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Lionel de Boisdeffre

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**Equilibrium with Asymmetric Information and Restricted  
Participation: The No-arbitrage Characterization**

Lionel De BOISDEFFRE

**2024.05**



EQUILIBRIUM WITH ASYMMETRIC INFORMATION AND RESTRICTED  
PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION

*Lionel de Boisdeffre*,<sup>1</sup>

***Abstract***

*The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010). Moreover, the definition and existence of the sequential equilibrium do not rely on Radner's (1972, 1979) rational expectation assumptions, which may be dropped.*

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information.

**JEL Classification:** D52

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<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: lionel.de.boisdeffre@wanadoo.fr

# 1 Introduction WITH ASYMMETRIC INFORMATION AND RESTRICTED

## PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION

The current paper extends Cornet-De Boisdeffre's (2002) step concept of purely financial equilibrium with asymmetric information to non-ordered preferences and restricted participation, and refines its existence characterization. Generalizing Cass (2006), it shows that equilibrium exists, not only for *one* collection of state prices, as in De Boisdeffre (2007), but for *every* collection of state prices in realizable states.

### *Abstract*

With nominal assets, the literature shows an essential real indeterminacy of equilibrium (see, e.g., Balasko-Cass, 1989, and Geanakoplos-Mas-Colell, 1989), while *The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result exists and are generically locally unique, but do not assess Cass' price equivalence. To my best knowledge, the first extension of Cass' (2006) equivalence to numeraire assets, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).* With numeraire assets, Geanakoplos and Polemarchakis (1986) show that equilibria exist and are generically locally unique, but do not assess Cass' price equivalence. To my best knowledge, the first extension of Cass' (2006) equivalence to numeraire assets and asymmetric information is provided in De Boisdeffre (2021).

*Moreover, the definition and existence of the sequential equilibrium do not rely on Radner's (1972, 1979) rational expectation assumptions, which may be dropped.*

With real or mixed assets, the existence of equilibrium is not guaranteed as shown by Hart (1975). Hart's counterexample is based on the collapse of the span of assets' payoffs, that occurs exceptionally at market-clearing prices. One response to that problem is to show, along Duffie and Shafer (1985), that it is indeed exceptional, in the sense that equilibria exist generically in payoffs and endowments. This argument

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When agents have asymmetric or incomplete information, they seek to learn in-  
<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital,  
 75013 Paris, France. E-mail: lionel.deboisdeffre@univ-paris1.fr

referred to as the *Radner-Arbitrage* is the foundation for the Radner (1979) ‘rational expectations equilibrium’ (REE) Generalized Gale as-called *Arrow-Debreu* models. These include, among others, McAllister’s (1990), Dutta-Morris’ (1997) and Ben Porath-Heifetz’ (2011).

Rational expectations, along Radner (1979) or CKRMC models, assume that agents are aware of a correspondence (or a map) between private information signals and the equilibrium prices observed on markets. Following Radner’s (1979) seminal **Abstract** paper, an abundant literature studied the (generic) existence and the information

revealed by equilibrium prices (see, e.g., Allen, 1981; Mischel, Polemarchakis and Siconolfi, 1990; Polemarchakis and Siconolfi, 1993; Pietra and Siconolfi, 1997 and 1998; or Citanna and Villanacci, 2000a and 2000b, to quote only a few). I drop this assumption to show, along Cornet and De Boisdeffre (2009), that agents observing arbitrage price and equilibrium asset price coincide. Namely, any collection of state markets with no price model may refine their information until financial markets prices in realizable states is proved to support a sequential equilibrium. This result preclude arbitrage. The relevance of the latter inferences is argued in Section 6.

To show the equivalence between *Radner-Arbitrage* and *Equilibrium Prices* in financial markets with symmetric information, Cass (2006) uses a *trick* by which Radner’s (1972, 1979) *Arrow-Debreu* expectation assumption at the first step by which other agents are constrained a la Radner (1972), that is, in every state of nature. That ‘trick’ implies that the first agent has a full access to financial markets. Not surprisingly, Balasko, Cass and Siconolfi (1990) show that the equivalence between no-arbitrage and equilibrium prices fails under restricted participation, in general.

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information. assets’ payoffs by agents, which guarantees the latter equivalence under restricted participation. The current paper proposes an alternative - in my view milder - ac-

<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l’Hôpital, 75013 Paris, France, under which the latter equivalence holds. Namely, it shows

that any collection of state prices in realizable states supports a sequential equilibrium. The latter paper is then compared in Section 5 with the symmetric information case.

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None of Radner's (1972 and 1979) rational expectation assumptions are assumed in the current model. The sequential equilibrium is proved to exist, even when these standard assumptions are dropped. This outcome has an interest of its own. For expositional purposes, I present a two-period model. Its extension to multiple periods is standard and straightforward. But it is deferred as requiring additional nodes and notations into the model, which would hamper its simplicity and focus.

### **Abstract**

*The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends the (2006) De Boisdeffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).*

Throughout, a pure-exchange economy is considered, with two periods,  $t \in \{0, 1\}$ . Moreover, the definition and existence of the sequential equilibrium do not rely on an uncertainty, at  $t = 0$ , upon which state of nature will randomly prevail Radner's (1972, 1979) rational expectation assumptions, which may be dropped. at  $t = 1$ . Agents exchange consumption goods, on spot markets of both periods,

and assets, on financial markets at  $t = 0$ , which transfer wealth across periods and states. The economy is finite, in the sense that the sets,  $I$ ,  $S$ ,  $L$  and  $J$ , respectively, of consumers, states of nature, consumption goods and assets are all finite. The

observed state, at  $t = 0$ , is denoted by  $s = 0$  and  $\Sigma'$  denotes  $\{0\} \cup \Sigma$ , whenever  $\Sigma \subset S$ .

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information.

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Agents consume or exchange (the same) consumption goods,  $l \in L$ , on spot

<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. E-mail: lionel.deboisdeffre@wanadoo.fr. I privately the correct



of asset EQUILIBRIUM WITH ASYMMETRIC INFORMATION AND RESTRICTED PARTICIPATION: CONDITIONAL PAYOFFS ACROSS STATES, PERFECT FORESIGHT AND THE EXCHANGE OF ASSETS MAY BE RESTRICTED. For each agent  $i \in I$ , her budget set,  $Y_i \subset \mathbb{R}^J$ , or the  $i^{\text{th}}$  agent's set of admissible portfolios, is henceforth given. Restrictions to trade are typically encountered on financial markets. Some may be institutional, such as short sale constraints, transaction costs, bid-ask spreads or ratio regulations (see Elsinger and Summer's 2001 overview). Others may be behavioural or informational.

**Abstract**

**2.2 The consumer's behaviour and concept of equilibrium**

The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences, and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information.

Each agent,  $i \in I$ , receives an endowment,  $e_i := (e_{i,s})$ , granting the commodity bundles,  $e_{i,0} \in \mathbb{R}_+^L$  at  $t = 0$ , and  $e_{i,s} \in \mathbb{R}_+^J$ , in each state,  $s \in S_i$ , if it prevails. Given the observed market prices at  $t = 0$ , namely,  $p \in \mathcal{P}$ , for goods, and  $q \in \mathbb{R}^J$ , for assets, and her expectation of future prices,  $p_i := (p_{i,s}) \in P_i$ , the generic  $i^{\text{th}}$  agent's consumption set is  $X_i := \mathbb{R}_+^L \times \mathbb{R}_+^J$ , and her budget set is defined as follows:

$B_i(p, q) := \{(x := (x_s), z) \in X_i \times Y_i : p \cdot (x_0 - e_{i,0}) \leq -q \cdot z \text{ and } p_{i,s} \cdot (x_s - e_{i,s}) \leq V(s) \cdot z, \forall s \in S_i\}$ .

Moreover, this definition is consistent with the complete ordering,  $\prec_i$ , dominated by

for all  $x \in X_i$ , by the set,  $P_i(x) := \{y \in X_i : x \prec_i y\}$ , of consumptions that she strictly

prefers to  $x$ . To simplify presentation, the latter set only depends on the agent's own consumption (and not on other agents' consumptions).

The economy is denoted by  $\mathcal{E}_{(p_i)} = \{(I, S, L, J), V, (S_i), (p_i), (e_i), (\prec_i)\}$ . Henceforth, **Key words:** incomplete markets, restricted participation, arbitrage, existence of the structural characteristics,  $\{(I, S, L, J), V, (S_i), (e_i), (\prec_i)\}$ , of the economy are fixed and always referred to. Only agents' exogenous expectations,  $(p_i) \in P^{S_i \times I}$ , may vary and distinguish economies. When agents share the same beliefs, a sequential

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1 University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: elisabeth.deboisdeffre@wanadoo.fr



**Definition 1** Given the Assumptions (A1)-(A5) and the state beliefs (i.e.,  $p_{is} = P_{is}$ ), the no-arbitrage condition  $\mathbb{N} \cap \mathbb{A} = \emptyset$ , and decisions,  $(x_i := (x_{is}), z_i) \in B_i(p, q)$ ,  $i \in I$ , is a (sequential) equilibrium of the economy,  $\mathcal{E}_{(p_i)}$ , if the following Assertions hold:

- (a)  $\forall i \in I, (x_i, z_i) \in B_i(p, q)$  and  $P_i(x_i) \times \mathbb{R}^J \cap B_i(p, q) = \emptyset$ ;
- (b)  $\forall s \in \underline{S}, \sum_{i \in I} (x_{is} - e_{is}) = 0$ ;
- (c)  $\sum_{i \in I} z_i = 0$ .

**Abstract**

Remark 1 Since no state  $s \notin \underline{S} := \cap_{i \in I} S_i$  may prevail, no spot market may open or clear in any state  $s \in S \setminus \underline{S}$ . This explains Condition (b) of Definition 1, and why the paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result does not need to assume that common expectations are self-fulfilling. This outcome is natural and contradicted by no specification in a pure exchange economy, such as  $\mathcal{E}_{(p_i)}$ . It lets agents' decisions coincide ex ante and ex post. It tends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information, such as Martini de Rocca and Triantis (2005) or Cornet and Copeland (2010).

Moreover, the definition and existence of the sequential equilibrium do not rely on

- $Z_i := \{z \in \mathbb{R}^J : V(s) \cdot z = 0, \forall s \in S_i\}$ , and its orthogonal,  $Z_i^\circ = \sum_{s \in S_i} \mathbb{R}V(s)$ ;
- $Z^\circ := \{z \in \mathbb{R}^J : V(s) \cdot z = 0, \forall s \in \underline{S}\}$ , and its orthogonal,  $Z = \sum_{s \in \underline{S}} \mathbb{R}V(s)$ ;
- $Z^\circ := \sum_{s \in S_i} Z_i^\circ$ , its orthogonal complement,  $Z = \cap_{i \in I} Z_i$ , and  $Z^* := Z \cap Z^\circ$ .

To the generic  $i^{th}$  agent,  $Z_i^\circ$  is the subjective space of redundant (or worthless) portfolios. The latter is always included in the actual set of redundant (or worthless) portfolios,  $\underline{Z}$ . All other vector spaces above are defined relative to the former and will serve in the proof of Section 4. Similarly,  $\pi : \mathbb{R}^J \rightarrow \underline{Z}$  will henceforth denote the orthogonal projection of  $\mathbb{R}^J$  on  $\underline{Z}$ , and serve later.

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information.

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<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: [isabelle.tchet.debois@univ-paris1.fr](mailto:isabelle.tchet.debois@univ-paris1.fr) and the following Assumptions:

- ~~A1 (strict non satiation)~~ ~~ASYMMETRIC INFORMATION AND RESTRICTED PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION~~
- **A2** (*strong survival*):  $\forall i \in I, e_i \in \mathbb{R}_{++}^{S_i}$ ;  
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- **A3** for every  $i \in I, \prec_i$  is lower semi-continuous, convex-open-valued and such that  $x \prec_i x + \lambda(y - x)$ , for every  $(x, y, \lambda) \in X_i \times P_i(x) \times ]0, 1]$ ;
- **A4** for every  $i \in I, Y_i$  is closed, convex and contains zero;
- **A5**  $A := \{(z_i) \in \mathbb{R}^{J \times I} : \sum_{i \in I} z_i \in Z^o, V(s_i)z_i \geq 0, \forall (i, s_i) \in I \times S_i\} \subset \times_{i \in I} Y_i$   
**Abstract**  
and  $Z^* \subset \cap_{i \in I} \text{cone}(Y_i)$ ;

• **A6** (*riskless asset*):  $\exists \text{ a riskless asset } p \in \text{exchange economy}_i^+$  where  $e_i \in S$  and  $S$

*Remark 2* Assumptions A1-A2-A3 are standard. Assumption A1, retained for financial markets. Under standard conditions, it shows that the two concepts of non-convenience, could be weakened to the non-satiation of agents in any state. Assumption A4, made, e.g., by Siconolfi (1989) and Cornet-Gopalan (2010), is a minimal condition of most portfolio choice models with financial constraints (see Elsingher and Summer, 2001). Assumption A5 is technical, and dropped when one agent has full information (see Section 5). Under symmetric information, it would typically and eliminate, redundant portfolios. It is then weakened to a standard condition of

restricted participations models, namely:  $Z^o \subset \cup_{i \in I} Y_i$ . The latter condition always holds if redundant assets are eliminated ( $Z^o = \{0\}$ ). Why is Assumption A5 met ?

Because financial intermediaries, possibly better informed, typically emit and pro-

pose riskless clearing-market claims to agents when they exist (possibly freely), and other limited claims against their idiosyncratic risks. When it is met, agents typically refine their information from observing exchange opportunities on financial

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markets, until no arbitrage remains (see Claim 2). Along Assumption A6, agents

<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: lionel.deboisdeffre@univ-paris1.fr

### 3 The model's arbitrage concepts and properties

#### PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION

This Section defines no-arbitrage prices and shows their existence.

**Definition 2** For each  $i \in I$ ,  $As(Y_i) := \{z \in \mathbb{R}^J : \{z\} + Y_i \subset Y_i\}$  is the asymptotic cone of  $Y_i$ . A price  $q \in \mathbb{R}^J$  is a no-arbitrage price if the following equivalent Conditions hold:

- (i)  $\forall (i, z) \in I \times As(Y_i)$ ,  $(-q \cdot z \geq 0 \text{ and } V(s) \cdot z \geq 0, \forall s \in S_i) \Rightarrow (z \in Z_i^o \text{ and } q \cdot z = 0)$
- (ii)  $\forall i \in I$ ,  $\exists \lambda_i := (\lambda_{is}) \in \mathbb{R}_{++}^{S_i}$ ,  $q \cdot z_i = \sum_{s \in S_i} \lambda_{is} V(s) \cdot z_i$ ,  $\forall z_i \in As(Y_i)$ .

Let  $\mathcal{NA}$  be the set of no-arbitrage prices and  $\mathcal{NA}(\lambda) := \{q \in \mathcal{NA} : \pi(q) = \sum_{s \in \underline{S}} \lambda_s V(s)\}$ , for every  $\lambda := (\lambda_s) \in \mathbb{R}_{++}^{\underline{S}}$ , be given. Financial markets, are said to be arbitrage-free if

The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdreffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010). Markets are frictionless arbitrage-free if there exists a frictionless no-arbitrage price. Moreover, the definition and existence of the sequential equilibrium do not rely on

**Remark 3** The equivalences between Conditions (i) and (ii), and Conditions (iii) and (iv), of the above Definition 2 follow from a standard separation argument. A frictionless no-arbitrage price is akin to a no-arbitrage price on frictionless financial markets, that is, where agents' participation is unrestricted ( $Y_i = \mathbb{R}^J$ , for each  $i \in I$ ).

*Radner's (1972, 1979) rational expectation assumptions, which may be dropped.*

Martins-da-Rocha and Triki (2005) provide a sufficient condition for no-arbitrage prices to be frictionless. They require markets to be collectively frictionless, in the sense that  $\cup_{i \in I} Y_i = \mathbb{R}^J$ . A standard economy yields another interesting result:

Martins-da-Rocha and Triki (2005) provide a sufficient condition for no-arbitrage prices to be frictionless. They require markets to be collectively frictionless, in the sense that  $\cup_{i \in I} Y_i = \mathbb{R}^J$ . A standard economy yields another interesting result:

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**Claim 1** In a standard economy,  $\mathcal{E}_{(p_i)}$ , financial markets are arbitrage-free if and only if there exists a frictionless no-arbitrage price.

**Proof** Let us start with the economy  $\mathcal{E}_{(p)}$ . If financial markets are frictionless, arbitrage-free, then they are obviously arbitrage-free. To show the converse, on the economy,  $\mathcal{E}_{(p)}$ , is henceforth assumed to have arbitrage-free markets. From Assumptions A4-A5, the relation  $A := \{(z_i) \in \mathbb{R}^{J \times I} : \sum_{i \in I} z_i \in Z^o, V(s_i)z_i \geq 0, \forall (i, s_i) \in I \times S_i\} \subset \times_{i \in I} Y_i$  holds and implies, since  $A$  is a cone:  $\times_{i \in I} Z_i^o \subset A \subset \times_{i \in I} As(Y_i)$ .

Let  $q \in \mathcal{NA}$  be given. Then, the following relation (I) holds:

$$((z_i) \in \times_{i \in I} As(Y_i), \sum_{i \in I} z_i = 0 \text{ and } V(s_i) \cdot z_i \geq 0, \forall (i, s_i) \in I \times S_i) \Rightarrow ((z_i) \in \times_{i \in I} Z_i^o)$$

Indeed, assume by contraposition that there exists  $(z_i) \in \times_{i \in I} As(Y_i)$ , such that  $\sum_{i \in I} z_i = 0$  and  $V(s_i) \cdot z_i \geq 0$ , for every  $(i, s_i) \in I \times S_i$ , with at least one strict inequality. Then, from Condition (ii) of Definition 2 (applied to  $q \in \mathcal{NA}$ ), the relation  $q \cdot \sum_{i \in I} z_i > 0$  holds, with  $\sum_{i \in I} z_i = 0$ . This contradiction proves relation (I), namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information.

Indeed, assume by contraposition, that there exist  $(z_i) \in \times_{i \in I} As(Y_i)$ , such that  $\sum_{i \in I} z_i = 0$  and  $V(s_i) \cdot z_i \geq 0$ , for every  $(i, s_i) \in I \times S_i$ . Then, the definition of Radner's (1972, 1979) rational expectation implies  $(z_i) \in \times_{i \in I} As(Y_i)$  and  $\sum_{i \in I} z_i = 0$ , which contradicts the opposite relation,  $(z_i) \in \times_{i \in I} \mathbb{R}^{J \times I} \setminus \times_{i \in I} Z_i^o$ , assumed above. This contradiction proves that relation (II) holds, which characterizes frictionless arbitrage-free markets, from Cornet and De Boisdeffre (2002, p. 401).  $\square$

**Remark 4** Claim 1 does not imply that all no-arbitrage prices are frictionless. **Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information, and two assets, such that:  $I = \{1, 2\}$ ,  $S = \{1, 2, 3\}$ ,  $S_1 = \{1, 2\}$ ,  $S_2 = S$ ,  $V(1) = (1, 1)$ ,  $V(2) = (0, 1)$ ;  $V(3) = (-1, 1)$ ,  $Y_1 = \{0\} \times \mathbb{R}$  and  $Y_2 = \mathbb{R}^2$ . Then, obviously,  $p = (1, 4)$  and

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<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. E-mail: [francois.boisdeffre@univ-paris1.fr](mailto:francois.boisdeffre@univ-paris1.fr) and [boisdeffre@univ-paris1.fr](mailto:boisdeffre@univ-paris1.fr)

The right sets of words are and frictionless no arbitrage. In general, any standard economy can be assumed to have arbitrage-free financial markets. This property generalizes Cornet and De Boisdeffre's (2009) to the economy  $\mathcal{E}_{(p_i)}$ :

**Claim 2** *In a standard economy, the information structure,  $(S_i)$ , admits a coarsest refinement,  $(S_i^*)$ , which makes financial markets arbitrage-free. Agents with no price model can infer the refinement  $(S_i^*)$  by ruling out unlimited arbitrage opportunities.*

**Proof** Let a standard economy,  $\mathcal{E}_{(p_i)}$ , be given, whose information structure is denoted by  $(S_i^0) = (S_i)$ . Assume, that markets fail to be arbitrage-free. Then, from the proof of the above Claim 1, the following relation (I) fails:

*The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-*

*arbitrage price and equilibrium asset price coincide. Namely, any collection of state Let  $B^0 = \{(z_i) \in \times_{i \in I} A_i(Y_i) : \sum_{i \in I} z_i = 0 \text{ and } V(s_i) \cdot z_i \geq 0, \forall (z_i, s_i) \in I \times S_i^0\}$  be a non-empty cone. For every  $j \in I$ , let  $S_j = \{s \in S_j : \exists (z_i) \in B^0, V(s) \cdot z_j > 0\}$  be the  $j$ -th agent's (possibly empty) set of arbitrage states. Since relation (I) fails,  $\cup_{j \in I} S_j$  is non-empty.*

*tends. Cass (2006), De Boisdeffre (2007), and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010). Along Assumption A5, financial intermediation permits to implement exchanges; Moreover, the definition and existence of the sequential equilibrium do not rely on which reveal to agents their arbitrage states, when they exist. Those agents should Radner's (1972, 1979) rational expectation assumptions, which may be dropped. rightly infer that such arbitrage states cannot prevail. As a result, agents infer the*

refinement  $(S_i^1) = (S_i^0 \setminus \bar{S}_i^1)$  of their initial information structure,  $(S_i^0) = (S_i)$ . This refinement process proceeds as long as financial markets fail to preclude arbitrage.

After the initial sets above, we define by induction, on  $n \in \mathbb{N}$ , the sets:  $(S_i^n) = (S_i^{n-1} \setminus \bar{S}_i^n)$ . **Key words:** incomplete markets; restricted participation; arbitrage; existence of equilibrium; rational expectations; perfect foresight; asymmetric information. The decreasing sequence  $\{(S_i^n)\}_{n \in \mathbb{N}}$  of finite sets is stationary and converges to a minimal element, say  $(S_i^*)$ . Since the refinement process stops at  $(S_i^*)$ , the following relation (II) holds from above:

University of Paris 1 Panthéon - Sorbonne, 100-112 Boulevard de l'Hopital, 75013 Paris, France. Email: lionel.de.boisdeffre@wanadoo.fr

$((z_i) \in \text{EQUILIBRIUM WITH ASYMMETRIC INFORMATION AND RESTRICTED PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION})$

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By the same token as for the proof of Claim 1, relation (III) implies the following:  
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$((z_i) \in \mathbb{R}^{J \times I}, \sum_{i \in I} z_i = 0, V(s_i) \cdot z_i \geq 0, \forall (i, s_i) \in I \times S_i^* \Rightarrow (V(s_i) \cdot z_i = 0, \forall (i, s_i) \in I \times S_i^*),$

which characterizes frictionless arbitrage-free markets, from Cornet-De Boisdeffre (2002, p. 401). Moreover, by construction, the refinement  $(S_i^*)$  inferred by rational agents is the coarsest refinement of  $(S_i)$  making financial markets arbitrage-free.

**Abstract**

Thus, arbitrage-free markets at the outset reveal no information and let  $(S_i^*) = (S_i)$ . □

*The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information.*

**Claim 3** For every  $\lambda \in \mathbb{R}_+^{\mathbb{S}}$ , the set  $\mathcal{NA}(\lambda)$  of Definition 2 is non-empty, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).

Moreover, the definition and existence of the sequential equilibrium do not rely on Radner's (1972, 1979) rational expectation assumptions, which may be dropped.

**Proof** Let  $\lambda := (\lambda_s) \in \mathbb{R}_+^{\mathbb{S}}$  be given. Since markets are arbitrage-free, there exists  $q \in \mathcal{NA}$  and  $(\lambda_{is}) \in \times_{i \in I} \mathbb{R}_+^{\mathbb{S}_i}$ , such that  $q = \sum_{s \in S_i} \lambda_{is} V(s)$ , for each  $i \in I$ , which implies  $q \cdot z = 0$  whenever  $z \in Z^o$ . For each  $i \in I$ , let  $q_i \in \underline{Z}$  be defined by  $q_i = \sum_{s \in S_i} \lambda_{is} \pi(V(s))$ , if  $S_i \neq \underline{S}$ , and  $q_i = 0$  otherwise.

Since  $q_i \in \underline{Z} = \sum_{s \in \underline{S}} \mathbb{R} V(s)$ , for every  $i \in I$ , there is a vector  $(\mu_{is}) \in \mathbb{R}^{\underline{S}}$ , such that

**Key words:** Incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information, JEL Classification: D52 -  $\frac{\mu_{is}}{N}$ , for every  $s \in \underline{S}$ . By construction, the state prices,

$\gamma_i := (\gamma_{is}) \in \mathbb{R}_+^{\mathbb{S}_i}$  defined for all  $i \in I$ , support some no-arbitrage price  $q \in \mathcal{NA}(\lambda)$ .

<sup>1</sup>University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hopital, 75013 Paris, France. Email: lionel.de.boisdeffre@wanadoo.fr

## 4 The existence Theorem

### PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION

Arbitrary vectors,  $\lambda = (\lambda_s) \in \mathbb{R}_{++}^{\underline{\mathbf{S}}}$  and  $\bar{p}_i \in P^{S_i \setminus \underline{\mathbf{S}}}$ , for  $i \in \{i \in I : S_i \neq \underline{\mathbf{S}}\}$ , are  
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henceforth set as given. Under Assumptions A1 to A6, the following Theorem 1 guarantees the existence of a sequential equilibrium, consistent with the state prices,  $(\lambda_s)$ , and the subjective expectations  $(\bar{p}_i)$ :

**Theorem 1** *Let arbitray vectors of state prices,  $\lambda := (\lambda_s) \in \mathbb{R}_{++}^{\underline{\mathbf{S}}}$ , and subjective expectations,  $\bar{p}_i := (\bar{p}_{i,s}) \in P^{S_i \setminus \underline{\mathbf{S}}}$ , of un-*

*Abstract*  
*be given. Under Assumptions A1 to A6, there exists a collection of market prices,  $(p, p^1 = (p^1_s)_{s \in \underline{\mathbf{S}}}) \in P \times P^1 \times \mathcal{N}^A(\lambda)$ , and decisions  $(x_i, z_i) \in B_i(p, q)$ , defined for each  $i \in I$ , such that  $(p, q, [(x_i, z_i)])$  is a sequential equilibrium of the economy  $\mathcal{E}_{(p_i)}$ , defined by*

*metric information, non-ordered preferences and a restricted access to purely fi-*

*pancial markets. Under standard conditions, it shows that the two concepts of no-*  
*arbitrage price and equilibrium asset price coincide. Namely, any collection of state*  
*The proof applies Gale-Mas-Colell's (1975, 1979) fixed-point-like theorem to prices in realizable states is proved to support a sequential equilibrium. This result ex-*  
*lower semi-continuous correspondences representing agents' behaviours in an aux-*  
*tends Cass (2006), De Boisdeffre (2007) and similar results of symmetric informa-*  
*liary compact economy, and derives the equilibrium from a pseudo fixed-point.*  
*tion, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).*

*Model. An auxiliary compact economy with modified budget sets not rely on*  
*Radner's (1972, 1979) rational expectation assumptions, which may be dropped.*

Let  $Q := \{q \in Z^* : \|q\| \leq 1\}$  and  $P^* := \{p \in \mathbb{R}^L : \|p\| \leq 1\}^{\underline{\mathbf{S}}}$  be given convex compact sets. From Assumptions A4-A5, for every  $i \in I$ , the portfolio set  $Y_i$  contains  $Z_i^o$  and its orthogonal projection on  $Z_i$  (hence, equal to  $Y_i \cap Z_i$ ). Therefore, the following convex sets are well defined for every  $(p, p^1 = (p^1_s)_{s \in \underline{\mathbf{S}}}) \in P^*$  and every  $q \in Q$ :

$B_i^1(p, p^1, q) := \{ (x, z) \in X_i \times Y_i \cap Z_i : p \cdot (x_0 - e_{i0}) + q \cdot z + \sum_{s \in \underline{\mathbf{S}}} \lambda_s p^1_s \cdot (x_s - e_{is}) \leq 1,$   
**Key words:** incomplete markets, restricted participation, arbitrage, existence of  
 $p^1_s \cdot (x_s - e_{is}) \leq V(s) \cdot z, \forall s \in \underline{\mathbf{S}},$  and  $\bar{p}_{i,s} \cdot (x_s - e_{is}) \leq V(s) \cdot z, \forall s \in S_i \setminus \underline{\mathbf{S}} \}$ ;  
 equilibrium, rational expectations, perfect foresight, asymmetric information.  
 $A(p, p^1, q) := \{ [(x_i, z_i)] \in \times_{i \in I} B_i^1(p, p^1, q) : \sum_{i \in I} (x_{is} - e_{is}) = 0, \forall s \in \underline{\mathbf{S}} \text{ and } \sum_{i \in I} z_i \in Z^o \}$ .  
**JEL Classification:** D52

<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital,  
 75013 Paris, France (E-mail) lionel.de.boisdeffre@vtm.unipari.fr,  $\sum_{i \in I} (\|x_i\| + \|z_i\|) < r$

**Proof** See the Appendix ASYMMETRIC INFORMATION AND RESTRICTED PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION □

PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION

Along Lemma 1, let  $X_i^* := \{(x_s) \in X_i : \|x\| \leq r\}$  and  $Z_i^* := \{z \in Y_i \cap Z_i : \|z\| \leq r\}$ , and

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define, for every  $(i, (p, p^1 := (p_s^1)), q) \in I \times P^* \times Q$ , the following convex bounded sets:

$$B'_i(p, p^1, q) := \{ (x, z) \in X_i^* \times Z_i^* : p \cdot (x_0 - e_{i0}) + q \cdot z + \sum_{s \in \underline{S}} \lambda_s p_s^1 \cdot (x_s - e_{is}) \leq \gamma_{(p, p^1, q)} \\ p_s^1 \cdot (x_s - e_{is}) \leq V(s) \cdot z, \forall s \in \underline{S}, \text{ and } \bar{p}_{is} \cdot (x_s - e_{is}) \leq V(s) \cdot z, \forall s \in S_i \setminus \underline{S} \};$$

$$B''_i(p, p^1, q) := \{ (x, z) \in X_i^* \times Z_i^* : p \cdot (x_0 - e_{i0}) + q \cdot z + \sum_{s \in \underline{S}} \lambda_s p_s^1 \cdot (x_s - e_{is}) < \gamma_{(p, p^1, q)} \text{ and} \\ p_s^1 \cdot (x_s - e_{is}) < V(s) \cdot z, \forall s \in \underline{S}, \text{ and } \bar{p}_{is} \cdot (x_s - e_{is}) < V(s) \cdot z, \forall s \in S_i \setminus \underline{S} \};$$

**Abstract**

where  $\gamma_{(p, p^1, q)} = 1 - \min(1, \|p\| + \|p^1\| + \|q\|)$ .

The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets.

**Claim 4** For every  $i \in I$ ,  $B'_i$  is upper semicontinuous, for having a closed graph in a compact set.

**Proof** Let  $i \in I$  be given. The correspondences  $B'_i$  is, as standard, upper semicontinuous, for having a closed graph in a compact set. □

prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).

**Claim 5** The following Assertions hold, for each  $i \in I$ :  
 (i)  $\forall (p, p^1) \in P^* \forall q \in Q, B''_i(p, p^1, q) \neq \emptyset$ .  
 (ii) the correspondence  $B''_i$  is lower semicontinuous (l.s.c.).  
 Moreover, the definition and existence of the sequential equilibrium do not rely on Radner's (1972, 1979) rational expectation assumptions, which may be dropped.

**Proof** Assertion (i): let  $(i, (p, p^1), q) \in I \times P^* \times Q$  be given. Assume, that  $(p, p^1) \neq 0$  or that  $\gamma_{(p, p^1, q)} = 1$ . Then, from Assumptions A2-A4-A6, there exists  $(\lambda, x) \in \mathbb{R}_{++} \times X_i^*$ , such that  $(x, \lambda z_i^+) \in B''_i(p, p^1, q)$ . Assume, alternatively, that  $(p, p^1) = 0$  and  $q \in Z^* \setminus \{0\}$ . The portfolios of  $Z^* \subset \underline{Z}^\circ$  yield zero payoffs on  $\underline{S}$ . It follows from Assumptions A2-A4-A5-A6 that there exists  $(\lambda, \mu) \in \mathbb{R}_{++}^2$ , small enough, such that:  $(0, \lambda z_i^+ - \mu q) \in B''_i(p, p^1, q)$ . □

equilibrium, rational expectations, perfect foresight, asymmetric information.

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information.

**JEL Classification:** D52  
 (i)  $(p, p^1), q) \in I \times P^* \times Q$ , an open subset,  $V$ , of  $X_i^* \times Z_i^*$ , and  $(x, z) \in V \cap B''_i(p, p^1, q)$ . From the definition of  $B'_i$ ,  $(x, z) \in X_i^* \times Z_i^*$  be given such that  $(x, z) \in V \cap B''_i(p, p^1, q)$ . From the definition of  $B'_i$ ,  $(x, z) \in X_i^* \times Z_i^*$  be given such that  $(x, z) \in V \cap B''_i(p, p^1, q)$ .

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information.

**JEL Classification:** D52  
 (i)  $(p, p^1), q) \in I \times P^* \times Q$ , an open subset,  $V$ , of  $X_i^* \times Z_i^*$ , and  $(x, z) \in V \cap B''_i(p, p^1, q)$ . From the definition of  $B'_i$ ,  $(x, z) \in X_i^* \times Z_i^*$  be given such that  $(x, z) \in V \cap B''_i(p, p^1, q)$ . From the definition of  $B'_i$ ,  $(x, z) \in X_i^* \times Z_i^*$  be given such that  $(x, z) \in V \cap B''_i(p, p^1, q)$ .

University of Paris 1 Panthéon-Sorbonne, 106 Boulevard de l'Hopital, 75013 Paris, France. Email: lionel.de.boisdeffre@wanadoo.fr



there exists a price  $(p, p^1, q)$  in  $P^* \times Q$  such that  $(p, p^1, q)$  is an equilibrium price and yield Claim 6.  $\square$

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## 4.2 The fixed-point-like argument

A convex compact set,  $\Theta := P^* \times Q \times (\times_{i \in I} X_i^* \times Z_i^*)$ , and l.s.c. correspondences are introduced. Namely, for each  $i \in I$  and every  $\theta := ([p, p^1 := (p_s^1)], q, [(x_i, z_i)]) \in \Theta$ , we let:

$$\Psi_i(\theta) := \begin{cases} B'_i(p, p^1, q) & \text{if } (x_i, z_i) \notin B'_i(p, p^1, q) \\ B''_i(p, p^1, q) \cap P_i(x_i) \times Z_i^* & \text{if } (x_i, z_i) \in B'_i(p, p^1, q) \end{cases}$$

$$\Psi_0(\theta) := \{ (p', p'^1, q') \in P^* \times Q :$$

The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-

**Lemma 2** For each  $i \in I \cup \{0\}$ , the correspondence  $\Psi_i$  is lower semicontinuous. arbitrage price and equilibrium asset price coincide. Namely, any collection of state

prices  $(p, p^1, q)$  that support a sequential equilibrium. This result extends

*Cass (2006), De Boisdeffre (2007) and similar results of symmetric information.*

The above correspondences display a remarkable element,  $\theta^* \in \Theta$ , which meets the properties of Claim 6, and is henceforth set as given and always referred to: Moreover, the definition and existence of the sequential equilibrium do not rely on

**Claim 6** (1972, 1979) a rational expectation  $(p^*, p^{*1}, q^*)$ , which may be dropped that:

$$(i) \forall (p, p^1 := (p_s^1)) \in P^*, \forall q \in Q,$$

$$(q^* - q) \cdot \sum_{i \in I} z_i^* + (p^* - p) \cdot \sum_{i \in I} (x_{i0}^* - e_{i0}) + \sum_{s \in \mathbb{S}} (p_s^{1*} - p_s^1) \cdot \sum_{i \in I} (x_{is}^* - e_{is}) \geq 0;$$

$$(ii) \forall i \in I, (x_i^*, z_i^*) \in B'_i(p^*, p^{*1}, q^*) \text{ and } B''_i(p^*, p^{*1}, q^*) \cap P_i(x_i^*) \times Z_i^* = \emptyset.$$

**Proof** Quoting Gale-Mas-Colell (1975, 1979): “Given  $X = \times_{i=1}^m X_i$ , where  $X_i$  is a non-empty compact convex subset of  $\mathbb{R}^n$ , let  $\varphi_i : X \rightarrow X_i$  be  $m$  convex (possibly empty) valued correspondences, which are lower semicontinuous. Then, there exists  $x :=$

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$(x_i) \in X$  such that, for each  $i$ , either  $x_i \in \varphi_i(x)$  or  $\varphi_i(x) = \emptyset$ ”. The correspondences

<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: lionel.deboisdeffre@univ-paris1.fr and yield Claim 6.  $\square$

**Claim 7 (EQUILIBRIUM WITH ASYMMETRIC INFORMATION AND RESTRICTED PARTICIPATION):** Let  $q :=$

$(q^* + \bar{q}) :=$  (PARTICIPATION): *Then, the following assertions hold:*

- (i)  $\sum_{i \in I} z_i^* \in Z \oplus Z^o$  and  $\sum_{i \in I} (x_{is}^* - e_{is}) \in Z^o$ , for every  $s \in \underline{S}$ ;
- (ii)  $(x_i^*, z_i^*) \in B'_i(p^*, p^{*1}, q^*)$  and  $B'_i(p^*, p^{*1}, q^*) \cap P_i(x_i^*) \times Z_i^* = \emptyset$ , for every  $i \in I$ ;
- (iii)  $\sum_{i \in I} z_i^* \in Z^o$  and we let  $(z_i^o) \in \times_{i \in I} Z_i^o$  and  $(z_i) = (z_i^* - z_i^o)$  be such that  $\sum_{i \in I} z_i = 0$ ;
- (iv)  $(p^*, p^{*1}) \in P \times P^{\underline{S}}$  and  $(p^*, q, [(x_i^*, z_i)])$  is a sequential equilibrium of the economy  $\mathcal{E}_{(p_i)}$ , defined by  $p_{is} = \bar{p}_{is}$ , for  $(i, s) \in I^* \times S_i \setminus \underline{S}$ , and  $p_{is} = p_s^{*1}$ , for  $(i, s) \in I \times \underline{S}$ ;
- (v)  $q \in \mathcal{NA}(\lambda)$ .

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**Proof**

The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010), and  $0 < p^* \cdot \sum_{i \in I} (x_{i0}^* - e_{i0}) + q^* \cdot \sum_{i \in I} z_i^* + \sum_{s \in \underline{S}} \lambda_s p_s^* \cdot \sum_{i \in I} (x_{is}^* - e_{is}) \leq \gamma_{(p^*, p^{*1}, q^*)} = 0$ , the budget constraints,  $p^* \cdot (x_{i0}^* - e_{i0}) + q^* \cdot z_i^* + \sum_{s \in \underline{S}} \lambda_s p_s^* \cdot (x_{is}^* - e_{is}) \leq \gamma_{(p^*, p^{*1}, q^*)} = 0$ , Radner's (1972, 1979) rational expectation assumptions, which may be dropped also hold, for each  $i \in I$ . Summing them up (for  $t \in I$ ) yields a contradiction from above, namely:  $0 < p^* \cdot \sum_{i \in I} (x_{i0}^* - e_{i0}) + q^* \cdot \sum_{i \in I} z_i^* + \sum_{s \in \underline{S}} \lambda_s \sum_{i \in I} p_s^{*1} \cdot (x_{is}^* - e_{is}) \leq 0$ .  $\square$

Assertion (ii): from Claim 6, it suffices to show:  $B'_i(p^*, p^{*1}, q^*) \cap P_i(x_i^*) \times Z_i^* = \emptyset$ . By contraposition, assume that there exists a strategy  $(x_i, z_i) \in B'_i(p^*, p^{*1}, q^*) \cap P_i(x_i^*) \times Z_i^*$ . Let  $(x'_i, z'_i) \in B''_i(p^*, p^{*1}, q^*)$  be given, along Claim 5. From Assumptions 4<sup>o</sup>, 4<sup>o</sup> and convexity arguments, both relations  $(x'_i, z'_i) \in (\frac{1}{n})(x'_i, z'_i) + (1 - \frac{1}{n})(x_i, z_i) \in B''_i(p^*, p^{*1}, q^*)$  and  $(x'_i, z'_i) \in B'_i(p^*, p^{*1}, q^*) \cap P_i(x_i^*) \times Z_i^*$  would hold, for  $n \in \mathbb{N}$  big enough, contradicting Claim 6.  $\square$

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<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: [tonel.de.boisdeffre@wanadoo.fr](mailto:tonel.de.boisdeffre@wanadoo.fr)

$(x_i^*, z_i^*) \in B_i(p^*, q^*)$  for all  $i \in I$ . Summing the budget constraints yields, from Assertion (i)  $\sum_{i \in I} (x_i^* - e_{is}) = \sum_{s \in S} \lambda_s \sum_{i \in I} p_s^* \cdot (x_i^* - e_{is}) = \#I \gamma_{(p^*, q^*)}$ . The latter relations are written  $\sum_{i \in I} z_i^* \in Z^o$  which holds from Assertion (i):  $\sum_{i \in I} z_i^* \in Z^o$ .  $\square$

Assertion (iv): summing up the binding (from above) budget constraints at  $t = 0$  yields, from Assertions (i)-(ii),  $0 = p_0^* \cdot \sum_{i \in I} (x_{i0}^* - e_{i0}) + q^* \cdot \sum_{i \in I} z_i^* + \sum_{s \in S} \lambda_s \sum_{i \in I} p_s^* \cdot (x_{is}^* - e_{is}) = \#I \gamma_{(p^*, q^*)}$ . The relations  $(x_i^*, z_i) \in B_i(p^*, q)$ , for each  $i \in I$ , in the economy  $\mathcal{E}_{(p_i)}$  (defined in Assertion (iv)), follow from the definitions, Assumptions A4-A5 and

**Abstract**

the relation  $q^* + \bar{q} \in Z^* \oplus \underline{Z}$ , where  $q := (q^* + \bar{q}) := q^* + \sum_{s \in S} \lambda_s V(s)$  and  $(z_i) \in \times_{i \in I} Y_i$  is defined along Assertion (iii). From Assertions (i)-(iii) and above, the collection,

*The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial collection.* Under Conditions of Definition 1 the intertemporal (arbitrary) price and equilibria from Assertion (iv) exist. Namely, we can identify, state prices and reliable, state independent support for sequential equilibrium. This result extends Gosselin (2000) Assertion (iv) and Assumption A1. A result of contract theory that there exists a market as Recha and Frikhis (2005) or Kohler and Gosselin (2010) Assumptions A2-A4, and the strategy of the sequential equilibrium that will hold get

Constraint (1972, 1979) rational expectation assumptions, the relations  $(\text{app}) := [\frac{1}{n}(x'_i, z'_i) + (1 - \frac{1}{n})(x_i^*, z_i^*)] \in B_i(p^*, p^{*1}, q^*) \cap P_i(x_i^*) \times Z_i^*$  would hold, from the definitions, Assumptions A3-A4 and above. The latter relations contradict Assertion (ii).  $\square$

Assertion (v): Assertion (v) states the relations  $\pi(q) = \bar{q}$  and  $q \in \mathcal{N.A}$ . The former follows from the definitions, which imply:  $q := q^* + \bar{q} \in Z^* \oplus \underline{Z}$ . The latter is the

**Key words:** incomplete markets, restricted participation, arbitrage, existence of standard no-arbitrage property of equilibrium prices: if there existed  $(i, z) \in I \times As(Y_i)$ , equilibrium, rational expectations, perfect foresight, asymmetric information, such that  $-q \cdot z \geq 0$  and  $V(s) \cdot z \geq 0$  for every  $s \in S_i$ , with one strict inequality,

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the strategy  $(x_i^*, z_i)$  would not (from Assumption A1) be optimal in  $B_i(p^*, q)$ , in

<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. E-mail: henri.henbois@univ-paris1.fr. Theorem 1 is proved completely.  $\square$

## 5 The model's outcomes when one agent is fully informed

### PARTICIPATION : THE NO-ARBITRAGE CHARACTERIZATION

Throughout this Section, one agent, denoted  $i_0 \in I$ , is fully informed (i.e.,  $S_{i_0} = \mathbf{S}$ ).

All structural characteristics of a standard economy,  $\{(I, S, L, J), V, (S_i), (e_i), (\prec_i)\}$ , are unchanged from Section 2, except Assumption A5, which is dropped without changing the result of Theorem 1. With one informed agent, the relation  $\underline{Z}^\circ = Z^\circ$  holds, and implies:  $\mathcal{NA} \subset \underline{Z} = Z$ . Hence,  $\pi(q) = q$ , for every  $q \in \mathcal{NA}$ . Thus, the

no-arbitrage prices in the current economy identify to the standard no-arbitrage prices on frictionless markets of symmetric information. Theorem 1 states that the latter coincide with equilibrium prices and may be supported by any collection of state prices of the informed agent. Assumption A5 is now replaced by a standard assumption of restricted participation models with symmetric information, namely:

**A5'**  $Z^\circ \subset Y_{i_0}$ , or agents have symmetric information and, either  $Z^\circ \subset \cup_{i \in I} Y_i$ , or  $F := \cup_{i \in I} Y_i$  is a vector space.

tends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).

*Remark 5* The condition  $Z^\circ \subset \cup_{i \in I} Y_i$  of Assumption A5' is equivalent, from A4, to  $Z^\circ \subset \cup_{i \in I} As(Y_i) := \cup_{i \in I} \{z \in \mathbb{R}^J : \{z\} + Y_i \subset Y_i\}$  (Cornet and Gopalan, 2010, p. 247). Radner's (1972, 1979) rational expectation assumptions, which may be dropped.

Under Assumptions A1-A2-A3-A4-A5'-A6, Section 5.1 proves that sequential equilibria exist consistently with any state prices in realizable states, and any given expectations in the other states. Section 5.2 relates this result to the literature.

#### 5.1 Equilibrium with one fully informed agent.

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information.

There exists an abundant literature on restricted participation. To quote only

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a few contributions, the reader may refer, e.g., to Balasko-Cass-Siconolfi (1990),

<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: lionel.de.boisdeffre@wanadoo.fr

established in particular with a stronger restriction on the trade of securities may lead to a real indeterminacy of equilibrium in a similar context of incomplete markets. The present focus is different. It is on the *De Boisdeffre* and steps used to extend Cass' (2006) price equivalence theorem, and, in particular, to restricted participation. As shown by Balasko, Cass and Siconolfi (1990), that price equivalence fails with restricted participation, in general. Few contributions have investigated this issue. Martins-da-Rocha and Triki (2005) and Cornet and Gopalan (2010) are exceptions.

### **Abstract**

In the latter papers, a weak form of equilibrium is proved to exist first, which lets portfolios clear in value only. Namely, with two periods, Condition (c) of the *The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-arbitrage price and equilibrium asset price coincide. Namely, any collection of state prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), De Boisdeffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).* above Definition 1 ( $\sum_{i \in I} z_i = 0$ ) would be replaced by:  $q \cdot \sum_{i \in I} z_i = 0$ . The latter equilibrium is called a 'weak equilibrium' by Martins-da-Rocha and Triki (2005) and an 'accounts clearing equilibrium' by Cornet and Gopalan (2010). Their proofs derive the existence of the weaker equilibrium from a technical condition on agents' prices in realizable states is proved to support a sequential equilibrium. This result extends Cass (2006), *De Boisdeffre (2007) and similar results of symmetric information, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).* yields a sequential equilibrium, consistent with any collection of given state prices.

Moreover, the definition and existence of the sequential equilibrium do not rely on Hereafter, Assumption A5' replaces A5 to suggest an alternative, direct, less *Radner's (1972, 1979) rational expectation assumptions, which may be dropped.* demanding proof (in my view) of Cass' price equivalence on purely financial markets, when agents, with one informed, may have non-ordered preferences and restricted participation. No other assumptions than  $A1-A2-A3-A4-A5'-A6$  are required to prove the existence of a sequential equilibrium and the desired price equivalence.

**Key words:** incomplete markets, restricted participation, arbitrage, existence of *th* agent's portfolios were restricted to the set  $Y_i \cap Z_i$ . This restriction was possible equilibrium, rational expectations, perfect foresight, asymmetric information.

**JEL Classification:** D92

possibility vanishes, which may pose a boundary problem (see the Appendix). This University of Paris 1 - Panthéon - Sorbonne, 106-H2 Boulevard de l'Hopital, 75013 Paris, France. Email: lionel.de.boisdeffre@wanadoo.fr

problem disappears when the ~~assumptions are eliminated (and restricted)~~.<sup>2</sup> Yet, Won and ~~Palfi (2003)~~ show that ~~reducing assets can contribute to risk sharing.~~

Lionel de Boisdeffre,<sup>1</sup>

Moreover, restricted participation models typically keep the set  $Z^o$  non-trivial, so, I leave this possibility open. To circumvent the above problem, I introduce an auxiliary economy, which only changes portfolio sets. The latter are replaced, for each  $i \in I$ , by:  $Y'_i := Y_i + Z_i^o$ . When one agent is fully informed, both relations  $Z^* = \{0\}$  and  $A := \{(z_i) \in \mathbb{R}^{J \times I} : \sum_{i \in I} z_i = 0, V(s_i)z_i \geq 0, \forall (i, s_i) \in I \times S_i\} \subset \times_{i \in I} Z_i^o \subset \times_{i \in I} Y'_i$  hold, **Abstract** from the definitions and the above assumption that markets are arbitrage-free.<sup>3</sup>

~~The paper presents a pure exchange economy, where consumers may have asymmetric information, non-ordered preferences and a restricted access to purely financial markets. Under standard conditions, it shows that the two concepts of no-~~  
~~latter is denoted by  $\mathcal{E}_{(p_i)}$ , for every price expectations,  $(p_i) \in P^{S_i \times I}$ . It resumes all~~  
~~arbitrage price and equilibrium asset price coincide. Namely, any collection of state~~  
~~definitions and notations of Section 2, except budget sets' notations, which are now:~~  
~~prices in realizable states is proved to support a sequential equilibrium. This result ex-~~  
~~tends Cass (2006), De Boisdeffre (2007) and similar results of symmetric informa-~~  
~~tion, such as Martins-da-Rocha and Triki's (2005) or Cornet and Gopalan's (2010).~~

~~Moreover, the definition and existence of the sequential equilibrium do not rely on~~  
**Claim 8** ~~Let  $\lambda \in \mathbb{R}_{++}^S$  and subjective expectations,  $p_i := (p_{is}) \in P^{S_i \setminus \underline{S}}$ , for every~~  
~~Radner's (1972, 1979) rational expectation assumptions, which may be dropped.~~  
 ~~$i \in I^* := \{i \in I : S_i \neq \underline{S}\}$ , be given. Along Theorem 1, let a sequential equilibrium,~~  
 ~~$(p, q, [(x_i, z_i)]) \in P \times \mathcal{NA}(\lambda) \times (\times_{i \in I} B'_i(p, q))$  of an auxiliary economy,  $\mathcal{E}'_{(p_i)}$ , be given, such~~  
~~that  $p_{is} = \bar{p}_{is}$ , for every  $(i, s) \in I^* \times S_i \setminus \underline{S}$ . Then, there exist portfolios,  $(z_i^*) \in \times_{i \in I} Y_i$ ,~~  
~~such that  $(p, q, [(x_i, z_i^*)])$  is a sequential equilibrium of the initial economy,  $\mathcal{E}_{(p_i)}$ .~~

**Key words:** ~~incomplete markets, restricted participation, arbitrage, existence of~~  
~~equilibrium, asymmetric information, perfect foresight, asymmetric information.~~  
~~From the definition,  $Z^o$  is always included in  $Z^o$ . In the current economy, the relations  $Z^* = \{0\}$  and  $Z^o = Z^o$~~   
**JEL Classification:** ~~D52 holds, Assumption A5 is implied by A4.~~

<sup>3</sup> The latter assumption is unnecessary with symmetric information, since in that case the relation  $A \subset Z^o = Z^o$  holds from the definition.  
<sup>1</sup> University of Paris - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: lionel.de.boisdeffre@wanadoo.fr

given. Along with the  $(\text{ASYM})$  and  $(\text{METRIC})$  conditions, let  $(\text{INFORMATION})$  and  $(\text{REP})$  be a sequential equilibrium participation,  $\mathcal{E}'_{(p_i)}$  such that arbitrage, for every  $(p, q) \in \mathcal{Z}$ . From the definition of budget sets, there exists  $B_i(p, q)$  defining  $Z_i^\circ$ , such that  $z^\circ := \sum_{i \in I} z_i^\circ \in Z^\circ$  and  $(x_i, z_i + z_i^\circ) \in B_i(p, q)$ , for all  $i \in I$ , since  $q \in \mathcal{N}\mathcal{A} \subset Z = Z^{\circ\perp}$  holds from above.

Assume, from Assumption  $A5'$  and Remark 5, that the relation  $Z^\circ \subset Y_{i_0}$  holds or that  $Z^\circ \subset \cup_{i \in I} A_s(Y_i)$  and agents have symmetric (full) information. Then, there exists a fully informed agent,  $i' \in I$ , such that  $(z_{i'} + z_{i'}^\circ - z^\circ) \in Y_{i'}$ . Let  $(z_i^*) \in \times_{i \in I} Y_i$  be defined by  $z_{i'}^* := (z_{i'} + z_{i'}^\circ - z^\circ)$  and  $z_i^* := z_i + z_i^\circ$  for every  $i \in I \setminus \{i'\}$ . Then,  $\mathcal{C} := (p, q, [(x_i, z_i^*)])$

is a sequential equilibrium of the economy  $\mathcal{E}_{(p_i)}$ . Indeed,  $\mathcal{C}$  meets conditions (b)-(c)

The paper presents a pure exchange economy, where consumers may have asymmetric information, by construction. Moreover, the relation  $q \in Z$  and the optimality of  $(x_i, z_i)$  in  $B_i(p, q) \supset B_i(p, q)$ , imply that  $(x_i, z_i^*) \in B_i(p, q)$  in  $B_i(p, q)$ , for each  $i \in I$ .

Under standard conditions, it shows that the two concepts of non-arbitrage sequential equilibrium and asset price coincide for a complete market and symmetric information. The paper contains a sequential equilibrium. This result extends the results of Gale (2006), De Boisselier (2007) and similar results of symmetric information, such as Martini, de Boisselier and Strub (2001) or Cornet and Gopalan (2010). Moreover, the definition and existence of this sequential equilibrium is identical to the Radner's (1972, 1979) rational expectation assumptions, which may be dropped.

**Remark 6** It follows from the above proof, resuming its notations, that the equilibrium,  $\mathcal{C}' := (p, q, [(x_i, z_i)])$ , of the economy  $\mathcal{E}'_{(p_i)}$ , along Claim 8, yields an accounts clearing equilibrium of the economy  $\mathcal{E}_{(p_i)}$ , namely,  $\mathcal{C}^\circ := (p, q, [(x_i, z_i + z_i^\circ)])$ . The equilibrium,  $\mathcal{C} := (p, q, [(x_i, z_i^*)])$ , of the economy,  $\mathcal{E}_{(p_i)}$ , is derived from the latter,  $\mathcal{C}^\circ$ , owing to

Assumption  $A5'$ . The condition to apply Proposition 1 (p. 234) in Cornet-Gopalan (2010), which derives an equilibrium from an accounts clearing equilibrium, is exactly the same: Assumption  $A5'$ . The latter paper builds the existence of its accounts clearing equilibrium on an additional condition on the accessibility of asset

**Key words:** incomplete markets, restricted participation, arbitrage, existence of equilibrium, rational expectations, perfect foresight, asymmetric information.

**JEL Classification:** D52  
 counts clearing equilibrium on an additional condition on the accessibility of asset  
<sup>1</sup> University of Paris 1 - Panthéon - Sorbonne, 106-112 Boulevard de l'Hôpital, 75013 Paris, France. Email: [deboissel@univ-paris1.fr](mailto:deboissel@univ-paris1.fr)

## 5.2 The above results with reference to previous ones of the literature

The literature is abundant on asymmetric information and restricted participation. When it introduces asymmetric information, agents would have the perfect beliefs, and make the Radner inferences, described in De Boisdeffre (2022). The two conditions are linked and highly theoretical. The current paper prefers to tackle asymmetric information with the Cornet-De Boisdeffre's (2009) inferences, which require no price model nor structural knowledge on the economy. On restricted participation, the literature typically assesses conditions under which equilibrium exists, the limits to Cass' (2006) theorem, and conditions leading, either to the generic regularity of equilibrium, or to its indeterminacy (see Section 1 and above).

The current paper has a different focus. It aims to extend Cass' (2006) theorem to a fairly general model of restricted participation, where agents may have non-ordered preferences, asymmetric information and no price expectations a la Radner or CKRMC models. Since the same issue was investigated, under symmetric information, by Martins-da-Rocha-Triki (2005) and Cornet-Gopalan (2010) it is relevant to compare the latter papers and the current one when agents are fully informed.

Cornet-Gopalan (2010) weakens slightly Martins-da-Rocha-Triki's (2005) assumptions for the existence of equilibrium prices, but uses a similar approach for the proof, which is not so far from the current paper's. The main differences stem from the fact that the former drop Assumption *A6*, at a significant cost. Technically, the authors have to insure that auxiliary budget sets, similar to those in Section 4, have non-empty interiors, which is far more complex than proving Claim 5, above. Replacing Assumption *A6*, Cornet-Gopalan (2010) assumes that the closed cone spanned by agents' accessible payoffs is a vector space, and Martins-da-Rocha-Triki (2005), that all available payoffs in the aggregate are accessible in reduced amount



to an agent. Those technical assumptions are sufficient conditions for the existence of the weaker equilibrium, but neither necessary nor transparent conditions.

With symmetric information, the current paper replaces the latter accessibility conditions by Assumption A6, which is met on all actual markets. Doing so yields a direct existence proof along Theorem 1, in the sense that no weak form of equilibrium is required as a first step. An auxiliary economy can be used instead, where an accounts clearing equilibrium is only implicit (Remark 6). The notion of sequential equilibrium does not require that frictions be technically restricted as above, but that an agent might purchase any accessible portfolio, which she desires at market prices. In either models, equilibrium prices adapt to insure that condition.<sup>4</sup> Prices would prevent accessible transfers to take place, which cannot clear under frictions.

Another issue, which deserves attention, is the notion of no-arbitrage price. Definition 2 presents two standard concepts: the frictionless no-arbitrage price, and the no-arbitrage price, to which Martins-da-Rocha and Triki (2005) refer. Cornet and Gopalan (2010) introduce an intermediate concept of ‘*aggregate no-arbitrage price*’, which refers to the vector space,  $\langle \cup_{i \in I} Y_i \rangle \subset \mathbb{R}^J$ , spanned by  $\cup_{i \in I} Y_i$ . A price  $\bar{q} \in \mathbb{R}^J$  is said to be ‘*aggregate arbitrage-free*’ if there exists a vector of state prices,  $\lambda := (\lambda_s) \in \mathbb{R}_{++}^S$ , such that  $\bar{q} \cdot z = \sum_{s \in S} \lambda_s V(s) \cdot z$ , for every  $z \in \langle \cup_{i \in I} Y_i \rangle$ . Cornet-Gopalan’s theorem states the equivalence between equilibrium prices and aggregate no-arbitrage prices, under the payoff accessibility condition above.

Theorem 1 states, more precisely, the equivalence between the sets of equilibrium prices (on markets with or without frictions) and of *frictionless* no-arbitrage prices (i.e.,  $\{\sum_{s \in S} \lambda_s V(s) : (\lambda_s) \in \mathbb{R}_{++}^S\}$ ), which also define no-arbitrage prices and

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<sup>4</sup> Consider the economy of Remark 4, where  $s_1 = s_2 = s$  instead. This economy meets Cornet and Gopalan’s accessibility condition, above. But no trade in the first asset may take place at equilibrium. Prices need to insure that condition.

aggregate no-arbitrage prices through a vector of state prices,  $(\lambda_s) \in \mathbb{R}_{++}^S$ . Up to mild changes, Theorem 1 thus embeds the latest developments on Cass' price equivalence given by Martins-da-Rocha and Triki (2005), that no-arbitrage prices characterize equilibrium prices provided financial markets be locally collectively frictionless ( $\text{cone } \cup_{i \in I} VY_i = \text{Im } V$ ), or by Cornet and Gopalan (2010), who state that aggregate no-arbitrage prices characterize equilibrium prices whenever the closed cone spanned by agents' accessible payoffs is a linear space. Theorem 1 mainly replaces the above accessibility conditions by Assumption *A6*.

## 6 Concluding remarks

Section 5 shows that Theorem 1 and Claim 8 complement the existing results of the literature of symmetric information on Cass' price equivalence. Theorem 1 also refines and extends De Boisdeffre's (2007), which establishes that an equilibrium with asymmetric information on frictionless purely financial markets exists and is supported by *one* (unknown) collection of state prices. Theorem 1 now states that *any* collection of state prices in realizable states supports a sequential equilibrium, whether agents had a restricted or full access to financial markets. Finally, Theorem 1 proves the existence of a sequential equilibrium without assuming any form of rational expectations a la Radner (1972 and 1979). Agents' expectations are private exogenous in the current paper, as in the standard temporary equilibrium model.

The latter result is of self-interest. First, it contradicts the view that the sequential and temporary equilibria are two dichotomous concepts (Grandmont, 1982). The sequential equilibrium of Definition 1 is a temporary equilibrium, which lets agents' demands clear markets ex post.

Second, it suggests a response to the standard criticism of Radner's (1972) perfect foresight assumption, stated, e.g., by Kurz and Wu (1996): "*agents need to know the maps from states at future dates to prices in the future and it is entirely unrealistic to assume that agents can find out what this sequence of maps is.*" Radner (1982) himself acknowledges that perfect price foresight "*seems to require of the traders a capacity for imagination and computation far beyond what is realistic.*" Theorem 1 makes perfect foresight unnecessary to insure the existence of sequential equilibria. This result had been intuitive so far, but, to my best knowledge, never proved.

Under asymmetric information, Theorem 1 avoids agents to detain the structural knowledge of the ex ante characteristics of the economy and of the other agents' beliefs. This assumed knowledge is typically used to justify the Radner (1979) inferences in rational expectations' and CKRMC models (see De Boisdeffre, 2022). Moreover, given the indeterminacy of equilibrium with nominal assets, the inferences in Radner (1979) are questionable from the theoretical viewpoint also. The latter are based on the assumption that there exists a map between the state of nature to be revealed and one selected equilibrium price to be observed - a map which all agents agree upon and reveals information.

With many other possible outcomes, how can one particular revealing price emerge as a market equilibrium outcome, Polemarchakis and Siconolfi (1993) ask, when alternative equilibrium prices are non-revealing ? Why not agree on one of the latter prices, which would reveal no information ? CKRMC models typically let agents anticipate finitely many possible equilibrium prices in any state of nature (instead of one price with certainty) and apply the same inferences as Radner's to the correspondence relating states to multiple selected prices. But this new selection would not change the conundrum, when there are infinitely many equilibria.

In the current paper, agents endowed with no price model, but with private exogenous expectations, infer information from observing financial markets along the Cornet-De Boisdeffre (2009) inferences. The latter demand very little in terms of rationality. Agents only have to realize that a lasting unlimited arbitrage opportunity cannot exist, and, therefore, to rule out the states, which would offer a free lunch. The latter inferences and their properties extend to arbitrary financial structures and state spaces (possibly infinite). They seem to describe the actual refinement of information, which takes place on most financial markets (De Boisdeffre, 2016).

The current model drops rational expectations completely, that is, both the perfect foresight and Radner inference assumptions. But this does not solve one major problem of the standard sequential equilibrium concept, which stems from the degenerate (i.e., unique) price expectations. Because of insufficiently many observed parameters, no rational agent could infer, in general, which particular spot prices might obtain tomorrow, amongst all equilibrium possibilities. In case of indeterminacy, uncountably many market-clearing prices exist. Yet, the standard definition of sequential equilibrium in the literature lets agents commonly expect one price on every spot market. The common selection of a unique equilibrium price, amongst uncountably many, is a mysterious process. It could only be explained, theoretically, if the unique spot prices were the necessary outcomes of the economy's ex ante characteristics and agents' beliefs, under their common knowledge of rationality and unlimited forecast abilities. Such an assumption is, indeed, at the core of CKRMC models, but would not lead to degenerate expectations in such models.

In the current model, no such rationality is assumed. Future market-clearing prices always exist, from Theorem 1. But the probability that agents had expected them uniquely ex ante (as required at equilibrium), say 'by chance', is as negligible

as above. This problem vanishes when agents anticipate exogenous sets of spot prices (endowed with a probability distribution), which always contain (but may not be limited to) the true equilibrium price. Facing uncountably many equilibrium possibilities, rational agents should also have uncountable anticipation sets. When the latter always contain the true spot price, agents are said to have correct foresight.

Correct foresight may replace perfect foresight in the definition of the sequential equilibrium. With smooth preferences, the set of all market-clearing prices is shown to be non-empty, fixed and bounded above zero (whatever agents' beliefs). Having no price model, and no awareness of other agents' characteristics and beliefs, the latter set is argued, in De Boisdeffre (2022), to represent agents' minimum uncertainty about the future spot prices. To agents, the location of tomorrow's spot price within the minimum uncertainty set is undetermined until it is observed. In the latter paper, a correct foresight equilibrium is shown to exist whenever agents' anticipation sets embed the minimum uncertainty set. By a same technique of proof, the result of Theorem 1 could be used and extended to an economy, where agents' degenerate anticipations are replaced by anticipation sets. The extended theorem would state that any collection of state prices in realizable states supports a '*correct foresight*' sequential equilibrium, in an economy which fully discards rational expectations. The latter equilibrium would exist when agents' anticipation sets embed the minimum uncertainty set. Theorem 1 is also a step in the proof of this outcome.

## Appendix

**Lemma 1**  $\exists r > 0 : \forall ((p, p^1), q) \in P^* \times Q, \forall [(x_i, z_i)] \in \mathcal{A}(p, p^1, q), \sum_{i \in I} (\|x_i\| + \|z_i\|) < r,$

where  $\mathcal{A}(p, p^1, q) := \{[(x_i, z_i)] \in \times_{i \in I} B_i^1(p, p^1, q) : \sum_{i \in I} (x_{is} - e_{is}) = 0, \forall s \in \underline{S}, \sum_{i \in I} z_i \in Z^o\}.$

**Proof** Referring to the notations of Theorem 1, let  $\delta := (1 + \sum_{i \in I^*} \|\bar{p}_i\|) \sum_{i \in I} \|e_i\|$  be given. From the market-clearing conditions in  $\mathcal{A}(p, p^1, q)$ , the relations  $x_{is} \in [0, \delta]^L$  hold, for every  $(p, p^1, q) \in P^* \times Q$ , every  $[(x_i, z_i)] \in \mathcal{A}(p, p^1, q)$  and every  $(i, s) \in I \times \underline{S}'$ . Moreover, the idiosyncratic expectations,  $(\bar{p}_{is})$ , for  $i \in I^*$ , have a positive lower bound. Lemma 1 will, hence, be implied by the following Assertion:

$$\exists r' > 0, \forall (p, p^1, q) \in P^* \times Q, \forall [(x_i, z_i)] \in \mathcal{A}(p, p^1, q), \sum_{i \in I} \|z_i\| < r'.$$

Assume, by contraposition, that the above Assertion fails, that is, for every  $k \in \mathbb{N}$  ( $k > 0$ ), there exist prices,  $(p^k, p^{1k}, q^k) \in P^* \times Q$ , and strategies,  $[(x_i^k, z_i^k)] \in \mathcal{A}(p^k, p^{1k}, q^k)$ , such that  $\alpha_k := \sum_{i \in I} \|z_i^k\| \geq k$ . Then, the relations  $z^k := (z_i^k) \in \times_{i \in I} Y_i \cap Z_i$  and  $z'^k := z^k / \alpha_k := (z_i^k / \alpha_k) \in \times_{i \in I} Y_i \cap Z_i$  hold, for every  $k \in \mathbb{N}$ , from the definition of  $\mathcal{A}(p^k, p^{1k}, q^k)$  and Assumption A4. The bounded sequence,  $\{z'^k\}$ , may be assumed to converge in a closed set, say to  $z := (z_i) \in \times_{i \in I} Y_i \cap Z_i$ , such that  $\|z\| = 1$ . The relations  $[(x_i^k, z_i^k)] \in \mathcal{A}(p^k, p^{1k}, q^k)$  hold, for every  $k \in \mathbb{N}$ , and imply, for every  $(i, s) \in I \times S_i$ :

$$V(s) \cdot z_i^k \geq -\delta, \text{ that is, } V(s) \cdot z_i'^k \geq -\frac{\delta}{k}, \text{ hence, in the limit, } V(s) \cdot z_i \geq 0, \text{ for each } s \in S_i, \\ \text{while } \sum_{i \in I} z_i^k \in Z^o \text{ implies } \sum_{i \in I} z_i'^k \in Z^o, \text{ hence, in the limit, } \sum_{i \in I} z_i \in Z^o.$$

Let  $(z_i^o) \in \times_{i \in I} Z_i^o$  satisfy  $\sum_{i \in I} z_i^k = \sum_{i \in I} z_i^o \in Z^o$  and define  $(z_i^*) = (z_i - z_i^o)$ . Then, the relations  $V(s) \cdot z_i^* \geq 0$ , for each  $(i, s) \in I \times S_i$ , and  $\sum_{i \in I} z_i^* = 0$  hold and imply, from Cornet and De Boisdeffre (2002, p. 401) and the fact that markets are frictionless arbitrage-free along Claim 1:  $(z_i^*) \in \times_{i \in I} Z_i^o$ . The relation  $(z_i) \in \times_{i \in I} (Z_i^o \cap Z_i) = \{0\}$  follows by construction, contradicting the above:  $\|z\| = 1$ . This completes the proof.  $\square$

**Lemma 2** For each  $i \in I \cup \{0\}$ ,  $\Psi_i$  is lower semicontinuous.

**Proof** The correspondence  $\Psi_0$  is lower semicontinuous for having an open graph.

We now set as given  $i \in I$  and  $\theta := (p, p^1, q, [(x_i, z_i)]) \in \Theta := P^* \times Q \times (\times_{i \in I} X_i^* \times Z_i^*)$ .

- Assume that  $(x_i, z_i) \notin B'_i(p, p^1, q)$ . Then,  $\Psi_i(\theta) = B'_i(p, p^1, q)$ .

Let  $V$  be an open set in  $X_i^* \times Z_i^*$ , such that  $V \cap B'_i(p, p^1, q) \neq \emptyset$ . It follows from the convexity of  $B'_i(p, p^1, q)$  and the non-emptiness of the open set  $B''_i(p, p^1, q) \subset B'_i(p, p^1, q)$  that  $V \cap B''_i(p, p^1, q)$  is non-empty. From Claim 5, there exists a neighborhood  $U$  of  $(p, p^1, q)$ , such that  $V \cap B'_i(p', p'^1, q') \supset V \cap B''_i(p', p'^1, q') \neq \emptyset$ , for every  $(p', p'^1, q') \in U$ .

Since  $B'_i(p, p^1, q)$  is non-empty, closed, convex in the compact set  $X_i^* \times Z_i^*$ , there exist two open sets,  $V_1$  and  $V_2$ , in  $X_i^* \times Z_i^*$ , such that  $(x_i, z_i) \in V_1$ ,  $B'_i(p, p^1, q) \subset V_2$  and  $V_1 \cap V_2 = \emptyset$ . From Claim 4, there exists a neighborhood,  $U_1 \subset U$ , of  $(p, p^1, q)$ , such that  $B'_i(p', p'^1, q') \subset V_2$ , for every  $(p', p'^1, q') \in U_1$ . Let  $W = U_1 \times (\times_{j \in I} W_j)$ , where  $W_i := V_1$ , and  $W_j := X_j^* \times Z_j^*$ , for each  $j \in I \setminus \{i\}$ , be a neighbourhood of  $\theta$  in  $\Theta$ . Then, the joint relations,  $\Psi_i(\theta') = B'_i(p', p'^1, q')$  and  $V \cap \Psi_i(\theta') \neq \emptyset$ , hold, for every  $\theta' := (p', p'^1, q', [(x'_i, z'_i)]) \in W$ , and imply that  $\Psi_i$  is lower semicontinuous at  $\theta$ .  $\square$

- Assume that  $(x_i, z_i) \in B'_i(p, p^1, q)$ . Then,  $\Psi_i(\theta) = B''_i(p, p^1, q) \cap P_i(x_i) \times Z_i^*$ .

The lower semicontinuity of  $\Psi_i$  at  $\theta$  results from the definition if  $\Psi_i(\theta) = \emptyset$ . Assume, now, that  $\Psi_i(\theta) \neq \emptyset$ . The correspondence  $P_i$  is lower semicontinuous with open values, from Assumption A3, and  $B''_i$  has an open graph in  $P^* \times Q \times X_i^* \times Z_i^*$ . As a corollary, the correspondence  $(p', p'^1, q', [(x'_i, z'_i)]) \in \Theta \mapsto B''_i(p', p'^1, q') \cap P_i(x'_i) \times Z_i^* \subset B'_i(p', p'^1, q')$  is lower semicontinuous at  $\theta$ ; and so is also  $\Psi_i$ , from the latter inclusions.  $\square$

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