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Essays on macroeconomics of hysteresis and climate change

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Essays on macroeconomics of hysteresis and climate change

**Thèse d'Habilitation à Diriger des Recherches de
l'Université Paris-Saclay**

**présentée et soutenue à Paris-Saclay,
le 12 Octobre 2021, par**

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General Introduction

Back in prelude of the modern business cycle theory, [Lucas \(1976\)](#) posited the controversial observation that “*all business cycle alike*”. Aggregate variables, such as output or employment, undergo repeated fluctuations around trend with essentially the same patterns over time. These cyclical oscillations also exhibit regular co-movements among different time series, countries, time, sectors and markets. The origin, transmission and persistence of these regular cyclical variations went at the center of the research agenda. Prior this agenda, it was commonplace following the Keynesian Revolution in macroeconomics to interpret fluctuations through the lens of general equations (e.g. Okun, Phillips, consumption equations) governing the structural interactions across macroeconomic time series. Yet, the decision-process at an agent level leading to these macroeconomic relations was not part of the research agenda. The intellectual revolution, desired by Lucas, suggests to interpret economic fluctuations from the behavior of consumers, firms, and financial intermediaries, when present, is formally derived from microfoundations. The challenge of the modern business cycle theory is therefore to provide an unified framework organized around a microfounded common core.

Modern models of business cycles, pioneered by [Lucas Jr \(1978\)](#) and [Kydland and Prescott \(1982\)](#), interpret economic fluctuations as endogenous and coordinated responses from agents to exogenous shifts in economic fundamentals. The main ingredients of the workhorse macroeconomic model typically include representative agents, exogenous shocks, rational expectations, market clearing conditions and micro-foundations. This class of models – the so-called dynamic stochastic general equilibrium (DSGE) models – is very convenient as it provides an unified framework among macroeconomists to interpret economic fluctuations from a general equilibrium perspective. Many, if not most, fields of macroeconomics have yet been colonized by this new analytical framework of fluctuations. Since then, it is commonplace to build on this framework, by further questioning the core hypothesis to provide always more relevant economic mechanisms.

While many see modern macroeconomics as a mature science, others think macroeconomics went wrong. In particular, a disagreement among macroeconomists emerged following the financial crisis when benchmark DSGE models performed poorly during the financial turmoil. [Stiglitz \(2018\)](#) and [Krugman \(2018\)](#) (among many others) recognize these waves of criticism as a possible paradigm shift that would translate into discarding DSGE models from the toolkit of macroeconomists. Others, such as [Blanchard \(2018\)](#), actually see this as an opportunity to improve the existing toolkit in a

way to portray economic fluctuations in a more realistic manner.

My research lies in the continuity of the Blanchard's view, designed to improve the existing toolkit. Broadly defined, my research focuses on the analysis of business cycles, from its origin to its transmission into aggregate fluctuations, with the final aim to develop and enhance stabilization policies. In particular, the implementation of these policies requires a clear understanding of the origins of business cycles, as well as the transmission mechanisms from one market to another. For the achievement of this research objective, I am building on structural business cycle models that allow rigorous quantification of shocks and transmission mechanisms through Bayesian techniques in the same way as [Smets and Wouters \(2003, 2007\)](#). My research takes into account new phenomena in macroeconomics such as social interactions in expectations, international banking or climate change.

This *Habilitations à diriger des Recherches* (HDR, hereafter) is an opportunity to synthesize ongoing and future work. In the following, I first discuss the scientific background characterizing my research. Next, I provide a summary of my research agenda during my Ph.D. studies. Finally I discuss my ongoing work on two challenging topics in macroeconomics that I find relevant for my research agenda: macroeconomics of hysteresis and macroeconomics of climate change.

SCIENTIFIC BACKGROUND

This HDR actually builds on a set of contributions that can be gathered and summarized as two complementary blocks: (i) an empirical quantification of the forces driving the business cycles; (ii) a welfare-based evaluation of macroeconomic policies. This section describes these two devices from an historical perspective.

A quantification of the forces operating in the economy. The decomposition of business cycles can only be addressed through the lens of empirical models that decompose macroeconomic time-series into relevant informations for econometricians and policymakers.¹ Back in the 80s, DSGE models were unable to perform such decomposition exercises. At that time “deep” structural parameters were inferred through the match of a limited number of first and second order moments. The very first inference of structural parameters with the Maximum Likelihood Estimator (MLE, henceforth) can be dated back in [Fair and Taylor \(1983\)](#), while [Ireland \(2004\)](#) provides the conventional and modern routine to take to the data linearized macroeconomic models. In a nutshell, structural disturbances are set so as to exactly replicate macroeconomic time series through the lens of a statistical filter (e.g. kalman, particle

¹For example, monetary policy response to supply shocks differs with demand shocks. The conduct of monetary policy is thus contingent on the set of shocks that currently drives the business cycles.

or inversion). Assuming that errors of predictions are Gaussian, it is possible to construct and maximize the joint probability distribution of the sample. Then, the goal of MLE is to make inference about structural parameters that are the most likely to have generated the sample.

While MLE has appealing features, experience shows that it is rather difficult to estimate model with this method. The main reason is that the data are not informative enough which makes the likelihood function flat in some directions. These weak identification patterns are a curse for numerical methods that are not able to pursue the optimization. Therefore, this suggests other sources of informations should be employed like prior informations. Following [Smets and Wouters \(2003\)](#) and [An and Schorfheide \(2007\)](#), it is commonplace to curb the likelihood function with prior informations to improve the identification, to reduce the misspecification and to reduce the posterior uncertainty of inferred structural parameters.

Prior Bayesian techniques for DSGE models, one of the state-of-art forecasting model in macroeconometrics was the Bayesian Vector-Autoregressive (BVAR, henceforth) model. The BVAR model uses prior information to reduce the parameter uncertainty and, by-product, improves its forecasting accuracy. If this methods provides accurate predictions, it however suffers from overfitting: the number of parameters to be estimated increases exponentially in the number of observable variables and lags. In a path-breaking contribution, [Smets and Wouters \(2007\)](#) show that this BVAR benchmark can be outperformed by an a DSGE model estimated with Bayesian techniques. This performance is remarkable on two aspects. First, a theoretical model provides a more accurate representation of the data than the benchmark atheoretical model. Second, a DSGE model – that features a very low number of parameters (30 structural parameters in the Smets-Wouters) – outperforms the benchmark BVAR model (244 parameters with 4 lags). Therefore, DSGE became very popular as they could perform fit exercises with an empirical relevance close to best other forecasting tools.

Almost all my research builds on these full-information methods to put my structural models to the data. The main advantage of these methods is to rigorously decompose the net effect of forces operating on different parts of the economy. As summarized by [Smets et al. \(2010\)](#) and [Christiano et al. \(2018\)](#), a central challenge facing policymakers is to gauge the relative strength of those forces. The DSGE approach is particularly suitable to this aspect, in particular for contrasting the macroeconomic outcome under alternative policies. The general equilibrium structure lends itself to telling economically coherent stories and structuring forecast-related discussions around it.

A welfare-based evaluation of macroeconomic policies. Part of the success of DSGE models lies their empirical performance in providing an accurate representation of the data. However this success is also corroborated by their usefulness for policy analysis, in particular for monetary authorities. [Clarida et al. \(1999\)](#) and [Woodford \(2003\)](#) provide the foundations of the modern analysis of inflation targeting policies in a stylized model of business cycles. This monetary policy framework, referred to as *New Keynesian*, radically departs from the initial framework of [Kydland and Prescott \(1982\)](#) on two main aspects.

First, the New Keynesian framework assumes that prices do not immediately ensure market clearing, but gradually adjust to short term fluctuations. As a consequence, market clearing operates inefficiently through a joint adjustment of prices and quantities. [Rotemberg \(1982\)](#) and [Calvo \(1983\)](#) provide the two most common micro-founded devices to introduce nominal rigidities into a dynamic rational expectation model. Second, given the presence of inefficient fluctuations in prices, [Fisher \(1911\)](#) argued that the natural authority to deal with these fluctuations is monetary policy. The modern interpretation of the Fisherian approach translates into an inflation targeting regime characterized by a central bank that anchors inflation expectations through the proportional adjustment of its interest rate to inflation. This statement is actually grounded by a strong body of evidence that highlights inflation targeting reduces inflation volatility ([Svensson, 1997](#)), dampens the effect of nominal shocks ([Mishkin, 2004](#)) and encourages the anchoring of inflation expectations ([Gürkaynak et al., 2010](#)).

The most convenient manner to introduce monetary policy is through the lens of a monetary policy rule in the same way as [Taylor \(1993\)](#). This rule typically connects the response of the nominal rate to fluctuations in inflation and the output gap. Given this rule, what would be the optimal conduct of a monetary policy? The characterization of an optimal policy is akin the determination of the rule's coefficients that maximizes the objective function of policy makers. Therefore, the missing ingredient is a criterion, consistent with the model, that would pin down the optimal policy coefficients of the rule. [Woodford \(2001\)](#) argues that it is possible to motivate a quadratic loss function for monetary policy as a second order Taylor-series approximation to the expected utility of the economy's representative household. DSGE models became very popular in central banks, and in most policymaking institutions, as they easily provide policy recommendations. Each policy option can be ranked and interpreted in terms of permanent consumption that would be added or subtracted with respect to some benchmark (see [Lucas \(2003\)](#)). Combined with Bayesian techniques, they are also amenable for counterfactual analysis by providing the macroeconomic outcome that would have been achieved through alternative policies.

CROSS-BORDER BANKING IN A MONETARY UNION

When I started my Ph.D. in 2011, the Eurozone debt crisis was the world's greatest threat. The main concern in policymaking institutions were to avoid a grexit –Greece's potential withdrawal from the monetary union– as it would threaten the integrity of the Eurozone. The 2008 financial turmoil and the sovereign crisis highlighted that the financial connections across members of the monetary union are so high that it has become impossible to leave a monetary union without facing large costs. Therefore, a natural starting point for my research was to study the role of international connection of bank balance sheet in the building up of the crisis.

My first paper (available in [Part A](#)) focuses on the consequences of the rise in cross-border loan flows observed since the adoption of the Euro in 1999 on the international propagation of asymmetric shocks. Cross-border lending is a distinguishing feature of financial integration in the Eurozone: it has been multiplied by 3 in 9 years, reaching a peak value of 120% of GDP in 2008, before experiencing a 25% decrease after the recent financial crisis. Taking a closer look at the data, this cross-border phenomenon is heterogeneous: it affects mainly interbank lending while cross border corporate lending is much small and cross-border lending to households is negligible. In [Poutineau and Vermandel \(2015\)](#), we provide a general equilibrium interpretation of this phenomenon through a two-country DSGE model of a monetary union with cross-border interbank and corporate lending facilities. To keep the model tractable, we analyze cross-border loans through home bias in the borrowing decisions concerning interbank and corporate loans using CES function aggregates. In addition to standard trade linkages, global banking provides an alternative mechanism of transmissions of shock across the national borders of the monetary union. As an example, a rise in productivity for firms living in one country of the monetary union reduces their probability of default. Financial intermediaries are thus more willing to provide loans at lower rates, which translates –through cross-border linkages– into lower rates for the monetary union as whole, proportionally to the degree of banking integration.

To gauge the quantitative potential of the cross-border banking channel in the international propagation of shocks, we infer the structural parameters as well as the sequence of innovations using Bayesian techniques. As explained previously, this method allows to disentangle the relative forces that have driven output and credit fluctuations in the Euro Area since the creation of the monetary union. We use the core-periphery dichotomy and infer the structural parameters using country-specific macroeconomic time series. We find evidence of the role of cross-border lending channel as an amplifying mechanism for the transmission of asymmetric shocks, in particular we find that peripheral economies are more affected by the 2008 financial

crisis through a deeper impact on interbank loan shortage.

The presence of banks across borders is a distinguishing features of the monetary union that poses some serious concerns for national authorities when they implement macroprudential measures. The next section investigates how the social planner can internalize the possible cross-border spillovers from national macroprudential policies in the Euro Area.

FINANCIAL STABILITY WITH INTERNATIONAL BANKING

In 2012, the Basel accords III were embedded into European Union laws. This new legal framework allows both national competent institutions (such as the Banque de France) as well as supra-national authority (ECB-SSM) to take macroprudential measures in order to ensure the financial stability of the monetary union. Despite this step forward a more integrated monetary union, a large uncertainty remained about the conduct and coordination of macroprudential across Euro Area members. In absence of explicit framework to examine the question at hand, policymaking institutions had little knowledge about the conduct of macroprudential policy and the possible presence of cross-border spillovers from national policies. Given this policy uncertainty, a chapter of my Ph.D. published in [Poutineau and Vermandel \(2017a, 2017b\)](#) (and available in [Part B](#)) investigates how macroprudential policy can efficiently ensure financial stability in the Euro Area when cross-border banking is high. Much of the question is to gauge whether a macroprudential policy in one country can have unintended effects on other members of the monetary union. If so, the international coordination of macroprudential policy must be addressed by a supra-national authority such as the ECB.

On policy aspects, the disruption in cross-border financial relations in 2008 across EU countries set the basis for the adoption of macroprudential policies in advanced economies such as the Eurozone. In the Euro Area, the implementation of such measures remains fragmented along national lines while the coordination and internalization of cross-border spillovers are achieved through the actions of the European Systemic Risk Board (ESRB, henceforth). We therefore question how sizable cross-border lending flows should be treated in the definition of national macroprudential policies in the Euro Area. We more particularly assess whether cross-border bank lending should explicitly be considered in the setting of coordinated national macroprudential measures or whether national regulators should only focus on the sole national financial stance to contribute to the financial stability of the Eurozone.

We thus consider an extension of the two country model in [Poutineau and Vermandel \(2015\)](#) in which banks face capital adequacy regulation. Macroprudential policy

affects financial intermediaries through a countercyclical capital buffer (CCB, henceforth) rate that determines the share of equity that banks must hold. This instrument has become one of the leading facets of prudential regulation since the adoption of Basel III accords (2010) by building up a bank capital buffer during periods of excessive credit growth that can be released when systemic risks abate. The international dimension of banks offered by our setting allows us to contrast different CCB rules based on the targeting of loan supply or demand union-wide loans. at a domestic or international level.

To rank alternative macroprudential policies, we follow [Woodford \(2012\)](#) and assume that the financial stability authority minimizes a stylized loss function that mimics the objective of a conventional macroprudential authority. The latter typically aims at limiting financial system-wide distress by smoothing credit-to-GDP fluctuations at a business cycle frequency. Therefore, we translate this objective as a loss that increases in both credit and output volatilities.

By minimizing the stylized loss function, we obtain three main results. First, targeting a national credit-to-GDP ratio should be favored to federal averages as this rule induces better stabilizing performances in front of important divergences in credit cycles between core and peripheral countries. Second, policies reacting to the evolution of national credit supply should be favored as the transmission channel of macroprudential policy directly impacts the marginal cost of loan production and, by so, financial intermediaries. Third, the interest of lifting up macroprudential policymaking to the supra-national level remains questionable for admissible value of international lending between Eurozone countries. Indeed, national capital buffers reacting to the union-wide loan-to-GDP ratio only lead to the same stabilization results than the one obtained under the national reaction if cross-border lending reaches 45%. However, even if cross-border linkages are high enough to motivate the implementation of a federal adjusted solution, the reaction to national lending conditions remains remarkably optimal.

THE NEXT RESEARCH AGENDA

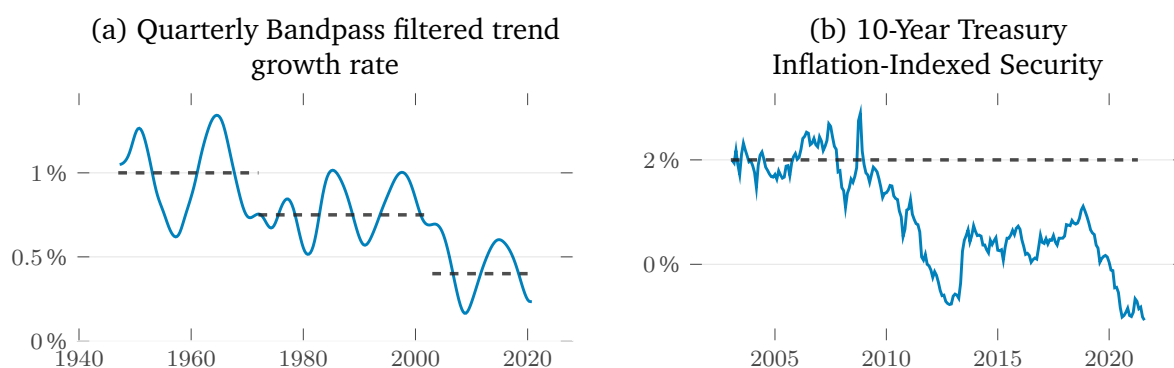
While the implementation of macroprudential policy was a hot topic debate during my Ph.D. studies, much of the interest vanished when new generations of models with financial frictions were able to ground the conduct of macroprudential policies within the Euro Area. From my own perspective, the policy agenda gradually shifted toward new issues for macroeconomics. Among these issues, two of them particularly raised my attention: *hysteresis* and *climate change*. This HDR provides a discussion on the ongoing work on these two challenging topics in macroeconomics.

1. HYSTERESIS IN MACROECONOMICS

Two stylized facts related to hysteresis are hard to unfold within our state-of-art DSGE models. First, economic growth is stalling in developed economies: the US economy was growing at a rate twice higher in 1960 than today. This persistent reduction in economic growth, that we can refer to hysteresis, took place at a frequency lower than the usual spectrum of the business cycle. In the **Part I** of this HDR entitled “*Endogenous Trends*”, I investigate how our benchmark DSGE models can be improved to take into account medium terms swings in macroeconomic times series. The second stylized fact is related to inflation in the aftermaths of the financial crisis: inflation has remained remarkably low while output was expanding. This fact conflicts with the usual design of the New Keynesian Phillips curve that posits a positive co-movement between price and quantities. In **Part II**, we capture this inflationless recovery by introducing heterogenous expectations in an otherwise standard new Keynesian model. During a recession in a liquidity trap, a fraction of the population updates their expectations downward. These pessimistic expectations spread within the population and remain even if the recessionary shock is gone, thus creating hysteresis effects. Inflation may remain low in absence of active communication from the central bank to coordinate inflation expectations back on the target.

An estimated medium term cycle model. In a seminal speech at the IMF, Larry Summers argued that the global economy had entered a long-term economic slump, referred to as *Secular Stagnation*. To illustrate Larry Summers’ argument, let us consider the US growth rate without the business cycle component, as depicted in **Figure 1a**. Since WWII, the US has experienced two drifts in the average rate of economic growth. The last one occurred after the 2001 dot-com boom and bust, suggesting the US has entered since then in a low growth era.

This concept of permanent downturn in economic growth is not new and can be traced back to the “Secular Stagnation” view of **Hansen (1939)**. The latter argued that the US would face a large and persistent economic depression, as US population growth was declining and that this would lead to a large fall in investment. If the concept was empirically motivated in 1939 as the US recovery from the great depression remained fragile, the emergence of Keynesian macroeconomics probably contributed to restore the US prosperity. Nowadays, the economic outlook is however fairly different with respect to 1939. Institutional changes between 1939 up to now give a predominant role to fiscal and monetary policy to address economic imbalances. The key question in macroeconomics is to know whether this slump is the natural path of any advanced economies experiencing technological exhaustion, or a stagnation trap that could be avoided with further stimulus.



Notes: Output growth is based on US real output annual data from FRED. The business cycle component is subtracted using the bandpass filter of [Christiano and Fitzgerald \(2003\)](#), the remaining component is interpreted as the medium and long term components of the data. For the expected inflation rate, it is proxied by Treasury Inflation-Protected Securities (TIPS) from FRED, based on treasury bond data. TIPS, are securities whose principal is tied to the Consumer Price Index (CPI). The principal increases with inflation and decreases with deflation. When the security matures, the U.S. Treasury pays the original or adjusted principal, whichever is greater.

Figure 1: The persistent slump characterized by an always lower slope of growth and inflation expectations below 2% inflation target

In [Vermandel \(2020\)](#) (referred to as **Part I** in the HDR), I address this question of low growth by extending the mainstream business cycle DSGE allowing permanent drifts in output. In particular, conventional business cycle analysis interprets economic fluctuations as high frequency variations around an exogenous trend. In contrast to this approach, I include two sources of endogenous growth (technological change and human capital) to determine the endogenous trend of an economy, and examine its quantitative potential in a standard medium scale New Keynesian model. I estimate this model on the US data between 1950q1-2018q4 with an occasionally binding constraint on the nominal rate. I find that the endogenous trend has been sharply declining since 1970, thus corroborating the secular stagnation theory. This dynamic is captured by a slowdown in the accumulation technology reflecting the low productivity of the R&D sector, thus highlighting a technological exhaustion. While the contribution of human capital has been remarkably stable, the financial crisis damaged the accumulation of skills.

Social interactions in expectations as a source of hysteresis. In the wake of a low growth environment, the conduct of monetary policy can be threatened if the anchorage of inflation expectations weakens. The term “anchored” actually refers to expectations that do not react to incoming data ([Bernanke, 2007](#)). During the Great moderation –the period of low macroeconomic volatilities in the 80s up to 2007– the plain vanilla new Keynesian provided a very accurate description of monetary policymaking. The conventional wisdom from the macro textbook is the following. The presence of sticky prices prevents firms to immediately re-adjust their selling price to changes in their costs of production. If inflation expectations are high, firms actually anticipate that the cost of inputs (such as the nominal wage) is likely to

increase in the next periods. Firms therefore already hedge against a risk of a peak in inputs costs by rising their current selling price. To anchor inflation expectations, a central bank must therefore increase its nominal rate until expected future input costs lowers consistently with an inflation path close to 2%. Therefore, inflation targeting policy was always the same: an increase in expected inflation must be compensated by an increase in nominal rates.

During the great moderation, this narrative worked accurately well as regular surges in inflation expectations were compensated by further tightening in monetary policy. However, this principle stalled when inflation expectations went below the inflation target, as reported in [Figure 1b](#). The starting point of this joint decline in expectations and interest rates is the 2008 financial crisis. Monetary policy became accommodative to dampen the crisis, but probably not sufficiently to anchor inflation expectations back to their pre-crisis level. The post-2007 recovery was unusual with respect to previous recessions: current and expected inflation remained low while the output gap was high. This type of recovery is hard to unfold within the standard New Keynesian model, as the latter predicts through the New Keynesian Phillips curve that imbalances in goods markets are compensated by higher prices.

To interpret this mechanism in a benchmark New Keynesian setup, in [Arifovic et al. \(2020\)](#) (referred here to as [Part II](#)), we develop a model in which the economy is directly influenced by how pessimistic or optimistic economic agents are about the future. Agents may hold different views and update them as new economic data become available. A large shock –similar to the 2007 turmoil– endogenously makes agents on average more pessimistic. In a liquidity trap, agents may on average expect deflation to remain. The latter increases real rates and depresses aggregate demand, thus creating an alternative source of hysteresis with respect to the part I of this HDR. In this

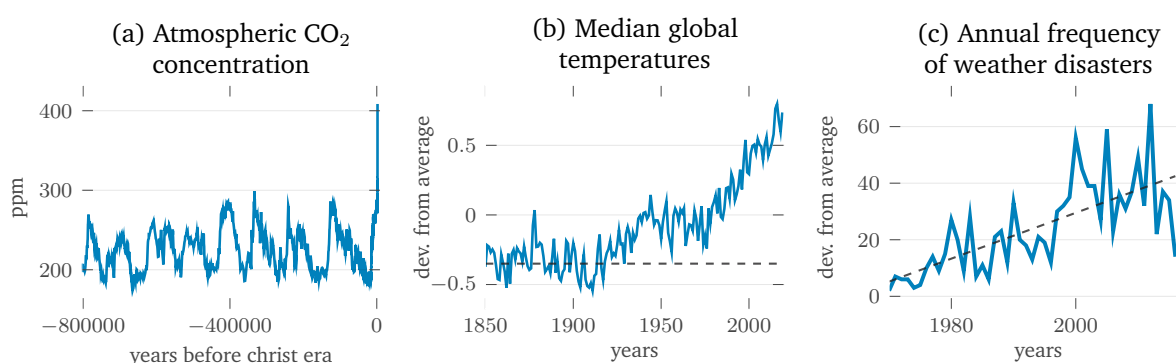
Using US data, the model accounts for several key features of macroeconomic time series from expectation surveys of professional forecasters. In particular, the model reproduces prolonged episodes of below-target inflation coupled with near-zero interest rates resembling the recent economic experience since the 2008 crisis. Following an external shock that decreases output or inflation, agents lower their inflation and output outlooks. Their pessimistic views become self-confirming and inflation is locked into a below-target level.

We find that the lack of coordination in expectations entails large welfare losses. Therefore to avoid welfare costs, central banks can release information to help agents coordinate their economic views on higher inflation levels. However, this communication comes with a risk of credibility loss. On the one hand, if actual inflation is

low, agents may not believe a central bank’s message about higher future inflation or targets. On the other hand, true forecasts of low future inflation can create a risk of self-confirming pessimism: the agents will adopt the central bank’s views and keep on expecting below-target inflation. Either way, communicating reduces the heterogeneity of agents’ views but does not solve the issue of persistently low inflation and interest rates.

2. THE MACROECONOMICS OF CLIMATE CHANGE

Beyond the problem of current low growth detailed in the previous section, the prospects of future growth are also threatened by permanent changes in climatic conditions. Therefore the second issue in macroeconomics, that is also part of my research agenda, is the macroeconomics of climate change. It took billions of years for natural systems to absorb and store carbon dioxide in soil. Fossil fuels are the result of this natural process, which led our atmosphere to be favorable to life.² In contrast to this very low frequency process of carbon sequestration, it took only 250 years for anthropogenic activities to release hundred of gigatons of carbon back into the atmosphere. While oceans and plants are actually able to capture half of anthropogenic CO₂ emission, the remaining share unintendedly remains in the atmosphere. The carbon cycle exhibits a very low frequency spectrum, from its release in the atmosphere to its capture into a carbon sunk. The duration of carbon cycle oscillates between 80 up to 140 years. Therefore, the level of carbon that we observe today in the atmosphere have been liberated back in 1900 from coal consumption.



Notes: Average concentration of carbon dioxide (CO₂) in the atmosphere data are taken from Bereiter et al. (2015). The median temperature anomaly data are taken from ?. The weather data are taken from EM-DAT.

Figure 2: Carbon cycles and their macroeconomic implications for temperatures and extreme weather events

The accumulation of carbon in the atmosphere is not a free lunch. Scientists have

²The rise in atmospheric oxygen started more than 2 billions of years ago via the emergence of primitive bacteria that produced oxygen during photosynthesis.

been collecting data on carbon concentration and temperatures spanning between 800,000 years BC up to now as shown in [Figure 2a](#). The relationship between temperatures and carbon concentration is well established in the literature and grounded by empirical evidence. The bottom line of this analysis is that more carbon concentration leads to warmer temperatures on average. [Figure 2b](#) highlights that the rise in anthropogenic emissions coincides with a $+1^\circ$ increase in temperatures with respect to the 1750 pre-industrial average (black dashed line).

If we have accurate measures of the relationship between carbon concentration and temperatures, however many uncertainties remains on climate change. Carbon concentration always oscillated between 180 and 300 ppm for more than 800,000 years as reported in [Figure 2a](#). In contrast, the levels observed today are unprecedentedly high (above 400 ppm). In those areas, there are no data available that could ground a quantification of carbon-climate interactions. In such high levels of CO₂ concentration, it is likely that some tipping points would translate into critically a non-linear increase in temperatures. The consequences could be large, exemplified by extreme and more frequent weather shocks. While records on temperatures averages span a very long period, the variations around the mean, i.e. distribution, still remain unknown.

Over the last decades, economists took a benign view about the role of climate change and how it could durably affect economic growth. The macroeconomics of climate change can be traced by to [Nordhaus \(1992\)](#) with his quantitative framework, the so-called Dynamic Integrated Climate-Economy (DICE, henceforth) model. In a nutshell, this canonical model of climate change is an extension of Ramsey–Cass–Koopmans model. The extension takes the form of an endogenous Total Factor Productivity (TFP, henceforth) that is directly connected to the carbon concentration: more emissions, stronger radiative forcing, higher temperatures and more severe output losses. In this setup, climate change just takes the form a deterministic problem in which the cumulative sum of current and past emissions determines the loss in TFP. Therefore, the future path of emissions as well as the effort to reduce it through a carbon tax entail the future path of TFP. Climate change becomes an optimal control problem based on the trade-off between the cost of reducing CO₂ emissions today and the cost of temperatures-induced damages in the future.

If this canonical became very popular for macroeconomists as well as for policy-making institutions,³ it is also facing many critiques. [Pindyck \(2017\)](#) argues that the DICE model has crucial flaws that make it “close to useless” as tools for policy analysis. Among these flaws, two of them rose my attention (i) functional forms and parameter

³The quantitative results and policy recommendation from the Intergovernmental Panel on Climate Change (IPCC) are typically obtained from DICE-type models.

values in DICE models are arbitrary and not empirically grounded, (ii) catastrophic climate outcome, such as extreme weather shocks, are omitted from DICE models. In the Blanchard's view about the toolkit of macroeconomists, the Pindyck's critique is an opportunity for environmental economists to question the core hypothesis of the DICE model to improve it. While DICE models typically feature deterministic dynamics, some economists have put many efforts to respond to the critique by introducing uncertainty and business cycle analysis into the DICE models. This branch of the literature, exemplified by Heutel (2012) or Cai and Lontzek (2019), is referred to as the Environmental DSGE models (E-DSGE, henceforth).

This HDR is part of research agenda of the E-DSGE approach that aims at improving our understanding of climate issues at a business cycle frequency. This HDR develops two contributions. First, in Part III of this HDR I develop a business cycle model that can capture how weather shocks to agricultural land entail possibly large fluctuations for output, consumption, exports and the real exchange rate for New Zealand. By increasing the variance of the weather, we are able to measure how climate change will affect the macroeconomic volatilities for developed economies. Second, we explore in Part IV of this HDR the asset pricing implications when the marginal utility of consumption is affected by pollution. The rest of this section develops these two parts.

The case of a more volatile weather. In Gallic and Vermandel (2020), we gauge how much weather shocks matter in explaining business cycle fluctuations. The literature addresses this question in two isolated ways: either by looking at long-term effects through the prism of calibrated theoretical models, or by focusing on both short and long terms through the lens of empirical models. We propose a framework that reconciles these two approaches by taking the theory to the data in two complementary ways. We first document the propagation mechanism of a weather shock using a Vector Auto-Regressive model on New Zealand Data.

To explain the mechanism, we build and estimate a general equilibrium model with a weather-dependent agricultural sector to investigate the weather's business cycle implications. First, we find that weather shocks explain about 35% of GDP and agricultural output fluctuations in New Zealand. This is remarkably high, in particular for a developed economy that heavily rely on its agricultural sector. Weather shocks should be even more costly for developing countries that do not relies on irrigation and fertilizers to dampen adverse weather. Second, we find that these weather shocks entail a welfare cost of 0.30% of permanent consumption. This number is critically high with respect to Lucas (1987) as weather shocks durably affects the productivity of land, which in turn force farmers to divert a fraction of resources toward the mitigation of the extreme weather. Consumption remains low and entail large welfare costs.

Third, we find that weather shocks critically increases the macroeconomic volatility under climate change, resulting in a higher welfare cost peaking to 0.46% in the worst case scenario of climate change. This kind of consequences from climate change are typically omitted from DICE models, and if incorporated, could lead to recommend even more stringent abatement policies.

Carbon and asset pricing. Conventional macroeconomic models of climate change assume that climate change does not directly affect the utility of agents. This assumption conflicts with a strong body of evidence showing that agents compensate from the consequences of CO₂ emissions. To illustrate the compensation effect, empirical evidence suggests that emissions raise the demand for goods that are used to mitigate the effect of pollution, such as air purifiers (Ito and Zhang, 2020) or medical products (Deschênes et al., 2017). If we observe a positive co-movement between emissions and consumption at business cycle frequency, it strongly suggests that the marginal utility of consumption is positively affected by emissions. To capture this effect, we model this compensation effect of climate change via an approach similar to that in the seminal contribution of Campbell and Cochrane (1999). In our case, however, it is the current stock of CO₂ emissions rather than past levels of consumption that raises marginal utility. We next derive the asset pricing implication, and the interaction with a carbon tax.

This work shows how to design a carbon tax that is optimal from a welfare perspective. We firstly use asset pricing theory to derive the implicit market price of CO₂ emissions. We then show that the optimal carbon tax is determined by this implicit price. Next, we use our methodology to compute an estimate of the optimal carbon tax over the business cycle. In our framework, the optimal environmental policy is procyclical. It is therefore optimal to use the carbon tax to “cool down” the economy during periods of booms and to stimulate it in recessions.

Our second main result is that the environmental externality can affect financial markets. In our framework, we find that climate risk reduces the natural rate of interest. This result is relevant for monetary policy because a low natural rate increases the likelihood of hitting the effective lower bound. The reason is that households become more risk averse when firms fail to internalize the damage caused by their emissions. In our simulated economy, low interest rates are thus a consequence of the uncertainty induced by climate change.

A main takeaway is that the effectiveness of the policy critically depends on the ease at which emissions can be abated. The welfare gains from the optimal tax are of a much lower magnitude if the abatement technology is not efficient. Without a well-developed technology, the decline in risk premiums induced by the policy is also

much smaller. The success of the policy may therefore critically depend on the timing of implementation. Improving the existing emission abatement technology should probably come first. Once available, an efficient technology would in turn help to mitigate the side effects of the tax, thereby maximizing the welfare gains from the policy.

Curriculum Vitae

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RESEARCH INTERESTS

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CURRENT AND PREVIOUS POSITIONS

2015-present	Assistant Professor, Paris-Dauphine & PSL Research Universities, Paris (France)
2020-present	Consultant, Banque de France, Financial Economics Research Division, Paris (France)
2017-2020	Scientific Advisor, France Stratégie (French Prime Minister Research Unit), Paris (France)
2014	Ph.D Research Intern at European Central Bank DG-Macro-Prudential Policy and Financial Stability, Macro-Financial Linkages Division, under the supervision of Christoffer Kok.
2012-2015	Teaching and Research Assistant, CREM-CNRS, University of Rennes (France)
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EDUCATION

- 2011-2014 | **Ph.D in Economics** (Completion: 3rd December 2014)
University of Rennes and University of Bordeaux, France
Dissertation: *Essays on Cross-Border Banking and Macroprudential Policy.*
Supervision: J.C. Poutineau, University of Rennes, S. Maveyraud, University of Bordeaux.
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- 2011 | **Master in Mathematical, Statistical and Economic Engineering (with honors)**
University of Bordeaux - Bordeaux, France
Master Thesis: *Financial Integration and Welfare in EMU: a two-country DSGE analysis* (in french)

WORKING PAPERS

1. **Green Asset Pricing**(2020), joint with G. Benmir, I. Jaccard, *ECB WP Series.*
2. **Social Learning about Monetary Policy at the Effective Lower Bound** (2020), joint with J. Arivofic, A. Grimaud, I. Salle, *Bank of Canada WP.*
3. **Endogenous Trends** (2021).

PUBLISHED ARTICLES

IN ENGLISH:

- **Weather shocks** (2020), joint with E. Gallic, *European Economic Review.*
- **Global Banking and the Conduct of Macroprudential Policy in a Monetary Union** (2017), joint with J-C Poutineau, *Journal of Macroeconomics.*
- **A Welfare Analysis of Macroprudential Policy Rules in the Euro Area** (2017), joint with J-C Poutineau, *Revue d'Economie Politique.*
- **Financial Frictions and the Extensive Margin of Activity** (2015), joint with J-C Poutineau, *Research in Economics.*
- **The analytics of the 3-equation New Keynesian Model** (2015), joint with Jean-Christophe Poutineau and Karolina Sobczak, *Economics and Business Review*, vol. 15, no. 2.

- **Cross-border Banking Flows Spillovers in the Eurozone: Evidence from an Estimated DSGE Model** (2015), joint with J-C Poutineau, *Journal of Economic Dynamics and Control*, vol. 51, 378-403.
- **A Primer on Macroprudential Policy** (2015), joint with J-C Poutineau, *Journal of Economic Education*, vol. 46(1), 68-82.

IN FRENCH:

- **Améliorer la réglementation peut-il réduire le chômage structurel ?** (2019), joint with Cédric Crofils, *France Stratégie*.
- **Quelle prise en compte des caractéristiques nationales dans les mesures macro-prudentielles en zone euro?** (2015), joint with Jean-Christophe Poutineau, *Revue Française d'Economie*.
- **Intégration bancaire et conjoncture macroéconomique dans une union monétaire hétérogène** (2013), joint with Jean-Christophe Poutineau, *Brussels Economic Review*, vol. 56(3-4), pages 241-260.

PHD SUPERVISION

2017-present	Josselin Roman (Dauphine University): Secular Stagnation and Climate Change in Macroeconomics
2019-present	Ghassane Benmir (LSE and Dauphine University): Asset Pricing under Climate Change.
2019-present	Louise Narbonne (Rennes University): Income Inequality in Open Economies.
2020-present	Cédric Crofils (Dauphine University): Weather Shocks in Developing Economies.
2021-present	Côme Poirier (Dauphine University): Sectoral Business Cycles.

PHD DEFENSE COMMITTEE

2019	Caroline Bozou (Université Paris 2) on « Risk aversion and banking competition »
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GRANTS/PRIZES

2021	EFA 2021 Best Paper Prize in Responsible Finance
2021-2025	French Research Ministry Grant for PhD supervision(PEDR)

VISITINGS

Feb 2019 | ECB Research Department in Frankfurt. Invited by Ivan Jaccard.
April 2018 | University of Amsterdam. Invited by Professor Cars Hommes.

Part I

Endogenous Trends

INTRODUCTION

In modern models of the business cycle, economic fluctuations are interpreted as high frequency fluctuations around a trend growing at an exogenous rate (either deterministic or stochastic). This conception of business cycles is questionable given the strong body of evidence in empirical macroeconomics showing that the trend of the US economy is time-varying (Nelson and Plosser (1982)) and reducing over time.⁴ Despite this evidence, most of recent medium scale macroeconomic models assumes either a fixed slope of growth (e.g., Smets and Wouters (2007)) or exogenous drifts to productivity (e.g., Christiano et al. (2014)). The resulting interpretation of business cycles is at odd with the evidence of structural changes in the long run growth of an economy observed over the last decades. In particular, the underlying factors that are jointly driving low frequency changes in macroeconomic time series are usually swept out by business cycle filters, or erroneously captured by exogenous disturbances.

The main goal of this paper is therefore to develop a quantitative model that features an endogenous slope of growth, referred to as an *endogenous trend*.⁵ Guided by the endogenous growth theory, the trend at which the economy is growing at a low frequency is determined by two growth engines based on the accumulation of technologies and knowledge. For the first engine of growth based on the accumulation of technologies, the endogenous productivity mechanism we develop is based on Comin and Gertler (2006), which uses the approach to connect business cycles to growth. This model of Comin and Gertler (2006) is itself a variant of Romer (1990)'s expanding variety model of technological change, modified to include a friction on the endogenous probability of technology adoption. We include a sticky rate of adoption to capture a congestion externality in the diffusion of new technologies. The second engine of growth is based on the accumulation of knowledge (i.e., “experience” or “skill”), through a model of human capital à la Lucas Jr (1988). Each period, firms engage a fraction of their labor inputs into vocational training in order to produce human capital. We modify the Lucas framework to allow for an endogenous rate of adoption of new skills, along with endogenous human capital formation. By doing so, we are able to allow for empirically reasonable diffusion lags but still generate endogenous medium-term swings in productivity of labor.

We then estimate the model with endogenous trend on a sample spanning from 1950q1 up to 2018q4 using Bayesian techniques. The solution method employed to estimate the model features an occasionally binding constraint on the nominal rate. We then use the model to assess the slowdown of long term growth, in particular

⁴For a long run perspective on growth, see Antolin-Diaz et al. (2017) For recent papers after the Great Recession documenting the slowdown of the US economy, see Fernald and Jones (2014) and (Gordon, 2012, 2017).

⁵These cyclical movement are interpreted by Comin and Gertler (2006) as medium term fluctuations. In this paper, I interpret these fluctuations as persistent changes in the growth rate of the economy that affects key macroeconomic variables.

following the onset of the Great Recession. Based on the estimated model, our key result is that we corroborate the thesis of a strong decline in the long term trend of the US economy. Among the two sources of growth examined in the paper, the slowdown mainly is induced by the technology engine reflecting a decline in the productivity of R&D spending since 1960. This finding tends to favor the [Gordon \(2012\)](#) theory stating that the US growth has strongly declined since 1970. In addition, we find that a standard macro-model with exogenous growth erroneously captures low frequency changes in economic growth by highly persistent macroeconomic shocks. In contrast, the model featuring an endogenous trend successfully captures this low frequency fluctuations. This endogenous persistence is key, as it allows the model to outperform the forecasting performance of a DSGE model with an exogenous trend.

In addition to the literature cited above, there are several other papers related to our analysis. [Anzoategui et al. \(2019\)](#) estimates a macroeconomic model with one source of growth for the US economy. They evaluate the role of R&D in the productivity slowdown following the financial crisis, they find that the reduction in productivity is induced by a reduction in the adoption rate of technology. [Moran and Queralto \(2017\)](#) complete this analysis by including the role of monetary policy, in particular when the ZLB is binding. Both [Queralto \(2019\)](#) and [Bianchi et al. \(2019\)](#) inspect the role of financial frictions on knowledge accumulation to capture the recent slowdown in economic growth for the US. An alternate approach of [Garcia-Macia \(2017\)](#) stresses misallocation between tangible and intangible capital following a financial crisis. [Annicchiarico and Pelloni \(2016\)](#) inspect the implications of the endogenous growth on the optimal conduct of monetary policy.

This paper is also related to a literature that puts endogenous growth mechanism into real business cycles models. [Hercowitz and Sampson \(1991\)](#) is probably the first paper that attempts to connect the business cycles and endogenous growth. They estimate their model as a VAR process and find that endogenous growth successfully accounts for the persistence of output growth. [Boileau \(1996\)](#) evaluates how in an open economy context the endogenous growth mechanism helps the model in replicating salient business cycle statistics. [Barlevy \(2004\)](#) revisited the welfare cost of business under endogenous growth and finds that the presence of endogenous growth exacerbates the welfare cost of business cycles. Similarly, [Wu and Zhang \(1998\)](#) revisited the welfare cost of inflation under endogenous growth. While most of this literature addresses the question of growth as an expanding variety effect, [Lucas Jr \(1988\)](#), [Collard \(1995\)](#) and [Jones et al. \(2005\)](#) originally consider growth as an accumulation of human capital. [Matheron et al. \(2004\)](#) examines the interactions of the growth engine in a non-walrasian labor market.

The rest of the paper is organized as follows. [section I.1](#) presents a New Keynesian Model with two sources of endogenous growth. [section I.2](#) is devoted to the estimation of the model using Bayesian econometrics. [section I.3](#) evaluates the consequences an endogenous rate of growth on the transmission of TFP shock, and the role of a time-varying trend on the cross-correlation of observable variables. [section I.4](#) studies the contribution of the accumulation of technologies and knowledge in the historical evolution of the long run growth rate of the US economy since 1950. [section I.5](#) evaluates the role of the zero lower bound on the economic contraction during the financial crisis. [section I.7](#) concludes.

I.1 A NEW KEYNESIAN MODEL WITH TWO SOURCES OF ENDOGENOUS GROWTH

This section describes the theoretical framework, consisting of a standard medium-sized New Keynesian model augmented to include endogenous creation and adoption of new technologies and human capital, respectively denoted A_t and H_t . The slope of growth of the economy, denoted Γ_t , is thus a function of these two engines of growth, with $\Gamma_t = f(A_t, H_t)$. Sub-sections I.1.2.3 and I.1.4 constitute the main departures from other medium-sized DSGE models found in the literature. The rest of the sub-section provides the conventional ingredients of the a New Keynesian model similar to Smets and Wouters (2007).

I.1.1 HOUSEHOLDS

The preferences of the j^{th} family are given by:

$$E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \left[\frac{(c_{jt+\tau} - hc_{jt-1+\tau})^{1-\sigma_C}}{1-\sigma_C} \exp \left(\left(\frac{\sigma_C - 1}{1 + \sigma_L} \right) \chi_L l_{jt+\tau}^{1+\sigma_L} \right) \right] \right\} \quad (I.1)$$

where E_t denotes the expectation operator and $\beta \in [0, 1)$ is the discount factor. The consumption index c_{jt} is subject to external habits governed by parameter $h \in [0; 1)$ while $\sigma_C > 0$ is the risk aversion parameter on consumption. Parameter $\sigma_L > 0$ shapes the consumption-leisure trade-off, while $\chi_L > 0$ is a shift parameter pinning down the steady state amount of hours worked.

Household j face a budget constraint:

$$c_{jt} + b_{jt} = b_{jt-1} \underline{r}_{t-1} / \pi_t + w_t h_{jt} l_{jt} + \Pi_{jt} - t_{jt}, \quad (I.2)$$

The income of the representative household is made of labor income with real wage w_t combined with human capital h_{jt} (or skills) and hours worked l_{jt} , total firm profits Π_{jt} , real interest payments $\underline{r}_{t-1} / \pi_t$ from riskless bonds b_{jt} , with inflation rate $\pi_t = P_t / P_{t-1}$. Nominal rate \underline{r}_{t-1} is subject to an effective zero lower bound that limits its fluctuations above zero, $\underline{r}_t = \max(r_t, 1)$ with r_t the unconstrained counterpart.

I.1.2 INTERMEDIATE FIRMS

I.1.2.1 Intermediate goods composite

There exists a continuum of measure A_t of monopolistically competitive intermediate goods firms that each makes a differentiated product x_{it} with $i \in [0, A_t]$. The endogenous variable A_t is the stock of types of intermediate goods adopted in production, i.e., the stock of adopted technologies. We assume that one firm produced one type of variety such that $i \in [0, A_t]$ both refers to a good or an intermediate firm. Each firms produces produces output x_{it} at a selling price p_{it}^x . Intermediate goods are packed into one homogenous good X_t that is sold to final firms. The intermediate goods composite

is the following CES aggregate of individual intermediate goods:

$$X_t = \left[\int_0^{A_t} x_{it}^{(\vartheta-1)/\vartheta} di \right]^{\vartheta/(\vartheta-1)}, \quad (I.3)$$

where parameter $\vartheta > 1$ is the degree of imperfect substitution between varieties allowing intermediate firms to make profits. The aggregate price index is given by: $P_t^x = [\int_0^{A_t} (p_{it}^x)^{1-\vartheta} di]^{1/(1-\vartheta)}$. The optimal demand for the i -th varieties is given by:

$$x_{it} = (p_{it}^x/P_t^x)^{-\vartheta} X_t. \quad (I.4)$$

I.1.2.2 Production technology

There is a continuum of i firms that produces an intermediate goods by combining labor inputs, capital inputs and technology. The i^{th} firm has the following Cobb-Douglas technology:

$$x_{it} = \varepsilon_t^A [(1 - e_{it}) l_{it}^d h_{it}^\omega]^\alpha [u_{it} k_{it-1}]^{1-\alpha} \quad (I.5)$$

where (exogenous) AR(1) technology is ε_t^A , hours worked demand l_{it}^d , human capital h_{it} , e_{it} the fraction of the labor supply involved in the accumulation of knowledge, u_{it} is the utilization rate of physical capital and k_{it-1} is the physical capital. Parameter $\alpha \in [0, 1]$ measures the labor intensity in the firms technology. Workers may spend a fraction e_{it} of their time acquiring skills. That is, they can learn to use more advanced capital goods. Parameter $\omega \geq 0$ is the internal effect of human capital which benefits to the overall economy. According to [Mincer \(1974\)](#), an additional year of schooling or an additional year of experience should increase wages proportionally. To incorporate this mechanism in the model, we assume increasing returns on human capital, with elasticity $\omega > 1$.

Real profits are given by:

$$d_{it}^x = \frac{P_{it}^x}{P_t} x_{it} - w_t h_{it} l_{it}^d - \varepsilon_t^I \left(1 + S_I \left(\frac{i_{it}}{\gamma_t l_{it-1}} \right) \right) i_{it} - \frac{P_t^I}{P_t} \left(1 + S_H \left(\frac{z_{it}^H}{z_{it-1}^H} \right) \right) z_{it}^H - \frac{P_t^I}{P_t} s_{it}^H h_{it}^u, \quad (I.6)$$

where $\varepsilon_{i,t}^I$ is a stochastic process which captures exogenous changes in the value of physical capital, regarding adjustment cost functions $S_a(x_t) = \chi_a (x_t - \bar{x})^2$ with $\chi_a \geq 0$ is the adjustment cost parameter.

For clarity purpose, we separate production and labor decisions in the following subsections.

I.1.2.3 Production and adoption of skills

As in [Lucas Jr \(1988\)](#), we assume that there firms can spent a fraction e_{it} of working to the accumulation of human capital while $(1 - e_{it}) l_{it}^d$ is the skill-weighted man-hours devoted to current production. The rise in more skilled worker does not necessary translate into immediate growth of output. We capture this pattern by assuming that all skills in the economy are not necessarily adopted by firms. More specifically, the

stock of unadopted human capital, denoted h_{it}^u , is given by:

$$h_{it}^u = (1 - \delta_H) \left[F_H(e_{it-1}, z_{it-1}^H) + (1 - p^H(s_{t-1}^H, s_{it-1}^H)) h_{it-1}^u \right], \quad (\text{I.7})$$

where δ_H is the obsolescence rate of a skill, $F_H(\cdot)$ is the production function of new human capital and $p^H(\cdot)$ is the endogenous probability of adoption of a skill by the i -th firm. Regarding the adoption probability of a skill, our goal is to capture the notion that adoption takes time on average, but allow for adoption intensities to vary procyclically. These considerations lead us to the following formulation for the functional form: $p^H(s_{t-1}^H, s_{it-1}^H) = \varsigma_t^H (s_{it}^H)^{\zeta^H}$, where ς_t^H is a scaling factor that pins down the steady state in the balanced growth path, s_{it}^H are the adoption expenditures in units of final goods. Note that an increase in skill adoption effort s_{it}^H by firm i reduces the stock of un-used skills, boost the stock of adopted skills which creates endogenous growth.

As in [Jones et al. \(1993\)](#), human capital creation is a Cobb-Douglas function that combines education hours e_{it} and education expenditures z_{it}^H :

$$F_H(e_{it}, z_{it}^H) = \xi_t^H (e_{it})^{1-v} (z_{it}^H)^v, \quad (\text{I.8})$$

where ξ_t^H is a productivity parameter that pins down the steady state in the balanced growth path, v is a technology parameter determining the intensity of education expenditures in the production of knowledge. For $v = 0$, the model reads as in Lucas, while for $v > 0$, the model is similar to the setup of [Jones et al. \(1993\)](#).

The law of motion of adopted skills, or effective human capital, is given by:

$$h_{it} = (1 - \delta_H) [p^H(s_{t-1}^H, s_{it-1}^H) h_{it-1}^u + h_{it-1}]. \quad (\text{I.9})$$

Intermediate firms maximize their profits under [Equation I.6](#), the supply constraint [I.5](#), the demand constraint [I.4](#) and law of motions [I.7](#) and [I.9](#). Letting v_t^U and v_t^H denote the Lagrangian multipliers associated with laws of motion of unadopted and adopted human capital respectively. They represent the current marginal value of unadopted and adopted skills, respectively.

The optimal fraction of hours worked spent in education e_{it} is given by:

$$\frac{p_t^x}{\mu_\vartheta} \alpha \frac{x_{it}}{(1 - e_{it})} = (1 - \delta_H) E_t \{ m_{t,t+1} V_{t+1}^U \} F_{H,t}'^e, \quad (\text{I.10})$$

where $F_{H,t}'^e$ is the derivative in education of the production function of knowledge and V_t^U is the value of unadopted skills. The left hand side of [Equation I.10](#) is the productivity loss of increasing e_{it} , while the right hand side denotes the expected marginal product of unadopted skills. Parameter $\mu_\vartheta = \vartheta / (\vartheta - 1)$ is the markup over the marginal cost of producing intermediate goods.

The optimal education spending z_{it}^H reads as:

$$1 + \frac{\partial S_{H,t} z_{it-1}^H}{\partial z_{it}^H} + E_t \left\{ m_{t,t+1} \frac{\partial S_{H,t+1} z_{it}^H}{\partial z_{it}^H} \right\} = (1 - \delta_H) E_t \{ m_{t,t+1} V_{t+1}^U \} F_{H,t}'^z, \quad (\text{I.11})$$

Similarly to the optimal education, the left hand side denotes the marginal cost of education spending and the right hand side is the expected marginal product of unadopted skills.

In addition, the optimal amount of adopted human capital h_{it} is given by:

$$V_t^H = \frac{p_t^x}{\mu_\vartheta} \omega \alpha \frac{y_{it}}{h_{it}} - w_t l_{it}^d + (1 - \delta_H) E_t \{m_{t,t+1} V_{t+1}^H\}. \quad (\text{I.12})$$

The current value of human capital V_t^H is determined by its marginal productivity, net of wage payments, and the expected value of the adopted skill if the human capital does not depreciate.

The value of unadopted skills:

$$V_t^U = (1 - \delta_H) E_t \left\{ m_{t,t+1} \left[p_t^H V_{t+1}^H + (1 - p_t^H) V_{t+1}^U \right] \right\}. \quad (\text{I.13})$$

Finally the optimal demand for s_{it}^H is given by:

$$(1 - \delta_H) E_t \left\{ m_{t,t+1} \left[V_{t+1}^H - V_{t+1}^U \right] \right\} p_t^{H'} = 1, \quad (\text{I.14})$$

where $p_t^{H'}$ is the derivative of the probability of adoption with respect to the quantity of goods s_{it}^H spent in adoption of skills. The term on the right is the marginal gain from adoption expenditures: the increase in the adoption probability p_t^H times the discounted difference between the value of an adopted versus an unadopted skill. The right side is the marginal cost. The term $V_{t+1}^H - V_{t+1}^U$ is pro-cyclical, given the greater influence of near term profits on the value of adopted skills relative to unadopted ones.

I.1.2.4 Capital decisions

Intermediate firms maximize their profits under [Equation I.6](#) under the supply constraint [I.5](#) and the demand constraint [I.4](#) and the following law of motion of capital:

$$k_{it} = i_{it} + (1 - \delta(u_{it})) k_{it-1}, \quad (\text{I.15})$$

where $\delta(u_{it}) = \delta_c + \frac{b}{1+\psi} u_{it}^{1+\psi}$. In this function $\delta_c > 0$ is the fixed part of the depreciation, while the time-varying part is a function of the utilization rate of capital. $\psi \geq 0$ is the elasticity of the depreciation with respect to utilization. Parameter $b \geq 0$ is a shift parameter which allows to pin down the steady state utilization rate.

The first order condition determining the shadow value of investment goods is given by:

$$q_t = \varepsilon_t^I \left(1 + \frac{\partial i_{it} S(i_{it}/i_{it-1})}{\partial i_{it}} \right) + m_{t,t+1} \varepsilon_{t+1}^I \frac{\partial S(i_{it+1}/i_{it}) i_{it+1}}{\partial i_{it}}, \quad (\text{I.16})$$

where q_t is the Lagrangian multiplier associated with the law of motion of physical capital.

The optimal demand for physical capital is given by:

$$q_t = E_t \left\{ m_{t,t+1} \left[\frac{p_{t+1}^x}{\mu^\vartheta} (1 - \alpha) \frac{y_{it+1}}{k_{it}} - (1 - \delta(u_{it+1})) q_{t+1} \right] \right\}. \quad (\text{I.17})$$

The optimal utilization rate is given by:

$$(1 - \alpha) \frac{y_{it}}{u_{it}} \frac{p_t^x}{\mu^\vartheta} = \delta'(u_{it}) q_t. \quad (\text{I.18})$$

I.1.3 FINAL FIRMS

A continuum of mass unity of final firms produce final output Y_t using the intermediate output X_t as input. Each producer $j \in [0, 1]$ simply purchases intermediate output X_t , differentiate it into a variety y_{jt} and sell them to final output consumers. In equilibrium, the total demand of final firms must reach the total supply of intermediate inputs as follows $\int_0^1 y_{jt} dj = X_t$. Final output is a CES composite of differentiated varieties:

$$Y_t = \left[\int_0^1 y_{jt}^{(\epsilon-1)/\epsilon} dz \right]^{\epsilon/(\epsilon-1)}. \quad (\text{I.19})$$

where y_{jt} is the output by final firm $z \in [0, 1]$ with an associated price denoted p_{jt} . The parameter ϵ , satisfying $\epsilon > 1$, governs the extent of imperfect substitutability across final goods varieties. Cost minimization by final goods consumers implies a downward sloping demand curve for each variety of final good

$$y_{jt} = (p_{jt}/P_t)^{-\epsilon} Y_t, \quad (\text{I.20})$$

where P_t is the aggregate price index determined by the zero profit condition in this market: $P_t = [p_{jt}^{(1-\epsilon)/\epsilon} dz]^{1/(1-\epsilon)}$. To introduce nominal rigidities, we employ a Calvo pricing scheme. In particular, a fraction of final firms is not allowed to re-optimize its selling price with probability θ but price increases by $\xi \in [0, 1)$ with respect to the previous period's rate of price inflation, $p_{jt} = \pi_{t-1}^\xi \bar{\pi}^{1-\xi} p_{jt-1}$. The z^{th} firm allowed to update its selling price p_{jt}^* with a probability $1 - \theta$ maximizes the following discounted sum of profits

$$\max_{\{p_{jt}^*\}} E_t \left\{ \sum_{s=0}^{\infty} \theta^s m_{t,t+s} \left[\frac{p_{jt}^*}{P_{t+s}} \Xi_{t+s} - \varepsilon_{t+s}^P \frac{P_{t+s}^x}{P_{t+s}} \right] y_{zt+s} \right\},$$

$$s.t. \quad y_{zt+s} = (p_{jt}^* \Xi_{t+s} / P_{t+s})^{-\epsilon} Y_{t+s}$$

where ε_t^P is an *ad hoc* cost-push shock to the inflation equation following an AR(1) process which captures exogenous changes in input costs of final firms. Variable Ξ_t captures the contribution of the indexation rule to the firm's future profits, $\Xi_{t+s} = \prod_{j=1}^s \pi_{t-1+j}^\xi \bar{\pi}^{1-\xi}$ for $s > 0$, while $\Xi_{t+s} = 1$ for $s = 0$.

I.1.4 INNOVATORS

We model technology following [Comin and Gertler \(2006\)](#), which is in turn based on the expanding-variety framework due to [Romer \(1990\)](#). Innovations in the model take the form of new patents Z_t which are discovered endogenously as a result from private R&D spending. As [Comin and Gertler \(2006\)](#), patents are subject to a “time-to-adop” friction: a new technology does not necessarily give birth immediately to a new variety of intermediate goods. Converting a patent into a new variety is costly for innovators and create a lag between the creation of a new technology and its translation into an stronger rate of growth for the economy.

Assuming that among the family members of each household, there is a fraction $j \in [0; \eta]$ of innovators that creates new technologies.⁶ Each innovator owns a stock of existing patents, denoted Z_{jt} , representing the technological frontier in the economy. These technologies are subject to exogenous obsolescence, which occurs with probability δ_A . Letting x_{jt}^A denotes R&D expenditures (in units of growth-oriented investment goods) devoted to the creation of a new patent, denoted $v(x_{jt}^A)$, the law of motion of patents (or the “technological frontier”) are given by:

$$Z_{jt} = (1 - \delta_A) (Z_{jt-1} + F_A(x_{jt-1}^A)). \quad (\text{I.21})$$

Here, both existing and new patents are subject to the obsolescence shock, this implies that some new technologies are abandoned and never translate into intermediate varieties. Regarding the production of a new technology, we assume $F_A(x_{jt}^A) = \varepsilon_t^A \xi_t^A (x_{jt}^A)^{\alpha_A}$ where α_A is a technology parameter, and ξ_t^A pins down the growth rate of technology in the balanced growth path. As suggested by [Griliches \(1990\)](#), the production of new patent has decreasing return to scale (*i.e.* $\alpha_A < 1$) that captures a congestion effect that raises the cost of developing new products as the aggregate level of R&D intensity increases. This effects is usually referred to as the “stepping on toes”: *i.e.* the obvious new ideas are discovered first and it gets increasingly difficult to find the next new one (see [Jones \(2005\)](#) for a discussion).

Recall that A_{jt} is the number of varieties of intermediate goods, thus any point on the real line between 0 and A_{jt} represents a distinct variety of intermediate goods. With a time-to-adopt assumption, there is a gap between numbers of available and adopted technologies, $Z_{jt} - A_{jt} > 0$. This gap, denoted A_{jt}^u , is referred to as the stock of unadopted technologies and has the following law of motion:

$$A_{jt}^u = (1 - \delta_A) (F_A(x_{jt-1}^A) + (1 - p_A(s_{jt-1}^A)) A_{jt-1}^u). \quad (\text{I.22})$$

In this expression, $p_A(\cdot)$ denotes the speed of adoption of an unadopted technology, that is an increasing function of R&D spending s_{jt-1}^A in units of growth-oriented investment goods.⁷ If the adopter is not successful, he may try again in the next period. Thus, under our formulation there is slow diffusion of technologies on average that varies positively with the intensity of adoption expenditures. This endogenous

⁶The number of innovator pins down the steady state of R&D spending-to-GDP ratio.

⁷The functional form for $p_A(s_{jt-1}^A) = \varsigma_t^A (s_{jt-1}^A)^{\varkappa_A}$ is taken from [Comin and Gertler \(2006\)](#), parameter \varkappa_A is the elasticity of adoption with R&D spending s_{jt-1}^A while ς_t^A is a scaling factor pinning down the steady state in the balanced growth path of the model.

mechanism of adoption reproduces the cyclicity of technology diffusion that is observed in the micro data, as shown by [Anzoategui et al. \(2016\)](#).

The remaining set of technologies A_{jt} that are effectively converted into an intermediate goods are given by the following law of motion:

$$A_{jt} = (1 - \delta_A) (p_A (s_{jt-1}^A) (1 - S_A (p_{jt}^A/p_{jt-1}^A)) A_{jt-1}^u + A_{jt-1}), \quad (\text{I.23})$$

where $S_A (p_{jt}^A/p_{jt-1}^A)$ denotes an adjustment cost on rising the probability of adoption with $S_A (x_t) = 0.5\chi_A (x_t - \bar{x})^2$ similar to [Christiano et al. \(2005\)](#). This cost function is new with respect to the literature and has two goals. First, it captures another congestion externality *à la* [Romer \(1990\)](#) on the adoption of a new technology: firms trying to get a new product to market face a lower probability success. Second, this cost aims at capturing the low frequency nature of A_{jt} : an higher value for χ_A implies a lower frequency for the growth of technology χ_A . We are thus free to estimate this cost parameter to match the evidence by setting a diffuse prior distribution on this parameter. The fit exercise of [Moran and Queraltó \(2017\)](#) shows that adjustment cost on R&D expenditures are much larger than for investment goods.

The real profit of the innovator is given by:

$$\Pi_{jt}^A = A_{jt} \frac{\Pi_{jt}^X}{P_t} - \frac{P_t^I}{P_t} x_{jt}^A - \frac{P_t^I}{P_t} A_{jt}^u s_{jt}^A, \quad (\text{I.24})$$

where Π_{jt}^X is the monopoly rent that the innovator obtain from selling an amount A_{jt} of varieties of intermediate goods. At every stage of the innovation process, the innovator successfully adopting a new technology exploits the competitive advantage and monopolize the market as in [Aghion and Howitt \(1996\)](#). The innovator must pay cost of adoption $A_{jt}^u s_{jt}^A$ and R&D expenditures x_{jt}^A in units of growth-oriented investment goods at market price P_t^I/P_t .

Each period maximizes the discounted sum of profits [Equation I.24](#) using control variables x_{jt}^A , s_{jt}^A , A_{jt} , A_{jt}^u and p_{jt}^A under technology law of motions [Equation I.22](#) and [Equation I.23](#). Anticipating symmetry, and letting J_t^U and J_t^A denotes the real shadow values of unadopted and adopted technologies respectively, the value of adopted technologies is the present discounted value of profits from producing the good:

$$J_t^A = \Pi_t^X/P_t + (1 - \delta_A) E_t \{m_{t,t+1} J_{t+1}^A\}. \quad (\text{I.25})$$

While the value of unadopted technologies is determined by:

$$J_t^U = -\frac{P_t^I}{P_t} s_t^A + (1 - \delta_A) E_t \{m_{t,t+1} [J_{t+1}^U (1 - p_t^A) + J_{t+1}^A p_t^A (1 - S (p_{t+1}^A/p_t^A))]\}. \quad (\text{I.26})$$

Firm invest x_t^A units of growth-oriented investment goods in R&D until the expected marginal product of discovering a new patent reaches the marginal cost of production:

$$\frac{P_t^I}{P_t} = (1 - \delta_A) \Phi'_A (x_t^A) E_t \{m_{t,t+1} J_{t+1}^U\}. \quad (\text{I.27})$$

The marginal cost of rising the adoption rate, denoted q_t^A , reads as follows:

$$\frac{P_t^I}{P_t} A_t^u = q_t^A p_A'(x_t^A). \quad (\text{I.28})$$

Finally, optimal adoption rate is given by:

$$\frac{q_t^A}{A_t^u} + \Psi_t = (1 - \delta_A) E_t \{ m_{t,t+1} [J_{t+1}^A - J_{t+1}^U] \}. \quad (\text{I.29})$$

The left hand side of this equation reflects the current marginal cost of adopting a technology,⁸ while the right hand side is the discounted benefits in the next period. Innovators increases their adoption expenditures until the marginal cost of adopting is equal to the expected marginal gain. As [Comin and Gertler \(2006\)](#), this marginal gain is $J_{t+1}^A - J_{t+1}^U$ is pro-cyclical, given the greater influence of near term profits on the value of adopted technologies relative to unadopted ones

I.1.5 AUTHORITIES

Concerning federal monetary policy, the general expression of the central bank's rate follows a standard Taylor rule:

$$r_t = r_{t-1}^\rho \left[\bar{r} \left(\frac{\pi_t}{\bar{\pi}} \right)^{\phi_\pi} \right]^{(1-\rho)} \left(\frac{Y_t}{Y_{t-1} \bar{\gamma}} \right)^{\phi_G} \varepsilon_t^R, \quad (\text{I.30})$$

where ε_t^R is a monetary policy shock, $\phi_\pi \geq 1$ is the inflation stance, ϕ_G is another stance on deviations of production growth from its steady state $\bar{\gamma}$. Recall that here, changes in the medium term component, denoted γ_t , is affecting the nominal rate as long as $\phi_G \neq 0$. Following [Gust et al. \(2017a\)](#), the smoothing of the rule is based on the shadow rate rather than the effective interest rate to allow the shadow rate to go below one.

However, a zero lower bound constraint on the nominal rate generates a wedge between the desirable interest rate for the economy and the effective one. The effective rate, denoted \underline{r}_t , determining the rate of return of government bonds reads as:

$$\underline{r}_t = \max(r_t, 1). \quad (\text{I.31})$$

Regarding the government, it consumes G_t units of final goods. The government supports these expenditures by issuing one-period debt securities, b_t , and charging a lump-sum tax to household, T_t . The government budget balance reads as: $G_t + b_{t-1} \underline{r}_{t-1} / \pi_t = b_t + T_t$. We assume that along the balance growth path, the share of government purchases in output, denoted s_g , is constant over time. To this end, we impose $G_t = \Gamma_t s_g \bar{Y} \varepsilon_t^G$, where Γ_t is the time-varying trend of output, $s_g \bar{Y}$ is the detrended steady state of public spending and ε_t^G is an exogenous AR(1) capturing exogenous changes in aggregate demand. The presence of Γ_t maintains the balanced

⁸The term Ψ_t denotes the adjustment cost that must be paid by the innovator that makes the adoption rate sluggish : $\Psi_t = v_t^A (1 - \delta_A) p_{t-1}^A \frac{\partial S(p_t^A/p_{t-1}^A)}{\partial p_t^A} \frac{A_{t-1}^u}{A_t^u} + (1 - \delta_A) v_{t+1}^A m_{t,t+1} \frac{\partial p_t^A S(p_{t+1}^A/p_t^A)}{\partial p_t^A}$.

growth path by making the share of public spending stable as the economy grows.

I.1.6 MARKET CLEARING CONDITIONS

The aggregate constraint on final goods market is given by:

$$\frac{Y_t}{\Delta_t^P} = C_t + I_t \varepsilon_t^I (1 + S_I(\cdot)) + \frac{P_t^I}{P_t} (I_t^H + I_t^A) + G_t, \quad (\text{I.32a})$$

where Δ_t^P is the price dispersion term induced by the Calvo pricing scheme.

Aggregate expenditures in R&D and educations are given by:

$$I_t^H = Z_t^H (1 + S_H(\cdot)) + H_t^u S_t^H \text{ and } I_t^A = \eta (Z_t^A + A_t^u S_t^A). \quad (\text{I.33})$$

The equilibrium on the intermediate market is given by the demand function:

$$Y_t = A_t^{\vartheta-1} \varepsilon_t^A ((1 - e_t) H_t^\omega L_t)^\alpha (u_t K_{t-1})^{1-\alpha},$$

where $K_{t-1} = \int_0^{A_t} K_{it-1} di$ and $L_t = \int_0^{A_t} L_{it} di$. Here, Y_t is interpreted as an average firm given by $Y_t = X_t/A_t$.

I.1.7 BALANCED GROWTH PATH

Empirically, the growth of education and R&D expenditures have both been secularly increasing twice faster than output. To capture this upward trend in the expenditure side of the GDP without structurally modifying key supply side ratios in output, we introduce a common investment-specific trend, denoted, Υ_t , which grows at a fixed gross rate $\bar{\gamma}_X = \Upsilon_t/\Upsilon_{t-1}$. These investment goods I_t^H and I_t^A are produced from final goods by means of a linear technology whereby $1/\Upsilon_t$ units of final goods yield one unit of investment goods. The slope of this investment-specific trend crucially appears in the measurement equation of the model and is estimated in the fit exercise.

This economy features three sources of permanent growth: two are endogenous (A_t and H_t) and one is exogenous Υ_t . As a result, a number of variables, such as output, are not stationary. We therefore perform a change of variable in order to obtain a set of equilibrium conditions that involves only stationary variables. Along the balanced growth path, per capita output $\{Y_t\}$, per capita expenditure categories $\{Z_t^H, Z_t^A, A_t^u X_t^A, I_t, C_t\}$ and per capita capital stocks $\{K_{t-1}\}$, per capita income categories $\{W_t\}$ and government expenditures and lump sump transfers $\{T_t\}$ grow at the same rate. This growth rate is equal to:

$$\gamma_t = \Gamma_t/\Gamma_{t-1} \text{ with } \Gamma_t = [A_t^{\vartheta-1} H_t^{\mu\alpha}]^{1/\alpha}. \quad (\text{I.34})$$

The growth rate given in Equation I.34 depends on technology parameters μ , α and γ ; competition parameter in intermediate markets ϑ ; and stocks of adopted technologies A_t and skills H_t . These stocks grow both at rates $g_{H,t} = H_t/H_{t-1}$ and $g_{A,t} = A_t/A_{t-1}$.

I.2 ESTIMATION

I.2.1 SOLUTION METHOD

To take into account the zero lower bound constraint on the nominal rate, we employ the solution method developed by [Guerrieri and Iacoviello \(2015\)](#). It applies a first order perturbation approach in a piecewise fashion in order to handle occasionally binding constraints. In this model, the presence of the ZLB is treated as a second regime that occasionally binds when the state variable in [Equation I.30](#) is below zero, otherwise the constraint is slack. The piecewise linear solution method maps these two different regimes in the same model by using first order approximation of each regime around the same steady state. The solution of the model is non-linear as decision rules parameters depend on the value of the nominal rate. Unlike global methods, this piecewise solution is fast enough to allow the estimation with full information methods of models with many state variables.

Because the solution is state-dependent, the Kalman filter cannot be employed to compute the smoothed sequence of shocks. We follow the estimation method of [Guerrieri and Iacoviello \(2017\)](#) by replacing the Kalman filter by an inversion filter in order to construct the log-likelihood function. Pioneered by [Kollmann \(2013\)](#), this filter extracts the sequence of innovations recursively by inverting the observation equation. One of the drawbacks of this approach lies in the number of shocks that has to be exactly the same as the number of innovations to allow the recursive inversion of the observation equation.⁹ Given this limitation, the model is estimated on 8 observable macroeconomic time series and are jointly replicated by the model through the joint realization of 8 corresponding innovations.

I.2.2 DATA

The model is estimated with Bayesian methods on US quarterly data over the sample time period 1950Q1 to 2018Q4 and are all taken from FRED. Our sample spans an extended period of time to capture US growth patterns.

Concerning the transformation of series, the point is to map non-stationary data to a stationary model (namely, the GDP, consumption, investment, R&D and educations expenditures). Following [Smets and Wouters \(2007\)](#), data which exhibit a trend or unit root are made stationary in two steps. First, we divide the sample by the working age population. Second, data are taken in logs and we use a first difference filtering to obtain growth rates. Real variables are deflated by GDP deflator price index. Following [Chang et al. \(2002\)](#), who underline the limited coverage of the nonfarm business sector compared to GDP, we multiply the index of average hours for the nonfarm business sector (all persons) by civilian employment. The inflation rate is computed

⁹Another drawback concerns the accuracy of this solution method, in particular to capture the precautionary effect that emerges when the ZLB is expected to bind. [Atkinson et al. \(2019\)](#) compare the estimation accuracy of the Occbin solution method with the inversion filter versus a fully non-linear model with a particle filter. They naturally find that the non-linear model is more accurate, but the overall gain does not compensate the computational burden induced by the solution method and the filter. Given the large number of state variables and observables, the Occbin solution with inversion provides enough tractability to deal with the ZLB.

from the log variations of the GDP deflator, while the nominal rate is measured by the effective fund rate. The latter is divided by 4 to be in a quarterly basis. Interest rate data prior 1955 are taken from [Olson and Enders \(2012\)](#). The effective FF rate is not the central bank target, but an average interest rate charged by depository institutions on money market. The use of this series with no prior transformation rules out the ZLB, as the FF rate never exactly reached zero but remained slightly above. In addition, the piece-wise solution method does not capture the precautionary effects, so the likelihood that the ZLB binds in the future would have no effects on consumption. To portray more accurately the ZLB, we set the nominal rate data to zero when the lower limit of the federal funds target established by the Federal Open Market Committee reached zero.

To measure the empirical contribution of endogenous growth, we use a cost-based approach by including two new time series with respect to the benchmark model of [Smets and Wouters \(2007\)](#). First, R&D expenditures are observable which allows to characterize the unobserved growth of technology. We use the nonresidential gross fixed private domestic investment in intellectual property products from FRED. Second, we measure investment in education through personal consumption expenditures in education services. However this series is in an annual basis, so we apply the temporal disaggregation method of [Fernandez \(1981\)](#). This method makes the use of the information obtained from related indicators observed at the desired higher frequency. We use health expenditure as the latter is the most correlated time series with education expenditures among all sub-elements constituting personal consumption expenditures. Finally, these two new time series are transformed using the same scheme as output.

Measurement equations are given by:

$$\begin{bmatrix} \text{Output Growth} \\ \text{Hours} \\ \text{Consumption Growth} \\ \text{Investment Growth} \\ \text{Inflation} \\ \text{Interest Rate} \\ \text{R\&D Investment} \\ \text{Education Expenditures} \end{bmatrix} = \begin{bmatrix} 100 \times \log \bar{\gamma} \\ 0 \\ 100 \times \log \bar{\gamma} \\ 100 \times \log \bar{\gamma} \\ 100 \times \log \bar{\pi} \\ 100 \times \log \bar{r} \\ 100 \times \log (\bar{\gamma} \cdot \bar{\gamma}_X) \\ 100 \times \log (\bar{\gamma} \cdot \bar{\gamma}_X) \end{bmatrix} + \begin{bmatrix} \hat{\gamma}_t \\ 0 \\ \hat{\gamma}_t \\ \hat{\gamma}_t \\ 0 \\ 0 \\ \hat{\gamma}_t \\ \hat{\gamma}_t \end{bmatrix} + \begin{bmatrix} \Delta \hat{y}_t \\ \hat{l}_t \\ \Delta \hat{c}_t \\ \Delta \hat{i}_t \\ \hat{\pi}_t \\ \hat{r}_t \\ \Delta \hat{i}_t^A \\ \Delta \hat{i}_t^H \end{bmatrix}, \quad (\text{I.35})$$

where the hat over the variables' names denotes the percentage deviations of these variables from their steady state, while those with a bar denotes the steady state. A striking feature of this model with respect to other estimated macroeconomic models is the existence of a common endogenous trend. We note that $\Delta \hat{y}_t$, $\Delta \hat{c}_t$, $\Delta \hat{l}_t$, $\Delta \hat{i}_t^A$ and $\Delta \hat{i}_t^H$ are cointegrated with $\hat{\gamma}_t$, thus the endogenous determination of $\hat{\gamma}_t$ is key as it jointly affects most of observed variables.

I.2.3 CALIBRATION AND PRIOR DISTRIBUTIONS

Calibrated parameters are reported in [Table I.1](#). As [Christiano et al. \(2014\)](#), the discount factor is set as to 0.9989, the depreciation rate of physical capital is 2.5% and the government spending to GDP ratio is 20%. As in most real business cycles mod-

	Calibrated parameters	Values
β	Discount factor	0.9989
\bar{l}	Labor supply	1/3
s_A	Public spending share in output	0.20
\bar{u}	Capital utilization rate	1
α	Labor intensity	0.80
ε_A	Patent production function	0.60
ϑ	Substitution intermediate goods	3.85
$\delta(\bar{U})$	Capital depreciation rate	0.025
\bar{U}	Utilization rate in steady state	1
$p_A(\bar{x}^A)$	Technology adoption rate	0.20/4
$p_H(\bar{x}^H)$	Skill adoption rate	0.33/4
\bar{I}^A/\bar{Y}	R&D expenditures to GDP	0.0131
ω	Skill premium	1.15
ϵ	Substitution final goods	10

Table I.1: Calibrated parameter values (quarterly basis)

els, steady state working hours are given a value of 1/3. Given the high value of the discount factor, we impose $\alpha = 0.8$ for the labor intensity parameter in the production function to obtain an investment to GDP ratio close to 20%. Substitution on final goods market is set to 10 as in [Smets and Wouters \(2007\)](#) thus implying a 11% percent steady state markup. For intermediate goods, the elasticity of substitution is set to 3.85 as [Anzoategui et al. \(2016\)](#) to be in line with the estimate of [Broda and Weinstein \(2006\)](#). Steady state adoption rate for technology is set to 0.2/4 as [Anzoategui et al. \(2016\)](#) to get an average time lag to adopt of five years. The calibration of human capital adoption rate is more problematic as human capital is an unobservable variable. We impose an adoption rate of 0.33/4 in order to mimic the graduation of a bachelor degree in 3 years. Regarding the elasticity of patents creation to R&D expenditures, we follow the calibration strategy of [Comin and Gertler \(2006\)](#) by borrowing the lower bound interval value estimated by [Griliches \(1990\)](#). R&D expenditures in GDP are set to 1.31% to match postwar US data. Finally, regarding the skill premium ω , [Alon et al. \(2018\)](#) finds that this parameter lies at 95% in the interval [1;2] for the US economy. Consistently with this estimate, we assign a value of 1.15 to match the education spending to GDP over the same sample period.

[Table I.2](#) and [I.3](#) report prior distributions of shock and structural parameters, respectively. Common parameters with [Smets and Wouters \(2007\)](#) are given prior distributions similar or close to this benchmark paper. Regarding the adoption elasticity to final goods inputs, papers featuring an endogenous technology such as [Comin and Gertler \(2006\)](#) typically calibrate this parameter to 0.95. To get an estimated parameter in the same range, we impose a beta distribution with prior mean of 0.8 and standard deviation of 0.05. We impose the same prior distribution for human capital. For congestion costs on adopting new technologies, [Moran and Queralto \(2017\)](#) argue that adjustment cost of R&D are higher than those of investment, unlike these authors we do not make any strong prior assumption on this cost by setting the same prior information as investment adjustment costs. This prior is not informative and will let

the data be informative about their posterior values. For the percentage growth rate of human capital $\bar{\gamma}_H$, Lucas Jr (1988) calibrates this parameter to 0.014 using the estimation of Denison Edward (1962). This would correspond to a 0.35% quarterly growth rate that would abnormally drive all the contribution to the growth in the model.¹⁰ We thus impose on $\bar{\gamma}_H$ a gamma distribution with mean of 0.2 and standard deviation of 0.15: this prior is diffuse enough to allow the data to decide whether one engine drives all the observed growth of output. The remaining set of parameters are which are not estimated nor calibrated are determined endogenously in the deterministic steady state of the model.

		Prior distributions			Posterior distributions mean	
		Shape	Mean	Std.	Mean [0.050;0.950]	
					Endogenous Trend	Exogenous Trend
Std. productivity	$100 \times \sigma_Z$	IG_2	0.1	0.5	0.702 [0.651;0.756]	0.807 [0.715;0.902]
Std. premium	$100 \times \sigma_B$	IG_2	0.1	0.5	0.193 [0.171;0.218]	0.242 [0.207;0.286]
Std. markup	$100 \times \sigma_P$	IG_2	0.1	0.5	1.807 [1.520;2.127]	4.380 [2.928;4.944]
Std. investment	$100 \times \sigma_I$	IG_2	0.1	0.5	1.327 [1.172;1.515]	1.194 [1.050;1.367]
Std. spending	$100 \times \sigma_G$	IG_2	0.1	0.5	3.033 [2.833;3.268]	3.208 [2.957;3.485]
Std. monetary policy	$100 \times \sigma_R$	IG_2	0.1	0.5	0.281 [0.255;0.312]	0.247 [0.226;0.274]
Std. patent	$100 \times \sigma_A$	IG_2	0.1	0.5	3.640 [3.051;4.862]	-
Std. human capital	$100 \times \sigma_H$	IG_2	0.1	0.5	1.953 [1.653;2.290]	-
AR(1) productivity	ρ_Z	B	0.5	0.2	0.964 [0.951;0.975]	0.977 [0.969;0.984]
AR(1) premium	ρ_B	B	0.5	0.2	0.916 [0.892;0.940]	0.926 [0.901;0.948]
AR(1) markup	ρ_P	B	0.5	0.2	0.928 [0.909;0.946]	0.978 [0.908;0.993]
AR(1) investment	ρ_I	B	0.5	0.2	0.963 [0.942;0.978]	0.945 [0.926;0.960]
AR(1) spending	ρ_G	B	0.5	0.2	0.986 [0.980;0.991]	0.987 [0.979;0.992]
AR(1) patent	ρ_A	B	0.5	0.2	0.982 [0.966;0.992]	-
AR(1) human capital	ρ_H	B	0.5	0.2	0.918 [0.881;0.949]	-
Marginal log-likelihood					-800.2968	-176.1236

Table I.2: Prior and Posterior distributions of shocks

I.2.4 POSTERIOR DISTRIBUTIONS

In addition to priors distributions, Table I.2, I.3 and Figure I.3 also report posterior distributions drawn from four parallel chains of 100,000 iterations of the Metropolis-Hasting algorithm, with an acceptance ratio close to 25%. To contrast the result with the fixed trend assumption of Smets and Wouters (2007), an alternative version of the model was estimated with the same prior distribution but with a fixed trend. Two time series and shocks related to the two engines of growth are thus discarded from the estimation, while the ZLB is preserved. This difference in the number of observable time series between the two models does not allow us to compare likelihood ratios. Figure I.3 shows that data were all informative as the posterior distribution of each parameter is fairly different its posterior distribution.

Regarding the model with endogenous trends, standard parameters from the workhorse New Keynesian model are rather consistent with previous findings such as Christiano et al. (2005) and Smets and Wouters (2007). Regarding parameters specific to the two endogenous growth engines, we find that shocks which are the most persistent are those related to the accumulation of technologies and knowledge, these shocks are probably the main source of persistence in the model with endogenous trends,

¹⁰This result is not surprising as Lucas' model only include one source of growth.

		Prior distributions			Posterior distributions mean	
		Shape	Mean	Std.	Mean [0.050;0.950]	
					Endogenous Trend	Exogenous Trend
Consumption aversion	σ_C	G	1	0.35	0.981 [0.905;1.084]	1.425 [1.244;1.584]
Labor Disutility	σ_L	G	2	0.5	0.973 [0.709;1.247]	1.829 [1.357;2.282]
Consumption habits	h	B	0.5	0.2	0.074 [0.030;0.124]	0.240 [0.169;0.314]
Calvo price lottery	θ	B	0.5	0.1	0.663 [0.627;0.697]	0.863 [0.810;0.882]
Price indexation rate	ξ	B	0.5	0.2	0.041 [0.012;0.096]	0.033 [0.009;0.076]
Capital utilization elasticity	ψ	B	4	1	3.995 [3.245;4.815]	0.082 [0.024;0.205]
Investment cost	χ_I	N	4	1	0.928 [0.743;1.156]	1.246 [0.956;1.626]
MPR smoothing	ρ	B	0.75	0.1	0.822 [0.800;0.841]	0.867 [0.847;0.885]
MPR inflation	ϕ_π	N	1.5	0.25	2.693 [2.529;2.863]	2.746 [2.562;2.951]
MPR output growth gap	$\phi_{\Delta y}$	G	0.5	0.25	0.136 [0.106;0.167]	0.146 [0.122;0.176]
Patents obsolescence rate	$\delta_A \times 100$	G	2.5	0.4	0.876 [0.682;1.168]	-
Skills obsolescence rate	$\delta_H \times 100$	G	2.5	0.4	0.366 [0.252;0.522]	-
Adoption rate elasticity	\varkappa_A	B	0.7	0.07	0.944 [0.929;0.958]	-
Adoption rate elasticity	\varkappa_H	B	0.7	0.07	0.665 [0.630;0.703]	-
Adoption congestion cost	χ_A	N	4	1.5	8.608 [6.837;10.34]	-
Adoption congestion cost	χ_H	N	4	1.5	8.605 [7.085;9.794]	-
Goods intensity in skills	ν	B	0.2	0.05	0.548 [0.429;0.616]	-
Trend slope	$100 \times \log \bar{\gamma}$	G	0.4	0.15	0.277 [0.196;0.329]	0.474 [0.456;0.491]
Human capital trend slope	$100 \times \log \bar{\gamma}_H$	G	0.2	0.05	0.104 [0.040;0.145]	-
Investment specific slope	$100 \times \log \bar{\gamma}_X$	G	0.4	0.15	0.661 [0.625;0.696]	-
Nominal rate	$100 \times \log \bar{r}$	G	1	0.10	1.449 [1.393;1.524]	1.947 [1.815;1.996]
Marginal log-likelihood					-800.2968	-176.1236

Table I.3: Prior and Posterior distributions of structural parameters.

and generate desired low frequency variations for the endogenous trend. For parameters related to technology, the obsolescence rate of technology is 0.75% in a quarterly basis, which is consistent with the 3% annual obsolescence rate of [Comin and Gertler \(2006\)](#). In the same vein, the adoption rate elasticity is strikingly the close to the one of [Comin and Gertler \(2006\)](#). Regarding the sluggishness of the adoption rate, the cost parameter is much higher than for investment goods as suggested by [Moran and Queraltó \(2017\)](#).

Next we turn to the parameter related to the accumulation of knowledge. First, we find a quarterly obsolescence rate of knowledge of 0.4% that lies in the ballpark of the 1.5% annual rate of [Jones et al. \(1993\)](#), while the technology of skill creation is more intensive in goods compared to the same benchmark paper. In addition, the external effect of knowledge is twice lower than the one computed by [Lucas Jr \(1988\)](#). Finally by comparing the models with endogenous versus exogenous growths, we find that low frequency fluctuations are not correctly accounted by the exogenous growth model, and are thus captured by more persistence in the shocks processes.

I.3 MACROECONOMIC IMPLICATIONS OF ENDOGENOUS TRENDS

I.3.1 INSPECTING THE PROPAGATION MECHANISM

To understand how the two endogenous trends affect the propagation mechanism, we contrast the impulse response functions of our model with those obtained with the exogenous trend model. We use the same calibration for the two models based on the posterior mean of the endogenous growth model. We thus examine the propagation following a standard productivity shock and a cost-push shock. We consider a cost-

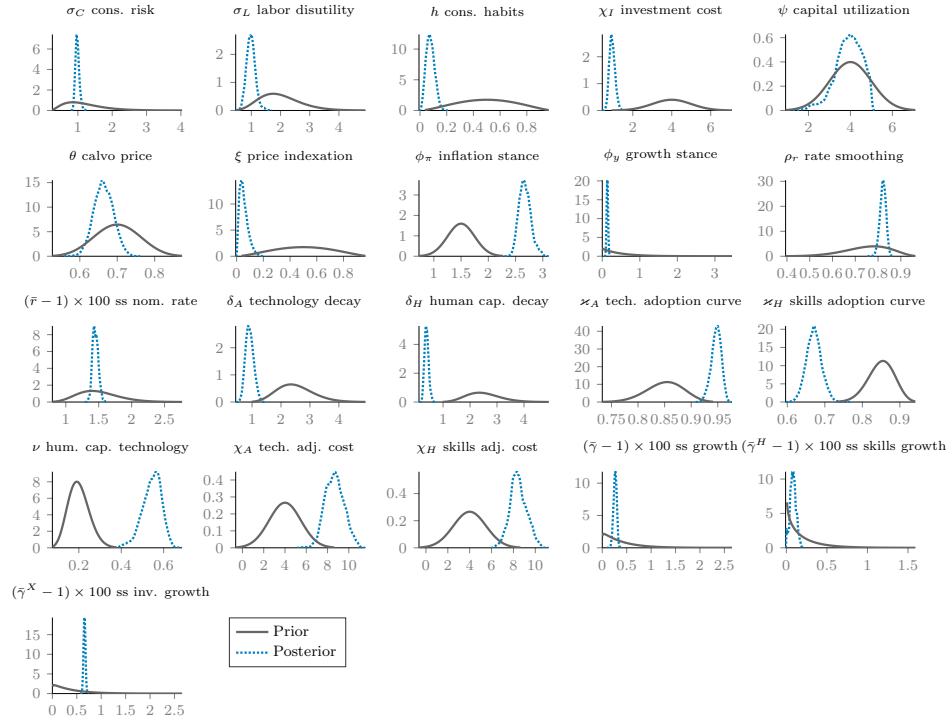


Figure I.3: Prior and posterior distributions of the model with endogenous growth

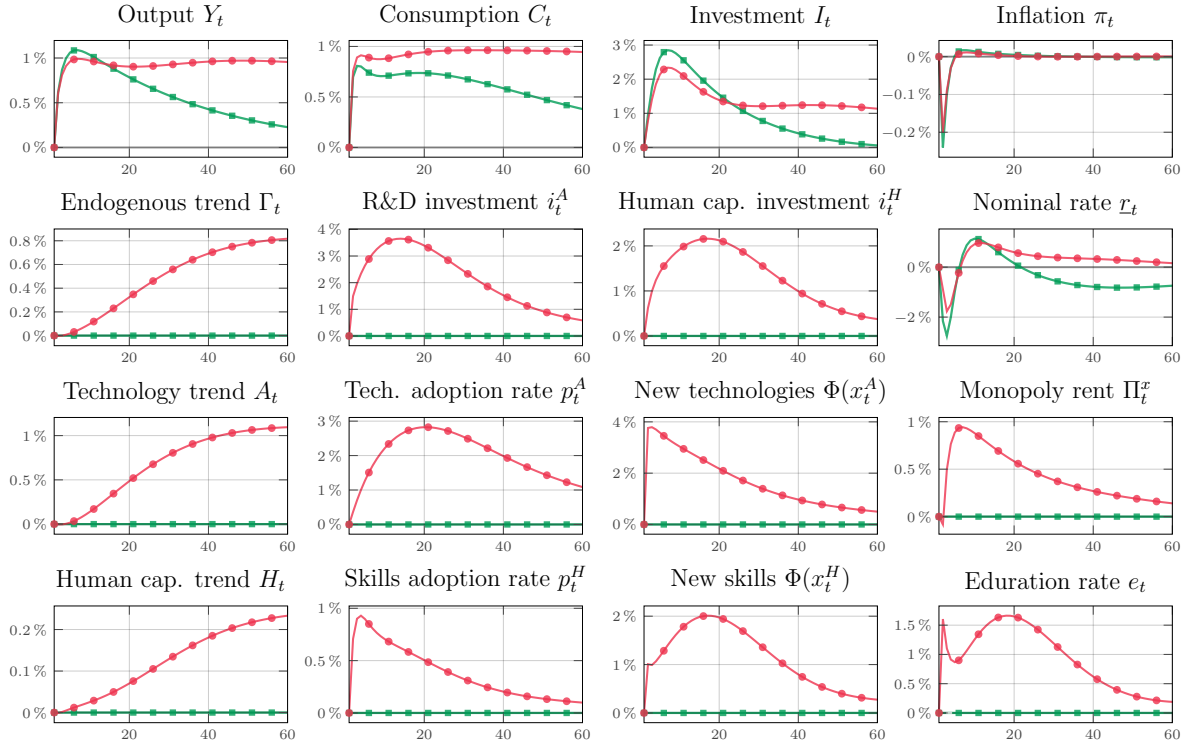
push shock, given his importance in shaping the monetary policy trade-off.

I.3.1.1 A productivity shock

Figure I.4 reports the impulse response functions of the model following a standard productivity shock in the production function of firms. In the short term, the IRFs between the two models are remarkably the same: the rise in productivity makes both labor and physical capital more productive, leading to a decline of the inflation rate combined with a rise in the rate of growth of output. As in the workhorse New Keynesian model, monetary reacts to the decline in inflation by lowering the nominal rate. The decline in the real rate lowers the incentive to save for households, and thus rises in turn consumption expenditures. In the meantime, the cost of physical is lower and allows intermediate firms to investment more.

However after about 10 periods, the IRFs between the two models seriously diverge. This divergence originates from the higher persistence featured by the endogenous trends. The rise in productivity increases the marginal product of human capital, and in turn enhances the value of unadopted skills. Firms thus engage their employees into vocational training which rises the share of the labor force into education. Accumulating one effective unit of human capital takes on average 3 years which makes the adoption of a new skill very sticky. The resulting consequence of this persistence mechanism lies in the fact that education efforts takes time to translate into effective units of human capital. The trend of adopted skills gradually rises which drives the endogenous persistence of output, consumption and investment above their linear trends for an extended period of time.

For the growth of technology engine, the propagation of a TFP shock features simi-



Notes: Variables with a trend are detrended using a linear trend. Both models are calibrated using the posterior mean of the endogenous trends model.

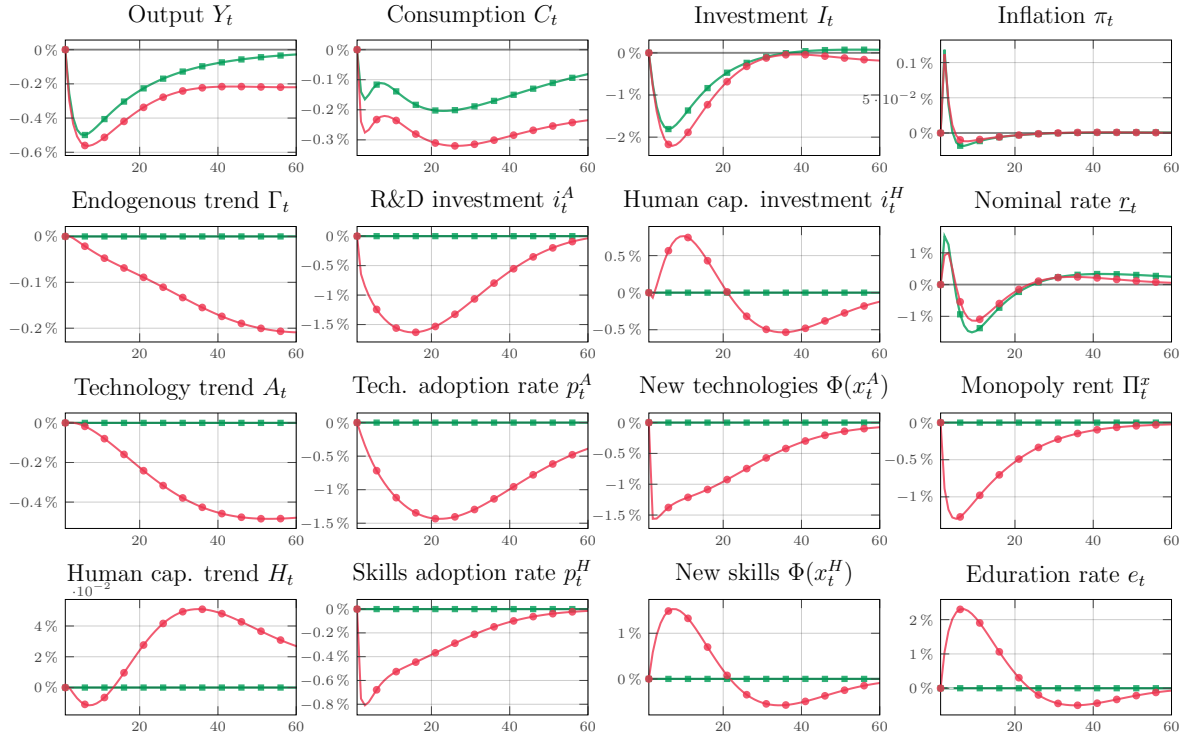
Figure I.4: Propagation of an unit productivity shock in the endogenous and exogenous trends models.

lar dynamics with respect to the knowledge engine of growth. Following a TFP shock, firms are more profitable as they produce more with less inputs. Higher profits increases the monopoly rent for innovators which through a Schumpeterian effect drives upward the shadow value of adopted technologies. Innovators have more incentive to innovate and adopt new technologies to monopolize the rent, in turn they rise their R&D spending which enhances the demand for final goods. As for human capital, this engine of growth features important delays in the propagation of a TFP shock. The trend of technologies thus requires up to 60 quarters to peak drives output growth.

I.3.1.2 A cost push shock

Figure I.5 reports the response following a cost push shock to the marginal cost of production of intermediate firms. This shock typically increases inflation and reduces real production, which creates a trade-off for monetary policy between output and prices stabilization. As in the standard New Keynesian model with exogenous growth, a cost push shocks reduces output, consumption and investment, while monetary policy rise the interest rates to dampen inflationary pressures in the economy.

However, the presence of endogenous trends affects the persistence of the cost push shock. This shock deteriorates the monopoly rent of intermediate firms, innovators thus have in turn less incentive to engage into R&D spending as prospects of future profits sinks. Innovators thus reduce their R&D spending, which in turn exacerbates the recession under endogenous growth. The production of new patent



Notes: Variables with a trend are detrended using a linear trend. Both models are calibrated using the posterior mean of the endogenous trends model.

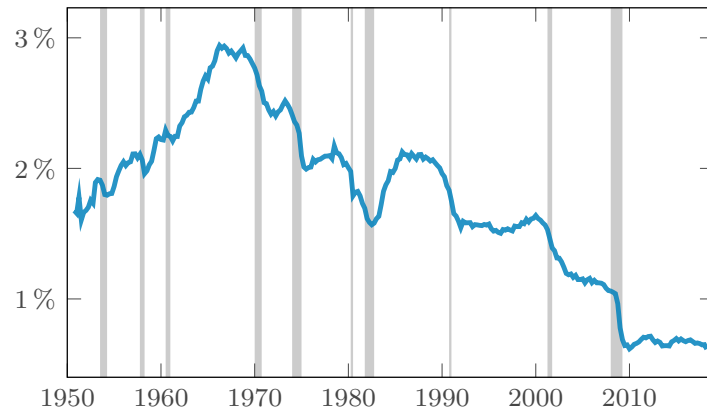
Figure I.5: Propagation of an unit cost-push shock in the endogenous and exogenous trends models.

and the adoption rate of technologies both declines, as a consequence the endogenous trend of technology is below its linear trend level for an extended period of time. As output, consumption and investment are co-integrated with the endogenous trend.

In contrast, the endogenous trend of knowledge features a different dynamic with respect to the technology trend. The cost push shock deteriorates the marginal product of labor, the opportunity cost of being in education (rather than employment) declines sharply. As a result, during a recession firms cope with the crisis by increasing their efforts in education as its opportunity cost is lower.¹¹ However, these education efforts don't materialize immediately into adopted units of human capital, as higher inflation increases the adoption costs. The decline in the adoption rate of skills dominates the positive effect of more education effort. However after 15 periods, the accumulation of new skills are finally adopted which drives the human capital above its linear trend.

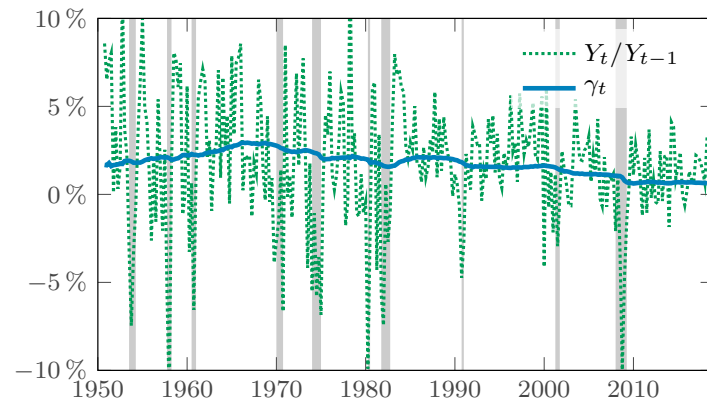
I.4 WHY ECONOMIC GROWTH HAS DECLINED SINCE WWII?

¹¹This result is not new in the literature. Opportunity cost models of growth have argued that recessions are times when firms engage in productivity-improving activities because of intertemporal substitution. See Saint-Paul (1993) for an empirical evaluation of these models.



Notes: The shaded areas represent the recessions as dated by the NBER.

Figure I.6: Historical path of the endogenous trend between 1950q1 to 2018q4.



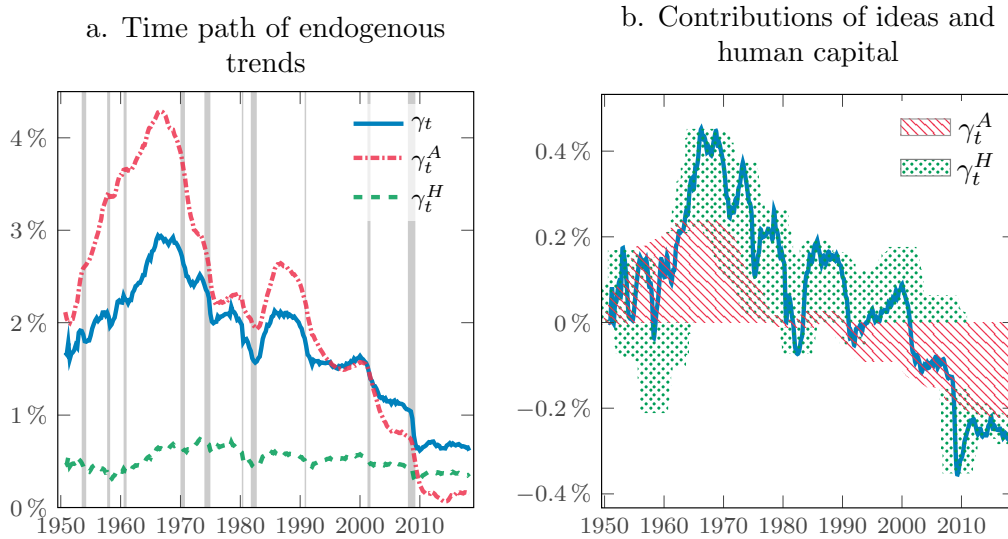
Notes: The shaded areas represent the recessions as dated by the NBER.

Figure I.7: Comparison of the trend with output growth

I.4.1 THE ROLE OF ENDOGENOUS TRENDS

Figure I.6 reports the time-path of this medium term component measured by the estimated model. Recall that this component jointly rise growth rates of key macroeconomic aggregates such as consumption, investment and GDP. Over the postwar, the US economy has experienced sizable medium frequency oscillations. From 1950 up to 1970, the endogenous trend has been continually increasing upward despite small recessionary episodes. This period is characterized by a persistent increase in R&D expenditures, thus leading the trend to peak up to 3% at the end of the 60s. In decades following the 70s, the trend growth rate have been declining synchronously with the different recessions hitting the US economy.

To gauge the relative importance of the trend with respect to the business component of the data, Figure I.7 reports both the observed path of output and the medium run component of the data measured by the DSGE model. Recessions induced by oil price shocks in the 70s and the Great Recession clearly damaged the engine of growth. If at first sight the trend to be volatile, Figure I.7 shows that these fluctuations are less volatile compared to the annual fluctuations of real output. Thus the endogenous trend clearly replicates a fraction of the low frequency volatility in



Notes: The shaded areas represent the recessions as dated by the NBER.

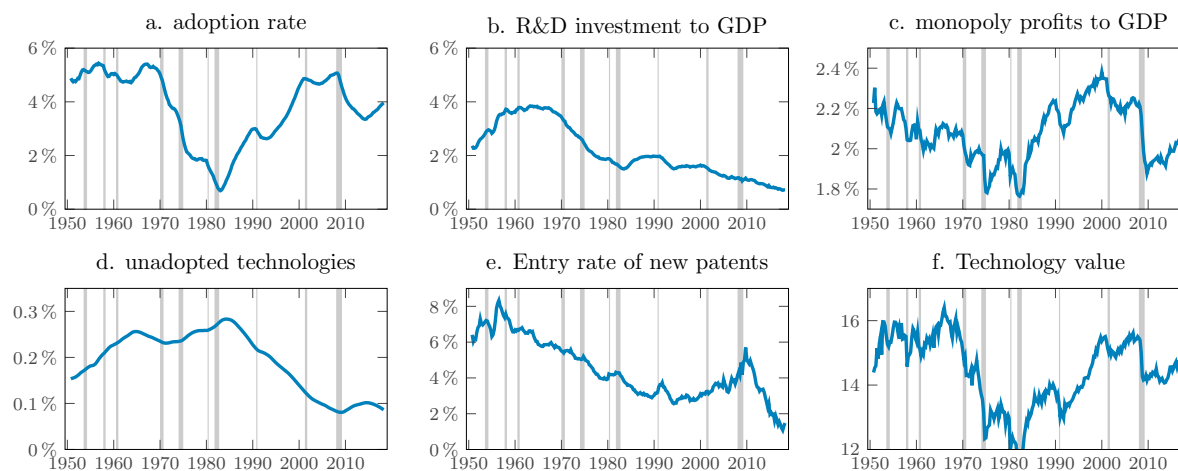
Figure I.8: A breakdown of the drivers of the endogenous trend

macroeconomic time series. [Antolin-Diaz et al. \(2017\)](#) employs a dynamic factor model to track changes in the long run growth rate of GDP, by separating them from their cyclical counterpart. Their sample span a period as long as the one used in the fit exercise and thus allows us to examine any similarity between their estimates of the long run growth with the endogenous trend. Both models seems to generate close estimates of the long run growth, which confirms that the endogenous growth model is able to successfully capture low frequency variations.

Why has the trend reduced over time? Unlike [Anzoategui et al. \(2016\)](#) who consider only one source of growth, the present framework allows to disentangle contributions induced by the growth of knowledge (human capital) from those induced by the growth of ideas (technology). [Figure I.8](#) reports on the left the common endogenous trend of the economy γ_t , that is a non-linear function of γ_t^A and γ_t^H (see [Equation I.34](#)). Over the sample period, it's striking to notice that the growth of skills has been remarkably stable over time while the main source of variations of the endogenous trend since 1950 has been the growth of technology. From 1950, the growth rate of technology peaked up to 4.5% but started to decline prior to the two coming recessions induced by rising oil prices.

We next explore the relative importance of the two sources of growth on the common trend. [Figure I.9.b](#) reports the percentage contribution of each source of growth using a linear approximation of [Equation I.34](#). This figure confirms that the R&D engine accounts for much of the cyclical variation in the endogenous trend, as it has contributed on average up to 60% of the variation of the trend. The downward pressure on the trend has clearly been driven by variations in technology since 1970. While before the financial crisis, the growth of knowledge was driving up the trend, the financial crisis worryingly reversed the contribution of human capital.

How does the model account for the decline in the growth rate of technology? [Figure I.9](#) plots the detrended evolution of main state variables determining the ag-



Notes: The shaded areas represent the recessions as dated by the NBER. These figures are generated by feeding the smoothed errors into the model's policy function to obtain unobservable state variables.

Figure I.9: Contribution of human capital and new varieties on the trend

gregate evolution of technology. During the first twenty years of the sample, the growth of R&D investment has been high enough (figure b) to fuel an high rate of entry of new patents (figure e), the latter were mostly effectively adopted and thus converted into new intermediate goods (figure a). In the meantime, the monopoly was declining but did not translated into lower expected technology value (figure f) as expectations about future monopoly rents were high. However the 70s recessions irrevocably damaged the main engine of growth and announce the beginning of a slowdown.

For the post 70s period, (Gordon, 2012, 2017) argues that technological advancement has been slowing and translates into slower growth over time. The model tends provides a theoretical formulation of Gordon's narrative that explains this reduction in the rate of growth of the US economy. The model captures this decline in the growth rate by a reduction in the entry rate of new patents, that measures the productivity of innovators during 1950. This result is corroborated by the estimated model of Anzoategui et al. (2016) that finds a similar path for the R&D productivity. Recessions in the 70s reduced the monopoly rents (figure c), and thus reduced the value of adopted technologies (figure f). Thus, the incentive for the innovator to adopt a technology became low (figure a). This reduction in the adoption rate of technology rose the stock of unadopted technologies until 1985. After this date, the stock of unadopted technologies has been critically falling, mainly because the creation rate of new patent and the R&D expenditures were declining. Despite an improvement of the situation in the 2000s, the financial crisis broken this recovery in the engine of growth through a large contraction of the monopoly rent.

I.4.2 THE ROLE OF LABOR PRODUCTIVITY

The model is able to disentangle the driving forces of productivity growth. Growth accounting provides further perspective on the forces driving labor productivity growth over the sample period. The expression of labor productivity growth, defined as growth in real output per hour, is directly obtained by dividing the production function by

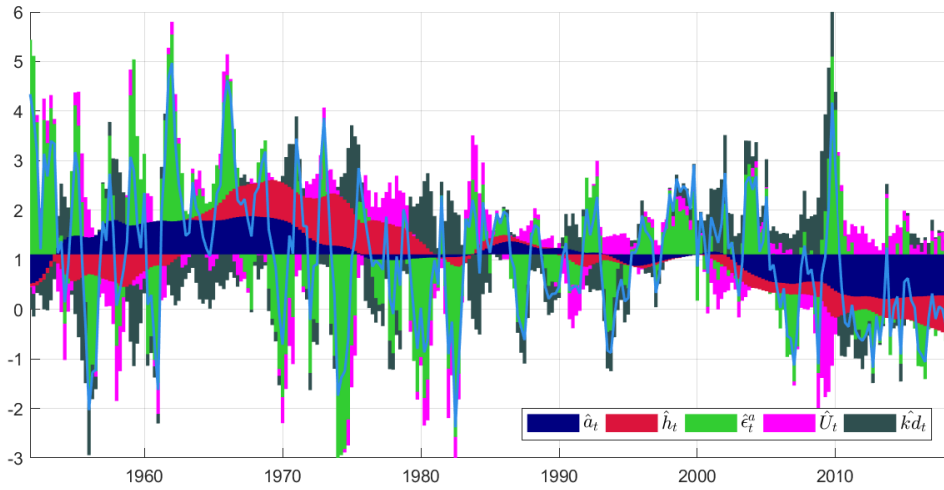


Figure I.10: Growth accounting of US labor productivity (y-o-y basis) between 1951Q3 to 2019Q3.

hours worked:

$$\frac{Y_t}{L_t} = \underbrace{\left[A_t^{(\vartheta-1)} H_t^\mu \right]^{1/\alpha}}_{\text{endogenous TFP}} \times \underbrace{\varepsilon_t^Z}_{\text{Exogenous TFP}} \times \underbrace{(1 - e_t)^\alpha (u_t)^{1-\alpha}}_{\text{inputs utilization}} \times \underbrace{\left(\hat{K}_{t-1} / L_t \right)^{1-\alpha}}_{\text{capital deepening}}. \quad (\text{I.36})$$

Our model offers four different sources of productivity growth. In a similar growth accounting exercise as [Fernald \(2015\)](#), labor productivity is explained by TFP, variable inputs utilization and capital deepening. A few differences with [Fernald \(2015\)](#) are worth to be discussed. First, Fernald interprets inputs utilization as variations in capital's workweek and labor effort. In our model there is no labor effort but education effort e_t , the latter behaves very similarly through its countercyclical aspects: in a recession firms increase their education efforts as the opportunity cost of being in vocational training rather than working reduces. As a result, our measure of inputs utilization includes both education efforts and capital utilization rate. Secondly, unlike Fernald who considers TFP as a Solow residual from a growth accounting exercise, our model provides three different sources of TFP growth. TFP is determined by the standard TFP shock from the real business cycle theory, denoted ε_t^Z , and by two endogenous sources based on the accumulation of ideas A_t and knowledge H_t . Our approach slightly differs from [Anzoategui et al. \(2019\)](#) as our measure of TFP includes the role of human capital. Human capital is likely to be important, and can be interpreted as the labor quality of Fernald's growth accounting.

Differentiating logarithmically (where hats are log-changes) yields the expression of labor productivity growth:

$$\hat{y}_t - \hat{l}_t = \frac{(\vartheta - 1)}{\alpha} \hat{a}_t + \frac{\mu}{\alpha} \hat{h}_t + \hat{\varepsilon}_t^A + \hat{U}_t + (1 - \alpha) kd_{t-1}. \quad (\text{I.37})$$

[Figure I.10](#) reports the contribution of ideas, knowledge, exogenous TFP, inputs utilization and capital deepening on the annual labor productivity growth of the US

economy. Strikingly, the endogenous components of TFP plays a non-trivial role on the observed fluctuations of productivity. Labor productivity has been largely fuelled by the accumulation of technologies and knowledge during the 60s. At their peak in 1970, endogenous trends increased by 1.5% of the growth of labor productivity. However, oil price shocks in the 70s reduced the role of endogenous trends in driving labor productivity. During the Great Moderation period, the role of endogenous trends was modest. In the 2000s, the contribution has become strongly negative, thus corroborating the findings of [Anzoategui et al. \(2019\)](#) who found that technical change accounted for an important slowdown of the labor productivity. In our setup, this slowdown is explained by both the decline in the accumulation of ideas and knowledge. Regarding the financial crisis, we find that the decline in productivity is fueled by the drop in capital intensity as underlined by [Hall \(2015\)](#) and [Anzoategui et al. \(2019\)](#).

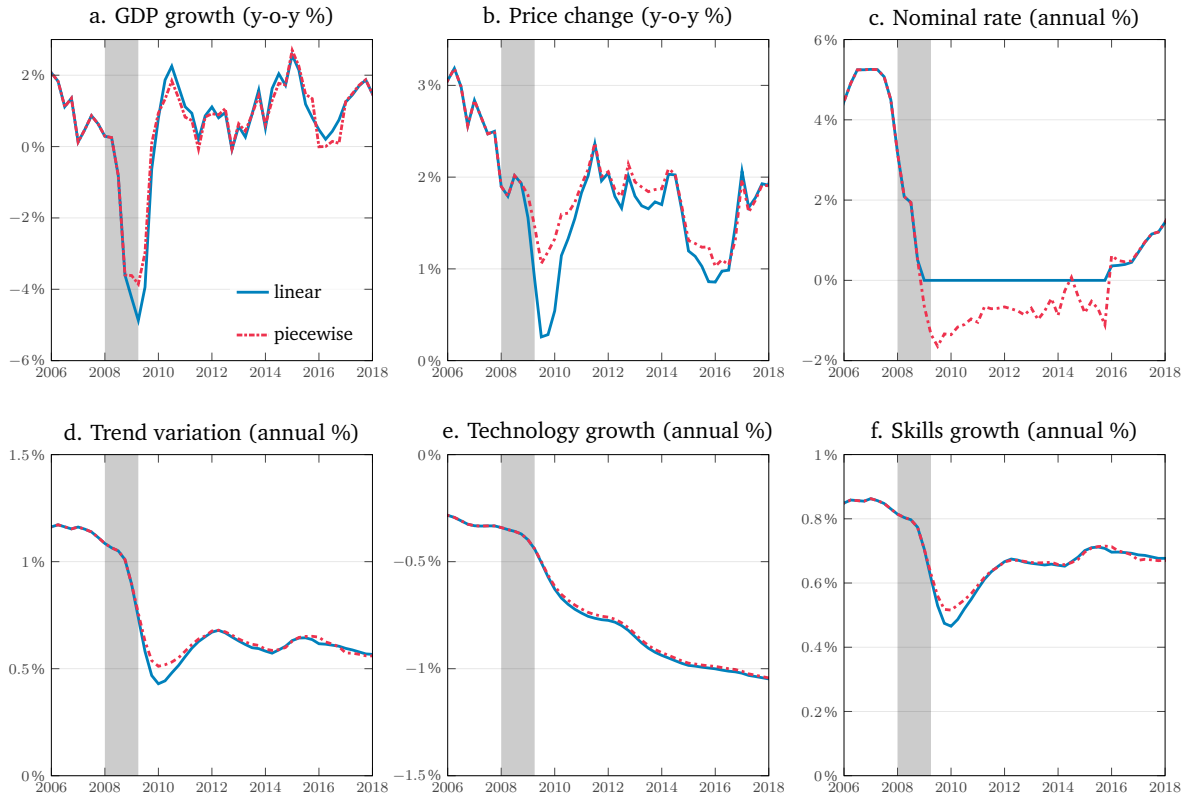
I.5 QUANTIFYING THE EFFECT OF THE ZERO LOWER BOUND

We now explore how important was the presence of the zero lower bound on the economic contraction of the US economy during the Great Recession. [Figure I.11](#) compares the observed data against the outcome from the same model without the constraint on the nominal rate.

When the ZLB started to bind in 2009, monetary policy could not accommodate further the nominal rate to dampen the recession. As a consequence, real interest rates were abnormally high i which, through the Euler equation, artificially increased both the marginal utility of consumption and the cost of capital renting, and in turn it weakened aggregate demand. Our results show that without the ZLB the annual growth rate of output would have been 1.5% higher in 2009. In addition, the ZLB has amplified the deflation mechanism, this translates into year-on-year inflation differential in 2009 of 1%, and 0.3% in 2014 and 2016.

Using the estimated model, we can also gauge the effect of the zero lower bound on the two engines of growth in the economy. [Figure I.11](#) provides the annualized growth rate of the medium term component in [Equation I.34](#). This component is itself a combination of adopted technologies and adopted skills depicted in subfigures e and d. According to the model, the role of the ZLB on growth is trivial as the trend decline was quick and negligible: the trend reduced of 0.1 pp in 2009 before recovering quickly with no persistent effect. The main contributor to this modest drop is the accumulation process of human capital that is temporary damaged by the high rates.

A natural question at hand is to the possible causal relation between the slowdown in economic growth and the zero lower bound. As [Orphanides \(2003\)](#) emphasized, real-time misperceptions about the long-run growth of the economy can play a large role in monetary policy mistakes. In the standard workhorse New Keynesian, monetary policy stabilizes short run fluctuations of output and inflation without having any concerns about possible long-term changes in the growth patterns of the economy. Here, we perform a counterfactual exercise to examine whether the monetary policy reaction to long term growth strongly has lead the nominal rate to reach the zero lower bound. [Figure I.12](#) reports a counterfactual interest rate that does not re-



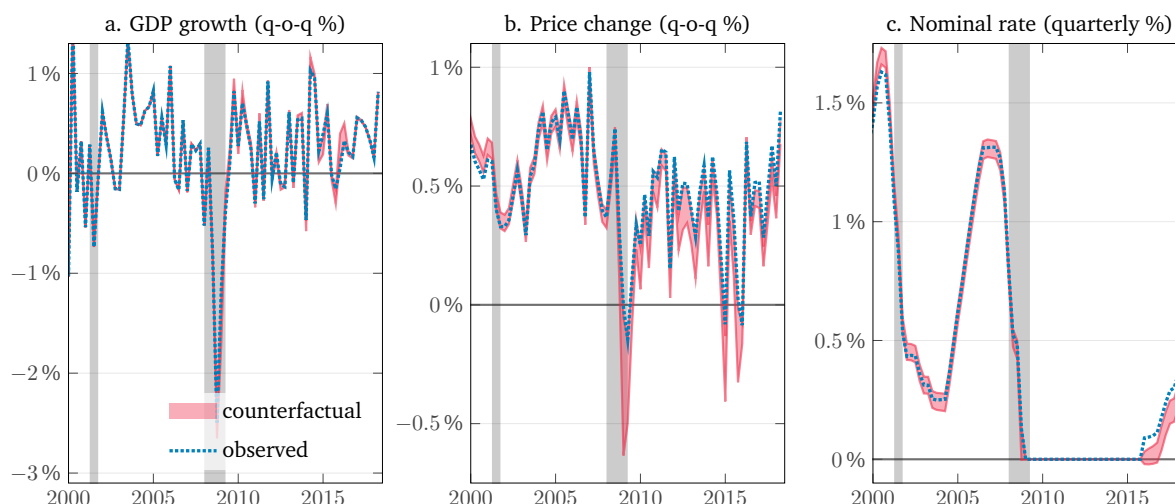
Notes: The shaded areas represent the recessions as dated by the NBER. The simulation shows the filtered series for 6 variables from the estimated model. The dashed line show their paths feeding in the same shocks but in absence of the zero lower bound on the nominal rate.

Figure I.11: Macroeconomic implications of the zero lower bound during the Great Recession.

spond to the endogenous trend. When monetary policy does not respond to long term change in the growth rate, the nominal rate is higher. An higher interest rate induces a reduction of inflation and actually increases the zero lower bound probability. Thus the macroeconomic situation is worse when monetary policy does not respond to long term growth as a ZLB binds for more quarters.

I.6 FORECASTING PERFORMANCE

DSGE models has been criticized for not being able to anticipate the slow growth in output after the financial crisis. In this section, we investigate whether our model with endogenous growth is the missing ingredient of current state-of-art forecasting models. We compute out-of-sample forecasts between 2003Q1 up to 2018Q2 based on model parameters estimated only on revised data available at the date of the forecast. We assess the accuracy of each forecast through the root the mean square errors (RMSE). **Figure I.13** display RMSE at forecast horizon lying between 1 up to 12 quarters. We compare the RMSEs of our baseline model with those implied by the exogenous trend model as well as RMSEs implied by a Bayesian Vector Autoregres-



Notes: The shaded areas represent the recessions as dated by the NBER. The simulation shows the filtered series for 6 variables from the estimated model. The dashed line show their paths feeding in the same shocks but in absence of monetary policy reaction to long term changes in output. Uncertainty in red is drawn from by randomly picking 500 draws from the posterior distribution.

Figure I.12: Counterfactual path in a model with no reaction to the endogenous trend

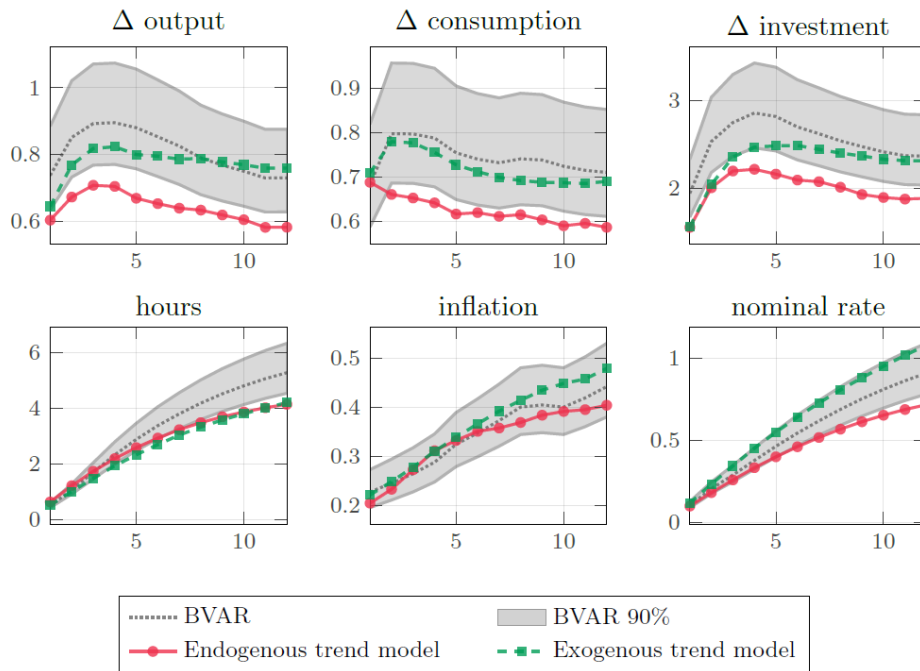
sion.¹² Forecasts of the BVAR are based on the posterior means of the parameters updated each quarter while those of the DSGE are based on the posterior modes. The grey area is uncertainty on the RMSE of the BVAR. As [Christiano et al. \(2014\)](#), this uncertainty is constructed so that if the RMSE of our baseline model lies in the grey area for a particular variable and forecast horizon, then the classical null hypothesis that the two RMSE are actually the same fails to be rejected at the 90 percent level.

With the exception of inflation and the nominal rate, we find that the DSGE models are performing better than the BVAR model. This corroborates the findings of [Smets and Wouters \(2007\)](#) that theoretical models are serious alternatives to atheoretical models for forecasting exercises. Strikingly, our model compares very well against the two alternatives. Except for inflation, RMSE of the endogenous trends model is statistically different than the BVAR model. This shows that the endogenous growth mechanism is able to improve the forecasting performance of the model, even during the financial crisis episode.

I.7 CONCLUSION

We have estimated a non-linear DSGE model that originally features a time varying trend driven by two sources of endogenous growth. We then used the model to assess the slowdown of long term growth, in particular following the onset of the Great Recession. Based on the estimated model, our key result is that we corroborate the thesis of a strong decline in the long term trend of the US economy. Among the two sources of growth examined in the paper, the slowdown mainly is induced by the technology engine reflecting a decline in the productivity of creation of new technolo-

¹²As [Smets and Wouters \(2007\)](#), our BVAR includes four lags and its parameters follow the Minnesota priors.

Figure I.13: RMSE comparison for three forecasting models

Notes: The shaded areas and red dashed lines represent 90% intervals of the RMSE from the BVAR model assuming that $N * RMSE/\sigma^2$ follows a chi-squared distribution with N degrees of freedom.

gies since 1960. This finding tends to favor the [Gordon \(2012\)](#) theory stating that the US growth has strongly declined since 1970. In addition, we find that a standard macro-model with exogenous growth erroneously captures low frequency changes in economic growth by highly persistent macroeconomic shocks. In contrast, the model featuring an endogenous trend successfully captures this low frequency fluctuations.

Part II

Social learning in a liquidity trap

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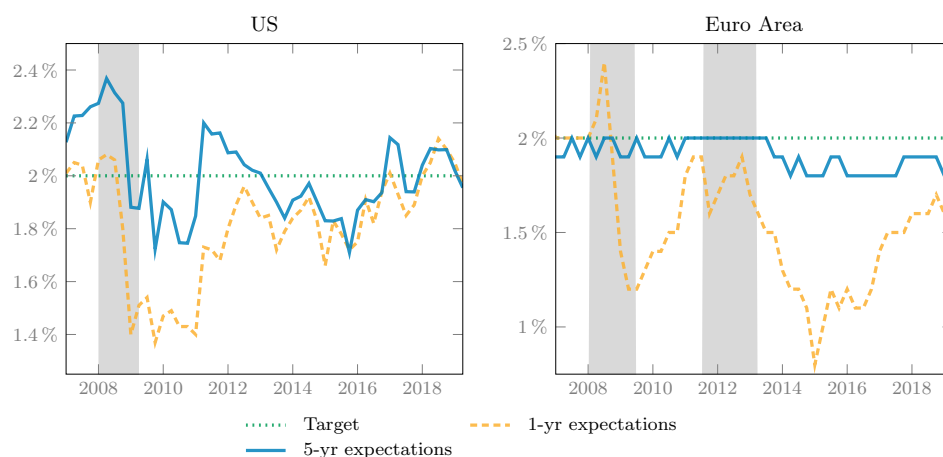
INTRODUCTION

The Great Recession in the US and Europe and the ensuing monetary policy reactions have given way to a ‘new normal’ in economic conditions: interest rates have remained at historically low levels. This situation is particularly acute in Europe, where interest rates have remained at the effective lower bound (ELB) ever since. Yet, no substantial changes in the price levels have been recorded, neither in the wake of the downturn – despite the severity of the recession – nor along the recent output growth episode, which then resembles an inflation-less recovery. Meanwhile, inflation expectations have remained consistently below target, as depicted in [Figure II.1](#), which puts at risk the long-run anchorage of expectations. Low inflationary pressures pushed a number of major central banks (CBs) to further ease monetary policy, before the COVID-19 pandemic brought interest rates further down to their ELB.

This low-inflation narrative is hard to unfold within the standard macroeconomic model – namely the New Keynesian (NK) class of models – for at least two reasons. First, zero interest rates generate implausible macroeconomic dynamics in those models. Under rational expectations (RE), the dynamics are indeterminate at the ELB ([Benhabib et al., 2001](#)), which implies excess volatility in inflation that is clearly at odds with the recent experience. This puzzle is clearly visible from survey data, which have been lying in the indeterminacy region of the inflation-output state space since the financial crisis, as depicted in [Figure II.2](#). Replacing RE by boundedly rational and learning agents induces diverging deflationary spirals at the ELB, which does not match the current situation either ([Evans et al., 2008](#)), or does so at the cost of the introduction of exogenous floors on deflation– motivated e.g. by a subsistence production level ensured by the government ([Evans et al., 2020](#)).

Second, the assumption of complete information and common beliefs leaves little room for expectations to be persistently off the target and play any autonomous role in driving business cycles. In those models, recessive episodes are typically generated by exogenous and persistent technology or financial shocks.¹³ Not only does this concep-

¹³There are some recent exceptions, e.g. [Angeletos et al. \(2018\)](#), who investigate the role of strategic



Notes: The shaded areas represent the recessions as dated by the National Bureau of Economic Research (NBER) and Centre for Economic Policy Research (CEPR), and the green dashed lines the inflation targets. Data are from the Survey of Professional Forecasters (SPF) and the European Central Bank (ECB).

Figure II.1: Inflation expectations in the US and Euro Area 2008–2019

tion of expectations conflict with the empirical evidence of unanchored and dispersed forecasts,¹⁴ but it also does not leave any room for monetary policy to influence or coordinate heterogeneous private expectations through communication.

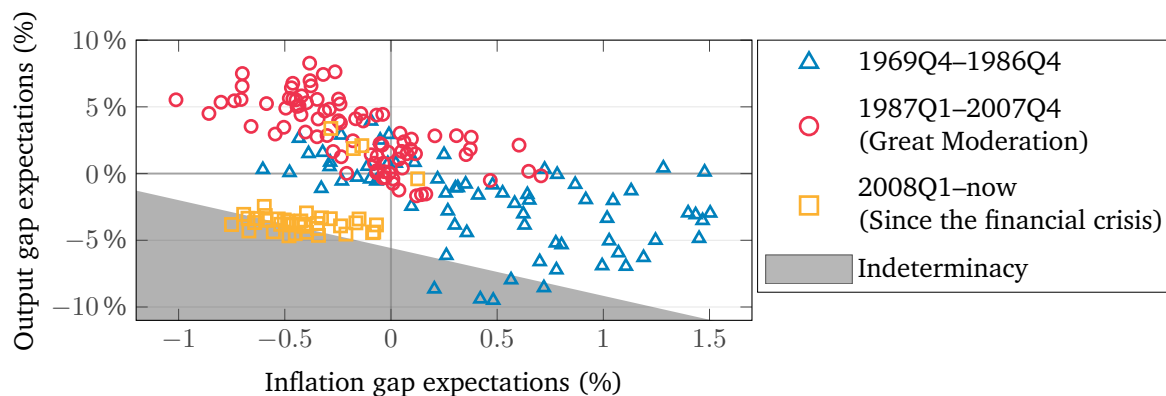
Therefore, the main contribution of this paper is to address these challenges by developing a model in which time-varying heterogeneity in expectations endogenously produces ELB dynamics so as to account for the recent economic experience. The use of heterogeneity and learning in agents' forecasts is not anecdotal given the above-mentioned large literature documenting pervasive heterogeneity in real-world expectations.

We develop a micro-founded NK model featuring inflation and output dynamics to which we add a parsimonious evolutionary learning process that specifies the dynamics of expectations and nests the RE homogeneous agent benchmark. This latter feature, together with the sole use of white noise fundamental shocks, isolates learning as the only source of persistence in the endogenous variables and allows us to identify the amplifier role of expectations in driving business cycles. In our model, agents use steady-state learning, i.e. they form beliefs about the long-run values of inflation and output, which easily translates into the issue of expectation anchorage.

Specifically, we choose a social learning (SL) process. Our choice is motivated by the parsimony of this class of learning models, their ability to match experimental findings and the evolutionary role of heterogeneity in the adaptation of the agents. In these models, agents collectively adapt to an ever-changing environment in which their own expectations contribute to shape the macroeconomic variables that they are

uncertainty in the presence of heterogeneous information within a general equilibrium model. However, those authors use a real business cycle (RBC) model, which implies that monetary policy is left out.

¹⁴See, *inter alia*, Mankiw et al. (2003) in survey data from professional forecasters; and Branch (2004) from households. Coibion et al. (2019) show that more than half of the surveyed firms and households do not know the value of the Fed inflation target. One-year-ahead households' inflation expectations are on average 1.5 percentage points (p.p.) above the target, and the cross-sectional dispersion reaches up to 3 p.p. (Coibion et al., 2020).



Notes: The shaded area denotes the region of the state space that violates the Blanchard-Kahn determinacy condition under RE and leads to diverging deflationary spirals under recursive learning. The white area denotes the determinate region of the state space, which is also the basin of attraction of the target under recursive learning. Data on expectations are taken from the SPF. The output gap is computed using a linear trend. Calibration of the NK model is taken from Galí (2015).

Figure II.2: (Ir)relevance of the New Keynesian model with rational expectations since the ‘new normal’

trying to forecast. This feature is well suited to self-referential economic systems such as standard macroeconomic models. SL expectations also find an intuitive interpretation that is reminiscent of the idea of epidemiological expectations where ‘expert forecasts’ only gradually diffuse across the entire population (Carroll, 2003).

In a novel effort within the related literature,¹⁵ we take our stylized model to the data and show that it is able to *jointly* replicate ten salient business cycle moments from the Survey of Professional Forecasters (SPF) and the main US macroeconomic time series, including the frequency of ELB episodes, major dimensions of heterogeneity in expectations and a substantial share of the persistence in output and inflation data. This empirical exercise is already a remarkable result given the parsimony of the model. Our empirical exercise adds to the literature (i) an estimation routine of a non-linear model under heterogeneous expectations and (ii) an empirical discipline device to learning models by offering estimated values to the learning parameters for which there are no observable counterparts.¹⁶

A second major contribution is to show that our model *endogenously* produces stable dynamics at the ELB. Those stable dynamics correspond to inflation-less recoveries as recently experienced, *i.e.* inflation persists for an extended period of time below its target, the ELB binds, but output expands. Hence, our simple framework jointly accounts for the missing deflation in the wake of the crisis and the missing inflation in the wake of the recovery. In our model, recessive dynamics arise endogenously when agents coordinate on pessimistic expectations following a series of adverse fun-

¹⁵Del Negro and Eusepi (2011) attempt to replicate expectation data with RE models. Milani (2007) fits an adaptive learning NK model to macroeconomic time series only. Closer to our contribution, Slobodyan and Wouters (2012a,b) estimate an NK model on both macroeconomic and expectation times series. However, the authors use exogenous autocorrelated shocks on expectations to reproduce the observed persistence in the data.

¹⁶Hence, our empirical exercise does not aim to compare the matching abilities of the SL model regarding macroeconomic time series with those of an RE counterpart. For a meaningful comparison, the SL model would need to compete with an RE version of the model with sunspot dynamics at the ELB. While certainly an interesting exercise, it is beyond the scope of the present paper.

damental shocks. From there, the transition back to the target can be particularly long if expectations have become unanchored and, per their self-fulfilling nature, nurture the bust. Hence, we offer a reading of the recent economic experience as a long-lasting coordination of agents on pessimistic expectations rather than as the result of persistent and exogenous shocks.

Given that our model nests the RE homogeneous-agent benchmark, we interpret the dispersion of expectations as a friction and quantify the ensuing welfare loss with respect to the RE outcome. We find that heterogeneous expectations entail a consumption loss of almost 3.3% with respect to the RE allocation. From there, a natural follow-up analysis is to introduce an additional monetary policy instrument, namely CB communication, and investigate whether it may offset the costs of forecast dispersion. To address this question, we exploit the flexibility of the SL model, which enables us to integrate CB communication into the learning process of the agents. From two simple communication examples, we show the critical role of a strong credibility for the CB's announcements to reshape expectations. The CB may lose credibility whenever the announcements become decoupled from the actual realizations of inflation. Moreover, accurate but below-target inflation forecasts may turn self-defeating. In light of these observations, we discuss recent policy debates, such as the forward-guidance puzzle or the adoption of (temporary) higher inflation targets.

RELATED LITERATURE

Our treatment of communication adds to the existing literature on communication under learning by modeling endogenous credibility.¹⁷ The closest to our concept of endogenous credibility is the work by [Hommes and Lustenhouwer \(2019\)](#), who derive the stability conditions of the targeted equilibrium in an NK model with ELB where agents' expectations switch to follow past inflation, should the target be missed.

While we stress the simplicity of our framework, there exist larger-scale and considerably more complex DSGE models in which non-linearities play a key role in accounting for the recent economic experience; see, *inter alia*, [Gust et al. \(2017b\)](#); [Lindé and Trabandt \(2019\)](#). Our work is particularly related to the NK models with multiple equilibria where the persistent slump after the Great Recession is understood as an exogenously driven regime switch from the targeted equilibrium to the deflationary steady state ([Schmitt-Grohé and Uribe, 2017](#); [Aruoba et al., 2017](#); [Arifovic et al., 2018](#); [Lansing, 2020](#)). However, the coordination mechanism generating liquidity traps in our model is fundamentally different from the one used in the above-cited contributions.

In the context of our model, agents never coordinate on the low-inflation steady state, nor do they contemplate the possibility of a regime switching between the two steady states. The target is the only stable equilibrium under SL and expectations always remain within its basin of attraction, which is shown to be larger under SL than the determinacy region under RE. As a result of a series of adverse fundamental shocks, SL agents may 'pick up' a downward trend and their expectations may travel

¹⁷The learning literature usually concludes that communication is stabilizing under learning in models where communication imposes model-consistent restrictions on the forecasting model used by the agents; see *e.g.* [Eusepi and Preston \(2010\)](#). However, in these models, communication is fully credible.

to regions of that basin from where inflation and output gaps stagnate below their target and the convergence back to target takes a very long time. This is because in those regions of the state space, the ZLB binds and the pessimistic expectations are self-defeating per the self-fulfilling nature of the expectations in the NK model.

In particular, while we borrow from Arifovic et al. (2013, 2018) a similar SL mechanism to model expectations, our work differs substantially.

Importantly, those two theoretical contributions study the asymptotic stability of the NK model under SL, while we focus on the short-term fluctuations arising from the interplay between fundamental shocks and learning dynamics and their empirical performances. Arifovic et al. (2018) interpret liquidity trap episodes as the coordination of expectations on the low inflation steady-state that is stable under their learning mechanism. By contrast, our agents have a finite memory and our empirical calibration differs from theirs, which does not allow us to generalize their result to our setup. In fact, in our model, the low-inflation state is unstable under SL as it belongs to the basin of attraction of the target: if expectations shift on the low-inflation state, they will eventually converge back to the target, but after a considerable amount of time.

The rest of the paper proceeds as follows. In Section II.1, we develop the model; the estimation is presented in Section II.2; the dynamic properties of the model are analyzed in Section II.3; Section II.4 discusses CB communication; and Section II.5 concludes.

II.1 THE MODEL

We first describe the building blocks of the model, then discuss the solution under the RE benchmark and finally explain our implementation under SL.

II.1.1 A PIECEWISE LINEAR NEW KEYNESIAN MODEL

Our model builds on the workhorse three-equation NK model. All variables below are expressed in deviation from their steady-state level that corresponds to the target of the CB.

The first equation, the IS curve, describes aggregate demand:

$$\hat{y}_t = \mathbb{E}_{j,t}^* \hat{y}_{t+1} - \sigma^{-1}(\hat{v}_t - \mathbb{E}_{j,t}^* \hat{\pi}_{t+1}) + \hat{g}_t, \quad (\text{II.1})$$

where \hat{y}_t is the output gap, \hat{v}_t the nominal interest rate set by the CB, $\hat{\pi}_t$ the deviation of the inflation rate from the target (hence, $\hat{v}_t - \mathbb{E}_t \hat{\pi}_{t+1}$ is the real interest rate), $\sigma > 0$ the inter-temporal elasticity of substitution of consumption (based on a CRRA utility function), and $\mathbb{E}_{j,t}^*$ the (possibly boundedly rational) expectation operator based on information available at time t . The subscript j is introduced to suggest the possibility of heterogeneous expectations, where each agent-type $j = 1, \dots, N$ forms her own expectation (with N the number of agent-types).¹⁸ \hat{g} is an exogenous real disturbance.

¹⁸We follow here most of the learning literature and introduce heterogeneity in the reduced-form models rather than in the micro-foundations (see, *inter alia*, Bullard and Mitra (2002), Arifovic et al. (2013), Hommes and Lustenhouwer (2019)). We are well aware of the conceptual limitation of this

Second, the forward-looking NK Phillips curve summarizes the supply side:

$$\widehat{\pi}_t = \beta \mathbb{E}_{j,t}^* \widehat{\pi}_{t+1} + \kappa \widehat{y}_t + \widehat{u}_t, \quad (\text{II.2})$$

where $0 < \beta < 1$ represents the discount factor, $\kappa > 0$ a composite parameter capturing the sensitivity of inflation to the output gap and \widehat{u}_t an exogenous cost-push shock.

Monetary policy implements a flexible inflation-targeting regime subject to the ELB constraint, which results in the following non-linear Taylor rule:

$$\widehat{r}_t = \max\{-\bar{r}; \phi^\pi \mathbb{E}_{j,t}^* \widehat{\pi}_{t+1} + \phi^y \mathbb{E}_{j,t}^* \widehat{y}_{t+1}\}, \quad (\text{II.3})$$

where ϕ^π and ϕ^y are, respectively, the reaction coefficients to the inflation and the output gaps, and $\bar{r} \equiv \pi^T + \rho$ the steady-state level of interest rate associated with the inflation target π^T and the households' discount rate $\rho \equiv -\log(\beta)$. The forward-looking rule translates the emphasis of CBs on expectations as contemporaneous variables are not instantaneously observable.

We now solve the model under the RE benchmark and then detail how we introduce SL in the expectation formation process of the agents.

II.1.2 THE MODEL UNDER RATIONAL EXPECTATIONS

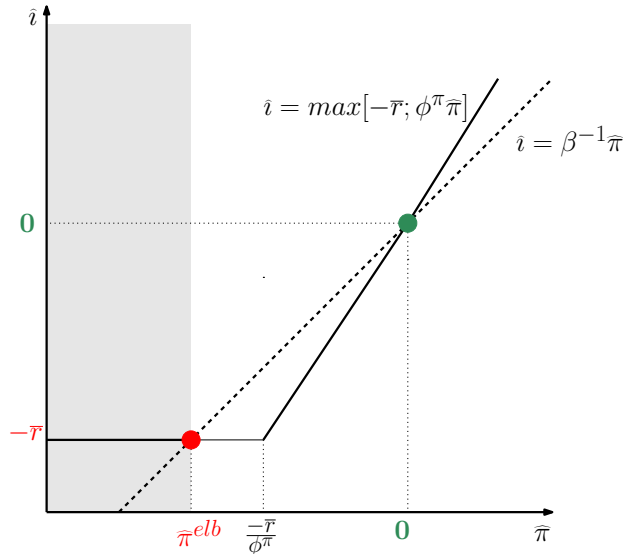
In this section, we consider RE and impose $\mathbb{E}_{j,t}^*(\cdot) = E(\cdot | I_t)$ to be the RE operator given the information set I_t common to all agents in period t . We solve for the Minimal State Variable (MSV) solution using the method of undetermined coefficients).

It is well known that the ELB introduces a non-linearity in the Taylor rule and generates an additional deflationary steady-state next to the target (Benhabib et al., 2001). Hence, expressing the model in reduced form is challenged by this non-linearity, and we need to disentangle two pieces, one around the target and one when the ELB is binding.¹⁹

A short digression through the one-dimensional Fisherian model easily illustrates this configuration. Figure II.3 displays inflation and interest rate dynamics, abstracting from the production side: the inflation target corresponds to $\widehat{\pi} = 0$ and the deflationary steady state to $\widehat{\pi}^{elb}$. Provided that $\widehat{\pi}^{elb} \leq 0 \leq \pi^T$, the two equilibria co-exist.

approach. Nonetheless, while the complications of the alternative are clear (see e.g. Woodford (2013)), the benefits in terms of qualitative results remain uncertain. For instance, in an asset-pricing model, Adam and Marcet (2011) show that under a sophisticated form of adaptive learning, the infinite-horizon pricing equation reduces to a myopic mean-variance equation. Bearing in mind those caveats, we proceed within the reduced-form model.

¹⁹We follow here the related NK literature and impose the ELB constraint in the log-linearized model around the targeted steady-state to describe the dynamics around the low inflation state, see, *inter alia*, Guerrieri and Iacoviello (2015). This method gives a second-best estimate of the dynamics around the deflationary steady-state. A first-best would be to log-linearize the model around this second steady-state but would result in an MSV solution involving extra additional state variables (Ascari and Sbordone, 2014) and, hence, additional coefficients to learn under SL (see Section II.1.3). However, the benefits in terms of qualitative results are unlikely to outweigh the costs of such a complication of the learning process of the agents.



Notes: We can write the log-approximated Fisher equation as follows: $\hat{i} = \beta^{-1}\hat{\pi}$. At the targeted steady-state (in green), no deviation occurs: $\hat{i} = \beta^{-1}\hat{\pi} = 0$. At the ELB (in red), we can derive an equilibrium such that $-\bar{r} = \beta^{-1}\hat{\pi}^{elb} \Leftrightarrow \hat{\pi}^{elb} = -\bar{r}\beta$. Provide that $\hat{\pi}^{elb} \leq \pi^T$, the two equilibria co-exist. The shaded area is indeterminate under RE and unstable under adaptive learning (Evans et al., 2008).

Figure II.3: Co-existence of two steady states under the ELB constraint

Coming back to the two-dimensional model, we have to specify a process for the exogenous shocks.

In the rest of the paper, we consider white noise shocks only, so \hat{g} and \hat{u} are non-observable *i.i.d.* processes. In this case, the MSV solution boils down to a noisy constant without persistence. The presence of a floor on the nominal rate makes this solution piece-wise linear:

$$\hat{z}_t = [\hat{y}_t \ \hat{\pi}_t]' = \begin{cases} a^T + \chi^g \hat{g}_t + \chi^u \hat{u}_t, & \text{if } i_t > 0 \\ a^{elb} + \chi^g \hat{g}_t + \chi^u \hat{u}_t, & \text{if } i_t = 0, \end{cases} \quad (\text{II.4})$$

where the first case is the law of motion when the ELB is not binding (denoted by a ‘T’ superscript) and the second case when the ELB is binding (denoted by a ‘elb’ superscript). Note that as variables are expressed in deviation from their steady-state values at the target, we have $a^T = (0 \ 0)'$.

We now introduce expectations under SL.

II.1.3 EXPECTATIONS UNDER SOCIAL LEARNING

Under SL, we relax the assumption of homogeneous agents endowed with RE and consider instead a population J of N heterogeneous and interacting agents, indexed by $j = 1, \dots, N$. We now define $\mathbb{E}_{j,t}^*(\cdot) = E_j^{SL}(\cdot | I_{j,t})$ to be the expectation operator under SL given the information set $I_{j,t}$ available in period t to agent j . The information set is agent-specific as it contains, besides the history of past inflation and output gaps up until period $t - 1$, the current and past individual forecasts that need not be shared with the whole population.

Individual forecasting rules Following Arifovic et al. (2013, 2018), we assume that agents are endowed with a forecasting rule that involves the same variables as the MSV solution. The form of the rule is the same across agents, but with agent-specific coefficients that they revise over time. In any period t , each agent j is therefore entirely described by a two-component forecast $[a_{j,t}^y, a_{j,t}^\pi]'$ and her expectations read as:

$$E_{j,t}^{SL} \{\hat{z}_{j,t+1}\} = \begin{bmatrix} E_{j,t}^{SL} \{\hat{y}_{t+1}\} \\ E_{j,t}^{SL} \{\hat{\pi}_{t+1}\} \end{bmatrix} = \begin{bmatrix} a_{j,t}^y \\ a_{j,t}^\pi \end{bmatrix}. \quad (\text{II.5})$$

Those forecast values find an appealing interpretation. In the absence of shocks, they correspond to her long-run output and inflation gap forecasts. In the presence of *i.i.d.* shocks, those values correspond to her average output gap and inflation gap forecasts. Under either of those interpretations, the forecasts $[a_{j,t}^y, a_{j,t}^\pi]'$ represent agents' beliefs about the steady-state values of the inflation and output gaps, which allows us to intuitively measure expectation (un)anchorage using their distance to the targeted values (*i.e.* zero).²⁰ On empirical grounds, heterogeneous coefficients $[a_{j,t}^y, a_{j,t}^\pi]$ capture the disagreement among forecasters observed in survey data. In particular, dispersed coefficients on inflation $a_{j,t}^\pi$ can be interpreted as disagreement about the CB's target, as the latter coincides with the inflation steady state in the NK model.

Under learning, the model is solved sequentially so as to obtain a temporary equilibrium in each period, which makes it straightforward to account for the non-linearity induced by the ELB. Figure II.4 summarizes the sequence of events within a period under SL. Let us now detail each step.

Aggregation of individual forecasts Following Arifovic et al. (2013, 2018), individual expectations (II.5) are aggregated using the arithmetic mean as:

$$E_t^{SL} \hat{z}_{t+1} = \frac{1}{N} \sum_{j=1}^N \mathbb{E}_{j,t}^{SL} \hat{z}_{t+1}, \quad (\text{II.6})$$

Note that under this aggregation procedure, agents have the same relative weight in expectations formation, thus one agent cannot influence market expectations when the number of agents N is large enough. To have a sizable effect on market expectations and generate expectation-driven fluctuations, a sentiment or news must spread to a large enough fraction of the population.

Computation of the endogenous variables Given the aggregate expectations $E_t^{SL} \hat{z}_{t+1}$ and the realization of the shocks, the piece-wise linear Taylor rule (II.3) sets the nominal interest rate: if the shadow rate is negative, the nominal interest rate is set to zero. Given the nominal rate, the expectations and the shock g , the IS curve (II.1) then determines the output gap and finally, the Phillips curve (II.2) determines the inflation gap given inflation expectations, the output gap and the shock u .

We now detail how SL agents form their individual forecasts. Specifically, this class of learning models utilizes two operators.

Mutation The first one is a stochastic innovation process, or mutation, that allows

²⁰In the sequel, we denote by Ω such an indicator of expectation anchorage. Specifically, we use the average squared distance of individual expectations to zero: $\Omega_t^{\mathbb{E}\pi} = \frac{1}{N} \sum_{j=1}^N \mathbb{E}_{j,t}^{SL} \{\hat{\pi}_{t+1}\}^2$ and $\Omega_t^{\mathbb{E}y} = \frac{1}{N} \sum_{j=1}^N \mathbb{E}_{j,t}^{SL} \{\hat{y}_{t+1}\}^2$. The lower those values, the stronger the anchorage of expectations.

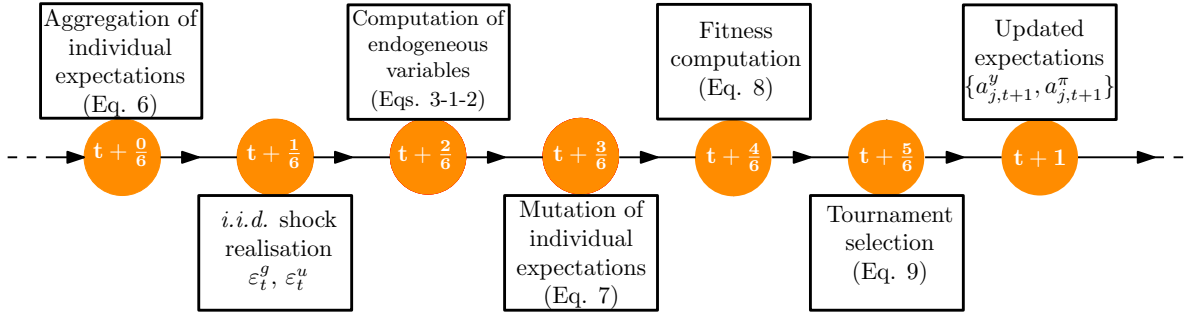


Figure II.4: Intra-period timing of events in the model under SL

for a constant exploration of the state space outside the existing population of forecasts. In each period, each agent's forecasts are modified by an idiosyncratic shock with exogenously given probabilities. Her output gap forecast is modified with probability μ_y and her inflation gap forecast with probability μ_π . In short, her forecasts of any variable $x = \{y, \pi\}$ is modified in any period as follows:

$$a_{j,t+1}^x = \begin{cases} a_{j,t}^x + \iota_{j,t} \xi^x & \text{with probability } \mu_x \\ a_{j,t}^x & \text{with probability } 1 - \mu_x, \end{cases} \quad (\text{II.7})$$

with $\iota_{j,t}$ an idiosyncratic random draw from a standard normal distribution with standard deviation ξ^x . The larger parameters ξ^x , the wider the neighborhood to be explored around the existing forecasts. Mutation can be interpreted as an innovation, a trial-and-error process or a control error in the computation of the corresponding expectations.

Tournament and computation of forecasting performances The second operator, the tournament, is the selection force of the learning process and allows better-performing forecasts to spread among the population at the expense of lower-performing ones. Forecast performance is evaluated using the forecast errors over the whole past history of the economy given the stochastic nature of the environment (see [Branch and Evans 2007](#)).

To each forecast $a_{j,t}^x$, $x = \{y, \pi\}$, of each agent j is assigned a so-called fitness, computed as:

$$F_{j,t}^x = - \sum_{\tau=0}^t (\rho^x)^\tau (\hat{x}_{t-\tau} - a_{j,t}^x)^2. \quad (\text{II.8})$$

The terms $\hat{y}_{t-\tau} - a_{j,t}^y$ and $\hat{\pi}_{t-\tau} - a_{j,t}^\pi$ correspond, respectively, to the output and inflation gap forecast errors that agent j would have made in period $t - \tau - 1$, had she used her current forecasts $a_{j,t}^y$ and $a_{j,t}^\pi$ to predict the output and inflation gaps in period $t - \tau$. The smaller the forecast errors, the higher the fitness.

Parameter $\rho^x \in [0, 1]$ (for $x = y, \pi$) represents memory. In the nested case where $\rho^x = 0$, the fitness of each forecast is completely determined by the forecast error on

the most recent observable data. For any $0 < \rho^x \leq 1$, all past forecast errors impact the fitness but with exponentially declining weights while, for $\rho^x = 1$, all past errors have an equal weight in the computation of the fitness. This memory parameter allows the agents to discriminate between a one-time lucky draw and persistently good forecasting performances.

In the tournament, agents are randomly paired (the number of agents is conveniently chosen even), their fitness on inflation and output gap forecasts are each compared and the agent with the lowest fitness copies the forecast of the other. There are two separate tournaments: one for inflation gap forecasts $\{a_{j,t}^\pi\}_{j \in J}$ and one for output gap forecasts $\{a_{j,t}^y\}_{j \in J}$.²¹ Formally, for each pair of agents $k, l \in J$, $k \neq l$, with individual forecasts $a_{k,t}^x$ and $a_{l,t}^x$ ($x \in \{\pi, y\}$), the tournament operates an imitation of the more successful forecasts as follows:

$$\begin{aligned} a_{k,t+1}^x &= a_{l,t+1}^x = a_{k,t}^x && \text{if } F_{k,t}^x > F_{l,t}^x \\ a_{k,t+1}^x &= a_{l,t+1}^x = a_{l,t}^x && \text{if } F_{k,t}^x \leq F_{l,t}^x \end{aligned} \quad (\text{II.9})$$

The tournament occurs after the mutation operator in order to screen out bad-performing forecasts stemming from mutation. This allows the model to be less sensitive to the parameter values tuning mutation than if mutation were to take place after the tournament selection, and all newly created forecasts were to determine aggregate expectations without consideration of their performances. This way, the mutation process can be more frequent and of wider amplitude so as to allow for a faster adaptation of the agents to new macroeconomic conditions, while limiting the amount of noise introduced by the SL algorithm.

Simulation protocol We study the dynamics of the model using numerical simulations. Throughout the rest of the paper, we proceed as described in Arifovic et al. (2013, 2018). We generate a history of 100 periods along the law of motion of the economy around the target (see Eq. (II.4)) and introduce a population of SL agents in $t = 100$. Their initial forecasts are drawn from the same support as the one used in the mutation process, *i.e.* from a normal distribution with standard deviation ξ^x , $x = \pi, y$. The first 100 periods are used to provide the agents with a history of past inflation and output gaps in order to compute the fitness of their newly introduced forecasts. In the simulation exercises in the next section, we vary the initial average of the normal distribution to tune the degree of pessimism in the economy. The further below zero the initial average forecasts are, the more pessimistic views the agents hold about future inflation and output gaps.

Finally, it is important to recognize that the RE representative agent benchmark is nested in our heterogeneous-agent model: as soon as the inflation and output gap expectations of all agents are initialized at the targeted values and mutation is switched off (*i.e.* $\xi^y, \xi^\pi = 0$), the dynamics boil down to the RE benchmark. Under SL, our model involves a few parameters, namely the probabilities of mutation, the sizes of those mutations and the memory of the fitness function. We now detail how we estimate those parameter values.

²¹This assumption will turn out useful in the empirical exercise below while not being restrictive: Arifovic et al. (2013) show that results are robust to a single tournament.

II.2 ESTIMATION UNDER SOCIAL LEARNING

We jointly estimate the learning parameters and the structural parameters of the model. We first describe our choice and construction of the dataset, then discuss our estimation method and, finally, present the results.

II.2.1 DATASET

Macroeconomic US time series for output, price index and nominal rates are taken from the FRED database. Forecast data come from the SPF of the Federal Reserve of Philadelphia. This choice is usual in the related literature, as it is argued that those data provide a good approximation of the private sector expectations that are involved in the NK model (Del Negro and Eusepi, 2011).

SPF data span the period from 1968 to 2018 on a quarterly basis. To make the dataset stationary, we divide output by both the working age population and the price index. In order to obtain a measurement of the output gap, we compute the percentage deviations of the resulting output time series from its linear trend. The inflation rate is measured by the growth rate of the GDP deflator.

As heterogeneity in expectations is essential to the dynamics of the SL model, we construct an empirical measure of that heterogeneity in the survey data. We use the cross-sectional dispersion of the individual forecasts, measured by the standard deviation of the individual forecasts among all participants in each period, to obtain time series of forecasts' heterogeneity.

II.2.2 ESTIMATION METHOD

With those data at hand, we proceed by matching the statistics from empirical moments with their simulated counterparts under SL. In short, we use the Simulated Moments Method (SMM), which provides a rigorous basis for evaluating whether the model is able to replicate salient business cycle properties.²²

To avoid identification issues, we take the number of estimated parameters to be equal to the number of matched moments. Hence, we first reduce the number of dimensions of the matching problem and calibrate some of the parameters, namely the monetary policy and the preference parameters, as is standard in the related macroeconomic literature, and the number of agents (see Table II.1).

We are left with four structural parameters from the NK model, namely the size of the fundamental shocks σ^g and σ^u , the slope of the NK Phillips curve (parameter κ) and the natural rate \bar{r} . As we have calibrated the value of the discount factor β (see

²²Due to the non-linearity introduced by the ELB, we may not apply the Kalman filter and would need to use a non-linear filter to estimate the model with Bayesian full-information techniques. Given that this paper is the first attempt to bring such a heterogeneous-expectation model to the data, we encountered additional difficulties in estimating the SL model with an SMM. In particular, the SL algorithm brings an additional non-linearity into the piecewise-linear model and an additional source of stochasticity next to the fundamental shocks. Hence, we have left the perspective of Bayesian estimation for future research.

		Values	Sources
σ	intertemporal elasticity of substitution	1	Galí (2015)
ϕ^π	policy stance on inflation gap	1.50	Galí (2015)
ϕ^y	policy stance on output gap	0.125	Galí (2015)
β	discount factor	0.995	Jarociński and Maćkowiak (2018)
N	number of agents	300	Arifovic et al. (2018)

Table II.1: Calibrated parameters (quarterly basis)

Table II.1), we estimate the value of the inflation trend over the period considered, which uniquely determines the value of \bar{r} .²³ As for the SL parameters, we need not estimate common values for the inflation and the output gap expectation processes as the two tournaments are separated and the two time series are likely to behave differently and exhibit different properties, both in reality and in the model. For instance, estimating inflation and output gap-specific memory parameters ρ^π and ρ^y may translate the fact that agents can learn that one variable may display more persistence than the other. Hence, we estimate six learning parameters, namely the mutation sizes and frequencies ξ^x and μ_x as well as the memory of the fitness measures ρ^x for $x = \{\pi, y\}$.

We now discuss the mapping between those parameters and the empirical moments to match. First, the standard deviations of the shocks σ^g and σ^u naturally capture the empirical volatility of output and inflation. Second, the inflation trend $\bar{\pi}$ aims to match the ELB probability. To see why, recall that a higher natural rate \bar{r} mechanically decreases the probability of hitting the ELB, as the latter is defined as $\hat{v}_t = -\bar{r}$, which is strictly decreasing in the value of the inflation target. Finally, the slope of the Phillips curve κ determines the correlation between the output and inflation gaps per Eq. (II.2).

As for the SL parameters, the memories of the fitness function ρ^y and ρ^π tune the sluggishness of the expectations because they determine the weights on recent versus past forecast errors in the computation of the forecasting performances. The higher ρ^y and ρ^π , the longer the memory of the agents, the less reactive the learning process to recent errors and the more sluggish the expectations. As sluggishness in expectations is the only source of persistence in the model once we consider *i.i.d.* shocks, parameters ρ^y and ρ^π are matched with the autocorrelation of, respectively, the output and the inflation gaps.

The remaining four learning parameters control the mutation processes that are the source of the pervasive heterogeneity in expectations in the SL model. We understandably use those parameters to match four moments characterizing heterogeneity in the SPF data: the average dispersion of the output and the inflation gap forecasts over the time period considered, denoted respectively by $\Delta^{\mathbb{E}y}$ and $\Delta^{\mathbb{E}\pi}$, and their first-order autocorrelations, denoted by $\rho(\Delta_t^{\mathbb{E}y}, \Delta_{t-1}^{\mathbb{E}y})$ and $\rho(\Delta_t^{\mathbb{E}\pi}, \Delta_{t-1}^{\mathbb{E}\pi})$. In line with intuition, sensitivity analyses of the objective function of the matching problem with respect to those learning parameters have reported the following associations: the mutation sizes ξ^y and ξ^π capture a substantial share of the empirical dispersion of output and inflation gap forecasts, while the mutation frequencies μ^y and μ^π match

²³Strictly speaking, \bar{r} is associated with the inflation target per Eq. (II.3) but no such target existed in the US for most of the time period considered. Therefore, we estimate an inflation trend over that period.

Matched moments	Moments		Empirical
	Empirical \mathcal{M}_O	Simulated \mathcal{M}_S	Conf.int.
$\sigma(\hat{y}_t)$ - output gap sd.	4.38	4.39	[3.97 - 4.83]
$\rho(\hat{y}_t, \hat{y}_{t-1})$ - output gap autocorr.	0.98	0.22	[0.98 - 0.99]
$\sigma(\hat{\pi}_t)$ - inflation gap sd.	0.6	0.66	[0.54 - 0.66]
$\rho(\hat{\pi}_t, \hat{\pi}_{t-1})$ - inflation gap autocorr.	0.9	0.56	[0.87 - 0.92]
$\rho(\hat{\pi}_t, \hat{y}_t)$ - inflation-output correlation	0.08	0.097	[-0.07 - 0.21]
$\Delta^{\mathbb{E}y}$ - av. forecast dispersion of output gap	0.36	0.4	[0.31 - 0.41]
$\Delta^{\mathbb{E}\pi}$ - av. forecast dispersion of inflation gap	0.25	0.20	[0.22 - 0.28]
$\rho(\Delta_t^{\mathbb{E}y}, \Delta_{t-1}^{\mathbb{E}y})$ - autocorr. of forecast disp. of output gap	0.76	0.63	[0.70 - 0.82]
$\rho(\Delta_t^{\mathbb{E}\pi}, \Delta_{t-1}^{\mathbb{E}\pi})$ - autocorr. of forecast disp. of inflation gap	0.64	0.4	[0.55 - 0.72]
$P(\hat{t}_t > 0)$ - probability not at the ELB	0.86	0.83	[0.81 - 0.91]
Objective function	×	0.85	×

Notes: The values in brackets are the confidence interval at 99% of the empirical moments.

Table II.2: Comparison of the (matched) theoretical moments with their observable counterparts

Estimated Parameters	Prior Distributions			Posterior Results	
	Shape	Mean	STD	Mean	STD
σ^g - real shock std	Invgamma	.1	5	3.8551	5.1e-06
σ^u - cost-push shock std	Invgamma	.1	5	0.4232	4.1e-06
$\bar{\pi}$ - quarterly inflation trend	Beta	.62	.1	0.829	7.7e-06
κ - Phillips curve slope	Beta	.05	.1	0.0095	4e-06
μ^y - mutation rate for $\mathbb{E}y$	Beta	.25	.1	0.2467	4.6e-06
μ^π - mutation rate for $\mathbb{E}\pi$	Beta	.25	.1	0.2748	6e-06
ξ^y - mutation std. for $\mathbb{E}y$	Invgamma	.1	2	0.8547	3.3e-06
ξ^π - mutation std. for $\mathbb{E}\pi$	Invgamma	.1	2	0.7406	1.9e-06
ρ^y - fitness decay rate for $\mathbb{E}y$	Beta	.5	.2	0.8301	9.4e-06
ρ^π - fitness decay rate for $\mathbb{E}\pi$	Beta	.5	.2	0.5465	5.4e-06

Notes: The low values of the standard deviation of the estimated parameter values only indicate that the algorithm has converged; they do not translate into confidence intervals.

Table II.3: Estimated parameters using the simulated moment method matching the SPF data (1968–2018)

most of their autocorrelation.

Finally, in the same vein as Ruge-Murcia (2012), we impose prior restrictions on the estimated parameters and treat them as additional moments in the objective function. The priors for the structural NK parameters are taken from the literature on Bayesian estimation of DSGE models (Smets and Wouters, 2007) and we choose priors for the learning parameters that are in line with the values used in the SL literature such as Arifovic et al. (2013) (see Table II.3).

II.2.3 ESTIMATION RESULTS

Table II.2 reports the matched moments and their empirical counterparts (in p.p.). Table II.3 gives the corresponding estimated values of the parameters.

It is first striking to see that the simple two-dimensional model accounts for a substantial share of all ten moments. For half of them, the simulated moments even

fall within the confidence interval of their empirical counterparts, which means that our model replicates those moments fully. We succeed in capturing not all, but a non-negligible part, of the persistence in macroeconomic variables with a model that employs only white-noise shocks.²⁴ We shed further light on the source of that persistence in Section II.3.1, but at that stage, we can state that learning acts as an endogenous propagation mechanism that amplifies the effects of *i.i.d.* shocks and accounts for 22% of the empirical output gap persistence and even 63% of the inflation persistence found in the data.

Furthermore, all simulated correlations are of the same sign as their observed counterparts. Our model succeeds in producing positive autocorrelation in forecast dispersion. This result is an important step forward in the modeling and estimation literature as we show that our simple framework can address the empirical heterogeneity in expectations that is not part of the RE material.

The model also matches particularly well the probability of the ELB on nominal interest rates to bind despite the relatively modest amplitude and *i.i.d.* structure of the fundamental shocks. Those ELB episodes are not the result of large exogenous shocks but are an endogenous product of the interplay between learning and those small *i.i.d.* shocks, as detailed in the next section.

All our estimated values are consistent with empirical values and usual estimates. For instance, the estimated (yearly) inflation trend is 3.4%, which nicely falls into the range between the average inflation rate over the time span considered that includes the 1970s (4.3%) and the Fed inflation target that was adopted later (2%). Next, given the calibrated discount factor β , the implied value for the (yearly) natural interest rate is 5.45%, which is close to the average federal funds rate over the sample (namely 5.2%).

As for the estimated values of the mutation parameters of SL, we can see that they are all in line with the values usually employed in numerical simulations in the related literature (Arifovic et al., 2013). The estimated values of ρ^y and ρ^π imply that agents' memory is bounded,²⁵ which is highlighted by experimental evidence (Anufriev and Hommes, 2012) and empirical estimates from micro data (Malmendier and Nagel, 2016).

We conclude with our first major contribution: our parsimonious model is able to jointly and accurately reproduce ten salient features of macroeconomic time series and survey data, including the ELB duration and the pervasive heterogeneity in forecasts, while using plausible parameter values. We now proceed to the analysis of the underlying propagation mechanism in the model induced by SL.

²⁴Matching all the persistence would not be a realistic or desirable objective: it is unlikely that all macroeconomic persistence stems from learning in expectations and our model ignores all other fundamental sources of persistence in the economy. We rather provide a measure of the share of the persistence that could be attributed to learning.

²⁵If one discards observations weighting less than 1%, we have $0.83^{25} < 0.01$ and $0.54^7 < 0.01$, which implies that agents' memory amounts to roughly 25 quarters for forecasting the output gap and 7 quarters for forecasting the inflation gap.

II.3 DYNAMICS UNDER SOCIAL LEARNING

This section first analyzes the stability properties of the targeted steady state under SL. To unravel the dynamics of expectations at the ELB, we analyze one transitory path to the target as an illustration. Next, we systematically compare the business cycles properties under SL and RE and assess the welfare loss entailed by heterogeneous expectations with respect to the RE representative agent benchmark.

II.3.1 STABILITY ANALYSIS

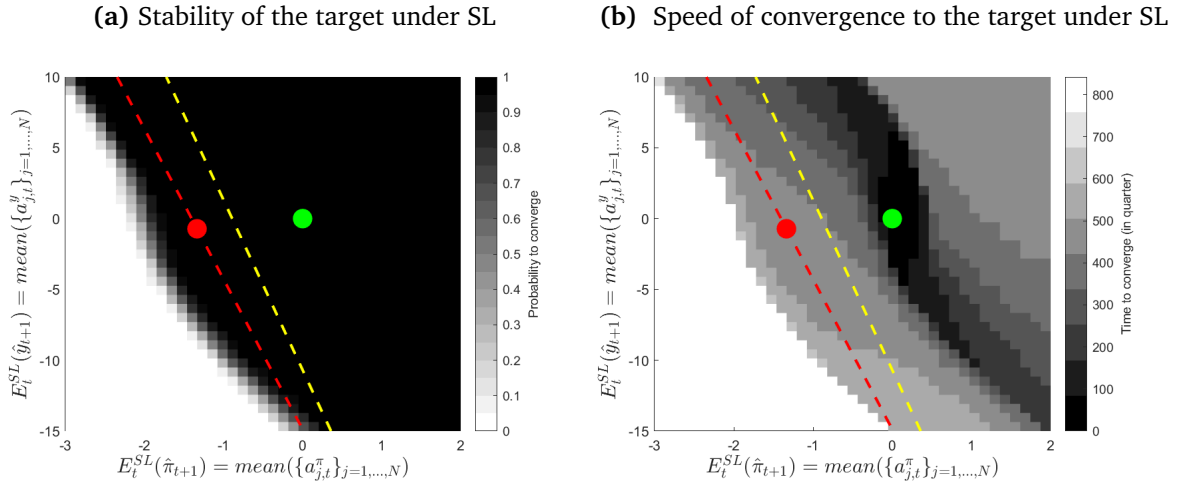
We examine here the asymptotic behavior of the model over the entire state space of the endogenous variables $(\hat{\pi}, \hat{y})$, as utilized in the introduction (see, again, Fig. II.2). We proceed through Monte Carlo simulations. Figure II.5 represents the phase diagram of the model where the average inflation gap expectation (*i.e.* the average of the $\{a_j^\pi\}$ values across agents) is given on the x-axis and the average output gap expectation (*i.e.* the average of the $\{a_j^y\}$ values) on the y-axis. The initial strategies are drawn around each point of the state space, and we repeat each initialization configuration 1,000 times with different seeds of the Random Number Generator (RNG). We obtain the phase diagram by imposing a one-time expectational shock from the target to each point of the state space and then assess whether inflation and output gaps converge back on the targeted steady-state (see Fig. II.5a) and if so, at which speed (see Fig. II.5b). The two figures show that the model either converges to the target (in gray-shaded areas) or diverges along a deflationary spiral (in white areas).

The main message from that exercise is that the basin of attraction of the target under SL is larger than the determinacy region of the targeted steady state under RE. To see that, notice that there is a considerable locus of points on the left-hand side of the stable manifold associated with the saddle point under recursive learning (red dashed line in Fig. II.5) from where the model converges back to the target under SL.²⁶ By contrast, we know from the related literature that this manifold marks the frontier between (local) determinacy and indeterminacy under RE. It also marks the frontier between (local) E-stability and divergence under adaptive learning (see Evans et al. (2008)).

A wider stability region of the target under SL than under recursive learning is due to a key difference between the two expectation formation mechanisms. SL expectations are heterogeneous at any point in time. Among that diversity, only the forecasts that deliver the lowest forecast errors over the past history (and not just the most recent period) survive and feed back into the dynamics of the endogenous variables. Hence, under SL, history matters and a single inflation and output gap data point in the unstable region caused by a one-time pessimistic shift of the *average* forecasts is

²⁶Per consequence, in our model, the low-inflation state is unstable under SL as it belongs to the basin of attraction of the target: if expectations shift on the low-inflation state, they will eventually converge back to the target. Hence, the stability result in Arifovic et al. (2018), that is obtained under infinite memory in the fitness function, does not generalize to our setup where agents discard past observations. Intuitively, the difference in dynamics is akin to the differences observed in the adaptive learning literature between constant and decreasing gain algorithms.

Figure II.5: Global dynamics under social learning



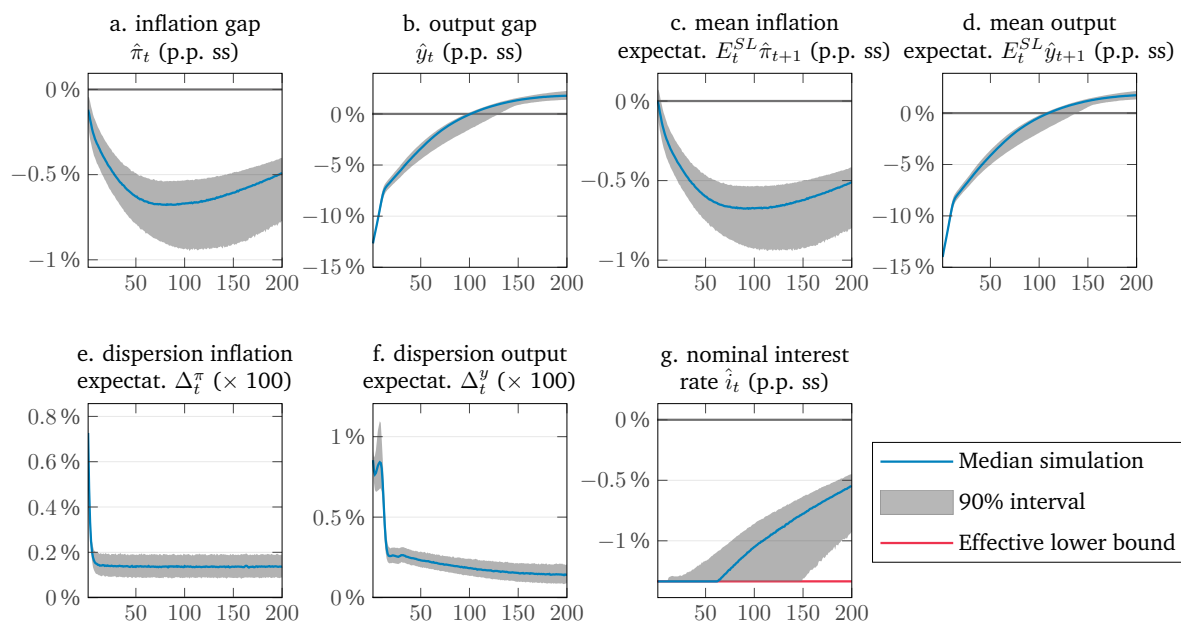
Notes: See explanations at the end of Section II.1.3. We perform 1,600,000 Monte-Carlo simulations over 1000 periods. The targeted steady state is denoted by the green dot, and the deflationary steady state by the red one. The ELB frontier (yellow dashed line) is the locus of points for which $-\bar{r} = \phi^{\pi}\hat{\pi} + \phi^y\hat{y}$: on the left-hand side, the ELB binds. The stable manifold associated with the saddle low inflation steady-state (red line) is computed under recursive learning and corresponds to the stable eigenvector of B^{elb} : on the left-hand side, the model is indeterminate under RE and E-unstable. The empty area represents pairs of expectation values for which the model diverges along a deflationary spiral. We define convergence as ϵ -convergence, i.e. inflation and output respectively enter and do not exit the neighborhood $[-\epsilon_{\pi}, \epsilon_{\pi}]$ and $[-\epsilon_y, \epsilon_y]$ with $\{\epsilon_{\pi}, \epsilon_y\} = \{0.1\%, 0.5\%\}$. Results are robust to tighter convergence criteria. Left: The darker, the higher the probability to converge back to the steady state. Right: The darker, the faster the convergence back to the steady state.

not enough to steer the whole population of forecasts beyond the stable manifold, along a deflationary path. Even after a strong pessimistic shift, as soon as some individual forecasts remain above the red line in Fig. II.5 (albeit below the target), they deliver lower forecast errors than the more pessimistic ones when it comes to forecasting *on average over the whole history*, which includes pre-shock dynamics. Hence, those less pessimistic forecasts spread out and steer the economy back to the target.

By contrast, an adaptive learning algorithm is not concerned with alternative forecasting solutions. A single forecast in the indeterminacy region would result in a negative forecast error, *i.e.* realized inflation and output gaps decline even further below their expected values as they diverge in that region of the state space. This negative forecast error causes agents to revise down their expectations even further, which eventually drives the economy along a deflationary spiral. Yet, our model may also lead to self-sustaining deflationary spirals when shifts in expectations are large enough to throw the entire population of strategies beyond the stable manifold. However, for this to happen, as shown by the white area in Figure II.5a, the one-time shift in expectations has to be implausibly large in light of where the actual data lie, as depicted by Fig. II.2.

Another related interesting observation is given by Figure II.5b. Using the same state space as Figure II.5a, the figure reports the speed of convergence to the target for each pair of initial average expectations. The darker the area, the faster the convergence. It is striking to see that the closer expectations to the targeted steady-state, the faster the convergence. In general, there is a locus of points spiraling around the target where convergence is fast, which is consistent with the complex eigenvalues

Figure II.6: Illustrative transitory path of the estimated model after an expectation shock



Notes: From the top left to the bottom right, transitory path of inflation gap, output gap, average inflation gap expectations, average output gap expectations, nominal interest rate, standard deviation of individual inflation expectations and standard deviation of individual output gap expectations. The blue plain line represents the median realization and the dotted lines are the 5% and 95% confidence intervals over 1,000 Monte Carlo simulations. All plots report the zero line. The lower horizontal line next to $\hat{\pi}_t$ is the ELB.

associated with that steady state.

Most interestingly, the area in the southwest side from the target, beyond the stability frontier, is depicted in light gray. This means that for those severely pessimistic inflation and output gap expectations, the model under SL does converge back to the target, but does so at a particularly slow speed. This area is beyond the ELB frontier (yellow dashed line), which indicates that the ELB is binding yet the model does not diverge along a depressive downward spiral.

Those observations show that our model can produce persistent but non-diverging episodes at the ELB, and heterogeneity in expectations plays an essential role in generating those dynamics. To shed more light on these dynamics, we now focus on a single expectational shock as simulated for Fig. II.5 and study how it propagates in the model.

II.3.2 ILLUSTRATION OF PERSISTENT DYNAMICS AT THE ELB

Fig. II.6 illustrates the persistent dynamics at the ELB by plotting the path from one particular point of the state space II.5 back to the target.²⁷

²⁷In the simulations, expectations travel to pessimistic regions of the state space as a result of the combination of SL and a series of adverse fundamental shocks. Here, the exercise is solely illustrative so we consider an arbitrary shift in expectations and use the point (0, -14)% as an example of a starting point on Fig. II.5, without claiming any empirical realism as to the size or the nature of the shock.

Such a shock produces a *prolonged* depressive episode at the ELB: inflation and interest rates exhibit considerable persistence below their target while output gap recovers faster, and even temporarily overshoots the steady state. These dynamics entailed under SL are empirically much closer to the recent economic experience discussed in the introduction than the excess volatility in the indeterminacy region under RE or the diverging deflationary paths under adaptive learning.

Let us now unravel the underlying forces at play under SL that deliver those empirically appealing dynamics. The initial deviations from steady state are triggered by the pessimistic shock only, while the resulting prolonged low inflation and ELB environment stems entirely from the sluggish dynamics of expectations under SL and their self-fulfilling nature in the NK model.

As explained in Section II.3.1, right after the shock, the elimination of the most negative forecasts rules out the possibility of deflationary spirals and generates the ‘missing disinflation’ along the bust. Per their self-fulfilling nature, below-target forecasts nurture the downturn, which triggers an accommodating response from the CB. This stimulating monetary policy has the largest impact on output gap, which eventually turns positive.

In particular, the paths of inflation and the average inflation expectations almost perfectly overlap, which means that low inflation forecasts are almost self-fulfilling and deliver near-zero forecast errors, which allows them to diffuse among the agents. This selection mechanism explains the considerable persistence in inflation and inflation forecasts depicted in Figure II.6. Inflation and inflation expectations cannot converge back on target until the conjugated force of positive output gaps and low interest rates become strong enough to overcome the almost self-fulfilling force of low inflation expectations.²⁸ Those dynamics generate the inflation-less recovery. This prolonged period of positive output gaps may also suggest that the economy may settle back to equilibrium only after full tapering by the CB.

Finally, it is interesting to note that our model reproduces another stylized fact discussed in Mankiw et al. (2003): a recession is associated with an increase in the dispersion of forecasts among agents or, in other words, the level of disagreement between agents – in our estimated model, the correlation between output gap and output gap forecast dispersion is in fact significant and reaches -0.34. Indeed, Figure II.6 reports how the dispersion of individual expectations spikes in the aftermath of the shock. The rise in forecast dispersion does not last: this is because the selection pressure of the SL algorithm pushes the agents to adapt to the ‘new normal’ in the aftermath of the shock. The level of heterogeneity between agents then returns to its long-run value, which is dictated by the size of the mutations.

We conclude that our simple model offers a stylized representation of the observed loss of anchorage of long-run inflation expectations depicted in Figure II.1 and, more

²⁸Admittedly, the number of periods before convergence back on target appears implausibly large but the model does a good job once one bears in mind that the only policy in our simple model is a Taylor rule constrained by the ELB; hence, our model abstracts from many empirically relevant dimensions of policy that would be likely to play a role in fostering the recovery. The simple structure of the model depicts inflation as almost entirely expectation-driven. It also ignores many other empirically relevant determinants of inflation which could also entail a quicker inflation response. Lastly, as explained above, this exercise is only meant for illustrative purposes, not to match any empirical counterpart in recent history. Hence, one should refrain from drawing an explicit time interpretation.

generally, of the inflation dynamics in the wake of the Great Recession and the ensuing recovery as discussed in the introduction. With this model, we offer a reading of this state of affairs as the consequence of the coordination of agents' expectations on pessimistic outlooks.

From an allocation perspective, the coordination of expectations on large and persistent recessive paths leaves out the economy into second-best equilibria with respect to the benchmark representative-agent model under RE.²⁹ Hence, SL expectations can be envisioned as a friction with respect to the RE representative-agent allocation, which may imply a substantial welfare cost, as we now demonstrate.

II.3.3 WELFARE COST OF SOCIAL LEARNING EXPECTATIONS

To evaluate this cost, we use the welfare function, which has become the main micro-founded criterion, to compare alternative policy regimes. Following [Woodford \(2001\)](#), we consider a second-order approximation of this criterion and use the unconditional mean to express this criterion in terms of inflation and output volatility. The corresponding welfare function reads as:

$$E[\mathcal{W}_t] \simeq \bar{\mathcal{W}} - \lambda^y E[\hat{y}_t^2] - \lambda^\pi E[\hat{\pi}_t^2], \quad (\text{II.10})$$

where $\bar{\mathcal{W}}$ is the steady-state level of welfare and λ^π and λ^y are, respectively, the elasticities of the loss function with respect to the variance of the inflation gap $E[\hat{\pi}_t^2]$ and the output gap $E[\hat{y}_t^2]$. It is straightforward to notice that macroeconomic volatility reduces the welfare of households.

While in representative-agent models the loss function is unique, it may be expressed in an agent-specific manner in a heterogeneous-agent framework. Since the aggregation of agents is performed within the linearized model, we proceed in the same way with the welfare function and linearize it up to the second order. The welfare criterion provides a metric to compare macroeconomic performances under SL and under RE. Comparing these two allocations results in a measurement of the welfare cost of expectation miscoordination, which can be expressed in permanent consumption equivalents ([Lucas, 2003](#)). Using a standard no-arbitrage condition between the SL and the RE allocations, the fraction of consumption λ that SL households are willing to pay to live in an RE world solves the following conditions on utility streams:

$$\sum_{\tau=0}^{\infty} \beta^\tau \frac{1}{N} \sum_{j=1}^N \mathcal{U}((1+\lambda)C_{j,t+\tau}^{SL}, H_{j,t+\tau}^{SL}) = \sum_{\tau=0}^{\infty} \beta^\tau \mathcal{U}(C_{t+\tau}^{RE}, H_{t+\tau}^{RE}), \quad (\text{II.11})$$

where x_t^{SL} and x_t^{RE} denote any endogenous variable x resulting from the same sequence of shocks under the two different expectation schemes.

Table II.4 compares the major business cycles statistics under RE and under SL using the estimated parameters given in Table II.3. This exercise allows us to disentangle the contribution of exogenous fluctuations in the RE-NK model from those

²⁹We refer to the RE counterpart of the NK model as the first-best equilibrium because we do not study the welfare implications of the price rigidities vs. the first-best allocation under flexible prices.

Table II.4: Business cycles statistics and welfare under RE and SL using estimated parameters

<i>Moments</i>		<i>Expectations scheme</i>	
		<i>RE</i>	<i>SL</i>
$var(\hat{\pi}_t)$	- inflation gap variance	0.1775 (0.002)	0.462 (0.029)
$var(\hat{y}_t)$	- output gap variance	14.8159 (0.159)	19.645 (0.644)
Δ_t^π	- inflation gap forecast dispersion	—	0.2 (0.001)
Δ_t^y	- output gap forecast dispersion	—	0.399 (0.002)
$E[\mathcal{W}_t]$	- welfare	-88.099 (0.001)	-94.6 (0.09)
λ	- welfare cost	—	3.303%
$P[r_t=1]$	- ELB probability	0 (0)	0.17 (0.026)

Notes: Average statistics (and standard errors between brackets) over 9,400 Monte Carlo simulations of 200 periods under SL (94 series of shocks repeated 100 times) and over the same series of shocks under RE.

additionally induced by SL.

Table II.4 shows that SL expectations induce considerably more macroeconomic volatility than under RE, especially by inducing endogenous ELB episodes, as explained above. These self-fulfilling recessions substantially deteriorate the welfare of households in comparison to the RE benchmark. By contrast, under the assumption of *i.i.d.* shocks, the rational forecasts of inflation and output gaps boil down to their targeted values (see Section II.1.2). Therefore, under RE, expectations remain anchored, self-fulfilling ELB episodes cannot occur and macroeconomic volatility is negligible.

The resulting cost of SL expectations with respect to RE reaches up to 3.3% of permanent consumption, which is far from negligible with respect to the related literature. This cost creates room for additional monetary policy instruments, especially communication, to enforce the additional objective of coordinating the private sector on the target.

II.4 CENTRAL BANK COMMUNICATION

We first introduce a simple form of CB communication to develop intuition on its anchoring effect on expectations and then discuss how these insights may inform a broader range of topical communication policy debates.

II.4.1 MODELING COMMUNICATION UNDER SL

We represent communication as an announcement, which we denote by A_t^{CB} , made by the CB at the end of any period t . In the model, this is an announcement about inflation in the next period ($t + 1$). We focus on inflation because it is the main objective under an inflation targeting regime.

To introduce the CB announcements into the SL algorithm, we follow Arifovic et al. (2019), albeit in a simpler game. Besides her output and inflation gap forecasts ($a_{j,t}^y$ and $a_{j,t}^\pi$), each agent j now carries a probability $\psi_{j,t} \in [0, 1]$ of incorporating the CB

announcement into her inflation forecast in any period t . If she does so, her inflation forecast in $t+1$ is simply the CB announcement. Conversely, with a probability $1 - \psi_{j,t}$, she ignores the announcement and sets her inflation forecast equal to her forecast $a_{j,t}^\pi$ as before. The determination of her output gap forecasts remains unchanged and equal to $a_{j,t}^y$.

Formally, in the presence of announcements, the expectation formation process of the agents given by (II.5) is modified as:

$$\begin{aligned} E_{j,t}^{SL} \{\hat{\pi}_{t+1}\} &= \begin{cases} A_t^{CB} & \text{with probability } \psi_{j,t} \\ a_{j,t}^\pi & \text{with probability } 1 - \psi_{j,t} \end{cases} \\ E_{j,t}^{SL} \{\hat{y}_{t+1}\} &= a_{j,t}^y. \end{aligned} \quad (\text{II.12})$$

The communication-augmented inflation forecast $\{(\psi_{j,t}, a_{j,t}^\pi)\}_{j \in J}$ undergoes the same mutation and tournament processes as the output gap forecast $a_{j,t}^y$ (see Section II.1.3).³⁰ The only difference from the algorithm used so far lies in the computation of the fitness of inflation forecasts, where Eq. (II.8) is modified as follows:

$$F_{j,t}^\pi = -\psi_{j,t} \sum_{\tau=0}^t (\rho^\pi)^\tau (\hat{\pi}_{t-\tau} - A_{t-\tau-1}^{CB})^2 - (1 - \psi_{j,t}) \sum_{\tau=0}^t (\rho^\pi)^\tau (\hat{\pi}_{t-\tau} - a_{j,t}^\pi)^2,$$

where the first (resp. second) term now corresponds to the discounted sum of squared forecast errors had the agent followed (resp. ignored) the announcements of the CB.

The probabilities $\{\psi_j\}$ can be easily interpreted as the credibility of the announcements. If agents following the announcements (*i.e.* agents with a relatively high value of ψ_j) have lower forecast errors than agents ignoring the announcements (*i.e.* agents with a relatively low value of ψ_j), the following strategy shall spread among agents, which means that the average value of ψ across agents shall increase. The opposite shall hold if following the announcements performs more poorly than ignoring them. Thus, SL agents endogenously build trust or distrust in the communication of the CB as a function of the relative forecasting performances of each alternative. We now develop two simple examples of announcements to show how communication affects expectations.

II.4.2 TWO SIMPLE COMMUNICATION EXAMPLES

We consider the following two communication examples under SL.

The CB announces the inflation target We then have $A_t^{CB} = 0$ (as the model is written in deviations from steady state). It should be noted that the target corresponds to the RE inflation forecasts in our simple model. The announcement of the CB is therefore consistent with the conduct of monetary policy under RE. Hence, the inflation target is redundant information to RE agents, but this piece of information

³⁰In the simulations below, the initial credibility $\{\psi_{j,0}\}$ is drawn from a normal distribution centered around 0.5 with a standard deviation equal to 0.25, a value that is also taken to dictate the mutation process of the probabilities $\{\psi_{j,0}\}$. Results are insensitive to alternatives.

Table II.5: Business cycles statistics under RE, under SL and with CB communication about the inflation target and the inflation forecasts

<i>Expectations:</i>	<i>RE</i>	<i>SL</i>	<i>SL</i>	<i>SL</i>
<i>Type of Communication</i>	×	×	target	VAR(8) forecast
<i>Macroeconomic variability:</i>				
$var(\hat{\pi}_t)$	0.178 (0.002)	0.463(0.029)	0.215 (0.010)	0.2445 (0.01)
$var(\hat{y}_t)$	14.816 (0.159)	19.645 (0.644)	17.181 (0.310)	17.118 (0.304)
<i>ELB occurrence:</i>				
$P[i_t=0]$	0.000 (0.000)	0.17 (0.026)	0.038 (0.012)	0.002 (0.001)
<i>Expectation dispersion:</i>				
Δ_t^π	×	0.2 (0.001)	0.138 (0.004)	0.155(0.0045)
Δ_t^y	×	0.399 (0.002)	0.396 (0.002)	0.397 (0.002)
<i>Expectation anchorage:</i>				
Ω_t^π	0.000 (0.000)	0.887 (0.107)	0.108 (0.02)	0.176 (0.0028)
Ω_t^y	0.000 (0.000)	10.031 (1.22)	6.433 (0.336)	6.31 (0.359)
<i>Welfare:</i>				
$E[\mathcal{W}_t]$	-88.099 (0.001)	-94.599 (0.089)	-88.92 (0.02)	-89.693 (0.0017)
λ (%)	×	3.303%	0.411%	0.8%

Notes: See Table II.4.

may play a non-trivial role under SL.

The CB announces its own inflation forecasts for the next period We assume that the policy authority estimates a commonly used VAR forecasting model that is recursively updated with new observations in each period (see Eusepi and Preston (2010) for a similar assumption). Note that assuming VAR forecasting amounts to assuming that the CB is aware of agents being boundedly rational and, therefore, includes past realizations of the endogenous variables in its forecasting model to account for the propagation mechanism induced by learning. Indeed, such a forecasting model would be misspecified should the agents have RE and, hence, the economy evolve according to the MSV solution. In this second communication scenario, the announcement of the CB is therefore consistent with the conduct of monetary policy under SL.³¹

We now develop intuitions on how communication affects agents' expectations under SL.

First, Table II.5 compares the business cycles statistics of the model under RE and SL – for ease of reading, the first two columns recall the statistics in Table II.4 – and under the two communication scenarios, i.e. when the target and the inflation forecasts are announced.

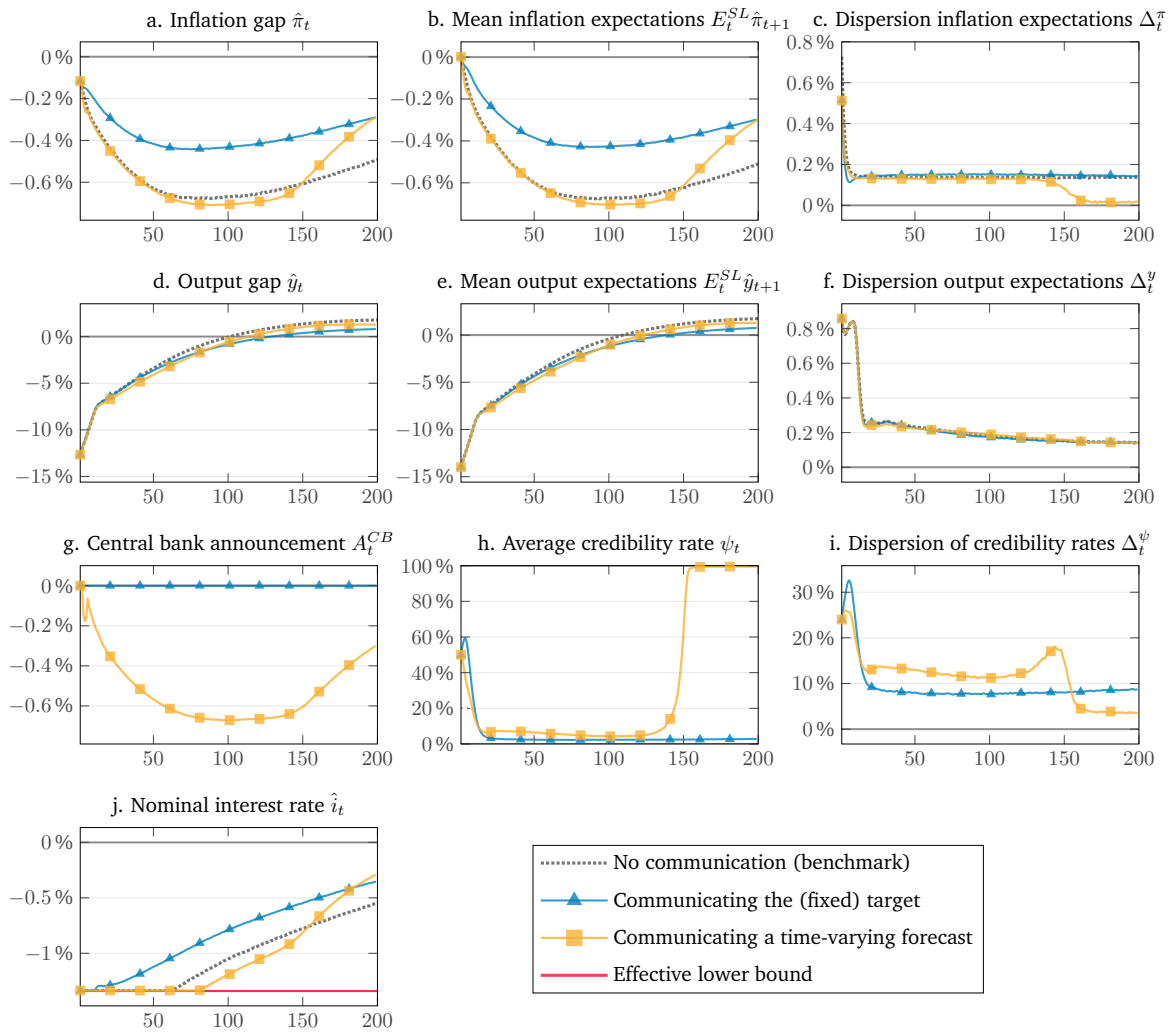
The first three rows of Table II.5 indicate that communication significantly im-

³¹The MSV solution under SL is a complicated and non-linear function of all the states in the system, including those pertaining to the SL process, and an explicit form is not available. We claim that the best the CB can do in such an environment is to estimate the law of motion of the economy with an atheoretical model, such as a VAR. We use 8 lags, in line with the memory of the agents that is implied by the estimated value of the fitness memory on inflation (see, again, Table II.3) but results are robust to more or fewer lags and to assuming a decreasing gain.

proves macroeconomic stabilization with respect to the baseline SL model: the volatility of inflation decreases by more than 50% and the risk of ELB episodes drops considerably.

A look at the next four lines of Table II.5 reveals that not only are expectations better coordinated (*i.e.* disagreement between agents is reduced) in the presence than in the absence of communication, but coordination occurs around the CB objectives (*i.e.* expectations are better anchored at the target).

Hence, we first conclude that in our model, CB communication acts as an anchor for heterogeneous expectations and, by improving their coordination, communication contributes to macroeconomic stabilization. This effect translates into a narrower, yet positive, welfare gap with respect to the RE representative agent benchmark.



Notes: See Figure II.6. The median realizations over 1,000 Monte Carlo simulations are reported. The red dashed-dotted line corresponds to the baseline SL model without communication. The blue lines report the scenarios with communication about the CB forecasts (with triangles) and the target (with squares).

Figure II.7: Illustrative transitory path of the estimated model to a one-period -14% output gap expectation shock under various communication scenarios

Next, we consider the same illustrative transitory path as in Section II.3.2 with communication; see Figure II.7. In the wake of the shock, both communication scenarios result in a loss of credibility. As a consequence, both types of announcement

temporarily lose their anchoring power on agents' inflation expectations; see Figure 7h, where credibility invariably drops towards zero right after the shock. When announcing the target, this credibility loss stems from the actual realizations of inflation drifting away from the target. When announcing forecasts, the credibility loss results from the inaccuracy of the announced forecasts, as the pessimistic shock is unexpected – to see that, look at the discrepancy between the plunging inflation and the near-target announcements immediately after the shock (Fig. 7a vs. 7g). In both cases, the forecasting performances of the followers deteriorate and a large fraction of the agents stop following the CB's announcements.

This credibility loss leads us to the second conclusion: in our model, agents need to 'see it to believe it'. In other words, if the CB's announcements are decoupled from the actual inflation dynamics, they lose their anchoring power on expectations.

Next, as time goes forward, the CB, by updating its model, provides more accurate forecasts – to see that, notice the similarity between the announcements and actual inflation some periods after the shock – and regains credibility. In parallel, this coordination on the forecast announcements leads to a reduction in expectation heterogeneity – to see that, notice the drop in inflation forecast dispersion (Fig. 7c) as credibility increases (Fig. 7h). By contrast, if announcing the target, the CB only regains its credibility once inflation has converged back to the target, which may take a considerable amount of time, as discussed in Section II.3.

Yet, announcing forecasts is not a panacea: it also accentuates the downturn. Indeed, inflation dives deeper, the ELB binds for a longer period (Fig. 7j) and output overshoots further (Figs. 7d-7e) than when the CB announces its target. This observation illustrates an important pitfall of communication: by extrapolating the bust, the announced forecasts may turn self-defeating per the self-fulfilling nature of expectations and contribute to driving expectations away from the target. This striking effect is illustrated in the three graphs of the average inflation forecasts (Fig. 7b), the CB forecast announcements (Fig. 7g) and the actual inflation (Fig. 7a) which all almost overlap.

II.4.3 A BROADER POLICY PERSPECTIVE

From those two simple communication examples, we can inform a broader range of monetary policy issues. For instance, announcing CB's forecasts can be integrated to inflation-forecast targeting (IFT). IFT is based on the principle that, given a long-term inflation objective, the CB's own inflation forecasts act as time-varying intermediate targets because such a forecast path embodies all the relevant information available to the policy makers. IFT requires a strong communication policy with the public to ensure credibility of the CB's announcements. It has been conceived as a way to circumvent the rigidity of a purely rule-based reaction function while avoiding expectations drifts that may result from a discretionary approach (Woodford, 2007).

One good example where IFT has turned particularly useful is the case of transition economies upon adoption of an inflation-targeting regime; see, e.g. Clinton et al. (2017) for the case of Czech Republic. The CB aims to bring inflation to the newly announced target and anchor expectations at this target. It does so by announcing inflation forecasts that gradually converge to the target in an attempt to coordinate

expectations on these forecasts and gradually steer inflation and inflation expectations towards the target. Perhaps closer to our current experience, in a low-inflation environment, our communication exercise shows how IFT allows the CB to coordinate inflation expectations despite the indeterminacy generated by the neutralization of the interest rate feedback at the ELB.

Let us now consider another recent policy discussion, namely the ‘forward-guidance puzzle’ (Carlstrom et al., 2012): under RE, any CB announcement about the future is immediately incorporated into agents’ expectations and optimal decisions and triggers dramatic effects right from the time of the announcement, while empirical evidence contradicts such a strong effect; see, *inter alia*, (Del Negro et al., 2012; Campbell et al., 2016). Based on our model, imperfect credibility may play a central role in explaining this puzzle.³² If agents need to ‘see it first to believe it’, announcements that are at odds with the actual inflation dynamics have a milder effect on expectations and hence actual decisions than under the full credibility assumption underlying RE.

One more example of a topical policy debate that can be informed by our work is the recent inflation targeting reviews undertaken by a number of major CBs. As persistently low inflationary pressures, exacerbated by the ELB constraint, have resulted in price levels falling behind the paths consistent with the inflation target, policy makers and academic circles have been discussing alternatives, such as a higher inflation target, average-inflation targeting or price-level targeting. Under these proposals, the CB would allow (temporary) higher inflation in the future to compensate for the price effects of past low inflation. At the 2020 Jackson Hole monetary policy symposium, the Fed announced such a policy shift. However, per the self-fulfilling nature of inflation expectations, such a shift needs to be accompanied by higher inflation expectations; in other words, the ability of the CB to deliver higher future inflation needs to be credible. Such a rise in inflation expectations is non-trivial if agents, accustomed to a decade of low inflation, need first to see higher inflation to revise upward their expectations. Those considerations reinforce the rationale for intensifying the CBs’ efforts to communicate beyond market participants and reach the general public, as most recently emphasized by a number of major CBs.

II.5 CONCLUSION

This paper develops a model that features expectation-driven business cycles. The key mechanism works through heterogeneous expectations that may lose their anchorage to the target and persistently coordinate on below-target paths, which triggers prolonged ELB episodes. Heterogeneous expectations are introduced via an SL process into an otherwise standard two-equation macroeconomic model with a constrained Taylor rule. Our model nests the RE representative agent benchmark. In particular, we use white noise fundamental shocks to identify the propagation mechanism stemming from expectations in the formation of business cycles.

³²Other solutions in the literature rely on weakening the effect of expected real interest rates on consumption by adding frictions such as liquidity constraints, limited asset-market participation or habit formation Del Negro et al. (2012). Related contributions explain the puzzle by weakening the expectation channel if agents use k-level reasoning (Farhi and Werning, 2019) or pay limited attention (Gabaix, 2020).

Our first contribution is to bring such a model to the data and estimate jointly its fundamental and learning parameters by matching moments from both US inflation and output gaps and the SPF. Our parsimonious model is able to account for ten stylized facts, including properties related to heterogeneity in forecasts, persistence in macroeconomic variables and endogenous occurrence of ELB episodes.

We then analyze the dynamics of the model and show that the basin of attraction of the target under SL is larger than the determinacy region under RE. In the context of our model, ELB episodes are episodes where expectations have coordinated on pessimistic outlooks following a series of adverse fundamental shocks and have visited regions of that basin from where the transition back on the target does occur but at a particularly slow pace. Our second major contribution is then to provide a framework that can account for the recent inflation dynamics that are challenging to capture in standard simple macroeconomic models. In particular, our model accounts for the ‘missing disinflation’ along the Great Recession per its stable but below-target dynamics and extensive ELB episodes. It also accounts for the ‘inflation-less recovery’ per consequence of the combination of unanchored inflation expectations that put downward pressure on inflation and the boosting effect of low interest rates on output.

Finally, we extend our model to illustrate how CB communication may influence expectations. In our model, the credibility of the announcements is not *a priori* granted but rather follows the same evolutionary process as the forecasts of the agents. From two simple examples, we show that this endogenous credibility plays a central role in reshaping expectations: in our model, agents need to ‘see it to believe it’. Moreover, pessimistic inflation forecasts may turn self-defeating per the self-fulfilling nature of inflation expectations. From those observations, we discuss broader policy implications to shed light on recent debates such as the forward-guidance puzzle or inform topical policy proposals such as temporary higher inflation targets.

Our model offers a simple framework that yet opens up the possibility for analyzing a rich set of monetary policy alternatives. As for our estimation routine, it may be applied to a wide range of standard workhorse models that could then be explored under heterogeneous expectations. Those research avenues are left for further work.

Part III

Weather shocks

This is a joint work with

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INTRODUCTION

Among the many shocks and disturbances driving business cycles, weather shocks have received little attention as a serious source of economic fluctuations in modern macroeconomic models. Yet over the last 40 years, heat waves and droughts have been causing significant damages at global level peaking to a total value of US\$25 billion in 2012.³³ Both the frequency and the intensity of these adverse events tend to follow an upward trend, suggesting that weather shocks are likely to become a more frequent source of business cycles in the coming years. This growing source of macroeconomic fluctuations is emerging as one of the most important facets of global warming, in particular for agricultural-based countries. In such economies, weather shocks generate detrimental fluctuations in the agricultural sector that can spread to the rest of the economy.

The economic literature has devoted considerable efforts to quantify the effects of the weather on economic activity in two isolated ways: either by looking at long-term effects through the prism of theoretical models, or by focusing on both short and long-term effects using empirical analysis.³⁴ Theoretical models exemplified by Nordhaus (1991) have been strongly criticized in particular regarding their lack of empirical foundations (see Pindyck, 2017). If these models are useful to rationalize long term implications of climate, they become irrelevant for short run analysis at a business cycle frequency. In contrast, empirical models provide important quantitative insights on the short term transmission channels. However, these channels have never been yet interpreted from a general equilibrium perspective. The lack of an economic framework that tackles the short-term dimension of the weather is a major issue in a climate change context, as policymakers are expected to more frequently cope with short-term adverse weather events with important implications (e.g., food insecurity, recessions, currency depreciation, etc.).

Therefore, the main objective of the paper is to fill the gap by providing a quantitative framework that directly addresses the short term dimension of the weather. This paper contributes to the current literature by reconciling theoretical models of

³³The cumulative sum of estimated damage caused by droughts or extreme temperatures worldwide is calculated from EM-DAT data, and set in real 2012 terms using the US GDP deflator.

³⁴See Acevedo et al. (2017) for a survey on weather shocks, Nordhaus (2018) for a summary of the evolution of the canonical model of climate change over the three decades, and Deschenes and Greenstone (2007) for an empirical assessment of long term effects of climate change on agricultural output.

the weather with the data, using the weather as an observable variable. The resulting framework is able to disentangle the contribution of weather shocks from alternative sources of business cycles and originally price this contribution into consumption-equivalent welfare losses. Most of the literature considers climate change solely as a trend phenomenon, leaving the cost of weather fluctuations as a second order issue. Hence, this paper also contributes to the literature by quantifying climate change through a rise in the variability of weather events.

In this paper, our methodology follows a two-step strategy. In a first step, we document the transmission mechanisms of weather shocks using an a-theoretical model. Since the time-varying productivity of agricultural land is directly measurable from soil moisture observations, we build a weather index at a macro level that captures unsatisfactory levels of soil moisture for New Zealand.³⁵ This aggregate measure of the weather is included in a Structural Vector Auto-Regressive (SVAR) model, alongside seven macroeconomic series from New Zealand. The impulse response functions analysis documents the transmission mechanism of weather shocks in a small-open economy environment and provides a benchmark for the development of a general equilibrium model. In a second step, we enrich a Dynamic Stochastic General Equilibrium (DSGE) model with a weather-dependent agricultural sector facing exogenous weather.³⁶ Entrepreneurs involved in the agricultural sector (i.e., farmers) are endowed with land with a time-varying productivity determined by both economic and weather conditions. The model is estimated through Bayesian techniques on the same sample as the SVAR model to provide a complementary representation of the data. In addition to its empirical relevance, the estimated model provides a detailed understanding of how weather shocks propagate in the economy and yields several predictions on climate change from a general equilibrium perspective.

We get three main results from the aforementioned strategy. First, both the SVAR and the DSGE models provide a similar picture about the transmission of an adverse weather shock through a large and persistent contraction of agricultural production, accompanied by a decline in consumption, investment and a rise in hours worked. At an international level, a weather shock causes current account deficits and a depreciation of the domestic currency. Second, we find that weather shocks play a non-trivial role in driving the business cycles in New Zealand. On the one hand, the inclusion of weather-driven business cycles strikingly improves the statistical performance of the model. On the other, weather shocks drive an important fraction of the unconditional variance, in particular for GDP, consumption and agricultural output. The resulting consequence is a high welfare cost of business cycles induced by weather shocks. In particular, we find that households would be willing to give up 0.30% of their un-

³⁵We use New Zealand data for two reasons. First, New Zealand has faced many weather shocks, in particular droughts, which have caused severe damages to its agricultural sector. Second, the size of the country is relatively small compared to other countries such as the United States. So when a drought strikes New Zealand, most of the regions are affected at the same time. The choice to rely on such data leads to a specific modeling strategy for the SVAR and DSGE models.

³⁶Treating the weather as an exogenous process is a main departure from Nordhaus (1991)-type models, but this departure is necessary to avoid most of the critiques raised by Pindyck (2017). The empirical and theoretical grounds motivating the feedback loop effect between human activity and CO₂ emissions are considered as very fragile in these models, because we yet know little about climate sensitivity to temperatures changes. Treating weather as exogenous is thus a conservative approach with respect to the current practice in the literature.

conditional consumption to rule out weather shocks, which is remarkably high with respect to other sources of disturbances in our model. A third result concerns an original counterfactual analysis on climate change. We increase the volatility of weather shocks in accordance with IPCC (2014)'s climate change projections for 2100, and evaluate how these structural changes in the distribution of weather shocks affect macroeconomic volatility. We find that climate change critically increases the variability of key macroeconomic variables, such as GDP, agricultural output or the real exchange rate. The corollary of this structural change is an increase in the welfare cost of weather driven business cycles peaking up to 0.46% in the worst-case climate change scenario.

Our work contributes to the literature that connects the macroeconomy with the weather through the lens of theoretical models. This literature is mainly dominated by integrated assessment models (IAMs) pioneered by Nordhaus (1991). In a nutshell, economic activity generates a negative externality through greenhouse gas emissions that adversely change temperatures. Higher temperatures deteriorate aggregate production through a damage function in the production technology of firms. The externality resulting from greenhouse gas emissions is not properly taken into account by firms. This market failure motivates a kind of Pigouvian tax that internalizes the social cost of the externality by setting a price on an additional ton of emissions. This price is estimated so that it is equal to the social marginal damages resulting from that additional ton of emissions. Nordhaus' pioneer models can be classified into two categories: those with a single region (DICE models – Dynamic Integrated model of Climate and the Economy) and those with several regions (RICE models – Regional Integrated model of Climate and the Economy). The literature is obviously not limited to Nordhaus' models. As noted by Hassler and Krusell (2012), the increased interest in climate change in the 1990s led to the development of many models, 21 of which are listed by Kelly and Kolstad (1999b). A classification into five broad categories is suggested by Santon et al. (2009): welfare optimization, general equilibrium, partial equilibrium, simulation, and cost minimization. In this literature, the conception of climate faces two main critiques initiated by IPCC (2014) and Pindyck (2017). The first critique is conceptual and concerns the deterministic nature of these models. Under this assumption, agents have perfect knowledge about future states of climate and economic fundamentals. This leaves no role for uncertainty, economic fluctuations and their possible costs in terms of welfare. In addition, the IAMs' assessment of climate change only accounts for shifts in the mean of climate variables but not in their variability, resulting in a possibly large underestimation of the cost of climate change. The second critique is empirical. The usual practice in this literature is to calibrate the model without estimating structural parameters. In absence of explicit empirical foundations, Pindyck (2017) argues that these models can be used to provide any result one desires. Stern (2016) points out shortcomings in the consideration of certain risks in many IAM models, leading these models to underestimate the impacts of climate change. Some recent IAM models, however, incorporate uncertainties (see for example Kelly and Kolstad, 1999a; Leach, 2007; Gerlagh and Liski, 2017). Some alternative models including DSGEs, as mentioned by Stern (2016), also have the ability to take uncertainty into account. In particular, Golosov et al. (2014a) develop such a model to derive an analytical formula for the social cost of carbon. We complement this literature by tackling the short-term dimension of the weather, and

evaluate their social costs in a context of climate change. Most of this literature consider climate change as an increase in the mean of climate variables, in this paper we analyze climate change from a different perspective by considering an increase in the variance of climate variables. We find that the implications from a rise in the variability of climate is non-trivial and should be more considered in the literature of climate change. Unlike IAMs models that limit the analysis to a calibration exercise, we also take the model to the data by estimating the structural parameters of the model to avoid Pindyck's critique.

Another strand of the literature employs empirical models to examine the short run effects of the weather on economic activity. In particular, some authors focus on the relationship between temperatures and productivity. [Dell et al. \(2012\)](#) show that high temperatures have a detrimental effect on economic growth, but only in poor countries. These results are contrasted by the empirical study of [Burke et al. \(2015\)](#) who show that the relationship between high temperatures and productivity is non-linear, for both poor and rich countries. The studies of [Acevedo et al. \(2017\)](#) and [Mejia et al. \(2018\)](#), conducted on larger samples, confirm these results. In addition, [Fomby et al. \(2013\)](#) show that in the case of developed countries, droughts have a negative effect on growth, in particular for the agricultural sector. Our analytical framework builds on these studies to model how climate can affect economic activity, but from a general equilibrium perspective. We also rely on the results of empirical studies that focus more on the weather and the economy at business cycle frequency. For example, [Buckle et al. \(2007\)](#) and [Kamber et al. \(2013\)](#) underline the importance of weather variations as a source of aggregate fluctuations, along with international trade price shocks, using a structural VAR model for New Zealand. [Bloor and Matheson \(2010\)](#) find evidence of the importance of the weather, more particularly the occurrence of El Niño events, on agricultural production and total output in New Zealand. [Cashin et al. \(2017\)](#) also investigate the effects of El Niño on the world economy, using a country-by-country analysis. More specifically, they find evidence of negative effects of an El Niño shock on real output growth in New Zealand. Lastly, in a recent study, [Donadelli et al. \(2017\)](#) propose a framework related to ours. In a real business cycle model, they introduce temperature levels as an explanatory factor of productivity for the US economy. In their model, productivity is affected by the unpredictable component of temperatures. Their results show that a one-standard deviation temperature shock causes a 1.4 percentage point decrease in productivity growth. The authors emphasize the importance of temperature shocks regarding welfare costs. Our article complements this study by taking a theoretical model to the data, instead of limiting the analysis to a calibration exercise. In addition, our measure of the weather is not limited to temperatures, as our weather index also includes the role of rainfalls as a determinant of agricultural productivity.

The remainder of this article is organized as follows: [section III.1](#) provides empirical evidence regarding the impact of weather shocks on macroeconomic variables. [section III.2](#) and [section III.3](#) sketch the DSGE model and present its estimation, respectively. [section III.4](#) provides evidence on the importance of introducing weather shocks in the model. [section III.5](#) analyzes the short-term effects of weather shocks. [section III.6](#) illustrates how the parameters of the weather-dependent agricultural sector affect our results. [section III.7](#) assesses the fluctuations and welfare costs induced by weather shocks under different climate scenarios. [section III.8](#) concludes.

III.1 BUSINESS CYCLE EVIDENCE ON WEATHER SHOCKS

How do we measure the weather? In most of the models in environmental economics, weather and climate measurements are solely based on temperature records. In agricultural economics these measures are often supplemented by rainfall observations in order to characterize agricultural returns patterns. In this paper, the weather is measured through soil moisture deficits. Soil moisture deficits depict the balance ratio between rainfalls and temperatures. Rainfalls typically boost the productivity of the land by favoring crop growth, and conversely the evapotranspiration process induced by higher temperatures reduces land productivity.³⁷ Based on observations of soil moisture deficits, we build a macroeconomic index³⁸ that aims at providing an accurate measure of land productivity in New Zealand. A graphical representation of this index is provided in [Figure III.1](#). By construction, the index values range from -4 to +4, where positive values indicate a soil moisture deficit, while negative ones indicate an excess of moisture.

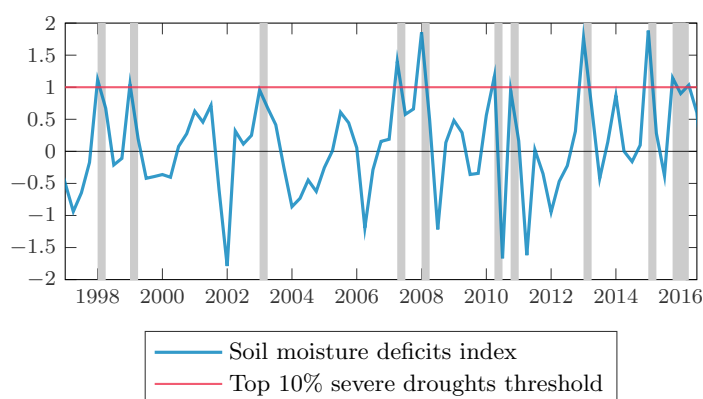


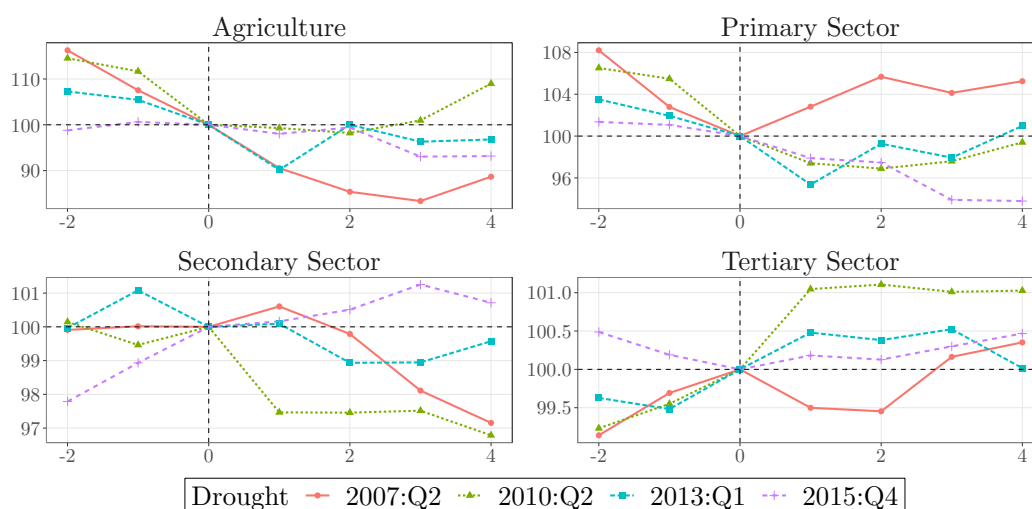
Figure III.1: Weather index measuring soil moisture deficits in New Zealand.

As shown in [Figure III.1](#), New Zealand has experienced cyclical changes in its soil water deficits index over the last two decades, oscillating between periods of high volumetric water content in soils and periods of droughts. Assuming a normal distribution of the weather, the 10th percent of the most severe episodes can be inferred directly from the time series when the soil moisture deficits index peaks above 1. In the same way as for NBER recessions, the index allows to easily date and monitor severe weather events which are very likely to be costly for the agricultural sector as shown by [Kamber et al. \(2013\)](#) and [Mejia et al. \(2018\)](#). In recent years, New Zealand has undergone numerous episodes of severe droughts of various intensities that have disrupted its economy to a greater or lesser extent, most notably in 2007, 2010, 2013 and 2015.

What is the supply-side adjustment of New Zealand following a severe drought? A preliminary assessment of these extreme events on the sectoral reallocation is performed through the examinations of changes in the relative share of each sector in the total production of New Zealand. [Figure III.2](#) reports these changes in the shares

³⁷See [Doorenbos and Kassam \(1979\)](#) and [Narasimhan and Srinivasan \(2005\)](#) for a analysis of soil moisture on crop yields.

³⁸More details on the construction of the index can be found in the online appendix.



Notes: The lines show the evolution before and after a drought for each sector's share in total production, after normalizing the sector's share to 100 at the time of the drought.

Figure III.2: Sectoral re-allocations following severe weather shocks.

of agriculture, primary, secondary, and tertiary sectors in total activity, two quarters before and four quarters after the four most severe droughts. For convenience, each sector's share of the total activity is normalized to 100 at the time of the drought. Each line corresponds to a drought episode reported by the index at hand. After a drought shock, the share of the agricultural sector in total output declines substantially although temporarily. A similar pattern is observed for the primary sector, although the magnitude of the reaction is naturally not as important as for agriculture because the primary sector includes mining and fishing which are less sensitive to the weather. Regarding the secondary sector, the result is unclear suggesting that there is no salient effects. As for the tertiary sector, it tends to experience a relative expansion, in accordance with [Mejia et al. \(2018\)](#), suggesting that weather shocks possibly generate positive spillover effects.

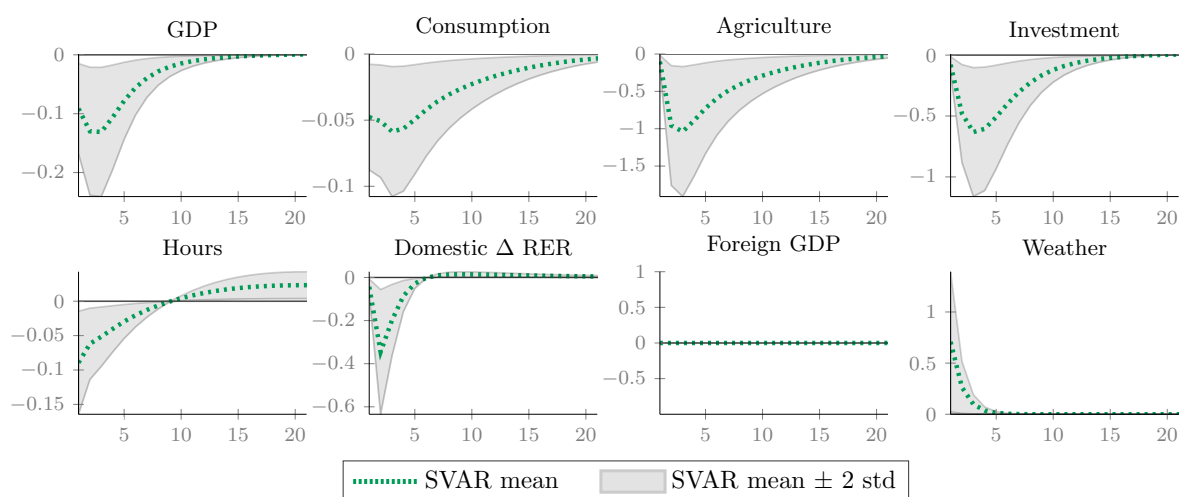
	correlation	t-stat	p-value	95% Confidence interval
Agriculture Only	-0.31	-2.99	0.00	[-0.48, -0.10]
Primary Sector	-0.25	-2.41	0.02	[-0.44, -0.04]
Secondary Sector	-0.10	-0.91	0.37	[-0.30, 0.11]
Tertiary Sector	0.39	3.90	0.00	[0.19, 0.55]

Notes: The significance of correlations is tested using the Pearson test.

Table III.1: Correlations of Sectoral GDP with the weather index.

To complete the assessment, we compute correlations between the time series of the weather and the relative share of different sectors used in the previous figure. [Table III.1](#) also corroborates the presence of possible sectoral adjustments. In particular, the share of the agricultural sector is negatively correlated with the weather index, as is, to a lesser extent, the GDP of the primary and secondary sectors. On the other hand, the activity of the tertiary sector is positively correlated with the drought measure.

To investigate further the interactions between the weather and other standard macroeconomic time series, a structural vector autoregressive model is employed. A few constraints on the VAR's equations are necessary to portray New Zealand's specific situation: (i) we impose an exogenous weather (i.e., the weather is not Granger caused by any other variable),³⁹ (ii) we force domestic variables to have no effect on foreign variables as Cushman and Zha (1997).⁴⁰ The VAR includes 8 observable variables. Six of them represent the domestic block: GDP, agricultural production, hours worked, consumption, investments, and variations of the real effective exchange rate. The foreign block contains a measure of GDP for the rest of the world.⁴¹ All these variables are taken in real terms and expressed in percentage deviations from a log-linear trend. In addition, the restricted VAR model is estimated with one lag, as suggested by both Hannan-Quinn and Schwarz criteria. Once the restricted VAR is estimated, some further restrictions on the contemporaneous effects of the covariates are imposed to estimate the Structural VAR.⁴²



Notes: The green dashed line is the Impulse Response Function. The gray band represents 95% error band obtained from 10,000 Monte-Carlo simulations. The response horizon is in quarters. Time horizon is plotted on the x-axis while the percent deviation from the steady state is plotted on the y-axis.

Figure III.3: SVAR impulse response to a 1% weather shock (drought) in New Zealand.

³⁹As the historical data only cover a restricted period of time, we assume that human activities do not significantly affect the occurrence of droughts.

⁴⁰In particular, a first constraint concerns the small open economy nature of New Zealand with respect to its trading partners. Letting New Zealand be the domestic country and NZ trading partners be the foreign country, we prevent both domestic shocks and variables to cause fluctuations on foreign variables. We follow Cushman and Zha (1997) and create an exogenous block for the variables from the rest of the world. We impose a second constraint on the VAR's equations concerning the weather itself. In particular, exogeneity is also imposed for the weather variable, so that it can affect the domestic macroeconomic variables, and so that neither domestic nor foreign macroeconomic variables can affect the weather variable. More details are given in the paper's online appendix.

⁴¹We use a weighted average of GDP for New Zealand's top trading partners, namely Australia, Germany, Japan, the United Kingdom and the United States, where the weights are set according to the relative share of each partner's GDP in the total value.

⁴²Specifically, we disable the correlation link between the shock on the weather and foreign variables to be consistent with the small open economy situation. More details on the estimation strategy can be found in the online Appendix.

To investigate the effects of an adverse weather shock, we examine the impulse responses to a one-standard-deviation of the drought variable. A lower triangular Choleski decomposition of the error variance-covariance matrix is used to derive the orthogonal impulse responses. The results are depicted in [Figure III.3](#), where each panel represents the response of one of the variables to a weather shock. Overall, a shock to the weather equation generates a contraction of New Zealand's economy in the similar magnitude as [Buckle et al. \(2007\)](#): a rise in soil moisture deficits implies a contemporaneous 0.12% contraction of agricultural production, as already suggested by the two previous assessments. The depression in agricultural production reaches a peak decline of 1.27% after three periods. It is simultaneously followed by a 0.05% decline in consumption and a 0.1% decline in investment. The adjustment of the labor market is naturally slower and materialize first through a contraction in hours worked, followed by a late rise occurring 10 quarters after the realization of the weather shock, thus suggesting that the weather mimics the dynamic patterns of a TFP shock. The weather shock vanishes five periods after its realization, although its effects on the economy are strikingly very persistent, in particular for the labor market. This underlines the presence of an unusual propagation mechanism inherent to the weather which is to be taken into account in the modeling of the DSGE presented in the remainder of the article. More specifically, the presence of a slow adjustment effect will require a specific friction for the farmer problem.

III.2 THE MODEL

Our model is a two-sector, two-good economy in a small open economy setup with a flexible exchange rate regime.⁴³ The home economy, i.e., New Zealand, is populated by households and firms. The latter operate in the agricultural and the non-agricultural sectors. Workers from the agricultural sector face unexpected weather conditions that affect the productivity of their land. Households consume both home and foreign varieties of goods, thus creating a trading channel adjusted by the real exchange rate. The general structure of the model is summarized in [Figure III.4](#). The remainder of this section presents the main components of the model.

III.2.1 AGRICULTURAL SECTOR

The economy is populated by a unit mass $i \in [0, 1]$ of infinite living and atomistic entrepreneurs. A fraction n_t of these entrepreneurs are operating in the agricultural sector while the remaining fraction $1 - n_t$ operates in the non-agricultural sector. We allow any of the entrepreneurs to switch from one sector to another assuming that the fixed portion of agricultural firms is subject to an exogenous shock: $n_t = n \times \varepsilon_t^N$ where

⁴³Our small open economy setup includes two countries. The home country (here, New Zealand) participates in international trade but is too small compared to its trading partners to cause aggregate fluctuations in world output, price and interest rates. The foreign country, representing most of the trading partners of the home country, is thus not affected by macroeconomic shocks from the home country, but its own macroeconomic developments affect the home country through the trade balance and the exchange rate.

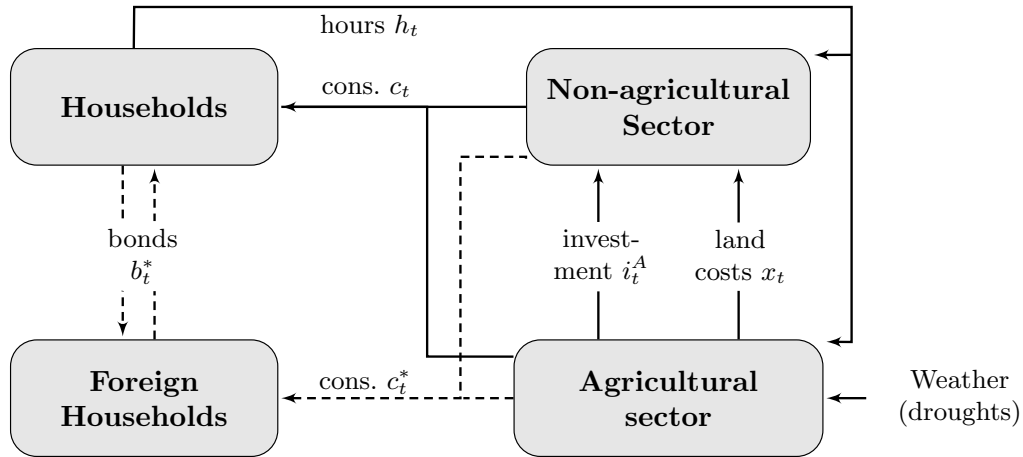


Figure III.4: The theoretical model.

ε_t^N is a stochastic $AR(1)$ process.⁴⁴ The fraction $i \in [0, n_t]$ of entrepreneurs operating in the agricultural sector is referred to as farmers.

To investigate the implications of variations of the weather as a source of aggregate fluctuations, a weather variable denoted ε_t^W is introduced in the model. More specifically, this variable captures variations in soil moisture that affect the production process of agricultural goods. To be consistent with the SVAR model, we assume that the aggregate drought index follows an autoregressive process with only one lag:

$$\log(\varepsilon_t^W) = \rho_W \log(\varepsilon_{t-1}^W) + \sigma_W \eta_t^W, \quad \eta_t^W \sim \mathcal{N}(0, 1), \quad (\text{III.1})$$

where $\rho_W \in [0, 1)$ is the persistence of the weather shock and $\sigma_W \geq 0$ its standard deviation. In the model, shock processes are all normalized to one in the steady state so that a positive realization of η_t^W – thus setting ε_t^W above one – depicts a possibly prolonged episode of dryness that damages agricultural output. The stochastic nature of the model imposes that farmers are surprised by contemporaneous and future weather shocks. We do not consider the perspective of news shocks about the weather, as the usual forecast horizon for farmers about weather shocks lies between 1 and 15 days.⁴⁵

Treating the weather as an exogenous process is rather conservative with respect to the canonical setup of Nordhaus (1991). As argued by Pindyck (2017), empirical and theoretical grounds motivating the feedback loop effect between human activity and CO_2 emissions are considered as very fragile in these models. Under the weather exogeneity assumption, there is no CO_2 feedback loop, instead we just let the data be informative about the distribution of weather shocks.

The outcome of farmers' activity in the agricultural sector encompasses a large variety of goods such as livestock, vegetables, plants, or trees. All of these agricul-

⁴⁴More specifically, the $AR(1)$ shock is given by: $\log(\varepsilon_t^N) = \rho_N \log(\varepsilon_{t-1}^N) + \sigma_N \eta_t^N$, with $\eta_t^N \sim \mathcal{N}(0, 1)$ and $0 \leq \rho_N \leq 1$.

⁴⁵For example, in New Zealand the NIWA provides forecast services to farmers about weather shocks at a high frequency level (1 or 2 days ahead), medium frequency level (6 days ahead) and probabilistic forecast out of fifteen days.

tural goods typically require land, labor and physical capital as input to be produced. The general practice in agricultural economics is to explicitly feature the input-output relationship by imposing a functional form on the technology of the agricultural sector.⁴⁶ Among many possible functional forms, the Cobb-Douglas production function has become popular in this economic field following the contribution of [Mundlak \(1961\)](#).⁴⁷ We accordingly assume that agricultural output is Cobb-Douglas in land, physical capital inputs, and labor inputs:

$$y_{it}^A = [\Omega(\varepsilon_t^W) \ell_{it-1}]^\omega \left[\varepsilon_t^Z (k_{it-1}^A)^\alpha (\kappa_A h_{it}^A)^{1-\alpha} \right]^{1-\omega}, \quad (\text{III.2})$$

where y_{it}^A is the production function of the intermediate agricultural good that combines an amount of land ℓ_{it-1} (subject to the weather $\Omega(\varepsilon_t^W)$ through a function described later on), physical capital k_{it-1}^A , and labor demand h_{it}^A . Production is subject to an economy-wide technology shock ε_t^Z following an $AR(1)$ shock process affecting the two sectors. The parameter $\omega \in [0, 1]$ is the elasticity of output to land, $\alpha \in [0, 1]$ denotes the share of physical capital in the production process of agricultural goods, and $\kappa_A > 0$ is a technology parameter endogenously determined in the steady state.⁴⁸ We include physical capital in the production technology, as, in developed countries the agricultural sector heavily relies on mechanization. Because of the delays in the settlement of physical capital and land, these two variables naturally embody “time to build” features à la [Kydland and Prescott \(1982\)](#).

Each farmer owns a land ℓ_{it} that is subject to changes depending both on economic and meteorological conditions. During the production process of agricultural goods between $t - 1$ and t , land ℓ_{it-1} is subject to the unexpected realization of the weather ε_t^W . Agricultural production is tied up with exogenous weather conditions through a damage function $\Omega(\cdot)$ in the same spirit as the Integrated Assessment Models literature pioneered by [Nordhaus \(1991\)](#). We opt for a simple functional form for this damage function:⁴⁹

$$\Omega(\varepsilon_t^W) = (\varepsilon_t^W)^{-\theta}, \quad (\text{III.3})$$

where θ determines elasticity of land productivity with respect to the weather. Imposing $\theta = 0$ shuts down the propagation of weather-driven business cycles. The effective

⁴⁶See [Chavas et al. \(2010\)](#) for a survey about the building of theoretical models in agricultural economics over the last century.

⁴⁷We refer to [Mundlak \(2001\)](#) for discussions of related conceptual issues and empirical applications regarding the functional forms of agricultural production. In an alternative version of our model based on a CES agricultural production function, the fit of the DSGE model is not improved, and the identification of the CES parameter is weak.

⁴⁸This parameter has the same interpretation as [Restuccia et al. \(2008\)](#): as long as $\kappa_A > 1$, the productivity of land in the agricultural sector is below the productivity of non-agricultural firms. Since capital and labor are perfectly mobile in the deterministic steady, κ_A allows marginal products of physical capital and labor to be equal across sectors.

⁴⁹The literature on IAMs traditionally connects temperatures to output through a simple quadratic damage function in order to provide an estimation of future costs of carbon emissions on output. However, [Pindyck \(2017\)](#) raised important concerns about IAM-based outcome as modelers have so much freedom in choosing a functional form as well as the values of the parameters so that the model can be used to provide any result one desires. To avoid the legitimate criticisms inherent to IAMs, our model is solved up to a first approximation to the policy function. This does not allow us to exploit the non-linearities of the damage function which critically drives the results of IAM literature through a quadratic term in the damage function.

units of land in the production function are denoted $\Omega(\varepsilon_t^W) \ell_{it-1}$.

In addition to being contemporaneously impacted by weather fluctuations, agricultural production is also subject to effects that spread over time, which we call *weather hysteresis effects*. These hysteresis effects that imply atypical supply dynamics have been well established in the economic literature. For the case of cattle breeding for example, Rosen et al. (1994) document the persistence of livestock induced by the biological process of gestation and maturation of dairy cattle. In the presence of weather shocks, prolonged severe droughts entail early liquidation of stocks combined with a drop in the fertility rate. These changes in the population size and characteristics have permanent effects in the future production of agricultural goods. Kamber et al. (2013) have shown that beyond the immediate rise in slaughter, there tends to be slightly less slaughter for several following years, as stock levels are rebuilt. Hysteresis effects are not limited to the production of animal stocks. Crops are also subject to specific cycles. For example, Narasimhan and Srinivasan (2005) have shown that soil moisture deficits exhibit persistence that is directly connected to the interaction between rainfalls and evapotranspiration, as lands require several months to recover their average productivity levels. In addition, the crop growth process spans over multiple periods. A drought occurring at a specific stage of the process (e.g., during pollination⁵⁰) may entail a critical loss on the final crop yield at harvest time. This temporal gap between the drought and the harvest period needs a specific device that captures this well documented persistence mechanism. To do so, we relax the standard assumption in agricultural economics of fixed land and assume that the productivity of land is possibly time-varying. In particular, each farmer owns land with a productivity (or efficiency) following an endogenous law of motion given by:

$$\ell_{it} = \left[(1 - \delta_\ell) + v(x_{it}) \right] \ell_{it-1} \Omega(\varepsilon_t^W), \quad (\text{III.4})$$

where $\delta_\ell \in (0, 1)$ is the rate of decay of land productivity that features the desired persistence effect. We assume that the marginal product of land is increasing in the accumulation of land productivity. This is captured by assuming that land expenditures x_{it} yield a gross output of new productive land $v(x_{it}) \ell_{it-1}$ with $v'(\cdot) > 0$, $v''(\cdot) \geq 0$. More specifically, x_{it} can be viewed as agricultural spending on pesticides, herbicides, seeds, fertilizers and water used to maintain the farmland productivity.⁵¹ In a presence of a drought shock, the farmer can optimally offset the soil dryness by increasing field irrigation or the feeding budget, as the feed rationing of cattle is based on the use of local forage produced by country pastures. There is yet no micro-evidence about the functional form of land costs $v(x_{it})$, so we adopt here a conservative approach by imposing the functional form: $v(x_{it}) = \frac{\tau}{\phi} x_{it}^\phi$ where $\tau \geq 0$ and $\phi \geq 0$. For $\phi \rightarrow 0$, land productivity exhibits constant return, while for $\phi > 0$ land costs exhibits increasing returns. The parameter τ allows here to pin down the amount of *per capita* land in the deterministic steady state.

⁵⁰See Hane et al. (1984) for an evaluation of the relationship between water used by crops at various growth stages.

⁵¹Cropping costs consist of charges for fertilizers, seeds and chemicals; for pasture these costs concern fence and watering equipment; while for animal production costs, these include purchased feed and bedding as well as medical costs.

The law of motion of physical capital in the agricultural sector is given by:

$$i_{it}^A = k_{it}^A - (1 - \delta_K) k_{it-1}^A, \quad (\text{III.5})$$

where $\delta_K \in [0, 1]$ is the depreciation rate of physical capital and i_{it}^A is investment of the representative farmer.

Real profits d_{it}^A of the farmer are given by:

$$d_{it}^A = p_t^A y_{it}^A - p_t^N \left(i_{it}^A + S \left(\varepsilon_t^i \frac{i_{it}^A}{i_{it-1}^A} \right) i_{it-1}^A \right) - w_t^A h_{it}^A - p_t^N x_{it}, \quad (\text{III.6})$$

where $p_t^A = P_t^A/P_t$ is the relative production price of agricultural goods, the function $S(x) = 0.5\kappa(x-1)^2$ is the convex cost function as in [Christiano et al. \(2005\)](#) which features a hump-shaped response of investment consistently with VAR models, and ε_t^i is an investment cost shock making investment growth more expensive. It follows an $AR(1)$ shock process:

$$\log(\varepsilon_t^I) = \rho_I \log(\varepsilon_{t-1}^I) + \sigma_I \eta_t^I, \quad (\text{III.7})$$

where $\rho_I \in [0, 1)$ denotes the root of the $AR(1)$, and $\sigma_I \geq 0$ the standard deviation of the innovation.

We assume that a representative farmer is a price taker. The profit maximization he or she faces can be cast as choosing the input levels under land efficiency and capital law of motions as well as technology constraint:

$$\max_{\{h_{it}^A, i_{it}^A, k_{it}^A, \ell_{it}, x_{it}\}} E_t \left\{ \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} d_{it+\tau}^A \right\}, \quad (\text{III.8})$$

where E_t denotes the expectation operator and $\Lambda_{t,t+\tau}$ is the household stochastic discount factor between t and $t + \tau$.

The original equation that is worth commenting is the optimal demand for intermediate expenditures:

$$\frac{p_t^N}{v'(x_{it}) \ell_{it-1} \Omega(\varepsilon_t^W)} = E_t \left\{ \Lambda_{t,t+1} \left(\omega \frac{y_{it+1}^A}{\ell_{it}} + \frac{p_{t+1}^N}{v'(x_{it+1}) \ell_{it}} \left[(1 - \delta_\ell) + v(x_{it+1}) \right] \right) \right\}. \quad (\text{III.9})$$

The left-hand side of the equation captures the current marginal cost of land maintenance, while the right-hand side corresponds to the sum of the marginal product of land productivity with the value of land in the next period. A weather shock deteriorates the expected marginal benefit of lands and rise the current cost of land maintenance. The shape of the cost function $v(x_{it})$ critically determines the response of agricultural production following a drought shock. A concave cost function, i.e., $v''(x_{it}) < 0$, would generate a negative response of land expenditures and a decline in the relative price of agricultural goods, which would be inconsistent with the VAR model. Therefore, a linear or convex cost function with $\phi \geq 0$ is preferred to feature an increase in spending x_{it} following an adverse weather shock. A second reason motivating increasing returns is the stability of land productivity dynamics: if a farmer decreases her land maintenance expenditures when land productivity is already low, this further deteriorates land productivity to reach zero.

III.2.2 HOUSEHOLDS

There is a continuum $j \in [0, 1]$ of identical households that consume, save and work in the two production sectors. The representative household maximizes the welfare index expressed as the expected sum of utilities discounted by $\beta \in [0, 1]$:

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\frac{1}{1-\sigma} (C_{jt+\tau} - bC_{t-1+\tau})^{1-\sigma} - \frac{\chi \varepsilon_{t+\tau}^H}{1+\sigma_H} h_{jt+\tau}^{1+\sigma_H} \right], \quad (\text{III.10})$$

where the variable C_{jt} is the consumption index, $b \in [0, 1)$ is a parameter that accounts for external consumption habits, h_{jt} is a labor effort index for the agricultural and non-agricultural sectors, and $\sigma > 0$ and $\sigma_H > 0$ represent consumption aversion and labor disutility coefficients, respectively. Labor supply is affected by a shift parameter $\chi > 0$ pinning down the steady state of hours worked and a labor supply $AR(1)$ shock ε_t^H that makes hours worked more costly in terms of welfare.

Following [Horvath \(2000\)](#), we introduce imperfect substitutability of labor supply between the agricultural and non-agricultural sectors to explain co-movements at the sector level by defining a CES labor disutility index:

$$h_{jt} = \left[(h_{jt}^N)^{1+\iota} + (h_{jt}^A)^{1+\iota} \right]^{1/(1+\iota)}. \quad (\text{III.11})$$

The labor disutility index consists of hours worked in the non-agricultural sector h_{jt}^N and agriculture sector h_{jt}^A . Reallocating labor across sectors is costly and is governed by the substitutability parameter $\iota \geq 0$. If ι equals zero, hours worked across the two sectors are perfect substitutes, leading to a negative correlation between the sectors that is not consistent with the data. Positive values of ι capture some degree of sector specificity and imply that relative hours respond less to sectoral wage differentials.

Expressed in real terms and dividing by the consumption price index P_t , the budget constraint for the representative household can be represented as:

$$\sum_{s=N,A} w_t^s h_{jt}^s + r_{t-1} b_{jt-1} + rer_t^* r_{t-1}^* b_{jt-1}^* - T_t \geq C_{jt} + b_{jt} + rer_t^* b_{jt}^* + p_t^N rer_t \Phi(b_{jt}^*). \quad (\text{III.12})$$

The income of the representative household is made up of labor income with a real wage w_t^s in each sector s ($s = N$ for the non-agricultural sector, and $s = A$ for the agricultural one), real risk-free domestic bonds b_{jt} , and foreign bonds b_{jt}^* . Domestic and foreign bonds are remunerated at a domestic rate r_{t-1} and a foreign rate r_{t-1}^* , respectively. Household's foreign bond purchases are affected by the foreign real exchange rate rer_t^* (an increase in rer_t^* can be interpreted as an appreciation of the foreign currency). The real exchange rate is computed from the nominal exchange rate e_t^* adjusted by the ratio between foreign and home price, $rer_t^* = e_t^* P_t^* / P_t$. In addition, the government charges lump sum taxes, denoted T_t . The household's expenditure side includes its consumption basket C_{jt} , bonds and risk-premium cost $\Phi(b_{jt}^*) = 0.5 \chi_B (b_{jt}^*)^2$ paid in terms of domestic non-agricultural goods at a relative mar-

ket price $p_t^N = P_t^N/P_t$.⁵² The parameter $\chi_B > 0$ denotes the magnitude of the cost paid by domestic households when purchasing foreign bonds.

We now discuss the allocation of consumption between non-agricultural/agricultural goods and home/foreign goods. First, the representative household allocates total consumption C_{jt} between two types of consumption goods produced by the non-agricultural and agricultural sectors denoted C_{jt}^N and C_{jt}^A , respectively. The CES consumption bundle is determined by:

$$C_{jt} = \left[(1 - \varphi)^{\frac{1}{\mu}} (C_{jt}^N)^{\frac{\mu-1}{\mu}} + (\varphi)^{\frac{1}{\mu}} (C_{jt}^A)^{\frac{\mu-1}{\mu}} \right]^{\frac{\mu}{\mu-1}}, \quad (\text{III.13})$$

where $\mu \geq 0$ denotes the substitution elasticity between the two types of consumption goods, and $\varphi \in [0, 1]$ is the fraction of agricultural goods in the household's total consumption basket. The corresponding consumption price index P_t reads as follows: $P_t = [(1 - \varphi)(P_{C,t}^N)^{1-\mu} + \varphi(P_{C,t}^A)^{1-\mu}]^{\frac{1}{1-\mu}}$, where $P_{C,t}^N$ and $P_{C,t}^A$ are consumption price indexes of non-agricultural and agricultural goods, respectively.

Second, each index C_{jt}^N and C_{jt}^A is also a composite consumption subindex composed of domestically and foreign produced goods:

$$C_{jt}^s = \left[(1 - \alpha_s)^{\frac{1}{\mu_s}} (c_{jt}^s)^{\frac{(\mu_s-1)}{\mu_s}} + (\alpha_s)^{\frac{1}{\mu_s}} (c_{jt}^{s*})^{\frac{(\mu_s-1)}{\mu_s}} \right]^{\frac{\mu_s}{\mu_s-1}} \text{ for } s = N, A \quad (\text{III.14})$$

where $1 - \alpha_s \geq 0.5$ denotes the home bias, i.e., the fraction of home-produced goods, while $\mu_s > 0$ is the elasticity of substitution between home and foreign goods. In this context, the consumption price indexes $P_{C,t}^s$ in each sector s are given by: $P_{C,t}^s = [(1 - \alpha_s)(P_t^s)^{1-\mu_s} + \alpha_s(e_t^* P_t^{s*})^{1-\mu_s}]^{\frac{1}{1-\mu_s}}$, for $s = N, A$. In this expression, P_t^s is the production price index of domestically produced goods in sector s , while P_t^{s*} is the price of foreign goods in sector s .

Finally, demand for each type of good is a fraction of the total consumption index adjusted by its relative price:

$$C_{jt}^N = (1 - \varphi) \left(\frac{P_{C,t}^N}{P_t} \right)^{-\mu} C_{jt} \text{ and } C_{jt}^A = \varphi \left(\frac{P_{C,t}^A}{P_t} \right)^{-\mu} C_{jt}, \quad (\text{III.15})$$

$$c_{jt}^s = (1 - \alpha_s) \left(\frac{P_t^s}{P_{C,t}^s} \right)^{-\mu_s} C_{jt}^s \text{ and } c_{jt}^{s*} = \alpha_s \left(\frac{e_t^* P_t^{s*}}{P_{C,t}^s} \right)^{-\mu_s} C_{jt}^s \text{ for } s = N, A. \quad (\text{III.16})$$

III.2.3 NON-AGRICULTURAL SECTOR

There exists a continuum of perfectly competitive non-agricultural firms indexed by $i \in [1, n_t]$, with $1-n_t$ denoting the relative size of the non-agricultural sector in the total production of the economy. These firms are similar to agricultural firms except in their technology as they do not require land inputs to produce goods and are not directly

⁵²This cost function aims at removing a unit root component that emerges in open economy models without affecting the steady state of the model. We refer to [Schmitt-Grohé and Uribe \(2003\)](#) for a discussion of closing open economy models.

affected by weather. Each representative non-agricultural firm has the following Cobb-Douglas technology:

$$y_{it}^N = \varepsilon_t^Z (k_{it-1}^N)^\alpha (h_{it}^N)^{1-\alpha}, \quad (\text{III.17})$$

where y_{it}^N is the production of the i^{th} intermediate goods firms that combines physical capital k_{it-1}^N , labor demand h_{it}^N and technology ε_t^Z . The parameters α and $\alpha - 1$ represent the output elasticity of capital and labor, respectively. Technology is characterized as an $AR(1)$ shock process:

$$\log(\varepsilon_t^Z) = \rho_Z \log(\varepsilon_{t-1}^Z) + \sigma_Z \eta_t^Z, \quad (\text{III.18})$$

where $\rho_Z \in [0, 1)$ denotes the $AR(1)$ term in the technological shock process and $\sigma_Z \geq 0$ the standard deviation of the shock. Technology is assumed to be economy-wide (i.e., the same across sectors) by affecting both agricultural and non-agricultural sectors. This shock captures fluctuations associated with declining hours worked coupled with increasing output.⁵³

The law of motion of physical capital in the non-agricultural sector is given by:

$$i_{it}^N = k_{it}^N - (1 - \delta_K) k_{it-1}^N, \quad (\text{III.19})$$

where $\delta_K \in [0, 1]$ is the depreciation rate of physical capital and i_{it}^N is investment from non-agricultural firms.

Real profits are given by:

$$d_{it}^N = p_t^N y_{it}^N - p_t^N \left(i_{it}^N + S \left(\varepsilon_t^i \frac{i_{it}^N}{i_{it-1}^N} \right) i_{it-1}^N \right) - w_t^N h_{it}^N, \quad (\text{III.20})$$

Firms maximize the discounted sum of profits:

$$\max_{\{h_{it}^N, i_{it}^N, k_{it}^N\}} E_t \left\{ \sum_{\tau=0}^{\infty} \Lambda_{t,t+\tau} d_{it+\tau}^N \right\}. \quad (\text{III.21})$$

under technology and capital accumulation constraints.

III.2.4 AUTHORITY

The public authority consumes some non-agricultural output G_t , issues debt b_t at a real interest rate r_t and charges lump sum taxes T_t . Public spending is assumed to be exogenous, $G_t = Y_t^N g \varepsilon_t^G$, where $g \in [0, 1)$ is a fixed fraction of non-agricultural goods g affected by a standard $AR(1)$ stochastic shock:

$$\log(\varepsilon_t^G) = \rho_G \log(\varepsilon_{t-1}^G) + \sigma_G \eta_t^G, \quad \eta_t^G \sim \mathcal{N}(0, 1), \quad (\text{III.22})$$

where $1 > \rho_G \geq 0$ and $\sigma_G \geq 0$. This shock captures variations in absorption which are not taken into account in our setup such as political cycles and international demand in intermediate markets.

⁵³The lack of sectoral data for hours worked does not allow to directly measure sector-specific TFP shocks.

The government budget constraint equates spending plus interest payment on existing debt to new debt issuance and taxes:

$$G_t + r_{t-1}b_{t-1} = b_t + T_t. \quad (\text{III.23})$$

III.2.5 FOREIGN ECONOMY

Following the literature on estimated small open economy models exemplified by [Adolfson et al. \(2007\)](#), [Adolfson et al. \(2008\)](#) and [Justiniano and Preston \(2010b\)](#), our foreign economy boils down to a small set of key equations that determine New Zealand exports and real exchange rate dynamics. The foreign country is determined by an endowment economy characterized by an exogenous foreign consumption:⁵⁴

$$\log(c_{jt}^*) = (1 - \rho_C) \log(\bar{c}_j^*) + \rho_C \log(c_{jt-1}^*) + \sigma_C \eta_t^C, \quad \eta_t^C \sim \mathcal{N}(0, 1), \quad (\text{III.24})$$

where the $0 \leq \rho_C < 1$ is the root of the process, $\bar{c}_j^* > 0$ is the steady state foreign consumption and $\sigma_C \geq 0$ is the standard deviation of the shock. The parameters σ_C and ρ_C are estimated in the fit exercise to capture variations of the foreign demand. A rise in the demand triggers a boost in the exportation of New Zealand goods, followed by an appreciation of the foreign exchange rate.

Each period, foreign households solve the following optimization scheme:

$$\max_{\{c_{jt}^*, b_{jt}^*\}} E_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \varepsilon_{t+\tau}^E \log(c_{jt+\tau}^*) \right\}, \quad (\text{III.25})$$

$$s.t. \quad r_{t-1}^* b_{jt-1}^* = c_{jt}^* + b_{jt}^*. \quad (\text{III.26})$$

where variable ε_t^E is a time-preference shock defined as follows:

$$\log(\varepsilon_t^E) = \rho_E \log(\varepsilon_{t-1}^E) + \sigma_E \eta_t^E, \quad (\text{III.27})$$

with $\eta_t^E \sim \mathcal{N}(0, 1)$. This shock temporarily raises the household's discount factor and drives down the foreign real interest rate and naturally leads capital to flow to New Zealand. Regarding the budget constraint, it comprises consumption and domestic bonds purchase, the latter are remunerated at a predetermined real rate r_{t-1}^* . In absence of specific sectoral shocks, all sectoral prices of the foreign economy are perfectly synchronized, i.e., $P_t^* = P_t^{A*} = P_t^{N*}$. In addition, the small size of the domestic economy implies that the import/exports flows from the home to the foreign country are negligible, thus implying that $P_t^* = P_{C,t}^{A*} = P_{C,t}^{N*}$.

⁵⁴For simplicity, our foreign economy boils down to an endowment economy à la [Lucas Jr \(1978\)](#) in an open economy setup where consumption is exogenous. Most of the parameters and the steady states are symmetric between domestic and the foreign economy. Consistently with the restricted VAR model featuring a small open economy, the foreign economy is only affected by its own consumption shocks but not by shocks of the home economy.

III.2.6 AGGREGATION AND EQUILIBRIUM CONDITIONS

After aggregating all agents and varieties in the economy and imposing market clearing on all markets, the standard general equilibrium conditions of the model can be deduced.

First, the market clearing condition for non-agricultural goods is determined when the aggregate supply is equal to aggregate demand:

$$(1 - n_t) Y_t^N = (1 - \varphi) \left[(1 - \alpha_N) \left(\frac{P_t^N}{P_{C,t}^N} \right)^{-\mu_N} \left(\frac{P_{C,t}^N}{P_t} \right)^{-\mu} C_t + \alpha_N \left(\frac{1}{e_t^*} \frac{P_t^N}{P_{C,t}^{N*}} \right)^{-\mu_N} \left(\frac{P_{C,t}^{N*}}{P_t^*} \right)^{-\mu} C_t^* \right] + G_t + I_t + n_t x_t + \Phi(b_t^*), \quad (\text{III.28})$$

where the total supply of home non-agricultural goods is given by $\int_{n_t}^1 y_{it}^N di = (1 - n_t) Y_t^N$, and total demands from both the home and the foreign economy read as $\int_0^1 c_{jt} dj = C_t$ and $\int_0^1 c_{jt}^* dj = C_t^*$, respectively, with $1 - \alpha_N$ and α_N the fraction of home and foreign home-produced non-agricultural goods, respectively. Aggregate investment, with $\int_{n_t}^1 i_{it}^N di = (1 - n_t) I_t^N$ and $\int_0^{n_t} i_{it}^A di = n_t I_t^A$, is given by: $I_t = (1 - n_t) I_t^N + n_t I_t^A$. Turning to the labor market, the market clearing condition between household labor supply and demand from firms in each sector is $\int_0^1 h_{jt}^N dj = \int_{n_t}^1 h_{it}^N di$ and $\int_0^1 h_{jt}^A dj = \int_0^{n_t} h_{it}^A di$. This allows us to write the total number of hours worked: $H_t = (1 - n_t) H_t^N + n_t H_t^A$. Aggregate real production is given by:

$$Y_t = (1 - n_t) p_t^N Y_t^N + n_t p_t^A Y_t^A.$$

In addition, the equilibrium of the agricultural goods market is given by:

$$n_t Y_t^A = \varphi \left[(1 - \alpha_A) \left(\frac{P_t^A}{P_{C,t}^A} \right)^{-\mu_A} \left(\frac{P_{C,t}^A}{P_t} \right)^{-\mu} C_t + \alpha_A \left(\frac{1}{e_t^*} \frac{P_t^A}{P_{C,t}^{A*}} \right)^{-\mu_A} \left(\frac{P_{C,t}^{A*}}{P_t^*} \right)^{-\mu} C_t^* \right], \quad (\text{III.29})$$

where $\int_0^{n_t} y_{it}^A di = n_t Y_t^A$. In this equation, the left side denotes the aggregate production, while the right side denotes respectively demands from home and foreign (i.e., imports) households.

Given the presence of intermediate inputs, the GDP is given by:

$$gdp_t = Y_t - p_t^N n_t X_t. \quad (\text{III.30})$$

The law of motion for the total amount of real foreign debt is:

$$b_t^* = r_{t-1}^* \frac{rer_t^*}{rer_{t-1}^*} b_{t-1}^* + tb_t, \quad (\text{III.31})$$

where tb_t is the real trade balance that can be expressed as follows:

$$tb_t = p_t^N [(1 - n_t) Y_t^N - G_t - I_t - n_t X_t - \Phi(b_t^*)] + p_t^A n_t Y_t^A - C_t. \quad (\text{III.32})$$

The general equilibrium condition is defined as a sequence of quantities $\{Q_t\}_{t=0}^{\infty}$

and prices $\{\mathcal{P}_t\}_{t=0}^{\infty}$ such that for a given sequence of quantities $\{\mathcal{Q}_t\}_{t=0}^{\infty}$ and the realization of shocks $\{\mathcal{S}_t\}_{t=0}^{\infty}$, the sequence $\{\mathcal{P}_t\}_{t=0}^{\infty}$ guarantees simultaneous equilibrium in all markets previously defined.

III.3 ESTIMATION

The model is estimated using Bayesian methods and quarterly data for New Zealand. We estimate the structural parameters and the sequence of shocks following the seminal contributions of [Smets and Wouters \(2007\)](#) and [An and Schorfheide \(2007\)](#). In a nutshell, a Bayesian approach can be followed by combining the likelihood function with prior distributions for the parameters of the model to form the posterior density function. The posterior distributions are drawn through the Metropolis-Hastings sampling method. We solve the model using a linear approximation to the model's policy function, and employ the Kalman filter to form the likelihood function and compute the sequence of errors. For a detailed description, we refer the reader to the original papers.

III.3.1 DATA

The Bayesian estimation relies on the same sample as the one used by the VAR model over the sample period 1994Q2 to 2016Q4.⁵⁵ Therefore, each observable variable is composed of 91 observations. The dataset includes 8 times series: output, consumption, investment, hours worked, agricultural production, foreign production, variations of the real effective exchange rate and the drought index.

Concerning the transformation of the series, the point is to map non-stationary data to a stationary model. Observable variables that are known to have a trend (namely here, output, investment and foreign output) are made stationary in three steps. First, they are divided by the working age population. Second, they are taken in logs. And third, they are detrended using a quadratic trend. We thus choose to neglect the low frequency component (i.e., the trend) in all empirical variables for two main reasons: (i) the sample employed here is too short to observe any trend effects on the weather making the use of trend on the weather irrelevant;⁵⁶ (ii) dealing with trends in open economy models is challenging when economies are not growing at the same rate, the solution adopted in estimated open economy models is simply to neglect trends as in [Justiniano and Preston \(2010b\)](#). For hours worked, the correction method of [Smets and Wouters \(2007\)](#) is applied: it consists of multiplying the number of paid hours by the employment rate. Finally, turning to the weather index, daily data from weather stations are collected and then spatially and temporally aggregated to compute an index of soil moisture for each local state composing New Zealand.⁵⁷

⁵⁵Series for world output and hours worked for the period 1989-Q2 and 1993-Q4 are not available. This incomplete sub-sample is, however, used to initialise the Kalman filter. Only time periods after the presample enter the actual likelihood computations.

⁵⁶In the IAM literature, the time horizon considered is usually higher than 100 years, which allows to measure long-terms effects from trends.

⁵⁷The index is computed following [Kamber et al. \(2013\)](#). More details are provided in the online appendix.

The local values of the index are then aggregated at the national level by means of a weighted mean, where the weights are chosen according to the relative size of the agricultural output in each state. The resulting index is, by construction, zero mean.

The vector of observable is given by:

$$\mathcal{Y}_t^{obs} = 100 \times [\hat{y}_t, \hat{c}_t, \hat{i}_t, \hat{h}_t, \hat{y}_t^A, \hat{y}_t^*, \Delta \widehat{rer}_t, \hat{\omega}_t]', \quad (\text{III.33})$$

where \hat{y}_t is the output gap, \hat{c}_t is the consumption gap, \hat{i}_t is the investment gap, \hat{h}_t is an index of hours worked, \hat{y}_t^A is the agricultural production gap, \hat{y}_t^* is the foreign production gap, \widehat{rer}_t is New Zealand real exchange rate and finally $\hat{\omega}_t$ is the drought index.

The corresponding measurement equations are given by:

$$\mathcal{Y}_t = 100 \times [\widehat{gdp}_t, \tilde{C}_t, \tilde{p}_t^N + \tilde{I}_t, \tilde{H}_t, \tilde{n}_t + \tilde{p}_t^A + \tilde{Y}_t^A, \tilde{C}_t^*, -\Delta \widetilde{rer}_{t+1}^*, \tilde{\varepsilon}_t^W]', \quad (\text{III.34})$$

where all these variables are expressed in percentage deviations from their steady state: $\tilde{x}_t = \log(x_t/\bar{x})$. Note that in the model, the real exchange rate corresponds to the price of the foreign currency, we thus take the minus of the growth rate of the real exchange rate to get the real exchange rate of New Zealand.

III.3.2 CALIBRATION AND PRIOR DISTRIBUTIONS

Table III.2 summarizes the calibration of the model. We fix a small number of parameters that are commonly used in the literature of real business cycle models, including $\beta=0.9883$, the discount factor; $\bar{H}^N = \bar{H}^A = 1/3$, the steady state share of hours worked per day; $\delta_K=0.025$, the depreciation rate of physical capital; $\alpha=0.33$, the capital share in the technology of firms; and $g=0.22$, the share of spending in GDP.

The portfolio adjustment cost of foreign debt is taken from **Schmitt-Grohé and Uribe (2003)**, with $\chi_B = 0.0007$.⁵⁸ The current account is balanced in steady state assuming $\bar{b}^* = \bar{ca} = 0$. Regarding the openness of the goods market, our calibration is strongly inspired by **Lubik (2006)**, with a share α_N of exported non-agricultural goods set to 25% and to 45% for agricultural goods α_A in order to match the observed trade-to-GDP ratio of New Zealand. Turning to agricultural sector, the share of agricultural goods in the consumption basket of households is set to $\varphi = 15\%$, as observed over the sample period. In addition, the land-to-employment ratio $\bar{\ell}=0.4$ is based on the hectares of arable land (hectares per person) in New Zealand (FAO data provided by the World Bank).

The rest of the parameters are estimated using Bayesian methods. **Table III.3** and **Figure III.5** report the prior (and posterior) distributions of the parameters for New Zealand. Overall, our prior distributions are either relatively diffuse or consistent with earlier contributions to Bayesian estimations such as **Smets and Wouters (2007)**. In particular, priors for the persistence of the $AR(1)$ processes, the labor disutility curvature σ_H , the consumption habits b and the investment adjustment cost κ are directly

⁵⁸The value of this parameter marginally affects dynamics of endogenous variables, but it allows to remove an unit root component induced by the open economy setup.

taken from [Smets and Wouters \(2007\)](#).⁵⁹ The standard errors of the innovations are assumed to follow a Weibull distribution with a mean of 1 and a standard deviation of 2. The Weibull distribution is more diffuse than the Inverse Gamma distribution (both type 1 and 2), has a positive support and provides a better fit in terms of data density. Substitution parameters μ , μ_N , and μ_A are each assumed to follow a Gamma distribution with a mean of 2 and a standard deviation of 1 in order to have a support that lies between 0 and 5. The risk aversion parameter σ_C is assumed to follow a Normal distribution with a mean of 2 and a standard deviation of 0.35 in the same vein as [Smets and Wouters \(2007\)](#). The labor sectoral cost ι follows a diffuse Gaussian distribution with a mean of 1 and a standard deviation of 0.75, as the literature of two-sector models suggests that this parameter is above zero to get a positive correlation link across sectors.

Regarding priors for the agricultural sector, the land efficiency decay rate parameter δ_ℓ is assumed to follow a Beta distribution with prior mean and standard deviation of 0.2 and 0.07, respectively. This prior is rather uninformative as it allows this decay rate to be either close to 0 or 0.40, the latter would imply an annual decay rate of 180%. Regarding the land share in the production function ω , first, under decreasing return this parameter must be below 1, second, the economic literature suggests that this parameter is close to 20%,⁶⁰ we thus impose accordingly a Beta distribution on ω with a mean of 0.2 and standard deviation 0.12 to allow this parameter to be either close to 0 or 0.50. The land cost parameter ϕ is also assumed to follow a diffuse Gaussian distribution with prior mean and standard deviation both set to 1, so that the response of output is consistent with that of the VAR model. One of the key parameters in the paper is the damage function parameter θ that is possibly subject to controversy. The literature on IAMs traditionally connects temperatures to output through a simple quadratic damage function in order to provide an estimation of future costs of carbon emissions on output. However, [Pindyck \(2017\)](#) raised important concerns about IAM-based outcome as modelers have so much freedom in choosing a functional form as well as the values of the parameters so that the model can be used to provide any result one desires. To avoid the legitimate criticisms inherent to IAMs, we adopt here a conservative approach on the value of this key parameter of the damage function and set a very diffuse prior with a uniform distribution with zero mean and standard deviation 500. This very flat prior only allows the data be informative about the posterior distribution of this parameter.

III.3.3 POSTERIOR DISTRIBUTIONS

In addition to the prior distributions, [Table III.3](#) reports the estimation results that summarize the means and the 5th and 95th percentiles of the posterior distributions,

⁵⁹Note that for any shock process, we divide by 100 the standard deviation of the stochastic disturbance in order to use the same prior distribution as [Smets and Wouters \(2007\)](#) for the estimated standard deviation: $\log(\varepsilon_t^m) = \rho_m \log(\varepsilon_{t-1}^m) + \frac{\sigma_m}{100} \eta_t^m$ with $m = \{Z, G, I, H, W, N, C, E\}$.

⁶⁰The share of land ω in the production function is estimated at 15% for the Canadian economy by [Echevarría \(1998\)](#), while [Restuccia et al. \(2008\)](#) calibrates this parameter 18% for the US economy.

Variable	Interpretation	Value
β	Discount factor	0.9883
δ_K	Capital depreciation rate	0.025
α	Share of capital in output	0.33
g	Share of spending in GDP	0.22
φ	Share of agricultural goods in consumption basket	0.15
$\bar{H}^N = \bar{H}^A$	Hours worked	1/3
$\bar{\ell}$	Land per capita	0.40
α_N	Openness of non-agricultural market	0.25
α_A	Openness of agricultural market	0.45
χ_B	International portfolio cost	0.0007

Table III.2: Calibrated parameters on a quarterly basis.

		Prior distributions			Posterior distribution	
		Shape	Mean	Std.	Mean [5%:95%]	
SHOCK PROCESS $AR(1)$						
Economy-wide TFP (SD)	$\sigma_Z \times 100$	\mathcal{W}	1	2	2.09	[1.77:2.39]
Hours supply (SD)	$\sigma_H \times 100$	\mathcal{W}	1	2	6.13	[4.78:7.45]
Spending (SD)	$\sigma_G \times 100$	\mathcal{W}	1	2	4.00	[3.49:4.49]
Investment (SD)	$\sigma_I \times 100$	\mathcal{W}	1	2	6.19	[4.85:7.48]
Sector reallocation (SD)	$\sigma_N \times 100$	\mathcal{W}	1	2	8.85	[6.88:10.69]
Weather (SD)	$\sigma_W \times 100$	\mathcal{W}	1	2	0.81	[0.71:0.91]
Foreign time-preference (SD)	$\sigma_E \times 100$	\mathcal{W}	1	2	5.33	[4.47:6.10]
Foreign consumption (SD)	$\sigma_C \times 100$	\mathcal{W}	1	2	0.69	[0.6:0.77]
Economy-wide TFP (AR term)	ρ_Z	\mathcal{B}	0.5	0.2	0.33	[0.18:0.47]
Labor supply (AR term)	ρ_H	\mathcal{B}	0.5	0.2	0.88	[0.82:0.94]
Spending (AR term)	ρ_G	\mathcal{B}	0.5	0.2	0.85	[0.79:0.91]
Investment (AR term)	ρ_I	\mathcal{B}	0.5	0.2	0.40	[0.23:0.56]
Sector reallocation (AR term)	ρ_N	\mathcal{B}	0.5	0.2	0.85	[0.79:0.92]
Weather (AR term)	ρ_W	\mathcal{B}	0.5	0.2	0.38	[0.24:0.52]
Foreign time-preference (AR term)	ρ_E	\mathcal{B}	0.5	0.2	0.23	[0.08:0.37]
Foreign consumption (AR term)	ρ_C	\mathcal{B}	0.5	0.2	0.95	[0.91:0.98]
STRUCTURAL PARAMETERS						
Risk consumption	σ_C	\mathcal{N}	2	0.35	1.27	[0.89:1.65]
Labor disutility	σ_H	\mathcal{N}	2	0.75	4.27	[3.33:5.20]
Land expenditure cost	ϕ	\mathcal{N}	1	1	2.57	[1.64:3.46]
Share of land in agricultural output	ω	\mathcal{B}	0.2	0.1	0.12	[0.03:0.19]
Consumption habits	b	\mathcal{B}	0.7	0.1	0.59	[0.47:0.71]
Labor sectoral cost	ι	\mathcal{N}	1	0.75	2.85	[2.00:3.66]
Substitutability by type of goods	μ	\mathcal{G}	2	1	5.96	[4.05:7.74]
Substitutability home/foreign	μ_A	\mathcal{G}	2	1	1.16	[0.83:1.49]
Substitutability home/foreign	μ_N	\mathcal{G}	2	1	0.78	[0.65:0.92]
Land efficiency decay rate	$\delta_{\bar{\ell}}$	\mathcal{B}	0.2	0.07	0.05	[0.03:0.08]
Investment cost	κ	\mathcal{N}	4	1.5	2.44	[1.41:3.57]
Land-weather elasticity	θ	\mathcal{U}	0	500	20.59	[5.34:36.19]
Marginal log-likelihood					-1443.84	

Notes: The column entitled ‘‘Shape’’ indicates the prior distributions using the following acronyms: \mathcal{N} describes a normal distribution, \mathcal{G} a Gamma, \mathcal{U} an Uniform, \mathcal{B} a Beta, and \mathcal{W} a Weibull.

Table III.3: Prior and posterior distributions of structural parameters and shock processes.

while the latter are illustrated in Figure III.5.⁶¹ According to Figure III.5, the data were fairly informative, as their posterior distributions did not stay very close to their priors. However, we assess the identification of our parameters using methods developed by Iskrev (2010), these identification methods show that sufficient and necessary conditions for local identification are fulfilled by our estimated model.

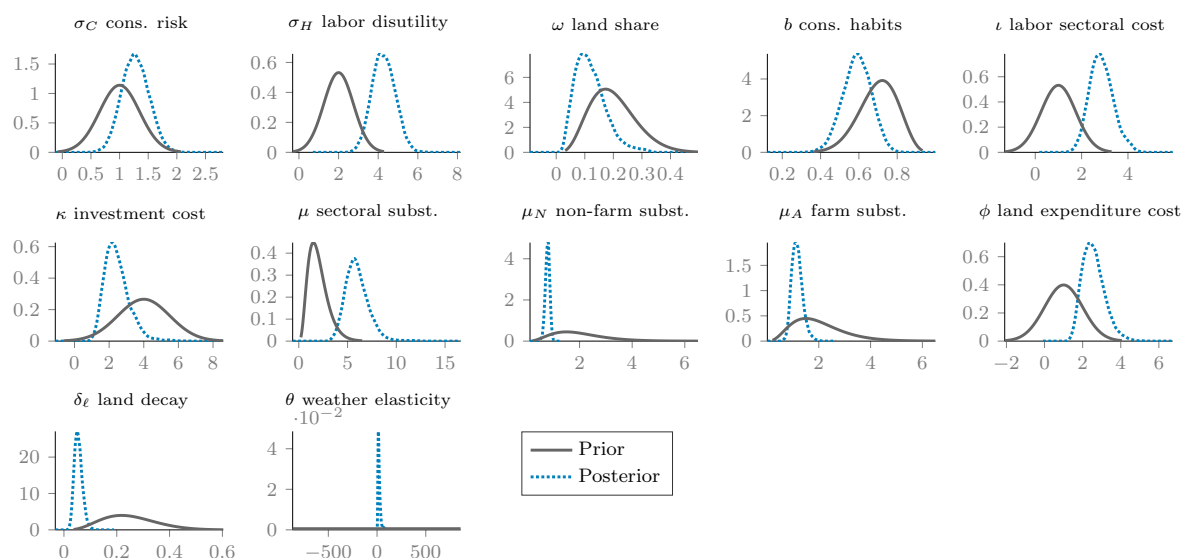


Figure III.5: Prior and posterior distributions of structural parameters for New Zealand (excluding shocks).

While our estimates of the standard parameters are in line with the business cycle literature (see, for instance, Smets and Wouters (2007) for the US economy or Lubik (2006) for New Zealand), several observations are worth making regarding the means of the posterior distributions of structural parameters. Strikingly, the land-weather elasticity parameter θ has a high posterior value that is clearly different from 0. This implies that even with loose priors, the model suggests that variable weather conditions matter for generating business cycles consistently with empirical evidence of Kamber et al. (2013) and Mejia et al. (2018). The land expenditure cost ϕ suggests that the returns to scale for land expenditures lies between quadratic and cubic functional form. Substitution seems to be an important pattern of consumption decisions of households, especially at a sectoral level. However, the substitution between home and foreign goods appears to be rather low for both sectors. Regarding the labor reallocation parameter ι in the utility function of households, the data favor a costly labor reallocation across sectors, which is in line with the findings of Iacoviello and Neri (2010) for the housing market.

Notice that the value of θ may appear critically high, and thus could question the relevance of the damage function and its resulting interpretation of the weather cycles

⁶¹The posterior distribution combines the likelihood function with prior information. To calculate the posterior distribution to evaluate the marginal likelihood of the model, the Metropolis-Hastings algorithm is employed. We compute the posterior moments of the parameters using a total generated sample of 800,000, discarding the first 80,000, and based on eight parallel chains. The scale factor was set in order to deliver acceptance rates close to 24%. Convergence was assessed by means of the multivariate convergence statistics taken from Brooks and Gelman (1998). We estimate the model using the dynare package Adjemian et al. (2011).

on economic activity. To gauge the relative magnitude of the damage function on land productivity, we first examine the unconditional mean of the damage function. Recall that the weather volatility from [Table III.3](#) is $\sigma_W = 0.81/100$. Therefore since all random shocks are Gaussian, the unconditional mean of weather shocks is given by: $E(\varepsilon_t^W) = e^{0+0.5\sigma_W^2/(1-\rho_W^2)} = 1.000038$. The resulting average damage, $\Omega(E(\varepsilon_t^W)) = 0.99921$, is actually reasonable: weather shocks through the damage function actually generate a permanent loss to land productivity that is below 0.0008%. Second, we also examine the contemporaneous effects of a weather shock on land productivity. The realization of an average weather shock (i.e., $\sigma_W\eta_t^W = 0.81$) slashes land productivity by 15% *ceteris paribus*, while the realization of an extreme drought (i.e., $\sigma_W\eta_t^W = 1.5$) cuts land productivity by 26% through the damage function. These results are rather reasonable as an extreme drought would directly reduce agricultural production by 3.6%.⁶² We thus conclude that a high value for θ is necessary to map the changes in the weather index and agricultural production cycles.

To assess how well the estimated model captures the main features of the data, we report in [Table III.4](#) and [Table III.5](#) both the moments simulated by the model and their empirical counterpart. First, the model does a reasonably job through its steady state ratios in replicating the observed mean. The model performs quite well in terms of volatility for most of observable variables, except for total output and consumption as both are clearly overstated by the model while the theoretical volatility of foreign output is understated. The model performs very well at replicating the persistence of all observable variables. Finally regarding the correlation with GDP, the model replicates the sign of all the correlations, but not their full magnitude. In particular, the correlation with the foreign GDP is not captured by the model, this is a well known puzzle in international economic that can be easily solved by imposing a positive correlation across shocks in the model's covariance matrix.

III.4 DO WEATHER SHOCKS MATTER?

A natural question to ask is whether weather shocks significantly explain part of the business cycle. To provide an answer to this question, two versions of the model are estimated – using the same data and priors. In an alternative version of the model, which we consider as a benchmark, the damage function given in [Equation III.3](#) is neutralized by imposing $\theta = 0$. Under this assumption, any fluctuation in the weather has no implication for agriculture and thus does not generate any business cycles. In contrast, we compare the benchmark model with the version presented previously in the model section, characterized by the presence of weather-driven business cycles with $\theta \neq 0$.

[Table III.6](#) reports for the two models the corresponding data density (Laplace approximation), posterior odds ratio and posteriors model probabilities, which allow us to determine the model that best fits the data from a statistical standpoint. Using a uninformative prior distribution over models (i.e., 50% prior probability for each

⁶²An extreme drought materializes in the model as: $\varepsilon_t^W = 1 + 1.5/100$, the damage on land productivity is given by $\Omega(\varepsilon_t^W) = .7343$, the direct effect on agricultural production is given by $\Omega(\varepsilon_t^W)^\omega = 0.9636$. These calculations neglect previous realizations of weather shocks (and any other disturbances), and neglect any reactions from farmers in terms of labor demand and land expenditures.

model), we compute both posterior odds ratios and model probabilities taking the model $\mathcal{M}(\theta = 0)$, i.e., the one with no weather damage as the benchmark.⁶³ We conduct a formal comparison between models and refer to Geweke (1999) for a presentation of the method to perform the standard Bayesian model comparison employed in Table III.6 for our two models. Briefly, one should favor a model whose data density, posterior odds ratio and model probability are the highest compared to other models.

We examine the hypothesis $H_0: \theta = 0$ against the hypothesis $H_1: \theta \neq 0$. To do this, we evaluate the posterior odds ratio of $M(\theta \neq 0)$ on $M(\theta = 0)$ using Laplace-approximated marginal data densities. The posterior odds of the null hypothesis of no significance of weather-driven fluctuations is 118.97:1 which leads us to strongly reject the null, i.e., weather shocks do matter in explaining the business cycles of New Zealand. This result is confirmed in terms of log marginal likelihood and posterior odds ratio. This is an important result from the model that highlights the non-trivial role of the weather in driving the business cycles of New Zealand.

III.5 WEATHER SHOCKS AS DRIVERS OF AGGREGATE FLUCTUATIONS

This section discusses the propagation of a weather shock and its implications in terms of business cycle statistics.

III.5.1 PROPAGATION OF A WEATHER SHOCK

We first report the simulated Bayesian system's responses of the main macroeconomic variables following a standard weather shock in Figure III.6.⁶⁴ We also report the responses from the SVAR estimation for observable variables which are common between the SVAR and the DSGE model. Unlike the SVAR model, the DSGE model provides the underlying micro-founded mechanisms that drives the propagation of a weather shock.

From a business cycle perspective, this shock acts as a standard (sectoral) negative supply shock through a combination of rising hours worked and falling output. Consistently with the SVAR model, a drought event strongly affects business cycles through a large decline in agricultural output (1.5%), as the weather influences land input in the production process of agricultural goods. Land productivity is strongly negatively affected by the drought. This result is in line with Kamber et al. (2013), as New Zealand's farmers rely extensively on rainfall and pastures to support the agricultural sector. A drought shock decreases land productivity by 22% in the model. To compensate for this loss, farmers can use more non-agricultural goods as inputs to reestablish their land productivity. For instance, dairy or crop producers may require more water to irrigate their grasslands or cultures to offset the dryness. Farmers may

⁶³As underlined by Rabanal (2007), it is important to stress that the marginal likelihood already takes into account that the size of the parameter space for different models can be different. Hence, more complicated models will not necessarily rank better than simpler models, and $\mathcal{M}(\theta \neq 0)$ will not inevitably be favored to the benchmark model.

⁶⁴The impulse response functions (IRFs) and their 90% highest posterior density intervals are obtained in a standard way when parameters are drawn from the mean posterior distribution, as reported in Figure III.5.

Variable	Interpretation	Model	Data
\bar{C}/\bar{Y}	Ratio of consumption to output	0.55	0.57
\bar{I}/\bar{Y}	Ratio of investment to output	0.23	0.22
$400 \times (\bar{r} - 1)$	Real interest rate	4.72	4.75
$(1 - \varphi) \alpha_N + \varphi \alpha_A$	Goods market openness	0.28	0.29
$n\bar{Y}^A/\bar{Y}$	Ratio of agricultural production to GDP	0.08	0.07

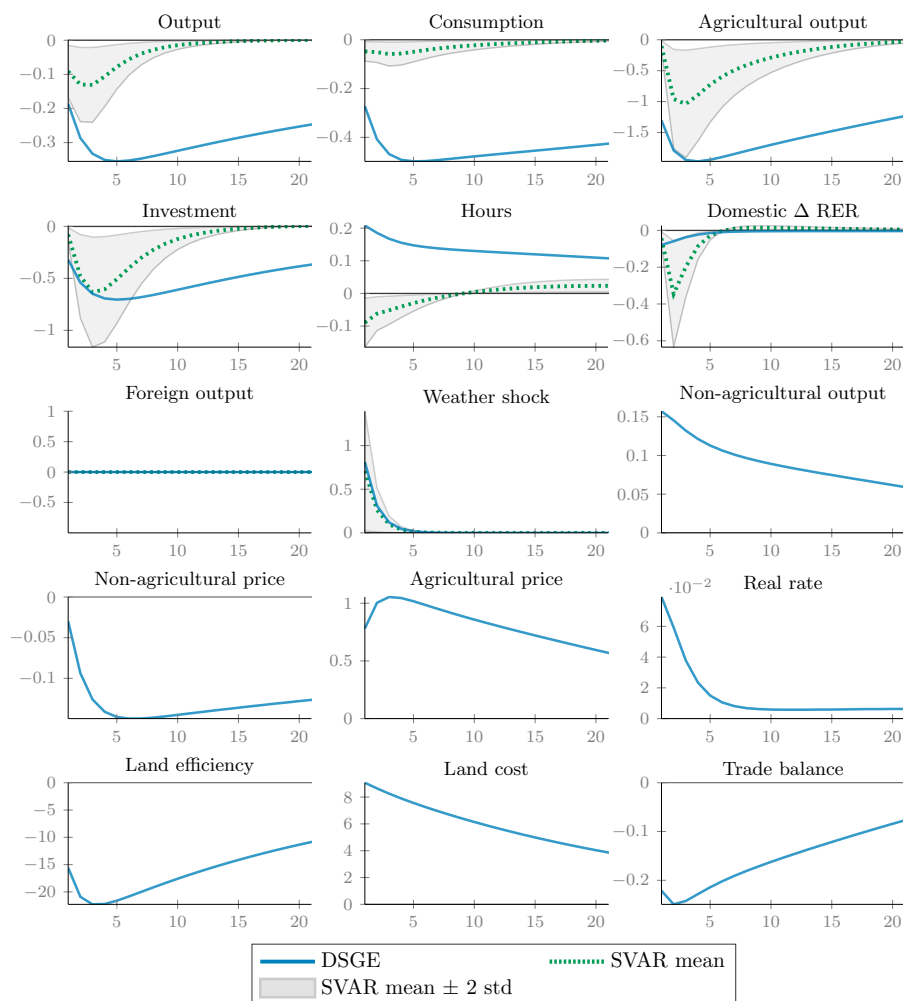
Table III.4: Steady state ratios (empirical ratios are computed using data between 1990 to 2017).

	Standard Deviation		Autocorrelation		Correlation w/ output	
	Model	Data	Model	Data	Model	Data
Total output	3.18	[2.38;3.19]	0.89	[0.92;0.96]	1.00	[1.00;1.00]
Consumption	4.75	[2.18;2.92]	0.93	[0.85;0.93]	0.69	[0.60;0.80]
Hours	2.75	[2.45;3.28]	0.84	[0.95;0.98]	0.24	[-0.15;0.26]
Investment	9.63	[10.4;14.0]	0.88	[0.91;0.96]	0.71	[0.55;0.78]
Agricultural output	13.50	[11.6;15.6]	0.91	[0.88;0.95]	0.53	[0.22;0.56]
Foreign output	2.11	[3.01;4.03]	0.95	[0.97;0.99]	0.15	[0.51;0.75]
RER variations	3.36	[3.14;4.22]	0.27	[0.06;0.45]	0.03	[-0.14;0.27]
Weather	0.88	[0.75;1.01]	0.38	[0.18;0.54]	-0.11	[-0.22;0.19]

Table III.5: Comparison of theoretical business cycles moments with their empirical counterpart at a 95% confidence interval.

Model type	$\mathcal{M}(\theta = 0)$	$\mathcal{M}(\theta \neq 0)$
Model description	No Weather Damage Model	Weather-Driven Business Cycles
Damage function $\Omega(\varepsilon_t^W)$	1	$(\varepsilon_t^W)^{-\theta}$
Prior probability	1/2	1/2
Laplace approximation	-1449.597	-1443.841
Posterior odds ratio	1.000000	118.9675
Posterior model probability	0.003151	0.996849

Table III.6: Prior and posterior model probabilities



Notes: Blue lines are the Impulse Response Functions (IRFs) generated when parameters are drawn from the mean posterior distribution, as reported in Figure III.5. IRFs are reported in percentage deviations from the deterministic steady state. Dotted green lines are the means of the distributions of the Impulse Response Functions (IRFs) of the SVAR model and gray areas are their 90 confidence intervals.

Figure III.6: System response to an estimated weather shock η_t^W for the estimated DSGE and SVAR model (when available).

also use more pesticides, as droughts are often followed by pest outbreaks (Gerard et al., 2013). The demand effect for these non-agriculture goods is captured in the model by a rise in inputs x_{it} in Equation III.4, which results in a dramatic increase in land costs. The surge in non-agriculture goods has a positive side effect on non-agriculture output. Both the drop in the agricultural production and the rise in non-agriculture output alter the sectoral price structure. As the drought causes a reduction in the agricultural production and a rise in land costs, the relative price in the agricultural sector rises through a market clearing effect. Since relative prices are negatively correlated across sectors, the price of non-agricultural goods declines in response, thus fueling the demand for non-agricultural goods. With respect to the SVAR model, the DSGE model predicts a higher contraction of economic activity η_t combined with a weaker response of the real exchange rate. In addition, the SVAR model predicts a weaker response of consumption following a weather shock while for labor supplies, a drought induces a reduction in hours worked followed by a persistent increase.

From an international standpoint, the decline in domestic agricultural production generates trade balance deficits. Two factors might explain this. First, around fifty percent of New Zealand's merchandise exports are accounted for by agricultural commodities over the sample period. As both output and price competitiveness of the agricultural sector are deteriorated, New Zealand exports decline. However, the decline price in relative price of non-agricultural fuels the external demand for non-agricultural, thus explaining why this sector experiences a boom. Taken together, the effect of the agricultural sector outweighs the other sector, through a fall in the trade balance and the current account. In the meantime, the domestic real exchange rate depreciates driven by the depressed competitiveness of farmers, which helps in restoring their competitiveness. This reaction of the exchange rate is consistent with the prediction of the SVAR model in Figure III.3.

III.5.2 THE CONTRIBUTION OF WEATHER SHOCKS ON AGGREGATE FLUCTUATIONS

Figure III.7 reports the forecast error variance decomposition for four observable variables, i.e., aggregate real production (gdp_t), real agricultural production (Y_t^A), real consumption (C_t) and hours worked (H_t). Five different time horizons are considered, ranging from two quarters ($Q2$), to ten ($Q10$) and fifty quarters ($Q50$) along with the unconditional forecast error variance decomposition ($Q\infty$). In each case, the variance is decomposed into four main components related to supply shocks (technology, labor supply and sectoral reallocation shock), demand shocks (government spending, household preferences and investment shocks), foreign shocks (consumption and foreign preferences), and obviously the weather shocks.

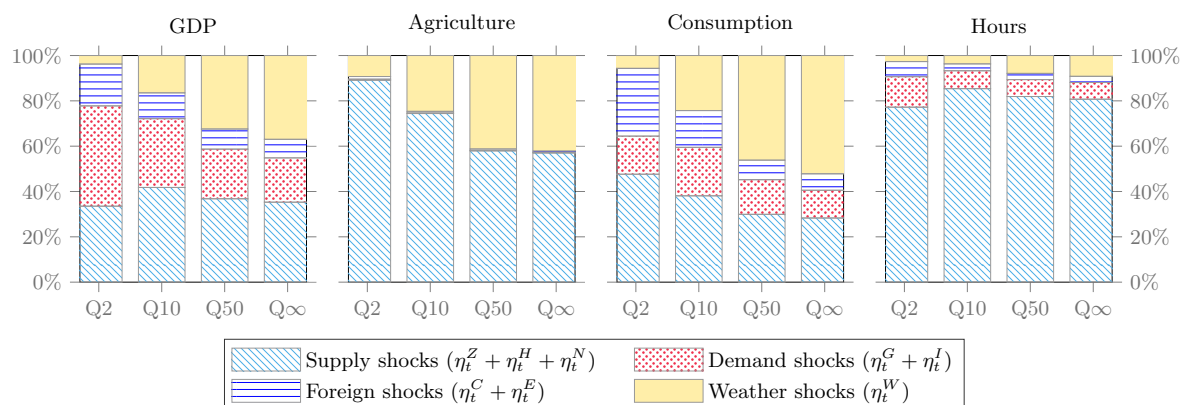


Figure III.7: Forecast error variance decomposition at the posterior mean for different time horizons (one, ten, forty and unconditional) for four observable variables.

For GDP (gdp_t), supply shocks are the main drivers of the variance in both the short and the long term, followed by demand and foreign shocks. Interestingly, we find that foreign shocks are a sizable driving force of output in the short run by contributing up to 18% of the volatility of GDP. Unlike Justiniano and Preston (2010a) who find

a trivial contribution of foreign shock in small open economy models, our model is able to capture the key role of foreign shock as a driver of economic fluctuations. Foreign shocks play a non-negligible role: they account for 18.4% of New Zealand's production in the short run, and 8.1% in the long run. By increasing the time horizon, the contribution of demand and foreign shocks tends to reduce and are gradually replaced by weather shocks, starting from 3.7% at two-quarter horizon to 35% for the unconditional variance.

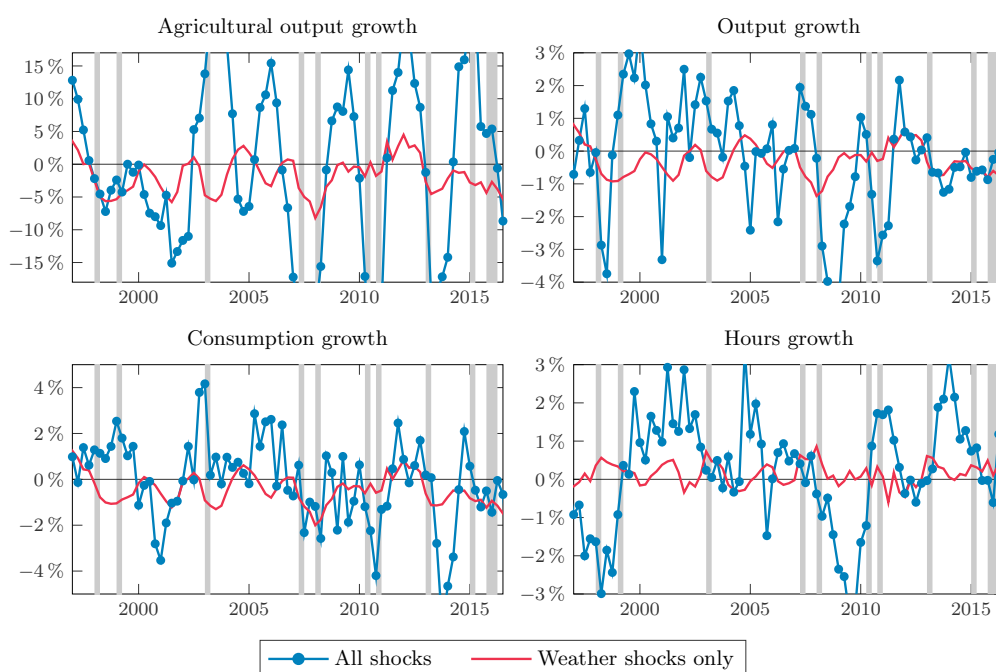
Turning to agricultural production, supply shocks account for most fluctuations in the short run. They are responsible for 89% of the variance of agricultural production at two-quarter horizon. Domestic and foreign demand shocks play a trivial role in the volatility of agricultural production. The importance of supply shocks declines in the long run, although remaining non-negligible, explaining 57% of agricultural production for the unconditional variance. Weather shocks remarkably drive the variance of agricultural production after a time lag of two quarters. In addition, increasing the time horizon magnifies this result. Thus the weather is a key determining factor of agricultural fluctuations according to the theoretical representation of the data by our model. Concerning the variance of consumption, it is mainly affected, in the short term, by foreign shocks. Weather shocks play a significant role in the same way as for agricultural production, starting from a more distant time horizon. Finally for working hours, they are only slightly affected by weather shocks. Supply shocks are the main drivers of the variance of hours worked as they drive most of the variance of hours.

Overall, we find that weather shocks cause important macroeconomic fluctuations. The increasing contribution of the weather in the time horizon highlights an interesting persistence mechanism which can be associated to the weather hysteresis effects discussed in the business cycle evidence section.

III.5.3 HISTORICAL DECOMPOSITION OF BUSINESS CYCLES

An important question one can ask of the estimated model is how important were weather shocks in shaping the recent New Zealand macroeconomic experience. [Figure III.8](#) displays the year-over-year growth rate in per capital of real agricultural production, GDP, consumption and hours worked. The blue dotted line is the result of simulating our model's response to all of the estimated shocks and to the initial conditions. The dotted line shows the result of this same simulation when we feed our model only the weather shock.

A notable feature of agricultural production is the important contribution of the weather to its fluctuations. More specifically, this weather contribution oscillates between +4% and -6% over the sample period. During periods of good soil moisture, land productivity is enhanced, which fuels the higher supply of agricultural goods. In contrast, drought periods are associated with lower levels of agricultural output. Severe droughts coincide with a sharp drop in agricultural production driven by the weather shock. In particular, one fourth of agricultural slowdown following the most severe drought in 2008 is strikingly accounted by the weather shock. In 2016, a prolonged episode of drought also contributed by 5% to the contraction of the agricultural supply.



Notes: All data are demeaned. Blue line and red lines are annual growth rates of selected observable variables. The blue line results of feeding the model with all shocks (i.e., the actual data), while the red line results of feeding the model only with the weather shock. The red line depicts the contribution of the weather shock to the corresponding deviation. Shaded area indicates the 10th percent of the most severe drought episodes, as inferred from the time series of the weather index.

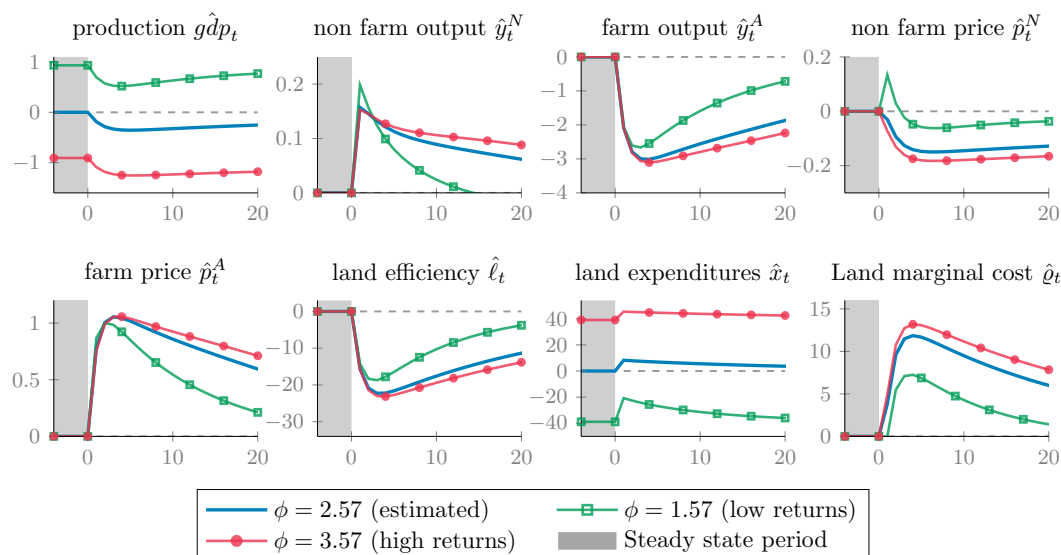
Figure III.8: The role of weather shocks on selected variables.

The weather contribution is not limited to the supply of agricultural goods, the remaining panels in **Figure III.8** show that real output, consumption and hours growth rates are also affected by the weather, but the absolute contribution is on average lower than for agricultural production. For GDP and consumption, the weather's contribution to the growth rate of these variables oscillates between +2% to -2%. Regarding the labor market, the model suggests that weather-driven changes in aggregate labor demand oscillates between -0.5 to 0.9% on the sample period. There is an overall clear spillover mechanism from the agricultural sector to the rest of the economy, which allows the weather to propagate and generate business cycles. Weather-driven fluctuations in agriculture are translated to other selected variables and contribute to their fluctuations. Severe droughts also have important implications for these variables, as the 2008 and 2016 droughts entailed a joint 1% contraction in GDP and consumption while labor supplied increased by 1%.

III.6 INSPECTING THE PROPAGATION MECHANISM

The originality of the model lies in the introduction of a weather-dependent agricultural sector that relies on a set of structural parameters driving the response of the economy following a weather shock. In this section, we investigate how critical these parameters are by contrasting the responses of the model under different calibrations

for three key parameters: the land expenditure cost ϕ , the labor sectoral cost ι , and the land efficiency decay rate δ_ℓ . Each parameter is likely to affect both the propagation and the steady state of the model. To disentangle the short run from the long run, we draw the steady state of the model prior to the realization of the shock in $t = 1$. All the IRFs are expressed in percentage deviations from the steady state of the estimated model.

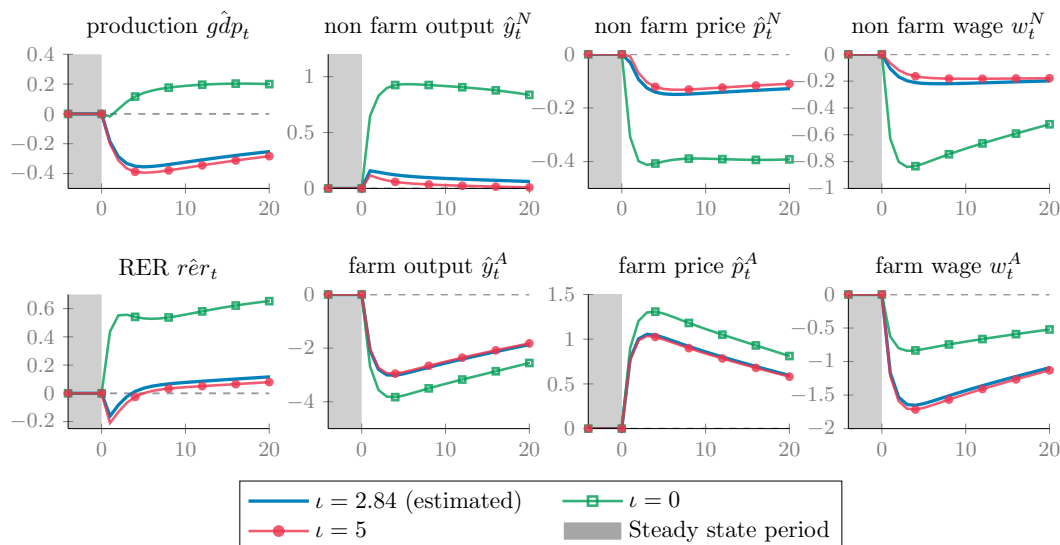


Notes: IRFs are expressed in percentage deviations of the estimated model's steady state. Prior to period 1 (shaded area), the model is at its deterministic steady state. The weather shock occurs at $t = 1$.

Figure III.9: Impulse response functions (in percentage deviations from steady state of the estimated model) for different values of the land expenditure cost ϕ following a weather shock in $t=1$.

We first consider the parameter ϕ shaping the land cost function (see Equation III.6). This cost function critically determines the marginal cost of rising the land productivity. Recall that a stability condition for land productivity dynamics imposes that $\phi > 1/(1 - \delta_\ell)$, as a result this condition does not allow to examine dynamics under decreasing and constant returns to scale. In Figure III.9, we thus contrast the IRFs from the model using the estimated cost curvature ($\phi = 2.57$) with lower ($\phi = 1.57$) and higher ($\phi = 3.57$) increasing returns. The value of this parameter clearly affects the propagation mechanism of a weather shock. Under highly increasing returns, the marginal cost of land costs (e.g., fertilizers and water) rises after a drought, while it tends to be less responsive with a lower ϕ . The main implication of lower returns lies in the response of the agricultural sector, through a positive spike of its relative price generating a strong recession in this sector, before quickly adjusting back to steady state. This relative price distortion across sectors clearly reshapes the response of the non-agricultural sector and total production by creating a quick recession that is not consistent with empirical evidence of the SVAR model. The steady state of the model is also affected. A rise in ϕ increases land expenditures, since the latter are accounted as intermediate consumption, a increase in land expenditures mechanically reduces the GDP (through Equation III.30).

We next turn to the labor substitutability parameter ι from the labor disutility index

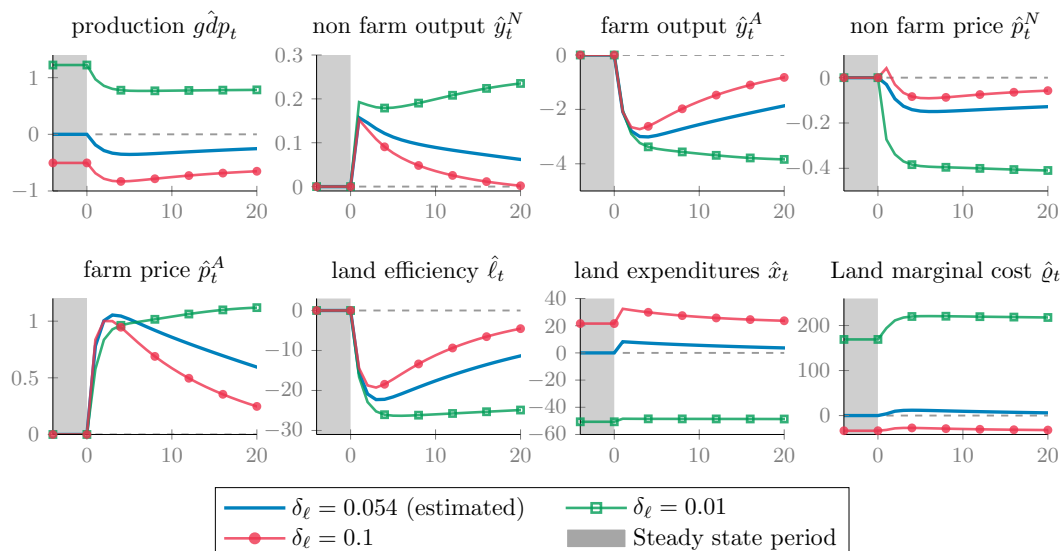


Notes: IRFs are expressed in percentage deviations of the estimated model's steady state. Prior to period 1 (shaded area), the model is at its deterministic steady state. The weather shock occurs at $t = 1$.

Figure III.10: Impulse response functions (in percentage deviations from steady state) following a weather shock for various degrees of labor substitution across sectors $\iota = 0, 2.84$ and 5 .

(Equation III.11). This parameter determines the household labor supply substitution across sector, we thus contrast a situation with a perfect labor mobility across sectors at a business cycle frequency versus a very costly substitution. Figure III.10 reports the IRFs under a linear substitution index ($\iota = 0$) versus the estimated value ($\iota = 2.9$) and a high substitution cost ($\iota = 5$). When $\iota = 0$, households face no cost of adjusting their labor supply to sectoral wages differentials so that during a weather event, the households increase their labor supply in the non-agricultural sector as the equilibrium wage is higher in this sector. Labor supply is thus flowing to the sector with the highest wage, thus boosting the non-agricultural one. At a macro level, the perfect reallocation generates a strong negative correlation link between sector, and translates into an expansion of the economy. This propagation mechanism is clearly at odd with the SVAR model. In contrast, the increase in the cost of labor reallocation reduces this substitution mechanism and amplifies the recession. The steady state, however, is not affected by this parameter.

Finally, we investigate how the rate of decay of land productivity, denoted δ_ℓ (see Equation III.4), shapes the responses of the model by contrasting 3 different calibration from low to high decay rates. Figure III.11 reports the corresponding IRFs. This parameter determines the hysteresis effect of the weather by ruling how quickly the land (and thus the economy) returns to its steady state following a drought shock. For a low value of the decay rate, macroeconomic fluctuations are amplified and more persistent, as land productivity requires more time to recover from a drought. Conversely, a higher value reduces the persistence, but mechanically increases the steady state intermediate expenditures in land productivity.



Notes: IRFs are expressed in percentage deviations of the estimated model's steady state. Prior to period 1 (shaded area), the model is at its deterministic steady state. The weather shock occurs at $t = 1$.

Figure III.11: Impulse response functions (in percentage deviations from steady state) following a weather shock for various decay rates of land efficiency $\delta_\ell = 0.01, 0.054$ and 0.10 .

III.7 CLIMATE CHANGE IMPLICATIONS

We now turn to the implications of climate change for aggregate fluctuations and welfare. The IPCC defines climate change as “a change in the state of the climate that can be identified (e.g., by using statistical tests) by changes in the mean and/or the variability of its properties, and that persists for an extended period, typically decades or longer” (IPCC, 2014). In our framework, climate is supposed to be stationary, which makes our setup irrelevant for analyzing changes in mean weather values. However unlike standard models, our framework is able to tackle the implications of a change in the variance of weather shocks. As a first step, we assess the change in the variance of the weather shock by estimating it under different climate scenarios. Then, in a second step, we use the estimates of these variances for each scenario and investigate the effects on aggregate fluctuations.

III.7.1 CLIMATE CHANGE AND MACROECONOMIC VOLATILITY

We use the estimated DSGE model to assess the effects of a shift in the variability of the weather shock process. We do so in a two-step procedure. First, the simulations are estimated with the value of the variance of the weather shock that is estimated during the fit exercise, which corresponds to historical variability. Second, new simulations are made after altering the variability of the weather shock so it corresponds to the one associated with climate change (we refer to the online appendix for a description of the method determining the change in weather variability). Hence, we proceed to four different alterations of the variance of the weather process.

To measure the implications of climate change on aggregate fluctuations of a repre-

sentative open economy, we compare the volatility of some macroeconomic variables under historical weather conditions (for the 1989–2014 period) to their counterpart under future climate scenarios (for the 2015–2100 period), normalizing the values of the historical period of each variable to 100. [Table III.7](#) report these variations for some key variables.

The first scenario, with regard to the volatility of the weather shock for New Zealand is clearly optimistic, as the variance of drought events is declining by 8.03%. As a result, macroeconomic fluctuations in the country naturally decrease. Agriculture output is particularly affected by this structural change, with a 3.39% decrease of its variance. In contrast, the other scenario for which the rise in variance of the weather shock ranges between 14.11% for the less pessimistic scenario to 51.91% for the most pessimistic one, exhibit a strong increase in the volatility of macroeconomic variables. As a matter of facts, the variance of total output rises by 5.22% under the RCP 4.5 scenario, and by 19.19% under the RCP 8.5 scenario. Agricultural production volatility experiences an important shift of 21.88% under the worst-case scenario. We also observe a dramatic increase in the variance of consumption of 27.09%, relative price of agricultural goods of 16.8%, net foreign asset of 31.86%. The variance of the current account rises by 11.45% while the variance of the real exchange rate rises by 7.54%. For the remaining macroeconomic variables, the changes are relatively smaller.

We therefore find some important changes in the volatility of key macroeconomic variables induced by climate change, which could be very critical, especially for developing economies. [Wheeler and Von Braun \(2013\)](#) find similar effects of climate change on crop productivity which could have strong consequences for food availability for low-income countries. Adapting our setup to a developing economy by increasing the relative share of the agricultural sector, and reducing the intensity of the capital, would critically exacerbate the results reported in [Table III.7](#).

III.7.2 THE WELFARE COST OF WEATHER-DRIVEN BUSINESS CYCLES UNDER CLIMATE CHANGE

To get a welfare perspective on climate change, we compute how much consumption households are willing to abandon to live in an economy free of weather shocks. We compute the path of the economy contrasting two regimes using a second order approximation to the welfare index to obtain an accurate of the welfare cost.⁶⁵ The regime *a* is free of weather shocks (i.e., $\sigma_W = 0$ in [Equation III.1](#)) while regime *b* includes weather shocks as estimated in the fit exercise. We introduce λ as the fraction of consumption that the household would be willing to give up to live in the regime *a* rather than the *b*. Put differently λ denotes the welfare cost of weather shocks and is computed as:

$$E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \mathcal{U} \left((1 - \lambda) [C_{t+\tau}^a - bC_{t-1+\tau}^a], h_{t+\tau}^a \right) = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \mathcal{U} \left(C_{t+\tau}^b - bC_{t-1+\tau}^b, h_{t+\tau}^b \right). \quad (\text{III.35})$$

The last two rows of [Table III.7](#) report the corresponding welfare mean and cost

⁶⁵See the online appendix for the closed-form expression of the welfare index up to second order.

		1994-2016	2100 (projections)			
		Historical	RCP 2.5	RCP 4.5	RCP 6.0	RCP 8.5
$\text{Var}(\eta_t^W)$	Weather shock	100	91.97	114.11	119.44	151.91
$\text{Var}(gdp_t)$	GDP	100	97.03	105.22	107.19	119.19
$\text{Var}(C_t)$	Consumption	100	95.81	107.36	110.15	127.09
$\text{Var}(p_t^N I_t)$	Investment	100	99.24	101.34	101.85	104.94
$\text{Var}(p_t^A Y_t^A)$	Agriculture	100	96.61	105.95	108.20	121.88
$\text{Var}(p_t^A)$	Agricultural price	100	97.40	104.56	106.29	116.80
$\text{Var}(H_t)$	Hours	100	99.26	101.29	101.78	104.77
$\text{Var}(R_t)$	Real interest rate	100	99.99	100.01	100.02	100.05
$\text{Var}(rer_t)$	Exchange rate	100	98.83	102.05	102.82	107.54
$\text{Var}(tb_t)$	Trade balance	100	98.23	103.11	104.29	111.45
$\text{Var}(b_t^*)$	Net Foreign Asset	100	95.31	108.24	111.36	130.32
$E(\mathcal{N}_t)$	Welfare	-624.4944	-624.4629	-624.5497	-624.5707	-624.6980
λ (%)	Welfare cost	0.3035	0.2791	0.3464	0.3626	0.4613

Notes: The model is first simulated as described in section III.3. Theoretical variances of each variable are then estimated and normalized to 100. Then, variances of weather (η_t^W) shocks are modified to reflect different climate scenarios (compared to the reference 1994–2016 period, changes in the volatilities are as follows: RCP 2.5, -8.03% ; RCP 4.5, $+14.11\%$; RCP 6.0, $+19.44\%$; RCP 8.5, $+51.91\%$). New simulations are estimated using the modified variances of these shocks, and the theoretical variances of the variables of interest are then compared to those of the reference period.

Table III.7: Changes in variances of simulated observables under climate change scenarios.

computed under alternative scenarios. First of all, the simulations show that today, New Zealanders would be willing to give up to 0.3% of their unconditional consumption in order to live in an economy free of droughts. The magnitude of this cost is not negligible, as our model evaluates the welfare costs of business cycles induced by productivity shocks to 0.08%, 0.05% for spending shocks, 0.06% for investment shocks, 0.75% for labor supply shocks, 0.12% for sector reallocation shock, 0.003% for foreign consumption shock and 0.06% for foreign discount factor.⁶⁶ In the literature of uncertainty in macroeconomics, the welfare cost of business cycles is typically low under a CRRA utility specification (e.g., Lucas (1987, chap. 3) and Lucas (2003, section II)). This literature usually concludes that business cycles induce trivial welfare costs. However in presence of weather shocks, we find that the welfare cost of the weather is non-trivial, even with a variant of a CRRA utility function.⁶⁷ This conflicting result with the standard macroeconomic literature is directly connected to the weather hysteresis effect: when an adverse weather shock deteriorates land productivity, agricultural output is low for an extended period of time as livestock and crops need time to recover. The resulting consequence is an higher uncertainty for households on their agricultural consumption which naturally drives the welfare cost of business cycles. By disabling the persistence mechanism of land productivity

⁶⁶On average, these costs lie in the ballpark of estimates obtained in the RBC literature, see for example Otrok (2001) except for the labor supply shock. The latter generates important welfare costs as it directly affects utility function.

⁶⁷In this paper, the utility function is not exactly the same as Lucas (1987, chap. 3) as it also features consumption habits. However by disabling consumption habits b , the welfare cost of the weather remains high up to 0.08% in the baseline scenario.

in Equation III.4, the welfare cost of business cycles becomes trivial by representing 0.009% of permanent consumption. The magnitude of these results can be contrasted with those of Donadelli et al. (2017) who consider temperature shocks and who find an even larger welfare cost peaking to 18.1%.

We approximate climate change by increasing the variance of weather shocks. The results from this exercise are illustrative as we do not account for crop and livestock adaptation. Therefore, these costs can be interpreted as a maximum bound of the feasible welfare costs. In all our scenarios except for the optimistic RCP 2.5, households would be worse off under the new weather conditions in which the volatility of droughts has increased. Under the optimistic scenario, they would only abandon only 0.30% of their permanent consumption. In the worst-case scenario, this fraction would reach 0.46%. With respect to the benchmark situation over the 1994-2016 period, the welfare cost increased by 0.09, from 0.19 for the historical period to 0.28% for the worst-case scenario. This suggests that there is a strong non-linear relationship between the variance of the shock and the welfare cost as exemplified by Donadelli et al. (2017) for temperature shocks.

III.8 CONCLUSION

In this paper, we have investigated how the weather can play an autonomous role in generating business cycles. We have developed and estimated a DSGE model for a small open economy, New Zealand. Our model includes an agricultural sector that faces exogenous weather variations affecting land productivity, and in turn the production of agricultural goods. We find from a statistical standpoint that weather shocks do matter in explaining the business cycles of New Zealand. Both the SVAR and the DSGE models find that a weather shock generates a recession through a contraction of agricultural production and investment combined with a rise in hours worked. Our business cycle decomposition exercises also show that weather shocks are an important driver of agricultural production and, in a smaller proportion, of the GDP. Finally, we use our model to the analysis of climate change by increasing the variance of weather shocks consistently with projections in 2100. The rise in the variability of weather events leads to an increase in the variability of key macroeconomic variables, such as output, agricultural production or the real exchange rate. In addition, we find important welfare costs incurred by weather-driven business cycles, as today households are willing to pay 0.30% of their unconditional consumption to live in a world with no weather shocks, and this cost is increasing in the variability of weather events.

The analysis of weather-driven business cycles is a burgeoning research area given the important context of climate change. In this paper, we have analyzed the importance of weather shocks on the macroeconomic fluctuations of a developed economy. However, the application of our framework to developing countries could highlight the high vulnerability of their primary sectors to weather shocks. In addition, from a policymaker's perspective, our framework could be fruitfully employed to evaluate the optimal conduct of monetary policy to mitigate the destabilizing effects of weather shocks for different scenarios of climate change. Fiscal policy could also play a role in a low-income country, for instance by providing disaster payments, which may be

seen as insurance schemes paid by the tax payers. These disaster payments may make sense in the absence of well-functioning insurance markets. Another possibility could be the introduction of trends in the model, which could be affected by weather events both in the short and in the long run. This would provide a scope for crop adaptation and environmental policies aiming at offsetting the welfare costs of weather. Finally, weather shocks could also have implications for financial markets, through a possible rise in the equity premium as predicted by the risk disaster theory in asset pricing.

Part IV

Green asset pricing

This is a joint work with

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INTRODUCTION

Current evidence shows that the mean temperature is 1 degree higher than it was in the pre-industrial era. In recent years, this increase in temperature has accelerated and temperatures are currently estimated to rise by about 0.2 degrees per decade.⁶⁸ The link between carbon-dioxide emissions (CO₂) and climate change is by now clearly established. CO₂ emissions are about 20 times higher than they were at the beginning of the 20th century. Moreover, evidence from Antarctic ice cores shows that CO₂ emissions have not only risen rapidly, current levels are also the highest in over 400,000 years.⁶⁹

CO₂ emissions are not only a low-frequency phenomenon, they also exhibit large cyclical fluctuations. A decomposition between trend and cyclical components reveals that CO₂ emissions are procyclical and more volatile than GDP (e.g. [Doda, 2014](#); [Heutel, 2012](#)). Against the background of the ongoing debate over emission taxes, these large cyclical fluctuations raise several important questions. In particular, are these strong cyclical fluctuations desirable from a welfare perspective? And how should the optimal carbon tax vary over the business cycle?

This paper addresses these questions by considering the optimal carbon tax in the presence of an environmental externality. The novelty of our approach is to investigate the link between asset-pricing theory—in particular the stochastic discount factor (SDF)—and climate policies. The SDF is a key building block of modern asset-pricing theory (e.g. [Cochrane, 2011](#)). Our main contention is that it also has a critical impact on the design of the optimal carbon tax.

Following [Stokey \(1998\)](#), [Acemoglu et al. \(2012\)](#) and [Golosov et al. \(2014b\)](#), among others, environmental considerations are captured by introducing an externality into the utility function. Apart from a few exceptions (see for instance [Michel and Rotillon, 1995](#)), most papers in this literature use a separable specification that implies no direct link between the environment and the marginal utility of consumption. Our innovation is to study a model in which the presence of an environmental externality raises households' willingness to consume goods.

⁶⁸[Pachauri et al. \(2014\)](#).

⁶⁹The Economist (2019). "Briefing Climate Change", Sept. 21st-27th.

Our approach can be motivated by the effect of climate change on consumption. As documented by [Abel et al. \(2018\)](#) and [Mansur et al. \(2008\)](#), one perverse effect of climate change is to increase the use of electricity. Higher levels of emissions cause climate change, which in turn increases the need to consume electricity to cool homes. This complementarity between climate change and consumption can be illustrated by the exponential increase in the use of air-conditioning in recent decades.⁷⁰ Projections by the International Energy Agency also suggest that this is only the beginning, as the demand for air-conditioning is expected to triple by 2050.⁷¹ This latter result is consistent with the US findings in [He et al. \(2020\)](#). Using data for a large sample of consumers, they show that pollution, which is highly correlated with CO2 emissions, increases electricity consumption.

Apart from electricity consumption, there is evidence that emissions also increase other types of expenditure. [Deschênes et al. \(2017\)](#) show that air pollution increases the consumption of medical products. There is moreover evidence that emissions raise the demand for goods that are used to mitigate the effect of pollution, such as air purifiers (e.g. [Ito and Zhang, 2020](#)). Climate change also increases investment in adaptation measures (e.g. [Fried, 2019](#); [Gourio and Fries, 2020](#)).

Overall, the evidence therefore suggests the existence of a compensation effect of climate change (e.g. [Michel and Rotillon, 1995](#)). As Greenhouse Gas emissions rise, the need to consume electricity as well as other goods to mitigate the effect of climate change becomes more pressing. In other words, the presence of environmental externalities could raise the marginal utility of consumption.

From a finance perspective, this non-separability between consumption and the environmental externality has key implications. Indeed, the SDF—the ratio of future to current marginal utility—is at the core of modern asset-pricing theory. Consequently, if environmental factors modify agents' marginal utility of consumption, they will also affect the pricing of risky and safe assets. This compensation effect of climate change therefore implies a potential role for green factors in asset-pricing models.

We model this compensation effect of climate change via an approach similar to that in the seminal contribution of [Campbell and Cochrane \(1999\)](#). In our case, however, it is the current stock of CO2 emissions rather than past levels of consumption that raises marginal utility. Moreover, following [Heutel \(2012\)](#), the stock of emissions is a slow-moving variable whose level depends on the quantity of emissions. As in [Campbell and Cochrane \(1999\)](#), this specification implies that risk aversion increases as the distance between consumption and the externality, or “surplus consumption” in the case of habits, declines. One advantage of this particular specification is that it will allow us to generate realistic fluctuations in the SDF without introducing too many degrees of freedom.

Relative to the endowment economy approach (e.g. [Lucas Jr, 1978](#)), another difference is that we analyze the environmental externality in a production economy, following the seminal contribution of [Jermann \(1998\)](#). We then derive the optimal tax by comparing the decentralized equilibrium to the planner's problem, as is usually

⁷⁰The Economist (2018). "Air-conditioners do great good, but at a high environmental cost". August 25th.

⁷¹International Energy Agency (2018). "Air conditioning use emerges as one of the key drivers of global electricity-demand growth". News, May 15th 2018.

the case in the environmental literature (e.g. Xepapadeas, 2005) or in Ljungqvist and Uhlig (2000) for the case of a consumption externality.

Following Nordhaus (2008) and Heutel (2012), among others, we introduce an abatement technology that firms can use to reduce their carbon footprint. Even when available, firms do not use this technology if emissions are not taxed. The abatement technology diverts resources from production. Consequently, profit-maximizing firms have no incentive to reduce emissions unless they are forced to do so.

Our first main result is that the optimal tax is determined by the shadow value of CO₂ emissions. We show that this implicit price can be expressed as the infinite discounted sum of the marginal disutility caused by emissions. This discounted sum is in turn critically affected by the SDF used by agents to price assets. This result therefore highlights the importance of asset-pricing considerations for the design of an optimal environmental tax.

This link between the optimal tax and the SDF breaks the macro-finance separation (e.g. Cochrane, 2017; Tallarini, 2000). The reason is that the model's ability to reproduce basic asset-pricing moments, such as the bond premium for example, has a crucial impact on the SDF. As the optimal tax is in turn determined by the SDF, the model's financial-market implications affect the design of environmental policies, and hence welfare. In contrast, with a separable preference specification we find that the dichotomy between climate policies and finance is close to perfect.

Imposing a tax on emissions restores the first-best allocation by encouraging firms to use the abatement technology. Abating carbon emissions is costly for firms. From the point of view of the social planner, it is therefore optimal to set the cost of abating emissions that firms face to its implicit market price.

Our second main result is that slow movements in the stock of CO₂ can have significant financial-market implications. Of particular relevance to Central Banks is the finding that environmental externalities affect the natural rate of interest. Climate change reduces the natural rate of interest.

The intuition behind this result is that the environmental externality generates time-variation in risk aversion, as in a model with external habits. In other words, when firms fail to internalize the damage caused by their emissions, households become more risk-averse. This rise in risk-aversion raises the risk premium demanded by investors, and induces precautionary saving. This stronger precautionary motive in turn explains the effect on the natural rate of interest.

We next show that introducing an optimal environmental tax reduces risk premia and increases the natural rate of interest. Under our baseline scenario, the tax reduces the premium on a long-term bond by half, and increases the natural rate by around 2 percentage points.

This result can be explained by the effect of the optimal policy on risk aversion. A tax on production reduces output, and hence consumption as well as emissions. The key is that the decline in emissions causes a fall in the externality that exceeds the drop in consumption. The resulting increase in this distance between consumption and the externality in turn reduces risk aversion.

Although consumption declines, the optimal tax generates large welfare gains. Under our benchmark calibration, this result is explained by the large fall in emissions

induced by the policy. The magnitude of this gain in turn depends on how firms react to the carbon tax. A profit-maximizing firm increases abatement until the marginal cost of abating emissions equals the marginal benefit. Under the optimal policy, the tax incentivizes firms to use the abatement technology to reduce the burden of the tax. This incentive to reduce emissions therefore lies behind the large welfare gain that we obtain.

The effect on welfare critically depends on the efficiency of the abatement technology available in the economy. If the technology is not sufficiently well-developed, the distortion caused by the tax can be sizeable: if firms cannot circumvent the tax by abating emissions, their only choice is to reduce production. In this case, the tax generates a smaller drop in emissions, which in turn reduces the policy's welfare gains.

The effect of the optimal policy on asset prices also depends crucially on the abatement technology. In this model, this can be explained by the impact of the tax on risk aversion. A less-developed technology reduces the decline in the stock of emission induced by the carbon tax. Consequently, a smaller increase in the distance between consumption and the externality can result if the technology is inefficient. This in turn implies a smaller drop in risk aversion, which causes higher risk premia and lower real interest rates.

Our third main result is that the optimal tax is pro-cyclical. As in [Ljungqvist and Uhlig \(2000\)](#), it is therefore optimal to “cool down” the economy during booms and to stimulate it in recessions. Estimating the model using higher-order perturbation methods allows us to estimate the implicit price of carbon. Our approach can therefore be used to provide an estimate of the optimal carbon tax over the business cycle. As illustrated in [Figure IV.2](#), it would have been optimal to progressively increase the tax in the run-up to the financial crisis and to reduce it sharply when the financial shock hit.

The intuition here is that the externality produces excessive fluctuations in risk aversion. As in a model with external habits and time-varying risk aversion (e.g. [Campbell and Cochrane, 1999](#)), the externality is beyond the agents' control. By internalizing the effect of emissions on utility, the policy allows the planner to find an optimal trajectory for both consumption and the stock of emissions. Controlling both variables at the same time in turn reduces the variations in “surplus consumption” that are unnecessary from a welfare perspective. These lower fluctuations in turn imply more moderate variations in risk aversion.

During recessions, this optimal trajectory involves lowering the carbon tax. A decline stimulates consumption. This effect helps to reduce risk aversion by increasing the distance between consumption and the externality. The key is that, as in the data, the stock of emissions moves very slowly over time. As the impact of the policy on consumption is more immediate, a tax cut generates a rise in consumption that exceeds the increase in the stock of emissions. The optimal policy therefore allows the planner to mitigate the surge in risk aversion that occurs in recessions.

As pointed out by [Bansal et al. \(2019\)](#) and [van den Bremer and van der Ploeg \(2019\)](#), there is evidence that climate-change risk could already be reflected in current equity prices. In [Bansal et al. \(2019\)](#), this link is explored in a model in which climate change is a source of long-run risk (e.g. [Bansal and Yaron, 2004](#)). The long-run risk

approach relies on Epstein-Zin-Weil preferences (e.g. [Epstein and Zin, 1989](#); [Weil, 1989](#); [Weil, 1990](#)).

The results in [Bolton and Kacperczyk \(2020\)](#) also suggest that exposure to carbon emission is already priced-in by investors. They find that the increase in stock returns caused by higher emissions is economically significant. In [Van der Ploeg et al. \(2020\)](#), the optimal carbon tax is derived in an endogenous-growth model. They also find that the natural rate of interest is lower under *laissez-faire*.

[Bauer and Rudebusch \(2020\)](#) show that the decline in the natural interest rate observed over the last decade implies a dramatic increase in the social cost of climate change. Our findings are also related to [Gollier \(2021\)](#) who highlights the role of abatement technologies and their efficiency in shaping carbon pricing. Following [Piazzesi et al. \(2007\)](#), we analyze the asset-pricing implications of a nonseparable utility function. [Piazzesi et al. \(2007\)](#) show that variations in the relative share of housing in agents' consumption baskets is a significant source of risk. In our case, it is the slow movements in the environmental externality that affect marginal utility. A review of the macro-financial implications of climate change is provided by [Van der Ploeg \(2020\)](#).

Our approach also builds on [Heutel \(2012\)](#), which is one of the first papers to consider environmental externalities from a business-cycle perspective. Relative to [Heutel \(2012\)](#), the model is estimated and generates a bond premium of about 1 percent. Reproducing a bond premium of this magnitude is a challenge for standard macroeconomic models (e.g. [Rudebusch and Swanson, 2008](#); [Rudebusch and Swanson, 2012](#)). Recent improvements in this literature for instance includes the work of [Andreasen et al. \(2018\)](#), which studies feedback effects from long-term bonds to the real economy within a model that matches the level and variability of the term premium.

In our case, environmental factors affect financial markets through the effect of the externality on attitudes towards risk. All else equal, the key is that an increase in the stock of emissions increases risk aversion. While it is difficult to test this hypothesis in the data, recent results in the psychology literature provide some indirect support.

First, in this literature, it is well-established that air pollution tends to increase anxiety. A recent review of the evidence on the link between air pollution and anxiety is provided in [Lu \(2020\)](#). Air pollution is in turn strongly correlated with CO2 emissions. Second, there is evidence that anxiety and risk aversion are tightly linked. For instance, according to [Charpentier et al. \(2017\)](#), more-anxious individuals exhibit a reduced propensity to take risks. The authors argue that this result is driven by risk aversion, and not loss aversion.

This kind of effect of air pollution on risk aversion is also consistent with the findings in [Levy and Yagil \(2011\)](#) of a negative correlation between air pollution and stock returns. Their interpretation is that air pollution has negative mood effects. As experimental work in Psychology in turn has related bad mood to increased risk aversion, they argue that air pollution could affect stock returns.

IV.1 THE MODEL

Consider a business-cycle model characterized by discrete time and an infinite-horizon economy populated by *firms* and *households*, which are infinitely-lived and of measure one. In this setup, production by firms produces an environmental externality via emissions, and these latter affect the household welfare by reducing the utility stemming from the consumption of goods. Firms do not internalize the social cost from their emissions of CO₂. As such there is market failure, opening the door to optimal policy intervention.

As the contribution of the paper lies in the role of the environmental externality in shaping investors' risk behavior, we start by presenting the accumulation of emissions in the atmosphere. We then explain how this environmental externality affects households' behavior.

IV.1.1 BALANCED GROWTH

Given that one objective of this paper is to estimate the model, we need to take into account that emissions grow at a different rate from output. In the context of our model, this difference in growth rates can be explained by introducing a rate of Green technological progress.

As is standard in the literature, macroeconomic variables are also assumed to grow along the balanced growth path. This is achieved by introducing labor-augmenting technological progress, denoted by Γ_t . The growth rate of labor-augmenting technological progress is γ^Y , where:

$$\frac{\Gamma_{t+1}}{\Gamma_t} = \gamma^Y$$

We denote Green technological progress in the growing economy by Ψ_t . The growth rate of Green progress γ^E is as follows:

$$\frac{\Psi_{t+1}}{\Psi_t} = \gamma^E$$

This trend is necessary to capture the long-term process of the decoupling of output growth from emission growth. As documented by [Newell et al. \(1999\)](#), this trend can be interpreted as an energy-saving technological change that captures the adoption of less energy-intensive technologies in capital goods. An improvement in the technology therefore implies a value for γ^E that is below 1. As in [Nordhaus \(1991\)](#), we assume that this trend is deterministic.

In the following sections, we present the de-trended economy. The detailed derivation of this de-trended economy appears in Appendix C.

IV.1.2 FIRMS AND EMISSIONS

Following standard integrated assessment models (IAM) (see [Nordhaus \(1991\)](#) and [Nordhaus and Yang \(1996\)](#)), a large part of the accumulation of Carbon Dioxide and other Greenhouse Gases (GHGs) in the atmosphere results from the human activity of economic production. We therefore employ a similar law of motion as in IAM to describe the concentration process of Carbon Dioxide in the atmosphere:

$$\gamma^X x_{t+1} = \eta x_t + e_t, \quad (\text{IV.1})$$

where x_{t+1} is the concentration of gases in the atmosphere, $e_t \geq 0$ the inflow (in kilotons) of Greenhouse Gases at time t , and $0 < \eta < 1$ the linear rate of continuation of CO₂-equivalent emissions that enter the atmosphere on a quarterly basis.⁷² Anthropogenic emissions of CO₂ result from both economic production and exogenous technical change:

$$e_t = (1 - \mu_t) \varphi_1 y_t^{1-\varphi_2} \varepsilon_t^X. \quad (\text{IV.2})$$

Here, the variable $1 \geq \mu_t \geq 0$ is the fraction of emissions abated by firms, y_t the aggregate production of goods by firms, and variable ε_t^X an AR(1) exogenous shock.

This functional form for emissions allows us to take into account both low- and high-frequency variations in CO₂ emissions. For the high-frequency features of the emissions data, the term $\varphi_1 y_t^{1-\varphi_2}$ denotes the total inflow of pollution resulting from production, prior to abatement. In this expression, $\varphi_1, \varphi_2 \geq 0$ are two carbon-intensity parameters that respectively pin down the steady-state ratio of emissions-to-output and the elasticity of emissions with respect to output over the last century. While φ_2 is set to 0 in [Nordhaus \(1991\)](#), we follow [Heutel \(2012\)](#) and allow this parameter to be positive to capture potential nonlinearities between output and emissions. Note that for $\varphi_2 < 1$, the emissions function exhibits decreasing returns.

In the de-trended economy, the presence of both Green and labor-augmenting technological progress introduces an adjustment into equation (IV.1), where γ^X is given as follows:

$$\gamma^X = \gamma^E (\gamma^Y)^{1-\varphi_2}$$

The remaining set of equations for firms is fairly standard, and similar to [Jermann \(1998\)](#). In particular, the representative firm seeks to maximize profit by making a trade-off between the desired levels of capital and labor. Output is produced via a Cobb-Douglas production function:

$$y_t = \varepsilon_t^A k_t^\alpha n_t^{1-\alpha}, \quad (\text{IV.3})$$

where k_t is the capital stock with an intensity parameter $\alpha \in [0, 1]$, n_t is labor, and ε_t^A is a total factor productivity shock that evolves as follows: $\log(\varepsilon_t^A) = \rho_A \log(\varepsilon_{t-1}^A) + \eta_t^A$, with $\eta_t^A \sim N(0, \sigma_A^2)$. The capital-share parameter is denoted by α . Firms maximize

⁷²One limitation is that we do not consider emissions from the Rest of the World (ROW). At the same time, US and ROW emissions are strongly correlated at the business-cycle frequency. Moreover, the US accounts for 1/3 of total anthropogenic emissions.

profits:

$$d_t = y_t - w_t n_t - i_t - f(\mu_t) y_t - e_t \tau_t \quad (\text{IV.4})$$

The real wage is denoted by w_t , $f(\mu_t)$ is the abatement-cost function, and $\tau_t \geq 0$ a potential tax on GHG emissions introduced by the fiscal authority. Investment is denoted by i_t and the accumulation of physical capital is given by the following law of motion:

$$\gamma^Y k_{t+1} = (1 - \delta) k_t + \left(\frac{\chi_1}{1 - \epsilon} \left(\varepsilon_t^I \frac{i_t}{k_t} \right)^{1-\epsilon} + \chi_2 \right) k_t \quad (\text{IV.5})$$

where $\delta \in [0, 1]$ is the depreciation rate of physical capital and ε_t^I an exogenous shock process, as in [Christiano et al. \(2014\)](#). This can be interpreted as an investment shock that captures financial frictions associated with asymmetric information or costly monitoring. As in [Jermann \(1998\)](#), χ_1 and χ_2 are two scale parameters that are calibrated to ensure that adjustment costs do not affect the deterministic steady state of the economy. The elasticity parameter $\epsilon > 0$ measures the intensity of adjustment costs.

The abatement-cost function is taken from [Nordhaus \(2008\)](#), where $f(\mu_t) = \theta_1 \mu_t^{\theta_2}$. In this expression, $\theta_1 \geq 0$ pins down the steady state of the abatement, while $\theta_2 > 0$ is the elasticity of the abatement cost to the fraction of abated GHGs. This function $f(\mu_t)$ relates the fraction of emissions abated to the fraction of output spent on abatement, where the price of abatement is normalized to one.

IV.1.3 HOUSEHOLDS AND THE ENVIRONMENTAL EXTERNALITY

We model the representative household via a utility function where the household chooses consumption expenditures as well as its holdings of long-term government bonds. Following [Stokey \(1998\)](#), [Acemoglu et al. \(2012\)](#) and [Golosov et al. \(2014b\)](#), we introduce the environmental externality into the utility function. However, instead of considering an additive specification, we assume that the marginal utility of consumption is affected by the externality.

Given our focus on asset prices, we choose a specification similar to that employed in the seminal contribution of [Campbell and Cochrane \(1999\)](#). As will become clear, adopting this particular specification will dramatically improve the model's ability to generate realistic asset-pricing implications. The difference relative to [Campbell and Cochrane \(1999\)](#) is that it is the disutility caused by pollution rather than past consumption that affects the marginal utility of consumption. As the evolution of x_t is determined by the environmental block of the model (e.g. [Nordhaus \(1991\)](#)), we refer to this preference specification as Campbell and Cochrane/Nordhaus (CCN) preferences.

The utility of the representative agent depends on the distance between consumption and the externality:

$$E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{(c_t - \phi x_t)^{1-\sigma}}{1-\sigma}, \quad (\text{IV.6})$$

where E_0 is the expectations operator conditioned on information at time 0, $\tilde{\beta}$ the

time discount factor adjusted for growth,⁷³ and $\sigma > 0$ the curvature parameter. The parameter ϕ represents the sensitivity of utility to a rise in CO₂ concentration in the atmosphere, which is denoted by x_t .⁷⁴ This can also be interpreted as the proportion of consumers affected by the damage caused by CO₂ emissions. Furthermore, the externality is a predetermined variable that moves slowly over time. This is to account for the possible long-term effects of decisions made in the past, which have possibly irreversible future consequences. This assumption has important implications for optimal choices, which we discuss in the following paragraphs.

First, from a consumer's perspective, consumption and the stock of CO₂ emissions can be interpreted as complements. As a result, the marginal utility of consumption increases in CO₂ concentration, so that households are more willing to consume when GHG concentration is high. This mechanism, pioneered by Michel and Rotillon (1995), is referred to as the *compensation effect*: households consume as a result of the change in marginal utility following an increase in emissions.

Second, this environmental externality in the utility function has important asset-pricing implications. To illustrate, we define, as in Campbell and Cochrane (1999), the consumption surplus ratio, $s_t = (c_t - \phi x_t) / c_t$. When the surplus falls in cyclical downturns, investors require a higher expected return compared to a standard CRRA utility function with $\phi = 0$. Under these preferences, the coefficient of relative risk aversion is given by $-(u''_c / u'_c) c_t = \sigma / s_t$. As such, a higher emissions stock reduces the surplus, which in turn increases risk aversion.

The budget constraint of the representative household is as follows:

$$w_t n_t + b_t + d_t = c_t + p_t^B (b_{t+1} - b_t) + t_t \quad (\text{IV.7})$$

where the left hand-side refers to the household's different sources of income. Total income is firstly comprised of labor income (with inelastic labor supply n_t). Every period, the agent also receives income from holding a long-term government bond, b_t . As the representative agent owns firms in the corporate sector, there is last dividend income of d_t .

On the expenditure side, the representative household first spends its income on consumption goods, c_t . The price at which newly-issued government bonds are purchased is p_t^B , and the quantity of new government bonds purchased during the period is $b_{t+1} - b_t$. Finally, we assume that the government levies a lump-sum tax of t_t .

⁷³Where $\tilde{\beta} = \beta \gamma^{1-\sigma}$. See Appendix C for a derivation of the effect of growth on the subjective discount factor.

⁷⁴Note that c_t and x_t do not grow at the same rate in the deterministic steady-state of the model. To obtain a stationary utility function, we assume that, in the growing economy, the preference parameter Θ_t is affected by labor-augmenting and Green technological progress. As we show in Appendix C, this implies the following relationship between ϕ and Θ_t :

$$\Theta_t = \phi \frac{(\Gamma_t)^{\varphi_2}}{\Psi_t}$$

IV.1.4 GOVERNMENT AND MARKET CLEARING

The government finances its expenditures by issuing a bond and collecting taxes. The government budget constraint is as follows:

$$g_t + b_t = p_t^B(b_{t+1} - b_t) + t_t + \tau_t e_t, \quad (\text{IV.8})$$

where public expenditure is denoted by g_t and t_t is a lump-sum tax. The revenue is composed of newly-issued government bonds $b_{t+1} - b_t$ on financial markets to households, while $\tau_t e_t$ denotes the revenues obtained from the implementation of an environmental tax on emissions. In this expression, e_t and τ_t are the level of emissions and the tax, respectively. As in any typical business-cycle model, government spending is exogenously determined and follows an AR(1) process: $g_t = \bar{g}\varepsilon_t^G$, with $\log \varepsilon_t^G = \rho_G \log \varepsilon_{t-1}^G + \eta_t^G$, $\eta_t^G \sim N(0, \sigma_G^2)$, and \bar{g} denoting the steady-state amount of resources that is consumed by the government. This shock accounts for changes in aggregate demand driven by both changes in public spending and the trade balance.

The resource constraint of the economy reads as follows:

$$y_t = c_t + i_t + g_t + f(\mu_t) y_t. \quad (\text{IV.9})$$

Finally, for the asset-pricing variables, we calculate the risk-free rate and the conditional risk premium respectively as:

$$1 + r_t^F = \{E_t m_{t,t+1}\}^{-1}, \quad (\text{IV.10})$$

$$E_t(r_{t+1}^B - r_t^F) = E_t((1 + p_{t+1}^B)/p_t^B - (1 + r_t^F)), \quad (\text{IV.11})$$

where $m_{t,t+1} = \beta^Y \{\lambda_{t+1}/\lambda_t\}$ is the stochastic discount factor, and the modified discount factor β^Y is as follows:

$$\beta^Y = \tilde{\beta}/\gamma^Y$$

IV.2 WELFARE THEOREMS WITH ENVIRONMENTAL PREFERENCES

In this section, we derive the optimal tax by comparing the decentralized equilibrium to the planner's problem.

IV.2.1 THE CENTRALIZED ECONOMY

We start by characterizing the first-best allocation and consider the optimal plan that the benevolent social planner would choose so as to maximize welfare. This equilibrium provides the benchmark against which the allocation obtained in the decentralized economy should be compared.

Definition 1 *The optimal policy problem for the social planner is to maximize total welfare in Equation IV.6 by choosing a sequence of allocations for the quantities $\{c_t, i_t, y_t, \mu_t, e_t, k_{t+1}, x_{t+1}\}$,*

for given initial conditions for the two endogenous state variables k_0 and x_0 , that satisfies equations (IV.1), (IV.2), (IV.3), (IV.5), and (IV.9).

Define λ_t as the time t marginal utility of consumption, q_t as the shadow value of capital and ϱ_t as the Lagrangian multiplier on the production function (note that both q_t and ϱ_t are expressed in terms of the marginal utility of consumption). The first-order conditions for this problem are as follows:

$$\lambda_t = (c_t - \phi x_t)^{-\sigma}, \quad (\text{IV.12})$$

$$1 = \chi_1 \varepsilon_t^I q_t \left(\varepsilon_t^I \frac{i_{t+1}}{k_{t+1}} \right)^{-\epsilon}, \quad (\text{IV.13})$$

$$q_t = \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} q_{t+1} \left[(1 - \delta_K) + \frac{\chi_1}{1 - \epsilon} \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} + \chi_2 - \chi_1 \left(\varepsilon_{It+1} \frac{i_{t+1}}{k_{t+1}} \right)^{1-\epsilon} \right] + \beta^Y E_t \frac{\lambda_{t+1}}{\lambda_t} \alpha \frac{y_{t+1}}{k_{t+1}} \varrho_{t+1} \quad (\text{IV.14})$$

where:

$$\beta^Y = \tilde{\beta} / \gamma^Y$$

Letting v_{Et} denote the Lagrange multiplier (expressed in units of marginal utility of consumption) on equation (IV.2), the first-order conditions with respect to the firm's optimal choice of output and abatement are given as follows:

$$\varrho_t + f(\mu_t) + v_{Et} (1 - \varphi_2) e_t / y_t = 1, \quad (\text{IV.15})$$

$$v_{Et} e_t / (1 - \mu_t) = f'(\mu_t) y_t. \quad (\text{IV.16})$$

The Lagrange multiplier ϱ_t is usually interpreted as the marginal cost of producing a new good, while v_{Et} is the social planner's value of abatement. Equation (IV.15) thus highlights the key role of emissions in shaping price dynamics: the production of one additional unit of goods increases firm profits but is partially compensated by the marginal cost from abating emissions. The planner also takes into account the marginal cost from emitting GHGs in the atmosphere. Notice that if abatement effort is zero, the marginal cost of production is one, as in the standard real business-cycle model. The second equation (IV.16) is a standard cost-minimizing condition on abatement: abating CO2 emissions is optimal when the resulting marginal gain (the left-hand side of equation IV.16) is equal to its marginal cost (the right-hand side of the same equation).

Two remaining first-order conditions on each of the environmental variables, namely

x_t and e_t , are necessary to characterize the decision rules of the social planner:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} (\phi + \eta v_{Xt+1}) \quad (\text{IV.17})$$

$$v_{Et} = v_{Xt}. \quad (\text{IV.18})$$

where:

$$\beta^X = \tilde{\beta} / \gamma^X$$

Recall that v_{Et} is the Lagrange multiplier on emissions in equation (IV.2), while v_{Xt} is the Lagrange multiplier on the law of motion of GHGs in equation (IV.1). The variable v_{Xt} can be interpreted as the implicit price of carbon. Equation (IV.17) shows that this implicit price can be considered via an asset-pricing formula. The first term $(\beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} \phi)$ is the discounted utility loss incurred by society from a marginal increase in the stock of emissions in the atmosphere. The second term $(\eta \{E_t \frac{\lambda_{t+1}}{\lambda_t} v_{t+1}^X\})$ is the continuation value of the discounted utility loss caused by emissions, which remain in the atmosphere with probability η . The second equation is the internal cost of GHG emissions for firms, where v_{Et} is the marginal cost for a firm of emitting one kiloton of carbon. In the first-best allocation, this cost must be exactly equal to the price of carbon emissions v_{Xt} .

Definition 2 *The inefficiency wedge induced by the environmental externality is defined as the gap between the price of carbon emissions and this marginal cost:*

$$\varpi_t = v_{Xt} - v_{Et}. \quad (\text{IV.19})$$

When the social cost of carbon is perfectly internalized by society, optimal abatement in (IV.18) is such that the marginal cost of emissions equals their price. In this case, it is optimal for firms and society to spend a fraction of resources to reduce CO2 emissions by using the abatement technology $f(\mu_t)$.

Proposition 3 *In a centralized equilibrium, the social cost of carbon is perfectly internalized by the planner. The marginal cost of emissions is therefore equal to the price of carbon emissions. This implies (from the previous definition) a first-best allocation with an inefficiency wedge $\varpi_t = 0$.*

The resulting equilibrium is optimal, as the social cost of the externality is perfectly internalized by society. As a consequence, the inefficiency wedge from carbon emissions is zero. In the following section, we show that this optimum is not reached in a *laissez-faire* equilibrium with profit-maximizing firms.

IV.2.2 THE COMPETITIVE EQUILIBRIUM

We now describe the competitive equilibrium resulting from economic decisions taken by households and firms separately, with no centralization. This decentralized economy is also referred to as the competitive or *laissez-faire* equilibrium, where social

preferences for carbon are different across firms and households. We propose the following definition to characterize this economy.

Definition 4 *The laissez-faire equilibrium is defined as a competitive equilibrium in which the environmental tax on carbon emissions τ_t is set to 0. Households maximize utility in Equation IV.6 under constraints (IV.7) and (IV.5). Firms maximize profits (IV.4) under constraints (IV.2) and (IV.3).*

Relative to the efficient equilibrium, the difference here is that firms maximize profits and no longer consider the stock of CO2 emissions as a control variable. This implies that firms and households exhibit different preferences regarding carbon emissions. As a result, the price of carbon for firms differs from that obtained in the centralized economy. Since emissions are costly to abate, and given that firms do not internalize the effect of their emissions on consumers, the cost of carbon emissions for firms is zero. In contrast, the price of carbon for households, which we denote v_{Xt} , is given as follows:

$$v_{Xt} = \beta^X E_t \frac{\lambda_{t+1}}{\lambda_t} (\phi + \eta v_{Xt+1}) \quad (\text{IV.20})$$

We here have a market failure, as the social value of carbon differs between the emitters of carbon and the agents who experience the social loss.

As emissions are not taxed, the shadow cost for a firm to emit CO2 in the atmosphere is zero:⁷⁵

$$v_{Et} = 0. \quad (\text{IV.21})$$

In this setup, firms simply cost-minimize by optimally choosing zero abatement spending: with a cost of releasing CO2 of zero, firms have no incentive to allocate resources to use the abatement technology $f(\mu_t)$ to reduce emissions. The socially-optimal level of abatement is not implemented, as the equilibrium abatement share is zero in the *laissez-faire* equilibrium:

$$\mu_t = 0. \quad (\text{IV.22})$$

Consequently, the marginal cost of production ϱ_t is similar to that obtained in any typical real business-cycle model. In terms of the notation introduced in definition **Theorem 4**, this produces an environmental inefficiency wedge that differs from zero:

$$\varpi_t = v_{Xt} - v_{Et} = v_{Xt}. \quad (\text{IV.23})$$

CO2 emissions therefore create a market failure via an environmental externality. As a result, the first welfare theorem breaks down as the competitive equilibrium does not coincide with the social planner's outcome. The externality, measured by the inefficiency wedge ϖ_t , distorts the equilibrium and gives rise to a deadweight loss proportional to v_{Xt} . Note that the first welfare theorem applies only if the environmental policy has no effect on preferences, which is the case only if $\phi = 0$.

⁷⁵The optimality conditions corresponding to the *laissez-faire* equilibrium are derived in Appendix D.

IV.2.3 ENVIRONMENTAL POLICY

In the presence of the environmental externality reflected in $\varpi_t > 0$, the social value of carbon differs across agents. This market failure opens the door for government policy to address this externality and render the *laissez-faire* allocation the same as that of the social planner. In particular, the government can introduce a tax, τ_t , on GHG emissions to be paid by firms. This policy tool has two interpretations. It first can be considered as a tax on carbon emissions, in the same spirit as a standard Pigouvian tax that aims to force firms to internalize the social cost of carbon emissions on household utility, thereby correcting the market failure (i.e. the negative externality) by setting the tax equal to the price of carbon emissions.

An alternative interpretation is that the government creates a market for carbon emissions (i.e. a carbon-permits market). Here the government regulates the quantity of emissions. The optimal value for this instrument can be directly computed from a Ramsey optimal problem. Comparing the social planner's solution to the competitive equilibrium, we make the following proposition:

Proposition 5 *The first-best allocation can be attained by using the instrument τ_t in order to close the inefficiency gap (i.e. $\varpi_t = 0$). This condition is achieved by setting the carbon tax such that:*

$$\tau_t = v_{X_t}.$$

As shown in Appendix D, setting the environmental tax to v_{X_t} ensures that the first-order conditions under the competitive and centralized equilibria coincide. This result is fairly intuitive. In the absence of an environmental policy, abatement reduces profits, and firms will not be willing to bear this cost unless an enforcement mechanism is implemented. The government can impose a price on carbon emissions by choosing the optimal tax (either quantity- or price-based, as discussed in [Weitzman \(1974\)](#)) to produce the desired level of abatement. This environmental policy forces firms to internalize the effect of emissions, which in turn leads to a better integration of economic and environmental policies.

Furthermore, as argued in both the public economics and environmental literatures ([Goulder \(1995\)](#)), either a tax or a permit policy would generate revenue that could be used as a "double dividend" to not only correct the externality but also reduce the number of distortions due to the taxation of other inputs, such as labor and capital. Moreover, an equivalence between the tax and permit policies holds when the regulator has symmetric information about all state variables for any outcome under the tax policy and a cap-and-trade scheme ([Heutel \(2012\)](#)).

IV.3 ESTIMATION

In this section, we estimate the structural parameters of the model using Bayesian methods. For a presentation of the method, we refer to the canonical papers of [An and Schorfheide \(2007\)](#) and [Smets and Wouters \(2007\)](#). As the U.S. has not implemented any environmental policy, we propose to estimate the *laissez-faire model*. The follow-

ing sub-sections discuss the non-linear method employed for the estimation, the data transformation and calibration, the priors and the posteriors.

IV.3.1 SOLUTION METHOD

Since we want to accurately measure higher-order effects of environmental preferences (e.g. precautionary saving, utility curvature), we consider a second-order approximation to the decision rules of our model. Taking higher-order approximated models to data remains a challenge as the nonlinear filters that are required to form the likelihood function are computationally expensive. An inversion filter has recently emerged as a computationally-cheap alternative to apply nonlinear models to data (e.g. [Guerrieri and Iacoviello 2017](#), [Atkinson et al. 2020](#)). Initially pioneered by [Fair and Taylor \(1983\)](#), this filter extracts the sequence of innovations recursively by inverting the observation equation for a given set of initial conditions. Unlike other filters (e.g. Kalman or particle),⁷⁶ the inversion filter relies on an analytic characterization of the likelihood function. [Kollmann \(2017\)](#) provided the first application of the inversion filter to second- and third-order approximations to the decision rules in a rational-expectations model.⁷⁷ To allow the recursion, this filter imposes that the number of fundamental shocks must be equal to the number of observable variables. Note that, for linearized models, this restriction is standard following [Smets and Wouters \(2007\)](#). For the relative gains of the inversion filter with respect to a particle filter, we refer to [Cuba-Borda et al. \(2019\)](#) and [Atkinson et al. \(2020\)](#).

The model is estimated using four observable macroeconomic time-series, which are jointly replicated by the model through the joint realization of four corresponding innovations. Note that we use the pruning state-space to obtain the matrices of the policy rule using the Dynare package of [Adjemian et al. \(2011\)](#). From this state-space representation, we reverse the observation equations to obtain the sequence of shocks. Unlike [Kollmann \(2017\)](#) who limits the analysis to a frequentist approach, we augment the likelihood function with prior information in the same spirit as [Smets and Wouters \(2007\)](#). This method requires a sampler, here Metropolis-Hastings, to draw the parametric uncertainty.

IV.3.2 DATA

The model is estimated with Bayesian methods on U.S. Quarterly data over the sample time period 1973Q1 to 2018Q4, which are all taken from FRED and the U.S. Energy Information Administration.

Concerning the transformation of series, the aim is to map non-stationary data to a stationary model (namely, GDP, consumption, investment and CO2 emissions). Following [Smets and Wouters \(2007\)](#), data exhibiting a trend or unit root are rendered stationary in two steps. We first divide the sample by the working-age population.

⁷⁶For a presentation of alternative filters to calculate the likelihood function, see [Fernández-Villaverde et al. \(2016\)](#).

⁷⁷[Kollmann \(2017\)](#) posits a modified higher-order decision rule in which powers of exogenous innovations are neglected to obtain a straightforward observation equation inversion. In this paper, we include these terms of the decision rule.

Second, data are taken in logs and we apply a first-difference filter to obtain growth rates. Real variables are deflated by the GDP deflator price index. The measurement equations mapping our model to the data are given by:

$$\begin{bmatrix} \text{Real Per Capita Output Growth} \\ \text{Real Per Capita Consumption Growth} \\ \text{Real Per Capita Investment Growth} \\ \text{Per Capita } CO_2 \text{ Emissions Growth} \end{bmatrix} = \begin{bmatrix} \log \gamma_A + \Delta \log(\tilde{y}_t) \\ \log \gamma_A + \Delta \log(\tilde{c}_t) \\ \log \gamma_A + \Delta \log(\tilde{i}_t) \\ \log \gamma_A^{1-\varphi_2} \gamma_E + \Delta \log(\tilde{e}_t) \end{bmatrix}, \quad (\text{IV.24})$$

where a variable with a tilde, \tilde{x}_t , denotes the de-trended version of a level variable, x_t .

IV.3.3 CALIBRATION AND PRIOR DISTRIBUTIONS

The calibrated parameters are reported in [Table IV.1](#). The calibration of the parameters related to business-cycle theory is standard: the depreciation rate of physical capital is set at 2.5 percent in quarterly terms, the Government spending to GDP ratio to 20 percent, and the share of hours worked per day to 20 percent. The environmental component parameters of the models, when not estimated, are set in a similar fashion as [Nordhaus \(2008\)](#) and [Heutel \(2012\)](#). We set the parameter φ_1 to match an average steady-state of pollution in laissez-faire equilibrium, which corresponds to the 2005 value of atmospheric carbon mass of 800 gigatons. The continuation rate of carbon in the atmosphere, denoted η , is set to match a roughly 139 years half time of atmospheric carbon dioxide, as in [Nordhaus \(1991\)](#).⁷⁸ Finally, for the abatement-cost function, we set $\theta_1 = 0.05607$ and $\theta_2 = 2.8$ as in [Nordhaus \(2008\)](#) and [Heutel \(2012\)](#).

For the remaining set of parameters and shocks, we employ Bayesian methods. [Table IV.2](#) summarizes the prior — as well as the posterior — distributions of the structural parameters for the U.S. economy. Let us first discuss the prior for structural disturbances. The prior information on the persistence of the Markov processes and the standard deviation of innovations are taken from [Guerrieri and Iacoviello \(2017\)](#). In particular, the persistence of shocks follows a beta distribution with a mean of 0.5 and a standard deviation of 0.2, while for the standard deviation of shocks we choose an inverse gamma distribution with mean 0.01 and standard deviation of 1.

For the parameters which have key asset-pricing implications, we translate some bound restrictions from the matching moments exercise of [Jermann \(1998\)](#) into prior distributions. In particular, the elasticity of Tobin's Q to the investment-capital ratio is assumed to follow a Gamma distribution with prior mean of 4 and standard deviation of 1. The latter implies a support for ϵ close to the bound $\epsilon \in [0.16; +\infty]$ of [Jermann \(1998\)](#). In addition, we set the capital intensity α to follow a Beta dis-

⁷⁸Let us assume that each unit of CO2 is subject to an idiosyncratic shock, denoted ω , and that the carbon is reused or sequestered in a carbon sink. This random variable is drawn from a binomial distribution, $\omega \sim B(n, p)$ with n the number of trials and p the probability of success $p = 1 - \tilde{\eta}$. We thus determine the number of trials, n , that are necessary on average for one unit of carbon to be sequestered. Recall that $E(\omega) = n.p$, by imposing $E(\omega) = 1$ we calculate that the average number of trials necessary for carbon sequestration is $n = 1/(1 - \tilde{\eta})$. On an annual basis, the latter becomes $n = 0.25/(1 - \tilde{\eta})$. Recall that in the balanced growth path the effective continuation rate of carbon is $\tilde{\eta} = \eta \gamma_A \gamma_E^{1-\varphi_2}$. Then imposing an average half time of carbon of 139, we deduce the value of η as $\tilde{\eta} = (1 - 0.25/139) (\gamma_A \gamma_E^{1-\varphi_2})^{-1}$.

tribution with mean of 0.25 and standard deviation 0.02 in order to be close to the value estimated by [Jermann \(1998\)](#). Note that we set a tight prior on this parameter in order to match the tight interval range of α that replicates the U.S. investment-to-output ratio. [Jermann \(1998\)](#) calibrates the risk aversion coefficient to 5 to be consistent with asset-pricing models. However, a high value for σ typically generates strong consumption-smoothing behavior in the Euler equation that is at odds with the data. Environmental economics typically favors values close to 2, while likelihood-estimated models usually find values below 2 (e.g. [Smets and Wouters \(2007\)](#)). To reconcile these three literatures, we propose to estimate this key parameter agnostically by imposing a rather diffuse information through a Gamma distribution with a prior mean of 2 and standard deviation of 0.35. This prior allows the parameter to be either high (i.e. close to 5), as in asset-pricing models, or lower (i.e. close to 2), as in the environmental models in [Stern \(2008\)](#) and [Weitzman \(2007\)](#), or low (i.e. equal to one), as in estimated business-cycle models. Unlike [Jermann \(1998\)](#), we cannot directly estimate $\beta\gamma_A^{-\sigma}$, because of weak identification when using full-information methods. We thus follow [Smets and Wouters \(2007\)](#) and estimate instead the term $(1/\beta - 1)100$: this allows to easily impose prior information based on a Gamma distribution with a mean of 0.5 and standard deviation 0.25. The resulting prior allows the discount factor to roughly lie between 0.99 and 0.9980.⁷⁹

Regarding the slopes of growth, we discuss first the productivity one (denoted $(\gamma_A - 1) \times 100$) that follows a Gamma distribution with a prior mean of 0.5 and a standard deviation of 0.04 in order to match the average 0.40 percent quarterly growth rate. For the (de)coupling rate (denoted $(\gamma_E - 1) \times 100$), we let the data be fully informative about the slope through a normal distribution with prior mean 0 and standard deviation 0.25. Finally, the last remaining parameter is the utility loss from cumulative CO2 emissions, ϕ . As in [Campbell and Cochrane \(1999\)](#), and given that we have several exogenous shocks, this parameter has to be restricted to ensure that surplus consumption always remains positive. This restriction ensures the non-negativity of the Lagrangian multiplier on the budget constraint (otherwise the budget constraint would not bind). We thus express this parameter in terms of steady-state consumption, $\phi\bar{c}/\bar{x}$, and impose an uninformative prior with a uniform distribution with mean 0.5 and standard deviation 0.285. This prior induces a bound restriction such that $\phi\bar{c}/\bar{x} \in [0; 1]$, this is rather conservative as, unlike Beta distributions, it does not favor any particular value within this interval.⁸⁰

IV.3.4 POSTERIOR DISTRIBUTIONS

In addition to prior distributions, [Table IV.2](#) reports the means and the 5th and 95th percentiles of the posterior distributions drawn from four parallel MCMC chains of 50,000 iterations each. The sampler employed to draw the posterior distributions is

⁷⁹Note in addition that our prior mean for $(1/\beta - 1)100$ is much higher than that in [Smets and Wouters \(2007\)](#) as our model is non-linear, and thus features the precautionary saving effect that drives down the real rate. With the prior information of [Smets and Wouters \(2007\)](#), we would obtain a real rate below zero; we thus re-adjust the prior information to render our non-linear model consistent with US real rate data.

⁸⁰Note that with the bounds $\hat{\phi} = \phi\bar{c}/\bar{x} \in [0; 1]$, the $MRS = \bar{c} - \phi\bar{x} = \bar{c} - \hat{\phi}\bar{c}$, as in any standard model featuring external consumption habits.

Model counterpart	Name	Values
\bar{N}	Labor supply	0.20
δ_K	Depreciation rate of capital	0.025
\bar{g}/\bar{y}	Public spending share in output	0.20
\bar{x}	Atmospheric carbon (gigatons) in laissez-faire	800
$[4(1 - \gamma_A \gamma_E^{1-\varphi_2} \eta)]^{-1}$	Half-life of CO2 in years	139
θ_1	Abatement cost	0.05607
θ_2	Curvature abatement cost	2.8

Table IV.1: Calibrated parameter values (Quarterly basis)

the Metropolis-Hasting algorithm with a jump scale factor so as to match an average acceptance rate close to 25-30 percent for each chain.

The results of the posterior distributions for each estimated parameter are listed in [Table IV.2](#) and [Figure IV.1](#). It is clear from [Figure IV.1](#) that the data were informative, as the shape of the posterior distributions is different from the priors. Our estimates of the structural parameters that are common with [Smets and Wouters \(2007\)](#) are mostly in line with those they find. The persistence of productivity and spending shocks are, for instance, very similar to theirs. The risk-aversion coefficient σ has a posterior mean of 4.2, which is lower than the value in [Jermann \(1998\)](#). It is however higher than the values reported in environmental macroeconomic and estimated DSGE models: for example, [Smets and Wouters \(2007\)](#) find a value of 1.38 for this parameter. Another key parameter that determines the consumption surplus is $\phi \bar{c}/\bar{x}$. We find a value of 0.67 which is very close to that estimated by [Smets and Wouters \(2007\)](#) in the case of external consumption habits (0.71). The corresponding value of ϕ , given the steady state ratio \bar{c}/\bar{x} , is 0.0004. Regarding the growth rate of productivity, our estimated value, 0.34, is lower than that in [Smets and Wouters \(2007\)](#), but this is unsurprising as economic growth is lower in our sample given that we exclude the 1960s and include the last decade. Regarding the last estimated parameter common with [Smets and Wouters \(2007\)](#), the data suggest a value for capital intensity α close to 0.41, which is higher than the estimated values of [Jermann \(1998\)](#) and [Smets and Wouters \(2007\)](#). This is important, as estimated DSGE models typically predict very low values for α that are at odds with data on both the capital structure of firms and the investment-to-output ratio. Finally for the discount rate, denoted $100(\beta^{-1} - 1)$, we find a posterior mean of 0.13 that generates a discount factor of 0.9987.

The last remaining parameters are not common with [Smets and Wouters \(2007\)](#). For the elasticity of Tobin's Q to the investment capital ratio ϵ , we find a posterior mean of 1.44 that is higher than that in [Jermann \(1998\)](#). The value of the elasticity of emissions to output, φ_2 , is 0.36, which is remarkably close to that estimated by [Heutel \(2012\)](#). Finally, for the decoupling rate we find that energy-saving technological change has caused reductions in CO2 of about 2% annually.

To assess the relevance of the estimated model, as in [Jermann \(1998\)](#), we compare the observable moments taken at a 90 percent interval versus the asymptotic moments generated by the model using a second-order approximation to the policy function. [Table IV.3](#) reports the results. We find that our model does a reasonably good job at

		Prior distributions			Posterior distributions
		Shape	Mean	Std.	Mean [0.050;0.950]
Shock processes:					
Std. productivity	σ_A	\mathcal{IG}_1	0.01	1	0.008 [0.007;0.009]
Std. spending	σ_G	\mathcal{IG}_1	0.01	1	0.035 [0.032;0.039]
Std. abatement	σ_X	\mathcal{IG}_1	0.01	1	0.020 [0.018;0.022]
Std. investment	σ_I	\mathcal{IG}_1	0.01	1	0.014 [0.012;0.016]
AR(1) productivity	ρ_A	\mathcal{B}	0.50	0.20	0.944 [0.930;0.955]
AR(1) spending	ρ_G	\mathcal{B}	0.50	0.20	0.953 [0.932;0.967]
AR(1) abatement	ρ_X	\mathcal{B}	0.50	0.20	0.896 [0.828;0.947]
AR(1) investment	ρ_I	\mathcal{B}	0.50	0.20	0.998 [0.998;0.999]
Structural parameters:					
Productivity growth rate	$(\gamma_A - 1) \times 100$	\mathcal{G}	0.50	0.04	0.340 [0.301;0.387]
Output-CO2 (de)coupling rate	$(\gamma_E - 1) \times 100$	\mathcal{N}	0	0.25	-0.45 [-0.538;-0.346]
Discount rate	$(\beta^{-1} - 1) \times 100$	\mathcal{G}	0.50	0.25	0.139 [0.051;0.343]
Capital intensity	α	\mathcal{B}	0.25	0.02	0.412 [0.374;0.453]
Capital-cost elasticity	ϵ	\mathcal{G}	4	1	1.448 [1.029;2.038]
Utility loss on emissions	$\phi \times \bar{c}/\bar{x}$	\mathcal{U}	0.50	0.285	0.677 [0.611;0.730]
Relative risk aversion	σ	\mathcal{G}	2.00	0.35	4.198 [3.681;4.740]
Output-CO2 elasticity	φ_2	\mathcal{B}	0.50	0.20	0.367 [0.138;0.633]
Log-marginal data density					-2124.0769

Notes: \mathcal{B} denotes the Beta, \mathcal{IG}_1 the Inverse Gamma (type 1), \mathcal{N} the Normal, and \mathcal{U} the uniform distribution.

Table IV.2: Prior and Posterior distributions of structural parameters

	Mean		Stand. Dev		Corr. w/ output	
	Data [5%;95%]	Model	Data [5%;95%]	Model	Data [5%;95%]	Model
$100 \times \Delta \log(y_t)$	[0.28;0.50]	0.34	[0.69;0.85]	0.81	[1.00;1.00]	1.00
$100 \times \Delta \log(c_t)$	[0.36;0.55]	0.34	[0.60;0.74]	0.90	[0.54;0.76]	0.58
$100 \times \Delta \log(i_t)$	[0.07;0.68]	0.34	[1.91;2.34]	2.58	[0.61;0.80]	0.72
$100 \times \Delta \log(e_t)$	[-0.53;0.07]	-0.24	[1.88;2.31]	2.12	[-0.01;0.35]	0.24

Table IV.3: Data moments vs. model moments (with parameters taken at their posterior means)

replicating some salient features of the data, as most of the moments simulated by the estimated model fall within the 95 percent confidence interval of the data.

The advantage of using Bayesian estimation is that the model can replicate the historical path of the observable variables that we introduce. Once the shock process parameters have been estimated, it is possible to simulate the model by drawing shocks from the estimated distribution. As illustrated in Table IV.3, however, this procedure does not ensure that the unconditional standard deviations observed in the data are matched perfectly.

Letting $u(c_t - C_t)$ denote the utility function, with C_t the reference variable to calculate the surplus consumption ratio, a natural question at this stage is how relevant is our specification of environmental preferences with respect to a standard consumption habits model à la Jermann (1998). Using an uninformative prior distribution over models (i.e. 50% prior probability for each model), Table IV.4 shows both the posterior odds ratios and model probabilities taking the consumption habits model $\mathcal{M}(C_t = \phi c_{t-1})$ as the benchmark model. We examine the hypothesis $H_0: C_t = \phi c_{t-1}$ against the hypothesis $H_1: C_t = \phi x_t$. The posterior odds of the null hypothesis of surplus based on lagged consumption is 8e17: 1, which leads us to strongly reject the null. The surplus consumption ratio is therefore more relevant when it is based

	Standard Cons. habits	Pollution externality
Utility function $u(c_t - C_t)$	$C_t = \phi c_{t-1}$	$C_t = \phi x_t$
Surplus parameter ϕ	0.21	0.67
Prior probability	0.50	0.50
Log marginal data density	2004.77	2045.99
Bayes ratio	1.000000000	8.02e17
Posterior model probability	0.000000000	1.000000000

Table IV.4: The comparison of prior and posterior model probabilities in the internal consumption habits and the environmental preferences models (with parameters taken at their posterior mode).

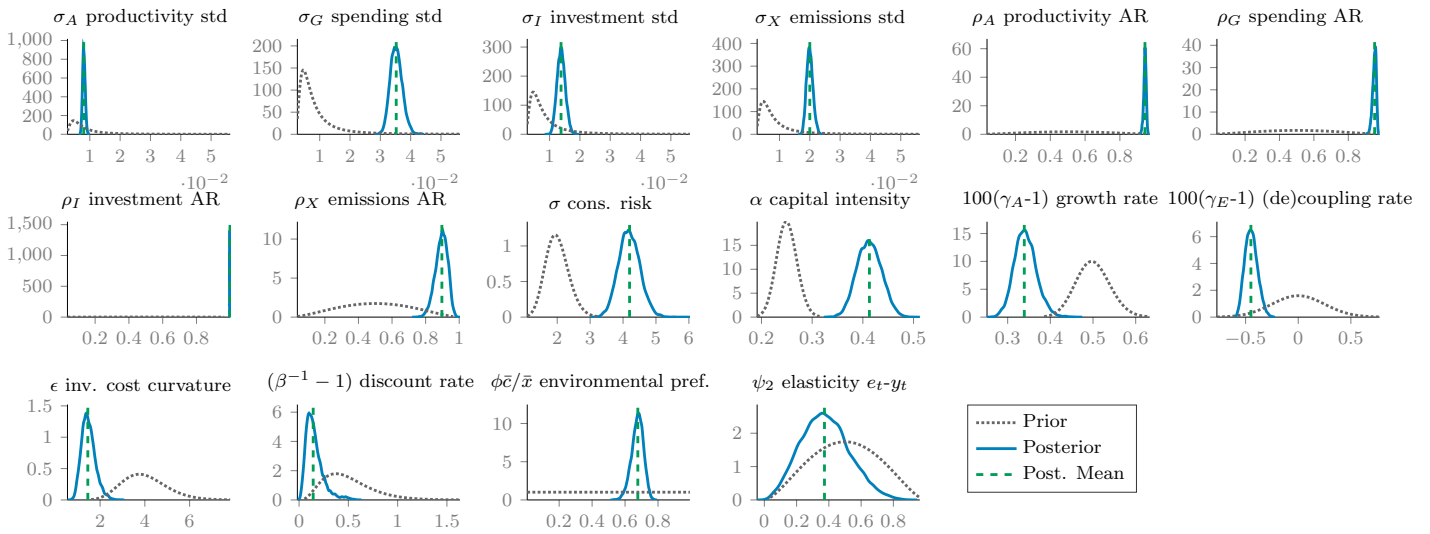


Figure IV.1: Prior and posterior distributions of the estimated parameters

on the stock of emissions rather than past consumption. This result should however be qualified, as prior distributions were selected here to estimate our model and do not necessarily fit the benchmark model of H_0 . This can diminish the empirical performance of the benchmark. The goal of this exercise is not to show that one model outperforms another, but to highlight that our model is least as consistent with the data as the standard habits-type model.

IV.4 RESULTS

Our main simulation results appear in [Table IV.5](#) below. The top panel of this table shows the average level of consumption and the stock of CO2 emissions, which are denoted by $E(c_t)$ and $E(x_t)$, respectively. The agent's lifetime utility, $E(\mathcal{W}_t)$, is our measure of welfare. The average tax chosen by the social planner is $E(\tau_t)$.

The asset-pricing implications appear in the middle panel, where $400E(r_t^F)$, $400E(r_{t+1}^B - r_t^F)$ and $std(\hat{\lambda}_t)$ are the mean real risk-free rate, the mean bond premium, expressed in annualized percent, and the standard deviation of marginal utility respectively. The average coefficient of relative risk aversion is $E(RRA_t)$ and $std(\widehat{rra}_t)$ is a measure of

its standard deviation (expressed in log-deviations from the steady state).

The bottom panel of [Table IV.5](#) first lists the share of emissions that firms choose to abate, $E(\mu_t)$. The average cost of abatement is $E(f(\mu_t))$, and $E(\tau_t e_t/y_t)$ is the average cost of the tax borne by firms as a share of GDP.

The first column shows these model implications in the decentralized *laissez-faire* equilibrium with a tax set to zero. Columns (2) to (4) show what happens once the optimal tax is introduced. The optimal-policy results are listed for three different values of the parameter θ_1 . This latter measures the efficiency of the abatement technology, with higher θ_1 corresponding to a less-efficient technology. As $\theta_1 = 0.05607$ is the value used in the literature (e.g. [Nordhaus 2008](#); [Heutel 2012](#)), the results in column (2) correspond to our baseline scenario.

IV.4.1 THE SIZE AND THE CYCLICALITY OF THE OPTIMAL TAX

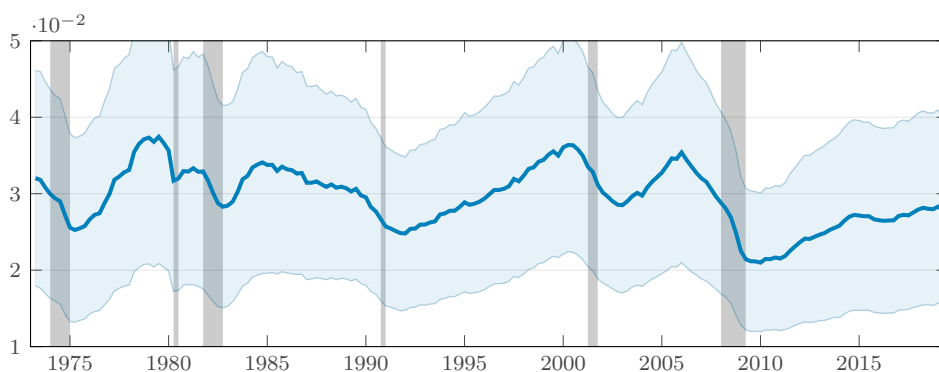
The first main takeaway from [Table IV.5](#) is that a small average carbon tax is sufficient to restore the first-best allocation. In our benchmark scenario, which corresponds to $\theta_1 = 0.05607$, the total tax bill is on average around two percent of GDP ($E(\frac{\tau_t e_t}{y_t}) = 0.02$).

As can be seen by comparing the total tax bill across columns 2 to 4, in the worst-case scenario, corresponding to a value for θ_1 implying a very-inefficient abatement technology, the total tax bill rises to 5.7 percent of GDP. In this adverse scenario, firms only manage to abate about 6 percent of all emissions, $E(\mu_t) = 0.0592$, once the tax is introduced.

One advantage of our method is that it can be used to construct counterfactual scenarios. In particular, we can answer the following question: What would the optimal tax τ_t have been in the United States from 1973 to 2018, had this optimal policy been implemented? [Figure IV.2](#) provides the answer. The optimal tax is time-varying, and rises in booms and falls during recessions. The optimal tax is thus strongly pro-cyclical, as illustrated by [Figure IV.3](#), so that the tax bill $\tau_t e_t/y_t$ falls during major recessions, like the global financial crisis.

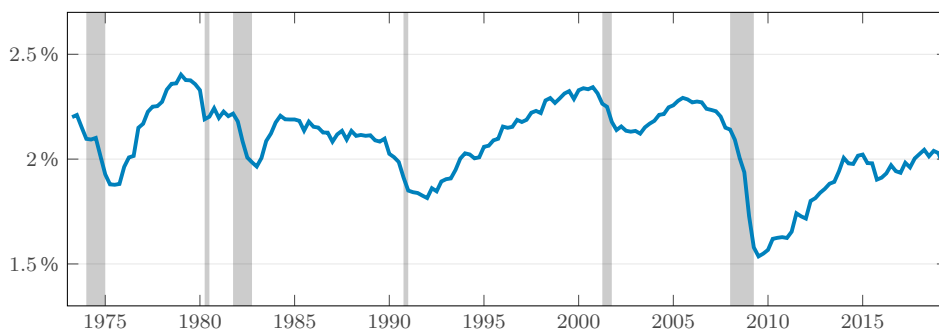
The optimal tax is pro-cyclical because the externality induces excessive fluctuations in risk aversion. As in a model with external habits and time-varying risk aversion (e.g. [Campbell and Cochrane 1999](#)), agents take the externality as given. As the optimal tax reproduces the first-best allocation, it eliminates this inefficiency by making firms internalize the effect of their production on consumers. Our analysis therefore provides a novel interpretation of the result in [Ljungqvist and Uhlig \(2000\)](#) for the case of habits. As shown in [Table IV.4](#), one motivation for our approach is that our specification is strongly supported by the data, especially relative to habits.

It is important to note that the fluctuations in risk aversion are essentially driven by consumption, not by the externality. In line with the evidence, we assume that the stock of CO2 depreciates very slowly over time. Whereas the flow of emissions can be volatile, the stock of emissions, and hence the externality, moves only very slowly over the business cycle.



Notes: The simulated path is expressed in levels. The blue shaded area is the parametric uncertainty at 95% confidence level, drawn from 1,000 Metropolis-Hastings random iterations. The blue line represents the mean of these 1,000 simulated paths. The gray shaded areas are NBER-dated recessions in the US.

Figure IV.2: Historical variations in the environmental tax



Notes: The simulated path is expressed in levels. The blue line represents the mean of 1,000 simulated paths of Metropolis-Hastings random iterations. The gray shaded areas are NBER-dated recessions in the US.

Figure IV.3: Historical variations in the tax bill in % of GDP, $\tau_t e_t / y_t$

IV.4.2 THE RISK PREMIUM AND THE RISK-FREE RATE IN THE LAISSEZ-FAIRE EQUILIBRIUM

As can be seen in column (1), the model generates an average bond premium, i.e. $400E(r_{t+1}^B - r_t^F)$, of about 1.3 percent. Although small, generating a bond premium of this magnitude remains a challenge for a large class of General-Equilibrium models with production. In our case, this relative success is due to our preference specification, which generates time-variation in risk aversion, as in [Campbell and Cochrane \(1999\)](#).

As in [Jermann \(1998\)](#), the positive bond premium that we obtain is due to interest-rate risk. The price of long-term bonds is determined by the term structure of interest rates. The key is that in this model short- and long-term interest rates are counter-cyclical. With interest rates rising during recessions, bond holders can expect capital losses to occur precisely during periods of low consumption and high marginal utility. Long-term bonds are therefore not good hedges against consumption risk. The positive bond premium is thus a compensation for holding an asset whose price declines during periods of low consumption.

In this model, the mean risk-free rate $400E(r_t^F)$ is critically affected by uncertainty.

As in [Jermann \(1998\)](#), a greater variance in marginal utility reduces the unconditional mean risk-free rate. The intuition is that a higher volatility of marginal utility implies more uncertainty about future valuations, and greater uncertainty in turn increases agents' willingness to build precautionary buffers. This effect therefore captures the impact of this precautionary motive on equilibrium interest rates.

IV.4.3 ASSET PRICES UNDER THE OPTIMAL POLICY

Relative to the *laissez-faire* equilibrium, the optimal tax has a sizeable effect on the mean risk-free rate. In the baseline scenario, under optimal taxation, our model predicts a rise in the average risk-free rate of around 2 percent. This effect on the risk-free rate can be better understood by comparing the volatility of marginal utility $std(\hat{\lambda}_t)$ in the two cases. One main effect of the tax is to reduce the volatility of marginal utility. Fluctuations in marginal utility provide a measure of uncertainty about future valuations. The lower volatility therefore reflects that agents face less uncertainty after the introduction of the tax. The higher mean risk-free rate can therefore be interpreted as reducing agents' precautionary saving motives.

The second effect of the tax is to reduce the risk premium. This can be explained by the effect of the tax on risk aversion. The carbon tax reduces both consumption and the stock of emissions, with the reduction in the latter being larger. The distance between consumption and the externality therefore rises. In this model, a larger gap between consumption and the externality in turn reduces risk aversion.

In contrast to an endowment economy, in our production economy lower risk aversion affects the dynamics of consumption as it implies a higher elasticity of intertemporal substitution (EIS). In other words, agents' consumption-smoothing motives are reduced under the optimal policy. This willingness to tolerate larger fluctuations in consumption has in turn asset-pricing implications. As agents are less reluctant to reduce consumption during recessions, there is less need to insure against such outcomes. Consequently, the premium required to compensate investors for holding an asset the price of which falls in recessions is also lower.

IV.4.4 WELFARE ANALYSIS

To assess the welfare implications of the optimal policy, Table 3 also shows agents' lifetime utility $E(\mathcal{W}_t)$, where:

$$E(\mathcal{W}_t) = E_0 \left\{ \sum_{t=0}^{\infty} \tilde{\beta}^t \frac{(c_t - \phi x_t)^{1-\sigma}}{1-\sigma} \right\}$$

As can be seen by comparing the value of $E(\mathcal{W}_t)$ across columns (1) and (2), the policy generates a sizeable rise in welfare. This welfare gain illustrates that the fall in the stock of emissions $E(x_t)$ more than compensates for the lower average consumption the tax produces. This result highlights the importance of the elasticity of emissions to a change in the tax. As this elasticity depends on firms' willingness to reduce emissions, we now discuss the role of the abatement technology.

	Laissez-faire	Optimal policy		
	Estimation (1972-2019)	$\theta_1 = 0.05607$	$\theta_1 = 0.28844$	$\theta_1 = 3.4996$
	(1)	(2)	(3)	(4)
<i>Business-cycle variables</i>				
$E(c_t)$	0.5502	0.5206	0.5310	0.5409
$E(x_t)$	848.9287	380.1978	632.2172	777.2627
$E(\mathcal{W}_t)$	-206778.4449	-10649.9577	-43607.6821	-124258.5811
$E(\tau_t)$	0.0000	0.0353	0.0390	0.0433
$std(\tau_t)$	0.0000	0.0063	0.0083	0.0101
<i>Asset-pricing implications</i>				
$400E(r_t^F)$	3.5870	5.4100	4.7417	4.0028
$400E(r_{t+1}^B - r_t^F)$	1.1542	0.6432	0.9176	1.1432
$std(\hat{\lambda}_t)$	2.4445	1.2753	1.7525	2.1893
$E(RRA_t)$	32.1922	12.9862	19.6120	27.1408
$std(\widehat{rra}_t)$	0.5837	0.3045	0.4185	0.5228
<i>Abatement technology</i>				
$E(\mu_t)$	0.0000	0.5269	0.2234	0.0592
$E(f(\mu_t))$	0.0000	0.0094	0.0044	0.0013
$E(\frac{\tau_t c_t}{y_t})$	0.0000	0.0233	0.0423	0.0566

Notes: The first column is the estimated model under the laissez-faire equilibrium, with no abatement and no environmental tax. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match an abatement share $\bar{\mu}$ of 20% and 5%. Note that $E(\mu_t) \neq \bar{\mu}$ in columns (3) and (4), due to the contribution of future shocks to the asymptotic mean of these variables.

Table IV.5: In column (1), the model simulations correspond to the laissez-faire equilibrium. The simulations under the optimal environmental policy are shown in columns (2) to (4). Columns (2) to (4) correspond to different abatement costs, ranging from low to high.

IV.4.5 THE ROLE OF THE ABATEMENT TECHNOLOGY

The purpose of columns (3) and (4) is to illustrate that the effect of the optimal tax critically depends on the efficiency of the abatement technology. In the laissez-faire equilibrium, the externality not being internalized leads firms to spend nothing on abatement. By forcing firms to internalize the externality, the tax incentivizes firms to use the abatement technology to reduce the burden of the tax.

In our preferred scenario, about 55 percent of emissions are abated once the optimal tax is introduced. As shown in the bottom panel of Table IV.5, when θ_1 is above 0.056, less-efficient technology reduces the share of emissions abated $E(\mu_t)$. Note that as abatement-technology efficiency declines, the planner also chooses to allocate a larger fraction of resources to consumption. This reflects that this model embeds a trade-off between consumption and the abatement technology. The marginal cost of renouncing a unit of consumption should equal the marginal benefit from abating one unit of emissions. Consequently, the planner finds it optimal to allocate more resources to consumption as abatement-technology efficiency falls.

As can be seen by comparing $E(\mathcal{W}_t)$ across columns (2) to (4), the size of the welfare gain depends critically on the abatement technology. This illustrates that the distortion caused by the tax can be sizeable if the technology is not sufficiently well-developed. If emissions are costly to abate, the policy has a stronger negative impact on production, as it is more difficult for firms to circumvent the tax. In this case, the

tax generates a smaller drop in emissions, which in turn reduces the policy's welfare gains.

Comparing the effect of the optimal tax on $400E(r_t^F)$ and $400E(r_{t+1}^B - r_t^F)$, the effect on asset prices also depends crucially on θ_1 . Relative to the first-best scenario, the effect of the tax on the risk premium is more muted when the abatement technology is less efficient.

This illustrates that part of the reduction in uncertainty is due to the additional margin provided by the abatement technology. The effect of θ_1 is therefore akin to the adjustment-cost parameter in [Jermann \(1998\)](#). The more efficient is the abatement technology, the easier it is for agents to insure against unexpected shocks. This greater flexibility makes the economy less risky from a consumption-smoothing perspective, which reduces the risk premium and increases the risk-free rate.

IV.4.6 THE COEFFICIENT OF RELATIVE RISK AVERSION

[Table IV.5](#) also lists the average level of risk aversion, where risk aversion is defined as follows:

$$RRA_t = -\frac{u_c''}{u_c'} c_t$$

In the *laissez-faire* equilibrium, this average level is 32. Once the tax is introduced, this falls to around 13. The main effect of the tax is then to increase the distance between consumption and the externality. As in [Campbell and Cochrane \(1999\)](#), risk aversion in our model is determined by "surplus consumption". A greater distance between consumption and the externality therefore implies a lower coefficient of relative risk aversion.

IV.4.7 CLIMATE POLICY AND ASSET PRICES WITH STANDARD PREFERENCES

In many models, the EIS mainly affects quantities, whereas asset-pricing implications are driven by risk aversion (e.g. [Cochrane 2017](#); [Tallarini 2000](#)). In contrast, the financial and macroeconomic implications of our model are tightly linked. The specification with CCN preferences creates this interaction between finance and the environmental policy. This point is illustrated in [Table IV.6](#), which repeats the experiment shown in [Table IV.5](#) using a separable specification. We analyze the effect of the optimal policy in a model in which preferences are as follows:

$$\mathcal{W}_t = E_0 \sum_{t=0}^{\infty} \tilde{\beta}^t \left(\log c_t - \phi \frac{x_t^\chi}{\chi} \right)$$

where, following [Stokey \(1998\)](#), χ is set to 1.2. To ensure comparability, the parameter ϕ is calibrated to imply an optimal tax similar to that obtained in the case of CCN preferences.

With constant relative risk-aversion, the model is no longer able to generate a realistic risk premium in the *laissez-faire* equilibrium. Relative to the case of CCN

preferences, the risk premium falls from about 1.2 percent to essentially 0. In this case, the dichotomy between climate policies and finance is also close to perfect. Indeed, as illustrated in Table IV.6 the introduction of the optimal tax essentially has no effect on the risk-free rate and risk premium. In a model in which risk plays no role, one may therefore be tempted to conclude that climate risk and environmental policies have a negligible effect on financial markets.

The log utility case ($u = \log c_t - \phi \frac{x_t^x}{x}$)

	Optimal policy			
	Laissez-faire	$\theta_1 = 0.05607$	$\theta_1 = 0.48164$	$\theta_1 = 6.4039$
	(1)	(2)	(3)	(4)
Business-cycle variables				
$E(c_t)$	0.5274	0.5136	0.5178	0.5210
$E(x_t)$	804.3029	348.5493	629.2131	745.8746
$E(\mathcal{W}_t)$	-1102.7147	-673.4293	-921.8315	-1043.4189
$E(\tau_t)$	0.0000	0.0389	0.0530	0.0581
Asset-pricing implications				
$400E(r_t^F)$	6.2415	6.2430	6.2425	6.2421
$400E(r_{t+1}^B - r_t^F)$	0.0715	0.0696	0.0704	0.0709
$std(\hat{\lambda}_t)$	0.2194	0.2135	0.2130	0.2134
$E(RRA_t)$	1.0000	1.0000	1.0000	1.0000
$std(\widehat{rra}_t)$	0.0000	0.0000	0.0000	0.0000
Abatement technology				
$E(\mu_t)$	0.0000	0.5563	0.1999	0.0500
$E(f(\mu_t))$	0.0000	0.0111	0.0055	0.0015
$E(\frac{\tau_t e_t}{y_t})$	0.0000	0.0239	0.0597	0.0781

Notes: The first column shows the results in the laissez-faire (counter-factual) equilibrium, where we use the estimated values obtained for non-separable utility. We calibrate $\phi = 5.7105e - 05$ in order to match the optimal tax obtained in the case of non-separable utility. Column (2) is the equilibrium under an environmental tax with θ_1 set as in the literature. Columns (3) and (4) are equilibria under alternative values of θ_1 that match abatement shares of $\bar{\mu}$ of 20% and 5%. Note that $E(\mu_t) \neq \bar{\mu}$ in columns (3) and (4) due to the contribution of future shocks to the asymptotic mean of these variables.

Table IV.6: Counter factual robustness check – The case of separable utility.

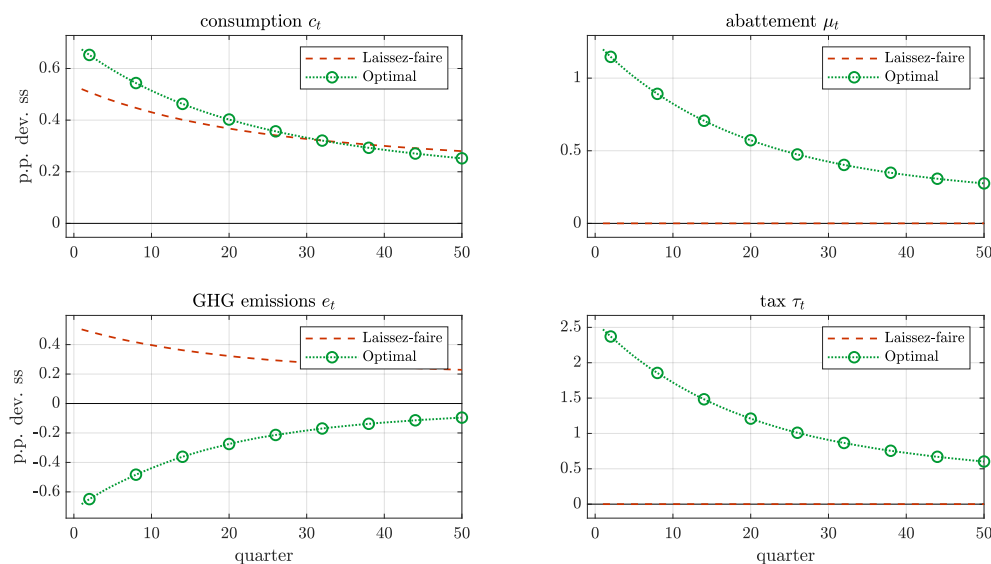
Since we use a log utility specification for consumption, we also tried to increase the curvature coefficient from 1 to 20. We find that increasing curvature has a negligible impact on the risk premium but generates a very large increase in the mean risk-free rate. With a high curvature coefficient, the optimal policy also has no effect on the model's asset-pricing implications. Therefore, the dichotomy between climate policies and finance cannot be broken by a very high value of the curvature coefficient.

IV.4.8 THE RESPONSES TO SHOCKS

Figure IV.4 compares the response of consumption c , abatement μ , emissions e and the optimal tax τ following a positive technology shock. As can be seen by comparing the red crosses to the green circles in the upper-left panel, the first key difference is that the response of consumption on impact is stronger under the optimal policy. This can be explained by the lower EIS. In models with habits, relative risk aversion and the EIS are connected. As the tax reduces risk aversion, it also increases the EIS.

As illustrated in the upper-right panel of [Figure IV.4](#), the second key difference is that the quantity of emissions that firms choose to abate increases sharply during boom periods. Once the optimal policy is introduced, firms therefore find it optimal to use the abatement technology to reduce the burden of the tax.

Figure IV.4: Impulse responses from an estimated TFP shock



Notes: The IRFs are generated using a second-order approximation to the policy function and are expressed as percentage deviations from the deterministic steady state. Estimated parameters are taken at their posterior mean.

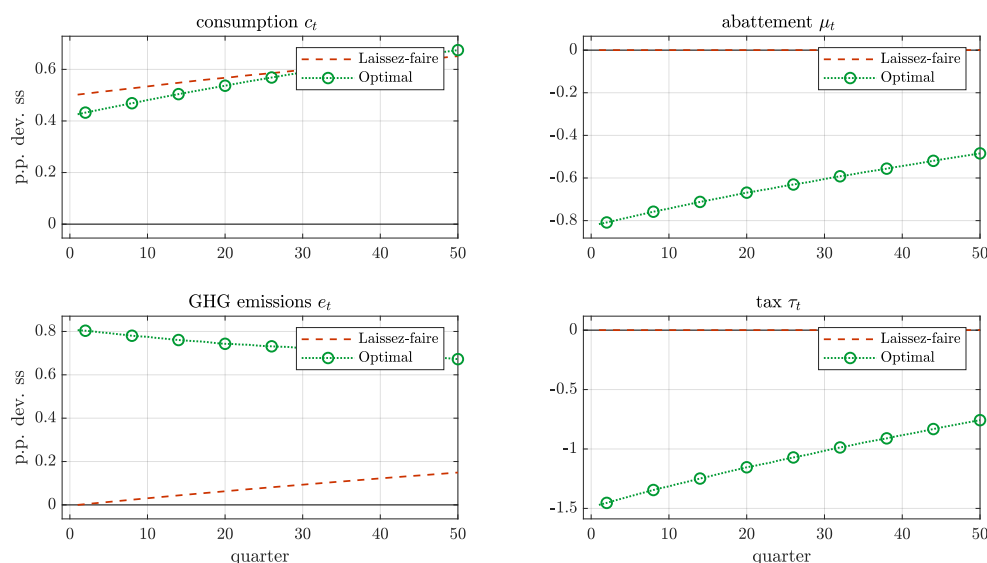
The lower left panel of [Figure IV.4](#) shows that the pro-cyclical response of the abatement technology implies lower emissions under the optimal policy. In contrast to the *laissez-faire* equilibrium, emissions therefore become counter-cyclical once the optimal tax is introduced.

Finally, the lower-right panel of [Figure IV.4](#) depicts the response of the optimal tax, which is constant and equal to zero in the *laissez-faire* equilibrium. As in [Ljungqvist and Uhlig \(2000\)](#), the optimal tax is pro-cyclical when the economy is hit by a technology shock. Relative to the decentralized equilibrium, the planner therefore chooses to cool down the economy during booms.

The response to an investment-specific technology shock is shown in [Figure IV.5](#). This shock generates a negative co-movement between consumption and investment. Relative to the *laissez-faire equilibrium*, the optimal policy attenuates the fall in investment by reducing the tax as well as abatement. Introducing this shock reduces the volatility of investment, which in turn explains the lower value of the adjustment-cost parameter that we find compared to [Jermann \(1998\)](#).

The response to a government spending shock is shown in [Figure IV.6](#). In both cases, a positive government-spending shock reduces consumption. In our model, this can first be explained by the negative wealth effect from the shock. On impact, the shock has no effect on production, but increases the share of output allocated to government spending. On impact, consumption and investment therefore have to fall.

This negative wealth effect is reinforced by a negative substitution effect. As in models with habits and adjustment costs, this reflects the increase in the real interest

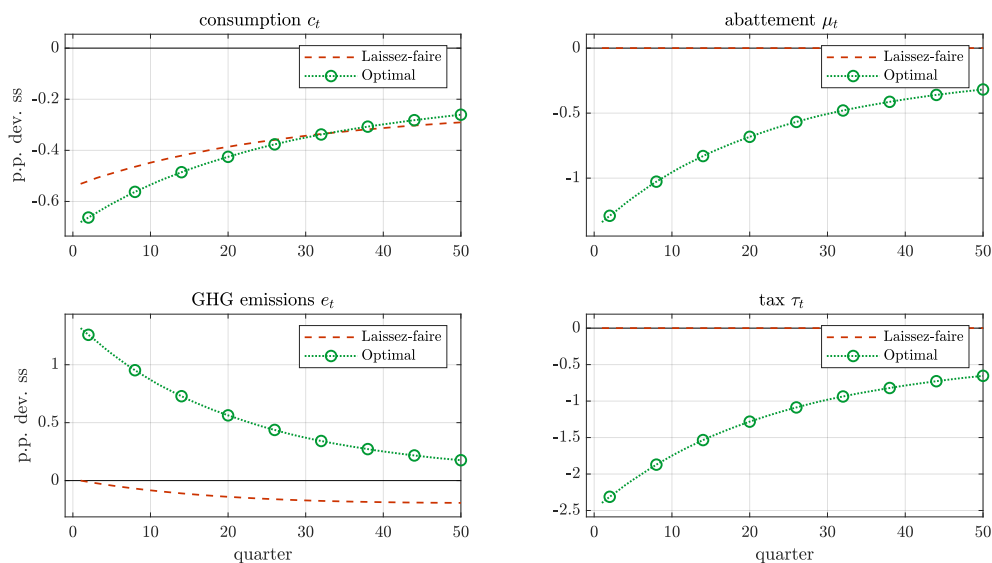
Figure IV.5: Impulse responses from an investment-specific technology shock

rate generated by the shock. As agents become more reluctant to save as consumption falls, the real interest rate has to rise to restore equilibrium.

This illustrates the trade-off between environmental protection and macroeconomic stabilization in this model. Whereas emissions decline in the *laissez-faire* case, the social planner chooses to increase the stock of pollution. The social planner internalizes that the shock reduces the resources available for consumption. It is therefore optimal to mitigate the effect of the shock by lowering abatement as well as the tax (see the upper-right and lower-right panels of Figure IV.6). When the consumption cost is too large, environmental policy is used to mitigate the adverse effect of the shock. In this case, the planner chooses macroeconomic stabilization over environmental protection.

Relative to a standard business-cycle model, the main innovation is the introduction of emission shocks. In the *laissez-faire* equilibrium, consumption falls on impact and then increases above its steady-state level (see the upper-left panel of Figure IV.7). As emission shocks do not affect output, their main effect is to reduce “surplus consumption”. The only way to mitigate the effect of this rise in the emissions stock is then to increase consumption. The problem is that to do so income has to rise first. The only way of raising income in this model is to accumulate capital. This explains why on impact consumption needs to fall. This fall is necessary to finance an increase in investment, which in turn allows agents to increase output. A few quarters after the shock, as the higher investment raises output, consumption gradually increases. The short-term decline in consumption is therefore compensated by a rise in the medium-term. As illustrated by the red dotted line in the upper-left panel of Figure IV.7, consumption initially declines and then increases above its steady state a few periods after the shock.

As can be seen by comparing the red-dotted and green-circled line, the response of consumption and emissions is very different under the optimal policy. The planner chooses to allocate a large fraction of resources to the abatement technology. It is

Figure IV.6: Impulse responses from a government-spending shock

therefore optimal to reduce consumption and investment to finance abatement to prevent emissions from rising.

As illustrated in the lower-right panel, the social planner also chooses to reduce the tax. The tax reduction helps to mitigate the fall in consumption and investment that is necessary to finance abatement.

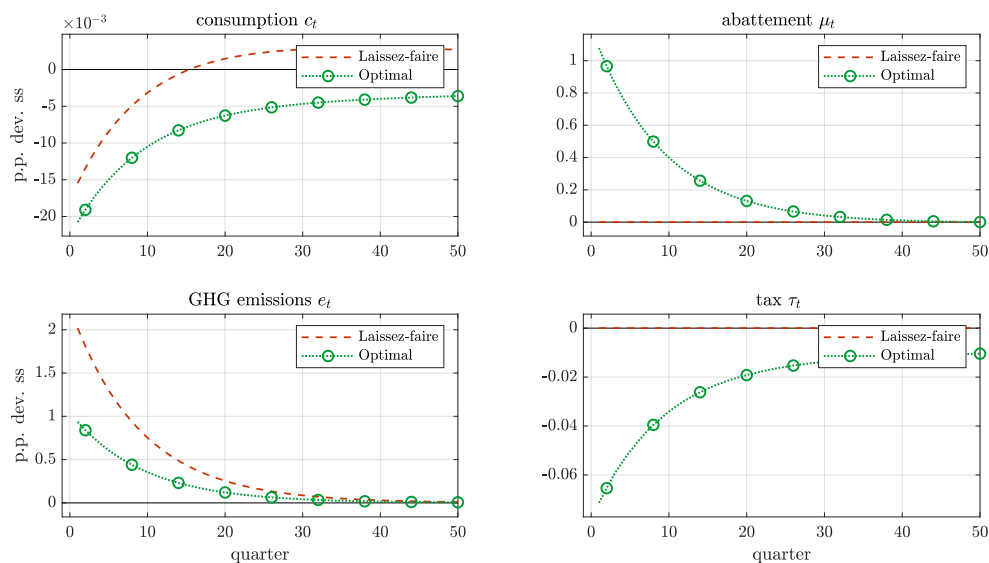
IV.5 ROBUSTNESS CHECKS

This section discusses two robustness checks. First, asset-pricing models are not only evaluated in terms of their ability to match asset market facts. Reproducing the volatility of macroeconomic aggregates, such as consumption, is also an important test for this class of models. Second, since we use a solution method that is relatively novel, we compare it to other nonlinear methods that are more widely-used in the literature.

IV.5.1 THE VOLATILITY OF CONSUMPTION

As discussed in subsection IV.3.4, the model overstates the volatility of consumption when simulated. Using consumption as an observable variable ensures that the model can perfectly reproduce the historical path of consumption growth over the estimation period. However, when simulated using the estimated values for the shock parameters, and as shown in Table IV.3, we obtain that consumption is more volatile than output, which does not fit the facts. This naturally raises the concern that our model's ability to generate realistic asset-pricing facts comes at the cost of implausibly-large fluctuations in consumption growth.

This section shows that this counterfactual implication does not affect the main message of the paper. To illustrate, we consider a simplified version of the model in Section 2 in which technology shocks are the only source of business-cycle fluctuations

Figure IV.7: Impulse responses from an emissions shock

and where all variables grow at the same rate. Then, following the analysis in [Jermann \(1998\)](#), we calibrate the main model parameters to maximize its ability to match a set of moments that includes the volatility of consumption.

To ease the comparison with [Jermann \(1998\)](#), we target the same stylized facts, with one exception, and calibrate a similar set of parameters using the simulated method of moments. In our case, the five parameters are: (i) the adjustment-cost parameter, ϵ ; (ii) the marginal-damage parameter, ϕ ; (iii) the subjective discount factor, β ; (iv) the technology-shock standard deviation, σ_A ; and (v) the shock-persistence parameter, ρ_A . The first four moments to match are the standard deviations of output, consumption and investment, and the mean risk-free rate. Since the model in [Jermann \(1998\)](#) tends to generate excessive risk-free rate variations, we target a risk-free rate standard deviation of 5 percent instead of a 6.18 percent risk premium. The loss function is minimized for the following combination of parameter values:

ϵ	ϕ	β^Y	σ_A	ρ_A
0.36	0.0028	0.993	0.01	0.96

All other parameter values are kept at their estimated values. The moments corresponding to the laissez-faire economy appear in the first column of [Table IV.7](#). Compared to [Jermann \(1998\)](#), the model generates a lower risk-free rate standard deviation and is still able to reproduce the low mean risk-free rate as well as the volatility of macroeconomic aggregates. As regards the moments that were not targeted, shown in the last two rows of [Table IV.7](#), the model generates a bond premium of 3.4 percent. As the carbon tax is zero in the laissez-faire economy, the abatement chosen by firms is constant at a value of zero.

The second column of [Table IV.7](#) lists the simulated moments when the optimal tax is introduced. As in the previous section, we first consider a scenario in which firms are able to abate around 50 percent of all emissions under the tax. The moments in

this scenario appear under the column $\mu = 0.5$. Comparing the laissez-faire economy to the optimal-tax case, the risk-free rate rises by about one percentage point, and the risk premium falls under the optimal policy. The effect on the risk premium is particularly large, as the tax generates a fall of about 2.4 percentage points. Moreover, relative to the analysis from the previous section, this sizeable effect is obtained in a model with one single source of shocks. To sum up, the optimal tax also has sizeable asset-pricing implications in a version of the model that reproduces the fact that consumption is half as volatile as output.

	Laissez-faire Economy	Optimal Policy	
	$\mu = 0.0$	$\mu = 0.5$	$\mu = 0.1$
$std(y)$	1.0	1.0	1.0
$std(c)$	0.5	0.3	0.8
$std(i)$	2.5	0.7	1.5
$400E(r_t^F)$	0.9	1.9	0.8
$400std(r_t^F)$	5.2	2.3	4.8
Non-targeted moments			
$400E(r_{t+1}^B - r_t^F)$	3.4	1.0	3.2
$E(\mu)$	0	0.5	0.1

Table IV.7: Laissez-faire vs. Optimal Policy

The second main takeaway is that this robustness analysis confirms the importance of the abatement technology in our results. If firms can only abate 10 percent of emissions following the tax, the effect on the risk-free rate and the bond premium becomes negligible. This scenario corresponds to the case with an inefficient abatement technology.

This result also confirms that the asset-pricing effect that we obtain depends critically on the additional margin that is activated by the optimal policy. Once the optimal environmental tax is introduced, the abatement technology is used to reduce the amount of consumption risk in the economy. If sufficiently flexible, this margin helps agents to smooth consumption, which in this class of models not only reduces precautionary savings but also the compensation for holding a risky asset such as a long-term bond.

IV.5.2 COMPARISON WITH THE PARTICLE FILTER

In this section, we investigate whether our results continue to hold with alternative filtering methods other than the inversion filter. In the asset-pricing literature, the natural benchmark for non-linear models is particle filtering, as the latter allows likelihood-based inference of nonlinear and/or non-normal macroeconomic models (e.g. van Binsbergen et al., 2012; Andreasen, 2012). The inversion and particle filters are algorithms that recursively update and estimate the state and find the innovations driving a stochastic process given a set of observations.

The inversion filter does so by inverting the model’s recursion rule, while the particle filter uses a sequential Monte Carlo method. Both estimation methods require the use of numerical approximation techniques that introduce error between the “true” value of the parameter and its estimate.

In the implementation of the particle filter, it is common to posit that the data-generating process (DGP) includes measurement errors. As underlined by [Cuba-Borda et al. \(2019\)](#), the presence of measurement error may seem to be an innocuous way of getting around degeneracy issues when choosing a computationally-manageable number of particles. As the number of innovations must be the same as the number of observables, the inversion filter may exhibit misspecification errors if measurement errors are part of the DGP. It is nonetheless standard to assume no measurement errors for linearized models, following [Smets and Wouters \(2007\)](#).

Sample: Filter:	Historical Data		Artificial Data
	(1) Particle	(2) Inversion	(3) Inversion
ESTIMATED PARAMETERS			
Productivity AR(1)	0.9714 [0.9459;0.9851]	0.9727	0.9632
Productivity std	0.0074 [0.0067;0.0080]	0.0076	0.0075
PREMIUM			
Premium laissez-faire	0.7500 [0.6230;0.9118]	0.8412	0.7867
Premium tax policy	0.3516 [0.2851;0.4232]	0.3774	0.3759

Notes: 25,000 iterations of the random-walk Metropolis-Hastings algorithm are drawn for the posterior uncertainty for each model. The maximization of the mode is carried out via simplex optimization routines. The confidence intervals in column(1) are drawn from the posterior uncertainty from 1,000 draws from the Metropolis-Hastings algorithm. The artificial data in column (1) are obtained from 1,000 simulations of the estimated model with the particle-filtering method.

Table IV.8: Outcomes from the particle vs. inversion filters under historical and simulated data

To gauge how much our results are robust to misspecification errors, we estimate our model solved up to the second order with innovations to productivity estimated with output growth as an observable variable. We limit ourselves to productivity shocks as these are the main driver of the risk premium. The rest of the parameters are set to the posterior mean taken from the previous estimation in [Table IV.2](#). We consider three situations: (1) the particle filter algorithm as described in [Fernández-Villaverde and Rubio-Ramírez \(2007\)](#) estimated on US data;⁸¹ (2) the inversion filter estimated on US data; and (3) the inversion filter estimated on 1,000 simulated output-growth data from the particle filter from column (1) that includes measurements error. The latter allows us to see whether measurement errors affect the inference of structural parameters when using the inversion filter. Table 5 shows the results.

The comparison of columns (1) and (2) shows whether the inversion filter and particle filter outcomes differ. The two filters provide a very similar measure of the

⁸¹We use 10,000 particles to approximate the likelihood, and set the variance of the measurement errors to 10% of the sample variance of the observables to help estimation. These values are very standard in the literature.

likelihood function, as the differences in the inference of structural parameters are only minor. In particular, the outcome from the inversion filter always lies in the confidence interval of that from the particle filter, both for the estimated structural parameters and the premium effects. The fact that the lower risk premium from environmental policy is very similar across estimation methods is also reassuring, and suggests that our results may remain similar under alternative filtering methods.

To make sure that the robustness of our results to measurement errors holds unconditionally in larger samples, we follow [Fernández-Villaverde and Rubio-Ramírez \(2005\)](#) and simulate 1,000 output-growth data from the model in column (1). We estimate the model on this artificial data using the inversion filter and list the outcomes in column (3). The inversion filter infers a value that is close to the true parameter values, despite the presence of measurement errors.

IV.6 CONCLUSION

Drawing from the macroeconomic, financial, and environmental literatures, this paper introduces an environmental externality into the neoclassical growth model. Our first main takeaway is that the optimal carbon tax is determined by the implicit price of CO₂ emissions. We then show how to use asset-pricing theory to estimate the optimal carbon tax over the business cycle.

In our economy, risk aversion is higher when firms do not internalize the damage caused by emissions. We show that this higher risk aversion in turn raises risk premia and lowers the natural rate of interest by increasing precautionary saving. In the *laissez-faire* equilibrium, the key is that a fraction of these variations in risk aversion are excessive. The optimal policy therefore eliminates inefficient fluctuations in risk aversion.

The main policy implication is that the effectiveness of the policy critically depends on the abatement technology, so that policy success may depend on the timing of implementation. Clearly, improving the existing emission-abatement technology should come first. Once available, an efficient technology would help to mitigate the side effects of the tax, thereby maximizing the welfare gains from the policy.

As our study focuses primarily on tax policy, future research could investigate how a permits market could affect asset prices and welfare, either by considering the case of asymmetric information,⁸² or by developing a framework where both households and firms are affected by the externality. This type of framework would allow for multi-policy evaluation, such as the comparison of tax and cap-and-trade policies.

Another important limitation of our analysis is that the deterministic growth rate of the economy is given exogenously. On the contrary, abatement choice is endogenously determined, and as we are primarily interested in the cyclicity of the carbon tax, our analysis focuses on business-cycle frequency. Addressing this question in a unified framework in which long-term growth and business cycle fluctuations can be jointly analyzed would be a major step forward.

⁸²Asymmetric information breaks the equivalence between the tax and the permit policy ([Heutel 2012](#)).

We also restrict our analysis to the case of fiscal policy, and do not study the interaction between the carbon tax and other policy instruments. Understanding how the optimal carbon tax will affect the conduct of monetary and macro-prudential policies is another important avenue for further research (e.g. [Benmir and Roman, 2020](#)).

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Part A

International banking in a monetary union

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A.1 INTRODUCTION

By eliminating currency risk, the adoption of the euro in 1999 generated forces for a greater economic and financial integration. The single currency reshaped financial markets and international investment patterns by enhancing cross-border banking activity between the members of the European Monetary Union (EMU). This phenomenon can be measured along various complementary dimensions such as the increase of FDI in bank activities, the diversification of bank assets and liabilities between countries, the access of local banks to international financial sources or through the increase of banks' lending via foreign branches and direct cross-border lending.

This paper focuses more specifically on the consequences of the rise in cross-border loan flows observed since the adoption of the Euro in 1999. Cross-border lending is a distinguishing feature of financial integration in the Eurozone:⁸³ it has been multiplied by 3 in 9 years, reaching a peak value of 120% of GDP in 2008, before experiencing a 25% decrease after the recent financial crisis. Taking a closer look at the data, this cross-border phenomenon is heterogeneous: it affects mainly interbank lending while cross border corporate lending is much small and cross-border lending to households is negligible⁸⁴.

We develop a two-country DSGE model to document how the transmission of asymmetric shocks in the Eurozone has been affected with a banking system that provides cross-border interbank and corporate lending facilities. This solution is original with respect to the existing literature of monetary policy issues in a monetary union. Indeed, most papers related to this topic can roughly be separated in two strands. On the one hand, one-country models such as [Gerali et al. \(2010\)](#), [Darracq-Pariès et al. \(2011\)](#) and [Christiano et al. \(2010\)](#), assume complete banking integration so that all countries are impacted in the same way by the ECB monetary policy. On the other hand, two-country models such [Kollmann et al. \(2011\)](#) ignore the possibility of cross-border funds. In the meanwhile, the fewer models that adopt a middle of the road solution by assuming an imperfect integration of the loan market ([Faia \(2007\)](#));

⁸³See [Figure A.1](#) in the text below.

⁸⁴As underlined by [Figure A.2](#), European banks mainly finance foreign banks on the interbank market and foreign firms on the corporate credit market while mortgage and deposit markets remain strongly segmented in the Eurosystem.

Dedola and Lombardo (2012); Ueda (2012); Dedola et al. (2013)) do not account for the above mentioned heterogeneity in Eurozone cross-border loan flows.

Our paper brings theoretical and empirical contributions. To keep the model tractable, we analyze cross-border loans through home bias in the borrowing decisions concerning interbank and corporate loans using CES function aggregates⁸⁵. Cross-border banking flows are introduced analogously to standard trade channel assuming CES function aggregates. This modeling strategy is flexible as it allows treating in a more compact way two levels of cross-border lending related to interbank loans and corporate loans. The heterogeneity between national financial systems is accounted for through different interest rate set by financial intermediaries. In our setting, bonds are mainly used, as in the intertemporal macroeconomics literature, to allow households to smooth intertemporally consumption and countries to finance current account deficits. Thus, our model does not truly introduce banking but rather reinterpret the financial accelerator from a banking perspective⁸⁶.

To enhance the empirical relevance of the model we introduce a set of nominal, financial and real rigidities. We estimate the model on quarterly data using Bayesian techniques over a sample time period running from 1999Q1 to 2013Q3. The estimation procedure is implemented by splitting the Eurozone in two groups of countries, the core and the periphery. According to our estimates, we find that accounting for cross-border loans strongly improves the fit of the model.

In this setting, we find evidence of the role of cross-border lending channel as an amplifying mechanism for the transmission of asymmetric shocks. First, using Bayesian impulse response functions, we get two main results. In all cases, cross-border lending leads to more diverging investment cycles following either real or financial shocks and, as a consequence, clearly affects the dynamics of the current account with respect to the segmentation of the loan market. Furthermore, by affecting the liquidity of national banking systems, cross-border loans amplify the transmission of a negative financial shock on aggregate activity in the Eurozone. Second, an analysis of the historical variance decomposition shows that for most variables cross-border lending has reduced the impact of national financial shocks on national variables while it has increased the effect of financial shocks on the bilateral current account between core and peripheral countries. Third, we perform a counterfactual exercise to evaluate the effect of cross-border banking in the transmission of the financial crisis between the two groups of countries. We find that peripheral countries have been much more affected by the crisis through a deeper impact on interbank loan shortage and that the degree of cross-border banking affects the time path of the

⁸⁵Home bias in the borrowing decisions catches up some extra costs involved by cross-border activities, such as increasing monitoring costs due to the distance, differences in legal systems and payments, etc. These iceberg costs are closely related to home bias as underlined by Obstfeld and Rogoff (2001).

⁸⁶As a first modeling choice, we do not attempt to model explicitly the balance sheet of the banking system but we try to capture the key elements relevant to our analysis, namely the way the accelerator is affected by cross-border lending. We thus depart from some recent papers where the balance sheet of the banking system lies at the heart of the analysis such as Angeloni and Faia (2013) (that provide an integrated framework to investigate how bank regulation and monetary policy interact when the banking system is fragile and may be subject to runs depending on their degree of leverage) or Gertler and Karadi (2012) (where financial intermediaries face endogenously determined balance sheet constraints to evaluate the effects of unconventional monetary policy decisions to dampen the effect of the financial crisis).

main national macroeconomic indicators.

The rest of the paper is organized as follows: [Appendix A.2](#) presents some stylized facts and a quick summary of the related literature. [Appendix A.3](#) describes the financial component of model. [Appendix A.4](#) presents the real component of the model. [Appendix A.5](#) presents the data and the econometric method. [Appendix A.6](#) uses Bayesian IRFs to evaluate the consequences of cross-border bank lending on the transmission of asymmetric real and financial shocks. [Appendix A.7](#) provides a quantitative evaluation of the consequences of cross-border flows on the volatility of representative aggregates. [Appendix A.8](#) concludes.

A.2 STYLIZED FACTS AND RELATED LITERATURE

A.2.1 CROSS-BORDER LENDING IN THE EUROZONE

Cross-border lending is a distinguishing feature of financial integration in the Eurozone. As reported in [Figure A.1](#), between 1999Q1-2012Q1, cross-border loans have increased much more between participating counties than between the Eurozone and the European Union, and even much more than with countries outside Europe. The rise in cross-border loans is peaking in 2008, where cross-border loans represented 300% of the value initially observed in 1999. The financial crisis is characterized by a 25% drop in cross-border lending between Euro partners. In 2008, cross-border lending represented around 120% of GDP for Eurozone countries at its peak value.

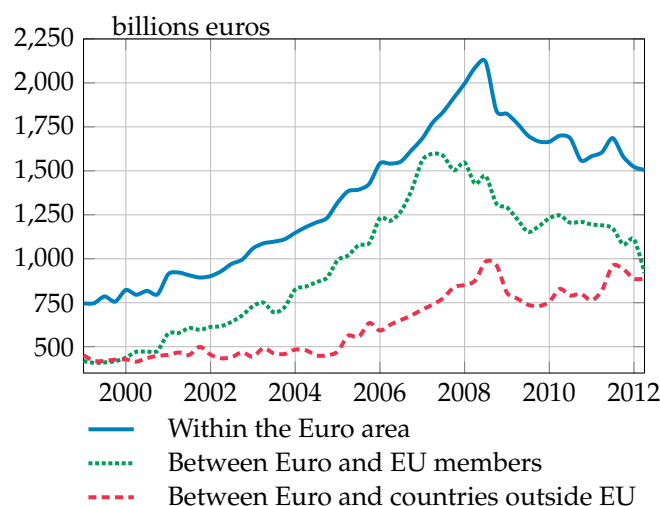


Figure A.1: Cross-border loans of MFIs residing in the euro area between 1999 and 2013 (*Sources ECB*)

However, a closer view at the data underlines the heterogeneity of bilateral flows within the Eurozone. In [Figure A.2](#), we split the Eurozone in two groups: core countries and peripheral countries. In the first group we aggregate data for Germany and France, while in the second group, we aggregate data for Spain, Greece, Ireland, Italy and Portugal. We summarize the main stylized fact by contrasting interbank loans (in panel (a)), corporate loans (in panel (b)) and loans to households (in panel

(c)). Cross border loans are reported as the percentage of loans exported to the other economies either by core countries (plain lines) or peripheral countries (dotted lines). Thus, each curve represents the percentage points of loans exported by the relevant group of countries towards the rest of the Monetary Union.

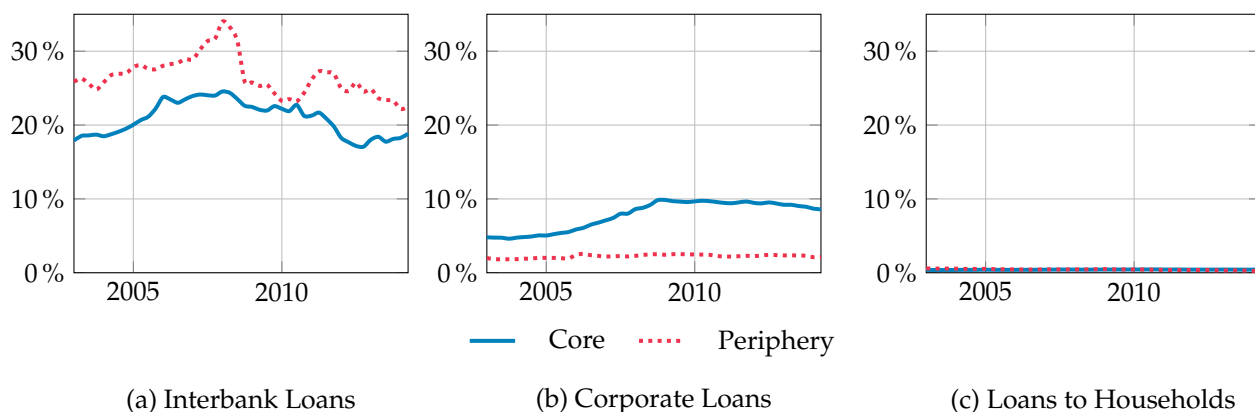


Figure A.2: Share of Cross-Border Loans between EMU participants in the assets of Core and Peripheral Banks

The picture clearly shows the main contribution of interbank loans to cross border lending in the Eurozone, as they represent 25% on average over the sample period for peripheral countries and 20% for core countries. The financial crisis of 2008 had a clear depressing impact on cross-border lending from peripheral countries while it left cross-border lending from core countries almost unchanged. Peripheral countries cross-border lending to firms is low and remains constant over the sample period (averaging 2% of total loan creation) while it has clearly increased for core countries before the propagation of the financial crisis in the Eurozone (from 5% of total national loans in 2003 to 10% in 2008). The financial crisis has affected cross-border lending by stabilizing its level at around 10% over these last years for core countries, while having no noticeable effect for peripheral countries. Finally, cross-border lending to households is almost negligible over the sample period: it represents a constant value of 0.4% of total household loans for core countries and almost the same value on average (with a monotone downward trend) for peripheral countries. As a consequence, cross border interbank lending is clearly the key channel to consider when building our analytical framework

A.2.2 A QUICK SUMMARY OF THE RELATED LITERATURE

Recently a few authors have proposed DSGE models with cross-border lending features to assess the relevance of financial factors in the international transmission of shocks. Moreover, these models fall in one of the three following categories: international financial accelerators (Dedola and Lombardo (2012) and Ueda (2012)), global banks (Kollmann et al. (2011) and Kalemli-Ozcan et al. (2013)) and international borrowing constraint (Faia and Iliopoulos (2011) and Guerrieri et al. (2012)). However, none of these papers analyses the heterogeneity in cross-border lending flows combining corporate and interbank loans.

Concerning the first category of models of international credit cycles, [Dedola and Lombardo \(2012\)](#) suggest that cross-border spillovers result from holding foreign assets by domestic agents. In their model, entrepreneurs solve an endogenous portfolio choice problem composed by home and foreign assets. A variation of asset prices in one economy has side effects on the other economy, as investors sell or buy both domestic and foreign assets. Analytically, this model has to be solved using a second-order approximation to the policy function⁸⁷. In the same vein, [Ueda \(2012\)](#) extends the financial accelerator in a two-country framework and imposes a credit constraint for both entrepreneurs and banks using a financial accelerator mechanism. Under banking globalization, the cost of capital in the economy depends on the capital to net wealth ratio of home and foreign entrepreneur and banks. The model of Ueda is close to Dedola and Lombardo, as entrepreneurs and banks maximize profits that combine an average of home and foreign funds calibrated in steady state.

Turning to the second category, both [Kollmann et al. \(2011\)](#) and [Kalemli-Ozcan et al. \(2013\)](#) consider a two-country environment with a global banking sector. When the capitalization of global banks declines, it reduces credit supply and depresses economic activity in both countries. In their setting, financial frictions are reinforced by the fact that bank losses raise intermediation costs in both countries, triggering synchronized business fluctuations. However, these models consider a homogeneous banking system in the Eurozone while we introduce asymmetries in lending rate settings and financial shocks between the core and the periphery to account for financial heterogeneity in the Eurozone.

[Faia and Iliopoulos \(2011\)](#) develop a small open economy DSGE model with durable and non-durable goods sectors where households face a collateral constraint on the foreign level of debt. The model offers a reduced form of the banking system and concentrates on housing that is financed through foreign lending. We do not use this model for our purposes given the marginal flows of cross-border loans for house purchases encountered in Eurozone data as showed in [Figure A.2](#). Furthermore, as a small open country model, it cannot be kept for our analysis that requires a two-country model. Finally, in the model developed by [Guerrieri et al. \(2012\)](#), banks grant loans to firms and invest in bonds issued by home and foreign government. The model is calibrated on the Euro area. In a two-country set-up, there are core and peripheral countries where large contractionary shocks trigger sovereign default. This model is well suited to analyze the diffusion of sovereign default risk in the Eurozone as shock to the value of peripheral bonds have side effects on the core economy. The model is also very rich in terms of financial frictions. However, the model is aimed at evaluating the diffusion of a sovereign debt crisis, a topic not covered in this paper.

One of the novelties of our analysis is to provide a simple way to model cross-border lending activity to account for the previous stylized facts. To take our two-country model to the data easily, we assume that the banking system determines the loan interest rate while the quantity of loans that is contracted is determined by loan demand. Thus, in this paper, rather than assuming that loans result of optimal portfolio choices from the supply side of the credit market, we suppose that the cross-border

⁸⁷Bringing the model to the data is very challenging as if the estimated variance of the shocks is too big (which is mainly the case with financial data), the IRFs may diverge with a second order approximation. The solution we adopt in this paper does not need a second order approximation contrary to their framework.

decisions arise from the demand side of credit market. International financial linkages are analogous to the external trade channels, assuming that a CES function aggregates domestic and international lending. This choice - that borrows from the New Open Economy Macroeconomics (NOEM) - remains quite simplistic but offers an interesting feature when going to the empirical estimation of the model and a simple reinterpretation of the financial accelerator from a banking perspective.

A.3 A MONETARY UNION WITH CROSS-BORDER LOANS

We describe a two-country world. The two countries are equal in size and share a common currency. Each country $i \in \{h, f\}$ (where h is for home and f for foreign) is populated by consumers, labor unions, intermediate and final producers, entrepreneurs, capital suppliers and a banking system. Regarding the conduct of macroeconomic policy, we assume national fiscal authorities and a common central bank. As in [Christiano et al. \(2005\)](#) and [Smets and Wouters \(2003, 2007\)](#), we account for several sources of rigidities to enhance the empirical relevance of the model. The set of real rigidities encompasses consumption habits, investment adjustment costs, loan demand habits. Regarding nominal rigidities, we account for stickiness in final goods prices, wages and loan interest rates.

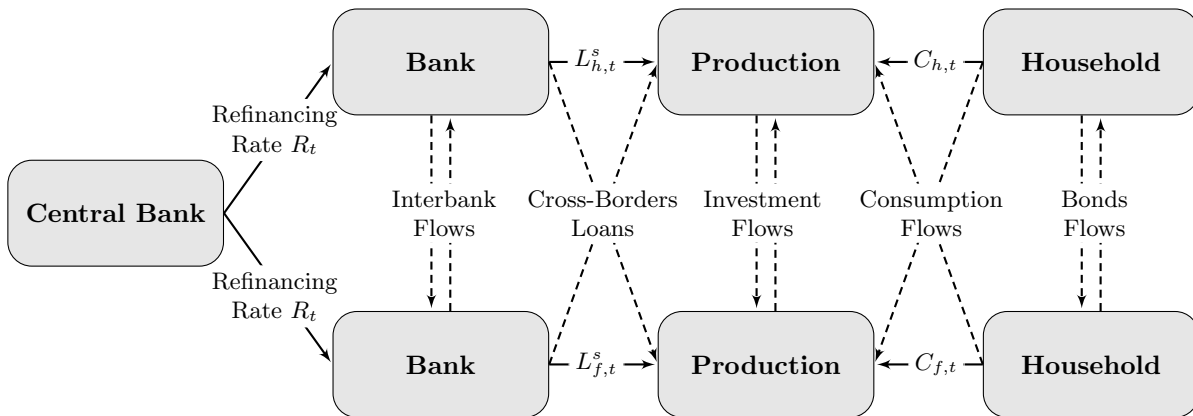


Figure A.3: The model of a two-country monetary union with international bank loan flows

The general structure of the model is summarized in [Figure A.3](#). For expository purposes, this section describes the financial component of the model. We first outline the structure of the banking system that gives rise to cross-border interbank loan, then we describe the origin of cross-border corporate loans. The standard new Keynesian and RBC components of the model are presented afterward in [Appendix A.4](#).

A.3.1 AN HETEROGENOUS BANKING SYSTEM

In each country, the banking sector finances investment projects to home and foreign entrepreneurs by supplying one period loans. The banking system is heterogeneous with regard to liquidity, and banks engage in interbank lending at the national and

international levels. Thus, cross-border loans are made of corporate loans (between banks and entrepreneurs) and interbank loans.

To introduce an interbank market, we suppose that the banking system combines liquid and illiquid banks. Normalizing the total number of banks in each economy to 1, we assume that banks distributed over $[0, \lambda]$ are illiquid (*i.e.* credit constrained), while the remaining banks distributed over share $[\lambda, 1]$ are liquid and supply loans to entrepreneurs and to illiquid banks. We assume that a liquid bank is characterized by her direct accessibility to the ECB fundings. Conversely, an illiquid bank does not have access to the ECB fundings. This assumption is empirically motivated: in the Eurosystem, only a fraction of the 2500 banks participates regularly to the bidding process in main refinancing operations of the ECB while the others rely on interbank funding, as underlined by [Gray et al. \(2008\)](#). Extending this assumption in an international perspective, illiquid banks can borrow from both domestic and foreign liquid banks, which gives rise to cross-border interbank lending flows.

Illiquid Banks: The representative illiquid bank $b \in [0, \lambda]$ in country i operates under monopolistic competition to provide a quantity of loans $L_{i,t+1}^s(b)$ to entrepreneurs that is financed by interbank loans $IB_{i,t+1}(b)$ from the interbank market (with a one period maturity) at a rate $P_{i,t}^{IB}$. The balance sheet of the bank writes,

$$L_{i,t+1}^s(b) = IB_{i,t+1}^H(b) + BK_{i,t+1}(b) + liab_{i,t}, \quad (\text{A.1})$$

where $L_{i,t+1}^s(b)$ is the loan supply of borrowing banks, $IB_{i,t+1}^H(b)$ is the interbank loans supplied by liquid banks subject to external habits, $BK_{i,t+1}(b)$ is the bank capital and $liab_{i,t}$ are other liabilities in the balance sheet of the bank that are not considered in the model⁸⁸. We suppose that the demand for interbank funds are subject to external habits at a degree h_i^{ib} where, $IB_{i,t+1}^H(b) = IB_{i,t+1}^d(b) - h_i^{ib} \left(IB_{i,t+1}^d(b) - \overline{IB}_i^d \right)$. These habits are deemed necessary to catch up the high autocorrelation observed in the supply of loans⁸⁹.

This bank engages in corporate loans. In this setting, we assume that there is no discrimination between borrowers, so that the representative and risk-neutral bank serves both domestic and foreign entrepreneurs without taking into account specificities regarding the national viability of projects. Bank default expectation regarding entrepreneurs' projects is defined as, $\eta_{i,t+1} \equiv (1 - \alpha_i^L) \eta_{h,t+1}^E + \alpha_i^L \eta_{f,t+1}^E$, where $\eta_{i,t+1}^E$ is the default rate in country $i \in \{h, f\}$ of entrepreneurs and $(1 - \alpha_i^L)$ measures the home bias in corporate loan distribution. Thus, the marginal cost of one unit of corporate loan $MC_{i,t}^{ill}(b)$ provided by the illiquid bank is the solution of the expected profit $\mathbb{E}_t \Pi_{i,t+1}^B(b)$ optimization problem,

$$\max_{L_{i,t+1}^s(b)} \mathbb{E}_t \eta_{i,t+1} MC_{i,t}^{ill}(b) L_{i,t+1}^s(b) - P_{i,t}^{IB} \left(L_{i,t+1}^s(b) - BK_{i,t+1}(b) - liab_{i,t}(b) \right). \quad (\text{A.2})$$

⁸⁸We suppose that they follow an exogenous $AR(1)$ shock process $\varepsilon_{i,t}^B$ such that, $liab_{i,t} = e^{\varepsilon_{i,t}^B} \overline{liab}_i$, this shock captures some aggregate movements in the capital constraint of banks.

⁸⁹In the fit exercise, DSGE models with banking are estimated on the outstanding amount of loans contracted in the economy. Since DSGE models only include one-period maturity loans, external habits are a tractable way to catch up the high persistence in the loan contracts without modifying the steady state. [Guerrieri et al. \(2012\)](#) develops a similar financial friction in the borrowing constraint of entrepreneurs.

The marginal cost of one unit of loan, denoted $MC_{i,t}^{ill}(b)$, is the same across illiquid banks,

$$MC_{i,t}^{ill}(b) = MC_{i,t}^{ill} = \frac{P_{i,t}^{IB}}{\mathbb{E}_t \eta_{i,t+1}}, \quad (\text{A.3})$$

so that each bank decides the size of the spread depending on the expected failure rate of its customers $\mathbb{E}_t \eta_{i,t+1}$. The bank has access to domestic and foreign interbank loans to meet its balance sheet. The total amount borrowed by the representative bank writes,

$$IB_{i,t+1}^d(b) = \left((1 - \alpha_i^{IB})^{1/\xi} IB_{h,i,t+1}^d(b)^{(\xi-1)/\xi} + (\alpha_i^{IB})^{1/\xi} IB_{f,i,t+1}^d(b)^{(\xi-1)/\xi} \right)^{\xi/(\xi-1)}, \quad (\text{A.4})$$

where parameter ξ is the elasticity of substitution between domestic and foreign interbank funds, α_i^{IB} represents the percentage of cross-border interbank loan flows in the monetary union and $IB_{h,i,t+1}^d(b)$ (resp. $IB_{f,i,t+1}^d(b)$) the amount of domestic (resp. foreign) loans demanded by borrowing bank b in country i . The total cost incurred by illiquid banks to finance interbank loans, $P_{i,t}^{IB}$, is thus defined according to the CES aggregator,

$$P_{i,t}^{IB} = \left((1 - \alpha_i^{IB}) (R_{h,t}^{IB})^{1-\xi} + \alpha_i^{IB} (R_{f,t}^{IB})^{1-\xi} \right)^{1/(1-\xi)}, \quad (\text{A.5})$$

where $R_{h,t}^{IB}$ (resp. $R_{f,t}^{IB}$) is the cost of loans obtained from home (resp. foreign) banks in country i . The decision to borrow from a particular bank is undertaken on the basis of relative interbank national interest rates,

$$IB_{h,i,t+1}^d(b) = (1 - \alpha_i^{IB}) \left[\frac{R_{h,t}^{IB}}{P_{i,t}^{IB}} \right]^{-\xi} IB_{i,t+1}^d(b), \text{ and } IB_{f,i,t+1}^d(b) = \alpha_i^{IB} \left[\frac{R_{f,t}^{IB}}{P_{i,t}^{IB}} \right]^{-\xi} IB_{i,t+1}^d(b).$$

Here, cross-border lending is measured through the values undertaken by $IB_{h,f,t+1}^d(b)$, (*i.e.*, interbank loans contracted by liquid foreign banks from domestic overliquid banks) and symmetrically by $IB_{f,h,t+1}^d(b)$ (*i.e.*, interbank loans contracted by liquid domestic banks from foreign overliquid banks). Finally following [Hirakata et al. \(2009\)](#), the bank capital accumulation process of illiquid banks ($BK_{i,t+1}(b)$) is determined by,

$$BK_{i,t+1}(b) = (1 - \tau^B) \Pi_{i,t}^B(b), \quad (\text{A.6})$$

where τ^B is a proportional tax on the profits of the bank.

Liquid Banks: The representative liquid bank $b \in [\lambda; 1]$ in country i operates under monopolistic competition to provide a quantity of loans $L_{i,t+1}^s(b)$ to entrepreneurs. It also provides a quantity of interbank loans $IB_{i,t+1}^s(b)$ to illiquid banks. We suppose that the intermediation process between liquid and illiquid banks is costly: we introduce a convex monitoring technology à la [Cúrdia and Woodford \(2010\)](#) and [Dib \(2010\)](#) with a functional form $AC_{i,t+1}^{IB}(b) = \frac{\chi_i^{IB}}{2} (IB_{i,t+1}^s(b) - \overline{IB}_i^s(b))^2$ where parameter χ_i^{IB} is the level of financial frictions between liquid banks in country i and home and foreign illiquid banks⁹⁰. Loans created by the liquid bank are financed by one-

⁹⁰Contrary to [Cúrdia and Woodford \(2010\)](#) but in the same vein of [Dib \(2010\)](#), the monitoring technology does not alter the steady state of the model to keep the estimation of χ_i^{IB} as simple as possible. Several papers refer to monitoring technology functions in the intermediation process of

period maturity loans from the central bank ($L_{i,t+1}^{ECB}(b)$) at the refinancing interest rate R_t . Finally, the bank's balance sheet is defined by,

$$L_{i,t+1}^s(b) + IB_{i,t+1}^s(b) = L_{i,t+1}^{ECB}(b) + BK_{i,t+1}(b) + liab_t(b).$$

According to the behavior of illiquid banks, we assume that there is no discrimination between borrowers. The marginal cost of one unit of loan $MC_{i,t}^{liq}(b)$ solves the profit ($\Pi_{i,t}^B(b)$) maximization problem,

$$\max_{L_{i,t+1}^s(b), IB_{i,t+1}^s(b)} \mathbb{E}_t \eta_{i,t+1} MC_{i,t}^{liq}(b) L_{i,t+1}^s(b) + R_{i,t}^{IB}(b) IB_{i,t+1}^s(b) - R_t L_{i,t+1}^{ECB}(b) - AC_{i,t+1}^{IB}(b). \quad (\text{A.7})$$

The marginal cost of one unit of loan is the same for all liquid banks,

$$MC_{i,t}^{liq}(b) = MC_{i,t}^{liq} = \frac{R_t}{\mathbb{E}_t \eta_{i,t+1}}. \quad (\text{A.8})$$

Similarly to the illiquid bank, bank capital evolves according to [Equation A.6](#)⁹¹.

Loan interest rates: There are two interest rates to be determined: the interest rate on the interbank market and the interest rate on corporate loans. First, on a perfectly competitive market, the interbank rate in country i is determined from the problem [\(A.7\)](#),

$$R_{i,t}^{IB}(b) = \chi_i^{IB} (IB_{i,t+1}^s(b) - \overline{IB}_i^s(b)) + R_t, \quad (\text{A.9})$$

where, χ_i^{IB} is a cost parameter, $IB_{i,t+1}^s(b)$ is the amount of interbank loans contracted in period t with a one period maturity and $\overline{IB}_i^s(b)$ is the steady state value of interbank loans.

Second, the interest rate charged by banks of country i on corporate loans accounts for the liquidity of the national banking system. Anticipating over symmetric issues at the equilibrium to improve the tractability of the model, we assume that all banks belonging to a national banking system share the same marginal cost of production, reflecting the average liquidity degree of national banks. Thus, aggregating over each group of banks, we get, $\int_0^\lambda MC_{i,t}^{ill}(b) db = MC_{i,t}^{L,ill}$, and $\int_\lambda^1 MC_{i,t}^{liq}(b) db = MC_{i,t}^{L,liq}$. Aggregate marginal cost $MC_{i,t}^L$ combines outputs from liquid and illiquid banks of country i according to⁹²,

$$MC_{i,t}^L = (MC_{i,t}^{ill})^\lambda (MC_{i,t}^{liq})^{(1-\lambda)} = \frac{(P_{i,t}^{IB})^\lambda (R_t)^{(1-\lambda)}}{\mathbb{E}_t \eta_{i,t+1}}. \quad (\text{A.10})$$

Thus, the representative bank $b \in [0; 1]$ of country i operates under monopolistic competition to provide a quantity of loans $L_{i,t+1}^s(b)$ incurring a marginal cost $MC_{i,t}^L$. The marginal cost is the same for all banks b and depends on the expected failure rate of borrowers' projects and the central bank refinancing rate. [Equation A.10](#) taken in logs

banks, see for example [Goodfriend and McCallum \(2007\)](#) or [Casares and Poutineau \(2011\)](#).

⁹¹The accumulation of bank capital is necessary to close the model but it is not binding for liquid banks as they are not credit constrained.

⁹²We borrow this aggregation procedure from the solution introduced by [Gerali et al. \(2010\)](#), to aggregate borrowing and saving households labor supply.

becomes,

$$\widehat{mc}_{i,t}^L = \frac{1}{(1 - \bar{N}/\bar{K})} \left[(1 - \alpha_i^L) (1 - \varkappa_i) \widehat{kn}_{i,t} + \alpha_i^L (1 - \varkappa_j) \widehat{kn}_{j,t} \right] + (1 - \lambda) \hat{r}_t + \lambda \hat{p}_{i,t}^{IB}, \quad \forall i \neq j \in \{h, f\},$$

where, $\widehat{kn}_{i,t}$ is the capital to net worth ratio of entrepreneurs and \bar{N}/\bar{K} is the steady state net worth to capital ratio. Under Calvo pricing with partial indexation, banks set the interest rate on loans contracted by entrepreneurs on a staggered basis as in [Darracq-Pariès et al. \(2011\)](#). A fraction θ_i^L of banks is not allowed to optimally set the credit rate⁹³ and index it by ξ_i^L percent of the past credit rate growth, $R_{i,t}^L(b) = (R_{i,t-1}^L/R_{i,t-2}^L)^{\xi_i^L} R_{i,t-1}^L(b)$. Assuming that it is able to modify its loan interest rate with a constant probability $1 - \theta_i^L$, it chooses $R_{i,t}^{L*}(b)$ to maximize its expected sum of profits,

$$\max_{\{R_{i,t}^{L*}(b)\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i^L \beta)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \eta_{i,t+1+\tau} \left[(1 - \tau^L) R_{i,t}^{L*}(b) \Xi_{i,t,\tau}^L - MC_{i,t+\tau}^L \right] L_{i,t+1+\tau}(b) \right\},$$

subject to, $L_{i,t+1+\tau}(b) = (\Xi_{i,t,\tau}^L R_{i,t}^{L*}(b) / R_{i,t+\tau}^L)^{-\mu_{i,t+\tau}^L / (\mu_{i,t+\tau}^L - 1)} L_{i,t+1+\tau}$, $\forall \tau > 0$, where $\Xi_{i,t,\tau}^L = \prod_{k=1}^{\tau} (R_{i,t+k-1}^L / R_{i,t+k-2}^L)^{\xi_i^L}$ is the sum of past credit rate growth and $L_{i,t}(b)$ denotes the quantity of differentiated banking loans b that is used by the retail banks⁹⁴. The time-varying markup is defined by, $\mu_{i,t}^L = \mu_L + \varepsilon_{i,t}^L$, so that an increase in $\varepsilon_{i,t}^L$ can be interpreted as a cost-push shock to the credit rate equation⁹⁵. As [Benigno and Woodford \(2005\)](#), we introduce a proportional tax τ^L on profits that restores the first-best allocation in the steady state. Allowing for a partial indexation of credit interest rates on their previous levels (where $\xi_i^L \in [0; 1]$ is the level of indexation that catches some imperfect interest rate pass-through with θ_i^L), and imposing symmetry, the log equation of the real loan interest rate in country i is set according to,

$$\hat{r}_{i,t}^L = \frac{1}{1 + \beta(1 + \xi_i^L)} \left(\begin{aligned} & (1 + \xi_i^L(1 + \beta)) \hat{r}_{i,t-1}^L - \xi_i^L \hat{r}_{i,t-2}^L + \beta \mathbb{E}_t \hat{r}_{i,t+1}^L \\ & + \beta \theta_i^L \mathbb{E}_t \hat{\pi}_{i,t+2}^c - (1 + \beta \theta_i^L) \mathbb{E}_t \hat{\pi}_{i,t+1}^c + \hat{\pi}_{i,t}^c \\ & + \frac{(1 - \theta_i^L)(1 - \theta_i^L \beta)}{\theta_i^L} \left[\widehat{mc}_{i,t}^L - \hat{r}_{i,t}^L \right] \end{aligned} \right) + \varepsilon_{i,t}^L. \quad (\text{A.11})$$

Solving this equation forward, one can see that past, current and expected future marginal cost of loans are driving today's loan interest rate. With fully flexible rates ($\theta_i^L = 0$), the loan interest rate $\hat{r}_{i,t}^L$ is a function of the interest rate and the expected profitability share of investment projects, that is $\hat{r}_{i,t}^L = \widehat{mc}_{i,t}^L + \varepsilon_{i,t}^L = \hat{r}_t - E_t \hat{\eta}_{i,t+1} + \varepsilon_{i,t}^L$. Since credit risk is measured by the level of firm leverage in the economy, credit rates reflect both past and future risk in the economy caught up by parameters ξ_i^L and θ_i^L .

⁹³This parameter, once estimated in the next section, will serve as a measure to measure the flexibility of national banking systems in the transmission of interest rate decisions.

⁹⁴Retail banks are perfectly competitive loan packers, they buy the differentiated loans and aggregate them through a CES technology into one loan and sell them to entrepreneurs.

⁹⁵Differentiated loans are imperfect substitutes, with elasticity of substitution denoted by $\frac{\mu_L}{(\mu_L - 1)}$.

A.3.2 ENTREPRENEURS AND CORPORATE LOANS

Cross-border corporate loans occur between entrepreneurs and banks. In each economy, the representative entrepreneur $e \in [0, 1]$ finances the capital renting of intermediate firms. In period t , entrepreneur e conducts a great number of heterogenous projects with total value $Q_{i,t}K_{i,t+1}(e)$, (where $Q_{i,t}$ is the price of capital and $K_{i,t+1}(e)$ is the amount of capital financed). These projects are financed by his net wealth and by loans from the banking system ($L_{i,t+1}^d(e)$). The balance sheet of the representative entrepreneur writes,

$$Q_{i,t}K_{i,t+1}(e) - N_{i,t+1}(e) = L_{i,t+1}^H(e). \quad (\text{A.12})$$

where $L_{i,t+1}^H(e) = L_{i,t+1}^d(e) - h_i^L(L_{i,t}^d - L_i^d)$ denotes external demand habits for loans⁹⁶. The entrepreneur has access to domestic and foreign banks to meet its balance sheet. The total amount borrowed by the representative entrepreneur writes,

$$L_{i,t+1}^d(e) = \left((1 - \alpha_i^L)^{1/\nu} L_{h,i,t+1}^d(e)^{(\nu-1)/\nu} + (\alpha_i^L)^{1/\nu} L_{f,i,t+1}^d(e)^{(\nu-1)/\nu} \right)^{\nu/(\nu-1)}, \quad (\text{A.13})$$

where parameter ν is the elasticity of substitution between domestic and foreign loans, α_i^L represents the percentage of cross-border loan flows in the monetary union and $L_{h,i,t+1}^d(e)$ (resp. $L_{f,i,t+1}^d(e)$) the amount of domestic (resp. foreign) loans demanded by entrepreneur e in country i . The total cost of loans, $P_{i,t}^L$, is thus defined according to,

$$P_{i,t}^L(e) = \left((1 - \alpha_i^L) R_{h,t}^L(e)^{1-\nu} + \alpha_i^L R_{f,t}^L(e)^{1-\nu} \right)^{1/(1-\nu)}, \quad (\text{A.14})$$

where $R_{h,t}^L(e)$ (resp. $R_{f,t}^L(e)$) is the cost of loans obtained from home (resp. foreign) banks by entrepreneur e in country i . The decision to borrow from a particular bank is undertaken on the basis of relative national interest rates,

$$L_{h,i,t+1}^d(e) = (1 - \alpha_i^L) \left[\frac{R_{h,t}^L(e)}{P_{i,t}^L(e)} \right]^{-\nu} L_{i,t+1}^d(e), \quad \text{and,} \quad L_{f,i,t+1}^d(e) = \alpha_i^L \left[\frac{R_{f,t}^L(e)}{P_{i,t}^L(e)} \right]^{-\nu} L_{i,t+1}^d(e).$$

The investment projects undertaken by the entrepreneur are risky and differ with respect to their individual returns. To model individual riskiness, we assume that each project has an individual return equal to $\omega R_{i,t}^k$, i.e. that the aggregate return of investment projects in the economy $R_{i,t}^k$ is multiplied by a random value ω (drawn from a Pareto distribution⁹⁷). Defining the value for a profitable project by $\bar{\omega}_{i,t}(e) = E(\omega | \omega \geq \omega_{i,t}^C(e))$ (where $\omega_{i,t}^C(e)$ is the critical value of ω that distinguishes profitable and non profitable projects), the profit function of entrepreneur e after aggregating

⁹⁶These lending demand habits are deemed necessary to replicate the dynamic of loans. In the estimation exercise, we use the total stock of loans, they are of different maturities implying a strong autocorrelation. Simply by introducing loan demand habits, taking into account the high autocorrelation of loans becomes tractable easily and does not change the steady state of the model.

⁹⁷With respect to the canonical framework standardly used in the literature (Bernanke et al., 1999), we assume that the heterogeneity in the return of investment project undertaken by firms is modeled using a Pareto distribution. This device commonly used in other branches of the economic literature provides a series of interesting features in the analysis and allows an easier estimation of the financial amplification effect.

all projects writes,

$$\Pi_{i,t+1}^E(e) = \begin{cases} \bar{\omega}_{i,t+1} R_{i,t+1}^k Q_{i,t} K_{i,t+1}(e) - P_{i,t}^L(e) L_{i,t+1}^H(e) & \text{with probability } \eta_{i,t+1}^E, \\ 0 & \text{with probability } 1 - \eta_{i,t+1}^E, \end{cases} \quad (\text{A.15})$$

where $\eta_{i,t+1}^E$ is the time-varying expected share of gainful projects. Since entrepreneurs cannot screen the value of $\bar{\omega}_{i,t+1}(e)$ *ex ante*, $\omega_{i,t}^C(e)$ cannot be a control variable of the financial contract between borrowers and lenders contrary to [Bernanke et al. \(1999\)](#). To introduce a financial accelerator mechanism, we borrow a concept of [De Grauwe \(2010\)](#) applied in a different context, by assuming that entrepreneurs' forecasts regarding the aggregate profitability of a given project $\bar{\omega}_{i,t}(e)$ are optimistic (*i.e.*, biased upwards)⁹⁸. The perceived *ex ante* value of profitable projects is defined by the isoelastic function,

$$g(\bar{\omega}_{i,t+1}, \varepsilon_{i,t}^Q) = \gamma_i (\bar{\omega}_{i,t+1})^{\frac{\varkappa_i}{(\varkappa_i-1)}} \left(e^{\varepsilon_{i,t}^Q} \right)^{\frac{1}{(\varkappa_i-1)}},$$

where $\varepsilon_{i,t}^Q$ is an $AR(1)$ process⁹⁹, \varkappa_i is the elasticity of the external finance premium¹⁰⁰ and γ_i is a scale parameter¹⁰¹. In this expression, the exogenous shock is affected by exponent $1/(\varkappa_i - 1)$ to normalize to unity the impact of the financial shock $\varepsilon_{i,t}^Q$ in the log deviation form of the model. Thus, *ex-ante* the entrepreneur chooses a capital value of $K_{i,t+1}(e)$ that maximizes its expected profit defined as,

$$\max_{\{K_{i,t+1}(e)\}} \mathbb{E}_t \left\{ \eta_{i,t+1}^E \left[g(\bar{\omega}_{i,t+1}, \varepsilon_{i,t}^Q) R_{i,t+1}^k Q_{i,t} K_{i,t+1}(e) - P_{i,t}^L(e) L_{i,t+1}^H(e) \right] \right\}. \quad (\text{A.16})$$

Using the characteristics of the Pareto distribution, the expected spread required by representative entrepreneur e to undertake the decision to finance firms' investment is,

$$S_{i,t}(e) = \frac{\mathbb{E}_t R_{i,t+1}^k}{P_{i,t}^L(e)} = \gamma_i^{\varkappa_i-1} \left[\frac{\kappa}{\kappa-1} \left(1 - \frac{N_{i,t+1}(e)}{Q_{i,t} K_{i,t+1}(e)} \right) \right]^{\varkappa_i} e^{\varepsilon_{i,t}^Q}. \quad (\text{A.17})$$

The size of the accelerator is determined by the elasticity of the external finance premium \varkappa_i . For $\varkappa_i > 0$, the external finance premium is a positive function of capital to net wealth ratio, $Q_{i,t} K_{i,t+1}(e) / N_{i,t+1}(e)$, so that an increase in net wealth induces a reduction of the external finance premium. This phenomenon disappears if $\varkappa_i = 0$. Concerning the exogenous movements of the external finance premium, a positive realization of $\varepsilon_{i,t}^Q$ means that entrepreneurs require a higher expected profitability of capital $E_t R_{i,t+1}^k$ to finance investment for a given level of lending conditions $P_{i,t}^L$. Furthermore,

⁹⁸Assuming optimistic firms is motivated empirally, [Bachmann and Elstner \(2013\)](#) find evidence of such expectations for German firms using microdata. The optimistic expectations hypothesis of the private sector is very close to the utility functions introduced by [Goodhart et al. \(2005\)](#) for bankers. In our setting, the financial accelerator does not result from a moral hazard problem but rather from a bias in the expectations of the private sector.

⁹⁹This shock affects the expected profitability of financial projects by rising in exogeneously the risk premium implying an increase in the cost of capital and hence a reduction in investment as underlined by [Gilchrist et al. \(2009b\)](#) for the US economy.

¹⁰⁰The elasticity of the external finance premium expresses the degree of bias in estimating the expected rentability of entrepreneurs' projects such that if $\bar{\omega} > 1$ and $\varkappa_i > 0$ then $g(\bar{\omega}) > \bar{\omega}$. Expressed *à la* [De Grauwe \(2010\)](#), $\mathbb{E}_t^{opt} \bar{\omega}_{i,t+1} = \mathbb{E}_t \gamma_i (\bar{\omega}_{i,t+1})^{\varkappa_i/(\varkappa_i-1)}$ where \mathbb{E}_t^{opt} is the expectation operator of optimistic entrepreneurs.

¹⁰¹This parameter is needed to make the steady state independent of \varkappa_i , such that $\gamma_i = \bar{\omega}^{1/(1-\varkappa_i)}$.

a shock that hits the entrepreneur net wealth $N_{i,t+1}(e)$ will also affect the rentability of the physical capital in the economy. As the rentability of capital is a cost for the intermediate sector, a variation in the net wealth will have aggregate consequences on goods supply through the channel of the capital market as underlined by [Gilchrist et al. \(2009b\)](#). The amount of capital of non-profitable entrepreneurs' investment projects is consumed in terms of home final goods $P_{i,t}(1 - \eta_{i,t}^E) \underline{\omega}_{i,t}(e) R_{i,t}^k Q_{i,t-1} K_{i,t}(e)$. Thus the net wealth of the entrepreneur in the next period is equal to,

$$N_{i,t+1}(e) = (1 - \tau^E) \frac{\Pi_{i,t}^E(e)}{e^{\varepsilon_{i,t}^N}}, \quad (\text{A.18})$$

where $\varepsilon_{i,t}^N$ is an exogenous process of net wealth destruction and τ^E is a proportional tax on the profits of the entrepreneur. Anticipating symmetry between entrepreneurs in equilibrium and aggregating, the log-linear expression of the external finance premium $S_{i,t}$ (i.e. $\hat{s}_{i,t}$) writes as in [Bernanke et al. \(1999\)](#),

$$\hat{s}_{i,t} = \mathbb{E}_t \hat{r}_{i,t+1}^k - \hat{p}_{i,t}^L = \varkappa_i \left(\hat{q}_{i,t} + \hat{k}_{i,t+1} - \hat{n}_{i,t+1} \right) + \varepsilon_{i,t}^Q. \quad (\text{A.19})$$

A.4 THE REST OF THE MODEL

This section describes the real component of the model: households, labour unions, firms, capital suppliers, the authorities and the general equilibrium conditions.

A.4.1 HOUSEHOLDS

In each economy there is a continuum of identical households who consume, save and work in intermediate firms. The total number of households is normalized to 1. The representative household $j \in [0, 1]$ maximizes the welfare index,

$$\max_{\{C_{i,t}(j), H_{i,t}(j), B_{i,t+1}(j)\}} \mathbb{E}_t \sum_{\tau=0}^{\infty} \beta^\tau e^{\varepsilon_{i,t+\tau}^\beta} \left[\frac{(C_{i,t+\tau}(j) - h_i^c C_{i,t-1+\tau}(j))^{1-\sigma_i^c}}{1 - \sigma_i^c} - \chi_i \frac{H_{i,t+\tau}^{1+\sigma_i^L}(j)}{1 + \sigma_i^L} \right], \quad (\text{A.20})$$

subject to,

$$\frac{W_{i,t}^h}{P_{i,t}^c} H_{i,t}(j) + R_{t-1} \frac{B_{i,t}(j)}{P_{i,t}^c} + \frac{\Pi_{i,t}(j)}{P_{i,t}^c} = C_{i,t}(j) + \frac{B_{i,t+1}(j)}{P_{i,t}^c} + \frac{T_{i,t}(j)}{P_{i,t}^c} + \frac{P_{i,t}}{P_{i,t}^c} A C_{i,t}^B(j). \quad (\text{A.21})$$

Here, $C_{i,t}(j)$ is the consumption index, $h_i^c \in [0, 1]$ is a parameter that accounts for consumption habits, $H_{i,t}(j)$ is labor effort, $\varepsilon_{i,t}^\beta$ is an exogenous $AR(1)$ shock to household preferences. The income of the representative household is made of labor income (with nominal wage, $W_{i,t}^h$), interest payments for bond holdings, (where $B_{i,t}(j)$ stands for the bonds subscribed in period $t-1$ and R_{t-1} is the gross nominal rate of interest between period $t-1$ and period t), and earnings $\Pi_{i,t}(j)$ from shareholdings¹⁰².

¹⁰²The nominal amounts of dividends received from final good producers $\Pi_{i,t}^y(j)$ and labor unions $\Pi_{i,t}^w(j)$ writes $\Pi_{i,t}(j) = \Pi_{i,t}^y(j) + \Pi_{i,t}^w(j)$.

The representative household spends this income on consumption, bond subscription and tax payments (for a nominal amount of $T_{i,t}(j)$). Finally, he has to pay quadratic adjustment costs to buy new bonds (Schmitt-Grohé and Uribe, 2003), according to the function, $AC_{i,t}^B(j) = \frac{\chi^B}{2} (B_{i,t+1}(j) - B_i(j))^2$, where $B_i(j)$ is the steady state level of bonds. The first order conditions that solve this problem can be summarized with an Euler bond condition,

$$\frac{\beta R_t}{1 + P_{i,t} \chi^B (B_{i,t+1}(j) - B_i(j))} = \mathbb{E}_t \left\{ \frac{e^{\varepsilon_{i,t}^\beta}}{e^{\varepsilon_{i,t+1}^\beta}} \frac{P_{i,t+1}^c}{P_{i,t}^c} \left(\frac{(C_{i,t+1}(j) - h_i^c C_{i,t})}{(C_{i,t}(j) - h_i^c C_{i,t-1})} \right)^{\sigma_i^c} \right\}, \quad (\text{A.22})$$

and a labor supply function,

$$\frac{W_{i,t}^h}{P_{i,t}^c} = \chi_i H_{i,t}(j)^{\sigma_i^L} (C_{i,t}(j) - h_i^c C_{i,t-1})^{\sigma_i^c}. \quad (\text{A.23})$$

The consumption basket of the representative household and the consumption price index of country i are, $C_{i,t}(j) = \left((1 - \alpha_i^C)^{1/\mu} C_{h,i,t}(j)^{(\mu-1)/\mu} + (\alpha_i^C)^{1/\mu} C_{f,i,t}(j)^{(\mu-1)/\mu} \right)^{\mu/(\mu-1)}$ and $P_{i,t}^c = \left((1 - \alpha_i^C) P_{h,t}^{1-\mu} + \alpha_i^C P_{f,t}^{1-\mu} \right)^{1/(1-\mu)}$ where μ is the elasticity of substitution between the consumption of home ($C_{h,i,t}(j)$) and foreign ($C_{f,i,t}(j)$) goods and α_i^C is the degree of openness of the economy. In this model, we assume home bias in consumption, so that $\alpha_i^C < \frac{1}{2}$.

A.4.2 LABOR UNIONS

Households provide differentiated labor types, sold by labor unions to perfectly competitive labor packers who assemble them in a CES aggregator and sell the homogeneous labor to intermediate firms. Each representative union is related to an household $j \in [0; 1]$. Assuming that the trade union is able to modify its wage with a probability $1 - \theta_i^w$, it chooses the optimal wage $W_{i,t}^*(j)$ to maximize its expected sum of profits,

$$\max_{\{W_{i,t}^*(j)\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i^w \beta)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \left[(1 - \tau^w) \frac{W_{i,t}^*(j)}{P_{i,t+\tau}^c} \prod_{k=1}^{\tau} (\pi_{i,t+k-1}^c)^{\xi_i^w} - \frac{W_{i,t+\tau}^h(j)}{P_{i,t+\tau}^c} \right] H_{i,t+\tau}(j) \right\},$$

subject to the downgrade sloping demand constraint from labor packers, $H_{i,t+\tau}(j) = \left(W_{i,t}^*(j) / W_{i,t+\tau} \prod_{k=1}^{\tau} (\pi_{i,t+k-1}^c)^{\xi_i^w} \right)^{-\mu_{i,t+\tau}^w / (\mu_{i,t+\tau}^w - 1)} H_{i,t+\tau}$, $\forall \tau > 0$, where $H_{i,t}(j)$ denotes the quantity of differentiated labor types j that is used in the labor packer production with time-varying substitutability $\mu_{i,t}^w / (\mu_{i,t}^w - 1)$ between different labor varieties. The first order condition results in the following equation for the re-optimized real wage,

$$\frac{W_{i,t}^*(j)}{P_{i,t}^c} = \frac{\mu_{i,t+\tau}^w}{(1 - \tau^w)} \frac{\mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{(\theta_i^w \beta)^\tau}{(\mu_{i,t+\tau}^w - 1)} \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \frac{W_{i,t+\tau}^h(j)}{P_{i,t+\tau}^c} H_{i,t+\tau}(j)}{\mathbb{E}_t \sum_{\tau=0}^{\infty} \frac{(\theta_i^w \beta)^\tau}{(\mu_{i,t+\tau}^w - 1)} \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \prod_{k=1}^{\tau} \frac{(\pi_{i,t+k-1}^c)^{\xi_i^w}}{\pi_{i,t+k}^c} H_{i,t+\tau}(j)} \quad (\text{A.24})$$

The markup of the aggregate wage over the wage received by the households is taxed by national governments (at rate τ_i^w that cancels the markup in steady state (Benigno and Woodford, 2005)).

A.4.3 FIRMS

This sector is populated by two groups of agents: intermediate firms and final firms. Intermediate firms produce differentiated goods i , choose labor and capital inputs, and set prices according to the Calvo model. Final goods producers act as a consumption bundler by combining national intermediate goods to produce the homogenous final good¹⁰³.

Concerning the representative intermediate firm $i \in [0, 1]$, it has the following technology, $Y_{i,t}(i) = e^{\varepsilon_{i,t}^A} K_{i,t}^u(i)^\alpha H_{i,t}^d(i)^{1-\alpha}$, where $Y_{i,t}(i)$ is the production function of the intermediate good that combines (an effective quantity of) capital $K_{i,t}^u(i)$, labor $H_{i,t}^d(i)$ and technology $e^{\varepsilon_{i,t}^A}$ (an AR(1) productivity shock)¹⁰⁴. Intermediate goods producers solve a two-stage problem. In the first stage, taking the input prices $W_{i,t}$ and $Z_{i,t}$ as given, firms rent inputs $H_{i,t}^d(i)$ and $K_{i,t}^u(i)$ in a perfectly competitive factor markets in order to minimize costs subject to the production constraint. The first order condition leads to the marginal cost expression,

$$MC_{i,t}(i) = MC_{i,t} = \frac{1}{e^{\varepsilon_{i,t}^A}} \left(\frac{Z_{i,t}}{\alpha} \right)^\alpha \left(\frac{W_{i,t}}{(1-\alpha)} \right)^{(1-\alpha)}. \quad (\text{A.25})$$

From the cost minimization problem, inputs also satisfy, $\alpha H_{i,t}^d(i) W_{i,t} = Z_{i,t} K_{i,t}^u(i) (1-\alpha)$.

In the second-stage, firm i sets the price according to a Calvo mechanism. Each period, firm i is not allowed to reoptimize its price with probability θ_i^p but price increases of $\xi_i^p \in [0, 1]$ at last period's rate of price inflation, $P_{i,t}(i) = \pi_{i,t-1}^{\xi_i^p} P_{i,t-1}(i)$. The firm allowed to modify its selling price with a probability $1 - \theta_i^p$ chooses $\{P_{i,t}^*(i)\}$ to maximize its expected sum of profits,

$$\max_{\{P_{i,t}^*(i)\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i^p \beta)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \left[(1 - \tau^y) P_{i,t}^*(i) \prod_{k=1}^{\tau} \pi_{i,t+k-1}^{\xi_i^p} - MC_{i,t+k} \right] Y_{i,t+\tau}(i) \right\},$$

under the demand constraint, $Y_{i,t+\tau}(i) = \left(\prod_{k=1}^{\tau} \pi_{i,t+k-1}^{\xi_i^p} P_{i,t}^*(i) / P_{i,t+\tau} \right)^{-\varepsilon_p} Y_{i,t+\tau}^d$, $\forall \tau > 0$, where $Y_{i,t}^d$ represents the quantity of the goods produced in country i , τ^y is a proportional tax income on final goods producers' profits which removes the steady state price distortion caused by monopolistic competition (Benigno and Woodford, 2005), $\lambda_{i,t}^c$ is the household marginal utility of consumption. The first order condition that

¹⁰³Final good producers are perfectly competitive and maximize profits, $P_{i,t} Y_{i,t}^d - \int_0^1 P_{i,t}(i) Y_{i,t}(i) di$, subject to the production function $Y_{i,t}^d = (\int_0^1 Y_{i,t}(i)^{(\varepsilon_p-1)/\varepsilon_p} di)^{\varepsilon_p/(\varepsilon_p-1)}$. We find the intermediate demand functions associated with this problem are, $Y_{i,t}(i) = (P_{i,t}(i)/P_{i,t})^{-\varepsilon_p} Y_{i,t}^d$, $\forall i$. where $Y_{i,t}^d$ is the aggregate demand.

¹⁰⁴As in Smets and Wouters (2003, 2007), we assume that capital requires one period to be settled so that, $K_{i,t}^u(i) = u_{i,t} K_{i,t-1}(i)$ given a (variable) level of capital utilization of capital $u_{i,t}$, and a quantity of capital $K_{i,t}(i)$ provided to the intermediate firm in the previous period. Both the level of $u_{i,t}$ and the quantity $K_{i,t}(i)$ are determined below by capital suppliers.

defines the price of the representative firm i is,

$$P_{i,t}^*(i) = \frac{\epsilon_p}{(\epsilon_p - 1)(1 - \tau^y)} \frac{\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i^p \beta)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} MC_{i,t+\tau} Y_{i,t+\tau}(i) \right\}}{\mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} (\theta_i^p \beta)^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} \prod_{k=1}^{\tau} \pi_{i,t+k-1}^{\xi_i^p} Y_{i,t+\tau}(i) \right\}}. \quad (\text{A.26})$$

A.4.4 CAPITAL SUPPLIERS

Capital suppliers are homogeneous and distributed over a continuum normalized to one. The representative capital supplier $k \in [0; 1]$ acts competitively to supply a quantity $K_{i,t+1}(k)$ of capital. Investment is costly, *i.e.* the capital supplier pays an adjustment cost $AC_{i,t}^I(k)$ on investment, such that $AC_{i,t}^I(k) = \frac{\chi_i^I}{2} (I_{i,t}(k)/I_{i,t-1}(k) - 1)^2$. The capital stock of the representative capital supplier thus evolves according to, $K_{i,t+1}(k) = (1 - AC_{i,t}^I(k)) I_{i,t}(k) + (1 - \delta) K_{i,t}(k)$. The capital supplier produces the new capital stock $Q_{i,t} K_{i,t+1}(k)$ by buying the depreciated capital $(1 - \delta) K_{i,t}(k)$ and investment goods $I_{i,t}(k)$, where $I_{i,t}(k) = \left((1 - \alpha_i^I)^{1/\mu} I_{h,t}(k)^{(\mu-1)/\mu} + (\alpha_i^I)^{1/\mu} I_{f,t}(k)^{(\mu-1)/\mu} \right)^{\mu/(\mu-1)}$. In this expression, parameter μ is the elasticity of substitution between domestic and foreign goods in investment and α_i^I measures the degree of investment diversification in the monetary union between home and foreign countries. We assume a national bias in investment choices so that, $\alpha_i^I < 0.5$. The price index of investment is, $P_{i,t}^I = \left((1 - \alpha_i^I) (P_{h,t})^{1-\mu} + \alpha_i^I (P_{f,t})^{1-\mu} \right)^{1/(1-\mu)}$. The representative capital supplier chooses $I_{i,t}(k)$ to maximize profits,

$$\max_{\{I_{i,t}(k)\}} \mathbb{E}_t \left\{ \sum_{\tau=0}^{\infty} \beta^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c} [Q_{i,t} (1 - AC_{i,t}^I(k)) - P_{i,t}^I] I_{i,t}(k) \right\}, \quad (\text{A.27})$$

where $\beta^\tau \frac{\lambda_{i,t+\tau}^c}{\lambda_{i,t}^c}$ is the household stochastic discount factor. The price of capital renting thus solves,

$$Q_{i,t} = P_{i,t}^I + Q_{i,t} \frac{\partial (I_{i,t}(k) AC_{i,t}^I(k))}{\partial I_{i,t}(k)} + \beta \mathbb{E}_t \frac{\lambda_{i,t+1}^c}{\lambda_{i,t}^c} Q_{i,t+1} \frac{\partial (I_{i,t+1}(k) AC_{i,t+1}^I(k))}{\partial I_{i,t}(k)}. \quad (\text{A.28})$$

As in [Smets and Wouters \(2003, 2007\)](#), capital requires one period to be settled so that, $K_{i,t}^u = u_{i,t} K_{i,t-1}$ given a level of capital utilization of capital $u_{i,t}$. Thus, the return from holding one unit of capital from t to $t + 1$ is determined by,

$$\frac{\mathbb{E}_t R_{i,t+1}^k}{1 + P_{i,t} \chi^B (B_{i,t+1}(j) - B_i(j))} = \mathbb{E}_t \left[\frac{Z_{i,t+1} u_{i,t+1} - P_{i,t+1} \Phi(u_{i,t+1}) + (1 - \delta) Q_{i,t+1}}{Q_{i,t}} \right] \quad (\text{A.29})$$

where $\Phi(u_{i,t+1})$ is the capital utilization cost function. Thus, the optimal capital utilization determines the relationship between capital utilization and the marginal production of capital is defined in logs by, $\frac{\psi_i}{1 - \psi_i} \hat{u}_{i,t} = \hat{z}_{i,t}$, where $\psi_i \in [0, 1]$ is the elasticity

of utilization costs with respect to capital inputs¹⁰⁵.

A.4.5 AUTHORITIES

National governments finance public spending by charging proportional taxes on profits arising from imperfect competition to compensate price distortions in the steady state and from entrepreneurs net wealth accumulation. Governments also receive a total value of taxes from households. The total amount of public spending, $P_{i,t}G_{i,t}$, is entirely home biased in the i^{th} economy¹⁰⁶ and evolves according to an AR(1) exogenous shock process $P_{i,t}\bar{G}\varepsilon_{i,t}^G$. The balance sheet of governments writes,

$$\begin{aligned} P_{i,t}\bar{G}\varepsilon_{i,t}^G &= \int_0^1 T_{i,t}(j) \, dj + \tau^y \int_0^1 P_{i,t}(i) Y_{i,t}(i) \, di + \tau^w \int_0^1 W_{i,t}(j) H_{i,t}(j) \, dj \\ &+ \tau^L \int_0^1 L_{i,t+1}^s(b) R_{i,t}^L(b) \, db + \tau^E \int_0^1 N_{i,t}^E(e) \, de + \tau^B \int_0^1 BK_{i,t}(b) \, db \end{aligned}$$

$G_{i,t}$ is the total amount of public spending in the i^{th} economy that follows and AR(1) shock process, $\tau^y = (1 - \epsilon_p)^{-1}$, $\tau^w = (1 - \mu^w)$ and $\tau^L = (1 - \mu^L)$ are taxes that mitigate the negative effects of monopolistic competition in steady states.

The central banks reacts to fluctuations in union wide measures of price and activity growths. The general expression of the interest rule implemented by the monetary union central bank writes,

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^\rho \left[(\pi_{h,t}^c \pi_{f,t}^c)^{\phi^\pi} \left(\frac{Y_{h,t} Y_{f,t}}{Y_{h,t-1} Y_{f,t-1}} \right)^{\phi^{\Delta y}} \right]^{\frac{1}{2}(1-\rho)} e^{\varepsilon_t^R} \quad (\text{A.30})$$

where ε_t^R is a AR(1) monetary policy shock process, ϕ^π is the inflation target parameter and $\phi^{\Delta y}$ is the GDP growth target.

A.4.6 EQUILIBRIUM CONDITIONS

In this model, there are in total 8 country specific structural shocks and one common shock in the Taylor rule. For $i \in \{h, f\}$, exogenous disturbances follow a first-order autoregressive process, $\varepsilon_{i,t}^s = \rho_i^s \varepsilon_{i,t-1}^s + \eta_{i,t}^s$ for $\forall s = \{\beta, A, Q, N, L, B\}$ and one common shock in the Taylor rule, $\varepsilon_t^R = \rho^R \varepsilon_{t-1}^R + \eta_t^R$. For the spending shock process, it is affected by the productivity shock as follows, $\varepsilon_{i,t}^G = \rho_i^G \varepsilon_{i,t-1}^G + \eta_{i,t}^G + \rho_i^{ag} \eta_{i,t}^A$; this assumption is empirically motivated as spending also includes net exports, which may be affected by domestic productivity developments (Smets and Wouters, 2007). The wage mark-up disturbance is assumed to follow an ARMA(1,1) process, $\varepsilon_{i,t}^W = \rho_i^W \varepsilon_{i,t-1}^W + \eta_{i,t}^W - u_i^W \eta_{i,t-1}^W$,

¹⁰⁵When households do not take capital supply decisions, the optimal capital utilization is determined by solving, $\max_{u_{i,t}} (Z_{i,t} u_{i,t} - \Phi(u_{i,t})) K_{i,t}$. The utilization choice is defined by the first order condition, $\Phi'(u_{i,t}) = Z_{i,t}$, up to a first-order approximation in deviation from steady states, $\frac{\Phi''(u)}{\Phi'(u)} \hat{u}_{i,t} = \hat{z}_{i,t}$.

¹⁰⁶National public spending are entirely home biased and consists of home varieties, i.e., $P_{i,t}G_{i,t} = P_{i,t}(\int_0^1 G_{i,t}(i)^{(\epsilon_p-1)/\epsilon_p} di)^{\epsilon_p/(\epsilon_p-1)}$. The government demand for home goods writes, $G_{i,t}(i) = \left(\frac{P_{i,t}(i)}{P_{i,t}} \right)^{-\epsilon_p} G_{i,t}$.

where the MA term u_i^W is designed to capture the high-frequency fluctuations in wages. Finally, to catch up the co-moment in financial time series, we add common financial shocks η_t^s for $\forall s = \{Q, N, L, B\}$. We denote by $\rho_i^\beta, \rho_i^A, \rho_i^G, \rho_i^Q, \rho_i^N, \rho_i^L, \rho_i^W, \rho_i^B$ and ρ^R the autoregressive terms of the exogenous variables, $\eta_{i,t}^\beta, \eta_{i,t}^A, \eta_{i,t}^G, \eta_{i,t}^Q, \eta_{i,t}^N, \eta_{i,t}^L, \eta_{i,t}^W, \eta_{i,t}^B$ and $\eta_t^Q, \eta_t^N, \eta_t^L, \eta_t^B, \eta_t^R$ are standard errors that are mutually independent, serially uncorrelated and normally distributed with zero mean and variances $\sigma_{i,\beta}^2, \sigma_{i,A}^2, \sigma_{i,G}^2, \sigma_{i,Q}^2, \sigma_{i,N}^2, \sigma_{i,L}^2, \sigma_{i,W}^2, \sigma_{i,B}^2$ and $\sigma_Q^2, \sigma_N^2, \sigma_L^2, \sigma_B^2, \sigma_R^2$ respectively. A general equilibrium is defined as a sequence of quantities $\{Q_t\}_{t=0}^\infty$ and prices $\{P_t\}_{t=0}^\infty$ such that for a given sequence of quantities $\{Q_t\}_{t=0}^\infty$ and the realization of shocks $\{S_t\}_{t=0}^\infty$, the sequence $\{P_t\}_{t=0}^\infty$ guarantees the equilibrium on the capital, labor, loan, intermediate goods and final goods markets.

After (i) aggregating all agents and varieties in the economy, (ii) imposing market clearing for all markets, (iii) assuming that countries are mirror images of one another in terms of market openness¹⁰⁷, (iv) substituting the relevant demand functions, the resource constraint for the home country reads as follows,

$$\begin{aligned} \frac{Y_{h,t}}{\Delta_{h,t}^p} &= (1 - \alpha^C) \left(\frac{P_{h,t}}{P_{h,t}^c} \right)^{-\mu} C_{h,t} + \alpha^C \left(\frac{P_{h,t}}{P_{f,t}^c} \right)^{-\mu} C_{f,t} \\ &+ (1 - \alpha^I) \left(\frac{P_{h,t}}{P_{h,t}^I} \right)^{-\mu} (1 + AC_{h,t}^I) I_{h,t} + \alpha^I \left(\frac{P_{h,t}}{P_{f,t}^I} \right)^{-\mu} (1 + AC_{f,t}^I) I_{f,t} \\ &+ \bar{G} \varepsilon_{h,t}^G + AC_{h,t}^B + (1 - \eta_{h,t}^E) \omega_{h,t} Q_{h,t} K_{h,t} + \Phi(u_{h,t}) K_{h,t-1}, \end{aligned} \quad (\text{A.31})$$

where $\Delta_{h,t}^p = \int_0^1 (P_{i,t}(i)/P_{i,t})^{-\epsilon_p} di$ is the price dispersion term¹⁰⁸. The aggregation of prices of the final goods sector leads to the expression,

$$P_{i,t}^{1-\epsilon_p} = \theta_i^p \left(P_{i,t-1} \pi_{i,t-1}^{\xi_i^p} \right)^{1-\epsilon_p} + (1 - \theta_i^p) (P_{i,t}^*)^{1-\epsilon_p}. \quad (\text{A.32})$$

Concerning unions, the aggregation of unions allowed and not allowed to reoptimize leads to the following expression of the aggregate wage index,

$$W_{i,t}^{\frac{1}{1-\mu_{i,t}^w}} = \theta_i^w \left[W_{i,t-1} (\pi_{i,t-1}^C)^{\xi_i^w} \right]^{\frac{1}{1-\mu_{i,t}^w}} + (1 - \theta_i^w) (W_{i,t}^*)^{\frac{1}{1-\mu_{i,t}^w}}, \quad (\text{A.33})$$

and the equilibrium on this market reads, $\int_0^1 H_{i,t}(j) dj = \Delta_{i,t}^w \int_0^1 H_{i,t}^d(i) di$, where $\Delta_{i,t}^w = \int_0^1 \left(\frac{W_{i,t}(j)}{W_{i,t}} \right)^{-\mu_{i,t}^w / (\mu_{i,t}^w - 1)} dj$ is the wage dispersion term between different labor types.

¹⁰⁷i.e, $\alpha_h^s = \alpha^s \Leftrightarrow \alpha_f^s = (1 - \alpha^s)$ for markets $s = C, I, L, IB$ and the two countries are of equal size.

¹⁰⁸To close the model, additional costs are entirely home biased, i.e. adjustment costs on bonds $AC_{i,t}^B = \left(\int_0^1 AC_{i,t}^B(i)^{(\epsilon_p-1)/\epsilon_p} di \right)^{\epsilon_p/(\epsilon_p-1)}$, insolvent investment projects of entrepreneurs and capital utilization costs from capital suppliers $K_{i,t} = \left(\int_0^1 K_{i,t}(i)^{(\epsilon_p-1)/\epsilon_p} di \right)^{\epsilon_p/(\epsilon_p-1)}$. The demands associated with the previous costs are, $AC_{i,t}^B(i) = (P_{i,t}(i)/P_{i,t})^{-\epsilon_p} AC_{i,t}^B$, $K_{i,t}(i) = (P_{i,t}(i)/P_{i,t})^{-\epsilon_p} K_{i,t}$.

The equilibrium on the home loan market reads,

$$L_{h,t+1}^s = \left((1 - \alpha^L) \left[\frac{R_{h,t}^L}{P_{h,t}^L} \right]^{-\nu} L_{h,t+1}^d + \alpha^L \left[\frac{R_{f,t}^L}{P_{f,t}^L} \right]^{-\nu} L_{f,t+1}^d \right) \Delta_{h,t}^L,$$

where $\Delta_{i,t}^L = \int_0^1 \left(\frac{R_{i,t}^L(b)}{R_{i,t}^L} \right)^{-\mu_{i,t}^L / (\mu_{i,t}^L - 1)} db$ is the dispersion term of credit rates in the economy. The aggregation of loan prices writes,

$$(R_{i,t}^L)^{\frac{1}{1-\mu_{i,t}^L}} = \theta_i^L \left(R_{i,t-1}^L \left(\frac{R_{i,t-1}^L}{R_{i,t-2}^L} \right)^{\xi_i^L} \right)^{\frac{1}{1-\mu_{i,t}^L}} + (1 - \theta_i^L) (R_{i,t}^L)^{\frac{1}{1-\mu_{i,t}^L}}.$$

On the perfectly competitive interbank market, the market clears when the following condition holds,

$$IB_{h,t+1}^s = \frac{\lambda}{1-\lambda} \left((1 - \alpha^{IB}) \left[\frac{R_{h,t}^{IB}}{P_{h,t}^{IB}} \right]^{-\xi} IB_{h,t+1}^d + \alpha^{IB} \left[\frac{R_{f,t}^{IB}}{P_{f,t}^{IB}} \right]^{-\xi} IB_{f,t+1}^d \right)$$

Asset market equilibrium implies that the world net supply of bonds is zero, the same applies to current accounts excess and deficits, $B_{h,t+1} + B_{f,t+1} = 0$ and $CA_{h,t} + CA_{f,t} = 0$, where home current account dynamic reads as follow,

$$CA_{h,t} = (B_{h,t+1} - B_{h,t}) + [(L_{h,f,t+1} - L_{h,f,t}) - (L_{f,h,t+1} - L_{f,h,t})] \\ + [(IB_{h,f,t+1} - IB_{h,f,t}) - (IB_{f,h,t+1} - IB_{f,h,t})].$$

A.5 ESTIMATION

A.5.1 DATA

The model is estimated with Bayesian methods on Euro Area quarterly data over the sample period 1999Q1 to 2013Q3. The dataset includes 17 time series: real GDP, real consumption, real investment, the ECB refinancing operation rate, the HICP, the real unit labor cost index, the real index of notional stocks of corporate and interbank loans, and the real borrowing cost of non-financial corporations. Data with a trend are made stationary using a linear trend and are divided by the population. We also demean the data because we do not use the information contained in the observable mean. [Figure A.4](#) plots the transformed data.

A.5.2 CALIBRATION AND PRIOR DISTRIBUTION OF PARAMETERS

We fix a small number of parameters commonly used in the literature of real business cycles models in [Table A.1](#). These include the quarterly depreciation rate δ , the quarterly discount factor β , the capital share in the production α , the steady state of government expenditures in output \bar{G}/\bar{Y} and the adjustment cost on portfolio χ^B

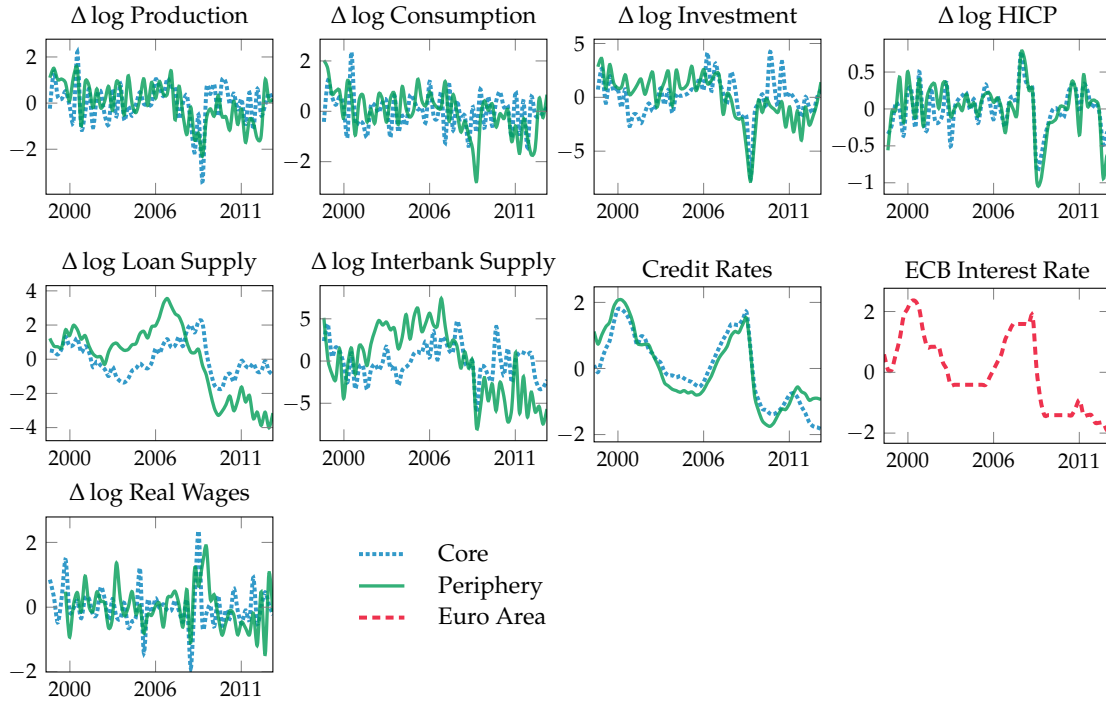


Figure A.4: Observable variables used in the estimation

(Schmitt-Grohé and Uribe, 2003). Under this calibration, the share of consumption and investment in the GDP is 56% and 20% respectively which is consistent with the Euro Area.

Regarding financial parameters, we fix the net worth to capital ratio of the private sector on the findings of Gerali et al. (2010), while the spread between the lending rate and the refinancing rate is calculated on the average observable variables used in the estimation for France and Germany and has a value of 200 points basis annually. We suppose that in steady state, the interbank rate in the Euro Area is equal to the refinancing rate $R = R_h^{IB} = R_f^{IB}$. Recall that following the Pareto distribution $\omega \sim \mathcal{P}(\kappa; \omega_{\min})$ where κ is the shape parameter and ω_{\min} the minimum value of $\omega \in [\omega_{\min}, +\infty[$. When ω^C hits the lower bound ($\omega^C = \omega_{\min}$), the economy is riskless implying $R^k = R^L = R$ so that when $\omega^C > \omega_{\min}$ there are financial frictions and defaulting entrepreneurs projects in the steady state. Given the first order condition of banks $R^L = R/\eta$, the conditions $E[\omega] = 1 = \frac{\omega_{\min}^\kappa}{\kappa - 1}$ and the definition of the share of gainful projects $\eta = (\omega_{\min}/\omega^C)^\kappa$, we compute κ and ω_{\min} via the following condition $\omega_{\min} = (\kappa - 1)/\kappa = 1 - \bar{N}/\bar{K}$. Calibrating the model without financial frictions ($\omega = \omega_{\min}$) and without loans ($L = 0$) makes the model really close to the Smets and Wouters model in a two-country set-up. From the previous calibration, we get the quarterly entrepreneur failure rate of $\eta = 1.2\%$, which is comparable to Bernanke et al. (1999).

Our priors are listed in Table A.3. Overall, they are either consistent with the previous literature or relatively uninformative. For a majority of new Keynesian models' parameters, i.e. σ_i^c , σ_i^L , h_i^c , θ_i^p , θ_i^w , ξ_i^p , ξ_i^w , χ_i^I , ψ_i , ϕ^π , $\phi^{\Delta y}$ and shocks processes parameters, we use the prior distributions chosen by Smets and Wouters (2003, 2007). Concerning international macroeconomic parameters, our priors are largely inspired

Parameter	Value	Description
β	0.995	Discount factor
δ	0.02	Depreciation rate
α	0.25	Capital share
\bar{H}	1/3	Share of hours worked per day
χ^B	0.07%	Portfolio adjustment costs
\bar{G}/\bar{Y}	0.24	Spending to GDP ratio
\bar{N}/\bar{K}	0.40	Net worth to capital ratio
\bar{TB}/\bar{L}	0.20	Interbank funds to lending ratio
\bar{BK}/\bar{L}	0.10	Bank capital to lending ratio
$\bar{R}^L - \bar{R}$	0.02/4	Loan spread

Table A.1: Calibration of the model (all parameters are quarterly)

by Lubik and Schorfheide (2006) for substitution parameters μ , ν and ξ . Regarding market openness, we use priors that are close to the observed degrees of openness: α^C , α^I , α^L and α^{IB} have a beta prior of means 0.10, 0.08, 0.08 and 0.20 and standard deviations of 0.04, 0.04, 0.04 and 0.07 respectively. For Calvo credit rates parameters, our priors are the same as the Calvo price priors. We set the prior for the elasticity of the external finance premium \varkappa_i to a normal distribution with prior mean equal to 0.10 and standard deviation 0.05 consistent with previous financial accelerator estimations (De Graeve, 2008; Gilchrist et al., 2009a). For loan demand habits for firms and banks, we chose a very uninformative prior of mean 0.50 and standard deviation 0.20 with a beta distribution. Finally, the monitoring cost is set to a normal distribution with mean 0.50 and variance 0.20 which is consistent with Cúrdia and Woodford (2010).

		Prior distributions			Posterior distribution [5%:95%]		
		Shape	Mean	Std.	CORE	PERIPHERY	EURO
Productivity std	σ_i^A	\mathcal{IG}	0.10	2	0.88 [0.67:1.08]	0.76 [0.58:0.92]	-
Gov. spending std	σ_i^G	\mathcal{IG}	0.10	2	1.64 [1.37:1.90]	1.66 [1.39:1.93]	-
Preferences std	σ_i^β	\mathcal{IG}	0.10	2	1.59 [1.08:2.07]	2.12 [1.29:2.90]	-
Net Wealth std	σ_i^N	\mathcal{IG}	0.10	2	0.14 [0.08:0.21]	0.15 [0.06:0.23]	0.14 [0.08:0.19]
External Finance std	σ_i^Q	\mathcal{IG}	0.10	2	0.46 [0.03:0.78]	0.42 [0.03:0.79]	0.82 [0.64:1.01]
Bank cost-push std	σ_i^L	\mathcal{IG}	0.10	2	0.27 [0.03:0.48]	0.22 [0.02:0.47]	0.68 [0.45:0.90]
Bank Liab. std	σ_i^B	\mathcal{IG}	0.10	2	2.28 [1.77:2.78]	2.43 [1.80:3.03]	0.08 [0.02:0.16]
Wage cost-push std	σ_i^W	\mathcal{IG}	0.10	2	0.92 [0.56:1.24]	1.65 [0.86:2.47]	-
Monetary policy std	σ_i^R	\mathcal{IG}	0.10	2	-	-	0.09 [0.07:0.11]
Productivity AR	ρ_i^A	\mathcal{B}	0.85	0.10	0.99 [0.98:1.00]	0.99 [0.98:1.00]	-
Gov. spending AR	ρ_i^G	\mathcal{B}	0.85	0.10	0.93 [0.88:0.98]	0.91 [0.83:0.99]	-
Preferences AR	ρ_i^β	\mathcal{B}	0.85	0.10	0.81 [0.72:0.91]	0.54 [0.38:0.70]	-
Net Wealth AR	ρ_i^N	\mathcal{B}	0.85	0.10	0.96 [0.92:0.99]	0.96 [0.93:1.00]	-
Riskiness AR	ρ_i^Q	\mathcal{B}	0.85	0.10	0.47 [0.35:0.59]	0.59 [0.47:0.71]	-
Bank cost-push AR	σ_i^L	\mathcal{B}	0.85	0.10	0.99 [0.98:1.00]	0.90 [0.84:0.96]	-
Bank Liab. AR	ρ_i^B	\mathcal{B}	0.85	0.10	0.90 [0.83:0.97]	0.93 [0.88:0.99]	-
Wage cost-push AR	σ_i^W	\mathcal{B}	0.85	0.10	0.99 [0.99:1.00]	0.99 [0.99:1.00]	-
Wage MA term	u_i^W	\mathcal{B}	0.85	0.10	0.51 [0.26:0.77]	0.28 [0.07:0.49]	-
Taylor AR	ρ_i^R	\mathcal{B}	0.85	0.10	-	-	0.41 [0.29:0.52]
Productivity-Spending	ρ_i^{ag}	\mathcal{B}	0.85	0.10	0.84 [0.69:0.99]	0.83 [0.69:0.99]	-

Table A.2: Prior and Posterior distributions of shock processes

A.5.3 POSTERIOR ESTIMATES

The methodology is standard to the Bayesian estimation of DSGE models¹⁰⁹. **Figure A.5** reports the prior and posterior marginal densities of the parameters of the model, excluding the standard deviation of the shocks and the parameters driving the shocks processes. In **Figure A.5**, the data were relatively informative except for a small numbers of parameters for which the posterior distribution stay very close to the chosen priors. These parameters are the risk consumption parameter σ_i^c , the elasticity of the external premium for peripheral countries \varkappa_f , the inflation and GDP growth penalization degrees in the Taylor rule ϕ^π and $\phi^{\Delta y}$, the elasticity for loans ξ , ν and the

¹⁰⁹Interest rates data are associated with one-year maturity loans, we take into account this maturity by multiplying by 4 the rates in the measurement equation. The number of shocks is higher (or equal) to observable variables to avoid stochastic singularity issue. Recalling that $i \in \{h, f\}$, the vectors of observables $\mathcal{Y}_t^{obs} = [\Delta \log \tilde{Y}_{i,t}, \Delta \log \tilde{C}_{i,t}, \Delta \log \tilde{I}_{i,t}, R_t, \Delta \log HICP_{i,t}, \Delta \log W_t, \Delta \log \tilde{L}_{i,t}^s, R_{i,t}^L, \Delta \log \tilde{I}B_{i,t}^s]'$ and measure-

ment equations $\mathcal{Y}_t = [\hat{y}_{i,t} - \hat{y}_{i,t-1}, \hat{c}_{i,t} - \hat{c}_{i,t-1}, \hat{i}_{i,t} - \hat{i}_{i,t-1}, 4 \times \hat{r}_t, \hat{\pi}_{i,t}^c, \hat{w}_t - \hat{w}_{t-1}, \hat{l}_{i,t}^s - \hat{l}_{i,t-1}^s, 4 \times \hat{r}_{i,t}^L, \hat{i}b_{i,t}^s - \hat{i}b_{i,t-1}^s]'$

where Δ denotes the temporal difference operator, \tilde{X}_t is per capita variable of X_t . The model matches the data setting $\mathcal{Y}_t^{obs} = \bar{\mathcal{Y}} + \mathcal{Y}_t$ where $\bar{\mathcal{Y}}$ is the vector of the mean parameters, we suppose this is a vector of all 0. The posterior distribution combines the likelihood function with prior information. To calculate the posterior distribution to evaluate the marginal likelihood of the model, the Metropolis-Hastings algorithm is employed. To do this, a sample of 400,000 draws was generated, neglecting the first 50,000. The scale factor was set in order to deliver acceptance rates of between 20 and 30 percent. Convergence was assessed by means of the multivariate convergence statistics taken from **Brooks and Gelman (1998)**.

		Prior distributions			Posterior distribution [5%:95%]		
		Shape	Mean	Std.	CORE	PERIPHERY	EURO
Cons. aversion	σ_i^c	\mathcal{N}	2	0.30	2.00 [1.56:2.45]	2.04 [1.57:2.49]	-
Cons. inertia	h_i^c	\mathcal{B}	0.7	0.10	0.32 [0.21:0.42]	0.57 [0.46:0.68]	-
Labour disutility	σ_i^L	\mathcal{G}	1	0.30	0.59 [0.30:0.88]	0.66 [0.36:0.96]	-
Calvo prices	θ_i^p	\mathcal{B}	0.5	0.10	0.56 [0.47:0.65]	0.56 [0.47:0.65]	-
Indexation prices	ξ_i^p	\mathcal{B}	0.5	0.2	0.07 [0.01:0.14]	0.08 [0.01:0.16]	-
Calvo wages	θ_i^w	\mathcal{B}	0.5	0.10	0.67 [0.56:0.78]	0.6 [0.49:0.72]	-
Indexation wages	ξ_i^w	\mathcal{B}	0.5	0.10	0.46 [0.16:0.76]	0.36 [0.07:0.64]	-
Calvo banks rates	θ_i^L	\mathcal{B}	0.5	0.10	0.29 [0.20:0.38]	0.31 [0.18:0.43]	-
Indexation bank rates	ξ_i^L	\mathcal{B}	0.5	0.15	0.12 [0.01:0.22]	0.21 [0.05:0.37]	-
Investment adj. costs	χ_i^I	\mathcal{N}	4	1.5	0.63 [0.38:0.86]	1.87 [1.16:2.58]	-
Monitoring cost	χ_i^{IB}	\mathcal{N}	0.5	0.2	0.48 [0.28:0.68]	0.23 [0.00:0.48]	-
Capital utilization	ψ_i	\mathcal{B}	0.5	0.15	0.66 [0.47:0.85]	0.68 [0.51:0.86]	-
EF. Premia Elasticity	\varkappa_i	\mathcal{B}	0.1	0.05	0.05 [0.01:0.09]	0.08 [0.02:0.15]	-
Firms loans habits	h_i^L	\mathcal{B}	0.5	0.2	0.96 [0.93:0.99]	0.95 [0.92:0.98]	-
Interbank loans habit	h_i^B	\mathcal{B}	0.5	0.2	0.20 [0.05:0.34]	0.21 [0.07:0.35]	-
Illiquid bank share	λ	\mathcal{B}	0.5	0.08	-	-	0.25 [0.20:0.29]
MPR smoothing	ρ	\mathcal{B}	0.85	0.10	-	-	0.84 [0.81:0.88]
MPR Inflation	ϕ^π	\mathcal{N}	2	0.15	-	-	1.85 [1.60:2.10]
MPR GDP	$\phi^{\Delta y}$	\mathcal{N}	0.12	0.05	-	-	0.15 [0.08:0.23]
Cons. openness	α^C	\mathcal{B}	0.10	0.04	-	-	0.17 [0.11:0.23]
Investment openness	α^I	\mathcal{B}	0.08	0.04	-	-	0.06 [0.01:0.10]
Corporate openness	α^L	\mathcal{B}	0.08	0.04	-	-	0.09 [0.03:0.15]
Interbank openness	α^{IB}	\mathcal{B}	0.2	0.07	-	-	0.11 [0.05:0.17]
Subst. final good	μ	\mathcal{G}	1	0.75	-	-	4.43 [3.09:5.75]
Subst. corporate loan	ν	\mathcal{G}	1	0.75	-	-	2.02 [0.03:4.07]
Subst. interbank loan	ξ	\mathcal{G}	1	0.75	-	-	0.87 [0.02:1.75]

Table A.3: Prior and Posterior distributions of structural parameters

financial openness α^L for the corporate sector¹¹⁰. We investigate the sources of non identification for these parameters using methods developed by Saltelli et al. (2008), Andrieu (2010) and Iskrev (2010). We find that the low identification of parameters driving the risk aversion coefficient σ_i^c and the substitutions of loans ν and ξ is due to their small impacts on the likelihood. As An and Schorfheide (2007), we find that the Taylor rule smoothing ρ is the best identified parameter, and that it strongly interacts with other parameters in the Taylor rule ϕ^π and $\phi^{\Delta y}$. Indeed, using the brute force search à la Iskrev (2010), we note a correlation link that involves ϕ^π , $\phi^{\Delta y}$ with ρ ¹¹¹. We also find a partial confounding with the elasticity of the external premium \varkappa_f and the credit rate stickiness θ_f^L : since the external finance premium is mainly driven by the monetary policy shock (De Graeve, 2008), the introduction of Calvo credit rate weakens the identification of the external finance premium as monetary policy shocks are dampened when rates are sticky. Finally, even if the market openness α^L stay very close to the prior, identification methods show that α^L is accurately identified (even better than α^C)¹¹².

The posterior parameters' differences or similarities between core and peripheral countries provide the microfoundations for national asymmetries as shown in Table A.3 and in Table A.2. Concerning shocks parameters, there is one noticeable difference between core and periphery in the persistence of riskiness shocks, any change in the expectations of the private sector lasts more in periphery. Regarding structural parameters, real frictions are more important in periphery as consumption habits, capital utilization elasticity and investment costs are higher. Turning to nominal frictions, core and periphery face similar price rigidities as Calvo and indexation parameters for prices are nearly equal, while wage rigidities are more pronounced in core countries. Finally for financial frictions, the pass-through of the policy rate is not surprisingly better in core countries as Calvo and indexation parameters for credit rate are lower than in periphery. According to the elasticity of external premium, peripheral firms are more optimistic than core. The monitoring technology is better for peripheral liquid banks. For the Euro area parameters, the banking system is composed of 25% of banks in need of interbank funds. Concerning market openness parameters, they are consistent with the data and close to the findings of ? for α^C and α^I while the market openness of the interbank market is slightly lower than its value observed in Figure A.2.

Since we are interested in finding evidence that cross-border loans significantly explain a part of the business cycles, we consider θ the vector of the estimated parameters of the model $\mathcal{M}(\theta)$ presented in Appendix A.3 and we estimate $\mathcal{M}(\theta)$ under two scenarios: in $\mathcal{M}_1(\theta)$ there is no cross-border lending flows between countries so that, $\alpha^L = \alpha^{IB} = 0$, $\nu = \xi = 0$; in $\mathcal{M}_2(\theta)$ we introduce cross-border lending flows between countries by estimating $\alpha^L, \alpha^{IB} \in [0, 1]$, $\nu, \xi \geq 0$. At last, we are interested in finding evidence that cross-border loans significantly explain a part of the business cycles of the Eurozone. Put differently, we examine the hypothesis $H_0: \alpha^L = \alpha^{IB} = 0$,

¹¹⁰The elasticity of intertemporal substitution, inflation weight and output growth in the monetary policy rule are parameters that are frequently not well identified, see for example An and Schorfheide (2007) or Kolasa (2008).

¹¹¹See An and Schorfheide (2007) for further explanations on this correlation link.

¹¹²Parameter α^L does not involve any important correlation link with other parameter, its log-likelihood is not flat and is not a weak element of the parameter set.

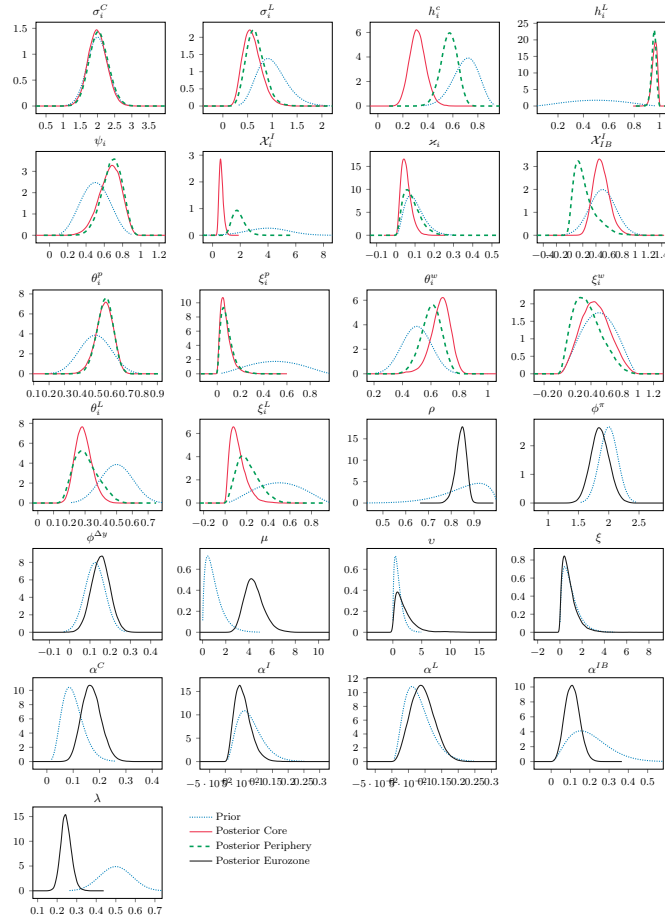


Figure A.5: The priors and posteriors distributions of the model parameters with cross-border banking flows.

$\nu = \xi = 0$ against the hypothesis $H_1: \alpha^L, \alpha^{IB} \in [0, 1), \nu, \xi > 0$, to do this we evaluate the posterior odds ratio of $\mathcal{M}_2(\theta)$ on $\mathcal{M}_1(\theta)$ using Laplace-approximated marginal data densities. The posterior odds of the null hypothesis of no significance of banking flows is 13.5:1 which leads us to strongly reject the null, *i.e.* cross-border lending flows do matter in explaining the business cycles of the Euro Area. This result is confirmed in terms of log marginal likelihood. When the models are estimated without common financial shocks, then cross-border flows have an even more important role in explaining the business cycles as the posterior odds ratio becomes $4.7 \times 10^8:1$.

A.6 THE CONSEQUENCES OF CROSS-BORDER LOANS

Once the model has been estimated, we evaluate the consequences of cross-border lending on the national and international transmission of asymmetric shocks. We report the Bayesian IRFs obtained from linearized models \mathcal{M}_1 and \mathcal{M}_2 . We concentrate on three main shocks that affect the core countries: an asymmetric productivity shock affecting firms, an asymmetric financial shock that reduces the net worth of entrepreneurs and a positive shock affecting the liquidity situation of the banking system.

	w/ common financial shocks		w/o common financial shocks	
	$\mathcal{M}_1(\theta)$	$\mathcal{M}_2(\theta)$	$\mathcal{M}_1(\theta)$	$\mathcal{M}_2(\theta)$
	<i>autarky</i>	<i>globalization</i>	<i>autarky</i>	<i>globalization</i>
Prior probability	1/2	1/2	1/2	1/2
Laplace approximation	-1392.0	-1389.5	-1432.1	-1412.1
Posterior odds ratio	1.00	13.5	1.00	4.7×10^8
Posterior model probability	0.07	0.93	0	1.00

Table A.4: Prior and posterior model probabilities

A.6.1 A POSITIVE SHOCK ON TOTAL FACTOR PRODUCTIVITY

Figure A.6 reports the simulated responses of the main macroeconomic and financial variables following a positive shock to $\varepsilon_{h,t}^A$ equal in size to the standard deviation of total factor productivity estimated in Table A.3.

In the benchmark situation (dotted line), loan markets are segmented. As standardly documented in the literature, this productivity shock increases production, consumption and investment while decreasing the inflation rate in the core countries (Smets and Wouters, 2003). This shock is transmitted to peripheral economies through the terms of trade, the current account and the reaction of the central bank interest rate. The deterioration of the core countries' terms of trade increases the relative competitiveness and the exports of core countries goods towards peripheral economies. The decrease of the relative price of core countries goods depresses peripheral activity and investment. The average union wide rate of consumption price inflation decreases, which leads the central bank to reduce the refinancing interest rate (?). As observed, corporate loans increase in both countries. In core countries, entrepreneurs contract more loans to finance new investment flows after the positive supply shock. Central bank reaction affects the banking system through the decrease of the interest rate. This, in turn, lowers the interest rate on loans and increases corporate loan demand. As observed, interbank lending also rises to allow illiquid banks to meet the increased corporate loan demand. This increase in corporate loan demand dampens the decrease in investment as all the new loans remain in the periphery. However the rise in firm leverage increases the failure rate of investment projects in both countries and by so, the interest rate served by banks increases after 5 quarters. Thus, the segmentation of the loan market has a clear dampening effect in the periphery with regards to the transmission of core countries' productivity shocks.

The possibility of banks to engage in cross-border lending (plain lines) acts as a mechanism that mainly increases the dispersion of investment cycles in the monetary union. As cross-border lending improves the international allocation of financial resources in the monetary union, it amplifies the positive impact on investment in core countries and the negative impact in the peripheral economies, while leaving unaffected the dynamics of consumption and activity in both part of the monetary union. As a consequence, the current account adjustment (that reflects net savings) is significantly affected by the assumption regarding the degree of cross-border banking. Part of the increase in domestic investment is fuelled by foreign loans: the increase in foreign lending increases (partly financed by an increase in interbank lending in the peripheral countries). This implies a net increase in foreign loan supply after 5 quar-

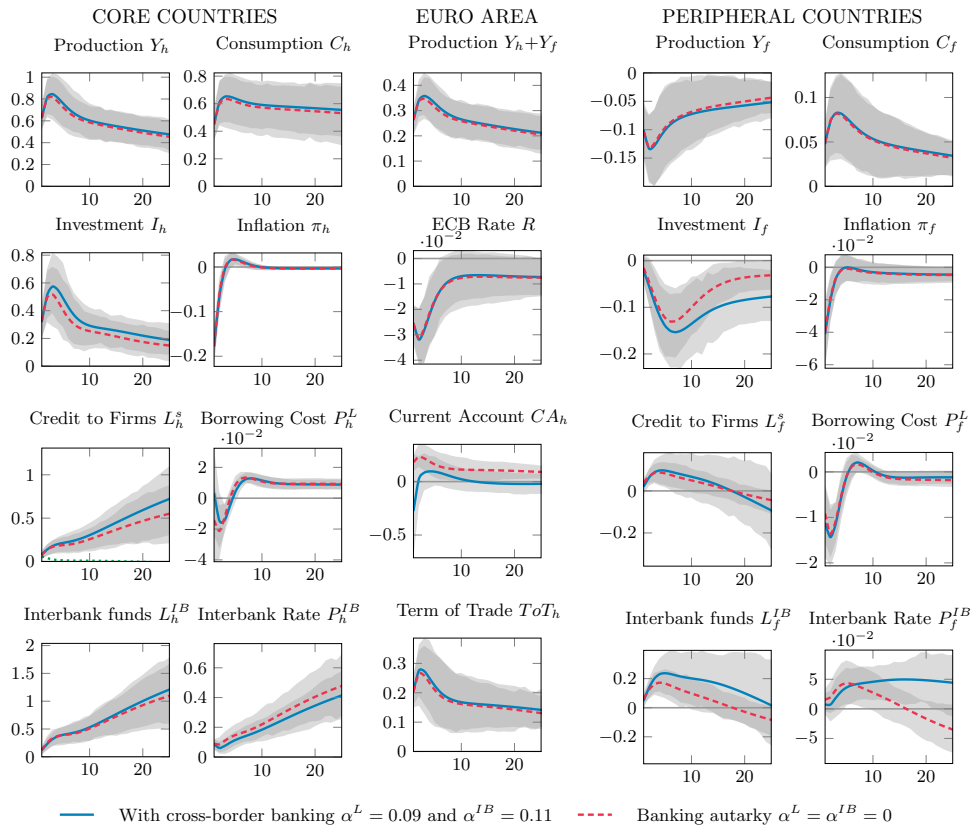


Figure A.6: Bayesian system response to an estimated positive productivity shock in core countries under banking globalization ($\alpha^L = 9\%$, $\alpha^{IB} = 11\%$) and autarky ($\alpha^L = \alpha^{IB} = 0\%$)

ters with regard to the segmented situation. By lending to more productive domestic firms, the foreign banking system has access to more reliable borrowers. Cross-border lending clearly impacts negatively the foreign macroeconomic performance, as more lending resources are diverted towards the domestic economy. This is clearly shown by the increased slump in peripheral countries' investment. With cross-border relations, the increase in interbank lending is reflected by a decrease in core countries' loan supply. Part of the liquidity of domestic banks comes from peripheral banks, through cross-border interbank lending.

Cross-border lending significantly affects the dynamics of the current account. Ignoring cross-border banking, the adjustment of the current account is standard, as the domestic economy experiences a surplus of net exports, that depicts the intertemporal allocation of the increase in national resources over a sample time period of thirty quarters. Cross-border loans clearly deteriorate core countries' current account with respect to the benchmark situation (dotted lines). As activity and consumption remain unaffected by the integration of the loan market, and as the increase of investment is higher in the core countries (while the decrease in investment is higher in peripheral countries), the current account of core countries deteriorates with respect to the segmented situation, to reflect the increased dispersion of investment cycles. Finally, the IRFs of the terms of trade and of the central bank interest rate are unaffected by cross-border lending.

A.6.2 A NEGATIVE SHOCK ON FIRMS NET WORTH

The second set of IRFs, reported in [Figure A.7](#), describes the consequences of a negative shock on core countries' firm net worth $\varepsilon_{h,t}^N$. This negative shock can be thought of as an overnight decrease in the value of investor capital (following, for example a stock exchange collapse).

Without cross-border loans (dotted lines), a reduction in firms' net worth depresses investment and production and is deflationary in core countries. The reduction of activity is driven by the decrease in investment decisions. The central bank reacts to deflation by decreasing the interbank interest rate. This, in turn, increases consumption and dampens the negative impact on core countries' investment after 4 periods. As investment decreases more than activity, consumption increases and, as core countries' inflation rate decreases more than peripheral inflation rate, the domestic terms of trade deteriorates. As a consequence, more domestic goods are exported and consumed by peripheral households. The production of foreign goods decreases which, in turn, implies a decrease in investment decisions. In the meanwhile, following the decrease in the ECB central bank interest rate, peripheral consumption increases (this increase falls on imported goods). Initially, the negative wealth shock increases the probability of insolvent projects, and leads to higher interest rates on loans, despite the reduction of the central bank interest rate. This, in turn depresses investment. As observed, since banks engage in less corporate loans, their liquidity situation improves and interbank lending decreases. The improvement in core countries' current account reflects the increase in net savings coming from the decrease of investment in this part of the monetary union. Finally, interbank loans remain almost unaffected in peripheral countries.

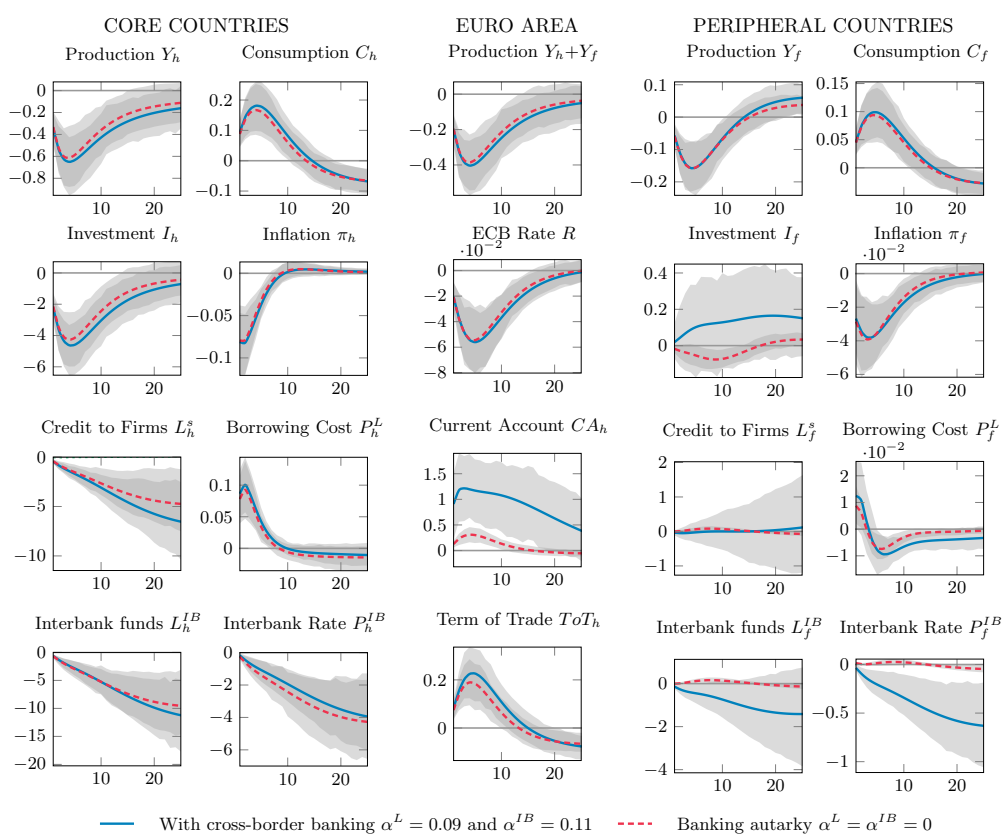


Figure A.7: Bayesian system response to an estimated negative net wealth shock in core countries under banking globalization ($\alpha^L = 9\%$, $\alpha^{IB} = 11\%$) and autarky ($\alpha^L = \alpha^{IB} = 0\%$)

Cross-border bank activity (plain lines) acts as a mechanism that amplifies the negative financial shock in core countries while it improves the situation of the periphery. The positive impact observed on peripheral investment comes from the fact that part of core countries' loans are diverted towards the periphery. This net inflow of loans in the periphery fuels firm investment. The second phenomenon is the drop in interbank loans: now part of corporate loans is directly distributed by core countries' banks to peripheral entrepreneurs. This phenomenon is reinforced by the fact that the borrowing cost of corporate loans is cheaper in the periphery with cross border loans, with respect to the segmented situation. Thus, as banks engage in cross-border loans towards peripheral firms, they lend less to core countries' firms, which furthermore depresses investment and activity in this part of the monetary union.

Taken globally, the macroeconomic performance of the currency union worsens with cross-border bank lending, as activity slightly decreases with respect to the segmented situation. The combination of a higher deterioration of the core countries terms of trade and decrease in domestic investment improves the current account of core countries. Thus cross-border lending clearly amplifies the diffusion of a negative net worth shock in the monetary union. It also increases the heterogeneity of investment cycles and the dispersion of current account positions. Negative consequences of net worth shocks have already been studied in the literature with financial globalization (Ueda, 2012) and without (Hirakata et al., 2011). Our results are in line with this literature.

A.6.3 A POSITIVE SHOCK ON BANK RESOURCES

The third set of IRFs, reported in Figure A.8, describes the consequences of a positive shock on bank resources $\varepsilon_{h,t}^B$. This positive shock represents an increase in the resources of the core countries' banking system. It improves the liquidity situation of the core countries' banking system which in turn implies the creation of more corporate loans and reduces the need for interbank loans.

Without cross-border loans (dotted lines), this shock induces an increase in core countries' investment, which in turn leads to more activity. However, as it generates a positive demand shock (the increase in investment is higher than that of activity), it leads to more inflation and to an initial improvement in the terms of trade of core countries. This last phenomenon deteriorates the price competitiveness of core countries' goods while increasing that of peripheral goods. As a consequence, activity increases in the peripheral economies. In the meanwhile as this shock implies an increase in the average inflation rate of the monetary union, the central bank reacts by increasing its interest rate, which in turn depresses consumption and leads to a decrease in activity and inflation after 5 periods in core countries. Lending decisions in the periphery can be explained as follows: the increase in the interest rate of the central bank affects positively the interest rate on loans which in turn depresses loan demand in the periphery. As a consequence, the increase in peripheral investment is financed by peripheral entrepreneur net wealth.

Cross-border bank activity (plain lines) acts as a mechanism that amplifies the transmission of the core country shock on peripheral countries' investment. Now, peripheral entrepreneurs have access to domestic corporate loans which increases

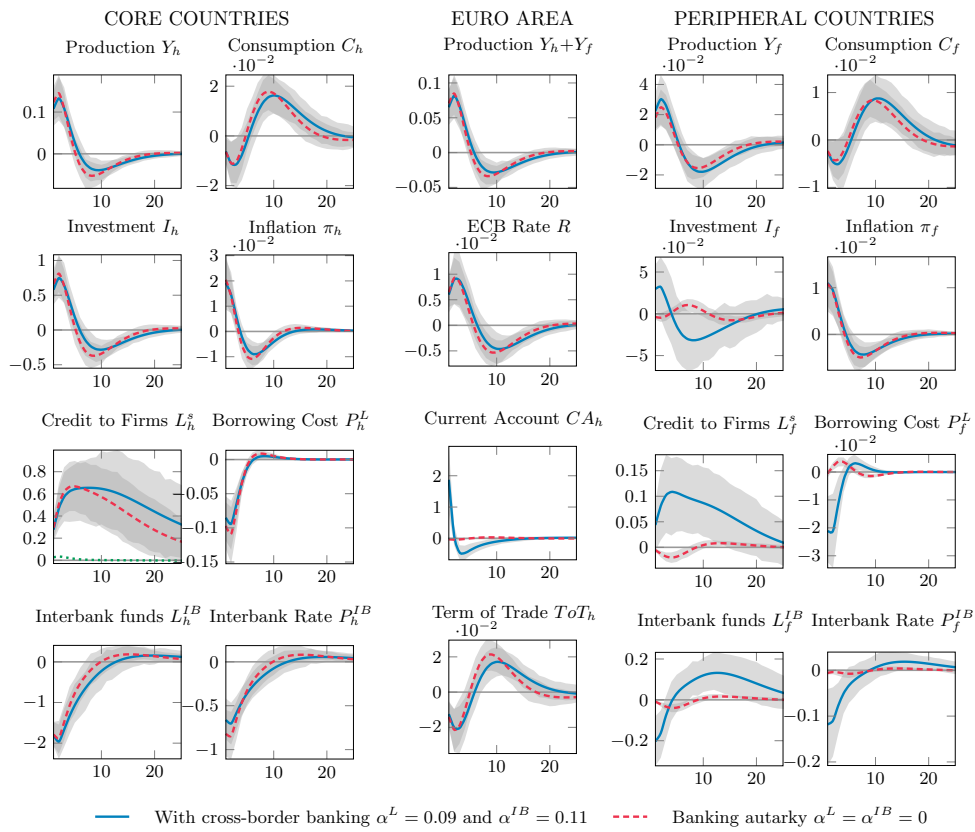


Figure A.8: Bayesian system response to an estimated positive liabilities shock in the balance sheet of core countries banks under banking globalization ($\alpha^L = 9\%$, $\alpha^{IB} = 11\%$) and autarky ($\alpha^L = \alpha^{IB} = 0\%$)

peripheral investment. With cross-border lending, interbank developments should be understood as follows: the further decrease in interbank loans and higher increase in peripheral countries interbank lending clearly fuels investment in the periphery. The increase in the supply of corporate loans is channelled through the increase in interbank lending and the increase in cross-border bank lending (more decrease in the core countries' interbank loans). Initially more corporate loans are distributed in the periphery. However, as it leads to an increase in the leverage ratio of firms, it increases the probability of unproductive projects and, by so, increases the interest rate on loans after 4 periods. This, in turn, depresses peripheral investment after 4 periods.

Cross border banking impacts the dynamics of the bilateral current account. As the dynamics of investment in the periphery is reversed between segmented and integrated situations, it clearly affects the time path of investment and, by so, the time path of the current account. This reaction of investment more than compensates the impact of the terms of trade improvement that leads to the current account deficit in the situation with segmented loan markets.

A.7 THE DRIVING FORCES OF BUSINESS AND CREDIT CYCLES

A.7.1 THE HISTORICAL VARIANCE DECOMPOSITION

Table A.5 reports the posterior variance decomposition of the main aggregates (rate of growth of activity, consumption, investment and loan supply), the average interest rate paid by borrowers, the interest rate of the central bank and the current account. To evaluate the consequences of cross-border interbank and corporate loans, we contrast the variance decomposition reported for model \mathcal{M}_2 with corresponding benchmark figures reported for \mathcal{M}_1 under loan market segmentation. To see the role played by the shocks on these evolutions, we decompose each aggregate variations into the proportions explained by supply shocks (we aggregate $\eta_{i,t}^A$ and $\eta_{i,t}^W$), demand shocks (we aggregate $\eta_{i,t}^\beta$ and $\eta_{i,t}^G$), financial shocks (we aggregate $\eta_{i,t}^N$, $\eta_{i,t}^Q$, $\eta_{i,t}^B$ and $\eta_{i,t}^L$) and the monetary policy shock (η_t^R).

As reported, most of the variance of the growth rate of activity, consumption and inflation is explained by real supply shocks, while the variance of investment, loan supply and interest rates are mainly affected by national financial shocks. These results are in line with the ones reported by Hirakata et al. (2011). Remarkably, the contribution of financial shocks to the fluctuations of the interbank and corporate loan growth rates are comparable (respectively around 46% and 48%).

Besides these general features, our model reports national heterogeneities regarding financial and real supply shocks. On average, financial shocks have a higher impact on core countries' variables than on peripheral variables. As an example, the contribution of the financial shocks accounts respectively for 14.6% and 1.7% of the variance of activity and consumption in core countries, while it accounts for respectively 10.0% and 1.2% in the periphery. As a main noticeable exception core investment is less affected by financial shock than peripheral investment (50.1% instead of 51.4%). For activity and consumption, real supply shocks have a stronger impact on

	CORE			PERIPHERY			EURO	
	Supply	Demand	Financial	Supply	Demand	Financial	Common Financial	Monetary Policy
<i>With cross-border flows</i>								
$V(\Delta Y_{core})$	47.2	12.5	14.6	0.9	1.9	0.6	20.4	1.9
$V(\Delta C_{core})$	70.5	18.2	1.7	0.9	0.6	0.8	4.4	2.8
$V(\pi_{core})$	24.9	18.6	12.6	1.5	2.2	5.4	20.1	14.7
$V(\Delta I_{core})$	1.2	0.3	50.1	0.1	0	0.4	47.5	0.4
$V(\Delta L_{core}^s)$	1.3	0.1	46.6	0.2	0.1	1	50.2	0.5
$V(R_{core}^L)$	18.6	0.5	26.8	1.6	0.1	1.1	49.2	2.2
$V(\Delta IB_{core}^s)$	0.9	0.1	48.7	0.2	0.1	1	48.6	0.5
$V(P_{core}^{IB})$	15	0	37.3	0.4	0	0.8	46.5	0
$V(\Delta Y_{peri})$	1.2	2	1.1	53	14.8	10	17	0.8
$V(\Delta C_{peri})$	0.4	0.5	0.4	76.3	17.4	1.2	2.7	1.1
$V(\pi_{peri})$	1.5	3.1	3.9	24.4	19.3	10.5	21.8	15.6
$V(\Delta I_{peri})$	0.1	0	0.1	1.5	0.2	51.4	46.6	0.1
$V(\Delta L_{peri}^s)$	0.1	0.1	0.3	2	0.1	46.1	50.8	0.4
$V(R_{peri}^L)$	0.4	0.1	0.3	51.1	0.3	15.3	31.2	1.2
$V(\Delta IB_{peri}^s)$	0.1	0.1	0.5	1.7	0.1	48.9	48.3	0.3
$V(P_{peri}^{IB})$	0.1	0	0.8	47.9	0	23	28.2	0
$V(R)$	6	9.3	11.8	5.9	5.3	14	45.6	2.1
$V(CA)$	0.4	0.6	32.3	1.7	0.2	58.9	6	0
<i>Without cross-border flows</i>								
$V(\Delta Y_{core})$	45	10.3	16.3	0.5	1.3	0.9	23.8	1.7
$V(\Delta C_{core})$	75.3	13.1	2	0.4	0.5	1.2	4.8	2.6
$V(\pi_{core})$	20.4	15.9	15.6	1	2	6.4	23.4	15.3
$V(\Delta I_{core})$	1.3	0.3	48.9	0	0	0	49.1	0.4
$V(\Delta L_{core}^s)$	1.3	0.1	48	0.1	0.1	0.1	49.9	0.3
$V(R_{core}^L)$	18.9	0.2	30.4	0.3	0.1	0.1	48.5	1.5
$V(\Delta IB_{core}^s)$	1.2	0.1	57	0.1	0.1	0.1	41.2	0.3
$V(P_{core}^{IB})$	19.6	0	34.6	0.2	0	0	45.5	0
$V(\Delta Y_{peri})$	0.8	1.7	1.4	49.9	13.5	10.9	21.1	0.8
$V(\Delta C_{peri})$	0.2	0.5	0.6	78.3	14.7	1.5	3	1.1
$V(\pi_{peri})$	1.1	3	5.2	19.6	16.2	12.3	26.2	16.4
$V(\Delta I_{peri})$	0	0	0	1.4	0.2	48.7	49.5	0.1
$V(\Delta L_{peri}^s)$	0.1	0.1	0.1	1.4	0.2	43.3	54.7	0.3
$V(R_{peri}^L)$	0.2	0	0.1	46.7	0.2	10.6	41.4	0.8
$V(\Delta IB_{peri}^s)$	0.1	0.1	0.1	1.3	0.2	50.9	47.3	0.2
$V(P_{peri}^{IB})$	0.1	0	0	33	0	38.5	28.5	0
$V(R)$	3.8	8.6	13.8	3.6	4.5	17.2	46.6	1.9
$V(CA)$	9.8	16.3	20.5	8	8.3	33.4	3.7	0

Notes: The supply group gathers productivity ($\eta_{i,t}^A$) and wage cost-push ($\eta_{i,t}^W$) shocks, demand group gathers preferences ($\eta_{i,t}^\beta$) and spending ($\eta_{i,t}^G$), financial group gathers collateral ($\eta_{i,t}^N$) riskiness ($\eta_{i,t}^Q$) rate cost-push ($\eta_{i,t}^L$) and banks liabilities ($\eta_{i,t}^B$) and common financial group gathers collateral (η_t^N) riskiness (η_t^Q) rate cost-push (η_t^L), banks liabilities (η_t^B) and monetary policy (η_t^R).

Table A.5: The unconditional variance decomposition is the share of variance accounted for by each shock of the model estimated with and without cross-border flows.

peripheral aggregates.

As observed, fluctuations in financial indicators related to lending decisions are deeply affected by financial shocks. Contrasting the relative contribution of national, cross-border and common financial shocks, we find that with the noticeable exception of investment (mainly affected by national shocks), variables are mainly affected by common shocks. In contrast, cross-border shocks have only a marginal contribution to the fluctuations of the variables of interest. In most cases, cross-border financial shocks have a higher contribution than cross-border real supply and demand shocks. The main impact of cross-border shocks is observed for financial shocks on financial variables. As an example, peripheral financial shocks account for 7.9% of the fluctuation of core countries' corporate loan interest rate. As reported, leaving aside activity, core countries variables are more altered by cross-border shocks than peripheral variables.

We evaluate how the transmission of shocks is affected by cross-border loans by contrasting these general findings with the lower part of [Table A.5](#). As reported, cross-border banking has a stabilizing effect on financial shocks for activity and consumption (this affects all dimensions of the financial shocks) but increases the contribution of the financial shocks on the fluctuations of the growth rate of investment. Evidence is mixed for other national variables. However has reported, core countries get a higher profit from the integration of the loan market in the Eurozone, as we observe more reduction in the contribution of national financial shocks on national indicators (for 12 variables instead of 9 for peripheral countries). Thus, cross-border lending has a *per-se* impact on the diffusion of financial shocks as it significantly reduces the contribution of national financial shocks on the fluctuations of national variables.

Finally, contrasting the two models, we find that cross-border lending reduces the contribution of financial shocks to the fluctuations in the central bank interest rate (it explains 71.4% of the interest rate fluctuations instead of 77.6% if the loan market is segmented).

A.7.2 UNDERSTANDING THE TIME PATH OF THE CURRENT ACCOUNT

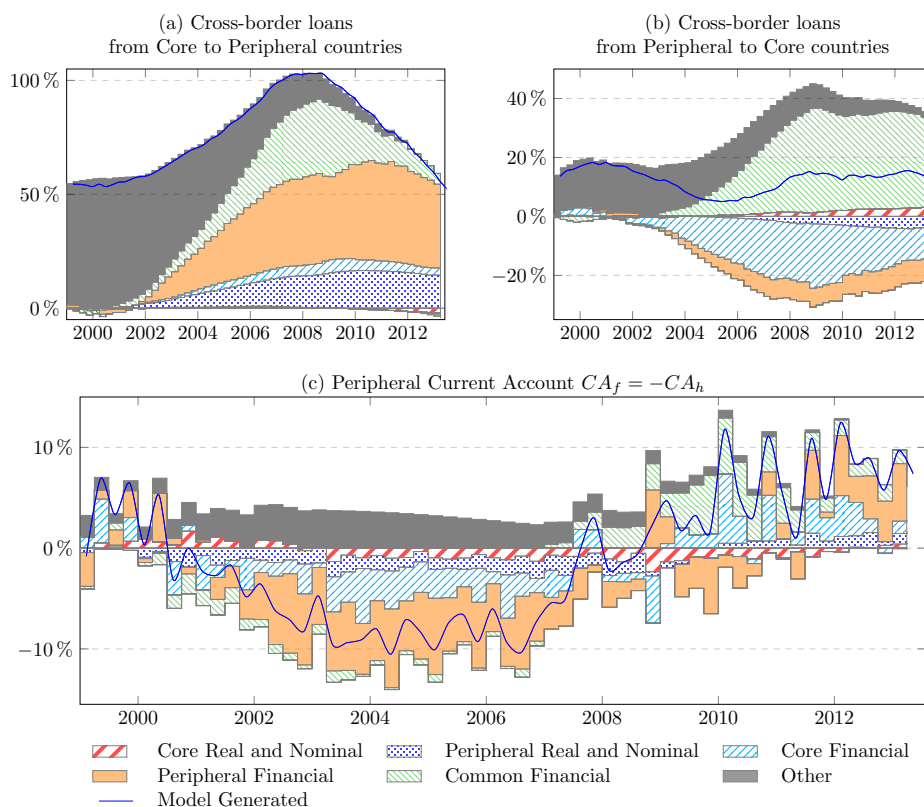
The last rows devoted to the models \mathcal{M}_2 and \mathcal{M}_1 in [Table A.5](#) present the variance decomposition of current account fluctuations. As reported, it is clearly affected by the integration of the loan market: under loan market segmentation, financial shocks contribute to 57.6% of current account fluctuations, while they account for 91.8% with cross-border banking. Remarkably, the contribution of both national and common shocks increases. The main contribution to the current account fluctuations comes from peripheral countries' financial shocks, that account for 58.9% of the current account fluctuations over the time period. Thus, despite the increased contribution of common shocks, the current account fluctuations are more closely related to national financial developments: the need for cross-border lending in this fit exercise is not a substitute for common shocks.

We document in panel (c) of [Figure A.9](#) the time path of the current account on a quarter-on-quarter basis by taking the point of view of peripheral countries (that has been characterized by a persistent current account deficit between 2001Q2 and 2007Q4). The solid line depicts the time path of the current account in deviation from

the steady state as reported by the data, while bars depict the size of shocks in the corresponding deviation. As observed, the explanatory power of the model is quite high (the darker component in the figure that accounts for other factors not taken into account by the model has only a marginal contribution to the current account deficit) and we can link the time path of the peripheral countries' current account to shocks in a rather reliable way. Over the considered time period, the fluctuations in the current account are quite high (between -10% and +10%). As already noticed for the historical variance decomposition, the contribution of real shocks (originating from both the core and the periphery) is marginal. The contribution of core real and nominal developments is almost constant over the sample period and negatively contributes to the current account surplus. In contrast, peripheral real and nominal shocks that initially contributed to the deficit have a positive (although marginal) contribution to the peripheral current account surplus after the occurrence of the financial crisis in the Eurozone.

Overall, financial shocks are the main drivers of the time path of the current account over the considered period. The deterioration of the current account between 2001Q2 and 2007Q4 is clearly linked to the jointly negative contribution of peripheral, core and common financial shocks, even if the former plays a key role in the reported time path. The transmission of the financial crisis in 2008 on the current account surplus appears as a combination of common and core countries' financial shocks. Leaving aside the first two quarters of 2009, the contribution of peripheral financial shocks remained negative long after the beginning of the financial crisis (at least up to the middle of 2011). They have a clear positive impact on the bilateral current account only since 2011Q3.

In [Figure A.9](#), we also document the time path of cross-border interbank loans on a quarter-on-quarter basis. In panel (a) we report the cross-border loans from core to peripheral countries while in panel (b) we present the time path of cross-border loans from peripheral to core countries. As observed, we get a better fit of the model for the latter phenomenon, especially after 2004Q1. Contrasting the two panels, our model clearly shows that financial innovations are the main drivers of cross border loans, as real and nominal factors contribute only marginally to the reported time path in both cases. Common financial factor affecting both regions of the Eurozone have had a positive impact on cross-border lending in both cross-border directions all over the time period. In particular they are the main driver of cross-border loans from peripheral countries. The financial situation of core countries leads to more cross-border loans towards the periphery (they affect positively the reported time path) while it affects negatively cross-border loans from the periphery. In contrast, financial shocks in the periphery contribute positively to cross-border lending from the core and negatively to the core. Putting pieces altogether clearly shows that peripheral countries have benefited from cross border lending over the sample time period: they received more loans than they exported, and this phenomenon has mainly been fuelled by peripheral financial shocks, as already been noted above for the time path of the bilateral current account.



Notes: The solid blue line depicts the quarterly growth rate in real GDP and Investment (per capita), expressed in percentage point deviations from the model's steady state. The colored bars depict the estimated contributions of the various groups of shocks (Real and Nominal: productivity, wage cost-push, spending, preferences asymmetric shocks; Financial: external finance premium, credit cost push, net worth and liabilities asymmetric shocks; Common Financial: external finance premium, credit cost push, net worth, bank liabilities and monetary policy shocks).

Figure A.9: The peripheral current account (quarter-on-quarter % change generated by the model with cross-border flows).

A.7.3 COUNTERFACTUAL ANALYSIS

We report in [Figure A.10](#) the propagation of the financial crisis of 2009 on the model. We represent it as the sum of all the shocks that affected each economy in 2009Q1. In this figure, plain lines represent the adjustment of the corresponding variable estimated by the model, dotted lines report the IRFs computed with perfect banking integration and dashed lines report the IRFs computed without cross-border lending between the two regions of the Eurozone. We get two main findings from this counterfactual exercise: (i) peripheral countries have been much more affected by the crisis than core countries and (ii) the degree of cross-border banking affects the time path of the main national macroeconomic indicators (consumption being the main exception).

First, we find a deeper impact of the financial crisis on interbank loans (reported figures for the IRFs are almost twice the value of corporate loans for each part of the Eurozone). There is a sharp reduction in both corporate and interbank loans in the periphery while, in the core country group, the reduction of interbank loans 'only' diminishes by up to -10% (instead of -40% for the periphery) and the reduction in

corporate loans reaches 'only' -3% (instead of -20% for the periphery). The persistency of the shock is also much higher on peripheral loans. Cross-border banking has deteriorated the evolution of interbank lending in core countries while it has mildly improved the situation of interbank lending in the periphery. In contrast, the degree of cross-border banking estimated by the model between the two groups of countries did not have a significant effect on the time path of corporate loans or on the cost of borrowing faced by firms.

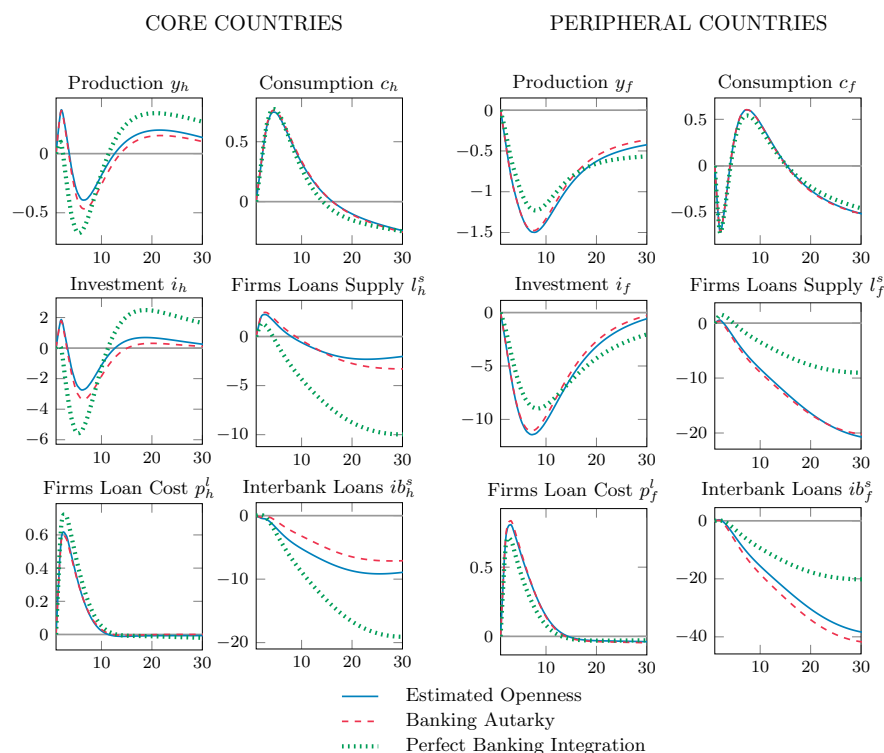


Figure A.10: The system response during the financial crisis (2009Q1) under different levels of financial openness

Second, the size of the fluctuations in macroeconomic aggregates are in line with the behavior of financial variables, as the fluctuations in activity and investment are much more pronounced in the periphery than in the core countries. However, in both parts of the world, the time path of activity mimics that of investment. In the periphery, the financial crisis has clearly led to a sharper and more persistent decrease in investment (reaching a maximum decrease of -12% after 8 quarters instead of -2% after 5 quarters and going back to equilibrium after 11 periods for the core country group) and activity (reaching a maximum decrease of -1.5% after 8 quarters instead of -0.4% after 5 quarters and going back to equilibrium after 11 periods for the core country group). The contribution of cross border banking to the observed dynamics of output and investment underlines that cross-border lending has reduced the negative consequences of the financial crisis for core countries, with respect to a segmentation of the loan market, while it has mildly deteriorated the situation of the periphery.

As a final counterfactual exercise, we find that a perfect banking integration of the Eurozone (in dotted lines) would have amplified the fluctuations of all core countries'

variables, while dampening that of peripheral countries. This complete integration would have led to a transfer of volatility between the two components of the Eurozone, contributing to a better mutualization of the negative consequences of the financial crisis over the region.

A.8 CONCLUSION

In this paper, we have developed and estimated a two-country DSGE model with banking that accounts for interbank and corporate cross-border loan flows. Using Bayesian econometrics, we have found evidence of the key role of this cross-border channel as an amplifying mechanism in the diffusion of asymmetric shocks. In particular, our model reveals that under banking globalization, most national variables are less sensitive to national financial shocks while investment and current account imbalances are more affected. In a counterfactual analysis, we have analyzed how cross border lending has affected the transmission of the recent financial crisis between the two groups of countries.

Our model contributes to the New Open Economy Macroeconomics literature by underlying the critical contribution of and interbank cross-border lending in the Eurozone to account for both the transmission of asymmetric shocks and the effect of monetary policy decisions. In particular, they suggest the importance of cross border loans to assess the impact of the financial accelerator in models of the Eurozone.

Looking forward, our analysis outlines several areas for future research. A natural extension of this analysis should concentrate on the effect of interbank cross border lending on the implementation of unconventional monetary policy measures such as the ones aimed at providing ample funding to the banking system after the recent financial crisis. Our model could also be used as a framework to assess alternative way of supervising lending decisions (based on the home or host principle) and thus contribute to the current discussion of the best way of implementing macroprudential measures in the European Monetary Union.

Part B

Macroprudential policy with global banks

This is a joint work with

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B.1 INTRODUCTION

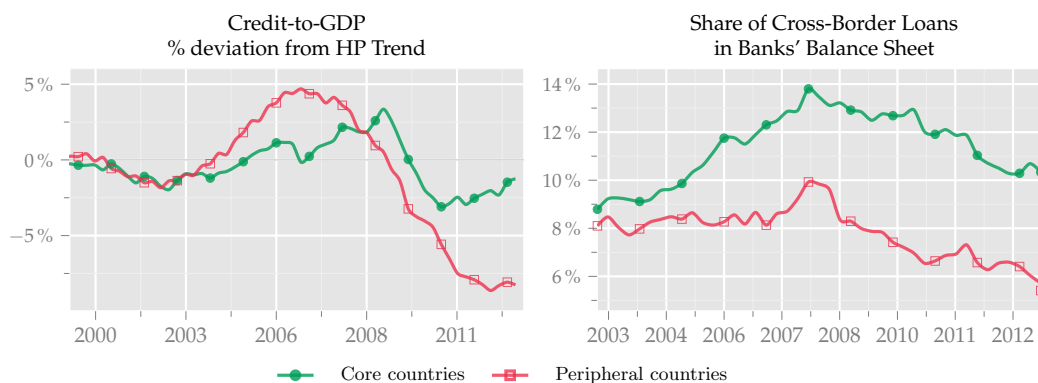
The disruption of financial relations that followed the 2007 subprime crisis set the basis for the adoption of macroprudential policies in most countries.¹¹³ In the Euro Area, the implementation of such measures remains fragmented along national lines while the coordination and internalization of cross-border spillovers are achieved through the actions of the European Systemic Risk Board (ESRB, henceforth). This federal organization accounts for two conflicting features of the Eurozone that can be approached by contrasting core and peripheral countries.¹¹⁴ Panel (a) of [Figure B.1](#) shows that financial cycles (as measured by the credit to GDP ratio in percentage deviation from HP trend) remain clearly national, which militates for a decentralized definition and implementation of macroprudential measures. However, as reported in panel (b) of [Figure B.1](#), these two regions are closely linked by cross-border banking activities (as measured by the share of loans lent to a foreign agent residing in another Euro Area country) and the international spillovers of national macroprudential policies may be harmful for the monetary union. The remaining uncertainties on undesirable side-effects of self oriented macroprudential policies have thus put global banks at a central stage in the on-going debate related to the conduct of macroprudential policies.¹¹⁵

This paper questions how sizable cross-border lending flows should be treated in the definition of national macroprudential policies in the Euro Area. We more particularly assess whether cross-border bank lending should explicitly be considered in the setting of coordinated national macroprudential measures or whether national regulators should only focus on the sole national financial stance to contribute to the financial stability of the Eurozone.

¹¹³In a nutshell, macroprudential policy aims at completing monetary policy to enhance the resilience of the financial system and contain the procyclicality of financial factors on activity.

¹¹⁴In the first group we aggregate data for countries with a current account surplus and low government bond yields over the sample period (Austria, Belgium, Germany, Finland, France, Luxembourg and Netherlands), while in the second group, we aggregate data for countries with a current account deficit and high government bond yields over the sample period (Spain, Greece, Ireland, Italy and Portugal).

¹¹⁵For example, regarding issues related to macroprudential policy with global banking, we refer to the IMF (2013, key issues, p31), the [ESRB handbook](#) (2014), ECB (2015, Financial Stability Review, May), Bank of England (2015, Staff Working Paper).



NOTE: Cross-border lending refers to any financing arrangement that crosses national borders between a domestic bank and a foreign borrower. The share of cross-border loans is computed here as the ratio between loans to euro area excluding the domestic area and the loans to euro area (i.e. cross-border loans between core countries are included in the calculation of the share of international loans). Sources: ESRB and ECB statistics.

Figure B.1: Stylized facts characterizing the Eurosystem banking system: credit cycles remain clearly national while cross-border lending experienced an important growth

We build and estimate a two-country DSGE model that accounts for two major aspects to address the question at hands. First, we extend the setup of [Poutineau and Vermandel \(2015\)](#) - featuring cross-border banking on the corporate and inter-bank loan markets¹¹⁶ - to account for bank capital regulation and thus to contrast the effectiveness of macroprudential policy from banking autarky to perfect integration. Second, in line with the actual organization of macroprudential policy,¹¹⁷ we focus on the joint-optimization of macroprudential policy rules in each country using the countercyclical capital buffer (CCB, henceforth) rate as an instrument. This solution has become one of the leading facets of prudential regulation since the adoption of [Basel III accords \(2010\)](#) by building up a bank capital buffer during periods of excessive credit growth that can be released when systemic risks abate. The international dimension of banks offered by our setting allows us to contrast different CCB rules based on: (i) the federal or the national credit-to-gdp targeting, (ii) the loan demand (from firms) or supply (from banks) to GDP targeting, and (iii) the loan inflows-to-GDP ratio targeting as envisaged by [Rey \(2015\)](#).

The methodology employed in this paper comprises three steps. First, we build and estimate a two-country DSGE model for the Euro Area with only monetary policy (as there are no observations for an estimation of a macroprudential rule). Second, we compute the optimal policy rules (both monetary and macroprudential policy) given the estimated parameters assuming a two-stage game where monetary policy is

¹¹⁶In this paper, we omit the mortgage market and concentrate on corporate and interbank loans. Given the insignificant size of cross-border housing loans in the portfolio of banks (the share of cross-border loans is below 1% in the Euro Area according to ECB internal data), this omission does not seem to be important for the analysis conducted here.

¹¹⁷We refer to [Carboni et al. \(2013\)](#) for a discussion regarding the macroprudential policy mandate in the Euro Area shared between European Central Bank and the Single Supervisory Mechanism, national competent authorities and coordinated by the European Systemic Risk Board.

the leader.¹¹⁸ Third, we examine implications of cross-border lending on the optimal design of macroprudential rules across country members of the Eurosystem using the optimal monetary policy rule as a benchmark.

The main result of the paper suggests that self oriented macroprudential national policies reacting to the evolution of home country loan creation should be favored even with high amounts of cross-border lending flows: First, targeting a national credit-to-gdp ratio should be favored to federal averages as this rule induces better stabilizing performances in front of important divergences in credit cycles between core and peripheral countries. Second, policies reacting to the evolution of national credit supply should be favored as the transmission channel of macroprudential policy directly impacts the marginal cost of loan production and, by so, financial intermediaries. Third, the interest of lifting up macroprudential policymaking to the supra-national level remains questionable for admissible value of international lending between Eurozone countries. Indeed, national capital buffers reacting to the union-wide loan-to-GDP ratio only lead to the same stabilization results than the one obtained under the national reaction if cross-border lending reaches 45%. However, even if cross-border linkages are high enough to justify the implementation of a federal adjusted solution, the reaction to national lending conditions remains remarkably optimal.

Additionally, we outline some particularities regarding the conduct of macroprudential policies for peripheral countries. We find that adjusting the macroprudential instrument to capital inflows-to-GDP is a promising tool for these countries that have experienced a large amount of loan inflows. Furthermore, disentangling the demand/supply of credit has implications for macroprudential policymaking as it is preferable to target credit suppliers for core countries and borrowers for peripheral economies.

Our approach is partly related to a set of papers examining macroprudential measures in the Eurozone with a closed economy setup. Notably, [Darracq-Pariès et al. \(2011\)](#) and [Angelini et al. \(2014\)](#) build a DSGE model of the Eurozone close to [Gerali et al. \(2010\)](#) with both corporate and housing credit markets and evaluate the optimal mix between monetary and macroprudential policy using loss functions. As a key contribution to the literature, they suggest that time-varying capital requirements can improve macroeconomic stability by supporting monetary policy actions. Our analysis can thus be considered as an extension to these papers, by accounting for the heterogeneity between Euro Area participants and the existence of national macroprudential policies with cross-border spillovers.

Our paper also contributes to macroprudential policy analysis in open economies. As an example, [Quint and Rabanal \(2014\)](#) account for financial asymmetries between participating countries and focus on the interaction between financial and housing cycles without considering cross-border flows between countries. By omitting cross-border lending, they naturally find that there are no important spillover effects of regulation from one member state to another via an estimated two-country DSGE model of the Eurozone. Additionally, [Jeanne \(2014\)](#) employs a static open economy

¹¹⁸A important branch of the literature analyzed the interaction between monetary policy and financial stability, a topic not covered in the paper as we concentrate here on interactions between national prudential authorities. We refer to [Woodford \(2012\)](#) for a summary of policy challenges and results offered by the existing literature concerning the role of monetary policy in providing financial stability.

model to evaluate the effectiveness of macroprudential and capital control measures. Contrary to [Quint and Rabanal \(2014\)](#), he finds that these prudential policies generate important global spillovers even with international coordination.

The paper is organized as follows: [Appendix B.2](#) describes the financial sector of the model. [Appendix B.4](#) takes the model to the data. [Appendix B.5](#) discusses the performance of macroprudential policy. [Appendix B.6](#) provides a sensitivity analysis to assess the robustness of our results. [Appendix B.7](#) concludes.

B.2 THE FINANCIAL SECTOR

The economy is composed of two countries of unequal size and populated by households, firms and banks. This first section describes the banking component of the model while the rest of the framework (standard to the literature) is presented in appendix.

B.2.1 THE FINANCIAL SECTOR IN A NUTSHELL

[Figure B.2](#) provides a broad picture of the financial sector and summarizes its interaction with the rest of the economy. Banks engage in interbank lending/borrowing relations and provide corporate loans to entrepreneurs and deposit services to households. Authorities affect the decisions of the banking sector through monetary and macroprudential policies.

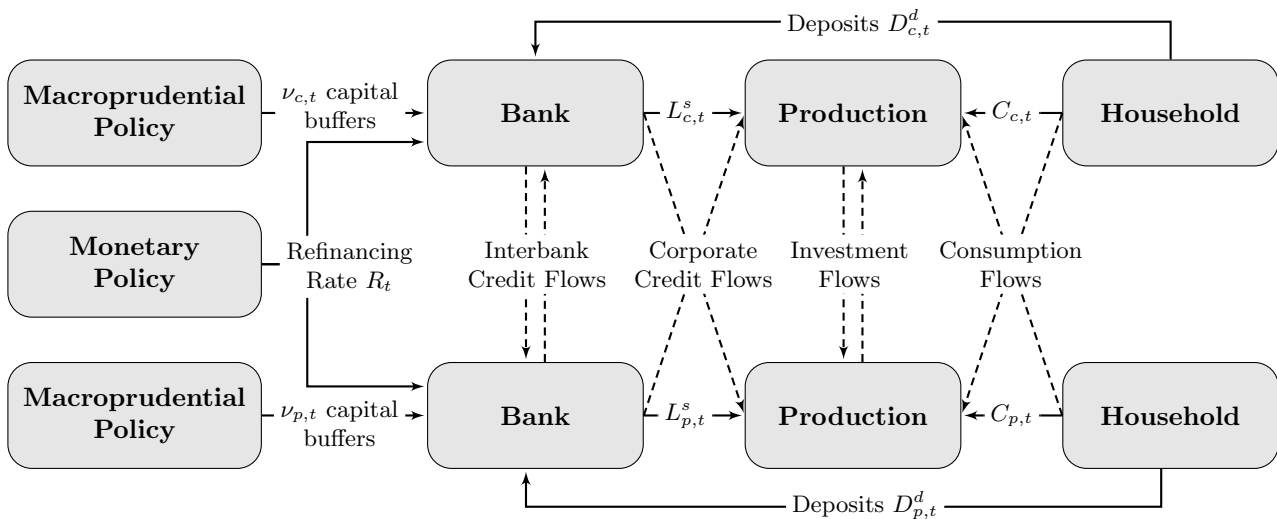


Figure B.2: Macroprudential policy and cross-border banking in a New Keynesian Framework

To introduce an interbank market, we assume that banks are heterogenous in terms of liquidity. This feature gives rise to an interbank market where liquid banks provide interbank loans to both home and foreign banks. This feature is line with the current European banking system characterized by banks relying on wholesale fundings as illustrated by [Giannone et al. \(2012\)](#). In our setup, the distinction between

liquid and illiquid banks lies in the direct access of liquid banks to ECB fundings which allow intra-financial sector flows between financial intermediaries.¹¹⁹ Extending this assumption to an international perspective, illiquid banks can borrow from both domestic and foreign liquid banks, which gives rise to cross-border interbank lending flows. The decision of the banking system regarding the provision of deposit services to households and loans to the corporate sector affects the rest of the economy through the setting of deposit and lending interest rates.¹²⁰ In line with the EMU situation, we do not consider cross-border deposit nor cross-border lending to households. The international flow of loans between economies is thus a consequence of interbank liquidity provision and borrowing choices undertaken by entrepreneurs (following a comparison between the relative interest rates of domestic and foreign corporate loans).

This paper adopts a macroeconomic perspective to focus on the effect of cross-border lending on the conduct of macroprudential policy in a heterogeneous monetary union. As a consequence, the financial sector is combined with a standard two-country DSGE model accounting for short run rigidities in goods prices and nominal wages. In what follows, we outline the main assumptions regarding the functioning of the financial sector that are deemed necessary to improve both the tractability of the analysis and the estimation of the many behavioral parameters of the DSGE structure. Some modelling choices have been done in line with the DSGE literature that contrast with a more standard description of the behavior of the banking sector as summarized by Freixas and Rochet (2008) and VanHoose (2009). As in the initial contribution of Gerali et al. (2010), this macro superstructure is augmented with a highly simplified banking model. A host of assumptions should be invoked that effectively splinter a bank's decisions into independent choices about different portions of its balance sheet.¹²¹

This paper extends Poutineau and Vermandel (2015) to account for deposit decisions and for macroprudential consideration in the balance sheets of financial intermediaries. The stickiness in both deposit and loan interest rates is a key ingredient of the framework. The setting of interest rate mimics the way other sticky nominal variables such as prices and wages are set in the model by adopting a Calvo-type

¹¹⁹This assumption is empirically motivated: in the Eurosystem, only a fraction of the 2500 banks participates regularly to the bidding process in main refinancing operations of the ECB while the others rely on interbank funding.

¹²⁰For tractability reasons we assume that even if banks differ in their ability to raise funds from the central bank, their loan and deposit supply decisions remain homogenous after aggregation. In a real life situation, illiquid banks may face more difficulties in attracting households deposits requiring banks to set higher deposit rates to compensate their default risk. Regarding corporate loans provision, the tighter funding constraint of illiquid may diminish their loan supply compared to liquid banks.

¹²¹First, portfolio separation holds (Baltensperger (1980) and Santomero (1984)), which means (Sealey and Lindley (1977) and Sealey (1985)) that a number of assumptions have been invoked. For instance, either shareholder unanimity is assumed for all banks in the model, or risk neutrality has been assumed to render shareholder unanimity a non-issue. In addition, it must be assumed that banks' costs of real resources utilized in their operations are separable from resource costs for others of the banks' assets and liabilities at during each period and across periods if interperiod adjustment costs are taken into account. Finally, banks must have access to a market in which they can both borrow and lend at exactly the same interest rate. Only when all such assumptions are invoked, it is legitimate for each bank to be able to make separable decisions about balance-sheet choices as assumed in this model.

mechanism. This device, shared by most DSGE models with a banking sector, partly contrasts with the literature developed from the banking industry perspective. Indeed, most of the banking literature has, following [Flannery \(1982\)](#) original work on deposits as quasi-fixed factors, focused on intertemporal quantity adjustment costs. It is also worth noting that the substantial banking literature on this topic has proposed alternative ways of approaching this question, including [Cosimano and Van Huyck \(1989\)](#), [Cosimano \(1987, 1988\)](#), and [Elyasiani et al. \(1995\)](#) and [Abo-Zaid \(2015\)](#). Furthermore, sluggish and even asymmetric variations in bank retail rates have been documented in the empirical literature as in [Van Leuvensteijn et al. \(2013\)](#) through imperfect competition among banking systems, while [Kopecky and Van Hoose \(2012\)](#) rely on intertemporal quantity adjustment costs together with imperfect competition to explain such observations. The adoption of a Calvo mechanism combined with monopolistic competition has been employed here in a macro-perspective for credit and deposit interest rates, as this solution allows us to consider the sluggishness in the adjustment of all the nominal variables of the economy (prices, wages and interest rates) through the estimation of a "Calvo lottery parameter".

As a second major noticeable difference from [Poutineau and Vermandel \(2015\)](#), we account for endogenous leverage of financial intermediaries, thus reflecting the riskiness in the balance sheet of banks. We use time-varying capital requirements as the macroprudential instrument. As underlined by [Angelini et al. \(2014\)](#), capital buffers have taken a center stage in the ongoing debate on regulatory reform and have become one of leading facet of macroprudential regulation. Specifically in the European Union, a number of macro-prudential policy instruments including countercyclical buffers are embedded in the legislative texts transposing the Basel III regulatory standards into EU law.¹²² To account for this compulsory macroprudential instrument, we borrow the modelling device of [Darracq-Pariès et al. \(2011\)](#) and [Angelini et al. \(2014\)](#) by assuming that each type of bank must pay a quadratic cost when its risk weighted assets ratio deviates from the time-varying ratio fixed by the macroprudential authority in country i according to the systemic risk arising within the financial system. The decision to penalize banks for keeping equity-capital positions below the official benchmark is easy to understand, as undercapitalized banks make the banking sector more fragile and in turn subject to bank runs ([Diamond and Rajan \(2001\)](#)). Symmetrically, the decision to impose costs on banks for having equity-capital positions above the required levels may be understood in a macroeconomic perspective: by keeping more equity capital levels than required by the official regulation, the banking sector diverts resources and, in turn, creates credit rationing for both entrepreneurs and illiquid banks. This may create lower than desired banking activity, reduce investment in the economy and incur inefficiencies.¹²³

¹²²Namely the new Capital Requirements Directive (CRD IV) and the Capital Requirements Regulation (CRR). We refer to [Carboni et al. \(2013\)](#) for a discussion regarding the macroprudential policy mandate in the Euro Area shared between ECB/SSM, national competent authorities and coordinated by the ESRB.

¹²³[Van den Heuvel \(2008\)](#) finds using a general equilibrium model that increasing capital requirements induces high welfare costs in terms of unconditional consumption, suggesting that capital requirements should be lower than in the current adequacy framework. [Clerc et al. \(2014\)](#) highlight the presence of a tradeoff using a financial accelerator model between too high and too low capital requirements.

B.2.2 INTERBANK RELATIONS

In each country the banking system consists of two distinct branches: a continuum of monopolistic banks and financial packers. Monopolistic banks provide different types of loans and deposit services and set interest rates on a Calvo basis. The financial intermediary is a CES packer that produces one homogenous loan and deposit service.¹²⁴ A share λ of banks is illiquid (*i.e.* credit constrained), while the remaining share of banks $1-\lambda$ is liquid and supplies interbank loans to illiquid banks.

The representative share λ of illiquid banks b in country i has the following balance sheet,

$$L_{i,t}^s = IB_{i,t}^H + BK_{i,t}^{ill} + D_{i,t} + liab_{i,t}^{ill}, \quad (\text{B.1})$$

where $L_{i,t}^s$ is the loan supply of borrowing banks, $IB_{i,t}^H$ is the interbank loans supplied by liquid banks subject to external habits, $BK_{i,t}^{ill}$ is the bank capital, $D_{i,t}$ are deposit services to households and $liab_{i,t}$ are other liabilities in the balance sheet of the bank that are not considered in the model.¹²⁵ To close the model, we assume that the cost of these liabilities is set by the central bank through its refinancing rate. We suppose that the demand for interbank funds are subject to external habits at a degree h_i^B where $IB_{i,t}^H = IB_{i,t}^d - h_i^B (IB_{i,t-1}^d - \overline{IB}_i^d)$. These habits captures the empirical autocorrelation of interbank funding. In addition, these habits are empirically documented in the interbank network literature: [Finger et al. \(2014, 2015\)](#) find at a bank level that bilateral links between banks are persistent as banks heavily rely on well-established business relations, thus exhibiting some habits in borrowing/lending decisions.

The one-period stream of profits of the b -th illiquid bank is given by:

$$\begin{aligned} \Pi_{i,t}^{ill} = & \left[1 - \mu^B (1 - \mathbb{E}_t \{ \eta_{i,t+1} \}) \right] (1 + R_{i,t}^L) L_{i,t}^s - (1 + R_{i,t}^D) D_{i,t} - (1 + P_{i,t}^{IB}) IB_{i,t}^H \\ & - (1 + R_t) liab_{i,t}^{ill} - F (rwa_{i,t}^{ill} - v_{i,t}) BK_{i,t}^{ill}, \end{aligned} \quad (\text{B.2})$$

where $\mu^B \in [0, 1]$ denotes the loss-given-default (*i.e.* the percentage of the amount owed on a defaulted loan that the bank is not able to recover), $1 - \mathbb{E}_t \{ \eta_{i,t+1} \}$ is the expected average default rate of the bank's home and foreign customers,¹²⁶ $R_{i,t}^D$ is deposit rate, $P_{i,t}^{IB}$ is the borrowing cost on the interbank, R_t the interest rate set by the

¹²⁴The financial packer acts as a loan and deposit bundler in a perfectly competitive market. Banks supply differentiated types b of deposits $D_{i,t}(b)$ and loans $L_{i,t}^s(b)$ bundled by financial packers. Their packing technology for deposit services and loans reads as, $D_{i,t}^d = [(1/n_i)^{1/\epsilon_D} \mathcal{G}(D_{i,t}(b)^{(\epsilon_D-1)/\epsilon_D})]^{\epsilon_D/(\epsilon_D-1)}$, and $L_{i,t}^d = [(1/n_i)^{1/\epsilon_L} \mathcal{G}(L_{i,t}^s(b)^{(\epsilon_L-1)/\epsilon_L})]^{\epsilon_L/(\epsilon_L-1)}$. It maximizes profits, $R_{i,t}^D D_{i,t}^d + R_{i,t}^L L_{i,t}^d - \mathcal{G}(R_{i,t}^D(b) D_{i,t}(b)) - \mathcal{G}(R_{i,t}^L(b) L_{i,t}^s(b))$, subject to their two technology curves. Here, $L_{i,t}^d$ is the loans demand from home and foreign entrepreneurs, $D_{i,t}^d$ is the deposit services demand from domestic households and $\mathcal{G}(\cdot)$ is the aggregator function. Deposits and loans are imperfect substitute with elasticity of substitution $\epsilon_D < -1$ and $\epsilon_L > 1$. The corresponding demand functions associated from the previous problem are, $D_{i,t}(b) = (1/n_i)(R_{i,t}^D(b)/R_{i,t}^D)^{-\epsilon_D} D_{i,t}^d$ and $L_{i,t}^s(b) = (1/n_i)(R_{i,t}^L(b)/R_{i,t}^L)^{-\epsilon_L} L_{i,t}^d$. The aggregate price index of all varieties in the economy is given by, $R_{i,t}^D = [(1/n_i)\mathcal{G}(R_{i,t}^D(b)^{1-\epsilon_D})]^{1/(1-\epsilon_D)}$ and $R_{i,t}^L = [(1/n_i)\mathcal{G}(R_{i,t}^L(b)^{1-\epsilon_L})]^{1/(1-\epsilon_L)}$.

¹²⁵We suppose that they follow an exogenous $AR(1)$ shock process $\varepsilon_{i,t}^B$ such that, $liab_{i,t} = e^{\varepsilon_{i,t}^B} \overline{liab}_i$, this shock captures some aggregate movements in the funding constraint arising from the wholesale funding market, see for instance [Pérignon et al. \(2017\)](#) for an analysis of liquidity runs on the French unsecured market of certificates of deposits.

¹²⁶To simplify both the steady state and the log-linear version of the model, the bank default expecta-

central bank and $F_i(\cdot)$ denotes the capital requirement cost function. This cost function is taken from [Gerali et al. \(2010\)](#) and is defined as $F_i(x) = 0.5\chi^k x^2$ where $\chi^k \geq 0$ is the cost of capital adequacy framework paid in term of bank capital.¹²⁷ This cost function is a shortcut that makes bank capital more costly than any source of financing, and allows in turn to mimic the response of credit rates and credit to a capital requirement tightening consistently with empirical evidence (see for instance [Fraisie et al. \(2013\)](#) for an empirical measure of this elasticity). When the bank capital-to-risky-asset ratio $rwa_{i,t}^{ill}$ is below the policy target v_{it} , the bank is penalized by regulatory rules that affect the borrowing rates in the monetary union and in turn damage output. This penalization replicates the market discipline imposed by investors on low capitalized banks, forcing the latter to boost their retained earnings though higher credit rates. The risk is evaluated through fixed weights on assets, based on the type of the borrowers (1 for corporate exposure and 0.20 for interbank exposure between OECD banks as defined in Basel accords) as defined in Basel I accords. Since illiquid banks are only exposed to corporate risk, the risk weighted assets ratio is given by $rwa_{i,t}^{ill} = BK_{i,t}^{ill}/L_{i,t}^s$. In addition, the financial intermediary has access to domestic and foreign interbank loans to meet its balance sheet. The modelling device to introduce international borrowing is analogous to trade channels through a CES as in [Poutineau and Vermandel \(2015\)](#) and [Brzoza-Brzezina et al. \(2015\)](#). The total amount borrowed by the representative bank reads as follows:

$$IB_{i,t}^d = \left((1 - \alpha_i^{IB})^{1/\xi} (IB_{hi,t}^d)^{(\xi-1)/\xi} + (\alpha_i^{IB})^{1/\xi} (IB_{fi,t}^d)^{(\xi-1)/\xi} \right)^{\xi/(\xi-1)}, \quad (\text{B.3})$$

where parameter $\xi > 0$ is the elasticity of substitution between domestic and foreign interbank funds, α_i^{IB} represents the percentage of cross-border interbank loan flows in the monetary union and $IB_{hi,t+1}^d$ (resp. $IB_{fi,t+1}^d$) the amount of domestic (resp. foreign) loans demanded by borrowing bank b in country i . This existence of an home bias on the interbank market is empirically motivated, [Fricke and Lux \(2015\)](#) find, using Italian bank-level data, that Italian banks tend to trade with each other rather than with foreign banks, in particular after the financial turmoil. More broadly in the literature of finance, the home bias in portfolio was first documented by [French and Poterba \(1991\)](#).

The total cost incurred by illiquid banks to finance interbank loans, $1 + P_{i,t}^{IB}$, is thus defined according to the CES aggregator:

$$1 + P_{i,t}^{IB} = ((1 - \alpha_i^{IB}) (1 + R_{h,t}^{IB})^{1-\xi} + \alpha_i^{IB} (1 + R_{f,t}^{IB})^{1-\xi})^{1/(1-\xi)}, \quad (\text{B.4})$$

where $1 + R_{h,t}^{IB}$ (resp. $1 + R_{f,t}^{IB}$) is the cost of loans obtained from home (resp. foreign) banks in country i . Finally following [Gerali et al. \(2010\)](#), the bank capital accumula-

tion regarding entrepreneurs' projects is defined by a geometric average of home and foreign surviving rates of entrepreneurs, $\eta_{i,t} = (\eta_{h,t}^E)^{1-\alpha_h^L} (\eta_{f,t}^E)^{\alpha_h^L} \bar{\eta}^{\alpha_h^L - \alpha_j^L}$ where $\eta_{i,t+1}^E$ is the default rate of entrepreneurs operating in country $i \in \{c, p\}$. The expression $\bar{\eta}^{\alpha_h^L - \alpha_j^L}$ ensures the detesministic steady state remains symmetric between Core and Periphery without affecting the dynamic of the model up to a first order approximation.

¹²⁷The quadratic nature of this cost has been discussed in the previous subsection.

tion process of illiquid banks ($BK_{i,t}^{ill}$) is determined by:

$$BK_{i,t}^{ill} = (1 - \delta_i^{ill}) \Pi_{i,t-1}^{ill}, \quad (\text{B.5})$$

where $\delta_i^{ill} \in [0, 1]$ measures resources used in managing bank capital and conducting the overall banking intermediation activity and is determined endogenously by the steady state of the model. Given the functional form of $F_i(\cdot)$, the first order condition on loans which determines the marginal cost of supplying an additional unit of loans to home and foreign entrepreneurs is:

$$1 + MC_{i,t}^{ill} = \frac{1 + P_{i,t}^{IB} + \chi^k (v_{i,t} - rwa_{i,t}^{ill}) (rwa_{i,t}^{ill})^2}{1 - \mu^B (1 - \mathbb{E}_t \{\eta_{i,t+1}\})}. \quad (\text{B.6})$$

From this equation, we observe that an increase (reduction) in the CCB rate $v_{i,t}$ (risk weighted assets ratio $rwa_{i,t}^{ill}$) imposes on banks to accumulate more equity via retained earnings through a rise in credit rates. Parameter χ^k determines the elasticity of interest rates to capital regulation change.¹²⁸ During phases of expansion, banks have incentives to increase their leverage away from the target in order to boost their profits. This risk taking by banks is addressed in our model though the cost function that forces banks to control their capital structure.

The fraction $1 - \lambda$ of remaining liquid banks has the following balance sheet:

$$L_{i,t}^s + IB_{i,t}^s = L_{i,t}^{ECB} + BK_{i,t}^{liq} + D_{i,t} + liab_{i,t}^{liq}, \quad (\text{B.7})$$

where $L_{i,t}^s$ is the lending supply to entrepreneurs, $IB_{i,t}^s$ is the supply of funds on the interbank market, $L_{i,t}^{ECB}$ is the amount of refinancing operations obtained by the liquid bank, $BK_{i,t}^{liq}$ is the amount of bank capital, $D_{i,t}$ are deposits collected from domestic households and $liab_{i,t}$ are exogenous liabilities as explained previously. The one-period profit of the bank $\Pi_{i,t}^{liq}$ is defined as:

$$\begin{aligned} \Pi_{i,t}^{liq} = & (1 - \mu^B (1 - \mathbb{E}_t \{\eta_{i,t+1}\})) (1 + R_{i,t}^L) L_{i,t}^s + (1 + R_{i,t}^{IB}) IB_{i,t}^s - (1 + R_{i,t}^D) D_{i,t} \\ & - (1 + R_t) liab_{i,t}^{liq} - (1 + R_t) L_{i,t}^{ECB} - F(rwa_{i,t}^{liq} - v_{it}) BK_{i,t}^{liq}. \end{aligned} \quad (\text{B.8})$$

Here, $R_{i,t}^{IB}$ is the interest rate set by liquid banks to home and foreign illiquid banks, R_t is the refinancing rate of the central bank and $F_i(\cdot)$ denotes the same Basel cost function as for illiquid banks: $F_i(x) = 0.5\chi^k x^2$. Interbank claims affect the amount of equity held by banks and are given a risk weight at 20%. The risk weighted asset ratio for liquid bank incorporating corporate and bank exposures is given by $rwa_{i,t}^{liq} = BK_{i,t}^{liq} / (L_{i,t}^s + 0.2IB_{i,t}^s)$. According to the illiquid bank, bank capital of liquid banks evolves according to

$$BK_{i,t}^{liq} = (1 - \delta_i^{liq}) \Pi_{i,t-1}^{liq}, \quad (\text{B.9})$$

where $\delta_i^{liq} \in [0, 1]$ is similar to the illiquid bank and measures the fraction of capital used during the intermediation process that cannot be re-invested next period. The

¹²⁸Empirically, [Fraisie et al. \(2013\)](#) find at a bank level that one percentage increase in capital requirements leads to a reduction in lending by approximately 10%.

first order condition on loans determining the marginal cost of loans of the liquid bank b is:

$$1 + MC_{i,t}^{liq} = \frac{1 + R_t + \chi^k (v_{i,t} - rwa_{i,t}^{liq}) (rwa_{i,t}^{liq})^2}{1 - \mu^B (1 - \mathbb{E}_t \{\eta_{i,t+1}\})}, \quad (\text{B.10})$$

and the second first order condition on interbank loans determines the interbank rate set by banks operating in country i :

$$R_{i,t}^{IB} = R_t + 0.2\chi^k (v_{i,t} - rwa_{i,t}^{liq}) (rwa_{i,t}^{liq})^2. \quad (\text{B.11})$$

Here again, an increase in bank capital requirements raises the bank's cost of lending, and in turn increases both interbank and corporate interest rates. This result is consistent with standard business cycle models and is referred to the bank capital channel as in Van den Heuvel (2008), Meh and Moran (2010), Darracq-Pariès et al. (2011) and Angelini et al. (2014).

B.2.3 INTEREST RATE SETTING

We assume that interest rates on deposits and corporate credit loans are sticky. In particular, sluggish and even asymmetric variations in bank retail rates have been documented in the empirical literature as in Kopecky and Van Hoose (2012) and Van Leuvensteijn et al. (2013) through imperfect competition among banking systems. The setting of interest rate mimics the way other sticky nominal variables such as prices and wages are set in the model. As in Darracq-Pariès et al. (2011), we introduce a Calvo model for credit rates to firms and deposit rates while the interbank rate is left flexible as banks operate under perfect competition on the interbank market. Banks must solve a two-stage problem. In the first stage, banks minimize the cost of managing their funds on a competitive input markets by computing the marginal cost of supplying an additional loan to borrowers and a deposit service to households. The computation of these marginal costs has already been performed in the previous subsection. In a second stage, they operate under monopolistic competition by applying a markup (markdown) on their commercial loan (deposit) rate, and set the interest rate on a staggered basis. Using a Calvo nominal rigidity device, each period a random fraction θ_i^L (θ_i^D) of banks is unable to update its lending (deposit) rate, $R_{i,t}^L = R_{i,t-1}^L$ ($R_{i,t}^D = R_{i,t-1}^D$), creating an imperfect transmission of monetary policy decisions to borrowers and savers living in the monetary union. The bank that it is able to modify its loan interest rate (with a constant probability $1 - \theta_i^L$) chooses $R_{i,t}^{L*}$ to maximize its expected stream of profits adjusted by the risk of default:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\theta_i^L)^\tau \Lambda_{i,t+s} (1 - \mu^B (1 - \eta_{i,t+1+\tau})) [R_{i,t}^{L*} - \exp(\varepsilon_{i,t+s}^L) MC_{i,t+s}^L] L_{i,t+s}^s, \quad (\text{B.12})$$

where $\varepsilon_{i,t}^L$ is an *ad-hoc* markup AR(1) shock to the credit rate equation, $\theta_i^L \in [0, 1)$ is the Calvo lottery coefficient determining the degree of nominal rigidity and $MC_{i,t}^L$ is the aggregate marginal cost combining outputs from liquid and illiquid banks of country i . We aggregate loans from liquid and illiquid banks and their respective marginal costs before applying the markup for tractability purposes: this device is useful to compute a single Phillips curve as well as an unique credit rate for both

liquid and illiquid banks. We borrow this shortcut procedure from [Gerali et al. \(2010\)](#) adapted in a different context, i.e. all banks belonging to a national banking system share the same marginal cost of production, reflecting the average liquidity degree of national banks: $1 + MC_{i,t}^L = (1 + MC_{i,t}^{ill})^\lambda (1 + MC_{i,t}^{liq})^{(1-\lambda)}$. In addition, the banking spread reflecting the level of financial distress is given by $S_{i,t}^L = (1 + R_{i,t}^L)/(1 + R_t)$.

In a similar fashion for deposit rates, assuming that it is able to modify its interest rate with a constant probability $1 - \theta_i^D$, the representative bank chooses $R_{i,t}^{D*}$ to maximize its expected stream of profits, by applying a markdown on the refinancing rate of the central bank R_t :

$$\mathbb{E}_t \sum_{\tau=0}^{\infty} (\theta_i^D)^\tau \Lambda_{i,t+s} [R_{t+s} \exp(\varepsilon_{i,t+s}^D) - R_{i,t}^{D*}] D_{i,t+s}, \quad (\text{B.13})$$

where $\varepsilon_{i,t}^D$ is an *ad-hoc* time-varying AR(1) markdown shock to the deposit rate equation and $\theta_i^D \in [0, 1)$ is the Calvo lottery parameter.

B.2.4 MACROPRUDENTIAL POLICY

Macroprudential policy affects the general equilibrium of the economy through the policy instrument $v_{i,t}$ that contributes to the marginal cost of commercial banks' loans. As a consequence, a macroprudential policy tightening is associated with higher lending rates, and lower bank credit growth and asset prices. We assume that the macroprudential authority sets the time-varying capital requirement $\nu_{i,t}$ according to:

$$v_{i,t} = (1 - \rho_i^v) \bar{v} + \rho_i^v \nu_{i,t-1} + \phi_i (\mathcal{T}_{i,t} - \bar{\mathcal{T}}_i), \quad (\text{B.14})$$

where $\rho_i^v \in [0, 1)$ is the smoothing coefficient of the rule, $\mathcal{T}_{i,t}$ is the macroprudential target, $\phi_i \geq 0$ is the macroprudential weight to the target in country i and $\bar{\mathcal{T}}_i$ is the steady state of the target. In our specification, capital requirements are expected to increase when the target deviates from its steady state. The choice of the target $\mathcal{T}_{i,t}$ is a key aspect of the paper that will be discussed below.

The ESRB has developed a buffer guide to choose the CCB rate based on the credit-to-gdp gap.¹²⁹ However, the global nature of the European banking system introduces many possibilities for the definition of the credit-to-gdp ratio taken into account by national authorities. Indeed, the CCB rate may be adjusted to the credit supply (of banks) or the credit demand (of entrepreneurs),¹³⁰ either on a national or on a federal basis. Our framework with international bank flows allows us to distinguish between five operational targets as listed in [Table B.1](#).

The first set of credit targeting rules is oriented towards the supply of credit using either a federal (1.a) or a country-specific aggregate (1.b). A macroprudential policy based on credit supply aims at stabilizing lenders by focusing more on the stabilization of financial shocks hitting lenders rather than demand and supply shocks hitting

¹²⁹Other indicators (such as early warning variables) are included in the CCB guide which are not implementable in our model.

¹³⁰In an open economy context where banks can lend across borders, banks supply credit to both home and foreign, which creates a gap between the domestic supply and the domestic demand for loans. This distinction between demand and supply is easy to see on the market clearing conditions of interbank ([Equation B.37](#)) and corporate markets ([B.36](#)).

Schemes	Target
LOAN SUPPLY TARGETING	
1.a Union-wide loan supply	$\mathcal{T}_t = (L_t^s + (1 - \lambda)IB_t^s)/Y_t$
1.b National loan supply	$\mathcal{T}_{i,t} = (L_{i,t}^s + (1 - \lambda)IB_{i,t}^s)/Y_{i,t}$ for $i \in \{c, p\}$
LOAN DEMAND TARGETING	
2.a Union-wide loan demand	$\mathcal{T}_t = (L_t^d + \lambda IB_t^d)/Y_t$
2.b National loan demand	$\mathcal{T}_{i,t} = (L_{i,t}^d + \lambda IB_{i,t}^d)/Y_{i,t}$ for $i \in \{c, p\}$
CAPITAL INFLOWS TARGETING	
3 Capital Inflows	$\mathcal{T}_{i,t} = (L_{i,t}^d - L_{i,t}^s + \lambda IB_{i,t}^d - (1 - \lambda)IB_{i,t}^s)/Y_{i,t}$ for $i \in \{c, p\}$

Note: variables without country subscript such as x_t denote union-wide averages computed as a weighted sum of each country $x_t = nx_{c,t} + (1 - n)x_{p,t}$.

Table B.1: Various Macroprudential Policy Schemes in terms of Target (average in the monetary union, national supply or national target) and in terms of policy stance (common or national-adjusted)

borrowers. Given the scale of cross-border loans in the Eurozone, the decisions of the national supervisor has side effects on countries where a national bank has a subsidiary or branches or where this bank lends to may favor a federal definition of the ratio. Thus to handle these pecuniary externalities, we evaluate the possibility of an union-wide targeting system (1.a) against a national targeting system (1.b), the latter being expected to create more externalities (positive or negative) as it affects the foreign banking system without taking into account its financial developments.¹³¹

The second set of credit targeting rules concentrates on the demand of credit emanating from entrepreneurs.¹³² The interest of a CCB rate tailored to borrowers is that it may provide more stabilization following real and nominal shocks hitting households and firms at the expense of financial shocks affecting banks. This solution seeks at internalizing the social cost of entrepreneurs' over-borrowing that may arise given their biased expectations. As this policy regime inefficiently affects foreign borrowers through cross-border lending, spillovers effects may be dampened by a federal targeting (2.a) rather than a national one (2.b).

We also evaluate the interest of adopting provisional measures to affect cross border lending directly, through targeting capital inflows in the CCB. This solution, as envisaged by [Jeanne and Korinek \(2010\)](#), [Brunnermeier et al. \(2012\)](#) and [Rey \(2015\)](#), is relatively similar to a capital control measure. The main insight behind this scheme would rely on the fact that persistent capital account imbalances induce financial stability risks and may have implications for the sustainability of net external asset positions. In particular since the creation in the Eurozone, global banking has experienced an explosive growth helping to fuel unsustainable credit booms in peripheral

¹³¹For further discussions of these cross-border issues, we refer to [Beck et al. \(2016\)](#).

¹³²A loan demand targeting is feasible in a real life situation, the ECB already disentangles the credit demand and supply by collecting the domestic and cross-border positions of Euro area monetary financial institutions since 1999 for each participant of the monetary union. Regarding the demand side of credit markets, the bank lending survey published by the ECB on a quarterly basis provides an analysis of the driving forces of the demand of credit in the Euro Area. For the supply side, both the ECB and the BIS collect domestic and cross-border positions of euro area monetary financial institutions.

economies such as in Spain and in Ireland, followed by a sudden stop in capital inflows compensated by unconventional measures. Macroprudential policies can play a key role to contain this problem by imposing targeted regulations on banks engaged in cross-border activities. When borrowing to other European banks is increasing faster with respect to the GDP, a national authority can rise the CCB rate to affect banks' balance sheet management and reduce their exposure to international borrowing. In addition when system risks abate in one economy, leading to capital flow reversals, national authorities may release the buffer thus loosening the banks' funding constraint to address the procyclicality of capital flows.

B.3 THE NON-BANKING PART OF THE MODEL

We extend the model of [Poutineau and Vermandel \(2015\)](#) to account for the conduct of macroprudential policy in an heterogeneous monetary union such as the Euro Area. Our model describes a monetary union made of two asymmetric countries $i \in \{c, p\}$ (where c is for core and p for periphery). Each part i of the monetary union is of a relative size n_i .¹³³ As shown in [Figure B.2](#), each country is populated by consumers, intermediate and final producers, entrepreneurs, capital suppliers and a banking system. Regarding the conduct of macroeconomic policy, we assume national fiscal authorities and a common central bank. The implementation of the macroprudential policy is left open, and will be discussed below in another section. Our model is confronted to the data using Bayesian econometrics and it encompasses several sources of rigidities to enhance its empirical relevance. The set of real rigidities accounts for consumption habits, investment adjustment costs and loan demand habits. Regarding nominal rigidities, we account for stickiness in final goods prices and loan interest rates.

B.3.1 HOUSEHOLDS AND LABOR UNIONS

The preferences of the j^{th} household are given by:

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \exp(\varepsilon_{i,t+s}^U) \left(\log (C_{i,t+s} - h_i^C C_{i,t-1+s}) - \frac{\chi_i}{(1 + \sigma_i^H)} H_{i,t+s}^{1+\sigma_i^H} \right), \quad (\text{B.15})$$

where \mathbb{E}_t denotes the expectation operator, $\beta \in (0, 1)$ is the discount factor, parameter $\sigma_i^H > 0$ shapes the utility function of the j^{th} household associated to hours worked $H_{i,t}$. The consumption index $C_{i,t}$ is subject to external habits with degree $h_i^C \in [0; 1)$ with $C_{i,t-1}$ the aggregate lagged consumption, while $\chi_i > 0$ is a shift parameter allowing us to pin down the steady state amount of hours worked. The discount factor is affected by a time-preference shock $\varepsilon_{i,t}^U$ following an AR(1) stochastic process that exogenously changes the household's intertemporal allocation of consumption over the cycle.

¹³³Normalizing the size of the monetary union to unity, the relative size of the core are is n and the relative size of the peripheral area is $1 - n$.

Household j^{th} period budget constraint is given by:

$$w_{i,t}^h H_{i,t} + D_{i,t-1}^d \frac{(1 + R_{i,t-1}^D)}{(1 + \pi_{i,t}^C)} + \Pi_{i,t} = C_{i,t} + D_{i,t}^d + t_{i,t} + p_{i,t} AC_{i,t}^D. \quad (\text{B.16})$$

The income of the representative household is made of labor income with the desired real wage $w_{i,t}^h$,¹³⁴ interest payments for deposit services $D_{i,t}^d$ and real earnings $\Pi_{i,t}$ from shareholdings of firms and unions. The interest rate is deflated by the consumer price inflation rate $1 + \pi_{i,t}^C = P_{i,t}^C/P_{i,t-1}^C$. The representative household spends this income on consumption, deposits and tax payments for a real amount of $t_{i,t}$. Finally, we assume that the household has to pay quadratic adjustment costs to buy new deposits,¹³⁵ these costs are paid in terms home goods with relative price $p_{i,t} = P_{i,t}/P_{i,t}^C$ where $P_{i,t}$ is the production price index of home produced goods while $P_{i,t}^C$ is the consumption price index. Households consume both home and foreign goods and their corresponding consumption basket follows a standard CES function:

$$C_{i,t} = \left((1 - \alpha_i^C)^{1/\mu} C_{hi,t}^{(\mu-1)/\mu} + (\alpha_i^C)^{1/\mu} C_{fi,t}^{(\mu-1)/\mu} \right)^{\mu/(\mu-1)}, \quad (\text{B.17})$$

where parameter $\mu > 0$ is the elasticity of substitution between domestic and foreign final goods and $\alpha_i^C \in [0, 1/2]$ measures the fraction of goods bought abroad. The corresponding price index is, $P_{i,t}^C = ((1 - \alpha_i^C) P_{h,t}^{1-\mu} + \alpha_i^C P_{f,t}^{1-\mu})^{1/(1-\mu)}$.

Households delegate the wage negotiation process to unions. Households provide differentiated labor types, sold by labor unions to perfectly competitive labor packers who assemble them in a CES aggregator and sell the homogenous labor to intermediate firms.¹³⁶ Unions negotiate the real margin between the real desired wage of households $w_{i,t}^h$ and the real marginal product of labor $W_{i,t}/P_{i,t}^C$. Using a Calvo wage nominal rigidity device, each period a random fraction θ_i^W of unions is unable to re-negotiate a new wage. Assuming that the trade union is able to modify its wage with a probability $1 - \theta_i^W$, the j^{th} union chooses the nominal optimal wage $W_{i,t}^*$ to maximize its expected sum of profits:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\theta_i^W)^s \Lambda_{i,t+s} \left[\frac{W_{i,t}^*}{P_{i,t+\tau}^C} \prod_{k=1}^s (1 + \pi_{i,t+k-1}^C)^{\xi_i^W} - \exp(\varepsilon_{i,t+s}^W) w_{i,t+s}^h \right] H_{i,t+s}, \quad (\text{B.18})$$

where $\Lambda_{i,t+\tau}$ is household's stochastic discount factor, $\varepsilon_{i,t}^W$ is an *ad-hoc* wage-push shock to the real wage equation following an AR(1) process which captures exogenous fluc-

¹³⁴As explained below, the desired wage is negotiated by a trade union.

¹³⁵This cost is almost neutral on the dynamic of the model and is necessary to remove a unit root component which is standardly induced by the international nature of our model. See [Schmitt-Grohé and Uribe \(2003\)](#) for an extensive discussion and solutions regarding this issue. The functional form we choose is: $AC_{i,t}^D(j) = 0.5\chi_D(D_{i,t}^d(j) - \bar{D}_i)^2/\bar{D}_i$, where \bar{D}_i is the steady state level of deposits and $\chi_D > 0$ is the cost parameter.

¹³⁶Labor packers are perfectly competitive and maximize profits, $W_{i,t} H_{i,t}^d - \mathcal{G}(W_{i,t}(j) H_{i,t}(j))$, under their packing technology constraint, $H_{i,t} = [(1/n_i)^{1/\epsilon_W} \mathcal{G}(H_{i,t}(j))^{(\epsilon_W-1)/\epsilon_W}]^{\epsilon_W/(\epsilon_W-1)}$. Here, $W_{i,t}$ is the production price, $H_{i,t}^d$ is the labor demand and ϵ_W is a substitution parameter. The first order condition which determines the optimal demand for the j^{th} labor type is, $H_{i,t}(j) = (1/n_i)(W_{i,t}(j)/W_{i,t})^{-\epsilon_W} H_{i,t}^d, \forall j$. Thus the aggregate wage index of all labor types in the economy emerges from the zero-profit condition: $W_{i,t} = [(1/n_i)\mathcal{G}(W_{i,t}(j))^{1-\epsilon_W}]^{1/(1-\epsilon_W)}$.

tuations in the wage margin negotiated by unions and affects in turn the productivity of the economy.

B.3.2 FIRMS

Intermediate firms produce differentiated goods, decide on labor and capital inputs on a perfectly competitive inputs market and set prices according to the Calvo model. The i^{th} firm has the following Cobb-Douglas technology:

$$Y_{i,t} = \exp(\varepsilon_{i,t}^A) (K_{i,t}^u)^\alpha (H_{i,t}^d)^{1-\alpha}, \quad (\text{B.19})$$

where $Y_{i,t}$ is the standard production function that combines (utilized) physical capital $K_{i,t}^u$, labor demand $H_{i,t}^d$ to household and (exogenous) technology $\varepsilon_{i,t}^A$. Intermediate firms solve a two-stage problem. In the first stage, taking the real input prices $w_{i,t}$ and $z_{i,t}$ as given, firms rent inputs $H_{i,t}^d$ and $K_{i,t}^u$ in a perfectly competitive factor market in order to minimize costs subject to the production constraint (B.19) to determine the real marginal cost $mc_{i,t}$.

In the second-stage, the intermediate firm i sets prices according to a Calvo mechanism. Each period firm i is not allowed to re-optimize its price with probability θ_i^P but price increases by $\xi_i^P \in [0; 1)$ with respect to the previous period's rate of price inflation, $P_{i,t} = (1 + \pi_{i,t-1})^{\xi_i^P} P_{i,t-1}$. The i^{th} firm allowed to modify its selling price with a probability $1 - \theta_i^P$ chooses $P_{i,t}^*$ to maximize its discounted sum of profits:

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\theta_i^P)^s \Lambda_{i,t+s} \left[\frac{P_{i,t}^*}{P_{i,t+s}^C} \prod_{k=1}^s (1 + \pi_{i,t+k-1})^{\xi_i^P} - \exp(\varepsilon_{i,t+s}^P) mc_{i,t+s} \right] Y_{i,t+s}, \quad (\text{B.20})$$

where $\varepsilon_{i,t}^P$ is an *ad-hoc* cost-push shock to the inflation equation following an AR(1) process which captures exogenous inflation pressures.

Once goods are produced and prices are set, final firms act as goods packers: they combine differentiated goods to produce the homogenous final good sold mainly to households.¹³⁷

B.3.3 ENTREPRENEURS

The capital required by the intermediate firm in the production process is financed by an entrepreneur that belongs to the same business unit i . The balance sheet of the i^{th} entrepreneur is given by:

$$q_{i,t} K_{i,t} = L_{i,t}^H + N_{i,t}. \quad (\text{B.21})$$

Defining $Q_{i,t} K_{i,t}$ as the amount of capital to be financed by entrepreneur i , $q_{i,t} = Q_{i,t}/P_{i,t}^C$ is the real shadow value of capital goods. This quantity $q_{i,t} K_{i,t}$ is financed

¹³⁷Goods packers are perfectly competitive and maximize profits, $P_{i,t} Y_{i,t}^d - \mathcal{G}(P_{i,t}(i) Y_{i,t}(i))$, under their packing technology constraint, $Y_{i,t}^d = [(1/n_i)^{1/\epsilon_P} \mathcal{G}(Y_{i,t}(i)^{(\epsilon_P-1)/\epsilon_P})]^{\epsilon_P/(\epsilon_P-1)}$. Here, $P_{i,t}$ is the production price, $Y_{i,t}^d$ is the aggregate demand (or the resource constraint) and ϵ_P is a substitution parameter. The first order condition which determines the optimal demand for the i^{th} good is, $Y_{i,t}(i) = (1/n_i)(P_{i,t}(i)/P_{i,t})^{-\epsilon_P} Y_{i,t}^d, \forall i$. Thus the aggregate price index of all varieties in the economy emerges from the zero-profit condition: $P_{i,t} = [(1/n_i) \mathcal{G}(P_{i,t}(i)^{1-\epsilon_P})]^{1/(1-\epsilon_P)}$.

by the entrepreneur through two means: its net wealth $N_{i,t}$ and the real amount borrowed from the banking system, $L_{i,t+1}^H$. Formally, loan demands are subject to external habits as follows: $L_{i,t}^H = L_{i,t}^d - h_i^L(L_{i,t-1}^d - \bar{L}_i^d)$ with the habit degree $h_i^L \in [0, 1)$, $L_{i,t-1}^d$ the aggregate average level of loans of the previous period and \bar{L}_i^d the steady state stock of loans.¹³⁸ Empirically, firms and banks operating in the Euro Area choose longer debt maturities than the standard one-period contract usually used in real business cycle models. Then the term $h_i^L L_{i,t-1}^d$ allows for slow adjustment over time of the balance sheet constraint, to capture the idea that in practice borrowers do not readjust their outstanding amount of loans every quarter. This approach of introducing slow adjustment of credit is close to [Iacoviello \(2015\)](#), employed here in a context of a financial accelerator model. During phases of recession characterized by asset price collapses of $q_{i,t}$, this friction prevents the total stock of loans to fall at the same rate as the price of financial assets, thus making credit less procyclical consistently with empirical evidence. Since these habits don't directly affect the first order condition of the entrepreneur (as the overall problem of the entrepreneur can be expressed in terms of physical capital directly), their implications on entrepreneurs' profits is rather minor but large for financial intermediary facing a persistent demand for loans.

To introduce corporate cross-border lending, we follow [Poutineau and Vermandel \(2015\)](#) and [Brzoza-Brzezina et al. \(2015\)](#) by adopting a CES function that bundles domestic and foreign loans offered by banks operating in the monetary union:¹³⁹

$$L_{i,t}^d = \left((1 - \alpha_i^L)^{1/v} (L_{hi,t}^d)^{(v-1)/v} + (\alpha_i^L)^{1/v} (L_{fi,t}^d)^{(v-1)/v} \right)^{v/(v-1)}. \quad (\text{B.22})$$

Here, parameter $v \geq 0$ is the elasticity of substitution between domestic and foreign interbank funds, $\alpha_i^L \in [0, 1/2]$ represents the percentage of cross-border interbank loan flows in the monetary union and $L_{hi,t}^d$ (resp. $L_{fi,t}^d$) the amount of domestic (resp. foreign) loans demanded by borrowing entrepreneurs living in country i . As a consequence, the borrowing cost is a CES aggregate of home and foreign credit rates defined as: $1 + P_{i,t}^L = ((1 - \alpha_i^L)(1 + R_{h,t}^L)^{1-v} + \alpha_i^L(1 + R_{f,t}^L)^{1-v})^{1/(1-v)}$.

Regarding financial frictions, we reinterpret the financial accelerator *à la* [Bernanke et al. \(1999\)](#) from a banking perspective in order to have state-contingent lending rates needed to introduce macroprudential measures.¹⁴⁰ To do so, we follow the modelling device of [Poutineau and Vermandel \(2015\)](#) that provides a micro-foundation for

¹³⁸In the estimation exercise, we use the total stock of loans, they are of different maturities implying a strong autocorrelation. Simply by introducing loan demand habits, taking into account the high autocorrelation of loans becomes tractable easily and does not change the steady state of the model. For instance in 1999, loans with a maturity above one year represented 64% of the outstanding stock of loans in the Eurosystem.

¹³⁹[Kollmann et al. \(2011\)](#) provides a complementary way of introducing cross-border lending through global banks. However, this approach assumes a perfect credit market integration between Euro participants that is not consistent with the data. Alternatively, [Dedola and Lombardo \(2012\)](#) introduce cross-border loans through a portfolio problem that requires a second order approximation to the policy function, which poses an issue when putting the model to the data.

¹⁴⁰The pathbreaking contribution of [Bernanke et al. \(1999\)](#) focuses on the demand side of credit market through a moral hazard problem but neglects its supply side and in turn the possibility to introduce macroprudential measures that could affect the macroeconomic outcome. Their model is closed assuming that lending rates are pre-determined.

the financial accelerator mechanism relying on biased expectations of entrepreneurs instead of a standard moral hazard problem. The investment projects undertaken by the entrepreneur are risky and differ with respect to their individual returns. To model individual riskiness, we borrow from [Bernanke et al. \(1999\)](#) and assume that each project has an individual return equal to $\omega (1 + R_{i,t}^k)$, *i.e.* that the aggregate return of investment projects in the economy $1 + R_{i,t}^k$ is multiplied by a random value ω . The representative entrepreneur conducts a mass ω of diversified investment projects and the profit of the ω^{th} investment project is given by:

$$\Pi_{i,t}^E(\omega) = \omega \mathbb{E}_t \{1 + R_{i,t+1}^k\} q_{i,t} K_{i,t}(\omega) - (1 + P_{i,t}^L) L_{i,t}^H(\omega), \quad (\text{B.23})$$

In order to acquire a loan, entrepreneurs have to engage in a financial contract before the realization of ω .¹⁴¹ After engaging in the financial contract, entrepreneurs recognize *ex post* the value of $\omega_{i,t}^C$ which separates the default space ($\omega < \omega_{i,t}^C$) from the space of gains ($\omega \geq \omega_{i,t}^C$). Thereby the *ex post* threshold separating the default space from the profitable space is computed through the zero profit condition on [Equation B.23](#):

$$\omega_{i,t}^C (1 + R_{i,t}^k) q_{i,t-1} K_{i,t-1} = (1 + P_{i,t-1}^L) L_{i,t-1}^H. \quad (\text{B.24})$$

Following [Helpman et al. \(2004\)](#), we adapt the Pareto distribution to model the productivity of firms in a financial context. Investment projects are drawn from a Pareto distribution $\omega \sim \mathcal{P}(\kappa)$ with support $\omega \in [\omega_{\min}, +\infty)$ where $\kappa > 1$ is the shape parameter and $\omega_{\min} > 0$ is the lower bound of the distribution. Given the characteristics of the distribution, it is possible to compute the share of profitable projects, denoted $\eta_{i,t}^E = (\omega_{\min}/\omega_{i,t}^C)^\kappa$, and their aggregate value, $\bar{\omega}_{i,t} = \kappa/(\kappa-1)\omega_{i,t}^C$.¹⁴² When the entrepreneur is underwater with an investment project value below the cost of credit, she endogenously defaults on her loan and abandons her investment project.

To introduce a financial accelerator mechanism, we assume that entrepreneurs have short term distorted expectations regarding the aggregate profitability of their aggregate investment projects $\bar{\omega}_{i,t}$, thus creating a financial friction in the economy with dynamic properties close to [Bernanke et al. \(1999\)](#). The perceived *ex ante* value of profitable projects $\bar{\omega}_{i,t+1}$ is defined by the CES function:¹⁴³

$$g(\bar{\omega}_{i,t+1}) = \bar{\omega}_i^{1/(1-\alpha_i)} (\bar{\omega}_{i,t+1})^{\alpha_i/(\alpha_i-1)}, \quad (\text{B.25})$$

¹⁴¹The individual return ω is also referred as an idiosyncratic shock in the financial accelerator literature. The debt contract is concluded before the idiosyncratic shock is recognized which generates unexpected losses for the entrepreneurs and lenders.

¹⁴²Using the characteristics of the Pareto distribution $F(\omega)$, the distribution of stochastic investment projects ω has a positive support, is independently distributed (across entrepreneurs and time) with unitary mean $E[\omega] = 1$, and density function $f(\omega)$. Investment projects above the cut-off value, $\omega > \omega_{i,t}^C$, have positive profits $\Pi_{i,t}^E(\omega) \geq 0$ which allows entrepreneurs to repay its loans to the bank. The share of profitable projects $1 - F(\omega)$ is computed as, $\eta^E = \Pr[\omega \geq \omega^C] = \int_{\omega^C}^{\infty} f(\omega) d\omega = (\omega_{\min}/\omega^C)^\kappa$ while the conditional expectation of ω when entrepreneur's project is gainful is, $\eta^E \bar{\omega} = \int_{\omega^C}^{\infty} \omega f(\omega) d\omega$ with $\bar{\omega} = E[\omega | \omega \geq \omega^C] = \frac{\kappa}{\kappa-1} \omega^C$.

¹⁴³There is a rich literature providing evidence that entrepreneurs are more optimistic compared to the general population; for some recent studies see, e.g., [Landier and Thesmar \(2009\)](#), [Puri and Robinson \(2013\)](#), [Dawson and Henley \(2013\)](#).

where $\varkappa_i \in [0, 1)$ is the elasticity of the external finance premium and $\bar{\omega}_i$ is the steady state of $\bar{\omega}_{i,t+1}$. During phases of expansion characterized by high aggregate returns above the steady state $\bar{\omega}_{i,t+1} > \bar{\omega}_i$, entrepreneurs' forecasts regarding the aggregate profitability are optimistic with $g(\bar{\omega}_{i,t+1}) > \bar{\omega}_{i,t+1}$. In contrast for low expected realizations of $\bar{\omega}_{i,t+1}$ below its steady state, entrepreneurs tend to hold pessimistic expectations about their returns with $g(\bar{\omega}_{i,t+1}) < \bar{\omega}_{i,t+1}$. Finally in steady state, there is no expectation bias, $g(\bar{\omega}_i) = \bar{\omega}_i$.¹⁴⁴ Any shock driving financial returns above or below the steady state will trigger an acceleration of the business cycles through these biased expectations for $\varkappa_i > 0$.

Aggregating all profitable investment projects (i.e. above $\omega_{i,t}^C$) that the entrepreneur does not abandon, it chooses a capital value of $K_{i,t}$ that maximizes its profit (before the realization of ω) defined as:

$$\Pi_{i,t}^E = \mathbb{E}_t \left\{ \eta_{i,t+1}^E \left[g(\bar{\omega}_{i,t+1}) (1 + R_{i,t+1}^k) q_{i,t} K_{i,t} - (1 + P_{i,t}^L) L_{i,t}^H \right] \right\}. \quad (\text{B.26})$$

Taking the first order condition combined with Equation B.24, the financial accelerator principle emerges through the external finance premium expression:

$$\frac{1 + R_{i,t+1}^k}{1 + P_{i,t}^L} = \frac{1}{\bar{\omega}_i} \left[\frac{\kappa}{\kappa - 1} \left(1 - \frac{N_{i,t}}{q_{i,t} K_{i,t}} \right) \right]^{\varkappa_i}. \quad (\text{B.27})$$

Up to a first order, $\hat{r}_{i,t+1}^k - \hat{p}_{i,t}^L \simeq \varkappa_i (\hat{q}_{i,t} + \hat{k}_{i,t} - \hat{n}_{i,t})$, the spread is a positive function to the capital-to-net wealth ratio. Under this assumption, the balance sheet of borrowers affects the borrowing conditions and magnifies the financial cycle. The size of the accelerator is determined by the degree of bias \varkappa_i of borrowers.

Finally, the law of motion of the net wealth is given by profit obtained at the end of period t-1:

$$N_{i,t} = (1 - \delta^E) \Pi_{i,t-1}^E \exp(\varepsilon_{i,t}^N), \quad (\text{B.28})$$

where $\delta^E \in [0, 1]$ is the net wealth decay that is related to the default rate of entrepreneur (endogenously determined in steady state) and $\varepsilon_{i,t}^N$ is an AR(1) shock to the net wealth of entrepreneurs which captures exogenous variations in stock prices and the demand for loans.

B.3.4 CAPITAL GOODS PRODUCERS

The capital producer rents and refurbishes the capital stock used by intermediate firms and financed by the entrepreneurs on a competitive market. The law of motion of physical capital is determined by:

$$K_{i,t} = (1 - S((\exp(\varepsilon_{i,t}^I) I_{i,t} / I_{i,t-1}))) I_{i,t} + (1 - \delta) K_{i,t-1}, \quad (\text{B.29})$$

where $\varepsilon_{i,t}^I$ is a stochastic process which captures exogenous changes in asset price fluctuations and the adjustment cost function taken from Smets and Wouters (2007) reads as: $S(x_t) = \chi_i^I (x_t - 1)^2$ with $\chi_i^I \geq 0$ is the adjustment cost. In addition, investment

¹⁴⁴It is important to stress that function $g(\bar{\omega}_{i,t+1})$ only affects expected returns of $\mathbb{E}_t \{\bar{\omega}_{i,t+1}\}$, while for *ex post* values (e.g., $\bar{\omega}_{i,t}$ and $\bar{\omega}_{i,t-1}$) the entrepreneur recognizes the true value of her return.

is composed of domestic and foreign goods summarized by a CES function:

$$I_{i,t} = \left((1 - \alpha_i^I)^{1/\mu} I_{hi,t}^{(\mu-1)/\mu} + (\alpha_i^I)^{1/\mu} I_{fi,t}^{(\mu-1)/\mu} \right)^{\mu/(\mu-1)}, \quad (\text{B.30})$$

where parameter μ is the elasticity of substitution between domestic and foreign goods in investment and α_i^I measures the degree of investment diversification in the monetary union between home and foreign countries. The corresponding price index of investment is, $P_{i,t}^I = \left((1 - \alpha_i^I) (P_{h,t})^{1-\mu} + \alpha_i^I (P_{f,t})^{1-\mu} \right)^{1/(1-\mu)}$.

The representative capital supplier chooses $I_{i,t}$ to maximize its real discounted profits:

$$\max_{\{I_{i,t}\}} \mathbb{E}_t \sum_{s=0}^{\infty} \Lambda_{i,t+s} \left(q_{i,t+s} \left(1 - S(\exp(\varepsilon_{i,t}^I) I_{i,t} / I_{i,t-1}) \right) - p_{i,t+s}^I \right) I_{i,t+s}, \quad (\text{B.31})$$

where $q_{i,t} = Q_{i,t} / P_{i,t}^C$ stands for the real shadow value of investment goods and $p_{i,t}^I = P_{i,t}^I / P_{i,t}^C$ is the relative price of investment goods.

Finally regarding capital utilization, the optimal rate of utilization is given by:¹⁴⁵

$$a'(u_{i,t}) = Z_{i,t}, \quad \text{with } a(u_{i,t}) = \bar{Z}(u_{i,t} - 1) + 0.5\psi_i / (1 - \psi_i) \bar{Z}(u_{i,t} - 1)^2, \quad (\text{B.32})$$

where $\psi_i \in (0, 1)$ is the elasticity of utilization costs with respect to capital inputs. As in [Smets and Wouters \(2003, 2007\)](#), capital requires one period to be settled so that utilized capital is defined as, $K_{i,t}^u = u_{i,t} K_{i,t-1}$.

B.3.5 MONETARY AND FISCAL POLICY

National governments finance public spending by charging lump-sum taxes to households $t_{i,t}$. The total amount of taxes finance public spending without contracting public debt such that the public budget is always balanced. As in [Smets and Wouters \(2003, 2007\)](#), the level of spending $G_{i,t}$ is exogenously determined as a constant fraction of output $g\bar{Y} \exp(\varepsilon_{i,t}^G)$, where $g\bar{Y}$ is the fix component and $\varepsilon_{i,t}^G$ is the time-varying component of spending that follows a standard AR(1) shock process. Parameter $g \in [0, 1)$ is the steady state spending-to-GDP ratio. It is important to mention that we omit seigniorage revenues that could be given to fiscal authorities as their inclusion generates an explosive path of deposits.¹⁴⁶ This could be a limitation of our model, however [Leeper \(1991\)](#) indicates seigniorage represents a small fraction of government revenues for developed economies thus showing that its omission does not seem to be important for the analysis conducted here.

Concerning federal monetary policy, the general expression of the interest rule implemented by the monetary union central bank follows the linear rule:

$$R_t - \bar{R} = \rho (R_{t-1} - \bar{R}) + (1 - \rho) (\phi^\pi \pi_t^C + \phi^{\Delta y} (Y_t - Y_{t-1})) + \varepsilon_t^R, \quad (\text{B.33})$$

where ε_t^R is a monetary policy shock common to the monetary union members, $\phi^\pi \geq 1$

¹⁴⁵When households do not take capital supply decisions, the optimal capital utilization is determined by, $\max_{u_{i,t}} (Z_{i,t} u_{i,t} - a(u_{i,t})) K_{i,t}$.

¹⁴⁶This assumption is common for recent macroeconomic models as underlined by [Curdia and Woodford \(2011\)](#). We refer to [Gerali et al. \(2010\)](#) or [Adam \(2011\)](#) for macro-models without seigniorage.

is the inflation target parameter, $\phi^{\Delta y}$ is the GDP growth target. As monetary policy responds to the aggregate evolution of inflation and activity in the monetary union, recall that $\pi_t^C = n_c \pi_{c,t}^C + n_p \pi_{p,t}^C$ and $Y_t = n_c Y_{c,t} + n_p Y_{p,t}$.

B.3.6 STOCHASTIC SHOCK PROCESSES

To be in line with the benchmark model of [Smets and Wouters \(2003\)](#) for the Euro Area, all our random processes follows an AR(1) specification. Each part of the Eurozone includes 10 country-specific shocks for $s = \{A, G, U, I, P, W, N, D, B, L\}$ such that:

$$\varepsilon_{i,t}^s = \rho_i^s \varepsilon_{i,t-1}^s + \eta_{i,t}^s \text{ with } \eta_{i,t}^s \sim N(0, \sigma_i^s). \quad (\text{B.34})$$

Finally both part of the monetary union are affected by a common monetary policy shock in [Equation B.33](#) defined as, $\varepsilon_t^R = \rho^R \varepsilon_{t-1}^R + \eta_t^R$ with $\eta_t^R \sim N(0, \sigma^R)$. Markups shocks for $m = \{P, W, D\}$ are normalized to one, as implicitly assumed in [Smets and Wouters \(2003\)](#), by dividing their stochastic innovations η_{it}^m by the elasticity of the linear New Keynesian Phillips curve, $(1-\beta\theta_i^m)(1-\theta_i^m)/\theta_i^m$.

B.3.7 AGGREGATION AND MARKET CLEARING

The general equilibrium of the model is set as follows. After (i) aggregating all agents and varieties in the economy, (ii) imposing market clearing for all markets, (iii) substituting the relevant demand functions, (iv) normalizing the total size of the monetary union ($n_c + n_p = 1$) such that the size of the core area is n and the peripheral area size is $1 - n$, we get the general equilibrium conditions of the model. We can express the aggregation function of variable $X_t(x)$ as: $\mathcal{G}(X_{i,t}(x)) = \int_0^n X_{i,t}(x) dx$ for $i = c$ and $\mathcal{G}(X_{i,t}(x)) = \int_n^1 X_{i,t}(x) dx$ for $i = p$.

Thus, replacing the demand functions of foreign and home goods (consumption and investment), we finally obtain the home final goods market equilibrium:

$$\begin{aligned} Y_{c,t}/\Delta_{c,t}^P &= (1 - \alpha_c^C) [P_{c,t}/P_{c,t}^C]^{-\mu} C_{c,t} + (1 - \alpha_c^I) [P_{c,t}/P_{c,t}^I]^{-\mu} I_{c,t} \\ &+ (1 - n)/n \left(\alpha_p^C [P_{c,t}/P_{p,t}^C]^{-\mu} C_{p,t} + \alpha_p^I [P_{c,t}/P_{p,t}^I]^{-\mu} I_{p,t} \right) \\ &+ a(u_{c,t}) K_{c,t-1} + g\bar{Y} \exp(\varepsilon_{c,t}^G) + AC_{c,t}^D, \end{aligned} \quad (\text{B.35})$$

where $\Delta_{i,t}^P = \mathcal{G}(P_{i,t}(i)/P_{i,t})^{-\varepsilon_P}$ denotes the price dispersion term, which is induced by the assumed nature of price stickiness.

Concerning the corporate loan market, recall that entrepreneurs borrow to domestic and foreign banks with varieties b produced by liquid illiquid banks, leading to the following equilibrium for each country:

$$\begin{aligned} L_{c,t}^s/\Delta_{c,t}^L &= (1 - \alpha_c^L) [(1 + R_{c,t}^L)/(1 + P_{c,t}^L)]^{-\nu} L_{c,t} \\ &+ n/(1 - n) \alpha_p^L [(1 + R_{c,t}^L)/(1 + P_{p,t}^L)]^{-\nu} L_{p,t}, \end{aligned} \quad (\text{B.36})$$

where $\Delta_{i,t}^L$ is the credit rate dispersion term.

Turning to the interbank market, the perfect competition involves no interest rate

dispersion between loan varieties. It clears when the following condition between liquid banks and home and foreign illiquid banks holds:

$$IB_{c,t}^s(1-\lambda)/\lambda = (1-\alpha_c^{IB}) \left[(1+R_{c,t}^{IB})/(1+P_{c,t}^{IB}) \right]^{-\xi} IB_{c,t}^d + n/(1-n)\alpha_p^{IB} \left[(1+R_{c,t}^{IB})/(1+P_{p,t}^{IB}) \right]^{-\xi} IB_{p,t}^d \quad (\text{B.37})$$

B.4 ESTIMATION STRATEGY

We fit the previous two country DSGE to Eurozone data over the sample time period 1999Q1-2013Q4 using Bayesian techniques. We estimate structural parameters and the sequence of shocks by following the seminal contributions of [Smets and Wouters \(2003, 2007\)](#) and [Christiano et al. \(2005\)](#). For a detailed description, we refer to the original papers.

B.4.1 DATA

We split the Eurozone in two groups adopting the core-periphery dichotomy as in [Quint and Rabanal \(2014\)](#) and [Poutineau and Vermandel \(2015\)](#). Core countries gather Austria, Belgium, Germany, Finland, France, Luxembourg and Netherlands while peripheral countries include Spain, Greece, Ireland, Italy and Portugal. The model is estimated with Bayesian methods on Eurozone quarterly data over the sample period 1999Q1 to 2013Q4, which makes 60 observations for each observable variable. Concerning the transformation of series, the point is to map non-stationary data to a stationary model. Data which are known to have a trend (namely GDP, consumption, investment, corporate loan and interbank supply) or unit root are made stationary in two steps. First, we divide the sample by the population. Second, data are taken in logs and we use a first difference filtering to obtain growth rates. In addition, real variables are deflated by the HICP price index and we remove the seasonal component in the data using a multiplicative decomposition. Furthermore, we demean the data as we do not use the information contained in the observable mean. Interest rates are set on a quarterly basis by dividing them by 4. Since hours worked are not observable for the Euro Area, we adopt the same modelling strategy as [Smets and Wouters \(2003\)](#) to identify TFP shocks using employment as a proxy for hours worked. Employment is divided by the working population index, taken in logs and demeaned. To map employment to hours worked in our model, we introduce an auxiliary equation for each country which states that only a share $\theta_i^E \in [0, 1)$ of firms is allowed to adjust its level of employment $\hat{e}_{i,t}$ to its optimal labor demand $H_{i,t}^d$:

$$\hat{e}_{i,t} = \beta \hat{e}_{i,t+1} + (1 - \beta \theta_i^E) (1 - \theta_i^E) / \theta_i^E (\log(H_{i,t}^d / \bar{H}^d) - \hat{e}_{i,t}). \quad (\text{B.38})$$

The vector of observable variables reads as:

$$\mathcal{Y}_t = 100[\Delta \hat{y}_{i,t}, \hat{e}_{i,t}, \Delta \hat{c}_{i,t}, \Delta \hat{i}_{i,t}, \hat{\pi}_{i,t}^C, \Delta \hat{w}_{i,t}, \hat{r}_{i,t}^D, \Delta \hat{l}_{i,t}^s, \Delta \hat{b}_{i,t}^s, \hat{r}_t] \text{ for } i = \{c, p\}.$$

B.4.2 CALIBRATION, PRIORS AND MODEL ASSUMPTIONS

We fix a small number of parameters commonly used in the literature of real business cycles models which are weakly identified. The discount factor β is set at 0.99, the depreciation rate δ at 0.025, the capital share α at 0.38, the share of steady state hours worked \bar{H} at 1, the spending to GDP ratio g at 24%.¹⁴⁷ Concerning ϵ_P and ϵ_W (the substitutability between final goods and labor), we consider the calibration at 10 as in [Smets and Wouters \(2007\)](#). Regarding financial parameters, we fix \bar{N}/\bar{K} (the net worth to capital) ratio to 0.40 to be consistent with the observed debt-to-financial assets ratio of non-financial corporations which fluctuates between 50% and 65% since 1999. The steady state value of spreads and the bank balance sheet are calibrated on their averages observed over the sample period in the Euro Area: $\bar{R}-\bar{R}^D=1.66/400$, $\bar{R}^L-\bar{R}^D=3.67/400$, $\bar{D}/\bar{L}^s=0.46$, $\bar{r}\bar{w}\bar{a}=\bar{v}=0.10$ and $\bar{I}\bar{B}^d/\bar{L}^s=0.20$. The capital regulation cost χ^k is set at 11 as in [Gerali et al. \(2010\)](#) to replicate the response of credit and interest rate to a capital requirement rise.

For substitution parameters for corporate and interbank loans v and ξ as well as for the fraction of illiquid banks λ , to our knowledge there are no empirical analysis using bank level data that provides an estimation of these parameters. We rely on the previous fit exercise of [Poutineau and Vermandel \(2015\)](#) by calibrating λ at 0.38 and v, ξ at 1.1. The latter calibration for substitution parameters is rather conservative by allowing very low substitution effects between home and foreign loans.¹⁴⁸ The quarterly share of defaulting firms' projects $1 - \bar{\eta}^E$ is fixed at 0.025/4,¹⁴⁹ and the auditing cost μ^B at 0.10,¹⁵⁰ those values are very similar to [Bernanke et al. \(1999\)](#). We compute the parameter governing the relative size of the core area n at 0.58 as in [Kolasa \(2009\)](#), which is the share implied by nominal GDP levels averaged over the period 1999-2013. We calibrate symmetrically the adjustment cost on deposits χ_i^D at 0.0007 as in [Schmitt-Grohé and Uribe \(2003\)](#) to remove an unit root component generated by the two-country set-up. Finally, the lower bound ω_{\min} and the shape κ of the Pareto distribution are endogenously determined by the model equations assuming a risk-free economy with no spread and default, we obtain: $\omega_{\min}=1-\bar{N}/\bar{K}$ and $\kappa=\bar{K}/\bar{N}$. Our calibration delivers for the main endogenous variables the following steady state: $\bar{\omega}^C=0.6015$, $\varepsilon^D=-2.41$, $\varepsilon^L=4.37$, $\bar{r}^L=0.0192$ and $\bar{r}^K=0.0166$.

Our priors are listed in [Table B.2](#). Overall, they are either relatively uninforma-

¹⁴⁷This calibration offers a consumption-to-output ratio of 55.45% (vs 57.31% in the data) and investment-to-output ratio of 20.55% (vs 20.70% in the data).

¹⁴⁸In contrast, [Brzoza-Brzezina et al. \(2015\)](#) assign a value of 6 to their substitution parameter, which is rather high with respect to the literature of trade. In general, substitution parameters for goods market are rather low and usually remain between 1 and 2 as in [Quint and Rabanal \(2014\)](#) or [Poutineau and Vermandel \(2015\)](#).

¹⁴⁹This is consistent with corporate default statistics from Moody's, the rating agency, which show an average default rate on (non-US) non-financial corporate bonds of 0.75% for the period 1989-2009, as shown by [Darracq-Pariès et al. \(2011\)](#). The other rating agency Standards & Poor's evaluates the rate of default for the period 1991-2014 to 0.58%. We consider a default rate of 0.63% which is in the ballpark of the numbers found by rating agencies.

¹⁵⁰The auditing cost cannot be observed as few data on loan losses are publicly available for reasons of confidentiality. [Dermine and De Carvalho \(2006\)](#) find using bank level data that these costs critically depends on the size of the loans: recovery costs on smaller loans are substantially higher than on large loans, 4.1% vs. 0.9%. In addition, once the contentious department has to rely on external lawyers, the recovery costs rise to 10.4%.

tive or consistent with earlier contributions to Bayesian estimations. For a majority of new Keynesian models' parameters, *i.e.* σ_i^L , h_i^C , θ_i^P , ξ_i^P , θ_i^W , ξ_i^W , θ_i^E , χ_i^I , ψ_i , ϕ^π , $\phi^{\Delta y}$ and shock processes parameters, we use the prior distributions close to [Smets and Wouters \(2003, 2007\)](#). Calvo probabilities for rates have the same uninformative priors as for prices/wages while loans habits are given a prior mean 0.5 with standard deviation 0.2. Our priors for openness parameters are based on their observed average over the sample period. Substitutabilities between home/foreign credit and final goods are set to 2 with standard deviations of 0.50. We set the prior for the elasticity of the external finance premium \varkappa_i to a beta distribution with prior mean equal to 0.05 and standard deviation 0.02 consistent with prior information of [Gilchrist et al. \(2009a\)](#). Finally, in order to catch up the correlation and co-movements between countries' aggregates, we estimate the cross-country correlation between structural shocks, associated priors are inspired by in [Jondeau et al. \(2006\)](#) and [Kolasa \(2009\)](#), we set the mean of the prior distribution for shock correlations between core countries and peripheral countries at 0.2 with a standard deviation at 0.2.

Finally, regarding bank capital regulation for the fit exercise, we disable the macroprudential instrument by fixing the CCB rate to its deterministic steady state value:

$$\nu_{i,t} = \bar{\nu}. \quad (\text{B.39})$$

This assumption is reasonable for two main reasons. First over the sample period, capital regulation has been mainly dominated by the Basel I Accords characterized by fixed capital requirement ratios. Second, even through the adoption of the Basel III Accords allows Euro Area countries to employ the countercyclical capital buffer as a shield against the build up of financial imbalances, it has not been yet employed by a participant of the monetary union.¹⁵¹

B.4.3 ESTIMATION RESULTS

The methodology employed is standard to the Bayesian estimations of DSGE models.¹⁵² [Table B.2](#) reports estimation results which summarizes the means and the 5th and 95th percentiles of the posterior distributions while the latter are drawn in ???. According to this figure, prior and posterior distributions are relatively different showing that the data were fairly informative. Several parameters are well identified for one country but weakly for the other economy, we decide to keep these parameters in the fit exercise after checking that their weak identification does not affect our estimations

¹⁵¹The ESRB offers on its website an interactive map of the Euro Area on countercyclical capital buffers. To this date, only Sweden and Norway have activated the CCB rate in the European Union but both of these countries are not Euro Area participants.

¹⁵²The posterior distribution combines the likelihood function with prior information. To calculate the posterior distribution to evaluate the marginal likelihood of the model, the Metropolis-Hastings algorithm is employed. We compute the posterior moments of the parameters using a sufficiently large number of draws, having made sure that the MCMC algorithm converged. To do this, a sample of 250,000 draws was generated for four chains through parallelization, neglecting the first 50,000. The scale factor was set in order to deliver acceptance rates of between 20 and 30 percent for each chain. Convergence was assessed by means of the multivariate convergence statistics taken from [Brooks and Gelman \(1998\)](#). We estimate the model using the dynare package of [Adjemian et al. \(2011\)](#). We provide in the online appendix the bayesian IRF of the model which are all fairly consistent with VAR-type models evidence.

Table B.2: Prior and Posterior distributions of structural parameters and shock processes.

Parameter		Prior distributions	Posterior distribution [5%:95%]				
		Shape[mean;std]	CORE		PERIPHERY	EURO	
SHOCK STANDARD DEVIATIONS							
σ_i^A	Productivity	$\mathcal{IG}[0.10,2.00]$	0.82	[0.64:1.00]	0.79	[0.43:1.11]	-
σ_i^G	Government spending	$\mathcal{IG}[0.10,2.00]$	1.43	[1.20:1.65]	1.39	[1.15:1.63]	-
σ_i^U	Preferences	$\mathcal{IG}[0.10,2.00]$	1.24	[0.76:1.73]	1.52	[0.98:2.06]	-
σ_i^I	Investment costs	$\mathcal{IG}[0.10,2.00]$	2.55	[1.83:3.26]	2.57	[1.78:3.47]	-
σ_i^F	Firms markup	$\mathcal{IG}[0.10,2.00]$	0.10	[0.06:0.14]	0.29	[0.17:0.41]	-
σ_i^W	Unions markup	$\mathcal{IG}[0.10,2.00]$	0.45	[0.33:0.57]	0.71	[0.58:0.85]	-
σ_i^N	Firms net wealth	$\mathcal{IG}[0.10,2.00]$	0.36	[0.28:0.45]	0.37	[0.26:0.47]	-
σ_i^D	Deposit markdown	$\mathcal{IG}[0.10,2.00]$	0.30	[0.23:0.37]	0.64	[0.48:0.79]	-
σ_i^B	Bank liabilities	$\mathcal{IG}[0.10,2.00]$	5.89	[4.63:7.10]	9.75	[7.95:11.57]	-
σ_i^L	Credit markup	$\mathcal{IG}[0.10,2.00]$	2.31	[1.78:2.81]	2.09	[1.61:2.57]	-
σ_i^R	Monetary policy	$\mathcal{IG}[0.10,2.00]$	-	-	-	-	0.09 [0.07:0.10]
SHOCK PROCESS AR(1)							
ρ_i^A	Productivity	$\mathcal{B}[0.50,0.20]$	0.98	[0.97:0.99]	0.96	[0.91:1.00]	-
ρ_i^G	Government spending	$\mathcal{B}[0.50,0.20]$	0.87	[0.82:0.93]	0.63	[0.44:0.82]	-
ρ_i^U	Preferences	$\mathcal{B}[0.50,0.20]$	0.29	[0.08:0.49]	0.82	[0.67:0.96]	-
ρ_i^I	Investment costs	$\mathcal{B}[0.50,0.20]$	0.79	[0.70:0.88]	0.68	[0.50:0.90]	-
ρ_i^F	Firms markup	$\mathcal{B}[0.50,0.20]$	0.99	[0.99:1.00]	0.76	[0.54:0.95]	-
ρ_i^W	Unions markup	$\mathcal{B}[0.50,0.20]$	0.49	[0.21:0.77]	0.16	[0.02:0.30]	-
ρ_i^N	Firms net wealth	$\mathcal{B}[0.50,0.20]$	0.86	[0.81:0.90]	0.91	[0.87:0.95]	-
ρ_i^D	Deposit markdown	$\mathcal{B}[0.50,0.20]$	0.88	[0.84:0.93]	0.90	[0.84:0.95]	-
ρ_i^B	Bank liabilities	$\mathcal{B}[0.50,0.20]$	0.94	[0.90:0.99]	0.96	[0.93:0.99]	-
ρ_i^L	Credit markup	$\mathcal{B}[0.50,0.20]$	0.71	[0.55:0.86]	0.68	[0.54:0.82]	-
ρ_i^R	Monetary policy	$\mathcal{B}[0.50,0.20]$	-	-	-	-	0.36 [0.25:0.48]
INTERNATIONAL SHOCK CORRELATION							
$corr_i^A$	Productivity	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.20 [0.01:0.40]
$corr_i^G$	Government spending	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.13 [-0.06:0.33]
$corr_i^U$	Preferences	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.07 [-0.12:0.26]
$corr_i^I$	Investment costs	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.45 [0.28:0.63]
$corr_i^F$	Firms markup	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.28 [0.10:0.47]
$corr_i^W$	Unions markup	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.24 [0.08:0.42]
$corr_i^N$	Firms net wealth	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.33 [0.15:0.51]
$corr_i^D$	Deposit markdown	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.68 [0.56:0.81]
$corr_i^B$	Bank liabilities	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.00 [-0.21:0.22]
$corr_i^L$	Credit markup	$\mathcal{N}[0.20,0.20]$	-	-	-	-	0.81 [0.72:0.90]
STRUCTURAL PARAMETERS							
σ_i^H	Labour disutility	$\mathcal{G}[2.00,0.50]$	0.79	[0.41:1.15]	1.96	[1.17:2.73]	-
h_i^C	Consumption habits	$\mathcal{B}[0.70,0.10]$	0.55	[0.42:0.69]	0.48	[0.34:0.62]	-
θ_i^P	Calvo price	$\mathcal{B}[0.50,0.10]$	0.72	[0.67:0.78]	0.72	[0.62:0.82]	-
ξ_i^P	Price indexation	$\mathcal{B}[0.50,0.15]$	0.22	[0.08:0.37]	0.23	[0.07:0.38]	-
θ_i^W	Calvo wage	$\mathcal{B}[0.50,0.10]$	0.85	[0.79:0.91]	0.89	[0.85:0.93]	-
ξ_i^W	Wage indexation	$\mathcal{B}[0.50,0.15]$	0.51	[0.35:0.67]	0.18	[0.10:0.25]	-
θ_i^E	Employment elasticity	$\mathcal{B}[0.50,0.10]$	0.84	[0.81:0.86]	0.52	[0.18:0.74]	-
θ_i^L	Calvo credit rate	$\mathcal{B}[0.50,0.10]$	0.71	[0.64:0.78]	0.74	[0.69:0.79]	-
θ_i^D	Calvo deposit rate	$\mathcal{B}[0.50,0.10]$	0.79	[0.76:0.81]	0.73	[0.69:0.77]	-
χ_i^I	Investment costs	$\mathcal{N}[0.50,1.50]$	6.59	[4.63:8.41]	7.83	[5.08:10.3]	-
ψ_i	Utilization elasticity	$\mathcal{B}[0.50,0.10]$	0.71	[0.62:0.80]	0.70	[0.58:0.82]	-
\varkappa_i	External finance elasticity	$\mathcal{B}[0.05,0.02]$	0.13	[0.06:0.19]	0.09	[0.05:0.13]	-
h_i^L	Loan demand habits	$\mathcal{B}[0.50,0.20]$	0.79	[0.70:0.89]	0.91	[0.85:0.97]	-
h_i^B	Interbank habits	$\mathcal{B}[0.50,0.20]$	0.28	[0.07:0.48]	0.14	[0.02:0.26]	-
α_i^C	Goods market openness	$\mathcal{B}[0.12,0.05]$	0.08	[0.04:0.13]	0.14	[0.07:0.22]	-
α_i^I	Inv. market openness	$\mathcal{B}[0.08,0.03]$	0.05	[0.02:0.09]	0.08	[0.03:0.13]	-
α_i^L	Credit market openness	$\mathcal{B}[0.10,0.04]$	0.05	[0.02:0.08]	0.12	[0.05:0.18]	-
α_i^B	Interbank openness	$\mathcal{B}[0.20,0.05]$	0.38	[0.30:0.46]	0.32	[0.24:0.4]	-
μ	Substitutability goods	$\mathcal{G}[1.50,0.50]$	-	-	-	-	1.42 [0.99:1.83]
ρ	MPR smoothing	$\mathcal{B}[0.50,0.20]$	-	-	-	-	0.84 [0.80:0.89]
ϕ^π	MPR inflation	$\mathcal{N}[2.00,0.15]$	-	-	-	-	1.65 [1.37:1.92]
$\phi^{\Delta y}$	MPR GDP growth	$\mathcal{N}[0.12,0.05]$	-	-	-	-	0.08 [0.03:0.12]
Marginal log-likelihood					-1199.2		

NOTE: \mathcal{IG} denotes the Inverse Gamma distribution, \mathcal{B} the Beta, \mathcal{N} the Normal, \mathcal{G} the Gamma.

(i.e. calibrating these parameters and re-estimating the model provides very similar results). While our estimates of the standard parameters are in line with the literature (see for instance Smets and Wouters (2003) and Quint and Rabanal (2014)), several observations are worth making by commenting the mean of the posterior distribution of structural parameters.

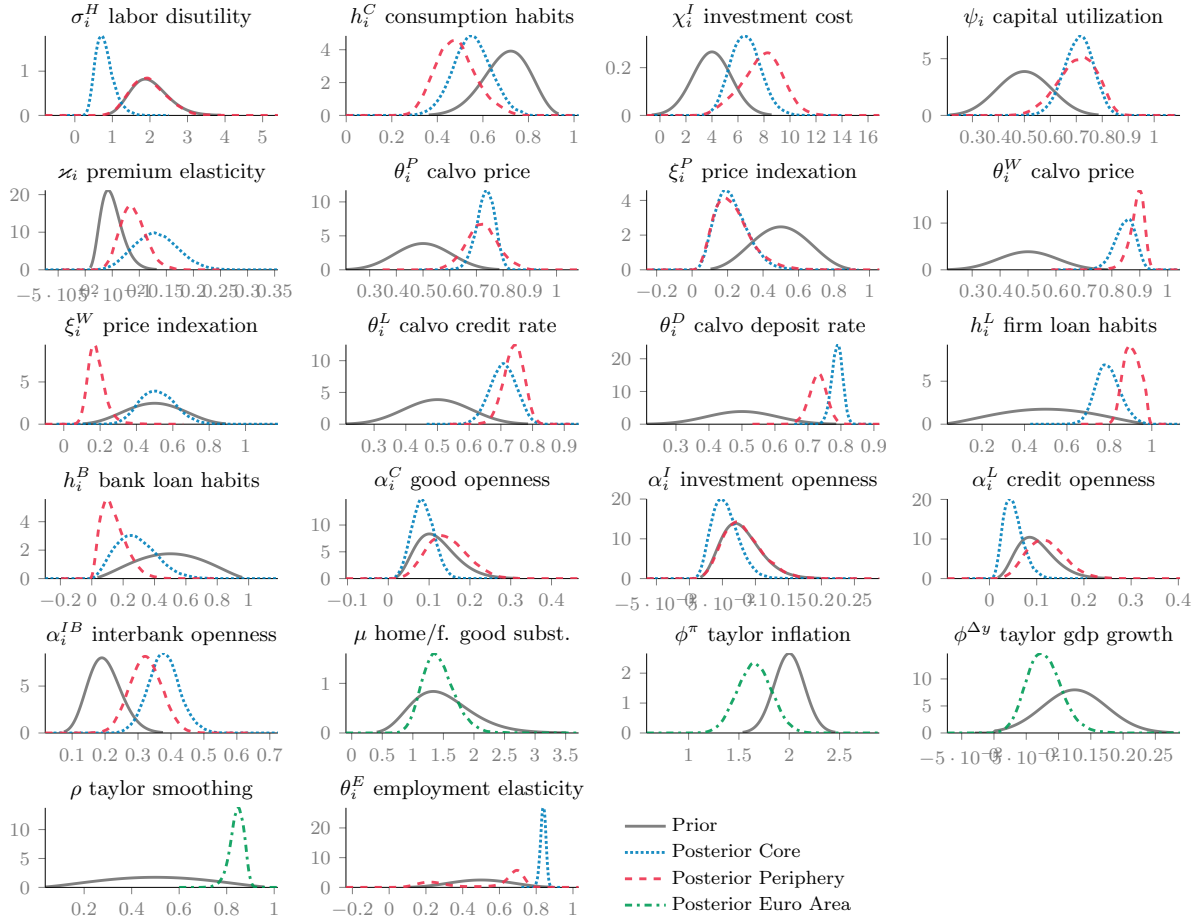


Figure B.3: Prior and posterior distributions of structural parameters.

First regarding asymmetries in business and credit cycles between the core and the periphery, they are mainly driven by the standard deviation of shocks which are larger in peripheral economies. In particular, inefficiency shocks for wages and prices are more volatile in periphery which may constitute an issue in the implementation of a single monetary policy. In the same vein for macroprudential regulation, the presence of heterogeneous financial shocks in terms of volatility questions the perspective of a single federal macroprudential authority.

Second turning to structural parameters, we find an important difference between countries regarding parameter θ_i^E that determines the adjustment of employment to the demand of hours worked: core countries observe a sluggish response of employment to the cycle while the mirror image is seen for periphery. Still regarding the labor market, wage rigidity and indexation parameters are also higher in core countries suggesting that core countries are farther from the optimal allocation characterized by flexible wages and prices. However this interpretation is nuanced by Galí (2013) showing that wage rigidities can, in some particular situations, play a stabilizing role for the economy. One of these particular situations exposed by Galí (2013) is a monetary policy weakly oriented toward inflation which can be observed when monetary policy has hit its lower bound. In the light of this new reinterpretation that meets the current situation of the Euro Area, wages and employment rigidities of core countries may have been stabilizing frictions since the financial crisis episode in 2009.

Third, the results related to market integration are in line with the standard empirical evidence. In particular, peripheral economies are more open and dependent to the core area than the opposite, except for interbank facilities. This latter result is hard to reconcile with the empirical evidence as, before the financial crisis, peripheral economies were net recipient of interbank loans that fueled the property boom. This could be a limitation of the analysis conducted here, however by summing both the net entry of corporate and interbank loans, our model predicts that peripheral economies were net recipient of loans consistently with the historical experience of the Euro Area.

B.5 THE PERFORMANCE OF MACROPRUDENTIAL POLICY

B.5.1 THE SUBOPTIMALITY OF THE FEDERAL SOLUTION

The countercyclical capital buffer (CCB, henceforth), as defined in the [Basel III accords \(2010\)](#) and [ESRB handbook \(2014\)](#), is an instrument designed to contain the procyclicality of the financial sector. It is aimed at building up a capital buffer when threats to resilience are high or during periods of excessive credit growth and can be released when systemic risks abate. The ESRB has selected the credit-to-gdp gap as a leading indicator to signal upcoming crises that the CCB is meant to mitigate. A natural translation of the CCB's objective in our setup corresponds to the minimization of the variance of the credit-to-gdp ratio in the monetary union:¹⁵³

$$\mathcal{L} = \sigma_{L/Y}^2 + \lambda_Y \sigma_Y^2 + \lambda_\nu \sigma_\nu^2, \quad (\text{B.40})$$

where $\sigma_{L/Y}^2$, σ_Y^2 and σ_ν^2 denote respectively the unconditional variance of the credit-to-gdp ratio, output and policy tool $\nu_{i,t}$ while parameters λ_Y and λ_ν are weights on output and CCB. This *ad-hoc* loss function \mathcal{L} borrowed from [Angelini et al. \(2014\)](#) is obtained as a weighted average of national loss functions for each area. It is defined as, $\mathcal{L} = n\mathcal{L}_c + (1 - n)\mathcal{L}_p$, where for each country the national loss is given by, $\mathcal{L}_i = \sigma_{i,L/Y}^2 + \lambda_Y \sigma_{i,Y}^2 + \lambda_\nu \sigma_{i,\nu}^2$. Noticeably, as our model features an interbank market, the credit-to-gdp ratio is given by the aggregate credit supply divided by output: $ctg_{i,t} = (L_{i,t}^s + (1 - \lambda)IB_{i,t}^s)/Y_{i,t}$. As [Angelini et al. \(2014\)](#), we assume that $\lambda_\nu = 0.10$ and $\lambda_Y = 0$, however in a robustness section we investigate whether our results are sensitive to this calibration.

Using the criterion (B.40), we are able to perform a similar exercise as [Angelini et al. \(2014\)](#) by ranking macroprudential policies selecting CCB rule's coefficients $[\rho_c^v, \rho_p^v, \phi_c, \phi_p]$ that deliver the smallest loss. We search over a four-dimensional grid over parameters ranges $[0,1)$ for ρ_i^v and $[0,5]$ for ϕ_i . As a benchmark for compar-

¹⁵³We are aware that the minimization of a loss function rather than a micro-founded welfare criterion is a limitation of our analysis. However, it is also well-known that the usual welfare criterion weakly portrays the trade-off faced by macroprudential authorities between macroeconomic and financial stabilization. A macroprudential policy maximizing the welfare index reduces inflation to the detriment of the financial system which experiences higher volatilities for credit supply and spreads. In response, [Woodford \(2012\)](#) employs an *ad hoc* loss function that fairly portrays the objective of macroprudential policy. Most of the literature follows Woodford's approach, such as [Darracq-Pariès et al. \(2011\)](#) and [Angelini et al. \(2014\)](#).

ing our scenarios for CCB implementation, we consider the optimal monetary policy situation characterized by the optimized Taylor rule that maximizes the welfare of households living in the monetary union. Put differently, the interaction between monetary and macroprudential policy follows a Stackelberg game where monetary policy is leader by removing nominal inefficiencies in the Euro area through the refinancing rate, followed afterward by macroprudential policy which dampens financial cycles. Optimal monetary policy is based on a second order approximation to equilibrium conditions of the model as in [Schmitt-Grohé and Uribe \(2007\)](#) using estimated parameters of [Table B.2](#).¹⁵⁴ Optimal weights in the Taylor rule are respectively $\rho=0.99$, $\phi^\pi=4.38$, $\phi^{\Delta y}=0.5$.

Finally, the minimization of the variance of the credit-to-GDP gap can be re-interpreted through an allocation problem for authorities. Entrepreneurs' distorted beliefs generate overborrowing decisions which inefficiently amplify the cycle. By so, entrepreneurs do not internalize their contribution to the financial amplification. Authorities thus implement a capital requirement policy which can be seen as a Pigouvian tax on banks aiming at internalizing the increase of the social cost through higher lending rates to entrepreneurs.¹⁵⁵ Thus the financial amplification is measured here through the variance of the credit-to-GDP ratio.

We evaluate the stabilization performance of each macroprudential policy scheme by minimizing the second order loss function defined in [Equation B.40](#) subject to linear equilibrium conditions of the estimated model.

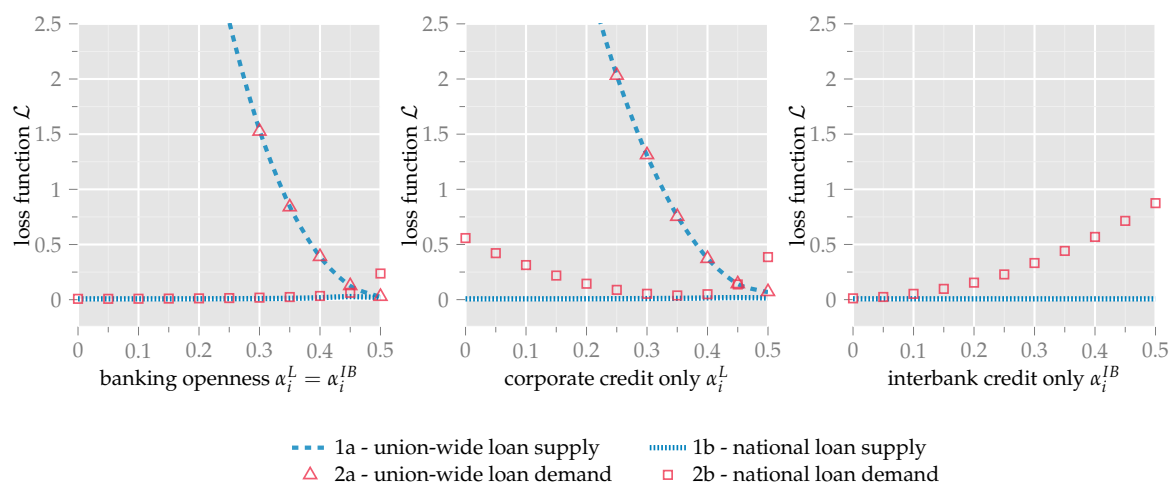
Table B.3: Loss-based ranking of different macroprudential policy implementation schemes

Scheme	Optimal Stances				Loss		
	ρ_c^v	ϕ_c	ρ_p^v	ϕ_p	\mathcal{L}	\mathcal{L}_c	\mathcal{L}_p
LOAN SUPPLY TARGETING							
1.a Union-wide loan supply	0.59	5.2	0.28	4.6	5.4076	3.9097	7.4761
1.b National loan supply	0.96	2.2	0.91	1.97	0.0071	0.0078	0.0062
LOAN DEMAND TARGETING							
2.a Union-wide loan demand	0.46	2.64	0.49	1.25	5.4787	3.9577	7.5791
2.b National loan demand	0.16	2.55	0.96	2.79	0.67336	0.46427	0.9621
CAPITAL INFLOWS TARGETING							
3. Capital Inflows	0.53	2.36	0.15	0.57	19.7407	13.5852	28.241

?? reports the policy stance and the stabilizing performances for each implementation scheme. The optimal stabilization of the financial system critically depends on the target selected by macroprudential authorities. Unsurprisingly, we observe a clear ranking favoring operational instruments reacting to national loan developments

¹⁵⁴In the quantitative simulation, we first search for weights attached to inflation ϕ^π , GDP growth $\phi^{\Delta y}$ and the smoothing degree ρ in the Taylor rule that gives the highest unconditional welfare of households from [Equation B.15](#). Based on the grid search by 0.01 unit, we limit our attention to policy coefficients in the interval $(1, 5]$ for ϕ^π , $[0, 0.5]$ for $\phi^{\Delta y}$ ([Schmitt-Grohé and Uribe, 2007](#)), and in the interval $[0, 0.99]$ for ρ to speed up optimization routines. We take into account the zero-lower bound by adding a penalty term in the welfare index associated to the variance of the nominal interest rate following the calibration of [Woodford \(2003\)](#).

¹⁵⁵We refer to [Jeanne and Korinek \(2013\)](#) for the implementation of macroprudential measures through a social planner problem.



NOTE: for each value of the share of foreign loans in the portfolio of entrepreneurs (denoted α_i^L) and banks (denoted α_i^{IB}), we compute the optimal macroprudential policy for four different schemes. The loss function is an average between core and periphery detailed in ???. Capital inflows-adjusted policy is not reported as its loss is too high compared to alternative schemes.

Figure B.4: The role of cross-border banking in the scheme ranking.

(schemes 1.b and 2.b) that outperforms solutions based on federal loan developments (schemes 1.a and 2.a).

A natural question is thus to determine the degree of mutual financial cross-border lending flows that should be observed to affect this main conclusion. Indeed, as underlined by [Cecchetti and Tucker \(2016\)](#) and [Beck et al. \(2016\)](#) a higher banking integration should require a common prudential standard (here, the targeting rule) applied appropriately to all parts of the financial system. As a consequence, the efficiency of federal targeting rules (*i.e.* schemes 1.a and 2.a) is expected to increase with the share of cross-border loans while national adjusted should be less efficient. To investigate this question, [Figure B.4](#) reports minimized loss functions for different levels of cross-border loans. Three component are presented related to an increase in total (namely the sum of corporate and interbank) cross-border loans in the left panel, in corporate loans only in the center panel and in interbank loans only in the right panel.

We can draw three main conclusions from [Figure B.4](#). First, the interest of conducting federal based definition of the credit-to-gdp ratio unsurprisingly increases with the size of cross-border flows. As reported in the first panel, the relative interest of implementing a national adjusted rule (such as 1.b and 2.b) is magnified with respect to the federal adjusted rule for lower values of α_i^L and α_i^{IB} . However for values of these parameters higher than 25%, the gap in the loss function values tends to decrease significantly. Nevertheless, macroprudential rules based on a federal definition of the credit to GDP ratio becomes only interesting for a mutual cross-border lending openness lying around 45%. This figure is rather high with respect to the current value of cross-border lending, which makes this solution not optimal for the moment.

Second, this policy outcome regarding the reduction in the loss function under a federal definition of the credit-to-gdp ratio is mainly driven by the mutual openness of the corporate credit markets. As reported in the center and right panels, interbank

cross-border lending credit has no noticeable impact on the relative ranking of policy solutions, while the integration of the corporate loan segment determines the slope of the decrease in the loss function under the federal solution.

Third, even if banking integration clearly enhances the stabilization performances of federal-adjusted schemes, a macroprudential solution targeting the national credit supply remains remarkably efficient with a global banking system. For the all spectrum of values of α_i^L and α_i^{IB} displayed in panels of [Figure B.4](#), CCB reaction to a national definition of the credit-to-gdp ratio determines the lowest value for the loss function. Thus, our experiments suggest that even if cross-border linkages are high enough to justify the implementation of a federal adjusted solution, the reaction to national lending conditions remains optimal.

B.5.2 CONTRASTING NATIONAL SOLUTIONS

As underlined in ?? our numerical results suggest that the best outcome for the loss function value is obtained when macroprudential policy targets the national supply of loans instead of the national demand for loans (*i.e.*, accounts for the national and foreign nature of loans contracted in the economy). The interest of targeting loan supply is easily understandable, as the transmission channel of macroprudential policy directly impacts the marginal cost of loan production and, by so, financial intermediaries. If macroprudential policy targets loan demand, this direct channel is dampened, which leads to a lower reduction of the loss function. National macroprudential policies reacting to federal averages do not target the origin of financial imbalances as regional divergences in credit cycles are too important to have a single federal target. The solution focusing on cross-border lending developments (3), is clearly dominated by all the other implementation schemes: in this case, the loss function reaches its highest value, revealing that targeting external imbalances is not appropriate as it does not take into account the financial roots of the problem.

To understand these results we simulate the dynamic responses to a negative productivity shock in core countries and a negative net wealth shock in peripheral economies.¹⁵⁶ We concentrate on these two shocks as they are leading drivers of the loan-to-gdp ratio that authorities aim at stabilizing through capital buffer measures.

First, [Figure B.5](#) reports the IRFs after a negative productivity shock for each CCB rule with respect to the optimal monetary policy situation. Under the benchmark of an optimal monetary policy (dashed lines), a negative home productivity shock depresses investment and activity and implies inefficient fluctuations in the credit-to-gdp ratio. This shock translates to the peripheral region through trade channels, cross-border lending, monetary policy reaction and shock correlation. The introduction of national macroprudential measures has a clear stabilizing effect for business cycles of the monetary union. The release of the buffer eases the bank capital constraint which in turn lowers credit spreads and investment fluctuations. However, the targeting regime determining the CCB rate critically affects the outcome the economy that does not experience the shock and explains the effectiveness of national credit targeting regimes over federal ones. In a federal targeting regime (1.a and 2.a), both countries react to

¹⁵⁶As underlined by [Angelini et al. \(2014\)](#), supply shocks may dominate in normal times, while financial shocks are important in exceptional times.

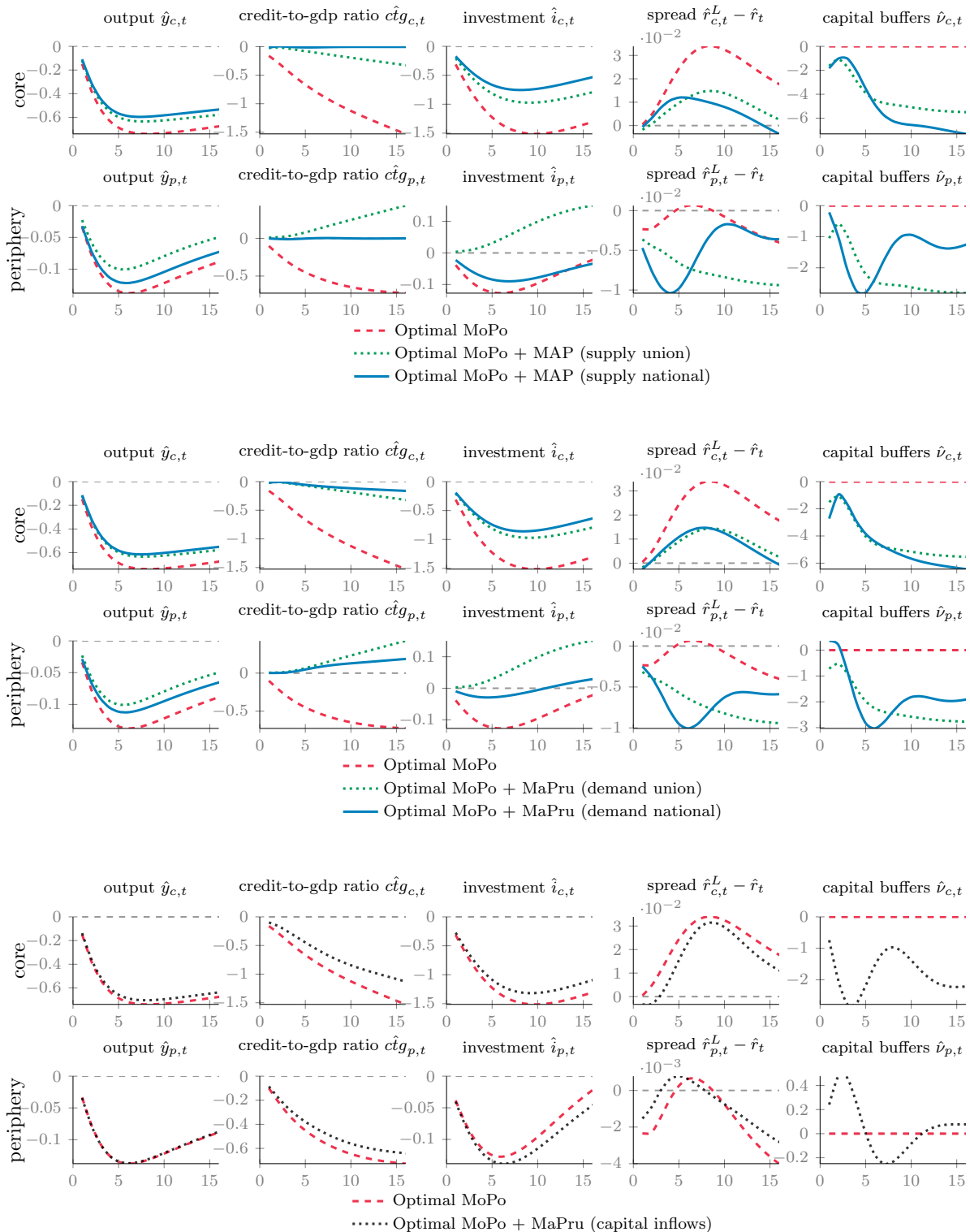


Figure B.5: System response to an estimated negative productivity shock in core countries $\eta_{c,t}^A$ measured in percentage deviations from steady state under different macroprudential policy rules (domestic or union-wide supply/demand/inflows targeting).

a common average credit-to-gdp ratio which leads the foreign country to react procyclically to foreign shocks. In addition, we do not find clear differences between targeting national credit demand or supply. Finally CCB rates adjusted to capital inflows fail at providing macroeconomic stability in particular for the peripheral country. The shock in the core country generates a re-allocation of credit from core to peripheral economies and authorities in peripheral economies procyclically tighten the capital constraint which inefficiently amplifies the crisis.

Second, [Figure B.6](#) depicts the IRFs after a negative stock market shock in peripheral economies. Under the optimal monetary policy benchmark (dashed lines), this shock deteriorates the borrowing conditions of entrepreneurs, thus incurring a large decline in output and investment through the external finance premium channel. Consequently, the credit-to-gdp gap experiences a large decline inefficiently driven by the biased expectations of entrepreneurs. Our main results regarding the implementation of macroprudential measures are similar to the productivity shock. National credit targeting is preferred to a federal one as the latter exacerbates fluctuations for the country that does not experience the shock, creating a spillover effect. The same procyclical mechanism is observed for the capital inflows targeting scheme. Finally, targeting the demand or supply of credit provides very similar responses.

Table B.4: Macroeconomic performances of different implementation schemes in comparison to the optimal policy benchmark

Scheme	Standard deviations (%)								Correlation
	Core				Periphery				$\text{corr}(\hat{y}_{c,t}, \hat{y}_{p,t})$
	$\hat{y}_{c,t}$	$\hat{i}_{c,t}^s$	$\hat{i}b_{c,t}$	$\hat{s}_{c,t}^L$	$\hat{y}_{p,t}$	$\hat{i}_{p,t}^s$	$\hat{i}b_{p,t}$	$\hat{s}_{c,t}^L$	
MONETARY POLICY ONLY									
Benchmark	100	100	100	100	100	100	100	100	0.15
LOAN SUPPLY TARGETING									
1.a Union-wide loan supply	91.57	91.95	75.11	127.47	103.88	113.45	73.86	105.96	0.15
1.b National loan supply	79.67	76.60	74.48	130.46	95.06	86.19	73.93	120.19	0.46
LOAN DEMAND TARGETING									
2.a Union-wide loan demand	91.58	92.06	75.1	127.29	103.79	112.92	73.96	105.58	0.16
2.b National loan demand	82.89	92.94	71.73	137.08	76.24	63.42	76.19	116.41	0.44
CAPITAL INFLOWS TARGETING									
3 Capital Inflows	93.42	138.3	73.27	156.67	96.33	79.92	88.94	90.46	0.37

Accounting for all shocks of the model, ?? reports the standard deviation of activity, corporate and interbank loans and interest rate spread under alternative policy schemes. We contrast our results with respect to the optimal monetary policy (without prudential regulation) to measure how the conduct of macroprudential measures have decreased/increased the standard deviation of endogenous variables for each country. This exercise measures how the stabilizing gains are distributed between countries. We also report business cycle synchronization statistics, as measured by the correlation of output between economies, to evaluate whether the scheme is able to smooth the heterogeneity between Euro Area participants.

Overall, the highest gains can be obtained by adopting macroprudential policy measures reacting to national developments in the credit-to-gdp ratio. The reaction of the macroprudential instrument to other measures of the credit-to-gdp ratio (based on either loan demand or federal averages) leads to less reduction in the standard deviation of these aggregates. However, the implementation of macroprudential policy is not a free lunch since the building up of a capital buffer mechanically increases the

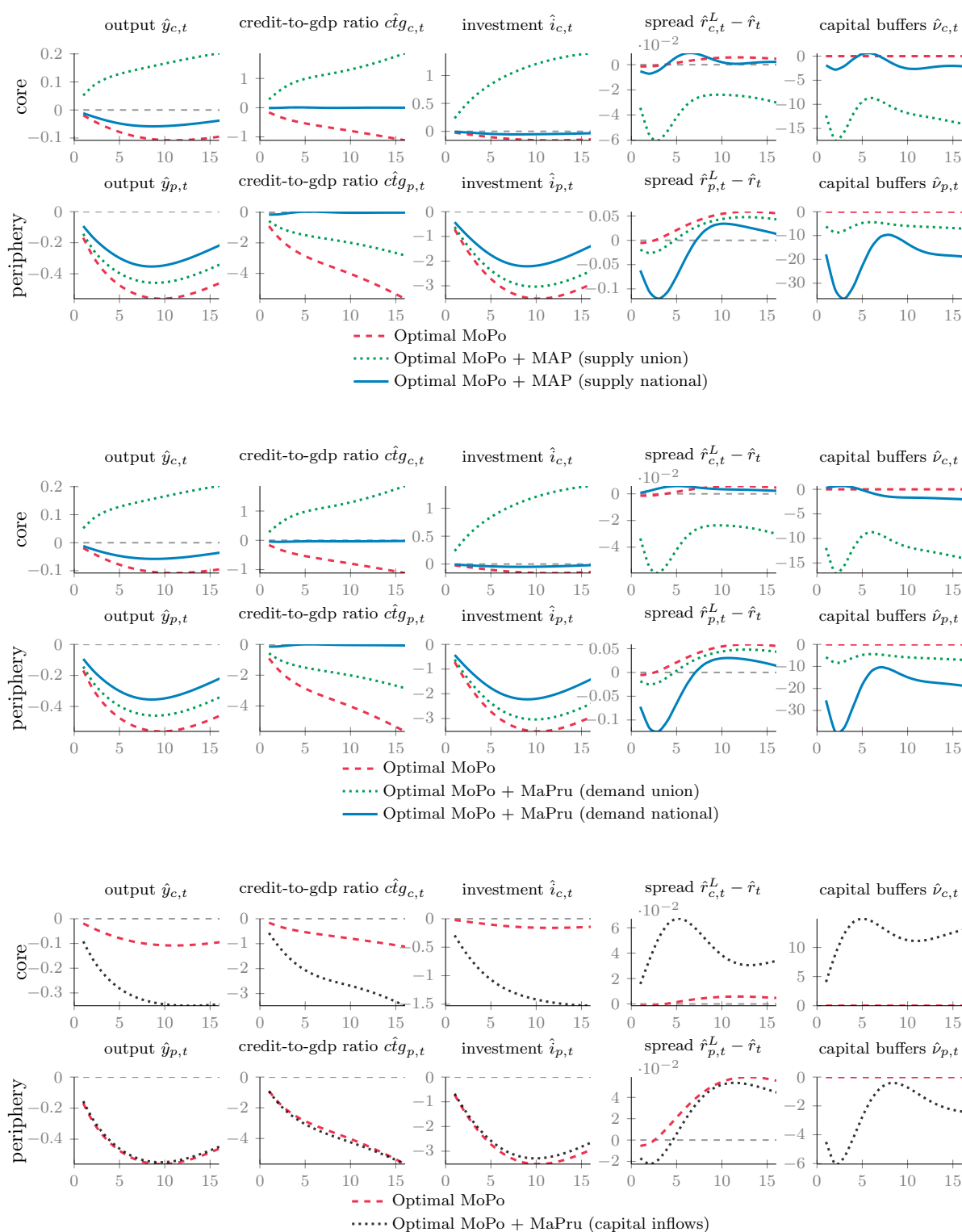


Figure B.6: System response to an estimated negative firms net wealth shock in peripheral countries $\eta_{p,t}^N$ measured in percentage deviations from steady state under different CCB regulation schemes (domestic or union-wide supply/demand/inflows targeting).

volatility of the spread when stabilizing the debt-to-GDP ratio.¹⁵⁷

In addition, we observe a natural link between loan-to-GDP stabilization and business cycle synchronization, showing that the implementation of national-adjusted macroprudential policies smooths the heterogeneity across regions. Such a result is interesting for monetary policy makers, as the effectiveness of a single monetary policy critically depends on business cycle synchronization between monetary union participants. Thus the enhanced cycle synchronization partially solves the Euro Area's problem of a "one-size-fits-all" monetary policy.

Contrasting the national demand and national supply targeting solutions, we find that their effectiveness are clearly different according to the country considered. As an example, the supply side oriented policy fits the situation of core economies, while the one oriented towards the demand of credit meets the situation of peripheral economies in terms of macroeconomic stabilization. Core countries should thus focus on the stabilization of its banks while peripheral economies should stabilize its borrowers. Having asymmetric targets between regions of the Eurozone could be an interesting perspective to implement stabilization policies.

Finally the capital inflows targeting solution fits well peripheral economies that were net recipient of foreign claims before the 2009 crisis. However, this policy is harmful for core countries, affected by an increase in the volatility of loans and of the credit spread. Over the sample time period, core countries were net exporter of loans by fueling property booms in peripheral economies through interbank lending, this capital outflow involves an inefficient and durable reduction of the CCB rate enhancing the volatility of credit domestically. While capital controls appears to be a promising tool for Periphery, it is clearly unsuited to countries experiencing capital outflows.

B.6 ADDITIONAL SENSITIVITY ANALYSIS

This section assesses the robustness of our results with respect to some key parameters of the model and to the nature of shocks encountered in the economy.

B.6.1 LOSS FUNCTION CALIBRATION

First, ?? reports the sensitivity analysis of the main results to the calibrated value of some underlying parameters. The first experiments focus on the weight parameters of the loss function of macroprudential authorities. As reported, the ranking of policies remain unaffected by the value of these parameters. An increase in the policymakers preferences for output (denoted λ_y) or the penalization of the variance of capital requirements (denoted λ_ν) increases the loss. Turning to structural parameters (namely the degree of substitutability between different varieties of loans ν and n the share of core countries in the monetary union) the sensitivity analysis does not alter the ranking of macroprudential decisions. As observed, an increase in the size of the core

¹⁵⁷The variability of the lending spread is a leading indicator of financial distress, Woodford (2012) sets its stabilization as an objective for monetary policy making with financial frictions.

Table B.5: Sensitivity analysis of scheme ranking to different calibrated parameters

		Euro area loss \mathcal{L}				
		1.a	1.b	2.a	2.b	3
Loss output stabilization	$\lambda_y = 0$	5.4076	0.0071	5.4787	0.67336	19.7407
	$\lambda_y = 5$	6.0138	0.4063	5.9196	1.0906	16.7659
	$\lambda_y = 10$	6.5325	0.8036	6.5119	1.5042	20.8989
Loss policy instrument	$\lambda_\nu = 0$	5.4063	0.0004	5.5147	0.66152	19.9706
	$\lambda_\nu = 5$	5.7099	0.28443	5.7186	0.97562	20.1045
	$\lambda_\nu = 10$	5.9328	0.53581	5.9252	1.1936	20.4273
Loan substitutability	$\nu, \xi = 0$	5.5082	0.0072	5.4190	0.67025	19.7962
	$\nu, \xi = 5$	5.5206	0.0068	5.5392	0.6914	20.2309
	$\nu, \xi = 10$	5.5372	0.0067	5.5298	0.7056	20.9558
Share of core countries	$n = 0.4$	5.5641	0.0069	5.5485	0.0868	19.0729
	$n = 0.5$	5.6804	0.0069	5.7378	0.3138	19.7849
	$n = 0.6$	5.4134	0.0071	5.4053	0.78181	20.2335
Flexible interest rates	$\theta_i^L = 0$	5.9139	0.0058	5.8096	0.65304	17.8474
	$\theta_i^D = 0$	5.8987	0.0074	5.8685	0.66579	16.6966
	$\theta_i^L = \theta_i^D = 0$	5.9067	0.0059	5.8446	0.66382	21.3504

NOTE: λ_y and λ_ν denote respectively weights on output and policy tool volatilities in the macroprudential loss function, ν is the substitution degree between home and foreign credit varieties and n denotes the share of core countries in terms of real GDP in the euro area. Losses are evaluated using the average of core and peripheral countries volatilities.

countries' group has opposite results on the value of the loss, depending on the dimension of the credit-to-gdp ratio that is taken into account in the reaction of macroprudential policy. The loss decreases for schemes based on a reaction to national loan developments while it increases when the macroprudential instrument reacts to the federal value of the ratio. However, the gap between the loss values remain so high that the ranking between national and federal solutions is left unaffected. Regarding the nominal rigidities on interest rates, thus reflecting the imperfect pass-through of both monetary and macroprudential policies, disabling this nominal friction does not affect the ranking too.

B.6.2 NATURE OF SHOCKS

Second, ?? reports the sensitivity analysis of the main results to the nature of shocks encountered in the economy. We distinguish between supply (productivity shocks), demand (gathering public spending shocks, preference shocks and investment shocks) and financial shocks (gathering shocks on the collateral of corporate lending, on riskiness of investment projects and cost push shocks on deposit). As underlined by [Angelini et al. \(2014\)](#), supply and demand shocks may dominate in normal times, while financial shocks are important in exceptional times. For each shock, we contrast the consequences of adopting one of the macroprudential scheme adopted for the definition of the credit-to-gdp ratio (1a to 3). As observed, the relative ranking of the policy scheme is not altered by the nature of shocks encountered in the economy, as the solution based upon the reaction of authorities to the fluctuations in the national loan supply to GDP dominates all the other possibilities. However, the value of the loss fluctuates and it is higher for financial shocks. Furthermore, a closer look at the macroprudential parameters shows that the nature of the shock affects the contemporaneous policy stance of regional authorities. As observed, for real shocks, the

Table B.6: Robustness check: optimal monetary and macroprudential Policies conditional on shocks

Scheme	Monetary Policy			Macroprudential Policy				Union	Loss Core	Periph	
	ρ	ϕ^π	$\phi^{\Delta y}$	ρ_c^v	ϕ_c	ρ_p^v	ϕ_p	\mathcal{L}_u	\mathcal{L}_c	\mathcal{L}_p	
SUPPLY SHOCKS											
1.a	Union-wide loan supply	0.94	5	0.5	0.82	0.30	0.41	3.86	1.3139	0.95417	1.8107
1.b	National loan supply	0.94	5	0.5	0.95	2.72	0.86	2.58	0.0027	0.0041	0.0007
2.a	Union-wide loan demand	0.94	5	0.5	0.90	3.54	0.39	0.50	1.3189	0.95679	1.819
2.b	National loan demand	0.94	5	0.5	0.46	2.49	0.48	2.46	0.065797	0.04746	0.091119
3	Capital Inflows	0.94	5	0.5	0.81	3.63	0.43	0.98	1.9538	1.5535	2.5065
DEMAND SHOCKS											
1.a	Union-wide loan supply	0.99	1	0.5	0.66	3.07	0.62	2.16	0.20432	0.14956	0.27993
1.b	National loan supply	0.99	1	0.5	0.94	2.52	0.64	2.52	0.0035	0.0039	0.0029
2.a	Union-wide loan demand	0.99	1	0.5	0.58	2.61	0.39	2.37	0.2051	0.1504	0.2806
2.b	National loan demand	0.99	1	0.5	0.05	2.35	0.80	2.69	0.7980	0.5139	1.1903
3	Capital Inflows	0.99	1	0.5	0.76	3.68	0.08	0.41	12.7015	11.5068	14.3513
FINANCIAL SHOCKS											
1.a	Union-wide loan supply	0	1.48	0.5	0.15	1.60	0.24	3.35	0.5895	0.4308	0.8086
1.b	National loan supply	0	1.48	0.5	0.92	2.21	0.94	1.66	0.0023	0.0014	0.0035
2.a	Union-wide loan demand	0	1.48	0.5	0.06	1.94	0.26	3.58	0.5900	0.4301	0.8107
2.b	National loan demand	0	1.48	0.5	0.98	1.72	0.85	1.37	0.0054	0.0038	0.0077
3	Capital Inflows	0	1.48	0.5	0.30	0.47	0.96	4.63	2.6168	2.1119	3.3141

NOTE: each group of shocks is composed of core and peripheral shocks and their associated cross-correlation. Supply shocks group gathers productivity shocks $\eta_{i,t}^A$; Demand shocks group gathers spending $\eta_{i,t}^G$, preferences $\eta_{i,t}^U$ and investment $\eta_{i,t}^I$; Financial shocks gathers collateral crunch $\eta_{i,t}^N$, riskiness $\eta_{i,t}^Q$ and deposit cost-push $\eta_{i,t}^D$ innovations.

contemporaneous reaction of core countries authorities tends to be higher for supply shocks while peripheral countries are more reactive for demand shocks. This latter feature is also observed for exceptional times.

B.6.3 STRUCTURAL FINANCIAL ASYMMETRIES

Table B.7: Sensitivity analysis of scheme ranking to financial structural asymmetries

		Euro area loss \mathcal{L}				
		1.a	1.b	2.a	2.b	3
Benchmark		5.4076	0.0071	5.4787	0.67336	19.7407
Firms rate of default	$1 - \bar{\eta}_p^E = 0.0125$	6.6312	0.0078	6.6535	0.58638	22.6661
Share of illiquid banks	$\lambda_p = 0.48$	6.0702	0.0078	6.3512	1.9242	21.7665
Corporate net wealth-to-assets ratio	$\bar{N}_p/\bar{K}_p = 0.2$	7.4030	0.0070	7.3528	0.55023	22.8376
Bank leverage ratio	$\bar{BK}_c/A_c = 0.06$	6.5421	0.0163	6.5513	0.64037	19.651

Third, we investigate whether structural asymmetries affect the ranking of the model, results are reported in ?? In the benchmark setup developed in the paper, we assumed that most of the endogenous variables in the deterministic steady state were symmetric between countries. However this assumption is questionable, in particular regarding the asymmetries in the financial sector which may be an important feature for macroprudential policymaking. As a first exercise, we examine whether the symmetry assumption on the default rate of entrepreneurs matters for the scheme ranking. Since we cannot observe the default rate of entrepreneurs, we use as a proxy the share of non-performing loans in the balance sheet of banks in BankScope database. We find that the share of non-performing loans is on average twice higher in Periphery and calibrate the defaulting share of entrepreneurs accordingly. We find that this structural asymmetry does not affect the ranking, however we observe a small reduction of the gap between the demand-adjusted and the supply-adjusted macroprudential policy.

We also investigate the implications of cross-country heterogeneity in the share of illiquid banks operating in the interbank market. We proxy this parameter through the number of banks borrowing on the unsecured money market provided by [Garcia-de Andoain et al. \(2014\)](#). We find that on average the share of banks borrowing on the interbank market is 25% higher in Periphery, we calibrate λ_P accordingly in our model. The new ranking obtained from the new set of simulations show no important difference, except for the national demand solution which becomes less efficient in stabilizing the credit-to-gdp ratio. We also investigate the implication of asymmetric steady state leverages of firms and of banks between countries. Core countries observed a lower net-worth-to-asset ratio than Peripheral economies for firms,¹⁵⁸ we take this feature into account by calibrating \bar{N}_p/\bar{K}_p at 20% as in Italy. For banks, we use the ECB's Risk Assessment Indicators (RAI) and find that Core banks are less capitalized on average, in particular because of Belgium, Germany and Netherlands's low equity to assets ratios. We calibrate the leverage ratio of core banks to 6% to incorporate this structural asymmetry and run the simulations. We observe no clear ranking change under these two asymmetries. Overall, these robustness exercises confirm that these structural aspects does not affect the ranking as second order statistics minimized in the loss function are rather independent of structural asymmetries.

B.7 CONCLUSION

This paper shows that international lending flows have mixed effects on the optimal conduct of macroprudential policy in the Eurozone. Contrasting alternative rules for countercyclical capital buffers, our results suggest that targeting a national credit-to-gdp ratio should be favored to federal averages as this rule induces better stabilizing performances in terms of output and loan volatility. The important divergences in credit cycles between core and peripheral countries reported in the data require a national orientation of macroprudential policy tailored to domestic financial developments. Our results have also underlined the reduced interest of lifting up macroprudential policymaking to the supra-national level. Indeed, national capital buffers reacting to the union-wide loan-to-GDP ratio lead to the same stabilization results than the one obtained under the national reaction when mutual cross-border lending reaches 45%. However, even if cross-border linkages are high enough to justify the implementation of a federal adjusted solution, the reaction to national lending conditions remarkably remains optimal. In addition, we find that adjusting the macroprudential instrument to capital inflows is a promising tool for countries experiencing loans inflows.

The analysis of cross-border lending on the conduct of macroprudential policy is a burgeoning research area. In this paper we focused on countercyclical capital buffers, and an interesting question for future research is to evaluate how this result favoring self-oriented macroprudential measures may be affected by the choice of alternative macroprudential instruments. The construction of an original welfare index, that features a trade-off between macroeconomic and financial stability, could be a next

¹⁵⁸There is a clear asymmetry between Core and Peripheral countries in terms of debt-to-financial assets ratios. For instance, France had a ratio of 40%, Germany 60% and Netherlands 60% while Italy had 80% and Spain 60%.

step of research. Finally, the analysis of the CCB rate through a Ramsey allocation problem could also be part of a future research agenda.

Titre: Essais sur la macroéconomie de l'hystérèse et du changement climatique

Mots clés: Macroéconomie, Hystérèse, Changement Climatique, Techniques Bayésiennes

Résumé: Cette thèse d'Habilitations à Diriger des Recherches est l'occasion de synthétiser les travaux en cours et futurs. Dans ce qui suit, je donne une description du contexte scientifique qui caractérise mes recherches. Ensuite, je résume mon programme de recherche pendant mon doctorat. Enfin, je discute de mon travail en cours sur deux sujets de la macroéconomie actuelle que je trouve pertinents pour mon programme de recherche: l'hystérésis et la macroéconomie du changement climatique. Les deux premiers chapitres de cette HDR traitent des problèmes d'hystérèse sur la productivité et les anticipations d'inflation dans le cadre de modèle d'équilibre général. Les deux derniers chapitres abordent les effets du changement climatique sur les fluctuations agricoles et l'aversion au risque des investisseurs.

Title: Essays on Macroeconomics of Hysteresis and Climate Change

Keywords: Macroeconomics, Hysteresis, Climate Change, Bayesian Techniques

Abstract: This Habilitations à diriger des Recherches is an opportunity to synthesize ongoing and future work. In the following, I first discuss the scientific background characterizing my research. Next, I provide a summary of my research agenda during my Ph.D. studies. Finally I discuss my ongoing work on two challenging topics in macroeconomics that I find relevant for my research agenda: hysteresis and the macroeconomics of climate change. The two first chapters of this document deal with hysteresis issues on productivity and inflation expectation in a general equilibrium perspective. The two last chapters question the role of climate change on agricultural cycles as well as on risk aversion of investors.