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# The Solow model under localized technical change\*

Mehdi Senouci<sup>†</sup>

June 27, 2024

## Abstract

Textbook growth models typically fail to account for the absence of global convergence in income per capita, for the growth effects of the investment rate, and for the existence of large swings in the labor share over the medium run. In this paper, I set a Solow model animated by a form of localized technical change. I assume that productivity growth is an increasing function of the capital-output ratio. I prove that the model has a globally stable balanced growth path. If technical change is locally biased, then the growth rate is a strictly increasing function of the saving rate, and the labor share slowly tends to zero along any balanced growth path.

**Keywords:** Solow model, biased technical change, steady growth theorem, convergence, labor share.

**JEL codes:** O33, O40.

## Introduction

Production is the process of turning inputs into output and is usually represented mathematically by a production function, and technical change consists of an alteration of the production function. General equilibrium theory has prompted economists to restrict to so-called ‘neoclassical production functions’, i.e. functions increasing in each argument, constant-returns-to-scale and concave. As a consequence, every form of technical change that turns one neoclassical production function into another is also compatible with competitive equilibrium. However, textbook models take quite a specific assumption according to which one so-called ‘core’ production function  $F$  is altered only by factor-*augmenting* technical change, meaning that if date- $t$  inputs are  $(X_t^1, \dots, X_t^n)$  then realized production is  $Y_t = F_t(X_t^1, \dots, X_t^n) = F(A_t^1 X_t^1, \dots, A_t^n X_t^n)$  for some factor-specific productivity indexes  $(A_t^1, \dots, A_t^n)$ . Of course, not all alterations of production functions can be represented that way.

The benefits and costs of modeling technical change as factor-augmenting are well understood. Benefits include (i) theoretical consistency with competitive market equilibrium,<sup>1</sup> (ii) straightforward analytical treatment, (iii) compatibility with balanced growth under purely-labor-augmenting technical change,<sup>2</sup> and (iv) straightforward projection on national production data, assuming pure and perfect competition, through growth accounting (Solow (1956, 1957)).

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<sup>1</sup> If function  $(X^1, \dots, X^n) \mapsto F(X^1, \dots, X^n)$  is a neoclassical production function, then so is function  $(X^1, \dots, X^n) \mapsto F(A^1 X^1, \dots, A^n X^n)$  for any  $(A^1, \dots, A^n)$ .

<sup>2</sup> A balanced growth path (BGP) is defined as a growth path where all real variables have constant growth rates.

Costs include a certain lack of flexibility. For instance, in a two-input framework, any positive productivity-index shock increases the marginal productivity of both inputs, although possibly unequally, making technical change unable to yield large redistributive consequences when modeled as factor-augmenting (Acemoglu (2010)). Furthermore, within this framework, balanced growth requires technical change to be purely labor-augmenting,<sup>3</sup> ruling out other directions of technical change as incompatible with the Kaldor facts.<sup>4</sup> The assumption of the existence of a fixed core production function also entails a seemingly arbitrary dichotomy: Why wouldn't technical change alter the core production function itself?

In response to these limitations, some scholars have developed alternative views and models of technical change. Some approaches remain within the traditional framework and assume that technical change somehow alters the core production function. Thus, Blanchard (1997, p. 114), Zuleta (2008), Zuleta and Young (2013), Peretto and Seater (2013) and Senouci (2014) assume that technical change alters the capital exponent of some Cobb-Douglas production function; and Kemnitz and Knoblach (2020) present a model where endogenous technical change alters the elasticity of substitution of the core production function.<sup>5</sup> While they provide new flexibility, these approaches to technical change are most often incompatible with balanced growth.

Other scholars have developed a more sophisticated framework of production and technical change, the 'task model',<sup>6</sup> greatly expanding possible macroeconomic consequences of technical change and also able to bring deeper understanding of its labor-market consequences. On the other hand, task models are typically significantly more complex than factor-augmenting technical change models, making them hardly a substitute for the textbook framework.

In this paper, I generalize the canonical, purely labor-augmenting technical change framework into a localized technical change approach. Specifically, I assume that the production function is altered by (exogenous) productivity gains which are *not* uniform across input mixes. The production function remains strictly concave at all dates, and it evolves through time in a way that cannot be represented by any form of factor-augmenting technical change. Consistently with existing cross-country literature, I assume that productivity gains are increasing in the capital-output ratio. I then set up a Solow model animated by localized technical change. It comes out that the model has a unique, stable balanced growth path where the growth rate is increasing in the saving rate, and along which the labor share slowly tends to zero.

Thus, technical change relatively localized at high-capital-output (or high capital-labor) ratio techniques accounts for the absence of global convergence as well as with the robust correlation between the saving/investment rate and income in cross-section.<sup>7</sup>

The rest of the paper is organized as follows. Section 1 presents our approach to localized technical change. Section 2 sets a Solow model animated by localized technical change and derives the main results. Section 3 concludes.

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<sup>3</sup>I.e., all productivity indexes to be constant except labor's, see Robinson (1938), Uzawa (1961), Acemoglu (2003), and Jones and Scrimgeour (2008).

<sup>4</sup>The factor-augmenting technical change framework has shown limits in the debate on the causes of the global decline of the labor share. For instance, Karabarbounis and Neiman (2014) have argued that the decline of the labor share can be thought of as the consequence of productivity growth within the investment-goods sector if capital and labor are strict substitutes at the macroeconomic level. This view has been widely dismissed on the ground that most empirical estimates of the elasticity of substitution fall significantly below unity (Chirinko (2008)). All in all, the fall of the labor share and the stagnation of median real wages in the last decades in the US challenge the representation of technical change as factor-augmenting.

<sup>5</sup>On the elasticity of substitution as an engine of growth, see also Klump and de La Grandville (2000). See Temple (2012) for a critique of modeling technical change as directly affecting the elasticity of substitution.

<sup>6</sup>See, among others, Zeira (1998), Autor and Acemoglu (2011), Acemoglu and Restrepo (2018), and Growiec (2022).

<sup>7</sup>On the robust correlation between the investment rate and the growth rate, see De Long and Summers (1991, 1992), Levine and Renelt (1992). Countless papers have investigated the positive growth effect of human capital accumulation (e.g. schooling).

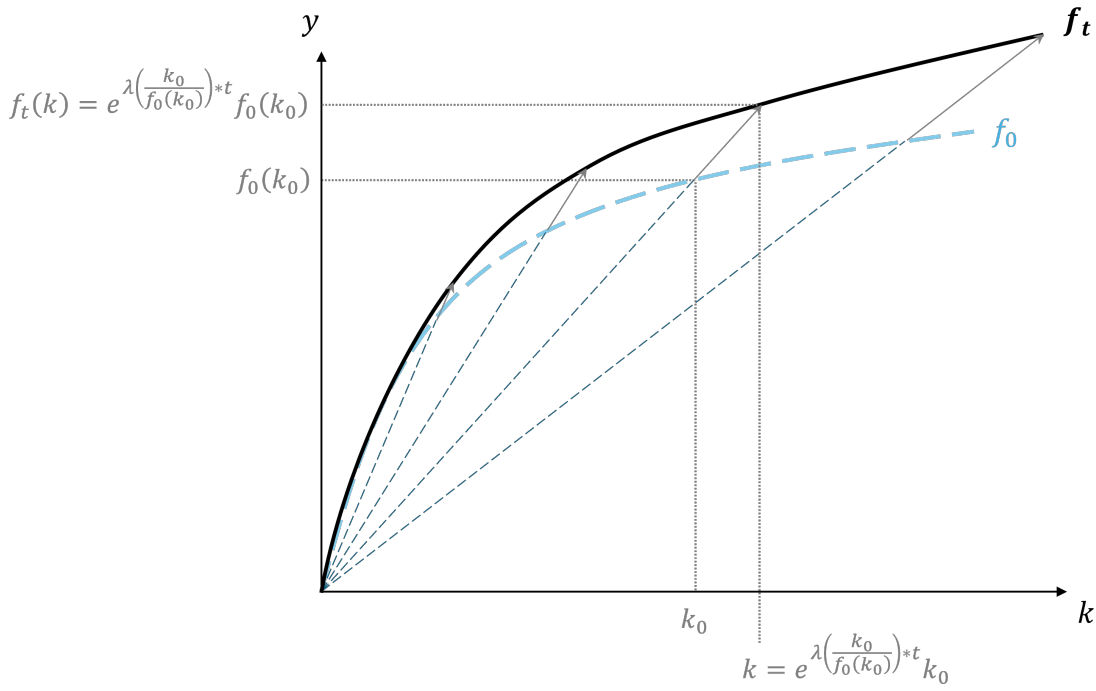


Figure 1: How  $f_t$  is obtained from functions  $f_0$  and  $\lambda$ .

## 1 A model of localized technical change

In this section, we call  $(F_t(\cdot, \cdot))_{t \geq 0}$  some sequence of two-input neoclassical production function,<sup>8</sup> and we call  $f_t$  the intensive-form production function corresponding to  $F_t$ .<sup>9</sup>

The labor-augmenting technical change (LATC) assumption is that there exists some sequence of positive numbers  $A_t$  such that for all  $(K, L)$ ,  $F_t(K, L) = F_0(K, A_t L)$ . A natural generalization of this framework into a localized technical change model consists in assuming that the state of technology at  $t$  cannot be summarized in one variable ( $A_t$ ), because productivity gains between dates 0 and  $t$  are uneven across the inputs-mix space. For example, productivity gains might be higher at high capital-labor ratios. However, to specify  $F_t(K, L) = F_0(K, A(\frac{K}{L}, t) * L)$  for some function  $A(\cdot, t)$  is too general a formula, as it might lead to non-neoclassical production functions. So, first, one wants to restrict to patterns of localized productivity growth – i.e., functions  $A(\frac{K}{L}, t)$  – such that function  $(K, L) \mapsto F_0(K, A(\frac{K}{L}, t) * L)$  is a neoclassical function.

Then, we also want a specification that might lead to balanced growth. Jones and Scrimgeour (2008) have shown that the capital-output ratio is constant along any balanced growth path with strictly positive gross investment; hence we also want to assume that the rate of change of  $A(\frac{K}{L}, t)$  depends only on the capital-output ratio, or:

$$\frac{\dot{A}(\frac{K}{L}, t)}{A(\frac{K}{L}, t)} = \lambda\left(\frac{K}{F_t(K, L)}\right) = \lambda\left(\frac{k}{f_t(k)}\right)$$

with  $k = K/L$  and where  $\lambda(\cdot)$  is some function representing localized technical change. If  $\lambda(\cdot)$  is a constant, and only in this case, the model lands back on the pure LATC framework. If function  $\lambda(\cdot)$  is increasing, then productivity growth is higher at higher capital-output ratios.

We can now set our definition of localized technical change. We will consider that the production function available at date  $t$  is given by formula:

$$\forall k > 0, \begin{cases} f_t(k) = e^{\lambda(\frac{k_0}{f_0(k_0)}) * t} f_0(k_0) \\ \text{where } \frac{k}{k_0} = e^{\lambda(\frac{k_0}{f_0(k_0)}) * t} \end{cases} \quad (1)$$

<sup>8</sup>For all  $t \geq 0$ ,  $F_t$  is assumed to be a function of  $[0, +\infty) \times [0, \infty[ \rightarrow [0, \infty)$ , increasing in each argument, constant-returns-to-scale and concave.

<sup>9</sup> $f_t$  is a function of  $[0, +\infty) \rightarrow [0, \infty)$ , defined by:  $\forall k \geq 0$ ,  $f_t(k) = F_t(k, 1)$ .  $f_t$  is increasing and concave (see Moysan and Senouci (2016)).

Figure 1 represents the construction of function  $f_t$  from functions  $f_0$  and  $\lambda$ . For a given  $k > 0$ , to determine the value of  $f_t(k)$  we first need to know what  $k_0$  is implicitly determined by the equation  $\frac{k}{k_0} = e^{\lambda\left(\frac{k_0}{f_0(k_0)}\right)t}$ . This  $k_0$  is the ‘ancestor’ capital-labor ratio of  $k$  as seen from date  $t$ . Then,  $f_t(k)$  is obtained by multiplying  $f_0(k_0)$  by the relevant growth rate between dates 0 and  $t$ , which is  $e^{\lambda\left(\frac{k_0}{f_0(k_0)}\right)t}$ .

The equation  $\frac{k}{k_0} = e^{\lambda\left(\frac{k_0}{f_0(k_0)}\right)t}$  might not have a closed-form solution, hence  $f_t(k)$  might not have an analytical formula. However, the mathematical treatment of implicit functions still permit to manipulate function  $f_t$ .

We will assume that for all  $t$ , function  $f_t$  defined by equation (1) is a neoclassical production function. This property is satisfied in a simple case, as we now prove:

**Proposition 1.** *If function  $f_0(\cdot)$  has everywhere an elasticity of substitution less than or equal to one, and if function  $\lambda(\cdot)$  is of the form  $\lambda\left(\frac{k}{y}\right) = \bar{g} + \theta \ln\left(\frac{k}{y}\right)$  where  $\bar{g}$  is a constant and  $\theta$  is a strictly positive constant; then function  $f_t(\cdot)$  defined in equation (1) is an intensive-form neoclassical production function.*

*Proof.* • Log-differentiating equation  $\frac{k}{k_0} = e^{\lambda\left(\frac{k_0}{f_0(k_0)}\right)t}$  with respect to  $k$  yields:

$$\forall k, \ln k = \ln k_0 + \lambda\left(\frac{k_0}{f_0(k_0)}\right)t \implies \frac{1}{k} = \frac{\partial k_0}{\partial k} \left( \frac{1}{k_0} + \frac{f_0(k_0) - k_0 f_0'(k_0)}{(f_0(k_0))^2} \lambda'\left(\frac{k_0}{f_0(k_0)}\right)t \right) \quad (2)$$

Hence:

$$\frac{\partial k_0}{\partial k} = \frac{k_0}{k} \frac{1}{\theta t (1 - \alpha_0(k_0)) + 1} \quad (3)$$

where  $\alpha_0(k_0) = \frac{k_0 f_0'(k_0)}{f_0(k_0)} \in (0, 1)$ . Remark that  $\frac{\partial k_0}{\partial k} > 0$ .

- We now log-differentiate the formula for  $f_t$  in (1) and integrate equation (2):

$$\forall k, \ln f_t(k) = \lambda\left(\frac{k_0}{f_0(k_0)}\right)t + \ln f_0(k_0) \implies \frac{f_t'(k)}{f_t(k)} = \frac{\partial k_0}{\partial k} \left( \frac{f_0(k_0) - k_0 f_0'(k_0)}{(f_0(k_0))^2} \lambda'\left(\frac{k_0}{f_0(k_0)}\right)t + \frac{f_0'(k_0)}{f_0(k_0)} \right) \quad (4)$$

This proves that  $f_t'(k) > 0$  for all  $k > 0$ . Using equation (3), the formula for function  $\lambda(\cdot)$  yields:

$$f_t'(k) = \frac{f_t(k)}{k} \frac{\theta t (1 - \alpha_0(k_0)) + \alpha_0(k_0)}{\theta t (1 - \alpha_0(k_0)) + 1} \quad (5)$$

- The definition of the elasticity of substitution associated to  $f_0$  is:  $\sigma_0(k_0) = -\frac{f_0'(k_0 - k_0 f_0')}{k_0 f_0 f_0''}(k_0)$ . Log-differentiating  $\alpha_0(k_0)$  with respect to  $k_0$  yields:

$$\frac{1}{\alpha_0} \frac{\partial \alpha_0}{\partial k_0} = \frac{1}{k_0} + \frac{f_0''(k_0)}{f_0'(k_0)} - \frac{f_0'(k_0)}{f_0(k_0)}$$

Dropping the arguments of functions  $\alpha_0(k_0)$  and  $\sigma_0(k_0)$ , this last equation reduces to:

$$\frac{\partial \alpha_0}{\partial k_0} = \frac{\alpha_0(1 - \alpha_0)}{k_0} \frac{\sigma_0 - 1}{\sigma_0} \quad (6)$$

- Log-differentiating equation (5) with respect to  $k$  gives:

$$\frac{f_t''(k)}{f_t'(k)} = \frac{f_t'(k)}{f_t(k)} - \frac{1}{k} + \frac{(1 - \theta t) \frac{\partial \alpha_0}{\partial k_0} \frac{\partial k_0}{\partial k}}{\theta t (1 - \alpha_0) + \alpha_0} - \frac{-\theta t \frac{\partial \alpha_0}{\partial k_0} \frac{\partial k_0}{\partial k}}{\theta t (1 - \alpha_0) + 1}$$

Incorporating equations (3) and (6) proves that:

$$\frac{f_t''(k)}{f_t'(k)} = -\frac{1}{k} \frac{1 - \alpha_0}{\theta t (1 - \alpha_0) + 1} + \frac{1}{k} \frac{\sigma_0 - 1}{\sigma_0} \frac{\alpha_0(1 - \alpha_0)}{(\theta t (1 - \alpha_0) + 1)^2 (\theta t (1 - \alpha_0) + \alpha_0)} \quad (7)$$

The first term is strictly negative. If  $0 < \sigma_0 \leq 1$ , the second term is negative, and so  $f_t''(k) < 0$  for all  $k > 0$ .<sup>10</sup>

□

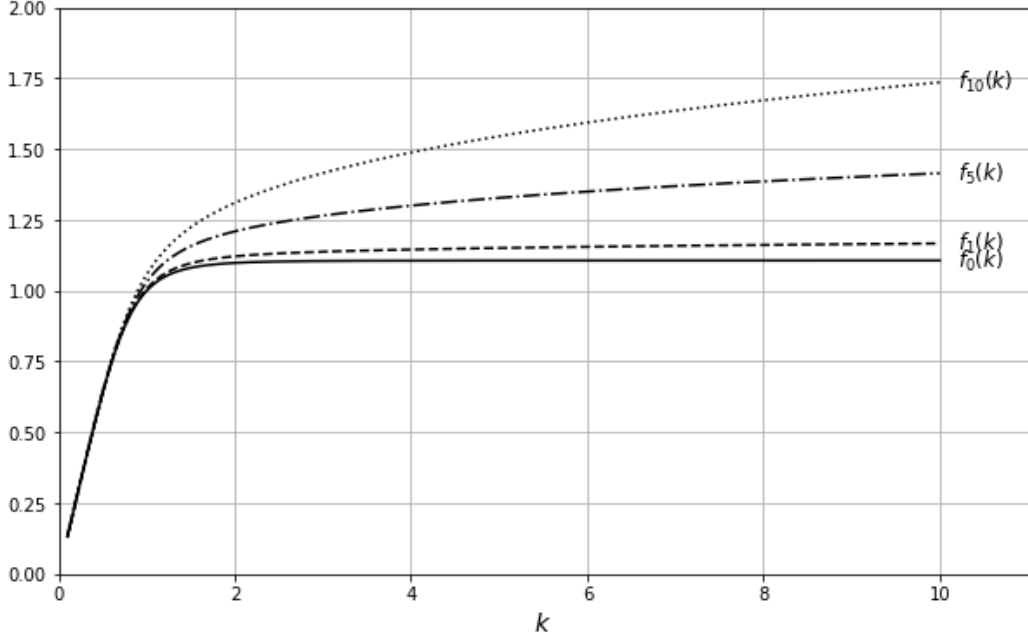


Figure 2:  $f_0$ ,  $f_1$ ,  $f_5$  and  $f_{10}$  for  $f_0(k) = \left(\frac{2}{3} + \frac{1}{3}k^{\frac{\sigma_0-1}{\sigma_0}}\right)^{\frac{\sigma_0}{\sigma_0-1}}$  for  $\sigma_0 = 0.2$ , and with  $\lambda\left(\frac{k}{y}\right) = 0.01 + 0.02 \ln \frac{k}{y}$ .

Proposition 1 proves that one can consistently think about exogenous, steady, localized technical change, while keeping the production function neoclassical. Figure 2 displays successive production functions obtained by this method.

## 2 A Solow model animated by high- $k$ technical change

### 2.1 Setup

In this section, we set a Solow model where the production function evolves through steady, localized technical change process. We consider the following model:

$$\begin{cases} y_t = f_t(k_t) \\ \dot{k}_t = s y_t - (\delta + n)k_t \\ k_0 \text{ given} \end{cases} \quad (8)$$

where  $f_t(\cdot)$  is given by equation (1) for some functions  $f_0(\cdot)$  and  $\lambda(\cdot)$ . We assume that  $f_t(\cdot)$  is continuously differentiable, strictly increasing and strictly concave for all  $t \geq 0$ .  $s \in (0, 1]$ ,  $\delta > 0$  and  $n > 0$  respectively stand for the investment rate, the instantaneous capital depreciation rate and the population growth rate. This version of the Solow model diverges from the textbook one only insofar as productivity gains are not uniform across the input mix space.

We now make an important assumption regarding function  $\lambda(\cdot)$ .

**Assumption 1.**  $\lambda(\cdot)$  is increasing, i.e.  $\lambda'(x) \geq 0$  for all  $x > 0$ .

Assumption 1 requires that productivity growth be higher or equal at higher capital-labor ratios, at all dates. This

<sup>10</sup>Remark that one can easily obtain a formula for the elasticity of substitution of  $f_t$  from equations (5) and (7). All simplifications done, we obtain:

$$\forall k > 0, \sigma_t(k) = -\frac{f_t'(f_t - k f_t')}{k f_t f_t''}(k) = \left(1 - \frac{\sigma_0 - 1}{\sigma_0} \frac{\alpha_0}{(\theta t(1 - \alpha_0) + 1)(\theta t(1 - \alpha_0) + \alpha_0)}(k_0)\right)^{-1}$$

where  $k_0$  is implicitly determined by equation  $k = k_0 e^{\lambda\left(\frac{k_0}{f_0(k_0)}\right)t}$ . So if  $f_0$  has an elasticity of substitution everywhere  $< 1$ , then  $f_t$  has an elasticity of substitution everywhere  $< 1$ .

assumption is in line with existing empirical literature<sup>11</sup> and is also taken by Basu and Weil (1998), who argue that “we (...) assume that  $A^*$  increases with  $K$ , which captures the idea that technologies have increasingly high potential at higher levels of development: the maglev train can be improved more than the oxcart” (p.1030). It is also a theoretical necessity: if  $\lambda$  is locally strictly decreasing somewhere, then after some date  $\tau$  the production functions defined in (1) would be strictly decreasing over some interval; hence productivity growth cannot steadily be strictly higher at lower capital-output ratios.

## 2.2 Dynamics of $k$ and $y$

We now investigate the existence of a BGP and its stability in this model. Consider the capital-output ratio  $x_t = \frac{k_t}{y_t}$ . Dropping time subscripts,  $x$  evolves according to equation:

$$\frac{\dot{x}}{x} = \frac{\dot{k}}{k} - \frac{\dot{y}}{y} = \frac{s}{x} - (\delta + n) - \frac{\dot{y}}{y} \quad (9)$$

The temporal variations of  $y$  are caused by productivity growth and capital accumulation. Considering some small time interval  $[t, t + dt]$ , we have:

$$y_{t+dt} = \left(1 + \lambda \left(\frac{k_t}{y_t}\right) dt\right) y_t + \left[k_{t+dt} - \left(1 + \lambda \left(\frac{k_t}{y_t}\right) dt\right) k_t\right] f'_{t+dt}(k_{t+dt})$$

Hence:

$$\frac{y_{t+dt} - y_t}{y_t} \equiv \frac{\dot{y}}{y} = \lambda \left(\frac{k_t}{y_t}\right) + \left[\frac{\dot{k}}{k} - \lambda \left(\frac{k_t}{y_t}\right)\right] \frac{k_t f'_{t+dt}(k_{t+dt})}{y_t}$$

At the first order,  $f'_{t+dt}(k_{t+dt}) = f'_t(k_t)$ ; so the dynamics of  $y$  can be summarized by:

$$\frac{\dot{y}}{y} = \lambda(x) + \left[\frac{s}{x} - \delta - n - \lambda(x)\right] \alpha \quad (10)$$

where  $\alpha = \alpha_t = \frac{k_t f'_t(k_t)}{y_t}$ .

Equations (9) and (10) lead to a non-autonomous differential equation in  $x$ :

$$\dot{x} = (1 - \alpha)x \left(\frac{s}{x} - \delta - n - \lambda(x)\right) \quad (11)$$

I can now state the main theoretical result on this model:

**Theorem 1.** *Let  $x^* > 0$  be the solution to equation:  $\frac{s}{x} - \delta - n - \lambda(x) = 0$ .*

- *There exists a unique BGP where  $\frac{k_t}{y_t}$  is constant at  $x^*$ , and where  $k_t$  and  $y_t$  both grow at the constant rate  $\lambda(x^*)$ ;*
- *For any initial condition  $k_0$ , the growth rates of  $k$  and  $y$  both tend to  $\lambda(x^*)$  as time tends to infinity.*

*Proof.*

- If  $x$  is constant at  $x^*$ , then  $\frac{\dot{k}}{k}$  is constant at  $g_k = \frac{s}{x^*} - \delta - n = \lambda(x^*)$ . Equation (10) proves that  $\frac{\dot{y}}{y}$  is constant at  $g_y = \lambda(x^*) = g_k$ . Hence, there exists a BGP where  $k$  and  $y$  grow at the constant rate  $\lambda(x^*)$ , and where the capital-output ratio is constant at  $x^*$ .

<sup>11</sup>One strand of empirical literature has employed Data Envelopment Analyses (DEAs) to investigate the dynamics of the world production frontier. Kumar and Russell (2002), with a database of 57 countries between 1965 and 1990, found that productivity growth was significantly higher in richer countries. Henderson and Russell (2005) and Jerzmanowski (2007) share this conclusion. Allen (2012), with 17 countries over 1820-1990, argues that productivity growth was localized at relatively high capital-labor ratios in the last two centuries.

The empirical results in Acemoglu and Zilibotti (2001), Caselli and Coleman (2006) can be interpreted as evidence of technical change biased towards high-human-capital techniques. Sunde et al. (2021) find that patience strongly correlates with income per capita at different levels of aggregation, with the elasticity rising with the level of aggregation. This finding (which the authors attribute to a human capital externality) is also consistent with high-human-capital technical change.

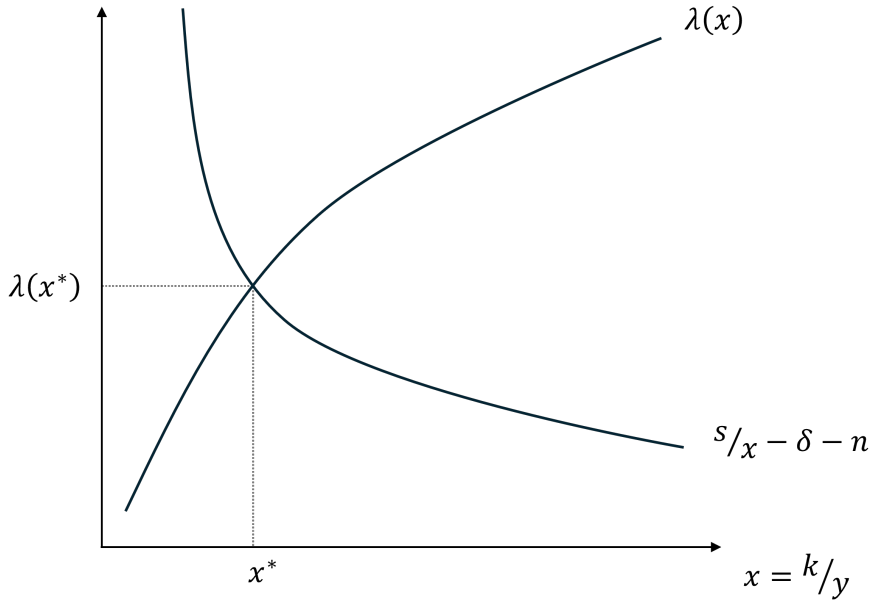


Figure 3: The joint determination of the capital-output ratio and the growth rate along the BGP.

- Since  $\lambda(\cdot)$  is increasing, function  $x \mapsto x\left(\frac{s}{x} - \delta - n - \lambda(x)\right)$  is strictly decreasing; strictly positive for  $x < x^*$  and strictly negative for  $x > x^*$ . Hence, for any initial condition, equation (11) proves that  $x_t \xrightarrow{t \rightarrow \infty} x^*$ . Because  $\frac{\dot{k}}{k} = \frac{s}{x} - \delta - n$ ,  $\frac{\dot{k}}{k} \xrightarrow{t \rightarrow \infty} \lambda(x^*)$ . Equation (10) proves that  $\frac{\dot{y}}{y} \xrightarrow{t \rightarrow \infty} \lambda(x^*)$ .

□

So  $k$  and  $y$  behave essentially like in a textbook Solow model animated by labor-augmenting technical change, except that their common growth rate along a BGP is an equilibrium variable determined jointly with the capital-output ratio. A sufficient condition for a globally stable BGP to emerge is that  $\lambda$  is increasing, i.e. that at each date productivity growth is higher at higher capital-labor ratios. The capital-output ratio  $x^*$  and the growth rate  $\lambda(x^*)$  are both increasing in  $s$  and decreasing in  $n + \delta$ . In the case where  $\lambda$  is locally constant around  $x^*$ , the parameters do not influence the growth rate (like in the textbook model). Figure 3 represents the joint determination of the BGP growth rate and capital-output ratio.

### 2.3 Dynamics of the factor shares over the BGP

I now turn to another aspect of the model: the behavior of the capital and labor shares along a BGP. Unlike labor-augmenting technical change, localized technical change distorts the production function across time, hence the factor shares might not behave like in the textbook model.

**Theorem 2.** *Let's assume that the initial condition  $k_0$  is such that the economy is on its BGP starting at date 0, i.e.  $x^* = k_0/f_0(k_0)$ .*

- If  $\lambda'(x^*) > 0$ , then  $\alpha_t \xrightarrow{t \rightarrow \infty} 1$ ;
- If  $\lambda'(x^*) = 0$ , then  $\alpha_t = \alpha_0$  for all  $t \geq 0$ .

*Proof.* Along the BGP,  $k_t = k_t^* = k_0 e^{\lambda(x^*)t}$  and  $y_t = y_t^* = f_0(k_0) e^{\lambda(x^*)t}$ . Hence, the date-0 ancestor capital-labor ratio of  $k_t^*$  is  $k_0$  for all  $t > 0$ . Using equations (2) and (4), the capital share at date  $t$  along the BGP can be expressed as:

$$\alpha_t \equiv \frac{k_t^* f_t'(k_t^*)}{k_t^*} = \frac{(1 - \alpha_0) x^* \lambda'(x^*) t + \alpha_0}{(1 - \alpha_0) x^* \lambda'(x^*) t + 1} \quad (12)$$

This quantity tends to 1 as time tends to infinity if and only if  $\lambda'(x^*) > 0$ . If  $\lambda'(x^*) = 0$ ,  $\alpha_t$  is constant at  $\alpha_0$ .

□



Thus, factor shares do have a trend along the BGP in this model if and only if technological change is locally biased, i.e. if  $\lambda'(x^*) > 0$ . In this case, technical change gradually increases the slope of the production function (i.e. the marginal productivity of capital) which converges to the inverse of the capital-output ratio. In the case where  $\lambda'(x^*) = 0$ , the model behaves locally like a Solow model animated by labor-augmenting technical change, and the factor shares along a BGP are constants that depend only on the shape of  $f_0$ .

### 3 Conclusion

The idea that technical change is localized rather than uniform across input mixes dates back to Atkinson and Stiglitz (1969). In this paper, I have set and solved a Solow model animated by a new form of technical change, whereby productivity gains are higher at higher capital-labor ratios (or capital-output ratios). To the best of my knowledge, this is the first attempt to represent localized technical change into a simple, neoclassical growth model with exogenous technical change.

It comes out that this model has a stable BGP and complies with the first, second and fourth Kaldor (1961) facts. The saving/investment rate has positive growth effects over real GDP per capita, and not only level effects, despite the production function being strictly concave at each date, hence it also reproduces the sixth fact, contrary to textbook growth models. It predicts a rise in the marginal product of capital and a decrease in the labor share along a BGP if and only if technical change is locally biased. If productivity gains are assumed to be constant above some capital-output threshold, the model reproduces the six stylized facts as well as the existence of convergence clubs.

The approach to technical change presented in this paper generalizes the textbook, purely labor-augmenting technical change framework. It pertains to the literature on local vs. global production functions.<sup>12</sup> By assuming steady and non-uniform technical change at the local production function level, a simple Solow model gains enough flexibility to account for the absence of global non-convergence.

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<sup>12</sup>See, among others, Jones (2005) and Growiec (2018).

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