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# Preliminary Remarks On Some Forms Radical Uncertainty May Take Some Lessons of the Elders

Talk at the Seminar

Radical Uncertainty: History, Theory, Concepts

<https://clerse.univ-lille.fr/detail-event/radical-uncertainty-history-theories-concepts-seminar>

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Humans thrive in conditions of radical uncertainty when creative individuals can draw on collective intelligence ...

[Tolstoy] understood uncertainty. We see through a glass, darkly. And communicate through narratives, not probabilities ..." Kay and King (2020, pp. 432, 433)

The present paper is just a preliminary sketch or even a first draft of a research project focusing on the historical development and philosophical foundations of the notion of radical uncertainty. I will proceed, somehow in reverse ,from the contemporary to medieval Arabic perspectives.

John Kay and Mervyn King (2020) propose a definition of radical certainty as a form of ontological uncertainty, rather than of epistemic uncertainty: the radical form is considered to be not resolvable.

Their notion of radical uncertainty can be likened to the notion of the 'unknown unknowns', which refers to the aspects of uncertainty that are not readily apparent or quantifiable.

Thus, instead of seeking solutions through probabilistic methods, Kay&King invite us to embrace this form of uncertainty and develop forms of reasoning within a framework where we do not know what we do not know, drawing inspiration from modal approaches to open futures (and even pasts).

Thus, in a sense, Kay&King endorse the inversion of Leibniz's dictum 'Probability is the number of the possible', focusing instead on the purely qualitative features of reasoning with the possible. These patterns of reasoning, known as modal reasoning, have been around since at least the time of Aristotle and are undergoing rapid development in contemporary logic.

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<sup>1</sup> <sup>1</sup>The research underlying the present paper has been *Funded by the European Union*. *Views and opinions expressed are however those of the author(s) only and do not necessarily reflect those of the European Union or European Research Council Executive Agency. Neither the European Union nor the granting authority can be held responsible for them.*"

Notwithstanding the thoroughness of Kay&King's dismissal of probabilistic methodologies, there persists a conspicuous absence of elucidation concerning a systematic characterization of the pattern or patterns of reasoning they advocate, in addition to a dearth of clarity regarding the precise conceptualisation of radical uncertainty within the confines of this particular paradigm of qualitative reasoning.

Moreover, their formulation of the notion of uncertainty is still epistemic. Indeed, they state: "We often do not even know the kinds of things that might happen."

Perhaps, as we will discuss further on, what they have in mind is that in the context of decision-making, the transition from one state of the 'world' to another is not causally ordered, but rather agentive choices produce both the new state and the transition to it – this makes agents *liable* in legal and moral contexts. Thus, according to this view, uncertainty is not rooted in not knowing but rather in the non-deterministic feature of agentive choices.

Be that as it may, it is evident that the concept of uncertainty proposed by Kay and King is intrinsically linked to the ancient Aristotelian notion of future contingencies and the framework for open futures concerning actions, their predictions and vindications.<sup>2</sup> Nonetheless, Kay and King, who utilise both the term *future contingent* and *open future*, do not appear to be acquainted with the ancient roots of their approach to uncertainty and with the host of ancient and contemporary studies referring to those notions. As I shall discuss further on, this precludes them from developing a finer-grained analysis of their notion

In this paradigm, abductive reasoning (which should lead them to endorse too non-monotonic reasoning) is prioritised over deductive reasoning.<sup>3</sup> Indeed, the text under scrutiny places shows a significant predilection for abductive forms of reasoning for dealing with future contingents involving radical uncertainty.

However, it must be noted that the precise links between future contingents, abduction and open futures remain largely underdeveloped within the context of the text.

Nevertheless, Kay and King identify several challenging issues concerning reasoning and decision-making in the context of unknowable futures.

These issues include the interplay between the particular and the general, practical and theoretical knowledge, and the need for what is termed 'collective intelligence' in order to create narratives – that is, create a structure of scenarios – to deal with decisions concerning, in principle, a particular future contingent concrete case.

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<sup>2</sup> As Claudio Majolino noted in the presentation of this paper, it is important to recall that Aristotle's renowned puzzle concerning tomorrow's sea battle pertains to actions and decision-making. The battle to which reference is being made is known as the Battle of Artemisium, which took place in 480 BCE. Indeed, a deliberation had previously occurred concerning the prospect of the Greeks following the counsel of General Themistocles and electing to engage in combat with the Persians, or whether they would instead heed the advice of Eurbyades and abstain from such a confrontation. The decision was in favour of Themistocles, and the battle was fought with an undecisive outcome – cf. Thomason (1984), Perloff&Belnap (2011), Adamson(2006), Tulenheimo (2020), Rahman&McConaughy (2024), Zarepour (2025a,b).

<sup>3</sup> Non-monotonic inferences, let us recall, are roughly those inferences where the addition of a new premise could alter the conclusion logically deduced from an initial set of premises.

So far so good, but telling us that radical uncertainty and the abductive processes it involves must be thought within a framework of open futures, and saying that it is of an unresolvable kind, leaves us indeed pretty much in the dark.

After all they posit that constructing narratives provides some rational means of dealing with them, even if they are dynamic and cannot be universalised in the way that monotonous logical truths can be.

What are the abductive processes at work that rely on *collective intelligence*, and are some of them specific to radical uncertainty, or do all abductive processes involve radical uncertainty? If so, we could focus only on abduction.

Moreover, in the event that there is a precise characterisation of radical uncertainty, it is imperative to ensure that not every uncertainty, including a moderate one, is reduced to a radical one.

So, let me start with recalling the main form of reasoning involving uncertainty in philosophy of logic.

In contemporary philosophy of logic, three main approaches to uncertainty can be identified, each with its own particular focus and associated philosophical underpinnings on the degree and scope of uncertainty.

As time is the element out of which unknown futures are forged, these approaches involve time either as a priori condition of knowledge, or as a context that relativizes truth, or both.

The primary lessons of the Elders that will be given due consideration in these preliminary reflections on decision-making under conditions of radical uncertainty are as follows:

1. In the context of radical uncertainty, we should recognise that the correctness of statements that inform decision-making processes depends on the execution of the actions encoded by these statements. It is the execution of an action that is subject to time not the propositions that express future contingent action types. This aligns with the Aristotelian dictum that time is not a substance! In summary, moments should not be conceived as carrying actions and events as their properties in their sleeves.
2. Fitch Paradox. To avoid dissolving all forms of certainty into radical certainty when articulating certainty by introducing modal operators, three possible responses have been advanced: (a) Knowledge is not an operator but a relation. (b) From an open proof no closed proof on knowability obtains. (3) The relevant notion of possibility is subject to generic plenitude. According to this notion, possibility is understood as the human capacity to be realised by the human species as a whole, but not necessarily by each individual. This appears to be linked to K&K's appeal to collective intelligence.
3. If radical uncertainty is to be conceived as expressing a form of future contingency within a framework for open futures, the question arises as to what form this contingency takes.<sup>4</sup> Different frameworks define different forms of

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<sup>4</sup> Kay&King's (2020) book, namely *Radical Uncertainty. Decision-making for an unknowable future* seems to be closely linked to Belnap&Perloff&Xu's book (2001) *Facing the Future. Agents and Choices in Our Indeterminist World*, which deals with Ockhamist approaches to branching time, though this book does not occur in their

uncertainty concerning the future. For instance, there are Peircean and Ockhamist structures of branching time (in fact the latter of which were conceived by Al-Fārābī almost 400 years before Ockham). Furthermore, the various types of uncertainty can also be situated within the realm of expectation, forming a 'dense' present (the specious present), which is considered pivotal for experiencing temporality by contemporary philosophers (a notion likely to have been anticipated by Suhrawardī's epistemology of presence) – note again that *certainty* seems to appeal to consciousness.

4. Dialectical patterns of parallel reasoning developed mainly by Arabic medieval thinkers, offer dynamic systems for reasoning under conditions of radical uncertainty that integrates defeasibility and perspectives focusing either in the particular or the general, by combining abductive with deductive moves

## I Time and the Unfolding of Uncertainty

Clearly, uncertainty of the unknown future is shaped by Time. Since the dawn of philosophy to the most contemporary studies philosophers distinguished time either

as a priori condition of knowledge, or  
as a context that relativizes truth and knowledge, or  
both.

The aforementioned distinctions also yield divergent forms of temporal awareness, a factor which is of pivotal significance in the delineation of certainty.

Indeed, the awareness of time enables the subject to establish a conscious relation with the knowledge of the truth of a proposition concerning processes such as actions and events.

### **Uncertainty and Time as a priori Condition of Knowledge. Time is not a Substance**

In constructive mathematics and logic inference systems are developed, wherein uncertainty is conceptualised as the absence of an actual verifier or proof, which does not preclude the possibility of such a verifier to be constructed. This approach instructs the practitioner to reason without the assumption of third-excluded (and thus, without double negation and *reductio ad absurdum*). Here time is close to the a priori notion of Kant: the time involved in all construction.

In some sense, constructivism can be regarded as a moderate form of uncertainty. The absence of a proof at this moment does not preclude the possibility of such a proof being subsequently constructed.

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bibliography. Belnap and collaborators different to Kay&King are acquainted with the ancient roots of the notion of future contingent– see Belnap&Green (1994), Perloff&Belnap (2011). Unfortunately, Belnap and his co-authors are unaware of the significant innovative developments on this issue within the Arabic tradition – see Adamson (2006) and Zarepour (2025a, b).

Nevertheless, it is also possible to interpret constructivism as a concept involving a certain special form of radical uncertainty, in the sense that some brands of constructivism understand uncertainty as not knowing *how* to achieve what we know we do not know

In order to emphasise this point, let me focus on the notion of existence within the domain of theoretical computer science.

The Curry-Howard isomorphism, further developed by Martin-Löf within the context of Gentzen natural deduction, is a result which establishes that the constructive notion of a proposition can be understood as a computer problem, with a proof of the proposition corresponding to the exhibition of a computer program that solves the problem.<sup>5</sup>

Let us assume that if a solution to some specific computer problem is not found, a contradiction will be deduced. For instance, given the problem on predicting the impact of tariffs in the world-economy let us assume that denying that a computer programme for solving that problem leads to a contradiction with say some principles of Economy.

In accordance with the classical conception of existence, it can be deduced that such a program exists, albeit with the utmost ignorance regarding its configuration and the methodology for its development.

Constructivists have historically eschewed such inferences to existence based on reductio – and rightly so, a standpoint that is likely to be endorsed by any computer scientist. Bluntly, constructivists claim that claiming a programme exists even when nobody knows how to write it is an empty unhelpful claim.

So far so good, but two main objections can be raised, focusing on the constructive notion of certainty and the forms of uncertainty constructivism generates:

1. How do we give meaning to the content of a statement if meaning involves the production of a proof, and we have no means yet of producing such a proof? For instance, it seems that we can grasp the meaning of the prediction: *Tariffs will affect many national economies*. Clearly, we understand its content though we might not yet determine either its truth or its falsity. Furthermore, suppose we could provide a theory that renders statements involving uncertainty meaningful within the fields of logic and mathematics. How would this help us deal with the empirical uncertainty of future contingencies discussed by K&K, which involve empirical actions?
2. Suppose we could provide a constructivist theory in the style of CTT (or Immanent Reasoning) that renders statements involving the kind of uncertainty discussed by K&K meaningful, why does not the constructivist notion of certainty collapse into a radical one, namely that all truths are known – thus preventing the formulation of radical uncertainty?

As for the first objection, we must distinguish between being able to express what might count as a proof and producing it. We grasp the meaning of the contradiction  $A \& \sim A$  even though we cannot produce a proof of it, namely its meaning amounts to being able to produce a proof of both members of the conjunction.

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<sup>5</sup> See Martin-Löf (1984).

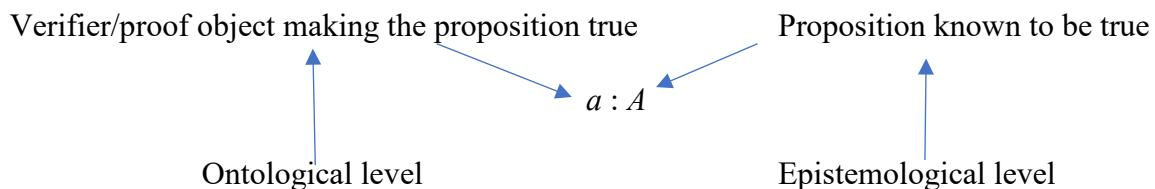
Moreover, there are some finer nuances, in the particular form of constructivism represented by the dialogical framework: to grasp the meaning of a contradiction, to stick to our example, amounts to knowing how to play a game of giving and asking for reasons for this conjunction, without having a method for developing a winning strategy for it. But the dialogical framework for constructivism might also add a deeper level: the uncertainty might involve the formation rules of the components of the conjunction, in other words: it might concern the material content at work.

In short, from the point of view of dialogical constructivism,<sup>6</sup> if radical uncertainty makes any sense, it is either a matter of **how** to build a winning strategy or **how** to play a game involving some material content – e.g. we grasp the meaning of “5 is an even number” because we know how to play a game where this statement is a thesis in contrast to” The square root of 25 is too arrogant” – cf Martin-Löf (2015, 2017).

Am I missing the point? K&K’s notion of radical uncertainty concerns future contingencies articulated within a framework of empirical time conditions (as set by a branching time structure), not the a priori time that shapes mathematical constructions.

Indeed, from a constructivist point of view, mathematical and logical propositions are made true by proof-objects that are the pure products of the priori time, which makes constructions possible.

However, we can extend this approach to cover empirical propositions if we distinguish between the proposition, which is the bearer of claims on knowledge, and the verifier, which provides ontological support for these claims. If we express this distinction in the object language in the style of Martin-Löf’s Constructive Type Theory (CTT) – cf. Sundholm (1998, 2012, 2013) and Martin-Löf (2014), the result is as follows:



Now, we can further distinguish the process of finding a verifier/proof object which takes of the form of demonstration or dialogical interaction shaped by the a priori time governing logical constructions and the actual execution or realization of the identified verifier which can be associated to some specific moment (empirically conceived as a state of the world).

According to this approach, we distinguish the actual action of talking from the proposition expressing the type of *me-taking*. The proposition is made true by my actual performance. Therefore, it is not the proposition that is directly qualified as bearing temporality, but rather my singular actions. Similarly, the proposition 'It is raining' is made true by actual precipitations. Actual actions/performance and events can be temporally contextualized: my talking at 14:00, the precipitation at 15:00.

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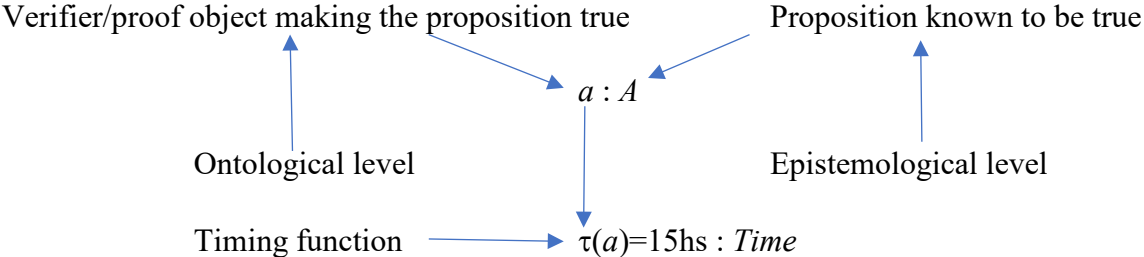
<sup>6</sup> I am referring to the framework called *Immanent Reasoning* developed in Lille, which provides a dialogical foundation of Constructive Type Theory – cf. Rahman et al. (2018) and Rahman&Gallois (2024).

The point is an Aristotelian one, historians of philosophy will remind us: time is not a substance: 14hs does neither bear the property of me talking nor of precipitating rather, action and events can be associated with a moment or span by a timing function.

Indeed, whereas the logical analysis of the proposition *Kay is an Economist*, results from substituting the variable  $x$  (ranging over the domain of human beings) occurring in the propositional function  $Economist(x)$ , the logical analysis of *Shahid's Talk at 14hs* cannot be rendered as resulting from *Shahid's Talk(t)* – where the variable  $t$  ranges over time moments.

Aristotle's warning should be clear: Kay possesses the attribute of being an Economist, yet the moment 14hs does not possess the attribute of being Shahid's Talk, unless the assumption is made that time-moments are of an ontological category equivalent to individuals of the human species.

It is regrettable that this erroneous assertion has gained widespread acceptance among contemporary logicians and linguists, who have disregarded Aristotle's prudent cautionary remarks on the matter. What we should instead do, is to analyse empirical propositions involving temporality as expressing that their verifiers are subject to time:



The cello-performance  $a$  by Pau Casals taking place at 15hs makes true the proposition  $A$ , namely the proposition *Pau Casals plays Bach's Suite for Cello Nr. 1*.<sup>7</sup>

Or more generally:

There is some cello-performance  $x$  taking place at 15hs that makes true the proposition that *Pau Casals plays Bach's Suite for Cello Nr. 1*:

$$(\exists x : \text{Pau Casals plays Bach's Suite for Cello Nr. 1}) \tau(x) =_{\text{Time}} 15\text{hs}$$

Which assumes that when  $A$  is a proposition made true by some  $x$ , the evaluation of the function  $\tau(x)$  renders some moment  $t_i$  constituting the set *Time*:

$$\text{Temporalization} \left\{ \begin{array}{l} A : \text{prop} \\ \tau(x) = t_i : \text{Time} \quad (x : A) \end{array} \right.$$

Or even more generally

<sup>7</sup> We can of course be more precise and distinguish hours, days, spans of time, spatial locations and so on, but for the sake of simplicity we focus on moments.

$$(\exists x : A) \tau(x) \equiv_{\text{Time } t} \text{prop}(t : \text{Time})$$

Thus we can now define uncertainty as concerned with time as an epistemological condition of knowledge and time as contextualizing the ontological component of an empirical proposition involving processes by temporalizing the actualizations that make those propositions true.

Still, the precise issue of radical uncertainty at work in future contingencies, such as predictions that influence actions triggered by decision-making processes, remains to be addressed. This issue will be discussed further on. However, let us discuss the threat of collapse mentioned above concerning the notion of moderate certainty inherent to constructivist approaches.

### **Fitch Paradox the Collapse of Moderate Certainty and Possibility to Know a Generic Capacity of the Human Species.**

Modal approaches (either epistemic or ontological or temporal), are adopted, where uncertainty is understood as relativizing truth to some states of information or states of the world. In this paradigm, we are taught to reason within a framework of states, shaped by a structure that determines the availability of information in a given state (or scenario) or the transitions from one state of the other determining the semantic of the modal operators in a given model. Thus  $\mathbf{K}A$  – the agent  $g$  knows  $A$  the information state  $s_i$  – iff for all information states  $s_j$  available to  $g$  from  $s_i$ ,  $A$  is true at  $s_j$ . Moreover, in order to distinguish in this framework knowledge from belief it is assumed that one of the states of information available to the agent  $g$  is the *actual, real* one.

In such a framework it has been proposed to formulate the constructivist notion of moderate certainty combining the modal ontological or metaphysical possibility operator  $\diamond$ ,<sup>8</sup> with the epistemic operator  $\mathbf{K}$  and produce the implication

$$A \supset \diamond \mathbf{K}A$$

In vernacular form:

If  $A$  is true then it is (at least) possible to know  $A$  or  
If  $A$  is true  $A$  is knowable

Which entails that there are no truths that are in principle not knowable. A principle generally endorsed by constructivists who associate meaning with the human capacity of being known.

Unfortunately, by the use of some modest modal principles, the implication logically forces us to accept that every true proposition is known

Or worse, in its paradoxical formulation known as Fitch-paradox: the implication that *if  $A$  is true,  $A$  is knowable* leads to the assertion that *I know that  $A$  is true but I do not know  $A$* :

$$\mathbf{K}(A \ \& \ \sim \mathbf{K}A)$$

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<sup>8</sup>  $\diamond A$  is true at the state of the world  $w$ , iff there is at least one state  $v$  accessible from  $w$  where  $A$  is true.

In fact, by applying basic modal principles it follows that  $\sim\mathbf{K}(A \ \& \ \sim\mathbf{K}A)$  is valid. From this it is embarrassingly straightforward to demonstrate that the implication, read as the substitution-schema  $\alpha \supset \Diamond\mathbf{K}\alpha$ , leads to the radical certainty implication  $\alpha \supset \mathbf{K}\alpha$ .

Indeed, assume that there is some proposition  $p$  that is true but not known to be so:  $p \ \& \ \sim\mathbf{K}p$ . Let  $\alpha$  stand for this conjunction. Then we obtain

by contraposition	$p \ \& \ \sim\mathbf{K}p \supset \Diamond\mathbf{K}(p \ \& \ \sim\mathbf{K}p)$
since $\sim\mathbf{K}(A \ \& \ \sim\mathbf{K}A)$ is valid in modal logic	$\sim(\Diamond\mathbf{K}(p \ \& \ \sim\mathbf{K}p)) \supset \sim(p \ \& \ \sim\mathbf{K}p)$
by modus ponens	$\sim(p \ \& \ \sim\mathbf{K}p)$
by definition of implication and double negation	$p \supset \mathbf{K}p$
which yields the schema	$\alpha \supset \mathbf{K}\alpha$

This result affects the notion of radical uncertainty since if every truth is known, then there is no room for unknown unknowns, or at least makes the notion of radical uncertainty tough to grasp since it would entail that no radical uncertainty could possibly be true.

There are several ways to respond to this challenge, hard-core constructivist simply reject the implication. Indeed, the constructivist meaning explanation of implication does not justify  $A \supset \Diamond\mathbf{K}A$ . For, from an *open* (hypothetical) proof of  $A$  – the crucial feature that distinguishes the meaning of an implication from a categorical inference – I cannot construct a proof of the proposition that  $A$  is knowable: there is no guarantee that there is a closed proof of  $A$ , but only if there is a closed proof of  $A$  is  $A$  knowable – cf. Sundholm (2014, pp. 20-21), Klev (2016, section 3).

A more fundamental rejoinder is to reject the whole idea of knowledge to be understood as an operator. Knowledge, and furthermore, Certainty is a relation between the subject and the content of an assertion. This relation cannot be encoded by a proposition, particularly if we understand Certainty as requiring consciousness of the content of knowledge.

Nevertheless, in the context of radical uncertainty, which, as we will discuss below, is formulated as involving predictions on future contingencies within a structure of branching time, we may still wish to follow Avicenna’s idea and render the possibility operator as a temporal one, where, as discussed above, the temporality affects the empirical realization of the empirical verifiers.<sup>9</sup>

Specifically, we could explain its meaning as follows:  $A$  is possibly true at a given moment,  $t$ , if and only if, for at least one sequence of actions/events occurring from  $t$  onwards,  $A$  is made true at some point in that sequence. Moreover, if we understand possibility as contingency, there might be a course of actions in which  $A$  is true at one moment and false at another for the same course of actions.

In order to avoid Fitch’s collapse or at least mitigate it, we can assume that the notion of contingency at work is subject to *generic plenitude*.

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<sup>9</sup> See Street(2000, 2002, 2010), Street&Germann (2013), Strobino (2015, 2018), Strobino&Thom(2016), Rahman (forthcoming).

This means that at least one agent will make a decision that renders  $A$  true, while another agent will make a decision that renders  $A$  false.

Therefore, if  $A$  represents an agent's ability, at least one agent can make a decision that renders  $A$  true. However, as it is contingent and not necessary, there must also be at least one agent that renders it false.

In such a context, the transition from a weak to a strong form of certainly is much more acceptable: it states that

if  $A$  represents a capacity that can be realised by a specific type of agent, such as the human species and is true – in ancient terms: if  $A$  is a *proprium* of the human species or of its genus, then at least one agent of that type knows how to make the proposition true. In other words, the truths envisaged are those within the scope of human capacity as a species or its genus, rather than as individuals. This is, of course, a restriction, but a sensible one in the context of decision-making<sup>10</sup>.

### Varieties of Unknown Futures within Branching Time

Let us expand upon the concept of unknow future which yields different types of uncertainty but always keep in mind that when we speak of temporal operator we are in fact thinking of temporalizing their verifiers.

K&K endorse the position that radical uncertainty regarding future contingencies is indicative of an open structure. In the event that this is indeed the case, the most efficacious manner in which to implement such a perspective is, as has been previously indicated, to hypothesise that future contingents are shaped by a branching-time structure.

In the domain of temporal logic, a range of structures has been delineated in the extant literature which yields different varieties of uncertainty – of which K&K, again, do not seem to be aware. A notable example is the hypothetical structure known as Peircean Temporal Logic, which posits possible course events (or histories) and proffers a prediction on a future contingent  $A$  as true iff there is a verifier for  $A$  in at least one moment of all the hypothesised courses of events that transpire post-utterance. Whereby a moment is conceived as a state of the world accessible by performances of actions, though no predicament is stated about which of the courses of event will actually realize.

Peircean temporal logic delineates an extreme form of uncertainty, as it is not difficult to formulate a scenario in which both  $\mathbf{F}A$  and  $\mathbf{F}\sim A$  are deemed to be false— thus, the disjunction  $\mathbf{F}A \vee \mathbf{F}\sim A$  will fail not because of constructivist epistemological time conditions but because of contextualising future contingent with a Peircean branching time structure. Moreover, we can also find in such a logic a counterexample to the implication *If  $A$  is made true now, then it has always been the case that  $A$  will become once true* ( $A \supset \mathbf{H}FA$ , whereby “**H**” stands for the temporal operator “It has always been the case”), since in the past there could have been a courses of events different to one in which  $A$  is now true.

One problem with this kind of approach is that it assumes that all and possible of courses of events have been identified, even if only hypothetically. An approach to branching time that

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<sup>10</sup> See Boussad (forthcoming).

yields a notion of uncertainty closer to the future contingency of actions, is the one often called Ockhamism. In such a framework, the verifier of a future contingent is not there since the relevant actions have not been yet performed. More precisely, neither the performances producing the changes in the actual state of the world, nor the result of these changes, namely the new state, have been realized. We can only wait. According to one approach to Ockhamist perspectives on future contingents, different to case of the past, contingent predictions require to add *duration* as a parameter for determining their truth value. This form of Ockhamism is a genuine logic of *becoming*.<sup>11</sup>

Another approach within Ockhamism focuses on *vindicating* retroactively once the verifier has “settled” the truth of the predicted proposition: predictions on future contingents of the form  $FA$  and  $F\sim A$  are in general neither true nor false at the moment of utterance, however, if we assume that the actions involved are after all decidable, in the sense that we can assume that after some finite time the actual course of events determine which of the predictions have been settled as true, then  $FA \vee F\sim A$  will hold.<sup>12</sup>

Is this what K&K have in mind? Perhaps but though the request to hypothesise underlying Peircean Temporal Logic, might be close to K&K’s appeal to narratives creating scenarios, the request of unknown unknowns in the realm of not yet performed actions, might be closer to some form of Ockhamist/ Al-Fārābīan temporal Logic.

In such a context radical uncertainty will result by not assuming that one of both will be settled at all. Logically, this amounts to add to the branching time structure the restriction of the a priori time of constructivists.

Furthermore, the various types of uncertainty can also be situated within the realm of expectation, forming a 'dense' present (the specious present), which is considered pivotal for experiencing temporality by contemporary philosophers. A notion likely to have been anticipated by Suhrawardī's epistemology of presence who referred to time as *the environment of the now* constituted by re-actualizations of the past and anticipations of the future.<sup>13</sup>

The point once more, that depending on the structure of the branching time, the specious present will contain either purely hypothetical scenarios or anticipations or expectations of “real courses of events”.

All this leaves room for unknown unknowns among agents who might rely on collective intelligence to create suitable scenarios for dealing with them.

However, we naturally want more, we wish to be able to design systems for reasoning under conditions of radical uncertainty that integrates defeasibility and perspectives focusing either in the particular or the general, by combining abductive with deductive moves.

In the following paragraphs I will mention some, among hundreds of the main forms of patterns of reasoning with **multiple degrees of uncertainty**, developed within the post-Avicennan tradition originally motivated by decision-taking in the context of legal reasoning.

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<sup>11</sup> See Tulenheimo (2020).

<sup>12</sup> See Rahman&McConaughy (2024).

<sup>13</sup> See Street (2008) and Rahman&Seck (2025).

## II Reasoning and the Unknown

### A Bold Proposal

#### Arguments by Parallel Reasoning involving Degrees of Uncertainty: The Intertwining of Abductive and Deductive Dialectical Procedures

Permit me now to present a concise exploratory working hypothesis that, in essence, seeks to expand the extensive framework for parallel reasoning developed within the Arabic Legal Disputation theory. The objective of this expansion is to establish a framework that enables the conceptualisation of reasoning processes encompassing a range of different degrees and kind of uncertainties associated with future contingencies.

In his PhD-dissertation *The Economy of Certainty*, Aron Zysow (2013, p. 159) proposed to study *uṣūl al-fiqh* (أصول الفقه), or **Islamic legal theory**, under an epistemological perspective, or more precisely as the *Islamic counterpart to [contemporary] philosophy of science*. Indeed, an epistemological perspective on *uṣūl al-fiqh* brings to the fore the fact that Islamic jurisprudence is deeply rooted in the task of pursuing rational knowledge and understanding with particular emphasis on reasoning under different degrees of certainty.

However, we should not lose sight of the point that *uṣūl al-fiqh* constitutes the body of knowledge and methods of reasoning that Islamic jurists deploy in order to provide solutions to *practical legal problems of decision making* linked to the dynamics of legal systems. Clearly, working out solutions to practical legal problems commits one to the practice of legal reasoning.

More precisely, since the ultimate purpose of such a kind of rational endeavour is to achieve decisions for new circumstances or cases not already established by the juridical sources, the diverse processes conceived within Islamic jurisprudence were aimed at providing both epistemological and practical tools able to deal with the evolution of the practice of *fiqh*. This dynamic feature animates Walter Edward Young's main thesis as developed in his book *The Dialectical Forge*.<sup>14</sup> In fact the main claim underlying the work of Young is that the dynamic nature of *fiqh* is put into action by both the dialectical understanding and the dialectical practice of legal reasoning. These already set out the motivations for the development of a dialectical framework such as the one we are aiming at in the present study.

The finest outcome of this approach to legal reasoning within *fiqh* is the notion of *qiyās* (قياس), known as *correlational inference*.<sup>15</sup> The aim of correlational inferences is to provide a rational ground for the application of a juridical ruling to a given case not yet considered by the original juridical sources. It proceeds by combining heuristic (and/or hermeneutic) moves with logical inferences. The simplest forms follow the following patterns:

- In order to establish if a given juridical ruling applies or not to a given case, called the *branch-case*, *al-far'* (الفرع), we look for a case we already know that falls under that ruling – the so-called *root-case*, *al-aṣl* (الأصل). Then we search for the property or set of properties upon which

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<sup>14</sup> Young (2017). See too Young (2021a,b,c).

<sup>15</sup> Cf. Young (2017, p. 10). The term quite often has a broader meaning which encompasses legal reasoning in general. However, Young's choice for its translation renders a narrower sense that stems from al-Shīrāzī's approach.

the application of the ruling to the source-case is grounded (the *ratio legis* or *legal cause* for that juridical decision).

If that grounding property (or set of them) is known, we ponder if it can also be asserted of the new case under consideration. In the case of an affirmative answer, it is inferred that the new case also falls under the juridical ruling at stake, and so the range of its application is expanded. When the legal cause is explicitly known (by the sources) or made explicit by specifying a relevant set of properties, the reasoning schema at work is called *qiyās al-‘illa* or correlational inference by the *occasioning factor*. Let us recall the classical example: Date liquor intoxicates, just as (grape) wine does, so that it is prohibited like wine. The canonical analysis identifies four elements in such an argument: the branch-case or case under consideration, date liquor; the root-case or case verified by the sources; wine; the character they have in common their power to intoxicate; and their common, legal qualification, prohibition (inferred in the case of date liquor, verified by the sources in the case of wine). The crucial step that underlies this form of argumentation is the identification of the occasioning factor, the *‘illa*, that lies behind its prohibition. The point here is that applying the general schema that *drinks that have the power to induce intoxication should be forbidden* to the case of date liquor *occasions* its interdiction.

When the grounds behind a given juridical ruling are neither explicit nor can they be made explicit, the reasoning schema at work is either *qiyās al-dalāla*, or correlational inferences by indication, or *qiyās al-shabah*, or correlational inferences by resemblance. Whereas the former are based on pinpointing specific relevant parallelisms between rulings (*qiyās al-dalāla*), the latter are based on resemblances between properties (*qiyās al-shabah*). Thus, *qiyās al-dalāla* and *qiyās al-shabah*, sometimes broadly referred to as arguments by analogy (or better, by the Latin denomination arguments *a pari*) are put into action when there is absence of knowledge of the occasioning factor grounding the application of a given ruling. The plausibility of a conclusion attained by parallelism between rulings (*qiyās al-dalāla*) is considered to be of a higher epistemic degree than the conclusion obtained by resemblance of the branch-case and the source-case in relation to some set of (relevant) properties (*qiyās al-shabah*). Conclusions obtained by either *qiyās al-dalāla* or *qiyās al-shabah*, have a lower degree of epistemic plausibility than conclusions inferred by the deployment of *qiyās al-‘illa*, where the occasioning factor can be identified.<sup>16</sup>

More generally, one interesting way to look at the contribution of the inception of the juridical notion of *qiyās* is to compare it with the emergence of European Civil-Law. Indeed, European Civil Law emerged as a system of general norms or rules that were thought to generalize the repertory of cases recorded mainly by Roman Law. The emergence of *qiyās* can be seen as the inception of an instrument to identify or grasp the general meaning behind the cases recorded

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<sup>16</sup> One striking example of the implementation of such a method is Arsyad al-Banjari's (1957; 1983) development of a dialectical model for integrating traditional Indonesian uses into Islamic Law. See Iqbal/Rahman (2019) and Iqbal (2019).

by the sources and the tradition. The dynamics triggered by implementing such an instrument “forges” the laws that structure Islamic Law.

Rahman&Iqbal&Soufi’s (2019) book, focused on Abū Ishāq al-Shīrāzī’s (393/1003 CE-476H/1083 CE) classification of *qiyās* as discussed in his *Mulakhkhaṣ fī al-Jadal (Epitome on Dialectical Disputation)*, *Ma’ūna fī al-Jadal (Aid on Dialectical Disputation)* and *al-Luma’ fī Uṣūl al-Fiqh (Refulgence of Islamic Legal Theory)* develops an examination based on a dialogical approach to Per Martin-Löf’s (1984) *Constructive Type Theory*, or CTT.

According to our view, such an approach provides both a natural understanding and a fine-grained instrument to stress three of the hallmarks of this form of reasoning:<sup>17</sup>

- (a) the interaction of hermeneutic, heuristic and epistemological processes with logical steps,
- (b) the dialectical dynamics underlying the meaning-explanation of the terms involved,<sup>18</sup>
- (c) the unfolding of parallel reasoning as similarity in action.

What the dialogical framework adds to the standard natural-deduction presentation of CTT is that this approach not only provides insights into the dynamics of meaning underlying the notion of *qiyās*, but it also leads to a conception of logic where logical rules too are understood as emerging from dialectical interaction. In other words, the dialogical reconstruction of the different forms of correlational inference is not to be conceived as the concatenation of a dialogical structure + logical rules + semantics + knowledge + jurisprudence, but rather as a unifying system where all those levels are constituted, or *forged* at once by argumentative interaction, they are *immanent* to a dialogue that makes reason and knowledge happen. For a discussion on *immanent reasoning* see part III.

Let us have a first glimpse at how this framework works out in the context of the traditional objections to *qiyās*.<sup>19</sup> The main objections can perhaps be summarized as follows<sup>20</sup>:

1. Within *fiqh* one very rarely finds attempts to deduce a general rule from the specific rule for each legal act. What we actually find in the legal writings more often than not, are specific rules.
2. Finding out the general rules by abduction or induction is not only pretentious but it also leads to uncertainty. How do we ever know that we identified the most appropriate or *relevant* properties? This casts doubt on even *qiyās al-‘illa*, purported to provide the most certain conclusion attained by legal reasoning.

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<sup>17</sup> Miller (1984) is one of the first to mention the dialogical framework of Lorenzen/Lorenz (1978) as a suitable approach for the study of Islamic argumentation. The dialogical approach to CTT is called *Immanent Reasoning*, see Rahman/McConaughy/Klev/Clerbout (2018). In fact there is ongoing work on deploying the dialogical setting in order to reconstruct logical traditions in ancient philosophy (see Castelnérac/Marion (2009), Marion/Rückert (2015), Crubellier/McConaughy/Marion/Rahman (2019)).

<sup>18</sup> The term *meaning-explanation* is due to Martin-Löf and has a natural dialogical reading – see Rahman/McConaughy/Klev/Clerbout (2018, chapters II and III), it amounts to setting the meaning of an expression by rules that establish how to challenge and defend it. Moreover, these rules also include formation-prescriptions, that is, rules that prescribe the type of an expression : is it an independent type?, like say the set of natural numbers; or is it a dependent type?, like a propositional function which renders a proposition from elements picked out from a relevant set; e.g. *French(x) : prop (x : Human)* – that can be glossed as “*the proposition that x is is French can be asserted from suitable candidates of the set of Humans*”.

<sup>19</sup> As we will discuss in our conclusion to the present book, some contemporary philosophers such as John Woods (2015, pp. 273-280) raised similar objections to the use of analogy in legal reasoning.

<sup>20</sup> For a thorough discussion on these points see Zysow (2013, pp. 160-191).

3. The uncertainty of the results of applying *qiyās* stems from the fact that understanding the general norm behind a specific juridical ruling requires the deployment of an interpretative process rather than of a dubious epistemological argument.

According to our reconstruction of al-Shīrāzī’s system of *qiyās*, the point on how to grasp the general meaning behind a specific law does not commit one to discovering Laws (legal laws are not “discovered”). The process involved consists in the ability to grasp that the specific rule instantiates a general one, by making apparent the meaning-constitution behind that specific rule.

Roughly, the generalization behind, which is very close to what Woods (2015, p. 278)<sup>21</sup> calls *generalization schema*, can be seen as a process of *exemplification*,<sup>22</sup> whereby one instance is grasped as exemplifying the whole (*pars pro toto*) – just as a sample of a carpet exemplifies the whole carpet. So the generalization schema exemplified by the case of wine can be formulated as follows:

The consumption of drink  $x$  is interdicted  
 Drink  $x$  has the property of inducing intoxication

This supports the assertion:

- The capability of drink  $x$  to induce intoxication leads to its interdiction

More precisely, given some ruling  $\mathcal{K}$  applied to some case  $b$ , i.e, given the specific ruling  $\mathcal{K}(b)$ , when we delve into the *meaning-explanation*<sup>23</sup> we might come to see that it is an instance of the following schema:

$\mathcal{K}(x)$  true ( $x : \mathcal{P}$ )  
 “it is true that ruling  $\mathcal{K}$  applies to  $x$ , provided  $x$  instantiates property  $\mathcal{P}$ ”,

which adds to the precedent schema the point not only that the inferential schema at stake has the form of a hypothetical judgement, but also that the interdiction is an interdiction *specific to objects* (in our example, drinks) *instantiating the property*  $\mathcal{P}$  (of inducing intoxication). Clearly, the interdiction of consuming some drink is different from the interdiction of, say, stealing. The legal consequences of the correspondent transgressions are certainly different.

Moreover, the *‘illa* is the application of the schema to a particular specific instance. Technically speaking, in our framework, the causative feature of the occasioning factor amounts to shaping it as an application of the function that instantiates the schema. This allows us to distinguish the property relevant for some specific juridical sanction from the actual procedure of triggering that sanction for some particular case instantiating the property. It is the triggering procedure that provides the notion of occasioning factor with its causal force.

Intuitively, given the schema  $\mathcal{K}(x)$  true ( $x : \mathcal{P}$ ),  
 if  $a$  instantiates  $\mathcal{P}$ ,

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<sup>21</sup> As we will discuss in the conclusion of the present book, Woods' (2015, pp. 273-280) take on reasoning by precedence in Common Law is strikingly close to al-Shīrāzī’s system of *qiyās al-‘illa*. The same applies to Brewer's (1996) *Exemplary Reasoning*.

<sup>22</sup> Here, we are using Goodman’s notion of exemplification – see Goodman (1976, chapter II).

<sup>23</sup> See above our footnote on the notion of *meaning-explanation*.

then, there is a method (in the practice a juridical procedure), encoded by the function  $b(x)$ , that when applied to  $a$  renders the specific ruling  $\mathfrak{H}(a)$ .

So far so good, but how do we know that this is the meaning behind  $\mathfrak{H}(a)$ ? How do we know that the chosen property is the relevant or the appropriate one? To come back to our canonical example, how do we know for certain that the property of inducing intoxication lead to the interdiction of consuming wine?

It is here that al-Shīrāzī's method of efficiency *ta'thīr* comes into action. In a nutshell, according to our analysis, al-Shīrāzī's notion of occasioning factor includes the following three main components:

1. *Wasf*, the property P relevant for a juridical sanction H, such that the latter is defined as being specific to the set of cases defined by P (e.g. those interdictions  $H(x)$  that apply to consuming those drinks that instantiate the set P of drinks inducing intoxication).
2. The efficiency feature or *ta'thīr*, that provides the means to test whether the property P purported to be relevant for the juridical sanction at stake is indeed so. The test declines into two complementary procedures: testing co-extensiveness or *ṭard* (if the property is present then the sanction too) and co-exclusiveness or *'aks* (if the property is absent then so is the juridical sanction – the consumption of vinegar is in principle not forbidden). While co-extensiveness examines whether sanction H follows from the verification of the presence of the property P, co-exclusiveness examines whether exemption from the sanction H follows from the verification of the absence of P.
3. The causal feature, i.e., the legal method encoded by the function  $b(x)$ , that when applied to some instance  $a$  of the relevant property P renders the ruling  $H(a)$  specific to that property. More precisely, when we focus on the causal feature of the occasioning factor, the function will be written as *'illa(x)*. The function *'illa(x)* admits the substitution *'illa(a)* for some case  $a$  (that satisfies the *wasf*), only after the efficiency of the property P has been verified by the test *ta'thīr*.

As pointed out by Zysow (2013, p. 215), the doctrine of efficiency represents an impressive attempt to answer the cardinal questions of those that opposed the deployment of *qiyās*. Notice that the method of efficiency not only tests the relevance but also responds to the point on the legal foundation of the general rules. The fact is that the general schema is both grounded and extracted from specific rulings found in the sources. Moreover, by means of *ta'thīr* the occasioning factor is identified as the application that yields a ruling grounded in the sources.

Still, Abū Hāmid al-Ghazālī, who vehemently defended the deployment of *qiyās*, points out that co-exclusiveness co-presence and co-extensiveness or co-absence do not always render the most appropriate or relevant (*munāsaba*) property for the ruling under consideration. His example involves the property of some particular smell, present when wine is present and absent when wine is absent, but these observations do not lead to the conclusion that that particular smell of wine is the relevant property for its interdiction.<sup>24</sup>

What al-Ghazali observes is that though the tests of *ṭard* and *'aks* pave the way for grasping the intention behind the norms given by the Lawgiver, this might not be enough: grasping the

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<sup>24</sup> Al-Ghazālī (1324H, pp. 307-308). Cf. Hallaq (1987, pp. 61-62).

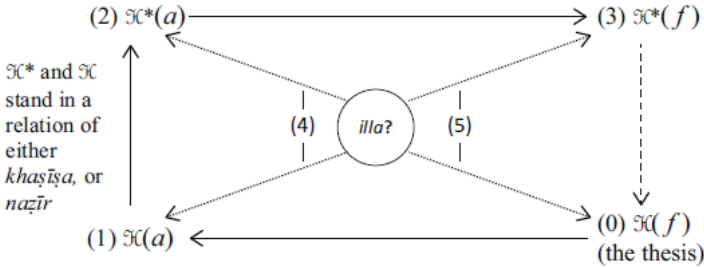
meaning might require additional hermeneutical procedures.<sup>25</sup> Nevertheless, formulating explicitly a claim on the precise form of a general schema implicit in the use of a specific ruling, brings this schema out into the open as liable to challenges and demands for justification.<sup>26</sup>

More generally, the idea is that the rational process invoked by the argumentative framework depends on the possibility of making explicit (in the form of claims) implicit commitments on the meaning-explanation of a ruling. In other words, the rational epistemological endeavour underlying *qiyās* consists in the possibility of publicly expressing claims concerning the general constitution of a ruling in order to subject them to ponderation and criticism.

***qiyās al-dalāla***

Assume now that there is quite of a higher degree of uncertainty with regard to the ratio legis behind a legal decision: in such a case we switch to another pattern of reasoning, involving rulings a pari as in the figure below.

I will explain it during the talk, but the point is to find another ruling associated to the one under scrutiny. The second ruling can either be a more general one or simply one that is a member of the set of rulings that also includes the one under consideration.



**Notational keys:**  
 “℔\*(a)” can be glossed as “Ruling ℔\* applies to the root-case”  
 “℔(a)” can be glossed as “Ruling ℔ applies to the root-case”  
 “℔\*(f)” can be glossed as “Ruling ℔\* applies to the branch-case”  
 “℔(f)” can be glossed as “Ruling ℔ applies to the branch-case”  
 “illa?” can be glossed as “Rulings ℔ and ℔\* are both dependent upon an unknown occasioning factor *illa*” The pointed arrows express the dependence of the indicators (i.e., the rulings ℔ and ℔\*) upon the *illa*.

**Schema 3.1** *Qiyās al-Dalāla*

According to our study; whereas from the perspective of Civil Law the point is to settle the emerging Law, within Common Law, as stressed by Woods (2015, p. 279) establishing, a

<sup>25</sup> Putting aside important differences, we might parallel al-Ghazālī’s point with Frege’s view that though concepts are ontological independent of the logical analysis that makes them explicit and publicly accessible to human understanding, this analysis clears the way to the grasping of those concepts. However, different to al-Ghazālī, Frege thinks that logical analysis is the *only way*.

<sup>26</sup> At this precise point, parallel reasoning in Common Law and *uṣūl al-fiqh* take different paths. Indeed, according to Woods (2015, p. 280), the generalization schema behind a parity argument is very rarely made explicit. We come back to this point in the conclusion.

general Law, is not the aim of an argument by precedent cases, despite the fact that the argument as such requires its formulation.

Thus, whereas as in the context of Civil Law the concrete specific precedent cases are instrumental to grasp the general Law behind, in the context of Common Law it is the general Schema that is instrumental.

If we generalize, we can say that parallel reasoning of the kind sketched above offers both perspectives, the focus on the individual case, whereby the instrumentalized universal Law can have quite of a restricted domain, or focus on the general Law, whereby the individual case is plays the role of an exemplification which provide a substitution schema

The proposal, which is arguably audacious and programmatic, entails the expansion of the aforementioned patterns of parallel reasoning with a view to integrating reasoning on future contingencies.

The leading idea is that while the branch case signifies a future contingent, the root case serves as a precedent case derived from the collective intelligence of the past.

The proposal is to acquire the ability to pose and respond to questions in various contexts and degrees of future unknowns drawing inspiration from the categorisation of numerous forms of objections and counter-objections. These objections and counter-objections might have as subject the very rules of reasoning itself, a concept developed within the framework of Arabic disputation theory.

### **Some Very Brief Conclusions:**

In order to comprehend the notion of uncertainty and its varying degrees, it seems useful to consider the a priori time in conjunction with the temporal indexation of verifiers.

This can be achieved by establishing a framework of branching time, wherein the presence of branches is not assumed, but rather the actions that ensue subsequent to our decisions. Through this approach, we can gain valuable insights into the concept of uncertainty and its various degrees.

It is possible to discern promising patterns for reasoning within such a framework in the Arabic tradition of parallel reasoning. This tradition combines hermeneutical, abductive and deductive perspectives, which are classified into a large number of categories and subcategories. These in turn are subclassified into hundreds of forms of objections and counter-objections, which concern the rules of the reasoning themselves.

Finally, it should be noted that the fusion of Avicennan Logic with parallel reasoning gave rise to a further fascinating historical development.

Avicenna's successors distinguished not only different degrees of certainty but also different degrees of causal entrenchment that motivated a legal decision. The objective was to develop Avicenna's distinction between strong and weak necessity in order to classify different forms

of causality, but this constitutes the commencement of a new talk and the conclusion of the existing one.

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