Sources of Economic Growth: Physical capital, Human Capital, Natural Resources, and TFP

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THESE

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Sources of Economic Growth: Physical Capital, Human Capital, Natural Resources and TFP

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To my Parents, Thu Thuy, and Bao Anh who made it happen

L'Université de Paris I n'entend donner aucune approbation ou improbation aux opinions émises dans cette thèse. Ces opinions doivent être considérées comme propre à leur auteur.

Contents

Acknowledgement

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General Introduction

In essence theories of economic growth are to show the nature of the exogenous variables which ultimately determine the rate at which the general level of production of an economy is growing, and thereby contribute to an understanding of the question of why some societies grow so much faster than others. There is general agreement that the critical factors determining the trend rate of growth are to be sought in the savings propensities of the community (which determine the rate of physical capital accumulation), the flow of invention or innovation (which determines the rate of growth of productivity), the growth of population and to some extent the endowment of natural resources. The Solow model (1956), a corner-stone of neoclassical growth models, contains two bases: first, steady state growth is independent of the savings rate; second, the main source of growth is technological change, rather than capital accumulation. However, the essential factor for economic growth in these models, namely technological progress, is however, exogenous to the model. This shortcoming inspires scholars such as Romer (1986, 1987, 1990), Lucas (1988), Rebelo (1991), Grossman and Helpman (1991), Aghion and Howitt (1992) and many others to develop new "endogenous" growth models which provide more insights into the Solow's residual. The new growth theory started with Romer's paper of 1986. This model explains persistent economic growth by referring to the role of externalities. This idea had been formalized earlier by Arrow (1962), who argued that externalities, arising from learning by doing and knowledge spillover, positively affect the productivity of labor on the aggregate level of an economy. Lucas (1988), whose model goes back to Uzawa (1965), stresses the creation of human capital, and Romer (1990) and Grossmann and Helpman (1991) focus on the creation of new knowledge as important sources of economic growth. The latter authors have developed an R and D model of economic growth. In the Romer model the creation of knowledge capital (stock of ideas) is the most important source of growth. In Grossman and Helpman, a variety of consumer goods enters the utility function of the household, and spillover effects in the research sector bring about sustained per capita growth. A similar model, which can be termed Schumpeterian, was presented by Aghion and Howitt (1992, 1998). In it the process of creative destruction is integrated in a formal model; the quality grades for a product are modeled as substitutes; in the extreme case the different qualities are perfect substitutes, implying that the discovery of a new intermediate good replaces the old one. Consequently, innovations are the sources of sustained economic growth. Recently, the growth performance of the East Asian newly industrialized economies (NIEs) gave rise to a broad and diversified literature aiming at explaining the reasons for such a long lasting period of expansion. Over the past thirty-five years Korea, Taiwan, Singapore, and Hong Kong, have transformed themselves from technologically backwards and poor, to relatively modern and affluent economies. Each has experienced more than a four fold increase of per capita incomes. Each now has a significant collection of firms producing technologically complex products competing effectively against rival firms based in the United States, Japan, and Europe. The growth performance of these countries has been unprecedented in history of economic growth in the world so far. Economists are not unanimous in identifying forces behind these high growth rates. On one hand, the supporters of the accumulation view stress that the high growth rates in NIEs were crucially driven by very high rate of investment. Consequently, the lack of technical progress will inevitably bound the engine of growth as a result of the diminishing returns affecting capital accumulation. On the other hand, the supporters of endogenous growth theory pinpoint productivity growth as the key factor of East Asian success. According to these authors, Asian countries have adopted technologies previously developed by more advanced economies (assimilation view) and "the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy" (Pack [1992]). In other words, the growth in NIEs can be sustained in the long run by learning-by-doing process.

These debates, theoretically and empirically, motivate us to explore more insights in the interactions between those essential forces namely, human capital, new technology, natural resources, and learning-by-doing with economic growth. Those interactions are not only contemplated in the steady state but on the whole dynamic growth process. We also emphasizes on transitional stages which is more applicable for developing countries.

The first chapter of this dissertation review neoclassical models which show the essential of TFP in long-run growth and the potential of being stuck in poverty trap. We highlight that TFP is not only essential for long-run growth but also important for a developing economy to escape a potential poverty trap. Then, based on Solow's model we discuss the so-called Solow-Krugman controversy about the "miracle growth" in Newly Industrialized Economies (NIEs). In effect, the "controversy" is not a real one. Krugman is right in short and medium term, while Solow is right in the long run.

In the second chapter we consider a developing country with three sectors in economy: consumption goods, new technology, and education. Productivity of the consumption goods sector depends on new technology and skilled labor used for production of the new technology. We show that there might be three stages of economic growth. In the first stage the country concentrates on production of consumption goods; in the second stage it requires the country to import both physical capital to produce consumption goods and new technology capital to produce new technology; and finally the last stage is one where the country needs to import new technology capital and invest in the training and education of high skilled labor in the same time. The third chapter shows that long-run economic growth can be sustained by learning-by-doing as claimed by accumulationists. However, using a CES production technology we can show that the growth model based purely on learning-by-doing is constrained by labor growth rate. If labor is constant in the long-run, then growth can not be sustained. In addition, we also explain why economic growth does not converge as predicted by Solowian models. We characterize four possible growth paths which are contingent on saving and elasticity between capital and labor. If the elasticity is smaller than 1, there are 3 possible scenarios: (i) if the saving rate is too low the economy will collapse in long run; (ii) if the saving rate is not very low but lower than the optimal level the economy can sustain its growth rate which is always lower than the potential rate; (*iii*) if the saving rate is high enough the economy converges asymptotically to its BGP which does not depend on saving but on the index of efficiency. If elasticity is higher than 1 the economy either converges to its BGP or its rate of growth decreasingly converges to a rate which is higher than the potential rate and does not depend on the index of efficiency. Finally, in the transitional stage savings always help growth to accelerate.

In the fourth chapter we study the optimal growth of a developing non-renewable natural resource producer. It extracts the resource from its soil, and produces a single consumption good with man-made capital. Moreover, it can sell the extracted resource abroad and use the revenues to buy an imported good, which is a perfect substitute of the domestic consumption good. The domestic technology is convex-concave, so that the economy may be locked into a poverty trap. We study the optimal extraction and depletion of the exhaustible resource, and the optimal paths of accumulation of capital and of domestic consumption. We show that the extent to which the country will escape from the poverty trap depends, besides the interactions between its technology and its impatience, on the characteristics of the resource revenue function, on the level of its initial stock of capital, and on the abundance of the natural resource. The last chapter devote for an empirical study on Vietnam's economic growth since *Doi Moi* (renovation). We found out that during last 22 years the high economic growth rates were driven mostly by physical capital accumulation. TFP contribute almost nothing to the growth. This implies that growth rate would slowdown if Vietnam failed to improve TFP in medium and long-term.

Chapter 1

Revision of economic growth models: TFP is essential

1.1 Introduction

In this chapter we review general equilibrium models for economic growth. These models essentially investigate four kinds of questions: ,(i) what are the sources of growth; (ii) how the agents determine their consumption and hence the saving which is necessary for investment; (iii) do the balanced growth paths exist in these models; and finally,(iv) stability of the balanced growth path, i.e., when the starting point of the economy is not on the balanced growth path, does this economy converge, in the long term, to the balanced growth path.

The Solow model (1956), a corner-stone of neoclassical growth models, contains two bases: first, steady state growth is independent of the savings rate; second, the main source of growth is technological change, rather than capital accumulation. However, the essential factor for economic growth in these models, namely technological progress, is however, exogenous to the model. This shortcoming inspires scholars such as Romer (1986, 1987, 1990), Lucas (1988), Rebelo (1991), Grossman and Helpman (1991), Aghion and Howitt (1992) and many others to develop new "endogenous" growth models which provide more insights into the Solow's residual. The new growth theory started with Romer's paper of 1986. This model explains persistent economic growth by referring to the role of externalities. This idea had been formalized earlier by Arrow (1962), who argued that externalities, arising from learning by doing and knowledge spillover, positively affect the productivity of labor on the aggregate level of an economy. Lucas (1988), whose model goes back to Uzawa (1965), stresses the creation of human capital, and Romer (1990) and Grossmann and Helpman (1991) focus on the creation of new knowledge as important sources of economic growth. The latter authors have developed an R and D model of economic growth. In the Romer's model the creation of knowledge capital (stock of ideas) is the most important source of growth. In Grossman and Helpman, a variety of consumer goods enters the utility function of the household, and spillover effects in the research sector bring about sustained per capita growth. A similar model, which can be termed Schumpeterian, was presented by Aghion and Howitt (1992, 1998). In it the process of creative destruction is integrated in a formal model; the quality grades for a product are modeled as substitutes; in the extreme case the different qualities are perfect substitutes, implying that the discovery of a new intermediate good replaces the old one. Consequently, innovations are the sources of sustained economic growth. In these models saving rate plays a crucial role but is exogenous. Ramsey (1928)present the way to endogenize the saving behavior. The growth performance of the East Asian newly industrialized economies (NIEs) gave rise to a broad and diversified literature aiming at explaining the reasons for such a long lasting period of expansion. On one hand, the supporters of endogenous growth theory pinpoint productivity growth as the key factor of East Asian success. According to these authors, Asian countries have adopted technologies previously developed by more advanced economies (assimilation view) and "the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy" (Pack [1992]). Implicitly, they admit that the TFP is one of the main factors of growth in accordance with the thesis developed by Solow [1957]. On the other hand, Krugman (1987) based on emprircal studies such as Young [1994, 1995], Kim and Lau [1994, 1996]concludes that Asian growth could mostly be explained by high saving rates, good education and the movement of underemployment peasants into the modern sector and these are one-time unrepeatable changes. Here, we share the view of Dollar [1993] that divergence between countries is also due to differences in TFP. Why is technology important? Because it can be simultaneously employed in different uses (public good and productive good as well). Dollar [1993] wrote "there are a number of pieces of evidence indicating that successful developing countries have borrowed technology from the more advanced economies". We think the so-called Solow-Krugman controversy is not really one. Krugman's view is correct in the short and mid terms. But in the long term, TFP is the main factor of growth. In this sense, Solow is right and his 1956 model is basically a long term growth model. Furthermore, Cross-countries empirical studies also show that development patterns differ considerably between countries in the long run (Barro and Sala-i-Martin [1995], Barro [1997]). These differences can be explained within a model of capital accumulation with convex – concave technology. In such a framework, Dechert and Nishimura [1983] prove the existence of threshold effect with poverty traps explaining alternatively "growth collapses" or taking-off. Azariadis and Drazen [1990] propose an elaboration of the Diamond model that may have multiple stable steady states because the training technology has many thresholds. They give an explanation to the existence of convergence clubs in Barro and Sala-i-Martin [1995], Barro [1997]. In this chapter using Romer model (1986) we also show that in the presence of fixed costs in production the poverty trap can be realized if the initial capital is below critical level.

1.2 The Solow Model (Solow, 1956)

We consider a simple intertemporal growth model for a closed economy.

$$
\begin{aligned}
C_t + S_t &= Y_t \qquad (1.1)\\
S_t &= sY_t,\ s \text{ is the exogenous saving rate}\\
K_{t+1} &= K_t(1-\delta) + I_t\\
L_t &= L_0(1+n)^t \qquad (1.2)\\
Y_t &= a(1+\gamma)^t K_t^{\alpha} L_t^{1-\alpha},\ 0<\alpha<1 \qquad (1.3)\\
I_t &= S_t
\end{aligned}
$$

$C_t, S_t, Y_t, K_t, I_t, L_t$ denote respectively the consumption, the saving, the output, the capital stock, the investment and the labour at period t. The labour force grows with an exogenous rate n. The Total Factor Productivity (TFP) grows at rate γ. It is easy to solve the model given above. Actually, we have

$$\forall t,\ K_{t+1} = (1-\delta)K_t + saK_t^{\alpha}L_t^{1-\alpha}(1+\gamma)^t \qquad (1.4)$$

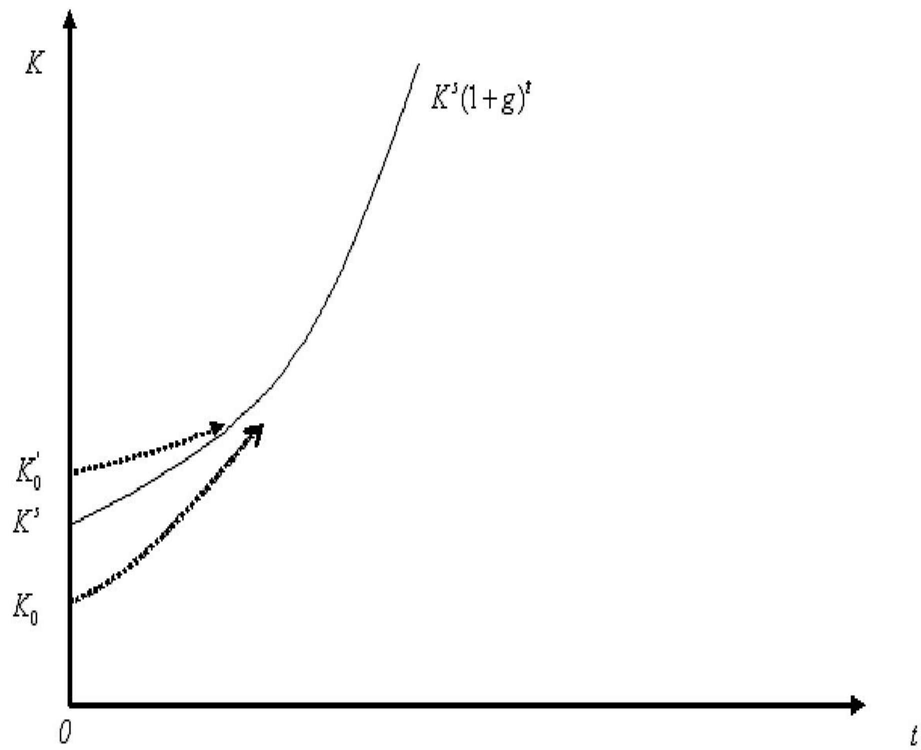
We can easily check that there exists a Balanced Growth Path (BGP) with rate g

$$(1+g) = (1+n)(1+\gamma)^{\frac{1}{1-\alpha}}$$

On the BGP, we have $K_t^* = K^s(1+g)^t,\ \forall t$, where $K^s = \left(\frac{sa}{g+\delta}\right)^{\frac{1}{1-\alpha}} L_0$. Given $K_0 > 0$, the path generated by equation (1.4) satisfies

$$\frac{K_t}{(1+g)^t} \to K^s$$

In other words, the path $\{K_t\}_t$ converges to the steady state K^s. It is interesting to notice that the rate of growth g is positively related to the rate of growth γ of the TFP.



Dynamic path of capital

1.3 The Ramsey Model (Ramsey, 1928)

Two criticisms may be addressed to the Solow Model. The first one is the saving rate is exogenous. The second one is the rate of growth is exogenous. In this section, we will endogeneize the rate of saving of the households. But we do not solve the question of the exogeneity of the rate of growth. This problem will be studied later with some endogenous growth models. The model we present here, is a discrete-time horizon version of the well-known Ramsey model (1928) which was formalized in continuous-time horizon. This model has been studied in more details by Cass (1965) and Koopmans (1965). The basic idea in the Ramsey model is to introduce an infinitely lived consumer who maximizes an intertemporal utility function of her intertemporal sequence of consumptions. At each date, her consumption is constrained by the maximum output produced by a stock of physical capital, and by the necessity of saving for obtaining a physical capital stock for the next period production process. The main results are that, under some conditions, optimal sequences of capital stocks and of consumptions exist, and converge to an optimal steady state. Moreover, the sequence of optimal capital stocks is monotonic.

We consider an economy in which there are, at each period t, L_t identical consumers. We denote by c_t the consumption, at period t, of one consumer. We assume that the number of consumers grow at rate n, i.e., $L_t = L_0(1+n)^t$, for every t. In this economy, there is a social planner whose task is to promote the welfare of its population . So, she wants to maximize the global utility of the consumers :

$$\max L_0 \sum_{t=0}^{\infty} (1/(1+\rho))^t (1+n)^t u(c_t)$$

Here, the function u is called the static utility function or instantaneous utility function and the parameter ρ is the positive time preference rate. A large value of ρ means that the consumers are more impatient and prefer the present to the future. At each date t, consumption c_t is subject to the constraint:

$$L_t c_t + I_t \leq F_t(K_t, L_t),$$

where I_t is the investment, F_t is the production function, K_t is the capital stock, L_t is the number of workers (we implicitly assume that the consumers and the workers are physically identical). The capital stock of period $t+1$ is defined by:

$$K_{t+1} = K_t(1-\delta) + I_t,$$

where $\delta \in]0,1[$ is the depreciation rate of the capital stock. Let us assume that the production function F_t exhibits constant returns to scale and let us introduce the per capita capital stock $k_t = K_t/L_t$. The constraint for each period, between consumption and investment becomes:

$$c_t + k_{t+1}(1+n) - (1-\delta)k_t \leq F_t(k_t, 1).$$

Assume that $F_t(k_t, 1) = A(1+\gamma)^t k_t^\alpha$, with $0 < \alpha < 1$. The parameter γ is the rate of growth of the productivity. We then obtain:

$$c_t + k_{t+1}(1+n) \leq A(1+\gamma)^t k_t^\alpha + (1-\delta)k_t.$$

If the utility function u is strictly increasing, then, at the optimum, the constraints will be binding at each period. If the optimal sequences of capital stock and consumption grow at rate g, i.e., for any t, $k_t = k_0(1+g)^t$, $c_t = c_0(1+g)^t$, we then have

$$(1+g)^{(1-\alpha)} = 1+\gamma.$$

In other words, the rate of growth of the economy is determined by the exogenous rate of growth of the productivity. Using the variables capital per capita k_t and consumption per capita c_t, the Ramsey model can be written as:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

under the constraints:

$$\forall t, c_t + k_{t+1}(1+n) \leq Ak_t^\alpha + (1-\delta)k_t,$$

and k_0 is given, and by definition, $\beta = (1+n)/(1+\rho)$. The parameter β will be called discount factor. If we assume, for simplicity, that $n = 0$, and if we define the function f by $f(k) = Ak^\alpha + (1-\delta)k$, then the Ramsey model will have the following compact form:

$$\max \sum_{t=0}^{\infty} \beta^t u(c_t)$$

under the constraints:

$$\forall t, c_t + k_{t+1} \leq f(k_t),$$

$$\forall t, c_t \geq 0, k_t \geq 0,$$

and $k_0 \geq 0$ is given. In the following, we will make use of this form. Notice that the production function is $F(k) = f(k) - (1-\delta)k$. The following assumptions will be maintained throughout this section.

H0 $0 < \beta < 1$.
H1 The function $u : R_+ \to R_+$, is twice continuously differentiable and satisfies $u(0) = 0$. Moreover, its derivatives satisfy $u' > 0$ (strictly increasing) and $u'' < 0$ (strictly concave).
H2 Inada Condition : $u'(0) = +\infty; u'(\infty) = 0$.
H3 The function $f : R_+ \to R_+$ is twice continuously differentiable and satisfies $f(0) = 0$. Its derivatives satisfy $f' > 0$ (strictly increasing), $f'' < 0$ (strictly concave), $\lim_{x\to+\infty} f'(x) < 1, f'(0) = M \leq +\infty$.

We get the following results:

Theorem 1 *Let* $r = \frac{1}{\beta} - 1$.
(1) If $F'(0) \leq \delta + r$, *then the optimal path* $\{k_t^*\}$ *will converge to* 0
(2) If $F'(0) > \delta + r$, *then the optimal path* $\{k_t^*\}$ *will converge to the steady state* k^s *defined by* $F'(k^s) = \delta + r$.

For a proof see e.g. Le Van and Dana (2003).
Following this results, if the countries have the same technology they will ″converge″ in the long term provided the initial capital stock is non null. In this case, the International Aid to developing countries helps them an initial endowment, even very small, then every country will reach in the long term the same stage of development. The reality is far to coincide with this claim. An explanation of the non-convergence between the countries may be found in the next section. Observe that one can relax the assumption $\lim_{x\to+\infty} f'(x) < 1$ and assume $f(k) = (A+1-\delta)k$. Assume $u(c) = \frac{c^\theta}{\theta}$ with $0 < \theta < 1$. If $\beta(A+1-\delta)^\theta < 1$ then the optimal solution to the Ramsey model is a BGP with rate of growth $g = [\beta(A+1-\delta)]^{\frac{1}{1-\theta}} - 1$. We see that the rate of growth is positively related the non-impatience of the consumer (large β) and the TFP A. The saving rate is constant $s = \frac{[\beta(A+1-\delta)]^{\frac{1}{1-\theta}}-1}{A}$ and positively related to β and A. We have a Solow model but we can explain why the saving rate is high (the consumer is patient, the technology is good).

1.4 The Convex-Concave Production Function

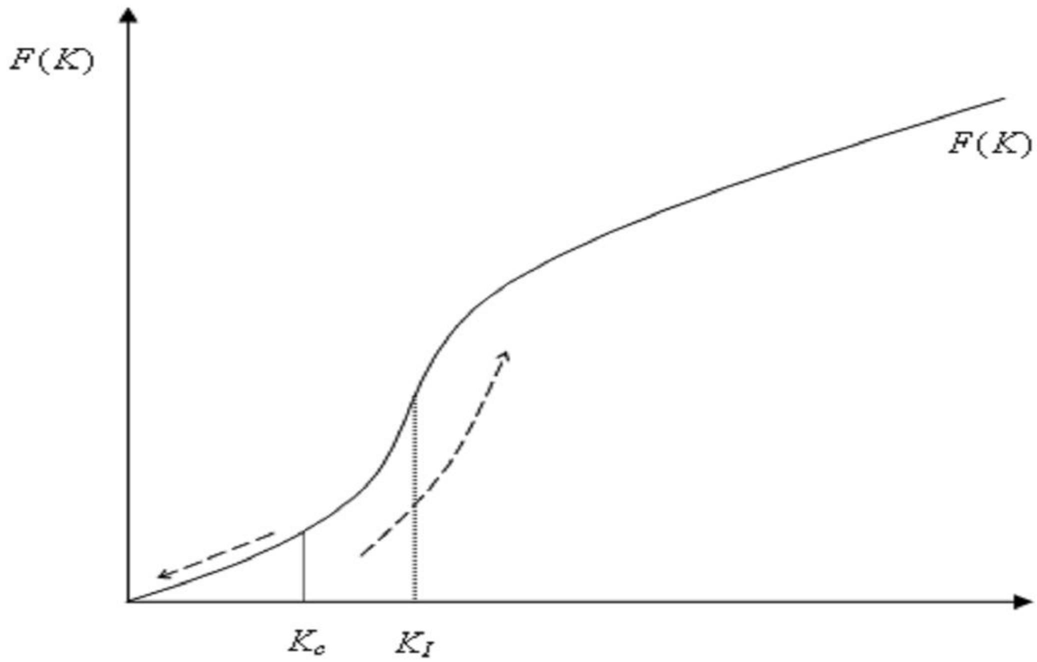
We change the assumption **H3** in Section 1.3. Assume

H3 The function $f : R_+ \to R_+$ is twice continuously differentiable and satisfies $f(0) = 0$. Its derivatives satisfy $f' > 0$ (strictly increasing). There is a point k_I such that $f''(k) < 0$ if $k > k_I$, and $f''(k) > 0$ if $k < k_I$. There exists a point $k_{max} > k_I$ such that $f(k_{max}) = k_{max}$ and $f(k) < k$ if $k > k_{max}$.

We then get the following result

Theorem 2 *[Dechert-Nishimura, 1983] Let* $r = \frac{1}{\beta} - 1$.
(1) If $F'(0) > \delta + r$, *then any optimal path* $\{k_t^*\}$ *will converge to the highest steady state* k^s *defined by* $F'(k^s) = \delta + r$.
(2) If $F'(0) < r + \delta < \max_{k>0}\{\frac{F(k)}{k}\}$, *then there exists a critical value* k^c *such that: (i) if* $k_0 < k^c$ *then any optimal path* $\{k_t^*\}$ *will converge to* 0*; (ii) if* $k_0 > k^c$, *then any optimal path* $\{k_t^*\}$ *will converge to the highest steady state* k^s *defined by* $F'(k^s) = \delta + r$.

King and Rebello (1993) calibrate, with the US Data [1948-1979] the Ramsey model with decreasing returns. They run simulations and show that the neoclassical dynamics can only play a minor role in explaining the observed growth rates. They conclude that their results point to the use of models which do not rely on exogenous technical change. We now present some models which endogeneize the rates of growth of the economy. They answer the concern: how to make growth endogenous, or more precisely, technical change endogenous?



1.5 The Solow-Krugman Controversy

The Solow [1957] implies that the TFP is the core factor of economic growth. If the economy bases merely on capital accumulation without technological progress, the diminishing returns on capital accumulation will eventually depress economic growth to zero. Accordingly, Solowian supporters attribute the miracle economic growths in Newly Industrialized Economies (NIEs) in second half of 20th century to adoption of technologies previously developed by more advanced economies. Pack [1992] suggests "the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy".

Empirically, however, Young [1994, 1995], Kim and Lau [1994, 1996] found that the postwar economic growth of the NIEs was mostly due to growth in input factors (physical capital and labor) with no increase in the total factor productivity. Moreover, the hypothesis of no technical progress cannot be rejected for the East Asian NIEs (Kim and Lau [1994]). Consequently, accumulation of physical and human capital seems to explain the major part of the NIEs' growth process. Krugman's [1994] concludes that "it (high growth rate) was due to forced saving and investment, and long hours of works...So if we are forced to save 40% of our income, and get only two weeks off a year of course a country will grow". Accordingly, due to diminishing returns the lack of technological progress will inevitably bound the growth engine of East Asian NIE.

In the following we will prove that the so-called Solow-Krugman controversy is not a real one.

Let's revisit the Solow model, from equation (1.4)we have:

$$\forall t,\ K_{t+1} = (1-\delta)K_t + saK_t^{\alpha}L_0^{1-\alpha}(1+\gamma)^t(1+n)^{t(1-\alpha)} \tag{1.5}$$

and $\{K_t\}$ converges to $\{K^s(1+g)^t\}$ where g is growth rate of capital stock and output at steady state and $1+g=(1+n)(1+\gamma)^{\frac{1}{1-\alpha}}$ and $K^s = [\frac{sa}{g+\delta}]^{\frac{1}{1-\alpha}}L_0$.

Notice that in Cobb-Douglas technology as defined in (1.2) the growth rate of out put is identical as growth rate of capital. Let's define this growth rate as follows:

$$\nu_t = \frac{K_t}{K_{t-1}}$$

From equation (5.4) we have:

$$\frac{K_t}{K_{t-1}} - (1-\delta) = saL_0^{1-\alpha}(1+\gamma)^{t-1}(1+n)^{(t-1)(1-\alpha)}K_{t-1}^{\alpha-1} \tag{1.6}$$

$$\nu_t - (1-\delta) = (1+\gamma)(1+n)^{1-\alpha}\nu_{t-1}^{\alpha-1}[\nu_{t-1} - (1-\delta] \tag{1.7}$$

Lemma 3 *Let* $\varphi(\nu) = [\nu-(1-\delta)]\nu^{\alpha-1}$ *with* $\nu > 0$ *then* φ *is increasing with* ν.

Proof.

$$\begin{aligned}
\varphi'(\nu) &= \nu^{\alpha-1} + [\nu-(1-\delta)](\alpha-1)\nu^{\alpha-2} \\
&= \nu^{\alpha-2}[\nu + (\alpha-1)\nu + (1-\delta)(1-\alpha)] \\
&= \nu^{\alpha-2}[\alpha\nu + (1-\delta)(1-\alpha) > 0]
\end{aligned}$$

■

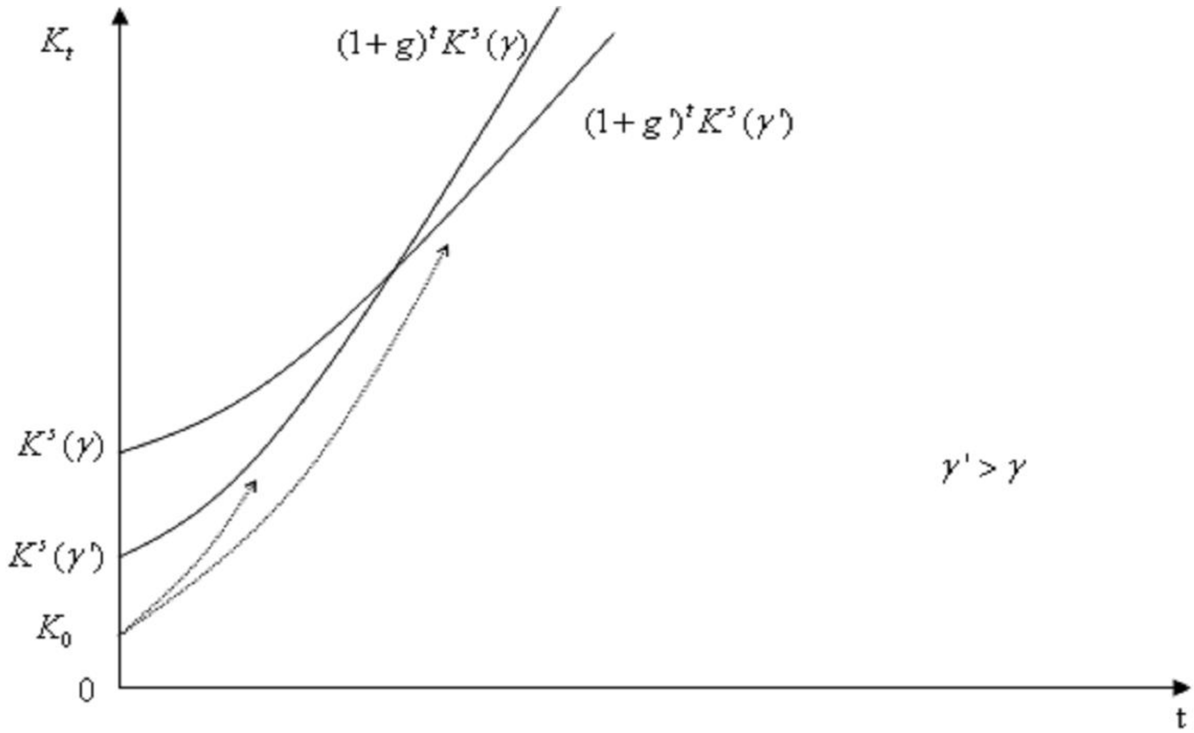
It is easy to check that

$$\begin{aligned} K_1 &= saK_0^{\alpha}L_0^{1-\alpha} + (1-\delta)K_0 \\ \nu_2 &= sa(1+\gamma)K_1^{\alpha-1}L_1^{1-\alpha} + 1 - \delta \end{aligned} \quad (1.8)$$

Lemma (12) and equation (1.8) imply that an increase in rate of technological progress will upgrade the growth rate of output in following periods. Put it differently, an economy with higher rate of technological progress not only has higher growth rate at steady state but also has higher growth rate in transitional period.

Similarly, though saving rate is neutral to growth rate at steady state, in dynamic transition an improvement of saving rate will also speed up the growth rate.

Now let's consider two economies which are identical in everything, except for rates of technological progress and rates of saving. The rates of technological progress and rates of saving in these two economies are (γ, s) and (γ', s') respectively. We assume that $\gamma < \gamma'$ and $s > s'$. It is obvious that: $\upsilon_t \rightarrow 1+g$ and $\upsilon_t' \rightarrow 1+g'$and $g < g'$.Therefore there exists a point T in time such that $\upsilon_t < \nu_t', \forall t \geq T$. In other words, in short run the impact of higher saving rate may be superior to the impact of better productivity $(\upsilon_t > \nu_t')$ however in the long run the better productivity always dominates in economic growth process.



If the economies initially operate below the steady state level (i.e. $K_0 < K^s$) we prove that the economy with higher rate of technological progress also converges faster to its own steady state than the other.

Let's define $\zeta_t = \frac{K_t}{K^s(1+g)^t}$ as speed of convergence, then $0 < \zeta_t < 1$ and $\zeta_t \to 1$ as $t \to \infty$.

Define $\hat{K}_t = \frac{K_t}{(1+g)^t}$ from equation (5.4) we have:

$$\zeta_{t+1} = \frac{1}{1+g}\left[(1-\delta)\zeta_t + saL_0^{1-\alpha}\zeta_t^{\alpha}\frac{1}{(K^s)^{1-\alpha}}\left(\frac{(1+n)^{1-\alpha}(1+\gamma)}{(1+g)^{1-\alpha}}\right)^t\right]$$

Since $1+g = (1+n)(1+\gamma)^{\frac{1}{1-\alpha}}$ and $K^s = [\frac{sa}{g+\delta}]^{\frac{1}{1-\alpha}}L_0$ then

$$\zeta_{t+1} = \frac{1}{1+g}\left[(1-\delta)\zeta_t + (g+\delta)\zeta_t^{\alpha}\right] \tag{1.9}$$

Take partial derivative equation (5.6) by g we get:

$$\frac{\partial \zeta_{t+1}}{\partial g} = \frac{1-\delta}{(1+g)^2}\left(\zeta_t^{\alpha-1} - 1\right) + \frac{\partial \zeta_t}{\partial g}\left(\frac{1-\delta}{1+g} + \frac{g+\delta}{1+g}\alpha\zeta_t^{\alpha-1}\right) \tag{1.10}$$

We can see that the first part of the LHS of equation (5.7) is positive since $0 < \zeta < 1$ hence $\left(\zeta^{\alpha-1} - 1\right) > 0$. Therefore if $\frac{\partial \zeta_t}{\partial g} > 0$ then $\frac{\partial \zeta_{t+1}}{\partial g} > 0$.Recall that $\zeta_0 = \frac{K_0}{K^s} = \frac{K_0}{\left[\frac{sa}{g+\delta}\right]^{\frac{1}{1-\alpha}} L_0}$ and then $\frac{\partial \zeta_0}{\partial g} > 0$. By induction we have $\frac{\partial \zeta_{t+1}}{\partial g} > 0, \forall t \geq 0$,which means that the economy whose rate of technological progress higher (then higher g) will converge faster to its own steady state.

It is easy to check that ξ_1 is negatively related to s, the equation (5.6) implies that ξ_t is negatively related to saving rate s for all t. The higher saving rate helps economy grow faster but converge slower to its own steady state.

Remark 1 *1. In short and medium term (transitional period), the saving rate (hence capital accumulation) does matter for growth rate. A permanent increase in saving rate not only raises the level of steady state but also increases the economic growth rate in transitional period.*

2. In development process, the rate of technological progress is dominant factor in long run. An economy with lower saving rate but higher growth rate of productivity than other can always overrun her contestants in long run.

3. The economy with higher rate of technological progress will converge faster to their own steady states; grow faster not only in steady state but also in transitional period. This result is consistent with findings of King and Rebelo (1993), who run simulations with neo-classical growth models and conclude that the transitional dynamics can only play a minor role in explaining observed growth rates. Furthermore, higher saving rate helps economy grow faster but converge slower to its own steady state.

4. The model also figures out the reason why there is no convergence in economic growth among developing economies (Barro and Sala-i-Martin 2004). The divergence in technological progress and saving rate among developing economies are factors which induce the divergence in development process among developing world.

1.6 Human Capital Growth Model (Lucas, 1988)

We present a simplified version of the Lucas model which is given in Stokey and Lucas (1989), p.111. In this version, there is no physical capital.

The consumption good is produced through a production function using only effective labor. At date t, effective labor is $\theta_t h_t N_t$ with N_t denoting the number of workers at date t and θ_t is the number of working hours. We assume that $N_t = 1,\ \forall t$. We assume that the accumulation of the human capital h is given by

$$h_{t+1} = h_t(1 - \delta + \lambda G(1 - \theta_t))$$

Where G satisfies $G(1) = 1,\ \lambda > 0$, $G(0) = 0$ and G is strictly increasing, continuous. In other words, we assume that without training ($\theta_t = 1$) the human capital depreciates with rate δ and if the worker devotes his whole time for training, his human capital will grow at rate λ. We assume that $\lambda > \delta$, and hence, the maximal rate of growth of human capital $\lambda - \delta$ is positive.

The model is

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t),$$

$$\text{such that } \forall t,\ 0 \leq c_t \leq A(h_t) f(\theta_t h_t),$$

$$h_{t+1} = h_t(1 - \delta + \lambda G(1 - \theta_t)),\ 0 \leq \theta_t \leq 1,$$

and $h_0 > 0$ is given.

We make the following assumptions:

(i) $u(c) = c^\mu,\ 0 < \mu < 1$,

(ii) $\beta > 0$,

(iii) $f(L) = L^\alpha,\ 0 < \alpha < 1$, $A(h) = h^\gamma$, $L = \theta h$,$\theta \in [0, 1]$

(iv) $\beta(1 + \lambda - \delta) < 1$.

We have the following result

Theorem 4 *The optimal path* $(h_t^*)_t$ *is :*

$$\exists u^* \in [1-\delta,\ \lambda-\delta],\ s.t.\ \forall t, h_t^* = h_0(u^*)^t$$

The optimal output is

$$y_t^* = (\theta^*)^\alpha (u^*)^{(\alpha+\gamma)t} h_0^{(\alpha+\gamma)}$$

where θ^* *is determined by*

$$u^* = 1-\delta+\lambda G(1-\theta^*)$$

The TFP $A(h_t^*)$ will growth at rate $(u^*)^\gamma$ which is endogenously determined. The parameter λ may be considered as an indicator of the quality of the human capital technology. The next proposition shows that the quality of the human capital technology will enhance the TFP and hence growth.

Proposition 1 *If* λ *increases then* u^* *increases.*

For a proof see e.g. Gourdel et al (2004).

1.7 The Romer Model (Romer, 1986)

A closed economy is considered. There are S identical consumers. Their preferences are globally represented by an intertemporal utility function $\sum_{t=0}^{+\infty} \beta^t u(c_t)$ where $\beta,\ u$ satisfy the assumptions **H0,H1** in section 1.3. We assume that the consumers own firms. The output of each firm is represented by a function $F(k_t, K_t)$ where k_t is the firm-specific knowledge at time t and K_t is the economy-wide knowledge at date t. At equilibrium we have $K_t = Sk_t$. We assume

F1: F is concave with respect to the first variable

F2: $F(k, Sk)$ is convex in k

By investing an amount I_t we obtain an additional knowledge $k_{t+1} - k_t = G(I_t, k_t)$. Assume

F3: G is concave and homogeneous of degree one.

Then

$$\frac{k_{t+1}-k_t}{k_t} = G(\frac{I_t}{k_t},1) = g(\frac{I_t}{k_t})$$

where $g(x) = G(x,1)$. Assume

F4: $g(0) = 0, g'(0) = +\infty, g'(x) > 0,\ \forall x$

For simplicity, we assume $S = 1$. Let $\mathcal{F}(k) = f(k,k)$. The problem becomes:

$$\begin{aligned} \text{Maximize} \qquad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \frac{k_{t+1}-k_t}{k_t} \quad \leq \quad & g(\frac{\mathcal{F}(k_t)-c_t}{k_t}),\ k_0 > 0 \text{ is given} \end{aligned}$$

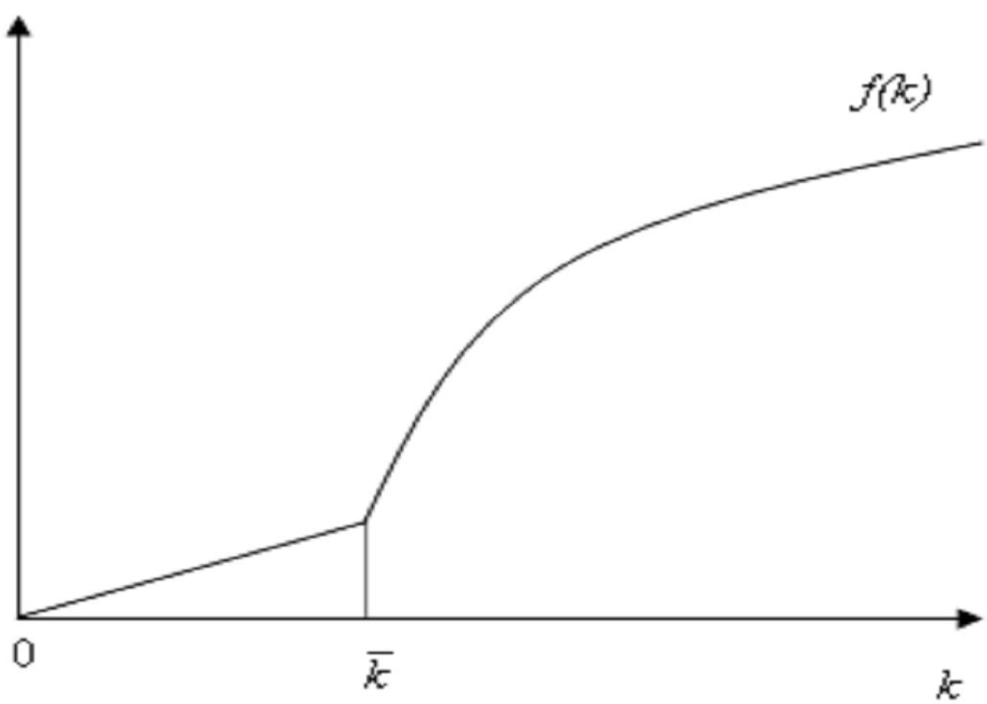
Assume

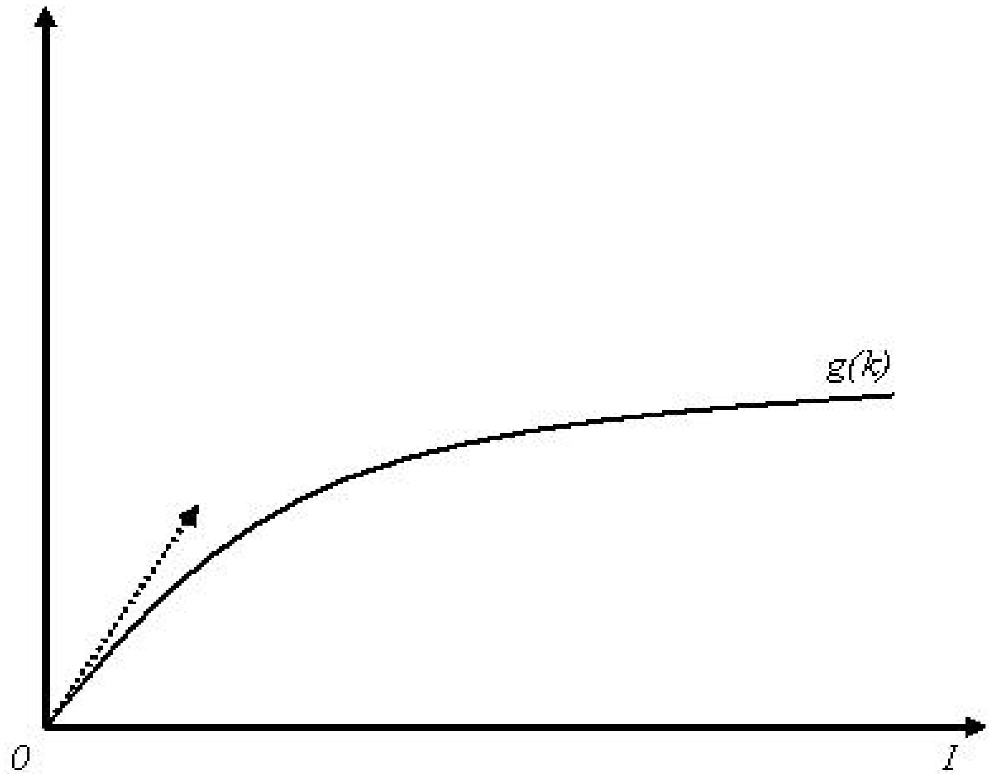
F5: $\mathcal{F}(k) \leq \mu + k^\rho,\ \rho > 1$, and $\mathcal{F}$ is C^1

F6: $0 \leq g(x) \leq \alpha, \forall x$

F7: $0 < \beta < 1$ and $\beta(1+\alpha)^\rho < 1$

We have the following result, the proof of which may be found in Le Van et al., 2002.





Theorem 5 *There exists an optimal path with grows without bound.*

This result is based on many crucial ingredients: (i) the private technology $f(.,K)$ is concave, the quality of the knowledge technology is very good ($g'(0) = +\infty$). Le Van and Saglam (2004) weaken these assumptions:

$\mathbf{F1'}$: $F(k,K) = f(k)h(K)$ where $f(k) = \delta k$ if $k \leq \overline{k}$, $f(k) = A + k^{\mu},\ 0 < \mu < 1$ if $k \geq \overline{k}$, $h(K) = K^{\rho},\ \rho > 0$

$\mathbf{F4'}$: $g(0) = 0, g'(0) = \lambda < +\infty, g'(x) > 0,\ \forall x$

We have the following result

Theorem 6 *1. Let $\lambda > 0$ be given. There exists k_c such that if $k_0 < k_c$ any optimal path $\{k_t\}$ will satisfy $k_t = k_0,\ \forall t$. If $k_0 > k_c$ then for any optimal path $\{k_t\}$ we have $k_t \to +\infty$.*
2. Given $k_0 > 0$, if the quality of knowledge technology increases (λ increases) then the tendency of the economy to take off will increase.
3. Given k_0 and λ, if the influence of fixed costs diminishes (i.e. δ increases or $\overline{k}$ decreases) then the tendency of the economy to take off will increase.

These results point out two factors: fixed costs in the production induce a poverty trap. The latter may be passed over if the quality of knowledge technology is good enough.

1.8 Conclusion

In development process physical capital accumulation can be a primary engine for economic growth in (perhaps prolonged) transitional period. During this period TFP may play a modest role and high rate of investment (saving) explains lion's share of high economic growth rate. However, in the long-run the role of high investment rate eventually fades out and growth can only sustained by improvement of TFP. Krugman, among others, was right when judged that East Asia' growth must slow down in future because of what he characterized as an excessive reliance on capital accumulation. However this pessimistic view may be not the case if after having crossed some developmental thresholds these economies start investing in human capital and new technology. Our examination on economic growth processes of developing countries and some East Asian economies supports our view. The improvement of TFP essentially requires investment in human capital, or new technology or both. The economy which possesses better quality of human capital and new technology will have higher TFP growth thus, grows faster not only in transitional period but also in the long term. Furthermore, the differences in qualities of human capital and new technology cause different rates of TFP. Accordingly the qualities of human capital and new technology are a good explanation for economic divergences among economies in the world.We also show that the presence of fixed costs may delay the growth process.

Chapter 2

New technology, Human capital and Growth for Developing Economies

2.1 Introduction

Technology and adoption of technology have been important subjects of research in the literature of economic growth in recent years. Sources of technical progress might be domestic or/and international though there always exists believes amongst economic professionals that there is an important difference between developed and developing countries, i.e. the first one innovates and exports technology while the second one imports and copies[1]. For developing countries, the adoption of technology from international market is vital since it might be the only way for them to improve their productivity growth and technical progress (Romer (1997, 1990)). But it is even more important to stress that these countries also need to care about their human capital (Lucas (1988)) which might be the key factor that determines whether a country, given their level of development, can take off or might fall into poverty trap.

This line of argument comes from the fact that the developing countries today are facing a dilemma of whether to invest in physical, technological, and human capital. As

[1] See among others: Baumol (1986), Dowrick and Nguyen (1989), Gomulka (1991), Young (1995), Lall (2000), Lau & Park (2003)), Barro and Sala-i-Martin (2004).

abundantly showed in literature (e.g. Barro (1997), Barro & Sala-i-Martin (2004), Eaton & Kortum (2000), Keller (2001), Kumar (2003), Kim & Lau (1994), Lau & Park (2003)) developing countries are not convergent in their growth paths and in order to move closer to the world income level, a country needs to have a certain level in capital accumulation.

Galor and Moav (2004) consider the optimisation of investment in physical capital and human capital on the view of suppliers (of capital). They assumed that technology of human capital production is not extremely good so that at initial stage of development when the physical capital is rare, rate of return to physical capital is higher than the return to human capital. Accordingly, at initial stage of development it is not optimal to invest in human capital but in physical capital. The accumulating physical capital progressively reduces rate of return to physical capital whereas increases rate of return to human capital. Consequently, there is some point in time investment into human capital becomes justified, then human capital accumulation gradually replaces physical capital accumulation as the main engine of growth.

Other than Galor and Moav (2004) we consider the optimal investments in human capital and physical capital on the demand (of capital) side. Furthermore, in Galor and Moav (2004) the source of growth is intergenerational transfer which has a threshold with respect to investment. In Bruno *et al.* (2008) and in this paper the source of growth is the ability of TFP generation which also has a threshold with respect to new technology input.

In their recent work, Bruno *et al.* (2008) point out the conditions under which a developing country can optimally decide to either concentrate their whole resources on physical capital accumulation or spend a portion of their national wealth to import technological capital. These conditions are related to the nation's stage of development which consists of level of wealth and endowment of human capital and thresholds at which the nation might switch to another stage of development. However, in their model, the role of education that contributes to accumulation of human capital and efficient use of technological capital is not fully explored[2].

[2] Verspagen (1991) testifies the factors that affect an economy's ability to assimilate knowledge spillovers in the development process and empirically shows that the education of the labor force is the most

In this paper we extend their model by introducing an educational sector with which the developing country would invest to train more skilled labors. We show that the country once reaches a critical value of wealth will have to consider the investment in new technology. At this point, the country can either go on with its existing production technology or improve it by investing in new technology capital in order to produce new technology. As soon as the level of wealth passes this value it is always optimal for the country to use new technology which requires high skilled workers. We show further that with possibility of investment in human capital and given "good" conditions on the qualities of the new technology, production process, and/or the number of skilled workers there exists alternatives for the country either to invest in new technology and spend money in training high skilled labor or only invest in new technology but not to spend on formation of human capital. Following this direction, we can determine the level of wealth at which the decision to invest in training and education has to be made. In this context, we can show that the critical value of wealth is inversely related to productivity of the new technology sector, number of skilled workers, and spill-over effectiveness of the new technology sector on the consumption goods sector but proportionally related to price of the new technology capital. In the whole, the paper allows us to determine the optimal share of the country's investment in physical capital, new technology capital and human capital formation in the long-run growth path. It is also noteworthy to stress that despite of different approach, our result on the replacement of physical capital accumulation by human capital accumulation in development process consist with those of Galor and Moav (2004).

Two main results can be pointed out: (1) the richer a country is, the more money will be invested in new technology and training and education, (2) and more interestingly, the share of investment in human capital will increase with the wealth while the one for physical and new technology capitals will decrease. In any case, the economy will grow without bound. Another point which makes our paper different from Bruno *et al.* (2008): we will test the main conclusions of our model with empirical data.

The paper is organized as follows. Section 5.3 is for the presentation of the one period

prominent one. (See also Baumol et al., 1989, on this matter) model and its results. Section 2.3 deals with the dynamic properties in a model with an infinitely lived representative consumer. Section 2.4 will look at some empirical evidences in some developing and emerging countries, particularly China, Korea and Taiwan. The conclusion is in Section 2.5. Appendices are in Sections 2.6, 2.7, 2.8. They are for the mathematical proofs, and for the tables on Inputs and Technical Progress in Lau and Park (2003).

2.2 The Model

Consider an economy where exists three sectors: domestic sector which produces an aggregate good Y_d, new technology sector with output Y_e and education sector characterized by a function $h(T)$ where T is the expenditure on training and education. The output Y_e is used by domestic sector to increase its total productivity. The production functions of two sectors are Cobb-Douglas, i.e., $Y_d = \Phi(Y_e)K_d^{\alpha_d}L_d^{1-\alpha_d}$ and $Y_e = A_e K_e^{\alpha_e} L_e^{1-\alpha_e}$ where $\Phi(.)$ is a non decreasing function which satisfies $\Phi(0) = x_0 > 0$, K_d, K_e, L_d, L_e and A_e be the physical capital, the technological capital, the low-skilled labor, the high-skilled labor and the total productivity, respectively, $0 < \alpha_d < 1, 0 < \alpha_e < 1$.[3]

We assume that price of capital goods is numeraire in term of consumption goods. The price of the new technology sector is higher and equal to λ such that $\lambda \geq 1$. Assume that labor mobility between sectors is impossible and wages are exogenous.

Let S be available amount of money for spending on capital goods and human capital. We have:

$$K_d + \lambda K_e + p_T T = S.$$

For simplicity, we assume $p_T = 1$, or in other words T is measured in capital goods.

Thus, the budget constraint of the economy can be written as follows

$$K_d + \lambda K_e + T = S$$

[3]This specification implies that productivity growth is largely orthogonal to the physical capital accumulation. This implication is confirmed by facts examined by Collins, Bosworth and Rodrik (1996), Lau and Park (2003)

where S be the value of wealth of the country in terms of consumption goods.

The social planner maximizes the following program

$$\max \ Y_d = \text{Max} \ \Phi(Y_e) K_d^{\alpha_d} L_d^{1-\alpha_d}$$

subject to

$$\begin{aligned}
& Y_e = A_e K_e^{\alpha_e} L_e^{1-\alpha_e}, \\
& K_d + \lambda K_e + T = S, \\
& 0 \leq L_e \leq L_e^* h(T), \\
& 0 \leq L_d \leq L_d^*.
\end{aligned}$$

Where h is the human capital production technology; L_e^* is number of skilled workers in new technology sector; L_e is effective labor; L_d^* is number of non-skilled workers in domestic sector.

Assume that $h(.)$ is an increasing concave function and $h(0) = h_0 > 0$ or Y_d is a concave function of education investment[4]. Let

$$\Delta = \{(\theta, \mu) : \theta \in [0,1], \mu \in [0,1], \theta + \mu \leq 1\}.$$

From the budget constraint, we can define $(\theta, \mu) \in \Delta$:

$$\lambda K_e = \theta S \ , K_d = (1 - \theta - \mu) S \text{ and } \ T = \mu S.$$

Observe that since the objective function is strictly increasing, at the optimum, the

[4]This assumption captures the fact that marginal returns to education is diminishing (see Psacharopoulos, 1994)

constraints will be binding. Let $L_e = L_e^* h$, $L_d = L_d^*$, then we have the following problem

$$\underset{(\theta,\mu)\in\Delta}{\text{Max}}\, \Phi(r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e})(1-\theta-\mu)^{\alpha_d} S^{\alpha_d} L_d^{*1-\alpha_d}.$$

where $r_e = \frac{A_e}{\lambda^{\alpha_e}} L_e^{*1-\alpha_e}$.

Let

$$\psi(r_e, \theta, \mu, S) = \Phi(r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e})(1-\theta-\mu)^{\alpha_d} L_d^{*1-\alpha_d}.$$

The problem now is equivalent to

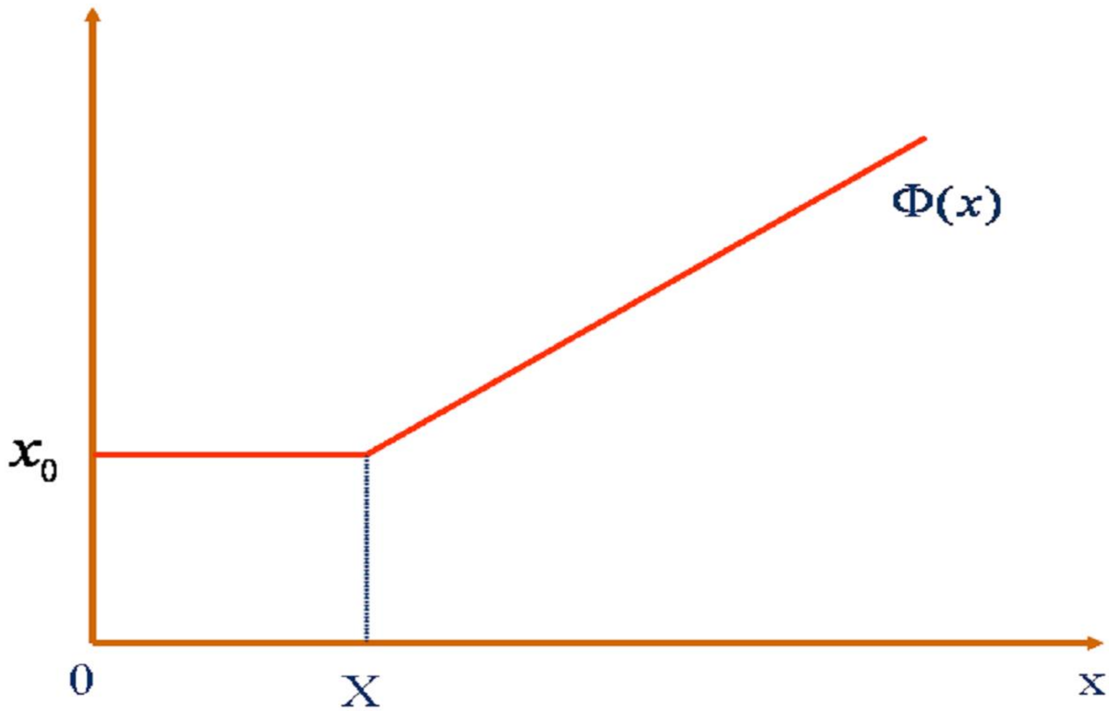
$$\underset{(\theta,\mu)\in\Delta}{\text{Max}}\, \psi(r_e, \theta, \mu, S). \tag{P}$$

Since the function ψ is continuous in θ and μ, there will exist optimal solutions. Denote

$$F(r_e, S) = \underset{(\theta,\mu)\in\Delta}{\text{Max}}\, \psi(r_e, \theta, \mu, S).$$

Suppose that function $\Phi(x)$ is a constant in an initial phase and increasing linear afterwards:

$$\Phi(x) = \begin{cases} x_0 \text{ if } x \leq X \\ x_0 + a(x - X) \text{ if } x \geq X, a > 0. \end{cases}$$



Then by Maximum Theorem, F is continuous and $F(r_e, S) \geq x_0 L_d^{*1-\alpha_d}$. The following proposition states that there exists a threshold.

Proposition 2 *There exists S^c such that, if $S < S^c$ then $\theta(S) = 0$ and $\mu(S) = 0$, and if $S > S^c$ then $\theta(S) > 0$.*

Proof. See appendix 1. ■

Remark 2 *If $S > S^c$ then $Y_e > X$ and $\Phi(Y_e) = x_0 + a(Y_e - X)$*

The following proposition shows that, when the quality of the training technology (measured by the marginal productivity at the origin $h'(0)$) is very high then for any $S > S^c$ the country will invest both in new technology and in human capital. When $h'(0)$ is finite, we are not ensured that the country will invest in human capital when $S > S^c$. But it will do if it is sufficiently rich. Moreover, if $h'(0)$ is low, then the country will not invest in human capital when S belongs to some interval $(S^c,\ S^m)$.

Proposition 3 *1. If $h'(0) = +\infty$, then for all $S > S^c$, we have $\theta(S) > 0, \mu(S) > 0$.*
2. Assume $h^{'}(0) < +\infty$. Then there exists S^M such that $\mu(S) > 0, \theta(S) > 0$ for every

$S > S^M$.

3. There exists $\alpha > 0$ such that, if $h'(0) < \alpha$, then there exists $S^m > S^c$ such that $\mu(S) = 0, \theta(S) > 0$ for $S \in [S^c, S^m]$.

Proof. See Appendix 1. ■

The following proposition states there exists a threshold for both $\theta(S)$ and $\mu(S)$ to be positive.

Proposition 4 *Assume $h'(0) < +\infty$. Then there exists $\widehat{S} \geq S^c$ such that:*

(i) $S \leq \widehat{S} \Rightarrow \mu(S) = 0$,

(ii) $S > \widehat{S} \Rightarrow \mu(S) > 0$, $\theta > 0$.

Proof. See Appendix 1. ■

Let us recall $r_e = \frac{A_e L_e^{*(1-\alpha_e)}}{\lambda^{\alpha_e}} = A_e L_e^*(L_e^*\lambda)^{-\alpha_e}$ where A_e is the productivity of the new technology sector, λ is the price of the new technology capital, α_e is capital share in new technology production sector, and L_e^* is number of skilled workers.

Recall also the productivity function of the consumption goods sector $\Phi(x) = x_0 + a(x - X)$ if $x \geq X$. The parameter $a > 0$, a spill-over indicator which embodies the level of social capital and institutional capital in the economy, indicates the effectiveness of the new technology product x on the productivity. We will show in the following proposition that the critical value S^c diminishes when r_e increases, i.e. when the productivity A_e, and/or the number of skilled workers increase; and /or the price of the new technology capital λ decreases; and/or the share of capital in new technology sector α_e decreases (more human-capital intensive); and /or the spill-over indicator a increases. Put it differently, the following conditions will be favorable for initiating investment in to new technology sector: (i) potential productivity in new technology sector; (ii) number of skilled workers in the economy; (iii) price of new technology; (iv) the intensiveness of human capital in new technology sector; and (v) level of spill-over effects. Except for price of new technology, if all or one of the above-mentioned conditions are/is improved, the economy will be more quickly to initiate investment in new technology sector.

Proposition 5 *Let* $\theta^c = \theta(S^c)$, $\mu^c = \mu(S^c)$. *Then*

(i) $\mu^c = 0$, θ^c *does not depend on* r_e.

(ii) S^c *decreases if* a *or/and* r_e *increases.*

Proof. See Appendix 1. ■

The following proposition shows that the optimal shares θ, μ converge when S goes to infinity. Furthermore the ratios of spendings on human capital to S and of the total of spendings on new technology capital and human capital formation to S increase when S increases.

Proposition 6 *Assume* $h(z) = h_0 + bz$, *with* $b > 0$. *Then the optimal shares* $\theta(S), \mu(S)$ *converge to* $\theta_\infty, \mu_\infty$ *when* S *converges to* $+\infty$. *Consider* $\widehat{S}$ *in Proposition 4. Then*

(i) Assume $x_0 < aX$. *If* ar_e *is large enough, then* $\mu(S)$ *and the sum* $\theta(S)+\mu(S)$ *increase when* S *increases.*

(ii) If $x_0 \geq aX$, *then* $\mu(S)$ *and the sum* $\theta(S) + \mu(S)$ *increase when* S *increases.*

Proof. For short, write θ, μ instead of $\theta(S), \mu(S)$. Consider $\widehat{S}$ in Proposition 4. When $S \leq \widehat{S}$,then $\mu = 0$ (Proposition 4).

When $S > \widehat{S}$. Then (θ, μ) satisfy equations (2.10) and (2.11) which can be written as follows:

$$\theta(\alpha_d + \alpha_e) = -\alpha_e\mu + [\alpha_e - \frac{\alpha_d(x_0 - aX){\alpha_e}^{-\alpha_e}}{ar_eS(1-\alpha_e)^{1-\alpha_e}b^{1-\alpha_e}}] \tag{2.1}$$

and

$$\theta(1-\alpha_e) = \alpha_e\mu + \frac{\alpha_e h_0}{bS} \tag{2.2}$$

We obtain

$$\theta(1+\alpha_d) = [\alpha_e - \frac{\alpha_d(x_0 - aX){\alpha_e}^{-\alpha_e}}{ar_eS(1-\alpha_e)^{1-\alpha_e}b^{1-\alpha_e}} + \frac{h_0\alpha_e}{bS}] \tag{2.3}$$

and

$$\mu = \theta(\frac{1}{\alpha_e} - 1) - \frac{h_0}{bS}$$

Thus

$$\theta + \mu = \frac{1}{1+\alpha_d}[1 - \frac{\alpha_d}{\alpha_e}\frac{(x_0 - aX){\alpha_e}^{-\alpha_e}}{ar_eS(1-\alpha_e)^{1-\alpha_e}b^{1-\alpha_e}}] - \frac{\alpha_d}{1+\alpha_d}\frac{h_0}{bS}.$$

Tedious computations give

$$\frac{\mu}{1-\alpha_e} = \frac{1}{1+\alpha_d}[1 - \frac{\alpha_d}{\alpha_e}\frac{(x_0 - aX)\alpha_e{}^{-\alpha_e}}{ar_eS(1-\alpha_e)^{1-\alpha_e}b^{1-\alpha_e}}] - [\frac{\alpha_d}{1+\alpha_d} + \frac{\alpha_e}{1-\alpha_e}]\frac{h_0}{bS}$$

If $x_0 \geq aX$, then $\theta + \mu$ and μ increase with S. If $x_0 < aX$, then when ar_e is large enough, then $\theta + \mu$ and μ are increasing functions in S.

When S converges to $+\infty$, then θ converges to $\theta_\infty = \frac{\alpha_e}{1+\alpha_d}$ and μ converges to $\mu_\infty = \frac{1-\alpha_e}{1+\alpha_d}$.

■

2.3 The Dynamic Model

In this section, we consider an economy with one infinitely lived representative consumer who has an intertemporal utility function with discount factor $\beta < 1$. At each period, her savings will be used to invest in physical capital or/and new technology capital and/or to invest in human capital. We suppose the capital depreciation rate equals 1 and growth rate of population is 0 and $L^*_{e,t} = L^*_e; L^*_{d,t} = L^*_d$.

The social planner will solve the following dynamic growth model

$$\begin{aligned}
\max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{s.t} \quad & c_t + S_{t+1} \leq \Phi(Y_{e,t}) K_{d,t}^{\alpha_d} L_{d,t}^{1-\alpha_d} \\
& Y_{e,t} = A_e K_{e,t}^{\alpha_e} L_{e,t}^{1-\alpha_e} \\
& K_{d,t} + \lambda K_{e,t} + T_t = S_t, \\
& 0 \leq L_{e,t} \leq L^*_e h(T_t), 0 \leq L_{d,t} \leq L^*_d. \\
& \text{ the initial resource } S_0 \text{ is given.}
\end{aligned}$$

The problem is equivalent to

$$\begin{aligned}
\max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\
\text{s.t} \quad & c_t + S_{t+1} \leq H(r_e, S_t), \forall t,
\end{aligned}$$

with

$$H(r_e, S) = F(r_e, S)S^{\alpha_d}.$$

where $r_e = \frac{A_e}{\lambda^{\alpha_e}} L_e^{*1-\alpha_e}$, β is time preference discount rate $0 \leq \beta \leq 1$ Obviously, $H(r_e, .)$ is continuous, strictly increasing and $H(r_e, 0) = 0$.

As in the previous section, we shall use S^c defined as follows:

$$S^c = \max\{S \geq 0 : F(r_e, S) = x_0 L_d^{*1-\alpha_d}\}$$

where

$$F(r_e, S_t) = \underset{0 \leq \theta_t \leq 1, 0 \leq \mu_t \leq 1}{\text{Max}} \psi(r_e, \theta_t, \mu_t, S_t).$$

We shall make standard assumptions on the function u under consideration.

H2. The utility function u is strictly concave, strictly increasing and satisfies the Inada condition: $u^{'}(0) = +\infty, u(0) = 0, u'(\infty) = 0$.

At the optimum, the constraints will be binding, the initial program is equivalent to the following problem

$$\max \sum_{t=0}^{\infty} \beta^t u(H(r_e, S_t) - S_{t+1})$$
$$\text{s.t} \quad 0 \leq S_{t+1} \leq H(r_e, S_t), \forall t.$$
$$S_0 > 0 \text{ given.}$$

By the same arguments as in Bruno *et al.* (2008), we have the following property

Proposition 7 *i) Every optimal path is monotonic*

ii) Every optimal trajectory (S_t^*) *from* S_0 *can not converge to* 0.

Let denote θ_t^*, μ_t^* be the optimal capital shares among technological capital stock and expenditure for human capital,

$$\lambda K_{e,t}^* = \theta_t{}^* S_t^* \text{ and } T_t^* = \mu_t^* S_t^*.$$

We then obtain the main result of this paper:

Proposition 8 *Assume* $h(z) = h_0 + bz$, *with* $b > 0$ *and* $\alpha_e + \alpha_d \geq 1$. *If* a *or/and* r_e *are large enough then the optimal path* $\{S_t^*\}_{t=1,+\infty}$ *converges to* $+\infty$ *when* t *goes to infinity. Hence:*

(i) there exists T_1 *such that*

$$\theta_t^* > 0 \ \forall t \geq T_1$$

(ii) there exists $T_2 \geq T_1$ *such that*

$$\theta_t^* > 0 \ , \mu_t^* > 0, \ \forall t \geq T_2$$

The sum $\theta_t^* + \mu_t^*$ *and the share* μ_t^* *increase when* t *goes to infinity and converge to values less than* 1.

Proof. See Appendix 2. ■

2.4 A Look At Evidence

There are numerous discusses in literature on the role of physical capital, human capital and technological progress in economic growth. King and Rebelo (1993) run simulations with neoclassical growth models and conclude that the transitional dynamics (contribution of physical capital accumulation) can only play a minor role in explaining observed growth rates. They suggest endogenous growth models such as human capital formation or endogenous technical progress. Hofman (1993) examines economic performances of Latin American countries, three Asian economies (S. Korea, Taiwan and Thailand), Portugal, Spain and six advanced economies (France, Germany, Japan, The Netherlands, UK and US) in the 20th century. The evidences show that growth in developing economies bases mainly on physical capital accumulation while growth in developed economies motivated essentially by human capital and technological progress. Young (1994), Kim and Lau (1994), Krugman (1994), Collins and Bosworth (1996) and Lau and Park (2003) all attribute the miracle growth in East Asia Economies mostly to physical capital accumulation and find no significant role of technological progress in miracle growth of East Asia Economies, which plays a crucial role in economic growth in Industrial Economies (see Table 2 in Appendix 3). Collins and Bosworth (1996) suggests "it is possible that the potential to adopt knowledge and technological from abroad depends on a country's stage of development. Growth in the early stages may be primarily associated with physical and human capital accumulation, and significant potential for growth through catchup may only emerge once a country has crossed some development thresholds". Lau and Park (2003) on the one hand, shows that the hypothesis of no technological progress in East Asia NIEs until 1986 can not rejected. On the other hand, since 1986 when these economies started investing heavily on R&D, technological progress plays significant role in growths of these economies. This evidence supports our model's prediction that there exists threshold for investing in new technology in process of economic development. Nevertheless, the question of threshold of investment in human capital is rarely raised in literature.

In this section we use pooled time-series aggregate data of educational attainment for 71 non-oil exporting, developing economies compiled by Barro and Lee (2000)[5] and real GDP per capita (y) (in PPP) of these countries in Penn World table 6.2, Heston, *et al.*, (2006) to find the correlation between human capital and level of development. In Barro and Lee (2000) we use five variables to measure human capital: percentage of labor force with completed primary school (l_1); with completed secondary school (l_2); with completed higher secondary school (l_3); and average schooling years of labor force (A). Those data are calculated for 5-year span from 1950 (if available) to 2000. Oil exporting countries are excluded from the sample because they enjoy peculiarly high level of GDP per capita regardless of production capacity of non-oil sectors. Some other developing countries whose data of human capital are available for two years also excluded.

[5] See Table 3 in appendix for list of economies

We run two simple OLS regression equations

$$\ln y = \alpha + \beta_1 l_1 + \beta_2 l_2 + \beta_3 l_3 \tag{2.4}$$

and

$$\ln y = \alpha + \gamma_1 A \tag{2.5}$$

These equations are tested for two sub-samples: the first with GDP per capita is less than 1000 (75 observations); and the second with GDP per capita more than 1000 (533 observations). The results are presented in table 1 below and show that when GDP per capita below 1000 USD (y in PPP and constant price in 2000) all hypotheses of no contribution of human capital to economic growth can not be rejected, while when $y > 1000$ those hypotheses are decisively rejected

Table 1: **Contributions of human capital to economic growth**

	Equation 2.4		**Equation 2.5**	
	$y \leq 1000$	$y > 1000$	$y \leq 1000$	$y > 1000$
R^2	4.7%	**46.6%**	2.1%	**54.3%**
$\overline{R^2}$	0.7%	**46.3%**	0.75%	**54.2%**
β_1	−0.015 (0.08)	0.002 (**0.000**)*		
β_2	0.002 (0.88)	0.050 (**0.000**)*		
β_3	0.040 (0.63)	0.042 (**0.000**)*		
γ_1			−0.03 (0.22)	0.25 (**0.000**)*
Obs	75	533	75	533

Note: the numbers in the parentheses are p-values of corresponding coefficients;

* Indicates statistically significant at the level of significance of 0.1%

Furthermore, when $y > 1000$ coefficients of variables: percentage of labor force with completed primary school (l_1), completed secondary school, and completed higher secondary school are all in expected sign and statistically significant at level of significance of 0.1%. The results of regression on equation (2.5) also solidly confirms the positive contribution of human capital when it is measured by average year of schoolings.

By contrast, when $y \leq 1000$, the values of adjusted R-square in both equations are nearly zero. There is no coefficient is statistically significant at level of significance of 5%. These results imply that human capital, by all means, plays no role in economic growth. Put it differently, they support our model's prediction that when income is lower than a critical level there is no demand for investing in human capital, or equivalently, there exists threshold for investing in human capital in process of development.

In the following we look closely at movement of expenditures on human capital and new technology in three economies, namely China, South Korea and Taiwan. The reasons to choose these economies are: (i) the availability of data; (ii) these economies have experienced high growth rates for long time from very low stage. The purpose of this section is to examine the our third point, that is the share of human capital and expenditure for new technology in total investment (S) in these economies shows the increasing trend in the examined periods and human capital increasingly becomes more important than two others.

Since the data for expenditure on human capital is not directly available, hence we follow Carsey and Sala-i-Martin (1995) to assume that wage paid to a worker consists of two parts: one for human capital and the other (non-skilled wage) for other things other than human capital. According to Carsey and Sala-i-Martin (1995) the latter part of wage depends on many factors such that: ratio of aggregate physical capital stock to human capital due to the complementary between physical capital and human capital; and change in relative supplies of workers. The former part depends not only on number of schooling years but also on others: on-the-job training, job experience, schooling quality, and technological level. Accordingly, this labor-income-based human capital that taking all these factors into account reflects the value of human capital more comprehensively than the conventional measurement that based on schooling years.

We assume further that minimum wage is the non-skilled wage. Consequently the expenditure for human capital can be calculated by following formula:

$$EHC_t = E_t * (AW_t - MW_t)$$

Where EHC is expenditure for human capital, E is total employed workers, AW is average wage, and MW is minimum wage. Recall that $AW - MW$ represents the part in the average wage which is rewarded for skill.

In our model, the new technological capitals are produced in R&D sector, then we use indicator of expenditure for R&D as a proxy for investment in technological capital (λK_e), and the fixed capital formation (if not available, then the gross capital formation) for expenditure on K_d.

Data

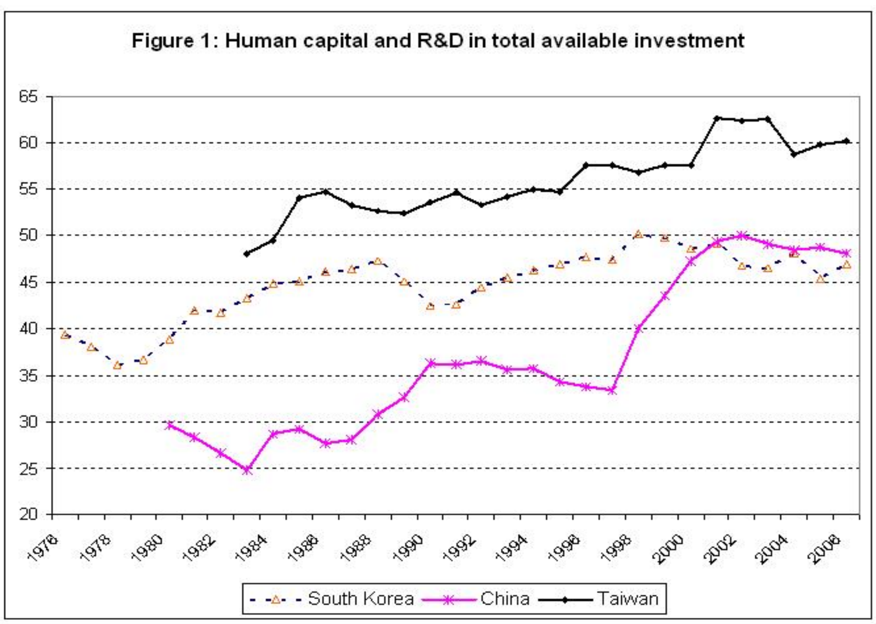
For China, the data of AW, GDP, and E are available in CEIC database from 1952 to 2006. The minimum wages in China vary from provinces and within province. Provinces and cities usually have multiple levels of minimum wage standards based upon different geographic locations and industries. The minimum wages for all provinces were only available discretely in period 2004-2006 from the Ministry of Labor and Social Security of China 2005 statistics[6]. Therefore we use average wage in sector of Farming, Forestry, Animal Husbandry & Fishery where use least human capital and physical capital as a proxy of minimum wage. All entries of this variables can be taken from CEIC database. Based on this series of indices we come up with an estimated time-series national minimum wage in China from 1980 to 2006. Since data of fixed capital formation in China are not available, we then use the data of gross capital formation, which are available in WDI database of World Bank. Finally, the statistics for R&D expenditure in period 1980-2006 are available in China statistical yearbook in various issues.

For Taiwan, the data for total compensation for employees ($E * AW$), employment (E), fixed capital formation, GDP, and average wage in manufacturing sector are available in

[6] Updates are based upon news reports prior to July 2006. Minimum wages listed as monthly-based

CEIC database in period 1978-2006. The minimum wage rates are only available in period 1993-2006 and in 1984 at US Department of State[7]. For missing data in period 1983-1992 we fill in by estimated ones. For that, we assume that minimum wage (MW) is a concave function of average wage in manufacturing sector (AW_m) or more specifically, the ratio of $\frac{MW}{AW_m}$ is linearly correlated with AW_m. The result of OLS regression strongly confirms our hypothesis. Based on coefficients of this OLS regression we come up with the estimations of missing data. The data of R&D expenditure is taken from National Science Council (2007) and Lau and Park (2003).

For South Korea, CEIC database provides data of employment (E), compensations for employees ($E*AW$), fixed capital formation, GDP, and nominal wage index. The minimum wages in period 1988-2006 are taken from GPN (2001) and US State Department website. If we assume that in period 1976-1987 the minimum wages proportionally change with nominal wage index, then we have the estimation of expenditure for human capital in the period 1976-1987. The data for R&D expenditure is taken from UNESCO.



[7]Cited at website: http://dosfan.lib.uic.edu/ERC/economics/commercial_guides/Taiwan.html and http://www.state.gov/g/drl/rls/hrrpt/2006/78770.htm

Figure 2: Share of Human Capital in Total available Investment

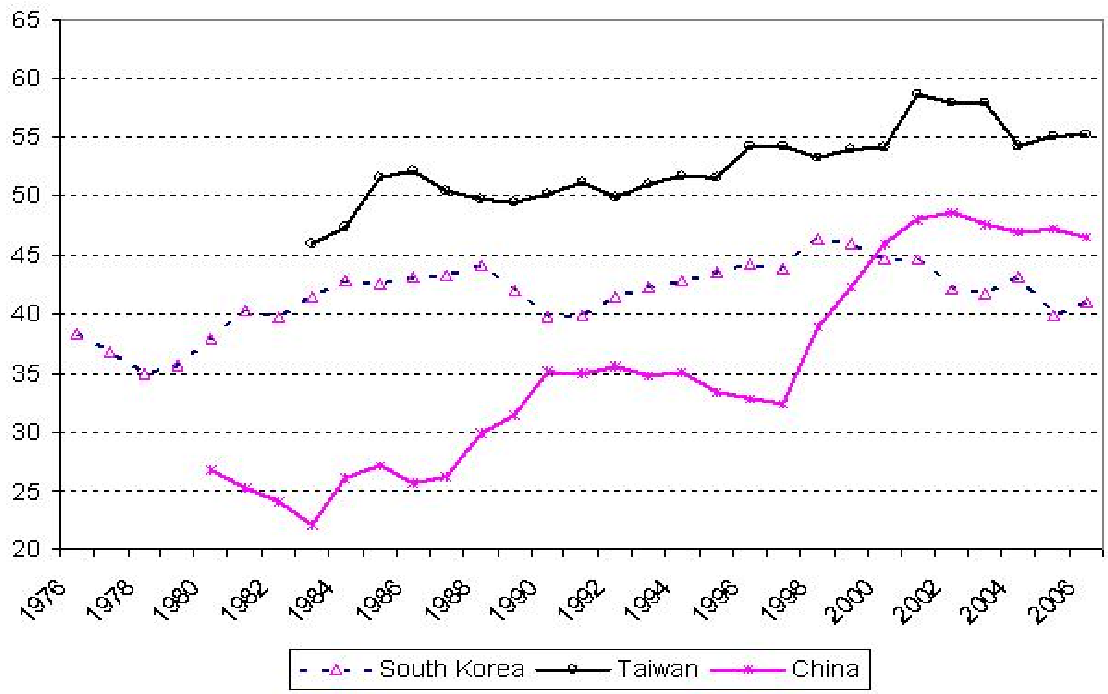
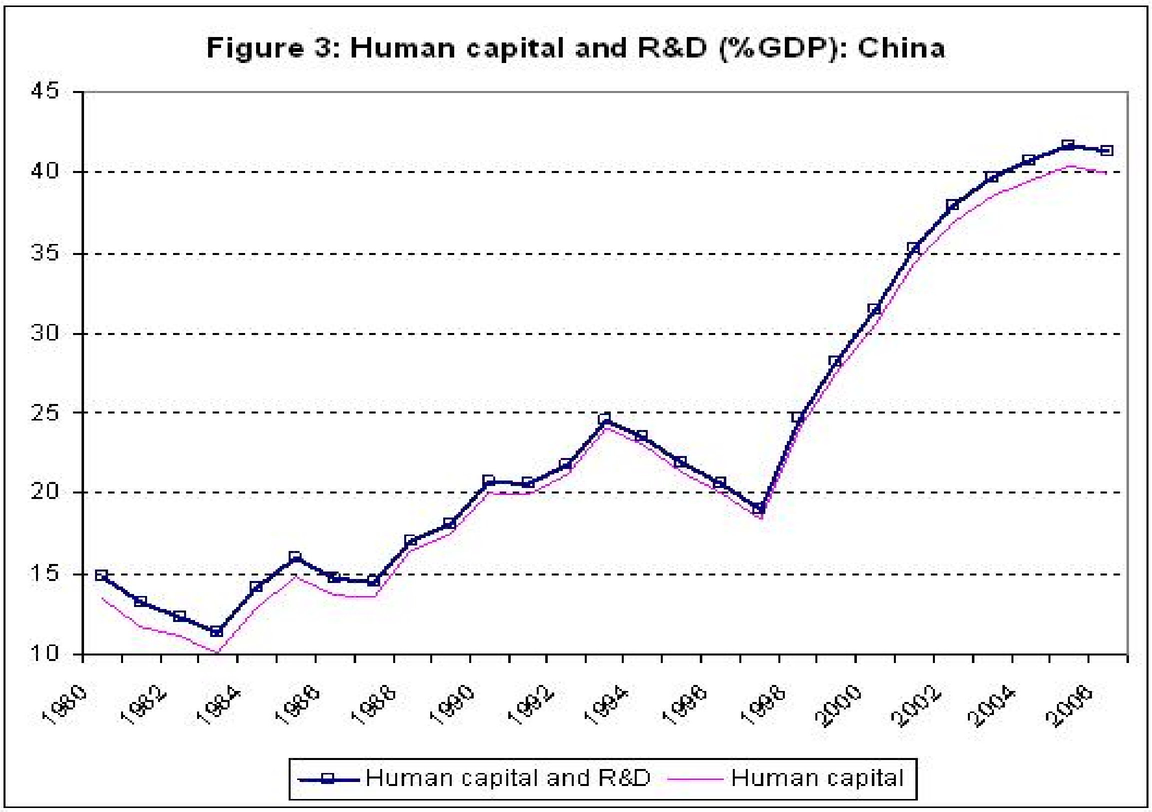
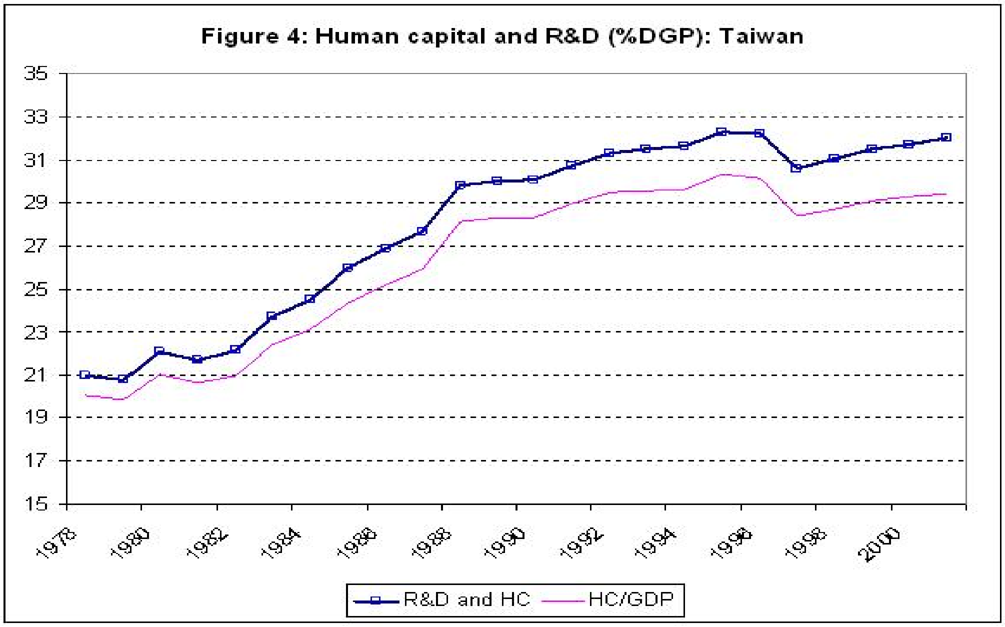


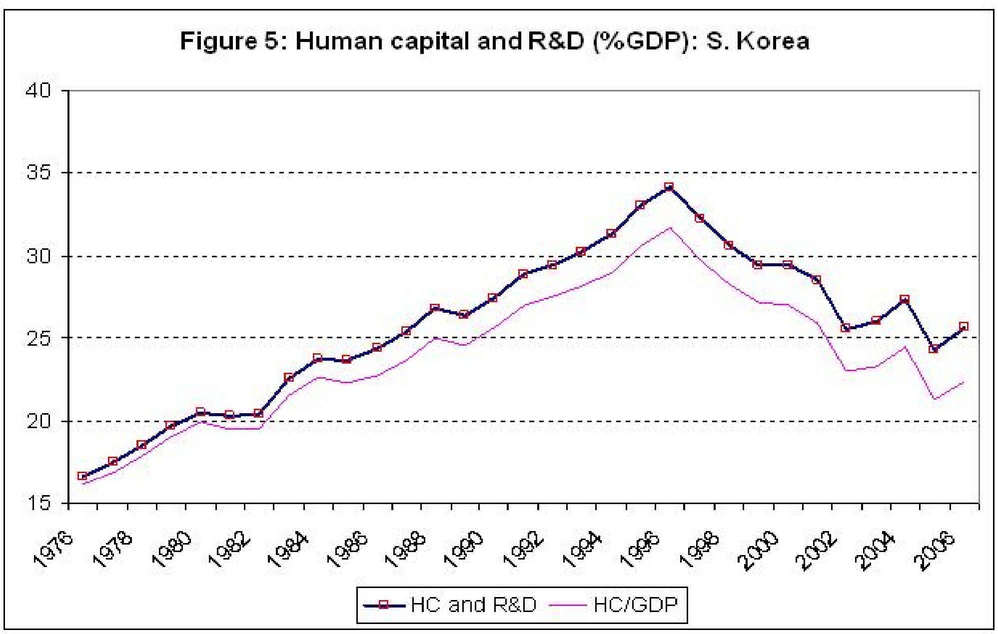
Figure 1 show the steadily increasing trend of shares of human capital and R&D in total available investment in all three economies in the examined periods. The movement of share of human capital in total available investment shown in figure 2 also show steadily increasing trend in Taiwan and China, while in South Korea the trend seems more fluctuant, nevertheless, increasing. Hence our predictions on the movements of the shares of human capital and of new technology on the one hand, and of physical capital on the other hand, cannot be rejected by evidences from these economies.

Let's consider the movements on another dimension. Assuming that the budget available (S) for total investment is positively related to GDP in the whole period. Thereby, the movement of ratios of λK_e and expenditure for human capital (T) to GDP are congruent to the movement of ratios of λK_e and T to S.

Figures below (3,4 and 5) all support our model's prediction, $\mu_t + \theta_t$, the sum of the share of human capital and R&D as well as share of human capital in GDP both increase. The figures also show the effects of Asian crisis in 1997 on investment in human capital and R&D these economies. China is the least affected and then quickly recovered the momentum investing activities. S. Korea, the most affected one and had to have recourse to IMF for help. Under pressure of IMF South Korea had to apply severely tightening expenditure policy. Even though South Korea started recovering since 1999 and GDP recovered high growth rate in following years, they remained tightening expenditure policy till early 2000s. That's why the figure 5 shows the declining trend of both variables, shares of human capital and R&D, and of human capital, since 1997.







2.5 Conclusion

We first summarize the main conclusions from our model.

1. At low level of economic growth this country would only invest in physical capital but

when the economy grows this country would need to invest not only in physical capital but also in first, new technology and then, formation of high skilled labor.

2. Under some mild conditions on the quality of the new technology production process and on the supply of skilled workers, the shares of the investments, respectively in human capital, and in new technology and human capital, will increase when the country becomes rich.

3. Thanks to New Technology and Human Capital, the TFP will increase and induces a growth process, i.e. the optimal path (S_t^*) converges to $+\infty$. In other words, the country grows without bound. In this case, the share of investment in new technology and human capital $(\theta_t^* + \mu_t^*)$ will increase while the one in physical capital will decrease. More interestingly, and in accordance with the results in Barro and Sala-i-Martin (2004), the share μ_t^* will become more important than the one for physical and new technology capitals when t goes to infinity. But they will converge to strictly positive values when time goes to infinity.

Second, the empirical tests seem confirm the results mentioned above.

1. They support our model's prediction that when income is lower than a critical level there is no demand for investing in human capital, or equivalently, there exists threshold for investing in human capital in process of development.

2. Our predictions on the movements of the shares of human capital and of new technology on the one hand, and of physical capital on the other hand, cannot be rejected by evidences from the economies of China, Korea and Taiwan.

2.6 Appendix 1

Proof of Proposition 2 The proof will be done in three steps.

Step 1 Define

$$B = \{S \geq 0 : F(r_e, S) = x_0 L_d^{*1-\alpha_d}\},$$

Lemma 7 *B is a nonempty compact set.*

Proof. It is easy, see e.g. Bruno *et al* (2008). ■

Remark 3 *Observe that* $F(r_e, S) \geq x_0 L_d^{*1-\alpha_d}$*. If the optimal value for* θ *equals* 0 *then the one for* μ *is also* 0 *and* $F(r_e, S) = x_0 L_d^{*1-\alpha_d}$.

Step 2 The following lemma shows that if S is small, then the country will not invest in new technology and human capital. When S is large, then it will invest in new technology.

Lemma 8 *i) There exists* $\underline{S} > 0$ *such that if* $S \leq \underline{S}$ *then* $\theta = 0$ *and* $\mu = 0$.

ii) There exists $\overline{S}$ *such that if* $S > \overline{S}$ *then* $\theta > 0$.

Proof. For any S, denote by $\theta(S)$, $\mu(S)$ the corresponding optimal values for θ and μ.

(i) Let $\underline{S}$ satisfies

$$r_e \underline{S}^{\alpha_e} h(\underline{S})^{1-\alpha_e} = X,$$

Then for any $(\theta, \mu) \in \Delta$, for any $S \leq \underline{S}$,

$$r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e} \leq X$$

and $(\theta(S), \mu(S)) = (0, 0)$.

(ii) Fix $\mu = 0$ and $\theta \in (0, 1)$. Then $\psi(r_e, \theta, 0, S) \to +\infty$ when $S \to +\infty$. Let $\overline{S}$ satisfy $\psi(r_e, \theta, 0, \overline{S}) > x_0 L_d^{*1-\alpha_d}$. Obviously, $F(r_e, \overline{S}) \geq \psi(r_e, \theta, 0, \overline{S}) > x_0 L_d^{*1-\alpha_d}$, and $\theta(\overline{S}) > 0$. If not, then $\mu(\overline{S}) = 0$ and $F(r_e, \overline{S}) = x_0 L_d^{*1-\alpha_d}$ (see Remark 3). ■

Step 3 : Proof of Proposition 2

Now, let us define

$$S^c = \max\{S \geq 0 : S \in B\}.$$

It is obvious that $0 < S^c < +\infty$, since $S^c \geq \underline{S} > 0$ and B is compact.

Note that for any $S \geq 0$ we have

$$F(r_e, S) \geq x_0 L_d^{*1-\alpha_d}.$$

If $S < S^c$ then for any $(\theta, \mu) \in \Delta$,

$$\psi(r_e, \theta, \mu, S) \leq \psi(r_e, \theta, \mu, S^c)$$

which implies

$$F(r_e, S) \leq F(r_e, S^c) = x_0 L_d^{*1-\alpha_d}.$$

Thus,

$$F(r_e, S) = x_0 L_d^{*1-\alpha_d}.$$

Let $S_0 < S^c$. Assume there exists two optimal values for (θ, μ) which are $(0, 0)$ and (θ_0, μ_0) with $\theta_0 > 0$. We have $F(r_e, S_0) = x_0 L_d^{*1-\alpha_d} = \psi(r_e, \theta_0, \mu_0, S_0)$. We must have $r_e \theta_0^{\alpha_e} S_0^{\alpha_e} h(\mu_0 S_0)^{1-\alpha_e} > X$ (if not, $\Phi(r_e, \theta_0, \mu_0, S_0) = x_0$ and $\theta_0 = 0, \mu_0 = 0$.)

Since $\theta_0 > 0$, we have $r_e \theta_0^{\alpha_e} (S^c)^{\alpha_e} h(\mu_0 S_0)^{1-\alpha_e} > r_e \theta_0^{\alpha_e} S_0^{\alpha_e} h(\mu_0 S_0)^{1-\alpha_e} > X$. Hence

$$\begin{aligned} x_0 L_d^{*1-\alpha_d} = F(r_e, S^c) &\geq \psi(r_e, \theta_0, \mu_0, S^c) \\ &> \psi(r_e, \theta_0, \mu_0, S_0) = x_0 L_d^{*1-\alpha_d} \end{aligned}$$

which is a contradiction.

Therefore, if $S > S^c$ then

$$F(r_e, S) > x_0 L_d^{*1-\alpha_d}$$

which implies $\theta(S) > 0$.

Proof of Proposition 3

1. Take $S > S^c$. From the previous proposition, $\theta(S) > 0$. Assume $\mu(S) = 0$. For short, denote $\theta^* = \theta(S)$. Define

$$F^0(r_e, S, \theta^*, 0) = \underset{0 \leq \theta \leq 1}{\mathrm{Max}} \psi(r_e, \theta, 0, S) = \Phi(r_e \theta^{*\alpha_e} S^{\alpha_e} h(0)^{1-\alpha_e})(1-\theta^*)^{\alpha_d} L_d^{*1-\alpha_d}.$$

and consider a feasible couple (θ, μ) in Δ which satisfies $\theta^* = \theta + \mu$. Denote

$$F^1(r_e, S, \theta, \mu) = \Phi(r_e \theta^{\alpha_e} S^{\alpha_e} h(\mu S)^{1-\alpha_e})(1-\theta^*)^{\alpha_d} L_d^{*1-\alpha_d}.$$

We then have

$$
\begin{aligned}
&\frac{F^1(r_e,S,\theta,\mu)-F^0(r_e,S,\theta^*,0)}{(1-\theta^*)^{\alpha_d}L_d^{*1-\alpha_d}} = \\
&\Phi(r_e\theta^{\alpha_e}S^{\alpha_e}h(\mu S)^{1-\alpha_e})-\Phi(r_e\theta^{*\alpha_e}S^{\alpha_e}h(0)^{1-\alpha_e}) \\
= \;& r_eS^{\alpha_e}[\theta^{\alpha_e}h(\mu S)^{1-\alpha_e}-\theta^{*\alpha_e}h(\mu S)^{1-\alpha_e}+\theta^{*\alpha_e}h(\mu S)^{1-\alpha_e}-\theta^{*\alpha_e}h(0)^{1-\alpha_e}].
\end{aligned}
$$

By the concavity of $h(x)$ and $f(x)=x^{\alpha_e}$, we obtain

$$
\begin{aligned}
&F^1(r_e,S,\theta,\mu)-F^0(r_e,S,\theta^*,0) \geq \\
&r_eS^{\alpha_e}\mu h(\mu S)^{-\alpha_e}[-\alpha_e h(\mu S)(\theta^*-\mu)^{\alpha_e-1}+S(1-\alpha_e)\theta^{*\alpha_e}h'(\mu S)].
\end{aligned}
$$

Let $\mu\to 0$. We have $h'(\mu S)\to +\infty$. The expression in the brackets will converge to $+\infty$, and we get a contradiction with the optimality of θ^*.

2. Assume that $\mu(S)=0$ for any $S\in\{S^1,S^2,...,S^n,...\}$ where the infinite sequence $\{S^n\}_n$ is increasing, converges to $+\infty$ and satisfies $S^1>S^c$. For short, denote $\theta=\theta(S)$. Then we have the following F.O.C.:

$$
\frac{ar_e\theta^{\alpha_e-1}S^{\alpha_e}h(0)^{1-\alpha_e}\alpha_e}{x_0+a[r_e\theta^{\alpha_e}S^{\alpha_e}h(0)^{1-\alpha_e}-X]}=\frac{\alpha_d}{1-\theta}, \tag{2.6}
$$

and

$$
\frac{ar_e\theta^{\alpha_e}S^{\alpha_e+1}h'(0)h(0)^{-\alpha_e}(1-\alpha_e)}{x_0+a[r_e\theta^{\alpha_e}S^{\alpha_e}h(0)^{1-\alpha_e}-X]}\leq\frac{\alpha_d}{1-\theta}. \tag{2.7}
$$

Equation (2.6) implies

$$
\frac{ar_e\theta^{\alpha_e-1}h(0)^{1-\alpha_e}\alpha_e}{\frac{x_0}{S^{\alpha_e}}+a[r_e\theta^{\alpha_e}h(0)^{1-\alpha_e}]}\leq\frac{\alpha_d}{1-\theta}. \tag{2.8}
$$

If $\theta\to 0$ when $S\to+\infty$, then the LHS of inequality (2.8) converges to infinity while the RHS converges to α_d: a contradiction. Thus θ will be bounded away from 0 when S goes to infinity.

Combining equality (2.6) and inequality (2.7) we get

$$
h'(0)(1-\alpha_e)S\leq h_0\alpha_e\theta^{-1}. \tag{2.9}
$$

When $S \to +\infty$, we have a contradiction since the LHS of (2.9) will go to infinity while the RHS will be bounded from above. That means there exists S_M such that for any $S \geq S_M$, we have $\mu(S) > 0$.

3. Let $S > S^c$. For short, we denote μ and θ instead of $\mu(S)$ and $\theta(S)$. If $\mu > 0$ then we have the F.O.C:

$$\frac{ar_e\theta^{\alpha_e-1}S^{\alpha_e}h(\mu S)^{1-\alpha_e}\alpha_e}{x_0 + a[r_e\theta^{\alpha_e}S^{\alpha_e}h(\mu S)^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta-\mu}, \tag{2.10}$$

and

$$\frac{ar_e\theta^{\alpha_e}S^{\alpha_e+1}h'(\mu S)h(\mu S)^{-\alpha_e}(1-\alpha_e)}{x_0 + a[r_e\theta^{\alpha_e}S^{\alpha_e}h(\mu S)^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta-\mu}. \tag{2.11}$$

Let θ^c and S^c satisfy the following equations

$$\frac{ar_e(\theta^c)^{\alpha_e-1}(S^c)^{\alpha_e}h(0)^{1-\alpha_e}\alpha_e}{x_0 + a[r_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e}h(0)^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta^c}, \tag{2.12}$$

and

$$(x_0 + a[r_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e}h(0)^{1-\alpha_e} - X])(1-\theta^c)^{\alpha_d} = x_0. \tag{2.13}$$

Equality (2.12) is the F.O.C. with respect to θ, while equality (2.13) states that $\psi(r_e, \theta^c, 0, S^c) = x_0 L_d^{*1-\alpha_d}$. If $h'(0) < \alpha = h(0)\frac{1}{\theta^c S^c}\frac{\alpha_e}{1-\alpha_e}$, $\theta^c > 0$ as defined in Bruno *et al.* (2008), then we get

$$\frac{ar_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e+1}h'(0)h(0)^{-\alpha_e}(1-\alpha_e)}{x_0 + a[r_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e}h(0)^{1-\alpha_e} - X]} < \frac{\alpha_d}{1-\theta^c}. \tag{2.14}$$

Relations (2.12), (2.13) and (2.14) give the the values of S^c and $\theta(S^c) = \theta^c$ and $\mu(S^c) = \mu^c = 0$. When $S > S^c$ and close to S^c, equality (2.12) and inequality (2.14) still hold. That means $\mu(S) = 0$ for any S close to S^c.

Proof of Proposition 4 The proof will be done in two steps.

Step 1

Lemma 9 *Assume $h'(0) < +\infty$. Let $S^1 > S^c$. If $\mu(S^1) = 0$, then for $S^2 < S^1$, we also have $\mu(S^2) = 0$.*

Proof. If $S^2 \leq S^c$ then $\mu(S^2) = 0$ since $\theta(S^2) = 0$ (see Proposition 2). For short, we write $\theta_1 = \theta(S^1),\ \theta_2 = \theta(S^2),\ \mu_1 = \mu(S^1),\ \mu_2 = \mu(S^2)$.

Observe that (θ_1, S^1) satisfy (2.6) and (2.7), or equivalently (2.6) and (2.9). Equality (2.6) can be written as

$$h_0^{1-\alpha_e} a r_e [\alpha_e \theta_1^{\alpha_e - 1} - (\alpha_e + \alpha_d)\theta_1^{\alpha_e}] = \frac{\alpha_d (x_0 - aX)}{S^{1\alpha_e}}. \tag{2.15}$$

If $x_0 - aX = 0$, then $\theta_1 = \frac{\alpha_e}{\alpha_e + \alpha_d}$. Take $\theta_2 = \theta_1$. If $S^2 < S^1$ then (θ_2, S^2) satisfy (2.6) and (2.9). That means they satisfy the F.O.C. with $\mu_2 = 0$.

Observe that the LHS of equation (2.15) is a decreasing function in θ_1. Hence θ_1 is uniquely determined.

When $x_0 > aX$, if (θ_2, S^2) satisfy (2.15), with $S^2 < S^1$, then $\theta_2 < \theta_1$. In this case, (θ_2, S^2) also satisfy (2.9), and we have $\mu_2 = 0$.

When $x_0 < aX$, write equation (2.15) as:

$$h_0^{1-\alpha_e} a r_e [\alpha_e \theta_1^{-1} - (\alpha_e + \alpha_d)] = \frac{\alpha_d (x_0 - aX)}{(\theta_1 S^1)^{\alpha_e}}. \tag{2.16}$$

If (θ_2, S^2) satisfy (2.15), with $S^2 < S^1$, then $\theta_2 > \theta_1$. Since $x_0 < aX$, from (2.16), we have $\theta_2 S^2 < \theta_1 S^1$. Again (θ_2, S^2) satisfy (2.15) and (2.9). That implies $\mu_2 = 0$. ■

Step 2 Proof of the proposition.

Let

$$\widetilde{S} = \max\{S_m : S_m \geq S^c,\ \text{and } S \leq S_m \Rightarrow \mu(S) = 0\},$$

and

$$\widetilde{\widetilde{S}} = \inf\{S_M : S_M > S^c,\ \text{and } S > S_M \Rightarrow \mu(S) > 0\}.$$

From Proposition 3, the sets $\{S_m : S_m > S^c,\ \text{and } S \leq S_m \Rightarrow \mu(S) = 0\}$ and $\{S_M : S_M > S^c,\ \text{and } S > S_M \Rightarrow \mu(S) > 0\}$ are not empty. From Step 1, we have $\widetilde{\widetilde{S}} \geq \widetilde{S}$. If $\widetilde{\widetilde{S}} > \widetilde{S}$, then take $S \in (\widetilde{S}, \widetilde{\widetilde{S}})$. From the definitions of $\widetilde{S}$ and $\widetilde{\widetilde{S}}$, there exist $S_1 < S$, $S_2 > S$ such that $\mu(S_1) > 0$ and $\mu(S_2) = 0$. But that contradicts Step 1. Hence $\widetilde{\widetilde{S}} = \widetilde{S}$. Put $\widehat{S} = \widetilde{\widetilde{S}} = \widetilde{S}$ and conclude.

Proof of Proposition 20

From Proposition 4, we have $\mu^c = 0$. In this case, θ^c and S^c satisfy equation (2.10) and, since $S^c \in B$, we also have $F(r_e, S^c) = \psi(r_e, \theta^c, 0, S^c) = x_0 L_d^{*1-\alpha_d}$.

Explicitly, we have

$$\frac{ar_e(\theta^c)^{\alpha_e - 1}(S^c)^{\alpha_e} h_0^{1-\alpha_e}\alpha_e}{x_0 + a[r_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e} h_0^{1-\alpha_e} - X]} = \frac{\alpha_d}{1-\theta^c}$$

and

$$(x_0 + a[r_e(\theta^c)^{\alpha_e}(S^c)^{\alpha_e} h_0^{1-\alpha_e} - X])(1-\theta^c)^{\alpha_d} = x_0 \tag{2.17}$$

Tedious computations show that θ^c satisfies the equation

$$\alpha_e[1 - \frac{x_0 - aX}{x_0}(1-\theta)^{\alpha_d + 1}] = \theta(\alpha_d + \alpha_e)$$

If $x_0 > aX$, then the LHS is a strictly concave function which increases from $\frac{\alpha_e aX}{x_0}$ when $\theta = 0$ to α_e when $\theta = 1$. The RHS is linear increasing, equal to 0 at the origin and to $\alpha_d + \alpha_e$ when $\theta = 1$. Therefore, there exists a unique solution $\theta^c \in (0,1)$.
If $x_0 < aX$, then the LHS is a strictly convex function which decreases from $\frac{\alpha_e aX}{x_0}$ when $\theta = 0$ to α_e when $\theta = 1$. The RHS is linear increasing, equal to 0 at the origin and to $\alpha_d + \alpha_e$ when $\theta = 1$. Therefore, there exists a unique solution $\theta^c \in (0,1)$.
If $x_0 = aX$, then $\theta^c = \frac{\alpha_e}{\alpha_e + \alpha_d}$.
In any case, θ^c does not depend on r_e. It is easy to show that θ^c is positively related with a if $x_0 \neq aX$. With higher value of spill-over indicator, a (*e.g.* better social capital and institutional capital), the economy in question not only invest in new technology earlier but also invest more initially.

Equation (2.17) gives:

$$ar_e(S^c)^{\alpha_e} = [x_0(\frac{1}{(1-\theta^c)^{\alpha_d}} - 1) + aX]\frac{1}{(\theta^c)^{\alpha_e} h_0^{1-\alpha_e}} \tag{2.18}$$

We see immediately that S^c is a decreasing function in a and r_e.

2.7 Appendix 2

Proof of Proposition 8 Let S^s be defined by

$$\alpha_d (S^s)^{\alpha_d - 1} x_0 L_d^{*1-\alpha_d} = \frac{1}{\beta}.$$

If $S_0 > \widehat{S}$ ($\widehat{S}$ is defined in Proposition 4) then $\theta_t^* > 0,\ \mu_t^* > 0$ for every t.
If $S_0 > S^c$ then $\theta_t^* > 0$ for every t. If S_t^* converges to infinity, then there exists T_2 where $S_{T_2}^* > \widehat{S}$ and $\theta_t^* > 0,\ \mu_t^* > 0$ for every $t \geq T_2$.
Now consider the case where $0 < S_0 < S^c$. Obviously, $\theta_0^* = 0$. It is easy to see that if a or/and r_e are large then $S^c < S^s$. If for any t, we have $\theta_t^* = 0$, we also have $K_{e,t}^* = 0\ \forall t$, and the optimal path (S_t^*) will converge to S^s (see Le Van and Dana (2003)). But, we have $S^c < S^s$. Hence the optimal path $\{S_t^*\}$ will be non decreasing and will pass over S^c after some date T_1 and hence $\theta_t^* > 0$ when $t \geq T_1$.
If the optimal path $\{S_t^*\}$ converges to infinity, then after some date T_2, $S_t^* > \widehat{S}$ for any $t > T_2$ and $\theta_t^* > 0, \mu_t^* > 0$.

It remains to prove that the optimal path converges to infinity if a or/and r_e are large enough.

Since the utility function u satisfies the Inada condition $u'(0) = +\infty$, we have Euler equation:

$$u'(c_t^*) = \beta u'(c_{t+1}^*) H_s'(r_e, S_{t+1}^*).$$

If $S_t^* \to \overline{S} < \infty$, then $c_t^* \to \overline{c} > 0$. From Euler equation, we get

$$H_s'(r_e, \overline{S}) = \frac{1}{\beta}.$$

We will show that $H_s'(r_e, S) > \frac{1}{\beta}$ for any $S > S^c$. We have

$$\begin{aligned} H_s'(r_e, S) &= F_s'(r_e, S) S^{\alpha_d} + \alpha_d F(r_e, S) S^{\alpha_d - 1} \\ &\geq F_s'(r_e, S) S^{\alpha_d}. \end{aligned}$$

From the envelope theorem we get:

$F_s^{'}(r_e, S)S^{\alpha_d} =$

$$[ar_e\theta^{*\alpha_e}(h(\mu^*S))^{-\alpha_e}(\alpha_e h(\mu^*S) + (1-\alpha_e)\mu^*Sh'(\mu^*S))S^{\alpha_d+\alpha_e-1}]$$
$$\times L_d^{*1-\alpha_d}(1-\theta^*-\mu^*)^{\alpha_d}$$

When ar_e is large, from Proposition 6, we have $\theta^* \geq \underline{\theta} = \min\{\theta^c, \theta_\infty\}$ and $\theta^* + \mu^* \leq \overline{\zeta} = \max\{\theta^c, \theta_\infty + \mu_\infty\}$. We then obtain

$$\begin{aligned} H_s^{'}(r_e, S) &\geq L_d^{*1-\alpha_d}(1-\theta^*-\mu^*)^{\alpha_d}[ar_e\theta^{*\alpha_e}(h^*(\mu S))^{1-\alpha_e}\alpha_e S^{\alpha_d+\alpha_e-1}] \\ &\geq L_d^{*1-\alpha_d}(1-\overline{\zeta})^{\alpha_d}[ar_e\underline{\theta}^{\alpha_e}(h^*(0))^{1-\alpha_e}\alpha_e(S^c)^{\alpha_d+\alpha_e-1}] \end{aligned}$$

since $h(x) \geq h(0)$ and $\alpha_d + \alpha_e - 1 \geq 0$.

If $\alpha_d + \alpha_e = 1$, then

$$H_s^{'}(r_e, S) \geq L_d^{*1-\alpha_d}(1-\overline{\zeta})^{\alpha_d}[ar_e\underline{\theta}^{\alpha_e}(h^*(0))^{1-\alpha_e}\alpha_e], \tag{2.19}$$

and when ar_e becomes very large, the RHS of inequality (2.19) will be larger than $\frac{1}{\beta}$.

Now assume $\alpha_d + \alpha_e > 1$. From equation (2.18), the quantity $ar_e(S^c)^{\alpha_e}$ equals

$$\gamma = [x_0(\frac{1}{(1-\theta^c)^{\alpha_d}} - 1) + aX]\frac{1}{(\theta^c)^{\alpha_e}h_0^{1-\alpha_e}}$$

and

$$S^c = (\frac{\gamma}{ar_e})^{\frac{1}{\alpha_e}}.$$

We now have

$$H_s^{'}(r_e, S) \geq L_d^{*1-\alpha_d}(1-\overline{\zeta})^{\alpha_d}\underline{\theta}^{\alpha_e}(h^*(0))^{1-\alpha_e}\alpha_e\gamma(\frac{\gamma}{ar_e})^{\frac{\alpha_d-1}{\alpha_e}}$$

It is obvious that, since $\alpha_d - 1 < 0$, when ar_e is large, we have $H_s^{'}(r_e, S) > \frac{1}{\beta}$.

2.8 Appendix 3

Table 2: Inputs and Technical Progress: Breaks in 1973 and 1985

Contributions (%) of the Sources of Growth

	Sample period	Physical capital	Labor	Human capital	Technical progress
(1) Pre-1973					
Hong Kong	66-73	68.37 (9.67)	28.50 (3.10)	3.13 (5.57)	0.00
S. Korea	60-73	72.60 (11.58)	21.87 (4.14)	5.53 (7.70)	0.00
Singapore	64-73	55.59 (12.73)	40.18 (7.56)	4.22 (9.17)	0.00
Taiwan	53-73	80.63 (13.21)	15.45 (2.63)	3.91 (6.73)	0.00
Indonesia	70-73	73.09 (11.09)	9.37 (2.15)	17.54 (19.50)	0.00
Malaysia	70-73	59.97 (9.56)	29.99 (4.32)	10.05 (12.64)	0.00
Philippines	70-73	39.79 (5.12)	49.97 (7.36)	10.24 (11.51)	0.00
Thailand	70-73	82.11 (10.96)	7.67 (0.57)	10.22 (11.44)	0.00
China	65-73	85.29 (13.51)	10.36 (3.19)	4.35 (7.01)	0.00
Japan	57-73	55.01 (11.43)	4.85 (0.82)	1.06 (2.87)	39.09
G-5	57-73	41.50 (4.62)	6.00 (4.24)	1.43 (1.70)	51.07
(2) 1974-85					
Hong Kong	74-85	64.31 (9.58)	32.73 (3.40)	2.96 (5.67)	0.00
S. Korea	74-85	78.08 (13.28)	18.10 (2.83)	3.81 (6.41)	0.00
Singapore	74-85	64.68 (9.94)	31.72 (3.42)	3.60 (5.48)	0.00
Taiwan	74-85	78.91 (11.89)	18.12 (2.23)	2.97 (4.98)	0.00
Indonesia	74-85	77.69 (12.22)	13.55 (2.65)	8.76 (10.20)	0.00
Malaysia	74-85	61.39 (10.76)	33.61 (4.94)	5.00 (8.15)	0.00
Philippines	74-85	62.59 (7.29)	29.28 (3.53)	8.13 (8.07)	0.00
Thailand	74-85	67.53 (8.69)	25.02 (3.55)	7.46 (8.96)	0.00
China	74-85	80.46 (9.44)	14.64 (2.53)	4.09 (6.37)	0.00
Japan	74-85	40.65 (6.73)	10.22 (0.93)	0.96 (1.69)	48.17
G-5	74-85	36.29 (2.65)	-14.55 (-0.42)	2.53 (1.90)	75.73

Note: The numbers in the parentheses are the average annual rates of growth of each of inputs.

Table 2 (*cont.*): Inputs and Technical Progress: Breaks in 1973 and 1985

Contributions (%) of the Sources of Growth

	Sample period	Physical capital	Labor	Human capital	Technical progress
(3) Post-1986					
Hong Kong	86-95	41.81 (7.56)	6.46 (0.53)	1.58 (3.10)	50.14
S. Korea	86-95	44.54 (11.90)	14.98 (2.76)	1.75 (4.15)	38.73
Singapore	86-95	37.01 (8.50)	31.30 (4.32)	1.52 (3.38)	30.17
Taiwan	86-95	43.00 (9.01)	10.46 (1.34)	1.38 (3.13)	45.16
Indonesia	86-94	62.79 (8.88)	15.91 (2.31)	5.69 (6.94)	15.61
Malaysia	86-95	42.87 (8.53)	33.41 (4.83)	3.25 (6.15)	20.47
Philippines	86-95	52.18 (3.77)	41.63 (2.96)	6.23 (5.09)	-0.03
Thailand	86-94	51.01 (11.27)	13.32 (2.72)	2.36 (5.25)	33.31
China	86-95	86.39 (12.54)	10.34 (1.92)	3.27 (4.54)	0.00
Japan	86-94	38.21 (4.86)	2.47 (0.11)	1.17 (1.44)	58.14
G-5	86-94	27.14 (2.70)	13.83 (5.37)	1.58 (1.36)	57.45

Note: The numbers in the parentheses are the average annual rates of growth of each of inputs.

G-5: France, W. Germany, Japan, UK and US

Source: Reproduced from Lau and Park (2003)

Table 3: List of Economies in the Sample of Human Capital

Economies	Range	Economies	Range
Algeria	1950-2000	Malaysia	1960-2000
Argentina	1950,1960-2000	Mali	1960-2000
Bangladesh	1960-2000	Malta	1950,1960-2000
Barbados	1960-2000	Mauritius	1950,1960-2000
Benin	1960-2000	Mexico	1950,1960-2000
Bolivia	1960-2000	Mozambique	1960-2000
Botswana	1960-2000	Nepal	1960-2000
Brazil	1960-2000	Nicaragua	1950,1960-2000
Cameroon	1960-2000	Niger	1960-2000
Central African Republic	1960-2000	Pakistan	1960-2000
Chile	1950,1960-2000	Panama	1950,1960-2000
China	1960-2000	Paraguay	1950,1960-2000
Colombia	1950,1960-2000	Peru	1960-2000
Congo, Dem. Rep.	1955-2000	Philippines	1950-2000
Congo, Republic of	1960-2000	Poland	1960-2000
Costa Rica	1950,1960-2000	Romania	1950,1960-2000
Cuba	1955-2000	Rwanda	1960-2000
Cyprus	1960-2000	Senegal	1960-2000
Dominican Republic	1960-2000	Sierra Leone	1960-2000
Ecuador	1950,1960-2000	Singapore	1960-2000
Egypt	1960-2000	South Africa	1960-2000
El Salvador	1950,1960-2000	Sri Lanka	1960-2000
Gambia, The	1960-2000	Sudan	1955-2000
Ghana	1960-2000	Swaziland	1960-2000
Guatemala	1950,1960-2000	Syria	1960-2000
Haiti	1950,1960-2000	Taiwan	1960-2000
Honduras	1960-2000	Thailand	1960-2000
Hungary	1960-2000	Togo	1960-2000
India	1960-2000	Trinidad &Tobago	1960-2000
Indonesia	1960-2000	Tunisia	1960-2000
Jamaica	1960-2000	Uganda	1960-2000
Jordan	1960-2000	Uruguay	1960-2000
Kenya	1960-2000	Venezuela	1950,1960-2000
Korea, Republic of	1955-2000	Zambia	1960-2000
Lesotho	1960-2000	Zimbabwe	1960-2000
Malawi	1960-2000		

Source: Extracted from Barro and Lee (2000)

Data is calculated at 5-years span and some economies data for 1955 are missing

Chapter 3

Total Factor Productivity, Saving and Learning-by-Doing in Growth Process

3.1 Introduction

The roles of capital accumulation and technological progress in economic growth are not new stories in the literature. The Solow model (1956) based on the classical assumption of diminishing returns to capital, states that without continuing improvement of technology per capita growth must eventually cease. The essential factor for economic growth, namely technological progress, is however, exogenous to the model. This shortcoming inspires scholars such as Romer (1986, 1987, 1990), Lucas (1988), Rebelo (1991), Grossman and Helpman (1991), Aghion and Howitt (1992) and many others to develop new "endogenous" growth models which provide more insights into the Solow's residual.

Recently, the spectacularly rapid growth of many Asian economies, especially the East Asian newly industrialized economies (NIEs) gave rise to a broad and diversified literature aiming at explaining the reasons for such a long lasting period of expansion (Kim and Lau [1994, 1996], Krugman [1994], Rodrik [1995], Worldbank [1993], Young [1994, 1995]). All these economies have experienced rapid growth of their physical capital stock and very high rate of investment in human capital.

On the one hand, the supporters of the accumulation view stress the importance of physical and human capital accumulation in the Asian growth process. Accordingly, the main engine of "miracle growth" in NIEs is simply, very high investment rates. Young [1994, 1995], Kim and Lau [1994, 1996] found that the postwar economic growth of the NIEs was mostly due to growth in input factors (physical capital and labor) with trivial increase in the total factor productivity. Moreover, the hypothesis of no technical progress cannot be rejected for the East Asian NIEs (Kim and Lau [1994]). Consequently, accumulation of physical and human capital seems to explain the lion's share of the NIEs' growth process. Krugman [1997] wrote that Larry Lau and Alwyn Young works suggested that Asian growth could mostly be explained by high investment rates, good education and the movement of underemployment peasants into the modern sector. Economists who take this point, implicitly assumed that adoption and mastering new technology and other modern practices could be done easily by trade.

"*Accumulationists seem to believe that the state of technological knowledge at any time is largely codified in the form of blueprints and associated documents and that, for a firm to adopt a technology that is new to it but not to the world, primarily involves getting access to those blueprints" (Nelson and Pack, 1998).*

Accordingly, any economy could have experienced high rates of growth like NIEs if it could also afford high investment rates. Krugman's [1994] interpretation of these results is very pessimistic since, in his opinion, the lack of technical progress will inevitably bound the growth engine of East Asian NIEs as a result of the diminishing returns affecting capital accumulation.

On the other hand, the supporters of endogenous growth theory pinpoint productivity growth as the key factor of East Asian success. According to these authors, Asian countries have adopted technologies previously developed by more advanced economies (assimilation view) and "*the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy*" (Pack [1992]). They admit that high rates of investment into physical and human capital is necessary to achieve high economic growth rate. However, as stressed by Nelson and Pack [1998] there is nothing automatic in learning about, in risking to operate and, in coming to master technologies and other practices that are new to the economy. These processes require searching and studying, learning, and innovating to master modern technologies and new practices. Thereby, the economy enhances its stock of knowledge and efficiency. Implicitly, they suggest that technological progress exists and does play a crucial role in NIEs' economic growth.

Empirically, Collins and Bosworth [1996] or Lau and Park [2003] show Total Factor Productivity (TFP) gains actually matter in Asian NIEs growth and that future growth can be sustained. For these authors, learning-by-doing in process of physical accumulation play an essential role in TFP growth in these economies.

In this paper we first prove that high saving rates may play an important role in "miracle growth" in NIEs in the short and mid terms, but in the long term TFP is the crucial factor of growth as claimed by Krugman. Specifically, in transitional stage the high saving rate induces high growth rate of output. This effect of high saving rate will die out in the long-run if the economy is not very elastic. If the economy is very elastic and the saving rate is high enough the growth rate will be decreasing but always higher than the growth rate that predicted at BGP regardless how efficient the economy in learning-by-doing is.

Second, we show that assimilationists are also right as claiming that learning-by-doing play an important role in TFP growth in NIEs. However, the growth model based purely on learning-by-doing is constrained by labor growth rate. If the labour is constant in the long-run, then the growth can not be sustained. In this sense, learning-by-doing is insufficient for growth in long run. To sustain growth other forms of TFP accelerating such as investment in human capital to release the labour constraints, new technology (*e.g.,* Bruno *et al* [2008], Le Van *et al* [2008], Aghion and Howitt [1992], Lucas [1988], *etc.,*) is needed.

In addition, we also explain why economic growth does not converge as predicted by Solowian models. We characterize four possible growth paths which are contingent on saving and elasticity between capital and labor. If the elasticity between capital and labour, ρ, is smaller than 1. Economically, it means that the structure of the economy is not elastic; it is not easy to switch from a labor-intensive technology to more capital-intensive technologies and *vice versa*, then saving plays a crucial role in economic growth. There are three possible scenarios. (i) If the saving rate is too low the economy will collapse in the long-run. (ii) If the saving rate is higher than a critical level but lower than an optimal level the economy can sustain its positive growth in long-run but lower than its potential level. In this case even if the economy possesses high efficiency of learning and spilling-over, it can not fully enjoy those in long run. In addition, in this case we show that the gap between the poor economy and the rich one can be widening if the saving rate of the poor is not superior than that of the rich. (iii) If the saving rate is high enough the economy converges asymptotically to its BGP which does not depend on saving but on the index of efficiency.

If $\rho > 1$, the economy either converges to its BGP or its rate of growth decreasingly converges to a rate which is higher than the potential rate and does not depend on coefficient of efficiency.

The organization of the paper is as follows. The general basic neoclassical model is presented in section 3.2. In section 5.3, we use a CES production function to take into account the process of knowledge accumulation through learning-by-doing and spillover effect in the growth of the economy. The next section summarizes the main results of the paper. Finally, we put in Appendix the proofs of our claims.

3.2 The Basic Neoclassical Model

In this section we set out the basic model of capital accumulation that will use in our analysis. The standard constant return to scale is defined as follows:

$$Y_t = F(A_t, K_t, L_t) \tag{3.1}$$

Where Y_t is output, K_t is physical capital, L_t is labour input, A_t is a parameter of technological progress. The production function, if A_t and L_t are constant, has positive and diminishing returns to the reproducible factor K_t. Mathematically, $\frac{\partial Y_t}{\partial K_T} > 0, \frac{\partial^2 Y_t}{\partial K_t \partial K_t} < 0$.

We follow Solow (1956) to assume that saving (net investment) is a fixed fraction s of income; the capital stock depreciates at a fixed rate δ, and the labor growth rate is constant at n. With these assumptions the transitional dynamics of the model is given by following program:

$$
\begin{aligned}
C_t + S_t &= Y_t = F(A_t, K_t, L_t) && \text{(3.2a)} \\
S_t &= sY_t, \ s \text{ is the exogenous saving rate} \\
K_{t+1} &= K_t(1-\delta) + sY_t && \text{(3.2b)} \\
L_t &= L_0(1+n)^t
\end{aligned}
$$

$C_t, S_t, Y_t, K_t, I_t, L_t$ denote respectively the consumption, the saving, the output, the capital stock, the investment and the labour at period t. The labour grows with an exogenous rate n. A_t denotes the technological level in the economy at time t. The growth rate of A_t is assumed to be identical with the growth rate of the Total Factor Productivity (TFP).

The accumulating knowledge through learning-by-doing as mentioned in Atkinson and Stiglitz (1969) and Nelsons and Pack (1998) is modelized in a CES production function in section 5.3.

3.3 Endogenous TFP: Learning-by-Doing

Atkinson and Stiglitz [1969] advocates that if a firm switches from one technique (say,

labour-intensive) to another one (e.g. capital-intensive), it requires a technological progress. Because the switching requires new knowledge to localize the technique; new knowledge to maneuver the production process; new knowledge to reorganize the production, *etc.*. Nelson and Pack [1998] explores further this ideas by arguing that the switching from labour-intensive economies to capital-intensive economies, as NIEs have done, can not be seen as simply "moving along production function". They admit, on the one hand, that developing economies can import technologies from developed economies. On the other hand, they argue further, only a small portion of what one needs to know to employ a technology is codified in the form of blueprints; much of it is tacit which requires an uncertain process of, searching and studying, learning-by-doing and using, restructuring production activities. In short-run this process, as proposed by Atkinson and Stigliz [1969] may be costly, however in long-run, it indeed improves knowledge stock and technological level of the economy. The effectiveness of this process is not automatic but contingent on efforts and effectiveness of learning-by-doing, of the restructuring of production activities, of searching and studying , and many other factors. However these analyses (Collins and Bosworth [1996], Nelson and Pack [1998] or Lau and Park [2003] *etc.*,) are essentially qualitative and thus the process of learning-by-doing seems to be insufficiently deliberated. Some important questions are still open: Whether is merely learning-by-doing sufficient to sustain growth in long run? Whether the impact of saving rate on growth vanish in the presence of learning-by-doing?

The concept of learning-by-doing was firstly incorporated into a macroeconomic model by Arrow [1962]. In his model, part of the technical change process does not depend on the passage of time as such but develops out of experience gained within the production process itself. Mathematically, the model assumes that a labour efficiency index associated with workers of a particular vintage is a strictly increasing function of cumulative output or gross investment. Such a relationship is expressed as .

$$A_t = A_0 E_t^{\beta}$$

where A_t is the level of technology of time t and A_0 is the initial level of technology. E is the index of experience and $\beta > 0$ is learning coefficient.

Arrow [1962] chooses cumulative gross investment as index of experience ($E_t = \Sigma I_t$) while other studies (*inter alia*, Bairam [1987], Stokey and Lucas [1989]) favoured cumulative output as an index ($E_t = \Sigma Q_t$). In this study we follow Arrow's argument that the appearance of new machines provides more stimulation to innovation while cumulative output is less inspiring to innovation. However, other than Arrow we assume that knowledge of obsolescent technology play negligible role in improving efficiency of using current technology. Accordingly, we choose stock of capital as index of experience rather than cumulative gross investment.

We endogenize the process of learning-by-doing in a CES model to give answers to above mentioned questions. The production function is presented in the following equation.

$$Y_t = [\alpha K_t^r + (1-\alpha)(K_t^\beta L_t)^r]^{\frac{1}{r}} \tag{3.3}$$

In this model we assume that:

(i) The stock of knowledge is accumulated through learning-by-doing and using. It works through each firm's investment. Specifically, an increase in a firm's capital stock requires firm to accumulate new knowledge of using, localizing and work organizing. As a result this increases the firm's stock of knowledge.

(ii) The knowledge accumulated can be internalized within the firm and then externalized to the whole economy through spillover effect to increase the total stock of knowledge of the economy as a whole

(iii) The effectiveness of the knowledge accumulation through learning-by-doing and spillover is given by parameter β which we call TFP coefficient. We assume that, this process complies with law of diminishing return, i.e. $0 < \beta < 1$. The higher β, the more effective the knowledge accumulation is, and then the faster technological level improves. The magnitude of β depends on the concentration and linkages of industries in the economy, the effectiveness of on-job training, *etc.*,.

L_t labour used in production, is assumed to grow at constant rate n. We define $r = \frac{\rho-1}{\rho}$, where ρ is elasticity of substitution between K and L. As common in literature we assume that $1 > r > -\infty$. The higher r, then the higher ρ which implies that the structure of the economy is more flexible; it is easier to switch from a labor-intensive technology to more capital-intensive technologies and *vise versa*.

In the following position we show that the production function (3.3) in long run will converge to its Balanced Growth Path (BGP).

Proposition 9 *Let us introduce condition* (C) : $\left[\frac{(1+n)^{\frac{1}{1-\beta}}-1+\delta}{s}\right]^r > \alpha$

(i) There exists a balanced growth path (BGP) $\{K_t^*\}$, *to which the capital path* $\{K_t\}$ *asymptotically converge, if and only if condition* (C) *holds. In this case it is unique. We have:* $\{K_t^* = K^s(1+g)^t\}$ *where* $1+g = (1+n)^{\frac{1}{1-\beta}}$ *and* K^s *is a steady state of capital stock and uniquely determined.*

(ii) Let $g_t = \frac{K_t}{K_{t-1}}, g_t^y = \frac{Y_t}{Y_{t-1}}$ *are growth rate of capital and output in period* t *respectively, then* $g_t \to g$ *and* $g_t^y \to g^y = g$, *when* $t \to \infty$ *and* $g = (1+n)^{\frac{1}{1-\beta}} - 1.$

Proof. See Appendix ■

Proposition 9 implies that at the steady state the growth rate of output is neutral to the saving while positively related to TFP coefficient, β. Without learning-by-doing and spillover the output growth rate conincides with the growth rate of labor, meaning output per capita is constant. It is noteworthy that if $n = 0$ then in long-run the growth will be ceased regardless how high TFP coefficient is. Moreover, if we take human capital improvement into account and replace raw labour by effective labour then n can always be positive. Put it differently, the growth based on learning-by-doing and investing to education to improve human capital can be sustained in the long-run.

Now let us consider two economies which are identical in everything, except for TFP coefficient and rates of saving. Let us denote (β, s) and (β', s') respectively the TFP coefficient and rates of saving in these two economies. We assume that $\beta < \beta'$ and $s > s'$. Let g^y (respectively $g^{y'}$) be the growth rates of the output Y_t associated with β (respectively β'). In the following proposition we show that in the short run the impact of higher saving rate may be superior to the impact of better productivity ($g_t^y > g_t^{y'}$) however in the long run the better productivity always dominates in economic growth process.

Proposition 10 *In transitional stage:* $\frac{\partial g_t}{\partial s} > 0$ *and* $\frac{\partial g_t^y}{\partial s} > 0$ *and* $\frac{\partial g}{\partial \beta} > 0.$

Proof. See Appendix ■

Proposition 16 implies that if $s > s'$ then in transitional stage $g_t' > g_t'$, $i.e.$,higher saving rate accelerates growth rate in transitional stage, However in the long run the effect of saving on growth will die out and only efficiency coefficient β, matters.

It is also noteworthy that equation (3.6) in the proof of proposition 9 indicates $\frac{\partial K^s}{\partial s} > 0$, $i.e.$, the economy with higher saving rate converges to higher steady state.

We make the following important remarks:

In transitional stage the economy with higher saving rate may enjoy higher growth rate of output, $g_t^y > g_t^{y\prime}$. *However, in the long run* $g_t^y \to g$ *and* $g_t^{y'} \to g'$ *where* $g < g'$ *(since* $\beta < \beta'$*). Therefore there exists a point* T *in time such that* $g_t^y > g_t^{y'}, \forall t \geq T$. *Notice that this point should be in transitional stage thus better ability of learning-by-doing and spilling-over knowledge not only helps the economy enjoy higher growth rate in long-run but also accelerates growth in short-run. In the first period of transition it is also easy to show that* $g_1^{y'} > g_1^y$.

In the following proposition we consider the economy where the elasticity of substitution is less that unity $(r < 0)$.We show that saving plays a crucial role in growth process in this case.

Proposition 11 *(i) if* $s \leq \frac{\delta}{\alpha^{\frac{1}{r}}}$ *then the economy will collapse in long-term*

(ii) if $\frac{(1+n)^{\frac{1}{1-\beta}}+\delta-1}{\alpha^{\frac{1}{r}}} \geq s > \frac{\delta}{\alpha^{\frac{1}{r}}}$ *then the economy may sustain its growth rate. However it always operates under its potential level,* $g = (1+n)^{\frac{1}{1-b}} - 1$. *In the long-run the growth rate of capital stock converges to* $\left[s\alpha^{\frac{1}{r}} + 1 - \delta\right]$, *which does not depend on TFP coefficient* β *but* s. *Moreover, if initial capital stock is high then the economy first contract to a critical level then expand with an increasing growth rate, however remains under its potential rate.*

In addition, in this case the gap between the rich (with high initial capital stock) and the poor (with low initial capital stock) keeps widening if the saving rate of the poor is not superior to the rich.

(iii) $s > \frac{(1+n)^{\frac{1}{1-\beta}}+\delta-1}{\alpha^{\frac{1}{r}}}$ *the economy will converge to its steady state*

Proof. See Appendix ■

We make the following remarks.

1. If saving is not high enough the economy may sustain its growth rate. However it always operates under its potential level. In the long-run the growth rate of capital stock converges to $\left[s\alpha^{\frac{1}{r}}+1-\delta\right]$*, which does not depend on TFP coefficient* β*. In other words, if saving is not high enough the economy in long-run is constrained from enjoying learning-by-doing and spillover effects.*

2. Consider two economies which are identical in every aspects except for the initial capital and saving rate. The gap between the poor economy with initial capital stock K_0' *and the rich with* K_0 *keeps widening if the saving rate of the poor is not higher than saving rate of the rich. In other words the poor economy will never catch-up and even be lagged behind further if its saving rate is not higher than the one of the rich economy.*

Figure 4 below shows three scenarios of growth with different saving rates. In the left of the panel, when saving is too low the economy will collapse in long run.

In the middle we have the case that saving is high enough but lower than a critical level. The economy first contracts then keeps on growing at rate below potential if initial capital stock is higher than a critical level $\underline{K}$. Economically, when initial capital stock is large and the saving rate is too low to offset depreciation, the capital stock will decrease in the first periods. In these periods, the capital stock is depreciated faster than the pace of decrease of saving. Thus there is some point in time where saving will overcome depreciation and then reverse the movements of growth of capital and output. If the initial capital is lower than $\underline{K}$ the growth rate is always positive but lower than potential level, g.

In the right hand side of the panel the economy always converges to its BGP.

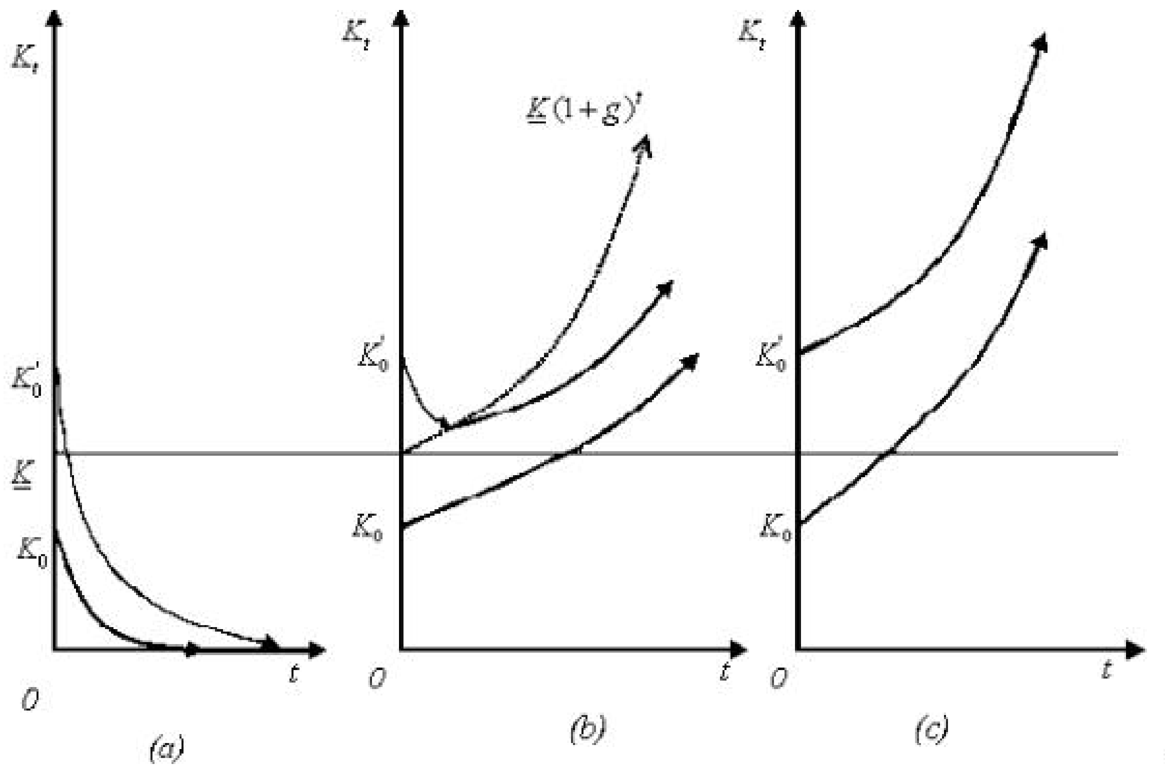


Figure 4: On the Left (a) $s \leq \frac{\delta}{\alpha^{\frac{1}{r}}}$; (b) in the middle $\frac{(1+n)^{\frac{1}{1-\beta}}+\delta-1}{\alpha^{\frac{1}{r}}} \geq s > \frac{\delta}{\alpha^{\frac{1}{r}}}$; and on the right (c) $\frac{(1+n)^{\frac{1}{1-\beta}}+\delta-1}{\alpha^{\frac{1}{r}}} < s$

Proposition 12 *If $r > 0$ and $\frac{(1+n)^{\frac{1}{1-\beta}}+\delta-1}{\alpha^{\frac{1}{r}}} < s$ then $g_t > g_{t+1} > g$ and the growth rate, g_t, converges to $s\alpha^{\frac{1}{r}} - \delta > g$ which does not depend on β.*

Proof. It is similar to the one of proposition 11. ■

Proposition 12 implies that if the elasticity of substitution between capital and labour in an economy is very elastic and its saving rate is also very high, the economy can sustain its growth rate in the long run which does not depend on β. However, it is noteworthy that the condition $\frac{(1+n)^{\frac{1}{1-\beta}}+\delta-1}{\alpha^{\frac{1}{r}}} < s$ is hardly met in reality. Economically, r can not be too high. In the investigation by Duffy and Papageorgiou (2000) for 4 groups of countries, the predicted r is smaller than 0.2, with α is in the range of 0.3 to 0.5, and $\delta > 4\%$ (Mc Quinn and Whelan [2007]) the condition $\frac{(1+n)^{\frac{1}{1-\beta}}+\delta-1}{\alpha^{\frac{1}{r}}} < s < 1$ can not be met.

In short, in this section we have shown that:

1. If the saving rate is good enough or the economy is elastic then in the long-run the growth rate depends positively on: the efficiency in accumulating knowledge, β (effectiveness of learning-by-doing, of spillover of knowledge and experience,etc.,), the growth rate of labour n. However, If the labour is constant the economy that based on importing technology and accumulating knowledge through learning-by-doing can not sustain its growth in long-term even though its process of knowledge accumulation is highly effective (high β). In this sense, learning by doing is insufficient for growth in long run.

2. If we take human capital improvement into account and replace raw labour by effective labour then n can always be positive. Put it differently, the growth based on learning-by-doing and investing to education to improve human capital can be sustained in the long-run. Accordingly, assimilationists are right when argue that the NIEs' economies may keep on growing based on learning-by-doing and importing new technology if they can afford high saving rates and invest heavily in education.

3. The saving rate does not affect the growth rate at steady state, however, the economy with higher saving rate grows faster in transitional stage and converges to a higher level of steady state. In other words Krugman is right when he ascribes the high growth rate in NIEs to high rate of saving.

4. If $r < 0$ the saving rate plays an important role in growth process. If this rate is lower than the critical level $(\frac{\delta}{\alpha^{\frac{1}{r}}})$, the economy will collapse in long term. If it is higher than the critical level but lower than the optimal level the economy always operates under its potential level. Furthermore, in this case the poor economy will be lagged behind if their saving rate is not superior than the rate of the rich economy.

5. When $r > 0$, if the saving rate is higher than the optimal level then the growth rate of the capital stock is decreasing and converges to $s\alpha^{\frac{1}{r}} - \delta > g$ which does not depend on β.

3.4 Conclusion

Krugman's view is correct in the sense that the high saving rate plays an important role in "miracle growth" in NIEs. Our model show that in transitional stage saving always play an important role in growth process. This effect of high saving rate will die out in the long-run if the economy is not very elastic, *i.e.,* $r < 0$. If the economy is very elastic (*i.e.*, $r > 0$) and the saving rate is high enough the growth rate will be decreasing but always higher than the growth rate that predicted at BGP regardless how efficient the economy in learning-by-doing is. However, as noted in the text the conditions for this case is rarely satisfied in the reality.

Second, if the economy is not very elastic *i.e.,* $r < 0$, which is a characteristic of developing economies as indicated by Duffy and Papageorgiou [2000]. Under this condition, savings play a crucial role in growth process. If the saving rate is too low, the economy will collapse in long-run. If the saving rate is higher than critical level and lower than optimal level the economy remain sustain its positive growth in long-run but lower than its potential level. In this case even if the economy possesses high efficiency of learning and spilling-over, it can not enjoy those in long run. Moreover, the better ability of learning-by-doing and spilling-over knowledge, the higher saving rate is required to enjoy fully these effects. In addition the poor economy will be lagged behind if its saving rate is not superior than that of the initially rich economy.

Finally, if $r < 0$, and saving is high enough or $r > 0$ and saving is not too high there is an unique BGP for these economies. In this case we show that assimilationists are right as claiming that learning-by-doing and spill-over play an important role in growth in NIEs. We also show, however, that the growth model based purely on learning-by-doing is constrained by labor growth rate. If the labour is constant in the long-run, then the growth can not be sustained. In this sense, learning by doing is insufficient for growth in long run. Notice that if labour in the model is defined as effective labour to include the effectiveness of human capital accumulation then coefficient n counts for growth rate of human capital which can be positive in the long-run. Equivalently, human capital would help the economy sustain its growth in the long run as proved by other previous studies such as Bruno *et al* [2008], Le Van *et al* [2008],Aghion and Howitt [1992], Lucas [1988] *etc.*,. In short, learning-by-doing can not replace the role of human capital and technological capacity.

3.5 Appendix

3.5.1 Proof proposition 9

(i) Let us define alternative path of capital $\{\hat{K}_t : \hat{K}_t = \frac{K_t}{(1+g)^t}\}$.

From equation (3.2b) we have:

$$\hat{K}_{t+1} = \frac{1-\delta}{1+g}\hat{K}_t + \frac{s}{1+g}\left[\alpha\hat{K}_t^r + (1-\alpha)\hat{K}_t^{r\beta}(1+g)^{rt(\beta-1)}(1+n)^{rt}L_0^r\right]^{\frac{1}{r}} \tag{3.4}$$

Replace $1+g = (1+n)^{\frac{1}{1-\beta}}$ into equation (5.10) we have:

$$\hat{K}_{t+1} = \Phi(\hat{K}_t) = \frac{1-\delta}{1+g}\hat{K}_t + \frac{s}{1+g}\left[\alpha\hat{K}_t^r + (1-\alpha)\hat{K}_t^{r\beta}L_0^r\right]^{\frac{1}{r}}$$

Let define:

$$\Psi(\hat{K}) = \frac{1-\delta}{1+g} + \frac{s}{1+g}\left[\alpha + (1-\alpha)\hat{K}^{r(\beta-1)}L_0^r\right]^{\frac{1}{r}} \tag{3.5}$$

$\Psi(\bullet)$ is strictly decreasing.

(a) if $r > 0$ then the assumption $\left[\frac{(1+n)^{\frac{1}{1-\beta}}-1+\delta}{s}\right]^r > \alpha$ is equivalent to $s\alpha^{\frac{1}{r}} < g+\delta$. We have:

$$\begin{cases} \hat{K} \to 0, \text{ then } \Psi(\hat{K}) \to +\infty \\ \hat{K} \to +\infty, \text{ then } \Psi(\hat{K}) \to \frac{1-\delta}{1+g} + \frac{s\alpha^{\frac{1}{r}}}{1+g} < 1 \end{cases}$$

(b) if $r < 0$ then the assumption $\left[\frac{(1+n)^{\frac{1}{1-\beta}}-1+\delta}{s}\right]^r > \alpha$ is equivalent to $s\alpha^{\frac{1}{r}} > g+\delta$.We have:

$$\begin{cases} \hat{K} \to 0, \text{ then } \Psi(\hat{K}) \to \frac{1-\delta}{1+g} + \frac{s\alpha^{\frac{1}{r}}}{1+g} > 1 \\ \hat{K} \to +\infty, \text{ then } \Psi(\hat{K}) \to \frac{1-\delta}{1+g} < 1 \end{cases}$$

Hence there always exists unique K^s such that $\Psi(K^s) = 1 \Longleftrightarrow \Phi(K^s) = K^s$. Notice that Φ is a concave function. Replace K^s in both sides of equation (5.10) we have

$$K^s = \left[\frac{(1-\alpha)L_0^r}{\left[\frac{(1+n)^{\frac{1}{1-\beta}} - 1 + \delta}{s} \right]^r - \alpha} \right]^{\frac{1}{r(1-\beta)}} \tag{3.6}$$

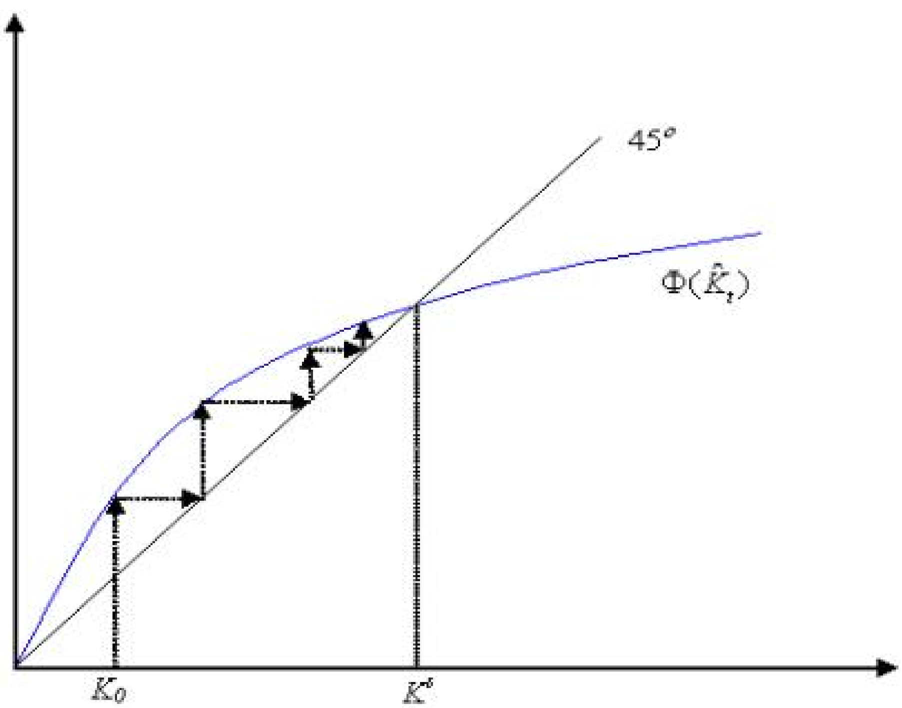


Figure 3

Hence, in the long-run capital stock path $\{K_t\}$ will converge to the BGP $\{K_t^* = K^s(1+g)^t\}$. The growth rate of capital stock at steady state:

$$g = (1+n)^{\frac{1}{1-\beta}} - 1 \tag{3.7}$$

At the steady state the growth rate of output is neutral to saving rate while positively related to TFP coefficient, β.

Hence, in the long-run capital stock path $\{K_t\}$ will converge to the BGP $\{K_t^* = K^s(1+$

$g)^t\}$. The growth rate of capital stock at steady state:

$$g = (1+n)^{\frac{1}{1-\beta}} - 1 \tag{3.8}$$

(ii) Figure 3 shows that if $K_0 < K^s$, then $\hat{K}_t$ monotonically increases to $K^s$and if $K_0 > K^s$ then $\hat{K}_t$ monotonically decreases to K^s. This property implies that:

$$if\ K_0 \ < \ K^s:\ 1+g_t = \frac{K_t}{K_{t-1}} = (1+g)\frac{\hat{K}_t}{\hat{K}_{t-1}} > 1+g, \forall t > 0. \tag{3.9}$$

$$if\ K_0 \ > \ K^s\ :1+g_t = \frac{K_t}{K_{t-1}} = (1+g)\frac{\hat{K}_t}{\hat{K}_{t-1}} < 1+g, \forall t > 0. \tag{3.10}$$

Where g_t is growth rate of capital stock in transitional stage which can be presented in another form:

$$g_t = \frac{K_t}{K_{t-1}} - 1 = s[\alpha + (1-\alpha)K_{t-1}^{(\beta-1)r}L_{t-1}^r]^{\frac{1}{r}} - \delta \tag{3.11}$$

Accordingly, we have:

$$\left(\frac{g_{t+1}+\delta}{s}\right)^r - \left(\frac{g_t+\delta}{s}\right)^r = (1-\alpha)K_{t-1}^{(\beta-1)r}L_{t-1}^r\left(\left(\frac{1+n}{(1+g_t)^{1-\beta}}\right)^r - 1\right) \tag{3.12}$$

From (5.12) we know that if $K_0 < K^s$, then $g_t > g, \forall t > 0 \Rightarrow \frac{1+n}{(1+g_t)^{1-\beta}} < 1, \forall t > 0$. Therefore, it can be infered from equation (3.12) that $g_{t+1} < g_t, \forall t > 0$. In other words the growth rate of capital stock decreases monotonically to its steady state.

By the same token, if $K_0 > K^s$, then the growth rate of capital stock increases monotonically to its steady state.

Now let us define g_t^y be the growth rate of output at period t. From Equations (3.3) and (3.11) we have:

$$g_t^y + 1 = \frac{Y_t}{Y_{t-1}} = \frac{\frac{Y_t}{K_t}}{\frac{Y_{t-1}}{K_{t-1}}}\frac{K_t}{K_{t-1}} = (g_t+1)\frac{\delta + g_{t+1}}{\delta + g_t} \tag{3.13}$$

It can be shown from the monotonicity of g_t and equation (3.13) that:

$$\begin{cases} If\ K_0 < K^s : g_{t+1} + 1 < 1 + g_t^y < g_t + 1 \\ If\ K_0 > K^s : g_{t+1} + 1 > 1 + g_t^y > g_t + 1 \end{cases} \quad \forall t > 0$$

Then by induction: $\begin{cases} If\ K_0 < K^s : g_t^y < g_{t-1}^y \\ If\ K_0 > K^s : g_t^y > g_{t-1} \end{cases} \quad \forall t > 0$

From part (i) we know that g_t and g_{t+1} both converge to g in long-run, therefore g_t^y also monotonically converges to its steady state $g^y = g$.

3.5.1.1 Proof of proposition 16

The proof of proposition 16 requires the following lemma:

Lemma 10 *Let us define* $g_t(s) = \frac{K_t(s)}{K_{t-1}(s)} - 1$ *and* $\tau(s) = \frac{\alpha K_t^\gamma(s)+C}{\alpha K_{t-1}^\gamma(s)+C}$. *For all* t,*if* $\frac{\partial g_t(s)}{\partial s} > 0$, *then* $\gamma\tau'(s) > 0$ *where* C *is a non-negative and constant number.*

Proof. Indeed, $\frac{\partial g_t(s)}{\partial s} > 0$ implies that

$$K_{t-1}(s)K_t'(s) - K_t(s)K_{t-1}'(s) > 0 \ and\ K_t'(s) > 0, \forall t > 0 \tag{3.14}$$

since $K_t(s) = K_0 \prod_{i=1}^{t} (1 + g_i(s))$.

If $\gamma < 0$, from equation (3.14) we have $(K_{t-1}(s))^\gamma$ and $\left(\frac{K_t(s)}{K_{t-1}(s)}\right)^\gamma$ both deacrease with s which further implies that $K_t^\gamma(s) - K_{t-1}^\gamma(s)$ decreases too. Mathematically, we have $\frac{\partial\left(K_t^\gamma(s) - K_{t-1}^\gamma(s)\right)}{\partial s} < 0$.

We have:

$$\begin{aligned} \tau'(s) \quad = \quad & \frac{\alpha^2\gamma\left(K_t(s)K_{t-1}(s)\right)^{\gamma-1}\left(K_{t-1}(s)K_t'(s) - K_t(s)K_{t-1}'(s)\ \right)}{\left(\alpha K_{t-1}^\gamma(s) + C\right)} \\ & + \alpha C \frac{\partial\left(K_t^\gamma(s) - K_{t-1}^\gamma(s)\right)}{\partial s} \end{aligned} \tag{3.15}$$

Both components of the RHS of equation (3.15) are negative which indicates that $\tau'(s) < 0$ if $\gamma < 0$.

By the same token we can prove that if $\gamma > 0$ then $\tau'(s) > 0$.Therefore $\gamma\ \tau'(s) > 0$ in general. ■

Proof of proposition 16

First we show that $\frac{\partial g_t}{\partial s} > 0, \forall t > 0$.

From (3.11) we have:

$$g_1 = sY_0 + 1 - \delta \ \Rightarrow \frac{\partial g_1}{\partial s} > 0$$

and

$$K_1(s) - sK_1'(s) = (1-\delta)K_0 > 0$$

Suppose that we have $K_t(s) - sK_t'(s) > 0$ and $\frac{\partial g_t}{\partial s} > 0$ we prove that $\frac{\partial g_{t+1}}{\partial s} > 0, \forall t \geq 1$.

Indeed, from equation (3.11) we have:

$$\begin{aligned}\frac{\partial g_{t+1}}{\partial s} &= (h(K_t, L_t))^{\frac{1}{r}} + s\,(h(K_t, L_t))^{\frac{1}{r}-1}\frac{1}{r}\frac{\partial h(K_t, L_t)}{\partial K_t}\frac{\partial K_t(s)}{\partial s} \\ &= (h(K_t, L_t))^{\frac{1}{r}-1}\left\{\alpha + (1-\alpha)K_t^{(\beta-1)r-1}L_t^r\left[K_t - (1-\beta)s\frac{\partial K_t(s)}{\partial s}\right]\right\} > 0\end{aligned}$$

By induction we have $\frac{\partial g_t}{\partial s} > 0, \forall t \geq 1$.This result also implies $\frac{\partial K_t}{\partial s} > 0, \forall t > 0$.

Second, we claim that $\frac{\partial g_t^y}{\partial s} > 0, \forall t \geq 1$

Actually we have:

$$\begin{aligned}(1+g_t^y)^r &= \frac{\alpha K_t^r + (1-\alpha)K_t^{\beta r}L_t^r}{\alpha K_{t-1}^r + (1-\alpha)K_{t-1}^{\beta r}L_{t-1}^r} \\ &= (1+g)^r\frac{\alpha \hat{K}_t^r + (1-\alpha)\hat{K}_t^{\beta r}L_0^r}{\alpha \hat{K}_{t-1}^r + (1-\alpha)\hat{K}_{t-1}^{\beta r}L_0^r} \\ &= (1+g)^r\left(\frac{\hat{K}_t}{\hat{K}_{t-1}}\right)^{(1-\beta)r}\frac{\alpha \hat{K}_t^{r-\beta r} + (1-\alpha)L_0^r}{\alpha \hat{K}_{t-1}^{r-\beta r} + (1-\alpha)L_0^r}\end{aligned} \tag{3.16}$$

Notice that $1 + g_t = \frac{K_t(s)}{K_{t-1}(s)} = (1+g)\frac{\hat{K}_t(s)}{\hat{K}_{t-1}(s)}$ .From first part of this proof we know that $\frac{\hat{K}_t(s)}{\hat{K}_{t-1}(s)}$ increases with s.

Applying lemma (14) into equation (3.16) we have:

$$\begin{cases} (1+g_t^y)^r \text{ increases with } s \text{ if } r > 0 \\ (1+g_t^y)^r \text{ decreases with } s \text{ if } r < 0 \end{cases}$$

Equivalently we have $\frac{\partial g_t^y}{\partial s} > 0, \forall t > 0$

3.5.2 Proof proposition 11

First, it is noteworthy that $g_t < g_t^y < g_{t+1}$, hence the movement of growth of output is of similar shape as of growth of capital. In the followings we only deal with the movement path of growth rate of capital stock, g_t which implies the similar path for growth rate of output.

Let us recall the function $\Psi(\bullet)$ which defined in proof of proposition 9. From (5.11) we know that Ψ is a decreasing function then

$$\Psi(\hat{K}_t) \leq \Psi(0) = \frac{s\alpha^{\frac{1}{r}} + 1 - \delta}{1+g}, \forall t \tag{3.17}$$

(i) if $s \leq \frac{\delta}{\alpha^{\frac{1}{r}}}$, then:

$$\begin{aligned} \frac{K_{t+1}}{K_t} &= (1+g)\frac{\hat{K}_{t+1}}{\hat{K}_t} = 1 - \delta + s\left[\alpha + (1-\alpha)\hat{K}_t^{r(\beta-1)} L_0^r\right]^{\frac{1}{r}} \\ &\leq 1 - \delta + s\alpha^{\frac{1}{r}} \leq 1, \forall t \end{aligned}$$

This means that $\{K_t\}$ is a decreasing sequences and converges to zero. Economically, the economy will collapse in long-term.

(ii) From condition $\frac{(1+n)^{\frac{1}{1-\beta}} + \delta - 1}{\alpha^{\frac{1}{r}}} \geq s$ and equation (3.17) we have $\Psi(\hat{K}_t) \leq 1$. This implies that $\{\hat{K}_t\}$ is a decreasing series from K_0 and converges to zero.

Let us define $\underline{K}$ such that: $(1+g)\Psi(\underline{K}) = 1$. The equation (5.11) gives:

$$\underline{K} = \left[\frac{(\frac{\delta}{s})^r - \alpha}{(1-\alpha)L_0^r}\right]^{\frac{1}{r(\beta-1)}}$$

a) If $K_0 \leq \underline{K}$, then $\hat{K}_t < \underline{K}, \forall t \geq 1$, since $\{\hat{K}_t\}$ is a decreasing sequence from K_0. This implies:

$$1 + g_{t+1} = \frac{K_{t+1}}{K_t} = (1+g)\Psi(\hat{K}_t) > (1+g)\Psi(\underline{K}) = 1$$

Hence, $\{K_t\}$ is an increasing sequence from K_0.

Moreover, since $0 < \hat{K}_t < \hat{K}_{t-1}$ hence

$$g_t < g_{t+1} < (1+g)\Psi(0) = 1 + \gamma$$

where

$$1 + \gamma = s\alpha^{\frac{1}{r}} + 1 - \delta$$

The growth rate of capital stock keeps rising in the whole growth process. However this growth rate is constrained by an upper bound γ, which is smaller than the potential level g.

(b) if $K_0 > \underline{K}$, then there exists $T \geq 0$ such that $\hat{K}_T > \underline{K} \geq \hat{K}_{T+1}$, since $\{\hat{K}_t\}$ is a decreasing series from K_0 and converges to zero. Then in first T periods capital stock K_t keep decreasing since $\frac{K_{t+1}}{K_t} < (1+g)\Psi(\underline{K}) = 1$. From period $T+1$ on:

$$(1+g)\Psi(0) > \frac{K_{t+1}}{K_t} > (1+g)\Psi(\underline{K}) \geq 1, \forall t > T$$

the capital stock keep rising in a trajectory similar to the one specified in part (a).

Finally, in the followings we will show that if $\frac{(1+n)^{\frac{1}{1-\beta}} + \delta - 1}{\alpha^{\frac{1}{r}}} \geq s > \frac{\delta}{\alpha^{\frac{1}{r}}}$ then the gap between the poor economy and the rich one will never be narrowed down if saving rate of the poor is not superior than the rich.

Let us define sequence $\{\varepsilon_t\}$such that:

$$K_t = \varepsilon_t (1+\gamma)^t$$

From equation (5.10) we have:

$$\varepsilon_{t+1} = \frac{1-\delta}{1+\gamma}\varepsilon_t + \frac{s}{1+\gamma}[\alpha\varepsilon_t^r + (1-\alpha)\varepsilon_t^{r\beta}(1+\gamma)^{(\beta-1)rt}(1+n)^{rt}L_0^r]^{\frac{1}{r}} \qquad (3.18)$$
$$\varepsilon_{t+1} = \varphi_t(\varepsilon_t(K_0));\ \varepsilon_0 = K_0$$

It is easy to see that $\frac{\varepsilon_{t+1}}{\varepsilon_t} < 1$. Hence the sequence $\{\varepsilon_t\}$ is decreasing and bounded below which implies that $\varepsilon_t \rightarrow \bar{\varepsilon}(K_0) > 0$[1].

Notice that $\varphi_t'(\bullet) > 0$ and $\varepsilon_1'(K_0) > 0$, therefore if $K_0 > K_0'$ then $\varepsilon_t > \varepsilon_t', \forall t > 0$. Accordingly, $\bar{\varepsilon}(K_0) > \bar{\varepsilon}(K_0')$. In addition, equation (3.18) gives $\frac{\partial \varepsilon_t}{\partial s} > 0$, which similarly results in $\varepsilon_t(s) > \varepsilon_t(s'), \forall t > 1$, if $s > s'$.

(iii) see proposition 9

[1]The proof of the argument $\bar{\varepsilon}(K_0) > 0$ tedious and will be fed in upon request

Chapter 4

With Exhaustible Resources, Can A Developing Country Escape From The Poverty Trap?

4.1 Introduction

The standard literature on growth and exhaustible resources, initiated by Dasgupta and Heal (1974) in the seventies, deals with developed economies, or a world economy, relying on a non-renewable natural resource as an input. Capital and resource are imperfect substitutes in the production process. The resource input is necessary in the sense that there is no production without it, but unessential in the sense that its productivity at the origin is unbounded. When the social planner adopts a social welfare function of the discounted utilitarian type, the shadow price of the resource stock follows the Hotelling rule, the resource is asymptotically depleted, and consumption asymptotically vanishes.

Our problematic is somewhat different. We are concerned here with a developing non-renewable natural resource producer –an oil producing country for instance–, which extracts the resource from its soil in its primary sector, and produces a single consumption good with man-made capital in its secondary sector. Moreover, it can sell the extracted resource abroad[1]. The revenues are then used to buy an imported good, which is a perfect substitute of the domestic consumption good. The resource is unnecessary in the Dasgupta and Heal's (1974) sense: domestic production is possible without it. We make the assumption that the country does not have any outside option. It does not have access to the international capital market, and consequently has no possibility of either borrowing against its resource stock or investing abroad. This restrictive assumption allows us to concentrate on the interplay between the ownership of natural resources, the technology, and development[2].

The question we want to address is the following: Can the ownership of non-renewable natural resources allow a poor country to make the transition out of a poverty trap? We suppose that the production function is convex for low levels of capital and concave for high levels. The conditions of occurrence of a poverty trap are then fulfilled (Dechert and Nishimura, [1983, Azariadis and Stachurski [2005]): the country, if initially poor, may be unable to pass beyond the trap level of capital, that is to say to develop. But the country can also extract its resource, sell it abroad, and use the revenues to import the good. The natural resource is a source of income, which, together with the income coming from domestic production, can be used to consume, or to accumulate capital. The idea is that a poor country with abundant natural resources could extract and sell an amount of resource which would enable it to accumulate a stock of capital sufficient to overcome the weakness of its initial stock. We want to know on what circumstances would such a scenario optimally occur.

Notice that additional resources could be generated through other mechanisms, such as foreign aid, which can also be used to consume or to accumulate capital. The main difference is that foreign aid is a windfall resource, while the decisions of extraction and selling of the non-renewable resource are endogenous. They are constrained by the finiteness of the stock, and a priori dependent on the size of the stock, the impatience of the economy and the characteristics of demand.

[1] In the same spirit, Eliasson and Turnowsky (2004) study the growth paths of a small economy exporting a renewable resource to import consumption goods, with a reference to fish for Iceland, or forestry products for New-Zealand.

[2] We discuss in the conclusion how the results would be modified if the country had an outside option.

We study in this paper the optimal extraction and depletion of the non-renewable resource, and the optimal paths of accumulation of capital and of domestic consumption. We take into account the characteristics of the domestic technology, the shape of the foreign demand for the non-renewable resource, and of course the initial abundance of the resource and the initial level of development of the country.

We show that in some cases, the ownership of the natural resource leads the country to give up capital accumulation, eat the resource stock and collapse asymptotically, while in others it allows the country to escape from the poverty trap. The outcome depends, besides the interactions between technology and impatience as in Dechert and Nishimura (1983) , on the characteristics of the resource revenue function, on the level of its initial stock of capital, and on the abundance of natural resource.

The remaining of the paper is organized as follows. Section 2 presents the model. Section 3 gives the properties of the optimal growth paths. Section 4 provides a summary of the main results and concludes by a discussion of how the model can embed the case where the country has access to international capital markets.

4.2 The model

We consider a country which possesses a stock of a non-renewable natural resource $\overline{S}$. This resource is extracted at a rate R_t, and then sold abroad at a price P_t, in terms of the numeraire, which is the domestic single consumption good. We consider a partial equilibrium set-up in which the demand side is simply modeled through an inverse demand function for the resource $P(R_t)$. The revenue from the sale of the natural resource, $\phi(R_t) = P(R_t)R_t$, is used to buy a foreign good, which is supposed to be a perfect substitute of the domestic good, used for consumption and capital accumulation. The domestic production function is $F(k_t)$[3], convex for low levels of capital and then concave. The depreciation rate is δ. We define the function $f(k_t) = F(k_t) + (1-\delta)k_t$, and we shall, in the following, name it for simplicity the technology. We are interested in the optimal growth of this country

[3]The labor input is supposed constant and is normalized to 1.

which, if its initial capital is low, can be locked into a poverty trap (Dechert and Nishimura [1983]). Will the revenues coming from the extraction of the natural resource allow it to escape from the poverty trap? Or, on the contrary, will the existence of the natural resource, which makes possible to consume without producing, destroy any incentive to accumulate?

Formally, we have to solve problem (P):

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t), \ \beta \ \in \ (0,1)$$

under the constraints

$$\begin{aligned} \forall t, c_t &\geq 0, k_t \geq 0, R_t \geq 0, \\ c_t + k_{t+1} &\leq f(k_t) + \phi(R_t), \\ \sum_{t=0}^{+\infty} R_t &\leq \overline{S}, \\ \overline{S} &> 0, \ k_0 \geq 0 \text{ are given.} \end{aligned}$$

We denote by $V(k_0, \overline{S})$ the value function of Problem (P). We make the following assumptions:

H1 The utility function u is strictly concave, strictly increasing, continuously differentiable in R_+, and satisfies $u(0) = 0$, $u'(0) = +\infty$.

H2 The production function F is continuously differentiable in R_+, strictly increasing, strictly convex from 0 to k_I, strictly concave for $k \geq k_I$, and $F'(+\infty) < \delta$. Moreover, it satisfies $F(0) = 0$.

H3 The revenue function ϕ is continuously differentiable, concave, strictly increasing from 0 to $\widehat{R} \leq +\infty$, and strictly decreasing for $R > \widehat{R}$. It also satisfies $\phi(0) = 0$.

Throughout this paper, an infinite sequence $(x_t)_{t=0,\dots,+\infty}$ will be denoted by $\mathbf{x}$. An optimal solution to Problem (P) will be denoted by $(\mathbf{c}^*, \mathbf{k}^*, \mathbf{R}^*)$. We say that the sequences

c, **k**, **R** are feasible from k_0 and $\overline{S}$ if they satisfy the constraints:

$$\begin{aligned} \forall t, c_t &\geq 0, k_t \geq 0, R_t \geq 0 \\ c_t + k_{t+1} &\leq f(k_t) + \phi(R_t), \\ \sum_{t=0}^{+\infty} R_t &\leq \overline{S}, \text{ and } k_0 \text{ is given.} \end{aligned}$$

Let $\Omega(k_0, \overline{S})$ denote the set of (**k**, **R**) feasible from k_0 and $\overline{S}$, i.e.,

$$\begin{aligned} \forall t,\ 0 \leq k_{t+1} &\leq f(k_t) + \phi(R_t),\ 0 \leq R_t \\ \sum_{t=0}^{+\infty} R_t &\leq \overline{S},\ k_0 \geq 0 \text{ is given.} \end{aligned}$$

We first list some preliminary results necessary for the main results of our paper.

Lemma 11 *The value function V is continuous in k_0, given $\overline{S}$.*

Proof. See the Appendix. ■

Lemma 12 *There exists a constant A which depends on k_0, $\widehat{R}$, and $\overline{S}$, such that for any feasible sequence $(\mathbf{c}, \mathbf{k}, \mathbf{R})$, we have $\forall t, 0 \leq c_t \leq A, 0 \leq k_t \leq A$.*

Moreover, Problem (P) has an optimal solution. If $k_I = 0$, then the solution is unique.

Proof. See the Appendix. ■

4.3 Properties of the optimal paths

We now study the properties of the optimal paths.

In the following, the superscript * denotes the optimal value of the variables.

We first show that along the optimal path consumption is always strictly positive and extraction always less than $\widehat{R}$, the extraction corresponding to the maximum of the revenue function (Proposition 13). In particular, a resource-rich economy ($\overline{S} > \widehat{R}$) could contemplate extracting the whole resource stock at the beginning of the path in order to accumulate a great amount of capital, that could allow it to overcome the weaknesses of its technology and initial capital stock. But such a development policy is never optimal. The amount of resource sold on the foreign market would be high enough to induce a sharp decrease of its price, and hence a low total revenue.

Proposition 13 *For any t, $c_t^* > 0$ and $R_t^* < \widehat{R}$.*

Proof. See the Appendix ■

4.3.1 Marginal revenue at origin and exhaustion

We now examine the properties of the revenue function, in order to rule out the unrealistic case in which the resource is never exhausted in finite time, whatever the technology, impatience and the initial capital stock.

Proposition 14 *If $\phi'(0) = +\infty$, then $R_t^* > 0$ for all t. Obviously, $R_t^* \to 0$ as $t \to +\infty$.*

Proof. See the Appendix. ■

We will favor in the remaining of the paper the case where the marginal revenue at the origin is finite:

H4 $\phi'(0) < +\infty$.

This case corresponds indeed to the existence of a finite choke price, the price at which the demand for the resource becomes nil because it is entirely transferred to a renewable (but expensive) substitute. It is the simplest way to implicitly recognize the existence of such a substitute to the non-renewable resource.

4.3.2 The Euler conditions and the Hotelling rule

We proceed with the optimality conditions of our problem (P).

Proposition 15 *Let $k_0 \geq 0$. We have the following Euler conditions:*

$$(i)\ \forall t,\ f'(k_{t+1}^*) \leq \frac{u'(c_t^*)}{\beta u'(c_{t+1}^*)} \qquad (E1)$$

with equality if $k^*_{t+1} > 0$,

$$(ii)\ \forall t,\ \forall t',\ \beta^t u'(c^*_t)\phi'(R^*_t) = \beta^{t'} u'(c^*_{t'})\phi'(R^*_{t'}), \qquad (E2)$$

if $R^*_t > 0,\ R^*_{t'} > 0,$ *and*

$$(iii)\ \forall t,\ \forall t',\ \beta^t u'(c^*_t)\phi'(R^*_t) \leq \beta^{t'} u'(c^*_{t'})\phi'(R^*_{t'}), \qquad (E2')$$

if $R^*_t = 0,\ R^*_{t'} > 0.$

Proof. (i) Given t, k^*_{t+1} solves:

$$\begin{aligned} \max_y \quad & \left[u(f(k^*_t) + \phi(R^*_t) - y) + \beta u(f(y) + \phi(R^*_{t+1}) - k^*_{t+2})\right] \\ \text{s.t. } 0 \ \leq \ & y \leq f(k^*_t) + \phi(R^*_t) \\ 0 \ \leq \ & y. \end{aligned}$$

Since $c^*_t = f(k^*_t) + \phi(R^*_t) - k^*_{t+1} > 0$, one easily gets $(E1)$.
(ii) Since $\overline{S} > 0$, there exists t with $R^*_t > 0$. Fix some T such that there exists $t \leq T$ with $R^*_t > 0$. Then $(R^*_0, ..., R^*_T)$ solve

$$\begin{aligned} \max_{(R_0,...,R_t)} \quad & \sum_{t=0}^{T} \beta^t u(f(k^*_t) + \phi(R_t) - k^*_{t+1}) \\ \text{s.t. } \sum_{t=0}^{T} R_t \ \leq \ & \overline{S} - \sum_{\tau=T+1}^{+\infty} R^*_\tau \\ 0 \ \leq \ & R_t, \forall t = 0, ..., T \\ k^*_{t+1} - f(k^*_t) \ \leq \ & \phi(R_t), \forall t = 0, ..., T. \end{aligned}$$

Since ϕ is concave and u is strictly concave, $(R^*_0, ..., R^*_T)$ will be the unique solution. Moreover, since $c^*_t > 0$ for every t, the third constraints system will not be binding. There exist therefore $\lambda \geq 0$ and $\mu_t \geq 0, t = 0, ..., T$ such that $(R_0^*, ..., R_T^*)$ maximize

$$\sum_{t=0}^{T} \beta^t u(f(k_t^*) + \phi(R_t) - k_{t+1}^*) - \lambda \left[\sum_{t=0}^{T} R_t - \overline{S} + \sum_{\tau=T+1}^{+\infty} R_\tau^* \right] + \sum_{t=0}^{T} \mu_t R_t,$$

with $\mu_t R_t^* = 0, \forall t = 0, ..., T$. One easily obtains $(E2)$ and $(E2')$. ■

Notice that in the case of an interior solution, equations $(E1)$ and $(E2)$ allow us to obtain the Hotelling rule:

$$\frac{\phi'(R_{t+1}^*)}{\phi'(R_t^*)} = f'(k_{t+1}^*). \tag{4.1}$$

It states that the growth rate of the marginal revenue obtained from the resource is equal to the marginal productivity of capital along the optimal path.

4.3.3 To accumulate or to "eat" the resource stock?

We have shown that consumption is always strictly positive along the optimal path (Proposition 13). But how is this consumption obtained? Does the country "eat" its resource stock or does it accumulate capital to produce the consumption good? We show in the following propositions that the answer depends on the characteristics of the technology compared to impatience and depreciation, and on the size of the non-renewable resource stock.

If the marginal productivity of capital at the origin $F'(0)$ is larger than the depreciation rate δ, i.e. if $f'(0) > 1$, the country accumulates capital from some date on and the resource stock is exhausted in finite time (Proposition 16). The country accumulates at any period provided that the marginal productivity at the origin is larger than the sum of the social discount rate and the depreciation rate, $\rho + \delta$, with $\rho = \frac{1}{\beta} - 1$, i.e. $f'(0) > \frac{1}{\beta}$ (Proposition 17). The country never accumulates if the marginal productivity is very low, such that its highest possible value is smaller than the depreciation rate, and the initial capital stock is small or the initial resource stock is large enough (Proposition 18). In these cases, the country does not exhaust its resource in finite time but consumes it and collapses asymptotically.

To prove that the natural resource will be exhausted in finite time if the marginal productivity of capital at the origin is high enough we introduce an intermediary step.

Consider Problem (Q), the same problem without natural resource:

$$U(k_0) = \max \sum_{t=0}^{+\infty} \beta^t u(c_t), \ \beta \in (0,1)$$

under the constraints

$$\begin{aligned} \forall t, c_t &\geq 0, k_t \geq 0, \\ c_t + k_{t+1} &\leq f(k_t), \\ k_0 &\geq 0 \text{ is given.} \end{aligned}$$

Let φ denote the optimal correspondence of (Q), i.e., $k_1 \in \varphi(k_0)$ iff we have $k_1 \in [0, f(k_0)]$ and

$$\begin{aligned} U(k_0) &= u(f(k_0) - k_1) + \beta U(k_1) \\ &= \max\{u(f(k_0) - y) + \beta U(y) : y \in [0, f(k_0)]\}. \end{aligned}$$

Next consider Problem $(Q_{\mathbf{a}})$ where $\mathbf{a}$ is a sequence of non-negative real numbers which satisfies $\sum_{t=0}^{+\infty} a_t < +\infty$:

$$W(k_0, (a_t)_{t\geq 0}) = \max \sum_{t=0}^{+\infty} \beta^t u(c_t), \ \beta \in (0,1)$$

under the constraints

$$\begin{aligned}\forall t, c_t &\geq 0, k_t \geq 0,\\ c_t + k_{t+1} &\leq f(k_t) + a_t,\\ k_0 &\geq 0 \text{ is given.}\end{aligned}$$

Obviously, $W(k_0, 0) = U(k_0)$, and $W(k_0, (a_t)_{t\geq 0}) \geq U(k_0)$. We also have the Bellman equation: for all k_0,

$$W(k_0, (a_t)_{t\geq 0}) = \max\{u(f(k_0) - y + a_0) + \beta W(y, (a_t)_{t\geq 1}) : y \in [0, f(k_0) + a_0]\}.$$

Let $\psi(., (a_t)_{t\geq 0})$ denote the optimal correspondence associated with $(Q_{\mathbf{a}})$, i.e., $k_1 \in \psi(k_0, (a_t)_{t\geq 0})$ iff $W(k_0, (a_t)_{t\geq 0}) = u(f(k_0) - k_1 + a_0) + \beta W(k_1, (a_t)_{t\geq 1})$ and $k_1 \in [0, f(k_0) + a_0]$. We have the following lemma, which basically ascertains, in the model without natural resources but with windfall foreign aid, the continuity of the optimal choices with respect to the initial capital stock k_0 and the sequence of aid $\mathbf{a}$..

Lemma 13 *Let $k_0^n \to k_0$ and $\mathbf{a^n} \to \mathbf{0}$ in l^∞ when n converges to infinity. If, for any n, $k_1^n \in \psi(k_0^n, \mathbf{a^n})$ and $k_1^n \to k_1$ as $n \to +\infty$, then $k_1 \in \varphi(k_0)$.*

Proof. See the Appendix. ■

Proposition 16 *Let $k_0 \geq 0$. Assume $f'(0) > 1$. Then there exists T and T_∞ such that for all $t \geq T$ we have $k_t^* > 0$, and for all $t > T_\infty$, we have $R_t^* = 0$.*

Proof. See the Appendix ■

We now show that the country will always accumulate, even without any capital endowment, if the marginal productivity at the origin is higher than the investment cost.

Proposition 17 *Let $k_0 \geq 0$. Assume $f'(0) > \frac{1}{\beta}$. Then $k_t^* > 0$ for any $t \geq 1$.*

Proof. See the Appendix. ■

Notice that when the initial capital stock is equal to 0, the same economy without natural resources never takes-off (Dechert and Nishimura [1983]).

Finally, we show that under adverse conditions the country may never accumulate in physical capital. It then does not exhaust its resource stock in finite time, but "eats" it and collapses asymptotically.

If the marginal productivity of capital is very low, and more precisely if $F'(k_I) \leq \delta$, i.e. $f'(k_I) \leq 1$, production will come to an end at some point in time (part (a) of Proposition 18). Notice that this case features an extremely bad technology, which net return is in fact *negative* whatever the level of capital.

We thus weaken the assumption and consider the case of low average productivity ($\max\{\frac{F(k)}{k} : k > 0\} \leq \delta$, i.e. $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$), due to very high fixed costs, compatible with large marginal productivity at some levels of capital. Then if the country's initial capital endowment is smaller than a certain threshold, it will never accumulate, whatever the level of the resource stock (part (b)). Moreover, for any given initial capital endowment, when impatience is high enough the country will never accumulate if the resource is very abundant (part (c)). One may then wonder whether a country endowed with very abundant natural resources will never accumulate. Part (d) of Proposition 18 shows that it is not true: for any given initial capital, when impatience is low enough, a country owning an abundant resource stock will indeed accumulate from period 1 on. The abundance of natural resources has opposite incentive effects depending on the impatience of the economy: it encourages a patient economy to invest in physical capital, whereas it discourages an impatient one from doing so. Moreover, the poorer the country (the smaller k_0) the larger the range of discount rates for which it does not accumulate. Finally, part (e) of the Proposition considers the case where the extraction giving the maximum revenue $\widehat{R}$ is infinite. This case is clearly not realistic, but is seen here at the limit of situations in which the country can at each period sell abroad very large amounts of resource without depressing the demand. We then show that the economy will never accumulate if the resource is abundant enough. Parts (d) and (e) highlight the importance of the maximum revenue that can be obtained by a resource-abundant economy, given the characteristics of demand. When it is finite, the supply of additional wealth that the country can obtain at each period is bounded, and capital accumulation is necessary, at least when the country is patient. When the supply of additional wealth is potentially infinite at the beginning of the development path, accumulation becomes unnecessary.

Proposition 18 *(a) Assume* $f'(k_I) \leq 1$. *Let* $k_0 \geq 0$. *Then there exists* T *with* $k_t^* = 0$ $\forall t \geq T$.

(b) Assume $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$ *and* $\widehat{R} < +\infty$. *Then there exists* $\varepsilon > 0$ *such that, if* $k_0 \leq \varepsilon$, *then* $k_t^* = 0$ $\forall t$.

(c) Assume $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$, $\widehat{R} < +\infty$ *and* $\beta < \frac{u'(f(k_0)+\phi(\widehat{R}))}{u'(\phi(\widehat{R}))}$. *Then* $k_t^* = 0$ $\forall t \geq 1$ *when* $\overline{S}$ *is large enough.*

(d) Assume $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$, $\widehat{R} < +\infty$, $u'(+\infty) = 0$ *and* $\beta > \frac{1}{f'(0)} \frac{u'(f(k_0)+\phi(\widehat{R}))}{u'(\phi(\widehat{R}))}$. *Then* $k_1^* > 0$ *when* $\overline{S}$ *is large enough.*

(e) Assume $\max\{\frac{f(k)}{k} : k > 0\} \leq 1$ *and* $\widehat{R} = +\infty$. *Then* $k_t^* = 0, \forall t \geq 1$ *when* $\overline{S}$ *is large enough.*

Proof. See the Appendix. ■

4.3.4 The long term: is it possible to escape from the poverty trap?

We now study the long term of our economy.

We know, from Dechert and Nishimura (1983), that in an economy without natural resource,

- if $f'(0) > \frac{1}{\beta}$ (good technology relatively to impatience), then any optimal path from $k_0 > 0$ converges to a steady state $k^s > k_I$ satisfying $f'(k^s) = \frac{1}{\beta}$;

- if $f'(0) < \frac{1}{\beta} < \max\{\frac{f(k)}{k} : k > 0\}$ (intermediate technology relatively to impatience), then there exists $k^c < \widetilde{k}$, with $\frac{f(\widetilde{k})}{\widetilde{k}} = \frac{1}{\beta}$, such that if $k_0 < k^c$ then any solution $\mathbf{k}$ to Problem (Q) converges to 0, and if $k_0 > k^c$, then it converges to a high steady state k^s fulfilling $f'(k^s) = \frac{1}{\beta}$;

- if $\max\{\frac{f(k)}{k} : k > 0\} < \frac{1}{\beta}$ (bad technology relatively to impatience), then if k^s is not an optimal steady state, any optimal path converges to 0, and if it is, there exists a critical value k^c with the same properties as in the case of an intermediate technology.

In the case of a good technology relatively to impatience, we will obviously have the same result as Dechert and Nishimura (1983), as the ownership of an additional natural resource cannot worsen the conditions of the country's development in this optimal growth set-up. The resource cannot be a curse, in the sense that a country is always better off with it than without. Notice however that we have extended Dechert and Nishimura (1983) result to the case $k_0 = 0$ (Proposition 17).

The interesting cases are those of intermediate and bad technologies relatively to impatience. When the economy does not own any additional natural resource, it can be prevented from developing by the poverty trap due to the shape of the technology, if its initial capital endowment is low. Intuitively, if the country owns a large stock of natural resource and can obtain high revenues from the extraction of a large amount of this stock at the beginning of its development path, it may be able to accumulate a stock of capital large enough to reach the concave part of the technology and escape the poverty trap. That is the point we want to investigate further.

We need a preliminary lemma, in which we study the case of an economy without natural resource, initially in the concave part of its production function, receiving an exogenous additional resource, an international aid for example, in periods 1 to T. We show that under some (mild) conditions the total resources available at any period t between 1 and T increase with the aid received at t along the optimal path, which is not *a priori* obvious as the expectation of aid could induce less capital accumulation in the previous periods. Hence, the economy is at period T still on the concave part of its production function, whatever the aid it has received before.

Lemma 14 *Consider the following problem:*

$$\max \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

under the constraints

$$\begin{aligned}
c_0 + k_1 &\leq f(k_0) \\
c_1 + k_2 &\leq f(k_1) + a_1 \\
&\ldots \\
c_T + k_{T+1} &\leq f(k_T) + a_T \\
c_t + k_{t+1} &\leq f(k_t) \quad t \geq T+1, \\
\forall t,\ 0 \leq c_t,\ 0 &\leq k_t,\ k_0 > k_I \text{ given}, \\
\text{with } a_t &\geq 0\ \forall t = 1, ..., T.
\end{aligned}$$

Assume $\frac{f(k_I)}{k_I} > \frac{1}{\beta}$ *and* $f'(0) < \frac{1}{\beta} < \max\{\frac{f(k)}{k} : k > 0\}$*. Then, for any* $\widetilde{a} = (a_1, ..., a_T) \geq 0$*, we have a unique solution* $\{k_t^*(\widetilde{a})\}_{t \geq 1}$*. Moreover,* $f(k_T^*(\widetilde{a})) + a_T > f(k_I)$*.*

Proof. See the Appendix. ■

We now show, in the case of an intermediate technology relatively to discounting, that the resource can allow the country to pass the poverty trap. We need to suppose that there exists a feasible (i.e. less than $\widehat{R}$) extraction level $\widetilde{R}$ which, if performed in one go and used to accumulate capital, leads the country to the concave part of its technology. In Proposition 19, we add the assumption that this extraction level is small ($\frac{\phi'(0)}{\phi'(\widetilde{R})} < f'(0)$), which implicitly means that the concave part of the technology is reached for a relatively small capital stock k_I. We drop this assumption in Proposition 20, and suppose instead that the initial stock of resource is very large.

Proposition 19 *Assume there exists* $\widetilde{R} \in (0, \widehat{R})$ *such that, if* k_0' *satisfies* $f(k_0') = \phi(\widetilde{R})$*, then* $k_0' > k_I$*. Assume moreover that* $\frac{f(k_I)}{k_I} > \frac{1}{\beta}$ *and* $\frac{\phi'(0)}{\phi'(\widetilde{R})} < f'(0) \leq \frac{1}{\beta} \leq \max\{\frac{f(k)}{k} : k > 0\}$*. Let* $k_0 \geq 0$*. The optimal sequence* $\mathbf{k}^*$ *converges to* k^s *as* $t \to +\infty$*.*

Proof. >From Proposition 16, there exists T_∞ such that:

$$\begin{aligned} c^*_{T_\infty-1} + k^*_{T_\infty} &= f(k^*_{T_\infty-1}) + \phi(R^*_{T_\infty-1}) \\ c^*_{T_\infty} + k^*_{T_\infty+1} &= f(k^*_{T_\infty}) + \phi(R^*_{T_\infty}) \\ c^*_t + k^*_{t+1} &= f(k^*_t),\ \forall t \geq T_\infty + 1. \end{aligned}$$

Case 1: $\exists t_0 \leq T_\infty$ such that $R^*_{t_0} \geq \widetilde{R}$.

Let $k^{*\prime}_0$ satisfy $f(k^{*\prime}_0) = f(k^*_{t_0}) + \phi(R^*_{t_0})$. Then, $k^{*\prime}_0 > k_I$. From Lemma 14, $f(k^*_{T_\infty}) + \phi(R^*_{T_\infty}) > f(k_I)$, and hence $k^*_{T_\infty+1} > k_I$. The optimal sequence $\{k^*_t\}_{t>T_\infty}$ converges therefore to the steady state k^s since $k_I > k^c$.

Case 2: $R^*_t < \widetilde{R}$ for all $t \leq T_\infty$.

We have, from the Euler conditions

$$f'(k^*_{T_\infty+1}) \leq \frac{\phi'(R^*_{T_\infty+1})}{\phi'(R^*_{T_\infty})} \leq \frac{\phi'(0)}{\phi'(\widetilde{R})} < f'(0).$$

Observe that $f'(k) > f'(0)$ for $k \in [0, k^s]$. Hence $k^*_{T_\infty+1} > k^s > k_I$. The optimal sequence $\{k^*_t\}_{t>T_\infty}$ converges therefore to k^s. ■

Proposition 20 *Assume there exists $\widetilde{R} \in (0, \widehat{R})$ such that, if k'_0 satisfies $f(k'_0) = \phi(\widetilde{R})$, then $k'_0 > k_I$. Assume moreover that $\frac{f(k_I)}{k_I} > \frac{1}{\beta}$ and $1 < f'(0) \leq \frac{1}{\beta} \leq \max\{\frac{f(k)}{k} : k > 0\}$. Let $k_0 \geq 0$. If $\overline{S} \to +\infty$, the optimal sequence $\mathbf{k}^*$ converges to k^s as $t \to +\infty$.*

Proof. >From Proposition 16, we know that there exists T_∞ such that for all $t > T_\infty$, $R^*_t = 0$. The Euler conditions give $\frac{\phi'(R^*_{t+1})}{\phi'(R^*_t)} \geq f'(k^*_{t+1}), \forall t \leq T_\infty$. Accordingly we have

$$\phi'(R^*_{T_\infty}) \geq \prod_{t=1}^{T_\infty} f'(k^*_t)\phi'(R^*_0).$$

Case 1: $\forall t \leq T_\infty$, $f'(k^*_t) \geq f'(0)$. Then

$$\phi'(R^*_{T_\infty}) \geq (f'(0))^{T_\infty} \phi'(R^*_0).$$

Besides,

$$\overline{S} = \sum_{t=0}^{T_\infty} R_t^* < T_\infty \widehat{R}.$$

We first claim that if $\overline{S} \to +\infty$, $R_0^* \to \widehat{R}$. When $\overline{S} \to +\infty$ and $\widehat{R}$ is finite, we have $T_\infty \to \infty$. If $\forall \overline{S}$, $R_0^*(\overline{S}) < \widehat{R}$, there exists a sequence $\left(\overline{S}_i\right)$ converging to infinity and a number $\alpha > 0$ such that $R_0^*(\overline{S}_i) < \widehat{R} - \alpha$. In this case, since $\phi'(R_{T_\infty}^*(\overline{S}_i)) \geq (f'(0))^{T_\infty(\overline{S}_i)} \phi'(R_0^*(\overline{S}_i)) > (f'(0))^{T_\infty(\overline{S}_i)} \phi'(\widehat{R} - \alpha)$, we have $\phi'(R_{T_\infty}^*(\overline{S}_i)) \to \infty$, which is impossible since $\phi'(R_{T_\infty}^*(\overline{S}_i)) \leq \phi'(0) < +\infty$. Hence, there exists $\overline{S}_{\min}$ such that $R_0^*(\overline{S}) = \widehat{R}$, $\forall \overline{S} \geq \overline{S}_{\min}$, and, moreover, $\forall \varepsilon > 0$ $\exists \widetilde{\overline{S}}$ such that $\forall \overline{S} \geq \widetilde{\overline{S}}$, $\widetilde{R} - R_0^*(\overline{S}) < \varepsilon$. Then

$$f(k_0) + \phi(R_0^*(\overline{S})) \geq \phi(\widetilde{R}) = f(k_0'),$$

with $k_0' > k_I$. From Lemma 14, $k_{T_\infty+1}^* > k_I$. The optimal sequence converges to k^s.

Case 2: There exists $t_0 \leq T_\infty$ such that $f'(k_{t_0}^*) < f'(0) < \frac{1}{\beta}$. Then $k_{t_0}^* > k^s$. From Lemma 14 again, $k_{T_\infty+1}^* > k_I$. The optimal sequence converges to k^s. ■

We have already noticed that in this optimal growth set-up the natural resource cannot be a curse, in the sense that the economy is always better off with this additional resource than without. In other words, the optimal value function of the model with resource is always higher than the one of the same model without resource. The natural resource may nevertheless be a curse in the very specific sense of Rodriguez and Sachs (1999) : in some cases, the economy may optimally overshoot its steady state, and then have, during the convergence towards the steady state, decreasing stock of capital and consumption and a negative growth rate. This happens in case 2 of the proof of Proposition 19, and in case 2 of the proof of Proposition 20. Proposition 21 below shows that it also happens when the extraction giving the maximum of the revenue function and the initial resource stock are very large.

Proposition 21 *Assume* $\widehat{R} = +\infty$, $u'(+\infty) = 0$, $\phi'(+\infty) > 0$ *and* $1 < f'(0) \leq \frac{1}{\beta} \leq \max\{\frac{f(k)}{k} : k > 0\}$. *Let* $k_0 \geq 0$. *Then when* $\overline{S}$ *is large enough, there exists* T *such that* $k_T^* > k^s$.

Proof. >From Proposition 16, we know that there exists T_∞ such that for all $t > T_\infty$, $R_t^* = 0$. This Proposition also implies that $R_{T_\infty}^* > 0$.

Suppose the statement is false, namely $k_t^* \leq k^s, \forall t \geq 0$. In this case, $f'(k_t^*) > f'(0) > 1, \forall t \geq 0$.

If there is $\tau < T_\infty$ such that $R_\tau^* = 0$ and $R_{\tau+1}^* > 0$, then from the Euler conditions we have:

$$1 < \frac{\phi'(0)}{\phi'(R_{\tau+1})} \leq \frac{\beta u'(c_{\tau+1}^*)}{u'(c_\tau^*)} \leq \frac{1}{f'(k_{\tau+1}^*)}.$$

This is a contradiction. Then $R_t^* > 0, \forall t \leq T_\infty$.

The Euler conditions give $\frac{\phi'(R_{t+1}^*)}{\phi'(R_t^*)} \geq f'(k_{t+1}^*) > 1, \forall t \leq T_\infty$. Accordingly we have:

$$R_0^* > R_1^* > ... > R_{T_\infty}^* > 0,$$

$$\overline{S} = \sum_{t=0}^{T_\infty} R_t^* < T_\infty R_0^*, \tag{4.2}$$

$$\forall t \leq T_\infty, \ \phi'(R_t^*) \geq \prod_{\tau=1}^{t} f'(k_\tau^*)\phi'(R_0^*). \tag{4.3}$$

We first claim that when $\overline{S} \to +\infty$ then $R_0^* \to +\infty$. If it is not the case, from equation (4.2) $T_\infty \to +\infty$. Then from equation (4.3) $\phi'(R_{T_\infty}) \to +\infty$ since $\phi'(R_{T_\infty}) \geq (f'(0))^{T_\infty} \phi'(R_0^*)$. It is impossible since $\phi'(R) \leq \phi'(0) < +\infty$.

Second, we claim that when $R_0^* \to +\infty$ then $R_1^* \to +\infty$ too. Recall that

$$c_0^* + k_1^* = f(k_0) + \phi(R_0^*).$$

Since $k_1^* \leq k^s$, we have $c_0^* \to +\infty$ when $R_0^* \to +\infty$.

From Euler relation:

$$u'(c_0^*) = \beta u'(c_1^*) f'(k_1^*) > \beta u'(c_1^*).$$

Hence when $c_0^* \to +\infty$ then $c_1^* \to +\infty$ because $u'(+\infty) = 0$. It clearly implies $R_1^* \to +\infty$ since $c_1^* = f(k_1^* + \phi(R_1^*)$.

Again, use Euler relation to set a contradiction:

$$f'(0) \leq f'(k_1^*) \leq \frac{\phi'(R_1^*)}{\phi'(R_0^*)} \to 1 < f'(0).$$

Therefore, there must be some T such that $k_T^* > k^s$. ■

4.4 Summary of the main results and conclusion

We summarize below the main results, in the cases where the country's technology is intermediate or bad relatively to its impatience, since it is mostly in theses cases that our results differ from those from Dechert and Nishimura (1983).

(a) *Intermediate technology relatively to impatience*

Assume $\delta < F'(0) \leq \rho + \delta \leq \max\{\frac{F(k)}{k} + 1 - \delta : k > 0\}$.

(a.1) The country accumulates from some date on and the stock of non-renewable resource is exhausted in finite time.

(a.2) When the concave part of the technology is relatively easy to reach or when the resource is very abundant, the country overcomes the poverty trap.

(a.3) In some cases, the economy may optimally overshoot its steady state k^s, before converging backwards towards it.

(b) *Bad technology*

(b.1) Assume $F'(k) < \delta\ \forall k$. Then the economy stops accumulating after some date.

(b.2) Assume $\max(\frac{F(k)}{k} : k \geq 0) \leq \delta$. Then the economy never accumulates if its initial capital stock is very small, whatever the resource stock.

(b.3) Assume again $\max(\frac{F(k)}{k} : k \geq 0) \leq \delta$. Then for any given initial capital stock k_0, when impatience is high enough the optimal capital path vanishes when the resource is very abundant.

(b.4) Keep the same assumption on $\frac{F(k)}{k}$, and assume moreover that the extraction giving the maximum revenue $\widehat{R}$ is infinite. Then the economy never accumulates if the resource is abundant enough.

Consider finally the case where the country is able to invest in international capital markets, or borrow against its resource stock. One could plausibly assume that if the country wants to borrow, it will face a debt constraint all the tighter since its resource stock is small. This framework would be particularly relevant for oil-exporting countries. Our model can easily embed this case.

Let m_t be net good imports, D_t0 net foreign lending or debt, and r the world interest rate, exogenous and constant for simplicity. The final good domestic market and the foreign market balances read respectively:

$$\begin{aligned} c_t + k_{t+1} &= f(k_t) + m_t \\ D_{t+1} + m_t &= (1+r)D_t + \phi(R_t). \end{aligned}$$

Let $W_t = k_t + D_t$ be total wealth. The resource constraint the country faces is then

$$c_t + k_{t+1} + D_{t+1} = \max_{k_t \geq 0, D_t \geq \chi(\overline{S})} \{f(k_t) + (1+r)D_t : k_t + D_t = W_t\} + \phi(R_t)$$

i.e.

$$\begin{aligned} c_t + W_{t+1} &= \max_{k_t \geq 0} \{f(k_t) - (1+r)k_t\} + (1+r)W_t + \phi(R_t) \\ &= \Psi(W_t) + \phi(R_t) \qquad \text{with } W_t \geq \chi(\overline{S}), \end{aligned}$$

where $\chi(\overline{S})$ is the debt constraint, depending on the initial resource stock and non-positive.

We consider by way of illustration the case of a technology satisfying $f'(0) < 1+r$ and $f'(k_I) > 1+r$. Extending the reasoning to other convex-concave technologies is straightforward. Then $\max_{k_t \geq 0} \{f(k_t) - (1+r)k_t\}$ admits a unique solution $\overline{k} > k_I$, satisfying $f'(\overline{k}) = 1+r$. Following Askenazy and Le Van (1999), define $\widetilde{k}_1$ and $\widetilde{k}_2$ by

$$\begin{aligned} f(\widetilde{k}_1) &= (1+r)\widetilde{k}_1 \\ f(\widetilde{k}_2) &= (1+r)\widetilde{k}_2 \\ 0 &< \widetilde{k}_1 < \overline{k} < \widetilde{k}_2. \end{aligned}$$

Then function Ψ will be as follows:

$$\begin{aligned}
\Psi(W) &= (1+r)W, \qquad 0 \le W \le \widetilde{k}_1 \\
\Psi(W) &= f(W), \qquad \widetilde{k}_1 \le W \le \overline{k} \\
\Psi(W) &= f(\overline{k}) + (1+r)W, \qquad \overline{k} \le W.
\end{aligned}$$

The extended technology Ψ is convex-concave. The most noteworthy difference from our model is that the return to wealth is constant for levels of wealth greater than $\overline{k}$, which will allow the country to grow without bounds if it is patient enough.

Appendix

Proof of Lemma 11

We first prove that the correspondence Ω is compact-valued and continuous in k_0, for the product topology, given $\overline{S}$.

To prove that $\Omega(k_0, \overline{S})$ is compact, take a sequence $\{\mathbf{k^n}\ , \mathbf{R}^n\}$ which converges to $\{\mathbf{k}\ , \mathbf{R}\}$ for the product topology. First, observe that for any feasible $\mathbf{k}$ we have

$$\forall t, 0 \le k_{t+1} \le f(k_t) + \phi(R_t) \le f(k_t) + \max\{\phi(\widehat{R}), \phi(\overline{S})\}.$$

Therefore, $\mathbf{k}$ will be in a compact set for the product topology (see e.g. Le Van and Dana [2003]). Second,

$$\forall n, \forall t, 0 \le k_{t+1}^n \le f(k_t^n) + \phi(R_t^n),$$

hence, by taking the limits we get

$$\forall t, 0 \le k_{t+1} \le f(k_t) + \phi(R_t).$$

We have proved that the set of feasible $\mathbf{k}$ is closed for the product topology. It is obvious that the set of feasible $\mathbf{R}$ belongs to a fixed compact set. To prove that this set is closed, observe that $\forall N, \forall n \sum_{t=0}^{N} R_t^n \leq \overline{S}$. Taking the limit we get $\forall N, \sum_{t=0}^{N} R_t \leq \overline{S}$. That implies $\sum_{t=0}^{+\infty} R_t \leq \overline{S}$. Summing up, we have proved that $\Omega(k_0, \overline{S})$ is compact.

It is easy to check that Ω is upper hemi-continuous in k_0. It is less easy for the lower hemi-continuity of Ω. We will prove that, actually, Ω is lower hemi-continuous. Let $k_0^n \to k_0$ as n goes to $+\infty$ and $(\mathbf{k},\ \mathbf{R}) \in \Omega(k_0, \overline{S})$. We have to show there exists a subsequence still denoted by $(\mathbf{k^n},\ \mathbf{R^n})$, for short, which converges to $(\mathbf{k},\ \mathbf{R})$ and satisfies $(\mathbf{k^n},\ \mathbf{R^n}) \in \Omega(k_0^n, \overline{S}), \forall n$. We have three cases.

Case 1:

$$
\begin{aligned}
0 \leq k_{t+1} &< f(k_t) + \phi(R_0),\ \forall t < T-1 \\
0 \leq k_t &\leq f(k_{t-1}) + \phi(R_{t-1}),\ \forall t \geq T.
\end{aligned}
$$

There exists N such that for any $n \geq N$, we have $k_1 < f(k_0^n) + \phi(R_0)$. Define, for any $n \geq N$, any t, $k_t^n = k_t$, $R_t^n = R_t$ and the proof is done.

Case 2:

$$
\begin{aligned}
k_{t+1} &= f(k_t) + \phi(R_t),\ \forall t \leq T-1, \\
k_{T+1} &< f(k_T) + \phi(R_T), \\
k_{t+1} &\leq f(k_t) + \phi(R_t),\ \forall t \geq T+1.
\end{aligned}
$$

Define, for $t = 0, ..., T-1$ and for any n, $k_{t+1}^n = f(k_t^n) + \phi(R_t)$. Obviously, $k_t^n \to k_t$ for $t = 0, ..., T-1$. Hence, there exists N such that for any $n \geq N$, $k_{T+1} < f(k_T^n) + \phi(R_T)$. The sequences $(k_0^n, k_1^n, ..., k_T^n, k_{T+1}, k_{T+2}, ...)$ and $\mathbf{R^n} = \mathbf{R}$, for every n, satisfy the required conditions.

Case 3:

$$\forall t, k_{t+1} = f(k_t) + \phi(R_t).$$

It suffices to take $k_{t+1}^n = f(k_t^n) + \phi(R_t)$ for every t, every n.

The second step is to prove that the intertemporal utility function is continuous on the feasible set for the product topology. But the proof is standard (see e.g. Le Van and Dana [2003]).

The third step is to apply the Maximum Theorem to conclude that V is continuous in k_0.

Proof of Lemma 12

It is obvious that $R_t \leq \overline{S}$, $\forall t$. Now, if $\widehat{R} < +\infty$ then for any t, we have $c_t + k_{t+1} \leq f(k_t) + \phi(\widehat{R})$. And if $\widehat{R} = +\infty$ then for all t, $c_t + k_{t+1} \leq f(k_t) + \phi(\overline{S})$. Since $f'(+\infty) < 1$, from Le Van and Dana, [2003], page 17, there exists a constant A which depends on $k_0, \widehat{R}$ (if $\widehat{R} < +\infty$) or on $k_0, \overline{S}$ such that $\forall t, 0 \leq c_t \leq A, 0 \leq k_t \leq A$.

We have already proved that the set of feasible sequences is compact for the product topology and the intertemporal utility function is continuous on the feasible set for the same topology. Hence, there exists a solution to Problem (P). When k_I equals 0, because of the strict concavity of the technology and the utility function u, the solution will be unique.

Proof of Proposition 13

Observe that the value function $V(k_0, \overline{S})$ is strictly positive for any $k_0 \geq 0$, since the sequence $\mathbf{c}$ defined by $c_0 = f(k_0) + \phi(\overline{S})$ and $c_t = 0$ for any $t > 0$ is feasible. Hence $V(k_0) \geq u(c_0) > 0$. That implies $c_t^* > 0, \forall t$, by the Inada condition $u'(0) = +\infty$.

Let us prove that $R_t^* < \widehat{R}$ for all t. If $\widehat{R} = +\infty$, the proof is obvious. So, assume $\widehat{R} < +\infty$. We cannot have $R_t^* > \widehat{R}$ for some t, since u is strictly increasing and ϕ is strictly decreasing for $R > \widehat{R}$. We cannot have $R_t^* = \widehat{R}$ for all t since $\sum_{t=0}^{+\infty} R_t^* = \overline{S}$. If there exists T with $R_T^* = \widehat{R}$, we can suppose $R_{T+1}^* < \widehat{R}$. Without loss of generality, take $T = 0$. So

$$
\begin{aligned}
c_0^* + k_1^* &= f(k_0) + \phi(\widehat{R}) \\
c_1^* + k_2^* &= f(k_1^*) + \phi(R_1^*), \text{ with } R_1^* < \widehat{R}.
\end{aligned}
$$

Choose $\varepsilon > 0$ small enough such that $R_1^* + \varepsilon < \widehat{R}$ and $\widehat{R} - \varepsilon > 0$. Let

$$\begin{aligned} c_0 + k_1^* &= f(k_0) + \phi(\widehat{R} - \varepsilon) \\ c_1 + k_2^* &= f(k_1^*) + \phi(R_1^* + \varepsilon) \\ \text{and } c_t &= c_t^*, \ \forall t \geq 2. \end{aligned}$$

Let $\triangle_\varepsilon = \sum_{t=0}^{+\infty} \beta^t u(c_t) - \sum_{t=0}^{+\infty} \beta^t u(c_t^*)$. We have

$$\begin{aligned} \triangle_\varepsilon &= u(c_0) - u(c_0^*) + \beta[u(c_1) - u(c_1^*)] \\ &\geq u'(c_0)[\phi'(\widehat{R} - \varepsilon)(-\varepsilon)] + \beta u'(c_1)[\phi'(R_1^* + \varepsilon)(\varepsilon)] \\ &\geq \varepsilon[\beta u'(c_1)\phi'(R_1^* + \varepsilon) - u'(c_0)\phi'(\widehat{R} - \varepsilon)]. \end{aligned}$$

Let $\varepsilon \to 0$. Then, as $\phi'(\widehat{R}) = 0$, $\lim_{\varepsilon \to 0} \frac{\triangle_\varepsilon}{\varepsilon} \geq \beta u'(c_1^*)\phi'(R_1^*) > 0$. Thus $\triangle_\varepsilon > 0$ for ε small enough. That is a contradiction to the optimality of $\mathbf{c}^*$.

Proof of Proposition 14

Consider the case $\phi'(0) = +\infty$. First assume $R_t^* = 0, \ \forall t$. Then let

$$\begin{aligned} c_0 &= f(k_0) - k_1^* + \phi(\overline{S}) > c_0^* \\ c_t &= f(k_t^*) - k_{t+1}^* = c_t^*, \text{ for } t \geq 1. \end{aligned}$$

Then $\sum_{t=0}^{+\infty} \beta^t u(c_t) > \sum_{t=0}^{+\infty} \beta^t u(c_t^*)$: a contradiction. Hence if $R_T^* = 0$ we can assume that $R_{T+1}^* > 0$. Without loss of generality, take $T = 0$. So

$$\begin{aligned} c_0^* &= f(k_0) - k_1^* \\ c_1^* &= f(k_1^*) - k_2^* + \phi(R_1^*), \text{ with } 0 < R_1^* < \widehat{R}. \end{aligned}$$

Let $\varepsilon \in (0, R_1^*)$. Define

$$
\begin{aligned}
c_0 &= f(k_0) - k_1^* + \phi(\varepsilon) \\
c_1 &= f(k_1^*) - k_2^* + \phi(R_1^* - \varepsilon) \\
c_t &= c_t^*, \ \forall t \geq 2.
\end{aligned}
$$

Then

$$
\begin{aligned}
\triangle_\varepsilon &= \sum_{t=0}^{+\infty} \beta^t u(c_t) - \sum_{t=0}^{+\infty} \beta^t u(c_t^*) \\
&= u(c_0) - u(c_0^*) + \beta[u(c_1) - u(c_1^*)] \\
&\geq u'(c_0)\phi(\varepsilon) + \beta u'(c_1)[\phi(R_1^* - \varepsilon) - \phi(R_1^*)] \\
&\geq [u'(c_0)\phi'(\varepsilon) - \beta u'(c_1)\phi'(R_1^* - \varepsilon)]\varepsilon.
\end{aligned}
$$

Notice that $\lim_{\varepsilon \to 0} \frac{\triangle_\varepsilon}{\varepsilon} = +\infty$ which implies $\triangle_\varepsilon > 0$ for ε small enough: a contradiction.

Proof of Lemma 13

We first prove that $W(k_0^n, \mathbf{a^n}) \to U(k_0)$ as $n \to +\infty$. We have:

$$
\forall n, \ W(k_0^n, (a_t^n)_{t \geq 0}) \geq U(k_0^n),
$$

hence

$$
\liminf_{n \to +\infty} W(k_0^n, (a_t^n)_{t \geq 0}) \geq \lim_{n \to +\infty} U(k_0^n) = U(k_0).
$$

We now prove that $\limsup_{n \to +\infty} W(k_0^n, (a_t^n)_{t \geq 0}) \leq U(k_0)$. Let $\alpha > 0$. There exists N such that, for any $n \geq N$, we have $f(k_0^n) + a_0^n \leq f(k_0) + \alpha$ and $k_0^n \leq \alpha$. Let $\bar{k}^\alpha$ be the largest value of k which satisfies $f(\bar{k}^\alpha) + \alpha = \bar{k}^\alpha$. Using the same argument as in Le Van and Dana [2003], page 17, one can show that, for any feasible sequences from k_0^n, $\mathbf{c^n}$, $\mathbf{k^n}$ of $(Q_{\mathbf{a^n}})$, for any $n \geq N$, any t, we have $c_t^n \leq \max\{\bar{k}^\alpha, k_0 + \alpha\}$, $k_t^n \leq \max\{\bar{k}^\alpha, k_0 + \alpha\}$. Let $\mathbf{c^{*n}}, \mathbf{k^{*n}}$ be the optimal sequences from k_0^n of Problem $(Q_{\mathbf{a^n}})$. Let $\varepsilon > 0$. There exists T such that

$$\forall n,\ W(k_0^n, (a_t^n)_{t\geq 0}) \leq \sum_{t=0}^{t=T} \beta^t u(c_t^{*n}) + \varepsilon.$$

For $t = 0, ...T$, we can suppose that $c_t^{*n} \to \bar{c}_t$ and $k_{t+1}^{*n} \to \bar{k}_{t+1}$. Since for $t = 0, ...T$, we have $c_t^{*n} + k_{t+1}^{*n} = f(k_t^{*n}) + a_t^n$, we obtain $\bar{c}_t + \bar{k}_{t+1} = f(\bar{k}_t)$ for $t = 0, ..., T$. Define $\bar{\mathbf{c}} = (\bar{c}_0, ..., \bar{c}_T, 0, 0, ..., 0, ...)$. We get

$$\limsup_{n\to+\infty} W(k_0^n, (a_t^n)_{t\geq 0}) \leq \sum_{t=0}^{t=T} \beta^t u(\bar{c}_t) + \varepsilon = \sum_{t=0}^{t=+\infty} \beta^t u(\bar{c}_t) + \varepsilon \leq U(k_0) + \varepsilon.$$

This inequality holds for any $\varepsilon > 0$. We have proved $\limsup_{n\to+\infty} W(k_0^n, \mathbf{a^n}) \leq U(k_0)$.

Now, let $k_1^n \in \psi(k_0^n, \mathbf{a^n})$ and suppose $k_1^n \to k_1$ as $n \to +\infty$. We have

$$W(k_0^n, (a_t^n)_{t\geq 0}) = u(f(k_0^n) - k_1^n + a_0^n) + \beta W(k_1^n, (a_t^n)_{t\geq 1}),$$

and $k_1^n \in [0, f(k_0^n) + a_0^n]$. Taking the limits we get

$$U(k_0) = u(f(k_0) - k_1) + \beta U(k_1),$$

with $k_1 \in [0, f(k_0)]$. That proves $k_1 \in \varphi(k_0)$.

4.4.1 Proof of Proposition 16

It will be done in many steps.

Step 1. Since $f'(0) > 1$, we can choose $\epsilon > 0$ such that $f'(0) > 1 + \epsilon$. Assume that there exists an infinite sequence $\{k_{t^\nu}^*\}_\nu$ such that $k_{t^\nu}^* = 0$, for any ν, and hence correspondingly $R_{t^\nu}^* > 0$. Because $\sum_{t=o}^{+\infty} R_t^* = \overline{S}$ we have $R_{t^\nu}^* \longrightarrow 0$ as $\nu \longrightarrow +\infty$. Since $R_{t^\nu}^* \longrightarrow 0$ and $R_{t^\nu-1}^*$ either equals 0 or converges to 0, there exists T such that $\frac{\phi'(R_{t^\nu}^*)}{\phi'(R_{t^\nu-1}^*)} < 1 + \epsilon$ if $t^\nu \geq T$.

We can write down the optimal consumptions at time t^ν and $t^\nu - 1$ as follows:

$$\begin{aligned} c^*_{t^\nu-1} &= \phi(R^*_{t^\nu-1}) + f(k^*_{t^\nu-1}) \\ c^*_{t^\nu} &= \phi(R^*_{t^\nu}) - k^*_{t^\nu+1} \end{aligned}$$

We have

$$u(c^*_{t^\nu-1} - y) + \beta u(c^*_{t^\nu} + f(y)) \leq u(c^*_{t^\nu-1})) + \beta u(c^*_{t^\nu}),$$

for all $y \in [0, c^*_{t^\nu-1}]$, thus

$$-u'(c^*_{t^\nu-1}) + \beta u'(c^*_{t^\nu}) f'(0) \leq 0,$$

and we get a contradiction:

$$1 + \epsilon < f'(0) \leq \frac{u'(c^*_{t^\nu-1})}{\beta u'(c^*_{t^\nu})} \leq \frac{\phi'(R^*_{t^\nu})}{\phi'(R^*_{t^\nu-1})} < 1 + \epsilon.$$

So, there must exist $T \geq 1$ such that $k^*_t > 0$ for all $t \geq T$.

Step 2. We will show that there exists T' such that $R^*_{T'} = 0$. If not, for any $t \geq T$ we have the Euler conditions:

$$\beta u'(c^*_{t+1}) f'(k^*_{t+1}) = u'(c^*_t),$$

$$\beta u'(c^*_{t+1}) \phi'(R^*_{t+1}) = u'(c^*_t) \phi'(R^*_t).$$

Hence

$$f'(k^*_{t+1}) = \frac{u'(c^*_t)}{\beta u'(c^*_{t+1})} = \frac{\phi'(R^*_{t+1})}{\phi'(R^*_t)}.$$

Since $\frac{\phi'(R^*_{t+1})}{\phi'(R^*_t)} \to 1$, we have $f'(k^*_{t+1}) \to 1$, as $t \to +\infty$. Under our assumptions there exists a unique $\widehat{k}$ which satisfies $f'(\widehat{k}) = 1$. Thus $k^*_{t+1} \to \widehat{k}$. In this case, for t large enough, $u'(c^*_{t+1}) > u'(c^*_t) \Leftrightarrow c^*_t > c^*_{t+1}$. The sequence $\mathbf{c}^*$ converges to $\bar{c}$. If $\bar{c} > 0$, we have $f'(\widehat{k}) = \frac{1}{\beta}$: a contradiction. So, $\bar{c} = 0$. Since

$$\forall t,\ c^*_{t+1} + k^*_{t+2} = f(k^*_{t+1}) + \phi(R^*_{t+1}),$$

we have $\widehat{k} = f(\widehat{k})$ with $f'(\widehat{k}) = 1$, and that is impossible. Hence, there must be T' with

$R^*_{T'} = 0$.

Step 3. Assume there exists three sequences $(c^*_{t^\nu})_\nu$, $(k^*_{t^\nu})_\nu$, $(R^*_{t^\nu})_\nu$ which satisfy

$$\begin{aligned} \forall \nu,\ c^*_{t^\nu-1} + k^*_{t^\nu} &= f(k^*_{t^\nu-1}) \\ c^*_{t^\nu} + k^*_{t^\nu+1} &= f(k^*_{t^\nu}) + \phi(R^*_{t^\nu}),\ \text{with } R^*_{t^\nu} > 0. \end{aligned}$$

Hence

$$\forall \nu,\ f'(k^*_{t^\nu}) = \frac{u'(c^*_{t^\nu-1})}{\beta u'(c^*_{t^\nu})} \le \frac{\phi'(R^*_{t^\nu})}{\phi'(0)} < 1.$$

Therefore, $\forall \nu$, $k^*_{t^\nu} > \widehat{k}$. Observe that there exists $\lambda > 0$ such that

$$\forall \nu, \beta^{t_\nu} u'(c^*_{t^\nu})\phi'(R^*_{t^\nu}) = \lambda.$$

This implies $c^*_{t^\nu} \to 0$ as $\nu \to +\infty$. From Lemma 12, $k^*_{t^\nu} \le A, \forall \nu$. One can suppose $k^*_{t^\nu} \to \bar{k} \ge \widehat{k} > 0$ and $k^*_{t^\nu+1} \to \underline{k} = f(\bar{k})$. From Lemma 13, $\underline{k} \in \varphi(\bar{k})$. This implies $c^*_{t^\nu} \to \bar{c} = f(\bar{k}) - \underline{k} = 0$. But, since $\bar{k} > 0$, we must have $\bar{c} > 0$ (see Le Van and Dana [2003]). This contradiction implies the existence of T_∞ such that for all $t \ge T_\infty$, we have $R^*_t = 0$.

4.4.2 Proof of Proposition 17

Assume $k^*_1 = 0$. Then we have

$$\begin{aligned} c^*_0 &= f(k_0) + \phi(R^*_0) \\ c^*_1 + k^*_2 &= \phi(R^*_1). \end{aligned}$$

The following Euler conditions hold:

$$\begin{aligned} -u'(c^*_0) + \beta u'(c^*_1) f'(0) &\le 0 \\ u'(c^*_0)\phi'(R^*_0) - \beta u'(c^*_1)\phi'(R^*_1) &\le 0. \end{aligned}$$

This implies

$$1 < \frac{1}{\beta} < f'(0) \leq \frac{u'(c_0^*)}{\beta u'(c_1^*)} \leq \frac{\phi'(R_1^*)}{\phi'(R_0^*)}.$$

$>$From these inequalities, we get $u'(c_0^*) > u'(c_1^*)$ and $\phi'(R_1^*) > \phi'(R_0^*)$, or equivalently $c_1^* > c_0^*$ and $R_0^* > R_1^*$. A contradiction arises:

$$\phi(R_1^*) \geq \phi(R_1^*) - k_2^* = c_1^* > c_0^* = f(k_0) + \phi(R_0^*) \geq \phi(R_0^*) > \phi(R_1^*).$$

Therefore, $k_1^* > 0$. By induction, $k_t^* > 0$ for all $t \geq 1$.

4.4.3 Proof of Proposition 18

(a) There must be t_0 with $R_{t_0}^* > 0$. We claim that $R_t^* > 0, \forall t > t_0$. Assume $R_{t_0+1}^* = 0$. Then we have the Euler conditions

$$f'(k_{t_0+1}^*) = \frac{u'(c_{t_0}^*)}{\beta u'(c_{t_0+1}^*)} \geq \frac{\phi'(0)}{\phi'(R_{t_0}^*)} > 1,$$

which is impossible. Hence $R_{t_0+1}^* > 0$. By induction, $R_t^* > 0, \forall t > t_0$. Thus, for $t \geq t_0$, we have the FOC:

$$f'(k_{t+1}^*) = \frac{u'(c_t^*)}{\beta u'(c_{t+1}^*)} = \frac{\phi'(R_{t+1}^*)}{\phi'(R_t^*)}, \text{ if } k_{t+1}^* > 0.$$

If there exists an infinite sequence $(k_{t_\nu+1}^*)_\nu$ with $k_{t_\nu+1}^* > 0, \forall \nu$, then from the previous F.O.C. we have $\lim_{\nu \to +\infty} f'(k_{t_\nu+1}^*) = 1$: a contradiction since $\forall \nu, f'(k_{t_\nu+1}^*) \leq f'(k_I) < 1$. Therefore, $k_t^* = 0$ for any t large enough.

(b) Consider Problem (R) in which capital accumulation never takes place:

$$S(k_0, \overline{S}) = \max \sum_{t=0}^{+\infty} \beta^t u(c_t)$$

under the constraints

$$\begin{aligned} 0 \leq c_0 &\leq f(k_0) + \phi(R_0) \\ \forall t \geq 1,\ 0 \leq c_t &\leq \phi(R_t),\ 0 \leq R_t \\ \sum_{t=0}^{+\infty} R_t &\leq \overline{S}. \end{aligned}$$

Let $(R_t^\sharp, c_t^\sharp)_t$ be the solution to this problem. We have $\sum\limits_{t=0}^{+\infty} R_t^\sharp = \overline{S}$, $c_t^\sharp = \phi(R_t^\sharp)\ \forall t \geq 1$ and $c_0^\sharp = f(k_0) + \phi(R_0^\sharp)$. Using the same argument as in Proposition 13, we get $c_t^\sharp > 0$ and $R_t^\sharp < \widehat{R}\ \forall t$. Then $R_t^\sharp > 0\ \forall t \geq 1$, while $R_0^\sharp \geq 0$.

There exists $\lambda > 0$ and $\mu_0 \geq 0$ such that

$$u'\left(f(k_0) + \phi(R_0^\sharp)\right)\phi'(R_0^\sharp) + \mu_0 = \lambda,\ \text{with } \mu_0 R_0^\sharp = 0, \tag{4.4}$$

and

$$\beta^t u'(\phi(R_t^\sharp))\phi'(R_t^\sharp) = \lambda\ \forall t \geq 1. \tag{4.5}$$

Let $(k_t^*, R_t^*)_t$ be the solution to the original problem. We have $\sum\limits_{t=0}^{+\infty} R_t^* = \overline{S}$.

Fix $T > 0$. Consider

$$\begin{aligned} \triangle_T &= u\left(f(k_0) + \phi(R_0^\sharp)\right) - u\left(f(k_0) + \phi(R_0^*) - k_1^*\right) \\ &\quad + \sum_{t=1}^{T+1} \beta^t \left[u(\phi(R_t^\sharp)) - u\left(\phi(R_t^*) + f(k_t^*) - k_{t+1}^*\right)\right]. \end{aligned}$$

We have

$$\begin{aligned} \triangle_T &\geq u'\left(f(k_0) + \phi(R_0^\sharp)\right)\left(\phi'(R_0^\sharp)(R_0^\sharp - R_0^*) + k_1^*\right) \\ &\quad + \sum_{t=1}^{T+1} \beta^t u'(\phi(R_t^\sharp))\left(\phi'(R_t^\sharp)(R_t^\sharp - R_t^*) + k_{t+1}^* - f(k_t^*)\right). \end{aligned}$$

>From equations (4.4) and (4.5), the right-hand side member of this inequality is equal to

$$(\lambda-\mu_0)(R_0^\sharp-R_0^*)+u'\left(f(k_0)+\phi(R_0^\sharp)\right)k_1^*+\lambda\sum_{t=1}^{T+1}(R_t^\sharp-R_t^*)+\sum_{t=1}^{T+1}\beta^t u'(\phi(R_t^\sharp))\left(k_{t+1}^*-f(k_t^*)\right)$$

and also, after re-arrangement,

$$\begin{aligned}&\mu_0 R_0^*+\lambda\sum_{t=0}^{T+1}(R_t^\sharp-R_t^*)+u'\left(f(k_0)+\phi(R_0^\sharp)\right)k_1^*\\&+\sum_{t=2}^{T+1}\beta^t u'(\phi(R_t^\sharp))\left(k_t^*-f(k_t^*)\right)+\beta^{T+2}u'(\phi(R_{T+2}^\sharp))k_{T+2}^*-\beta u'(\phi(R_1^\sharp))f(k_1^*).\end{aligned}$$

We have $\mu_0 R_0^*\geq 0$, $\beta^{T+2}u'(\phi(R_{T+2}^\sharp))k_{T+2}^*>0$, and, by assumption, $k_t^*-f(k_t^*)\geq 0$. Then

$$\begin{aligned}\triangle_T &\geq \lambda\sum_{t=0}^{T+1}(R_t^\sharp-R_t^*)+u'\left(f(k_0)+\phi(R_0^\sharp)\right)k_1^*-\beta u'(\phi(R_1^\sharp))f(k_1^*)\\&\geq \lambda\sum_{t=0}^{T+1}(R_t^\sharp-R_t^*)+\left[u'\left(f(k_0)+\phi(R_0^\sharp)\right)-\beta u'(\phi(R_1^\sharp))\right]f(k_1^*).\end{aligned}$$

Taking the limit, we obtain

$$\Delta_\infty=\lim_{T\to\infty}\Delta_T\geq\left[u'\left(f(k_0)+\phi(R_0^\sharp)\right)-\beta u'(\phi(R_1^\sharp))\right]f(k_1^*). \tag{4.6}$$

In the case $R_0^\sharp>0$, $\mu_0=0$, equation (4.6) reads $\Delta_\infty\geq\lambda\left[\frac{1}{\phi'(R_0^\sharp)}-\frac{1}{\phi'(R_1^\sharp)}\right]f(k_1^*)$. When $k_0=0$, equations (4.4) and (4.5) yield $R_0^\sharp>R_1^\sharp$. Then $\phi'(R_0^\sharp)<\phi'(R_1^\sharp)$ and $\Delta_\infty\geq 0$. By continuity, there exists $\varepsilon>0$ such that if $k_0\leq\varepsilon$ then $R_0^\sharp>R_1^\sharp$, which implies $\Delta_\infty\geq 0$.
In the case $R_0^\sharp=0$, $\mu_0>0$, equation (4.6) reads $\Delta_\infty\geq\left[u'\left(f(k_0)\right)-\beta u'(\phi(R_1^\sharp))\right]f(k_1^*)$. When $k_0=0$, that implies $\Delta_\infty=+\infty$ by the Inada condition $u'(0)=+\infty$. By continuity, $\Delta_\infty>0$ for k_0 small enough.

(c) Let $\Psi(R)=u'(\phi(R))\phi'(R)$ for $R\leq\widehat{R}$. Ψ is strictly decreasing, with $\Psi(0)=+\infty$ and $\Psi(\widehat{R})=0$. Recall that we have, from equations (4.4) and (4.5),

$$u'\left(f(k_0)+\phi(R_0^\sharp)\right)\phi'(R_0^\sharp)+\mu_0=\beta^t u'(\phi(R_t^\sharp))\phi'(R_t^\sharp),\ \forall t\geq 1.$$

We obtain

$$\begin{aligned}
\forall t \geq 1,\ R_t^\sharp &= \Psi^{-1}\left(\frac{u'\Big(f(k_0)+\phi(R_0^\sharp)\Big)\phi'(R_0^\sharp)+\mu_0}{\beta^t}\right) \\
\overline{S} &= R_0^\sharp + \sum_{t=1}^{\infty}\Psi^{-1}\left(\frac{u'\Big(f(k_0)+\phi(R_0^\sharp)\Big)\phi'(R_0^\sharp)+\mu_0}{\beta^t}\right), \\
\text{and } \mu_0 R_0^\sharp &= 0.
\end{aligned}$$

We claim that when $\overline{S} \geq \sum_{t=1}^{\infty}\Psi^{-1}\left(\frac{u'(f(k_0))\phi'(0)}{\beta^t}\right)$ we have $\mu_0 = 0$. Indeed, if $\mu_0 > 0$ then $R_0^\sharp = 0$ and $\overline{S} = \sum_{t=1}^{\infty}\Psi^{-1}\left(\frac{\mu_0+u'(f(k_0))\phi'(0)}{\beta^t}\right) < \sum_{t=1}^{\infty}\Psi^{-1}\left(\frac{u'(f(k_0))\phi'(0)}{\beta^t}\right)$ since Ψ^{-1} is strictly decreasing. We have a contradiction.

Now define Δ_T as in part (b) of the proof. We have as before equation (4.6). When $\overline{S} \geq \sum_{t=1}^{\infty}\Psi^{-1}\left(\frac{u'(f(k_0))\phi'(0)}{\beta^t}\right)$, we have:

$$\overline{S} = R_0^\sharp + \sum_{t=1}^{\infty}\Psi^{-1}\left(\frac{u'\left(f(k_0)+\phi(R_0^\sharp)\right)\phi'(R_0^\sharp)}{\beta^t}\right). \tag{4.7}$$

Relation (4.7) defines an increasing mapping $R_0^\sharp = \zeta_{k_0}(\overline{S})$ with $\zeta_{k_0}(0) = 0$ and $\zeta_{k_0}(+\infty) = \widehat{R}$. When $\overline{S}$ is large enough, $R_0^\sharp$ converges to $\widehat{R}$, which implies, from equation (4.4) with $\mu_0 = 0$ and equation (4.5), that $R_1^\sharp$ also converges to $\widehat{R}$. Then $u'\left(f(k_0)+\phi(R_0^\sharp)\right) - \beta u'(\phi(R_1^\sharp))$ converges to $u'\left(f(k_0)+\phi(\widehat{R})\right) - \beta u'(\phi(\widehat{R}))$. The additional assumption $u'\left(f(k_0)+\phi(\widehat{R})\right) > \beta u'(\phi(\widehat{R}))$ yields $\Delta_\infty \geq 0$.

(d) Assume that the optimal solution is the solution to problem $(\mathcal{R})$. We then have

$$\begin{aligned}
c_0^* &= f(k_0)+\phi(R_0^\sharp), \\
c_t^* &= \phi(R_t^\sharp), \quad t \geq 1.
\end{aligned}$$

Let $c_0 = c_0^* - \varepsilon$, $k_1 = \varepsilon$, $c_1 = f(\varepsilon)+\phi(R_1^\sharp)$ and $c_t = c_t^*$ for all $t \geq 2$.

Let

$$\Delta_\varepsilon = u(c_0) - u(c_0^*) + \beta\left(u(c_1) - u(c_1^*)\right).$$

We have

$$\begin{aligned}\Delta_\varepsilon &\geq u'(f(k_0)+\phi(R_0^\sharp)-\varepsilon)(-\varepsilon)+\beta u'(f(\varepsilon)+\phi(R_1^\sharp))f(\varepsilon)\\ &\geq \varepsilon\left[-u'(f(k_0)+\phi(R_0^\sharp)-\varepsilon)+\beta u'(f(\varepsilon)+\phi(R_1^\sharp))\frac{f(\varepsilon)}{\varepsilon}\right],\end{aligned}$$

and

$$\lim_{\varepsilon\to 0}\frac{\Delta_\varepsilon}{\varepsilon}\geq \beta f'(0)u'(\phi(R_1^\sharp))-u'(f(k_0)+\phi(R_0^\sharp)).$$

But, as we have previously shown in (c), $R_0^\sharp \to \widehat{R}$ and $R_1^\sharp \to \widehat{R}$ when $\overline{S}\to+\infty$. It follows that $\lim_{\varepsilon\to 0}\frac{\Delta_\varepsilon}{\varepsilon}>0$ when $\overline{S}$ is large enough, since by assumption $\beta f'(0)u'(\phi(\widehat{R}))>u'(f(k_0)+\phi(\widehat{R}))$.

(e) From (c), when $\overline{S}\to+\infty$ we have $R_0^\sharp\to\widehat{R}\to+\infty$. In this case, for $\overline{S}$ large enough, there exists $0<\varepsilon<1$ such that $\frac{\beta}{1-\varepsilon}<1$ and

$$(1-\varepsilon)u'\left(\phi(R_0^\sharp)\right)\phi'(R_0^\sharp)<u'(f(k_0)+\phi(R_0^\sharp))\phi'(R_0^\sharp)=\beta u'(\phi(R_1^\sharp))\phi'(R_1^\sharp),$$

which yields

$$\Psi(R_0^\sharp)<\frac{\beta}{1-\varepsilon}\Psi(R_1^\sharp)<\Psi(R_1^\sharp).$$

This implies $R_0^\sharp>R_1^\sharp$, and hence:

$$u'(f(k_0+\phi(R_0^\sharp))>\beta u'(\phi(R_1^\sharp)).$$

Therefore $\Delta_\infty\geq 0$.

Proof of Lemma 14

Let $\widetilde{a}=(a_1,...,a_T)$. We write $\widetilde{a}>0$ if $a_t\geq 0\ \forall t=1,...,T$, with strict inequality for some t.

When $\widetilde{a}=0$, we have $k_t^*(\widetilde{a})>k_0>k_I$ for any $t\geq 1$. Then when $\widetilde{a}>0$ and close to 0, it will still be true that $k_t^*(\widetilde{a})>k_0>k_I$ for any $t\geq 1$, and $f(k_T^*(\widetilde{a}))+a_T>f(k_I)$.

We say that $\widetilde{a}$ increases if no component decrease and at least one increases.

We have 3 cases.

Case 1: $k_I < k_0 < k^s$.

If V denotes the value function, then we have the Bellman equations

$$\begin{aligned} V(f(k_0)) &= \max_{0\le y\le f(k_0)} \{u(f(k_0)-y)+\beta V(f(y)+a_1)\} \\ V(f(k_1)+a_1) &= \max_{0\le y\le f(k_1)+a_1} \{u(f(k_1)+a_1-y)+\beta V(f(y)+a_2)\} \\ &\quad ... \\ V(f(k_T)+a_T) &= \max_{0\le y\le f(k_T)+a_T} \{u(f(k_T)+a_T-y)+\beta V(f(y))\}. \end{aligned}$$

For $\widetilde{a} > 0$ and close to 0, the value function V is concave.We have the following Euler relations:

$$\begin{aligned} u'(f(k_0)-k_1^*(\widetilde{a})) &= \beta V'(f(k_1^*(\widetilde{a}))+a_1)f'(k_1^*(\widetilde{a})) \\ u'(f(k_1^*(\widetilde{a}))+a_1-k_2^*(\widetilde{a})) &= \beta V'(f(k_2^*(\widetilde{a}))+a_2)f'(k_2^*(\widetilde{a})) \\ &\quad ... \\ u'(f(k_t^*(\widetilde{a}))+a_t-k_{t+1}^*(\widetilde{a})) &= \beta V'(f(k_{t+1}^*(\widetilde{a}))+a_{t+1})f'(k_{t+1}^*(\widetilde{a})) \\ &\quad ... \\ u'(f(k_T^*(\widetilde{a}))+a_T-k_{T+1}^*(\widetilde{a})) &= \beta V'(f(k_{T+1}^*(\widetilde{a})))f'(k_{T+1}^*(\widetilde{a})). \end{aligned}$$

We first claim that when $\widetilde{a}$ is close to 0 and increases, $f(k_t^*(\widetilde{a}))+a_t$ increases for any $t = 1, ..., T$.

Assume that $\widetilde{a}$ increases and $f(k_1^*(\widetilde{a}))+a_1$ decreases. It must then be the case that $k_1^*(\widetilde{a})$ decreases. Then the right-hand side of the first Euler relation increases since $V'(k)$ and $f'(k)$ are decreasing functions for $k > k_I$, and the left-hand side decreases since $u'(c)$ is a decreasing function. We have a contradiction. Hence $f(k_1^*(\widetilde{a}))+a_1$ increases when $\widetilde{a}$ is close to 0 and increases. The claim is true for $t = 1$.

Assume now it is true up to t. We prove it for $t+1$. Indeed if $k_{t+1}^*(\widetilde{a})$ increases, it is done. So assume $k_{t+1}^*(\widetilde{a})$ decreases. If $f(k_{t+1}^*(\widetilde{a}))+a_{t+1}$ decreases, then the RHS of the corresponding Euler relation increases. For the LHS, by induction $f(k_t^*(\widetilde{a}))+a_t$ increases. Since $k_{t+1}^*(\widetilde{a})$ decreases, this LHS will decrease: a contradiction, and our claim is true.

We now prove that actually, for any $t = 1, ..., T$, $f(k_t^*(\widetilde{a})) + a_t$ grows without bounds. We proceed by induction.

First consider $t = 1$. Assume there exists $\widetilde{\overline{a}}$ such that if $a_1 > \overline{a}_1$, then $f(k_1^*(\widetilde{a})) + a_1 < f(k_1^*(\widetilde{\overline{a}})) + \overline{a}_1$. Let $\widetilde{a}$ and $\widetilde{a}'$ be defined by $a_t = a_t' = \overline{a}_t$ $\forall t \neq 1$ and $a_1' < \overline{a}_1 < a_1$ with a_1 close to $\overline{a}_1$ and a_1' close to $\overline{a}_1$, such that $f(k_1^*(\widetilde{a})) + a_1 = f(k_1^*(\widetilde{a}')) + a_1'$. Consider the sequences $(k_t^*(\widetilde{a}))$, $(k_t^*(\widetilde{a}'))$ satisfying

$$
\begin{aligned}
c_0^*(\widetilde{a}) + k_1^*(\widetilde{a}) &= f(k_0) \\
c_1^*(\widetilde{a}) + k_2^*(\widetilde{a}) &= f(k_1^*(\widetilde{a})) + a_1 \\
c_t^*(\widetilde{a}) + k_{t+1}^*(\widetilde{a}) &= f(k_t^*(\widetilde{a})) \quad \text{for } t \geq 2,
\end{aligned}
$$

and

$$
\begin{aligned}
c_0^*(\widetilde{a}') + k_1^*(\widetilde{a}') &= f(k_0) \\
c_1^*(\widetilde{a}') + k_2^*(\widetilde{a}') &= f(k_1^*(\widetilde{a}')) + a_1' \\
c_t^*(\widetilde{a}') + k_{t+1}^*(\widetilde{a}') &= f(k_t^*(\widetilde{a}')) \quad \text{for } t \geq 2.
\end{aligned}
$$

Since $f(k_1^*(\widetilde{a}')) + a_1' = f(k_1^*(\widetilde{a})) + a_1$, the resources are the same at period 1 in the 2 cases, and the optimality principle implies $c_1^*(\widetilde{a}') = c_1^*(\widetilde{a})$. The following Euler relations hold:

$$
\begin{aligned}
u'(c_0^*(\widetilde{a})) &= \beta u'(c_1^*(\widetilde{a})) f'(k_1^*(\widetilde{a})), \\
u'(c_0^*(\widetilde{a}')) &= \beta u'(c_1^*(\widetilde{a}')) f'(k_1^*(\widetilde{a}')).
\end{aligned}
$$

But $k_1^*(\widetilde{a}') > k_1^*(\widetilde{a})$ since $a_1 > a_1'$, and hence $c_0^*(\widetilde{a}') < c_0^*(\widetilde{a})$ and we have a contradiction with the Euler relations. Hence $f(k_1^*(\widetilde{a})) + a_1$ grows without bounds with a_1.

Assume it is true up to $t - 1$. We will prove it for t. Assume there exists $\overline{a}_t$ such that if $a_t > \overline{a}_t$, then $f(k_t^*(\widetilde{a})) + a_t < f(k_t^*(\widetilde{\overline{a}})) + \overline{a}_t$. Construct as before $\widetilde{a}$ and $\widetilde{a}'$ with $a_s = a_s' = \overline{a}_s$ $\forall s \neq 1$ and $a_t' < \overline{a}_t < a_t$ with a_t' and a_t close to $\overline{a}_t$, and $f(k_t^*(\widetilde{a})) + a_t = f(k_t^*(\widetilde{a}')) + a_t'$. We have

$$
\begin{aligned}
c^*_{t-1}(\widetilde{a}) + k^*_t(\widetilde{a}) &= f(k_{t-1}(\widetilde{a})) + a_{t-1} \\
c^*_t(\widetilde{a}) + k^*_{t+1}(\widetilde{a}) &= f(k^*_t(\widetilde{a})) + a_t,
\end{aligned}
$$

and

$$
\begin{aligned}
c^*_{t-1}(\widetilde{a}') + k^*_t(\widetilde{a}') &= f(k_{t-1}(\widetilde{a}')) + a'_{t-1} \\
c^*_t(\widetilde{a}') + k^*_{t+1}(\widetilde{a}') &= f(k^*_t(\widetilde{a}')) + a'_t.
\end{aligned}
$$

Since $f(k^*_t(\widetilde{a}')) + a'_t = f(k^*_t(\widetilde{a})) + a_t$, we have, by the optimality principle, $c^*_t(\widetilde{a}') = c^*_t(\widetilde{a})$. We also have the following Euler relations:

$$
\begin{aligned}
u'(c^*_{t-1}(\widetilde{a})) &= \beta u'(c^*_t(\widetilde{a}))f'(k^*_t(\widetilde{a})), \\
u'(c^*_{t-1}(\widetilde{a}')) &= \beta u'(c^*_t(\widetilde{a}'))f'(k^*_t(\widetilde{a}')).
\end{aligned}
$$

But we have assumed that $f(k^*_{t-1}(\widetilde{a}'))+a'_{t-1} \leq f(k^*_{t-1}(\widetilde{a}))+a_{t-1}$. And since $k^*_t(\widetilde{a}') > k^*_t(\widetilde{a})$, we get $c^*_{t-1}(\widetilde{a}') < c^*_{t-1}(\widetilde{a})$. But a contradiction arises in the Euler relations because u' and f' are decreasing. Hence $f(k^*_t(\widetilde{a})) + a_t$ grows without bounds with a_t. We conclude that $f_T(k^*_T(\widetilde{a})) + a_T) \geq f(k_0) > f(k_I)$ for any $a_T \geq 0$.

Case 2: $k_0 > k^s$.

When $\widetilde{a} = 0$, from Dechert and Nishimura (1983) we have $k^*_t(\widetilde{a}) > k^s$ $\forall t$. We use the same technics as in case 1 to get that $f(k^*_T(a)) + a_T \geq k^s$ $\forall a_T \geq 0$.

Case 3: $k_0 = k^s$.

Actually $k^*_T(\widetilde{a})$ depends continuously on k_0, so we write $k^*_T(k_0, \widetilde{a})$ instead of $k^*_T(\widetilde{a})$. For $k_0 > k^s$, we have $f(k^*_T(k_0, \widetilde{a})) + a_T \geq k^s$ $\forall a_T \geq 0$. By continuity, $f(k^*_T(k^s, \widetilde{a})) + a_T \geq k^s$ $\forall a_T \geq 0$.

Chapter 5

Vietnam economic growth in 1986-2007: role of TFP and learning-by-doing

5.1 Introduction

Since *Doi Moi* (Renovation) launched in 1986 and especially since the 1989 reforms, the face of Vietnam's economy and society has changed significantly. Yet, it is now generally recognized that Vietnam is among the best developing countries in terms of achieving relatively high economic growth and reducing poverty incidence. Since *Doi Moi* in 1986 to 2007 the average growth rate of Vietnam is 7% and the level of the absolute poverty has dropped sharply from 75% of the population to 15.1% by the end of 2006. Studies on Vietnam's economic growth tend to attribute Vietnam's success to market-oriented institutional adjustments and especially prudent adjustments in the microeconomic foundations for supporting the private sector (Arkardie and Mallon (2003) and Joint Donor Report (2005), Vo and Nguyen (2008)). At the same time, some have argued that in international comparisons Vietnam's performance is not so spectacular and moreover, there remain many problems for sustaining economic growth and ensuring quality of development[1]. However, there are some warning signs such as high ICOR index, high investment to GDP ratio etc., which raise the question of sustainability of growth.

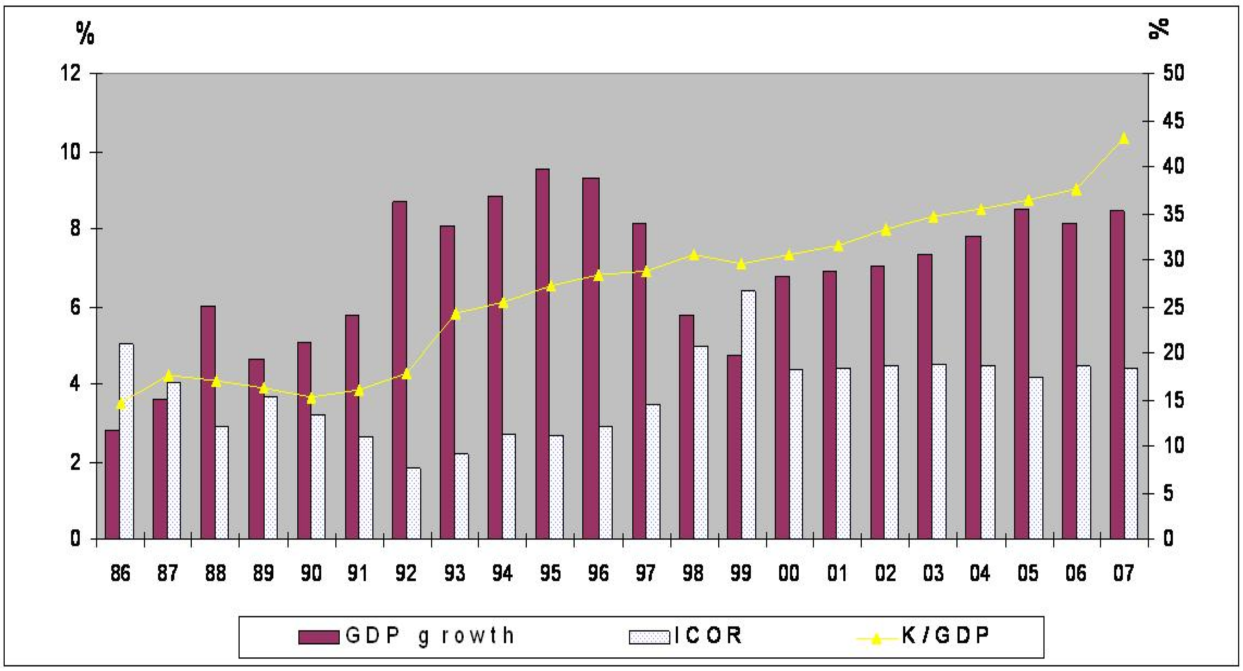


Figure 5.1: Vietnam Economic Growth, ICOR and Investment/GDP ratio 1986-2007

Source: CEIC data base 2008 and author's calculations. Note the values of GDP growth rate and ICOR refer to left axis and ratio of investment to GDP refers to right axis.

Figure 5.1 shows the salient feature of economic growth in Vietnam over last two decades: very high level of investment/GDP ratio. This feature is similar to what have been described for growth processes of first and second generation NIEs: S. Korea, Singapore, Malaysia and Thailand (See figure 5.2). The "miracle growth" in these economies in periods 1970s to the beginning of 1990s has given rise to a broad and diversified literature aiming at explaining the reasons for such a long lasting period of expansion (Kim and Lau

[1]See, for example, Dapice (2003). Vietnam also recognizes that economic growth during 2000-05 was under its potential and the competitiveness of the economy was quite low. According to the World Economic Forum, Vietnam's competitiveness is at the positioned 53/59 in 2000, 60/75 in 2001, 65/80 in 2002, 60/102 in 2003, 77/104 in 2004 and 81/117 in 2005

[1994], Krugman [1994], Rodrik [1995], Worldbank [1993], Young [1994, 1995]). All these economies have experienced rapid growth of investment into physical capital as well as into human capital.

On one hand, the supporters of the accumulation view stress the importance of physical and human capital accumulation in the Asian growth process. Accordingly, the main engine of "miracle growth" in NIEs is simply, very high investment rates. Young [1994, 1995], Kim and Lau [1994] found that the postwar economic growth of the NIEs was mostly due to growth in input factors (physical capital and labor) with trivial increase in the total factor productivity. Moreover, the hypothesis of no technical progress cannot be rejected for the East Asian NIEs (Kim and Lau [1994]). Consequently, accumulation of physical and human capital seems to explain the lion's share of the NIEs' growth process. Krugman [1997] wrote that Larry Lau and Alwyn Young works suggested that Asian growth could mostly be explained by high investment rates, good education and the movement of underemployment peasants into the modern sector. Economists who take this point implicitly assumed that adoption and mastering new technology and other modern practices could be done easily by trade.

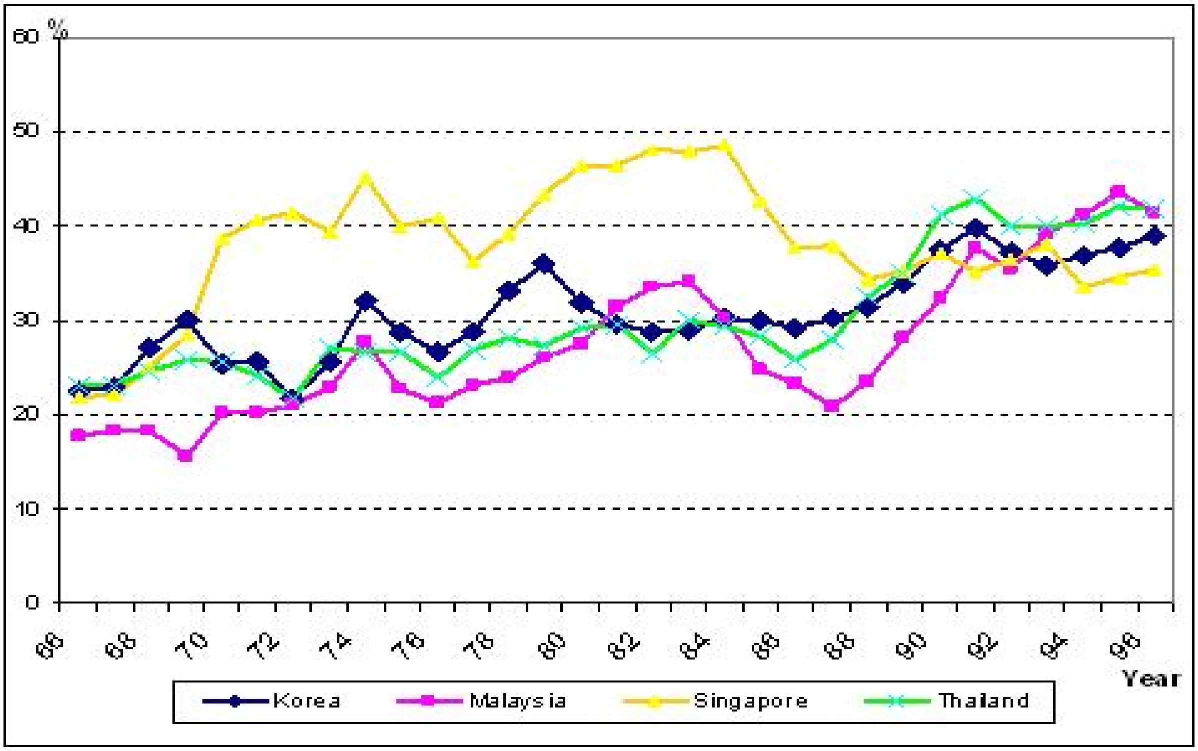


Figure 5.2: Ratios of Investment/GDP in selected Asian countries

Source: WDI, Worldbank

On the other hand, the supporters of endogenous growth theory pinpoint productivity growth as the key factor of East Asian success. According to these authors, Asian countries have adopted technologies previously developed by more advanced economies (assimilation view) and "*the source of growth in a few Asian economies was their ability to extract relevant technological knowledge from industrial economies and utilize it productively within domestic economy*" (Pack [1992]). They admit that high rates of investment into physical and human capital are necessary to achieve high economic growth rate. However, as stressed by Nelson and Pack (1998) there is nothing automatic in learning about, in risking to operate and, in coming to master technologies and other practices that are new to the economy. These processes require searching and studying, learning, and innovating to master modern technologies and new practices. Thereby, the economy enhances its stock of knowledge and efficiency. Implicitly, they suggest that technological progress exist and does play a crucial role in NIEs' economic growth.

Empirically, Collins and Bosworth [1996] or Lau and Park [2003] show Total Factor Productivity (TFP) gains actually matter in Asian NIEs growth and that future growth can be sustained. For these authors, learning-by-doing in process of physical accumulation play an essential role in TFP growth in these economies.

In recent paper (Le Van and Nguyen 2009) prove that in the short and mid terms one economy can enjoy high growth rates driven by highly accumulated physical capital. But in the long term, without TFP growth the growth will be ceased. Specifically, in transitional stage the high saving rate induces high growth rate of output however, in the long-run the impact of saving rate on output growth rate will vanish out. They show further that learning-by-doing may play an important role in TFP growth in NIEs. They also show, however, that the growth model based purely on learning-by-doing is constrained by labor growth rate. If the latter is constant in the long-run, then the growth can not be sustained. In this sense, learning by doing is insufficient for growth in long run. To sustain growth other forms of TFP accelerating such as investment in human capital, new technology (Bruno *at el (2008), Le Van et al. (2008), etc.,*) is needed.

The figure 5.1 shows that Vietnam's high growth rates in the last two decades have come with very high rates of investment. Does TFP play any role in high growth rates of Vietnam in the last two decades? Does learning-by-doing contribute anything to these growth rates? In this paper will examine available data in period 1986-2007 to answer these questions. The year 1986 is chosen because the *Doi moi* process started in this year. For this purpose annual time series data, 1985-2007, are used. Since data on capital stock is not available, based on available information of the annual capital consumption the author has established a series of capital stock for Vietnam in period 1985-2007. Using these data author first estimates an appropriate production function for Vietnamese economy in the period 1986-2007, then the contribution of learning-by-doing and Hicks-neutral technological progress. Finally, based on the estimates of capital share the contribution of TFP is calculated.

First, the examination significantly reject hypothesis of variable elasticity of substitution (VES) production function. It suggests Cobb-Douglass functional form is more appropriate for Vietnam economy in period 1986-2007.

Second, we find that economic growth in Vietnam in period 1986-2007 was driven essentially by physical capital accumulation, then by increase of labour. TFP contributes *negligibly* to the economic growth.

Third, conditional on availability of data we can say that in the transitional period 1986-2007 learning-by-doing proxied by cumulative output from 1975 negatively impacts on growth, while those are proxied under form of Hicks-technology positively contribute to growth. These two effects may cancel out each other and as a result, on aggregate TFP may play a trivial role in growth in this period. Intutively, it seems that in this transitional period on the one hand, economic reforms in Vietnam has improved productivity in terms of Hicks-neutral technological progress; on the other hand remnants of central-planning time in terms of institutions, legal frameworks, way of thinkings remain negatively affect.

The outline of the paper is followings: in section (5.2) we review milestones in Vietnam's economic reforms. Section (5.3) represents a VES model that used for examining Vietnam's economy in the period under consideration. Section (5.4) introduces sources of data and how to establish series of capital stock. Regression results and discussion are in section (5.5). Data for reference will be attached in appendix.

5.2 An overview of Vietnam's economic reforms

Since the reunification in 1975, the economic development and policy changes in Vietnam can be characterized by three periods

Before 1980s: Vietnam was essentially a centrally planned economy (CPE), following closely the Soviet model. Major characteristics of the economy included: (*i*) state or collective ownership of production means; (*ii*) government administered supply of physical input and output; (*iii*) lack of business autonomy, absence of factor markets, highly regulated goods and services markets; and (*iv*) a bias toward heavy industry in investments. Vietnam was also relatively autarkic, trading mostly with the former socialist countries.

With the poor incentives and restricted information flows, the resource allocation was heavily distorted. The problems were further compounded by an unfavorable geopolitical context because of the military conflict with Cambodia in late 1978 and China in 1979. By the late 1970s, Vietnam was facing a 'major economic crisis, with acute shortages of food, basic consumer goods, and inputs to agriculture and industry, and a growing external debt' (ADB [1997], p. 6). The failure of the centrally-planned system had become apparent and pressures for economic reforms increased substantially.

During the period 1980-87: the economy can be regarded as a modified-planned economy where some micro-reforms were undertaken to respond to depletion of the economy, but without any significant changes in macroeconomic management. De Vylder and Fforde (1988) have described the reform process as a "bottom up" one. It was firstly initiated through partial, unofficial relaxation of constraints on private activity and spontaneous moves towards production and trade outside of official/plan channels (for example "illicit contracting" in agriculture and "fence breaking" in manufacturing sector)[2], leading to eventual the Party's recognition of the role of the household sector in agriculture, handicrafts, and retail trading. In 1979, the Council of Ministers issued a decree providing scope for local state enterprises to operate outside the central plan once central plan targets had been realized[3]. In January 1981, a contract system was introduced in the agricultural sector[4], and the government issued a decision providing limited autonomy to state enterprises[5]. These micro-reforms enhanced voluntary and decentralized interactions between individual agents and created new incentives for producers in raising outputs during the period 1982-85. The economy became more dynamic and as a result, Vietnam enjoyed a rather high

[2] One interesting characteristic of the Vietnamese system is its pragmatic flexibility. This characteristic is believed to be built-up over three decades of fierce struggle against powerful enemies. This characteristic explains why such 'fence breaking' behaviors were more easily accepted in Vietnam than in other communist countries.

[3] Decree 279-CP (2/8/1979) 'On Work to Promote the Production and Circulation of Commodities not under State Management and the Supply of Inputs or Raw Materials and Waste and Low Quality Materials at the Provincial Level'.

[4] Directive No. 100 of the Party Central Committee, 13 January 1981, 'On Piece-work Contracts to Employee Groups and Individual Employees Working in Agricultural Cooperatives'

[5] Decision No. 25-CP (21/1/1981) on 'Several Directions and Measures to Enhance the Rights of Industrial State Enterprises to take Initiative in Production and Business and in Self-Financing.'

rate of economic growth in the first half of the 1980s.

Although those micro-reforms in the period 1979-1985 exhibited a trend towards liberalization and an undermining of the state planning system, they were not a transition in real terms. The fifth Party Congress 1982 initiated attempts to recentralize the economy and in 1983, administrative changes were made to control 'anarchy' in the market; the freedom of state enterprises to trade outside of official/plan channels was narrowed. These moves reflected considerable internal debate within the Party about future policy directions.

The improved economic growth was not to be sustainable. In September 1985, in a vain attempt to solve the problem of high prices in free market, the authorities increased state prices, introduced a new currency and the so-called 'price-salary-money reform'. These reforms were implemented without changing in fundamental problems of resource misallocation, trade restrictions and macroeconomic imbalances in the economy. As a result, these reforms failed to cut down inflation. In the mid-1980s, the inflation rate accelerated to several hundred percent.

The year 1986 is recorded as the beginning of the transition because it represented an irreversible change in ideology. The Sixth Party Congress in December 1986 publicly rejected the fiction of trying to implement the central planning model, and instead declared its intention to move towards some form of mixed market economy (a multi-ownership structure). This included the conclusion on the need for policy reforms aimed at reducing macroeconomic instability and accelerating economic growth, and that all 'economic levers' (price, wages, fiscal and monetary policies) were to be used to achieve these objectives.

From 1988-89 onwards, the economy has been an economy in transition, striving for industrialization and international integration. During 1988 and in early 1989, Vietnam adopted a radical and comprehensive reform package aimed at stabilizing and opening the economy, and enhancing freedom of choice for economic units and competition so as to change fundamentally its economic management system. The reforms included[6]:

[6] It is noteworthy that, Land Law of 1988 and 'Party Resolution 10', April 1988, abandoned the collective farming system that had been introduced in the 1960s; Resolution 27/HĐBT of March and Decision 16/NQTU of July 1988 officially encourage private enterprises; Law on foreign Investment 1987 to call for foreign investment.

- Recognition of private ownership, rights of doing business and competition: Land Law of 1988, and then Amended Law in 1993 is an important step towards the introduction property rights, recognized the private land-use rights. The Constitution 1992 acknowledges private ownership and provides guarantees against nationalization (Article 23). The property right and private ownership were further detailed in the first Civil Code 1995 (and modified in 2005). Law on foreign Investment 1987 to call for foreign investment; "Enterprise Law" 1999 (2005: "Unified Law on enterprise") officially acknowledges the right of doing business of people

- Relaxation of market entry restriction: Trade liberalization (presented later) allowed all business entities rights of trading, not only SOEs. Removing cumbersome administrative procedures for business registration and operation. Restructuring state-owned enterprises: From 1990 to 1994, the number of SOEs fell from 12000 to 6300; In period 2001- 2005, Vietnam restructured 3572 out of 5355 SOEs, of which 2378 SOEs restructured through equitization.

- Step-by-step building up market institutions: The mono-banking system was replaced by a two-tier system, which functioned in 1990. Monetary market gradually established: inter-bank market on domestic currency in 1993; inter-bank market on foreign currencies in 1994; bidding market for treasury bills in 1995. Bond market for short-term loans established in 1995. Stock market started functioning in 2000. Foreign invested banks stock holding banks and many financial institutions have been gradually being allowed to operate since 1997. Bankruptcy law approved December 1993. Labor code approved in 1994. Law on competition approved 2004, enacted July 2005. Common Law on Investment enacted July 2006. The law unified two previously promulgated laws: Laws on foreign investment (Approved 1987, amended In 1990, 1992, 1997, 2000) and Law on Promotion domestic investment (Approved 1994, amd. 1998). Law on Intellectual Property (approved Dec. 2005 and enacted July 2006).

- Opening and integrating the economy into the world: Vietnam signed a trade agreement with the European Union (EU) in 1992. In 1995 Vietnam joined ASEAN and committed to fulfill the agreements under the AFTA by 2006. Vietnam also applied for WTO membership in 1995 and attained membership status in November 2006. In 1998, Vietnam became a member of the APEC. In 2000, Vietnam signed the Bilateral Trade Agreement with the United States and the agreement became effective in December 2001. Since 2002, Vietnam has also joined regional integration clubs such as ASEAN +1. These moves have created huge market access for Vietnamese entrepreneurs and played a key role in booming exports which is the main engine for growth in Vietnam. These also opened a wide door for imports necessary for industrialization (88,9% of imports in 2000 was for industrial production) and for foreign investment as well.

These reforms have resulted in fairly high economic growth since the starting year 1986 (see table 1). In essence, these reforms on the one hand, remove barriers that set up in central-planning time; on the other hand, the reforms establish market institutions to promote all economic activities under market mechanism. Hence, these reforms are expected to increase Hicks-neutral productivity, which will be examined in section 5.5. In the model used for examining TFP growth and production specification will be presented in following section.

5.3 The Model

Consider an aggregate production function mapping capital (K) and labour (L), into output (Y). Assume that capital and labour are assumed internally homogeneous and continuously substitutable factors of production. The production function is assumed to be twice differentiable and linearly homogeneous.

$$Y_t = A_t F(K_{t,} L_t) \tag{5.1}$$

where Q, K, L are the level of output, capital stock and employed labour respectively, and A_t is level of technology at time t. $F(\bullet)$ is homogeneous degree one. The marginal rate of technical substitution (S) associated with (5.1) can be expressed as a function of the capital-labor ratio (k) in general form:

$$S = -\frac{\partial F_t/\partial K_t}{\partial F_t/\partial L_t} = f(k) \tag{5.2}$$

where $k = \frac{K}{L}$ and $f(k) > 0,\ f'(k) > 0$ for $\forall K > 0$ and $L > 0$. Notice that due to the neutrality of technical change and homogeneity of F, $f(\bullet)$ is independent of both t and Y. With these restrictions on $f(\bullet)$ the elasticity of substitution can be written:

$$\sigma(k) = \frac{dk/k}{dS/S} = \frac{f(k)}{kf'(k)} \tag{5.3}$$

Hence, the elasticity of substitution is a function of k alone and can be made constant or variable by appropriate specification of f.

In economic application, it is very often to assume the equation (5.1) taking Cobb-Douglas specification without any empirical verification. Using Cobb-Douglass production function implicitly describes a process with an elasticity of factor substitution equal to one. This study first hypothesize that the production function takes a more general form, variable elasticity of substitution (VES), which include the Cobb-Douglas function as a special case. Then based on the availability of data, the specification of production function of Vietnam's economy in period 1986-2007 is estimated. There is variety of forms of VES production functions. The choice in this study will be such that the selected production function is empirically manageable and economically insightful[7].

Ravankar (1971) and Bairam (1989) suggest the production function

$$Y_t = A_t K_t^{\alpha} L_t^{1-a} e^{\beta k_t} \tag{5.4}$$

β is defined as substitution parameter.

This specification (see Bairam [1989]) works well in estimating the production function of Japan's economy in industrializing period 1878-1939. Vietnam also started industrializ-

[7]Other VES specifications were developed by, *inter alia* Sato and Hoffman (1968), Revankar (1971), Lovell (1973).

ing her industry since 1986, hence in this study I also apply this specification to estimate Vietnam's production function in period 1986-2007.

From the production function (5.4) and (5.3) we have the variable elasticity of substitution between capital and labour:

$$\begin{aligned}\sigma_t &= \frac{(\beta k_t + \alpha)(1 - \alpha - \beta k_t)}{(\beta k_t + \alpha)(1 - \alpha - \beta k_t) - \beta k_t} \\ &= 1 - \frac{\beta k_t}{(\alpha + \beta k_t)^2 - \alpha}\end{aligned} \tag{5.5}$$

If $\beta = 0$, the production function is a Cobb-Douglass form, if $\beta \neq 0$, the production function is a VES one.

In this study the TFP growth is assumed to be driven by learning-by-doing and other exogenous factors. The concept of learning-by-doing was firstly incorporated into a macroeconomic model by Arrow (1962). In his model, part of the technical change process does not depend on the passage of time as such but develops out of experience gained within the production process itself. Mathematically, the model assumes that a labour efficiency index associated with workers of a particular vintage is a strictly increasing function of cumulative output or gross investment. Such a relationship is expressed as .

$$A_t = A_0 E_t^{\theta}$$

where A_0 is the initial level of technology. E_t is the index of experience at time t and $\theta > 0$ is the learning coefficient.

Arrow (1962) chooses cumulative gross investment as index of experience ($E_t = \Sigma I_t$) while other studies (Bairam (1987), Stokey and Lucas [1989]) favoured cumulative output as an index ($E_t = \Sigma Q_t$). Arrow (1962) argued that the appearance of new machines provides more stimulation to innovation while cumulative output is less inspiring to innovation. In this study both measures are used as proxies of experience. As mentioned above, technological progress is not assumed to be wholly the result of learning-by-doing but other exogenous factors. The technological change index, A_t, is specified as follows:

$$A_t = A_0 e^{\lambda t} E_t^{\theta} \tag{5.6}$$

where λ is Hicks-neutral rate of exogenous technological change which is a function of time.

In summing up, the VES production function in which technological progress is partly exogenous and partly the result of learning-by-doing can be presented by

$$Y_t = A_0 e^{\lambda t} E_t^{\theta} K_t^{\alpha} L_t^{1-a} e^{\beta k_t} \tag{5.7}$$

5.4 Data

The study requires annual time-series data on capital stock, working labour and output. The last two series are available at statistical yearbooks which published by Vietnam's General Statistics Office (GSO) in various years. The data of capital stock is not directly available and will be estimated next by the author.

Real GDP (at 1994 price) is the sum of value added of three sectors, namely agriculture, industry and service (Table: 5.7). These data are available in CEIC database, World Development Indicators (WDI) reported by the World Bank and statistical yearbooks published by GSO. Fortunately data from these sources are almost consistent. In case there are differences between these sources we use the data published by GSO since it is the official organization in Vietnam in charge of collecting and publishing data. Data on the total employment (see Table 5.8) can also be found in these sources and once again data from GSO is used priorly.

The WDI reports data of consumption of fixed capital in the period 1989-2006 as a percentage of GNI. These data can be taken as value of depreciation of capital stock. Using GDP deflator and data of GNI at current price we the data of real depreciation at

1994 price. The data of gross capital formation at constant price of 1994 are also available in WDI.

Let I_t, D_t, δ_t, and K_t and respectively denote gross capital formation, value of depreciation, depreciation rate and estimated capital stock at year $1988 + t$.

We have:

$$\begin{aligned}
K_1 &= \frac{D_1}{\delta_1} \\
K_2 &= I_2 + (1 - \delta_1)K_1 = K_2(\delta_1) \\
\delta_2 &= \frac{D_2}{K_2} = \delta_2(\delta_1) \\
K_3 &= I_3 + (1 - \delta_2)K_2 = K_3(\delta_1) \qquad (5.8) \\
& \textit{and so on....}
\end{aligned}$$

We have real data of D_t and I_t in period 1989-2006. For any value of δ_1 we can calculate the values of capital stock and depreciation rate in the whole period 1989-2006. In reality the depreciation rates are hardly fluctuated in a short period. Hence the best estimation of sequence $\{\delta_t\}$ is the one has the minimum standard deviation. Based on this criterion we can have the best estimation of depreciation rates and then of capital stock as show in table 5.1. The data of depreciation of capital stock in years 1985-1988 are not available, hence we can not estimate the depreciation rate by the above procedure. We take the average depreciation rate in period 1989-2006 as the estimation of depreciation rate in years of 1985-1988. The capital stock in this period is interpolated backward by

$$K_{t-1} = (K_t - I_t)/(1 - \delta)$$

It is noteworthy that Mankiw *et al.* (1992) based on US national accounts estimated depreciation rate of 3%. However since mid-1990s the Department of Commerce has significantly revised its capital stock estimates, with its new estimates on updated empirical evidence on depreciation for various types of assets. With these revisions this same calcu-

Year	D	I	δ	K
1985		12646	0.0322	113785.08
1986		16136	0.0322	126261.69
1987		19858	0.0322	142059.04
1988		20505	0.0322	157995.34
1989	6272.99	20434	0.0362	173348.12
1990	4851.86	20148	0.0259	187223.13
1991	6110.08	22366	0.0298	204737.28
1992	6655.82	27086	0.0295	225713.20
1993	8015.88	39862	0.0310	258919.38
1994	9362.11	45483	0.0316	296386.49
1995	12147.43	53249	0.0357	340273.38
1996	13847.69	60826	0.0356	388951.95
1997	15233.69	66529	0.0345	441633.26
1998	16109.82	74931	0.0321	501330.57
1999	17042.79	75830	0.0304	561050.75
2000	21822.60	83496	0.0348	627503.95
2001	23464.24	92487	0.0336	698168.36
2002	25395.83	104256	0.0326	778960.12
2003	27762.24	116623	0.0319	870187.28
2004	30550.12	128916	0.0315	971341.05
2005	33934.18	143291	0.0313	1084081.93
2006	37472.87	160247	0.0310	1210394.75
2007		199011		1371932.88

Source: D_t, I_t are from WDI, δ and capital stock are estimated by the author.

Table 5.1: Estimation of capital stock series 1985-2007 in bn. VND (Vietnam currency) at 1994 price

lation now produces a figure of about 4.5%[8]. Since the quality of data collected in Vietnam is not very good, the estimates of depreciation rates in Table 5.1 are around the estimate of Mankiw et al. 1992, justifiable.

5.5 Empirical results

[8]See Fraumeni (1997) for a discussion of the Commerce Department's methodology for constructing capital stocks. The data for these calculations were downloadable from the BEA's website at www.bea.doc.gov/bea/dn/home/fixedassets.htm.

This section we apply OLS procedures to estimate coefficients of equation (5.7). Dividing (5.7) by K_t and transforming logarithmically yields:

$$\ln(\frac{Y_t}{K_t}) = \ln A_0 + \lambda t + \theta \ln E_t + (\alpha - 1)\ln(k_t) + \beta k_t \tag{5.9}$$

where $k_t = \frac{K_t}{L_t}$.

The right hand side of equation (5.9) comprises four components that directly influence on the output-capital ratio: (*i*) exogenous technological change, λt; (*ii*) the learning-by-doing, $\theta \ln EL_t$; (*iii*) the capital-labour ratio $(\alpha - 1)\ln(k_t)$; and (*iv*) the influence of changing elasticity of substitution between capital and labour, βk_t. Consequently, if the estimated of β is significantly different from zero the hypothesis of Cobb-Douglas production function can be rejected in favour of the VES production function and *vice versa*. Thus specification (5.9) is sufficient to test the Cobb-Douglas hypothesis. The second and third components of RHS of (5.9) assume that technical progress is partly exogenous and partly the results of learning-by-doing. Similarly, if estimated θ is significantly different from zero the hypothesis of learning-by-doing can not be rejected. It is noteworthy that if estimated θ is negative, the economy was not learning but losing by doing in the period of study.

For the index of experience E_t, both specifications, namely $E'_t = \sum^t I_i$ and $E_t = \sum^t Y_i$ are tried. In which $\{I_t\}$ is sequence of gross capital formation. By these specification the cumulative data for starting year, say 1985, need specifying. Since data for capital formation before 1985 are not available, the capital stock of 1984 is used as cumulative gross capital formation up to 1985 instead

$$\begin{aligned} E'_{1985} &= K_{1984} = \frac{K_{1985} - I_{1985}}{1 - \delta} \\ E'_{1986} &= E'_{1985} + I_{1985}, \ E'_{1987} = E'_{1986} + I_{1986} \\ &\text{and so on...} \end{aligned}$$

In case $E_t = \sum^t Y_i$, the experience index in 1985 is assumed to be accumulated in the last 10 years, which means that:

$$E_t = \sum_{i=1975}^{t-1} Y_i, t \geq 1985$$

Details of data of E_t and E'_t are reported in appendix, table 5.9.

In addition, two alternative specifications (5.10) and (5.11) are also considered.

1. Technical progress is purely exogenous; learning-by-doing plays no role in technical progress:

$$\ln(\frac{Y_t}{K_t}) = \ln A_0 + \lambda t + (\alpha - 1)\ln(k_t) + \beta k_t \tag{5.10}$$

2. Technical progress is purely driven by learning-by-doing:

$$\ln(\frac{Y_t}{K_t}) = \ln A_0 + \theta \ln E_t + (\alpha - 1)\ln(k_t) + \beta k_t \tag{5.11}$$

First, applying the OLS regression for these five specifications yields statistical results which reported in table 5.2. Column 1 reports estimated parameters of specification (5.10). The parameters of specification (5.11) are estimated with two different proxies of experience index, $\sum I_t$ and $\sum Y_t$ and reported in columns 2a and 2b respectively. Similarly, results of regressing of specification (5.10) are reported in columns 3a ($\sum I_t$) and 3b ($\sum Y_t$)

Before looking at statistical significance of the estimated parameters in table 2, it is important to have a look at Durbin-Watson statistics and adjusted R^2. These regressions have very high $\overline{R}_2$ but too small Durbi-Watson statistics which implies that these regressions may suffer from serial-correlation and multicollinearity. Table 5.3 shows very high multicollinearity between regessors.

In order to correct for multicolinearity and serial-correlation we first rearrange equation (5.9) as follows

$$\Delta \ln(\frac{Y_t}{K_t}) = \lambda + \theta \Delta \ln E_t + (\alpha - 1)\Delta \ln(k_t) + \beta \Delta k_t \tag{5.12}$$

then using Prais-Winsten procedure to estimate equation (5.12). The rearrangement of equation (5.12) reduces sharply the multicollinearity between regressors (see table 5.4).

	1	2a	2b	3a	3b
$\ln A_0$	0.39**	3.68**	3.02**	10.09**	13.46**
	(2.24)	*(3.43)*	*(2.49)*	*(3.81)*	*(2.25)*
λ	-0.0167			0.0563**	0.084*
	(-1.5)			*(2.59)*	*(1.78)*
θ		-0.32**	-0.197*	-0.9034**	-0.956**
		(-2.84)	*(-1.96)*	*(-3.67)*	*(-2.18)*
α -1	-0.3486**	-0.0825	-0.314**	0.1177	-0.3954**
	(-2.68)	*(-0.51)*	*(-2.67)*	*(0.73)*	*(-3.28)*
β	0.0029	0.0135	0.0004	-0.0009	-0.0088
	(1.18)	*(0.59)*	*(0.16)*	*(-0.41)*	*(-1.5)*
DW	*0.5143*	*0.66*	*0.5016*	*1.043*	*0.452*
$\bar{R}^2$	0.9953	0.9957	0.9956	0.9967	0.9954

Note: figure in parentheses are t-statistics, DW is Durbin-Watson Statistics.
* and ** indicate coefficients are statistically significant at 90% and 95% confidence level respectively
confidence level respectively

Table 5.2: OLS regression without correcting for serial-correlation

The results of regression are reported in table 5.5.

In table 5.5 row 4a reports regression results of equation (5.12) with $E_t = \sum I_t$; row 4b reports results regressed on the same equation with $E_t = \sum Y_t$. The highlighted points can be seen from table 5.5:

First, the specification corresponding to row 4b is better that the specification corresponding to row 4a in terms of: adjusted R-square, Durbin-Watson statistics and statistcal significance of estimated parameters. This implies that cumulative output is a more appropriate proxy for learning index.

	$\ln \sum Y_t$	$\ln k_t$	t	k_t
$\ln \sum Y_t$	1.000			
$\ln k_t$	0.9956	1.000		
t	0.9989	0.9979	1.000	
k_t	0.9542	0.9703	0.9666	1.000

Table 5.3: Multicolinearity between regressors

	$\Delta \ln \sum Y_t$	$\Delta \ln k_t$	Δk_t
$\Delta \ln \sum Y_t$	1.000		
$\Delta \ln k_t$	-0.21	1.000	
Δk_t	-0.75	0.41	1.000

Table 5.4: Multicolinearity between regressors after rearranging

Second, in all possible specifications, the estimated substitution parameter β is not statistically significantly different from zero at 90% confidence level. This implies that the VES production function hypothesis is rejected and Cobb-Douglas function is more appropriate for Vietnam's economy in period 1985-2007. Rows 5a and 5b reports regressing results equation (5.11) without varible of substitution k_t. In row 5a cumulative of investment is used as proxy of learning index, while in 5b we use cumulative output instead. Both parameters in row 5a are not statistically significant while they are in row 5b. This again confirms that in context of Vietnam's economy in period 1986-2007 the cumulative investment is not a good proxy for learning index.

Third, the estimated α (share of capital) is around 0.50 which similar to those Kim and Lau (1994) estimated for NIEs.

Fourth, let us foucus on row 4b, both parameters λ and θ are well statistically significantly 95% confidence level. However the sign of learning-by-doing parameter, θ,while the sign of Hicks-neutral technological parameter, λ is positive as expect. This seems that the two main components that compose TFP in Vietnam's economic growth in period 1986-2007 act in opposite directions. On the one hand, Hicks-neutral technological progress contribute positively to growth; on the other hand, learning-by-doing contribute negatively. However, it should be more precise at this point. The negative sign of parameter θ may stem from bad proxy of learning-by-doing. The very high multicollinearity between variables t and $\ln \sum Y_t$ in table 5.3 indicate that not all effects of learning-by-doing are

	$\boldsymbol{\lambda}$	$\boldsymbol{\theta}$	$\boldsymbol{\alpha - 1}$	β	DW	$\bar{R}^2$
4a	-0.0048	0.0176	-0.6391**	0.1	1.527	0.37
	(-0.13)	*(0.06)*	*(-2.73)*	*(1.33)*		
4b	0.1104**	-1.2152**	-0.4967**	-0.0037	1.8	0.53
	(2.34)	*(-2.57)*	*(-2.65)*	*(-0.47)*		
5a	0.0038	-0.0677	-0.4775**		1.48	0.35
	(0.10)	*(-0.22)*	*(-2.33)*			
5b	0.0975**	-1.075**	-0.5448**		1.7947	0.55
	(2.66)	*(-3.04)*	*(-3.39)*			

Note: figure in parentheses are t-statistics, DW are Durbin-Watson Statistics. * and ** indicate coefficients are statistically significant at 90% and 95% confidence level respectivly.

Table 5.5: OLS regression: correcting for serial-correlation and multicollinearity

proxied by cumulative output, whereas some kinds of learning-by-doing can be embodied in Hicks-neutral technological progress.

Hence conditional on availability of data we can say that in the transitional period 1986-2007 learning-by-doing proxied by cumulative output from 1975 negatively impacts on growth, while those are proxied under form of Hicks-technology positively contribute to growth. These two effects may cancel out each other and as a result, on aggregate TFP may play a trivial role in growth in this period.

Finally, in row 5b all parameters are statistically significant at 95% level of confidence and sign of θ is negative again. The adjusted R-square is improved while DW statistics unchanged in comparison with row 4b, implies that dropping variable k_t fit better with the data. Using value of α estimated in this row we calculate the contributtion of TFP to growth by the following equation

$$GTFP = \frac{\Delta Y_t}{Y_t} - \alpha \frac{\Delta K_t}{K_t} - (1 - \alpha) \frac{\Delta L_t}{L_t}$$

and the results reported in table 5.6

The results in table 5.6 show that the main engine for Vietnam's economic growth in period 1986-2007 is physical capital accumulation and then labour. TFP plays an insignificant role in growth in this period. On average in 22 years the economy grew 7% yearly

Year	GDP Growth	Capital	Labour	TFP
1986	2.79	4.99	1.29	-3.49
1987	3.58	5.70	1.38	-3.49
1988	5.14	5.11	1.42	-1.39
1989	7.36	4.42	1.88	1.06
1990	5.10	3.64	1.90	-0.45
1991	5.96	4.26	1.03	0.67
1992	8.65	4.66	1.22	2.76
1993	8.07	6.70	1.33	0.05
1994	8.84	6.59	1.42	0.83
1995	9.54	6.74	1.34	1.46
1996	9.34	6.51	0.15	2.68
1997	8.15	6.17	1.18	0.81
1998	5.76	6.15	1.17	-1.56
1999	4.77	5.42	1.15	-1.80
2000	6.79	5.39	2.47	-1.08
2001	6.89	5.13	1.38	0.39
2002	7.08	5.27	1.33	0.48
2003	7.34	5.33	1.47	0.54
2004	7.79	5.29	1.36	1.14
2005	8.44	5.28	1.23	1.93
2006	8.23	5.30	1.04	1.88
2007	8.48	6.08	1.05	1.36

Table 5.6: GDP growth and contribution by components

in which physical capital and labour repectively accounted for 78% and 19%, and TFP accounted for the left: 3%. It is noteworthy that in the last 20 years Vietnam's economy has operated mainly in low technology industries (see figure 5.3 in appendix), hence then there is very limited scope for improving TFP. Furthermore, as we can see in figure 5.5 in appendix, Vietnam's productions are mainly for export and have to compete in the international market. Consequently, Vietnam's gains in learning-by-doing in low-tech industries if any, are outweighed by competitors' high productivity gained by better technologies. As a result TFP is hardly improved in the period under consideration.

5.6 Conclusion

Since *Doi moi* launched in 1986 Vietnam has consecutively grown at fairly high rate. Along the reform process, Vietnam has issued radical reform policies to improve economic performance. The current study first give an overview on economic growth and then to identify and quantify the contribution of some important factors namely labour, physical capital and TFP.

First, this study contends that the appropriate production function for Vietnam's economy in period 1986-2007 is not a VES production function. The Cobb-Douglas functional form is more appropriate for Vietnam's economy in the period under consideration.

Second, like NIEs in period 1965-1986, Vietnam economic growth in this period is essentially driven by high rate of capital accumulation. It seems that in transitional period, on the one hand economic reforms has improved productivity in terms of Hicks-neutral technological progress; on the other hand remnants of central-planning time in terms of institutions, legal frameworks, way of thinkings remain negatively affect. As a result, averagely TFP contribute negligibly to economic growth in the whole period.

Vietnam seems repeat the growth story of NIEs in period 1965-1986 which described by Krugman (1997) "it (high growth rate) was due to forced saving and investment, and long hours of works...". Krugman's [1997] interpretation of these results is very pessimistic since, according to him, the lack of technical progress will inevitably bound the engine of growth as a result of the diminishing returns affecting capital accumulation. However these signals should be taken as a warning not a worrying. Since for long period up to 1986 TFP contribute nothing to growth in NIEs, from 1986 on Lau and Park (2003) finds firm evidences of positive contribution of TFP to growth in these economies. "It is possible that the potential to adopt knowledge and technological from abroad depends on a country's stage of development. Growth in the early stages may be primarily associated with physical and human capital accumulation, and significant potential for growth through catch-up may only emerge once a country has crossed some development threshold" (Collins and Bosworth [1996]). It is obviously Vietnam now is in initial stage of development process. Negligible contribution of TFP to growth is justifiable. However, in the long run Vietnam needs to reverse this trend to sustain economic growth. The lessons from NIEs indicate that this reverse process essentially requires increasingly improved human capital and capacity of R&D.

5.7 Appendix

Year	VA in service	VA in Agriculture	VA in industry	GDP
1985	41149.78	36832.00	26396.00	104377.78
1986	40073.18	37932.00	29284.00	107289.18
1987	41872.86	37499.00	31762.00	111133.86
1988	44624.59	38867.00	33349.00	116840.59
1989	51371.34	41589.00	32485.00	125445.33
1990	56620.20	42003.00	33221.00	131844.20
1991	61003.22	42917.00	35783.00	139703.22
1992	65554.03	45869.00	40359.00	151782.03
1993	71207.98	47373.00	45454.00	164034.98
1994	78026.00	48968.00	51540.00	178534.00
1995	85698.00	51319.00	58550.00	195567.00
1996	93240.00	53577.00	67016.00	213833.00
1997	99930.41	55860.43	75474.00	231264.84
1998	104966.00	57866.00	81764.00	244595.99
1999	107330.00	60895.00	88047.00	256272.00
2000	113036.00	63717.00	96913.00	273666.00
2001	119931.00	65618.00	106986.00	292535.00
2002	127770.00	68352.00	117125.00	313247.00
2003	136016.00	70827.00	129399.00	336242.00
2004	145897.00	73917.00	142621.00	362435.00
2005	158276.00	76888.00	157866.99	393031.00
2006	171392.00	79722.00	174259.40	425373.40
2007	186272.93	82436.22	192733.84	461443.00

Source: WDI, The Worldbank

Table 5.7: Real GDP: billion VND in 1994 price

Year	Labour in (thousand)	Capital	K/L	Y/K
1985	26020	113785.1	4.372985	0.917324
1986	26636	126261.7	4.740265	0.849737
1987	27310	142059	5.201722	0.782308
1988	28023	157995.3	5.638059	0.739519
1989	28989	173348.1	5.97979	0.723661
1990	30002	187223.1	6.240355	0.704209
1991	30572	204737.3	6.696889	0.682354
1992	31258	225713.2	7.220974	0.672455
1993	32020	258919.4	8.086177	0.633537
1994	32857	296386.5	9.020498	0.602369
1995	33667	340273.4	10.10703	0.574735
1996	33761	388952	11.52075	0.549767
1997	34493	441633.3	12.80356	0.523658
1998	35233	501330.6	14.22901	0.487894
1999	35976	561050.8	15.59514	0.456772
2000	37609.6	627504	16.68467	0.436118
2001	38562.7	698168.4	18.10476	0.419004
2002	39507.7	778960.1	19.71667	0.402135
2003	40573.8	870187.3	21.44702	0.386402
2004	41586.3	971341.1	23.35724	0.373128
2005	42526.9	1084082	25.49168	0.362547
2006	43338.9	1210395	27.9286	0.351434
2007	44171.9	1371933	31.05895	0.336345

Source: Labour = number of annual employment in CEIC database and Statistical Yearbook of Vietnam in 2007 and 2006. Capital stock estimated by author.

Table 5.8: Labour, Capital stock and capital-labour ratio

Year	Capital formation	Cummulative Investment	Cummulative output
1985	12646	104926.9	819668
1986	16136	117572.9	924045.8
1987	19858	133708.9	1031335
1988	20505	153566.9	1142469
1989	20434	174071.9	1259309
1990	20148	194505.9	1384755
1991	22366	214653.9	1516599
1992	27086	237019.9	1656302
1993	39862	264105.9	1808084
1994	45483	303967.9	1972119
1995	53249	349450.9	2150653
1996	60826	402699.9	2346220
1997	66529	463525.9	2560053
1998	74931	530054.9	2791318
1999	75830	604985.9	3035914
2000	83496	680815.9	3292186
2001	92487	764311.9	3565852
2002	104256	856798.9	3858387
2003	116623	961054.9	4171634
2004	128916	1077678	4507876
2005	143291	1206594	4870311
2006	160247	1349885	5263342
2007	199011	1510132	5688715

Source: Author's calculation

Table 5.9: Capital formation, cumulative investment and cumulative output

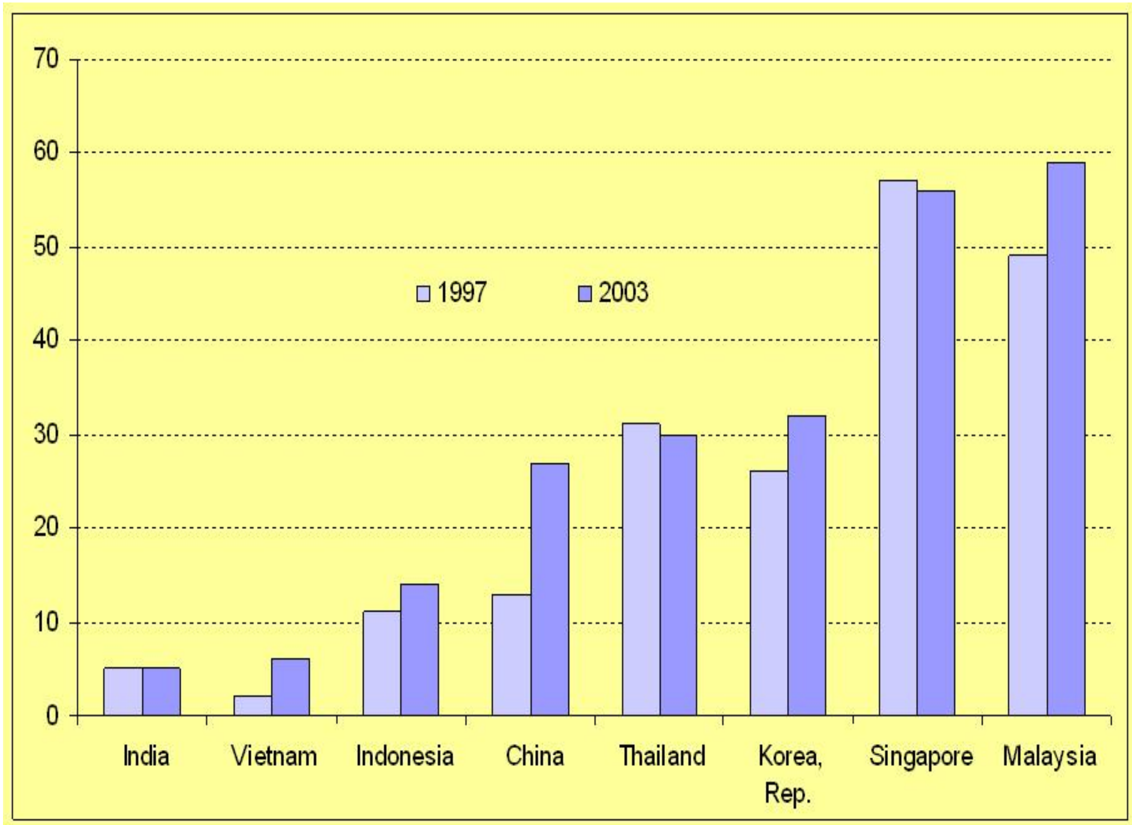


Figure 5.3: Proportion of high-tech products in exports of manufacture

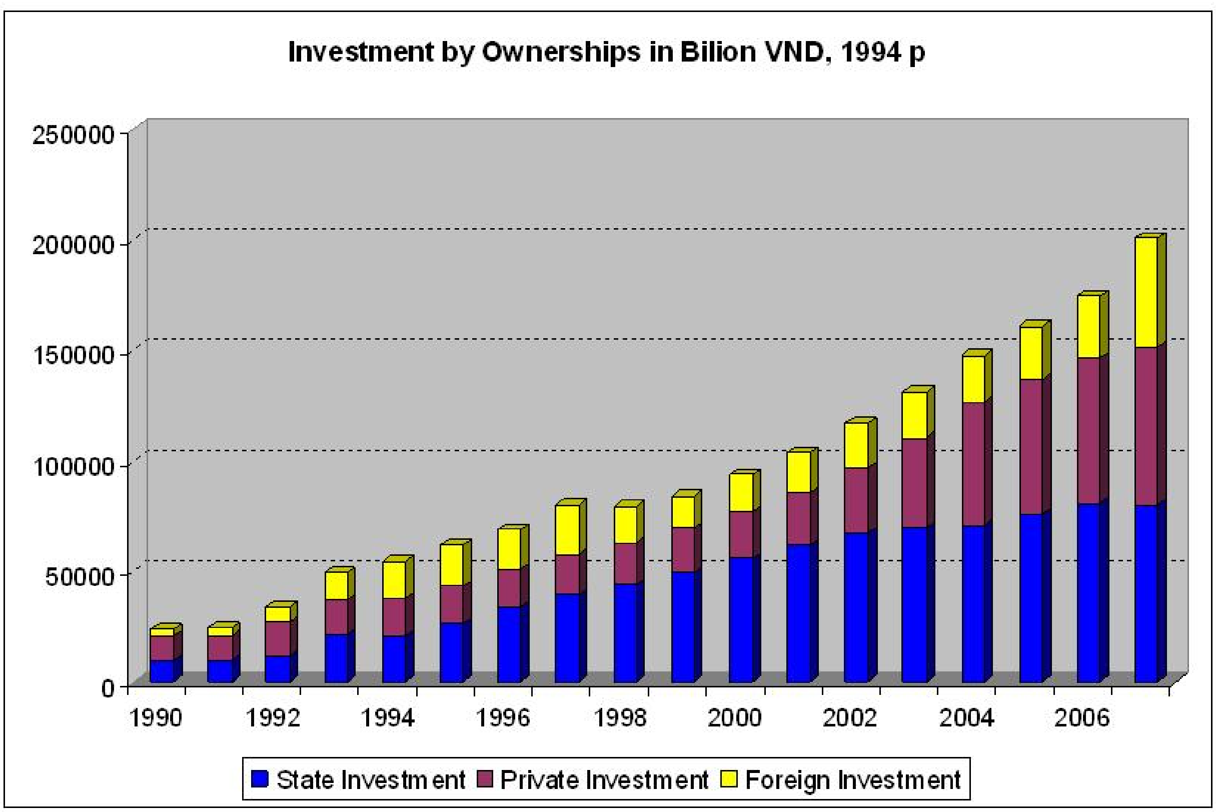


Figure: 5.4: Investment by ownership. *Source:* CEIC data base

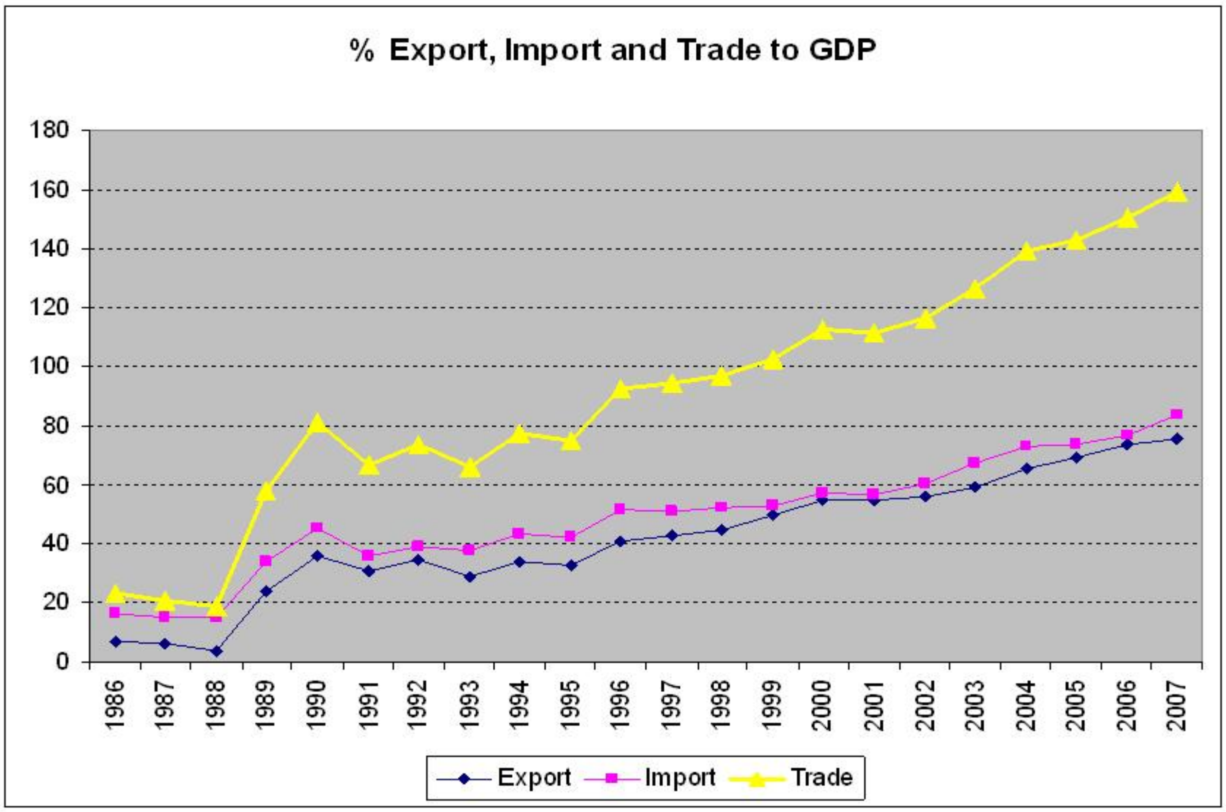


Figure 5.5: The openness of Vietnam's economy

Source: CEIC data base and Statistical yearbook 2007.

General Conclusion

In development process physical capital accumulation can be a primary engine for economic growth in (perhaps prolonged) transitional period. During this period TFP may play a modest role and high rate of investment (saving) explains lion's share of high economic growth rate. However, in the long-run the role of high investment rate eventually fades out and growth can only sustained by improvement of TFP.

Krugman, among others, was right when judged that East Asia' growth must slow down in future because of what he characterized as an excessive reliance on capital accumulation. However this pessimistic view may be not the case if after having crossed some developmental thresholds these economies start investing in human capital and new technology. Our examination on economic growth processes of developing countries and some East Asian economies supports our view. The improvement of TFP essentially requires investment in human capital, or new technology or both. The economy which possesses better quality of human capital and new technology will have higher TFP growth thus, grows faster not only in traditional period but also in the long term. Furthermore, the differences in qualities of human capital and new technology cause different rates of TFP. Accordingly the qualities of human capital and new technology are a good explanation for economic divergences among economies in the world.We also show that the presence of fixed costs may delay the growth process.

Our CES model of learning-by-doing also show that in transitional stage saving always play an important role in growth process. This effect of high saving rate will die out in the long-run if the economy is not very elastic, *i.e.,* $r < 0$. If the economy is very elastic *i.e.,* $r > 0$) and the saving rate is high enough the growth rate will be decreasing but always higher than the growth rate that predicted at BGP regardless how efficient the economy in learning-by-doing is. However, as noted in the text the conditions for this case is rarely satisfied in the reality.

If the economy is not very elastic *i.e.,* $r < 0$, which is a characteristic of developing economies as indicated by Duffy and Papageorgiou [2000]. Under this condition, savings play a crucial role in growth process. If the saving rate is too low, the economy will collapse in long-run. If the saving rate is higher than critical level and lower than optimal level the economy remain sustain its positive growth in long-run but lower than its potential level. In this case even if the economy possesses high efficiency of learning and spilling-over, it can not enjoy those in long run. Moreover, the better ability of learning-by-doing and spilling-over knowledge, the higher saving rate is required to enjoy fully these effects. In addition, the poor economy will be lagged behind if its saving rate is not superior than that of the initially rich economy.

Finally, if $r < 0$, and saving is high enough or $r > 0$ and saving is not too high there is an unique BGP for these economies. In this case we show that assimilationists may right as claiming that learning-by-doing and spill-over play an important role in growth in NIEs. We also show, however, that the growth model based purely on learning-by-doing is constrained by labor growth rate. If the labour is constant in the long-run, then the growth can not be sustained. In this sense, learning by doing is insufficient for growth in long run. Notice that if labour in the model is defined as effective labour to include the effectiveness of human capital accumulation then coefficient n counts for growth rate of human capital which can be positive in the long-run.

An empirical examination of Vietnam's economic growth over last 20 years also supports our results. During last 23 years Vietnam's economy has grown fast. However like NIEs in period 1965-1986, Vietnam economic growth in this period is essentially driven by high rate of physical capital accumulation. Contribution of TFP for growth is really negligible: only 3 percents on average. It is obviously Vietnam now is in initial stage of development process. Negligible contribution of TFP to growth is justifiable. However, in the long run Vietnam needs to reverse this trend to sustain economic growth. The lessons from NIEs indicate that this reverse process essentially requires increasingly improved human capital and capacity of Research and Development.

In relation with exhaustible resources we have shown that, under certain circumstances among which the most important is a high marginal productivity of capital at the origin compared with the social discount rate (technology is good), exhaustible resources can allow a developing economy to escape from the poverty trap. In this case resource is blessing. However, if technology is bad, possession of exhaustible resources may be a curse. In the sense that the abundance of resources deter the economy from accumulating.

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