial Times*, March 16, 2008). With the second convention of quantification (discontinuous representation of stock market fluctuations), however, discontinuity cannot be reduced to large variations, but remains present at small scales: discontinuity is a general property of the stock paths [34], not just a consequence of liquidity or other crises. It is then best to not separate large and small variations and to consider "quiet" periods, not as phases of continuity, but as a succession of micro-discontinuities (see Section 3.1 for details). Analysing discontinuities at small scales helps understanding the intrinsic fragility of a market and the occurrence of significant financial risks [20, 29]. The financial large scale risk management can be improved by extrapolating large discontinuities (large risks) from small moves (small discontinuities due to partial lack of liquidity, etc.).

In this framework, it is necessary to consider at least two independent factors governing price movements: in addition to volatility, which is a scale parameter, it is crucial to take into account the intensity of jumps, or *erraticity*, a factor which calibrates the statistical distribution of the sizes of jumps. In addition, because of the non-stationarity of the markets, it is essential to consider instantaneous versions of these two variates: an adequate understanding of risk requires measuring the local volatility and local intensity jumps. Empirical studies confirm the relevance of this view: typically, price changes exhibit jumps, and even an infinite number of jumps [1]; both the intensity of these jumps and volatility change over time; and all these market characteristics impact risk measures such as VaR [17].

## 2 Uncertainty modelling and risk of risk measures

In order to put the above elements in a form amenable to computations, we need to specify the characteristics of the quantification convention with a class of models for prices variations. Our aim here is not to determine the most adequate model but rather to evidence the impact of the presence of jumps on market risk. For reasons given in [1], we will choose infinite activity additive processes (that is, processes with infinite number of jumps) and we will base our discussion on the simplest class of these processes, which is the one of stable motions. Note however that more complex processes with infinite activity, such as CGMY ones, could be considered instead.

## 2.1 Recalls on stable motions

We briefly recall some basic notions on stable motions. These will be used in the empirical study of Section 2.2 as well as in Section 2.3. They will allow us to define and estimate the instantaneous scale parameter and stability exponent, which are respectively our local volatility and jump intensity measures.

Stable motions were introduced in financial modelling in [19, 34]. They have been since the subject of numerous studies, for instance in relation with pricing issues [7, 36, 40] or risk management [27, 35].

The main feature of interest to us is that paths of non-Gaussian stable motions almost surely display jumps. In fact, they are so-called “pure jump” processes, that is, they only move through jumps, and they possess almost surely a countably infinite number of jumps on any time interval. This allows one to account for the activity on markets in a way very different from the one provided by Brownian motion, or, more generally, continuous diffusions. More precisely, one may argue that in practice prices evolve in a discontinuous fashion, since their movements are quantized. Now, all pure jumps processes considered in financial modelling are such that “most” jumps are “small” (see below for a more precise statement), and thus akin to account for price changes even in “quiet” periods. For this reason, most of what is developed below would remain valid with other infinite activity pure jump processes.

A stable motion is a stochastic process with stationary and independent increments (as is Brownian motion), whose increments follow an  $\alpha$ -stable distribution. Such laws are described by their characteristic function, which takes the following form ( $\text{sign}(u)$  denotes the sign of  $u$ ) :

$$\varphi(u) = \begin{cases} \exp \left\{ i\mu u - \sigma^\alpha |u|^\alpha \left[ 1 - i\beta \text{sign}(u) \tan \left( \frac{\alpha\pi}{2} \right) \right] \right\} & \text{if } \alpha \neq 1 \\ \exp \left\{ i\mu u - \sigma |u| \left[ 1 + i\beta \text{sign}(u) \frac{2}{\pi} \ln(|u|) \right] \right\} & \text{if } \alpha = 1 \end{cases} \quad (1)$$

Choosing  $\alpha = 2$  above yields the characteristic function of a Gaussian RV, a case we exclude from now on. As the definition of  $\varphi$  shows, stable distributions are characterized by four parameters:

1. The number  $\alpha$  ranges between 0 and 2, and it quantifies the distribution of the size of jumps: within a given period of time, and for any integer  $j$ , the mean number of jumps of a stable motion whose increments follow a stable law with parameter  $\alpha$  with size of order  $2^j$  is proportional to  $2^{-j\alpha}$ . In particular, the mean number of jumps larger than any non-zero threshold

is always finite: “most” jumps are “small” as announced above. Besides, when  $\alpha$  is large (close to 2), the mean number of jumps decreases fast when their size increases, while, when  $\alpha$  is close to 0, it decreases slowly: a large  $\alpha$  corresponds to a small jump intensity, and vice-versa.

2. The positive real  $\sigma$  is a scale parameter: multiplying the RV by  $a > 0$  transforms  $\sigma$  into  $a\sigma$  (in the Gaussian case,  $2\sigma^2$  is the variance). One may thus identify  $\sigma$  as governing volatility.
3. The real number  $\mu$  is a location parameter: adding  $a$  to the RV transforms  $\mu$  into  $\mu + a$ . Also, when  $\alpha > 1$ ,  $\mu$  coincides with the expectation of the RV.
4. The real number  $\beta$ , which ranges in  $[-1, 1]$ , is a skewness parameter. A distribution that is symmetric around  $\mu$  has  $\beta = 0$ .

Our focus in this work is on the parameters  $\alpha$  and  $\sigma$ , which we recall account respectively for the jump intensity and volatility. As there is no reason to believe that these numbers remain constant in time, we will consider local versions, denoted  $\alpha(t)$  and  $\sigma(t)$ .

## 2.2 Empirical study

Figure 1 displays the local volatilities and jump intensities on daily data of the S&P 500, between 01/03/1928 and 02/01/2012. Volatilities and jump intensities were estimated using two classical statistical methods, namely the McCulloch and Koutrouvelis ones.

We first note that both methods give very similar results as far as  $\sigma$  is concerned. The difference between the estimated  $\alpha$  values is somewhat larger. This is however of little consequence for us, since both estimators yield roughly parallel curves: as our aim is to compare the evolutions of  $\sigma$  and  $\alpha$ , a constant shift in the stability exponent does not modify our conclusions.

A second fact is that, since 1960 or so, the jump intensity and volatility evolve in opposite directions: when volatility (that is,  $\sigma$ ) increases, the jump intensity decreases (since  $\alpha$  increases), and vice versa. In other words,  $\alpha$  and  $\sigma$  almost always move in the same way. Thus, as was announced above, when the market is less “rough”, or less “nervous”, it is more prone to large jumps.

A final noteworthy fact is that the evolution of the last years does confirm the general feeling of practitioners: volatility has significantly decreased, but at the expense of a notable increase in jump intensity. The market appears to be most of

the time “under control”, but the risk that it produces large jumps has increased. In [31], this is emphasized as follows: “the market is very quiet, except when it moves a lot”.

SP500 Index		Nikkei Index		Eurostoxx50 Index	
Period	$\rho$	Period	$\rho$	Period	$\rho$
12/30/63-03/12/73	0.595	03/31/50-09/27/61	0.613	12/30/86-05/12/92	0.424
03/13/73-05/27/82	0.014	09/28/61-03/28/73	0.625	05/13/92-09/25/97	0.285
05/28/82-08/12/91	0.902	03/29/73-09/26/84	-0.469	09/26/97-02/11/03	0.935
08/13/91-10/26/00	0.178	09/27/84-03/27/96	0.190	02/12/03-06/26/08	0.189
10/27/00-01/11/10	0.869	03/28/96-09/26/07	0.738	06/27/08-11/11/13	0.464
01/12/10-03/28/19	-0.787	09/27/07-03/28/19	0.219	11/12/13-03/28/19	0.828

Table 1: Correlation between the jump intensity and the volatility.

We complement the heuristic results that can be derived from Figure 1 by an analysis of the historical correlations between the local volatility  $\sigma$  and the regularity parameter  $\alpha$ , where we keep in mind that when  $\alpha$  is close to zero, extreme risks are frequent, and when  $\alpha$  is close to two, extreme risks are rare, so that a positive correlation between  $\sigma$  and  $\alpha$  is equivalent to a negative correlation between local volatility and the presence of extreme risks, and conversely.

The results, shown in Table 1, present the correlation  $\rho$  between  $\alpha$  and  $\sigma$  for the SP500, Eurostoxx50, and Nikkei indices. For each of the three indices, we collected a broad data set: specifically, we use daily data between 30-Dec-1963 and 28-Mar-2019 for the SP500 index, between 30-Dec-1986 and 28-Mar-2019 for the Eurostoxx50 index, and between 31-Mar-1950 and 28-Mar-2019 for the Nikkei index.

Each of these three databases is divided into six equal length subperiods, where  $\sigma$ ,  $\alpha$ , and  $\rho$  are computed. We find that five times out of six, the correlation between  $\alpha$  and  $\sigma$  is positive for the SP500 index, five times out of six, the correlation between  $\alpha$  and  $\sigma$  is positive for the Nikkei index, and six times out of six, the correlation between  $\alpha$  and  $\sigma$  is positive for the SP500 index.

Therefore, we find that most often in the past decades, the correlation between local volatility and alpha was positive - so the correlation between local volatility and extreme risks was negative - in the US, Europe, and Japan.

A legitimate question is then the following one: could it be the case that these coordinated evolutions of  $\sigma$  and  $\alpha$  be themselves, at least in part, consequences of the constraints imposed by certain prudential regulations under a given quantifica-

tion convention ? Indeed, these rules concentrate on volatility, aiming at reducing it. This effect is confirmed by our empirical study. However, it could well be that the very mechanism that reduces volatility also leads to a decrease in  $\alpha$ , that is, to an increase in jump risk. In such a scenario, we would have an example where regulation risk directly translates into a market risk, without being mediated by a model risk as in the situation considered in the next section. A study of the potential influence of prudential rules on the evolution of the jump intensity and volatility is presented in [30].

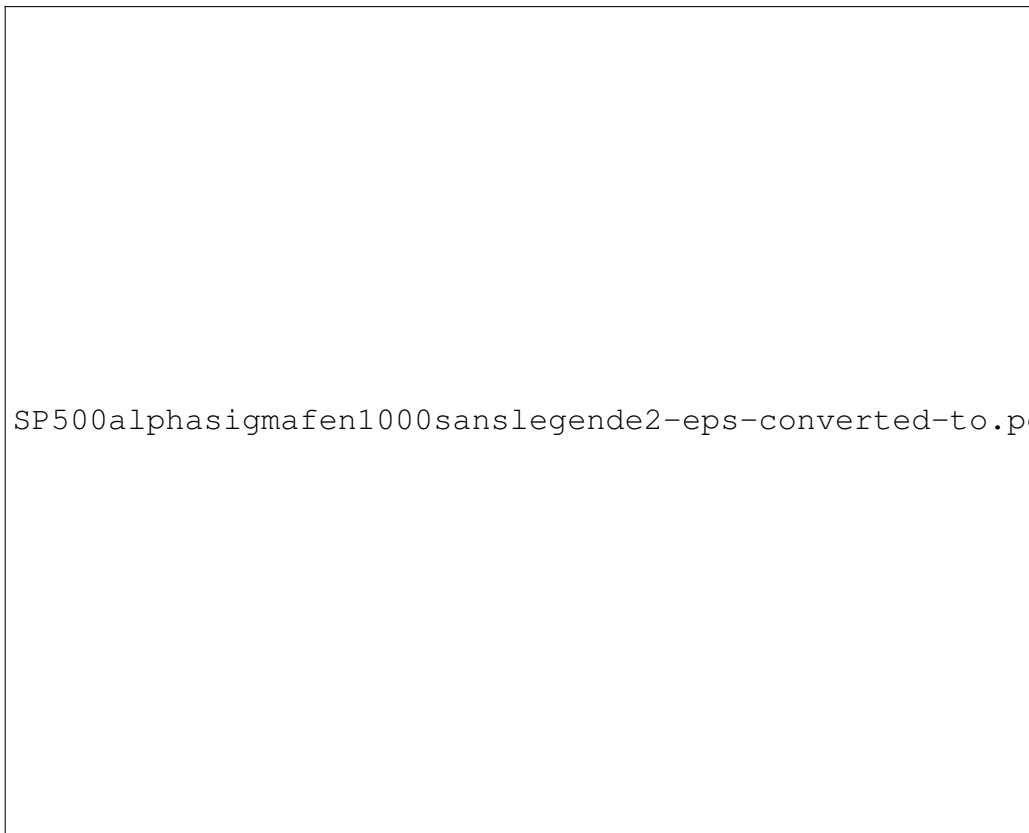


Figure 1: Compared evolutions of the local volatilities (top) and jump intensities (bottom). Estimations performed with the McCulloch (blue) and Koutrouvelis (black) methods on daily data of the S&P 500 between 01/03/1928 and 02/01/2012. Each value is estimated on a centred sliding window of size 2000 data points.

## 2.3 Consequences on risk measures

We now turn our attention to risk measures. According to the International Actuarial Association (IAA) note on the use of models for risk management [26], a risk measure is a numeric evaluator that can be used to determine the solvency capital requirement for an insurer. In this context, VaR is the main risk measure put forward by the regulator.

### 2.3.1 The coherency of risk measures

Recall that VaR at confidence level  $1 - p$  and horizon  $T$  is simply the amount such that the probability that losses at time  $T$  be smaller than VaR is equal to  $p$ , so that

$$\mathbb{P}(X_T < -\text{VaR}) = 1 - p.$$

VaR is simple to understand, and it adequately reflects certain aspects of risk. There is a huge quantity of works devoted to its study, that cannot be reviewed here. We just recall here some well-known shortcomings of VaR [13, 18, 26]. First, it is not a coherent measure of risk, in the sense that it is in particular not sub-additive: in general, the VaR of  $X_T + Y_T$  is not necessarily smaller than or equal to the sums of the VaRs of  $X_T$  and of  $Y_T$ . This is counter-intuitive for instance because it does not account for the reduction in risk typically entailed by diversification. In this respect, we note however that, in our frame where  $X_T$  follows an  $\alpha$ -stable distribution, this problem does not arise: as shown in [44], VaR is a coherent risk measure for sums of independent stable random variables when  $\alpha \geq 1$ .

Other limitations of VaR remain. For instance, it does not give any indication of what happens beyond VaR, although this is crucial information. This is why various other risk measures are considered to complement it. We shall be interested in the quantity called *Tail Conditional Expectation* (TCE), which was recommended by the IAA report [26] and has recently been adopted by the Basel regulation [4]. TCE at confidence level  $1 - p$  and horizon  $T$  is defined as:

$$\text{TCE} = \mathbb{E}(X_T \mid X_T < -\text{VaR}).$$

In words, TCE gives the mean loss beyond VaR. It is a coherent risk measure.

### 2.3.2 Risk measures and quantification conventions

We now turn our attention to quantification conventions. In the second quantification convention (discontinuous representation), when  $X_T$  follows an  $\alpha$ -stable

distribution, one can compute explicit asymptotic values of VaR and TCE when  $p$  tends to 1, that is, for very small risk levels. We note that, for given asset and time horizon, it may well be the case that, even for a high confidence level, say  $p = 99.5\%$ , one is still far from the region where the asymptotic holds. As a consequence, the computations presented below should be considered as indications of a general behaviour rather than as exact values.

Let:

$$C_\alpha = \frac{1 - \alpha}{\Gamma(2 - \alpha) \cos(\pi\alpha/2)}$$

where we assume for simplicity that  $\alpha \neq 1$ . Then, if  $X_T$  follows a stable distribution with parameters  $\alpha < 2, \sigma, \mu, \beta = 0$ , one has [42]:

$$\lim_{\lambda \rightarrow \infty} \lambda^\alpha \mathbb{P}(X_T < -\lambda) = \frac{C_\alpha}{2} \sigma^\alpha.$$

A similar formula holds in the non-symmetric case ( $\beta \neq 0$ ), but we restrict to  $\beta = 0$  for simplicity. Assuming that  $p$  is close enough to 1, and thus that VaR is sufficiently large, one thus has, by definition,

$$\text{VaR}^\alpha(1 - p) \simeq \frac{C_\alpha}{2} \sigma^\alpha,$$

or

$$\text{VaR} \simeq \sigma \left( \frac{C_\alpha}{2(1 - p)} \right)^{\frac{1}{\alpha}}.$$

One checks that, as is intuitively clear, VaR increases linearly with volatility. How it varies as a function of  $\alpha$  is less obvious from the formula. Figure 2 displays the evolution of VaR when  $\alpha$  varies between 1.1 and 1.9. As one can see, VaR decreases when  $\alpha$  increases, a fact that fits with intuition: larger jump intensity corresponds to a riskier market and thus a larger VaR.

Let us now deal with TCE. Assuming  $\alpha > 1$ , one can show [29, 48] that, when  $p$  is sufficiently close to 1,

$$\text{TCE} \simeq \frac{\alpha}{\alpha - 1} \text{VaR}. \quad (2)$$

The important point for us in the above formulas is the following one: assume that the market evolves in such a way that volatility decreases while the jump intensity increases (whether this joint evolution is the result of prudential regulations or not). This is what we have observed for recent years in our empirical

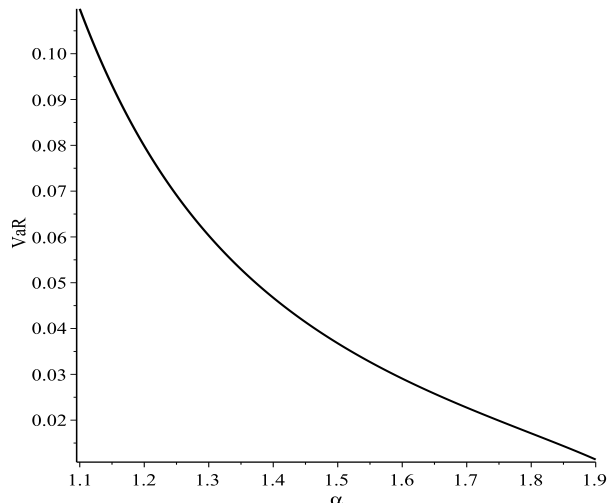


Figure 2: VaR as a function of  $\alpha$  for  $\sigma = 0.005$  and  $p = 0.99$ .

study. Then, at constant VaR<sup>1</sup>, one deduces from (2) and from the fact that the function  $\alpha \rightarrow \frac{\alpha}{\alpha-1}$  is decreasing, that TCE will increase. If, for instance,  $\alpha$  moves from 1.75 to 1.4, as it is observed empirically on Figure 1, then, although the VaR risk measure does not change, TCE is multiplied by 1.5. This is indeed a strong negative impact on financial companies. This simple mechanism highlights how model risk and regulation risk may combine to create a market risk.

### 3 Regulation risk in a conventionalist approach

We now conclude with the concept of regulation risk introduced at the beginning of this work. In the financial and statistical literature, model risk is traditionally understood as the existence of a discrepancy between a given probabilistic model and the ‘true’ nature of the financial phenomenon. This approach belongs to a classical epistemology scheme in which idealized models are facing the ‘true’ world, either with a descriptive or a prescriptive role. For instance, the following definition explicitly displays this understanding: a model is “an idealized representation of reality that highlights some aspects and ignores others” [38]. With

<sup>1</sup>The assumption of constant VaR is justified by the fact that the regulator imposes solvency capital requirements which are increasing functions of VaR. In order to maintain these requirements at a reasonable level, companies wish to control their VaR as much as possible



such an epistemology, it is possible to distinguish model risk (designer-based risk) and market risk (reality-based risk), because one can consider separately the model and the market, and thus the associated risks. This approach neglects an important component of model risk: the risk that the model itself can induce by influencing professional practices, either directly (tool-based influences) or through prudential rules (regulatory-based influences). This so called reflexive effect of models is crucial and must be taken into account in the management of financial risks [47]. For risk management issues, the ‘representational idiom’ of [38] has to be replaced by a ‘performative idiom’ [39]: it advocates for a move from an understanding of science as an attempt to represent ‘nature’ to an understanding of science as a mechanism creating ‘reality’.

The CARE report [25] definition of model specification risk is that the structure of the model itself is incorrect. The CARE report illustrates this model risk with the example of the erroneous use of the lognormal distribution when a Pareto distribution would have been a better representation of the underlying process. This is the sense in which we use the notion of model specification risk: the erroneous use of a continuity assumption when a discontinuity one would have led to a better representation of the underlying process. But we also say something more. Using the taxonomy introduced in the CARE report, we argue that the model specification risk combines with prudential rules to create a new type of risk that we term regulation risk, one which can be revealed with a conventionalist approach.

In section 1, we highlighted how financial data and quantification conventions of uncertainty join to co-construct a ‘reality’ of financial markets. We are now in a position to understand more precisely what this co-construction carries: a particular morphology of financial uncertainty. The so-called ‘observations’ are nothing but chains of mediation and human choices: for example, a chart of market path is the result of several steps of successive specific choices related to the relevant variables, the time scale, the length duration etc. In the same manner, risk metrology is the result of successive specific choices related to the convention of quantification that underpins prudential regulations. The first convention considers that prices movements are only due to volatility<sup>2</sup>, while an adequate model should distinguish at least two irreducible dimensions of risk, namely volatility and jump intensity.

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<sup>2</sup>Note that the danger of such a reduction has been emphasized rather early and has been studied in a stream of research that tries to evaluate volatility risk model. For instance, the fact that volatility alone is not sufficient for characterizing the dynamics of returns and how it may be complemented is analysed in [2, 21].

To summarize, the non-trivial relationships between the calculations and the institutions give birth to a morphology of uncertainty which may be different from the one that is desired. This fact was conjectured in [5] and our study is an example that makes precise the way it works. This negative spillover is exactly what we call regulation risk: by conveying the first quantification convention with prudential standards, economic incentives or regulatory constraints in order to reduce financial uncertainty, regulators create the uncertainty they seek to curb. This policymaking corresponds exactly to the political dimension of the quantification convention we introduced above. Specifically, regulators can provoke the very accidents that they want to avoid: here, in a market with both large and small jumps, imposing a VaR level increases TCE.

Furthermore, given its specific conventionalist characteristics, backtesting is a poor way to capture regulation risk. This new risk can be analysed either from an economic or statistical perspective [3, 12, 14] or from a sociological perspective [6, 32, 33, 37]. Both approaches yield the same findings: applying erroneous models has an effect on the phenomenon these models seek to describe because they shape professional practices through regulations and technical tools that rely on the conventions of quantification they carry. In this situation, our work shows that risk will also come from a bad shaping of business practices by an inadequate regulation, which itself rests on an erroneous representation of uncertainty, embedded in a particular quantification convention.

We suggest that the unexpected effect evidenced in this article is a special case of a class of regulation risks that remains to be explored.

## **Conclusion**

American and European insurance companies invest their assets in securities that are traditionally considered safe (and safer than the assets of other institutional investors): Government bonds, investment grade corporate bonds and high quality mortgage backed securities - and well-diversified (see the study of Klein et alii [28]). However, the ratio of kurtosis to standard deviation has historically been forty times bigger in the case of ten year treasury notes than in the case of the SP500 index, questioning the safety of fixed income securities compared with that of stocks and related securities, so questioning the safety of the assets of insurers compared with the assets of financial institutions. Furthermore, past performance may not be a good indicator of future performance: in the years between 2000 and 2007, investing in Greek Government bonds was considered a safe investment

by most European insurance companies and banks. From the viewpoint of year 2018, one may wonder whether it is safer for an Italian insurer to invest a huge part of its assets in Italian Government bonds or in a mix of Italian Government bonds and ENI stocks. To sum up, investing a lot in fixed income instruments can protect against volatility of the balance sheet but can as well expose an insurer to future potential high losses. Furthermore, looking at asset-liability management practices, we observe that many life and non-life insurers correlate their assets and liabilities using tools such as the duration or convexity. These tools are meaningful only in the first and second order cases and may not be suitable for the matching of the kurtosis of the assets and liabilities of an insurer. Therefore, additional risks may be incurred by using simplistic asset-liability management recipes. The exploration of such risks should be the topic of subsequent research.

Related to the cases discussed above is the situation of risks that jointly and adversely impact the assets and the liabilities of an insurance company. For instance, in the event of a hurricane, an insurer covering catastrophes may suffer losses on its real estate assets at the same time its liabilities sharply increase. More generally, insurers are subject to non-trivial systemic risks, as exposed in Cummins and Weiss [10], Grace [22], or Harrington [23]. A detailed study of how regulations can play a role in controlling these risks could be a direction of further research.

The fact that regulators and insurers have different objectives can create risks of their own. Indeed, regulators aim at ensuring the viability of insurers and at maintaining the costs of bankruptcy - should bankruptcy occur - within reasonable bounds. However, insurers not only aim at ensuring their own solvency, they also aim at producing a sufficiently high level of profitability and at reducing their cost of capital. Therefore, there is only a partial correspondence of the goals of insurers and regulators. Should insurers go too far in the direction set by regulators, or should regulators enforce too strong rules for ensuring solvency, insurers are likely to become less profitable, which in fact reduces their probability of ultimate solvency and goes against the objectives of regulators. There are many illustrations of this paradoxical situation: for instance, it could appear (and it has indeed very often been the case) safer to lead insurers to invest most of their assets in Government bonds. But doing so reduces the profitability of these insurers compared to other insurers subject to less stringent regulations. Ultimately, such recommendations could be detrimental to insurers. Also note that regulators must have in mind retroactions on consumers and markets when they set their targets. Indeed, if insurers cannot make enough profits, they will withdraw from some markets or stop issuing certain products.

Regulatory systems are likely to achieve their protection goals if they use risk

evaluation tools that: 1) take into account the fine structure of the tails of probability distributions (in this sense, the TCE is an incomplete but better tool than the VaR), 2) take into account the fact that probability distributions change with market conditions (in that sense, regime switching models can be useful), 3) integrate the fact that any model can be imperfect (in that sense, modeling uncertainty and not just risk is necessary), 4) take into account pro-cyclical effects (meaning that increasing reserves after a crisis can have the undesired effect of increasing bankruptcy risk), 5) take into account commercial risk (increasing too much the reserves of a company may make it less competitive worldwide), 6) take into account macro-economic retroactions (inciting insurance companies to massively invest their assets in Government bonds inflates a debt bubble that ultimately may be harmful to these companies). For instance, the Solvency 2 regulation: 1) uses VaR, 2) only partly takes into account time-changing conditions, 3) hardly considers (at least in its first pillar) model uncertainty, 4) starts taking into account pro-cyclical effects, 5) does not take into account commercial risk, 6) does not take into account macro-economic retroactions. Consequently, the Solvency 2 regulation, like most other regulatory systems, causes companies to massively concentrate their assets in fixed income instruments. Based on the past history of financial markets, this may look like a reasonably safe strategy. However, this may not be the best protective strategy in the forthcoming decades if the conditions and structure of markets change.

A key issue with the balance of volatility and tail risks is that they are related to fundamentally different time periods. In essence, tail risk, or the risk of violent jumps in securities prices, is an instantaneous or very short-term risk. Volatility risk is often regarded as a short-to-medium term risk but can also be understood and managed over long time horizons. Therefore, balancing volatility and tail risks can be done within frameworks that take into account the passage of time and that simultaneously take into account multiple time horizons. Unfortunately, most models and most regulations are not consistent with such frameworks. Moreover, it is very difficult to manage tail risk and volatility risk without a precise definition and understanding of time diversification. Indeed, most life insurance companies can survive, like pension funds, short-term economic depletions if they invest a reasonable share of their assets in stocks. This is because, in the long run, the volatility of the market induces price to rise again and to recoup harsh drawdowns. This example is an illustration that a lot remains to be achieved in models and regulatory frameworks to simultaneously consider various time-horizons and that this may be a key conceptual step allowing us to jointly deal with volatility and tail risks. On the statistical side, more research needs to be produced to establish

static and causal relationships between the moments of probability distributions, risk indicators, and regularity indicators, based on broad international stock data.

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