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INVENTORIES CYCLE IN AN AUGMENTED R.B.C. MODEL

Fabrice COLLARD

Jean-François JACQUES

*MAD*

# Inventories Cycle in an Augmented R.B.C. Model

Fabrice Collard\* & Jean-François Jacques<sup>†‡</sup>

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## Abstract

We propose an augmented version of the R.B.C. model that can take into account the existence of lags in observation of demand shocks. This hypothesis seems to be a good framework to account for inventories fluctuations. We find some evidence showing the introduction of lag in observation as well as the existence of monetary shocks improve the ability of the model in matching the observed fluctuations.

## Résumé

Nous proposons une version augmentée du modèle R.B.C. qui permet de prendre en compte l'existence de retard dans l'observation des chocs de demande. Ce cadre d'analyse semble adéquat pour étudier les fluctuations de stocks agrégés. Nous montrons que l'introduction de retard dans l'observation aussi bien que l'existence de chocs monétaires améliorent la capacité du modèle à reproduire les fluctuations observées.

**Keywords :** Inventories, Business Cycles, Demand shocks.

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\*M.A.D., unité associée CNRS-URA 0926, Université de Paris I, 90 rue de Tolbiac, 75634 Paris Cedex 13, France.

<sup>†</sup>M.A.D., CREST, 15 Bd Gabriel Péri, 92241 Malakoff CEDEX.

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## Introduction

Ignored during a long time, inventories fluctuations at the macroeconomic level have been widely explored and analysed during the eighties. The main reason for that renewal interest finds its explanation in the following paradox exposed by Blinder [1986]: inventories at the micro-economic level constitute a stabilizing factor, as they help to schedule production at each stage of the production process. Nevertheless at the aggregate level, inventories have been shown to magnify fluctuations. In particular, fluctuations in U.S. GNP are essentially amplified by inventories movements. In fact, as noted by Blinder and Holtz-Eakin [1984]:

“Inventory investment is the most volatile component of GNP. It plays a major role in business cycles, especially around turning points and during cyclical downswings, and is strongly procyclical. The facts are well established for the postwar U.S. economy. And everything we know from prewar data [...] suggests that the same stylised facts held in the prewar period as well.”

These facts are rather disturbing and cast doubts on the hypothesis of the “Production Smoothing Model” pioneered by Holt, Modigliani, Muth and Simon [1960]. This model predicts that the manager will optimally smooth production when marginal cost is increasing. This kind of behavior is well understood at the microeconomic level and can be extended to the macroeconomic level provided that the same technology hold in each firm. Other authors as Brunner, Cukierman and Meltzer [1983] argue that this smoothing effect exists at the macroeconomic level, but not at the level of the individual firm. In both case, inventories buffer demand shocks and are therefore depleted when a disturbance affecting sales occurs. Statistically the data should then reveal two main stylized facts. First of all, the variance of sales should exceed the output one. Secondly, the covariance of sales and changes in inventories should be negative.

The debate on the accuracy of that model is still opened. Among the large litterature on this subject, many authors as Blinder [1986], Eichenbaum [1984] or West [1990] have suggested that supply shocks can account for a substancial part of inventories fluctuations. The emergence of the Real Business Cycle (R.B.C.) theory in the eighties provides a peculiar framework for the analysis of the propagation mechanisms able to account for the role of supply shocks in inventories fluctuations.

In the lines of the seminal paper of Kydland and Prescott [1982], Christiano [1988] built a model in which preferences are affected by a taste shock, interpreted as a “demand” disturbance. The central question he adressed in his paper is :

“*Why do inventories fluctuate so much ?*” Do they buffer the imperfectly predicted shocks affecting the economy ? – in that case they play a residual role supporting the burden of bad shocks – Or in the contrary do they fluctuate when the shocks are perfectly known ?

Christiano's conclusion tends to indicate that the first effect dominates and can explain the smoothness of series. The role of inventories in buffering unexpected shocks can account for the volatility in inventory investment. It is possible to reinterpret his results as a test of imperfect information on the occurrence of the shocks. Indeed, in a constant return to scale environment, inventory behavior is not needed to respond to shocks. Inventory investment fluctuates in order to buffer unexpected changes in sales<sup>1</sup>.

In this contribution, we propose a model in the lines of that of Christiano [1988]. Inventories play a double role<sup>2</sup> :

- First of all, as inventories enters in the production function<sup>3</sup>, they play a direct role in the production process. As Christiano [1988] noted :

“all other things being equal, large inventory stocks probably do augment society's ability to produce goods. For example, spacial separation of the stages of production and distribution, together with economies of scale in transportation, implies that labor inputs can be conserved by transporting goods in bulk on holding inventories.”

- Secondly, they serve to smooth production, when certain disturbances are not perceived by the agents.

In order to capture the latter effect, two sources of demand disturbances are introduced : government expenditures and monetary shocks. Money is treated as in Cooley and Hansen [1989], through a cash in advance constraint in the program of the representative household. These two shocks are not perceived by the firm when it formulates its production plan, while the household observes the shocks as it chooses its consumption plan.

The aim of this paper is then to analyse whether this lag in observation is important to mimic inventories fluctuations and the cycle of inventories versus sales. Moreover, this model permits to bring some insight in the explanation of the inventory cycle by demand shocks. As the observation of data suggests, sales constitute a leading indicator of changes in inventories. The model we develop is simulated on U.S. data. It then seems that no smoothing is required to mimic the inventory cycle.

In a first section, we present and discuss the model, that is solved in a second section. The third section presents the calibration of the model and the stylized facts to be reproduced. The fourth section discusses the results.

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<sup>1</sup>Under the hypothesis that inventory stocks do not enter the production function.

<sup>2</sup>Though a stockout motive would have been able to capture the phenomenons we are interested in, we are not concerned with this mechanism .

<sup>3</sup>The production function does not induce any stockout effect because the level of inventories is always positive.

# 1 The model

This model is in the lines of Christiano [1988]. However, we consider the case of a government expenditures shock and we introduce a cash in advance constraint.

## 1.1 The households

The economy is populated by many identical infinitely-lived individuals acting in a competitive world. The population is normalized to unity and we assume there exists a representative agent with preferences over goods and leisure represented by :

$$U = E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(C_t, 1 - h_t; G_t) \right\}$$

where  $\beta$  is the discount factor,  $C_t$  is private consumption in period  $t$ , and  $h_t$  is the number of hours worked. The instantaneous utility function of the representative household is given by :

$$u(C_t, 1 - h_t; G_t) = \log(C_t + \tau G_t) + \theta(1 - h_t)$$

The choice of the households in terms of leisure is characterized by a Hansen [1985]'s specification, so that the leisure intertemporal substitution elasticity is infinite. This can be interpreted as the existence of indivisibilities in hours worked.  $\theta$  then represents the marginal utility of leisure.  $G_t$  denotes the government expenditures, which as in Barro [1981] affects the utility of households. The parameter  $\tau$  appears to be crucial in the determination of the impact of different shocks on macroeconomic variables.

- If  $\tau = 0$  then the marginal utility of private consumption remains unchanged with a government expenditures shock, which impact only operates through wealth effects.
- If  $0 < \tau < 1$  then the marginal utility of private consumption diminishes less after a government expenditures unit shock than after a unit increase in consumption.
- If  $\tau = 1$  then private consumption and government expenditures are perfect substitutes. A positive government expenditures shock then implies a complete crowding out on private consumption, the other variables remaining unchanged.

In each period, an individual faces a resource constraint which says that total use of commodity must not exceed the total output. Moreover, the household receives a lump sum monetary transfert  $N_t$ . The individual intertemporal resources constraint is then :

$$\frac{M_{t+1}}{P_t} + \int_{\Omega_t^*} \frac{\rho_{t+1}(\omega_{t+1}^*)}{\rho_t} B_{t+1}(\omega_{t+1}^*) d\mu(\omega_{t+1}^* | \omega_t^*) = b_t + \frac{W_t}{P_t} h_t + \frac{N_t}{P_t} + \frac{M_t}{P_t} - C_t - T_t$$

where  $T_t$  is a lump sum tax, and  $B_t$  an asset.  $M_t$  denotes the money stock, while  $N_t$  is a non consumable monetary lump sum transfert.  $\int_{\Omega^*} \frac{\rho_{t+1}(\omega_{t+1}^*)}{\rho_t} b_{t+1}(\omega_{t+1}^*) d\mu(\omega_{t+1}^* | \omega_t^*)$  is the price in period t of a contingent claim which insures a payment  $b_{t+1}$  in period t+1 whatever the state of Nature is.

The household is also submitted to a cash in advance constraint :

$$P_t C_t \leq M_t$$

At the aggregate level, using the government constraint ( $G_t = T_t$ ), the resources constraint is :

$$P_t C_t + P_t I_t + P_t G_t + M_{t+1} + P_t \Delta S_t \leq P_t Y_t + N_t + M_t$$

where  $Y_t$ ,  $I_t$ ,  $T_t$ ,  $M_t$  and  $\Delta S_t$  are respectively the output, the investment, government expenditures, the money supply and inventories changes.  $P_t$  denotes the general price level.

## 1.2 The government expenditures

The government expenditure process is given by  $G_t = \gamma_x^t g_t$ , where  $\gamma_x$  is the exogenous labor augmenting technical progress growth rate, and  $g_t$  is an exogenous stationary stochastic component. This stationary component is assumed to follow an exogenous first order autoregressive process :

$$\log(g_t) = \rho_g \log(g_{t-1}) + (1 - \rho_g) \log(\bar{g}) + \varepsilon_{g,t}$$

with  $0 < \rho_g < 1$ ,  $E(\varepsilon_g) = 0$  and  $E(\varepsilon_g^2) = \sigma_g^2$ .  $\log(\bar{g})$  denotes the unconditionnal mean of the process.

## 1.3 The money supply

The money supply evolves as in Cooley and Hansen [1989] :

$$M_{t+1} = \gamma_t M_t$$

The money growth factor  $\gamma_t$  follows the stochastic process :

$$\log(\gamma_t) = \rho_\gamma \log(\gamma_{t-1}) + (1 - \rho_\gamma) \log(\bar{\gamma}) + \varepsilon_{\gamma,t}$$

with  $0 < \rho_\gamma < 1$ ,  $E(\varepsilon_\gamma) = 0$  and  $E(\varepsilon_\gamma^2) = \sigma_\gamma^2$ .  $\log(\bar{\gamma})$  denotes the unconditionnal mean of the process.

## 1.4 The representative firm

There is a homogenous good that can be accumulated as physical capital or inventories and consumed. It is produced according to the following production function :

$$Y_t^G = A_t F(K_t, S_t, h_t) = A_t (h_t X_t)^{1-\alpha} \left( \sigma K_t^{-\nu} + (1 - \sigma) S_t^{-\nu} \right)^{-\frac{\alpha}{\nu}}, \quad 0 < \alpha, \sigma < 1, \nu > 0$$

where  $K_t$  and  $S_t$  are respectively the predetermined physical capital stock and inventories. The returns to scale are constant with respect to the remunerated factors,  $K_t$ ,  $S_t$  and  $(h_t X_t)$ . Since inventories are involved in the production process, the firm will always hold inventories, which are substitutes to physical capital. The elasticity of substitution between inventory stocks and physical capital is given by  $\frac{1}{1+\nu}$ .  $X_t$  is the harrodian neutral technical progress evolving as :

$$X_{t+1} = \gamma_x X_t, \quad \gamma_x > 1$$

$A_t$  is a technological shock assumed to follow an exogenous stationary first order autoregressive process :

$$\log(A_t) = \rho_a \log(A_{t-1}) + (1 - \rho_a) \log(\bar{A}) + \varepsilon_{a,t}$$

with  $0 < \rho_a < 1$ ,  $E(\varepsilon_a) = 0$  and  $E(\varepsilon_a^2) = \sigma_a^2$ .  $\log(\bar{A})$  denotes the unconditionnal mean of the process.

The physical capital accumulation process is traditionnally given as :

$$K_{t+1} = I_t + (1 - \delta_k)K_t, \quad 0 < \delta_k < 1$$

where  $\delta_k$  is the constant depreciation rate of the physical capital.

Inventories are defined by the following accounting identity :

$$S_{t+1} = \Delta S_t + S_t$$

The firm is also submitted to capital adjustment costs<sup>4</sup>, which are homogenous of degree one :

$$C^I(I_t, K_t) = \frac{b}{2} \frac{I_t^2}{K_t}$$

Net product is then given by :

$$Y_t = Y_t^G - C^I(I_t, K_t)$$

## 2 Solving of the model

This section is devoted to the computation of the decision rules of each agent and the characterization of the competitive equilibrium. As the model incorporates a growth factor, all the growing variables are stationarized by  $X_t$ . More over, the entire model is expressed in real variables, we then note  $m_t = \frac{M_t}{P_{t-1}}$  and  $\pi_t = \frac{P_t}{P_{t-1}}$  the inflation index.

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<sup>4</sup>This implies that the producer will be lead to adjust for inventories rather than capital, this induces a motive for production smoothing effect. Indeed, inventories play a lower role in the production function than capital does.



## 2.1 The representative firm problem

The firm maximizes its intertemporal profit, by mean of choosing its level of investment, changes in inventories and employment. This choice is formulated knowing the technological shock but neither government expenditures shocks nor monetary shocks<sup>5</sup>. The inventory stocks are modified exogeneously just after the demand shocks, firms do not exert a total control on its evolution. The Bellman equation associated to the program is :

$$V_0^F(\Delta_t^F) = \max_{(\Delta s_t, i_t, h_t)} \left\{ A_t F(k_t, s_t, h_t) - C^I(i_t, k_t) - \frac{w_t}{p_t} h_t - i_t - \Delta s_t \right. \\ \left. + \int_{\Omega^*} \frac{\rho_{t+1}(\omega_{t+1}^*)}{\rho_t} V_0^F(\Delta_{t+1}^F(\omega_{t+1}^*)) d\mu(\omega_{t+1}^* | \omega_t^*) \right\}$$

$$s.t. \begin{cases} \gamma_x k_{t+1} &= i_t + (1 - \delta_k) k_t \\ \gamma_x s_{t+1} &= \Delta s_t + s_t \end{cases}$$

$\Delta_t^F = (k_t, s_t, \Theta_t)$  the state vector<sup>6</sup>.  $\rho_t$  is the stochastic pricing kernel for contingent claims.

The optimality conditions are given by the traditionnal labor demand condition, the Tobin's Q evolution law and a standard equation saying that investing in inventories costs  $x_t$  in terms of profit flow but allows to increase the flow of profit by  $F_s(t+1) + x_{t+1}$ .

## 2.2 The household problem

The representative household maximizes his intertemporal utility subject to his intertemporal budget constraint and the cash in advance constraint. Whereas he chooses his labor supply conditionnaly to  $\Omega_t^*$ , he chooses his consumption flow knowing all the information on the disturbances.

**Labor Supply :** The representative household maximizes his intertemporal utility in the first sub-period of period t, and his choice alters its wealth in the first sub-period of period t+1. The Bellman equation is then given by :

$$V_0^M(\Delta_t^M) = \max_{(C_t, h_t)} \left\{ u(C_t, 1 - h_t; G_t) + \beta \int_{\Omega^*} V_0^M(\Delta_{t+1}^M(\omega_{t+1}^*)) d\mu(\omega_{t+1}^* | \omega_t^*) \right\}$$

$$s.t. \begin{cases} \gamma_x m_{t+1} + \gamma_x \int_{\Omega_t^*} \frac{\rho_{t+1}(\omega_{t+1}^*)}{\rho_t} b_{t+1}(\omega_{t+1}^*) d\mu(\omega_{t+1}^* | \omega_t^*) = b_t + \frac{w_t}{p_t} h_t + n_t + \frac{m_t}{\pi_t} - c_t - g_t \\ c_t \leq \frac{m_t}{\pi_t} (\xi_t) \end{cases}$$

$\Delta_t^M = (b_t, \Theta_t^*)$  is the state vector.

The optimality conditions attached to this program furnish the labor supply behaviour.

<sup>5</sup>We denote  $\Omega_t^*$  the information set including  $\{k_\tau\}_{\tau=0}^t$ ,  $\{s_\tau\}_{\tau=0}^t$ ,  $\{A_\tau\}_{\tau=0}^t$ ,  $\{g_\tau\}_{\tau=0}^{t-1}$  and  $\{\gamma_\tau\}_{\tau=0}^{t-1}$ .  $\Omega_t$  denotes the information set including  $\{k_\tau\}_{\tau=0}^t$ ,  $\{s_\tau\}_{\tau=0}^t$ ,  $\{A_\tau\}_{\tau=0}^t$ ,  $\{g_\tau\}_{\tau=0}^t$  and  $\{\gamma_\tau\}_{\tau=0}^t$ .  $\omega_t^*$  and  $\omega_t$  are events in respectively  $\Omega_t^*$  and  $\Omega_t$ .  $\mu^*(\cdot)$  and  $\mu(\cdot)$  are the associated measure.

<sup>6</sup> $\Theta_t^* = (A_t, g_{t-1}, \gamma_{t-1})$  and  $\Theta_t = (A_t, g_t, \gamma_t)$

**Consumption :** The representative household maximizes his intertemporal utility at the beginning of period  $t$ , and his choice alters its wealth in the first sub-period of period  $t+1$ . The Bellman equation is then given by<sup>7</sup> :

$$V_t^M(\Gamma_t^M) = \max_{(C_t, h_t)} \left\{ u(C_t, 1 - h_t; G_t) + \beta \int_{\Omega^*} V_0^M(\Delta_{t+1}^M(\omega_{t+1}^*)) d\mu(\omega_{t+1}^* | \omega_t) \right\}$$

$$s.t. \begin{cases} \gamma_x m_{t+1} + \gamma_x \int_{\Omega^*} \frac{\rho_{t+1}(\omega_{t+1}^*)}{\rho_t} b_{t+1}(\omega_{t+1}^*) d\mu(\omega_{t+1}^* | \omega_t) = b_t + \frac{w_t}{p_t} h_t + n_t + \frac{m_t}{\pi_t} - c_t - g_t \\ c_t \leq \frac{m_t}{\pi_t} \quad (\xi_t) \end{cases}$$

This program allows to derive the demand for consumption knowing that the household is constraint by the Cash in Advance, the consumption-saving arbitrage condition and an equation describing how the cash in advance constraint alters the intertemporal behavior of the household in terms of consumption.

## 2.3 The competitive equilibrium

In the equilibrium, the resources constraint is satisfied and  $n_t = m_{t+1} - \frac{m_t}{\pi_t}$ , so that :

$$y_t = c_t + i_t + g_t + \Delta s_t$$

The transversality conditions are satisfied. It can then be shown that the individual's wealth is given by the value of the firm :

$$b_t = V^F(\Delta_t^F)$$

This equation expresses the capital market equilibrium. Noting  $\phi_t = z_t q_t$ ,  $\lambda_t = z_t x_t$  and  $\psi_t = z_t \xi_t$ , the following proposition can be stated.

**Proposition :** *The competitive equilibrium of the economy is a set of policy rules :*

$$\begin{aligned} c_t &= C(k_t, s_t, m_t, A_t, g_t, \gamma_t) \\ i_t &= I(k_t, s_t, m_t, A_t, g_{t-1}, \gamma_{t-1}) \\ h_t &= n(k_t, s_t, m_t, A_t, g_{t-1}, \gamma_{t-1}) \\ \Delta s_t &= \Delta s(k_t, s_t, m_t, A_t, g_{t-1}, \gamma_{t-1}) \end{aligned}$$

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<sup>7</sup> $\Gamma_t^M = (b_t, m_t, \Theta_t^*)$

such that :

$$(1^E) \quad u_c(t) = \lambda_t + \psi_t$$

$$(2^E) \quad u_l(t) = \lambda_t F_h(t)$$

$$(3^E) \quad \phi_t = \lambda_t(1 + C_i^I(t))$$

$$(4^E) \quad \phi_t = \frac{\beta}{\gamma_x} E \left( \lambda_{t+1}(F_k(t+1) - C_k^I(t+1)) + (1 - \delta_k)\phi_{t+1} | \Omega_t^* \right)$$

$$(5^E) \quad \lambda_t = \frac{\beta}{\gamma_x} E \left( \lambda_{t+1}(1 + F_s(t+1)) | \Omega_t^* \right)$$

$$(6^E) \quad \lambda_t = \frac{\beta}{\gamma_x} E \left( \frac{\lambda_{t+1} + \psi_{t+1}}{\pi_{t+1}} | \Omega_t \right)$$

$$(7^E) \quad \psi_t \left( \frac{m_t}{\pi_t} - c_t \right) = 0$$

$$(8^E) \quad A_t F(k_t, s_t, h_t) - C^I(i_t, k_t) = c_t + i_t + g_t + \Delta s_t$$

$$(9^E) \quad \gamma_x k_{t+1} = i_t + (1 - \delta_k)k_t$$

$$(10^E) \quad \gamma_x s_{t+1} = \Delta s_t + s_t$$

$$(11^E) \quad \lim_{j \rightarrow \infty} E_t \{ \beta^{t+j} \phi_{t+j} k_{t+j+1} \} = 0$$

$$(12^E) \quad \lim_{j \rightarrow \infty} E_t \{ \beta^{t+j} \lambda_{t+j} s_{t+j+1} \} = 0$$

$$(13^E) \quad \lim_{j \rightarrow \infty} E_t \{ \beta^{t+j} \lambda_{t+j} m_{t+j+1} \} = 0$$

As one cannot obtain an analytical solution to this model, we apply the methodology described by King, Plosser and Rebelo [1988]. The first order optimality conditions are log-linearized around the steady-state of the stationarized model<sup>8</sup>. The dynamical system obtained admits a state-space representation that can be solved according to the methodology developed by Blanchard and Kahn [1980]. As two information sets exist, we must take each of them into account. The state-space form is first projected on  $\Omega_t^*$ , we then obtain the decision rules for  $i_t$ ,  $h_t$ ,  $k_t$ . We also obtain the decision rule for  $\Delta s_t$  (and  $s_t$ ) formulated by the firm when it chooses its production plan. Taking these decision rules as given, we then solve the model conditionally to  $\Omega_t$  in order to evaluate the decision rules for  $c_t$ ,  $s_t$  and sales<sup>9</sup>. The inventory investment and stocks are completely solved only after this second stage because of their role of buffering the disturbances.

<sup>8</sup>In the following we will note  $\hat{x}_t = \log \left( \frac{x_t}{\bar{x}} \right) \simeq \frac{x_t - \bar{x}}{\bar{x}}$

<sup>9</sup>This kind of technics has been already applied by Fairise and Langot [1994] in the case of labor hoarding.

### 3 Calibration and stylized facts

In this section, we describe the stylized facts the model has to mimic. In order to reproduce these characteristics, the model is calibrated for the U.S. economy.

#### 3.1 The choice of parameters

**Preferences :** The individual is assumed to discount time at a rate of 3% per year, which implies a value of  $\beta$  equal to  $(1.03)^{-\frac{1}{4}}$ . The parameter  $\tau$  is fixed to a value of 0.2 according to a study by Aschauer [1985]. The parameter  $\theta$  is chosen such that in equilibrium, the individual devotes 40% of its total time endowment to work.

The share of government expenditures in U.S. G.N.P is evaluated at 21%, which corresponds to the average value estimated for the post-war period. The parameters defining the government expenditures process are estimated to  $\rho_g = 0.9834$  for the persistence parameter and  $\sigma_g = 0.0107$  for the standard error.

The parameters defining the monetary shock are estimated to  $\rho_\gamma = 0.5682$  and  $\sigma_\gamma = 0.0081$ . The average growth rate of money is estimated to  $\bar{\gamma} = 1.01429$ , as in Cooley and Hansen [1989].

**Technology :** We adopt the same parametrization as Christiano [1988] for the production function in his second model<sup>10</sup>. The elasticity of output to the aggregate capital is equal to 0.3606.  $\sigma$  ruling the share of physical capital versus inventory in the aggregate capital factor is equal to 0.9987. Finally  $\nu$ , the parameter determining the elasticity of substitution between physical capital and inventory is set at a value of 1.3612. The depreciation rate of physical capital is set at 0.025 per quarter, which corresponds to an annual rate of 10%. The persistence parameter of the technological shock is estimated to  $\rho_a = 0.9564$  with a standard deviation of  $\sigma_a = 0.0079$ . The adjustment cost is calibrated such that the elasticity of output net of adjustment cost to investment equals 0.0336, according to an estimation by Shapiro [1986].

#### 3.2 The stylized facts

The data are extracted from the citibase databank for the sample 1950:1-1989:4. The capital stock is obtained applying the accumulation law with a depreciation rate equals to 0.025. The data are logged and then filtered by application of the Hodrick and Prescott [1980]'s filter, defining the cycle as the deviation of the series around the growth component.

We have estimated all the second order moments of the filtered series using GMM estimators. The results are reported in table (1).

The consumption appears to be less volatile than output, while investment is 3 times more variable. Concerning sales and inventories, it seems that inventories

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<sup>10</sup>This estimation is the less constrained, verifying the condition of a positive definite variance-covariance matrix.

Table 1: U.S.A. 1950:1 1989:4

series	$\sigma_{\hat{x}}$	$\sigma_{\hat{x}}/\sigma_{\hat{y}}$	$\sigma_{\hat{x}}/\sigma_{\hat{v}}$	AR(1)	Corr( $\hat{x}, \hat{y}$ )
$\hat{y}$	1.583 (0.9e-3)	1 (0)	-	0.814 (0.051)	1 (0)
$\hat{c}$	1.165 (0.7e-3)	0.736 (0.034)	-	0.796 (0.054)	0.746 (0.048)
$\hat{i}$	5.359 (0.3e-2)	3.385 (0.130)	-	0.891 (0.040)	0.766 (0.044)
$\hat{s}$	1.392 (0.7e-3)	0.879 (0.063)	0.905 (0.064)	0.864 (0.042)	0.510 (0.063)
$\widehat{\Delta s}$	0.723 (0.5e-3)	0.457 (0.033)	0.471 (0.037)	0.267 (0.088)	0.551 (0.061)
$\hat{v}$	1.538 (0.8e-3)	0.971 (0.029)	1 (0)	0.866 (0.044)	0.924 (0.013)

Note : Standard errors in parenthesis

stocks as well as changes in inventories are less volatile than output. Sales are also less variable than output<sup>11</sup>. Finally, inventories are less variable than sales. This pattern is fundamentally linked to the filtering method we adopt<sup>12</sup>. The Hodrick-Prescott filter cancels the low frequency phenomenons in the series and only the high frequency remains in the spectrum. This induces a particular view of the cycle that explain the stylized facts.

One can also note a strong correlation between inventories and output, indicating their procyclicality. This point goes against the production smoothing model as Blinder [1986] noticed, since inventories do not buffer anymore unexpected disturbances.

Table (2) presents the inventory cycle defined by the cycle existing between changes in inventories and sales.

Table 2: Inventory Cycle

	-4	-3	-2	-1	0	1	2	3	4
Corr( $\widehat{\Delta s}_t, \hat{v}_{t+i}$ )	-0.186 (0.095)	-0.107 (0.085)	0.068 (0.089)	0.205 (0.088)	0.316 (0.081)	0.331 (0.078)	0.244 (0.075)	0.142 (0.077)	0.047 (0.087)

Note : Standard errors in parenthesis

The maximum changes in inventories-Sales correlation is displayed at lead one. This mean that sales are a leading indicator of changes in inventories, this

<sup>11</sup>But, we cannot statistically reject the fact that inventory stocks and sales are as volatile as output at the 5% significant level. The same conclusion holds when we consider the relative volatility between inventory stocks and sales.

<sup>12</sup>Hodrick and Prescott [1980]'s filter is widely used in the R.B.C. literature.

tends to indicate that the production smoothing mechanism does not constitute the dominant force driving inventories dynamics. Indeed, as sales lead inventory investment with a significant positive correlation, inventories do not appear to buffer unexpected increases in sales. The lagged and advanced correlations are unambiguously different from zero, except for lags 2 and 3 and lead 4.

## 4 Simulation results

Each model presented in this study is simulated 100 times for a 120 quarters sample, which represents our benchmark sample. We generate three 120 disturbances sample for  $\varepsilon_{a,t}$ ,  $\varepsilon_{g,t}$  and  $\varepsilon_{\gamma,t}$ , drawn from a gaussian law. Then the processes of the shocks are built. The series are generated using the state-space form of the model. The series are then logged and HP-filtered. We then computed the second order moments of these series.

Four different models will be considered :

- M1 : Model including Money and Lags in observation of the shocks.
- M2 : Model including Money but No Lags in observation of the shocks.
- M3 : Model including No Money and Lags in observation of the shocks.
- M4 : Model including No Money Neither Lags in observation of the shocks.

We will focus on peculiar second order moment characterizing inventories fluctuations.

The results of the simulation for the different models are presented in table (3) and figure (1). The output volatility is well reproduced, since the theoretical value is statistically equal to the historical at a 95% significant level. The model is able to generate second order moments on inventories without altering the properties of the other aggregates obtained with the canonical R.B.C. model<sup>13</sup>. Concerning the stylized facts upon inventories, the results are a little bit statistically deceptive : the relative variability of inventories and changes in inventories is less than the value obtained on U.S. data whatever the model considered.

A striking qualitative fact emerges, the most satisfying model in terms of reproduction of relative volatilities seems to be the one including money and imperfect information upon demand disturbances (M1). This can be explained by the fact that taking money into account reinforces the volatility of stocks as long as a lag in observation is present in the model. This can be seen examining table (3) for changes in inventories. Excluding the hypothesis of imperfect information implies that the relative volatility of inventories is strongly reduced (0.34 to 0.19). Excluding money really alters the relative volatility of changes in inventory (from 0.34 to 0.17). But this latter effect would hold with another demand shock which separates sales from inventories. The relative variability of sales is well reproduced by each model.

The model M1 fails in reproducing the inventory cycle. The instantaneous correlation is slightly null illustrating the fact that the smoothing effect fully

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<sup>13</sup>The completed results are available from the authors upon request.

Table 3: Simulation results

moments	M1	M2	M3	M4
$\hat{\sigma}_y$	1.45	1.45	1.53	1.47
	(0.0017)	(0.0016)	(0.0023)	(0.0018)
$\hat{\sigma}_c/\hat{\sigma}_y$	0.82	0.83	0.59	0.60
	(0.0803)	(0.0758)	(0.0355)	(0.0483)
$\hat{\sigma}_i/\hat{\sigma}_y$	2.10	2.11	2.09	2.09
	(0.0769)	(0.0729)	(0.0350)	(0.0271)
$\hat{\sigma}_s/\hat{\sigma}_y$	0.53	0.42	0.49	0.32
	(0.0743)	(0.0011)	(0.0015)	(0.0010)
$\hat{\sigma}_{\Delta s}/\hat{\sigma}_y$	0.34	0.19	0.17	0.09
	(0.0412)	(0.0002)	(0.0003)	(0.0002)
$\hat{\sigma}_v/\hat{\sigma}_y$	0.90	0.89	0.84	0.86
	(0.0287)	(0.0140)	(0.0190)	(0.0016)
$\hat{\sigma}_s/\hat{\sigma}_v$	0.59	0.47	0.58	0.36
	(0.0896)	(0.0766)	(0.0877)	(0.0504)
$\hat{\sigma}_{\Delta s}/\hat{\sigma}_v$	0.38	0.22	0.20	0.11
	(0.0480)	(0.0242)	(0.0205)	(0.0058)
$Corr(\hat{s}, \hat{y})$	-0.07	-0.01	0.10	0.02
	(0.1422)	(0.1349)	(0.1320)	(0.0740)
$Corr(\widehat{\Delta s}, \hat{y})$	0.32	0.41	0.75	0.94
	(0.0854)	(0.0949)	(0.0619)	(0.0166)
$Corr(\hat{v}, \hat{y})$	0.95	0.98	0.99	0.99
	(0.0136)	(0.0044)	(0.0022)	(0.0003)
$AR(1)_{\hat{s}}$	0.79	0.88	0.93	0.94
	(0.0495)	(0.0338)	(0.0229)	(0.0220)
$AR(1)_{\widehat{\Delta s}}$	0.093	0.37	0.68	0.66
	(0.0912)	(0.0845)	(0.0581)	(0.0760)

operates in M1. This induces a countercyclicity that totally annuls the procyclicity due to the existence of inventory stocks in the production function. The pattern of the cycle is not reproduced by M1.

The model M2 allows to mimic the inventory cycle, since the theoretical cycle generated by the model is significantly equal to that of historical data. We obtain the instantaneous procyclicity of changes in inventories, because inventories do not play their residual role.

Results for model M3 and M4 concerning the inventory cycle are not totally satisfying. Although the pattern is quite well reproduced and inventory changes are well instantaneously procyclical, the instantaneous correlation between sales and changes in inventories is too high. It appears that contrary to the models including money, the smoothing effect seems to be required to obtain better statistical results. Only one period lagged and advanced correlations are not well reproduced.

It then appears that if the smoothing effect is necessary in order to mimic relative volatility of inventories, it seems to be rejected in reproducing the inventory cycle. But, assuming that the government expenditures shocks are more volatile or introducing another demand shock, it can be shown that the smoothing effect is necessary.

## Conclusion

In light of the Christiano [1988]'s work it seemed to us important to test whether his conclusion are or not reinforced by introducing monetary shocks. Indeed, knowing whether inventories constitute a relevant propagation channel of monetary disturbances, as Blinder and Fisher [1981] argue, seems to be an interesting challenge.

A cash in advance constraint introduced in the seminal R.B.C. model extending Christiano [1988]'s specification is a first stage of this program. Our results tend to indicate that lag in observation of the shocks is relevant for reproducing the variability of inventories. The results indicate how important are the (nominal) shocks inducing countercyclicity in inventory behavior. A better way of introducing monetary shocks should be found, notably through nominal rigidities mechanisms, in order to better match the data.

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Figure 1 : The Inventory Cycle

