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The Economics of the Informal Sector: New Estimates, New Implications for OECD countries

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Mamadou Lah. The Economics of the Informal Sector: New Estimates, New Implications for OECD countries. 2024. halshs-04721526

HAL Id: halshs-04721526

<https://shs.hal.science/halshs-04721526v1>

Preprint submitted on 4 Oct 2024

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PARIS SCHOOL OF ECONOMICS
ECOLE D'ECONOMIE DE PARIS

WORKING PAPER N° 2024-39

**The Economics of the Informal Sector:
New Estimates, New Implications for OECD countries**

Mamadou Lah

JEL Codes: O11, O17, O41, O43, O47, O50, H26, H41

Keywords: Informal sector, formal sector, economic growth, taxation, optimal policy

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**The Economics of the Informal Sector:
New Estimates, New Implications for OECD countries**

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Abstract

This paper explores the dynamics and implications of the informal sector within OECD countries, employing a novel estimation approach based on an endogenous growth model. The model integrates both formal and informal sectors, incorporating public services and key determinants of informality. By calibrating this model with OECD data from 1996 to 2016, we derive significant insights into the size and impact of the informal sector. Our findings reveal that, on average, the informal sector constitutes 19.98% of the economy. Policy adjustments influencing the informal sector exhibit a nonlinear effect on economic growth. Specifically, increased formal sector taxation, reduced penalties on informal activities, enhanced access to public goods, and improved quality of government institutions all dynamically interact to shape the informal sector's size and its economic consequences. This study underscores the necessity of a nuanced policy approach, considering the structural characteristics of the economy to optimize growth while managing informality effectively.

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Résumé

Ce papier explore les dynamiques et les implications du secteur informel dans les pays de l'OCDE, en utilisant une nouvelle approche d'estimation basée sur un modèle de croissance endogène. Le modèle intègre à la fois les secteurs formel et informel, en tenant compte des services publics et des principaux déterminants de l'informalité. En calibrant ce modèle avec des données de l'OCDE de 1996 à 2016, nous tirons des enseignements significatifs sur la taille et l'impact du secteur informel. Nos résultats révèlent qu'en moyenne, le secteur informel représente 19,98 % de l'économie. Les ajustements politiques influençant le secteur informel montrent un effet non linéaire sur la croissance économique. Plus précisément, l'augmentation de la fiscalité du secteur formel, la réduction des pénalités sur les activités informelles, l'amélioration de l'accès aux biens publics et l'amélioration de la qualité des institutions gouvernementales interagissent de manière dynamique pour façonner la taille du secteur informel et ses conséquences économiques. Cette étude souligne la nécessité d'une approche politique nuancée, tenant compte des caractéristiques structurelles de l'économie pour optimiser la croissance tout en gérant efficacement l'informalité.

Classification JEL: O11, O17, O41, O43, O47, O50, H26, H41

Mots-clés : Secteur informel, secteur formel, croissance économique, fiscalité, politique optimale

Table des matières

ABSTRACT	II
RESUME	III
LIST OF TABLES	V
LIST OF FIGURES	V
1. INTRODUCTION	1
2. LITERATURE REVIEW	3
3. MODEL	5
3.1. PRODUCTION	6
3.2. MAXIMIZATION OF PROFIT	9
3.3. EQUILIBRIUM	10
3.4. OPTIMIZATION OF UTILITY	13
4. CALIBRATION AND ECONOMIC IMPLICATIONS	14
4.1. CALIBRATION	14
4.2. EFFECT OF T AND Π ON INFORMAL SECTOR SIZE	15
4.3. EFFECT OF T AND Π ON ECONOMIC GROWTH	17
4.3.1. <i>Relation between economic growth and π for different value of τ</i>	17
4.3.2. <i>Relation between economic growth and τ for different value of π</i>	18
4.3.3. <i>Comparison with the Barro Model:</i>	19
4.4. EFFECT OF δ , T AND Π ON ECONOMIC GROWTH	19
5. ESTIMATED SIZES OF INFORMAL SECTORS IN OECD COUNTRIES	22
5.1. CHOICE OF PARAMETERS	22
5.2. RESULTS	23
6. CONCLUSION	24
REFERENCE	33
PROOF OF PROPOSITION 2.1	37
PROOF OF PROPOSITION 2.2	38

List of tables

TABLE 1 <i>VALUE OF THE PARAMETERS</i>	15
TABLE 2 <i>DATA COLLECTED FROM OECD (MEAN 1995-2016)</i>	30
TABLE 3 <i>ESTIMATED DATA OF TAX RATE ON FORMAL SECTOR AND PENALTY RATE ON INFORMAL SECTOR.</i>	31
TABLE 4 <i>ESTIMATED SIZE OF INFORMAL ECONOMY.</i>	32

List of figures

TABLE 1 <i>VALUE OF THE PARAMETERS</i>	15
TABLE 2 <i>DATA COLLECTED FROM OECD (MEAN 1995-2016)</i>	30
TABLE 3 <i>ESTIMATED DATA OF TAX RATE ON FORMAL SECTOR AND PENALTY RATE ON INFORMAL SECTOR.</i>	31
TABLE 4 <i>ESTIMATED SIZE OF INFORMAL ECONOMY.</i>	32

The Economics of the Informal Sector: New Estimates, New Implications

septembre 2024

1. Introduction

The informal economy, often associated with unregulated and untaxed economic activities, plays a significant role in many economies around the world. According to estimates, the informal economy accounts for between 20 and 40% of global GDP. Stylized facts show that the informal economy tends to be larger in developing countries and smaller in developed countries¹. Despite its prevalence, understanding its main determinants, its measurement, and its impact on economic growth and development remains a challenge. The purpose of this paper is to explore these aspects in a systematic way by developing a simple theoretical framework.

Our approach involves the development of an endogenous growth model, a type of economic model that explains growth as generated from within the system, specifically from the role of capital and investments. In our model, we consider two sectors: the formal and the informal sectors and using the main determinants of the informal sector. Formal sector agents pay a tax proportional to their income. This tax is collected by the government, which uses it to produce public goods. Informal agents have an exogenous probability of being detected. If detected, all their production is destroyed or confiscated by public authorities². Finally, we assume that public expenditures are part of the production function of both sectors as in Barro (1990); but because of its illegal status, the informal agent has access to only a fraction of public expenditures.

This theoretical model allows us to evaluate the sensitivity of the parameters and analyze the implications in terms of growth and in terms of economic policies. More precisely, the economic model integrates several variables that are considered as economic policy parameters.

¹ Medina and Schneider, 2019.

² This severe penalty aligns with the theoretical understanding that higher stakes increase the cost of engaging in informal activities, thereby providing a strong disincentive (Becker, 1968; Feige, 1990). Furthermore, can also be interpreted as the cost incurred to evade detection, measured as a fraction of their production. This incorporates the economic reality that informal agents might incur costs to reduce their probability of detection through various means such as bribery or investments in stealth technologies (Cowell, 1985; Schneider & Enste, 2000).

The probability of detection being exogenous reflects the unpredictability and uncontrollability of enforcement actions, simplifying the model while capturing the inherent risk associated with operating in the informal sector (Allingham and Sandmo, 1972; Slemrod, 2007).

These include the tax rate on formal activities, a technology for controlling the informal sector that leads to a penalty in case of detection. This technology has an efficiency parameter which is an indicator of the quality of institutions. The indicator of access to public goods (i.e., the extent to which the informal sector has access to public goods) is also an economic policy parameter.

We show that when we increase the tax parameter on formal activities, this results in increasing the relative share of the informal sector. If we increase the access parameter to public goods for informal activities, this leads to an increase in the relative share of the informal sector. Finally, an increase in the control parameter of the informal sector leads to a decrease in the relative share of the informal sector.

The model then show that any policy change leading to an increase in the relative size of the informal sector has a nonlinear impact on growth and is a function of the structural characteristics of the economy. More specifically, the model concludes to an increasing relationship between economic growth and the tax rate on formal activities when the latter is below a given threshold and a decreasing relationship when it is above this threshold.

As for the control parameter of informal activities, it also maintains a nonlinear relationship with economic growth. The model reveals the existence of two distinct thresholds. We observe an increasing relationship when this parameter exceeds the first threshold, a decreasing relationship when it is between these two thresholds, and finally an increasing relationship when it exceeds the second threshold. This suggests a complex interaction between taxation, control of informal activities, and economic growth.

Finally, the access parameter to public goods maintains a decreasing relationship with economic growth when the control parameter of informal activities is greater than or equal to the tax rate on formal activities. On the other hand, it maintains a nonlinear relationship with economic growth when the control parameter of informal activities is less than the tax rate on formal activities. This suggests that access to public goods and control of informal activities interact in a complex way with taxation to influence economic growth.

I then propose a new method of measuring the informal sector (by using panel data). This method uses public expenditure as a percentage of GDP (as a proxy for the tax rate), the growth rate of consumption, and a measure of the quality of public institutions to determine the level to which the penalty rate (i.e., the control parameter of informal activities) should correspond, so that the model matches the data. This exercise allows us to (1) measure the penalty rate, (2) measure the size of the informal sector, and (3) make recommendations in the area of public

finance, such as tax reforms and penalty rates. With our estimation method, we estimate the average penalty rate on informal activities to be 45% and the average size of the informal sector to be 19.98% for OECD countries.

The paper is organized as follows: the next section presents the literature review. Section 3 presents the model (the definition and the characterization of the equilibrium, as well as a comparative static). Section 4 presents the basic calibration and the quantitative evaluation of the model. Section 5 highlights suggestions for policy reforms. Section 6 is the conclusion.

2. Literature review

Before presenting this model, I briefly review some papers related to this topic. Schneider and Enste [2000] or, more recently, Antunes and Cavalcanti [2007] provide a fairly thorough review of the informal economy. Current literature estimates it averages in term of percentage of official GDP around 20% for OECD countries and 45% for the least advanced economies (up to 70% in some regions). this result is consistent with the estimate provided with our method for OECD countries. There is a variety of methods for measuring the size of the informal sector. These methods are often specific to one country, which limits their interest. Schneider and Buehn [2012], Medina and Schneider [2019], are among the few documents that have estimated the informal economy for several countries and over several years. They used the multiple indicators multiple causes (MIMIC) approach which is a particular methodology of structural equation models that allow the estimation of complex causal relationships between latent variables measured themselves through so-called manifest variables. It should be noted that this method is based on an ad hoc econometric specification. That being said, this result is consistent with the estimate provided with our estimation method for OECD countries. Unlike them, we contribute to the literature by also providing an estimate of the cost borne by agents operating in the informal sector. We estimate this cost to be 45% of their production.

Norman Loayza [1996] has been one of the first to relate the size of the informal sector to the level of growth. It presents an endogenous growth model in which the level of production depends on the share of public expenditure in total output. It concludes with a negative effect of informality on growth using the MIMIC approach for Latin American countries. The ad hoc nature of the equations of his model greatly limits its relevance. In contrast to him, we show that this relationship is not linear. We then show that any policy change leading to an increase

in the relative size of the informal sector has a nonlinear impact on growth and is a function of the structural characteristics of the economy.

More recently, Elgin and Oztunali [2012] developed a two-sector dynamic general equilibrium (DGE) model to estimate the size of the informal sector. Mauricio Prado [2011] uses a static version of the Ghironi and Melitz [2005] industry model to show that firms with a low level of productivity choose to move into the informal sector. They use cross-sectional data to estimate the cost of informal activities. It should be noted that his estimates are partly based on estimates provided by other papers. He uses, in particular, the estimate of the informal sector provided by Schneider (2007) and the cost of regulation provided by Djankov et al. [2002a]. This constitutes the main limitation of the results of his analysis. However, he provides an estimate quite close to ours in terms of the average cost borne by informal agents in OECD countries. He estimates this cost at 49%.

Antunes and Cavalcanti [2007] argue that reducing access to informal financing does not explain differences in the level of informality between countries. Scheinkman and De Paula [2010] looked at the role played by the value added tax. They find a correlation between the degree of informality existing between a firm and its suppliers or retailers. Djankov et al. [2002] point out that the costs of setting up an economic activity generate a large informal sector, while De Soto [1996] and Friedman et al. [2002] suggest that entrepreneurs decide to evolve in the informal sector not because of the tax but rather because of the bureaucracy and the weakness of the legal system. According to this latter understanding, the tax burden has two effects: (1) it encourages agents to evolve into informality; but (2) it can also increase tax revenues that can be a source of improvement in the quality of government institutions, leading to a reduction of the informal sector size. For them, these two effects are generally neutralized, and the relationship between the tax rate and the size of the informal sector of the countries is not statistically significant. In the same vein, Cavalcanti et al. [2007] identify cases where these costs due to the degree of government corruption and regulation can be major determinants of the size of the informal sector. Their conclusions partly coincide with our results.

We contribute to this literature by identifying the cases where these arguments hold. More specifically, we conclude that countries with a high tax burden have an interest in increasing the cost of informal activities through binding measures. Countries with low tax rates should ideally adopt incentives. The Rauch model [1991], based on that of Lucas [1978], suggests that entrepreneurial agents go to the informal because of the minimum income which is above the equilibrium wage of the market. For Dessy and Pallage [2003], legal entrepreneurs are

encouraged by their access to productive infrastructure on which production depends; on the same subject, Cavalcanti et al. [2007] consider that entrepreneurs are encouraged to operate legally because of access to external financing.

3. Model

We consider an economy in which two sectors coexist: the informal sector and the formal sector. The agents choose at each date to evolve in one of the two sectors. We index by (i) the informal sector and by (f) the formal sector. Formal sector agents pay a tax (τ) proportional to their income (Y_f) . This tax is levied by the government, which uses it to produce public goods (G) ³. Informal agents have an exogenous probability (π) of being detected. If detected, all their production (Y_i) detected is confiscated by public authorities⁴. This deduction from their income is used to finance the operating expenses of the tax administration (P) . We follow the literature, assuming that public expenditures (G) enter the production function of both sectors; however, due to their illegal status, informal agents only have access to a fraction (δ) of public expenditures. This assumption is supported by economic literature, which shows that public infrastructure increases overall productivity (Barro, 1990; Glomm and Ravikumar, 1994), but informal agents often face limited access to these public goods due to exclusion from formal distribution systems (Loayza, 1996; Ihrig and Moe, 2004). The fraction (δ) represents this partial exclusion of informal agents from public services (Dabla-Norris, Gradstein, and Inchauste, 2008; De Soto, 1989).

This difference in access to and use of public goods significantly impacts the formal and informal sectors. Informal agents only have access to a fraction of public goods, such as infrastructure (roads, health services, education, public security). For example, an informal

³ The formal sector includes businesses and workers officially registered with tax authorities and other government agencies. These entities comply with tax, labor, and other regulations in force. Tax revenues are the income that the state primarily collects through various taxes, including personal income tax, corporate tax, VAT (Value Added Tax), and various other taxes. A large and prosperous formal sector helps increase the state's tax revenues, thereby enabling the financing of public infrastructure, public services, and social security systems. Conversely, a large informal sector reduces tax revenues and may lead to difficulties for the state in adequately financing public expenditures, thus affecting the quality of services and the well-being of the population.

⁴ This severe penalty aligns with the theoretical understanding that higher stakes increase the cost of engaging in informal activities, thereby providing a strong disincentive (Becker, 1968; Feige, 1990). Furthermore, can also be interpreted as the cost incurred to evade detection, measured as a fraction of their production. This incorporates the economic reality that informal agents might incur costs to reduce their probability of detection through various means such as bribery or investments in stealth technologies (Cowell, 1985; Schneider & Enste, 2000).

The probability of detection being exogenous reflects the unpredictability and uncontrollability of enforcement actions, simplifying the model while capturing the inherent risk associated with operating in the informal sector (Allingham & Sandmo, 1972; Slemrod, 2007).

trader may not fully benefit from security services or transportation infrastructure funded by taxes, limiting their ability to operate effectively and safely. Additionally, due to their illegal status, informal agents may not be able to fully utilize the available public goods. They might avoid interactions with public authorities, thereby limiting their access to support programs or subsidies available to formal businesses. This inefficiency results in lower productivity in the informal sector, as agents cannot use public goods to enhance their operations in the same way formal agents can. Despite these limitations, informal agents may find it advantageous to operate in this sector to avoid high taxes in the formal sector. The savings on taxes can compensate for operational inefficiency. This phenomenon is heightened when tax rates on formal activities are high, making it more profitable for formal agents to switch to the informal sector despite a loss in productivity.

3.1. Production

Production in the formal sector takes place according to a Cobb-Douglas function.

$$Y_f = A_f G^\alpha k_f^{1-\alpha} \quad (1)$$

With A_f total factor productivity, and k_f capital.

We assume a zero-population growth rate ($\frac{\dot{L}}{L} = n = 0$). This assumption is standard in economic growth models to simplify the analysis and focus on other economic dynamics, such as capital accumulation or technological advancement. By assuming zero population growth, we eliminate demographic effects and achieve a clearer understanding of the economic mechanisms at play (Solow, 1956; Barro and Sala-I-Martin, 2004).

Additionally, the absence of labor in our analysis allows us to concentrate specifically on the impact of public policies, capital investments, and technological innovations. This approach is consistent with previous studies that have excluded labor to focus on other critical variables (Romer, 1990; Aghion and Howitt, 1992, Loayza 1996).

Assumption 1: Y_f is a function of class C^2 , homogeneous of degree one, increases with k_f and concave⁵.

These assumptions are standard in economic growth models (Barro and Sala-i-Martin, 2004; Solow, 1956; Romer, 1990; Aghion and Howitt, 1992).

⁵ In his endogenous growth model, Loayza [1996] assumed a production function in which the elasticity of output with respect to capital is equal to 1. This is a very strong assumption of his model. In contrast to Loayza [1996], we make a more realistic assumption by using a Cobb-Douglas production function (see assumption 1).

The net output level—net of the tax (τ)—in the formal sector is therefore:

$$y_f^n = (1 - \tau)A_f G^\alpha k_f^{1-\alpha} \quad (2)$$

Assumption 2: A fraction (η) of these levies is used by the government to provide public goods (G) and a fraction ($1 - \eta$) is used to finance the operation of the tax administration (P).

The budget constraint of the government⁶ is written:

$$G = \eta\tau Y_f \quad (3)$$

We assume that the government is subject to a balanced budget rule. This means that government expenditures are fully financed by tax revenues, ensuring no deficit financing. This assumption is often used in economic models to maintain fiscal discipline and to analyse the impact of public spending and taxation policies without the confounding effects of borrowing. Barro's work on balanced budget rules and their macroeconomic effects supports this approach, highlighting the importance of maintaining fiscal balance for economic stability.

Moreover, it is acknowledged that public infrastructure creates a positive externality for the private sector, which could theoretically lead to a suboptimal equilibrium growth rate. However, the model presented here focuses primarily on the dynamics between taxation, informality, and economic growth. Incorporating a social planner who maximizes a social welfare function to correct the gap between private and social returns on capital would be a relevant extension. This would allow for a deeper analysis of the impact of public externalities, but such an approach falls outside the scope of the current model.

The production function for the informal sector is written:

$$Y_i = A_i(\delta G)^\alpha k_i^{1-\alpha} \quad (4)$$

With $\delta \in]0, 1[$ and represents the share of public expenditure accessible to informal producers.

We assume that the elasticity of substitution is identical in both sectors. This assumption is based on several theoretical and empirical considerations. The assumption ensures theoretical consistency within the model, simplifies the analysis by maintaining uniformity in how inputs can be substituted across different sectors, and is supported by empirical observations which

⁶ As in Barro's model (1990), we assume that the government is subject to a balanced budget rule. This assumption is necessary for the sake of simplification. Even if this assumption is far from reality, it can be justified by the existence of Ricardian equivalence. Ricardian equivalence, introduced by David Ricardo and developed by Robert Barro, suggests that the method by which the government finances its spending (through taxation or borrowing) does not affect the consumption decisions of economic agents. According to this theory, agents anticipate that current borrowing will result in future taxes and adjust their savings accordingly, thereby neutralizing the impact of budget deficits on the real economy.

show that in many economies, the elasticity of substitution between capital and labor does not vary significantly across different sectors. This approach is common in multi-sector models to derive clear and comparable results across sectors, and it reflects the observed economic behaviour where sectors, despite their formal or informal nature, operate under similar production technologies⁷. This approach is supported by various studies, including Barro and Sala-I-Martin's (2004), which provides a comprehensive overview of growth models and their assumptions. Additionally, Arrow et al. (1961) and Antras (2004) provide empirical support showing that the elasticity of substitution does not vary significantly across sectors. Furthermore, Romer (1990) discusses the importance of simplifying assumptions in economic modelling to ensure clarity and analytical tractability. Although these references do not specifically address the informal sector, they collectively support the assumption of identical elasticity of substitution in both sectors⁸.

Unlike formal agents, the latter do not pay taxes but suffer a drain on their income. We denote (π) the effective penalty rate. We suppose that this activity carries the risk of being detected by the fiscal authorities, with an exogenous probability⁹ of (π) , where (π) is a value between 0 and 1 (exclusive). If detected, the entire production is confiscated or destroyed. Therefore, the level of net production of penalties is¹⁰:

⁷ This assumption is realistic for firms that decide not to declare all or part of their production. Indeed, in this case, the technologies used in the formal and informal sectors can be similar, which implies an identical elasticity of substitution. This is also the case if the factors of production, such as labor and capital, can easily move between sectors, the elasticity of substitution will tend to be identical. This assumption is common in the literature, but it is important to keep in mind its limitations. The elasticity of substitution can vary between the formal and informal sectors for several reasons: differences in technologies, market segmentation...

⁸ This assumption is realistic for firms that decide not to declare all or part of their production. Indeed, in this case, the technologies used in the formal and informal sectors can be similar, which implies an identical elasticity of substitution. This is also the case if the factors of production, such as labor and capital, can easily move between sectors, the elasticity of substitution will tend to be identical.

⁹ Informal agents are assumed to be rational and maximize their expected utility. In other words, they are risk-neutral and seek to maximize their profit. At the same time, the government invests resources to detect informal activities. This represents a cost for the government, as shown in equation 6. The assumption of a fixed probability of detection is a simplification that can be relaxed in future research. Modeling the behavior of the informal agent in a risky situation would allow for a more in-depth analysis of the impact of different types of sanctions. Comparing the effectiveness of financial sanctions with increased controls would be a valuable contribution to the literature on informal economy and tax evasion (Allingham et al. [1972]).

¹⁰ This assumption is common in the literature. It can be found in Loayza [1996] and Camacho et al. [2015], among others. Unlike the latter, we have linked this penalty to the fiscal recipe through hypothesis 3 to make the model more realistic. Another way to make the model more realistic would be to model the behavior of the informal agent in a risky situation who faces the probability of paying a fine in case of detection. This would allow to compare the effectiveness of financial sanctions with increased controls.

$$y_i^n = (1 - \pi)A_i(\delta G)^\alpha k_i^{1-\alpha} \quad (5)$$

This assumption is consistent with models of tax evasion and informal economies, where the risk of detection and the severity of penalties play crucial roles in shaping the behaviour of economic agents. Studies such as Allingham and Sandmo (1972) provide foundational insights into the role of detection probabilities and penalties in economic decision-making.

These penalties constitute a part of the resources of the tax administration and are also used to finance its operating expenses (P). So, we have:

$$P = \pi Y_i + (1 - \eta)\tau Y_f \quad (6)$$

Assumption 3¹¹: It is further assumed that (π) is an increasing and concave function of the relative share of the tax administration's (P) operating expenditure in informal production (Y_i), an increasing function of its degree of efficiency (λ).

So, we have:

$$\pi = \lambda \left(\frac{P}{Y_i}\right)^\beta \quad (7)$$

With $\beta \in]0,1[$.

Increasing the relative share of the operating expenses of the tax administration (P) in the informal production (Y_i) makes it possible to increase the detection rate or the effective penalty (π). But the marginal effectiveness of this measure is decreasing.¹²

By incorporating these assumptions, the model can focus more precisely on the dynamics between the formal and informal sectors, the effects of taxation, the provision of public goods, and the risks associated with informal activities without the additional complexity introduced by varying elasticities of substitution or deficit financing.

3.2. Maximization of profit

The formal producer chooses the level of capital (k_f) that maximizes its net profits:

¹¹ Although equation (7) presents π as a function of variables related to tax administration expenditures and their efficiency, π is treated as exogenous in the later stages of the model analysis. This is due to structural constraints in the model that do not allow π to be isolated as an independent variable without overly complicating the framework. Instead, it is the share of tax administration expenditures in informal production that is determined endogenously. These expenditures influence the effectiveness of detecting informal activities and, consequently, indirectly affect the penalty rate. This approach simplifies the model while preserving the key dynamics necessary to analyze the interactions between informality, taxation, and economic growth.

¹² Under assumptions (2) and (3), the government's total budget constraint is given by: $G + P = \tau Y_f + \pi Y_i$.

$$\left\{ \begin{array}{l} \max_{k_f} y_f^n - r_f k_f \\ \text{subject to} \\ y_f^n = (1 - \tau)Y_f \\ Y_f = A_f G^\alpha k_f^{1-\alpha} \end{array} \right. \quad (8)$$

First order condition involves:

$$r_f = (1 - \tau)(1 - \alpha)A_f G^\alpha k_f^{-\alpha} \quad (9)$$

The informal producer program is:

$$\left\{ \begin{array}{l} \max_{k_i} y_i^n - r_i k_i \\ \text{subject to} \\ y_i^n = (1 - \pi)Y_i \\ Y_i = A_i (\delta G)^\alpha k_i^{1-\alpha} \end{array} \right. \quad (10)$$

First order condition involves:

$$r_i = (1 - \pi)(1 - \alpha)A_i (\delta G)^\alpha k_i^{-\alpha} \quad (11)$$

3.3. Equilibrium

Assumption 4: We suppose a perfect mobility of capital between the two sectors¹³. So, we have:

$$r_f = r_i = r .$$

We note (K) the level of total capital available in the economy. We have:

$$K = k_f + k_i$$

¹³ This assumption is often used in the literature to simplify the analysis. Beyond this aspect, capital is often intangible, which implies that it is easily transferable. For example, a formal company that decides not to declare all its activity operates, by definition, in the informal sector.

Here are some additional points to consider:

The assumption of perfect capital mobility is a common simplification in economic models, as it allows for a focus on other aspects of the economy, such as the role of labor or institutions.

The transferability of intangible capital is a key factor that facilitates the movement of capital between the formal and informal sectors. Intangible capital includes things like knowledge, skills, and technology, which can be easily transferred between different types of businesses.

The existence of an informal sector is another factor that can contribute to capital mobility. The informal sector is often characterized by low entry barriers and limited regulation, which makes it easier for businesses to move between the formal and informal sectors. For example, a formal company that decides not to declare all of its activity operates, by definition, in the informal sector.

However, it is important to note that the assumption of perfect capital mobility is not always realistic.

There are several factors that can limit the mobility of capital between the formal and informal sectors, such as transaction costs, risks, and market imperfections.

Transaction costs include the costs of setting up a business, obtaining licenses, and complying with regulations. These costs can be significant, especially for small businesses.

Risks associated with the informal sector can also limit capital mobility. These risks include the risk of being caught by the authorities, the risk of losing assets, and the risk of being exploited.

Market imperfections, such as information asymmetry and credit constraints, can also limit the mobility of capital Gupta M R. [1993].

Proposition 3.1: Consider the above economy: at equilibrium,

i) the relative capital invested in the formal sector is given by:

$$\frac{k_f}{k_i} = \frac{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}}}{\delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (12)$$

ii) the share of formal capital (F_k) and informal capital (I_k) in the total capital of the economy (K) are given by:

$$F_k = \frac{k_f}{K} = \frac{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}}}{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (13)$$

$$I_k = \frac{k_i}{K} = \frac{\delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}}{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (14)$$

Corollary of Proposition 3.1: consider the above proposition, we can deduce:

- The higher the relative productivity of formal capital $\left(\frac{A_f}{A_i}\right)$, the greater the relative capital invested in the formal sector.
- The formal relative capital decreases with the tax rate (τ).
- The larger the penalties associated with the informal sector, the more attractive it is to allocate capital in the formal sector.
- The easier access is to public goods (i.e., the higher δ is), the more attractive it is to invest in the informal sector.

Proposition 3.2: consider the above economy, assumption 5 and proposition 2.1, at equilibrium:

i) the production levels (formal and informal) are given by:

$$Y_f = A_f^{\frac{1}{\alpha}} K \frac{(1 - \tau)^{\frac{1-\alpha}{\alpha}} \left[A_f^{\frac{1}{\alpha}} \tau (1 - \tau)^{\frac{1-\alpha}{\alpha}} + A_i^{\frac{1}{\alpha}} \delta (1 - \pi)^{\frac{1-\alpha}{\alpha}} \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right]^{\frac{\alpha}{1-\alpha}} \right]}{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (15)$$

$$Y_i = A_i^{\frac{1}{\alpha}} K \frac{\delta(1-\pi)^{\frac{1-\alpha}{\alpha}} \left[A_f^{\frac{1}{\alpha}} \tau(1-\tau)^{\frac{1-\alpha}{\alpha}} + A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right] \right]^{\frac{\alpha}{1-\alpha}}}{(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (16)$$

ii) the total production level is given by:

$$Y = Y_f + Y_i = K \cdot f(\tau, \pi, A_f, A_i, \delta, \lambda) \quad (17)$$

$$\text{With } f(\tau, \pi, A_f, A_i, \delta, \lambda) = \frac{\left[\delta(1-\pi)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}} + (1-\tau)^{\frac{1-\alpha}{\alpha}} A_f^{\frac{1}{\alpha}} \right] \left[A_f^{\frac{1}{\alpha}} \tau(1-\tau)^{\frac{1-\alpha}{\alpha}} + A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right] \right]^{\frac{\alpha}{1-\alpha}}}{(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}}$$

iii) the share of formal production (F_Y) and informal production (I_Y) in the total production (Y):

$$F_Y = \frac{Y_f}{Y} = \frac{A_f^{\frac{1}{\alpha}} (1-\tau)^{\frac{1-\alpha}{\alpha}}}{(1-\tau)^{\frac{1-\alpha}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (18)$$

$$I_Y = \frac{Y_i}{Y} = \frac{A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}}}{(1-\tau)^{\frac{1-\alpha}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (19)$$

iv) the relative part of formal and informal sector is given by:

$$\frac{I_Y}{F_Y} = \frac{Y_i}{Y_f} = \frac{A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}}}{A_f^{\frac{1}{\alpha}} (1-\tau)^{\frac{1-\alpha}{\alpha}}} \quad (20)$$

$$\frac{F_Y}{I_Y} = \frac{Y_f}{Y_i} = \frac{A_f^{\frac{1}{\alpha}} (1-\tau)^{\frac{1-\alpha}{\alpha}}}{A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}}} \quad (21)$$

Corollary of Proposition 3.2: consider the above proposition, we can deduce:

- **Tax rate:** The higher the tax rate (τ), the less attractive it is to operate in the formal sector.
- **Penalty rate:** The more important the penalties related to the informal sector are, the more desirable it is to produce in the formal sector.
- **Access to public goods:** The easier access is to public goods (i.e., the higher δ is), the more attractive it is to invest in the informal sector.

- **Relative Productivity:** *The greater the relative productivity of formal capital $\left(\frac{A_f}{A_i}\right)$, the greater is the relative capital invested in the formal sector, and the greater is the relative size of the formal sector.*

3.4. Optimization of utility

Each agent in the economy maximizes the value of its discounted (or expected) utility taking into account its constraint:

$$\begin{aligned} \max_c U &= \int_0^{+\infty} e^{-\rho t} \frac{c^{1-\theta}}{1-\theta} dt \\ \text{sc } \dot{k} &= Y - C \\ \dot{k} &= K \cdot f(\tau, \pi, A_f, A_i, \lambda) - c \end{aligned}$$

The Hamiltonian is:

$$\mathcal{H} = e^{-\rho t} \frac{c^{1-\theta}}{1-\theta} + \mu [K \cdot f(\tau, \pi, A_f, A_i, \lambda) - c]$$

The first order condition gives:

$$\begin{aligned} \frac{\partial \mathcal{H}}{\partial c} &= e^{-\rho t} c^{-\theta} - \mu = 0 \\ \dot{\mu} &= -\frac{\partial \mathcal{H}}{\partial K} = \mu f(\tau, \pi, A_f, A_i, \epsilon, \lambda, \Gamma) \\ \dot{K} &= \frac{\partial \mathcal{H}}{\partial \mu} = K \cdot f(\tau, \pi, A_f, A_i, \epsilon, \lambda, \Gamma) - c \end{aligned}$$

After differentiation, we get the growth rate of consumption:

$$\frac{\dot{c}}{c} = \frac{1}{\theta} [f(\tau, \pi, A_f, A_i, \epsilon, \lambda, \Gamma) - \rho] \quad (22)$$

This economy will jump directly to a constant rate trajectory. hence the interest of making comparative statics. To see how it varies according to the parameters.

The equilibrium structure of the model allows us to outline several implications in terms of policies. We will see how the government can use the instruments at its disposal to regulate the informal sector, at least its size. On the one hand, if the government only wants to reduce the size of the informal sector, that involves an increase in the penalty rate, a lower tax burden, and better control over access to public goods (which implies an improvement in the efficiency of public institutions). On the other hand, if the government's objective is to generate more

growth, we will see in the following sections that it must take into account the structural characteristics of its economy. The following section puts forward an analysis of these issues.

4. Calibration and economic implications

We note that, even if it is difficult to analytically determine the above expressions, they exist. If all the parameters of the model are known, we can calculate their values; then, using simulations, determine the sensitivity of the growth level and the size of the informal sector to the model parameters. This approach is supported by economic theory, which assures us of the existence of these expressions under certain conditions (Lucas, 1988; Barro and Sala-i-Martin, 2004). Additionally, parameter estimation methods allow for the calculation of these values using known parameters (Greene, 2012; OECD, 2008). Finally, simulations provide a robust method for sensitivity analysis, enabling a comprehensive understanding of the model's dynamics (Judd, 1998; Dixon and Parmenter, 1996).

4.1. Calibration

The model is calibrated. We assume that informal firms are relatively less productive than formal firms, following the literature. Studies indicate a 20–40% productivity difference between the two sectors (La Porta and Shleifer, 2014; Perry et al., 2007). Therefore, we set

$$\frac{\delta^{\alpha} A_i}{A_f} = 0.60.$$

The rate of preference for the present is $\rho = 0.05$. This rate of 0.05 is commonly used in economic literature and endogenous growth models. This value is well accepted by the academic community, allowing for consistent comparison with previous works. For example, Barro and Sala-I-Martin (2004) use similar rates in their economic growth models.

The inter-temporal elasticity of substitution is $\frac{1}{\theta} = \frac{1}{2}$. The chosen value is consistent with numerous empirical studies. For example, Hall (1988) finds a very low IES, close to zero, suggesting low responsiveness. However, other studies, such as Guvenen (2006), estimate the IES to be around 1, indicating greater sensitivity. An intermediate value of 1/2 represents a reasonable average between these extremes, aligning with the conclusions of many studies that estimate the inter-temporal elasticity of substitution to be between 0.3 and 0.5 (Guvenen, 2006; Deaton, 1992).

Table 1 summarizes the values taken by the different parameters of the benchmark model¹⁴.

Table 1 *Value of the parameters*

Parameters	α	β	λ	ρ	θ
Values	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{5}{100}$	2

We study the joint effect of π and τ on the growth and size of the informal sector. Simulation consists of studying how the growth rate and the size of the informal sector react when variables of government choice vary, more specifically, the penalty rate, the tax rate, and the indicator of access to public goods.

4.2. Effect of τ and π on informal sector size

4.2.1. Relation between Informal sector size and τ for different value of π

Figure 1¹⁵ shows the relationship between the informal sector and the tax rate on formal activities (τ) for different penalty rates (π) on informal activities. We run the simulation for three possible scenarios. In Scenario 1, with a penalty rate $\pi=0.2$ (blue curve), at low levels of τ , the informal sector is already relatively high. As τ increases, the size of the informal sector increases more rapidly. This indicates that when the penalties for informal activities are low, an increase in taxes on formal activities strongly encourages businesses or individuals to turn to the informal sector. In Scenario 2, with a penalty rate $\pi=0.4$ (orange curve), at low levels of τ , the informal sector is smaller than in Scenario 1. The increase in τ leads to an increase in the informal sector, but less rapidly than in Scenario 1. This shows that a moderate penalty rate makes the shift to the informal sector less attractive than with a low penalty rate. In Scenario 3, with a penalty rate $\pi=0.7$ (green curve), at low levels of τ , the informal sector is the smallest of the three scenarios. The increase in τ leads to an increase in the informal sector, but at an even

¹⁴ A value of $1/3$ for the capital share in the production function (α) is a commonly used convention in Cobb-Douglas production functions, based on empirical estimates of the capital share in GDP [Gollin, 2002]. The calibration of β and λ directly affects the model's predictions. A realistic choice allows for generating reliable and robust predictions, thereby enhancing the credibility of the model's results. Romer (1990) discusses the importance of calibrating parameters to ensure the robustness of results in endogenous growth models.

¹⁵ Horizontal axis (τ): It represents the tax rate on formal activities. An increase in τ means an increased tax burden on businesses or individuals operating in the formal sector.

Vertical axis: It shows the size or proportion of the informal sector relative to the entire economy. A higher value indicates a larger informal sector.

Colored curves: Each curve represents a different scenario based on the probability of detection:

- Blue curve ($\pi=0.2$): Low probability of detection.
- Orange curve ($\pi=0.4$): Moderate probability of detection.
- Green curve ($\pi=0.7$): High probability of detection.

slower rate than in Scenarios 1 and 2. A high penalty rate strongly discourages informal activities, even when taxes on formal activities increase. The graph clearly shows that the size of the informal sector is strongly influenced by the penalty rate on informal activities and the tax rate on formal activities. An increase in taxes on formal activities (τ) tends to increase the informal sector, but this increase is moderated by the penalty rate (π). The higher the penalty rate, the less economic actors are incentivised to turn to the informal sector in response to an increase in taxes on formal activities.

4.2.2. Relation between Informal sector size and π for different value of τ

Figure 2¹⁶ shows the relationship between the informal sector and the penalty rates (π) for different tax rate on formal activities (τ).

In Scenario 1, with a tax rate $\tau=0.2$ (blue curve), at low levels of π , the informal sector is relatively small. As π increases, the size of the informal sector slowly decreases. This indicates that when taxes on formal activities are low, an increase in penalties for informal activities gradually reduces the size of the informal sector. In Scenario 2, with a tax rate $\tau=0.4$ (orange curve), at low levels of π , the informal sector is larger than in Scenario 1. The increase in π leads to a faster decrease in the informal sector than in Scenario 1. This shows that a moderate tax rate encourages more people to stay in the formal sector when penalties on informal activities increase. In Scenario 3, with a tax rate $\tau=0.7$ (green curve), at low levels of π , the informal sector is the largest of the three scenarios. The increase in π leads to a rapid decrease in the informal sector. A high tax rate on formal activities initially encourages a shift to the informal sector, but high penalties for informal activities strongly discourage this transition, quickly reducing the size of the informal sector.

This graph clearly shows that the size of the informal sector is strongly influenced by the penalty rate on informal activities and the tax rate on formal activities. An increase in penalties on informal activities (π) tends to reduce the size of the informal sector, but this reduction is moderated by the tax rate (τ). The higher the tax rate, the larger the informal sector is initially, but an increase in penalties (π) more rapidly reduces the size of the informal sector.

¹⁶ Horizontal axis (π): It represents the penalty rate on informal activities. An increase in π means more severe penalties for those operating in the informal sector.

Vertical axis: It shows the size or proportion of the informal sector relative to the entire economy. A higher value indicates a larger informal sector.

Colored curves: Each curve represents a different scenario based on the tax rate (τ) on formal activities:

- Blue curve ($\tau=0.2$): Represents a low tax rate.
- Orange curve ($\tau=0.4$): Represents a moderate tax rate.
- Green curve ($\tau=0.7$): Represents a high tax rate.

4.3. Effect of τ and π on economic growth

4.3.1. Relation between economic growth and π for different value of τ

Figure 3 shows the relationship between the economic growth and the penalty rates (π) for different tax rate on formal activities (τ).

In Scenario 1 with $\tau=0.2$ (blue curve), the context is a low tax rate on formal activities. At low levels of π , economic growth is weak because the low taxes do not generate sufficient fiscal revenue to adequately fund public spending, and the informal sector remains significant, reducing overall tax contributions. As π increases, higher penalties for informal activities cause a shift towards the formal sector, increasing fiscal revenue. This improves public spending, thereby stimulating economic growth up to a certain point. However, at high levels of π , excessively high penalties can discourage overall economic activity, including in the formal sector, reducing fiscal revenue and public spending, which harms growth. In Scenario 2 with $\tau=0.4$ (orange curve), the context is a moderate tax rate on formal activities. At low levels of π , growth is stronger than in Scenario 1 because the moderate tax rate generates sufficient fiscal revenue for adequate public spending, although the informal sector is still present. As π increases, economic growth rises as more activities shift to the formal sector, increasing fiscal revenue and allowing for higher public spending, until a peak is reached. At high levels of π , excessively high penalties stifle economic activity, even in the formal sector, reducing fiscal revenue and public spending, leading to a decline in growth. In Scenario 3 with $\tau=0.7$ (green curve), the context is a high tax rate on formal activities. At low levels of π , economic growth is initially weak because the high tax rates discourage formal activity, pushing more businesses into informality. Fiscal revenue is low, limiting public spending. As π increases, there is pressure to return to the formal sector, increasing fiscal revenue and allowing for more robust public spending, thus stimulating growth. Growth peaks at higher penalty levels, where enough activities are incentivized to remain or return to the formal sector, maximizing fiscal revenue and public spending. However, excessively high penalties can also reduce overall economic activity.

The scenarios show that to maximize economic growth, it is crucial to find a balance between the tax rate on formal activities and the penalties on informal activities. Fiscal revenue must be high enough to adequately fund public spending without discouraging economic activity. Recommended Policies: Moderate Tax Rate (τ): A moderate tax rate appears optimal to maintain a stable tax base while encouraging formal economic activity. Reasonable Penalties

(π): Moderate penalties for informal activities are necessary to incentivize adherence to the formal sector without excessively stifling overall economic activity. In summary, the relationship between tax rates, penalties on informal activities, and economic growth heavily depends on balancing the creation of sufficient fiscal revenue and maintaining an environment conducive to formal economic activity.

4.3.2. Relation between economic growth and τ for different value of π

Figure 4 shows the relationship between the economic growth and the tax rate on formal activities (τ) for different penalty rates (π) on informal activities.

In Scenario 1 with $\pi=0.2$ (green curve), the context is a low penalty rate on informal activities. At low levels of τ , economic growth is low because the low taxes do not generate enough fiscal revenue to adequately fund public spending. As τ increases, economic growth rises because more fiscal revenue allows for better public spending, thus stimulating the economy. However, at high levels of τ , excessively high tax rates hinder economic activity, reducing fiscal revenue and consequently public spending, which harms growth. In Scenario 2 with $\pi=0.4$ (blue curve), the context is a moderate penalty rate on informal activities. At low levels of τ , economic growth is higher compared to Scenario 1 because moderate penalties encourage a shift to the formal sector, increasing fiscal revenue. As τ increases, economic growth peaks as optimal fiscal revenue allows for robust public spending. Yet, at high levels of τ , excessively high tax rates eventually discourage economic activity, even with moderate penalties, reducing economic growth. In Scenario 3 with $\pi=0.7$ (orange curve), the context is a high penalty rate on informal activities. At low levels of τ , economic growth is initially low because high tax rates discourage formal activity, and high penalties increase the cost of informality. As τ increases, economic growth rises as high penalties push informal activities to the formal sector, increasing fiscal revenue and public spending. However, at high levels of τ , as in the other scenarios, excessively high taxes eventually hinder economic activity, even with high penalties, reducing fiscal revenue and thus economic growth.

Each scenario demonstrates the importance of finding an optimal balance between the tax rate on formal activities and the penalties on informal activities to maximize economic growth. A tax rate that is too low does not generate sufficient fiscal revenue to fund necessary public spending for robust economic growth. A moderate tax rate allows for a peak in economic growth by balancing fiscal revenue and public spending. Conversely, a tax rate that is too high eventually discourages economic activity, even with high penalties on informal activities, thus

reducing economic growth. In summary, fiscal policy must be carefully balanced to maximize fiscal revenue without discouraging formal economic activity, while effectively deterring informal activities through appropriate penalties.

4.3.3. Comparison with the Barro Model:

Comparing our results with the Barro model reveals several important similarities and differences.

- Similarities:

Inverted U-Shaped Relationship: Like in the Barro model, our results show that there is an optimal tax rate for maximizing economic growth. This is manifested by increasing economic growth up to a certain point, after which higher tax rates start to reduce growth.

Impact of Distortionary Taxes: Both analyses demonstrate that excessively high tax rates have a negative effect on economic growth. In my graphs, this is reflected by an increase in the size of the informal sector at high tax levels, which reduces fiscal revenue and economic growth.

- Differences:

Parameter π : A key difference between my results and the Barro model is the inclusion of the parameter π , which represents penalties on informal activities. This parameter adds an additional dimension to the analysis by showing how penalties influence the decision of businesses to remain in the informal sector or formalize.

Informal Sector: The Barro model does not explicitly account for the informal sector, whereas my graphs highlight the importance of interactions between the formal and informal sectors. By observing the size of the informal sector, we can see how businesses react to changes in tax rates and penalties, providing a more comprehensive perspective on economic dynamics.

In conclusion, our results confirm certain aspects of the Barro model, notably the existence of an optimal tax rate and the negative effects of excessive taxation. However, by including penalties on informal activities, we can offer a more nuanced view of how businesses react to fiscal policies, emphasizing the importance of balancing tax rates and penalties to maximize economic growth.

4.4. Effect of δ , τ and π on economic growth

Figure 5 presents three sub-figures showing the relationship between economic growth ($\frac{\dot{c}}{c}$), the parameter δ (degree of access of informal activities to public goods), and the penalty rate π on

informal activities, for different levels of the tax rate τ on formal activities. Here is a detailed explanation of each sub-figure:

Public goods are entirely funded by the tax on formal activities. Thus, the tax rate (τ) plays a crucial role in the impact of δ and π on economic growth. Better utilization of public goods by informal producers (high δ) and penalties on informal activities (high π) can stimulate growth up to a certain point. However, very low or very high tax rates can mitigate or complicate this effect.

Each graph illustrates the necessity of finding an optimal balance between these parameters to maximize economic growth while effectively managing informal activities.

To study all possible combinations between (π) (probability of detection of informal activities), (τ) (tax rate on formal activities), and (δ) (degree of access to public goods for informal activities), we need to analyse their combined effects on economic growth ($\frac{G}{C}$). Let's start with the scenario where π is low, (τ) is low, and (δ) is low. In this case, the expected effects include low funding for public goods (τ low) and low deterrence of informal activities (π low). Informal producers do not efficiently use public goods (δ low), resulting in low growth since informal activities persist with a low contribution to growth due to a lack of funding and deterrence. Next, consider the scenario where π is low, (τ) is low, and (δ) is high. Here, the low funding for public goods persists, as does the low deterrence of informal activities, but informal producers efficiently use public goods. The impact on growth is slightly better than in the previous scenario but remains limited by the low funding for public goods. In the scenario where π is low, (τ) is high, and (δ) is low, public goods are well-funded (τ high), but the low deterrence of informal activities (π low) and the low utilisation of public goods by informals limit the positive impact, resulting in moderate growth. When π is low, (τ) is high, and (δ) is high, public goods are well-funded and efficiently used by informals, but the low deterrence of informal activities limits growth, which is moderate to high. If π is high, (τ) is low, and (δ) is low, we observe low funding for public goods, high deterrence of informal activities, and low utilisation of public goods by informals. This results in low to negative growth, as informal activities are highly deterred, but the low funding and usage of public goods limit growth. In the scenario where π is high, (τ) is low, and (δ) is high, the low funding for public goods and high deterrence of informal activities persists, but informal producers efficiently use public goods, resulting in low to moderate growth. For the scenario where π is high, (τ) is high, and (δ) is low, public goods are well-funded, but the high deterrence and low utilisation of public goods by informals limit growth gains, resulting in moderate to negative growth. Finally, when

π is high, (τ) is high, and (δ) is high, public goods are well-funded and efficiently used by informals, and the high deterrence of informal activities maximises the contribution to economic growth, which is potentially high.

To maximise economic growth while effectively managing informal activities, an optimal combination of π (probability of detection), (τ) (tax rate), and (δ) (access to public goods) is essential. A moderate to high probability of detection (π) can effectively deter informal activities without excessively destroying production, thereby encouraging a shift towards formal activities. A moderate to high tax rate (τ) is necessary to adequately fund public goods, which are crucial for supporting economic growth, especially when they are well-utilised by informal producers. Facilitating access to and the efficient use of public goods ((δ)) by informal producers is crucial, as this can enhance their productivity and contribution to economic growth. The analysis of different scenarios shows that adequate funding of public goods and moderate to high deterrence of informal activities, combined with efficient utilisation of public goods by informals, are key to achieving this objective.

To conclude, this section provides interesting insights. The growth rate and size of the formal sector increase with the penalty rate in all scenarios. But this increase becomes weaker as the penalty rate increases. Its variation is even negative when it is above 30% for the first two scenarios. This reflects the idea that the marginal effectiveness of the fight against the informal sector by binding means is negative for countries with a low or medium tax burden. This may seem paradoxical, but the underlying logic is that when the tax burden is low, agents are not encouraged to produce informally, especially when productivity is relatively lower. Most of those who decide to produce in this sector have an activity that requires little productivity and for which the payment of a tax makes the yield inefficient. Increasing the constraints related to informal production leads to the disappearance of these activities, which nevertheless participated in the economic activity of the country. For countries with a high tax burden, the marginal effectiveness of these constraints is positive, regardless of the level of the tax. Indeed, when the tax burden is very high, companies are all the more encouraged to produce informally. In this case, this incentive also applies to companies whose activities require a high level of productivity. In fact, the latter resort to a method called “semi-informal activity”; that is, they declare their activity only partially. There is here a voluntary aspect to be produced informally in order to benefit from the advantages of formal activity as well as those of informal activity. Increasing the constraints makes the benefits of the informal relatively small and reduces the effects of public spending congestion that is detrimental to growth.

To sum up, countries with a high tax burden have every interest in increasing the cost of informal activities through binding measures. Countries with low tax rates should ideally adopt incentives. These results lead us to conclude that there is a nonlinear relationship between the size of the informal sector and the growth rate.

Our simulations further show that $f(\tau, \pi, A_f, A_i, \delta, \lambda)$, F_Y , and F_K are decreasing functions of δ and increasing functions of π for all three scenarios (Figures 5). The more access to public goods is facilitated for informal businesses, the lower *are* $f(\tau, \pi, A_f, A_i, \delta, \lambda)$, F_Y , and F_K ; therefore, lower is growth. If a country's tax rate is low, fighting the informal sector through binding measures will have little effect on growth. This finding mitigates that of Loayza [1996], which suggests that any measure increasing the cost of informality has a positive impact on growth.

The next step of our analysis concerns the estimation of the informal sector size in OECD countries.

5. Estimated sizes of informal sectors in OECD countries

5.1. Choice of parameters

Using our model, we take a new measure of the size of the informal sector. In this section, we calibrate the model for OECD countries between 1995 and 2016. We use public expenditure panel data as a percentage of GDP (as proxy for the tax rate), the growth rate of consumption, and a measure of the quality of public institutions¹⁷ to determine the level at which the penalty rate must be established in order for the model to match the data. This exercise allows us to

¹⁷ To measure the quality of institutions, denoted by (λ) in the model, I chose a measure of institutional quality provided by the World Bank. This is an indicator that ranges from -2.5 to 2.5. The closer the value is to 2.5, the better the quality of institutions is considered to be. For each OECD country, i averaged the values between 1996 and 2016. In this sample, Denmark has the highest index. In the model, (λ) is assumed to be between 0 and 1. To respect this assumption of the model, I took the institutional quality index of Denmark as a reference. I then divided the institutional quality index of each country by that of Denmark.

provide (1) a measure of the penalty rate¹⁸ and (2) the measurement of the size of the informal sector.

5.2. Results

Therefore, the model is solved with a system of 8 equations—(3), (13), (14), (18), (19), (20), (21), and (22)—with 8 unknowns: τ , π , I_Y , F_Y , $\frac{I_Y}{F_Y}$, $\frac{F_Y}{I_Y}$, I_k , and F_k . The results are summarized in Table 2, Tables 3, Table 4 and figure 6.

On average, the size of the informal sector in OECD countries is estimated at 19.98% between 1995 and 2016. We can compare this with the estimate of Schneider and Buehn [2012], who found an average of 21% between 1999 and 2007. Tables 2, 3, 4 and figure 6 provide a country-by-country estimate. Unsurprisingly, countries with a high penalty rate and/or a low sampling rate are those with the lowest level of informality in the sample.¹⁹

These calculations provide a better understanding of the relationship between informal production and the different variables of the model. Consider, for example, the United States, Switzerland, Ireland, Australia, and Turkey, which have a similar tax rate. It is noted that except for Turkey, the size of the informal sector in these countries is exceptionally small. This is because Turkey's penalty rate (0.10%) is well below the average tax rate of these countries. In other words, a low tax rate alone is not enough to reduce the size of the informal sector; it is essential to implement measures that make the tax administration more efficient.

These countries are to be compared with countries with high tax rates, such as France, Austria, Belgium, Denmark, Italy, Norway, and Sweden. Intuitively, the size of the informal sector in these countries would be expected to be larger than that of the countries mentioned above. However, their penalty rate appears to be well above the sample average (44%).²⁰ This severely limits the growth of the informal sector in these countries despite a fairly high tax rate. Establishing a high tax rate certainly encourages informality, but if effective measures are implemented at the same time, the importance of the informal sector can be contained.

¹⁸ The equation (22) used to determine the penalty rate is estimated by assuming an identical total relative factor productivity across countries. This simplification is used because of a lack of robust data. However, the role of the latter is relatively marginal in the determination of the penalty rate.

¹⁹ But this observation hides some peculiarities highlighted below.

²⁰ The taxes collected can be used to increase the efficiency of the tax administration after deduction of public expenditure.

Finally, consider countries with a very low tax rate (around 17%) such as Mexico or Chile. These countries have a penalty rate well below the sample average. This naturally translates into a fairly low level of formal production despite a low tax rate. This last point clearly illustrates the point of view set out in the previous section. When the tax burden is low, agents are not encouraged to produce informally, especially when productivity is relatively lower. Most of those who decide to produce there have activities that require little productivity and for which the payment of a tax renders the yield inefficient. To increase the constraints related to informal production leads to the disappearance of these activities, which nevertheless participated in the economic activity of the country. This encourages some authorities to tolerate these activities informally.

6. Conclusion

This article presents a new method for estimating the size of the informal sector and analyses the complex relationship between the size of the informal sector and economic growth. We have developed an endogenous growth model using the determinants of the informal sector and using data for OECD countries between 1995 and 2016, we evaluated some implications of our model and estimated the size of the informal sector in these countries. Our results suggest:

- The average size of the informal sector in these countries is 19.98%. It is estimated that the largest informal sectors among OECD countries are found in Turkey, Italy, Slovenia, and Hungary and the smallest in Sweden, New Zealand, Finland, and Norway.
- Any policy change that increases the relative size of the informal sector has a nonlinear impact on growth. This concerns the increase in the tax rate on formal activities, the reduction of the penalty rate on informal activities, the increase in the indicator of access to public goods, and the decline in the quality of government institutions.

The tax rate is commonly singled out to explain the size of the informal sector or the concealment of assets. However, the tax rate alone cannot explain the degree of informality of an economy. We have just seen that the indicator of access to public goods, the penalty rate, and the level of regulation can be key vectors. Thus, any policy aimed at reducing the size of the informal sector must take into account all these vectors. Fighting the informal sector is a task that any public authority must tackle, but it is necessary to adapt the measures to the

structural characteristics of the economy in question—that is, taking into account economic, social, and even environmental factors.

Figure 1 Relation between Informal sector size and τ for different value of π

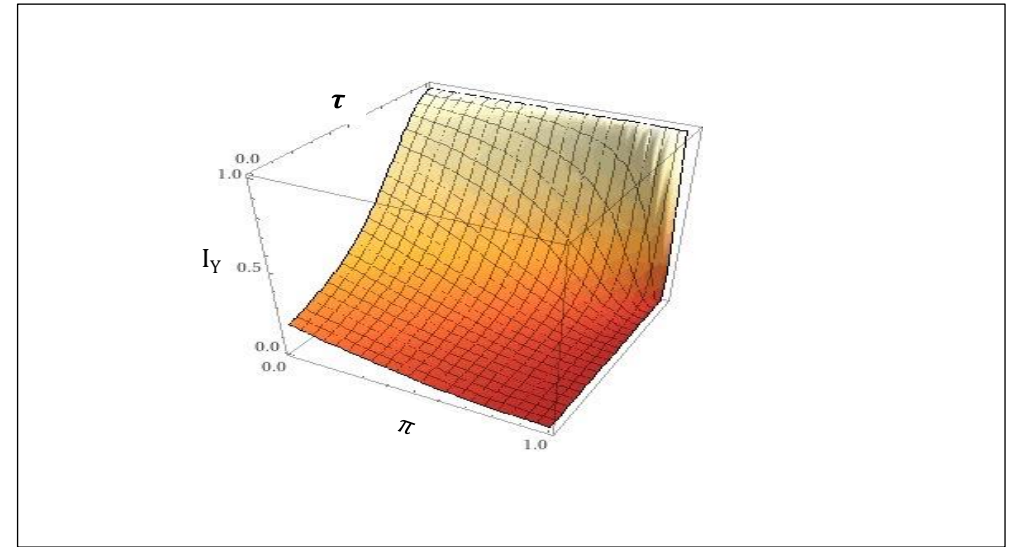
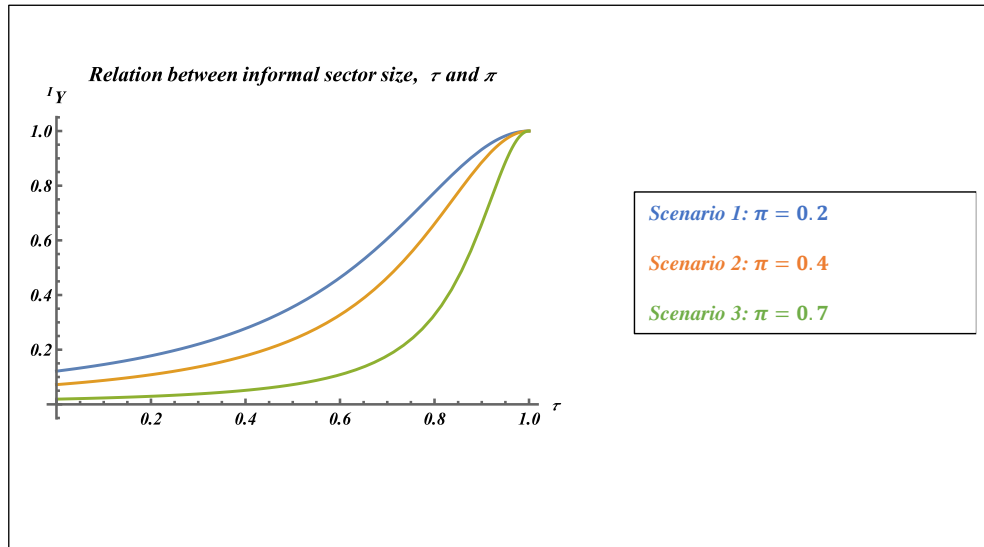


Figure 2 Relation between Informal sector size and π for different value of τ

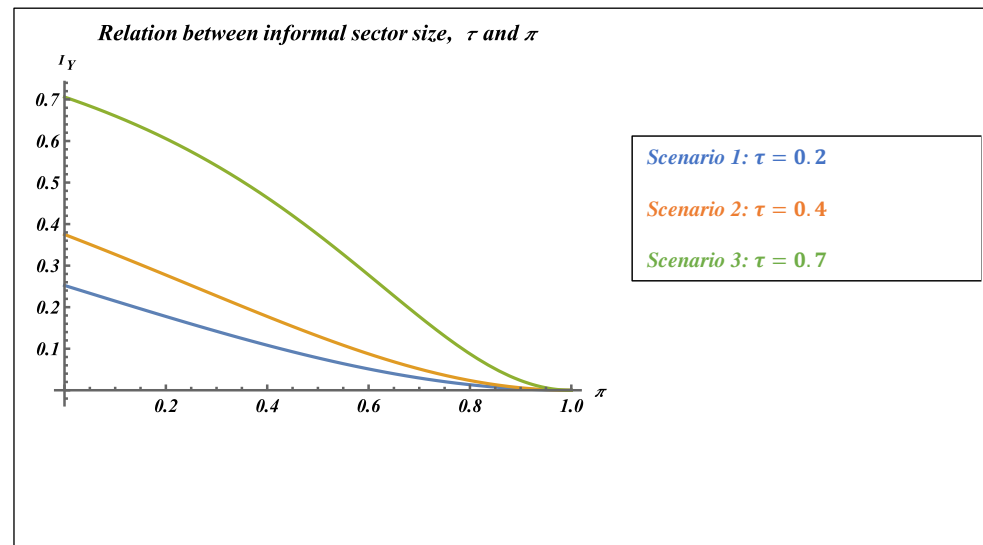


Figure 3 Relation between growth and π , for different value of τ

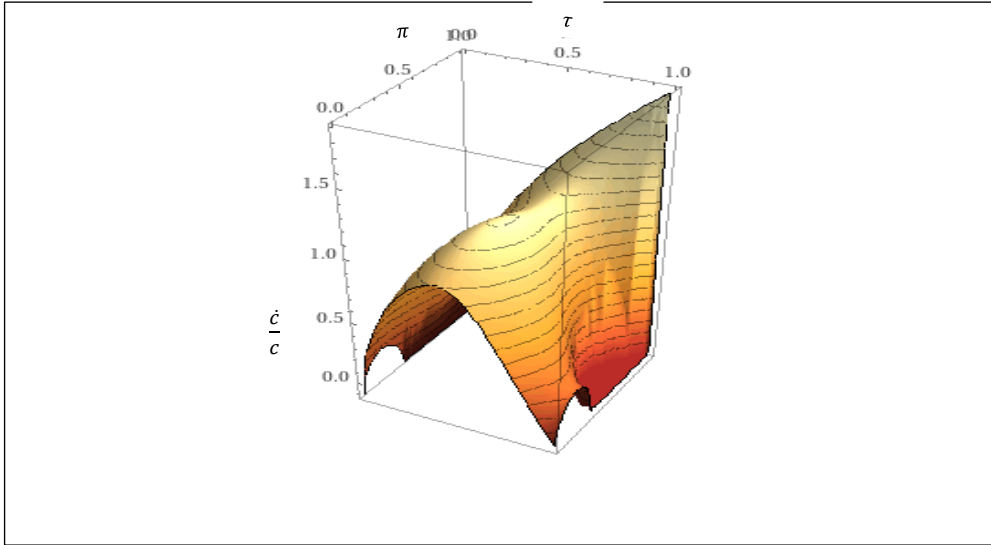
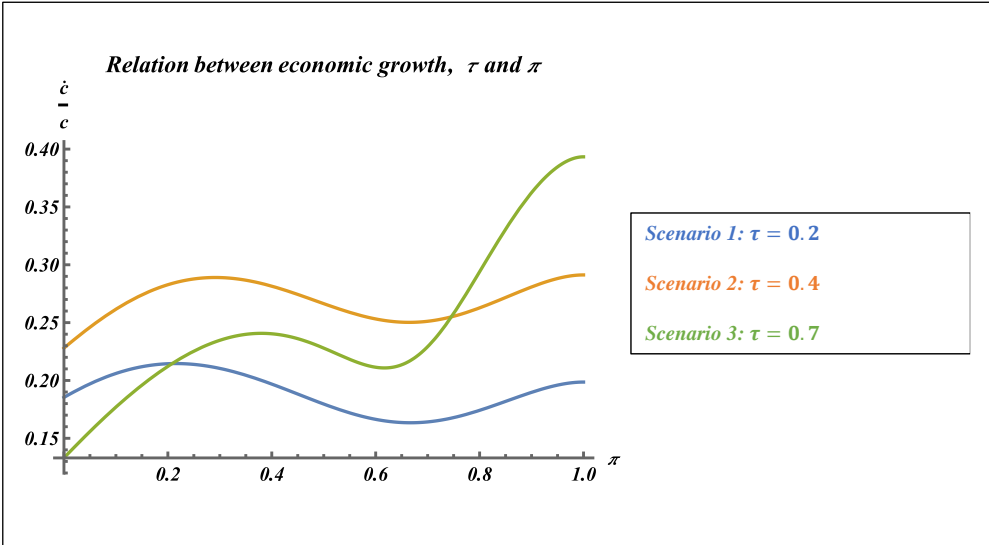


Figure 4 Relation between growth and τ , for different value of π

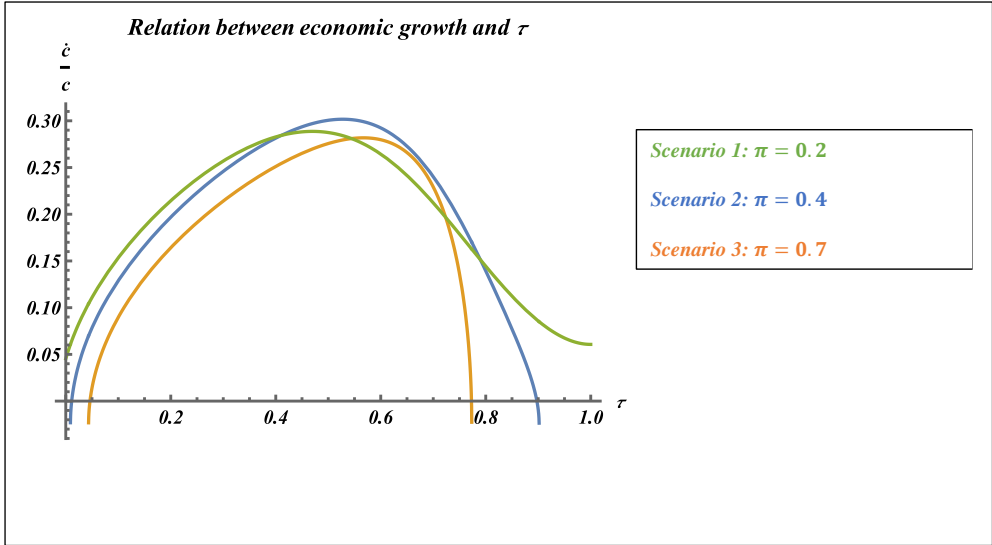


Figure 5 Relation between growth and δ and π for different value of τ

Figure 3.a: Relation between growth and δ and π . With $\tau = 0.33$

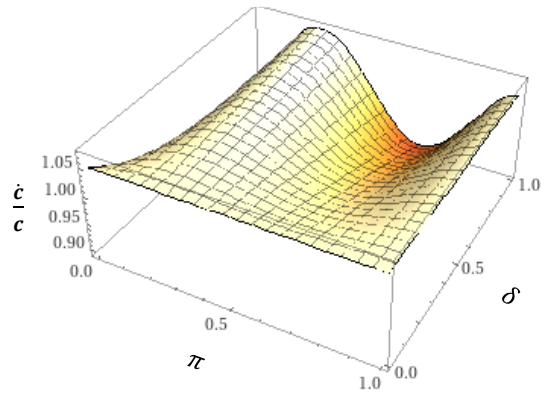


Figure 3.b: Relation between growth and δ and π . With $\tau = 0.1$

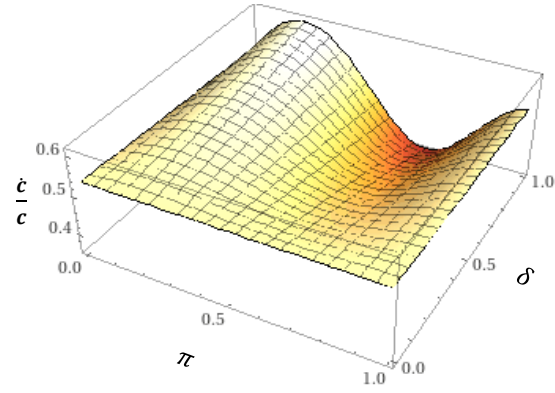


Figure 3.c: Relation between growth and δ and π . With $\tau = 0.70$

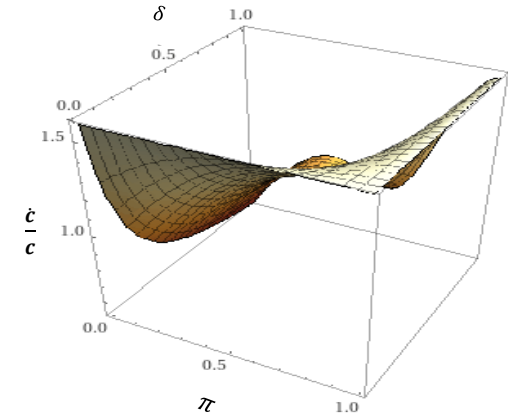
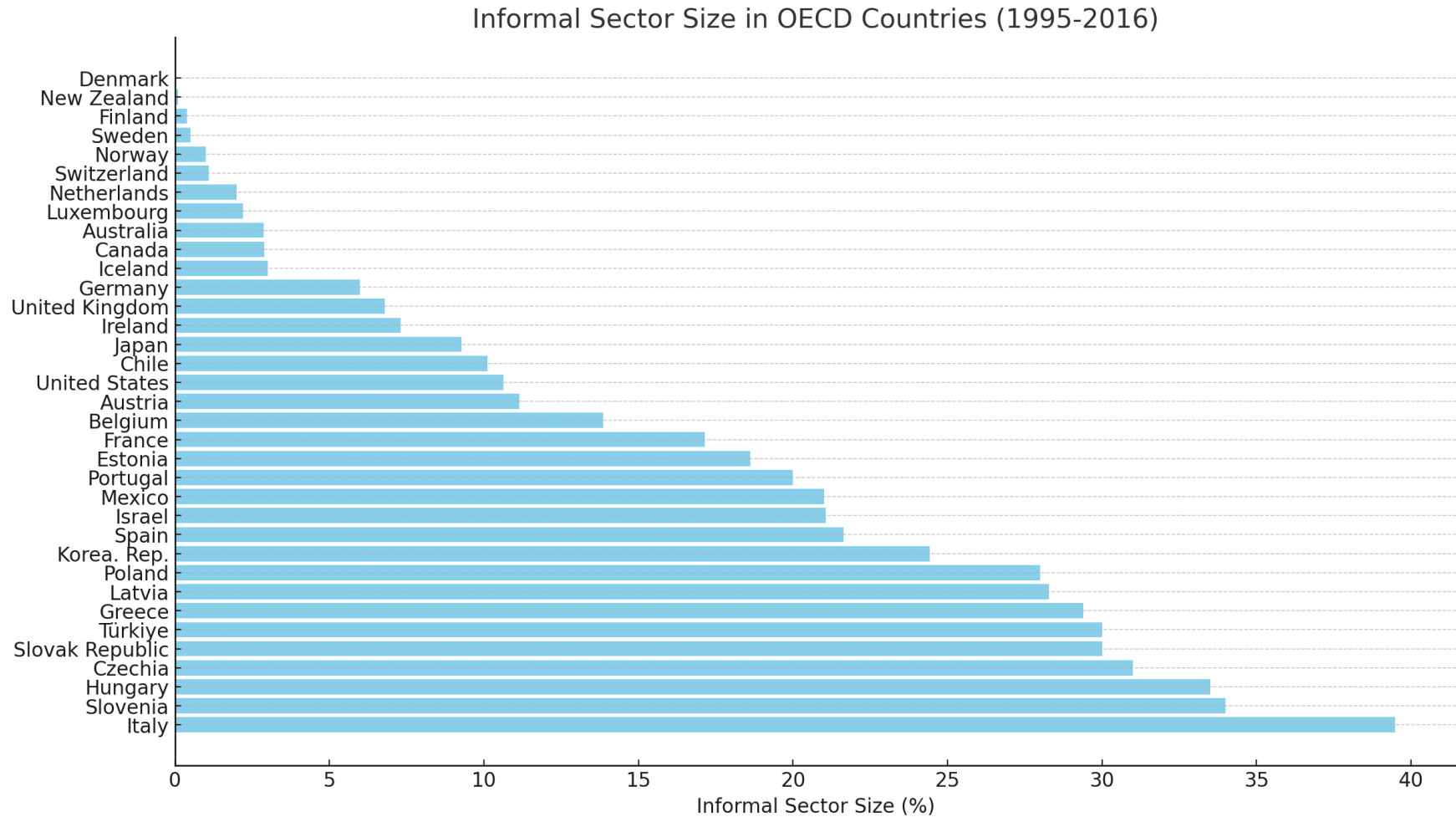


Figure 6 : Informal sector size in OECD countries (1995-2016)



Here is the graph showing average value of the size of the informal sector in OECD countries between 1995 and 2016. Each bar represents the proportion of the informal sector in the economy of each country, sorted in descending order.

Table 2 data collected from OECD (mean 1995-2016)

Source: Data: OECD (2017) For each country, we calculated the average between 1995 and 2016.
 Model: Our own calculations

Country Name	Data		
	$\frac{G}{F_y}$ %	$\frac{\dot{c}}{c}$	λ
Australia	27,26	3,57%	82,98%
Austria	42,3	1,41%	68,51%
Belgium	44,432	1,38%	65,11%
Canada	31,13	3,00%	83,40%
Chile	20,622	5,41%	60,43%
Czechia	33,516	2,21%	14,89%
Denmark	46,784	1,31%	100,00%
Estonia	32,112	5,01%	47,23%
Finland	43,232	2,55%	95,74%
France	44,74	1,75%	60,00%
Germany	36,398	1,03%	76,17%
Greece	35,46	0,82%	0,85%
Hungary	38,166	1,99%	14,47%
Iceland	36,354	2,90%	85,19%
Ireland	27	3,77%	69,79%
Israel	30,748	4,21%	37,45%
Italy	43,362	0,74%	5,53%
Japan	30,105	0,89%	64,26%
Korea. Rep.	24,616	3,57%	21,28%
Latvia	28,532	3,22%	13,62%
Luxembourg	38,02	2,46%	87,66%
Mexico	15,032	2,71%	20,43%
Netherlands	36,736	1,53%	88,09%
New Zealand	31,806	3,40%	98,72%
Norway	40,05	3,30%	90,64%
Poland	31,8725	3,83%	23,40%
Portugal	33,368	1,36%	43,40%
Slovak Republic	30,154	2,98%	9,79%
Slovenia	36,638	2,08%	9,79%
Spain	32,968	1,81%	39,57%
Sweden	42,82	2,38%	95,32%
Switzerland	27,118	1,57%	89,79%
Türkiye	28,706	4,28%	0,43%
United Kingdom	32,632	2,40%	72,77%
United States	25,174	2,79%	57,45%
Average	33,71	3%	54%

Source: Data: OECD (2017) For each country, we calculated the average between 1995 and 2016.
 Model: Our own calculations

Table 3 *Estimated data of tax rate on formal sector and penalty rate on informal sector.**Source: Data: OECD (2017) For each country, we calculated the average between 1995 and 2016.**Model: Our own calculations*

Country Name	Model	
	τ	π
Australia	27,26%	73,03%
Austria	42,30%	56,03%
Belgium	44,43%	52,04%
Canada	31,13%	74,39%
Chile	20,62%	42,69%
Czechia	33,52%	4,70%
Denmark	46,78%	99,00%
Estonia	32,11%	30,14%
Finland	43,23%	92,50%
France	44,74%	45,91%
Germany	36,40%	65,91%
Greece	35,46%	9,11%
Hungary	38,17%	5,56%
Iceland	36,35%	77,00%
Ireland	27,00%	55,60%
Israel	30,75%	23,00%
Italy	43,36%	1,55%
Japan	30,11%	51,93%
Korea. Rep.	24,62%	7,80%
Latvia	28,53%	3,40%
Luxembourg	38,02%	80,00%
Mexico	15,03%	5,00%
Netherlands	36,74%	81,00%
New Zealand	31,81%	95,00%
Norway	40,05%	87,00%
Poland	31,87%	9,00%
Portugal	33,37%	28,00%
Slovak Republic	30,15%	1,80%
Slovenia	36,64%	1,80%
Spain	32,97%	24,22%
Sweden	42,82%	91,00%
Switzerland	27,12%	81,00%
Türkiye	28,71%	0,10%
United Kingdom	32,63%	60,90%
United States	25,17%	44,50%
Average	33,71%	44,62%

*Source: Data OECD (2017) For each country, we calculated the average between 1995 and 2016.**Model: Our own calculations*

Table 4 *Estimated size of Informal economy.**Source: Data OECD (2017) For each country, we calculated the average between 1995 and 2016.**Model: Our own calculations*

Country Name	Model	
	I_Y	I_Y/F_Y
Australia	2,88%	2,97%
Austria	11,15%	12,54%
Belgium	13,86%	16,09%
Canada	2,90%	2,99%
Chile	10,12%	11,26%
Czechia	31,00%	44,93%
Denmark	0,01%	0,01%
Estonia	18,62%	22,88%
Finland	0,38%	0,38%
France	17,14%	20,69%
Germany	5,98%	6,36%
Greece	29,39%	41,62%
Hungary	33,50%	50,38%
Iceland	3,00%	3,09%
Ireland	7,30%	7,87%
Israel	21,07%	26,69%
Italy	39,49%	65,26%
Japan	9,27%	10,22%
Korea. Rep.	24,42%	32,31%
Latvia	28,29%	39,45%
Luxembourg	2,20%	2,25%
Mexico	21,00%	26,58%
Netherlands	2,00%	2,04%
New Zealand	0,10%	0,10%
Norway	1,00%	1,01%
Poland	28,00%	38,89%
Portugal	20,00%	25,00%
Slovak Republic	30,00%	42,86%
Slovenia	34,00%	51,52%
Spain	21,63%	27,60%
Sweden	0,50%	0,50%
Switzerland	1,10%	1,11%
Türkiye	30,00%	42,86%
United Kingdom	6,78%	7,28%
United States	10,62%	11,88%
Average	14,82%	19,98%
Variance	1,47%	3,36%
Standard-error	12,12%	18,32%
Max	39,49%	65,26%
Min	0,01%	0,01%

*Data source: OECD For each country, we calculated the average between 1995 and 2016.**Model: Our own calculations*

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Proof of proposition 2.1

i) We suppose a perfect mobility of capital between the two sectors. So, we have:

$$r_f = r_i = r.$$

Equalizing equations (10) and (11), we obtain:

$$\begin{aligned} (1 - \tau)(1 - \alpha)A_f G^\alpha k_f^{-\alpha} &= (1 - \pi)(1 - \alpha)A_i (\delta G)^\alpha k_i^{-\alpha} \\ \Leftrightarrow (1 - \tau)A_f k_f^{-\alpha} &= (1 - \pi)A_i \delta^\alpha k_i^{-\alpha} \\ \frac{k_f}{k_i} &= \frac{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}}}{\delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \end{aligned} \quad (12)$$

ii) Noting (K) the level of total capital available in the economy. We have:

$$K = k_f + k_i$$

Equations (12) allow us to obtain the share of formal capital (F_k) and informal capital (I_k) in the total capital of the economy (K):

$$F_k = \frac{k_f}{K} = \frac{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}}}{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (13)$$

$$I_k = \frac{k_i}{K} = \frac{\delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}}{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (14)$$

$$\frac{I_k}{F_k} = \frac{k_i}{k_f} = \frac{\delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}}{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}}}$$

$$\frac{F_k}{I_k} = \frac{k_f}{k_i} = \frac{(1 - \tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}}}{\delta(1 - \pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}}$$

Proof of proposition 2.2

- Net production levels (formal and informal) are deduced by using equation (13) and (14):

$$y_f^n = A_f^{\frac{1}{\alpha}} G^\alpha K^{1-\alpha} \frac{(1-\tau)^{\frac{1}{\alpha}}}{\left[(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}} \right]^{1-\alpha}} \quad (a)$$

$$y_i^n = A_i^{\frac{1}{\alpha}} G^\alpha K^{1-\alpha} \frac{\delta(1-\pi)^{\frac{1}{\alpha}}}{\left[(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}} \right]^{1-\alpha}} \quad (b)$$

- As well as production levels (formal and informal):

$$Y_f = A_f^{\frac{1}{\alpha}} G^\alpha K^{1-\alpha} \frac{(1-\tau)^{\frac{1-\alpha}{\alpha}}}{\left[(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}} \right]^{1-\alpha}} \quad (c)$$

$$Y_i = A_i^{\frac{1}{\alpha}} G^\alpha K^{1-\alpha} \frac{\delta(1-\pi)^{\frac{1-\alpha}{\alpha}}}{\left[(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}} \right]^{1-\alpha}} \quad (d)$$

- Equations (3), (7), (a), and (b) provide the expression of public expenditure (G) according to the parameters of the model:

$$G + P = \eta \tau Y_f + (1-\eta) \tau Y_f + \pi Y_i$$

$$\Leftrightarrow G = \tau Y_f + Y_i \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right]$$

$$G = K \frac{\left[A_f^{\frac{1}{\alpha}} \tau (1-\tau)^{\frac{1-\alpha}{\alpha}} + A_i^{\frac{1}{\alpha}} \delta (1-\pi)^{\frac{1-\alpha}{\alpha}} \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right] \right]^{\frac{1}{1-\alpha}}}{(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta (1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (e)$$

- Finally, (a), (b), (c) and (d) give:

$$Y_f = A_f^{\frac{1}{\alpha}} K \frac{(1-\tau)^{\frac{1-\alpha}{\alpha}} \left[A_f^{\frac{1}{\alpha}} \tau (1-\tau)^{\frac{1-\alpha}{\alpha}} + A_i^{\frac{1}{\alpha}} \delta (1-\pi)^{\frac{1-\alpha}{\alpha}} \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right] \right]^{\frac{\alpha}{1-\alpha}}}{(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta (1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (15)$$

$$Y_i = A_i^{\frac{1}{\alpha}} K \frac{\delta(1-\pi)^{\frac{1-\alpha}{\alpha}} \left[A_f^{\frac{1}{\alpha}} \tau(1-\tau)^{\frac{1-\alpha}{\alpha}} + A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right] \right]^{\frac{\alpha}{1-\alpha}}}{(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (16)$$

By noting (Y) the total production level, we have:

$$Y = Y_f + Y_i = K \cdot f(\tau, \pi, A_f, A_i, \delta, \lambda) \quad (17)$$

$$\text{With } f(\tau, \pi, A_f, A_i, \delta, \lambda) = \frac{\left[\delta(1-\pi)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}} + (1-\tau)^{\frac{1-\alpha}{\alpha}} A_f^{\frac{1}{\alpha}} \right] \left[A_f^{\frac{1}{\alpha}} \tau(1-\tau)^{\frac{1-\alpha}{\alpha}} + A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} \left[\pi - \left(\frac{\pi}{\lambda} \right)^{\frac{1}{\beta}} \right] \right]^{\frac{\alpha}{1-\alpha}}}{(1-\tau)^{\frac{1}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1}{\alpha}} A_i^{\frac{1}{\alpha}}}$$

- We can deduce the share of formal production (F_Y) and informal production (I_Y) in the total production (Y) :

$$F_Y = \frac{Y_f}{Y} = \frac{A_f^{\frac{1}{\alpha}} (1-\tau)^{\frac{1-\alpha}{\alpha}}}{(1-\tau)^{\frac{1-\alpha}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (18)$$

$$I_Y = \frac{Y_i}{Y} = \frac{A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}}}{(1-\tau)^{\frac{1-\alpha}{\alpha}} A_f^{\frac{1}{\alpha}} + \delta(1-\pi)^{\frac{1-\alpha}{\alpha}} A_i^{\frac{1}{\alpha}}} \quad (19)$$

$$\frac{I_Y}{F_Y} = \frac{Y_i}{Y_f} = \frac{A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}}}{A_f^{\frac{1}{\alpha}} (1-\tau)^{\frac{1-\alpha}{\alpha}}} \quad (20)$$

$$\frac{F_Y}{I_Y} = \frac{Y_f}{Y_i} = \frac{A_f^{\frac{1}{\alpha}} (1-\tau)^{\frac{1-\alpha}{\alpha}}}{A_i^{\frac{1}{\alpha}} \delta(1-\pi)^{\frac{1-\alpha}{\alpha}}} \quad (21)$$