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► **To cite this version:**

Ariane Lambert-Mogiliansky, Irénée Frérot. Deliberation Among Informed Citizens - The Value of Exploring Alternative Thinking Frames -. 2024. halshs-04725697v1

**HAL Id: halshs-04725697**

**<https://shs.hal.science/halshs-04725697v1>**

Preprint submitted on 8 Oct 2024 (v1), last revised 8 Nov 2024 (v2)

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**WORKING PAPER N° 2024-44**

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- The Value of Exploring Alternative Thinking Frames -**

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**JEL Codes: D71, D83, D91, C65**

**Keywords:**



# Deliberation Among Informed Citizens

- The Value of Exploring Alternative Thinking Frames -

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October 8, 2024

## Abstract

In this paper we investigate the potential of deliberation to create consensus among fully informed citizens. Our approach relies on a two cognitive assumptions i. citizens need a thinking frame (perspective) to consider an issue; ii. citizens cannot consider all relevant perspectives simultaneously only sequentially. Together this implies that citizens' opinions are intrinsically contextual i.e., they have quantum-like characteristics.

We capture contextuality in a simple quantum cognitive model. We find that in a binary voting problem, letting two citizens with alternative thinking frames and opposite voting intentions deliberate under the guidance of a benevolent facilitator allows reaching consensus. Opinion change occurs as the result of citizens' action in terms of "putting themselves in another citizen's shoes". The probability for reaching consensus depends on the correlation between perspectives and on their sophistication (dimensionality). Maximally uncorrelated sophisticated perspectives give the best chance for opinion change. With more than two citizens, multiple deliberation rounds with experts allow reaching consensus with significant probability.

A first central lesson is that with contextual opinions, the diversity of perspectives is beneficial and necessary to overcome initial disagreement. We also learn that well-design procedures managed by a facilitator are needed to increase the probability for consensus. An additional finding is that the richness of a thinking frame helps convergence toward a joint position. The optimal facilitator's strategy entails focusing deliberation on a properly reduced problem.

**JEL:** D71, D83, D91, C65

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<sup>†</sup>I want to thank François Dubois et Zeno Toffano for very useful discussions and suggestions in the early stages of the project.

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# 1 Introduction

Recently representative democracy has been questioned and is widely perceived as being in crisis in most developed countries. At the same time, more participative forms of democracy are gaining interest ([10]). They are most often framed as direct democracy (in restricted contexts) and based on sortition (in larger contexts) and they always rely on deliberations. "Deliberative democracy is now arguably the main theme in both democratic theory and the practice of democratic innovations" ([40]). Theories of collective decision-making are traditionally partitioned into two major fields: those dealing with issues related to the aggregation of diverse preferences (Social choice) and those dealing with citizen's active participation aiming at fostering reciprocal understanding and compromise and move toward consensus(see [38] for a review)<sup>1</sup>.

Social choice theory is repleted of impossibility theorems which have their foundations in the early work of Condorcet about voting cycles which was generalized by Arrow ([43]) fundamental results showing that every voting mechanism will fail to satisfy at least one among widely accepted conditions of fairness and rationality.<sup>2</sup> According to Jon Elster ([25]), social choice theory view the political process as instrumental and the decisive political act is a *private* action, an individual secret vote. The goal of politics is the optimal compromise between *given* and irreducibly opposed private interests. Social choice theories investigate the properties of aggregation rules and procedures. In contrast other views deny the private character of political behavior and its pure instrumental nature. According to Jurgen Habermas the goal of politics should be rational agreement and the decisive political act is that of engaging in public debate with the objective of reaching consensus. For participatory democrats from John Stuart Mills to Carole Pateman, the goal of politics is the transformation and education of participants so that politics is an end in itself ([25]). Data from Deliberative Polls support the hypothesis that people do change their opinion and this happens not only under the impact of better information ([37]; [29]). J. Dryzek writes that a "defining feature of deliberative democracy is that individuals participating in democratic processes are amenable to changing their minds and their preferences as a result of the reflection induced by deliberation." ([20])

Deliberative democratic theory is founded on the basic principle that "outcomes are democratically legitimate if and only if they could be the object of a free and reasoned agreement among equals" ([11]). The question is how does this process of presenting arguments leads to agreement among equals. Some scholars have argued that reasoned public deliberation lends legitimacy because the proposals that are sustained and survive through the process of deliberation are simply better in terms of their overall quality. This way of explaining the value of public deliberation and its connection to political justification presumes that there exist some procedure independent criteria of rightness or correctness. Many epis-

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<sup>1</sup>Deliberative institutional experimentation is flourishing throughout the world (a catalog is available at <https://participedia.net/>).

<sup>2</sup>When there are at least three alternatives and the domain of preferences is unrestricted, the following axioms are inconsistent: Pareto Efficiency, Independence of Irrelevant Alternatives, Nondictatorship, and Social Rationality.

temic democrats ([26]; [36]) hold this view. Other have proposed that post-deliberation outcomes are more justified than simple non-deliberative aggregated outcomes because the very procedure of reasoned public deliberation embodies or manifests core values of basic human morality and political justice, and it forces participants to be attentive toward the common good ([13]; [11]; [35]; [42]). Finally, a number of scholars have argued that reasoned public deliberation may also complement (or even nullify the need for) aggregative voting mechanisms: by generating unanimous agreement; by “inducing a shared understanding regarding the dimensions of conflict” ([34]); or by inducing “single peaked preferences” among the voters, which prevents majority rule from generating majority cycles ([21]; [37]). Bohman emphasizes both the transformative and epistemic benefits of confronting a diversity perspectives in deliberations.[5] Our approach is, in its spirit, close to that of Bohman’s. We view the process of deliberation as a procedure that invites citizens to explore alternative perspectives.

The central hypotheses of this paper is that i. to be able to consider an issue, people have to build a representation of that issue. Building a representation requires selecting a perspective or a thinking frame (a model), ii. there exist perspectives that people cannot consider simultaneously, they are incompatible in the mind. This has the crucial implication that no single perspective can aggregate all relevant information: opinions are contextual. In a close spirit Niemeyer et al. write “Deliberative reasoning as we characterize it, recognizes the possibility of identifying the set of relevant considerations, while falling short by failing actively to take all of them into account to capture the complete picture” ([40] p. 347). To focus on the evolution of opinions due to their contextuality, we consider deliberation exclusively as a process of confronting alternative models with no improvement in information. Kinder (2003) writes: “frames supply no new information. Rather, by offering a particular perspective, frames *organize - or better reorganize -* information that citizens already have in mind”[33]. Frames suggest how politics should be thought about, encouraging citizens to think in particular ways”.<sup>3</sup> To address the contextuality of opinions, we turn to the most widely recognized formal approach that features the co-existence of alternative representations of one and the same object: the Hilbert space model of Quantum Mechanics (QM).<sup>4</sup> Under the last decades, quantum-like models of contextuality have been developed in Social Sciences to explain a variety of behavioral anomalies.<sup>5</sup>(for overviews.) We briefly introduce the quantum cognition approach in section 1.2. We immediately reassure the reader that no prior knowledge of QM or Hilbert space is needed to read the present paper.

To fix idea consider a collective decision issue related to the introduction of Individual Carbon Budget (ICB). One perspective or thinking frame is the environmental one i.e., ICB properties to reduce green

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<sup>3</sup>This reminds of Aragonés et al. [1]. They establish that finding the best functional rule to explain a set of data is an NP-complete problem. Fully informed people may have different, mutually incompatible, predictions without it being possible to establish that some prediction is better than another. The truth of a prediction depends on the correlation that is being investigated i.e., it is “contextual”.

<sup>4</sup>In quantum Physics, the property values that define a system depend on how you measure them, the system is contextual to the measurement apparatus. There exists however of stable objective truth in the larger space that includes the system and all possible measurements which is formalized in the Hilbert space model of QM.

<sup>5</sup>See [8], [44], [41],

house gas (GHG) emissions . Another thinking frame relates to individual liberties i.e., ICB’s impact of individuals’ freedom of choice. People can very well consider those two perspectives in sequence. However, they may have difficulties to consider them simultaneously. As a consequence, they cannot aggregate information from the environmental and the liberty frames *in a stable way* instead a citizen’s opinion depends on the order in which the two (incompatible) perspectives are explored (probed). This instability is the expression of *intrinsic* contextuality i.e., an opinion does not exist independently of a frame (its context) and that it is impossible to integrate all frames into a unique ”super frame”.<sup>6</sup>

There exists, to the best of our knowledge, no formal approach that features the impact of the variety of thinking frames on the outcome of deliberations. However, recent theoretical and experimental works on quantum persuasion by Danilov et al.([14], [15]) can be relevant to the context of deliberation. In line with Danilov et al., we view a person’s opinion as a non-classical (quantum) object characterized by its state. Alternative thinking frames are modelled as alternative basis of the opinion space. The deliberative process is modelled as a structured sequence of measurements i.e., an ordered sequence of ”exploration” of different perspectives by the citizens. By force of the intrinsic contextuality of opinion, measurements move the opinion state in non-deterministic manner that reflects the correlations between the bases (the thinking frames). Given this technology, we investigate the properties of procedures that satisfy some requirements of deliberation put forward in the political philosophical literature. Deliberation should be a respectful, open-minded, fair and equitable communicative process which aims at achieving maximal consensus with respect to the decision at stake.

We consider a setting where there is a Yes or No decision to be made.<sup>7</sup> The objective of deliberation is to achieve democratic legitimacy in the following sens. First, the procedure should give a fair chance to everyone’s opinion to affect the decision. Second, full consensus is the overarching goal which translates in maximizing the support for the final decision. In this context, the paper addresses two central questions: 1. How can (fact-free i.e., without additional information) deliberation affect citizens’ voting behavior? 2. How should we structure deliberation to maximize the probability for consensus? A central assumption is that citizens are willing to explore alternative thinking frames before deciding how to vote. These alternatives frames can be provided by the citizens themselves and/or experts. The procedure is managed by a facilitator. We find that in the two-person case when starting from opposite voting intentions, fact-free deliberation procedure always achieves some extent of consensus. The largest chance of consensus obtains when the citizens’ perspective are maximally uncorrelated. <sup>8</sup> For the case the perspectives are two-dimensional consensus can be reached with probability 3/4. The results generalize to n-dimensional

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<sup>6</sup>Instead, there exists an irreducible multiplicity of alternative frames that are equally valid to characterize an issue. In physics this is a defining feature of sub-atomic particle i.e., some properties are Bohr complementary and cannot have a definite value simultaneously. In psychology and social sciences this is proposed as a defining feature of mental constructs like the representation of an issue.

<sup>7</sup>The voting issue is two dimensional implying that we are not addressing the impossibility theorems of collective choice (Arrow 1951).

<sup>8</sup>Perspective A and B are said to be distant (uncorrelated) whenever a citizen’s opinion in perspective A has no(little) predictive power for her opinion in perspective B.

perspectives where we show that the more fine-grained the opinion, the easier to reach consensus. A first central insight is that citizens' willingness to consider seriously an alternative perspective by "putting themselves in someone's shoes" can have a very significant impact on opinions. And secondly the more "distant" the considerations of the other citizen from the own perspective, the more powerful the impact. We next extend to a population of citizens and to multiple perspectives. An important finding is that to maximize the speed of convergence to consensus, the facilitator introduces some asymmetries between citizens and voting options while the role of experts becomes more prominent. In the Discussion section we address epistemic considerations and the performance of our model with respect to the ideal of deliberative democracy.

The main takeaways from our account of thinking frames in the analysis of fact-free deliberations are: i. The diversity of thinking frames among citizens not only is no obstacle but a necessary condition for deliberation to deliver consensus; ii. Deliberations can exhibit a powerful transformative power that hinges on a true willingness of participants to explore other thinking frames; iii. Well-designed procedures monitored by a facilitator are needed to move toward consensus.

This paper contributes to the literature on the value of pre-voting deliberation by providing a formalisation of opinion formation appealing to the intrinsic contextuality of opinions. Most formal approaches to deliberation belong to the epistemic tradition which postulate a single (common) correct decision. Among the most recent ones Dietrich and Spiekermann introduce some behavioral dimensions (sharing and absorbing) to the information theoretic approaches (see e.g., [22]). The other formal strand of literature is game-theoretic which emphasize incentive to share or withhold information (see e.g., [18]). The (quantum) contextuality revolution recasted the issue of objective truth and knowledge as witnessed by the wealth of the epistemological literature under the last century (see e.g., [17]). Our approach based on the most standard formalisation of intrinsic contextuality is closely related to Bohman's experiential perspective approach in [5]. He emphasizes the transformative and epistemic benefits of confronting the diversity perspectives in deliberations. We provide a formal mechanism for this transformation and derive procedures managed by a facilitator to increase consensus. Because our deliberation involves no improvement in information, it is complementary to classical epistemic approaches. Because intrinsic contextuality precludes the uniqueness of truth, it brings us close to procedural approaches to deliberation (see e.g., [2]) which recognize a value to deliberations in terms of actualizing basic values of democracy. Our results also contribute with new results with respect to the tension between efficiency and legitimacy within the process of deliberations itself([19]).

## 2 Contextuality

In this section, we illustrate with a story the kind of situation we have in mind and introduce the quantum cognitive approach.



Figure 1: What do you see?

## 2.1 A story

A community needs to decide whether or not to introduce an Individual Carbon Budget (ICB) scheme. To be able to relate to the issue, citizens build a mental representation using a thinking frame. Various aspects are of relevance to citizens, environmental efficacy, impact on individual liberties, legal feasibility etc... each corresponding to specific thinking frames. The citizens participating in the deliberations are open-minded. They are willing explore frames alternative to their own. An environmentalist discovers how she thinks about ICB in terms of individual liberties etc... As a result of exploration, the citizens discover that their initial opinion has evolved. They have not learned new facts but experiencing a new frame i.e., "putting themselves in the shoes of someone who thinks in different terms" modifies their state of mind. After some rounds of (guided) deliberation, they find that they agree to a significantly larger extent.

## 2.2 Quantum cognition

It is a common place that human beings are not capable of holding very complex pictures in mind. We consider reality focusing on one perspective (or thinking frame) at a time and show difficulties in combining perspectives in a stable way. This inability to seize reality in its full richness suggests that the process of developing an understanding of the world may not look like a puzzle that is assembled progressively. Instead, the human mind may exhibit structural "limitations" in terms of the incompatibility of perspectives in a way similar to properties in quantum mechanics. Ambiguous pictures of the kind provided in Fig. 1 provides a suggestive illustration of this phenomenon. You may see a duck or a rabbit (you may oscillate between the two as for the Necker cube) both are correct but you cannot see both simultaneously.



To many people it may appear artificial to turn to quantum mechanics (QM) when investigating human behavioral phenomena. However, the founders of QM, including Bohr and Heisenberg recognized an essential similarity between the two fields<sup>9</sup>. In both fields the object of investigation cannot (always) be separated from the process of investigation. QM and in particular its mathematical formalism was developed to respond to a general epistemological challenge: how can one study an object that is being modified by the measurement of its properties? QM is a general paradigm for intrinsic contextuality (i.e., non-separability between the object and the operation of investigation). It should, therefore, be viewed as truly legitimate to explore the value of the mathematical formalism of QM in the study of human behavioral phenomena - without reference to Physics.

The quantum paradigm has been proposed in decision theory and psychology to describe preferences, beliefs, attitudes, judgements and opinions (see [8], [44], [16], [41] among others). The approach has allowed providing a unified framework that accommodates a large variety of, so called, behavioral anomalies: preference reversal, disjunction effect, cognitive dissonance, framing effects, order effects etc. (see e.g. [32] and for empirical work [24],[9]).

In this paper, we recognize that our representation of the world is a mental object (a representation) which may exhibit non-classical features and we derive some implications of this hypothesis for the dynamics of opinion formation in the process of deliberation. The most important element that we borrow from QM is the notion of Bohr-complementarity applied to mental perspectives.<sup>10</sup> In line with QC, we propose that Bohr complementarity of perspectives captures the cognitive limitations that are responsible for our difficulties to synthesize information along different perspectives into a single stable picture. Just as in QM, the system (here our mental picture) makes discrete jumps when attempting to find a determinate value along distinct incompatible perspectives.

Recently theoretical and experimental applications to persuasion have shown how fruitful this approach could be ([14], [15],[9]). This paper is in continuation with those works. A citizen's opinion is a non-classical (quantum) object characterized by its state. Alternative thinking frames are modelled as alternative basis (incompatible properties) of the representation of the decision issue. Deliberation amounts to a sequence of measurements (probing arguments). Measurements moves the opinion state in a non-deterministic way that reflects the correlations between the bases (the thinking frames). With this formal description of the process of deliberation, we investigate procedures that satisfy some requirements on deliberation put forward in the literature.

We emphasize that the quantum cognition approach does not assume a quantum physical nature to the determinants of our opinions. Neither do we dwell into the psychology or neurology of the transformation of belief/opinion and preferences.<sup>11</sup> A presumption is that the correlations between perspectives which

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<sup>9</sup>In particular, Bohr was influenced by the psychology and philosophy of knowledge of Harald Höffding (see Bohr (1991) and the Introduction in Bitbol (2009) for an insightful discussion).

<sup>10</sup>It parallels the Bohr complementarity of properties for sub-atomic particles e.g., spin along different angles.

<sup>11</sup>The approach is an abstract way of capturing the fact that experiencing another perspective may have neurological, emotional and other impacts with consequences for opinion.

structure the mind exhibit some extent of regularity across individuals. As in Physics, the quantification of correlations is an empirical question.

## 3 Model

### 3.1 Basic structure

We shall formulate our model of deliberation in terms of a mediated communication game. There is a set  $\Omega$  of deliberator-citizens who may alternatively be in one of two roles R (Receiver) and S (Sender), one facilitator and a pool of experts. The interaction between the citizens and the facilitator is simplified by assumption 0 below. We are interested in deliberation aimed at influencing Receivers' vote over uncertain options which we model as quantum lotteries following Danilov et al. 2018 ([16]). The formal model shares significant features with the quantum persuasion model ([14], [15]).

**Assumption 1** *Citizens taking part in deliberation are willing to explore alternative thinking frames. They follow the recommendations of the facilitator.*

Citizens participating in deliberation are willing to engaged in real mental experiences with an open mind. This capture their respect for each other and their trust in the procedures as a legitimate way to reach a collective decision. The facilitator's recommendations relates exclusively to invitations to present an argument and invitation to probe a perspective (see below for definition). In this paper, we are not addressing possible incentive issues related to these operations. Instead, we consistently with the normative literature on deliberation assume goodwill from the side of citizens.

The description of a quantum system starts with the fixation of a Hilbert space  $H$  (over the field  $\mathbb{R}$  of real numbers or the field of complex number). Physicists usually work with the complex field  $\mathbb{C}$ . We, for simplicity, shall work with the real field  $\mathbb{R}$ , although everything goes with minor changes for the complex case. Let  $(\cdot, \cdot)$  denote the scalar product in Hilbert space  $H$  (in our case a finite dimensional space).

We shall be interested not so much in the Hilbert space as in operators, that is linear mappings  $A : H \rightarrow H$ . Such an operator  $A$  is Hermitian(or symmetric over  $\mathbb{R}$ ) if  $(Ax, y) = (x, Ay)$  for all  $x, y \in H$ . A Hermitian operator  $A$  is non-negative if  $(Ax, x) \geq 0$  for any  $x \in H$ .<sup>12</sup>

### 3.2 Opinion State and perspectives

Each individual  $j$  is characterized by her opinion state  $O^j$  and her thinking frame, operator  $P^j$  (perspective).

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<sup>12</sup>General Hermitian operators play the role of classical random variables. In fact for any Hermitian operator  $A$  and opinion  $O$ , we can define the 'expected value' of  $A$  in state  $O$  as  $\text{Tr}(AO)$ .The expected utility of voting action *yes* (to ICB) are presented by operator  $Y$  in an opinion state  $O$  is expressed as  $\text{Tr}(YO)$  and this number linearly depends on  $O$ . In this way, Receiver's preferences over voting options are determined by her opinion state  $O$ .

### *Opinion state*

With the help of the trace one can introduce the notion of state of a quantum system. The trace  $\text{Tr}$  of a matrix can be defined as the sum of its diagonal elements. It is known that the trace does not depend on the choice of basis. Let  $o$  (for opinion) be an element of  $H$  with length 1 (that is  $(o, o) = 1$ ). Let  $P_o$  be the orthogonal projector<sup>13</sup> on  $o$ , that is  $P_o(x) = (x, o)o$  for any  $x \in H$ ,  $\text{Tr}(P_o) = (o, o) = 1$ , therefore  $P_o$  is a state also denoted by operator  $O(= P_o)$ . Such states are called pure, we shall be exclusively dealing with pure states.<sup>14</sup> The non-negativity of the operator  $O$  is analogous to the non-negativity of a probability measure, and the trace 1 to the sum of probabilities which equals 1. This means that an opinion state is formally identical to a (subjective) belief state (cf. [15]).

### *Perspectives*

The formal account of thinking frames is a key building block of our theory. It is intimately linked with our cognitive assumptions:

#### **Assumption 2**

*i Citizens cannot address reality immediately. They need to build a representation of the voting issue using a thinking frame;*

*ii Citizens cannot resort to a "super frame" that aggregates all relevant aspects.*

Assumption 2 implies that, for the citizens, the voting issue admits a finite number  $k$  of equally valid (Bohr complementary) thinking frames, the *perspectives*. Formally, there is a set  $\mathcal{P}$  of perspectives (or thinking frames, we use the terms interchangeably). A perspective  $P = (P_1, \dots, P_n)$ ,  $P : H \rightarrow H$  is a Hermitian operator. We focus on a limited subset of operators referred to as direct (or von Neumann) measurements. These simple devices are sufficient for the purpose of the present paper. Such a device is given by a family of projectors  $(P_v, v \in V, V$  is the set of values of the device.) with the property  $\sum P_v = E$ , where  $E$  is the identity operator on  $H$ . The probability  $p_v$  to obtain outcome  $v$  (in a state  $o \in St$ ) is equal to  $\text{Tr}(P_v O)$  and if we obtained  $v$ , the revised opinion-state is  $O_v = P_v$  (because projector  $P_v$  is one-dimensional (that is a pure state)). If we repeat the measurement, we obtain the same outcome  $v$  and the state does not change.<sup>15</sup> An important feature that we want to emphasize is that the expected revised opinion  $O_{ex} = \sum_{v \in V} P_v O_v = \sum_{v \in V} P_v O P_v$  is generally different from the initial opinion-state  $O$ . That is, although the opinion state has the structure of a probability distribution, the revised opinion in the non-classical context is *not* subject to Bayesian plausibility as noted in [15]. This feature plays an important role in the analysis.

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<sup>13</sup>A projector is an Hermitian operator such that  $P^2 = P$

<sup>14</sup>This is by distinction with mixed states. Pure state are complete information state but because of intrinsic indeterminacy, the values along alternative (incompatible) perspectives can never be sorted out simultaneously, they remain stochastic.

<sup>15</sup>This type of measurement is repeatable or "first kind".

### *Probing a perspective*

Probing a perspective, is formalized as an operation whereby one applies some perspective  $P$  to an opinion state  $O$ . In practical terms, it corresponds to questioning oneself in the terms of perspective  $P$  i.e., which specific value do I agree with? We focus exclusively on *complete measurements* that is probing operations that fully resolves uncertainty with respect to the  $P$  perspective. The outcome of probing a perspective is one of its eigenvalues.

Assume citizen  $j$  is characterized by  $O$  and  $P^j$  such that  $O = P_i^j$ , probing an argument in an *alternative* (i.e., incompatible) perspective  $Q$  with arguments  $Q_i$  is called *challenging one's opinion*. Challenging an opinion with  $Q$  generates a new state  $O' \in \{Q_1, \dots, Q_n\}$  and, as a consequence, it upsets her previously held value  $v_i$  in  $P^j$ .<sup>16</sup> For instance the individual is confronted with the liberty perspective and reflects over whether she thinks that ICB is contrary to fundamental individual liberties  $Q_1$  or implies acceptable limitations  $Q_2$  or has no implication for individual liberty  $Q_3$ . After the process of probing, the revised opinion is either  $Q_1, Q_2$  or  $Q_3$ . A central feature of the model is that if the citizen started from  $O = P_2^j$  so she thinks/believes ICB are environmentally useless, probing argument  $Q$  and obtaining for instance  $i_Q = 1$  changes her previously held opinion into  $Q_1$ . So if the citizen probes  $P$  anew  $\text{Tr}(Q_1 P_i) > 0$  for  $P_i \neq 2$ , the citizen has changed her mind from initial  $P_2$  and now believes (with positive probability) that ICB are environmentally valuable  $P_1$ . This is the crucial property that generates a potential for opinions to evolve without additional information. This is illustrated in Fig. 2 in the two-dimensional case. With initial opinion state given by  $P_1$ , our citizen probes the liberty perspective  $Q$  and with probability  $\text{Tr}(P_1 Q_1)$  she finds that in that perspective her opinion is  $Q_1$  (and with the complementary probability  $Q_2$ ). She then update her opinion (to be able to evaluate the voting options see below) and with probability  $\text{Tr}(Q_1 P_2)$  she does not recover her initial opinion instead she now holds opinion  $P_2$ .

Throughout, we use the term *probing* for a measurement of an *alternative* perspective to the citizen's own perspective. While *updating* refers to the measurement operation of the *own* perspective.

### 3.3 Utility and voting

Voting is binary, a Yes or No choice. Citizens are endowed with preferences that allow evaluating the utility value of the two options given their individual opinion state. .

**Assumption 3** *Citizen  $j \in \Omega$  exclusively attributes utility to voting options in one of the eigenstates of her own frame i.e. in some state  $O^j \in \{P_i^j\}$ , where  $P^j$  is her own perspective.*

Assumption 3 captures a central feature of thinking in frames. A citizen is only capable to evaluate her utility in terms of her own perspective. Citizens can explore other perspectives and adopt any

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<sup>16</sup>In quantum mechanics, it is postulated that the state (of a measured system) changes in accordance with the von Neumann–Lüders postulate. More precisely, a system that was in state  $O$  transits to the state  $O' = POP/\text{Tr}(POP)$  as a result of performing a measurement that yields event  $P$ . In our simplified setting, we say that the opinion state transits into (pure state)  $P$ .

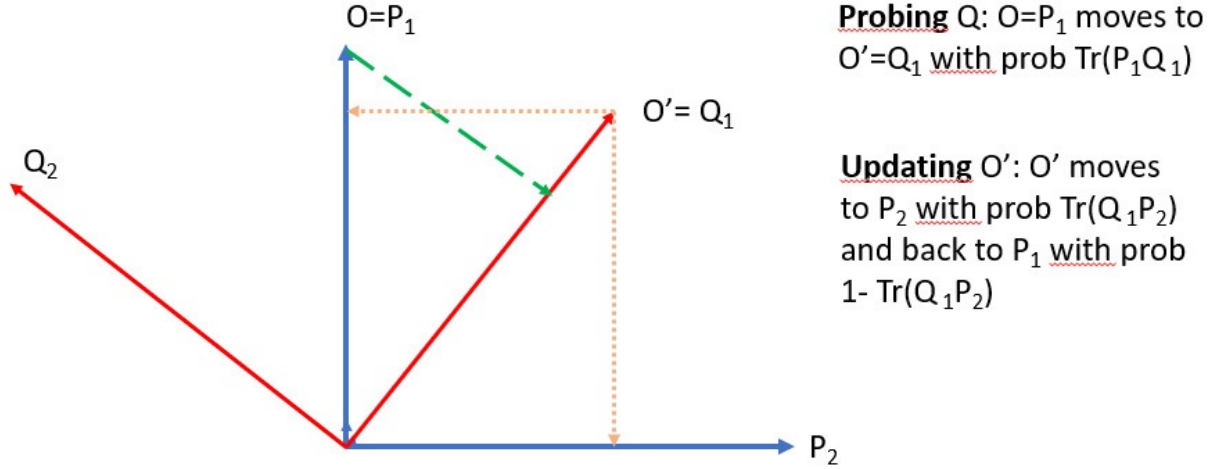


Figure 2: Probing perspectives

possible opinion. However, an opinion state can guide voting *only* when formulated in terms of the own perspective. The frame is an essential part of the citizen's identity, it is the language in which she can formulate the value of the voting options and make decisions.

Formally, consider a citizen with perspective  $\mathcal{P} = (P_1, \dots, P_n)$ . Her utility "function" is represented by a tuple  $\{u_1^Y, \dots, u_n^Y, u_1^N, \dots, u_n^N\}$  which associates a real number to option Yes respectively No to each possible opinion state. Generally, for any arbitrary opinion state  $O$ , we can formulate the expected utility for the two voting options as a "quantum lottery" (see [16]), with the probability for each of the states  $P_i$  given by  $\text{Tr}(OP_i)$ . The general formula for the expected utility is then

$$\text{EU}(Y; O) = \sum_i \text{Tr}(OP_i) u_i^Y \quad (1)$$

and similarly  $\text{EU}(N; O) = \sum_i \text{Tr}(OP_i) u_i^N$ . When  $\text{EU}(Y; O) \geq \text{EU}(N; O)$ , the citizen prefers the Yes option. Note that in order for the expression in Eq. (1) to guide voting behavior the citizens must be able to compute  $\text{tr}(OP_i)$  for any  $O$ . This is very demanding. In particular, it requires that citizens have knowledge about the correlations between all alternative perspectives and their own. We do not make that assumption, instead whenever in  $O \neq P_i$ , she will have to "go back" to her own thinking frame before voting (see below).

Citizens' utility for the voting options defines two complementary subsets:

$$\begin{aligned} \mathcal{Y} &\equiv \{P; \text{EU}(Y; P) \geq \text{EU}(N; P)\} \\ \mathcal{N} &\equiv \{P; \text{EU}(Y; P) \leq \text{EU}(N; P)\}. \end{aligned}$$

**Assumption 4** A citizen is endowed with a thinking frame that allows separating between voting options, namely  $\mathcal{Y}$  and  $\mathcal{N}$  are nonempty.

Assumption 4 excludes citizens whose voting decision is fixed and therefore cannot be affected by deliberation. For any citizen  $j$ , the own thinking frame  $P^j$ , allows separating among voting alternatives.

The voting behavior is most simple. We know by Assumption 3 that citizen  $j$  must be one of her perspective's eigenstates say  $P^j$  then if  $u(Y; P_i^j) \geq u(N; P_i^j)$ , citizen  $j$  cast a Yes vote otherwise she casts a No vote, i.e., voting is sincere (non-strategic). The winning option is the one that has obtained the largest number of votes.

### 3.4 Deliberation

Deliberation is modelled in terms of a multiple round mediated communication game. The facilitator is the sole true player in this game. By Assumption 1 the citizens follow the facilitator's recommendations. In each round, he chooses an argument  $P_i^k$  that will be exposed by citizen  $k$  and a selection of citizens (defined as being *active*) whom he invites to probe the corresponding perspective  $P^k$ . All other citizens remain "passive", they only listen. When hearing an argument, an active citizen not only decides for herself whether or not she agree with the specific argument  $P_i^k$ , she explores  $P^k$  by thinking in its terms so as to determine which argument she agrees with. Formally, the probing operation is a complete measurement resulting in a complete information (pure) state i.e. one of the eigenstates of  $P^k$ .<sup>17</sup> After challenging her opinion by probing  $P^k$ , the citizen "updates" her opinion by reassessing her position in her own perspective where she can evaluate the utility value of voting options.

#### *The facilitator strategy*

In each round the facilitator chooses two things: a perspective  $P \in \mathcal{P}$  ( $\mathcal{P}$  is the set of all perspectives including the experts') and  $\omega \subset \Omega$ , where  $\Omega$  is the set of all citizens, who are called to be active. The facilitator's objective is to maximize the support for the most supported voting option. Let  $\Omega^{Y^t}$  and  $\Omega^{N^t}$  be the subsets of citizens that support the Yes respectively the No vote in period  $t$ , with  $|\Omega^{Y^t}| (|\Omega^{N^t}|) = y^t (n^t)$  the number of Yes(No) supporters. Deliberation is a finite period game aimed at maximizing the probability for consensus among citizens before period  $T, T \geq 1$ . While consensus in larger groups is difficult to reach, it can be approached. We operationalize this with a "score function"  $s^t(\cdot) = \max\{y^t, n^t\}$ ,  $s^t(\cdot)$  is simply the score of the most supported option in period  $t$ . Next, in a multi-period game a strategy must generally describe what to do in each period  $t \leq T - 1$  after each possible history. A first crucial remark is that the vector of opinions  $\mathbf{O}^t = (P^{1t}, \dots, P^{Nt})$  captures all relevant information about history at time  $t$ . This follows from the fact that the probability for a citizen's opinion change,  $\text{Tr}(OP_i)$ , only depends on the current state of opinion - not on history. Next, we shall restrict attention to strategies that maximize the expected score *in each period*. This is not fully without loss of generality as we discuss in Section 4.3. Hence, a strategy is defined as a function from a vector of opinions  $\mathbf{O}^t$  to a pair  $(P, \omega)$  including a perspective  $P$  and a set of active citizens  $\omega : \mathbf{O}^t \rightarrow \mathcal{P} \times (2^N - 1)$ .

<sup>17</sup>This is more demanding than simply deciding to agree or reject the presented argument. Indeed if the argument is rejected, the revised state will not be an eigenstate of the perspective under exploration but a superposition of states.

The facilitator objective function in each period  $t \leq T - 1$

$$\max_{P \in \mathcal{P}, \omega \subset \Omega} Es^t(P, \omega; \mathbf{O}^{t-1}) = \max_{P \in \mathcal{P}, \omega \subset \Omega} \{\max\{Ey^t, En^t\}\} \quad (2)$$

where  $\mathbf{O}^{t-1}$  is the vector of opinions inherited from the previous period and  $Ey^t, En^t$  is the expected number of citizen who support the Yes respectively No vote at the end of the  $t$  round. We note that the formulation in Eq. (2) implies that facilitator has no preference over voting options. The facilitator's indifference is a non-innocuous assumption. We leave issues related to safeguards against possible manipulation for further research.

We make the following informational assumption:

**Assumption 5** *The deliberation facilitator has knowledge about all perspective and the correlations between them.*

The facilitator has an informational advantage compared to citizens in terms of knowing the full structure of the opinion state space. He is aware of all possible perspectives and how they correlate to each other. This is the critical resource that allows to him to optimize the deliberation process. We also assume that he has access to a pool of experts who can present arguments from any possible perspective  $P^e$ . Experts are simply "tools" that the facilitator can call upon whenever he wants. This implies that the facilitator is not constrained by the perspectives represented in the population of citizens. However, since the spirit of deliberation is (also) to give voice to citizens, we shall give particular attention to what can be achieve without appealing to experts.

## 4 Analysis

For the ease of presentation we start the analysis with deliberation between two citizens. This allows to derive basic results before moving to the case when deliberation involves larger groups of citizens.

### 4.1 2 citizens

We have 2 citizens Alice and Bob who face a binary collective decision about e.g., the introduction of a Individual Carbon Budget scheme. There exist two relevant aspects: environmental efficacy and individual liberty. These 2 aspects cannot be considered simultaneously by our citizens, they are assumed incompatible in the mind.

The perspectives are  $n$ -dimensional, the larger  $n$ , the finer the characterization of the opinions. In term of our lead example, the environmental perspective can have several possible values (each associated with its own eigenstate) e.g., ICB is the best solution ot reducing GHG (greenhouse gas), ICB is one among the better solutions to reduce GHG, ICB is a good solution etc.. until ICB is worthless to reduce GHG. So expanding the dimensionality corresponds to allowing for "finer" opinions.

The citizens are endowed with

- i. An opinion state  $O^{A(B)}$ ;
- ii. An own  $n$ -dimensional perspective. Denote  $A$  the operator representing Alice's perspective with eigenstates  $(A_1, \dots, A_n)$  and by  $B$  the Hermitian operator representing Bob's perspective with eigenstates  $(B_1, \dots, B_n)$  ;
- iii. A set of utility values associated with the own perspective's eigenstates:  $\{u^{A_1,Y}, \dots, u^{A_n,Y}, u^{A_1,N}, \dots, u^{A_n,N}\}$  for Alice, with  $u^{A_i,Y}$  the utility value for Alice corresponding to the Yes vote in opinion state  $A_i$ , similarly for  $u^{A_i,N}$  corresponding to the No vote. We define similarly  $u^{B_j,Y}, u^{B_j,N}, j = 1, \dots, n$  for Bob.

*Deliberation protocol.* – Before Bob and Alice start deliberating, they update their own opinion namely they probe their own perspective.<sup>18</sup> The initial opinion states are therefore always eigenstates of the own perspective: some  $A_i$  for Alice and some  $B_j$  for Bob. and similarly for Bob. We consider a process where the facilitator does not appeal to experts in the first 2 rounds (we call it the "voice phase").

*Timing*

$t = 0$  The facilitator asks for initial voting intentions. And if Alice and Bob agree, the procedure requires no deliberation, they vote; If they disagree the first round starts:

*Round 1*

- $t = 1$  A random draw determines who will present his/her argument first, say Alice;
- $t = 2$  Alice's argues for Bob in her terms (frame) and Bob responds by probing Alice's perspective;
- $t = 3$  Bob updates his opinion by probing his own perspective. If he now agrees with Alice, they vote.

If disagreement remains, the second round starts:

*Round 2*

- $t = 4$  Bob exposes his opinion in his own frame and Alice responds by probing Bob's perspective;
- $t = 5$  Alice updates her opinion by probing her own perspective.
- $t = 6$  Out of the resulting opinion states if they agree, they vote. If they do not the process 1-3 is repeated - possibly appealing to experts.
- $t = 8$  if disagreement persists at  $t = T$ , the decision is determined by a random device (to break the tie).

#### 4.1.1 The two-dimensional case ( $n = 2$ )

For the ease of presentation, we start with the case where the citizens' perspectives are 2-dimensional which allows establishing some results which we later extend. Since our facilitated communication game proceeds by (quantum) updating, and the facilitator maximizes the score in each round, we adopt a forward inductive approach.

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<sup>18</sup>This is most natural since they expect to be invited to expose their opinion



Alice's and Bob's respective opinion states are independent so the global system is characterized by the following 4 possible (combined) opinion states  $\{(A_1B_1), (A_1, B_2), (A_2, B_1), (A_2, B_2)\}$ .<sup>19</sup> By Assumption 4 and since Alice and Bob only have two possible opinions, we may, without loss of generality, define  $A_1, B_1$  as the opinion states leading to a Yes vote; and  $A_2, B_2$  as the opinion states leading to a No vote. The voting agreement is then entirely defined by the fact that either Alice and Bob have respective opinion states  $A_1, B_1$ , or they have respective opinion states  $A_2, B_2$ . In both cases the facilitator's score is maximal and equal to 2. When the vector of opinion states  $\mathbf{O} \notin \{(A_1B_1), (A_2, B_2)\}$ , to maximize the score, the facilitator needs one of the citizens to change his or her opinion. Maximizing the expected score is equivalent to maximizing the probability that this happens. In the two-dimensional case, the probability for Alice or Bob to change opinion when probing the other's perspective is entirely governed by a single parameter:  $x = \text{Tr}(A_1B_1)$ . Indeed, as  $A_1 + A_2 = E = B_1 + B_2$ , with  $E$  the identity operator, we have  $\text{Tr}(A_2B_1) = \text{Tr}(A_1B_2) = 1 - x$ , and  $\text{Tr}(A_2B_2) = x$ .

We consider deliberation starting from initial disagreement, say Alice and Bob have respective opinion states  $A_1, B_2$ . From the facilitator point of view the 2 two consensus states are fully symmetric. Let the initial random draw give Alice the initiative. The facilitator invites her to present her argument. Bob is then invited by the facilitator to probe Alice's perspective, and his opinion state changes to  $A_1$  with probability  $\text{Tr}(A_1B_2) = x$ , and to  $A_2$  with probability  $\text{Tr}(A_2B_2) = 1 - x$ . Bob then updates his opinion by probing his own perspective. The probability for reaching consensus is the probability that Bob's final state is now  $B_1$ , instead of his initial state  $B_2$ . This state is reached with probability:

$$\text{prob}(B_2 \rightarrow B_1) = \text{Tr}(B_2A_1)\text{Tr}(A_1B_1) + \text{Tr}(B_2A_2)\text{Tr}(A_2B_1) \quad (3)$$

$$= 2x(1 - x) \quad (4)$$

Since only Bob's opinion has been challenged, the other consensual state (namely,  $A_2$  for Alice and  $B_2$  for Bob) cannot have emerged.

*Maximally-uncorrelated perspectives.*— Generally,  $A$  and  $B$  perspectives are said to be maximally uncorrelated when  $\text{Tr}(B_iA_j) = 1/n$  for all  $i, j$  (with  $n = 2$  in the present case,  $n$  is the dimensionality of the perspective), probing the  $\mathcal{A}$  perspective gives Bob equal chance to move into any of the states  $A_i$  (here, Bob reaches  $A_1$  or  $A_2$  with probability  $1/2$ ). Updating then his opinion by probing the  $\mathcal{B}$  perspective, Bob reaches any of the opinion states  $B_j$  with equal probability. Effectively, Bob's initial opinion state has been completely randomized by the (intermediate) probing of Alice's completely uncorrelated perspective  $\mathcal{A}$ . In such a case, starting from initial disagreement, the chance for reaching consensus after one round is equal to  $1/2$ .

The second round following disagreement proceeds similarly, generating a probability for consensus in states  $A_2, B_2$  with the same probability  $2x(1 - x)$ . Hence after the two rounds the probability for reaching consensus is  $2x(1 - x) + (1 - 2x(1 - x))2x(1 - x) = 4x(1 - x)[1 - x(1 - x)]$ .

<sup>19</sup>Formally, we are dealing with tensor products so  $A_1B_1 = A \otimes B \in H^4$

We have the following

**Proposition 1** *Starting from disagreement on vote between two citizens,*

- i. fact-free deliberation with fully correlated perspectives has no impact at all;*
- ii. with distinct perspectives consensus is reached with strictly positive probability after a first round;*
- iii. The probability for consensus is largest when the perspectives are uncorrelated, it reaches 3/4 after two rounds.*

**Proof.** i. When the two perspectives are fully correlated, we have  $\text{Tr}(A_i B_j) \in \{0, 1\}$ : the opinion state are either equal ( $A_1 = B_1$  and  $A_2 = B_2$ ) or orthogonal ( $A_1 = B_2$  and  $A_2 = B_1$ ): Bob's and Alice's perspectives are formally indistinguishable. In this case, no transition  $B_1 \rightarrow A_i \rightarrow B_2$  or  $B_2 \rightarrow A_i \rightarrow B_1$  can ever occur by probing the intermediate  $\mathcal{A}$  perspective: letting Bob probe Alice's perspective has no impact whatsoever on Bob's opinion state ( $2x(1-x) = 0$ ). Initial disagreement cannot be overcome through deliberation.

ii. First note that from the point of view of the facilitator, the 2 two consensus states are fully symmetric  $\text{prob}(B_2 \rightarrow B_1) = \text{prob}(A_1 \rightarrow A_2) = 2x(1-x)$  so a random draw is optimal for the facilitator. The result in 1.ii. follows from Eq. (3), which shows that the probability to reach consensus is strictly positive whenever  $0 < x < 1$ , namely when perspectives  $\mathcal{A}$  and  $\mathcal{B}$  are distinct.

iii. The probability for consensus in the first round is maximal at  $x = .5$  where  $\frac{\partial}{\partial x} 2(1-x)x = 0, x \in [0, 1]$ . The total probability for success after the second round, conditional on failure in the first round, is  $4x(1-x)[1-2x(1-x)]$  which reaches its maximum for uncorrelated perspective as well i.e.,  $x = 1/2$ . For two rounds the maximum chance for consensus is 3/4. ■

Result 1.i. is quite remarkable because it shows that sharing the same thinking frame is an obstacle to achieving consensus when starting from disagreement. Indeed, within a common thinking frame, citizens can only update their opinion in response to new information (by Bayesian updating) which we preclude in this paper. However, when citizens are endowed with alternative perspectives new opportunities for opinion to evolve arise. By actively exploring a perspective incompatible with ones' own, intrinsic contextuality reveals its transformative power. Exploring an alternative perspective changes the opinion state because the possible outcomes of that operation do not exist in the own perspective. The opinion state is forced into a new state. This result about the value of diversity is truly novel and a main contribution of this paper. It is important at this point to recall 1, the transformative value of deliberation demands a true mental experience i.e., sincerely putting oneself in someone else shoes - the probing operation.

Result 1.ii quantifies how the diversity of perspectives allows opinions to evolve toward consensus. The weaker the correlation between perspectives  $x \rightarrow .5$ , the more impactful the probing operation. Starting from disagreement  $A_1 B_2$ , the probability that Alice changes her opinion from  $A_1$  to  $A_2$  is given by  $\text{Tr}(A_1 B_1) \text{Tr}(B_1 A_2) + \text{Tr}(A_1 B_2) \text{Tr}(B_2 A_2) = 2x(1-x)$  which tends to zero as  $x$  tends to 1 (or 0) and it is maximized at  $x = .5$ . The intuition is that the more closely related the perspectives the more likely that probing Bob's frame takes Alice to the closest opinion state in Bob's frame and when probing her own

perspective anew she is most likely confirmed in her initial opinion. Similarly for Bob, so disagreement is more likely to persist. Nevertheless with some positive probability at least one of the two citizens will end up having changed her/his mind which implies consensus on voting. Interestingly, the result that uncorrelated perspectives give the best chance for deliberation to achieve consensus, reminds of a result in quantum persuasion ([15]). They show that distraction understood as bringing attention to a perspective uncorrelated to the targeted state is an efficient way to persuade Receiver.

Result iii. says, without surprise, that starting from dissensus additional rounds following failure to reach agreement increase the probability for consensus. While a single round can already achieve consensus with probability  $2(1-x)x$ , with two rounds and uncorrelated perspective case ( $x = .5$ ), we reach consensus in 75% of the case. Of course we do not expect citizens to repeat the same argument from round to round if citizens are short of arguments that can be the time for experts with suitable perspectives to be called in.

**Corollary 1** *The first moving citizen has larger chance to see consensus on her initial voting preferences than the one who moves second.*

**Proof.** As earlier noted that the chance is the same whoever is selected:  $\text{prob}(A_1B_1; A_1B_2) = 2x(1-x) = \text{prob}(A_2B_2; A_1B_2)$ . But since the second is only probed in case of failure in the first round, the other consensual state has less chance to be selected in the vote. ■

The procedure gives more chance to the first selected citizen. The introduction of a random draw restores the equality of chance between citizens.

**Corollary 2** *When citizens's perspectives are correlated, relying on experts with perspective uncorrelated to the current round's active citizen increases the chance for reaching consensus in any given round.*

This follows from result ii. When Alice and Bob have correlated perspectives, the probability for reaching consensus when probing each other perspectives is lower than 75% after two rounds. The facilitator could choose instead the following strategy. First, a random draw designates the active citizen. The procedure then proceeds as above except that only experts are presenting arguments belonging to a perspective maximally uncorrelated with the active citizen's.

Corollary 2 implies that there exists a tension between letting citizen expose and probe each other's argument and the objective to maximize the score. This is not surprising given result 1.ii. In order to preserve the democratic character of deliberation so it gives voice to citizens, a mixture of citizen arguments and expert arguments can be chosen at the cost of delaying consensus however.

#### 4.1.2 Deliberations $n > 2$

We now consider the case when the perspectives have more than two dimensions that is we have a finer characterization of the opinions. We remind that probing a perspective corresponds to performing a

complete measurement i.e., asking oneself which of the several possible (orthogonal) opinions you agree with. The outcome of a probe is therefore always one of the possible eigenstates of the probed perspective. Our main interest is for the facilitator's choice of perspective to maximize the chance of consensus in such a context. By Assumption 4, citizens' utility for the voting options divides the  $n$ -dimensional state space into two subspaces. The Yes subspace and the No subspace:

$$\begin{aligned} Y^j &\equiv \{P_k; EU^j(Y; P_k) \geq EU^j(N; P_k)\} \\ N^j &\equiv \{P_l; EU^j(N; P_l) > EU^j(Y; P_l)\}, j = A, B, k, l \in \{1, \dots, n\} \end{aligned}$$

In contrast with the 2 dimensional case each citizen has a multiplicity of opinion eigenstates which are consistent with the same voting option.

Consider a situation where Alice and Bob initially disagree on the voting, say Bob votes Yes and Alice votes No. Assume that the random draw determines consensus is first sought on Bob's position. So the first round of deliberation is aimed at modifying Alice's voting intention from No to Yes. Alice's opinion states  $\mathcal{A} = (A_1, \dots, A_n)$  are such that all  $A_i$  are all pairwise orthogonal projectors in  $\mathbb{H}^n$  (that is:  $A_i A_j = 0$  for all  $i, j = 1, \dots, n$ ). We define the subspace of Alice's opinion states leading to a Yes vote as  $Y^A = (A_1, \dots, A_k)$  and similarly for opinion states leading to a No vote as  $N^A = (A_{k+1}, \dots, A_n)$ . Without loss of generality, we assume that Alice's initial opinion state is  $A_{k+1}$ . Hence, the aim of the facilitator is to lead Alice to change her opinion state from  $A_{k+1}$  (corresponding to an initial Yes vote), to any of the states  $A_i$  with  $i = 1, \dots, k$  (corresponding to a targeted No vote, in agreement with Bob's vote).

We shall characterize the optimal strategy for the facilitator in terms of the perspective that he invites Alice to probe. Typically, that will not be Bob's perspective but a one that can be deployed by an expert.<sup>20</sup> Because the expert's perspective is distinct (and incompatible) from Bob's, Alice is the only active citizen. Bob listens to the expert's argument but remains passive (see Lemma 1 below). The general strategy of the facilitator is to propose an (expert) perspective  $\mathcal{C} = (C_1, \dots, C_n)$ , incompatible with Alice's perspective (namely,  $[A_i, C_j] \neq 0$  for some of the opinion states  $A_i$  and  $C_j$ ). As in the  $n = 2$  case, there is a first transition  $A_{k+1} \rightarrow C_j$  with probability  $\text{Tr}(A_{k+1} C_j)$ ; then Alice updates her opinion leading to a second transition  $C_j \rightarrow A_i$  with probability  $\text{Tr}(C_j A_i)$ . The probability for  $A_{k+1} \rightarrow A_i$  is given by:

$$\text{prob}[A_{k+1} \rightarrow A_i] = \sum_{j=1}^n \text{Tr}(A_{k+1} C_j) \text{Tr}(C_j A_i) \quad (5)$$

where the sum is over the intermediate opinion states  $C_j$  in the perspective  $\mathcal{C}$  proposed by (the expert selected by) the facilitator.

We now investigate the optimal choice of perspective  $\mathcal{C}$  in order to maximize an opinion change  $A_{k+1} \rightarrow A_i$  with  $i = 1, \dots, k$  (namely, a change from an initial No vote to a target Yes vote). We

<sup>20</sup>We could see this exercise as following two unsuccessful rounds where the citizens probe each other's perspective. This is in order to preserve the ideal that citizens' are invited to present their own argument to each other.

below show that the facilitator can achieve such a change with probability  $p_{\text{success}} = \frac{k}{k+1}$ . This is obtained by choosing a perspective  $\mathcal{C} = (C_1, \dots, C_{k+1}, A_{k+2}, \dots, A_n)$ , with uniform transition probabilities  $\text{Tr}(A_{k+1}C_j) = \frac{1}{k+1}$  for all  $j = 1, \dots, k+1$  (see Appendix A for an illustration). In this case, we have:

$$\text{prob}[A_{k+1} \rightarrow A_i] = \sum_{j=1}^{k+1} \text{Tr}(A_{k+1}C_j)\text{Tr}(C_jA_i) + \sum_{j=k+2}^n \text{Tr}(A_{k+1}A_j)\text{Tr}(A_jA_i) \quad (6)$$

where the second sum vanishes thanks to the orthogonality of  $A_i$  projectors:  $\text{Tr}(A_{k+1}A_j) = 0$  for all  $j = k+2, \dots, n$ . Furthermore, we notice that the transitions  $A_{k+1} \rightarrow A_i$  with  $i = k+2, \dots, n$  are impossible, because in the first sum  $\text{Tr}(C_jA_i) = 0$  for  $j = 1, \dots, k+1$  and  $i = k+2, \dots, n$  (this property is a consequence of the context  $\mathcal{C}$  being composed of pairwise orthogonal projectors). Only transitions  $A_{k+1} \rightarrow A_i$  with  $i = 1, \dots, k+1$  are possible. Hence, Alice maintains her initial voting intention only when moving back to  $A_{k+1}$  which happens with probability:

$$\begin{aligned} \text{prob}[A_{k+1} \rightarrow A_{k+1}] &= \sum_{j=1}^{k+1} \text{Tr}(A_{k+1}C_j)\text{Tr}(C_jA_{k+1}) \\ &= \sum_{j=1}^{k+1} \left(\frac{1}{k+1}\right)^2 = \frac{1}{k+1} \end{aligned} \quad (7)$$

In this case, the facilitator fails to reach an agreement between Alice and Bob. In all other cases, Alice's new opinion state  $A_i$  with  $i = 1, \dots, k$  leads to a Yes vote, hence consensus is achieved, and this happens with probability  $p_{\text{success}} = 1 - 1/(k+1) = k/(k+1)$ .

Proposition 2 below collects these insights

**Proposition 2** *When citizens' perspective are  $n$ -dimensional and the dimensionality of the subspace of the projected consensus for the active citizen is  $k$ ,*

- i. consensus can be reached a success probability equal  $p_{\text{success}} = k/(k+1)$ ;*
- ii. the perspective  $\mathcal{C}$  that delivers  $p_{\text{success}} = k/(k+1)$  is of the form  $\mathcal{C} = (C_1, \dots, C_{k+1}, A_{k+2}, \dots, A_n)$ , with  $\text{Tr}(A_{k+1}C_j) = 1/(k+1)$  for all  $j = 1, \dots, k+1$ .*
- iii.  $\mathcal{C}$  is optimal within a natural class of perspectives.*

**Proof.** see Appendix B ■

Proposition 2.i and 2.ii establish that our results from Proposition 1 generalize to multidimensional perspectives. Indeed letting  $k = 1$ , we recover our previous result namely that  $x = \text{Tr}(A_{k+1}C_j) = 1/(k+1) = 1/2$  that is uniformly uncorrelated perspectives maximize the chance of consensus. We also find that the multiplicity of dimensions increases the chance of consensus when properly addressed. The probability of success is proportional to the dimensionality of the subspace associated with the projected consensus vote,  $k/(k+1)$ . So it is easier for people to reach agreement the finer their initial (own) perspective and the larger the set of opinion states consistent with the projected consensus vote (here Yes).

A new result in 2.ii is that the optimal perspective is characterized by effectively reducing the dimensionality of the problem. This is done by selecting  $C$  such that  $C_{k+1}$  to  $C_n$  are identical to  $A_{k+1}$  to  $A_n$  implying that there is no chance that deliberation results in a new opinion state that yields a No vote except for moving back to the initial state  $A_{k+1}$ . We can say that the dimensionality (in the probing operations) is effectively reduced from  $n$  to  $n - k$ . This reminds of an often heard argument saying that deliberation should aim at reducing the scope of disagreement.

To conclude that this strategy is globally optimal, one must show that other structures of the perspective  $C$  namely those allowing for transitions  $A_{k+1} \rightarrow A_i$  with  $i = k + 2, \dots, n$  cannot increase the probability to achieve a transition  $A_{k+1} \rightarrow A_i$  with  $i = 1, \dots, k$ . This conjecture is intuitive, but remains to be formally proved.

**Corollary 3** *To maximize the probability for consensus, the facilitator should ask the citizen who has the largest chance to change her voting intention to be the active one. This is the citizen whose alternative to her current voting intention is supported by a larger set of opinion states than the corresponding set for the other citizen.*

This means that in a multidimensional case, the two possible consensus states are not symmetric which contrast with the 2-dimensional case. In particular, if  $\dim Y^{Alice} < \dim N^{Bob}$ , it is easier to achieve consensus by having Bob change opinion from Yes to No than Alice from No to Yes:  $\frac{\partial}{\partial k} \left( \frac{k}{(k+1)} \right) > 0$ . We shall see below that asymmetry between voting options in terms of ease to reach consensus arise for different reasons. Therefore, we introduce a new feature that we call *projected consensus state* corresponding to the consensus state that the facilitator projects to reach.

**Definition**

We refer to as *projected consensus state*, the consensus state that is targeted by the facilitator.

Finally, One may want to question the practical value of the result in Proposition 2. First, it is of course very unlikely that a citizen's perspective exhibits the desired properties of the optimal perspective. Therefore, the facilitator should consider turning to an expert - possibly after two unsuccessful rounds of reciprocal probing between Alice and Bob. Constructing the optimal  $C$  which is a new perspective implies the "relabelling" of some opinion states from another perspective. Basically when  $C_1 = A_1$  it means that the two opinions while being represented by the same state are expressed "in two different languages".<sup>21</sup>

## 4.2 Deliberation in a population of citizens $|\Omega| > 2$

We next analyse deliberation in a population of voters like a citizen assembly. We consider the simpler but most relevant case when the population is divided into two groups each with its own frame of the

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<sup>21</sup>For instance, if  $A_1$  is opinion "ICB is the environmentally best way to reduce GHG",  $C_1$  could be "providing proper incentives to individuals is the most efficient way to reduce GHG".

voting issue. The ICB example is a suitable one as it relates to quite well established ideologies a left leaning social and environmental ideology ( $L$ ) and a right leaning conservative libertarian ideology ( $R$ ).

We first establish a simple Lemma

**Lemma 1** *Starting from consensus, one round of deliberation between two citizens with alternative frame leads to disagreement with positive probability.*

**Proof.** Consider starting from  $A_1B_1$  and letting Bob probe Alice's perspective, we are back in  $A_1B_1$  with probability equal to  $\text{Tr}(B_1A_1)\text{Tr}(A_1B_1) + \text{Tr}(B_1A_2)\text{Tr}(A_2B_1) = (1 - x)$  implying that consensus is lost with probability  $x > 0$ . ■

Lemma 1 shows that having citizens explore alternative perspectives can be harmful to existing agreement among citizens and thus to the facilitator's objective. This feature, as we show below, has implications for the strategy of the facilitator in the multiple citizens context. We note that it is consistent with much criticism appealing to cognitive biases that emphasizes that interacting politically can have "negative" impact on citizens' beliefs and preferences.([31], [25]).

Next, we note that since the vote is binary, and assuming the population is of an uneven magnitude, we always have a voting option that is majoritarian. The two voting options are not symmetric for the facilitator. A feature that we encountered in the  $n$ -dimensional perspective case already.

Let the two groups be of size  $l$  respectively  $r$ . We shall assume that in each group the citizens have determined themselves in their own frame between the 2 possible opinions so we have two distributions  $(l_1, l_2)$ ,  $l_1 + l_2 = l$  and similarly  $(r_1, r_2)$ ,  $r_1 + r_2 = r$ . A new feature compared with previous analysis is that there can be disagreement among people sharing the same perspective, i.e., between the  $L(R)$  people supporting the Yes vote  $l_1(r_1)$  and those support who the No vote  $l_2(r_2)$ . For the case citizens (rather than experts) present their own arguments, we have:

**Proposition 3** *Starting from disagreement within and/or between 2 groups, global consensus can be approached with selective deliberation in which the projected consensus is the standing majority vote say  $l_1 + r_1 > l_2 + r_2$ . In each round the facilitator invites disagreeing citizens to explore the other perspective while the remaining citizens listen but remain passive until the vote. After the first two rounds of reciprocal probing, the expected majority is  $l_1 + r_1 + l_2 \cdot 2x(1 - x) + r_2 \cdot 2x(1 - x)$  where  $x = \text{Tr}(L_1, R_1)$ .*

**Proof.** The proof follows immediately from earlier results in Proposition 1. ■

We believe that in view of previous results, the most interesting aspect of the population case is of practical nature. One would expect that in a larger population that the two first rounds in Proposition 3 fail to achieve global consensus. In order to speed up the process in the remaining  $T - 2$  rounds, we could imagine parallel "working groups". The facilitator constitutes two working groups of active citizen  $l_2$  and  $r_2$  with perspectives presented by experts with maximally uncorrelated perspective to  $L$  respectively to  $R$ . This is consistent with real life deliberative procedures that have been relying on interaction in smaller groups that address different issues.

We thus find that with a multiplicity of citizens, i. the asymmetry between voting option becomes a defining feature of deliberations; ii. In addition the facilitator heavily relies on differentiated treatment of citizens i.e., between active and passive as well as between different active citizens. The process clearly becomes more demanding for the facilitator and demand a larger number of rounds to reach global consensus. We discuss the implications for the democratic ideal in section 5.2.

### 4.3 Deliberation with more than two competing perspectives

Let us now consider the case with citizens representing more than two competing perspectives. Assume three citizens are endowed each with their own two-dimensional thinking frame. This context will also allow addressing the restriction on the facilitator’s strategies that asks him to maximize the score in each round.

Continuing on our ICB example, we now have Greg with a perspective concerned with legal/practical feasibility of ICB and two eigenstates e.g.,  $G_1$  : ICB can be integrated in the current set of laws and regulation with reasonable adaptation costs;  $G_2$  enforcing ICB requires major costly legal adaptation.

As in the population case, since the vote is binary, we initially always have at least two citizens who agree on voting. Hence the two consensus states, let them be  $(A_1B_1G_1)$  and  $(A_2B_2G_2)$ , are not symmetric with respect to the initial state. This feature was also encountered in the  $n$ -dimensional and population case.

#### 4.3.1 Maximizing the probability for consensus in each round

We first investigate the facilitator’s strategy when restricted as before to maximizing the score in each round – we thereafter relax that constraint and consider maximization over two rounds. In this context, it seems fair and common sense to demand that the same perspective not be presented twice. By Proposition 3, we expect the facilitator to select the majority voting option as the projected consensus state. If the initial opinion state is  $(A_1B_2G_1)$ , the natural candidate strategy entails focusing on the sole disagreeing citizen, Bob. As we focus on the voice phase where citizens expose their own views and appealing to Lemma 1, the disagreeing citizen (Bob) is the sole active citizen, Alice and Greg should never take *active* part in the deliberations. We have the following result:

**Proposition 4** *Assume that disagreement with respect to voting characterizes a group of three citizens belonging to three distinct perspectives. Consensus can be approached with selective targeted deliberation. It entails i. selecting the majority voting option as the projected consensus state in ii. targeted deliberation where only the disagreeing citizen is active in all rounds. To maximize the probability of consensus in the first round, the disagreeing citizen should be first exposed to the least correlated perspective. Yet, the order in which the two perspectives of the majority citizens are exposed to the disagreeing citizen is irrelevant to the overall probability of consensus.*



**Proof.** See appendix C. ■

The results in Proposition 4 invites two remarks. First, we find that the presence of more than two perspectives in a population of citizens does not change the facilitator’s problem significantly. He optimally orders the perspectives that he invites Bob to probe so as to start with the least correlated. So the multiplicity of perspectives neither facilitates the task of the facilitator nor is it an obstacle. As in Proposition 3, the deliberative process is skewed towards the opinion which is initially majoritarian. The process essentially amounts to persuading Bob to change opinion while Alice and Greg listen to new arguments but refrain from exploring any new perspective. Result 4.ii contradicts the principle of equity as Bob is never offered the opportunity to convince Alice and Greg to change their initial opinion, and reach the alternative consensus state  $A_2B_2G_2$ . We next consider relaxing the restriction on the facilitator. If his task is not to maximize the probability of consensus *in each round* but over a fixed number of rounds, may he choose to upset the standing majority? opening the way to reach the alternative consensus state  $A_2B_2G_2$ . We next consider the two-rounds case

### 4.3.2 Maximizing the consensus probability over two rounds

As in the previous subsection, we let the initial vector of opinion state be  $(A_1B_2G_1)$ . We consider an alternative strategy for the facilitator, where in the first round two citizens are active e.g., Bob and Greg who are invited to probe Alice’s perspective. There are four possible outcomes of this first round:  $(A_1B_1G_1), (A_1B_2G_1), (A_1B_2G_2), (A_1B_1G_2)$ . This implies that in the second round, where the facilitator maximizes the immediate probability for consensus, the two consensus states  $(A_1B_1G_1)$  and  $(A_2B_2G_2)$  may emerge. We have the following Proposition

**Proposition 5** *Assume that disagreement with respect to voting characterizes a group of three citizens belonging to three distinct perspectives. When the facilitator is aiming at reaching consensus within a 2-period time frame using the citizens’ perspective exclusively, shortsighted maximization is not always optimal. Instead, jeopardizing the standing majority may give higher chance for consensus and any one of the two voting options may prevail.*

**Proof.** See Appendix D. ■

In Appendix D we characterize the conditions under which letting Bob challenge both Alice and Greg can generate a better outcome than focusing on persuading Bob. The conditions corresponds to a case where Bob is closer (his perspective is more correlated) to both Alice and Greg than Greg is to Alice. This is illustrated with a numerical example

*Numerical illustration.*– For a concrete characterization of this situation in the quantum formalism, we introduce the angle  $\theta$  such that  $\text{Tr}(A_1B_1) = x = \cos^2(\theta/2)$ . We also assume that  $\text{Tr}(B_1G_1) = x$ ; and we assume that  $\text{Tr}(A_1G_1) = y = \cos^2(\theta)$ , namely, the unit vector representing  $A_1$  opinion state forms an angle  $\theta/2$  with the vector representing  $B_1$  opinion state, which forms itself an angle  $\theta$  with the vector representing the  $G_1$  opinion state. The whole correlations between Alice’s, Bob’s and Greg’s perspectives

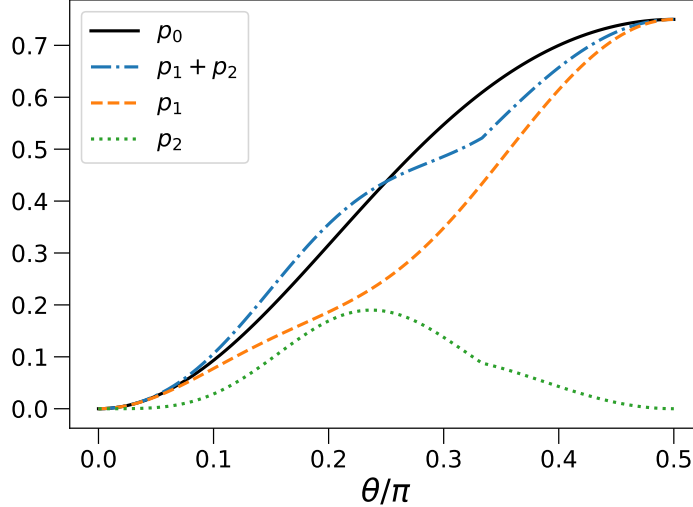


Figure 3: Two-rounds deliberation with three citizens in the two-dimensional case, starting from state  $(A_1B_2G_1)$ .  $p_i$  are probabilities to reach consensus states after two rounds in different scenarios. See text for a definition of  $p_0, p_1, p_2$ , and Appendix D for their relation to  $\theta$ .

are therefore entirely defined via a single parameter  $\theta$ , which allows for a simplified illustration of the different possibilities. We focus on the parameter regime  $\theta \in [0, \pi/2]$ , relevant to describe the two cases described above. We introduce  $p_0$  as the probability to reach consensus  $(A_1B_1G_1)$  by letting Bob probe first Alice’s and then Greg’s perspective.  $p_1$  is the probability to reach consensus  $(A_1B_1G_1)$  if both Bob and Greg actively probe Alice’s perspective in the first round; and  $p_2$  is the probability to reach consensus  $(A_2B_2G_2)$  in this same case. The dependence with  $\theta$  of  $p_0, p_1, p_2$  and  $p_1 + p_2$  is plotted on Fig. 3.

Our main finding is that for  $\theta \in [0, \pi/4]$ , we have  $p_1 + p_2 \geq p_0$ , namely it is better to let both Bob and Greg be active during the first round where Alice exposes her perspective. In this scenario, the majority (Alice and Greg) can be converted to the minority’s initial opinion (Bob’s) through the deliberation procedure with a sizable probability  $p_2$  (green-dotted line in Fig. 3); and this allows to reach consensus with a higher success rate ( $p_1 + p_2$ , blue dashed-dotted line) than insisting twice in converting the minority opinion (in our case, Bob) to the majority view (Alice and Greg) ( $p_0$ , black-solid line). On the other hand, for  $\theta \in [\pi/4, \pi/2]$ , the facilitator maximizes the consensus probability when trying twice to reverse Bob’s initial opinion, letting Greg and Alice play a less engaged role.

Proposition 5 shows that it can, under some conditions, be optimal to allow for disrupting the standing majority and give chance to the other voting option. As shown in the example (and in the proof of Proposition 5) this happens when the disagreeing citizen’s perspective is more closely correlated to both of the two agreeing citizens’ perspective than the agreeing citizens’ perspectives among themselves. We notice however that the most probable consensus state remains the one consistent with the initial majority (namely  $p_1 \geq p_2$ , respectively dashed-orange and green-dotted lines in Fig. 3).

## 5 Discussion

### 5.1 Learning under complete information

In this section, we wish to argue that while we developed the whole analysis in a context of complete information so that at each moment the citizens are endowed with a complete characterization of the issue at stake, some form of learning does nevertheless take place.

The central hypothesis of this paper, is that there exist perspectives that people cannot consider simultaneously, they are Bohr complementary. While incompatible perspectives can be considered in sequence, people cannot aggregate information from two incompatible frames *in a stable way*. This instability is the expression of intrinsic contextuality meaning that opinions do not "exist" (have a determinate value) independently of a thinking frame (perspective) and that it is impossible to integrate all frames into a unique "super frame". Instead, there exists a multiplicity of alternative frames that are equally valid to *fully* characterize an issue.

Nevertheless when probing an alternative perspective, the citizen learns something because the new state corresponding one of the eigenvalues of the perspective under exploration does not exist (as a pure state) in the initial perspective. An alternative frame is like a new language that can express something that could not be expressed in the original language (initial frame).<sup>22</sup> The exploration of an alternative frame comes at the cost of previously held information: the opinion state is modified by the probing operation. This is why we can speak about learning without learning *more*. We believe that our application to deliberation sheds a new light on the concept of "Bohr complementarity", more specifically on the *complementarity* of information from incompatible perspective. Niemeyer et al ([39]) speak about "Intersubjective representational framework" and "meta representation". Our approach allows capturing the meta representation as knowledge of the multiplicity of valid ways to structure one and the same issue. Physicist and philosopher, Michel Bitbol talks about the "second order objectivisation"<sup>23</sup>([4]). The meta-representation which emerges from our deliberation process corresponds to the state space (representing the issue) and the set of associated bases representing different incompatible perspectives together with the correlation between them. So citizens do learn something in deliberation namely how a problem can be considered in ways different from their own (alternative perspectives) and how these ways related to their own (correlation between perspectives) that is they learn elements of the "super structure" of the opinion space. We believe that apart from having one's own opinion evolving, this kind of more fundamental knowledge contributes to making deliberation a process that transform people as often reported. This also illustrates what some participatory democrats with John Stuart Mills (see Introduction) have in mind when assessing that deliberative democracy has a value in itself. It educates people as citizens of a democratic society. Learning the super structure of the opinion space from experiencing (and thus

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<sup>22</sup>Of course any state can be expressed in any perspective but except for the eigenstate, they will be mixtures of other (eigen)states.

<sup>23</sup>That is a procedure of objectivisation of the methodes of production and anticipation of phenomena that cannot be objectivised as properties belonging to objects. .

recognizing) a variety of perspectives, can be understood in this light.

This does not affect the epistemic value of the decision finally made, however. Our approach cannot provide an answer to that question. In the profession, beyond standard informational arguments, the question remains by large open (see Estlund and Landemore in [27]). With our approach, deliberation actualizes one of the deepest debates in QM (see [17]). Indeed even at the individual level, the "correctness" of a decision is contextual. What is the right decision from one perspective need not be the right decision for the same individual from another perspective, thus allowing deliberations to lead to changes in preferences. Therefore, the common aggregative argument has no bite on the epistemic value of the resulting decision. It is beyond the scope of this paper to formulate a precise relationship between the epistemic value of deliberations i.e., citizens learn about the super structure of the opinion space, and the epistemic value of the decision. We conjecture that a result in the spirit to L. Hong and S. Page "Diversity trumps ability" [28] is a promising path to go in a (quantum) non-classical context.

## 5.2 The deliberative democratic ideal

Our approach has been successful in characterizing the transformative character of deliberations: opinions evolve when exploring alternative thinking frames without improvement in information. The analysis shows not surprisingly that deliberations are subject to a tension between efficiency in terms of the probability to reach consensus and fairness in terms of equal treatment of the citizens. Traditionally, equality in the context of deliberation refers either the problem of including all concerned citizens [19] or of redressing situations of inequality in terms to social cultural resources that affect their ability to participate on equal footing in deliberative exchanges (see [3]). This is not our concerns here. We are interested in inequalities that arise because convergence to consensus may require breaking some symmetry in the treatment of participating citizens.

The approach allows characterizing two main aspects of this inequality: the asymmetry between voting options determined by the initial state of opinions and the distinction between active and passive citizens in the process of deliberation. As shown in proposition 5, under some circumstances it can be optimal to break the standing majority. Generally, however and when experts can be called upon, the initial asymmetry with one majoritarian voting option determines the outcome. As suggested in the text, the procedure could start with a "voice phase" where citizens are invited to probe each other's perspectives with no immediate demand to maximize consensus. As we show this can be even be optimal in case more than two perspective are present in the population. It would be followed by an "expert-led phase" where the facilitator maximizes the support of the most supported option. The latter phase which actualizes the distinction between active and passive citizens is clearly unequal. Only minority voters are subjected to persuasion. In defence of this unequal treatment of citizens is that it involves transparent and public operations of (quantum) persuasion. In addition, the participants would be invited to agree with the terms of the procedures before entering the whole process i.e., before knowing in which role they

might be.<sup>24</sup> Although not ideal, we would like to argue that with a benevolent facilitator, the optimal procedures that we depict are not irreconcilable with the ideal of fairness.

We believe that our results allows formulating in a novel way the tension between efficiency (here probability for consensus) and the ideal which puts every citizens on the same footing. We note however that full transparency (publicity of the debates) can be preserved because the quantum cognitive approach distinguishes between simply listening and "putting yourself in the other's shoes" which arguably is an effort.

#### *Caveats 1*

We have addressed deliberations in terms of the standard quantum formalism. As well-known this formalism entails quite a lot of formal constraints which may not all be meaningful in the context of social sciences. A more general framework e.g., appealing to POVM (positive operator-valued measure) could be more appropriate. We therefore propose the current analysis as an abstract approximation which we believe provides interesting insights and can be submitted to experimental testing.

#### *Caveats 2*

In our model the facilitator is fully benevolent. In view of his influence on the outcome, an important issue to investigate is how to secure proper incentives. We leave this important question for future research.

## 6 Conclusions

In this paper we have developed a formal approach to deliberation based on the behavioral premises and mathematical formalism of quantum cognition. The behavioral premises include i. people need a thinking frame (perspectives) to address reality, ii. Citizens cannot (always) combine all relevant aspects in a single thinking frame; iii. alternative thinking frames are Bohr complementary: they are both incompatible and equally valid perspectives on the issue at stake.

Deliberation is formulated in the context of complete information as a structured communication process managed by a facilitator with the aim of maximizing the probability for consensus in a binary collective choice problem. The process includes a sequence of rounds in which some citizen or experts develop an argument belonging to some perspective and other citizens are invited by the facilitator to probe that perspective. Probing involves a true action from citizens, they actively put themselves in someone else's shoes and decide how they position themselves in a perspective alternative to their own.

A first central result is that the incompatibility of perspectives, that is the diversity of view points between disagreeing citizens is what permits opinions to evolve. Our second central result is that the correlation between perspectives is the key property that determines the pace of evolution toward consensus. In the two citizens, two perspectives and two dimensional case, starting from disagreement, consensus is

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<sup>24</sup>The alternative to deliberation for those who reject its terms would be to participate to the vote directly.

reachable with 75% chance after two rounds only. This result generalizes to the case when perspectives have more than 2 dimensions. In that context, consensus is achievable after one round with a probability that can approach 1. The highest rate of convergence to consensus is achieved when the citizen's initial perspective and the one she is invited to probe are maximally uncorrelated that is when having an opinion in one perspective give equal chance for any opinion in the other perspective. In the multidimensional case, the strategy of the facilitator also involves reducing the dimensionality of the problem.

The results generalize to more than 2 citizens where the facilitator's strategy involves selective targeting in the sense that while deliberation remain fully public only a selected subset is invited to actively probe the presented perspective in each round. The others simply listen. When more than two thinking frame are present in the population, it may be optimal to jeopardize the standing majority in order to increase the chance for consensus on either voting option.

We thus find that the quantum cognition approach allows giving sense to a number of empirical features put forward in the literature. Most importantly, it delivers the transformative character of deliberation which goes far beyond Bayesian updating. In our model, people go through real (mental) experiences (probing) which transform their opinion and deepen their understanding of each others. While in the model they always are in a situation of complete information, they learn about how the issue at stake can be looked at from equally valid alternative perspectives and how these relate to their own. It is also consistent with the proposition often put forward that deliberation requires engagement and fosters the respect for each others as a practical school of democracy. In our context this is captured by the willingness to "put yourself in someone else's shoes". Our approach also characterizes the determinant role of the facilitator(s) who does play a central role in most actual experiments such as the Convention Citoyenne pour le climat in France. Finally and importantly, it offers an approach to reaching consensus in deliberation not based on improved information.

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## A Building an intermediate perspective

We illustrate here that it is indeed possible to build such a perspective  $\mathcal{C}$ . Formally, a change of perspective is given by a  $n \times n$  unitary matrix  $U$ , representing a change of orthonormal basis. The opinion states are then the projectors onto the corresponding basis states. If  $|a_i\rangle$  are the basis vectors for Alice's own perspective, and  $|c_j\rangle$  are the basis vectors for the intermediate perspective proposed by the facilitator, the property  $\text{Tr}(A_{k+1}C_j) = |\langle a_{k+1}|c_j\rangle|^2 = 1/(k+1)$  for all  $j = 1, \dots, k$  is for instance obtained if

$$|a_{k+1}\rangle = \frac{1}{\sqrt{k+1}} \sum_{j=1}^{k+1} |c_j\rangle. \quad (8)$$

## B Proof of proposition 2

Given perspective  $\mathcal{C}$ , the probability of failure is simply given by the probability that Alice maintains her initial opinion state, namely to make the transition  $A_{k+1} \rightarrow A_{k+1}$ . According to Eq. (7) Eq.(), this occurs with probability  $p_{\text{failure}} = \sum_{j=1}^{k+1} w_j^2$  where we introduced the notation  $w_j = \text{Tr}(A_{k+1}C_j)$ , which are such that  $\sum_{j=1}^{k+1} w_j = 1$ . Hence, we have to optimize the  $w_j$  such as to minimize  $p_{\text{failure}}$ , under the constraint  $\sum_j w_j = 1$ . Introduce a Lagrange multiplier  $\lambda$  to enforce the constraint, and introduce the function  $f(w_1, \dots, w_{k+1}) = \sum_j w_j^2 - \lambda(\sum_j w_j - 1)$ . The minimum of  $f$  is achieved when  $\frac{\partial f}{\partial w_j} = 0$  for all  $j$ , namely when  $w_j = \lambda/2$  for all  $j$ . That is: the minimum is achieved for a uniform choice of the  $w_j$ . Since we must have  $\sum_{j=1}^{k+1} w_j = 1$ , this leads to the result  $w_j = 1/(k+1)$ , and to the minimal failure probability  $p_{\text{failure}} \geq 1/(k+1)$ ; and correspondingly:  $p_{\text{success}} \leq k/(k+1)$ .

## C Proof of Proposition 4

Consider Bob disagreeing because he holds opinion  $B_2$  while Greg holds opinion  $G_1$  and Alice,  $A_1$ , they both vote Yes. Let the facilitator selects  $A_1B_1G_1$  as the projected consensus state. The total probability for success is  $\text{Prob}^{t=1}(B_2 \rightarrow B_1) + [1 - \text{Prob}^{t=1}(B_2 \rightarrow B_1)]\text{Prob}^{t=2}(B_2 \rightarrow B_1)$ . In the first round he chooses among Greg and Alice. Assume  $\text{Tr}(B_1A_1) > \text{Tr}(B_1G_1)$ , then he let's Alice develop her argument for Bob so he probes her perspective. Global consensus  $A_1B_1G_1$  is reached if Bob's opinion state changes from  $B_1$  to  $B_2$ . This happens with probability  $\text{Prob}^{t=1}(B_2 \rightarrow B_1) = \text{Tr}(B_2A_1)\text{Tr}(A_1B_1) + \text{Tr}(B_1A_2)\text{Tr}(A_2B_1) = 2x(1-x) = a$ , with  $x = \text{Tr}(A_1B_1)$ . In case consensus fails, the same reasoning applies for the second round: Greg is now invited to present his perspective to Bob, who changes opinion with probability  $\text{Prob}^{t=2}(B_2 \rightarrow B_1) = \text{Tr}(B_2G_1)\text{Tr}(G_1B_1) + \text{Tr}(B_1G_2)\text{Tr}(G_2B_1) = 2z(1-z) = b$ , with  $z = \text{Tr}(G_1B_1)$ . The total probability for success is  $\text{Prob}^{t=1}(B_2 \rightarrow B_1) + (1 - \text{Prob}^{t=1}(B_2 \rightarrow B_1))\text{Prob}^{t=2}(B_2 \rightarrow B_1) = a + (1-a)b$ . If instead Greg exposes first his perspective, and then Alice in case of failure, the total probability of success is  $b + (1-b)a = b + a - ab$ , namely the same result. Hence, the

order in which Alice and Greg expose their perspectives to Bob is irrelevant to the overall probability of success.

## D Proof of Proposition 5

We define  $a = 2x(1 - x)$ ,  $b = 2y(1 - y)$  and  $a' = 2z(1 - z)$  with  $x = \text{Tr}(A_1B_1)$ ,  $y = \text{Tr}(A_1G_1)$  and  $z = \text{Tr}(G_1B_1)$ .  $a$  is the probability that Alice (resp. Bob) changes opinion when exposed to Bob's (resp. Alice's) perspective;  $b$  is the probability that Greg (resp. Alice's) changes opinion when exposed to Alice's (resp. Greg's) perspective; etc. Recall that the initial state is  $(A_1B_2G_1)$ . We consider that Alice's perspective is exposed first, and compare the case where: 1) only Bob is active in the first round; and 2) both Bob and Greg are active in the first round.

*Only Bob is active in the first round.*— In this scenario, the probability to reach consensus (consensus state  $A_1B_1G_1$ , namely Bob has change his opinion to  $B_1$ ) at the first round is  $a$ . If his opinion remains  $B_2$ , then Bob is exposed to Greg's perspective and changes opinion with probability  $a'$ . Overall, the probability to reach consensus over two rounds is  $p_0 = a + (1 - a)a'$ , where the first term ( $a$ ) is the probability that Bob changes opinion at the first round, and the second term ( $(1 - a)a'$ ) is the probability that Bob changes opinion at the the second round, conditioned on the fact that he did not change opinion at the first round.

*Both Bob and Greg are active in the first round.*— The probability to reach consensus in the first round is  $a(1 - b)$ , which is obviously smaller than in the previous scenario, as there is now a non-zero probability that Greg also changes opinion, reaching the state  $(A_1B_1G_2)$  instead of  $(A_1B_1G_1)$ . But as we shall see, this may allow for a higher overall probability to reach consensus over the two rounds. If  $(A_1B_1G_1)$  is not reached at the first round, then the facilitator may choose to present either Bob's or Greg's perspective; and lets the minority citizen probe that perspective, offering her/him to change opinion. Let us enumerate the possibilities:

1. If the state after the first round is  $(A_1B_2G_1)$ , which happens with probability  $(1 - a)(1 - b)$ , then Greg presents his perspective to Bob, while Alice remains passive: consensus  $(A_1B_1B_1)$  is then reached with probability  $a'$ . The overall probability to follow this scenario is hence  $(1 - a)(1 - b)a'$ .
2. If the state after the first round is  $(A_1B_2G_2)$ , which happens with probability  $(1 - a)b$ , then the targeted consensus state is  $(A_2B_2G_2)$ : Alice is now the active citizen, and she is offered to probe either Bob's or Greg's perspective. She changes opinion with probability, respectively,  $a$  or  $b$ . Then facilitator hence chooses the perspective which maximizes this probability. Hence, overall, the probability to follow this scenario and end up in the  $(A_2B_2G_2)$  consensus state is  $(1 - a)b \max(a, b)$ .
3. Finally, the state after the first round may be  $(A_1B_1G_2)$ , namely both Bob and Greg change opinion while being exposed to Alice's perspective. This happens with probability  $ab$ . The targeted

consensus state remains  $(A_1B_1G_1)$ , and the facilitator has for only option to expose Greg to Bob's perspective, offering him the opportunity to turn his opinion back to  $G_1$ . This happens with probability  $a'$ , so that the overall probability for this scenario is  $aba'$ .

In summary, three of the above scenarios end up in the consensus state  $(A_1B_1G_1)$ , with total probability  $p_1 = a(1 - b) + (1 - a)(1 - b)a' + aba'$ . While the probability to reach consensus state  $(A_2B_2G_2)$  is given by  $p_2 = (1 - a)b \max(a, b)$ . The total probability to reach consensus over two rounds when both Bob and Greg are active in the first round is hence  $p_1 + p_2$ . Recall that if instead only Bob is active in the first round, the probability to reach consensus after two rounds is  $p_0 = a + (1 - a)a'$ .

*Comparison of both scenarios.*— We now investigate the possibility that  $p_1 + p_2 \geq p_0$ . To simplify the analysis, we consider the case  $a = a'$ , namely Bob's perspective is as much correlated with Alice's than with Greg's. We first consider and  $b > a$ , namely the Alice-Bob and Bob-Greg correlation (quantified by  $a$ ) is stronger than the Alice-Greg correlation (quantified by  $b$ ). A simple calculation shows that in this case,  $p_1 + p_2 = p_0 + b(1 - a)(b - 2a)$ . As both  $b \geq 0$  and  $1 - a \geq 0$ , we conclude that  $p_1 + p_2 \geq p_0$  if and only if  $b \geq 2a$ . When  $b < a$  we always find that  $p_0 \geq p_1 + p_2$ . In summary, when  $a = a'$ , it is optimal to insist twice on making Bob change opinion if  $0 \leq b \leq 2a$ ; while if  $b \geq 2a$ , it is optimal to have both Bob and Greg active in the first round, allowing to reach any of the consensus states  $(A_1B_1G_1)$  or  $(A_2B_2G_2)$ .