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**ECOLE NORMALE SUPERIEURE DE LYON
LABORATOIRE DE PHYSIQUE**

L'ECOLE DOCTORALE DE PHYSIQUE ET D'ASTROPHYSIQUE DE LYON

THÈSE

présentée pour obtenir

le GRADE de DOCTEUR EN SCIENCES

DE L'ECOLE NORMALE SUPERIEURE DE LYON

par

John MC BREEN

Sujet :

**AGENT-BASED SIMULATIONS IN URBAN ECONOMICS:
APPLICATIONS TO TRAFFIC CONGESTION AND HOUSING MARKETS**

Soutenu le 22 juin 2009 devant la Commission d'examen :

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Introduction

The agent-based modelling (ABM) paradigm has seen ever increasing use in the social sciences over the last decade (Tsfatsion and Judd, 2006; Amblard, 2007). Its methodological origins can be traced at least as far back as Clarkson and Simon (1960); Orcutt (1960). It remains however to be fully accepted as a standard scientific method in the social sciences even though its fundamental theoretical advantages are well understood (Bradburd et al., 2006; Colander et al., 2008). The main aim of this work is to evaluate the potential of agent-based modelling to enlarge our understanding of urban economic systems.¹

The great increase in computing power over the last few decades has brought about the possibility of modelling economic systems in a new way. The use of simulations has become standard practice in physics where they have achieved great success. The widespread use of simulation methods has been slower to develop in the social sciences in large part due to the greater complexity of the entities modelled. This has created a situation in which those who are specialised in the social sciences do not usually have the skills required to create simulation models of socio-economic systems. On the other hand specialists in simulation methods, including computer scientists and physicists, do not often have a solid background in the social sciences. It is important therefore, to bring together skills from different areas to further the development of simulation modelling of social systems. Much work has been done by physicists in modelling financial systems where the availability of enormous amounts of data permits a strongly empirical approach (Bouchaud, 2009; Rickles, forthcoming). Methods from physics can more easily be applied to urban systems when there is high quality data available (Jensen, 2006). In this thesis I model urban systems which, though often lacking the level of high-quality data that is available for financial markets, have many characteristics which make them suitable for the application of simulation methods.

In this Introduction, I will summarise the approach followed in the thesis. First, I present the original motivations for embarking on this project, namely

¹The research in this thesis has given rise to three papers: Mc Breen et al. (2006) upon which on chapter 1 is based. A slightly modified version of Chapter 2 has been submitted to the Journal of Artificial Societies and Social Simulation, under the title 'An Agent-based Model of a Monocentric Urban Rental Housing Market'. A paper based on chapter 3 has been submitted to The Journal of Real Estate Finance and Economics, under the title 'Information and Search on the Housing Market: An Agent-based Model'.

modelling the interactions between urban transport systems and the localisation of activities. I then explain the theoretical advantages and some of the practical difficulties encountered in using agent-based models to study socio-economic systems. The specific systems studied are presented thereafter. The process of distillation of model hypotheses that leads to a combination of object and method, which allows pertinent insights to be achieved, is emphasised. The characteristics of systems that lend themselves to successful multi-agent models are subsequently discussed as well as some general guidelines that help in achieving interesting results from such simulation models.

Original motivation

The original motivation of this project was to investigate the potential of multi-agent systems to furnish insights into the functioning of urban systems. Urban systems are of critical importance to the general economy, indeed most wealth is created in cities (Fujita and Thisse, 2003). Urban systems are complex evolving systems of large numbers of spatially distributed interacting heterogeneous agents. Hence it is an area in which simulations have great scope to surpass analytic models.

My intention was not to study the full range of multiagent systems. This extends from small scale ‘toy models’ such as Schelling (1978), to large scale integrated land use-transport models of entire urban areas (Salvini and Miller, 2005). These large engineering style models require a considerable amount of data, intensive calibration efforts and can never fully escape a black box aspect to the finished product, for a review see Timmermans (2003). My aim was to create models with motivated hypotheses, whose dynamics can be fully understood and which replicate the major dynamic features of real-world systems.

The initial focus was the interactions of transport systems and the localisation decisions of agents. Currently, models that predict the performance of the transport infrastructure starting from a given localisation of agents are quite well-developed (de Palma and Marchal, 2002; Arentze and Timmermans, 2004). The reverse approach, starting from the transportation infrastructure and attempting to predict the future urban form, that is the aggregation of localisation choices that are made in the long term, are understandably less developed. The final goal is to take into account the entire interaction loop between the joint evolution of the transport system and the localisation of population and activities. Such prospective models are in the development stage (Waddell, 2002; Timmermans, 2003). Agent models have the potential to model interactions often ignored in economic models but which are crucial to an understanding of urban development, see Glaeser et al. (2000).

In practice this initial motivation has led to the creation of three models of urban systems. I examine the stability of a standard congestion model of Vickrey

(1969) in Chapter 1. An agent-based model based on the classic monocentric urban economic model of Alonso (1964), which includes both a spatial and a market aspect, is developed in Chapter 2. Finally an aspatial model of the rental housing market, in which particular attention has been given to the search behaviours of agents is presented in Chapter 3.

Advantages and difficulties of agent-based models

The advantages of agent-based simulation methods are numerous and far-reaching. Among the principal advantages is the ability to model dynamic evolutions. Agent-based simulations offer great flexibility in modelling heterogeneous agents' characteristics and their interactions. The understanding of the emergence of macro forms and macro dynamics from the microscopic interactions of agents is a particular advantage of multi-agent systems. Indeed, agent-based models can be validated simultaneously by both the system-level and micro-level behaviour (LeBaron and Tesfatsion, 2008). Agent-based models also offer the possibility of modelling systems in a very natural and coherent way that is not possible for analytical models. For example, agents can meet and bargain for goods in imitation of real markets (Kirman and Vriend, 2001).

The dynamic nature of agent-based models permits the investigation of analytical equilibria and their stability. An example will be shown in chapter 1 through an agent-based simulation of the classic Vickrey (1969) model of traffic congestion. The stability of equilibria is a question often neglected in analytic economic models. My aim has been to use simple agent behaviours to explore the driving forces behind the modelled system's dynamics, with an emphasis on controlling their global stability.

The greater flexibility of simulations with respect to analytic models allows a much more realistic set of hypotheses to be made. These hypotheses relate to the entities themselves, their environments, and their interactions. Multiple interaction forms are possible and may be direct or indirect. In the latter case, agents interact with each other via their effects on their shared environment. A major advantage of agent-based models is the natural integration of heterogeneities in agents' characteristics, which is a crucial element for realistic economic models (Kirman, 1992). This will be seen particularly in Chapters 1 and 3 where heterogeneities in agents' characteristics are in fact necessary in order to obtain stable models. Blundell and Stoker (2005) present empirical methods for dealing with aggregation problems due to heterogeneities.

I have tested in this thesis the value of extending existing analytic models using agent-based simulations. One clear advantage of this approach is that it provides a solid theoretical basis for modelling decisions. We can also expect a better acceptance and diffusion of agent-based methods and insights when these are clearly founded in the existing economic theory. It is however certain that

agent-based models have the potential to move beyond the constraints of close connections to existing theory. These issues will be mainly dealt with in Chapter 3.

The difficulties associated with agent-based simulation methods are also numerous, and I shall now underline why considerable thought must be given to how to overcome or at least to attenuate these difficulties before embarking on a multi-agent modelling project. It is possible for the uninitiated to believe that agent-based methods can furnish insights into almost any system. In practice, there are many difficulties, notably in the validation and in the calibration of the simulation models to empirical data.

Data in the quantities and of the appropriate kinds required to calibrate large-scale multi-agent models are very hard to obtain. This leads to many difficulties in their calibration. Even with good data, its transformation into a usable form for the model and the subsequent calibration process are both extremely onerous. The calibration of large-scale complex simulation models of inherently unpredictable socio-economic systems appears to be a very problematic undertaking. Indeed, even the simplest multi-agent models are relatively rich in parameters whose calibration is not straightforward.

Every agent-based model can be considered as a small world of which every detail must be described. Models often have many more hypotheses than the number of parameters to calibrate would indicate. These potentially ‘hidden’ hypotheses can concern among others: the orders of interactions (random or sequential), the distributions of agents’ characteristics (e.g. uniform or Gaussian), the closed or open nature of the system, the nature of information available to agents (local or global) etc..

Finding existing theoretically or empirically founded hypotheses for agent behaviours is also a difficult task, see Roth (2007). Most of the economic literature has not been concerned with understanding the actual behaviours of human agents. Such research has mostly been conducted in psychology, cognitive science and recently in the new field of behavioural economics.

Using insights from psychology, behavioural economics has highlighted many of the erroneous assumptions used in analytical economic models (Camerer et al., 2003). While this work is encouraging for agent-based models, the integration of these insights is often not obvious. Insights from cognitive science are also a useful resource and agent-based methods are seen as a promising approach within cognitive science (Conte, 2002). Work on the real decisions made by human agents that are, contrary to neo-classical economic theory, neither fully rational nor omniscient nor completely selfish, is of huge potential use for agent-based models.

While basing simulation models upon existing analytic ones is often very productive, integration of some analytical assumptions into agent behaviours is not straightforward. Indeed, wishing to cling too tightly to analytical foundations

may hamper the full flowering of agent-based research. Different examples of this danger will be seen in Chapters 2 and 3. In different fields and for different specific models, analytical foundations, when they exist, shall have varying degrees of relevance. As the rest of the thesis shows, the true extent of the difficulties and advantages of agent-based modelling are only discovered in practice. I believe, and hope to have shown, that the advantages make the task worth the effort.

I shall now describe the specific models that I have studied with agent-based models.

Traffic congestion

In Chapter 1, I examine the stability of a well known traffic congestion model (Vickrey, 1969). The stability of this Nash equilibrium had not been established and traffic simulations inspired by this model, see de Palma and Marchal (2002), have shown instabilities when attempting to add day-to-day evolutions to the basic model. We wished to develop an agent-based model to examine the stability of the original model and to investigate the possibility of adding simple agent dynamics to large-scale traffic simulations. It has been argued persuasively that microscopic economic models are a necessary step towards a better understanding of the dynamics of transport systems generally (Arnott, 2001).

The classic Vickrey (1969) model of traffic congestion is a ‘within-day’ dynamic model, in which agents are assumed to optimise their departure time for the morning commute. In doing this, they trade off the cost of not arriving at the desired time and the cost of experiencing higher congestion levels. This is extended here to a ‘doubly dynamic’ agent-based model, in which the departure time distribution evolves from day-to-day. Agents learn through experimentation: with a certain probability each day, they try a new departure time. If their personal total cost reduces they adopt this new departure time. It is shown that with perfectly homogeneous agents that the original model’s Nash equilibrium solution cannot be found dynamically and the system is indeed highly unstable (Mc Breen et al., 2006).

In order to gain insights into the causes of the system’s instability, a number of extensions are explored. Firstly, the difficulty of using analytic methods to model such systems is demonstrated. This model inspired the investigation of a simple two-departure-time model. Stability is achieved through the introduction of heterogeneities in agents’ desired arrival times. This alters the structure of the model by reducing the intensity of competition for the same arrival times. An extension illustrating the possibility of studying this dynamic stochastic model with a deterministic approximation is also presented in Appendix B. These investigations evoke the intriguing possibility of using simulations as exploratory models that can subsequently be simplified to a corresponding analytical model

in order to conduct a more rigorous analysis (Varenne, 2008).

Monocentric Urban Model

In Chapter 2, I examine the potential of simple agent-based models to reproduce, and hence help to better understand, the spatial dynamics of urban real estate markets. The focus is on the interactions of transport systems and localisation decisions. In order to do this I decided to begin from the foundation provided by standard urban economic theory (Fujita, 1989), and add agent dynamics to this static analytic framework in the hope of generating interesting new results. Progress has already been made in this direction by Caruso et al. (2007) who build upon the monocentric urban model using cellular automata. The model developed is a spatial model that includes transport costs from an exogenous centre. These costs then determine the localisation of decisions of agents.

In our model, each cell in a simple grid represents a residence owned by a landlord who wishes to maximise her revenue. A certain number of potential tenants arrive each iteration and attempt to find the best possible housing. Each iteration, a certain number of tenants leave the town. The behaviours to be specified are the price setting decisions of landlords and the search mechanisms of tenants. The model is explored in both one and two dimensions.

It will be shown that the inclusion of a simple search mechanism leads to market frictions and a vacancy rate that is a well-known phenomenon in all search markets. The vacancy rate cannot be understood from the static standard model. A search market is a market in which a key Walrasian assumption of perfectly informed agents does not hold. Instead of a single market with a virtual auctioneer who decides prices so that demand equals supply, agents must search for trading opportunities, incurring costs. Decisions to trade depend upon the perceived utility of continued search. This is shown in Chapters 2 and 3 to be the main difference between this dynamic agent-based interaction model and the standard urban economic model.

However, the agents' behaviours in this spatial model are somewhat arbitrary. In Chapter 3, the focus is on the behaviours and interaction mechanisms of landlords and potential tenants in an aspatial rental housing market.

Housing Search Market

In order to generate pertinent and novel results I had learned from previous work that the choice of object is as important as the choice of method. Having assessed the strengths and weaknesses of agent-based models, I decided that a rental housing market model was probably the simplest search model with which to study the behaviour of agents on both sides of the market (Mc Breen et al., 2009). These behaviours give rise to complex correlations in the evolution of prices, times-on-the-market and traded volumes (Fisher et al., 2003). This

would hopefully lead to the subsequent reinjection of these modelled behaviours and interactions into a spatial model. I felt that this would be a more productive approach than developing the market and spatial aspects simultaneously. This approach allows greater freedom in studying dynamic market evolutions.

Real estate markets are clearly vitally important systems in a modern economy. Search frictions, that is the difficulty for tenants of finding suitable housing and for landlords of finding suitable tenants, present in rental real estate markets, are of general interest for all search markets. This is a key area in which a robust microscopic dynamic simulation model has considerable capacity for extensions, first of all to other real estate markets and also to labour markets.

I simulated a closed rental housing market with search and matching frictions, in which both landlord and tenant agents are imperfectly informed. Homogeneous landlords set rents to maximise revenue, using information on the market to estimate the relationship between posted rent and time-on-the-market (TOM). Tenants, heterogeneous in income, engage in undirected search, accepting residences based on their idiosyncratic tastes for housing and a reservation utility based on information on the distribution of offers. The steady-state to which the simulation evolves shows price dispersion, nonzero search times and vacancies.

I analyse the effects of increasing the information available to both categories of agent. When tenants see a larger part of the distribution of offers, they learn to refuse high rents and so their average utility rises. In contrast, when landlords are homogeneous and their information levels are reduced, their utilities can improve as their overestimations of the best posted rent increase these rents. Less surprisingly, the better informed landlords are better off when heterogeneities in information levels are introduced. The effects of a vacancy tax are also analysed, both statically and dynamically. It is found that, for certain parameter values, a level of taxation exists at which the welfare benefits of reduced vacancies, TOM and rents dominate the negative supply effects of the tax. The dynamic adjustment to large changes in the tax rate are shown to cause even larger short-term fluctuations in the market state. This highlights the potential perils of forcing abrupt adaptations in search markets, as the repercussions maybe difficult to predict with confidence.

Agent-based modelling guidelines

In the course of this exploratory project, progress should be measured not only by the results obtained but also by the new understanding of what types of objects are most suitable for this form of modelling. It will be seen in Chapter 2 that attempting to include too many dimensions (degrees of freedom) into a simple model limits the relevance of the results that can be extracted. I argue that this is not the optimal approach, and that when simpler separate models of parts of the system can be constructed, the insights gained can lead to a more satisfactory

composite model. It will be argued in Chapters 2 and 3 that it is preferable whenever possible to create initially a robust model of the simplest constituent parts of a system in order to better understand the composite system. In this thesis, this has meant concentrating on an aspatial housing market, in Chapter 3, rather than continuing to develop a spatial housing market model before the dynamics and interactions of agents within the market are fully understood.

It has been learnt in the course of this thesis that certain attributes are very often helpful in producing stable and well-behaved simulation models. In standard economic models with perfectly informed agents, results are obtained assuming the existence of the equilibrium. When dynamic interactions are included between explicitly modelled agents, this luxury is cast aside. It has been discovered that models that mimic closely standard analytic assumptions such as well-informed homogeneous agents, are prone to instability. This is because the excess resemblance between agents causes them to adopt similar behaviours in similar situations, and hence, system-wide instability. The introduction of stochasticity can also have a system-wide stabilising effect. This initially counterintuitive result occurs because the heterogeneity in agent behaviours causes a stable average distribution of behaviours to be adopted and hence avoids a situation in which all agents do the same thing at the same time. The potentially stabilising effect of heterogeneities has been remarked upon in Kirman (1992).

Modelling systems containing large number of agents helps to prevent any single agent, or small group of agents, excessively influencing the overall behaviour and stability of the model. Modelling the behaviour of complex individual agents in systems with a small number of agents is an approach that can be adopted, but is not one in which insights and intuitions coming from the study of physical systems can best be employed. This is why in all the systems modelled in this thesis there are a large number of agents who adopt relatively uncomplicated behaviours. It is hoped that, while the real variation in human behaviour in these systems is much greater than that modelled, this approach can succeed in capturing enough of the real determinants of agents' behaviours to make the macro-level insights pertinent.

The fact that in a world where everybody does the same thing, strange things happen is a result that comes naturally from simulations. The fact that more realistic assumptions very often lead to more realistic outcomes is an encouraging aspect of agent-based simulation methods. It underlines the advantages of adopting this intuitive and flexible approach to modelling socio-economic systems.

Traffic Congestion

1.1 Introduction

Traffic congestion is a major phenomenon in modern urban systems. There are two major dynamics in traffic congestion systems, the dynamics of traffic within a single day (or single rush-hour) and that between days. Correctly modelling the day-to-day evolution of the within day traffic variations is essential to capturing important dynamic effects that can come with modifying road infrastructure. For instance the effect of increasing the capacity of congested roads is not simply to reduce the travel time but also to shorten the duration of rush-hour. This happens because the increased capacity induces road users to depart closer to peak congestion times as the corresponding travel times have reduced. Conversely, the increase in peak congestion levels is less than may naively be expected when capacity is reduced, as road users alter their departure times to avoid the increased travel times.

The motivation for the models presented in this Chapter is essentially threefold. The primary motivation is to investigate the nature of the day-to-day stability of the equilibrium of the within-day-time dynamic congestion bottleneck model of Vickrey (1969). The second is to investigate if agent dynamics might be applicable in operational models in which aggregate dynamics have been shown to be unstable (de Palma and Marchal, 2002). Although real world road networks show day-to-day oscillations, models that are stable at the system level permit clearer policy comparisons.¹ The motivation for Section 1.5 of this chapter is to investigate if a doubly dynamic analytical solution can be found for this problem, in order to better understand the sources of instability. This investigation inspired the simulation of even simpler systems whose dynamics may be related to the bottleneck model and whose stability can more easily be explored.

We present firstly the agent-based study of the simple bottleneck congestion model.² This model uses simulated individuals or ‘agents’ to study the effect of departure time choice on the formation of traffic jams, taking into account

¹This is especially desirable when major sources of real-world oscillations are absent from the model, for example accidents and variations in the weather.

²The research in this chapter was conducted with Fabrice Marchal, and with Eric Bertin for Section 1.5 .

the cost of the journey and the cost of not arriving at exactly the desired time, known as the schedule delay cost. This model is based on the pioneering work of Vickrey (1969), who examined the effect of Departure Time Choice on congestion for a road with limited capacity. The behaviours of modelled agents reflect their desire to reduce the total cost of the morning commute. This costs include both congestion costs and the cost of not arriving at the desired time. Our agent model is scalable to city sized models.

Simulations with homogeneous agent systems, in which all agents wish to arrive at the same time, are shown to be unstable experimentally. The proportion of agents who review their departure times in the same iteration effects the stability of the system. When the agents are given a normal distribution of preferred arrival times the system is stabilised while the level of congestion remains significant. The variance of the distribution of agents' preferred arrival times and the reviewing rate are two important parameters that determine the qualitative behaviour of the model. A graph of the stability of the system against these two parameters highlights the parameter space of stable behaviour. The introduction of agents with different propensities to review their departure time is found to have a very slight stabilising effect.

A number of models have been proposed to examine within-day congestion on a single route network, for a review see Lindsey and Verhoef (2001). For instance, the instantaneous propagation model of Mahmassani and Herman (1984), where an increase in input flow is immediately absorbed by an equal increase in density everywhere along the road and the 'no-propagation' model introduced by Henderson (1977) where travel time depends only on the density at the moment of entry onto the road, are examples.

Deterministic day-to-day evolution processes have been studied with static within-day configurations by, for example, Friesz et al. (1994); Zhang and Nagurney (1996). Stochastic day-to-day evolution processes have been proposed by Cascetta (1989); Davis and Nihan (1993); Watling and Hazelton (2003), again without within-day dynamics. A doubly dynamic stochastic simulation model was developed by Cascetta and Cantarella (1991). Cantarella and Cascetta (1995) studied both deterministic and stochastic non-agent day-to-day evolution processes with the potential for extension to within-day dynamics. The major advantage of agent-based simulation models of transport is their capacity to study both dynamics simultaneously while including the heterogeneous characteristics of agents.

Departure time choice is also being studied currently in the context of activity-based models, see Ettema and Timmermans (2003). In activity-based models the demand for transport is derived from the desire to perform activities at different locations and times. Trip chaining through many activities is a feature of these models. The model presented in this chapter examines a prototype congestion model that could be included in an activity-based model.

The ultimate function of any congestion model is to be applied in a realistic network. Lim and Heydecker (2005) have studied dynamic departure time choice and route choice on a network. Peeta and Ziliaskopoulos (2001) present a review of dynamic traffic assignment.

This study is based on the work of Vickrey (1969), extended by Arnott et al. (1993). The Nash equilibrium is known for this analytic model, though the behavioural processes that could lead to this equilibrium have not, to the authors' knowledge, been studied before using an agent model. The agent-based modelling approach allows these processes to be investigated in more detail than in previous analytical and numerical studies of the single bottleneck model (de Palma et al., 1983; Ben-Akiva et al., 1984; Arnott et al., 1993).

It is shown that the system evolves towards the Vickrey equilibrium under certain conditions. The overall behaviour of the system can change qualitatively when the agents are heterogeneous instead of homogeneous. Three forms of heterogeneity were introduced in the model.

Section 1.2 presents a brief recall of the Vickrey model, which has been treated in greater detail elsewhere (Vickrey, 1969; Arnott et al., 1993; Marchal, 2001). Section 1.3 discusses the agent-based model, the results obtained with homogeneous agents and the cause of the oscillations. In Section 1.4, the effects of three forms of heterogeneity in the agent population are examined. In particular, we examine the interplay between two important parameters of the model, the reviewing rate and the level of heterogeneity. A graph of stability in the space of these two parameters shows the regions of qualitatively interesting behaviour. Section 1.5 presents an analytical specification of the doubly dynamic congestion model, and simulations of a simpler congestion system as well as a deterministic version of the agent-based model. Section 1.6 concludes the chapter.

1.2 Vickrey's model

In this model, a fixed number N of individuals wish to travel by car on a road with limited capacity to arrive at the same destination at the same time, denoted t^* . The model examines the departure time choice in this situation.

The capacity of the bottleneck is denoted, S (in cars per unit time), the optimal usage of the infrastructure is such that everybody passes the bottleneck in time $T = N/S$. If the departure rate is greater than the capacity, a traffic jam is created. This translates into an increased travel time with an associated increase in cost. There are also schedule delay costs, a cost for arriving early and a higher cost for arriving late, both of which increase linearly with time, see Figure 1.1.

Since the fixed travel costs don't change the dynamics of the model, they are normalised to zero. That is, if there is no congestion the arrival time is the same

as the departure time.

The Nash equilibrium is the situation in which no individual can reduce the cost they pay by changing their departure time. In order to better understand the departure rate function of the equilibrium we need first to examine how the level of congestion at any time is calculated and also how the total cost of a journey is calculated.

1.2.1 Congestion

Knowing the level of congestion at a certain time enables us to calculate at what time someone who departs at that time will arrive. The traffic jam introduces memory into the system because at any time the size of the jam depends on the number of people who have already left and at what time they departed. The amount of congestion (number of vehicles) encountered by someone who leaves at time t , is given by

$$Q(t) = \int_{t'=\tilde{t}}^t r(t') - S * (t - \tilde{t}). \quad (1.1)$$

where $r(t)$ is the departure rate (number of vehicles) at time t , S is the capacity, which is the rate at which commuters leave the traffic jam (vehicles per unit time), and \tilde{t} is the moment the traffic jam began. The first term on the right hand side in Equation (1.1) is the number of vehicles that have joined the traffic jam since it began, the second term is the number of vehicles that have left the traffic jam since it began. The travel time for someone who leaves at t_d is given by

$$tt(t_d) = \frac{Q(t_d)}{S}. \quad (1.2)$$

The arrival time, t_a , is given by $t_a = t_d + tt(t_d)$.

1.2.2 Cost

The cost $c(t_d)$ for an individual who departs at time t_d is a combination of the travel costs and the schedule delay costs. More precisely, it is the addition of the travel time multiplied by the value of time α , and either the length of time by which the individual arrives early multiplied by a constant β , or the length of time by which the individual arrives late multiplied by a constant γ . It is assumed, in accordance with empirical results (Small, 1982), that

$$\gamma > \alpha > \beta \quad (1.3)$$

In Figure 1.1 we can see that the cost of arriving late, γ increases more quickly than the cost of arriving early, β . α includes the value of time and physical transport costs.

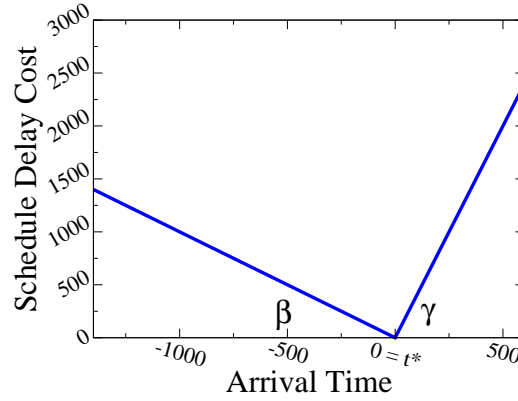


Figure 1.1: The schedule delay cost as a function of the within-day arrival time.

The cost function is

$$C(t_d) = \alpha t t(t_d) + \beta \{t^* - (t_d + t t(t_d))\}^+ + \gamma \{(t_d + t t(t_d)) - t^*\}^+ \quad (1.4)$$

where $\{a\}^+ = \max(0, a)$.

1.2.3 System optimum

The system optimum configuration is that which has the minimum possible total cost across all users. The departure rate of the system optimum is shown in Figure 1.2 and the corresponding cost function for each departure time in Figure 1.3.

1.2.4 Nash Equilibrium

The Nash equilibrium departure rate is shown in Figure 1.2 along with that of the system optimum, for which there is no congestion and the route is used at its full capacity. When the departure rate is greater than the route's capacity the level of congestion builds. Figure 1.2 shows that the congestion builds up to a moment at which the departure rate falls brutally and the congestion level begins to reduce. This is the moment at which the agent who arrives exactly at the desired time departs. This agent pays no schedule delay cost, but pays the highest congestion costs of all agents. The congestion level builds and then falls at constant rates, such that the associated changes in congestion costs compensate exactly the

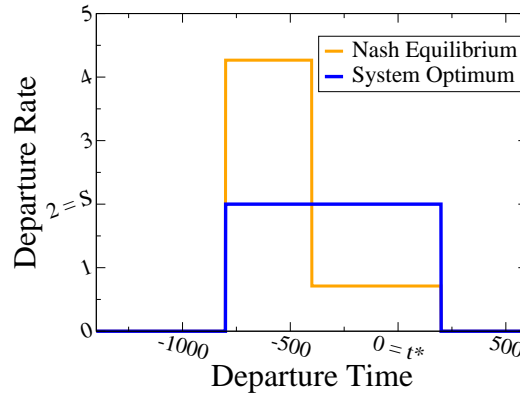


Figure 1.2: The departure rates for the Nash equilibrium and the system optimum. The capacity of the road is 2 cars per unit time. When this is exceeded a traffic jam forms.

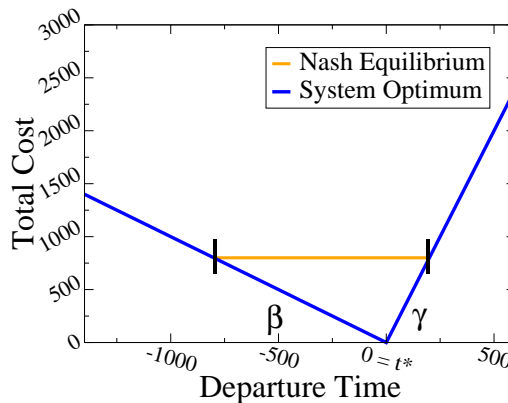


Figure 1.3: The cost functions for the Nash equilibrium, and the system optimum for which there is no congestion. The vertical lines show when the first and last drivers leave.

changing schedule delay costs. The flat Nash equilibrium cost function is drawn in Figure 1.3.

1.3 Agent-based model

Urban transport systems contain large numbers of heterogeneous agents who nonetheless make relatively simple strategic decisions. For example: What time to leave and what route to take? Agent-based models are very flexible with regard to the inclusion agent heterogeneities and also in the representation of transport networks. These facts combined with the straightforward nature of the trade-offs made by agents' mean that agent-based model are well adapted to

modelling road congestion.

In this model each simulated agent represents an individual and is given a simple behavioural rule which is followed in order to reduce the cost of the journeys undertaken. An agent who reviews his departure time calculates the cost of a randomly chosen departure time, and will change his departure time if this cost is sufficiently cheaper than his current cost. Simulations were performed with agents who had various sensitivities to cost differences.

Agents have a certain probability, or reviewing rate, of changing their departure time at each iteration. This reviewing rate is an important parameter of the model that must be carefully calibrated for operational models. The qualitative effects of changing this parameter are discussed below. In one version of the model tested, the propensity to review one's departure time varied across agents (Section 1.4.3).

At each iteration the agents who review their departure time are chosen randomly, and the new departure time tested is chosen randomly from a uniform distribution of size $2T$ centred on the current departure time, T being the minimum time in which all agents can pass through the bottleneck.³ All the agents who review their departure time calculate the cost of the new departure time assuming that no other agent changes their departure time.

We define one iteration as the time needed statistically for all agents to review their departure time once. These normalised iterations permit comparisons across varying reviewing rates and are used hereafter.

1.3.1 Homogeneous agents

Homogeneous agents all wish to arrive at the same time, t^* , and follow the same rules. For every agent the travel cost per unit time is $\alpha = 2$, the cost of arriving early is $\beta = 1$, and the cost of arriving late is $\gamma = 4$.

The first simulation is performed with 2000 agents who began with their departure times distributed so that overall the departure rate function is that of the system optimum. 5% of agents review their departure times each iteration. The within day time is broken into 2000 discrete units of time, therefore the minimum time in which all agents can pass the bottleneck is $T = 1000$. The agents who review their departure times choose a new departure time, at random, from a flat distribution between a time T before and a time T after their current departure time. The average travel cost for such a simulation can be seen in Figure 1.4. The agents in this simulation and in all other simulations presented here are assumed to be infinitely sensitive to reductions in cost. When the reduction in cost required for agents to change their departure time increased to 20 percent the only effect is to slow the overall evolution of the system. Instability is unaffected.

³This distribution is the same size as the domain of the simulation.

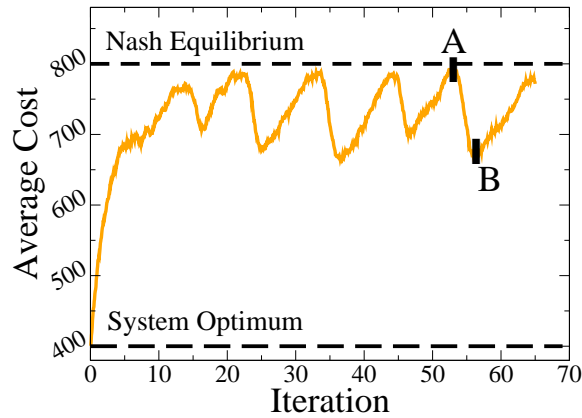


Figure 1.4: Average cost for 2000 homogeneous agents with a 5% reviewing rate at each iteration.

The average cost does not converge but oscillates below 800 which is the value of the equilibrium. Figure 1.5 shows two departure rates, one for which the average cost is close to the equilibrium value A, and one for which the average cost is significantly below the equilibrium value, B. The departure rate at A has a form closer to that of the equilibrium.

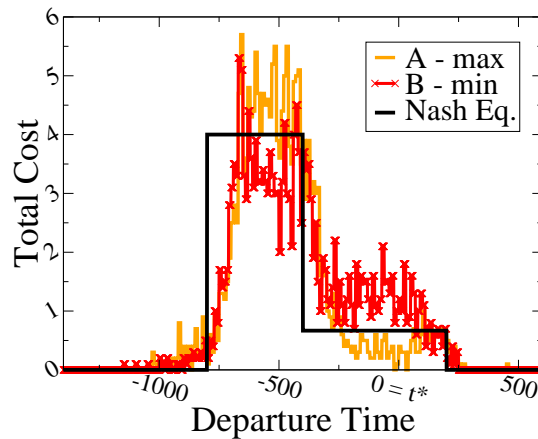


Figure 1.5: The departure rate functions at points A and B of Figure 1.4.

1.3.2 Explanation of the oscillations

The fundamental reason for the oscillations is that an agent who changes to reduce his own cost often has a much greater effect on the collective cost. When an agent changes from a departure time where he suffers no congestion to one where he encounters a traffic jam, he increases the travel time for all who join the traffic jam after him. These costs imposed on others are known as externalities.

An agent who changes to avoid the traffic jam reduces the cost for all who joined the traffic jam after him on previous days.

The trajectory of the global cost depends on the average evolution of the departure times:

- The effect of an agent who leaves earlier during rush hour is to increase the congestion experienced by those who leave between his new and old departure times.
- The effect of an agent who leaves later during rush hour is to decrease the congestion experienced by those who leave between his old and new departure times.

The oscillations are due to the competition between homogeneous agents who all have the same preferred arrival time t^* . We now examine the effect of introducing heterogeneous agents on the system's stability. In Section 1.5 we attempt to further understand the causes of the instability using an analytical specification.

1.4 Heterogeneous agents

Homogeneous agents is a very strong assumption to make. In order to add more realism and hopefully find a more stable global system we introduce heterogeneous agents to the simulation. The first heterogeneity introduced is in the schedule delay costs, that is the costs of arriving either early or late. Later the agents are given a distribution of preferred arrival times. The effect of assigning to agents different probabilities of reviewing their departure times are also investigated.

1.4.1 Distribution of schedule delay costs

We assume that the agents do not all have the same aversion to arriving early or late. Previously for homogeneous agents, the travel cost per unit time was $\alpha = 2$, the cost of arriving early was $\beta = 1$, and the cost of arriving late was $\gamma = 4$. Now, the schedule delay costs β and γ , see Equation (1.4), for each agent are multiplied by the agent's Schedule Delay Multiplier Ψ . The travel time costs α , see Equation (1.4), is the same for all agents.

$$\begin{aligned}\alpha' &= \alpha \\ \beta' &= \Psi\beta \\ \gamma' &= \Psi\gamma\end{aligned}$$

Studies by de Palma and Fontan (2001); Raux et al. (2003) have shown that there is a normal like distribution to the value of time among commuters. A study

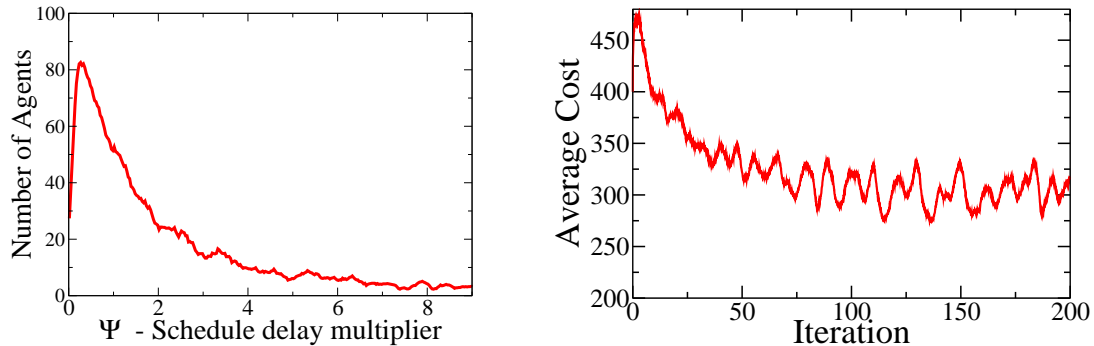


Figure 1.6: *Left*: Number of agents for each schedule delay cost multiplier for 6000 agents. The multipliers are distributed following a log-normal distribution with $\sigma = 1,7$ and $m = 2,4$. *Right*: The evolution of the average cost for the same distribution.

undertaken in Lyon by Raux et al. (2003) calculated this distribution to be given by the parameters $m = 2.4$ (mean) and $\sigma = 1.7$ (variance). The multiplier of schedule delay costs of each of the 6000 agents is assigned randomly from such a distribution, see Figure 1.6-*Left*. The schedule delay costs are calibrated so that on average the cost when the agents are distributed randomly at the system optimum is comparable to that for homogeneous agents i.e. 400.

We can see from Figure 1.6-*Right* that the average cost paid by the agents, who began with their departure times distributed so that the overall rate of departure was that of the system optimum, reduces. This is because the agents with high schedule delay costs arrive near the desired time while the other agents avoid the high levels of congestion around the preferred arrival time. There are still significant oscillations that would render it very difficult to calibrate the model with real world data.

1.4.2 Distribution of preferred arrival times

It is clearly unrealistic that everybody wishes to arrive at exactly the same time. Heterogeneity in preferred arrival times has already been explored analytically in a similar set-up (Arnott et al., 1988)⁴ Here each agent was given a preferred arrival time (PAT) chosen randomly from a normal distribution around t^* , many different variances of the normal distribution are tested below.

The amplitude of oscillations for a normal distribution of variance $\sigma = T/10$

⁴They studied the benefits of both time-varying congestion tolls, and capacity investments, with commuters that were heterogeneous in their preferred arrival times (PATs). They found that the estimated travel cost savings can be biased when computed using average travel cost parameters, rather than actual distributions. The benefits from capacity investments are underestimated in their model when commuters differ in preferred arrival times, while the savings from time-varying congestion tolls are overestimated.

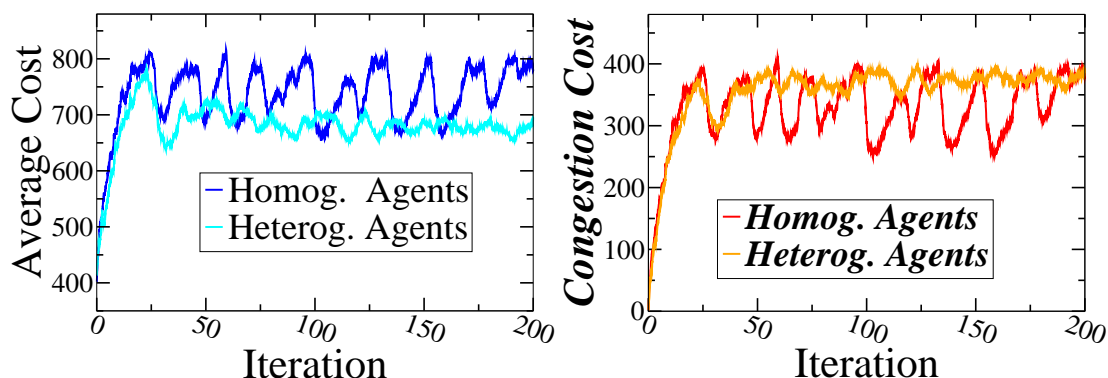


Figure 1.7: Comparison of average costs and congestion costs for homogeneous agents and agents with a Gaussian distribution of preferred arrival times of variance $\sigma = T/10$.

were significantly less than those found for homogeneous agents, see Figure 1.7-*Left*. The average costs are of roughly the same magnitude, though slightly reduced for heterogeneous agents, due to the reduction in the schedule delay costs. More importantly, the cost for heterogeneous agents is much more stable. The stability of the average cost for heterogeneous agents comes from the stabilisation of the congestion cost, Figure 1.7-*Right*.

The stability of the average cost is a result of the greater reluctance of the heterogeneous agents to change their departure times and the fact that the changes they do make are smaller in magnitude, see Figures 1.8 & 1.9. The agents tend to find a niche, a small range of departure times, that consistently give the lowest cost.

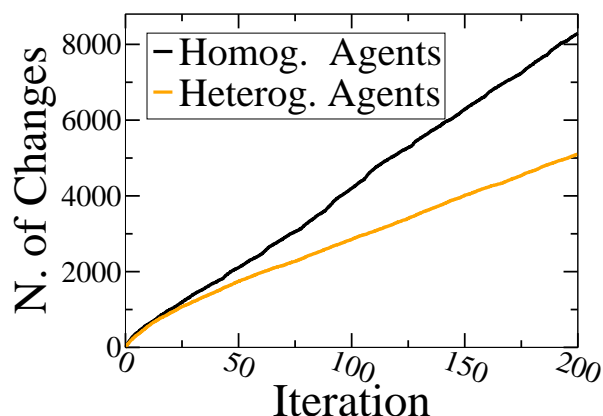


Figure 1.8: The cumulative number of changes of departure time for heterogeneous and homogeneous agents.

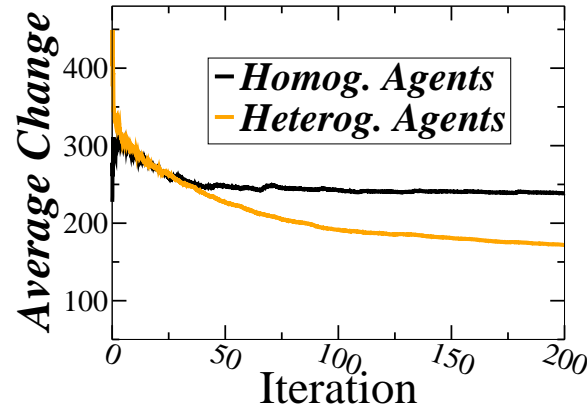


Figure 1.9: The average magnitude of changes since the beginning of the simulation for both types of agents.

Heterogeneity (in PAT) and the reviewing rate

In order to calibrate any model it is necessary to understand the qualitative effects of important parameters. In which regions of parameter space do we find macro level behaviour that resembles observations? Two parameters of this simple model that have important effects are the reviewing rate, the proportion of agents that try a new departure time at each iteration, and the level of heterogeneity, i.e. the variance of the distribution of PATs.

The stability is measured by the standard deviation between 400 normalised iterations and 800 normalised iterations. During this time the agents try, on average, a new departure time 400 times.

From Figure 1.10 we can see that the reviewing rate has a straightforward effect on the stability. When the reviewing rate is increased the system becomes less stable. It is also clear that increasing the variance of the Gaussian distribution of PATs increases the stability of the system. At low reviewing rates the system is effectively stable once the variance of PATs is greater than 50, which is one twentieth of the time in which all the agents can pass through the bottleneck, $T = 1000$. At higher reviewing rates the system reaches its maximum stability for a variance of PATs of around 100, or $T/10$.

Figure 1.11 shows that for each point in Figure 1.10 the level of congestion remains significant. The average level of congestion initially increases with greater agent heterogeneity because the level of congestion stabilises and collapses of the congestion cease to occur, see Figure 1.7.

The parameter space of stable behaviour, with a significant level of congestion, is contained between $T/20$ and $T/5$ for the variance of PATs with a reviewing rate of less than 15%. It remains to be established if the actual heterogeneity of real people corresponds to the space of stable behaviour of this model.

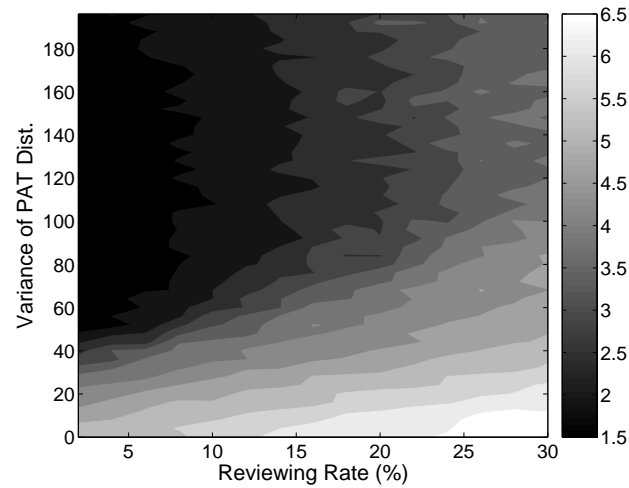


Figure 1.10: The standard deviation of the average cost over 400 normalised iterations, as a percentage of the average cost. The varying inputs are the variance of the Gaussian distribution of arrival times and the reviewing rate.

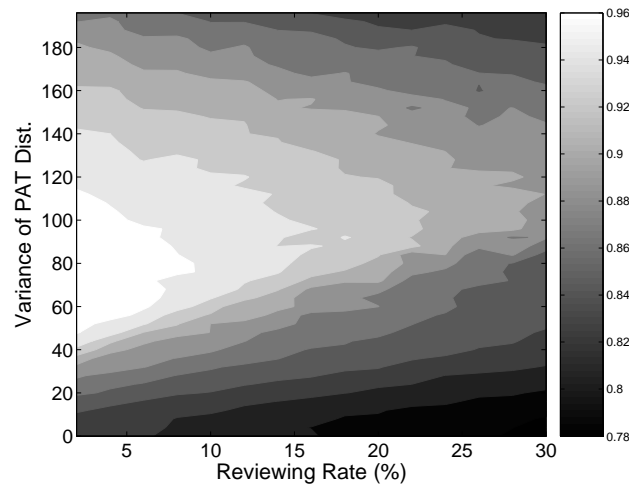


Figure 1.11: The average congestion of the system over 400 normalised iterations divided by the level of congestion of the equilibrium.

1.4.3 Heterogeneous reviewing rates

We investigate if assigning agents a distribution of reviewing rates as opposed to a common reviewing rate effects the stability of the model. The agents are assigned a reviewing rate chosen randomly from a Gaussian distribution around the central value. The variance of this distribution is $1/3$ of the central value, hence only a very small number of the agents never change their departure time, see Figure 1.12. We then compare the stability of the system to the case where there is one common reviewing rate equal to the mean of the distribution.

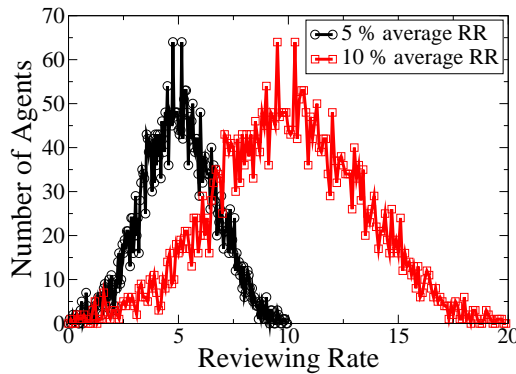


Figure 1.12: The number of agents assigned each reviewing rate.

This heterogeneity in the reviewing rates only marginally increases the stability, Figure not shown. On average the standard deviation of the average cost over 400 normalised iterations, for the same variance of the distribution of preferred arrival times, reduces by just under 3%.

1.5 An analytic investigation

In order to better understand the stationary state of the doubly dynamic model (within-day and day-to-day), we examine the possibility of creating a deterministic analytical analogue whose investigation could shed light on the origin of the oscillations in the agent-based model with homogeneous agents.

We begin by specifying an analytical analogue of the day-to-day dynamic evolution of the departure time distribution, Section 1.5.1. A notable feature of this is the congestion function proposed. The form of the schedule delay costs are also altered to be more analytically tractable by using a continuous function.

Secondly we investigate a one-dimensional deterministic iterative map (Bar-Yam, 1997) with two departure times, Section 1.5.2. The single dimension of this model is the proportion of agents departing at either time. When agents make a binary decision between the more expensive and less expensive departure time,

with no reference to the magnitude of the difference in cost, the system is unstable, but once agents propensity to change their departure time is proportional to the difference in cost, the system becomes stable, with homogeneous desired arrival times.

Thirdly a deterministic simulation analogue of the agent model with a full distribution of departure times is presented in Appendix B. It is found that the system oscillates for all parameters with both continuous and discontinuous schedule delay costs. Indeed, with discontinuous schedule delay costs for certain parameters the average cost is highly unstable, approaching chaos. This aspect of the model is not further investigated as the important insights into the causes of instability can be obtained from the work presented.

1.5.1 Analytic specification

The departure rate function $r_n(t)$ determines the congestion function $\hat{Q}_n(t)$ (in units of time), where t is within daytime and n specifies the day. $\hat{Q}_n(t)$ determines the arrival times corresponding to each departure time. Hence the total cost $c_n(t)$, on day n for each departure time t , can be deduced. It is simply the combination of the congestion and schedule delay costs, see Equation (1.4). The travel cost function $c_n(t)$ then determines the evolution of the departure rate function from day n to day $n + 1$.

The following specification is examined. At any moment the level of congestion changes following the equation

$$\frac{d\hat{Q}(t)}{dt} = r(t) - f(\hat{Q}) \quad (1.5)$$

a possible form of the function $f(\hat{Q})$ is given by $f(\hat{Q}) = 1 - e^{-\hat{Q}}$. This form was chosen for analytical convenience. On the right hand-side of Equation 1.5, the first term represents the arrival rate at the bottleneck and the second, $f(\hat{Q})$ the rate at which cars leave the bottleneck. The second term is zero when there is no queue, and rises quickly towards one as the size of the queue increases. Therefore, a queue will begin to form once $r(t)$ is positive, and this queue will dissipate at a near constant rate once it reaches a significant size. This is a close approximation to the congestion technology presented in Section 1.2.1, where no congestion forms at low departure rates, and cars leave any queue at a constant rate equal to the bottleneck's capacity. The solution to this system for $\hat{Q}(t)$ is⁵

$$\hat{Q}(t) = \ln\left[1 + \int_0^t e^{R(t)-R(t')-(t-t')} r(t') dt'\right] \quad (1.6)$$

where $R(t) = \int_0^t r(t) dt$. This is a means of translating the departure rate function to the corresponding function of the travel costs.

⁵Result due to Eric Bertin.

A first order continuous schedule-delay-cost functional form, more appropriate for an analytical study is given below. It should be noted that, for the coherence of the model, the cost function needs to be qualitatively parabolic and that the costs of arriving late must rise more quickly than the costs of arriving early. Such a function is $\tilde{k}(\tilde{t})$, which is the schedule delay cost for those that arrive at \tilde{t} ,

$$\tilde{k}(\tilde{t}) = e^{-A(\tilde{t}-t^*+L)} + e^{B(\tilde{t}-t^*+L)} \quad (1.7)$$

where t^* is the shared preferred arrival time and $B > A > 0$. The first term represents the costs of early arrival and the second term represents the costs of late arrival. Note that \tilde{t} is the arrival time corresponding to the departure time t and that $\tilde{t} = t + \hat{Q}(t)$. Hence the delay cost function in terms of the departure time is

$$k(t) = e^{-A(t+\hat{Q}(t)-t^*+L)} + e^{B(t+\hat{Q}(t)-t^*+L)}. \quad (1.8)$$

In order that $\tilde{k}(t^*)$ is a minimum of the function, we set

$$L = \frac{\ln(A/B)}{A+B}. \quad (1.9)$$

The total cost function is $c_n(t) = Q_n(t) + k_n(t)$ or explicitly

$$c_n(t) = \ln\left[1 + \int_0^\infty e^{R(t)-R(t')-(t-t')r(t')} dt'\right] + e^{-A(t+\hat{Q}(t)-t^*+L)} + e^{B(t+\hat{Q}(t)-t^*+L)}. \quad (1.10)$$

This is the cost function from which the evolution of the departure rate function must be determined:

$$\frac{dr_n(t)}{dn} = F(c_n(t)). \quad (1.11)$$

The system of Equations (1.10) & (1.11) is clearly not easily solved, even for the most simple reasonable function $F(c_n(t))$. They represent a non-linear integro-differential system with memory, due to the congestion term.

However, with a simpler system it might be possible to produce analytical results pointing out the fundamental factors contributing to the instability of the agent model, including notably the form of the schedule delay costs and the sensitivity to cost differences.

1.5.2 Two departure time iterative map

A simple iterative map with only two departure times was constructed in order to test the stability of as simple a system as possible, centred on a trade-off between travel costs and schedule delay costs.

Iterative maps are among the simplest dynamical systems. They are a function that is applied recursively at each time step in a simulation. We can contrast their behaviour with that of the second major branch of dynamical systems which

are known as continuous flows. In flows the system's evolution is represented by differential equations. Flow dynamical systems cannot have chaotic behaviour with less than three dimensions, among the best known of which is the Lorenz system. In contrast, the evolution of discrete *one* dimensional iterative maps can be chaotic. Iterative maps are systems whose rules of evolution can be straightforwardly stated but whose actual evolution can have very complex behaviours. The ability to depict their evolution on two-dimensional graphs facilitates the identification of fixed points and an analysis of the fixed points stability, as will be shown in Figures 1.14-*Right* & 1.16-*Right*. It is for this reason that we now examine a one-dimensional iterative map representing a simple system with congestion, and hence a trade-off between travel times and schedule delay costs.

The model determines the repartition between departure times $t_d(1)$ and $t_d(2)$. The map from one iteration to the next $x_i = F(x_{i-1})$ is

$$\begin{aligned} \text{if } c_{i-1}(1) > c_{i-1}(2) \quad \text{then } x_i &= x_{i-1} - \epsilon x_i \\ \text{if } c_{i-1}(2) > c_{i-1}(1) \quad \text{then } x_i &= x_{i-1} + \epsilon(1 - x_i) \end{aligned} \quad (1.12)$$

where x , belonging to $[0, 1]$, is the proportion of agents departing at $t_d(1)$, ϵ is the 'reviewing rate' and i indicates the iteration. This mapping means that if the cost for one departure time is less than at the other, a fraction ϵ of those departing at the more expensive time will change their departure time.

The two travel times, dropping the subscript for the iteration, are

$$\begin{aligned} tt(1) &= t_d(1) + x/S \\ tt(2) &= t_d(2) + (1 - x)/S + [t_a(1) - t_d(2)] \theta(t_a(1) - t_d(2)) \end{aligned} \quad (1.13)$$

where S is the capacity of the bottleneck (number of vehicles per unit time). $\theta(z - y) = 0$ if $z < y$ and $\theta(z - y) = 1$ if $z > y$. The third term on the right-hand side of the second travel time $tt(2)$, is the additional congestion experienced by those departing at this time due to congestion from the first group. The arrival times are given by $t_a(j) = t_d(j) + tt(j)$, where $j = (1, 2)$.

The cost functions are, similarly to Equation (1.4) for the agent model,

$$C(j) = \alpha tt(j) + \beta \{t^* - t_a(j)\}^+ + \gamma \{t_a(j) - t^*\}^+$$

where $\{a\}^+ = \max(0, a)$ and t^* is as before the preferred arrival time of all agents.

The discrete map

$$x_i = F(x_{i-1})$$

can be found straightforwardly for all x_{i-1} . If discretisation effects can be discounted (in the limit of large numbers), there will always be a fixed point once

$t_d(1)$ and $t_d(2)$ are close enough so that $c_1 = c_2$ for some x in $[0, 1]$. The stability of the fixed points is however uncertain.

The correct interpretation of ϵ is as the product of the probability to review one's departure time and the probability of accepting the new departure time tested. In short, it is the probability of changing departure time when the other departure time is cheaper.

$$\epsilon = \text{Prob}(\text{review}) * \text{Prob}(\text{accept}) = \text{Prob}(\text{change})$$

It should be noted that in the agent versions $\text{Prob}(\text{review})$ is constant while $\text{Prob}(\text{accept})$ varies as it depends on the overall configuration of departure times.

The setup and parameter values used to obtain the results shown in Figures 1.13 & 1.14 are presented here.⁶ The total number of commuters is normalised to one infinitely divisible agent. The within-day duration is normalised to 1 and the capacity S is 2. When all agents depart at the same time their travel time is $1/2$. The schedule-delay-cost form described in Section 1.2.2 was used. As in the agent-based model the travel cost is $\alpha = 2$, the cost of arriving early is $\beta = 1$ and the cost of arriving late is $\gamma = 4$. The parameters new to this model are $t_d(1) = 1/3$, $t_d(2) = 2/3$ and here $t^* = 6/10$ with $\epsilon = 1/10$.

Figure 1.13 for ϵ constant shows for one thousand uniformly distributed proportions x of agents leaving at $t_d(1)$, both the travel costs and the schedule delay costs. In Figure 1.13-*Left* the travel costs for those who leave at $t_d(2)$ are never zero, because when many agents depart at $t_d(1)$ there is still congestion at this time from those who left at $t_d(2)$. The $t_d(2)$ travel cost line is horizontal at higher x , because the total number of people in the traffic jam at this point in time is constant as x increases.

The total costs and the map from x to $F(x)$, see Equation (1.12), are shown in Figure 1.14. The fixed point, where $x_{i-1} = F(x_{i-1})$, can clearly be seen in Figure 1.14-*Right*. It is clear that $|\text{slope } F(x_i)| < 1$ at the fixed point, this is the criteria that determines whether or not the fixed point is stable. Figure 1.15 shows the evolution from $x_0 = 0.5$, and confirms the instability deduced from Figure 1.14-*Right*.

Two alterations extensions are made to this model, firstly a first-order continuous schedule delay cost function is tested and secondly the role of the ϵ parameter in the stability of the model is also examined.

Continuous schedule delay costs

The first order continuous schedule delay costs introduced in Section 1.5.1 are applied in this two departure time deterministic model. The travel cost is $\alpha = 2$,

⁶One thousand discrete possible proportions of agents departing at $t_d(1)$ uniformly distributed on the domain $[0,1]$ are presented in Figures 1.13 & 1.14 .

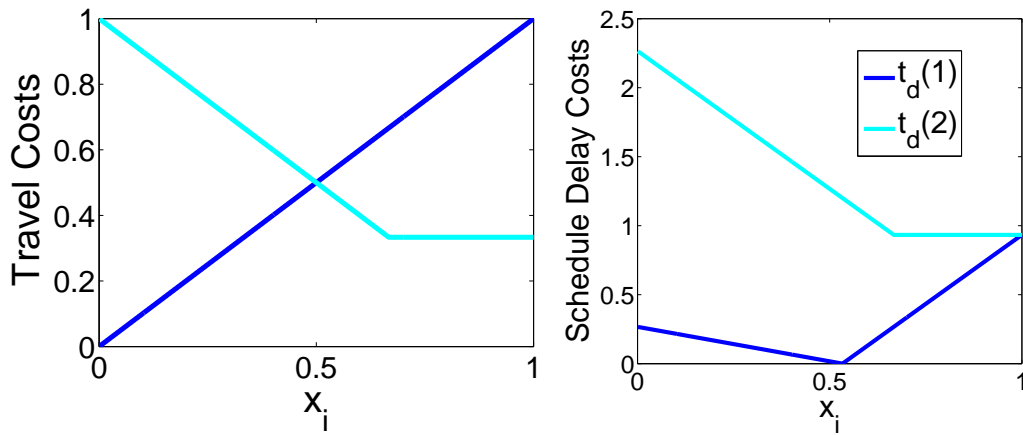


Figure 1.13: *Left*: Travel costs for both departure times. *Right*: The corresponding schedule delay costs.

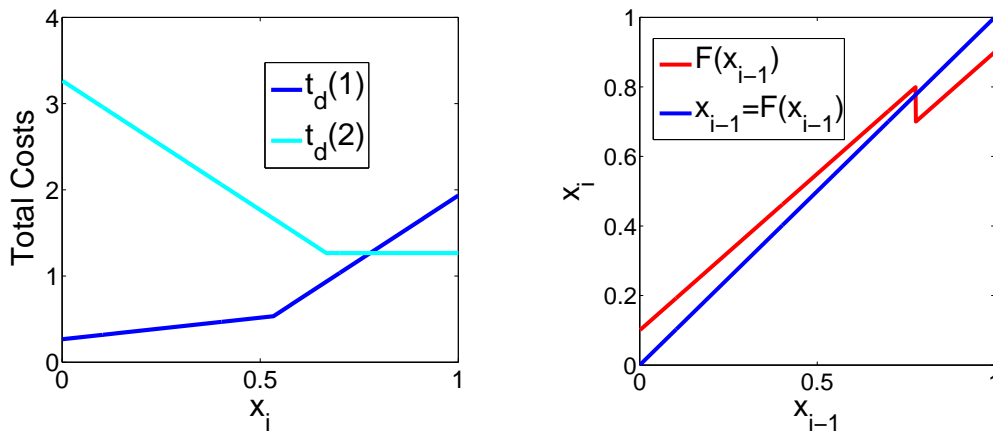


Figure 1.14: *Left*: The total costs for both departure times. *Right*: The one-dimensional map from x_i to $F(x_i)$.

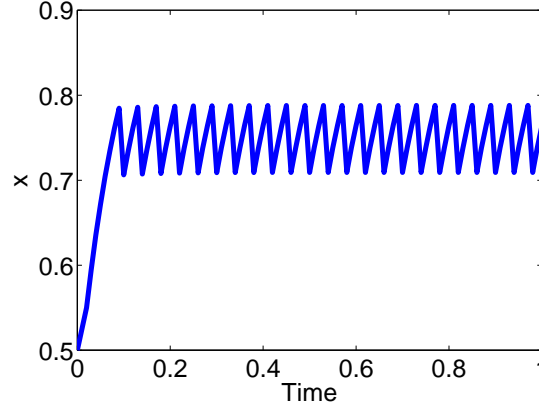


Figure 1.15: The evolution of the proportion of agents departing at $t_d(1)$, with $\epsilon = 0.1$ and 1st order discontinuous scheduling delay costs.

the new schedule delay cost parameters are $A = 1$ and $B = 4$, see Equation (1.8). The other parameters were unchanged.

The results can be seen in Figure 1.16. Figure 1.16-*Left* shows the schedule delay costs for both departure times and Figure 1.16-*Right* shows the map from x_{i-1} to x_i . We can see from Figure 1.16-*Right* that the system is unstable, and graphs of the system's evolution, not shown, confirm this.

Reduced cost sensitivity

In order to test the effects of reducing the sensitivity to cost the following form of the reviewing rate parameter was used, $\epsilon = |c_1 - c_2| / (c_1 + c_2)$. Figure 1.17 shows results with the first-order continuous schedule delay costs in Equation (1.8). In Figure 1.17-*Left* we see the one-dimensional map whose absolute slope is less than one at the fixed point, and in Figure 1.17-*Right* we see the evolution of x_i over time. The stability of the system is evident. The same qualitative effects were found with both forms of schedule delay costs.

A constant value for ϵ assumes that agents are infinitely sensitive to differences in cost. When this strong hypothesis is relaxed and agents become less likely to change departure times as the difference in cost is reduced, stability is brought to the system. It can be expected that the same qualitative effect would occur in the agent-based model if the probability of accepting a tested departure time was proportional to the difference in cost. In the agent model it has been tested with homogeneous agents, whether requiring agents' cost reductions to exceed a fixed threshold in order to change their departure time reduces stability. It was found that the evolution of the system was slowed but that the instability was unaffected, as mentioned in Section 1.3.1. It would be worth testing the effect of a *proportional* probability to accept tested departure times for agents.

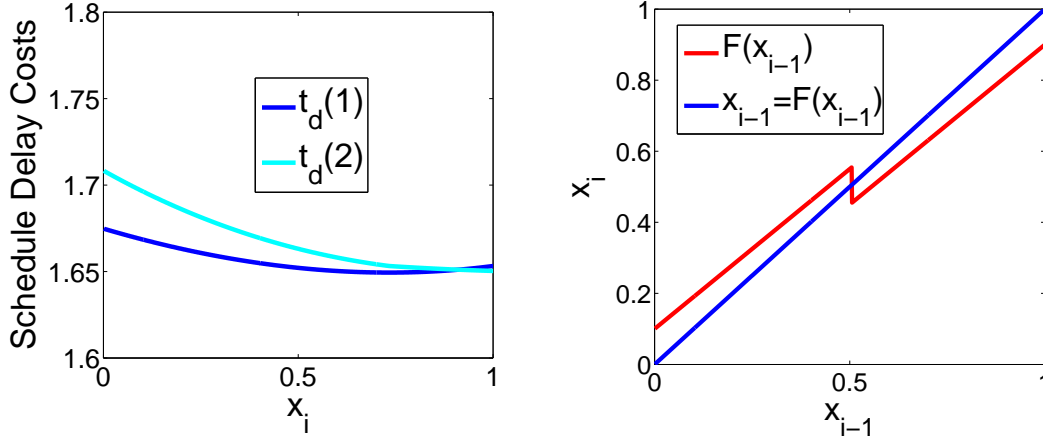


Figure 1.16: *Left*: The schedule delay costs for both departure times with $\epsilon = 0.1$ when the schedule delay costs are those presented in Section 1.5.1. *Right*: The corresponding one-dimensional map from x_i to $F(x_i)$.

Nevertheless, the essential insight is clear, hypersensitivity increases instability.

The nature of the oscillations in the two-departure-time model appears to be of a different nature from those observed in the agent model. In the agent model the oscillations are a result of competition over a distribution of departure times in which externalities play a major role. The oscillations are of a period that is much greater than one day. The two-departure-time model's instability is, in contrast, a classic unstable fixed point of an iterative map. The periods of oscillations are of the order of two days long. The stable configuration in the agent model with heterogeneous agents is not a fixed point, but a stable distribution of departure times with continuing micro level adjustments.

1.6 Conclusions

One aim of this research was to investigate the stability of the single bottleneck model. It was found that the Nash equilibrium distribution is never reached in simulations using simple agents with local information only. This is in part due to the fact that global knowledge is required to achieve the level of coordination required of agents to reach the theoretical distribution. It is unlikely that any reasonable disaggregated dynamic evolution mechanism can achieve this theoretical configuration.

Another aim of this research was to find a robust and convergent model of departure time choice that could subsequently be extended to a much more complex and realistic road network. It is clear that some level of heterogeneity is required in order to achieve stability. Basing the model on a network would add in

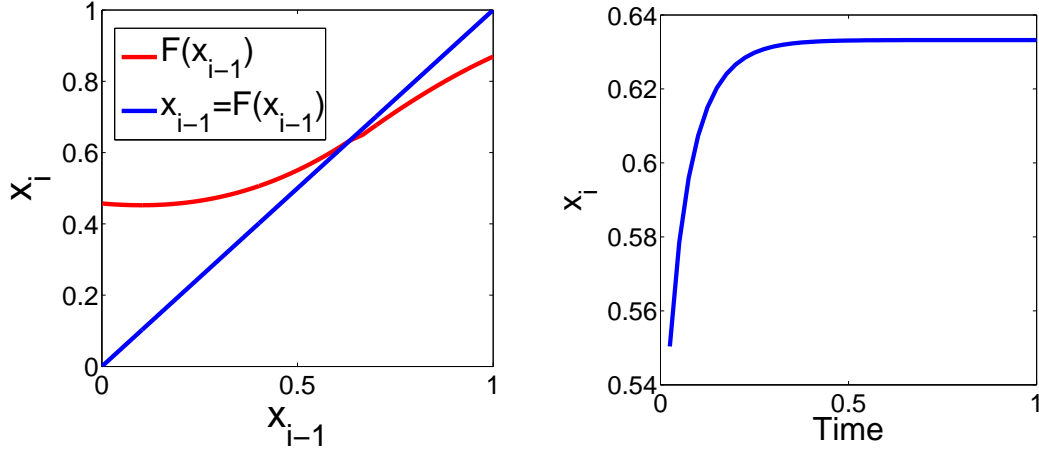


Figure 1.17: *Left*: The fixed point is stable when the propensity to change the departure time is proportional to the difference in cost. *Right*: It can be seen to converge very quickly to a fixed value.

itself an element of heterogeneity that could have a stabilising effect. Numerical experiments have shown that evolution mechanisms applied to networks can be more stable than single origin-destination models (Marchal, 2001). A behavioural rule that converges with sufficiently heterogeneous agents on a single road has been found.

The model of a traffic bottleneck has significant instabilities when implemented with homogeneous agents. These instabilities are due, in part, to the difference between the benefit an agent accrues from changing its departure time and the effect of this change on the overall system.

The explorations of the two departure time iterative and the deterministic analogue of the agent model lead to the following conclusion regarding the origins of the instability of the agent model with homogeneous itself. There are three fundamental causes of the instabilities: competition, externalities and sensitivity. The competition among homogeneous agents who all wish to arrive at the same time is central to the oscillations. The externalities, i.e. costs imposed on others, caused by agents' are a fundamental feature of congested systems. Indeed, competition and externalities together define congestion systems. The high sensitivity to cost differences in the agent-based model is a particular feature of this model that could be relaxed and hence reduce the instability. It is unlikely that this alone would eliminate the oscillations for reasonable sensitivities.

The structural change to the model brought about through the introduction of a distribution of preferred arrival times reduces the level of competition. This eliminates the oscillations. A tentative stab at a general rule for agent-based systems can be postulated from this: changes to behaviour within a constant model structure have less effect than changes to the model structure itself with

behaviours held constant. An example of this shall be seen again in Section 3.5, where heterogeneities are introduced in landlords' maintenance costs to stabilise the system.

Understanding the dynamics of the agent model on a simple example is a pre-requisite to incorporation in a more complex system where the same thorough analysis becomes impossible. We believe that we have found suitable agent dynamics that are sufficiently well 'controlled' and understood, to be incorporated in realistic simulations, such as the activity-based models MATSim (2009) or Arentze and Timmermans (2004).

The lessons on the causes of instability learned from this agent-based model and its deterministic extensions very probably apply in general to agent-based models. They are that large numbers of similar agents behaving in similar ways at the same time cause instability. Hence the addition of realism in terms of heterogeneities and stochastic elements produces often, not only conceptually more satisfactory models, but also more stable ones. In addition models with externalities by which the decisions of individual agents may affect many others are likely to be particularly prone to instabilities.

Monocentric Urban Model

2.1 Introduction

Agent-based models have the potential to model phenomena that are not tractable in analytic models. Urban systems have many spatially distributed, heterogeneous agents, with complex patterns of interaction. This makes it a field in which agent-based models have great potential with respect to the standard analytic models. A theoretically unlimited level of heterogeneity amongst agents' characteristics is possible, as amongst their levels of information and decision-making processes. Exploiting these advantages in practice is not straightforward. However, progress towards greater realism can certainly be made. Standard urban economic models contain perfectly homogeneous agents with perfect information and unlimited cognitive ability, see Fujita (1989).

The motivation for the work presented in this chapter is to investigate the potential of simple spatial agent-based models to move beyond the standard urban economic model. Many interesting empirical spatial phenomena have been observed in urban systems, see for example Anas et al. (1998). These include secondary centres, urban sprawl, income and racial segregation. Endogenous dynamics in the location of amenities have begun to be added to the standard model by Tivadar and Jayet (2006), building upon a static model of Brueckner et al. (1999). However this model serves to underline the much greater potential of simulations for modelling complex evolving systems. A secondary motivation is the possibility of developing agent dynamics that could be included in an integrated operational models of land-use and transport, see Hunt et al. (2005). Parker and Filatova (2008) review analytical, cellular and agent-based models of residential development. Filatova et al. (2009) present an extension of the monocentric urban economic model using negotiations between agents but without vacancies that are an endogenous feature of this model.

Our point of departure is the classic urban model of Alonso (1964), which is an adaptation of the model of von Thunen (1826). This model was later extended by Mills (1967); Muth (1969). In this model, commuting costs are directly proportional to the euclidean distance from the town centre. In the basic version this is the only transport cost that affects the utility of a tenant. The tenants are identical and the landlords all use the same rules of rent adjustment.

The heterogeneity of landlords is in the position of their residences.

Our agent model is aimed at reproducing the main results of the theoretical urban model while allowing to go a step further once we achieve to reproduce the benchmark theoretical situation. The focus of our analysis is on the rental market mechanism, namely the search behaviours of tenants looking for apartments and of landlords looking for tenants. We present an agent-based model of a town in which the rents charged emerge from the local interactions between potential tenants and landlords. The tenants maximise their utilities which depend on their income, rent and transport costs. The landlords adjust the rents in order to maximise their revenue. They have access to private information only. Their information on the demand for housing is the number of people who express an interest in their property when it is on the market. Increases in rent are proportional to the demand.

The mechanism of rent adjustment modelled is the essential dynamic ingredient of interest in the model. The rent adjustment mechanism remains applicable under alterations of the input demand, transport infrastructure and utility functions of agents. The explicit modelling of agents' search behaviours lead to an endogenous vacancy rate, this is in contrast to the static standard urban economic model. These search processes are the focus of Chapter 3, where a more thorough analysis, along with the relevant references to the existing search market literature, can be found. The dynamics of rent adjustment need to be efficient in order to be implemented in a simulation of a large urban system. This minimalist modular approach offers many methodological advantages. Note also that this model began on a two-dimensional grid, though the initial results presented shall be in one dimension. It shall be shown that modelling the emergence of non-trivial endogenous dynamic spatial configurations is not straightforward.

This is a stylised model of the effects of transport costs on the formation of urban rents. Transport is only one of many factors influencing residential location choice, others include schools, neighbourhood effects, job location and amenities, see Sheppard (1999) and references therein.

A brief summary of the analytical monocentric urban economic model is presented below, Section 2.2. Section 2.3 presents the agent-based model. Section 2.4 presents the results including parameter sensitivity tests. In Section 2.5 some brief extensions to heterogeneous agents are presented. Section 2.6 concludes. Results and parameter sensitivity tests for a two dimensional town are in Appendix C.

2.2 Theoretical benchmark

As discussed in the general introduction, it is important in the development of agent-based models to have a theoretical benchmark in order to test and compare

the simulation model in its simplest form. Our theoretical benchmark is an urban monocentric model of an open city. We consider that tenants consume a constant amount of housing, which is normalized to one. Everybody works in the city centre and the transport cost for an agent is proportional to her distance from the centre. The utility for a tenant living at distance d from the centre is:

$$U(d) = Y - R(d) - Td \quad (2.1)$$

where, Y is the income, $R(d)$ is the rent at distance d and T is the cost per unit distance of travelling to and from the centre.

At equilibrium, the utilities of all the inhabitants are equal. The analytical model shows that the equilibrium rents depend only on this distance d and they decline linearly with distance from the centre due to the linear form of the transport cost function. At the edge of the town, the rent $R(d_{max})$ is equal to the agricultural rent R_{agr} . The utility at any point in the town must also be greater than the outside utility U^* , that is, the utility level that tenants could enjoy in another town.

We assume here that tenants' income $Y = 1$. In order for the town to exist (Population $P > 0$), the space of possible normalisations for U^* and R_{agr} is given by $0 < U^* < Y - R_{agr}$ or $U^* + R_{agr} < 1$. For simplicity, we assume that $U^* = R_{agr} = 0$. Therefore, for a tenant living in the town centre the rent is given by $R(0) = Y - U^* = 1$. It follows that the edge of the town is at distance: $d_{max} = (Y - U^*)/T = 1/T$ from the centre.

In one dimension (and with one residence per unit distance), the theoretical population is $P = 2d_{max}$, the factor two is due to the fact that the centre is placed in the middle of the simulation domain. Similarly, the population of the two-dimensional town (with one residence per unit area) $P = \pi d_{max}^2$. This is a simplified version of the canonic monocentric city model of Alonso as the surface consumed by any tenant is constant.

2.3 Model

The model describes the interactions of two types of agents, tenants and landlords, who interact in an urban environment of homogeneous residences distributed on a 2D grid. There are no owner-occupiers in the model. In this Section the overall simulation procedure is presented and then the characteristics of both categories of agent are described.

2.3.1 Simulation procedure

At each iteration, a certain percentage L of tenants chosen randomly, leave the town. The landlords of those newly vacated houses update the rents.

Subsequently a number of potential tenants, determined exogeneously, arrive in town and search for accommodation.¹ Each new arrival chooses one residence to visit, from among those offering a positive utility as described in Section 2.3.2.

Those searching for accommodation visit their chosen residences.² The residences are attributed to the agents as follows. The first to visit an available house moves in. Those who visit a house that is already occupied cease searching, i.e. tenants visit only one residence. Landlords however can receive many visitors during the same iteration. Landlords who received visits, and hence have found a tenant, remember the number of visitors. Finally, landlords of houses on the market that remain vacant decrease the rent.

This mechanism models the flux of tenants that arrive and which residences they visit and the market frictions which lead to vacancies at the steady-state. The rents are adjusted accordingly. The numbers and types of tenants who arrive and depart are exogenous parameters of the model.

2.3.2 Tenants

The tenants aim to maximise their utility. The utility U that a tenant derives from a residence at distance d from the centre is given by

$$U = Y - R - Td \quad (2.2)$$

Real agents face many other costs. However, as these are not modelled dynamically in the model, they are normalised to zero. All tenants work in the city centre and the transport cost for an agent is proportional to her distance from the centre. When tenant agents arrive in town they choose one residence to visit from among those offering a positive utility. They make this choice using the probabilistic ‘logit’ function given in Equation (2.3). The ‘logit’ probability (Ben-Akiva and Lerman, 1985) of choosing a particular (positive utility) residence i is:

$$P_i = \frac{e^{U_i/\theta}}{\sum_j U_j/\theta} \quad (2.3)$$

θ is the parameter that determines the sensitivity of agents to differences in the utilities of the residences available. This function means that residences offering higher utilities to agents are more likely to be visited, but that searchers do not generally choose the same option when many apartments are available. This is

¹As a fixed number of homogeneous tenants search each iteration, we can consider that either, new searchers arrive every iteration or, that those that find a residence are replaced by a new searcher in the next iteration.

²With homogeneous agents, the order in which they do this does not count. Visit order is randomised in simulations with heterogeneous agents.

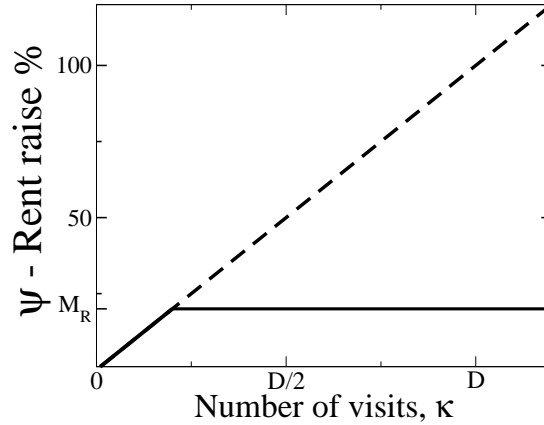


Figure 2.1: The rent increase function as a function of the number of visits κ . κ is expressed in terms of the rent increase parameter D . M_R is the maximum percentage increase. Here $M_R = 20\%$ is shown at its default value in one dimensional simulations.

a form of ‘directed search’ in the search market literature terminology. That is, searchers are more likely to visit attractive offers rather than seeing offers randomly as is the case in ‘undirected search’ models (Rogerson et al., 2005), and in the model in the subsequent chapter.

2.3.3 Landlords

Each landlord owns one residence and adjusts the rent according to the observed demand. Increases in rents are proportional to number of visitors who sought to rent the residence the last time it was on the market. Landlords reduce the asking rent of residences that remain vacant by a fixed percentage ρ each iteration.

When a tenant leaves a residence the landlord must decide what rent to ask. This decision is based on the current rent r_i and the number of potential tenants κ who showed an interest in the residence the last time it was on the market.³ It also depends on two parameters of the model: M_R the maximum percentage increase possible and D the rent increase parameter. The rent after a tenant has left is increased by a percentage Λ . Figure 2.1 shows the form of this function. Λ rises linearly with the number of visitors κ and is regulated by the parameter D . Formally

$$\Lambda = \min \left\{ M_R, \frac{\kappa}{D} \right\} \quad (2.4)$$

³Note that $\kappa \geq 1$ as at least the tenant who lived in the residence visited.

Note that when only one tenant visited the last time a residence was on the market, the rent shall increase by $(\frac{100}{D})\%$.

The rents are only increased after a tenant has left. If the residence is not taken in the first iteration, the landlords decrease their rent by a percentage ρ every iteration until a tenant is found. The formula for setting the rent of houses that are vacant after being on the market for at least one iteration is

$$r_{i+1} = r_i(1 - \rho/100) \quad (2.5)$$

This rule causes the rent of overpriced residences to decrease exponentially.

The rent increases of landlords can overshoot or underestimate the true market price for many reasons. Due to the stochastic nature of the tenants' visits, real demand may be over or under estimated. The market conditions can also evolve significantly between the moment the agents visit and the moment when the residence is once more put on the market.

Figure 2.2 shows how a landlord at a distance 100 from the town centre adjusts the rent under changing market conditions.⁴ The simulation began with the town at the standard theoretical urban economic equilibrium, i.e. no vacancies and all rents equal to tenant's willingness to pay.⁵ The unit distance transport costs were at their default value of $T = 5$ for a one-dimensional town from 0 to 300 iterations and were then increased to $T = 5.5$. This causes the maximum affordable rent for tenants at any point outside the town centre to decrease, see Equation (2.2). After 800 iterations the transport cost was reduced back to $T = 5$.

We can clearly see in Figure 2.2 that rents reduce quickly once tenants can no longer afford the rent due to the increased transport cost. The sharp reduction in rent is of almost 20% (from approximately 540 to 450). This takes place in less than 25 iterations as the rate of rent reductions per iteration ρ is 1%. However, rents are slow to increase after the transport cost reduction at 800 iterations. This is a result of landlords the relative rarity of their opportunities to raise rents, and to a lesser degree the result of having information on the demand for only one residence. They must have a number of tenants arrive and leave before they can fully adjust to the changed conditions. As all landlords follow the same behaviours the market takes a long time to reach its equilibrium. This asymmetry will be further explored in Section 2.4.1. We note also that after convergence to a steady-state the rent increases are mostly overshoots.

⁴The parameters were the default values for a one dimensional town given in Tables 2.1 & 2.2, except for the percentage of inhabitants who leave $L = 3\%$ and the number percentage reduction in vacant rents $\rho = 1\%$.

⁵That is, landlords charge rents that are equal to the maximum that tenants can afford.

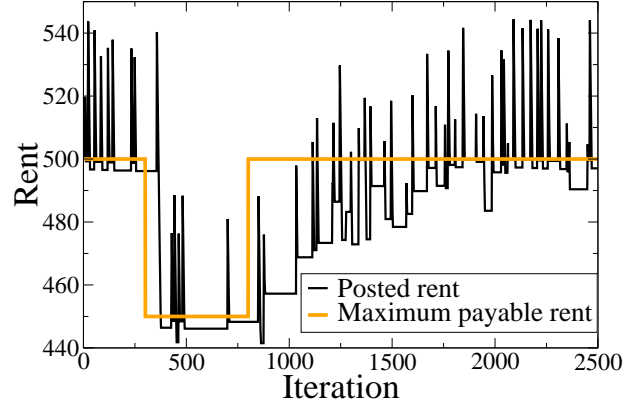


Figure 2.2: The rents asked by a landlord at distance 100 from the town centre. The transport cost was increased at 300 iterations from 5 to 5.5. The maximum affordable rent is the highest rent a tenant with income $Y = 1000$ can pay at this distance from the centre.

Table 2.1: Initialisation parameters of the model

| Symbol | Meaning | Default Value -1D | Default Value -2D |
|--------|---------------------------|-------------------|-------------------|
| Y | Income | 1000 | 1000 |
| R_I | Initial Rent | 500 | 300 |
| N_I | No of Initial Inhabitants | 400 | 800 |
| S | Size of simulation domain | 500 | 1235 (35*35) |

2.3.4 Parameters

In Tables 2.1 & 2.2 the parameters of the model and their default values in one and two dimensions are given. Tables 2.1 gives the initialisation parameters.

The dynamic parameters, shown in Table 2.2, are those that influence the evolution of the model. In the simulation results presented in the subsequent Sections the default values of the parameters were used unless otherwise specified.

2.4 Results

In this Section, results including parameter sensitivity tests are presented for a one-dimensional town. Results and parameter sensitivity tests for a two-dimensional town are presented in Appendix C.

Table 2.2: Dynamic parameters of the model

| Symbol | Meaning | Default Value-1D | Default Value-2D |
|-----------|------------------------------|------------------|------------------|
| D | Parameter of rent increases | 20 | 20 |
| M_R | Maximum rent increase | 20 | 100 |
| Λ | New arrivals per iteration | 60 | 120 |
| L | % who leave each iteration | 2 | 2 |
| ρ | % reduction of vacant rents | 1.5 | 1.5 |
| θ | Sensitivity to Utilities | 7 | 7 |
| T | Unit distance transport cost | 5 | 60 |

2.4.1 One-dimensional town

In the one dimensional town the exogenous centre is in the middle of the simulation domain.⁶

In Figures 2.3 & 2.4 we see that the model converges to an equilibrium quickly when $R_I = 900$, which is above the average theoretical rent of 500.⁷ The convergence is much slower when $R_I = 450$. This asymmetry in convergence speed from above and below the theoretical rent can be understood by examining landlords' rent adjustments shown in Figure 2.2. There we see that landlords who charge excessive rents quickly reduce them to the going market rate while increases occur over a much longer period of time. This is a direct consequence of landlords basing rent increase decisions on limited information and the limitation of the largest allowed increase. When a tenant leaves, they ignore the current market situation and use previously gathered information on the attractiveness of their residence. A model in which landlords use more up-to-date public information to decide what rent to offer is presented in Chapter 3.

Figure 2.3-*Right* shows that once the average utility is close to zero no further evolution occurs. Figure 2.4-*Left* shows the vacancy rate corresponding to the simulations shown in Figure 2.3. The vacancy rate is the percentage of residences that can offer positive utilities that are unoccupied. The evolutions of the average rent paid and the average transport cost to their steady-state values of 500 for a simulation initialised with $R_I = 450$ can be seen in Figure 2.4-*Right*. This is in agreement with theory, see Arnott and Stiglitz (1981), for which in a one dimensional town the aggregate land rents and transport costs are equal.⁸

⁶Similar results could also be obtained by considering the centre to be at one extreme of the 'town'.

⁷Those in the centre pay 1000 and those on the periphery pay nothing and hence in a one-dimensional town the average rent is 500 .

⁸Note that our simulations are also in agreement with the theory in two dimensions where

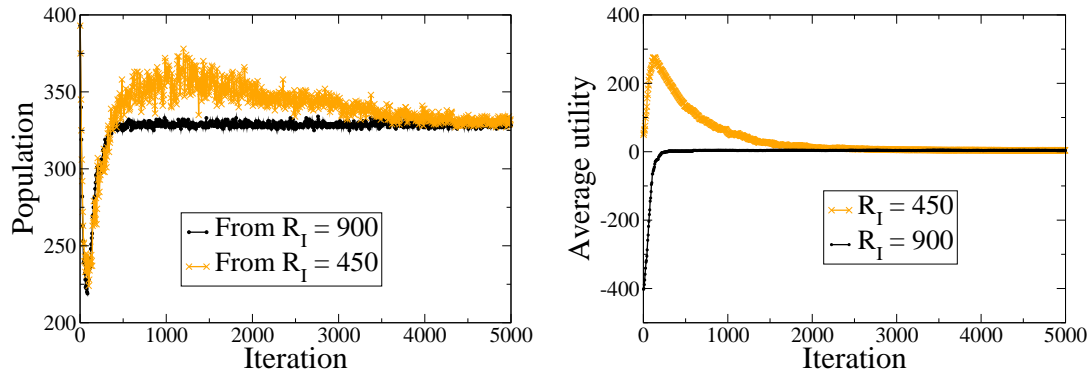


Figure 2.3: *Left*: Graph of the population in a one-dimensional town with the number of initial inhabitants $N = 400$ for initial rents of $R_I = 450$ and $R_I = 900$. *Right*: The average utility of tenants for the same simulations as *Left*.

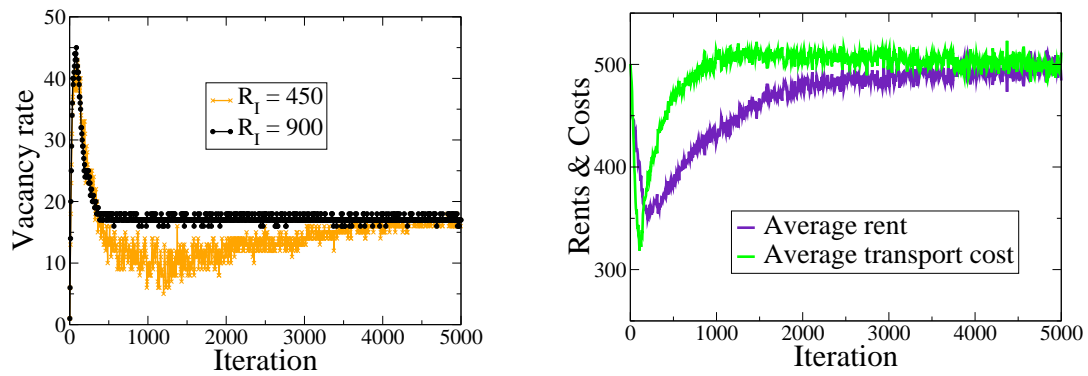


Figure 2.4: *Left*: Graph of the vacancy rate in a one-dimensional town with the number of initial inhabitants $N = 400$ for initial rents of $R_I = 450$ and $R_I = 900$. *Right*: The evolution of the average rent paid and the average transport cost with $R_I = 450$.

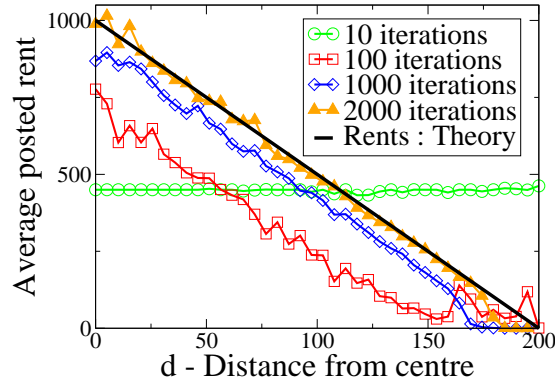


Figure 2.5: The form of the asking rents at various points in the revolution to their steady-state configuration for an initialisation with $R_I = 450$.

The evolution from the flat initial rent values to that of the steady-state configuration is shown in Figure 2.5 for an initialisation with $R_I = 450$. It can be seen that the *slope* of the rents adjusts relatively quickly (after just 100 iterations) towards that of the theory. Therefore residences offer similar utilities to tenants. Subsequently rents rise slowly to their theoretical values.

For the default parameters it is clear that the rents converge to those of the theoretical equilibrium. This is the case as long as the demand is sufficient, as will be shown in Section 2.4.2. However, the steady-state configuration differs from the theoretical equilibrium due to the presence of vacancies. Without these vacancies, rents would rise continually.

Once the simulation has converged, the reductions in rents due to these vacancies equilibrate with the increases due to the arrivals of new searchers. The steady-state configuration's vacancy rate is the rate at which the reductions in rent due to vacancies are balanced by the increases due to arrivals. This is explored further in Section 2.4.2.

2.4.2 Sensitivity Testing-1D

The sensitivity of the steady-state configuration to variations in all of the parameters in Table 2.2 in one dimension are presented in this Section. Every parameter other than that being varied is held at its default value, given in Table 2.2.

All the results presented are for simulations over 2000 iterations that began with $R_I = 900$. The main convergence criteria is the average utility of tenants. In Figure 2.5 we saw that rents are very close to the theoretical equilibrium values once the default simulation has converged. Figure 2.3-*Right* shows that for

the aggregate land rents are half the aggregate transport costs.

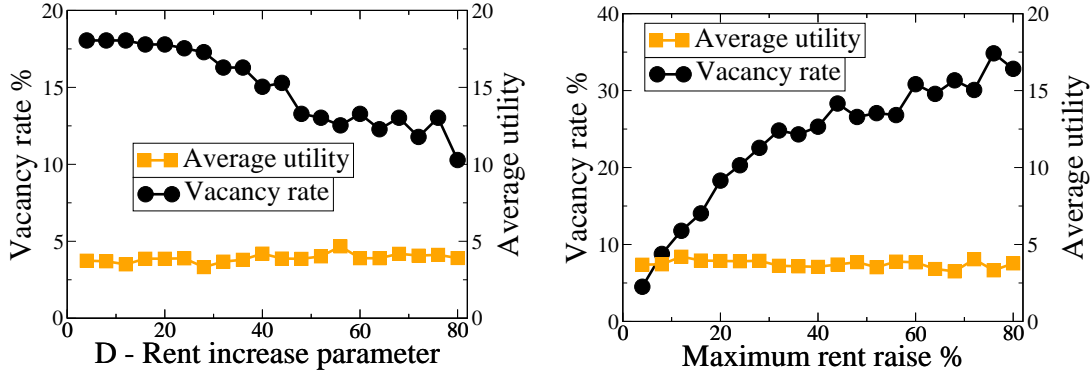


Figure 2.6: *Left*: The vacancy rate and average utility for variations in the rent increase parameter D , see Section 2.3.3. *Right*: The vacancy rate and average utility for variations in the maximum rent raise allowed M_R . Both Figures present results for a one-dimensional town.

a simulation that began with this high level of rents, with the default parameter values, the model converges after about 200 iterations. Parameter values for simulations in which, after 2000 iterations, average tenant utilities are significantly above zero, reflect a low demand regime in which both the vacancy rate and rents differ from the theoretical benchmark.

Figure 2.6-*Left* shows that as the rent increase parameter D rises, the vacancy rate falls. This is because rent increases are smaller and hence fewer excessive rents are posted. At low D rent increases are large and hence many rents are above affordable rates. The rents that are accepted are close to the equilibrium as can be seen by the low average utility.

In Figure 2.6-*Right* the effect of varying the maximum allowed rent raise M_R can be seen. The vacancy rate increases as a concave function of M_R . Higher values of M_R mean that there are a greater number of excessive rents demanded. The growth in the vacancy rate approaches an asymptotic value because for higher M_R less rent increases exceed its value.

Figure 2.7-*Left* shows that at very low demand, that is small Λ , rents are relatively low and the vacancy rate is high. That is because the landlords adjust to the low demand by reducing rents. The population then falls until the same number of agents arrive and leave. Here the default departure rate is $L = 2\%$, which means that 8 agents leave each iteration if the town is full. Note that the number of arrivals is a constant exogenous parameter while the number of departures depends on the population. Hence if the arrival rate is below this value the market adjusts through both increased vacancies and reduced rents. This is an interesting regime of the model as supply and demand determine both rents and vacancy rates, while at high demand the market adjusts through the vacancy rate only. In this regime the populated area is smaller as residences further from the centre are spurned by searchers.

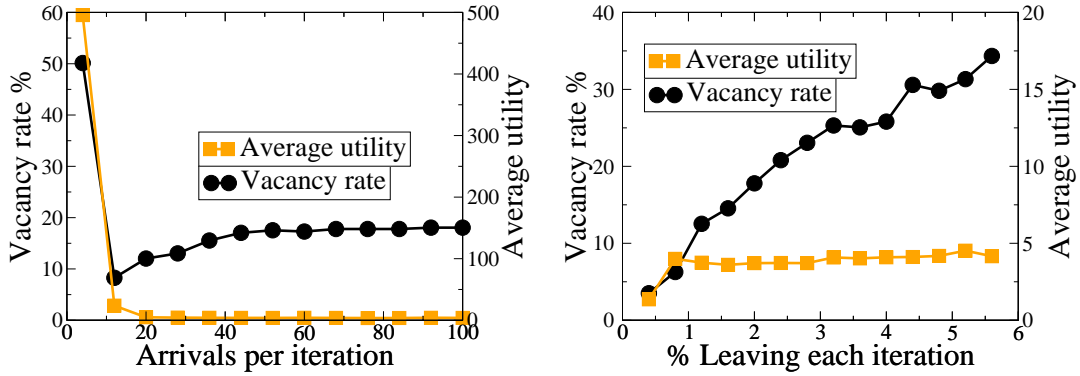


Figure 2.7: *Left*: The vacancy rate and average utility for variations in the number of searchers Λ arriving each iteration. *Right*: The vacancy rate and average utility for variations in the percentage of departures L . Both figures present results for a one-dimensional town.

Figure 2.7-*Left* shows that as demand rises rents approach their maximum affordable values (average utility is close to 0). Once the demand is sufficient to force rents close to the maximum value it is surprising that after this point, as the demand rises, so does the vacancy rate. This is due to the large demand causing very high rent increases. Landlords then take many iterations to bring prices back to affordable levels. This effect saturates due to the maximum rent raise parameter M_R which eventually limits the effect of additional demand.

In Figure 2.7-*Right* we see that increases in the percentage of departing tenants increases the vacancy rate, as is to be expected. At low departure rates tenants stay longer in residences. At very low departure rates the rents are closer to the maximum affordable rents, as seen by the lower average utility. This is due to the very low vacancy rate which increases effective demand for those residences on the market.

Figure 2.8-*Left* shows that there are two regimes for the effects of varying the reductions in rent ρ . At low levels increasing the rate reductions decreases the vacancy rate, as rents return more quickly to affordable values after they have been increased. However, the vacancy rate begins to rise once $\rho > 4$. The large rent reductions cause the rents charged in the town to reduce, as can be seen from the rising average utilities above this value of ρ . When the utilities available to tenants are large, they no longer search for residences in the town periphery. This is because these residences could not offer competitive utilities to tenants, even with zero rents. This causes the vacancy rate to rise.

In Figure 2.8-*Right* we see that increases in the sensitivity to differences in utility do not change the steady-state configuration above a value of 2. Below this value the vacancy rate rises as most tenants choose to visit the same apartments. An elevated vacancy rate means many landlords reducing their asking rents, and

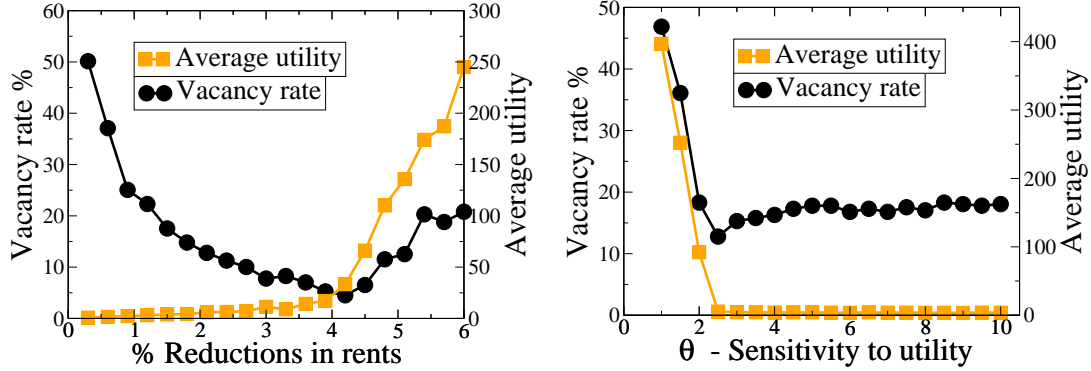


Figure 2.8: *Left*: The vacancy rate and average utility for variations in the percentage reductions of the rents of vacant residences ρ . *Right*: The vacancy rate and average utility for variations in searchers' sensitivity to differences in utility, θ . Both figures present results for a one-dimensional town.

so tenant utilities are higher at low θ .

Vacancy rates rise slightly for $\theta > 2$. At higher θ tenants choices of residence to visit are more random. This probably leads to less individual rent raises being limited by M_R and so the overall level of rent increases is higher. This means that rents take longer to return to affordable values, and the vacancy rate rises as a consequence for higher θ . This effect is more visible in the two-dimensional case, see Figure C.4-*Right* in Appendix C.⁹

It should be noted that these sensitivity tests were performed in the high demand limit, $\Lambda = 60$, this value was chosen to obtain a speedier convergence when market moves upwards. The role of theta in the low demand regime, $\Lambda < 15$ may be different. At low demand, high theta may increase the size of the population as residences with lower utilities further from the centre are accepted.

The size of the town increases for lower transport costs. Therefore sensitivity tests for the unit distance transport cost T are performed, with an arrival rate that is $\hat{\Lambda} = 15\%$ of the town size for each transport cost.¹⁰ Figure 2.9-*Left* shows that that the vacancy rate and average utility vary only slightly.

⁹The rent increase function, Equation (2.4), is linear in the number of visitors κ . This means that, at the steady-state, the overall increase in rents depends linearly on the number of searching tenant agents, unless M_R limits these rent raises. When θ is higher this happens less often and so rent increases are on average larger. This means that more residences are over-priced at any given moment and so the vacancy rate is greater.

¹⁰The default transport cost T is 5 and tenants' income Y is 1000. Hence furthest distance from the centre at which residences can offer positive utilities to tenants is 200, as $Y/T = 1000/5 = 200$. The maximum population of the town is therefore 400 as the town can extend in 2 directions from the centre. The default number of arrivals is $\Lambda = 60$ which is 15% of 400.

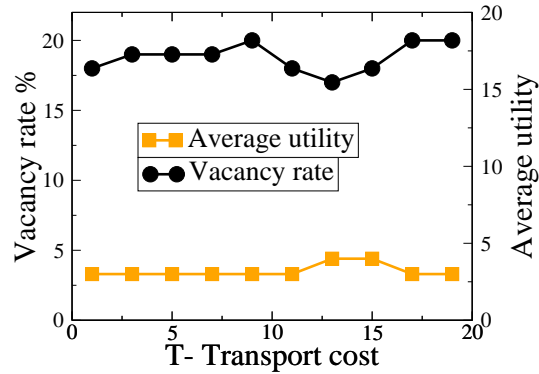


Figure 2.9: The vacancy rate and average utility for variations in the transport cost. This figures presents results for a one-dimensional town.

2.4.3 Comparison to theoretical benchmark

In order to emphasize the main differences between this dynamic simulation model and static analytical models, we compare the simplified version of the canonical monocentric city model to an implementation of this model in two dimensions.

Overall, the town at the steady-state is similar to the theoretical equilibrium. It is of the same size and the slope of the rents differs only in stochastic variations. The level of the rents are almost the exactly same.

The major difference to the standard *static* model is the presence of vacancies at the steady-state of this *dynamic* simulation, see Figure 2.10¹¹. Why is the population of the dynamic equilibrium less than that of the static equilibrium? The vacancy rate is due on the one hand to search frictions as tenants can visit only one residence per iteration and on the other hand to landlords' misreadings of the market which cause them to excessively increase the post rents which then take time to fall to affordable levels. The need for a certain number of empty residences reflects a real phenomenon of urban real estate rental housing markets, Rosen and Smith (1983). Indeed, this is a well known phenomenon more generally in markets involving costly search, see references the Sections 3.1 and 3.2.2 in the next Chapter.

2.5 Elements of heterogeneity

We now present two elementary forms of heterogeneity added to the basic two-dimensional model. Firstly, two types of tenants with different incomes and

¹¹It has been verified that discretisation effects are not responsible for the offset between the populations of the simulation and the theory.

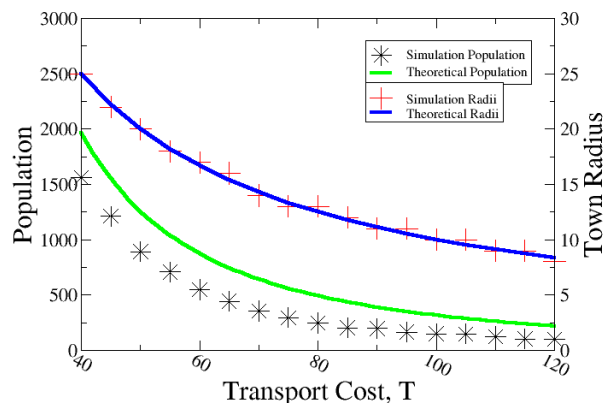


Figure 2.10: Graph of the population and town size for different values in the linear transport cost for both theory and simulation. In these simulations the town size was 51×51 .

transport costs. Secondly, a stylised high-speed transport link is used to demonstrate that the rent adjustment mechanism will translate any change in demand into the corresponding rents.

2.5.1 Heterogeneous tenants

We simulate a two-dimensional town in which tenants' transport costs are proportional to the linear distance from the centre. One group of tenants have an income, $Y=1000$ and transport cost $T=40$, for the second group $Y=2000$ and $T=160$. The searchers visit apartments in a random order each iteration. The theoretical 'bid rent theory' rents, see Fujita (1989), are shown in Figure 2.11, along with the simulated rents at the dynamic user equilibrium.

A visual representation of the town is shown in Figure 2.12, where we see the segregation of the two populations, with the wealthier agents living in the centre to avoid their high transport costs and the poorer agents living in the suburbs.

2.5.2 Heterogeneous transport infrastructure

A stylised 'high speed route' is added to the town transport infrastructure which modifies the simple proportional relation between linear distance and transport cost. We simulate the effect of a high-speed route from east to west through the town centre. Inside the shaded areas shown in Figure 2.13 the transport costs per unit distance to the centre were cheaper. Anywhere outside the shaded cone (and its extension) the transport costs are 1.3 times higher.

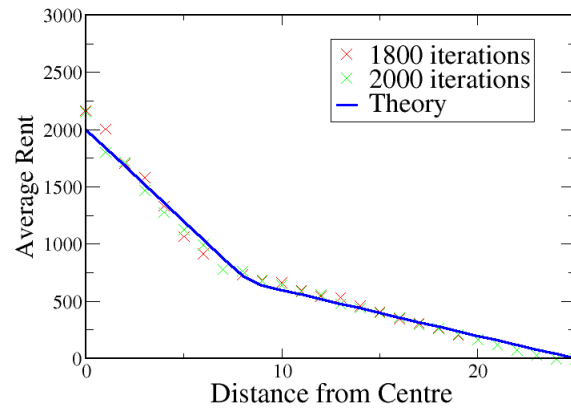


Figure 2.11: The theoretical rent values are shown in blue. We see clearly the point at which both populations of tenants are prepared to pay the same rent. The two populations are segregated either side of this point.

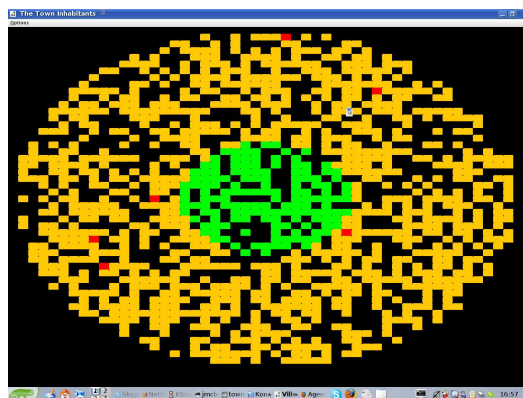


Figure 2.12: Positions of the two types of tenant agent in the town. The rich agents are green, the poor agents are orange while those who have just arrived are red.

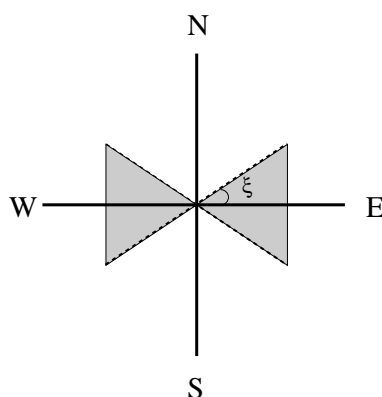


Figure 2.13: The shaded areas have lower unit transport cost to the centre. (The angle $\xi = \tan^{-1}(\frac{2}{3})$)

Figure 2.14-*Left* shows the initial utilities when all landlords charge the same rent. The high-speed route's effect on the transport costs can be seen clearly. This form of the transport cost was chosen to demonstrate visually the ability of the rent adjustment mechanism to adapt to any form of demand, as can be seen from the steady state rent configuration, see Figure 2.14-*Right*.

2.6 Conclusions

A model of the search behaviour of tenants and landlords has been presented. Tenants are more likely to visit high utility residences and landlords set the asking rent depending on privately observed demand. The major result of this model is the presence of an endogenous vacancy rate which is due to the explicit modelling of search behaviour. This discrepancy from the standard neo-classical model of economics is attributed in the literature on real estate and labour markets to search frictions (Wheaton, 1990; Rogerson et al., 2005). Here we have begun to explicitly model these search frictions, heretofore frequently subsumed into exogenous 'matching functions', (Rogerson et al., 2005). Agent-based models have a real methodological advantage in exploring these frictions.

The major weakness of this model is the asymmetric timescales of adjustments in rent, when the market goes up and down. This is the result of landlords basing rent increases on limited information on the market from previous periods and the constrained rent increases. A much more realistic hypothesis is to assume that landlords have information on the current market state each time they attempt to rent their residence, and optimise their asking rent based on this information. One factor inhibiting the use of such global information in this spatial set-up is

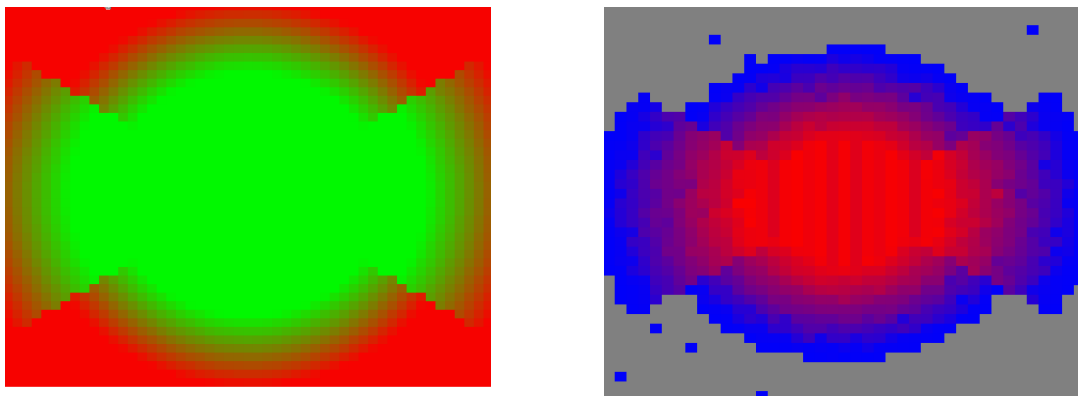


Figure 2.14: *Left*: The initial utilities when all rents are equal to 300. Green areas have higher than average utilities and red areas have lower than average utilities. The form is totally determined by the transport costs. *Right*: The equilibrium rents. Red areas represent high rents and blue areas have low rents, very low rents are grey.

the fact that landlords are heterogeneous by their position in the town. Therefore to exploit global information, they would need to be able to interpret the rents asked at different positions in the town into rents that they themselves could reasonably propose. Such mechanisms can be imagined¹². However, the solution chosen in the next chapter, for simplicity, was to concentrate on the market mechanism in an *aspatial* urban rental market.

The dynamics of the market adjustments need to be rendered more realistic before they can be integrated as a module within an integrated transport-land use modelling framework such as Waddell (2002) or an activities-based transport model such as MATSim (2009). These models hope to represent many aspects of the urban economy: mode choice, route choice, activity planning, demographic change, real-estate markets and also rental housing markets. In order to function in an integrated simulation, each module needs to be able to perform its function efficiently and robustly with a simple communication protocol with other modules. The input for this model is the list of potential tenants and their utility functions (Demand) and the list of available housing with their prices (Supply). The output is the modified list of available housing and the new rents. The values of the rent adjustment parameters for each landlord must be initialised at the beginning and could be altered during the simulation.

It is worth pointing out that the rent-adjustment mechanism can be applied to any rental market once we make the assumption that sellers have no information outside of their own personal experience. It is also possible that the mechan-

¹²Landlords need only have an idea of the income of tenants, of their transport costs and of the utilities available to tenants from other apartments on the market.

ism described here could be combined with another mechanism that takes into account some global information. The model of rent adjustment can easily be transferred to other rental markets where the owners lack perfect information on the market.

The mechanism of dynamic rent adjustment with utility-maximising tenants and ill-informed landlords is computationally efficient. The behaviour depends upon a few key dynamic parameters whose overall effect has been examined. It is a promising basis from which to explore more realistic models of housing markets and indeed search markets generally.

The conclusion is drawn that to best exploit the insights gleaned from this exploratory model, the emphasis should be put on modelling the market structure and the behaviours of agents. Attempting to model the emergence of spatial phenomena at the same time as exploring the intricacies of the market mechanism is not the most efficient approach at this stage. Indeed a well understood market mechanism is probably essential to beginning a serious agent-based study of the emergence of non-trivial spatial phenomena such as secondary urban centres (Anas et al., 1998) segregation by income or race (Fujita, 1989; Schelling, 1978) or the localisation of amenities (Brueckner et al., 1999).

Housing Search Market

3.1 Introduction

In the urban rental housing market two categories of agents meet. The first category consists of landlords who post rents. The second category consists of tenants who choose among offers. These markets are imperfect as can be seen by the existence of vacancies, price dispersion and nonzero search times for all agents. One source of imperfection is that both categories of agents are imperfectly informed about the characteristics of the market, and acquiring information is costly. Take for instance the tenants. Visiting more apartments gives them more information but is costly and so tenants may accept an offer quickly to avoid further search costs. Moreover the market state (rents, individual apartments on the market etc) changes over time, making previously acquired information less useful or even misleading. The landlords in turn observe times-on-the-market required to find a tenant for a given rent. For the reasons discussed above this observation does not give them perfect information about the preferences of the tenants.

These imperfections in comparison to the theoretical perfectly competitive market are referred to as frictions and more particularly as search frictions in the labour and housing market literatures. The impact of frictions has been extensively studied in search-matching models of the labour market. A survey of search theoretic models of the labour market by Rogerson et al. (2005) presents the various approaches to modelling the main search related frictions: how agents meet and how wages are determined.

The first major contribution to the housing market literature based on this pre-existing labour market literature is that of Wheaton (1990). He created a model of the owner-occupier market in which buyers are also sellers and the cost of search effort and its efficiency are defined by an exogenous matching function. This model was later extended to a spatial rental market by Desgranges and Wasmer (2000) who studied notably a tax on vacancies. The ‘thin’ nature of the housing rental market due to the heterogeneity of housing and tenants’ idiosyncratic tastes has been used to explain vacancies by Arnott (1989). The existence of vacancies is shown to have a social function as it expands the choice set of tenants. Further theoretical models of the housing market are discussed

in Section 3.2.1. All of these *static* analytical models contain strong simplifying assumptions which ignore important aspects of the housing market. Among these assumptions are the ‘law of one price’, the absence of heterogeneities and perfect information. Certain of these assumptions are relaxed in the models mentioned above but never all together in a general equilibrium model.

We propose a multi-agent model as a basis for relaxing many of the assumptions of analytical models in order to obtain a more realistic dynamic model of urban housing markets. Our aim is to build a model close enough to the existing search literature for its results to be comparable, while allowing the extension of these existing models in different directions.

We develop a simulation model of a closed urban housing market focusing on the role of information. Landlords deduce times-on-the-market (TOM) corresponding to posted rents from market signals and optimise their expected profit on this basis. Tenants with heterogeneous incomes have idiosyncratic preferences for housing units and engage in undirected search, selecting their reservation utility using imperfect information on the distribution of posted rents. Our agent model departs from theoretical models in that tenant agents do not know perfectly the rent offer distribution. More importantly, landlord agents do not know the acceptance probability of all tenants searching on the market. The discrepancy in the results which ensue from these differences is highlighted.

The evolution of urban rents is influenced by many factors including interest rates, conditions of credit, land supply and zoning, the economic environment, incomes, demographics etc. All of these factors cannot be easily integrated into a single model. The question for the modeller is which set of hypotheses can most faithfully capture this complexity while retaining a simplicity of conception that allows pertinent insights to be obtained from the model.¹ Here, we shall attempt to determine the generic effects that result from the fundamental market structure of heterogeneous agents searching in an uncertain market in order to maximise their utilities.

The sensitivity of the steady-state configuration to variations in model parameters are tested in Section 3.4 and Appendix D. The major results concern the different influences of the level of information on both sides of the market. Landlords are penalised when they are better informed, as when they are less well-informed underestimations of the TOM move the market to higher rents. However, when landlords are heterogeneous in information, the better informed are better off. The effects of alterations in the discount rate and tenant agents’ idiosyncratic preferences are also explored, as is the introduction of a tax on vacancies. When the vacancy tax is altered and the subsequent dynamic evolution of the market studied, overshoots in many market measures are seen. The market adjusts in a non-trivial way to the shock of the sudden introduction or

¹A notable and very detailed multi-agent model of the 1970’s French labour market was constructed by Ballot (2002).

abolition of the tax. This suggests that interventions in this market should be made gradually when possible.

Section 3.2 presents related theoretical, empirical and simulation literature. In Section 3.3 the model is presented. The comparative static results are presented in Section 3.4 and a dynamic model with a vacancy tax in Section 3.5. A comparison of these results with the literature is discussed in Section 3.6. Section 3.7 concludes.

3.2 Related literature

3.2.1 Theoretical literature

Read (1993, 1997) developed two models of the rental market with rent dispersion and vacancies. Read (1993) is a partial equilibrium model with rent dispersion and vacancies where searchers with heterogeneous information engage in directed search. Read (1997) is a partial equilibrium model where tenants, who have heterogeneous incomes, engage in stochastic search. The imperfect information of tenants leads to equilibrium rent dispersion. Yavas (2001) showed that rent dispersion can occur due simply to uncertainty in demand when rents are adjusted competitively and search is costless for tenants.

Two parallel paths have been followed in search-matching models of the labour market, which accounts for the large majority of the search literature in economics. Models aimed at analysing varying unemployment rates have focused on the matching function and search behaviours on the part of searching workers (Pissarides, 1990; Mortensen and Pissarides, 1994). Models aiming to explain the observed dispersion of wages have highlighted the behaviours of wage posting firms that benefit from the existence of frictions to offer wages below the Walrasian wage (Burdett and Mortensen, 1989, 1998; Mortensen, 1990). Mortensen (2000) builds a model merging the two approaches. He underlines the contribution of doing so: by allowing for a general equilibrium in which the wage distribution and the unemployment rate interact, this model allows the study of the welfare effect of a minimum wage or of an unemployment subsidy policy.

3.2.2 Empirical literature

We review here the empirical work on the main phenomena related to frictions in the housing market : agent's information, vacancies, time on the market and price setting, and price dispersion.

The role of information in the evolution of housing markets has received much attention. Fisher et al. (2003) studied the correlations in prices and liquidity changes over the housing market cycle. It is well known that when prices rise liquidity is high and when prices fall liquidity is low. They attributed this to

the differing rates at which agents update their beliefs. For example, when liquidity is low, there are few market signals for sellers to follow, causing them to be slow to update their beliefs. Clayton et al. (2008) examined a number of possible explanations of the correlations in price and liquidity changes, and found evidence supporting sellers slow rate for updating their beliefs. In our model, the rate at which information arrives affects the quality of landlord's estimation of the market state.

We now turn to the role of vacancies. Blanck and Winnock (1953) made a major contribution to the understanding of vacancies' role in determining urban housing price dynamics. They showed that price dynamics appear to be led by changes in the vacancy rate. A more thorough analysis was performed by Rosen and Smith (1983) who provided evidence, in a cross city analysis, of the existence of natural vacancy rates that are crucial in determining the strong correlations between fluctuations in the vacancy rate and the evolution of rents. Numerous authors report similar results including Gabriel and Nothaft (1988) in the rental housing market, Shilling et al. (1987); Grenadier (1995) in the office rental market and Hwang and Quigley (2006) in the purchasing housing market.

The price setting behaviour of imperfectly informed agents in search markets - who trade-off higher prices with longer selling times - is an important guide in model building. This has been closely studied, primarily in the residential purchasing market. Yavas and Yang (1995); Knight (2002); Anglin et al. (2003) have looked at the role of posted price on the final price obtained and the time on the market. Merlo and Ortalo-Magne (2004) have also looked at posted price changes and found that when reductions in asking price occur, they are generally substantial rather than incremental.

Price dispersion is a near universal aspect of search markets. Clear evidence has been presented by Leung et al. (2006), for the Hong Kong purchasing market. The duration of residence is another important element in any housing model. Empirical data in various rental housing markets in the USA has been presented by Deng et al. (2003) and across tenure types by de Una-Alvarez et al. (2008) for the Spanish market.

3.2.3 Simulation literature

Agent-based modelling is the analysis of complex systems through the explicit modelling of the individual behaviours of agents. The agents themselves, their behaviours and their possible interactions are defined initially and the simulation evolves independently in rounds of interaction. The results of the simulation are the sequence of decisions taken by agents and their outcome can be analysed using standard empirical methods. Aspects of the model specification can be altered and the corresponding changes in the outcome analysed. In this way, according to Bradburd et al. (2006) "...agent-based modeling permits formation

of testable hypotheses about the likely impacts of comparable changes in actual markets or systems”.

Simulation models have considerable advantages over analytical approaches when modelling dynamic processes and heterogeneities among agents. The difficulty of studying out-of-equilibrium dynamics in current analytical search models with price dispersion has been underlined by Postel-Vinay and Robin (2006). Bradburd et al. (2005, 2006) use agent-based simulations to relax the assumption of a single price with random matching and Nash bargaining in two rental housing models. They do not model search behaviour in their static models. They examine the distributional effects of rent controls (Bradburd et al., 2006) and ‘access discrimination’ (Bradburd et al., 2005), modelled as a reduced matching probability. In contrast, our dynamic simulation explicitly models search with rent posting, which is more realistic in the rental housing market.

3.3 Model

We present our rental housing search model in which *homogeneous* landlords post rents and make take-it-or-leave-it offers to the tenants, who are *heterogeneous* in income. These landlords face a trade-off between setting a higher rent and finding a tenant more quickly. Their optimising behavior is based on their knowledge of the market state, both in terms of rent offers and corresponding times-on-the-market. They withdraw from the market if their expected benefit from participation is negative.

Tenants are supposed to observe a sample of the offer distribution and to visit one randomly chosen residence each iteration. They accept offers based on an optimising behaviour that trade-offs a quicker match, and therefore reduced search costs, against a lower rent. Tenants must decide their reservation utility U_{res} , that is the minimum utility they are willing to accept from a residence. This reservation utility is chosen to determine whether a residence they have viewed is better in expectation than the outside opportunity, which is to continue searching with the associated costs.

We see in the top part of Figure 3.1, the timing of the tenant agents’ reservation utility decision and its consequences. At time zero tenant agents decide their reservation utility U_{res} , which determines their search time $T(U_{res})$ in expectation. While searching they experience a (negative) utility flow U_s^T . U_{res} also determines, in expectation, their utility flow once housed U_h^T . The expected duration of residence of tenants X is an exogenous constant, and hence $T(U_{res}) + X$ is the total time over which tenant agents optimise their expected utility.

The bottom part of Figure 3.1 shows the timing of the landlords rent posting decision and its consequences. Landlords choose their posted rent R at time zero, which determines their expected time-on-the-market $T(R)$ and clearly their

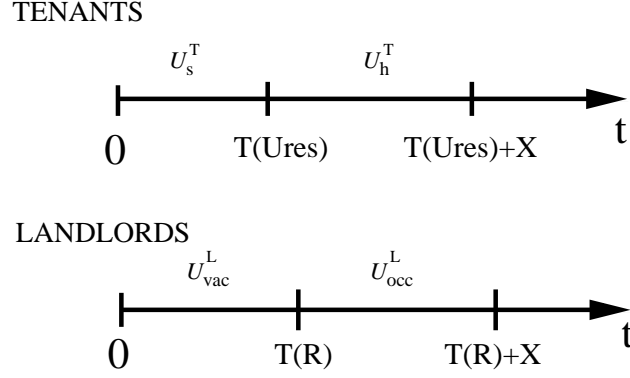


Figure 3.1: Tenants' and landlords' timing of decisions and their utilities. When landlords revise the rent of vacant residences the procedure is repeated.

utility flow once their residence is occupied. Landlords obtain the utility flow U_{vac}^L while their residence is vacant, and the utility flow U_{occ}^L once their residence is occupied. Note that if landlords review the posted rent of a vacant apartment, the moment of decision is taken once more to be time zero.

In the following, we outline more specifically the proposed agent behaviours, the simulation procedure as well as the dimensions of heterogeneity and the parameters whose influence on simulation outcomes shall be explored.

3.3.1 Tenants

We now describe in detail how tenant agents trade-off between, on one hand, minimising the cost of searching and on the other hand, maximising the eventual utility obtained from housing. Tenants decide their reservation utility U_{res} and obtain U_h from the market, after experiencing a negative utility U_s^T while searching.

We assume that each tenant has two utility functions. A housed utility function one that is defined below when housed and another utility function while searching, see Equation (3.2). Once housed each tenant has a separable utility function whose housing part is given by

$$U_h^T = Y - R(U_{res}) + \eta \geq 0 \quad (3.1)$$

where U_h^T is the instantaneous utility flow, Y is housing budget, R is the rent paid (which depends on U_{res}) and η is the idiosyncratic preference of an agent for an apartment, discovered by the agent once the apartment is visited. $\eta \sim N(0, \sigma)$, where σ is the variance of the normally distributed idiosyncratic preferences. σ is expressed as a percentage of the housing budget, as shown in Table 3.1 in Section 3.3.4.

While searching, the agent's instantaneous utility flow from housing is given by

$$U_s^T = Y - C_T < 0 \quad (3.2)$$

where C_T is the monetarised cost of searching, including both actual costs (temporary accommodation costs, transport costs, lost earnings and estate agency fees) and non-monetary inconveniences.² It is homogeneous across tenant agents, in order to simplify the model.

Tenant agents have heterogeneous incomes which translate into heterogeneous housing budgets for the purpose of this model. In the closed town unhoused tenant agents search every iteration. A searching agent sees one randomly chosen apartment from the distribution of offers, which is referred to as undirected search in the search literature. Searching agents have knowledge of a percentage S_T , of the full distribution of offers. Upon visiting an apartment, agents have to decide whether to accept it or keep searching. This will depend upon their idiosyncratic preference for the apartment, the posted rent, their housing budget and the cost of search.

U_{res} is optimised to yield the maximum utility per unit time over the expected period of search and residence. Tenant agents idiosyncratic preferences play no role in this decision in this version of the model.³ The expected benefit per unit time for a given reservation utility is given by

$$B_T(U_{res}) = \frac{U_s^T}{X + T(U_{res})} \int_0^{T(U_{res})} \exp(rt) dt + \frac{E[U_h^T]}{X + T(U_{res})} \int_{T(U_{res})}^{T(U_{res})+X} \exp(rt) dt \quad (3.3)$$

where U_s^T is the utility flow experienced while searching, X is the expected duration of residence, $T(U_{res})$ is the expected search time, r is the discount rate and $E[U_h^T]$ is the expected utility flow per iteration once housed if the chosen reservation utility is U_{res} , see Equation (3.5). Equation (3.3) is rewritten:

²Note that this negative utility can be interpreted as causing either reduced consumption of the composite good (all non-housing expenses) to cover temporary accommodation costs or that U_s^T represents a non-monetary disutility, or a combination of both.

³However the existence of idiosyncratic preferences should push reservation utility higher, as it becomes more beneficial to wait for an apartment with a particularly high idiosyncratic value, when the variance of preferences σ is higher. This effect is present even when preferences for apartments are on average zero.

$$\begin{aligned}
B_T(U_{res}) &= \frac{U_s^T}{X + T(U_{res})} [1 - \exp(-rT(U_{res}))] + \\
&\quad \frac{E[U_h^T]}{X + T(U_{res})} [\exp(-rT(U_{res})) * (1 - \exp(-rX))]. \quad (3.4)
\end{aligned}$$

The first line in Equations (3.3) & (3.4) are the total expected discounted search cost divided by the full expected duration of search and residency. The second line is the discounted total expected utility flow during residency divided by the full expected duration of search and residency. Equation (3.3) is similar to that used by Igarashi (1991) for the expected discounted housing costs of a searcher. This equation differs from that of Igarashi, as here we take the expected benefit per unit time. Also we do not explicitly include in the agents' optimisation the discounted expected housing utility upon reentering the market after the tenant eventually leaves the residence.

The expected probability of accepting the residence seen in any given iteration is simply the number of residences the agent is prepared to accept divided by the total number of residences, both quantities being taken from the residences that the agent sees. As this is a Poisson process, the expected search time $T(U_{res})$ in iterations is equal to the inverse of the probability of accepting a residence at each iteration. The expected utility flow per iteration once housed $E[U_h^T]$, is given by the average utility of residences which are expected to yield utilities larger than U_{res} :

$$E[U_h^T] = \frac{\sum U_h^T * \phi(U(U_{res}))}{\sum \phi(U(U_{res}))}, \quad (3.5)$$

where $\phi(U(a)) = 1$ if $U > a$ and $\phi(U(a)) = 0$ if $U \leq a$. It is assumed that $\eta = 0$, and the sums are over all offers seen. Note that the instantaneous utility enjoyed is often greater than U_{res} .

Observe that the probability of accepting an apartment increases when the outside opportunity is less promising. Once a residence is rejected it cannot be revisited, unless it remains vacant and is randomly reselected.

Each tenant is fully described by his *housing budget*, Y . In this simple model, this represents the maximum price she is willing to pay to rent an apartment (assuming neutral idiosyncratic preferences) and is a fixed percentage of income. The housing budgets in this model are uniformly distributed among fifty groups of tenants in the range [100,198]. Housed tenants have a constant probability of moving out of a residence in any given iteration; this is a reasonable approximation for the rental market, see de Una-Alvarez et al. (2008).

All tenant agents participate in the housing market. Searching tenant agents recalculate their reservation utility each iteration. When offered rents would im-

prove their utility, but this utility is still negative, tenants cannot accept the housing. Their negative search utility U_s^T is then interpreted as the inconvenience of long-term temporary accommodation (or even homelessness) rather than a monetary cost. These non-monetary disutilities are excluded from the average tenant welfare, i.e. negative utilities are not counted in the average tenant welfare.

3.3.2 Landlords

Landlords' have three possible states, having a tenant, on the market and off the market. The corresponding utilities are, $U_{occ}^L = R - C_L$ the rent minus the maintenance cost, $U_{vac}^L = -C_L$ the cost of a vacancy, and 0 their outside opportunity, that is the utility they can expect if they withdraw from the market.

Landlords' decision variable is what rent to post. In making this decision, they are assumed to trade-off speed of sale with rent procured. Landlords calculate their most advantageous rent, that is the rent that in expectation provides the highest benefit per iteration.

The function that gives this expected benefit is $B_L(R)$:

$$B_L(R) = \frac{-C_L}{X + T(R)} \int_0^{T(R)} e^{-rt} dt + \frac{R - C_L}{X + T(R)} \int_{T(R)}^{X+T(R)} e^{-rt} dt \quad (3.6)$$

where r is the discount rate, X is the exogenous expected time a tenant will stay in the apartment, C_L is the maintenance cost per iteration and $T(R)$ is the expected time-on-the-market, whose calculation is described below.

Equation 3.6 is rewritten:

$$B_L(R) = \frac{-C_L}{X + T(R)} [1 - \exp(-rT(R))] + \frac{R - C_L}{X + T(R)} [\exp(-rT(R)) * (1 - \exp(-rX))]. \quad (3.7)$$

The first line in Equations (3.6) & (3.7) is the total expected discounted costs incurred while searching for a tenant, divided by the full expected time-on-the-market and residency duration. The second term is the total expected discounted utility flow during occupancy, divided by the full expected time-on-the-market and residency duration. Average landlord welfare is the average utility over all landlords.

To avoid errors by landlords that would lengthen search times for tenant agents, the chosen rent cannot be more than $I_{max}\%$ above the highest accepted rent seen. The percentage of rent increases equal to the maximum allowed is near 2% for the default value of $I_{max} = 10$, see Figure D.2 in Appendix D. Note that the overall time over which the profit is determined varies with $T(R)$.

A landlord is fully described by their maintenance costs C_L (default value 100, see Table 3.1). The off-market cost associated with ownership is normalised to zero⁴ and hence a landlord is never willing to post a rent below C_L . Here, landlords are homogeneous in their maintenance costs. They may withdraw from the market if they do not expect to benefit from participation. Contracts are for fixed rents and have an exogenous probability $\frac{1}{X}$ of being terminated each iteration.

Expected time on the market

Landlords can be assumed to be generally aware of the market price for their class of apartment and to have a somewhat vaguer idea of the time required to sell an apartment, as this is harder to observe. Research has shown (Yavas and Yang, 1995; Knight, 2002; Anglin et al., 2003) that in the selling market a house with a higher asking price generally takes longer to sell, particularly in thick markets.

Landlords' perceptions of the market state are characterised by the expected search time required to find a tenant for a given asking rent. Landlords are assumed to have access to information on a certain percentage of residences on the market over the last F iterations. Concretely, they know for these residences for how many iterations they were on the market at their most recent market price, within the last F iterations. They also know whether or not they have been rented, as shown in Figure 3.2.

The above procedure generates two histograms, one of the cumulative times on the market within each rent interval (of size R_I , see Table 3.2) and another of the number of sales within each interval. This allows landlords to calculate the probability per iteration of finding a tenant for a range of price intervals, making the implicit assumption that the probability to sell was constant over the last F iterations. This probability is simply the number of agreed rents divided by the cumulative times on the market. The assumption of a constant hazard means that the landlords assume an exponential probability distribution for the time-on-the-market for a constant rent, see Kiefer (1988).

The probability of renting per iteration λ , is simply the inverse of the probable time required to rent $E(\hat{T})$ (where \hat{T} is time-on-the-market).

⁴This is equivalent to assuming that the expected utility flow in another activity, or 'outside opportunity' is equal to zero.

$$E(\hat{T}) = \int_0^{\infty} t\lambda \exp(-\lambda t) dt = \frac{1}{\lambda} \quad (3.8)$$

The landlords then calculate the expected TOM within each rent interval from the data and estimate the best least-squares fit of the exponential function for the expected TOM $T(R)$ as a function of the rent R ,⁵

$$T(R) = V \exp(ZR) \quad (3.9)$$

where V and Z are fit parameters.

The memory of recent sales is extended when necessary in order to keep the number of recent sales in memory above the minimum of 100 (default value). This means that landlords always have enough information on sale times to perceive the basic relation between time-on-the-market and posted rents.

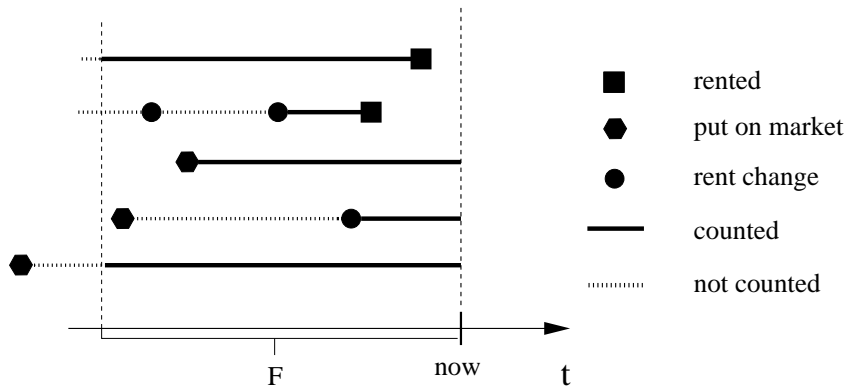


Figure 3.2: The periods of time-on-the-market known to landlords.

The behaviour of landlords is not a realistic representation of actual landlord price-setting decisions. However, we believe that it is an elegant way to simulate the trade-off between speed of sale and rent procured made by real landlords who have imperfect information on the market.

Rent revision

When a residence remains on the market at the end of an iteration, landlords review their chosen rent with probability $1/F$. They repeat the procedure described above and choose the rent that they believe will bring the maximum profit.

⁵Each point $T(R) = \omega$ is given a weight equal to the natural logarithm of the number of rentals $N(R)$ in the rent interval centred on R plus one, that is $weight = \ln(N(R) + 1)$. This weighting as opposed to a linear weighting gives greater importance to rarer information (higher agreed rents) and hence leads to a reduction in posted rents.

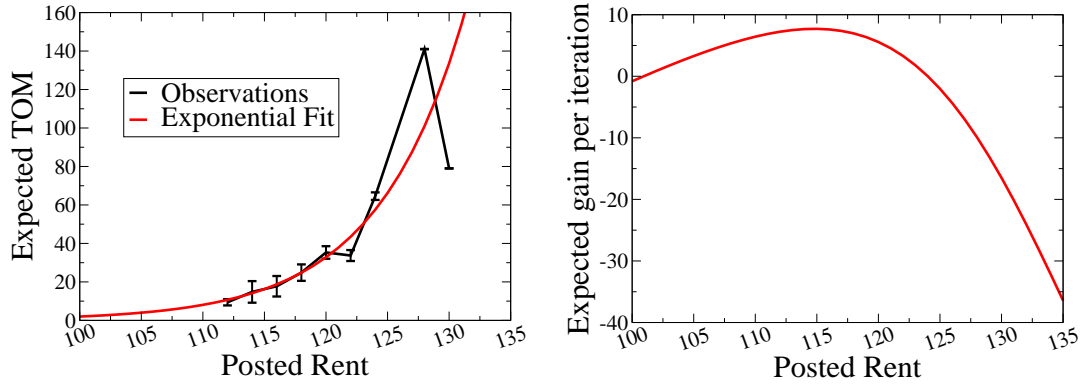


Figure 3.3: *Left*: Example of an estimated relation (red line) between asking rent and TOM per rental. The size of the ‘error bars’ is the statistical weight given to each point in the least-squares fit of the exponential. *Right*: The corresponding expected profits.

Withdrawing and returning to the market

If the expected profit estimated for a residence is negative, the residence is withdrawn from the market. Landlords who have withdrawn residences from the market review the market situation with probability $1/F$ every iteration, and return to the market if the expected profit is positive.

3.3.3 Simulation procedure

Our model repeats the following steps until a steady state is reached:

- Searchers visit a randomly chosen apartment, and accept or reject it.
- A portion of landlords ($1/F$) whose apartments remain vacant decide if they shall change their rent or withdraw from the market.
- A portion of landlords ($1/F$) who have withdrawn from the market decide if they shall return.
- A certain fraction ($1/X$) of tenants, randomly chosen, leave.
- Landlords of newly empty apartments choose their asking rents.
- The next iteration begins.

3.3.4 Parameters

Tables 3.1 & 3.2 along with the form of the distribution of the housing budgets of tenants defines a simulation run. In Table 3.1 the parameters directly involved

Table 3.1: Parameters determining agent decisions

| Symbol | Meaning | Default Value of Parameters |
|-----------|--|-----------------------------|
| | Landlords' parameters | |
| S_L | % of sales seen | 20% |
| F | Timescale rent changes (and memory) (iterations) | 15 |
| I_{max} | Maximum rent increase | 10% |
| C_L | Maintenance cost occupied | 100 |
| | Tenants' parameters | |
| X | Expected length of residence (iterations) | 240 |
| σ | Idiosyncrasy of tenants preferences (% Y) | 5 |
| S_T | Percentage of offers seen | 5% |
| C_T | Search costs | 200 |
| Y | Housing budget | [100-198] |
| | Shared parameters | |
| r | Discount rate (default 3% annual rate) | 0.0005 |

Table 3.2: Set-up parameters

| Symbol | Meaning | Default Value of Parameters |
|--------|-----------------------------------|-----------------------------|
| $size$ | Town size | 10000 |
| Z | Number of initialising iterations | 10 |
| R_I | Estimation rent interval size | 2 |

in the decisions of agents are listed, along with their default values. In Table 3.2 set-up parameters that do not play a direct role in agent behaviours are listed, along with their default values.

The default value of X , the expected length of residence, which can reasonably be assumed as being of the order of 4 years, see de Una-Alvarez et al. (2008), means that five iterations corresponds to one month and hence 60 iterations represents a year. Hence landlords' knowledge of recent rentals, F , stretches three months into the past and they change their rent in expectation every three months. The default discount rate is hence equivalent to an annual rate of 3%. It should be noted that these are simply default values. The effects of varying the parameters on the steady state configuration are discussed below and in Appendix D.

3.3.5 Initialisation

In order that landlords have some information on the market so that they can set rents, we use the following initialisation procedure. Ten thousand landlords and tenants are created. The landlords all have an on-the-market maintenance cost C_L of 100 and an initial asking rent randomly chosen in the interval 100-120. The tenant agents have a uniform distribution of housing budgets between 100 and 198 in 50 discrete groups.

Over the first Z iterations, tenant agents see five apartments and select the lowest asking rent if it offers the agent a positive utility. This preference for lower rent residences initialises the market in such a way that the information available to landlords indicates that higher rents mean longer waiting times. After the Z initialisation iterations are complete, the mechanism described in Section 3.3.3 is implemented, in which searchers see only one residence per iteration.

3.4 Results

3.4.1 Base case

We present simulation results for a 'city' with an inelastic supply of ten thousand apartments owned by ten thousand different landlords and ten thousand tenant agents wishing to be accommodated. All the parameters values are the default values presented in Tables 3.1 & 3.2.

The simulation converges to a steady state as can be seen in Figure 3.4-*Left*, for any initialisation. The rent and TOM fluctuate in a narrow range, see Figure 3.4-*Left*. The vacancy rate is about 15%, and the percentage of landlords who have withdrawn from the market is negligible. The 'Population' is the number of *housed* tenants (and therefore also the number of landlords with occupied residences). The number of *unhoused* tenants is equal to the difference between

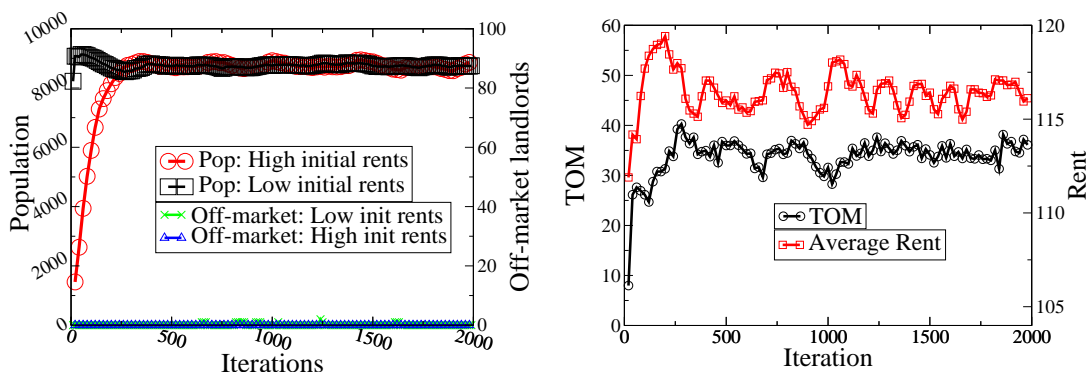


Figure 3.4: *Left*: The steady state Population and number of landlords off-the-market. The "Pop : high initial rents" curve is from an initialisation following the procedure in Section 3.3.5 with initial rents distributed between 180 and 200. The "Pop : low initial rents" curve corresponds to a standard initialisation as described in Section 3.3.5. *Right*: The average TOM of residences accepted over the last 15 iterations and their average rent for a standard initialisation.

the 'Population', shown in figures throughout this Section, and the total number of tenant agents (10,000). The number of landlords searching for tenants is equal to the difference between the 'Population' plus the number of landlords 'Off-the-market' and the total number of landlords (10,000).

The dispersion in both accepted and posted rents can be seen in Figure 3.5-*Left*. The curves are the cumulated distributions of rents set and accepted over 15 iterations. In Figure 3.5-*Right* we can see that, as expected, lower rents are more likely to be accepted. Figure 3.5-*Left* shows that most landlords offer rents close to the standard 'going rate'. The few who ask higher rents are less likely to find tenants. We can see that the basic model converges to a reasonable steady-state with a positive vacancy rate, rent dispersion and nonzero search times.

The simulation results show that a dispersion of rents occurs at steady-state. The heterogeneity of tenants' incomes and the presence of market frictions (un-directed search with a unitary arrival rate of offers) contribute to this dispersion. Additional factors contributing to the dispersion of prices are the idiosyncratic preferences of tenants and stochastic information effects: agents observe samples of market signals and therefore take different decisions. This is particularly true for low levels of information and for landlords, as will be discussed in Section 3.4.3.

The maximum rent posted by landlords is lower than the highest tenants' reservation rent, it is below 150 in Figure 3.5-*Left*. Landlords find that for higher rents it takes longer to find a tenant (because those tenants with low incomes can no longer afford it, and wealthier tenants prefer cheaper accommodation) and so offer relatively low rents. The lower bound of offered rent in our results is greater than the lowest of the tenants' housing budgets: although landlords

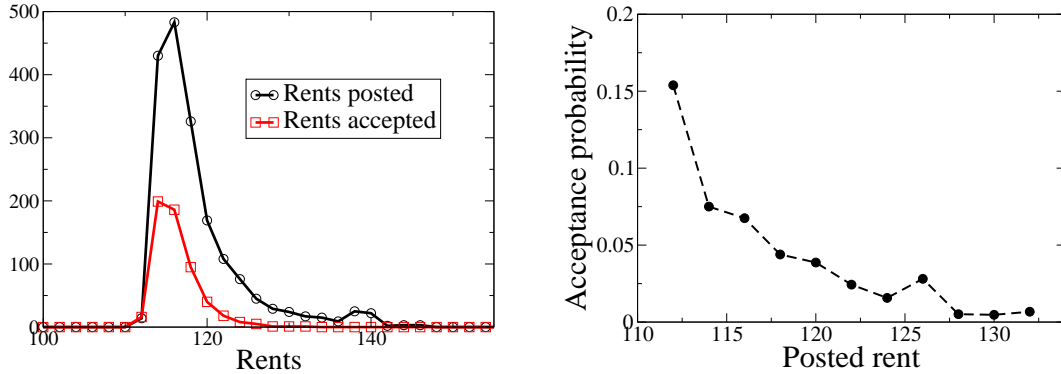


Figure 3.5: *Left*: Rents posted and accepted in last 15 iterations at the steady state, after 2000 iterations. *Right*: The probability of renting each iteration as a function of posted rent. There is some "noise" at the higher rents due to the small number of rentals.

might get profit with any rent strictly above 100, the minimum offered rent in our simulation is above 110, see Figure 3.5-*Left*, and the lowest income tenants are homeless. This is in part due to high search costs C_V compared to the expected benefit of having a residence on the market with a low posted rent. Landlords always trade-off longer search times for higher rents. The expected duration of residence is long compared to search times and hence search costs. For the lowest rents, the gains from minor reductions in search times are less than the loss from lower rents over the entire residence duration.⁶

3.4.2 Effects of agent's characteristics on the market state

After having shown that the basic model leads to a reasonable steady-state, we shall show its evolution upon changes in the information parameters, the discount rate, tenants' idiosyncratic preferences and landlords' vacancy costs. The effects of the other parameters (memory length, maximum rent increase, tenants' expected duration of residence, city size) are shown in Appendix D.

Note that at constant population, rent changes are predominately transfers in surplus between tenants and landlords. Changes in population or the withdrawal of landlords from the market are the major factors in changing the total welfare.

3.4.3 Varying information levels

Two very important parameters of this model are the percentages of offers seen by tenants S_T and the percentage of times-on-the-market of recent rentals and of

⁶Simulations have been run with a 70 cost of landlords. The lower bound of the distribution of market rents remains above a hundred. This is due to the inelastic supply of residences in this model.

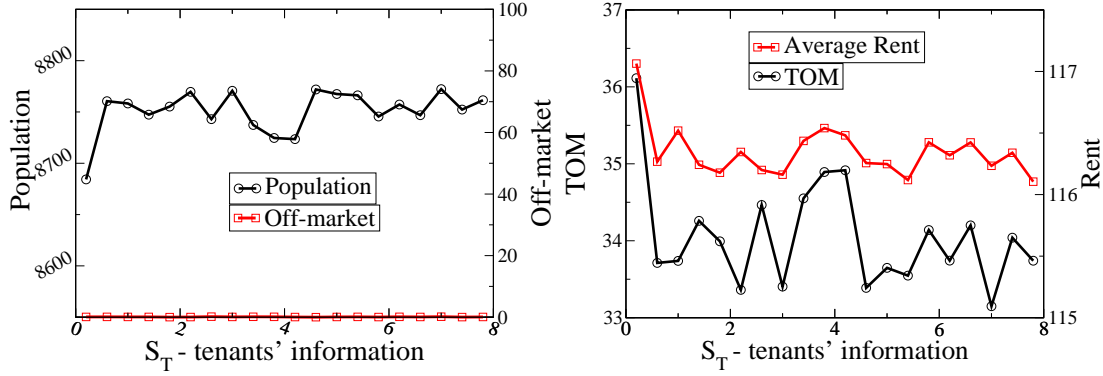


Figure 3.6: *Left*: The variations in population and the number of landlords off-the-market as a function of the percentage of offers seen by tenants. *Right*: The average rent and the average time-on-the-market (TOM).

vacant residences seen by landlords S_L . In order to avoid discrepancies between the optimisation decisions made by agents and the welfare measurement, we present here results obtained for a discount rate equal to zero. Figure 3.13, in Section 3.4.4, shows that the difference between simulation results with the default value $r = 3$ and $r = 0$ are not significant.

Tenants

The population is not greatly affected above a low threshold by alterations in the percentage of offers seen by tenants, S_T (Figure 3.6-*Left*). Information effects at low levels of S_T on the average rent and TOM can be seen in Figures 3.6-*Left* & *Right*, and on agents' welfares in Figure 3.7-*Left*. The percentage of offers seen by tenants ceases to alter the steady state configuration above a threshold value which can be seen to be approximately 0.5% or 5 observations from Figure 3.7-*Right*. It appears that a relatively low level of information gives an accurate impression of the market state to tenants. This is, in part at least, due to the range of the real distribution of offers being narrow. Moreover the fact that searchers renew their information every iteration excludes any persistence of erroneous perceptions. Search times of the majority of tenants, with the default level of information $S_T = 5\%$, are of the order of 3 or 4 iterations, as can be seen in Figure 3.7-*Right*. When tenants are very badly informed, they are likely to choose a reservation utility equal to the best offer seen. This is because they expect the additional waiting costs to be very low in comparison to the expected gain from waiting. We see in comparing Figures 3.6 & 3.7-*Left* with Figure 3.7-*Right* that once the expected search times of tenants cease to follow the increase in the distribution seen, the effect of increasing tenants' information is negligible.

Increasing tenants' information from very low levels improves their welfare as

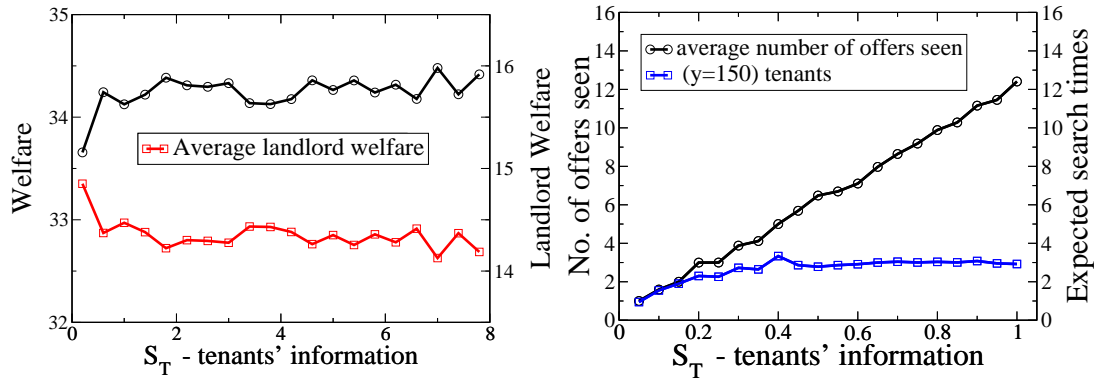


Figure 3.7: *Left*: The average welfares of tenant and landlord agents are shown as a function of the percentage of offers seen by tenants. *Right*: The number of residences seen by tenant agents with respect to the percentage of the distribution that they see, and the expected search times of ‘middle income’ tenants.

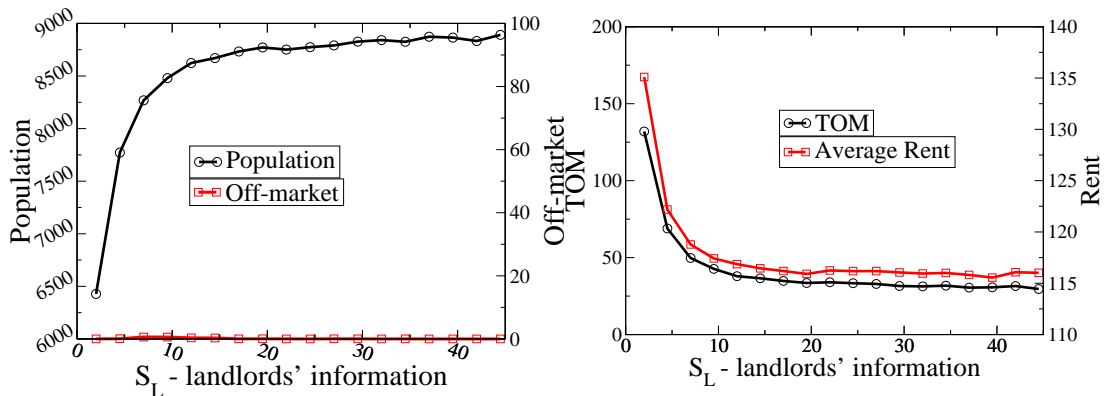


Figure 3.8: *Left*: The variations in population and the number of landlords off-market as a function of landlords' information. *Right*: The average rent and the average time-on-the-market (TOM).

they are more likely to refuse higher rents.

Landlords

Increasing landlords' information decreases their welfare as rents decrease (Figures 3.8-*Right* & 3.9-*Left*). The population also increases, as shown in Figure 3.8-*Left*.

Landlords' estimation of search times for different rents is based on a smoothing procedure (the exponential fit) over noisy data. Landlords need accurate two dimensional information to decide their asking rent, that is rents offered and their associated times-on-the-market. The times-on-the-market of many residences in each rent interval are required for accurate estimates of the rent/ times-on-the-

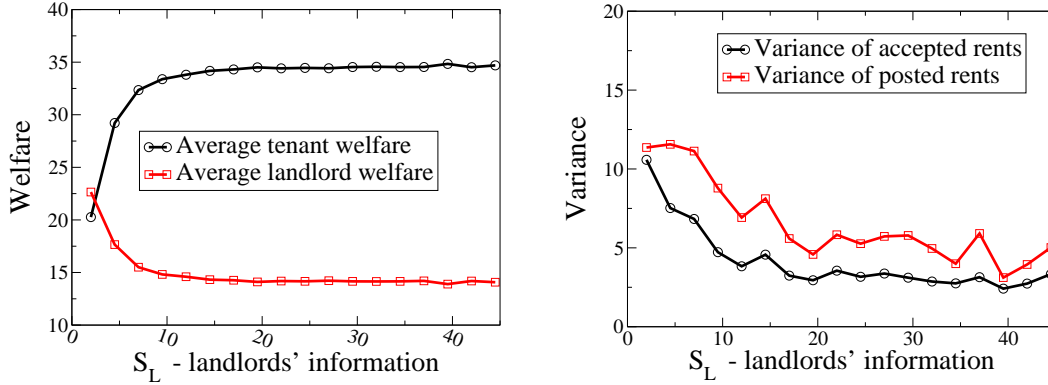


Figure 3.9: *Left*: The average welfares of tenant and landlord agents are shown for variations in the level of landlord's information. *Right*: The average variance of the distributions of posted and accepted rents over 1000 iterations, after convergence to a steady state.

market relationship. Therefore their information, which also depends on their memory length F , is not perfect even with $S_L = 100\%$. Figure 3.9-*Right* shows that the variance in posted and accepted rents decreases as landlords see larger (and hence more similar) information samples.

Figure 3.10 shows that the less informed are landlords, the greater their underestimation of TOM and hence their overestimation of the optimal rent. Errors made by ill-informed landlords tend to lead to higher asking rents because they are less likely to see the long waiting times (TOM) for higher rents. These are rare events which they are more likely to overlook due to their more limited vision. As the landlords are homogeneous, and make the same errors on average, this pushes the market prices upwards. Note that every high posted rent, if refused by tenants, increases their expected search times as tenants' search is undirected. This necessarily affects searchers' optimal reservation utilities, pushing the market towards higher rents. In contrast, increasing landlords' information makes them sharper competitors, leading to reduced rents.

Heterogeneous landlords

In order to further test the effect of landlords' information on the steady-state of the market we perform simulations with landlords who are heterogeneous in information. There are two types of landlord, those with the default level of information $S_L = 20$ and those with $S_L = 5$, values that were chosen because Figure 3.9 shows that the steady state changes significantly between these two values when they are shared by all landlords.

Figure 3.11-*Right* shows the increases in average rents and times-on-the-market with the proportion of ill-informed landlords. This is because, as de-

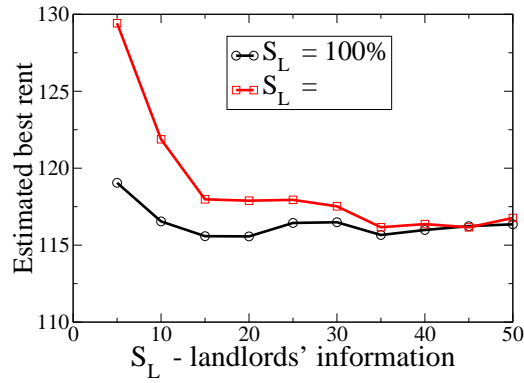


Figure 3.10: The average estimated best rent, over 1000 iterations after convergence to a steady-state, in simulations in which S_L for all agents has the value on the x-axis. The average best rent that an agent seeing 100% of available information is also shown.

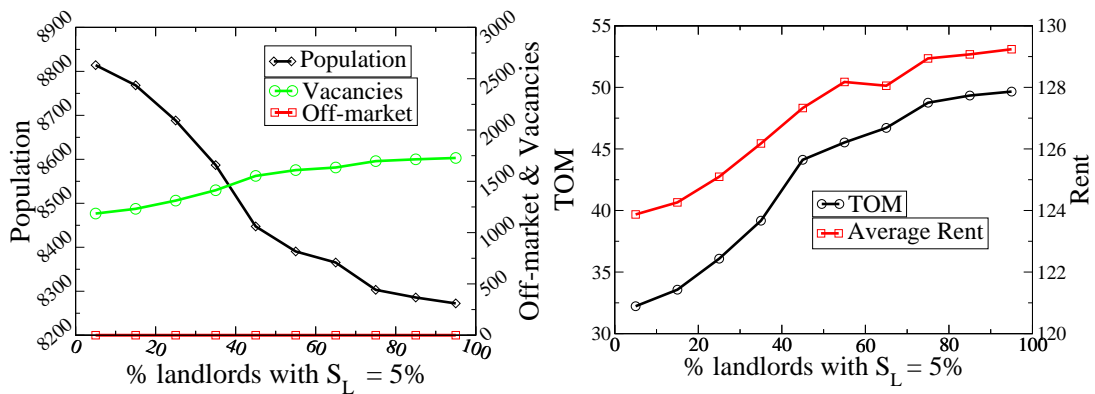


Figure 3.11: *Left*: The variations in population and the number of landlords off-the-market as a function of the percentage of landlords who see 5% of the available information while all other landlords see 20%. *Right*: The corresponding average rent and the average time-on-the-market (TOM).

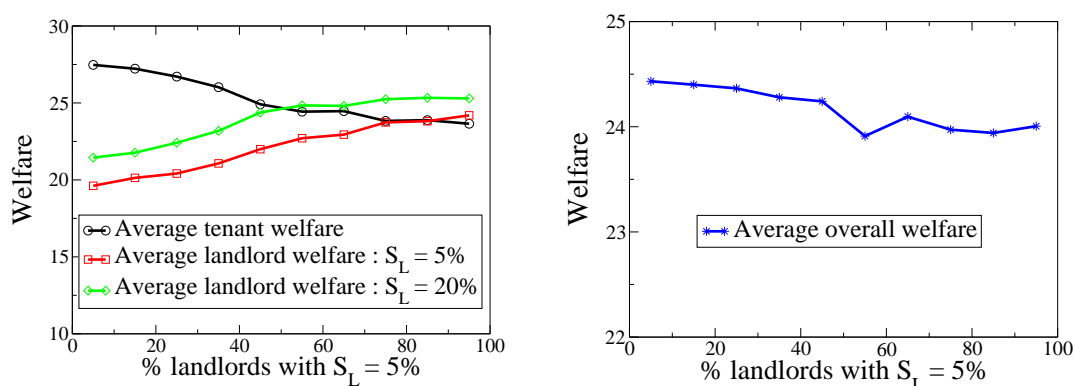


Figure 3.12: *Left*: The average welfares of tenants and both types of landlord as a function of the percentage of landlords who see 5% of the available information while all other landlords see 20%. *Right*: The corresponding average total welfare.

scribed in Section 3.4.3, errors made by ill-informed landlords tend to lead to higher asking rents. This changes the distribution seen by tenants who have no option but to lower their reservation utilities. As a consequence, well-informed landlords react to the reduced TOM by increasing their offers. Figure 3.11-*Left* shows that consequently the population falls as more landlords are ill-informed. Overall the welfare reduces because higher rents reduce the population, Figure 3.12-*Right*.

In Figure 3.12-*Left* we see that, following the rent increase, the welfare of both types of landlords increases as the proportion of ill-informed landlords rises. However, the better informed always have higher welfares. This results from their more accurate appreciation of the state of the market, see Figure 3.12-*Left*. In summary, there are positive externalities (or, more precisely, market effects) of ill-informed on well-informed landlords, the former moving the market rent upwards.

3.4.4 Discount rate

The discount rate was varied from less than 1% per annum to over 17% per annum. It should be noted that the effects of changing the discount rate arise from changes in all four terms in the right-hand sides of Equations (3.3) & (3.6) and the subsequent market interactions. Increasing the discount rate means that agents' impatience rises. Therefore, landlords have a tendency to post lower rents, while those tenants who have sufficiently high incomes are willing to accept higher rents. It is not obvious which of these contradictory effects should dominate. In general, one may anticipate that the effect of changes in the discount rate depends on the relative market power of the two kinds of agents.

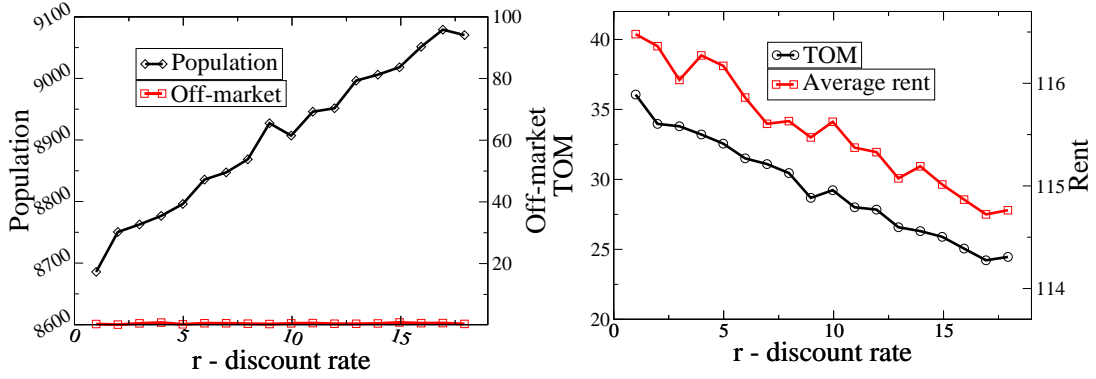


Figure 3.13: *Left*: The variations in population and the number of landlords off the market. *Right*: The average rent and the average time-on-the-market for residences rented over the last 15 iterations. Both graphs for a variation in the annual discount rate of both agent types.

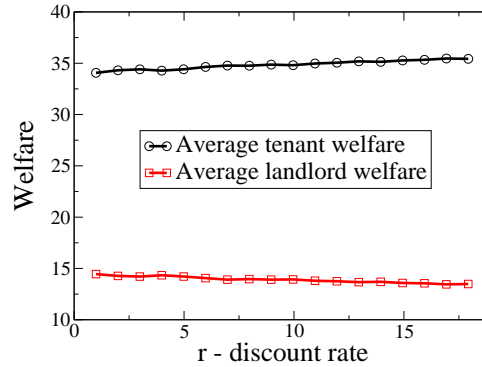


Figure 3.14: The average welfares of tenant and landlord agents are shown for variations in the annual discount rate.

For the default values of the other parameters, the average rent is lower with a higher discount rate. Indeed, Figures 3.13 show that increases in the discount rate lead to a reduction in average rents and TOM, and to an increase in population. Correspondingly, the average welfare of tenants is improved and that of landlords disimproved with increasing discount rates, as can be seen in Figure 3.14. This is consistent with the observation that in our setting, as was discussed in Section 3.4.3, changes in landlords' behaviour have a greater impact on market outcomes than do changes in tenants behaviour.

3.4.5 Idiosyncratic preferences

We now present the effect of varying tenants' idiosyncratic preferences for residences visited σ . Recall from Section 3.3.1 that the utility of housing is given

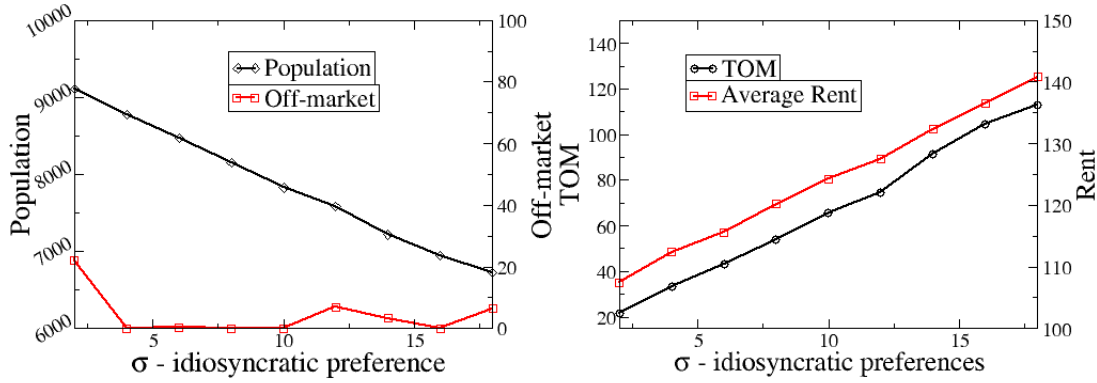


Figure 3.15: *Left*: The variations in population and the number of landlords off-the-market. *Right*: The average rent and the average time-on-the-market (TOM) for residences rented over the last 15 iterations. Both graphs for a variation in the idiosyncratic preferences of tenant agents.

in Equation (3.1) as $U_h = Y - R(U_{res}) + \eta$, where $\eta \sim N(0, \sigma)$, σ being the variance of the normally distributed idiosyncratic preferences and expressed as a percentage of the housing budget. As in Section 3.4.3 we present results with a discount rate of zero.⁷

The average rent increases with σ , as tenants agree to higher rents for apartments they particularly like, see Figure 3.15. Among accepted rents over the F iterations of the landlords' memory, there is a selection of lucky searchers (those who drew a high η). This reduces the average TOM of high rents. Therefore landlords post higher rents, which shifts the distribution of accepted rents upwards.

The rise in rents excludes many potential tenants from the market. This explains why the level of vacancies strongly increases with σ (the number of residences off the market is close to 0 while the city population decreases significantly). Hence the same number of landlords, all attracted by the high rents, search among a smaller number of tenants and so TOM rises. Note that on average, the city is populated by wealthier tenants, which sustains higher rents.

Figure 3.16 shows the result of two opposing effects on tenants' average welfare from changes in σ : firstly the selection of lucky tenants, who enjoy higher utility levels and secondly the reduction in population seen in Figure 3.15-*Left*. We see that at low σ the first effect dominates. However for higher σ , the increase in rent penalises tenants on average by more than they gain from being able to

⁷It would also be possible to consider tenant agents who decide not to search when their expected benefit from searching B is negative. η would then be discovered only when searching. We can anticipate that prior knowledge of σ , the variance of eta, would affect the participation decision.

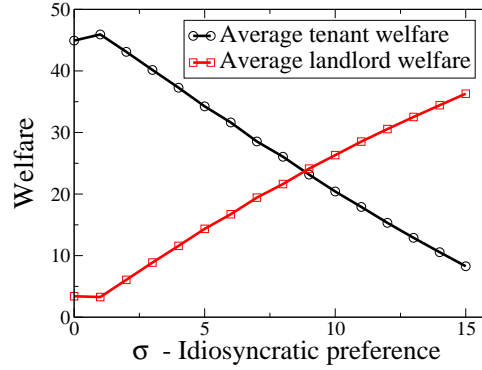


Figure 3.16: The variance of the average welfares of tenants and landlords for a variation in the idiosyncratic preferences of tenant agents.

choose apartments for which they have a greater personal preference.

3.4.6 Vacancy tax

A tax on vacancy housing in large cities (population >200,000) has been implemented in France as a means of increasing the housing supply. Taxes on vacant housing have been implemented or considered in many countries, for example Chen (2000). Our tax set-up is similar to that of Desgranges and Wasmer (2000) who examined a tax on vacant housing using a static equilibrium search model. They find reduced rents with vacancy taxes, which improve housed tenants' situation. However in the medium term, the tax decreases the welfares of landlords due to reduced rents and that of searchers due to a reduction in supply.

Here, we consider a tax τ that adds to landlords' maintenance costs C_L during the landlord's search period as shown in Equation (3.10).

$$B_L(R) = \frac{-(C_L + \tau)}{X + T(R)} \int_0^{T(R)} e^{-rt} dt + \frac{R - C_L}{X + T(R)} \int_{T(R)}^{X+T(R)} e^{-rt} dt \quad (3.10)$$

For this Section and Section 3.5 the results shown are with landlords who are heterogeneous in their maintenance costs C_L . The landlords were given uniformly distributed maintenance costs in 30 groups in the domain $[0,129]$. They all still have a zero utility outside option, and hence those with higher C_L are more likely to withdraw from the market. This heterogeneity was added to stabilise the model. When their only heterogeneity is in the stochastic information they receive, this results in large fluctuations. Figure 3.19-*Left* shows the relative stability of the current set-up.

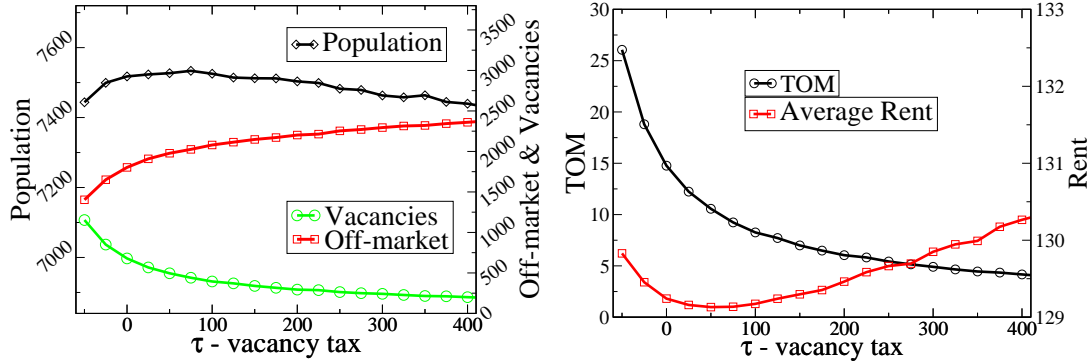


Figure 3.17: *Left*: The variations in population and the number of landlords off the market. *Right*: The average rent and the average time-on-the-market for residences rented over the last 15 iterations. Both graphs for variations in the vacancy costs, C_V of landlords.

Figures 3.17 show the effects on the steady-state of adding the tax τ . Small negative values of τ corresponding to a subsidy for participating in the market were also examined. For these simulations the discount rate $r = 5\%$, the average duration of residency $X = 180$ iterations (or 3 years), the variance of tenants' idiosyncratic preferences was $\sigma = 2\%$ of Y , their housing budget. All other parameters took their default values given in Tables 3.1 & 3.2.

We see in Figure 3.17-*Left*, that increasing the tax increases the population initially. This occurs because the increased costs of vacancy for landlords cause them to reduce rents, Figure 3.17-*Right*. Hence more tenants are able to afford housing. Figure 3.17-*Right*, also shows that the average time on the market also decreases with an increasing vacancy tax. However, as the tax becomes larger it impacts on the supply of housing, as landlords no longer find it beneficial to participate in the market. Those that do must increase rents to cover their increased costs due to the tax. The number of landlords who have withdrawn from the market can be seen to rise monotonically with the imposed tax, Figure 3.17-*Left*.

In Figure 3.18 the total welfare can be seen to follow the value of the population, and inversely that of the rent. Tenants' average welfare is initially improved as the subsidy is reduced and then the tax is increased. However, the reduction in supply penalises all agents at high tax levels. We can see that the increase in welfare for tenant agents dominates the reduction in welfare for landlords at certain tax levels. As a consequence there may exist in real world situations an optimal tax where the negative welfare effect of reduced supply is dominated by positive effects from lower rents and vacancies.

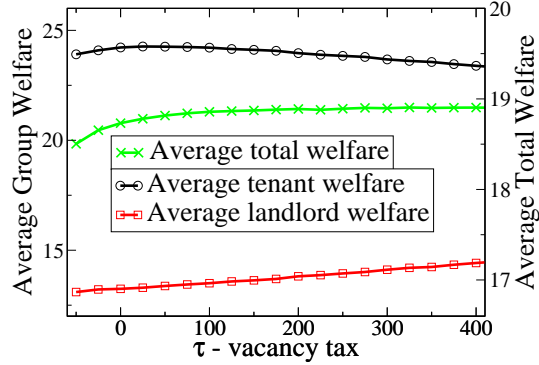


Figure 3.18: The welfare at the steady state corresponding to the results in Figures 3.17.

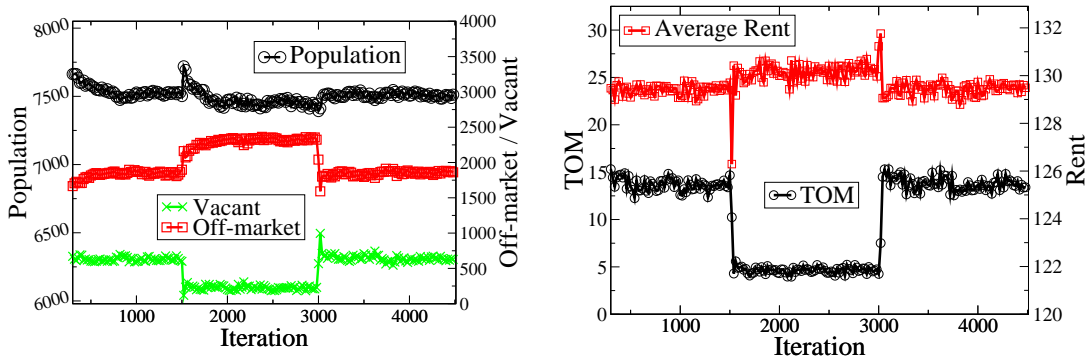


Figure 3.19: The landlords' vacancy costs are increased from $\tau = 0$ to $\tau = 300$ at 1500 iterations and decreased again from $\tau = 300$ to $\tau = 0$ at 3000 iterations. *Left*: The evolution of population, vacancies and the number of landlords off the market, after an initial convergence from the initialisation described in Section 3.3.5. *Right*: The evolution of the average rent and the TOM.

3.5 Dynamic variation of landlords vacancy costs

We simulate the introduction and then abrogation of a tax on vacancies, with landlords that are heterogeneous in their maintenance costs as in Section 3.4.6. The tax τ was raised from 0 to 300 at 1500 iterations and then reduced again from 300 to 0 at 3000 iterations, see Equation (3.10). Figure 3.19 shows the evolution of the population, the number of vacant residences and the number of residences off the market. An asymmetry in the two transitions can be seen.

Figure 3.19-*Right* shows the variation in times-on-the-market and average rents over the 4500 iterations. A significant overshoot in the average rents agreed can be seen at both 1500 and 3000 iterations. After the introduction of the tax landlords have higher costs, with initially unaltered perceptions of TOM. This

leads directly to a reduction in posted rents and an increase in the number of residences off the market, Figure 3.19-*Left*, as the profits that can be obtained from participation in the market have reduced. The population rises in the very short term as the rents have reduced.

After the abolition of the tax at 3000 iterations landlords have lower search costs (i.e. $TOM \times \text{vacancy costs}$) and so can afford to increase rents. This creates the initial spike. It is worth noting that landlords always have a memory of 15 iterations in their impression of the market state. When their vacancy costs (which include the tax) reduce they have the impression that the market will support much higher rents. Only after they have increased their rents and tenants react by rejecting a greater proportion of offers, can they learn the true market situation. The population adjustment is much smoother in the second transition. TOM appears to adjust swiftly and smoothly to changes in the tax rate Figure 3.19-*Right*.

Tenants can only select from among the offered rents and the richer tenants will normally select from the cheap part of the offered rents distribution. However poorer tenants will be unable to afford residences as rents rise. This effect is felt by landlords through increased times-on-the-market when they post higher rents. Eventually the competing forces of higher profits due to higher rents and lower profits due to longer waiting times that are the result of heterogeneities in tenant housing budgets, find an equilibrium, see Figure 3.19-*Right*.

The detail of the adjustments in TOM and in average rents after both transitions in landlords' vacancy tax rate can be seen in Figure 3.20. Figure 3.20-*Left* shows the significant downward adjustment in the values of both the average rent and TOM, after the vacancy tax is introduced at 1500 iterations. There are then a series of oscillations in the average rent. Falling TOMs then make higher rents seem optimal, and average rents rise again. There are concurrent adjustments in the numbers of residences, Figure 3.21-*Left*. The reduction in the number of vacant residences on the market pushes rents back up after the initial reduction. Once the number of vacancies has converged to its new steady-state value (at about 1525 iterations, Figure 3.21-*Left*) there are oscillations of decreasing amplitude in the average rent. Most of the adjustment is effectuated within the first 150 iterations (or two and a half years).

Figure 3.20-*Right* shows the considerable overshoot in rents demanded after landlord vacancy costs decrease at 3000 iterations. After the elimination of the tax, landlords have lower costs of vacancy: and so can afford to increase rents. This creates the initial spike in rents. Once a significant number of tenants begin to refuse these higher rents due to their housing budget constraint, landlords' TOM increase. This increase in TOM reduces the gains of landlords, who then reduce rents to decrease TOM. The TOM converges in about 60 iterations, corresponding to one year. The period over which landlords' collective misappreciation of the market causes them to post rents above the eventual steady-state

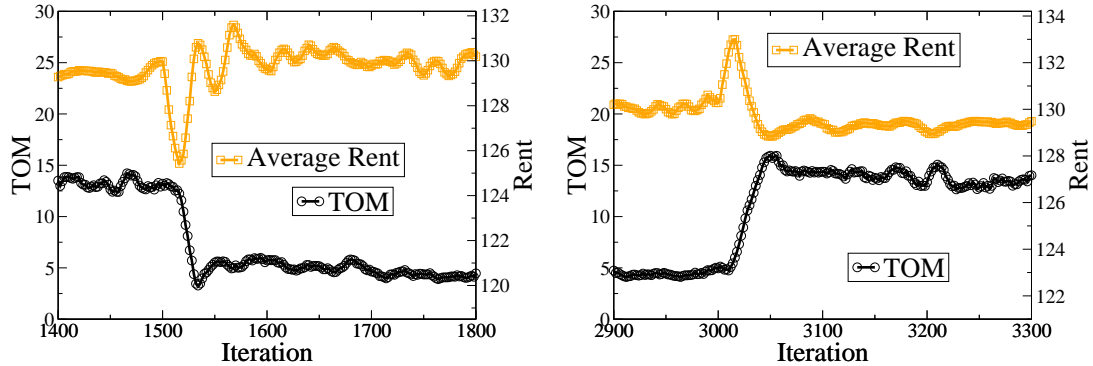


Figure 3.20: *Left*: The variations in TOM and average rents, around the transition of landlords' vacancy costs from $\tau = 0$ to $\tau = 300$ at 1500 iterations. *Right*: The variations in TOM and average rents, around the transition of landlords' vacancy costs from $\tau = 300$ to $\tau = 0$ at 3000 iterations.

is under 100 iterations (about $1\frac{1}{2}$ years). The increase in observed TOM causes a reduction in their offered rents as is seen in Figure 3.20-*Right*.

Figures 3.21 show in greater detail the transitions in population, vacancies and residences off the market after the changes in vacancy costs at 1500 and 3000 iterations. Figure 3.21-*Left* shows the initial increase in the number of landlords who remove their residence from the market. This appears to overshoot before begin a slow adjustment to the new steady-state. The number of vacancies decreases dramatically after the introduction of the vacancy tax at 1500 iterations. The full adjustment can be seen to take up to 200 iterations. This corresponds to slightly over 3 years with the default parameters.

Figure 3.21-*Right* shows the adjustment after the abolition of the vacancy tax. Note particularly that the number of vacant residences appears to overshoot when costs fall again at 3000 iterations. The system takes not much more than 100 iterations to fully readjust. With the default parameters this corresponds to just over $1\frac{1}{2}$ years. The asymmetry in adjustment speeds is due to the fact that most landlords with vacant apartments take into account in their posted rent the increase in vacancy costs in $F = 15$ iterations. However, when vacancy costs decrease the landlords who will eventually readjust their posted rents have to wait a time considerably longer, as the expected duration of tenants residency is $X = 240$.

An abrupt positive shock to landlords' market power can have exaggerated short term effects before the market finds its new equilibrium. More generally, shocks to the housing market may have the potential to provoke disproportionate effects due to agents basing their decision on outdated information. Such effects have been postulated to explain part of the dynamics of real estate markets, see Fisher et al. (2003) and Clayton et al. (2008). These results suggest that interventions on the housing market should be gradual when possible, to avoid

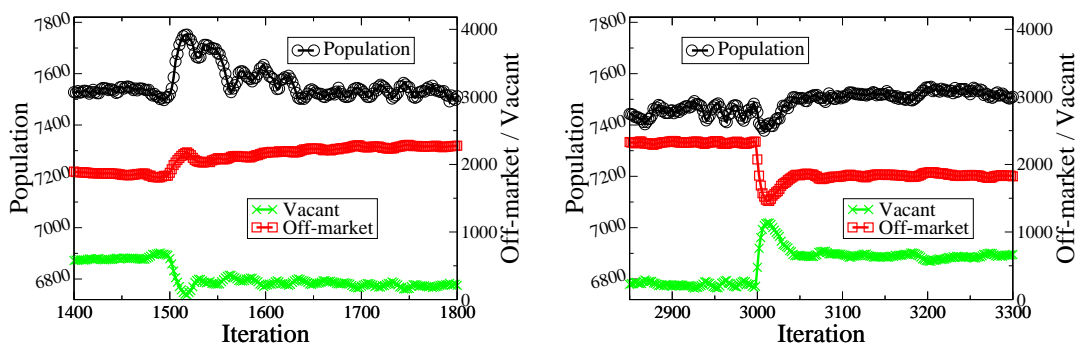


Figure 3.21: *Left*: The variations in population, vacancies and the number of landlords off the market, around the transition of landlords' vacancy costs from $\tau = 0$ to $\tau = 300$ at 1500 iterations. *Right*: The variations in population, vacancies and the number of landlords off the market, around the transition of landlords' vacancy costs from $\tau = 300$ to $\tau = 0$ at 3000 iterations.

abrupt over-reactions.

3.6 Discussion of results

We know that a perfectly competitive market should have rents offered at landlords' maintenance costs (Bertrand price competition): in a situation where every tenant would be able to choose the best offer among several of them, any landlord making an offer below others would very quickly attract a tenant. Rents would thus be pushed down to landlords' maintenance costs i.e. at 100. What drives the market away from this configuration is primarily the frictions due to the search process. Due to search costs, the matching of a tenant and a landlord produces a benefit to both of them. Because tenants can visit only one residence per time unit and accept or refuse it without recall, landlords have the power to extract a part of this benefit, which explains rents above landlords' maintenance costs.

We know from existing theoretical labour-market models that in a model with frictions and homogeneous workers, the equilibrium would be with a single wage, equal to the workers reservation wage. In this situation, firms are able to extract the whole surplus from the match and as a result, no searcher participates in the market, which is known as the Diamond paradox (Diamond, 1971). However, if workers differ in their reservation wages, due to heterogeneous outside options for instance, a distribution of wages emerges (Albrecht and Axell, 1984; Eckstein and Wolpin, 1990). This also ties in with the consumer search literature, where price dispersion has been shown to result from sellers playing mixed strategies against potential buyers who have different reservation prices, see McMillan and Rothschild (1994). Following an analogous mechanism, in our housing-market

model landlords facing tenants with heterogeneous reservation rents react by setting different offers

In contrast to those theoretical models, differences in reservation rents in our model result not only from the heterogeneity in the value of the outside option on the part of searchers. They also come from the existence of idiosyncratic preferences and from the heterogeneity in the information received by different agents: tenants seeing different samples from the distribution of offers decide different reservation rents.

The distribution of offered rents differs from the uniform distribution of reservation rents: although high income agents can accept any rent sufficiently below their reservation rent to cover the associated search costs, we observe that the maximum rent offered by landlords is lower than the maximum reservation rent. Landlords find that for higher rents it takes longer to find a tenant (because those tenants with low incomes can no longer afford it, and wealthier tenants prefer cheaper accommodation which they know to exist from their information on offered rents) and so offer relatively low rents compared to richer tenants housing budgets. Existing theoretical models can give us clues to better understand the distribution of offered and accepted rents. In a search model with heterogeneous workers, it has been shown that the distribution of offered wages differs from the distribution of reservation wages because unemployed individuals with high outside opportunity only flow to jobs with high wages (Eckstein and Van den Berg, 2007). Similarly in our model, high income agents can accept any rent that is sufficiently below their reservation rent to cover the associated search costs. In contrast, low budget searchers can only accept low rent offers. Therefore, the acceptance probability of low rent offers is higher, which distorts the distribution of offered rents with respect to the distribution of reservation rents.

Furthermore, Bontemps et al. (1999) demonstrate theoretically that with heterogeneous searching workers and heterogeneous firms, the minimum offered wage is higher than the minimum reservation wage. This parallels the observation in our model that the maximum rent offered by landlords is lower than the maximum reservation rent. Actually, landlords find that for higher rents it takes longer to find a tenant (because those tenants with low incomes can no longer afford it, and wealthier tenants prefer cheaper accommodation) and so offer relatively low rents compared to richer tenants housing budgets.

What is new in this model with respect to theoretical models is that landlords need sufficient information in two dimensions to have an accurate appreciation of the market state. The dramatic difference in the sensitivity to changes in the two information parameters (S_T for tenants and S_L for landlords) is due to the fundamental asymmetry in the market exposed in Sections 3.3.1 & 3.3.2. Landlords individually post prices which cannot be negotiated, while tenants decide whether or not to accept the offer received.

The idiosyncratic preferences of tenants are also absent from the search models cited in this Section, though they were introduced in housing search models by Arnott (1989). The addition of these preferences adds an important realistic element to the model and their stochastic nature has a stabilising influence.

3.7 Conclusion

Our dynamic model includes imperfect information and heterogeneous interacting agents. It leads to price dispersion, nonzero search times and vacancies, three essential ingredients of any realistic housing model. The matching probability depends endogenously on the posted price of apartments. It is a general equilibrium model that provides a basis for examining policy questions such as rent control and its welfare and distributional effects, the welfare effects of the taxation of vacant housing or the general equilibrium effect of providing social housing.

In our model landlords are Stackleberg leaders who set rents which tenant agents accept or refuse. This is a reasonable representation of the rental market. Landlords act upon their partial knowledge of the market in order to maximise their profits. Their knowledge of the market state is in the form of probable times-on-the-market before renting for the full range of possible rents. This allows the calculation of the expected profit and hence rational maximisation. The heuristics of real world agents are simulated here by a regression and profit calculation, with a larger number of individual information points than real agents normally know. Greater information for landlords disimproves their overall utility due to greater competition, see Section 3.4.3.

Tenants, with idiosyncratic preferences and heterogeneous in income, are also partially informed of the state of the market and use their information to decide the minimum utility they are willing to accept from a residence. Their search is undirected, that is they have equal probability of seeing any available offer in a given iteration. Greater information for tenants improves their overall welfare.

We have examined the comparative static and dynamic effects of a tax on vacancies on the market. The main aim of vacancy taxes is to benefit tenants by increasing supply in the short term and hence reduce rents. Here we find an optimal tax where the benefits of reduced steady-state vacancies and average rents dominate negative supply effects.

Making the tenant side of the market open - that is having a constant flow of arrivals of tenant agents - instead of a fixed number of searching tenants would allow the composition of the town to be more endogenous. This would also mean that the vacancy rate would represent market frictions only and not a mixture of market frictions and those tenants who are unable to pay market rates as is currently the case. Note that in Sections 3.4.6 & 3.5, part of the vacancy rate

is due to landlords with higher maintenance costs seeing no benefit in market participation. Transferring our model to one where tenant agents engage in directed search is also a promising extension.

One of the most interesting possible extensions is to alter the number of units owned by landlords. Heterogeneities in the level of information among landlords could represent the difference between large commercial landowners, who would also have greater private information, and small private owners. For example, large landlords are likely to be better able to absorb vacancies (as discussed by Blanck and Winnock (1953)), and more able to adapt their rental strategies to market conditions.

The current set-up allows the investigation of the distributive effects of policy decisions among tenant agents of varying incomes. Rent control is one possible example, as is the level of information among tenants. These two effects have been examined in static simulation models by Bradburd et al. (2005, 2006). It would be interesting to test whether the effects are similar in our more realistic model.

There is great scope to complexify the model for both types of agent. Tenants could be differentiated by their work places, housing preferences, household sizes, transportation modes etc. Landlords could own different numbers of apartments, and apply different rent setting rules.

The modelling of construction, demolition, depreciation, conversion and refurbishment of buildings would be an important extension for modelling price and liquidity dynamics. Coupling the rental market to the ownership market is another important possible extension. A simulation model of the ownership market needs to take into account the fact that the presence of the same agents on both sides of the market is a critical difference from the rental market.

Our main aim has been to construct a model that allows hypotheses on the functioning of the urban rental market to be investigated. We believe that a dynamic model based on straightforward micro-economic behaviours with imperfect information is a good approach. We believe we have found robust and simple agent dynamics (or rules) that reproduce the essential features of the rental housing market and results from analytical search models that have been developed to analyse labour as well as housing markets. This is an exploratory research project that suggests there is a real potential for dynamic and disaggregated simulation models to provide deeper insights into the functioning urban housing markets.

Conclusion

The original motivation for this thesis was to explore the potential of agent-based models to deepen our understanding of urban systems. As described in the general introduction to this thesis, agent-based models are well adapted to modelling the interactions of heterogeneous spatially dispersed agents in urban systems. Agent-based models are dynamic simulations that can easily incorporate heterogeneity in agents' characteristics. They can be combined with complex representations of spatial structures, such as those of geographical information systems (GIS), and realistic transport networks. The agent-based method of modelling agents is a very natural and intuitive one. However, capitalising on this potential is not a straightforward task. Windrum et al. (2007) have reviewed approaches to the difficult problem of agent-based model validation.

In this exploratory theoretical thesis, existing analytic models of urban systems have been taken as a starting point for a more thorough exploration of the systems' dynamics. Robust theoretical models of the dynamics of important urban systems, notably housing markets, are an important step in establishing a more constructive dialogue between theoretical and empirical research in urban economics.

In the model presented in Chapter 1, the equilibrium of the standard congestion model was shown to be unstable. The potential of heterogeneities to stabilise the system's behaviour was demonstrated. The presence of externalities, and competition between highly sensitive agents are responsible for the instability of the model. Further non-agent analysis of this model set-up clarified the potential of smoothing cost functions to reduce the instability. It was also established that replacing agents' sensitive binary decision-making criteria with probabilistic rules increases system level stability. These lessons are very probably applicable to many agent-based simulation models.

It should be noted that transport systems are particularly suited to simulation models as they are systems of large numbers of agents following relatively straightforward behaviours that repeat on a daily basis. Ample amounts of data are becoming available to calibrate these models, notably through the use of GPS tracking. It is for this reason that transport simulations are at the forefront of the progression of simulation methods in the social sciences

The model presented in Chapter 2 is a spatial rental market based upon the standard monocentric model of urban economics. Landlords base rent adjustments on privately observed demand and tenants engage directed search, i.e. they

are more likely to visit the apartment offering higher utilities. Heterogeneity in tenants' preferences is implicitly assumed. The major insight of this extension of the static monocentric model is the endogenous presence of vacancies due to the search frictions modelled.

The model suffers from problems in dynamic adjustments due to landlords using only private, and often out of date, information to update their beliefs on the state of the market. This led to most of the exploration of the model being conducted in a high demand regime, in which the dynamic behaviour is more reasonable. This model demonstrates the difficulties engendered by including many degrees of freedom in an exploratory model. It has also been established that modelling the endogenous development of nontrivial spatial phenomena is not a straightforward task. The lessons learned from this model were applied in the subsequent model of an aspatial housing search market where the focus is on the search behaviours and market interactions of agents.

Chapter 3 models the rental housing market using agent behaviours based on the search theory literature. Landlords use information on recent market evolutions to set their posted rent, trading-off the expected time required to finding a tenant and the revenues received. Tenants trade-off the utility received from housing against the associated search costs. The model has interesting information effects on the landlords' side of the market. When landlords are homogeneous, they are better off when ill-informed as their tendency to underestimate times-on-the-market causes them to raise rents in an effectively coordinated way. This counter-intuitive result no longer holds when landlords are heterogeneous, as then better information benefits individual landlords. The previous effect can still influence the market as is seen in the fact that with a higher proportion of ill-informed landlords, rents are higher. Tenants utilities are improved when their information is increased. However, in a system where landlords have market power due to the ownership of a scarce good, changes in their behaviours move the market in a much more significant way.

The effects of a vacancy tax were also explored. The presence of an optimal tax for certain model parameters was demonstrated. The potential of the modelled behaviours to produce complex dynamic effects was established when the introduction of a vacancy tax was simulated. There are nontrivial information updating effects, due to alterations in the number of rentals after changes in the tax level. This is a promising basis upon which to study the correlations of price adjustments, trading volumes and times-on-the-market in real estate markets.

The main extensions envisaged to the research presented in this thesis concern the third and most promising model. The theoretical justification of the assumptions used is well developed. Large real estate market databases exist, such that many intricate correlations between market variables are accessible to empirical study (Clayton et al., 2008). The current model has many features that can be adapted to the owner-occupier market. Following the research of

Stein (1995) it can be hoped that through the inclusion of credit constraints in the buying and selling decisions of agents it will be possible to capture the, at least partially endogenous, oscillations that appear to be a fundamental part of the housing market. Novy-Marx (2009) has shown the interest of including the inelastic adjustments of supply to changes in the market state. In standard search theory this elasticity is implicitly infinite. This effect probably plays a role in housing market oscillations that include intricate correlations between prices, times-on-the-market and trading volumes. Agents' heterogeneities and information effects play crucial roles in these dynamics. I hope to have shown in Chapter 3 that the complex correlations seen in the purchasing market can very likely be modelled as an extension of the approach applied in this thesis to rental housing markets.

I believe that it is probable that the major medium-term benefits of agent-based simulations shall be in their use as thought experiments whose theoretical insights shall give pointers towards areas or questions that can benefit from greater empirical study. This is due on the one hand, to the difficulty of calibrating agent-based simulations and the large data requirements discussed previously. On the other hand, it is also based on the simulations presented and the belief that they show a progression towards models that can provide useful insights into real-world systems. Theoretical models have often inspired empirical work in the past and it is also reasonable to hope that the extensions of this work on the housing market will inspire innovative empirical research.

As simulation techniques become more widely integrated in the social sciences the need for collaboration with hard science simulators may decrease. However, the different ways of thinking of complex systems of large numbers of heterogeneous interacting agents should always have cause for mutual inspiration.

I hope that the research presented here and its extensions, will play a role in helping economics to become a science that is more inspired by the actual actions and interactions of real-world agents, and less constrained by analytically convenient abstract hypotheses concerning behaviour. Much research has already moved in this direction notably in behavioural, experimental and neuro economics. The work presented in this thesis does not directly exploit the more evolved insights from these domains.

However by helping to open up the range of productive uses for simulations in economic modelling, I believe it has contributed to the advancement of more empirically inspired theoretical models in economics. This is an important step in developing a deeper understanding of complex evolving socio-economic systems.

Details of the agent simulation

The purpose of this Appendix is to give sufficient information to reproduce all the results presented in for the agent-based model in Chapter 1 .¹

A.1 Homogeneous agents

All simulations for homogeneous agents were performed with 2000 agents. The temporal size of the simulation was equal to the number of agents in all cases, i.e. there were 2000 discrete time units. The capacity of the bottleneck, S , was 2 cars per unit time, this is equal to the departure rate at the social optimum, see Figure 1.2.

For every simulation the agents were assigned their initial departure times such that the overall departure function was that of the social optimum, see Figure 1.2. The agents depart at the capacity of the bottleneck and the first and last to arrive pay the same schedule delay costs.

In every simulation the percentage of agents randomly chosen to try a new departure time was kept constant. The agents chosen each picked a "test" departure time chosen randomly from a flat distribution, the same size as the temporal domain of the simulation, centred on the current departure time. In order to calculate the cost of the new departure time the entire cost function was recalculated assuming that only this agent changed. It should be noted that in the limit of large numbers of agents this is unnecessary. If the cost of the new departure time was less then that paid in the previous iteration by the agent, the agent adopted this new departure time.

At each iteration the congestion experienced by those who departed at each discrete time were calculated, using Equations (1.1) & (1.2). The resulting congestion function was used to calculate the arrival time of every agent. The travel time on uncongested roads was normalised to zero as the constant portion of the travel cost does not affect the dynamics we wished to investigate. The total cost paid by each agent was then calculated by combining the congestion cost and the schedule delay cost as in Equation (1.4). The values of α , β and γ in Equation (1.4) were $\alpha = 2$, $\beta = 1$ and $\gamma = 4$.

¹Note that the code for all the models presented here is available from the author upon request: jmcgreen@ens-lyon.fr

A.2 Heterogeneous agents

In this section we specify how the simulations with heterogeneous agents differed from those with homogeneous agents.

A.2.1 Distribution of schedule delay costs

In the simulations performed with agents who had a distribution of schedule delay cost parameters, presented in Section 1.4.1, 6000 agents were used and there were 6000 discrete time units in the simulation.

When the agents were initialised, each was assigned a "schedule delay multiplier" chosen randomly from a log-normal distribution of mean, $m = 2.4$ and variance, $\sigma = 1.7$. In order to calculate the schedule delay cost for each agent the values of β and γ were multiplied by the "schedule delay multiplier" of each agent, the value of α remained 2 for all agents and the capacity S was unchanged. In order that the initial average cost should be comparable to that for homogeneous agents the schedule delay costs of each agent were divided by 32. This was also required to maintain the relevance of the congestion costs versus exploding schedule delay costs.

A.2.2 Distribution of preferred arrival times

All simulations with agents who had a distribution of preferred arrival times were performed with 2000 agents in simulations with 2000 discrete time units.

When the agents were initialised they were each assigned a preferred arrival time, t^* , from a Gaussian distribution whose variance varied from simulation to simulation. This value of t^* was then used in Equation (1.4) for the calculation of the schedule delay costs.

The values of α , β , γ and the capacity, S were the same as for homogeneous agents.

Deterministic congestion model

Another extension was to consider a deterministic simulation analogue of the agent-based model with homogeneous agents, described in Section 1.3. The set-up was quite similar to the two departure time model presented in Section 1.5.2 except that here, there is a full distribution of discrete departure times.

B.1 Agent reference model

In the agent reference model the following applies:

For each discrete departure time, the number of agents who will choose that departure time over their current departure time is proportional to the total number of agents who have departure times with higher costs. That is, the increase in the departure rate for a given departure time is proportional to the number of agents for whom this departure time represents an opportunity.

For the same departure time, the number of agents who will choose a different departure time is proportional to the total number of departure times that have lower costs. That is, the probability of changing, for an agent currently leaving at a given departure time, is proportional to the number of departure times that represent an opportunity.

B.2 Deterministic analogue

In terms of agents the hypotheses of this model are: there are N infinitely divisible agents¹ who are infinitely sensitive to cost. A certain proportion who leave at each departure time, review their departure time every iteration.

The simulation has N possible discrete departure times and everybody wishes to arrive at t^* . The capacity is such that the shortest time in which all the agents can pass the bottleneck is $T = N/2$. The capacity of the bottleneck is hence 2 agents per time unit.

¹That is level of departures at any time is a real number, and the integral of departures is a constant N . In the agent model there is necessarily an integer number of departures at each departure time.

The evolution of the departure rate at a given departure time follows closely that outlined above in Section B.1. It is shown formally below, where i stands for iteration and j is the departure time:

$$r_i(j) = r_{i-1}(j) - \frac{\epsilon}{100} r_{i-1}(j) + \frac{\epsilon N}{100} * \frac{\Theta_i(j)}{\Omega_i} \quad (\text{B.1})$$

where ϵ is the reviewing rate and $\Theta(j)$ is the total number of agents at other departure times who paid, at iteration $i - 1$, a total cost greater than that of departure time j . The normalising factor Ω_i is the cumulative number of agents who would like to move to each individual departure time:

$$\Omega_i = \sum_j^N \Theta_i(j)$$

and

$$\Theta_i(j) = \sum_{l \neq j}^N r_{i-1}(l) \theta(c_{i-1}(j) - c_{i-1}(l))$$

where $r(j)$ is the departure rate at time j , $\theta(z - y) = 0$, if $z < y$ and $\theta(z - y) = 1$ if $z > y$. In Equation (B.1) the second term is the constant proportion of those leaving at departure time j who review their departure time. The third term is the proportion of all those currently reviewing their departure time who shall choose departure time j . Note that it is possible here to review one's departure time and choose to depart again at the same departure time.

The evolution mechanism described above corresponds closely to that of the agent-based model. In both models agents are no more likely to try a good departure time over a bad one.²

Results shown in Figures B.2 & B.1 are for simulations that began with departure times distributed at the social optimum, i.e. with no congestion. The first order continuous schedule delay costs (SDCs) described in Section 1.5.1 were applied to obtain the results in Figure B.1. The travel cost is $\alpha = 2$, the schedule delay cost parameters are $A = 1$ and $B = 4$, see Equation (1.8).

We can see in Figure B.1-*Right* that the evolution of the average cost is unstable. Figure B.1-*Left* shows that there are repeated waves of more expensive departure times becoming progressively earlier (moving down from left to right). This is because the traffic jam begins earlier each day as agents avoid the peak congestion time and the costs of arriving late. This movement is halted when

²There there is however one other small difference, in the agent model those trying a new departure time chose a departure time from a uniform distribution equal in size to the simulation domain centred on their current departure time. In this deterministic version all departure times within the domain of model were equally likely to be tested.

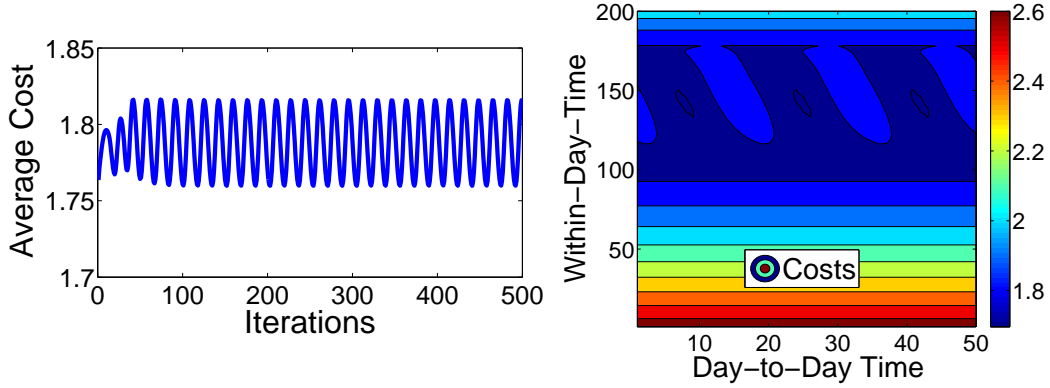


Figure B.1: *Left*: The evolution of the average costs through 500 iterations, with 1st order *continuous* SDCs. *Right*: The evolution of the total cost across departure times and from day-to-day over the last 50 iterations shown in Figure B.1*Left*.

later departure times become attractive, in a sufficient number, and so attract more agents to these later departure times. The total congestion level falls, as many agents move to the end of the congestion period and so impose less costs on the fewer agents who depart after them.

The evolution of the congestion level and hence the total cost depends upon the number of agents at each departure time and the number of opportunities that they see, leading to a non-trivial evolution pattern.

The 1st order discontinuous-schedule-delay-cost form described in Section 1.2.2 for the agent model, was used for the simulation results shown in Figure B.2. As in the agent-based and the two-departure-time models the travel cost is $\alpha = 2$, the cost of arriving early is $\beta = 1$ and the cost of arriving late is $\gamma = 4$.

Figure B.2-*Right* shows that the evolution of the average cost with schedule delay costs that are discontinuous in the first derivative is more unstable than in Figure B.1. Figure B.2-*Left* shows that the cheapest departure times again become progressively later in repeated waves, but this time their evolution is more complicated. Indeed, higher period oscillations of the average cost have been observed with first-order discontinuous schedule delay costs.

The lesson of this demonstration, which enables the system's dynamics to be more clearly seen, is on the one hand that continuous functions lead to more stable behaviour and on the other hand that the evolution of the congestion level for a given departure time depends in a nontrivial manner on the entire distribution of departure times.

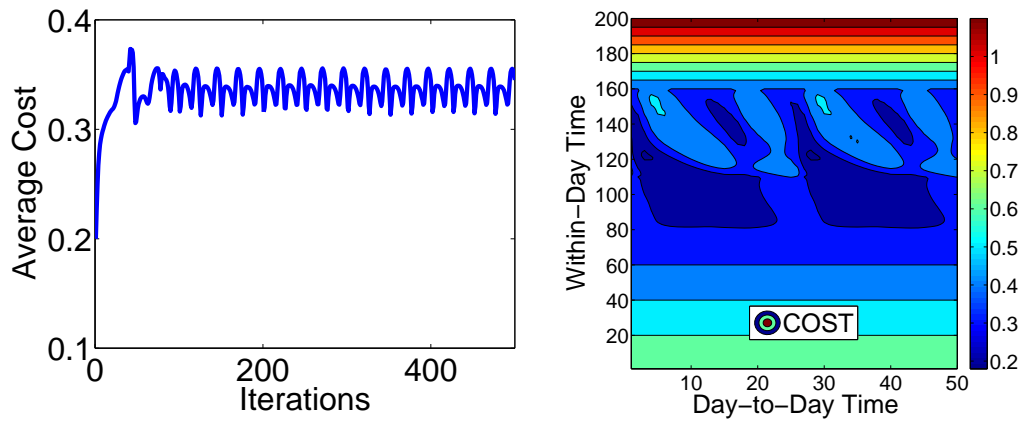


Figure B.2: *Left*: The evolution of the average costs through 500 iterations, with 1st order *discontinuous* SDCs. *Right*: The evolution of the total cost across departure times and from day-to-day over the last 50 iterations shown in Figure B.2Left

2D Monocentric urban model

In Figure C.1 we can see for the two-dimensional town a speedy convergence from high a initialisation $R_I = 500$ and a much slower convergence from a low initialisation $R_I = 250$.¹ This is similar to the one-dimensional case.

The two dimensional sensitivity tests are presented here. All the results presented are for simulations over 2000 iterations that began with $R_I = 500$. Recall that the average rent in the fully populated town of the theoretical equilibrium is 333. The main convergence criteria is the average utility of tenants. Figures C.1-*Left* & *Right* show that for the default parameters that began with this high level of rents, the simulation converges after about 500 iterations.

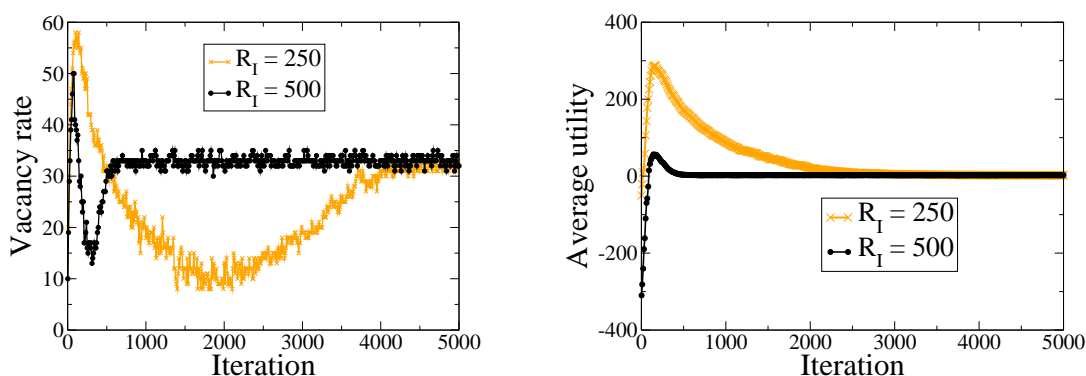


Figure C.1: *Left*: Graph of the vacancy rates in a two-dimensional town with initial rents of $R_I = 250$ and $R_I = 500$. *Right*: The average utility of tenants for the same simulations as *Left*.

Figure C.2-*Left* shows that as the rent increase parameter D rises the vacancy rate falls, this is as in the one-dimensional case.

In Figure C.2-*Right* the effect of varying the maximum allowed rent raise M_R can be seen. At very low M_R the average utility is relatively high as rent increases are severely limited. Rents need to be lower for the overall level of reductions

¹Note that for a two-dimensional city, the average rent paid at the theoretical equilibrium is $333\frac{1}{3}$. This can be found by dividing the total rent paid by the population πr_{max}^2 . The total rent is $\int_0^{r_{max}} \pi r^2 (Y - rT) dr$, which is the integral from the centre up to the maximum radius of the town of the population at distance r from the centre, multiplied by the rent charged at distance r .

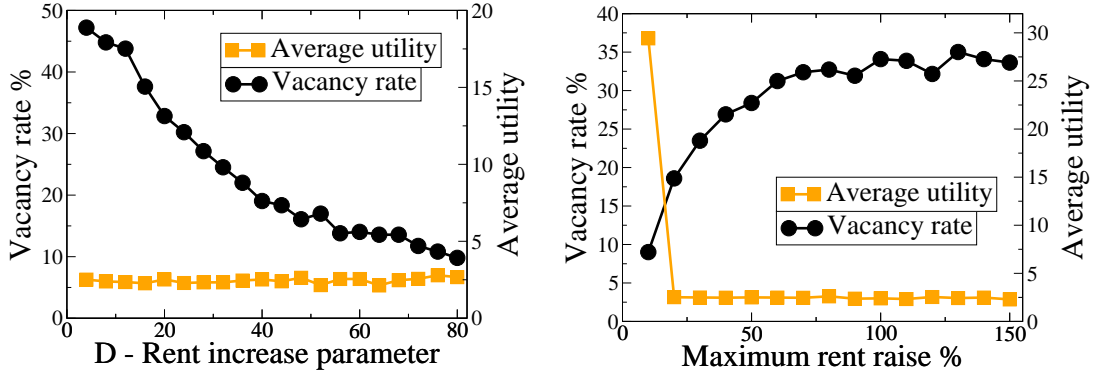


Figure C.2: *Left*: The vacancy rate and average utility for variations in the rent increase parameter D , see Section 2.3.3. *Right*: The vacancy rate and average utility for variations in the maximum rent raise allowed M_R . Both figures present results for a two-dimensional town.

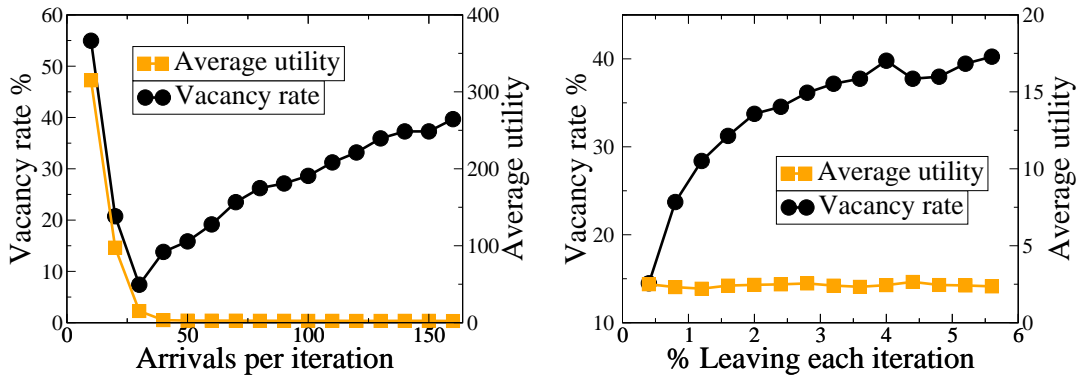


Figure C.3: *Left*: The vacancy rate and average utility for variations in the number of searchers Λ arriving each iteration. *Right*: The vacancy rate and average utility for variations in the percentage of departures L . Both figures present results for a two-dimensional town.

to equal the overall level of increases. It is clear that at very low M_R this effect must also exist in one dimension.

Figure C.3-*Left* shows that, as for the one-dimensional model at very low demand, that is small Λ , rents are lower and the vacancy rate is high. Here the default departure rate is $L = 2\%$, which means that approximately 16 agents leave each iteration if the town is full. Hence if the arrival rate is below this value the market adjusts through both increased vacancies and reduced rents. The increasing vacancy rate, with increasing demand in the high demand regime, is as in the one dimensional case.

In Figure C.3-*Right* we see that increases in the percentage of departing tenants increases the vacancy rate, as the case in one dimension.

Figure C.4-*Left* shows that there are two regimes for the effects of varying

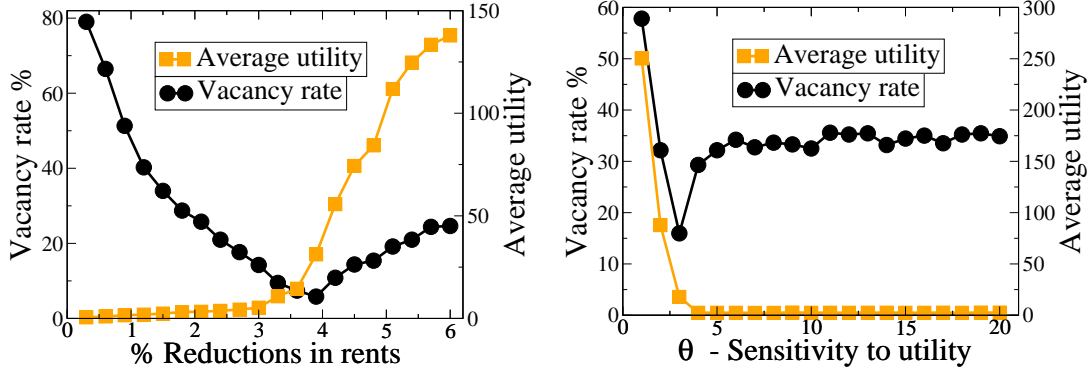


Figure C.4: *Left:* The vacancy rate and average utility for variations in the percentage reductions of the rents of vacant residences ρ . *Right:* The vacancy rate and average utility for variations in searchers sensitivity to differences in utility. Both figures present results for a two-dimensional town.

the reductions in rent parameter ρ , just as in the 1D case. Here, the vacancy rate begins to rise once $\rho > 4$.

In Figure C.4-*Right* we see that increases in the sensitivity to differences in utility do not change the steady-state configuration above a value of 7. The regime in which rising sensitivity to utility differences increases the vacancy rate due to less rent increases being restricted by M_R as described for the one-dimensional case in Section 2.4.2 can more clearly be seen in the two-dimensional case.

In Figure C.5 we see that with a fixed percentage of arrivals the effect on the steady-state of varying the transport cost is negligible in terms of the vacancy rate and the utilities of tenants (and therefore also in terms of rents). As in the one dimensional case, a fixed percentage of the town size $\hat{\Lambda} = 15\%$ arrives each iteration². The size of the town clearly varies with a changing transport cost as shown in Figure 2.10.

²The default transport cost in two dimensions T is 60. The corresponding town size is of the order of 800. The default number of arrivals in two dimensions is $\Lambda = 120$ which is 15% of 800.

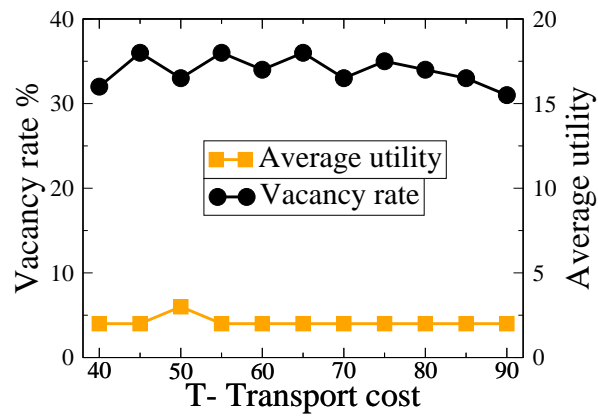


Figure C.5: The vacancy rate and average utility for variations in the transport cost T . This figure presents results for a two-dimensional town.

Sensitivity tests: Search market

D.1 Reviewing frequency

Alterations in the steady-state configuration due to changes in the reviewing frequency parameter, F , are shown in Figure D.1.

As F rises initially the population increases and rents fall. When landlords review their rents less frequently (higher F) they appear less able to extract a surplus from imperfectly informed tenants.

D.2 Maximum rent increase

Changes in the maximum allowed rent increase I_{max} do not have a dramatic effect on the steady-state configuration, see Figure D.2. Average rents initially increase slightly, as is to be expected. The percentage of maximum rent increases also decreases.

D.3 Occupation duration

Increases in the expected duration of residence X , increase the average rent as the profits from higher rents for landlords become greater, see Figure D.3. This has the effect of reducing the population as some tenants can no longer afford the market rents.

D.4 Size

The size parameter S plays an important role, as the level of information available to agents depends directly on the size of the city. As the city size increases, so does the occupation rate due to less errors being made by landlords in setting rents, and by tenants in refusing rents that they should normally accept. The information effect is present for both landlords and tenants, and in both cases pushes rents lower and therefore the population higher, with increasing size. As seen in section 3.4.3 the information effect is stronger for landlords. We note that even for a town size of 45,000 residences, the information effect has not yet

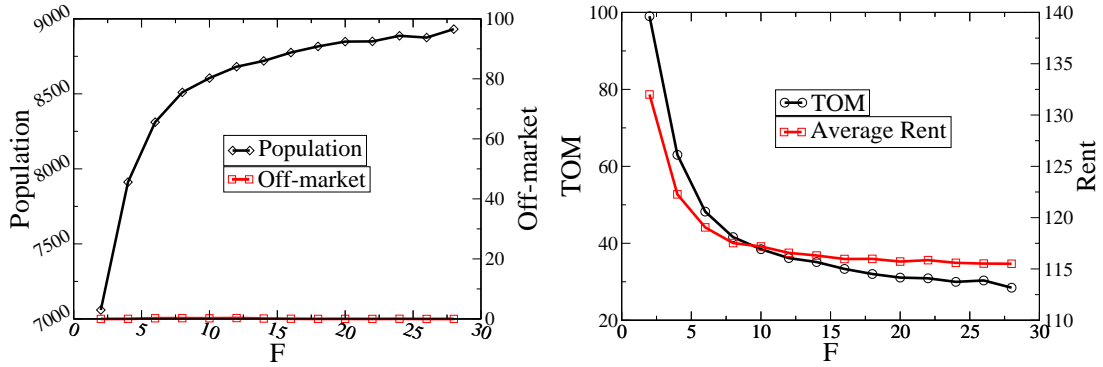


Figure D.1: *Left*: The variations in population and the number of landlords off-the-market as a function of the landlord memory and updating parameter F . *Right*: The average rent and the average time-on-the-market (TOM).

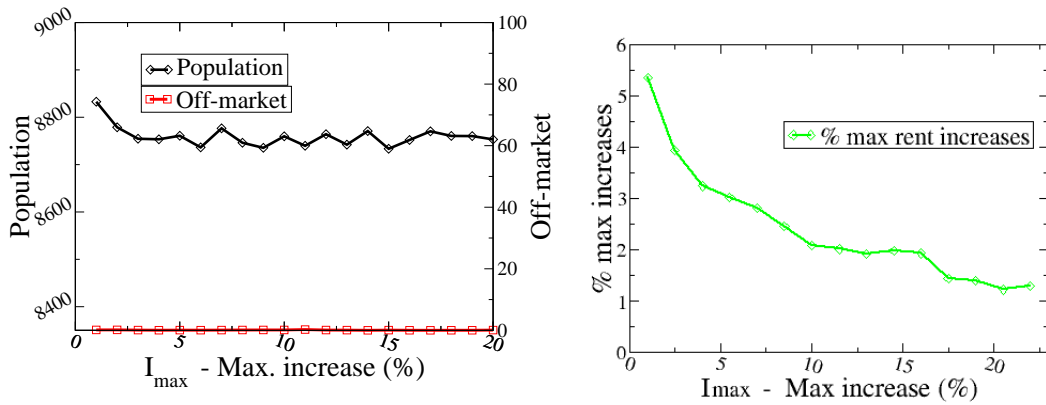


Figure D.2: *Left*: The steady-state levels of population and the number of landlords off-the-market, for changes in the maximum allowed increase of rent. *Right*: The percentage of maximum rent increases averaged over 1000 iterations.

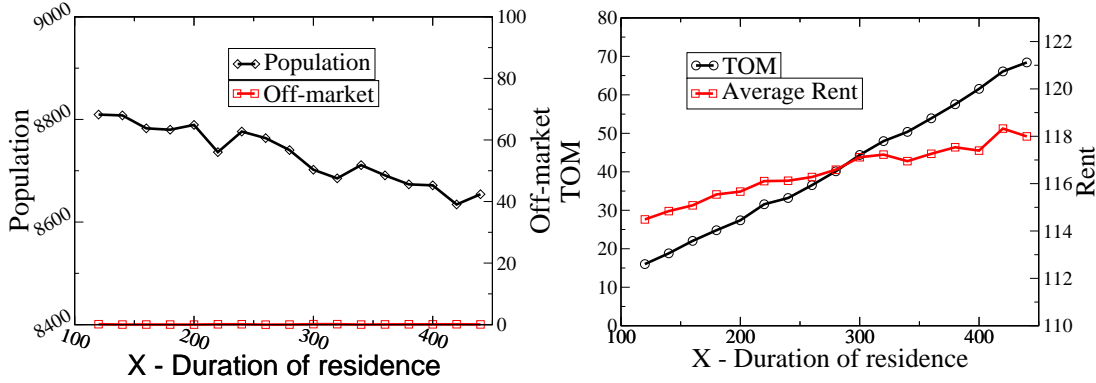


Figure D.3: *Left*: The variations in population and the number of landlords off-the-market as a function of the landlord expected duration of residence parameter X . *Right*: The average rent and the average time-on-the-market (TOM).

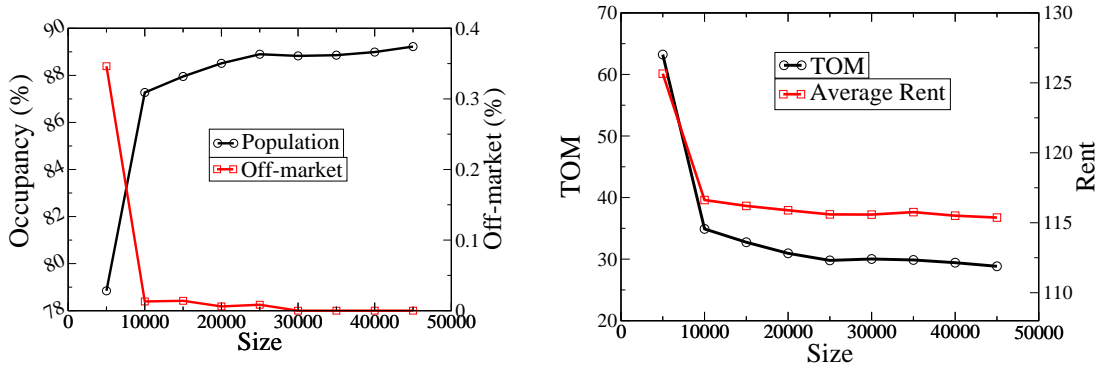


Figure D.4: *Left*: The variations in population and the number of landlords off-the-market as a function of town size. *Right*: The corresponding average rent and the average time-on-the-market (TOM).

converged to an asymptotic value. This value is likely to be above many urban rental markets' supply of close substitutes once questions of type and location are taken into account. We feel that the presence of finite size effects is a positive feature of the model.

D.5 Landlords' maintenance costs

As landlords' maintenance costs C_L rise from low levels the average rent falls as does the TOM. Rising maintenance costs, which are paid whether the apartment is occupied or not, both increase the cost of search and decrease the benefit of renting at a given rent. There are hence two adjusting forces in landlords' reactions to rising maintenance costs. The first is to reduce rents to reduce TOM. The second is to increase rents to raise the benefits of renting in order to cover

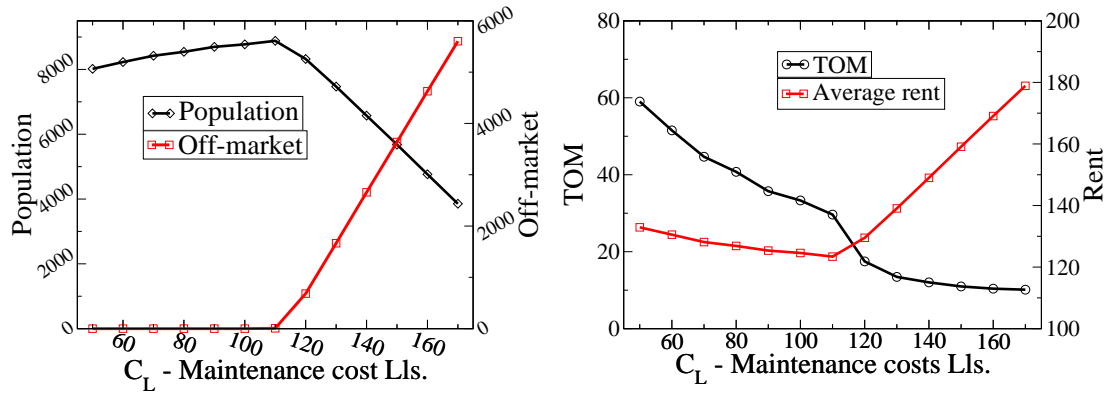


Figure D.5: *Left*: The variations in population and the number of landlords off-the-market as a function of landlords' maintenance costs. *Right*: The corresponding average rent and the average time-on-the-market (TOM).

the increasing costs. The first effect dominates in response to maintenance cost increases from low levels and the second dominates when maintenance cost are already high, Figures D.5-*Left* & *Right*.

The population variation follows that of the average rent, Figure D.5-*Left*. The number of landlords withdrawing from the market increases markedly once landlords' maintenance costs are above the average steady-state rent with the default parameters, which is about 117, see Figure 3.4-*Right*.

Resumé

Le paradigme du modèle multi-agent (Agent Based Modelling ABM) a vu s'accroître comme jamais son utilisation dans les sciences sociales au cours de la dernière décennie (Tesfatsion and Judd, 2006; Amblard, 2007). Ses origines méthodologiques remontent au moins aussi loin que Clarkson and Simon (1960); Orcutt (1960). Il lui reste cependant à être totalement accepté en tant que méthode scientifique standard dans les sciences sociales bien que ses avantages théoriques fondamentales sont bien compris Bradburd et al. (2006); Colander et al. (2008). Le but principal de ce travail est d'évaluer le potentiel du modèles multi-agents pour agrandir notre compréhension des systèmes économiques urbains.

La forte augmentation en puissance de calcul au cours des dernières décennies nous a permis d'appréhender la modélisation des systèmes économiques d'une nouvelle manière. L'utilisation des simulations est devenue une pratique standard en physique où elles ont obtenu un grand succès. La généralisation de l'utilisation de simulations a été plus lente à se développer dans les sciences sociales en grande partie à cause de la grande complexité des entités modélisées. Cela a créé une situation dans laquelle ceux spécialisés en sciences sociales n'avaient pas habituellement les compétences requises pour créer des modèles de simulations des systèmes socio-économiques. Alors que d'un autre côté, les spécialistes en méthodes de simulations, incluant les informaticiens et physiciens, n'ont que peu souvent une formation solide en sciences sociales. Il est donc important, de faire se rejoindre ces compétences provenant de différents domaines pour amener plus loin les modèles de simulations de systèmes sociaux. Beaucoup de travail a été fait par les physiciens dans la modélisation de systèmes financier où la disponibilité de grandes quantités de données a permis une approche très concrète (Bouchaud, 2009; Rickles, à paraître). Les méthodes physiques peuvent être appliquées encore plus facilement aux systèmes urbains quand des données de grande qualité sont disponibles (Jensen, 2006). Dans cette thèse, je modélise des systèmes urbains qui, bien que manquant souvent de données de grande qualité par rapport à celles disponibles pour les marchés financiers, ont beaucoup de caractéristiques qui les rendent convenable pour l'application de méthodes de simulations.

Dans cette introduction, je résumerai l'approche suivie pendant la thèse. D'abord je présenterai les principales motivations d'entreprendre ce projet, à savoir modéliser les interactions entre les systèmes de transports urbains et la localisation des activités. J'expliquerai ensuite les avantages théoriques et quelques-

unes des difficultés rencontrés dans l'utilisation des modèles multi-agents dans l'étude des systèmes socio économiques. Les systèmes spécifiques qui ont été étudiés sont présentés ensuite. Le procédé de distillation des hypothèses du modèle qui mène à une combinaison objet-méthode, permettant d'atteindre des résultats pertinents, est mis en valeur. Les caractéristiques des systèmes qui eux même contribuent à des modèles multi-agent réussis, sont par la suite discutés ainsi que quelques indications aidant à l'obtention de résultats intéressants issus de tels modèles.

La motivation initiale de ce projet était d'étudier le potentiel des systèmes multi-agent pour fournir un aperçu du fonctionnement des systèmes urbains. Les systèmes urbains sont d'une importance capitale pour l'économie générale, en effet la plupart des richesses est créée dans les villes (Fujita et Thisse, 2003). Les systèmes urbains sont des systèmes évolutifs complexes de grands nombres d'agents hétérogènes répartis spatialement et interagissant. C'est donc un domaine dans lequel les simulations ont un grand champ pour surpasser les modèles analytiques.

Mon intention n'était pas d'étudier une gamme complète de systèmes multi-agent. Cela s'étend de 'modèle jouet' petite échelle comme Schelling (1978) à des modèles grandes échelles qui intègre les interactions entre le transport et l'occupation des sols de la totalité des zones urbaines (Salvini et Miller, 2005). Ces grands modèles nécessitent une quantité de données considérable, d'intensifs efforts de calibration and ne peuvent jamais complètement échapper à un aspect 'boîte noire' au produit fini, pour étudier voir Timmermans (2003). Mon but était de créer des modèles avec des hypothèses motivées dont la dynamique peut être totalement comprise et qui reproduit les caractéristiques dynamiques majeures des systèmes du monde réel.

Le point principal était les interactions des systèmes de transports et les décisions de localisations des agents. Actuellement, les modèles qui prédisent les performances des infrastructures de transports partant d'une localisation donnée des agents sont assez bien développés (de Palma et Marchal, 2002; Arentze et Timmermans, 2004). L'approche inverse, partant d'une infrastructure de transport et essayant de prédire la future forme urbaine, qui est le regroupement des choix de localisations qui sont fait sur le long terme, sont de façon compréhensible moins développées. Le but final est de prendre en compte la boucle entière d'interaction entre l'évolution conjointe du système de transport et la localisation des populations et des activités. De tels modèles prospectifs sont en stade de développement (Waddell, 2002; Timmermans, 2003). Les modèles basés agent ont le potentiel de modéliser des interactions souvent ignorées dans les modèles économiques mais qui sont cruciaux pour la compréhension du développement urbain (Glaeser et al. 2000).

En pratique cette motivation initiale a abouti à la création de trois modèles de systèmes urbains. J'examinerai la stabilité d'un modèle de congestion standard de Vickrey (1969) au chapitre 1. Un modèle multi-agents sur le modèle économique urbain monocentrique classique d'Alonso (1964), qui inclut à la fois l'aspect spatial et l'aspect marché, est développé en chapitre 2. Enfin un modèle non spatial du marché des logements locatifs, dans lequel une attention particulière a été donnée aux comportements de recherche des agents, est présenté au chapitre 3.

Les avantages des méthodes de simulations multi-agents sont nombreux et considérables. Parmi les principaux avantages on trouve la capacité à modéliser des évolutions dynamiques. Les simulations multi-agents offrent une grande flexibilité dans la modélisation des caractéristiques d'agents hétéroclites et leurs interactions. La compréhension de l'émergence de formes macro et de dynamiques macro à partir des interactions microscopiques des agents est un avantage exceptionnel des systèmes multi-agents. En effet, les modèles multi-agents peuvent être validés simultanément par à la fois les comportements niveau système et le comportement niveau micro (LeBaron et Tesfatsion, 2008). Les modèles multi-agents offrent aussi la possibilité de modéliser les systèmes d'une manière naturelle et cohérente qui n'est pas accessible aux modèles analytiques. Par exemple, des agents peuvent se rencontrer et négocier des biens en imitation des marchés réels (Kirman et Vriend, 2001).

La nature dynamique des modèles multi-agents permet la recherche des équilibres analytiques et leur stabilité. Un exemple sera étudié au chapitre 1 à travers une simulation multi-agents sur le modèle classique de Vickrey (1969) des embouteillages. La stabilité des équilibres est une question souvent négligée dans les modèles économiques analytiques. Mon but a été d'utiliser des comportements simples d'agent pour explorer les forces motrices derrière les dynamiques du système modélisé, tout en mettant l'accent sur le contrôle de leur stabilité.

La plus grande flexibilité des simulations par rapport aux modèles analytiques permet un panel beaucoup plus réaliste d'hypothèses à faire. Ces hypothèses font le rapprochement entre les entités elles-mêmes, leurs environnements, et leurs interactions. Des formes multiples d'interactions sont possibles et peuvent être directes ou indirectes. Dans ce dernier cas, les agents interagissent entre eux via leur impacte sur leur environnement partagé. Un avantage majeur des modèles multi-agents est l'intégration naturelle des hétérogénéités des caractéristiques des agents, qui est un élément crucial pour des modèles économiques réalistes (Kirman, 1992). Cela sera abordé en particulier aux chapitres 1 et 3, où les hétérogénéités dans les caractéristiques des agents sont en fait nécessaires dans le but d'atteindre des modèles stables. Blundell et Stoker (2005) présentent des méthodes concrètes pour gérer les problèmes d'agrégation dus aux hétérogénéités.

J'ai testé au cours de cette thèse la validité de modèles analytiques existant utilisant des simulations multi-agents. Un avantage clair de cette approche est qu'elle fournit une solide base théorique pour la modélisation de décisions. On peut aussi s'attendre à une meilleure acceptation et diffusion des méthodes et aperçus multi-agents quand ceux-ci sont clairement fondés sur la théorie économique existante. Ces questions seront principalement traitées au chapitre 3.

Les difficultés associées aux méthodes de simulation basée agents sont aussi nombreuses, et je dois maintenant souligner pourquoi une grande considération doit être donnée sur comment surmonter ou au moins atténuer ces difficultés avant de se lancer dans un projet de modélisation multi-agents. Il est possible pour le non initié de croire que les méthodes multi-agents peuvent fournir des résultats dans presque tout système. En pratique, il y a beaucoup de difficultés, notamment dans la validation et dans la calibration des modèles de simulation aux données concrètes.

Les données en grande quantité et de types appropriés requises pour calibrer des modèles multi agents grande échelle sont très difficiles à obtenir. Cela entraîne beaucoup de difficultés dans leur calibration. Même avec de bonnes données, sa transformation sous forme utilisable pour le modèle et le procédé de calibration ultérieur sont extrêmement onéreux. La calibration de modèles de simulation complexes grande échelle de systèmes socio-économiques par nature imprévisibles apparaît être une entreprise très problématique. En effet, même les plus simples modèles multi-agents sont relativement riches en paramètres dont la calibration n'est pas simple.

Tous les modèles multi-agents peuvent être considérés comme un petit monde duquel chaque détail doit être décrit. Les modèles ont souvent beaucoup plus d'hypothèses que le nombre de paramètres à calibrer pourrait l'indiquer. Ces hypothèses potentiellement cachés peuvent concerner entre autres : l'ordre des interactions (aléatoire ou séquentiel), la distribution des caractéristiques des agents (par exemple, uniforme ou gaussienne), la nature fermée ou ouverte du système, la nature de l'information disponible aux agents (locale ou globale) etc.

La découverte d'hypothèses existantes établies théoriquement ou concrètement pour le comportement des agents est aussi une tâche difficile, voir Roth (2007). La plupart de la littérature économique ne s'est pas préoccupé de la compréhension des comportements actuels des agents humains. Une telle recherche est principalement conduite en psychologie, en science cognitive and récemment dans le domaine nouveau de l'économie comportementale.

Utilisant des indices de la psychologie, l'économie comportementale a mis en évidence beaucoup de suppositions erronées utilisées dans les modèles économiques analytiques (Camerer et al. 2003). Alors que ce travail est encourageant pour les modèles multi-agents, l'intégration de ces indices n'est pas souvent évidente. Les indices provenant des sciences cognitives sont aussi une ressource utile et les méthodes multi-agents sont vues comme une approche prometteuse à l'intérieure

des sciences cognitives (Conte 2002). Le travail sur les réelles décisions prises par les agents humains qui ne sont, contrairement à la théorie néo-classique de l'économie, ni complètement rationnel ou omniscient ou totalement égoïste, a un immense potentiel d'utilisation pour les modèles multi-agents.

Alors que baser les modèles de simulations multi-agents sur d'autres existants analytiques est souvent très productif, l'intégration de certaines suppositions analytiques à l'intérieur du comportements des agents n'est pas simple. En effet, souhaiter coller trop fortement aux bases analytiques peut entraver le complet épanouissement de la modélisation multi-agent. Différents exemples de ce danger seront abordés aux chapitres 2 et 3. Dans différents domaines et pour différents modèles spécifiques, les bases analytiques quand elles existent, doivent avoir des degrés variables de pertinence. Comme le montre le reste de la thèse, la étendue réelle des difficultés et avantages de la modélisation multi-agent sont seulement découverts en pratique. Je crois, et espère avoir montré, que les avantages valent les efforts dues à cette tâche.

Je dois maintenant décrire les modèles spécifiques que j'ai étudiés avec les modèles multi-agents.

Dans le premier chapitre, j'examine la stabilité d'un modèle bien connu de la congestion routière. La stabilité de cet équilibre de Nash n'a pas été établie et les simulations de circulation inspirées par ce modèle, voir de Palma et Marchal (2002), ont montré des instabilités lorsque l'on tente d'ajouter des évolutions au jour le jour au modèle de base. Nous avons souhaité développer un modèle multi-agents pour examiner la stabilité du modèle d'origine et examiner la possibilité d'ajouter des comportements dynamiques simples aux agents dans des simulations de circulation à grande échelle. Il a été argumenté de façon convaincante que les modèles économiques microscopiques sont une étape nécessaire vers une meilleure compréhension de la dynamique des systèmes de transport en règle générale (Arnott, 2001).

Le modèle standard d'embouteillage par Vickrey (1969) est un modèle dynamique 'intra-journalière', où les agents sont censés optimiser leur heure de départ pour la navette du matin. Pour ce faire, ils marchandent le coût de ne pas arriver à l'heure et le coût du pire embouteillage. Ceci est ici étendu à un modèle de 'dynamique double' multi-agents, dans lequel la distribution de l'heure de départ évolue de jour en jour. Les agents apprennent à-travers l'expérience : avec une certaine probabilité chaque jour, ils tentent une nouvelle heure de départ. Si leur coût total personnel réduit, ils adoptent cette nouvelle heure de départ. Il est montré qu'avec des agents parfaitement homogènes, la solution d'équilibre au modèle standard de Nash ne peut pas être obtenue dynamiquement : le système est alors instable (Mc Breen et al. 2006).

Afin de mieux comprendre les causes de l'instabilité du système, un certain

nombre d'extensions sont explorées. Tout d'abord, la difficulté de l'utilisation des méthodes analytiques pour modéliser de tel système est démontrée. Ce modèle a inspiré l'étude d'un modèle simple à deux heures de départ. La stabilité est atteinte grâce à l'introduction d'hétérogénéités dans les heures d'arrivée désirées par les agents. Cela modifie la structure du modèle en réduisant l'intensité de la concurrence pour les mêmes heures d'arrivée. Une extension pour illustrer la possibilité d'étude de ce modèle stochastique dynamique avec une approximation déterministe est aussi présentée dans l'Appendice B. Ces études évoquent l'intrigante possibilité d'utiliser des simulations comme des modèles d'exploration qui peuvent ensuite être simplifiée en un modèle analytique, en vue de procéder à une analyse plus rigoureuse (Varenne, 2008).

Dans le chapitre 2, j'examine le potentiel des modèles d'agents pour reproduire, et donc pour aider à mieux comprendre, les dynamiques spatiales des marchés de l'immobilier urbain. L'accent est mis sur les interactions des systèmes de transport et les décisions de localisation. Pour ce faire, j'ai décidé de commencer par la base fournie de la théorie économique standard urbaine (Fujita, 1989), et d'ajouter des dynamiques d'agents à ce cadre analytique statique dans l'espoir de générer des résultats intéressants nouveaux. Des progrès ont été réalisés dans ce sens par Caruso et al. (2007) qui s'appuient sur le modèle urbain à un centre en utilisant des automates cellulaires. Le modèle développé est un modèle spatial qui prend en compte les coûts de transport à partir d'un centre exogène. Ces coûts déterminent alors la localisation des décisions d'agents.

Dans notre modèle, chaque cellule d'une simple grille représente une résidence appartenant à un propriétaire qui souhaite maximiser ses revenus. Un certain nombre de locataires arrivent, à chaque itération, et tentent de trouver le meilleur logement. A chaque itération, un certain nombre de locataires quittent la ville. Les comportements à préciser sont le prix fixé par les propriétaires et les mécanismes de recherche des locataires. Le modèle est étudié à une et deux dimensions.

Il sera démontré que l'inclusion d'un mécanisme simple de recherche conduit à des logements vacants ; ce qui constitue un phénomène bien connu de toutes les marchés de recherche. Le taux de vacances ne peut être compris avec le modèle standard statique. Un marché de recherche est un marché pour lequel une des hypothèses Walrasiennes clés sur les agents économiques, d'être parfaitement informés, ne tiennent pas. Au lieu d'un marché unique, avec un commissaire priseur virtuel qui décide des prix de sorte que la demande soit égale à l'offre, les agents doivent chercher des débouchés commerciaux, ce qui a un coût. Les décisions d'échanges dépendent de la perception de l'utilité de poursuivre les recherches. Cela est montré être la principale différence entre ce modèle d'interaction dynamique d'agents et le modèle standard économique urbain (chapitres 2 et 3).

Toutefois, les comportements des agents dans ce modèle spatial sont quelque peu arbitraires. Dans le chapitre 3, l'accent est mis sur les comportements et les mécanismes d'interaction des propriétaires et des potentiels locataires dans un marché du logement locatif aspatial.

En vue de produire des résultats pertinents et nouveaux, j'avais appris à partir de travaux antérieurs, que le choix de l'objet est aussi important que le choix de la méthode. Après avoir évalué les forces et les faiblesses des modèles multi-agents, j'ai décidé qu'un modèle du marché locatif des logements est probablement le plus simple modèle de recherche pour lequel on peut étudier le comportement des agents des deux côtés du marché. Ces comportements donnent lieu à des corrélations complexes dans l'évolution des prix, des délais de marché et des volumes échangés (Fisher et al. 2003). Cela laisse espérer aboutir à la réinjection ultérieure de ces comportements modélisés et les interactions dans un modèle spatial. Je sentais que ce serait une approche plus productive que de développer les aspects spatiaux du marché simultanément. Cette approche permet une plus grande liberté dans l'étude des évolutions de marché dynamique.

Les marchés immobiliers sont clairement des systèmes d'une importance vitale pour l'économie moderne. Les frictions de recherche, qui sont la difficulté pour les locataires de trouver un logement et pour les propriétaires de trouver des locataires, présents dans les marchés immobiliers, sont d'intérêt général pour toutes les recherches de marché. Il s'agit d'un domaine clé dans lequel un modèle robuste de simulation, microscopique et dynamique, a une capacité d'extension, tout d'abord à d'autres marchés de l'immobilier, ensuite au marché du travail.

J'ai simulé un marché clos de logements locatifs avec recherche et désaccord correspondant, dans laquelle les deux agents (propriétaire et locataire) sont imparfaitement informés. Un ensemble homogène de propriétaires louent afin de maximiser leur revenu, utilisant de l'information sur le marché pour estimer la relation entre les loyers affichés et le temps sur le marché (TOM), (c.-à.-d. le temps qu'un propriétaire devraient normalement attendre avant de trouver un locataire). Les locataires, hétérogènes en termes de revenu, s'engagent dans une recherche aléatoire, en acceptant les résidences en fonction de leurs goûts particuliers pour le logement et une utilité de réservation basée sur l'information de la répartition des offres. L'état d'équilibre vers laquelle la simulation évolue montre une dispersion des prix, des temps de recherche non nul et des logements vacants.

J'analyse les effets de l'accroissement de l'information disponible pour les deux catégories d'agents. Lorsque les locataires voient une plus grande partie de la distribution des offres, ils apprennent à refuser des loyers élevés et leur utilité moyenne augmente. En revanche, lorsque les propriétaires sont homogènes et leur niveau d'information est réduit, leur bien-être peuvent s'améliorer tant

que leurs surestimations du meilleur loyer affiché augmentent les loyers. Moins surprenant, les propriétaires les mieux informés sont mieux lorsque des hétérogénéités dans les niveaux d'information sont introduites. Les effets d'une taxe sur les logements vacants sont également analysés, à la fois en terme statique et dynamique. Il a été constaté que, pour certaines valeurs de paramètre, un niveau de taxation existe pour lequel les bénéfices du bien-être de la réduction des logements vacants, TOM et loyers, dominent les effets négatifs de la taxe sur l'offre. L'ajustement dynamique aux grands changements dans le taux d'imposition est montré causer des fluctuations à court terme encore plus grandes sur l'état marché. Cela souligne les risques potentiels des interventions fortes, tant que les répercussions peuvent être difficiles de prédire avec certitude.

Dans le cadre de ce projet exploratoire, des progrès doivent être effectués, non seulement sur les résultats obtenus, mais aussi sur la nouvelle compréhension des types d'objets les plus adaptés à cette forme de modélisation. Il sera vu au chapitre 2 que tenter d'inclure un trop grand nombre de dimensions (degrés de liberté) dans un modèle simpliste limite la pertinence des résultats extraits. Je soutiens que ce n'est pas l'approche optimale, et que lorsque des modèles distincts plus simple de parties du système peuvent être construits, les connaissances acquises peuvent conduire à un modèle composite plus satisfaisant. Il sera soutenu dans les chapitres 2 et 3 qu'il est préférable, dans la mesure du possible, de créer d'abord un modèle robuste des éléments constitutifs les plus simples d'un système, en vue de mieux comprendre le système composite. Dans cette thèse, cela signifie se concentrer sur un marché aspatial de l'habitation, au chapitre 3, plutôt que de continuer à développer un modèle spatial de marché du logement, avant que les dynamiques et les interactions des agents dans le marché ne soient bien comprises.

Il a été appris au cours de cette thèse que certains attributs sont très souvent utile dans la production de modèles stables, ayant de bons comportements lors de la simulation. Dans les modèles économiques standard, avec des agents parfaitement informés, les résultats sont obtenus en supposant l'existence de l'équilibre. Lorsque les interactions dynamiques sont incluses explicitement entre agents modélisés, ce luxe est mis de côté. Il a été découvert que les modèles qui simulent étroitement les hypothèses analytiques standard, telles que les agents homogènes bien informés, sont sujettes à l'instabilité. La raison est que l'excès de ressemblance entre les agents les amènent à adopter des comportements similaires dans des situations similaires et, par conséquent, le système est instable dans son ensemble. L'introduction de stochasticité peut aussi avoir un effet stabilisateur sur l'ensemble du système. Ce premier résultat contre intuitif se produit en raison de l'hétérogénéité des comportements des agents, ce qui induit une distribution moyenne stable des comportements à adopter et, par conséquent, évite une situ-

ation dans laquelle tous les agents fassent la même chose au même moment. L'effet stabilisateur potentiel des hétérogénéités a été pointé par Kirman (1992).

Modéliser des systèmes contenant un grand nombre d'agents permet d'empêcher qu'un seul agent, ou bien un petit groupe d'agents, soient trop influent sur le comportement global et la stabilité du modèle. Modéliser le comportement d'agents individuels complexes dans les systèmes avec un petit nombre d'agents, est une approche que l'on peut adopter, mais ce n'est pas celui pour lequel les idées et les intuitions, qui viennent de l'étude des systèmes physiques, peuvent être les mieux utilisées. C'est pourquoi, dans tous les systèmes modélisés dans cette thèse, il existe un grand nombre d'agents qui adoptent des comportements relativement simples. Il est à espérer que, tandis que la variation réelle du comportement humain dans ces systèmes est beaucoup plus grande que celle modélisée, cette approche puisse réussir à capter suffisamment les déterminants des comportements réels des agents, pour se faire des idées pertinentes à un niveau plus global.

Le fait que dans un monde où tout le monde fait la même chose, des choses étranges se produisent est un résultat qui vient naturellement à partir des simulations. Le fait que des hypothèses plus réalistes conduisent très souvent à des résultats plus réalistes est un aspect encourageant des méthodes de simulation d'agents. Ceci souligne les avantages de l'adoption de cette approche souple et intuitive pour la modélisation des systèmes socio-économiques.

Bibliography

- Albrecht, J.W. and B. Axell. An equilibrium-model of search unemployment. *Journal of Political Economy*, 92(5):824–840, 1984.
- Alonso, W. *Location and Land Use*. Harvard University Press, 1964.
- Amblard, F. *Agent-based Modelling and Simulation in the Social and Human Sciences*. The Bardwell Press, 2007.
- Anas, A., R. Arnott, and K.A. Small. Urban spatial structure. *Journal of Economic Literature*, 36:1426–1464, 1998.
- Anglin, P.M., R. Rutherford, and T.M. Springer. The trade-off between the selling price of residential properties and time-on-the-market: The impact of price setting. *Journal of Real Estate Finance and Economics*, 26(1):95–111, 2003.
- Arentze, T.A. and H.J.P. Timmermans. A learning-based transportation oriented simulation system. *Transportation Research Part B - Methodological*, 38(7): 613–633, 2004.
- Arnott, R. The economic theory of urban traffic congestion: A microscopic research agenda. No 502, Boston College Working Papers in Economics, 2001. URL <http://fmwww.bc.edu/EC-P/WP502.pdf> (visited 20/04/2009).
- Arnott, R. Housing vacancies, thin markets, and idiosyncratic tastes. *Journal of Real Estate Finance and Economics*, 2:5–30, 1989.
- Arnott, R., A. de Palma, and R. Lindsey. Schedule delay and departure time decisions with heterogeneous commuters. *Transportation Research Record*, 1197: 56–67, 1988.
- Arnott, R., A. de Palma, and R. Lindsey. A structural model of peak-period congestion - a traffic bottleneck with elastic demand. *American Economic Review*, 83(1):161–179, 1993.
- Arnott, R.J. and J.E. Stiglitz. Aggregate land rents and aggregate transport costs. *The Economic Journal*, 91(362):331–347, 1981.

- Ballot, G. Modeling the labor market as an evolving institution: model artemis. *Journal of Economic Behavior and Organization*, 49(1):51–77, 2002.
- Bar-Yam, Y. *Dynamics of Complex Systems*. Westview Press, 1997.
- Ben-Akiva, M., M. Cyna, and A. de Palma. Dynamic-model of peak period congestion. *Transportation Research Part B - Methodological*, 18(4-5):339–355, 1984.
- Ben-Akiva, M. and S.R. Lerman *Discrete Choice Analysis: Theory and Application to Travel Demand*. The MIT Press, 1985.
- Blanck, D.M. and L. Winnock. The structure of the housing market. *Quarterly Journal of Economics*, 67:181–203, 1953.
- Blundell, R. and T.M. Stoker. Heterogeneity and aggregation. *Journal of Economic Literature*, 43:347–391, 2005.
- Bontemps, C., J.M. Robin, and G.J. Van den Berg. An empirical equilibrium job search model with search on the job and heterogeneous workers and firms. *International Economic Review*, 40(4):1039–1074, 1999.
- Bouchaud, J.-P. The (unfortunate) complexity of the economy. *arXiv:0904.0805v1*, 2009.
- Bradburd, R., S. Sheppard, J. Bergeron, E. Engler, and E. Gee. The distributional impact of housing discrimination in a non-walrasian setting. *Journal of Housing Economics*, 14(2):61–91, 2005.
- Bradburd, R., S. Sheppard, J. Bergeron, and E. Engler. The impact of rent controls in non-walrasian markets: An agent-based modeling approach. *Journal of Regional Science*, 46(3):455–491, 2006.
- Brueckner, J.K., J.-F. Thisse, and Y. Zenou. Why is central paris rich and downtown detroit poor? an amenity-based theory. *European Economic Review*, 43:91–107, 1999.
- Burdett, K. and D.T. Mortensen. Equilibrium wage differentials and employer size. Discussion Paper no. 860, Northwestern University, 1989.
- Burdett, K. and D.T. Mortensen. Wage differentials, employer size, and unemployment. *International Economic Review*, 39(2):257–273, 1998.
- Camerer, C.F., G. Loewenstein, and M. Rabin, editors. *Advances in Behavioral Economics*. Princeton University Press, 2003.

- Cascetta, E. A stochastic process approach to the analysis of temporal dynamics in transportation networks. *Transportation Research B*, 23(1):1–17, 1989.
- Cantarella, G.E. and E. Cascetta. A day-to-day and within-day dynamic stochastic assignment model. *Transportation Research A*, 25(5):277–291, 1991.
- Cantarella, G.E. and E. Cascetta. Dynamic processes and equilibrium in transportation networks: Towards a unifying theory. *Transportation Science*, 29(4):305–329, 1995.
- Caruso, G., D. Peeters, J. CavailhÃs, and M. Rounsevell. Spatial configurations in a periurban city. A cellular automata-based microeconomic model. *Regional Science and Urban Economics*, 37:542–567, 2007.
- Chen, Y.J. Modeling the impacts of vacancy taxes on the taiwan housing market. *Proceedings of the National Science Council*, 11(2):148–165, 2000.
- Clarkson, G.P.E. and H.A. Simon. Simulation of individual and group behavior. *American Economic Review*, 50:920–932, 1960.
- Clayton, J., G. MacKinnon, and L. Peng. Time variation of liquidity in the private real estate market: An empirical investigation. *Journal of Real Estate Research*, 30(2):125–160, 2008.
- Colander, D., P. Howitt, A. Kirman, A. Leijonhufvud, and P. Mehrling. Beyond dsge models: Toward an empirically based macroeconomics. *American Economic Review*, 98(2):236–240, 2008.
- Conte, R. Agent-based modeling for understanding social intelligence. *Proceedings of the National Academy of Sciences*, 99:7189–7190, 2002.
- Davis, G.A. and N.L. Nihan. Large population approximations of a general stochastic traffic assignment model. *Operations Research*, 41:169–178, 1993.
- de Palma, A. and C. Fontan. Choix modal et valeurs du temps en Ile-de-france. *Recherche Transports SÃcuritÃ*, 71(24), 2001.
- de Palma, A. and F. Marchal. Real cases applications of the fully dynamic METROPOLIS tool-box: An advocacy for global large-scale mesoscopic transportation systems. *Networks and Spatial Economics*, 2(4):347–369, 2002.
- de Palma, A., M. Ben-Akiva, C. Lefevre, and N. Litinas. Stochastic equilibrium-model of peak period traffic congestion. *Transportation Science*, 17(4):430–453, 1983.

- de Una-Alvarez, J., R. Arevalo-Tome, and M.S. Otero-Giraldez. Nonparametric estimation of households duration of residence from panel data. *Journal of Real Estate Finance and Economics*, 39(1):58–73, 2009.
- Deng, Y.H., S.A. Gabriel, and F.E. Nothaft. Duration of residence in the rental housing market. *Journal of Real Estate Finance and Economics*, 26(2-3):267–285, 2003.
- Desgranges, G. and E. Wasmer. Appariements sur le marche du logement. *Annales d'economie et de statistique*, (58):253–287, 2000.
- Diamond, P.A. Model of price adjustment. *Journal of Economic Theory*, 3(2):156–168, 1971.
- Eckstein, Z. and G.J. Van den Berg. Empirical labor search: A survey. *Journal of Econometrics*, 136(2):531–564, 2007.
- Eckstein, Z. and K.I. Wolpin. Estimating a market equilibrium search model from panel data on individuals. *Econometrica*, 58(4):783–808, 1990.
- Ettema, D. and H. Timmermans. Modeling departure time choice in the context of activity scheduling behavior. (1831):39–46, 2003.
- Filatova, T., D. Parker, and A. van der Veen. Agent-Based Urban Land Markets: Agent's Pricing Behavior, Land Prices and Urban Land Use Change. *Journal of Artificial Societies and Social Simulation*, 12(1):3, 2008.
- Fisher, J., D. Gatzlaff, D. Geltner, and D. Haurin. Controlling for the impact of variable liquidity in commercial real estate price indices. *Real Estate Economics*, 31(2):269–303, 2003.
- Friesz, T.L., D. Bernstein, N.J. Mehta, R.L. Tobin, and S. Ganjalizadeh. Day-to-day dynamic network disequilibria and idealized traveler information-systems. *Operations Research*, 42(6):1120–1136, 1994.
- Fujita, M. *Urban Economic Theory*. Cambridge University Press, 1989.
- Fujita, M. and J.-F. Thisse. *Economie des villes et de la localisation*. De Boeck and Larcier, 2003.
- Gabriel, S.A. and F.E. Nothaft. Rental housing markets and the natural vacancy rate. *American Real Estate and Urban Economics Association Journal*, 16(4):419–429, 1988.
- Glaeser, E.L., V. Henderson, and R.P. Inman. The future of urban research: Non-market interactions (with comments). *Brookings-Wharton Papers on Urban Affairs*, pages 101–149, 2000.

- Grenadier, S.R. Local and national determinants of office vacancies. *Journal of Urban Economics*, 38(1):57–71, 1995.
- Henderson, J.V. *Economic theory and the cities*. Academic Press, 1977.
- Hunt, J.D., E.J. Miller, and D.S. Kriger. Current operational urban land-use transport modeling frameworks. *Transport Reviews*, 25:329–376, 2005.
- Hwang, M. and J.M. Quigley. Economic fundamentals in local housing markets: Evidence from us metropolitan regions. *Journal of Regional Science*, 46(3): 425–453, 2006.
- M. Igarashi. The rent-vacancy relationship in the rental housing market. *Journal of Housing Economics*, 1:251–270, 1991.
- Jensen, P. Network-based predictions of retail store commercial categories and optimal locations. *Physical Review E*, 74:R035101, 2006.
- Kiefer, N.M. Economic duration data and hazard functions. *Journal of Economic Literature*, 26(2):646–679, 1988.
- Kirman, A.P. Whom or what does the representative individual represent. *Journal of Economic Perspectives*, 6(2):117–136, 1992.
- Kirman, A.P. and N.J. Vriend. Evolving market structure: an ace model of price dispersion and loyalty. *Journal of Economic Dynamics and Control*, 25: 459–502, 2001.
- Knight, J.R. Listing price, time on market, and ultimate selling price: Causes and effects of listing price changes. *Real Estate Economics*, 30(2):213–237, 2002.
- LeBaron, B. and L. Tesfatsion. Modelling macroeconomies as open ended dynamic systems of interacting agents. *American Economic Review*, 98(2):246–250, 2008.
- Leung, C.K.Y., Y.C.F. Leong, and S.K. Wong. Housing price dispersion: An empirical investigation. *Journal of Real Estate Finance and Economics*, 32(3): 357–385, 2006.
- Lim, Y. and B. Heydecker. Dynamic departure time and stochastic user equilibrium assignment. *Transportation Research Part B - Methodological*, 39(2): 97–118, 2005.
- Lindsey, R. and E. Verhoef. *Handbook of Transport Modelling*, chapter Congestion Modelling. Elsevier, 2001.

- Mahmassani, H. and R. Herman. Dynamic user equilibrium departure time and route choice on idealized traffic arterials. *Transportation Science*, 18(4):362–384, 1984.
- Marchal, F. *Contribution to Dynamic Transportation Models*. PhD thesis, University of Cergy-Pontoise, 2001.
- MATSim. A multi-agent transportation simulation, 2009. URL <http://www.matsim.org> (visited the 25/04/2009).
- Mc Breen, J., P. Jensen, and F. Marchal. An agent-based simulation model of traffic congestion. Proceedings of ATT2006, 4th Workshop on Agents in Traffic and transportation, 2006. URL <http://www.inf.ufrgs.br/bazzan/downloads/WS28ATT.pdf> (visited 20/04/2009).
- Mc Breen, J., F. Goffette-Nagot, and P. Jensen. An agent-based simulation of rental housing markets. HAL-SHS (Hyper Article en Ligne - Sciences de l’Homme et de la Société), 2009. URL <http://halshs.archives-ouvertes.fr/halshs-00374157/fr/> (visited 22/04/2009).
- McMillan, J. and M. Rothschild. *Handbook of Game Theory with Economic Applications - Volume 2*, chapter Search, pages 905–927. Elsevier, 1994.
- Merlo, A. and F. Ortalo-Magne. Bargaining over residential real estate: evidence from england. *Journal of Urban Economics*, 56(2):192–216, 2004.
- Mills, E.S. An aggregative model of resource allocation in a metropolitan area. *American Economic Review*, 57:197–210, 1967.
- Mortensen, D.T. *Panel Data and Labor Market Studies*, chapter Equilibrium wage distributions: a synthesis. Amsterdam: North Holland, 1990.
- Mortensen, D.T. *Panel Data and Structural Labor Market Models*, chapter Equilibrium Unemployment with Wage Posting. Amsterdam: Elsevier, 2000.
- Mortensen, D.T. and C.A. Pissarides. Job creation and job destruction in the theory of unemployment. *The Review of Economic Studies*, 61(3):397–415, 1994.
- Muth, R.F. *Cities and Housing*. The University of Chicago Press, 1969.
- Novy-Marx, R. Hot and cold markets. *Real Estate Economics*, 37(1):1–22, 2009.
- Orcutt, G.H. Simulation of economic systems. *American Economic Review*, 50(5):893–907, 1960.

- Parker, D. and T. Filatova. A conceptual design for a bilateral agent-based land market with heterogeneous economic agents. *Computers, Environment and Urban Systems*, 32454–463, 2008.
- Peeta, S. and A.K. Ziliaskopoulos. Foundations of dynamic traffic assignment: the past, the present and the future. *Networks and Spatial Economics*, 1(3): 233–265, 2001.
- Pissarides, C. *Equilibrium unemployment theory*. Oxford: Basil Blackwell, 1990.
- Postel-Vinay, F. and J.-M. Robin. *Advances in Economics and Econometrics: Theory and Applications, Ninth World Congress*, chapter Microeconomic Search-Matching Models and Matched Employer-Employee Data, pages 279–310. Cambridge University Press, 2006.
- Raux, C., M. Sdika, and V. Hermenier. Simulation de la dynamique du système de déplacements urbains : une plate-forme de modélisation. 2003. URL <http://www.innovations-transport.fr/IMG/pdf/210-R00MT61.pdf> (visited the 23/04/2009).
- Read, C. Tenants search and vacancies in rental housing markets. *Regional Science and Urban Economics*, 23(2):171–183, 1993.
- Read, C. Vacancies and rent dispersion in a stochastic search model with generalized tenant demand. *Journal of Real Estate Finance and Economics*, 15(3): 223–237, 1997.
- Rickles, D. *Handbook of the Philosophy of Science - Volume 10: Philosophy of Complex Systems*, chapter Econophysics and Financial Market Complexity. North Holland: Elsevier, forthcoming. URL <http://philsci-archive.pitt.edu/archive/00003851/> (visited the 21/04/2009).
- Rogerson, R., R. Shimer, and R. Wright. Search theoretic models of the labour market: A survey. *Journal of Economic Literature*, 63:959–988, 2005.
- Rosen, K.T. and L.B. Smith. The price-adjustment process for rental housing and the natural vacancy rate. *American Economic Review*, 73(4):779–786, 1983.
- Roth, C. Systèmes complexes sociaux et validation empirique. *Societes*, 98(4): 53–64, 2007.
- Salvini, P. and E.J. Miller. Ilute: An operational prototype of a comprehensive microsimulation model of urban systems. *Networks and Spatial Economics*, 5: 217–234, 2005.

- Schelling, T.C. *Micromotives and Macrobehavior*. Norton, New York, 1978.
- Sheppard, S. *Handbook of Regional and Urban Economics - Volume 3*, chapter Hedonic analysis of housing markets, pages 1595–1635. North-Holland, 1999.
- Shilling, J.D., C.F. Sirmans, and J.B. Corgel. Price adjustment process for rental office space. *Journal of Urban Economics*, 22(1):90–100, 1987.
- Small, K.A. The scheduling of consumer activities: Work trips. *American Economic Review*, 72:467–479, 1982.
- Stein, J. Prices and trading volume in the housing market : a model with down-payment effects. *The Quarterly Journal of Economics*, 110(2):379–406, 1995.
- Tesfatsion, L. and K.L. Judd, editors. *Handbook of computational economics - Volume 2: Agent-based educational economics*. Elsevier, 2006.
- Timmermans, H. The saga of integrated land use-transport modelling: How many more dreams before we wake up? *Conference keynote paper : 10th International conference on travel behaviour research*, 2003.
- Tivadar, M. and H. Jayet. Endogenous amenities and the spatial structure of cities. *European Regional Science Association (ERSA) Conference Papers*, 2006. URL <http://www-sre.wu-wien.ac.at/ersa/ersaconfs/ersa06/papers/734.pdf> (visited the 26/04/2009).
- Varenne, F. *Modèles, Simulations, Systèmes : no. 3 de la revue Matière Première*, chapter Modèles et simulations : pluriformaliser, simuler, remathématiser, pages 153–180. 2008.
- Vickrey, W.S. Congestion theory and transport investment. *American Economic Review*, 59(2):251–260, 1969.
- von Thunen, J.H. *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie*. F. Perthes, 1826.
- Waddell, P. Urbansim: Modeling urban development for land use, transportation and environmental planning. *Journal of The American Planning Association*, 68(3):297–314, 2002.
- Watling, D. and M.L. Hazelton. The Dynamics and Equilibria of Day-to-Day Assignment Models. *Networks and Spatial Economics*, 3:349–370, 2003.
- Wheaton, W.C. Vacancy, search, and prices in a housing market matching model. *Journal of Political Economy*, 98(6):1270–1292, 1990.

-
- Windrum, P., G. Fagiolo, and A. Moneta. Empirical validation of agent-based models: Alternatives and prospects. *Journal of Artificial Societies and Social Simulation*, 10(2):8, 2007.
- Yavas, A. Resolution of demand uncertainty and competitive rent adjustment. *Journal of Housing Economics*, 10:472–481, 2001.
- Yavas, A. and S. Yang. The strategic role of listing price in marketing real-estate - theory and evidence. *Real Estate Economics*, 23(3):347–368, 1995.
- Zhang, D. and A. Nagurney. On the local and global stability of a travel route choice adjustment process. *Transportation Research Part B - Methodological*, 30(4):245–262, 1996.

ABSTRACT

Simulations have considerable potential for the analysis of the evolution of economics systems, a subject often neglected by mainstream economics where the focus is on static equilibria. This thesis investigates the potential of this approach in urban economics. The purpose is to examine how global phenomena emerge from the interactions of economic agents. This is a promising method as classical economics, lacking the appropriate analytic tools, concentrates on the existence of equilibria and refrains from investigating their stability. This study demonstrates the potential of simulations in three models.

Firstly, in a standard model of traffic congestion it is shown that the Nash equilibrium is unstable and cannot be reached dynamically. Secondly, it is shown that simulations of the formation of urban land rents, reproduce elements of the theoretical equilibrium, and also endogenous vacancies, which are an important real-world phenomenon. Thirdly, an agent-based model of the housing market, which reproduces important empirical phenomena such as price dispersion, non-zero search times and vacancies, has been developed. The model provides a basis for the exploration of the complex dynamics of this market.

KEYWORDS

AGENT-BASED SIMULATIONS – URBAN ECONOMICS – URBAN DYNAMICS – TRAFFIC CONGESTION – STABILITY – REAL ESTATE – SEARCH MARKETS

RÉSUMÉ

Les simulations représentent une méthode de choix pour analyser la dynamique d'évolution des systèmes, que l'économie classique laisse de côté car elle s'intéresse essentiellement à l'équilibre. Dans ce travail, nous avons étudié l'intérêt d'une telle approche dans le cas des systèmes d'économie urbaine. Il s'agit de comprendre comment des interactions entre des agents économiques émergent dans l'état global du système, en s'intéressant notamment à la dynamique de convergence vers un éventuel équilibre. Cette méthode est prometteuse, car faute de méthodes analytiques performantes, l'économie classique ne s'intéresse en général qu'aux équilibres, sans se soucier des possibilités concrètes pour l'atteindre. Notre étude a montré l'intérêt des simulations dans trois cas précis.

D'abord, dans un modèle classique de la congestion routière il est démontré que l'équilibre prédit par la théorie économique était instable, et qu'il ne pouvait pas être atteint en pratique. Ensuite, il est démontré que des simulations de la formation de la rente foncière urbaine dans un modèle simple, permettaient de retrouver certains aspects de l'équilibre classique, mais rajoutaient de manière endogène un élément important : le taux de vacances observé dans la réalité. Enfin, nous avons développé un modèle multi-agents du marché immobilier, qui permet de retrouver des phénomènes empiriques importants telle que la dispersion des prix, des temps de recherche non nuls et des logements vacants. Le modèle autorise aussi une exploration de la dynamique complexe de ce marché.

MOTS-CLÉS

SIMULATIONS MULTI-AGENTS – ECONOMIE URBAINE – DYNAMIQUES URBAINES – CONGESTION ROUTIÈRE – STABILITÉ – MARCHÉ IMMOBILIER – MARCHÉS DE RECHERCHE