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Lise Clain-Chamosset-Yvrard

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Présentée et soutenue publiquement par

Lise CLAIN-CHAMOSSET-YVRARD

le 13 octobre 2015

en vue de l'obtention du grade de docteur d'Aix-Marseille Université

**PRIX D'ACTIFS, BULLES ET
FLUCTUATIONS MACROECONOMIQUES**

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Les différents chapitres de cette thèse sont issus d'articles de recherche rédigés en anglais et dont la structure est autonome. Ceci y explique la présence de terme "paper" ou "article" ainsi que l'éventuelle répétition de certaines informations.

Notice

The chapters of this dissertation are self-containing research articles. Consequently, terms "paper" and "article" are frequently used. This also explains that some informations are given in multiple places of the thesis.

A ma famille, ma chère Maman et Célestin

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Chapter 0

Introduction Générale

0.1 Préambule

Événement marquant de ce début du *XXI*^{ème}, la crise financière, apparue en août 2007 aux Etats-Unis, s'est rapidement transformée en une crise globale, se propageant tout d'abord à l'ensemble des marchés financiers nationaux et internationaux, puis affectant l'ensemble de l'économie réelle.

La chute vertigineuse des prix de l'immobilier aux Etats-Unis, associée à l'effondrement du marché des prêts hypothécaires à risque appelés "subprimes", se trouve être à l'origine de cette crise financière mondiale. La crise des subprimes s'est mutée en une crise bancaire internationale en raison de la globalisation financière et de la nature sophistiquée des instruments financiers, comme la titrisation. La titrisation est une technique de financement permettant aux institutions financières de construire des actifs financiers à partir de créances telles que les subprimes. Ainsi, les banques américaines ont pu sortir ces crédits risqués de leur bilan en vendant les actifs issus de la titrisation à d'autres institutions financières américaines et/ou étrangères. Celles qui détenaient ces actifs qualifiés de toxiques dans leur portefeuille ont subi des pertes en capital, et ont donc réduit à leur tour leurs financements. Le 15 septembre 2008, la crise s'intensifia

et devint une crise bancaire avec la faillite de la banque américaine Lehman Brothers, entraînant dans sa chute notamment des banques allemandes, anglaises et belges (e.g., Deutsche Bank, HBOS, Dexia). Les gouvernements et les autorités monétaires sont intervenus avec des plans de sauvetage afin de renflouer leurs systèmes financiers. Malgré ces interventions, la propagation à l'ensemble de l'économie n'a pu être évitée. La plupart des pays industrialisés entrent en 2008 dans une crise économique, si profonde qu'elle est nommée "Grande Récession" en référence à la Grande Dépression de 1929.

Les épisodes d'envolée et d'effondrement des prix des actifs financiers ou réels, qui sont souvent liés à l'existence et l'éclatement d'une bulle spéculative, rythment l'histoire économique : de la célèbre bulle des bulbes de tulipes de 1637 puis entres autres, celles des Compagnies des mers du Sud et du Mississippi en 1720, la crise boursière de 1987 aux Etats-Unis et la crise financière japonaise de 1990 qui entraîna le pays dans une longue phase de stagnation économique et de déflation. L'enchaînement des crises chez les pays émergents (Mexique, Argentine, Pays asiatiques, Russie) dans les années 1990, celle des valeurs "dotcom" en 2001, et dernièrement le krach boursier chinois montrent bien que la crise financière de 2007-2008 n'est ni la première, ni la dernière.

Or, bon nombre de décideurs politiques, organismes multilatéraux et d'universitaires furent surpris par cette crise, ou du moins ne s'attendaient pas à une telle ampleur. Ainsi, l'Organisation de Coopération et de Développement Économiques (OCDE) évalua en 2007 qu' "Après un ralentissement temporaire au premier trimestre 2007, la croissance pour l'ensemble de la zone de l'OCDE devrait dépasser son taux potentiel pendant le reste de l'année 2007 et au cours de l'année 2008, à la faveur du dynamisme des économies de marché émergentes et des conditions financières favorables."¹OECD (2007).

¹Dans son rapport de 2007, l'OCDE estima un taux potentiel avoisinant 2,7%. Or, le taux de croissance annuel en 2008 enregistré par l'OCDE pour la zone de l'OCDE fut de 0,25%.

Pourtant, les signes d’alertes étaient présents telles la hausse spectaculaire des prix sur le marché immobilier américain, laissant présager l’existence d’une bulle spéculative immobilière et une facilité d’accès des agents économiques au crédit. A cela vinrent s’ajouter une création excessive de liquidités notamment aux Etats-Unis financée par une épargne démesurée provenant des économies émergentes (e.g., Chine) et des pays exportateurs de pétrole, une politique monétaire trop accommodante menée par la Réserve Fédérale des Etats-Unis afin de limiter la récession de 2001-2002, avec des taux d’intérêt chutant jusqu’à 1%, et un manque certain de régulation financière.

Pour son manque d’anticipation, la profession de l’économie a essuyé de nombreuses critiques y compris de la part d’universitaires influents et de décideurs politiques. Paul Krugman, professeur à l’Université de Princeton et “prix Nobel d’économie” 2008, sermonna ses collègues macroéconomistes sur l’échec de leurs modèles à anticiper la crise dans un article intitulé “How Did Economists Get It So Wrong?” paru dans *The New York Times* en septembre 2009. Le précédent président de la Banque Centrale Européenne (BCE), Jean-Claude Trichet, affirma quant à lui que la théorie économique standard fut inutile durant la crise ²:

“[...]When the crisis came, the serious limitations of existing economic and financial models immediately became apparent.[...] Macro models failed to predict the crisis and seemed incapable of explaining what was happening to the economy in a convincing manner.³[...] The key lesson I would draw from our experience is the danger of relying on a single tool, methodology or paradigm.[...]” Trichet (2010).

Ce manque d’anticipation est d’autant plus étonnant qu’il existe des études approfondies sur les crises financières et économiques survenues au cours des siècles passés

²Ces paroles de Jean-Claude Trichet furent prononcées lors de la conférence des Banques Centrales organisée par la BCE en 2010.

³A cet endroit du texte, une référence est fait à l’article de Caballero (2010) intitulé “Macroeconomics after the crisis: Time to deal with the pretense-of-knowledge syndrome” dans lequel il pointe les limites et les nouveaux défis à relever de la macroéconomie actuelle.

et présents. Nous pouvons citer le célèbre ouvrage *Manias, Panics, and Crashes: A History of Financial Crises* écrit en 1978 de Charles Kindleberger, mais également les travaux de Minsky (1986), et plus récemment Kindleberger et Aliber (2005) et Reinhart et Rogoff (2009).⁴

Que pouvons-nous donc retenir des crises financières? Premièrement, à l'origine de ces crises, nous retrouvons souvent de larges fluctuations des prix d'actifs souvent attribuées à l'existence d'une bulle spéculative. La hausse spectaculaire des prix, puis leur effondrement sur le marché immobilier américain, point de départ de la dernière crise, en sont une parfaite illustration. Deuxièmement, les crises financières sont de nature contagieuse. Ce sont des phénomènes locaux qui peuvent se propager rapidement au reste du monde en raison de la mondialisation et de l'interdépendance des marchés financiers. Troisièmement, ces crises peuvent affecter profondément l'activité économique dans son ensemble, comme la Grande Dépression et la Grande Récession ont pu nous le montrer en 1929 et 2008. Il est clair que la présence de bulles au sein de l'économie peut affecter l'allocation réelle dans l'économie. Son éclatement peut détériorer les bilans des institutions financières, des entreprises et des ménages, mettant ainsi un frein à l'activité réelle. En raison de ces répercussions sur l'économie réelle, la mise en place de politiques économiques s'avère nécessaire. Événements exceptionnels aux conséquences notables, les crises financières nécessitent des réponses immédiates et perspicaces des politiques économiques. Elles peuvent même exiger la conception de nouvelles politiques monétaires et fiscales.

L'étude complète des crises survenues au cours de l'histoire économique n'est pas l'objet de cette thèse. Toutefois, elles soulèvent plusieurs questions quant aux interactions entre les sphères financière et réelle, par rapport aux faits exposés ci-dessus.

⁴Kindleberger et Aliber (2005) est la cinquième édition de *Manias, Panics, and Crashes: A History of Financial Crises* écrit par Kindleberger en 1978. Cette nouvelle édition intègre les crises survenues au cours des années 1980 et 1990.

Au regard de la Grande Récession de 2008 et des critiques qui ont suivi, il paraît crucial d'analyser plus en profondeur les interactions entre l'activité macroéconomique réelle et les marchés financiers, afin d'apporter les meilleures réponses possibles aux problèmes liés aux crises financières. Ainsi, les modèles macroéconomiques doivent accorder un rôle plus significatif à la finance. Les modèles d'équilibre général dynamique, reposant sur des fondements microéconomiques explicites, offrent un cadre approprié pour mener une telle analyse. Ils permettent de déterminer le prix des actifs et les conditions d'existence des bulles, mais également de considérer l'influence macroéconomique de ces actifs financiers et de ces bulles. C'est pourquoi, nous retiendrons ce cadre théorique dans cette thèse. Bien que des progrès aient déjà été réalisés ces dernières années, nous devons poursuivre nos efforts dans la compréhension des fluctuations des prix des actifs. Tel est l'objet de cette thèse : approfondir les connaissances théoriques sur le comportement des prix des actifs et leurs interactions avec l'activité économique réelle avec ou sans imperfections financières. Par ailleurs, nous voulons également apporter des recommandations en matière de politiques économiques à mener face aux crises financières.

Dans la première partie de l'introduction, nous reviendrons sur la volatilité des prix d'actifs et l'existence de bulles spéculatives notamment au sein des modèles d'équilibre général dynamique. Nous mettrons, par ailleurs, en exergue le rôle que peuvent jouer les anticipations dans ces derniers. La deuxième partie sera dédiée à la compréhension des interactions entre les sphères financière et réelle. Dans la troisième partie, nous aborderons le rôle des politiques économiques face aux crises. Enfin, nous terminerons cette introduction par la présentation de la thèse, ses enjeux et les différents chapitres.

0.2 Prix d'actifs, volatilité excessive et bulles spéculatives : le rôle des anticipations rationnelles

Comme indiqué au début de cette introduction, les marchés financiers ont connu des épisodes de fortes volatilités des cours boursiers lors de ces dernières décennies. Un exemple de ces épisodes, survenu sur le marché boursier américain, est donné à travers le graphique ci-dessous retraçant l'évolution de l'indice réel Standard & Poor's 500 (S & P 500) de 1980 à 2013 ⁵:

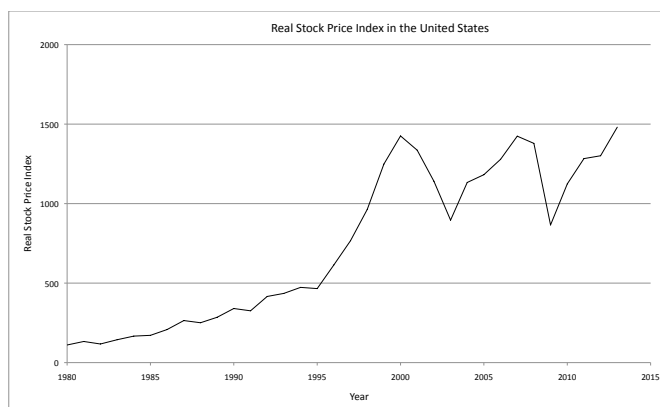


Figure 1: Evolution de l'indice réel S & P 500 de 1980 à 2013. *Source :* <http://www.econ.yale.edu/shiller/data.htm>.

Ces importants mouvements affectant les cours boursiers sont souvent associés à l'existence puis à l'éclatement d'une bulle spéculative. Celle-ci se définit comme la différence entre le prix de marché de l'actif et sa valeur fondamentale, c'est-à-dire la somme escomptée des dividendes futurs. Dans la conscience collective, les bulles sont généralement interprétées comme des phénomènes irrationnels. Cette vision est également partagée par des économistes. Le Président de la Réserve Fédérale des Etats-Unis

⁵S&P 500 est indice boursier basé sur 500 grandes sociétés cotées à la bourse américaine.

en 1996, Alan Greenspan, ira jusqu'à parler d'exubérance irrationnelle pour qualifier ces épisodes :

“[...] Clearly, sustained low inflation implies less uncertainty about the future, and lower risk premiums imply higher prices of stocks and other earning assets. We can see that in the inverse relationship exhibited by price/earnings ratios and the rate of inflation in the past. But how do we know when *irrational exuberance* has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade? [...]” Greenspan (1996).

Toutefois, l'existence et la nature des bulles spéculatives ne font pas consensus parmi les économistes.

0.2.1 La théorie des marchés efficients contre l'énigme de la volatilité excessive

La théorie des marchés efficients

A travers ces deux articles pionniers intitulés “Random Walks in Stock Market Prices” (1965) et “Efficient Capital Markets: A Review of Theoretical and Empirical Works” (1970), Eugene F. Fama montre que le prix d'un actif suit une marche aléatoire. L'évolution de ce dernier serait donc imprévisible. De ces travaux et ceux réalisés indépendamment par Paul Samuelson (1965) est née la théorie des “marchés efficients” (*Efficient Market Hypothesis*). Cette théorie postule que si les agents sur le marché des actifs sont rationnels et l'information pleinement disponible, le prix des actifs s'ajustera de manière quasi instantanée et correctement à l'arrivée de nouvelles informations sur les dividendes ou les taux d'intérêt futurs. Les marchés financiers sont donc efficients et le prix des actifs reflète toutes les informations privées et publiques connues. D'après

cette théorie, le prix d'un actif ne peut donc être qu'égal à sa valeur fondamentale. Une bulle ne peut exister sous l'hypothèse des marchés efficients.

Les travaux de Fama (1965, 1970) et Samuelson (1965) ont généré de nombreux modèles théoriques sur la valorisation des actifs financiers, fondateurs de la finance moderne. Grâce à une abondance de données sur les prix d'actifs, une pléthore d'études empiriques a vu le jour dans le but de tester cette théorie.⁶ L'approche en équilibre général de la valorisation des actifs basée sur la consommation développée notamment par Lucas (1978) reste, encore de nos jours, le cadre de référence en macroéconomie pour étudier le comportement des prix des actifs financiers.⁷ Le modèle de Lucas (1978) est intéressant dans la mesure où il s'agit d'un modèle avec agent représentatif à durée de vie infinie. Il permet ainsi de modéliser les prix d'actifs comme des prix d'équilibre, et de les relier ainsi à leurs déterminants fondamentaux, à la fois microéconomiques et macroéconomiques (risque individuel, taux de préférence pour le présent, dividendes, taxes ou politique monétaire). Dans ce modèle, il n'existe aucune imperfection et l'agent économique est rationnel (anticipations et comportement). Dans un tel contexte, l'hypothèse des marchés efficients s'applique. Le prix d'équilibre de chaque titre sera donc égal à sa valeur fondamentale. Le résultat de Lucas (1978) confirme la non-existence des bulles spéculatives soutenue par Fama (1965). Son résultat souligne également que l'équilibre est unique et Pareto-optimal. Par conséquent, les fluctuations des cours boursiers citées en début de cette section ne peuvent provenir que de chocs exogènes, dont l'origine peut être soit monétaire (Lucas, 1972), soit réelle (Kydland, Prescott, 1982). Dans la littérature macroéconomique, ces fluctuations sont qualifiées d'exogènes.

⁶Voir Campbell (1999), Cochrane (2005) et Mehra (2012) pour une revue de littérature détaillée sur le modèle d'évaluation d'actifs et leur pertinence empirique.

⁷Stiglitz (1970) et Breeden (1979) ont également contribué au développement de cette approche en équilibre général.

Toutefois, ce modèle échoue à expliquer certains faits stylisés observés dans les données du monde réel. Un premier fait concerne la prime de risque. A la suite d'une étude empirique des rendements sur un siècle, Mehra et Prescott (1985) ont tout d'abord montré que les actions avaient des rendements réels moyens d'environ 7% par an, contre 1% pour les bons du Trésor, donnant lieu à une prime de 6%. En utilisant des versions estimées et calibrées des modèles d'évaluation des actifs existants, Mehra et Prescott (1985) ont également mis en évidence que cette prime de risque était sous-estimée par la théorie standard. Dans la littérature, cette anomalie est nommée "l'énigme de la prime de risque" (*equity premium puzzle*).⁸ Le second fait stylisé mis en évidence dans la littérature est la volatilité excessive des cours boursiers, ou plus communément appelé "l'énigme de la volatilité excessive" (*excessive volatility puzzle*). Cette énigme s'avère être à l'origine de cette thèse. C'est pourquoi, nous nous concentrerons uniquement sur celle-ci et ses conséquences théoriques au cours de cette introduction.⁹

L'énigme de la volatilité excessive

L'énigme de la volatilité excessive trouve son origine dans les travaux empiriques de Shiller (1981, 1989), Grossman et Shiller (1981), LeRoy et Porter (1981) et Poterba and Summers (1988). Ceux-ci ont mis en lumière une volatilité particulièrement élevée des cours boursiers américains par rapport à leurs fondamentaux sous-jacents, tels que les dividendes ou les bénéfices d'entreprise. A la suite d'une étude empirique comparant l'historique du cours de l'indice S&P 500 et la valeur présente des dividendes réels futurs au taux d'intérêt constant (sur la période 1871-1979), Shiller (1981) révèle une volatilité des cours cinq fois supérieure à celle de la valeur présente. Ce résultat sur la volatilité implique qu'elle ne peut être attribuée uniquement à une quelconque informa-

⁸Pour une étude plus approfondie sur l'énigme de la prime de risque, le lecteur peut se rapporter aux revues littéraires écrites par Kocherlakota (1996), Campbell (2001), ou plus récemment par Mehra (2006).

⁹ Voir Shiller (1989) pour une revue détaillée théorique et empirique sur la volatilité excessive.

tion nouvelle sur les dividendes anticipés comme le soutient la théorie de l'efficience élaborée par E. Fama. Grossman et Shiller (1981) ont testé cette conjecture au modèle de Lucas (1978) en supposant une aversion relative au risque constante. Les résultats de Grossman et Shiller (1981) valide les conclusions de Shiller (1981), à savoir que les fondamentaux, à eux-seuls, ne peuvent expliquer la volatilité observée des prix d'actifs. Le graphique ci-dessous, tiré de Shiller (2014), illustre les différents résultats de Shiller (1981) et Grossman et Shiller (1981) ¹⁰:

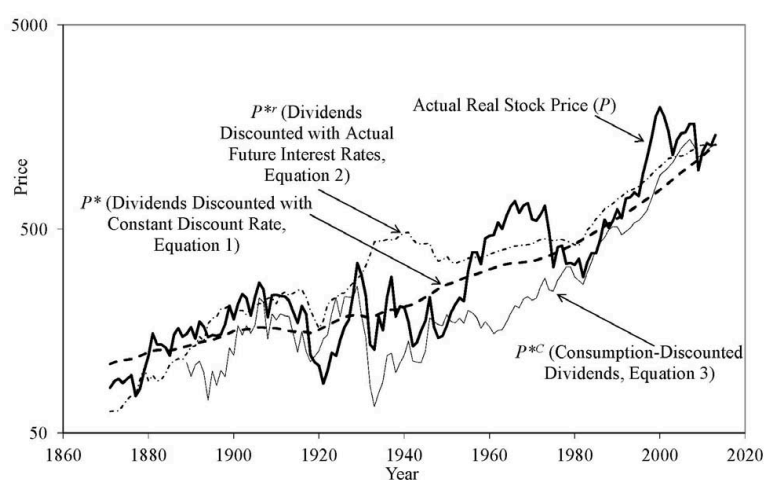


Figure 2: Evolution de l'indice réel composite Standard & Poors (P) et des trois valeurs présentes des dividendes réels futurs associées à cette indice (P^{*C} , P^* et P^{*r}). *Source: Shiller (2014).*

Diverses interprétations émergent de ces travaux. Par exemple, pour les partisans de la théorie des marchés efficients et ceux de l'approche des cycles exogènes, ces résultats montrent que le modèle de Lucas (1978) doit être amélioré en relâchant certaines hypothèses hormis la rationalité (voir Cochrane, 2005). Pour R. Shiller et la "finance comportementale", ces résultats soulignent l'existence de bulles irrationnelles abandonnant ainsi l'hypothèse d'anticipations rationnelles et/ou du comportement rationnel des agents. Dans le désordre, nous pouvons citer Camerer (1989), Shiller (1984) et Orléans

¹⁰Pour la construction du graphique, nous renvoyons le lecteur vers l'article de Shiller (2014).

(1989) qui introduisent des comportements de mimétisme pour expliquer la volatilité des bulles, Barsky et DeLong (1993) qui introduisent des agents bruyants (*noise traders*), DeLong *et al.* (1990) pour qui les agents présentent des biais optimistes et Bullard, Evans et Honkapohja (2010) qui considèrent des anticipations adaptatives.¹¹ Toutefois, Kamihigashi (2008) observe que : “Although such settings offer attractive explanations for bubbles, they are rather difficult to formulate in an aggregative framework in which the effects of stock market bubbles on output fluctuations can be analyzed relatively easily.”

Deux autres interprétations retiennent notre attention : d’une part, celle se basant sur l’existence d’équilibres multiples et à tache solaire, et d’autre part, celle se basant sur l’existence de bulles rationnelles. Le point fort de ces deux approches repose sur le fait qu’elles ne rejettent pas l’hypothèse de rationalité. C’est dans ce champ de recherche que s’inscrit cette thèse. Nous consacrons la section suivante à ces deux approches.

0.2.2 Équilibres multiples et bulles rationnelles : une réponse possible à la volatilité excessive

Équilibres multiples et à tache solaire

Des auteurs ont attribué la volatilité excessive à l’existence d’équilibres multiples et à tache solaire au sein de modèles d’équilibre général néoclassiques (Shell, 1977; Azariadis, 1981; Cass et Shell, 1983; Azariadis et Guesnerie, 1986). Cette multiplicité d’équilibres est souvent interprétée comme étant une manifestation de taches solaires, de prophéties auto-réalisatrices ou d’esprits animaux. Shell (1977) a mis en évidence que des phénomènes aléatoires déconnectés de l’économie, comme les taches solaires,

¹¹Pour une analyse plus détaillée des modèles avec bulles irrationnelles, voir Brunnermeier (2007) et Brunnermeier and Oehmke (2013).

peuvent avoir une influence sur des variables économiques telles que le prix d'une action. Par exemple, l'étude empirique de Saunders (1993) atteste que la météo à New York, phénomène déconnecté de tous fondamentaux, a un impact sur les cours boursiers.

Dans un tel contexte, les fluctuations des prix d'actifs sont le résultat des sauts consécutifs d'un équilibre à un autre, au gré de la volatilité des anticipations des agents. Ces fluctuations sont qualifiées d'endogènes car elles émergent dans un système économique soumis à aucun choc sur les fondamentaux (technologie, dotations, préférences).¹² De ce fait, l'exubérance irrationnelle énoncée par Greenspan (1996) serait compatible avec la rationalité des acteurs économiques.

La théorie des bulles rationnelles

L'approche des bulles rationnelles est une branche de la littérature sur les équilibres multiples, s'intéressant à la coexistence de deux types d'équilibres au sein des modèles d'équilibre général : l'un dans lequel le prix de l'actif est strictement égal à sa valeur fondamentale; l'autre dans lequel le prix contient un terme supplémentaire appelé bulle. Deux modèles néoclassiques s'offrent à nous pour l'étude de l'existence des bulles rationnelles et leur influence macroéconomique : le modèle avec agent représentatif à durée de vie infinie ou le modèle à générations imbriquées.¹³ Les premiers travaux à avoir déterminé les conditions d'existence de ces bulles dans ces modèles sont : Wil-

¹²Il existe plus précisément deux classes de fluctuations endogènes : d'une part, celle dont nous venons de citer et d'autre part, celle dont l'existence repose sur la non-linéarité des modèles. Cette dernière classe correspond à des trajectoires cycliques déterministes voire chaotiques. L'existence de ces cycles, qualifiés de déterministes, au sein des modèles en équilibre général ont été mis en évidence par Grandmont (1985). Pour une analyse historique des fluctuations endogènes, le lecteur pourra se rapporter à l'article de Farmer (2014).

¹³Le modèle à générations imbriquées a été développé indépendamment par Allais (1947) et Samuelson (1958) dans une version en économie d'échange, puis fut étendu à une économie de production par Diamond (1965).

son (1981), Tirole (1982, 1985), Weil (1987), Kocherlakota (1992) ainsi que Santos et Woodford (1997).¹⁴

Les anticipations sont l'élément-clé de cette approche. Une bulle rationnelle n'existe que si les agents anticipent qu'elle va perdurer, avec une probabilité non nulle, à la période suivante. Par ailleurs, Tirole (1982, 1985) montre également que les bulles peuvent exister uniquement si les agents anticipent qu'ils vont pouvoir revendre l'actif avec bulle à un prix supérieur à un autre agent. Les bulles ne peuvent donc apparaître dans le modèle avec un agent représentatif à durée de vie infinie. Le critère de Pareto-optimalité est également un argument d'élimination de bulles (Tirole, 1982, 1985). D'ailleurs, nous avons déjà pu prendre conscience de ce résultat, lors de la présentation du modèle de Lucas (1978) dans la section 0.2.1. Partant du constat qu'il est optimal pour un individu d'épuiser ses richesses à des fins de consommation, et donc qu'aucun agent ne voudra détenir un actif avec bulle, Kocherlakota (2008) fait la remarque suivante :

“However, despite the widespread belief in the existence of bubbles in the real world, it is difficult to construct model economies in which bubbles exist in equilibrium.”

La remarque de Kocherlakota (2008) est d'autant plus pertinente dans le cas des modèles à agent représentatif à horizon infini. Par exemple, dans un article récent, Kamihigashi (2015) définit un théorème de non-existence de bulles s'appliquant à une large catégorie de ces modèles. La cause principale de ce résultat est la présence de la condition de transversalité. Celle-ci, imposant que le prix de l'actif actualisé soit zero lorsque le temps tend vers l'infini, s'avère être satisfaite à l'équilibre dans ces modèles. Ainsi, des conditions supplémentaires affectant le comportement des agents sont nécessaires pour permettre aux bulles rationnelles d'exister dans les modèles à horizon infini.

¹⁴Pour une revue littéraire exhaustive sur les bulles rationnelles, voir Leroy (2004), Brunnermeier (2007), et plus récemment Miao (2014).

De par sa structure, le modèle à générations imbriquées offre un cadre d'équilibre général privilégié pour étudier l'existence de bulles rationnelles. D'une part, la condition de transversalité n'est pas nécessaire. D'autre part, à chaque période une nouvelle génération d'agents à durée de vie finie entre sur le marché. Dans un tel contexte, la bulle peut exister, puisque les agents peuvent se la transmettre de génération en génération. Dans son article fondateur, Tirole (1985) montre que les bulles peuvent exister et persister au sein d'une économie à condition que l'économie sans bulle est dynamiquement inefficace. Il prouve également que l'équilibre avec bulle est unique, et rétablit l'efficacité dynamique. En outre, Tirole (1985) considère des bulles ne pouvant éclater. Le problème d'éclatement sera résolu par Weil (1987) avec l'introduction de bulles stochastiques, c'est-à-dire des bulles pouvant éclater de manière aléatoire avec une probabilité non-nulle à chaque période.

Le modèle de Tirole (1985) présente deux faiblesses au regard de cette thèse : l'efficacité des bulles rationnelles et l'unicité de l'équilibre. L'efficacité de la bulle implique que la bulle évince une partie de l'investissement dans le capital physique (*crowd-out effect*), et ce faisant affaiblit la production. Or, ce résultat paraît inconciliable avec l'idée que nous pouvons avoir des crises financières, à savoir que la bulle promeut l'accumulation du capital (*crowd-in effect*), et *in fine* la croissance économique. Des études empiriques montrent, d'ailleurs, que l'investissement privé fixe non-résidentiel aux États-Unis a augmenté de manière significative au cours de la formation de la bulle internet durant les années 1990, puis a chuté lorsque celle-ci a éclaté, de même lors de la bulle immobilière américaine (Chirinko et Schaller, 2011; et IMF, 2015). L'unicité de l'équilibre implique qu'il ne peut exister de fluctuations endogènes persistantes dues à la volatilité des anticipations au sein de l'économie, ce qui *a priori* ne permet pas de retranscrire la volatilité des cours boursiers observée sur les marchés. À notre connaissance, très peu de travaux théoriques s'intéressent à l'existence de fluctuations en-

dogènes persistantes d'une bulle au sein des modèles à générations imbriquées (Grandmont, 1985; Weil, 1987; Michel et Wigniolle, 2003, 2005; Bosi et Seegmuller, 2010, 2013; Wigniolle, 2012). Les chapitres 1 et 2 chercheront à combler ce manque en apportant des explications supplémentaires à l'occurrence de ces fluctuations au sein de ces modèles.

Un temps abandonnée en faveur des bulles irrationnelles, la littérature sur les bulles rationnelles connaît un regain d'intérêt comme l'atteste la revue littéraire de Miao (2014). En offrant une approche en équilibre général, les modèles des bulles rationnelles permettent aux économistes d'analyser les effets macroéconomiques de la bulle, mais également d'étudier l'influence des politiques économiques sur cette dernière. Alors qu'il existe de nombreuses études sur les effets d'une bulle rationnelle sur l'activité économique, rares sont celles sur les effets d'une politique économique sur l'existence et/ou fluctuations d'une bulle. Par exemple, nous pouvons citer Michel et Wigniolle (2005), Bosi et Seegmuller (2010, 2013), Farhi et Tirole (2012), et Gali (2014) pour avoir étudié des politiques monétaires dans des modèles à générations imbriquées avec bulles, tandis que dans les modèles à horizon infini nous avons Kocherlakota (2009), Miao *et al.* (2014) et Hirano *et al.* (2015). Les chapitres 1 et 2 auront pour objectifs de combler ce manque, en analysant l'effet des certaines politiques monétaires et fiscales sur les fluctuations d'une bulle rationnelle.

De manière générale, cette thèse s'inscrit dans les champs de recherche dédiés à l'existence d'équilibres multiples et aux bulles rationnelles. Les chapitres 1, 2 et 3 s'appliqueront à utiliser ces deux approches pour apporter des connaissances théoriques nouvelles sur la volatilité d'une bulle spéculative et ses effets sur l'activité économique réelle, ainsi qu'étudier les implications des politiques économiques en terme de stabilisation. Le chapitre 4, quant à lui, s'intéressera à l'existence d'équilibres multiples au sein du modèle de Lucas (1978).

Dans cette première partie, nous avons rappelé que les anticipations rationnelles peuvent jouer un rôle important dans la valorisation et la volatilité des prix des actifs. Toutefois, nous avons également pu pointer que les modèles macroéconomiques en équilibre général standard, à savoir le modèle d'agents à durée de vie infinie et le modèle à générations imbriquées, échouaient à expliquer la volatilité excessive des prix des actifs et les effets sur la sphère réelle. C'est pourquoi dans la deuxième partie, nous porterons notre attention sur les différentes sources possibles à l'origine des crises financières afin d'améliorer notre compréhension de la valorisation des prix des actifs et des interactions entre les sphères financière et réelle.

0.3 Comprendre le lien entre les sphères financière et réelle

Au coeur des interactions entre le secteur financier et l'activité économique se trouvent les choix des investisseurs, à savoir les ménages et les entrepreneurs. Le comportement des prix des actifs est primordial dans la détermination de leurs décisions économiques, décisions qui en retour affecteront la valorisation des actifs. Par exemple, les prix des actifs impactent le choix entre consommer et épargner pour les ménages, ainsi que leur choix de portefeuille, c'est-à-dire le fait d'épargner sous forme de monnaie, de livrets d'épargne, d'actions, ou d'une maison, voire sous la forme d'oeuvres d'art. L'allocation des ressources résultant de ces choix impacte de manière réciproque les prix des actifs.

Or, les choix réalisés par les ménages dans le modèle de Lucas (1978) ou dans le modèle de Tirole (1985) ne permettent pas d'expliquer le comportement des prix des actifs et leur influence macroéconomique comme observée dans les données. Ainsi, les imperfections de marché et/ou les comportements des agents économiques pourraient améliorer notre compréhension de la valorisation et des fluctuations des prix des actifs, ainsi que de leur influence sur l'activité économique réelle.

0.3.1 Rôle des imperfections sur les marchés financiers

Les marchés financiers sont des lieux où se rencontrent des agents disposant d'un excédent de capitaux et ceux ayant besoin de financement. Les marchés financiers ont pour but d'allouer efficacement les capitaux des agents qui n'en font pas une utilisation productive vers ceux qui peuvent réaliser des investissements productifs ou qui leur sont nécessaires pour consommer. Les imperfections, dues à de l'asymétrie d'information ou à des limites à l'engagement, viennent entraver ce bon fonctionnement, et ce faisant, affectent les décisions économiques.

Les travaux fondateurs de Bernanke et Gertler (1989), Carlstrom et Fuerst (1997), Kiyotaki et Moore (1997), ainsi que Bernanke, Gertler et Gilchrist (1999) montrent que les imperfections financières jouent un rôle important dans la compréhension des cycles économiques.¹⁵ Depuis la crise des subprimes de 2007, il ne fait plus aucun doute que ces imperfections peuvent également jouer un rôle dans la formation de bulles sur les marchés financiers et/ou immobiliers, ainsi que dans la propagation de leur éclatement à la sphère réelle. Ceci nous amène à penser que la prise en compte de ces imperfections pour évaluer le rôle des politiques économiques semble nécessaire.

La littérature macroéconomique sur les bulles rationnelles accorde une place toute particulière aux imperfections financières. Des contributions théoriques mettent en évidence que l'introduction de frictions financières par le biais de contraintes d'emprunt, dépendant ou non du collatéral (la valeur des garanties financières dont disposent les agents), peut générer des bulles dans les modèles à horizon infini (Kocherlakota, 1992, 2008; Santos et Woodford, 1997; Hellwig et Lorenzoni, 2009; Miao et Wang, 2011; Le Van, Pham, et Vailakis, 2015). La présence de telles contraintes limite les possibil-

¹⁵Voir Brunnermeier *et al.* (2012) pour une revue détaillée sur les effets macroéconomiques des imperfections financières.

ités d'arbitrage des agents de sorte que les bulles ne peuvent pas être éliminées par la condition de transversalité.

Concernant les modèles à générations imbriquées, les frictions financières ne sont pas nécessaires pour l'existence de bulles dans ces modèles. Néanmoins, en intégrant des contraintes de crédit dans le modèle de Tirole (1985), Caballero et Krishnamurthy (2006), Farhi et Tirole (2012), ainsi que Martin et Ventura (2012) montrent que la bulle peut avoir un *crowd-in effect*, contrairement à Tirole (1985). Ce résultat est ainsi cohérent avec les évidences empiriques vues à la section 0.2.2. En présence d'imperfections financières, les marchés financiers ne permettent pas de transférer suffisamment de ressources vers les investissements productifs. La bulle aide à résoudre ce problème.

Une grande majorité des modèles à générations imbriquées, étudiant les effets des imperfections financières sur la bulle et son influence macroéconomique, considèrent uniquement des économies où ce sont les entrepreneurs qui sont soumis à des contraintes financières. Toutefois, de nombreuses études empiriques mettent en évidence que les ménages sont contraints par le crédit et que leur comportement est impacté par la valeur de leur collatéral (voir Campbell et Mankiw, 1989; Jappelli, 1990; Iacoviello, 2004; Crook et Hochguertel, 2005; Lustig and Van Nieuwerburgh, 2010). Au regard de la crise des subprimes et de ces études, il paraît crucial d'analyser les effets de ces contraintes affectant les ménages sur les bulles rationnelles, ainsi que sur les fluctuations de l'activité économique. Nous pouvons retrouver cette idée dans Bosi et Seegmuller (2010). Ces auteurs introduisent des imperfections sur le marché du crédit par le biais d'une contrainte de liquidité. L'hypothèse principale de cet article repose sur l'idée selon laquelle plus la valeur du collatéral dont dispose le ménage augmente, plus il pourra consommer à crédit, toutes choses égales par ailleurs. Cette hypothèse introduit des distorsions au sein de l'économie, et rend le choix de portefeuille des ménages

dépendant de leurs anticipations de telle sorte que des fluctuations peuvent apparaître dans une économie avec bulles à cause de la volatilité des anticipations, contrairement à Tirole (1985). Dans les chapitres 1 et 2, nous utiliserons la structure proposée par Bosi et Seegmuller (2010) pour compléter la compréhension du rôle des imperfections financières pesant sur les ménages dans l'existence et les fluctuations d'une bulle spéculative, et pour évaluer le rôle des politiques monétaire et fiscale dans un tel contexte.

0.3.2 Aspects psychologiques et/ou sociaux dans les décisions économiques

Les modèles de Lucas (1978) ou de Tirole (1985) supposent certaines propriétés sur les investisseurs qui animent leur décisions économiques. Comme nous l'avons indiqué dans la section 0.2, ces deux modèles échouent à reproduire le comportement des prix des actifs. Partant de ce constat, des auteurs se sont éloignés de la représentation standard des préférences dans ces modèles afin d'apporter une explication à la volatilité excessive des prix des actifs.

Par exemple, Wigniolle (2014) considère une fonction d'utilité espérée dépendant du rang dans un modèle à générations imbriquées avec bulles rationnelles. Ce type de fonction permet de rendre compte de certaines attitudes psychologiques chez les investisseurs comme le pessimisme et l'optimisme. Dans un tel contexte, Wigniolle (2014) montre que le pessimisme des agents favorise l'existence de bulles déterministes ainsi que l'existence d'équilibres multiples, source de fluctuations endogènes.

Pour comprendre la dynamique des prix des actifs, Shiller (1984) propose de prendre en compte également les aspects sociaux des décisions économiques d'un investisseur : "It is thus plausible that investors' behavior (and hence prices of speculative assets) would be influenced by social movements. Attitudes or fashions seem to fluctuate in many other popular topics of conversation, such as food, clothing, health, or politics.

These fluctuations in attitude often occur widely in the population and often appear without any apparent logical reason. It is plausible that attitudes or fashions regarding investments would also change spontaneously or in arbitrary social reaction to some widely noted events.” Shiller (1984).

Certains aspects sociaux, soulignés par Shiller (1984), peuvent être modélisés sous l’hypothèse de comportement rationnel comme les externalités de consommation. Par exemple, Gali (1994) suggère d’intégrer des externalités du type *keeping up with Joneses* dans le modèle Lucas (1978), tandis que Campbell et Cochrane (1999) introduisent des habitudes externes de consommation. Dans ces deux articles, les individus retirent de la satisfaction non seulement du niveau de leur propre consommation mais également lorsqu’ils la comparent à celle de ceux qui les entourent. Néanmoins, ces modèles ne parviennent pas à expliquer la volatilité excessive.

Des travaux empiriques mettent en lumière que les périodes caractérisées par une forte croissance des prix des actifs ont tendance à coïncider avec celles où les revenus sont fortement concentrés sur les ménages à hauts revenus, (voir Piketty et Saez, 2006 ; et Piketty, 2013). Ces études suggèrent qu’une augmentation de la part du revenu allant aux 1% des ménages les plus aisés pourrait expliquer la hausse des prix de certains actifs comme l’immobilier. Une des raisons soulevées est le fait que les ménages à haut revenus épargnent plus. Sur ce sujet, des études mettent en exergue le rôle des préférences pour la richesse dans le comportement d’épargne des ménages les plus aisés (voir Carroll, 2000; Dynan, Skinner, and Zeldes, 1996). Ces études révèlent que le comportement d’épargne de ces ménages ne peut pas être expliqué par des modèles dans lesquels le seul but de l’accumulation de richesses est de financer la consommation future. Ce résultat met en lumière l’hypothèse d’“Esprit de Capitalisme” développée par Max Weber en 1905 dans son ouvrage *L’éthique protestante et l’esprit du capitalisme*, selon laquelle le désir intrinsèque d’accumuler de la richesse est le fondement de toute

société capitaliste. En leur apportant du pouvoir et un statut social, la richesse influe directement sur le bien-être des individus. Des études théoriques soulignent aussi que de telles préférences peuvent s'avérer utiles pour comprendre le comportement des prix d'actifs et les fluctuations macroéconomiques au sein d'une économie (voir Bakshi et Chen, 1996; Boileau et Braeu, 2007 et Karnizova, 2010). Kamihigashi (2008), Airaudo (2012) ainsi que Zhou (2015) mettent l'accent sur le rôle des préférences pour la richesse dans l'existence de bulles au sein du modèle de Lucas (1978).

Ces différentes études laissent supposer que des facteurs psychologiques ou sociologiques peuvent apporter des éléments de réponse quant aux interactions entre les sphères financière et réelle. Dans le chapitre 4, nos agents auront des préférences pour la richesse et seront hétérogènes du point de vue de leurs préférences, de leur revenu et de leur richesse initiale. Dans un tel contexte, nous étudierons l'effet de l'hétérogénéité sur la volatilité des prix des actifs.

0.3.3 Rôle de l'intégration financière

Un autre élément à prendre en considération pour appréhender la dynamique de l'économie financière et son interaction avec l'économie réelle est l'intégration financière internationale. L'intégration financière est une situation dans laquelle les marchés financiers de pays voisins, régionaux et/ou mondiaux sont étroitement liés. Cette situation a été rendue possible notamment par la suppression de barrières comme le contrôle des changes permettant l'interconnexion des bourses de valeurs et les émissions sur les principales places financières mondiales. Du fait de cette intégration financière mondiale, les marchés financiers nationaux sont devenus extrêmement interdépendants. Par exemple, Tsutsui et Hirayama (2010) montrent, à travers une étude empirique, que la bourse de Tokyo réagit dans les six minutes à la bourse de New-York, alors que celle de New-York réagit dans les quatorze minutes.

Cette intégration financière peut s'avérer bénéfique pour les investisseurs dans la mesure où elle leur permet de diversifier le risque. Mais, elle peut aussi leur être néfaste en facilitant la transmission des crises entre les pays. Dans la littérature sur les crises, cette transmission est appelée contagion. Comme en épidémiologie, une contagion financière peut rendre un pays sain, malade. Cela nous pousse à penser que la prise en compte de cette dimension internationale est nécessaire pour comprendre les interactions entre les économies financière et réelle. Auparavant, il nous semble essentiel d'étudier l'existence de bulles spéculatives en économie ouverte, puisque les crises sont souvent étroitement liées à l'éclatement de celles-ci.

La littérature sur les bulles rationnelles a, pour l'heure, peu traité leur existence en économie ouverte et la transmission internationale de leur éclatement. Tandon et Wang (2003) ont étudié la substitution de monnaie dans une petite économie ouverte à générations imbriquées en analysant la dynamique d'une bulle stochastique, mais leur analyse a été restreinte à la dynamique déterministe de la bulle avant son éclatement. Ventura (2012) a examiné l'existence de bulles et leurs effets dans une économie ouverte à générations imbriquées, composée de plusieurs pays différents par rapport à leurs niveaux de productivité. Il montre que les bulles peuvent apparaître dans les pays où la productivité est faible et qu'elles ont un effet positif sur l'investissement à l'échelle mondiale. En outre, il met en évidence que l'éclatement d'une bulle dans un pays aura un effet néfaste sur la bulle d'un autre pays. Toutefois les marchés financiers sont complètement segmentés. Le chapitre 3 de cette thèse s'attachera à proposer un cadre d'analyse qui prend en compte l'intégration internationale des marchés financiers dans l'optique d'analyser la transmission internationale d'une crise financière à la suite de l'éclatement d'une bulle spéculative.

Dans la deuxième partie de l'introduction, nous avons évoqué les différentes sources possibles à l'origine des crises financières, à savoir des imperfections financières, des

préférences non-standard et l'intégration financière. Comme pour les chapitres 1 et 2, nous proposerons un cadre d'analyse avec imperfections financières pour évaluer le rôle des politiques économiques au sein d'une économie connaissant une bulle spéculative, la troisième partie de cette introduction est dédiée aux rôle des politiques économiques face aux crises financières et économiques.

0.4 Rôle stabilisateur des politiques monétaire et fiscale face aux crises

Face à l'ampleur de la crise de 2007-2008, il paraît légitime de s'interroger sur la manière dont les politiques monétaire et fiscale doivent être conduites pour minimiser les effets de ces crises, mais également les prévenir.

Au cours de cette introduction, il a été montré que les crises financières et économiques pouvaient être le résultat de fluctuations persistantes d'une bulle rationnelle et de l'activité économique, dues à la volatilité des anticipations des agents. C'est pourquoi, il nous semble important d'étudier les vertus stabilisatrices des politiques monétaire et fiscale. Nous définissons la stabilisation comme la possibilité de réduire voire d'éliminer la probabilité d'occurrence de ces fluctuations. Stabiliser l'économie a souvent été perçu comme inutile, le coût en bien-être des fluctuations économiques étant extrêmement faible, comme l'atteste R. Lucas (1987). Cela est vrai dans un monde où les marchés sont parfaits, et où il n'existe pas de distorsions. Or, nous avons vu dans la section 0.3.1 que les imperfections financières au sein d'une économie pouvaient être à l'origine de ces fluctuations et générer une allocation inefficace des ressources. Il existe donc une marge de manœuvre pour la conduite des politiques de stabilisation.

0.4.1 Quel rôle pour la politique monétaire?

La crise financière de 2007-2008 a fait resurgir d'anciennes questions quant au rôle de la politique monétaire dans la stabilité des marchés financiers : Quelle politique monétaire la Banque Centrale doit-elle mener dans la prévention d'un gonflement d'une bulle spéculative? Une Banque Centrale doit-elle tenir compte des prix des actifs dans sa conduite de la politique monétaire sous un régime de ciblage d'inflation flexible? Cette deuxième question fait écho à celles soulevées par Bernanke et Gertler en 1999 puis en 2001 à la suite des différentes crises financières puis économiques survenues dans les années 1990 : "Should Central Banks respond to movements in asset prices?"

Bernanke et Gertler (1999) s'intéressent à la manière dont la politique monétaire doit réagir envers l'inflation anticipée et une variation des prix des actifs afin d'assurer la stabilité financière et macroéconomique. Ils utilisent un modèle néo-keynésien avec des frictions nominales et financières dans lequel des bulles apparaissent suite à des chocs exogènes sur les prix d'actifs. Ces auteurs montrent que pour atteindre ces objectifs, les Banques Centrales doivent choisir une règle de politique monétaire agressive, c'est-à-dire réagissant fortement envers l'inflation, avec pour unique objectif la stabilité des prix des biens et des services. Malgré ce résultat, aucun consensus n'a émergé, à ce jour, quant au rôle de la politique monétaire face aux prix des actifs, et plus particulièrement face aux bulles. Par exemple, nous pouvons citer Carlstrom et Fuerst (2007) pour une réponse négative à la question de Bernanke et Gertler (1999, 2001) ou Cecchetti *et al.* (2000) et Nutahara (2010) pour une réponse positive.

Deux stratégies s'affrontent face au gonflement d'une bulle spéculative : d'une part, la stratégie "lean against the wind" en guise de prévention de la formation des bulles selon laquelle une politique monétaire devrait progressivement devenir restrictive, c'est-à-dire en augmentant les taux d'intérêt, pour contrer une bulle excessive sur les prix des actifs; d'autre part, la stratégie "clean" en guise de correction des effets de l'éclatement

des bulles : la Banque Centrale attend que la bulle explose et adopte alors une politique monétaire très expansionniste pour limiter les effets négatifs de l'éclatement de la bulle sur les sphères financière et réelle.¹⁶

Du fait de ce manque de consensus, l'ancien Président de la Réserve Fédérale Américaine Bernanke (2011) encourage plus de recherches sur ce sujet:

“In my view, the issue is not whether central bankers should ignore possible financial imbalances - they should not - but, rather, what “the right tool for the job” is to respond to such imbalances.” Bernanke (2011).

Toutefois, ces questions ont été peu traitées dans des modèles dans lesquels une bulle peut se former sans le recours à des chocs exogènes. A notre connaissance, le seul article retenant cette démarche est Gali (2014). Cet auteur s'intéresse à la question de savoir si les Banques Centrales doivent mener une politique de “lean against the wind”, et de ce fait, étudie la manière dont la politique monétaire doit répondre à une bulle sur le prix des actifs. Pour cela, il utilise un modèle à générations imbriquées avec rigidités nominales, dans lequel une bulle rationnelle peut apparaître. Ses résultats remettent en question la politique de “lean against the wind”, car celle-ci aurait tendance à accentuer la volatilité. Néanmoins, comme indiqué dans la conclusion de Gali (2014), ce modèle ne présente pas d'imperfections financières pouvant être pertinentes lors de la conception d'une politique monétaire dans une économie avec bulle.

Etant donné que des fluctuations endogènes de bulles spéculatives et de l'activité économique peuvent apparaître dans des modèles à générations imbriquées à cause d'imperfections financières, il nous paraît intéressant d'utiliser la structure de ces modèles pour étudier les vertus stabilisatrices d'une politique monétaire répondant aux prix des actifs.

¹⁶Voir Jones (2015) pour une revue détaillée sur les origines et les conséquences de ces deux stratégies.

0.4.2 Quel rôle pour la politique fiscale?

En réaction à la crise financière de 2007-2008, la Réserve Fédérale des Etats-Unis, la Banque Centrale Européenne, ainsi que la Banque d'Angleterre ont diminué leur taux d'intérêt nominal allant jusqu'à avoisiner la borne du zéro pour limiter les effets de la crise. Face à une situation de trappe à liquidité, la politique monétaire conventionnelle perd de son efficacité, laissant ainsi un rôle plus important à la politique fiscale pour stabiliser l'économie.

De ce fait, il serait intéressant de porter notre attention sur des questions relatives au rôle de la politique fiscale dans une économie qui connaît des bulles spéculatives, des crises financières et des fluctuations macroéconomiques. En outre, si ces fluctuations sont dues à l'existence d'imperfections financières comme dans les modèles à générations imbriquées (cf. section 0.3.1), la politique fiscale par le biais de la taxation pourrait jouer un rôle important dans l'allocation des ressources et réduire les inefficacités provenant de ces imperfections. Force est de constater que dans la littérature sur les bulles rationnelles, il existe très peu de contributions théoriques traitant des effets de la mise en place d'une taxation sur l'existence et les effets d'une bulle rationnelle. Nous pouvons citer Miao *et al.* (2014). Ces auteurs s'intéressent à l'existence d'une bulle immobilière dans un modèle à horizon infini avec production et contraintes de crédit pesant sur les entrepreneurs. Dans un tel contexte, ils montrent qu'une bulle peut exister, et que des taxations, telles qu'une taxation de propriété ou de transaction, suffisent à éliminer cette bulle. En revanche, de nombreuses contributions appartenant à la littérature sur les fluctuations endogènes se sont penchées sur les vertus stabilisatrices de différentes politiques de taxation.¹⁷ Nous voudrions étendre l'analyse de la politique

¹⁷Par exemple Guo et Lansing (1998), Lloyd-Braga *et al.* (2008), ainsi que Bosi et Seegmuller (2010b).

fiscale sur les fluctuations macroéconomiques aux modèles avec bulles rationnelles afin de pouvoir comparer avec les vertus stabilisatrices d'une politique monétaire.

0.5 Présentation de la thèse

Cette thèse a pour thème central l'étude des interactions entre les sphères financière et réelle de l'économie. Au sein de cette problématique générale, nous cherchons à répondre à des questions bien précises concernant des problèmes encore non résolus dans la littérature macroéconomique comme nous avons pu le voir lors de l'introduction. La première concerne la volatilité des prix des actifs financiers et l'existence de bulles spéculatives, ainsi que leurs implications sur les fluctuations macroéconomiques. La seconde question se rapporte à la transmission de l'éclatement d'une bulle spéculative à l'international. La dernière concerne le rôle des politiques monétaires et fiscales face à l'existence et aux fluctuations d'une bulle spéculative. A travers les chapitres de cette thèse, nous nous efforcerons d'apporter des éléments de réponses à ces questions.

Pour ce faire et pour mettre en évidence les interactions entre les sphères financière et réelle, nous aurons recours aux modèles d'équilibre général dynamique. Par ailleurs, les anticipations rationnelles vont jouer un rôle clé dans cette thèse : d'une part pour l'existence des bulles et d'autre part, pour les fluctuations. En effet, nous nous focaliserons uniquement sur l'existence de fluctuations endogènes dues à la volatilité des anticipations des agents.

Cette thèse s'articule au tour de 3 axes. Dans le premier axe (chapitres 1 et 2), nous étudierons l'existence et les fluctuations de bulles spéculatives rationnelles, comme source de la volatilité des prix d'actifs. Comme nous avons pu le voir dans la Section 0.3.1 de l'introduction, il paraît essentiel de tenir compte des imperfections financières dans la modélisation des choix des agents, et notamment des ménages. Puis, dans un tel contexte, nous analyserons le rôle stabilisateur des politiques fiscales et/ou monétaires.

Au regard de l'intégration des marchés financiers, il est opportun d'étudier l'existence de bulles dans une économie ouverte, et la transmission internationale de leur éclatement. Tel sera l'objet du deuxième axe (chapitre 3). Dans le dernier axe (chapitre 4), nous étudierons le rôle de l'hétérogénéité sur la dynamique des prix d'actifs et les inégalités lorsque les agents économiques ont des préférences pour la richesse.

Cette thèse est constituée de quatre articles, dont nous exposerons un résumé dans les sections suivantes.

0.5.1 Chapitre 1– Bulles rationnelles et fluctuations macroéconomiques : le rôle (dé-)stabilisateur des politiques monétaires.

Ce premier chapitre a pour objectif d'étudier l'existence et les fluctuations de bulles rationnelles ainsi que de contribuer au débat initié par Bernanke et Gertler (1999, 2001), à savoir si les banques centrales doivent réagir aux prix des actifs.

Pour cela, nous construisons une économie à générations imbriquées dans laquelle les agents réalisent un choix de portefeuille entre la détention d'une bulle représentée par un actif sans valeur fondamentale et de la monnaie due à une contrainte de liquidité saturée. Par le biais de cette contrainte de liquidité, nous introduisons les imperfections du marché du crédit. La part de la consommation achetée à crédit est positivement corrélée à la valeur de la bulle détenue par des agents : si les agents détiennent une bulle dont la valeur croît, alors ils peuvent accroître leurs chances d'obtenir des crédits et voir leur part de consommation achetée à crédit augmenter. Le choix de portefeuille entre la monnaie et la bulle est le mécanisme clé de ce chapitre à travers lequel les fluctuations émergent. Le choix de portefeuille dépend des anticipations des agents. Ceci signifie qu'un changement dans les anticipations génère une réallocation de l'épargne entre la monnaie et la bulle. Ce phénomène peut ainsi engendrer des fluctuations endogènes.

Tout d'abord, nous montrons qu'à cause des frictions sur le marché du crédit, l'existence de la bulle ne permet pas de rétablir l'efficacité dynamique contrairement à Tirole (1985). Ensuite, nous mettons en évidence la coexistence de plusieurs équilibres stationnaires avec bulles ainsi que l'existence de fluctuations pour de faibles distortions sur le marché du crédit et sous l'hypothèse de substitutabilité brute. Ce résultat est intéressant pour cinq raisons. Premièrement, il met en lumière les interactions entre les sphères financière et réelle. Comme nous considérerons un modèle d'équilibre général dynamique, les fluctuations de l'activité économique sont liées à celles de la bulle. Deuxièmement, il met en exergue le rôle déstabilisateur de la contrainte de crédit. Troisièmement, il répond à une critique récurrente concernant les modèles à générations imbriquées monétaires, à savoir que les fluctuations apparaissent souvent sous des conditions assez restrictives non vérifiées empiriquement. Quatrièmement, la multiplicité des états stationnaires fournit une évidence théorique de la volatilité excessive des prix des actifs à cause du problème de coordination des anticipations des agents. Enfin, ce problème de coordination justifie la mise en place d'une politique monétaire pour stabiliser l'économie.

Fort de ce contexte, nous analysons le rôle stabilisateur ou non d'une politique monétaire répondant aux prix d'actifs. Une politique est stabilisatrice si elle atténue voire élimine la probabilité d'apparition de fluctuations endogènes au sein de l'économie. Pour mener cette analyse, nous élargissons la règle de ciblage d'inflation proposée par Sorger (2005) en incluant directement le prix des actifs. Lorsque la politique monétaire dépend uniquement de l'inflation anticipée, nous montrons qu'une politique suffisamment agressive tend à déstabiliser. En revanche, lorsque la politique monétaire répond également aux mouvements des prix des actifs, elle peut avoir un rôle stabilisateur sur l'activité et restaurer l'unicité de l'état stationnaire. Ce résultat mitige les conclusions émises par Bernanke et Gertler (1999, 2001).

Dans ce premier chapitre, nous considérons une économie d'échange. Afin d'améliorer la modélisation des interactions entre les sphères financière et réelle, nous retenons une économie avec accumulation du capital dans le chapitre 2. Par ailleurs, dans le chapitre 1, nous supposons que la politique monétaire agit sur le taux de croissance de la masse monétaire et par le biais de transferts forfaitaires. Afin de modéliser au mieux la politique monétaire menée par les banques centrales actuelles, nous considérons une politique monétaire s'effectuant à travers des opérations d'open-market (dorénavant, OMO) dans le chapitre 2. Ces opérations consistent pour la banque centrale à acheter ou à vendre des titres en contrepartie de la monnaie. Modéliser de telles opérations nous permet d'introduire un taux d'intérêt comme instrument de la politique monétaire, et par conséquent considérer des règles à la Taylor.

0.5.2 Chapitre 2 – Les vertus stabilisatrices de la politique fiscale contre la politique monétaire sur les fluctuations endogènes d'une bulle

Dans ce deuxième chapitre, nous nous intéressons au rôle stabilisateur de la politique monétaire et de la politique fiscale.

Contrairement au premier chapitre, les ménages réalisent un choix de portefeuille entre trois actifs avec des rendements différents, capital, monnaie et bons du trésor. La demande de monnaie est rationalisée à travers une contrainte de liquidité; les bons du trésor servent de contrepartie monétaire dans la gestion des OMO; et le capital sert à la fois comme facteur de production et comme collatéral pour les ménages. Plus la valeur du capital physique dont dispose le ménage augmente, plus il pourra consommer une partie à crédit. A cause des imperfections sur le marché du crédit, ces trois actifs sont des substituts imparfaits, pouvant expliquer ainsi les choix de portefeuille entre les différents actifs, ainsi que leur écart de rendement.

Nous montrons d'une part que l'économie avec bulle est dynamiquement inefficace à cause des imperfections financières; et d'autre part qu'une multiplicité d'états stationnaires avec bulles ainsi que des fluctuations liées à la volatilité des anticipations peuvent apparaître sous l'hypothèse de substituabilité brute et des valeurs raisonnables de substitution entre les facteurs de production. Comme dans le chapitre 1, le choix de portefeuille est le mécanisme à travers lequel les fluctuations émergent. L'existence de ces fluctuations repose sur un arbitrage entre les différents actifs.

Nous analysons, par la suite, les vertus stabilisatrices d'une politique monétaire régie par une règle de Taylor répondant à l'inflation et les vertus d'une politique fiscale à travers une taxation progressive du capital. La politique monétaire nous paraît être une politique adéquate pour stabiliser l'économie dans la mesure où elle affecte le coût d'opportunité de la détention de monnaie par le biais de son instrument, à savoir le taux d'intérêt nominal. La politique fiscale peut également avoir un rôle à jouer dans la stabilisation de l'économie en réduisant les écarts de rendement entre le capital et les autres actifs. De cette analyse, nous tirons les conclusions suivantes. Une politique monétaire, menée sous une règle de Taylor, obtient des résultats mitigés en terme de stabilisation. Néanmoins, elle peut améliorer le bien-être à l'état stationnaire en augmentant son taux d'intérêt directeur. En revanche, une taxation progressive du capital peut stabiliser l'économie, mais également restaurer l'efficacité dynamique. Dans un tel cadre, il est préférable de recourir à la politique fiscale pour stabiliser l'économie.

Dans les précédents chapitres, nous considérons des bulles dites déterministes c'est-à-dire dont la probabilité d'exploser est nulle. Or, les crises financières sont étroitement liées à l'éclatement d'une bulle spéculative. De plus, les crises financières peuvent se propager rapidement à travers le monde, étant donné que les places financières mondiales sont connectées entre elles par le biais des portefeuilles des agents. Comme nous avons pu le voir dans la section 0.3.3, il existe peu de contributions théoriques à avoir

analysé la propagation de l'éclatement d'une bulle. Le troisième chapitre a donc pour but d'étudier la transmission internationale des éclatements de bulles.

0.5.3 Chapitre 3 - Transmission internationale des éclatements de bulles : Equilibres à taches solaires dans un modèle à générations imbriquées à deux pays

Dans ce chapitre, nous souhaitons comprendre l'effet de l'éclatement d'une bulle dans un pays sur la bulle d'un autre pays.

Pour mener cette analyse, nous élargissons le modèle à générations imbriquées de Weil (1987) à une économie ouverte composée de deux pays (domestique et étranger). Nous considérons des pays parfaitement identiques, et chacun émet un actif sans valeur fondamentale représentant une bulle. A l'ouverture, les marchés financiers sont parfaitement intégrés. Dans chaque pays, les agents réalisent un choix de portefeuille entre les deux actifs. Dans ce modèle, l'existence de la bulle est reliée à la réalisation d'une tache solaire. Ces taches solaires ont une certaine probabilité exogène et constante d'occurrence. Sur fond de croyances auto-réalisatrices, lorsqu'une tache solaire apparaît dans un pays, la bulle de ce pays explose. Nous étudions l'existence d'équilibres à tache solaire. Plus particulièrement, nous considérons deux types d'équilibre à tache solaire stationnaire.

Dans le premier type d'équilibre, nous supposons que les taches solaires apparaissent uniquement dans un pays, tandis que l'autre pays est immunisé des taches solaires. Les pays étant identiques, les bulles fournissent un rendement de 1 avant l'arrivée d'une tache solaire. A l'arrivée d'une tache solaire, le rendement de la bulle du pays non immunisé est 0 puisqu'elle a éclaté, tandis que le rendement de la bulle du pays immunisé peut être *a priori* positif ou nul. Dans un tel contexte, nous montrons que l'éclatement de la bulle dans le pays non immunisé suite à l'arrivée d'une tache solaire entraîne in-

évitablement l'éclatement de la bulle du pays immunisé. Par ailleurs, nous mettons en évidence que c'est l'éclatement de la bulle du pays non immunisé et non l'apparition de la tache solaire qui génère l'éclatement de la bulle du pays immunisé.

Dans le second équilibre, nous supposons que les taches solaires peuvent apparaître de manière indépendante avec des probabilités différentes dans les deux pays. Nous montrons que dans ce cas-là, une bulle peut persister après l'éclatement de l'autre bulle, puisque les agents prennent maintenant en compte le risque d'éclatement des bulles dans leurs décisions. Qui plus est, l'éclatement de la bulle peut avoir un effet positif sur l'autre bulle qui va persister, l'intuition reposant sur l'effet de taille de marché.

Dans les trois premiers chapitres, nous avons considéré un modèle à générations imbriquées pour analyser la volatilité des prix d'actifs. Dans le quatrième chapitre, nous changeons le cadre d'analyse et considérons un modèle à horizon infini, afin de mettre en lumière le rôle de l'hétérogénéité des agents économiques. Par ailleurs, dans les précédents chapitres, les sources de la volatilité des prix d'actifs reposaient sur les caractéristiques de marchés, à savoir des imperfections sur le marché du crédit dans les deux premiers chapitres, ou l'intégration financière dans le troisième. Dans le quatrième et dernier chapitre, nous avançons une explication alternative à savoir des préférences non-standard. Comme nous avons pu le voir dans la section 0.3.2, les aspects psychologiques et l'hétérogénéité parmi les consommateurs pourraient également expliquer la volatilité excessive observée sur les marchés financiers.

0.5.4 Chapitre 4 – Volatilité des prix d'actifs, distribution de richesse et esprit de capitalisme : Le rôle de l'hétérogénéité

Dans ce dernier chapitre, nous nous intéressons aux effets de l'hétérogénéité parmi les consommateurs sur la volatilité des prix d'actifs et les inégalités dans un modè-

le à horizon infini avec esprit de capitalisme. Comme indiqué dans la section 0.3.2, l'hypothèse d'esprit de capitalisme fut introduite par Max Weber (1905) et est formalisée à travers des préférences pour la richesse. En plus d'être soutenue empiriquement, ces préférences permettent de résoudre certaines énigmes comme le comportement d'épargne des ménages et la volatilité excessive des prix d'actifs (cf. section 0.3.2). A partir de cet environnement, nous souhaiterions savoir si une économie constituée de capitalistes et de travailleurs, est plus susceptible de connaître une volatilité sur ses marchés financiers, des fluctuations macroéconomiques et/ou des inégalités si elle est très hétérogène?

Pour ce faire nous étendons le modèle de Lucas (1978) à une économie où il existe deux types d'agents hétérogènes du point de vue de leurs préférences pour la richesse, de leurs revenus et de leur richesse initiale. Les agents avec un fort désir intrinsèque d'accumuler de la richesse sont qualifiés de capitalistes (au sens de Max Weber), les autres agents avec un plus faible désir d'accumulation de richesse sont qualifiés de travailleurs. Capitalistes ou travailleurs, les agents effectuent un arbitrage intertemporel entre la consommation et l'épargne en fonction de leurs préférences. L'épargne se fait à travers des actions qui versent, à chaque période, un dividende. Les agents sont incités à accumuler de la richesse même si leur taux d'escompte subjectif est plus élevé que le taux d'intérêt de l'épargne, et ce à cause de leurs préférences pour la richesse. C'est pourquoi à l'équilibre de long-terme tous les agents détiennent des actions; et les capitalistes détiendront plus d'actions. Par ailleurs, nous montrons qu'il n'existe pas de bulles à l'équilibre stationnaire, mais contrairement à Lucas (1978), il existe une infinité de trajectoires d'équilibre au voisinage de l'état stationnaire. Ainsi, des fluctuations peuvent émerger dues à la volatilité des anticipations. Les préférences pour la richesse sont l'élément-clé dans ce chapitre, car elles sont à l'origine des fluctuations.

L'hétérogénéité des préférences joue un rôle important dans ce chapitre, et ce pour deux raisons. D'une part, elle affecte les inégalités, le prix des actions et l'émergence de fluctuations au sein de l'économie. Par exemple, une économie dans laquelle il y aurait des capitalistes avec un très fort désir intrinsèque de richesse et des travailleurs avec un désir très faible, est une société avec des inégalités plus élevées et un prix des actions plus élevé à l'état stationnaire. Par ailleurs, ce sera une économie qui sera plus susceptible de connaître des fluctuations. Secondement, les inégalités de revenus ont un impact sur la dynamique de l'économie uniquement si les agents diffèrent en terme de préférence. Les inégalités de revenu peuvent notamment pousser le prix des actifs à la hausse à l'état stationnaire, et déstabiliser l'économie en favorisant l'émergence de fluctuations.

Pour conclure, ce dernier chapitre met en évidence le rôle que peut jouer l'hétérogénéité parmi les agents dans les inégalités, le niveau du prix des actions, ainsi que dans la déstabilisation de l'économie, lorsque les agents font preuve d'esprit capitaliste.

Chapter 1

Rational bubbles and macroeconomic fluctuations: The (de-)stabilizing role of monetary policy¹

1.1 Introduction

Over the last decades, financial markets have experienced periods of excessive volatility in asset prices with negative consequences on the real activity as illustrated during the subprime crisis. These large fluctuations in asset prices have triggered a renewed interest in a heated debate among policy practitioners and academics about the response of monetary policy to movements in asset prices. In this context, the present paper at-

¹This chapter bases on the published paper: Clain-Chamosset-Yvrard L. and T. Seegmuller (2015). Rational bubbles and macroeconomic fluctuations: The (de-)stabilizing role of monetary policy *Mathematical Social Sciences*, Vol. 75, pages 1-15.

tempts to explain this excessive volatility in a dynamic general equilibrium model and provide new insights for monetary policy responding to asset prices.

Some empirical studies, among other Shiller (1981, 1989, 2000), LeRoy and Porter (1981) and Campbell (2003), show that this excessive volatility in asset price does not result from economic fundamentals. Therefore, it can be interpreted as the existence and fluctuations of speculative bubbles in asset prices. The irrational behavior of agents is often the explanation given to justify such fluctuations (Greenspan, 1996; Shiller ;2000). Nevertheless, a large and growing literature highlights the existence of bubbles under the hypothesis of rational expectations (Tirole, 1985; Kocherlakota, 2008; Martin and Ventura, 2012; Wang and Wen, 2012; Bosi and Seegmuller, 2013; Hirano and Yanagawa, 2013).

In this paper, we provide a justification for asset price bubble fluctuations driven by the volatility of rational expectations, focusing on the existence of endogenous fluctuations. We refer to the concept of equilibrium indeterminacy. We argue that the existence of a portfolio choice and financial frictions faced by households promote the emergence of bubble fluctuations and endogenous business cycles.² This is motivated by some empirical evidence showing that households can be liquidity constrained (see Campbell and Mankiw, 1989; and Jappelli, 1990) and the presence of collateral in their portfolio can affect their consumption (see Iacoviello, 2004). We also contribute to the debate initiated by Bernanke and Gertler (1999, 2001) whether central banks should respond to asset prices, investigating the stabilizing virtues of monetary policy. Specifically, a monetary policy is stabilizing as it reduces the parameter conditions for equilibrium indeterminacy. No clear-cut conclusions have yet been provided to this question (see

²In contrast to us, most of the papers, which investigate the link between rational bubble and financial frictions, focus on frictions at the entrepreneur level (among others, Fahri and Tirole, 2012; Martin and Ventura, 2012; and Hirano and Yanagawa, 2013).

Carlstrom and Fuerst, 2007) for a negative conclusion, and Nutahara (2010) for a positive one both in settings with nominal rigidities).³

We consider a simple exchange overlapping generations (OLG) model with two period-lived households. Each household saves in her first period of life and realizes a portfolio choice between two assets: A pure bubble and money.⁴ The money demand is rationalized by a partial cash-in-advance (CIA) constraint on the second-period consumption: A fraction of second-period consumption is paid cash with monetary savings, whereas the other fraction is financed on credit in exchange for end-of-period income arising from non-monetary savings.⁵ In a simplified way, this constraint allows us to think about credit market imperfections at the household level in such a framework.

A key feature of the model is the assumption that the fraction of consumption financed on credit is positively correlated with the bubble. We implicitly suppose that households may pledge their non-monetary savings (bubble) to secure their loan in their second-period of life. Hence, the bubble serves as a collateral.⁶ To justify the positive correlation, we argue that because of some market imperfections as asymmetric information between lender and borrower, a household may increase her opportunities to obtain a credit from a financial institution by holding a higher amount of non-monetary savings in her portfolio, and thus may consume more on credit and less in cash. Thus, an increase in the value of the bubble helps households to relax their CIA constraint under this assumption. Our main results are connected to this feature.

³Interestingly, Nutahara (2010) embeds credit market imperfections in Carlstrom and Fuerst (2007) model.

⁴Following Tirole (1985), the bubble is represented by an asset without fundamental value.

⁵This is in line with Hahn and Solow (1995). For further details about CIA constraints in OLG settings, the reader can refer to Crettez *et al.* (1999) and Michel and Wigniolle (2003, 2005) for a model with bubble. In contrast to Michel and Wigniolle (2003, 2005), we focus on equilibria for which the CIA constraint is always binding. Thus, bubbles on real money balances are ruled out.

⁶Typically, a short term collateral loan is secured by a savings account or certificate of deposit. In our framework, monetary savings cannot be used as a collateral since money is only a medium of exchange. In contrast, the bubble being an end-of-period asset, and thus more illiquid than money, may be a credit guaranty.

Our model confirms previous findings but also exhibits new results with respect to the literature due to credit market features. Interestingly, these new results are derived for small credit market distortions. Even if the bubble helps households to relax their CIA constraint, it is not sufficient to counteract the distortion generated by this latter. Therefore, the bubble does not restore dynamic efficiency at the steady state unlike Tirole (1985). Nevertheless, we can show that the bubble improves the welfare of households at the steady state.

Typically, OLG models without collateral effect exhibit a unique stationary bubbly equilibrium (see Tirole, 1985; Weil, 1987 and Bosi and Seegmuller, 2013). In our paper, we highlight that the presence of collateral promotes the existence of multiple bubbly stationary equilibria. In such a framework, the multiplicity of bubbly steady states can be perceived as a form of global indeterminacy. Furthermore, the multiplicity of steady states provides a nice theoretical evidence of the excessive volatility in asset prices, since agents may coordinate their expectations on different dynamic paths converging to steady states characterized by different bubble values. We also show that one of the bubbly steady states is locally indeterminate, and therefore expectation-driven fluctuations of a rational bubble may appear. More importantly, these fluctuations occur for arbitrarily weak credit market imperfections and arbitrarily small degrees of utility concavity, that is when savings are a decreasing function of expected inflation. This result is also interesting since we provide an answer to a recurrent criticism about endogenous fluctuations in monetary OLG models: Fluctuations may emerge for low income effects and weak distortions.⁷ The portfolio choice between money and bubble is the key mechanism through which fluctuations occur. Because of credit market imperfections, the portfolio structure depends on agents' expectations. A change in expectations generates

⁷Expectation-driven fluctuations and two-period cycles under large income effects are also likely to occur in our model. However, this result is already known in the literature on endogenous fluctuations (Grandmont, 1985, 1986).

a reallocation of savings between monetary and non-monetary savings. Because of this phenomenon, expectation-driven fluctuations are likely to occur.

Within such a framework, the monetary authority might have a role to play coordinating expectations to stabilize the economy. Therefore, we pursue by analyzing the stabilizing role of monetary policy on the occurrence of fluctuations of the rational bubble. In our framework, a monetary policy is stabilizing as soon as it makes indeterminacy less likely to occur. We consider first the inflation forecast targeting rule proposed by Sorger (2005).⁸ We show that a rule, which strongly reacts to expected inflation, tends to destabilize the economy by promoting local indeterminacy. Furthermore, the inflation forecast targeting has no impact on the existence of the multiplicity of bubbly steady states. This means that this monetary policy fails to coordinate agents' decisions on the steady state chosen as target by the monetary authority.

Since the economy is characterized by a bubble, including a response to asset prices could solve this coordination issue. We therefore investigate whether a monetary rule responding to movements in asset prices has stabilizing virtues. We extend the previous rule by including asset prices. We show that a monetary rule responding to asset prices can locally and globally stabilize. Such a policy is powerful to rule out expectation-driven fluctuations and restore the uniqueness of the bubbly steady state. By responding to asset prices, the monetary authority is able to coordinate agents' expectations on a given steady state.

The rest of the paper is organized as follows. Section 1.2 discusses the contributions of our paper regarding the existing literature. Section 1.3 is devoted to the presentation of the model. In Section 1.4, the intertemporal equilibrium is defined. Steady states with a bubble are studied in Section 1.5. In Section 1.6, we show the existence of bubble fluctuations. Section 1.7 discusses the (de-)stabilizing role of monetary policies. A last

⁸Note that in a recent speech, Bernanke (2010) explains that the US monetary policy follows more closely such a rule than one based on observed inflation.

section provides concluding remarks, whereas some technical details are relegated to an Appendix.

1.2 Related Literature

Our paper contributes to two strands of literature. First, our paper contributes to the literature about rational bubbles that was pioneered by Samuelson (1958) and Tirole (1982, 1985) in OLG models. Since the seminal work of Tirole (1982, 1985), we know that the OLG model provides a simple and useful framework to deal with the existence of rational bubbles.⁹ Tirole (1985) proves also that in an environment without distortions, a rational bubble cannot experience any expectation-driven fluctuations.

To the best of our knowledge, the issue of fluctuations of a rational bubble within an OLG model has been addressed only in few contributions, among others Grandmont (1985), Weil (1987), Michel and Wigniolle (2003, 2005) and Wigniolle (2012). Recent contributions closely related to our paper underline the role of credit market imperfections in the emergence of endogenous fluctuations. For instance, Bosi and Seegmuller (2013) provide a survey about the existence and fluctuations of a rational bubble in a simple OLG framework without collateral. Our paper can be seen as an extension allowing for the role of collateral. Bosi and Seegmuller (2010) extend Tirole (1985) by adding a cash-in-advance constraint on the second-period consumption and assuming that non-monetary savings (capital and bubble) serve as collateral. Even if we do not consider capital accumulation, we generalize the analysis provided in Bosi and Seegmuller (2010) in two directions. First, we provide a deeper analysis of dynamics, and highlight new results related to the multiplicity of steady states with bubble. Second,

⁹However, the existence of rational bubbles could also be provided in infinitely-lived household models. For instance, Kocherlakota (1992) develops a model with heterogeneous agents and borrowing constraints, and Kamihigashi (2008) considers an economy with wealth in the utility function.

we consider sophisticated monetary policies that could fit better the current practices of central banks, instead of a constant growth rate of nominal balances. More recently, Clain-Chamosset-Yvrard and Seegmuller (2013) compare the stabilizing virtues of fiscal and monetary policies in an OLG model with capital accumulation. In our paper, we are concerned with a very different political issue. In an infinite-horizon framework, Hirano *et al.* (2013) analyze the role of bailout policies on business cycles when the economy experiences a stochastic bubble. They show that such a policy can stabilize the economy during bubble collapses by issuing a large amount of public funds. In our framework, we are rather interested in the role of policies on fluctuations with persistent bubbles. In a different approach, Gali (2014) studies the effect of monetary policy on a rational asset price bubble and on the amplitude of its fluctuations in an OLG model without financial frictions. In contrast to Gali (2014), we focus on the existence of endogenous fluctuations of a rational bubble and highlight that several steady states with bubble exist because of credit market imperfections raising a coordination failure of expectations. In such a framework, we analyze whether a monetary policy can be effective at stabilizing by preventing the excessive volatility of asset prices.

The question of whether monetary policy can stabilize endogenous fluctuations in a OLG setting has been essentially studied in models without portfolio choice and without collateral (Grandmont, 1985, 1986; Sorger, 2005). While Grandmont (1985, 1986) suggests monetary policies responding to previous inflation allowing for coordinating expectations, Sorger (2005) studies monetary rules based on inflation targeting which alter conditions for local indeterminacy, and shows that an active inflation forecast targeting rule destabilizes by increasing the range of parameter values for which local indeterminacy occurs.¹⁰ In our framework with a portfolio choice and collateral, we get the same

¹⁰As Sorger (2005), we adopt the terminology used by Benhabib *et al.* (2001) to qualify the degree of response of monetary policy with respect to targeting variable (inflation or asset prices). An active rule means that the monetary authority strongly reacts to the targeting variable, whereas a passive rule means that it weakly reacts.

conclusions. However, we show that the monetary policy can have stabilizing virtues when it responds to asset prices. Gali (2014) argues that the monetary authority should not lead a policy of “leaning against the wind” in a stabilizing perspective, and therefore lower the nominal interest rates when the bubble grows. He derives this conclusion focusing only on fluctuations which occur in a neighborhood of a steady state. As regards expectation-driven fluctuations based on local indeterminacy, our conclusion about the stance of the monetary policy in response to asset prices differs from Gali (2014): The monetary authority should lead either a sufficiently active or a sufficiently passive rule to locally stabilize. Since we also analyze the impact of our monetary policy on global indeterminacy, we claim that the monetary authority should not strongly react to asset prices, and therefore implement a passive rule to globally stabilize.

1.3 The model

We focus on an OLG exchange economy with identical two period-lived households, discrete time ($t = 0, 1, \dots, +\infty$) and three goods: A final good, money and an asset paper.

1.3.1 Households

There is no population growth and, at each date t , a generation of unit size is born. Each generation lives two periods.

In her first period of life, a household is endowed with $e \geq 0$ units of consumption good and receives a monetary transfer T_t from the monetary authority that she allocates between the purchase of the consumption good c_t at price p_t and savings in the form of nominal balances M_{t+1} and an asset paper without fundamental value, which corresponds to non-monetary savings B_t . Following Tirole (1985), Weil (1987) and more

recently Bosi and Seegmuller (2010, 2013), we assume that this asset is supplied in a constant amount normalized to one. If B_t denotes its monetary price and i_{t+1} the growth rate of this price, $B_{t+1} = (1 + i_{t+1})B_t$. This asset is called a bubble, when its price is strictly positive, i.e. $B_t > 0$. In her second period of life, since the household has neither endowments nor a monetary transfer, she uses her remunerated savings $M_{t+1} + (1 + i_{t+1})B_t$ to consume d_{t+1} at price p_{t+1} .

The preferences of a household are represented by an additively separable life-cycle utility function:

$$u(c_t) + \beta v(d_{t+1}) \quad (1.1)$$

where $\beta > 0$, and c_t and d_{t+1} respectively denote the consumption of final good in the first and second period of life.

Assumption 1. $u(c)$ and $v(d)$ are continuous functions defined on $[0, +\infty)$, C^2 on $(0, +\infty)$, strictly increasing ($u' > 0, v' > 0$) and concave ($u'' < 0, v'' < 0$). Moreover, $\lim_{c \rightarrow 0} u'(c) = +\infty$ and $\lim_{d \rightarrow 0} v'(d) = +\infty$. We define $\varepsilon_u(c) \equiv -c \frac{u''(c)}{u'(c)}$ and $\varepsilon_v(d) \equiv -d \frac{v''(d)}{v'(d)}$ as the degrees of concavity of $u(c)$ and $v(d)$ respectively.

Under perfect foresight, the representative household of a generation born at time t derives consumption plan and savings (money and asset paper) by maximizing the utility function (1.1) under the first and second-period budget constraints:

$$p_t c_t + M_{t+1} + B_t \leq p_t e + T_t \quad (1.2)$$

$$p_{t+1} d_{t+1} \leq M_{t+1} + (1 + i_{t+1})B_t \quad (1.3)$$

To get a diversified portfolio, we impose a partial cash-in-advance constraint on the second-period consumption. We consider a modified version of the constraint introduced by Hahn and Solow (1995), i.e. $\gamma p_{t+1} d_{t+1} \leq M_{t+1}$, allowing for the presence of

collateral.

$$\gamma(b_t) p_{t+1} d_{t+1} \leq M_{t+1} \quad (1.4)$$

where $b_t \equiv B_t/p_t$ is the real amount of non-monetary savings.

This binding constraint means that an old household has to pay a share $\gamma(b_t) \in (0, 1)$ of second-period consumption from money holdings, and the remaining share $1 - \gamma(b_t)$ by the resale of the bubble in the financial market at the end of the second period. As underlined by Rochon and Polemarchakis (2006), a partial cash-in-advance constraint in OLG settings implies an intra-period borrowing. As the bubble is an illiquid asset in the sense that it provides an end-of-period income, an old household can finance the fraction $1 - \gamma(b_t)$ of her consumption by borrowing from a financial institution an amount equal to value of the bubble at the end of the second period, i.e. $(1 + i_{t+1}) B_t$, that she will pay back after the resale. For further, we will refer to $1 - \gamma(b_t)$ as the credit share, because it corresponds to the fraction of consumption purchased on credit.¹¹ This constraint also means that an old household must make a down payment in cash, equivalent to $\gamma(b_t)d_{t+1}$, to consume and finance a fraction of her consumption on credit.

In addition, we assume that the credit share depends positively on the amount of non-monetary savings to capture indirectly the role of collateral on consumption behavior of the household highlighted by empirical studies (Iacoviello (2004)). We argue that real non-monetary savings (bubble) can be pledged as collateral to protect the financial institution against the default risk. This assumption indicates that if the household increases her non-monetary savings, then she can reduce her down payment in cash for a constant level of consumption d_{t+1} , and thus increase the fraction of second-period consumption purchased on credit. Because of credit market frictions, an old household

¹¹A similar idea can be seen in Lucas and Stokey (1987), where consumption purchased with money would correspond to “cash goods” and consumption purchased on credit to “credit goods”.

gets a bargaining power in the credit market holding collateral in her portfolio. If the collateral value increases, the old household can request a higher loan, which will be accepted by the financial institution, and thus relax her cash-in-advance constraint.¹² Our assumption allows us to think about credit market imperfections with a collateral effect in our framework.

Furthermore, since an household born at period t realizes its portfolio choice at t , we argue that the value of her real amount of non-monetary savings b_t , which may represent the value of her estate at t , can be pledged as a collateral instead of its expected value $b_{t+1} = [(1 + i_{t+1})/\pi_t] b_t$. In any way, the present value of a bubble asset embodies future values. Moreover, the choice of b_t rather than b_{t+1} as collateral has no consequences on the main result of our paper, i.e. the existence of the multiplicity of steady states with bubble. Our general conclusions on the existence of fluctuations and on the stabilizing role of monetary policy would be also true under small credit market distortions.¹³

Our way to introduce collateral is quite different to models with borrowing constraint *à la* Kiyotaki and Moore (1997), but the reason for credit as well. In these models, borrowing is used to finance investment project, whereas in our model it is to finance consumption.¹⁴

Assumption 2. $\gamma(b) \in (0, 1)$ is a continuous function defined on $[0, +\infty)$, C^2 on $(0, +\infty)$, decreasing ($\gamma'(b) \leq 0$) and such that $\gamma(0)$ cannot tend to 1. In addition,

¹²This assumption is also in accordance with some empirical studies underlining the positive correlation between money velocity and credit (see Gillman *et al.*, 1997). In our framework, $1/\gamma(b_t)$ can be interpreted as the velocity of money which is an increasing function of credit b_t .

¹³More details are available upon request.

¹⁴To highlight the main differences between our constraint and the usual borrowing constraint, let A_{t+1} be the amount of borrowing such that at the end of the second period $A_{t+1} = (1 + i_{t+1})B_t$. Using the second-period budget constraint, the CIA constraint implies $A_{t+1}/(p_{t+1}d_{t+1}) \leq 1 - \gamma(b_t)$. Unlike to the usual borrowing constraint, the borrowing's share of total consumption is bounded instead of the borrowing capacity itself. Second, the borrowing limit in these models are linear with collateral; whereas in our model the bound is nonlinear and increasing with collateral.

we define:

$$\eta_1(b) \equiv \frac{[1 - \gamma(b)]' b}{1 - \gamma(b)} \geq 0, \quad \eta_2(b) \equiv -\frac{[1 - \gamma(b)]'' b}{[1 - \gamma(b)]'} \quad (1.5)$$

$$\eta_\eta(b) \equiv \frac{\eta_1'(b) b}{\eta_1(b)} = 1 - \eta_1(b) - \eta_2(b) \quad (1.6)$$

Example. The following function satisfies Assumption 2:

$$\gamma(b) = \frac{A}{s} \exp(-sb), \quad \text{with } 0 < A < s \quad (1.7)$$

$\eta_1(b)$ and $\eta_2(b)$ are respectively given by:

$$\eta_1(b) = \frac{A \exp(-sb)}{1 - \frac{A}{s} \exp(-sb)} b \geq 0 \quad \text{and} \quad \eta_2(b) = sb > 0.$$

Notice that when collateral plays no role ($\eta_1(b) = 0$), and γ tends to 0, money is no longer needed and the credit market distortions disappear. When $\eta_1 = 0$ and $\gamma > 0$, the household is constrained to hold money to consume in her second period creating a distortion. When collateral matters ($\eta_1(b) > 0$), the household is aware of the credit share function in such a way that she takes into account its argument b_t in her decisions.¹⁵

Using $\pi_{t+1} \equiv p_{t+1}/p_t$ and introducing the real variables $m_t \equiv M_t/p_t$ and $\tau_t \equiv T_t/p_t$, the constraints (1.2)-(1.4) can be rewritten:

$$c_t + \pi_{t+1} m_{t+1} + b_t \leq e + \tau_t \quad (1.8)$$

$$d_{t+1} \leq m_{t+1} + \frac{1 + i_{t+1}}{\pi_{t+1}} b_t \quad (1.9)$$

$$\gamma(b_t) d_{t+1} \leq m_{t+1} \quad (1.10)$$

¹⁵This means that households internalize credit market distortions.

In our framework, we could add physical capital, and thus consider an economy with production. However, we should make some assumptions on its role for the consumer: Is it only a store of value or also a collateral? Hence, physical capital and bubble could be either perfectly or imperfectly substitutable from the point of view of consumer. This possible lack of substitutability will of course affect the dynamic properties of the economy and implies a loss of tractability of our model. Therefore, to keep the analysis tractable and highlight the role of credit market imperfections on fluctuations, we do not consider capital accumulation in our framework.¹⁶ As a weakness, it is not possible to study the effects of the bubble on the real economic activity.¹⁷

The representative household maximizes (1.1) under the budget and cash-in-advance constraints (1.8)-(1.10) to determine optimal consumption plan (c_t, d_{t+1}) and optimal real savings (m_{t+1}, b_t) . All constraints are binding if money is a dominated asset ($(1 + i_{t+1})/\pi_{t+1} > 1/\pi_{t+1}$), or equivalently the opportunity cost of holding money, the nominal interest rate i_{t+1} , is strictly positive ($i_{t+1} > 0$).

In the remaining of the paper, we only consider the case of a binding cash-in-advance constraint so that the portfolio choice is determined and only a bubble on the asset paper could appear. The asset paper holdings, whose the level depends on expectations about its future value, determine the money demand used for future consumption. Nonetheless, because of the endogeneity of the credit share, the portfolio choice is no longer constant. The trade-off between assets becomes endogenous and depends on the amount of collat-

¹⁶If we consider capital accumulation in our framework, it would be difficult to derive clear-cut and analytic results about the role of monetary policy when this latter responds to asset prices. Recent contributions study the existence of bubble fluctuations and the role of monetary policy in OLG model with capital accumulation (Bosi and Seegmuller, 2010 and Clain-Chamosset-Yvrard and Seegmuller, 2013). However, they consider simple monetary rules as a constant money growth rule or a traditional Taylor rule.

¹⁷Since the seminal papers of Tirole (1985) and Weil (1987), it is well known that bubbles can have a negative effect on the real activity by crowding out real investment when there are no distortions in the economy. However, recent contributions, studying effects of bubbles in models with credit market imperfections, show that bubbles can increase real economic activity by relaxing the borrowing constraint and/or promoting most efficient investments (see Kocherlakota, 2009; Martin and Ventura, 2012 and Hirano and Yanagawa, 2013).

eral held by the household implying the existence of a “portfolio effect”. This portfolio effect could be seen as a substitution effect within the portfolio following a change of the relative price between the two assets. Thereafter, we will see that this portfolio effect is a key mechanism through which expectation-driven fluctuations emerge.

Assumption 3. For all $t \geq 0$, we assume $i_t > 0$ and

$$\eta_1(b_t) < \frac{\gamma(b_t)}{1 - \gamma(b_t)} \frac{b_t}{\pi_{t+1} m_{t+1}} \quad (1.11)$$

Lemma 1. Under Assumptions 1 – 3, constraints (1.8)-(1.10) are binding.

Proof. See Appendix 1.9.1.

Inequality (1.11) puts an upper bound to the credit-share elasticity $\eta_1(b)$. It is specific to our model because of the presence of collateral. Since the right-hand side is strictly positive, inequality (1.11) is satisfied when the credit share is constant ($\eta_1 = 0$). Thereafter, we focus on the case where η_1 is not too large. In other words, we consider small distortions in the credit market.

Under Assumption 1 – 3, the optimal households’ behavior is summarized by the following equation:

$$\frac{u'(c_t)}{\beta v'(d_{t+1})} = \frac{1}{\pi_{t+1}} \frac{(1 + i_{t+1})/\pi_{t+1} - d_{t+1}\eta_1(b_t)[1 - \gamma(b_t)]/b_t}{[1 + i_{t+1}\gamma(b_t)]/\pi_{t+1} - d_{t+1}\eta_1(b_t)[1 - \gamma(b_t)]/b_t} > \frac{1}{\pi_{t+1}} \quad (1.12)$$

where the last inequality holds because money is a dominated asset ($1/\pi_{t+1} < (1 + i_{t+1})/\pi_{t+1}$).

Under a constant credit share ($\eta_1 = 0$), Eq. (1.12) rewrites:

$$\frac{u'(c_t)}{\beta v'(d_{t+1})} = \frac{1 + i_{t+1}}{\pi_{t+1}} \frac{1}{1 + i_{t+1}\gamma} \quad (1.13)$$

When γ tends to 0, the right-hand side would reduce to $(1 + i_{t+1})/\pi_{t+1}$, which is similar to the trade-off found in the monetary model by Samuelson (1958). There are no market distortions. When $\gamma > 0$, money demand implies an opportunity cost which lowers the real return on portfolio. More precisely, the household has to pay by cash a share γ to consume an extra-unit when she is old. The interest rate i_{t+1} entails an opportunity cost γi_{t+1} which reduces the purchasing power of savings in the form of asset paper. When collateral matters ($\eta_1(b_t) \neq 0$), the marginal effect of non-monetary savings on the credit share becomes an additional distortion, and counteracts the opportunity cost of money demand.

Furthermore, from the optimal behavior of the household, we deduce the following lemma¹⁸:

Lemma 2. *Let*

$$\tilde{\varepsilon}_u \equiv c \frac{(1 - \gamma)^2 / b}{[1 - \gamma + (1 + i)(\gamma - \eta_1)]^2} i \eta_1 \left(2 - \frac{\eta_2}{1 - \eta_1} \right)$$

*Under Assumptions 1 – 3, the second-order conditions are satisfied if $\varepsilon_u > \tilde{\varepsilon}_u$.*¹⁹

Proof. See Appendix 1.9.2.

1.3.2 Monetary rule

Let us introduce the money growth factor $\mu_t = M_{t+1}/M_t$. It can be rewritten:

$$\mu_t = \pi_{t+1} \frac{m_{t+1}}{m_t} \tag{1.14}$$

¹⁸For simplicity, the arguments of the functions and the time subscripts are omitted.

¹⁹This condition will be supposed satisfied at the steady state.

As in Sorger (2005), the money growth factor corresponds to the monetary instrument.²⁰ However, focusing on bubble fluctuations, we extend the rule proposed by Sorger (2005) by taking deviations of observed asset prices from the target into account.

The monetary policy is implemented through a rule of the following form:

$$\mu_t = \mu^* \left(\frac{\pi_{t+1}}{\mu^*} \right)^{-\alpha} \left(\frac{b_t}{b^*} \right)^{-\rho}, \quad (1.15)$$

with $\alpha \in (-1, +\infty)$ and $\rho \in (-1, +\infty)$.

We clarify that μ^* and b^* are respectively the targets of the money growth factor and the asset prices chosen by the monetary authority.

When $\alpha = 0$ and $\rho = 0$, the nominal money supply grows at the constant growth factor $\mu_t = \mu^*$. This policy corresponds to a regime of strict money growth targeting in which the nominal money growth rate is fixed at μ^* .

When $\alpha \neq 0$ and $\rho = 0$, Eq. (1.15) depicts a policy rule for inflation forecast targeting (Sorger, 2005). When the monetary authority sets the money growth factor μ_t , the inflation π_{t+1} has not been observed yet and can be seen as an inflation forecast. For $\alpha > 0$, the nominal money growth is a decreasing function of expected inflation. The nominal money supply is contracted if expected inflation is above the target μ^* and expanded if expected inflation is below. For $\alpha \in (-1, 0)$, the nominal money growth is an increasing function of expected inflation but less than proportional. As Benhabib *et al.* (2001) and Sorger (2005), we call a rule with $\alpha > 0$ an active one and a rule with $\alpha \in (-1, 0)$ a passive one.

When $\alpha \neq 0$ and $\rho \neq 0$, Eq. (1.15) depicts a monetary rule which takes into account the level of asset prices. This rule includes a response to the level of current

²⁰In recent work, Gaballo (2013) introduces a monetary rule, whose the monetary instrument is the money supply, to study the social value of information about the future in a OLG model of inflation.

asset prices b_t . For $\rho > 0$, the nominal money growth is a decreasing function of current asset prices. In other words, the nominal money supply is contracted if the level of asset prices is above b^* and expanded if the level is below. For $\rho \in (-1, 0)$, the nominal money growth rate is an increasing function of current asset prices but less than proportional. In the following, we call a rule with $\rho > 0$ an active one and a rule with $\rho \in (-1, 0)$ a passive one.

This rule is in accordance with Bernanke and Gertler (1999, 2001). In addition, several Governors of central bank or practitioners of monetary policy encourage more research on monetary policies which would react to credit booms (Yellen, 2009). By considering b_t as savings in the form of credit and thus as an index of the credit level, Eq. (1.15) could also be interpreted as a formalization of such a monetary policy.

Money is distributed by the monetary authority to young households through a lump-sum transfer $\tau_t = (M_{t+1} - M_t)/p_t$, or equivalently,

$$\tau_t = \pi_{t+1}m_{t+1} - m_t \tag{1.16}$$

Assuming that the monetary transfer is distributed in the first period of life and not in the second period, we closely follow Michel and Wigniolle (2005). This assumption seems to be appropriate to study the role of savings and the portfolio choice on dynamics. Actually, the monetary transfer distributed in the second period of life negatively affects the amount of individual savings.

1.4 Intertemporal equilibrium

Substituting Eq. (1.16) in the first-period budget constraint (1.8), we determine:

$$c_t + m_t + b_t = e \quad (1.17)$$

Given that $b_{t+1} = [(1 + i_{t+1}) / \pi_{t+1}] b_t$ and using Eqs. (1.9), (1.10) and (1.17), we can derive respectively the demand for real balances, the consumption levels at the first and second-period of life:

$$m_{t+1} = b_{t+1} \frac{\gamma(b_t)}{1 - \gamma(b_t)}, \quad c_t = e - \frac{b_t}{1 - \gamma(b_{t-1})} \quad \text{and} \quad d_{t+1} = \frac{b_{t+1}}{1 - \gamma(b_t)} \quad (1.18)$$

As $c_t > 0$, we can deduce an upper bound on b_t :

$$0 \leq b_t < [1 - \gamma(b_{t-1})] e \equiv \bar{b}(b_{t-1}) \quad (1.19)$$

If $b_t > \bar{b}(b_{t-1})$, then a young household would be unable to buy the bubble.²¹

Using Eqs. (1.14) and (1.18), we deduce the inflation factor and the rental rate on the asset paper:

$$\pi_{t+1} = \mu_t \frac{\gamma(b_{t-1})}{\gamma(b_t)} \frac{1 - \gamma(b_t)}{1 - \gamma(b_{t-1})} \frac{b_t}{b_{t+1}} \quad \text{and} \quad 1 + i_{t+1} = \mu_t \frac{\gamma(b_{t-1})}{\gamma(b_t)} \frac{1 - \gamma(b_t)}{1 - \gamma(b_{t-1})} \quad (1.20)$$

Substituting Eqs. (1.17)-(1.20) into Eq. (1.12), we obtain the consumers' intertemporal trade-off :

$$b_t \frac{u' \left(e - \frac{b_t}{1 - \gamma(b_{t-1})} \right)}{\beta v' \left(\frac{b_{t+1}}{1 - \gamma(b_t)} \right)} = b_{t+1} \frac{[1 - \eta_1(b_t)]}{1 - \gamma(b_t) + [\gamma(b_t) - \eta_1(b_t)] \mu_t \frac{\gamma(b_{t-1})}{\gamma(b_t)} \frac{1 - \gamma(b_t)}{1 - \gamma(b_{t-1})}} \quad (1.21)$$

²¹The value of the bubble would exceed the value of the resources available to young households after holding nominal balances needed for their future consumption.

At this stage, it is relevant to clarify that from Assumption 1, inequality (1.19) is satisfied.

From Eqs. (1.14), (1.15), (1.18) and (1.20), we obtain the expression of the money growth factor:

$$\mu_t = \mu^* \left[\frac{\gamma(b_{t-1})}{1 - \gamma(b_{t-1})} \frac{1 - \gamma(b_t)}{\gamma(b_t)} \frac{b_t}{b_{t+1}} \right]^{\frac{-\alpha}{1+\alpha}} \left(\frac{b_t}{b^*} \right)^{\frac{-\rho}{1+\alpha}} \quad (1.22)$$

Definition 1. *Under Assumptions 1-3, an intertemporal equilibrium with perfect foresight is a sequence (b_t) , with $0 \leq b_t < \bar{b}(b_{t-1})$, $b_{-1} = B_{-1}/p_{-1} > 0$ and $t = 0, 1, \dots, +\infty$, such that Eq. (1.21) is satisfied, where μ_t is given by Eq. (1.22).*

Note that b_0 is not predetermined in period 0. However, the second-order recurrence equation (1.21) has a predetermined variable at period t , b_{t-1} .

For further references, substituting Eqs. (1.18) into Eq. (1.11) allows us to rewrite Assumption 3:

$$1 < 1 + i_{t+1} < 1/\eta_1(b_t) \quad (1.23)$$

for $t = 0, 1, \dots, +\infty$, which ensures that the cash-in-advance constraint is binding.

In the following, we will use this definition of the equilibrium to show the existence of expectation-driven fluctuations of the rational bubble and study the (de-)stabilizing role of monetary policy. For this purpose, we first analyze the steady state. We will see that some new interesting results, as the multiplicity of steady states, based on credit market features will be obtained.

1.5 Steady state analysis

In this section, we show that several steady states characterized by a positive bubble coexist with the bubbleless one. The existence of two steady states, one

with bubble and another without bubble, in OLG models is not new (see Tirole, 1985; and Weil, 1987), and raises an issue of global indeterminacy. The novelty in our paper relies on the existence of the multiplicity of bubbly steady states, which can be seen as a different source of global indeterminacy.

From Eqs. (1.18) and (1.21), it is straightforward to prove the existence of a steady state without bubble. Without bubble, we have $b = 0$, $m = 0$, $c = e$ and $d = 0$. At this steady state, households hold no savings even in the monetary form in spite of the CIA constraint, because they receive no endowments in their second period of life. Such an allocation is not dynamically efficient, since the autarky interest rate of this economy, defined by $u'(c)/[\beta v'(d)] - 1$, tends to -1 .

All households can improve their allocation by holding the bubble. They may intertemporally smooth their consumption if they buy the bubble and resell it in the next period. In the following, we restrict our attention on the existence and properties of a steady state with a bubble ($b > 0$). A bubbly steady state $b > 0$ is a solution of the following equation:

$$\frac{u' \left(e - \frac{b}{1-\gamma(b)} \right)}{\beta v' \left(\frac{b}{1-\gamma(b)} \right)} = \frac{1 - \eta_1(b)}{1 - \gamma(b) + [\gamma(b) - \eta_1(b)] \mu(b)}, \quad (1.24)$$

where $\mu(b) \equiv \mu^* (b/b^*)^{-\rho/(1+\alpha)}$ is obtained from Eq. (1.22).

We first study the existence of the bubbly steady state. Second, we establish the existence of a normalized steady state. The issue of multiplicity of stationary solutions is finally addressed.

1.5.1 Existence and uniqueness

The following assumption helps us to prove the existence of a steady state with a positive bubble²²:

Assumption 4. *If $\rho > 0$, we assume:*

$$\varepsilon_v(0) < \frac{\eta_1(0)}{[1 - \eta_1(0)]} \frac{[1 - \gamma(0)] [\mu(0) - 1]}{\{1 - \gamma(0) + \mu(0) [\gamma(0) - \eta_1(0)]\}} \\ \left(2 - \frac{\eta_2(0)}{1 - \eta_1(0)}\right) + \frac{\rho}{1 + \alpha} \frac{\mu(0) [\gamma(0) - \eta_1(0)] / [1 - \eta_1(0)]}{1 - \gamma(0) + \mu(0) [\gamma(0) - \eta_1(0)]}$$

The next proposition proves the existence of a bubbly steady state and provides a result on uniqueness.

Proposition 1. *Let \bar{b} be defined by $e = \bar{b}/[1 - \gamma(\bar{b})]$. Under Assumptions 1-4, there exists a steady state characterized by a positive bubble, $b^* \in (0, \bar{b})$. The uniqueness of the steady state is ensured if the following condition holds $\forall b \in (0, \bar{b})$ ²³:*

$$\varepsilon_v(d) > \frac{b}{e[1 - \gamma(b)] - b} [\varepsilon_u^s(b) - \varepsilon_u(c)], \quad (1.25)$$

Proof. See Appendix 1.9.3.

Proposition 1 shows the existence of a steady state with a positive bubble. As we have seen earlier, this is in accordance with a steady state without bubble that is dynamically inefficient, as in Samuleson (1958) and Tirole (1985). Nevertheless, when collateral does not matter ($\eta_1 = 0$), the bubble does not restore the dynamic efficiency because of the presence of the CIA constraint. This result is in contrast to Tirole (1985) and still holds when η_1 is not too large. Even if the bubble serves

²²We will see in the next section that Assumption 4 is always satisfied for a certain class of utility functions.

²³ $\varepsilon_u^s(b)$ is given in Appendix 1.9.3 by Eq. (1.49).

as collateral and allows households to relax the constraint, it is not sufficient to counteract the negative effect of this latter. The bubbly steady state remains dynamically inefficient under small credit market distortions. Note, however, that the return of the economy is increasing with the size of the bubble, meaning that the bubble makes the economy less inefficient at the steady state. We can also show that the bubble improves the welfare of households at the steady state.²⁴

Furthermore, under a constant credit share of consumption purchases ($\eta_1 = 0$) and a passive rule on asset prices ($\rho \leq 0$), the bubbly steady state is unique, the condition (1.25) being always satisfied. This is also the case when the degrees of concavity of utility function are large enough. However, multiplicity of bubbly steady states may occur when the condition (1.25) is not satisfied for all $b \in (0, \bar{b})$. We clarify this in the next section.

We remind that the monetary authority specifies targets for inflation and asset prices, choosing the stationary values of an existing steady state. Since a steady state with bubble is always better than the bubbleless one regarding welfare and efficiency, we argue that the monetary authority targets a steady state with bubble. From Proposition 1, a steady state with bubble b^* always exists. We consider that the monetary authority selects this equilibrium b^* as the target.

1.5.2 Normalized steady state and multiplicity

In order to facilitate the analysis of the multiplicity of bubbly steady states and of local dynamics (Section 1.6), we establish the existence of a normalized steady state $b^* = 1$ (NSS). We follow the procedure introduced by Cazzavillan *et al.* (1998) and use the scaling parameter β to ensure the existence of such a steady state. Accordingly, we assume:

²⁴From the analyzing of the welfare at the steady state, we can show that the welfare is increasing with the size of the bubble. More details are available upon request.

Assumption 5. $e(1 - \gamma) > 1$,

which ensures that the first-period consumption at the normalized steady state is positive. Note that it is satisfied for a sufficiently large endowment e .

Proposition 2. *Under Assumptions 1-5, there exists a unique value $\beta^* > 0$ given by*

$$\beta^* = \frac{u' \left(e - \frac{1}{1-\gamma(1)} \right)}{v' \left(\frac{1}{1-\gamma(1)} \right) \frac{1-\eta_1(1)}{1-\gamma(1)+\mu(1)[\gamma(1)-\eta_1(1)]}}$$

such that $b^* = 1$ is a steady state of the dynamic equation (1.21).

Thereafter, we assume $\beta = \beta^*$ so that $b^* = 1$. At the normalized steady state, $\mu(1) = \mu^* = \pi$ (see Eq. (1.14)) and $\pi = 1 + i$ (see Eq. (1.20)). From inequality (1.23), Assumption 3 is satisfied if and only if:

$$1 < \mu^* < 1/\eta_1(1) \quad (1.26)$$

Furthermore, the second order conditions are satisfied for $\varepsilon_u > \tilde{\varepsilon}_u$, with $\tilde{\varepsilon}_u \equiv \frac{[e(1-\gamma)-1](1-\gamma)\eta_1(\mu^*-1)[2-\eta_2/(1-\eta_1)]}{[1-\gamma+\mu^*(\gamma-\eta_1)]^2}$. We further note ε_u and ε_v the degrees of concavity $\varepsilon_u(c)$ and $\varepsilon_v(d)$ evaluated at the steady state $b^* = 1$, $\varepsilon_u^s \equiv \varepsilon_u^s(1)$, $\gamma \equiv \gamma(1)$, $\eta_1 \equiv \eta_1(1)$, and $\eta_2 \equiv \eta_2(1)$.

We can now clarify the conditions for the multiplicity of steady states.

Proposition 3. *Under Assumptions 1-5, if the following condition holds:*

$$\varepsilon_v < \frac{1}{e(1-\gamma)-1} (\varepsilon_u^s - \varepsilon_u), \quad (1.27)$$

then there is a multiplicity of stationary equilibria with positive bubble. Their number is generically odd.

Proof. See Appendix 1.9.4.

Inequality (1.27) can be satisfied only if $\varepsilon_u \in (\tilde{\varepsilon}_u, \varepsilon_u^s)$. When $\rho = 0$, $\varepsilon_u^s > 0$ is satisfied if and only if $\eta_1 \in (0, 1 - \eta_2/2)$, which requires also $\eta_2 < 2$. This also implies $\tilde{\varepsilon}_u > 0$. Moreover, $\varepsilon_u^s > \tilde{\varepsilon}_u$ holds true when $\eta_1 < \gamma$. Thinking that the multiplicity of steady states is an interesting result in terms of indeterminacy issue, we ensure for the remainder of the paper that $0 < \tilde{\varepsilon}_u < \varepsilon_u^s$ when $\rho = 0$:

Assumption 6.

$$\eta_2 < 2 \text{ and } 0 < \eta_1 < \min\{\gamma; 1 - \eta_2/2\}$$

Under Assumption 6, the multiplicity of steady states appears under a small collateral effect (i.e. η_1 small enough) and small degrees of concavity on utility when the monetary policy rule does not depend on asset prices ($\rho = 0$). When the monetary rule takes into account asset prices ($\rho \neq 0$), inequality (1.27) is satisfied when the degrees of concavity ε_v and ε_u are small enough or for an active rule on asset prices ($\rho > 0$). Therefore, an active rule, which strongly reacts to asset prices, tends to promote the emergence of several bubbles in the economy.

With the multiplicity of steady states, convergence or not to a steady state may depend on agents' expectations.²⁵ The existence of multiple bubbly steady states may lead to global indeterminacy, which is a source of expectation-driven fluctuations of the bubble.

Furthermore, the multiplicity of steady states raises an issue for the monetary authority concerning the choice of the rule. If the monetary authority chooses to lead an active policy for the steady state $b^* = 1$, this does not mean that the policy

²⁵Note that this multiplicity will be related to local indeterminacy through the occurrence of a pitchfork bifurcation in the next section.

would be active at the other steady states. This issue of selection is addressed for instance by Benhabib *et al.* (2001). Nonetheless, in our model, the multiplicity of steady states arises without introducing any bound on the monetary rule.

Example. *The example given by Eq. (1.7) may fit all the requirements we impose on the function $\gamma(b_t)$ through Assumptions 2 – 6.*

Let $A_1 \equiv \frac{s(1-s)}{\exp(-s)}$ and $A_2 \equiv \frac{s(2-s)}{2+s}$. For $0 < s < 1$ and $0 < A < \min\{A_1; A_2; s\}$, the function $\gamma(b_t) = \frac{A}{s} \exp(-sb_t)$ satisfies Assumptions 2, 3 and 6.

With this example for $\gamma(b_t)$, Assumption 4 is always verified when $\rho > 0$ by imposing a restriction on the form of the utility function. Indeed, Assumption 4 can be written as follows:

$$\varepsilon_v(0) < \frac{\rho}{1 + \alpha} \quad (1.28)$$

For $\alpha < +\infty$, the right-hand side is strictly positive. As a consequence, an utility function whose the degree of concavity is increasing in consumption can satisfy the above inequality (1.28). A possible illustration of such an utility function is provided by the following formulation:

$$v(d) = \frac{d^{\delta_0}}{\delta_1 + d^{\delta_0}}, \text{ that is } \varepsilon_v(d) = 1 - \delta_0 + 2\delta_0 \frac{d^{\delta_0}}{\delta_1 + d^{\delta_0}}, \text{ with } \delta_0 < 1$$

For δ_0 sufficiently close to 1, inequality (1.28) holds.

1.6 Expectation-driven fluctuations and endogenous cycles

This section is devoted to the existence of expectation-driven fluctuations of a rational bubble. In his seminal paper, Tirole (1985) shows that there exist a

unique equilibrium path converging to a positive bubble and multiple paths where the bubble collapses to zero in the long run. In contrast, we show the existence of multiple equilibrium paths that converge to one or different steady states with positive bubbles. This means that the economy can experience persistent fluctuations driven by self-fulfilling expectations without involving the collapse of the bubble. Our explanation mainly relies on credit market features. More precisely, we show that the normalized steady state with a positive bubble can be locally indeterminate, and therefore expectation-driven fluctuations of the bubble can emerge, in particular under a sufficiently small degree of utility concavity and arbitrarily small credit market distortions. To highlight bubble fluctuations, we consider the model with a constant money growth ($\mu_t = \mu^*$). In the next section, we enrich the model with monetary rules based on inflation forecast targeting and on asset prices to study the (de-)stabilizing role of monetary policy.

1.6.1 Local dynamics: Preliminaries

To derive our different results, we start by linearizing the dynamic equation (1.21) around the steady state $b^* = 1$ to obtain the characteristic polynomial.

Assumption 7.

$$\varepsilon_v \neq \frac{1 - \gamma + \mu^* (\gamma - \eta_1) \frac{1}{1+\alpha}}{1 - \gamma + \mu^* (\gamma - \eta_1)} \equiv \bar{\varepsilon}_v$$

Using this assumption, we get:

Lemma 3. *Let*

$$\bar{\varepsilon}_u \equiv -[e(1 - \gamma) - 1] \frac{\mu^* (\gamma - \eta_1)}{1 - \gamma + \mu^* [\gamma - \eta_1]} \frac{1/\gamma}{1 + \alpha} \quad (1.29)$$

Under Assumptions 1-7, the characteristic polynomial, evaluated at the steady state $b^* = 1$, is defined by $P(X) \equiv X^2 - TX + D = 0$, where:

$$D = \frac{1}{\bar{\varepsilon}_v - \varepsilon_v} \frac{\eta_1}{e(1-\gamma) - 1} (\varepsilon_u - \bar{\varepsilon}_u) \equiv D(\varepsilon_v) \quad (1.30)$$

$$T = \frac{1}{\bar{\varepsilon}_v - \varepsilon_v} \left\{ \frac{\varepsilon_u}{e(1-\gamma) - 1} - \eta_1 \varepsilon_v + \frac{(1-\gamma)}{1-\gamma + \mu^*(\gamma - \eta_1)} \left[1 - (\mu^* - 1) \eta_1 \left(2 - \frac{\eta_2}{1-\eta_1} \right) \right] + \frac{\mu^*(\gamma - \eta_1)}{1-\gamma + \mu^*(\gamma - \eta_1)} \frac{1 - \rho + \eta_1/\gamma}{1 + \alpha} \right\} \equiv T(\varepsilon_v) \quad (1.31)$$

Proof. See Appendix 1.9.5.

As in Grandmont *et al.* (1998), we study the variations of the trace $T(\varepsilon_v)$ and the determinant $D(\varepsilon_v)$ in the (T, D) plane as one of the parameters of interest, namely ε_v , is made to vary continuously in its admissible range $(0, +\infty)$ (see Figure 1). The locus $\Sigma \equiv \{(T(\varepsilon_v), D(\varepsilon_v)) : \varepsilon_v \geq 0\}$ describes a part of a line that we call the Σ -line.

Along the line (AC) , one eigenvalue is equal to 1 ($D = T - 1$). Along the line (AB) , one eigenvalue is equal to -1 ($D = -T - 1$). Along the segment $[BC]$ ($|T| < 2, D = 1$), the characteristic roots are complex conjugates with modulus equal to 1. These lines divide the space (T, D) into three different types of regions. Inside the triangle ABC , the steady state is a sink, i.e. locally indeterminate ($|T| < 1 + D$ and $D < 1$). It is a saddle point if (T, D) lies on the right or left sides of both the lines (AB) and (AC) ($|1 + D| < |T|$). It is a source otherwise.

A (local) bifurcation arises when at least one eigenvalue crosses the unit circle, that is, when the Σ -line crosses one of the loci (AB) , (AC) or $[BC]$. According to the changes of the bifurcation parameter, a pitchfork bifurcation (generically)

emerges when the Σ -line crosses (AC) , as ε_v goes through ε_v^s .²⁶ A flip bifurcation (generically) occurs when the Σ -line crosses (AB) , as ε_v goes through ε_v^f . Finally, a Hopf bifurcation (generically) arises when the Σ -line crosses the segment $[BC]$, as ε_v goes through ε_v^h .²⁷

The Σ -line has a slope S given by:

$$S = \frac{\frac{\eta_1}{e(1-\gamma)-1} (\varepsilon_u - \bar{\varepsilon}_u)}{\frac{\varepsilon_u}{e(1-\gamma)-1} - \eta_1 \frac{\bar{\varepsilon}_u}{e(1-\gamma)-1} + \frac{(1-\gamma) \left[1 - \eta_1 - \eta_1 (\mu^* - 1) \left(2 - \frac{\eta_2}{1-\eta_1} \right) \right]}{1-\gamma + \mu^* (\gamma - \eta_1)} + \frac{1-\eta_1-\rho}{1+\alpha} \frac{\mu^* (\gamma - \eta_1)}{1-\gamma + \mu^* (\gamma - \eta_1)}} \quad (1.32)$$

We further note that the Σ -line is characterized by the endpoint $(T(+\infty), D(+\infty)) \equiv (\eta_1, 0)$ and the starting point given by:

$$T(0) = \frac{1}{\bar{\varepsilon}_v} \left\{ \frac{\varepsilon_u}{e(1-\gamma)-1} + \frac{(1-\gamma)}{1-\gamma + \mu^* (\gamma - \eta_1)} \left[1 - (\mu^* - 1) \eta_1 \left(2 - \frac{\eta_2}{1-\eta_1} \right) \right] + \frac{\mu^* (\gamma - \eta_1)}{1-\gamma + \mu^* (\gamma - \eta_1)} \frac{1-\rho + \eta_1/\gamma}{1+\alpha} \right\} \quad (1.33)$$

$$D(0) = \frac{1}{\bar{\varepsilon}_v} \left[\frac{\eta_1}{e(1-\gamma)-1} (\varepsilon_u - \bar{\varepsilon}_u) \right] \quad (1.34)$$

$$\text{with } 1 - T(0) + D(0) = \frac{1}{\bar{\varepsilon}_v} \frac{(1-\eta_1)}{e(1-\gamma)-1} (\varepsilon_u^s - \varepsilon_u) \quad (1.35)$$

$$\text{and } \varepsilon_u^s \equiv \frac{e(1-\gamma)-1}{1-\eta_1} \left\{ \eta_1 \frac{(1-\gamma)(\mu^*-1)}{1-\gamma + \mu^* (\gamma - \eta_1)} \left(2 - \frac{\eta_2}{1-\eta_1} \right) + \frac{\rho}{1+\alpha} \frac{\mu^* (\gamma - \eta_1)}{1-\gamma + \mu^* (\gamma - \eta_1)} \right\} \quad (1.36)$$

1.6.2 Fluctuations of a bubble

As already underlined, to prove the existence of bubble fluctuations, the monetary policy is assumed to neither depend on inflation forecasts, nor on asset prices (i.e. $\alpha = \rho = 0$).

²⁶Indeed, we (generically) have an odd number of steady states (see Section 1.5.2).

²⁷The bifurcation values of ε_v , ε_v^s , ε_v^f and ε_v^h , are given in Appendix 1.9.6.

To identify the indeterminacy properties of the steady state $b^* = 1$, we study how the Σ -line evolves in the (T, D) plane in function of η_1 . We assume:

Assumption 8. η_1 is not too large, such that:

$$1 > \frac{\eta_1}{1 - \eta_1} (\mu^* - 1) \left(2 - \frac{\eta_2}{1 - \eta_1} \right) \quad (1.37)$$

Assumption 8 appears reasonable since it implies a not excessively large effect of collateral on the share of consumption purchased on credit and is in accordance with Assumption 6.

From direct inspection of the different bifurcation values ε_v^f , ε_v^h and ε_v^s , we deduce that under Assumptions 1 – 8:

$$\varepsilon_v^s < \varepsilon_v^h < \varepsilon_v^f \quad (1.38)$$

Inequality (1.38) means that as $D(\varepsilon_v)$, $T(\varepsilon_v)$ is increasing in ε_v . Hence, $(T(\varepsilon_v), D(\varepsilon_v))$ goes rightward, crossing $\pm\infty$ when ε_v goes through $\bar{\varepsilon}_v$.

Let $S_C > 0$ be the critical value of S such that the Σ -line goes through the point C .²⁸ When $\eta_1 < \gamma$, it can easily be shown from Eq. (1.32) and inequality (1.38) that the Σ -line has a slope in $(0, S_C)$ under Assumptions 1 – 8. In addition, the Σ -line starts inside the triangle ABC for $\tilde{\varepsilon}_u < \varepsilon_u < \varepsilon_u^s$, then crosses (AC) as $\varepsilon_v = \varepsilon_v^s$, goes inside the triangle ABC by crossing (AB) as $\varepsilon_v = \varepsilon_v^f$, and ends at $(T(+\infty), D(+\infty))$ on the horizontal axis.²⁹ When $\varepsilon_u > \varepsilon_u^s$, the Σ -line starts outside the triangle ABC , on the right side of (AC) (see Figure 1.1).

Proposition 4. Under Assumptions 1-8, the following generically holds:

²⁸ $0 < S_C = 1/(2 - \eta_1) < 1$.

²⁹Because $\eta_1 < \gamma$, $1 - T(0) - D(0) > 0$ is satisfied for $\tilde{\varepsilon}_u < \varepsilon_u < \varepsilon_u^s$.

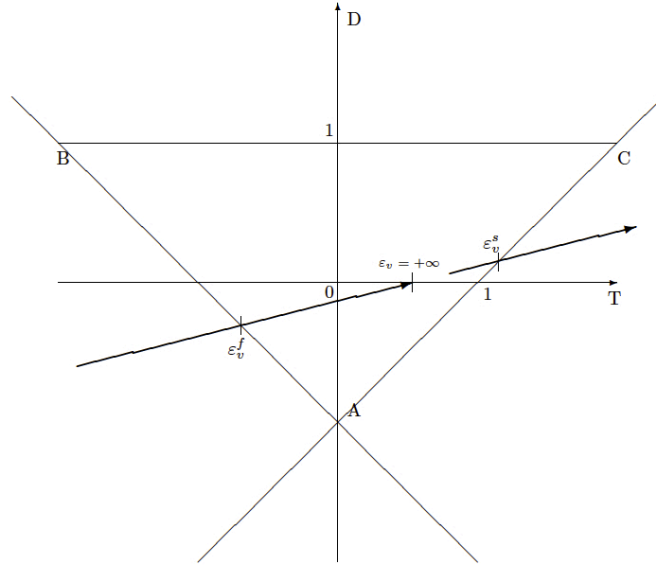


Figure 1.1: Fluctuations of a bubble

1. If $\varepsilon_u \in (\tilde{\varepsilon}_u, \varepsilon_u^s)$, then the steady state is a sink for $\varepsilon_v < \varepsilon_v^s$, undergoes a pitchfork bifurcation for $\varepsilon_v = \varepsilon_v^s$, is a saddle for $\varepsilon_v \in (\varepsilon_v^s, \varepsilon_v^f)$, undergoes a flip bifurcation for $\varepsilon_v = \varepsilon_v^f$, is a sink for $\varepsilon_v > \varepsilon_v^f$.
2. If $\varepsilon_u > \varepsilon_u^s$, then the steady state is a saddle for $\varepsilon_v < \varepsilon_v^f$, undergoes a flip bifurcation for $\varepsilon_v = \varepsilon_v^f$, is a sink for $\varepsilon_v > \varepsilon_v^f$.

Proposition 4.2 establishes that local indeterminacy and endogenous cycles could occur under sufficiently large income effects ($\varepsilon_v > \varepsilon_v^f$). This confirms the well-known result obtained by Grandmont (1985, 1986) even if there are a portfolio choice between two assets, a bubble and money held for transaction motive, and collateral in our model. We can note that when collateral does not matter ($\eta_1 = 0$), the local dynamic properties of the model correspond to Proposition 4.2.

More interesting is Proposition 4.1 which highlights the existence of endogenous fluctuations under sufficiently small income effects ($\varepsilon_v < \varepsilon_v^s$). Moreover, this last result is associated to the multiplicity of steady states through the pitchfork bifurcation. This can be seen as a form of global indeterminacy in our framework

(see Proposition 3). Global and local indeterminacy of equilibrium provide a theoretical explanation for large fluctuations, since agents can coordinate their expectations on dynamic paths that converge to different steady states. Figure 1.2 gives a qualitative illustration of an economy characterized by three steady states with different stationary values of bubble and different stability properties.³⁰ Global indeterminacy is not new in models with rational bubble, but is usually explained by the coexistence of a bubbly steady state and a bubbleless one. As shown by Tirole (1985) and Weil (1987), there is only one equilibrium path converging to the bubbly steady state and multiple paths converging towards the steady state without bubbles. In contrast, in our framework, there exist multiple equilibrium paths converging towards different steady states with different stationary values of the bubble, for a given past price of the bubble b_{-1} . Thus, shocks on expectations can explain the excessive volatility in asset prices observed in the data. This is a new result with respect to the literature.

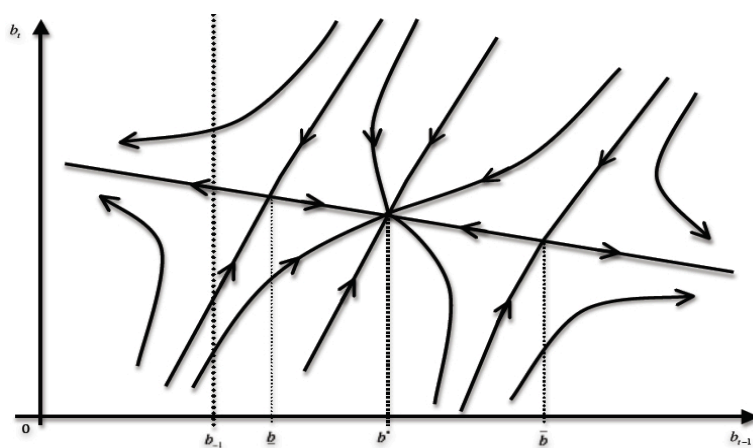


Figure 1.2: Dynamics

Our intuition for the existence of endogenous fluctuations relies on income effects and portfolio effects stemming from a change of inflation expectations. Income

³⁰ \underline{b} and \bar{b} are saddle-points and b^* a sink.

effects following an expected rise in inflation lead to a reduction of consumption in both the first and the second period, and thus to an increase in savings. Since the second-period consumption is decreasing, the return on savings must decrease also. An effective rise in inflation takes place.³¹ However, when income effects are small, this mechanism is no more relevant.

When collateral matters, a change in expectations generates a change in the portfolio structure creating portfolio effects. Portfolio effects following a rise in expected inflation induce a reallocation from monetary to non-monetary savings, increasing the share of the bubble in total savings. If the portfolio effects dominate, then the non-monetary savings increases. This implies a rise of collateral value which allows households to increase the share of consumption purchased on credit in the second period of life. However, as collateral effect is small enough (i.e. η_1 small), the consumption financed on credit decreases because of lower total savings. Since this consumption on credit is financed by the resale of the bubble at the end of second period, the return of the bubble must decrease. An effective rise in inflation takes place, and expectations are self-fulfilling.³²

We have shown that a rational bubble could experience endogenous fluctuations driven by the volatility of agents' expectations. We turn now to the analysis of the (de-)stabilizing role of monetary policy. A monetary policy is locally (globally) stabilizing in our framework as soon as it reduces the range of parameter values for which local (global) indeterminacy occurs.

³¹To make easier, suppose $\eta_1 = 0$. From the binding cash-in-advance constraint and the money growth factor (Eq.(15)), we obtain $\gamma d_{t+1} = (\mu^*/\pi_{t+1})m_t$. If d_{t+1} is decreasing and m_t increasing, μ^*/π_{t+1} must decrease.

³²From Eqs. (1.18) and (1.20), we get $d_{t+1} = (\mu^*/\pi_{t+1})(b_t/\gamma(b_t))\gamma(b_{t-1})/[1 - \gamma(b_{t-1})]$. Note that $b_t/\gamma(b_t)$ is increasing with b_t and $\gamma(b_{t-1})/[1 - \gamma(b_{t-1})]$ is considered as given at period t . If b_t is increasing and d_{t+1} decreasing, μ^*/π_{t+1} must decrease.

1.7 The (de-)stabilizing role of monetary policy

The question we address now is: Can a monetary policy protect the economy against such bubble fluctuations driven by the volatility of agents' expectations? The issue has been essentially explored within economies without collateral and portfolio choice (Grandmont, 1985, 1986; Sorger, 2005). In line with Sorger (2005), we first study an inflation forecast targeting rule which responds only to the expected inflation ($\alpha \neq 0$ and $\rho = 0$). Second, we analyze a rule taking into account asset prices. We extend the inflation forecast targeting rule by including directly asset price level ($\alpha \neq 0$ and $\rho \neq 0$). Considering such a monetary rule may be appropriate in an economy in which a bubble could persist and experience fluctuations. We will show that a rule including asset prices has better stabilizing virtues than a monetary rule which responds only to inflation. To do this, we examine how local dynamics are altered by the implementation of monetary policy. As previously, our analysis focuses on a not too large value of η_1 .

1.7.1 Inflation forecast targeting

Referring to Sorger (2005), we start by analyzing the stabilizing role of inflation forecast targeting on the emergence of fluctuations. We extend his analysis to a model with two assets, a bubble and money held for a transaction motive, and most importantly in which collateral matters.

The monetary authority conducts a monetary rule given by Eq. (1.15) with $\rho = 0$:

$$\mu_t = \mu^* \left(\frac{\pi_{t+1}}{\mu^*} \right)^{-\alpha}, \quad \text{with } \alpha \geq -1.$$

The purpose is to evaluate whether inflation forecast targeting stabilizes or rather destabilizes expectation-driven fluctuations around the bubbly steady state. As in the previous section, we analyze local dynamics focusing on geometrical arguments. We can easily check that under Assumptions 1 – 8, the slope S is always positive.

For any value of α , the Σ -line has a slope between $(0, S_C)$ and starts inside the triangle ABC for $\tilde{\varepsilon}_u < \varepsilon_u < \varepsilon_u^s$, then crosses (AC) as $\varepsilon_v = \varepsilon_v^s$, goes inside the triangle ABC by crossing (AB) as $\varepsilon_v = \varepsilon_v^f$, and ends at $(T(+\infty), D(+\infty))$ on the horizontal axis. When $\varepsilon_u > \varepsilon_u^s$, the Σ -line starts on the right side of (AC) . These configurations are very similar to Proposition 4.

To summarize, under Assumptions 1-8 local indeterminacy occurs if $\varepsilon_v < \varepsilon_v^s$ or if $\varepsilon_v > \varepsilon_v^f$. Furthermore, we recall that from Proposition 3 the multiplicity of steady states emerges for $\varepsilon_v < \varepsilon_v^s$.

To get a picture of the role of this monetary policy on the occurrence of local indeterminacy and the multiplicity of steady states, we only need to examine how these critical bifurcation values (ε_v^f and ε_v^s) vary in function of α .³³ See also Figure 1.3 for a qualitative illustration.³⁴

Proposition 5. *Under Assumptions 1-8, the following generically holds:*

1. *The more active inflation forecast targeting (α higher) is, the more destabilizing it is in the neighborhood of the steady state b^* for large income effects.*
2. *An inflation targeting monetary policy is neither destabilizing nor stabilizing in the neighborhood of the steady state b^* for small income effects.*

³³From the steady state analysis (see Section 1.4.2), we know that $\varepsilon_v < \varepsilon_v^s$ is a sufficient condition to obtain the multiplicity of steady states. Therefore, ensuring $\varepsilon_v > \varepsilon_v^s$ is a way to eliminate, at least, stationary solutions not too far from $b^* = 1$. This is what we call a globally stabilizing monetary policy.

³⁴Figure 1.3 qualitatively illustrates the case $\varepsilon_u \in (\tilde{\varepsilon}_u, \varepsilon_u^s)$. For $\varepsilon_u > \varepsilon_u^s$, the figure would be qualitatively the same but without the line ε_v^s and the region below this line.

3. An inflation forecast targeting has no impact on conditions for the existence of the multiplicity of steady states.

Proof. See Appendix 1.9.7.

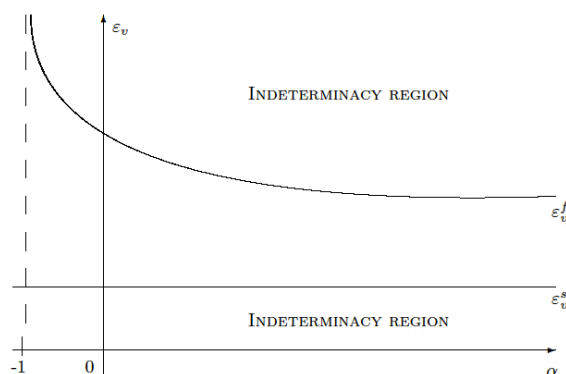


Figure 1.3: Stabilizing role of inflation forecast targeting

Proposition 5 suggests that an active inflation forecast targeting promotes the emergence of expectation-driven fluctuations by increasing the range of ε_v for local indeterminacy. Even if we consider an OLG setting with a portfolio choice and the money growth as monetary instrument, this result is in accordance with the literature about forward looking policy rules, showing that an active rule, responding to expected inflation instead of current inflation, is destabilizing (see among others Bernanke and Woodford, 1997; and Sorger, 2005).

Furthermore, we claim that an inflation forecast targeting does not affect conditions for global indeterminacy. Such a monetary rule is not able to restore the uniqueness of the steady state. This means that an inflation forecast targeting via the money growth factor fails to coordinate agents' decisions on the steady state b^* chosen as target. One possible explanation could come from the fact that the unique targeting variable of this monetary rule π_{t+1} is a (gross) rate. Hence, the long-term money growth factor is equal to μ^* at any steady state (see Eq. (1.22)).

Under an inflation forecast targeting, monetary policy does not help agents to discriminate between the different steady states. Therefore, global indeterminacy persists. Adding a targeting variable defined in level, as the asset prices, to the monetary rule could solve the coordination issue. Moreover, such a rule could also counteract the collateral effect, source of fluctuations.

1.7.2 Monetary rule with asset price level

To answer the question addressed in particular by Bernanke and Gertler (2001), “Should central banks respond to movements in asset prices?”, we enrich the inflation forecast targeting rule, including the asset price level. More precisely, the money growth factor also responds to asset price deviations from the stationary value b^* :

$$\mu_t = \mu^* \left(\frac{\pi_{t+1}}{\mu^*} \right)^{-\alpha} \left(\frac{b_t}{b^*} \right)^{-\rho}, \quad \text{with } \alpha \geq -1, \rho \geq -1.$$

We examine whether taking into account asset prices in the monetary rule significantly alters the (de-)stabilizing role of monetary policy. Thus, we extend our previous analysis to changes in ρ for a given value of α .

We focus again on geometrical arguments to analyze dynamics. When ρ increases from -1 to $+\infty$, the location of the Σ -line in the plane $(T(\varepsilon_v), D(\varepsilon_v))$ differs from the previous analysis.³⁵

Let $S_{\rho=0}$ be the slope of the Σ -line when $\rho = 0$, which is the slope studied in the previous section when the monetary policy does not respond to asset prices (Section 1.7.1). When $\rho = -1$, the configuration is quite similar to Proposition 4. Indeed, the slope S of the Σ -line belongs to $(0, S_{\rho=0})$ (see Eq. (1.32)) and $\varepsilon_v^s < \varepsilon_v^h < \varepsilon_v^f$. However, under some parameter restrictions ε_v^s is negative. As a result, when

³⁵Obviously, for $\rho = 0$, Propositions 5 applies.

$\rho = -1$, the Σ -line starts on the left side of (AC) ,³⁶ goes rightwards, then goes inside the triangle ABC by crossing (AB) between the points A and B as $\varepsilon_v = \varepsilon_v^f$, and ends at $(T(+\infty), D(+\infty))$ on the horizontal axis. As the slope S of the Σ -line is increasing in ρ , the Σ -line makes a counterclockwise rotation around the endpoint $(T(+\infty), D(+\infty))$ when ρ increases. Thus, the Σ -line first goes through C (i.e. $\varepsilon_v^s = \varepsilon_v^h$), second through A (i.e. $\varepsilon_v^s = \varepsilon_v^f$), then through B (i.e. $\varepsilon_v^f = \varepsilon_v^h$), and becomes flat close to zero.

To summarize, under Assumptions 1-8 local indeterminacy occurs if $\varepsilon_v < \min(\varepsilon_v^s, \varepsilon_v^f, \varepsilon_v^h)$ and if $\varepsilon_v > \max(\varepsilon_v^f, \varepsilon_v^s)$, and the multiplicity of steady states if $\varepsilon_v < \varepsilon_v^s$. See also Figure 1.4 for a qualitative illustration.³⁷

To derive results about this monetary policy, we study how these critical bifurcation values (ε_v^s , ε_v^f and ε_v^h) vary with respect to ρ for a given value of α .

Let ρ^A be the crossing point between ε_v^s and ε_v^f .³⁸ The next proposition summarizes the role of a monetary policy on local indeterminacy and the multiplicity of steady states:

Proposition 6. *Under Assumptions 1-8, the following generically holds:*

1. *If $-1 < \rho < \rho^A$, the more passive the monetary policy is, the more stabilizing it is in the neighborhood of the steady state b^* .*
2. *If $\rho^A < \rho$, the more active the monetary policy is, the more stabilizing it is in the neighborhood of the steady state b^* .*
3. *A more passive monetary policy rules out the multiplicity of steady states.*

³⁶When $\rho = -1$, the configuration with $\varepsilon_v^s > 0$ is quite similar to $\varepsilon_v^s < 0$, except that the Σ -line starts inside the triangle ABC . However, as regards the stabilizing virtues of monetary policy, our results are qualitatively the same.

³⁷Figure 1.4 depicts the stabilizing role of monetary policy with asset prices when $\varepsilon_u \in (\tilde{\varepsilon}_u, \varepsilon_u^s)$ for $\rho = 0$.

³⁸The value of ρ^A is given in Appendix 1.9.8. We can easily show that under Assumptions 1-8 $\rho^A > 0$.

Proof. See Appendix 1.9.8.

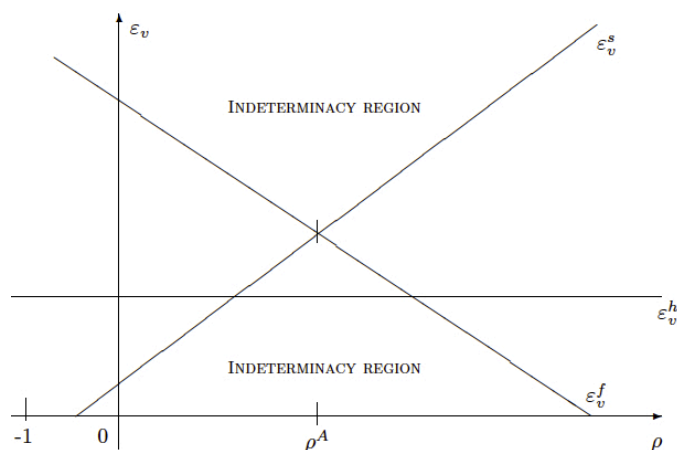


Figure 1.4: Stabilizing role of monetary policy with asset prices

A sufficiently active monetary policy taking asset prices into account (i.e. $\rho > \rho^A$) hardens the condition on ε_v , which leads to local indeterminacy. However, a passive monetary rule makes local and global indeterminacy less likely to emerge. This means that even if an active monetary rule can locally stabilize, a passive one may both locally and globally stabilize. The steady state b^* is getting determinate, and the multiplicity of steady states disappears when the policy is passive.

When asset prices are taken into account by the monetary authority, a monetary rule can stabilize for small income effects and rule out the multiplicity of steady states, which is not the case when the monetary rule responds only to expected inflation. By adding the targeting variable b_t to the rule, the long-term money growth factor differs according to the steady state, that is $\mu = \mu^*(b/b^*)^{-\rho/(1+\alpha)}$ (see Eq. (1.22)). Under a monetary policy responding to asset prices, agents are now able to discriminate between the steady states and coordinate their expectations on one dynamic path converging to the stationary equilibrium chosen as the target by the monetary authority in contrast to inflation forecast targeting.

Following the debate initiated by Bernanke and Gertler (1999, 2001) and recently advanced by several Governors of central bank or practitioners of monetary policy (Yellen, 2009; Bernanke, 2010, 2011), we provide a conclusion in favor of including asset prices in monetary policy rules. Although our theoretical framework differs from dynamic stochastic general equilibrium ones, our results are in accordance with some of other recent contributions (Nutahara, 2010): Under credit market imperfections, a monetary policy should respond to movements in asset prices to rule out expectation-driven fluctuations. Furthermore, our conclusions about the stance of the monetary policy with respect to bubble tend to mitigate the recommendation of Gali (2014). Gali (2014) analyzes the effect of the monetary policy on bubble fluctuations occurring in the neighborhood of a steady state in an OLG model with nominal rigidities, but without financial frictions. He shows that the monetary authority should not lead a policy of “leaning against the wind”, but should lower the interest rate when the bubble grows to locally stabilize. In our paper, we claim that the monetary authority should lead either a sufficiently active or a sufficiently passive rule to locally stabilize. Note that this result is based on a local analysis. An active policy stabilizing around the normalized steady state $b^* = 1$ also affects the stability of other steady states, by destabilizing them. However, we also provide an analysis about the impact of our monetary policy on global indeterminacy, and we argue that the monetary authority should not strongly react to asset prices, but should implement a passive rule to globally stabilize. Under a passive rule, the monetary authority can restore the uniqueness of the bubbly steady state, and thus prevent the existence of several bubbly equilibria in the economy.

1.8 Concluding remarks

We present a monetary overlapping generations model with two assets, bubble and money needed to transaction motives, and a collateral effect. The endogenous portfolio choice promotes expectation-driven fluctuations and the multiplicity of steady states, i.e. global indeterminacy, for small income effects and arbitrarily small distortions.

When monetary policy only depends on expected inflation, an active policy tend to be destabilizing, by promoting expectation-driven fluctuations occurring for large income effects. However, such a rule has no impact on the occurrence of expectation-driven fluctuations for small income effects and on the multiplicity of steady states.

In our model, a monetary policy including asset prices may rule out aggregate fluctuations. Indeed, a sufficiently active rule stabilizes fluctuations around the steady state both for small and large income effects. In addition, a monetary rule on asset prices, in particular a passive one, can locally and globally stabilize fluctuations unlike a monetary rule which only responds to inflation. Our results mitigate the conclusions pronounced by Bernanke and Gertler (1999, 2001), and reinforced, for instance, by Carlstrom and Fuerst (2007). Focusing on consumers' credit constraints and endogenous bubble fluctuations, including asset prices in monetary policy rules is powerful to rule out macroeconomic instability.

Finally, our qualitative results are obtained for all value of monetary growth factor's target μ^* . Therefore, these conclusions are compatible with a target μ^* which maximizes households' welfare at the steady state. When the monetary rule including asset prices is not too passive, $\mu^* = 1$ is an optimal rule. We recognize the Friedman rule: No money growth and a nominal interest rate equals to zero ($\mu^* = 1 + i = 1$). The intertemporal choices of households are no longer affected by

the distortions due to money holdings. From Eq. (1.24) at the normalized steady state, we obtain $u'(e - 1/[1 - \gamma(1)]) = \beta^* v'(1/[1 - \gamma(1)])$, which corresponds to the Phelps Golden rule in a monetary OLG model without population growth. Thus, a stabilizing monetary rule, which eliminates the cost of fluctuations, may be in accordance with welfare maximization at the steady state.

1.9 Appendix

1.9.1 Proof of Lemma 1

We maximize the Lagrangian function:

$$\begin{aligned}
 & u(c_t) + \beta v(d_{t+1}) \\
 & + \lambda_{1t}(e + \tau_t - \pi_{t+1}m_{t+1} - b_t - c_t) \\
 & + \lambda_{2t}\left(m_{t+1} + \frac{1 + i_{t+1}}{\pi_{t+1}}b_t - d_{t+1}\right) \\
 & + \lambda_{3t}(m_{t+1} - \gamma(b_t)d_{t+1})
 \end{aligned} \tag{1.39}$$

with respect to $(c_t, d_{t+1}, m_{t+1}, b_t, \lambda_{1t}, \lambda_{2t}, \lambda_{3t})$. Since $\lambda_{1t} = u'(c_t) > 0$, then Eq. (1.8) becomes binding. Because

$$\lambda_{2t} = \lambda_{1t} \frac{1 + \gamma'(b_t)d_{t+1}\pi_{t+1}}{\frac{1+i_{t+1}}{\pi_{t+1}} + \gamma'(b_t)d_{t+1}} \tag{1.40}$$

$$\lambda_{3t} = \lambda_{1t} \left(\pi_{t+1} - \frac{1 + \pi_{t+1}\gamma'(b_t)d_{t+1}}{\frac{1+i_{t+1}}{\pi_{t+1}} + \gamma'(b_t)d_{t+1}} \right) \tag{1.41}$$

strict positivity of λ_{2t} and λ_{3t} requires

$$\pi_{t+1} > \frac{1 + \gamma'(b_t)\pi_{t+1}d_{t+1}}{\frac{1+i_{t+1}}{\pi_{t+1}} + \gamma'(b_t)d_{t+1}} > 0 \tag{1.42}$$

Inequality $1 + \gamma'(b_t) \pi_{t+1} d_{t+1} > 0$ is equivalent to inequality (1.11). Moreover, $i_{t+1} > 0$ implies $(1 + i_{t+1}) + \gamma'(b_t) \pi_{t+1} d_{t+1} > 1 + \gamma'(b_t) \pi_{t+1} d_{t+1}$, which ensures that inequality (1.42) holds. \square

1.9.2 Proof of Lemma 2

We compute the Hessian matrix of the Lagrangian function (1.39) with respect to $(c_t, d_{t+1}, m_{t+1}, b_t, \lambda_{1t}, \lambda_{2t}, \lambda_{3t})$ ³⁹:

$$H \equiv \begin{bmatrix} u'' & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & \beta v'' & 0 & -\lambda_3 \gamma' & 0 & -1 & -\gamma \\ 0 & 0 & 0 & 0 & -\pi & 1 & 1 \\ 0 & -\gamma' \lambda_3 & 0 & -\gamma'' \lambda_3 d & -1 & \frac{1+i}{\pi} & -\gamma' d \\ -1 & 0 & -\pi & -1 & 0 & 0 & 0 \\ 0 & -1 & 1 & \frac{1+i}{\pi} & 0 & 0 & 0 \\ 0 & -\gamma & 1 & -\gamma' d & 0 & 0 & 0 \end{bmatrix} \quad (1.43)$$

In order to get a strict local maximum, we need to check the negative definiteness of H over the set of points satisfying the constraints. Let p and n the numbers of constraints and variables. If the determinant of H has sign $(-1)^n$, and the last $n - p$ diagonal principal minors have alternating signs, then the optimum is a regular local maximum. In our case $n = 4$ and $p = 3$. Therefore, we simply require

³⁹For simplicity, the arguments of the functions and the time subscripts are omitted.

$\det H > 0$, that is,

$$\begin{aligned} \det H = & -u'' \left[\pi \left(\gamma' d + \frac{1+i}{\pi} \gamma \right) + (1-\gamma) \right]^2 \\ & - \beta v'' \left(\gamma' d + \frac{1+i}{\pi} \right)^2 \\ & + \lambda_3 (1-\gamma) \left[2 \left(\gamma'^2 d + \gamma' \frac{1+i}{\pi} \right) + \gamma'' d (1-\gamma) \right] > 0 \end{aligned} \quad (1.44)$$

Using Eqs. (1.9) and (1.10), we find $d_{t+1} = \frac{b_t}{1-\gamma(b_t)} \frac{1+i_{t+1}}{\pi_{t+1}}$. Substituting in Eq. (1.44), we get:

$$\det H = - \left(\frac{1+i}{\pi} \right)^2 \left[\chi_0 + \chi_1^2 u'' + \beta v'' (1-\eta_1)^2 \right] > 0 \quad (1.45)$$

where

$$\begin{aligned} \chi_0 & \equiv \lambda_3 \eta_1 \frac{(1-\gamma)^2}{b} [2(1-\eta_1) - \eta_2] \frac{\pi}{1+i} \\ \chi_1 & \equiv \pi (\gamma - \eta_1) + \frac{1-\gamma}{1+i} \pi \end{aligned}$$

$\chi_0 + \chi_1^2 u'' < 0$ is a sufficient condition for $\det H > 0$. Hence, the second order conditions are satisfied if:

$$\varepsilon_u > \frac{c(1-\gamma)^2/b}{[1-\gamma + (1+i)(\gamma - \eta_1)]^2} i \eta_1 \left(2 - \frac{\eta_2}{1-\eta_1} \right) \equiv \tilde{\varepsilon}_u \quad (1.46)$$

whatever the value of $\varepsilon_v \geq 0$ □

1.9.3 Proof of Proposition 1

A steady state b is a solution of $g(b) = h(b)$, with:

$$g(b) \equiv \frac{u' \left(e - \frac{b}{1-\gamma(b)} \right)}{\beta v' \left(\frac{b}{1-\gamma(b)} \right)} \quad (1.47)$$

$$h(b) \equiv \frac{1 - \eta_1(b)}{1 - \gamma(b) + [\gamma(b) - \eta_1(b)] \mu(b)} \quad (1.48)$$

Since $\frac{b}{1-\gamma(b)}$ is increasing in b and $c > 0$, then $b < \bar{b}$, where $\bar{b} > 0$ is such that $e = \frac{\bar{b}}{1-\gamma(\bar{b})}$.⁴⁰ Therefore, all the stationary solutions b belong to $(0, \bar{b})$.

To prove the existence of a stationary solution b , we use the continuity of $g(b)$ and $h(b)$. From Eqs. (1.24), (1.47) and (1.48), we determine the boundary values of $g(b)$ and $h(b)$:

$$\lim_{b \rightarrow 0} g(b) = \frac{u'(e)}{\beta v'(0)} = 0^+, \quad \lim_{b \rightarrow \bar{b}} g(b) = +\infty$$

$$\lim_{b \rightarrow 0} h(b) = \frac{1 - \eta_1(0)}{1 - \gamma(0) + [\gamma(0) - \eta_1(0)] \mu(0)} \geq 0$$

$$\lim_{b \rightarrow \bar{b}} h(b) = \frac{1 - \eta_1(\bar{b})}{1 - \gamma(\bar{b}) + [\gamma(\bar{b}) - \eta_1(\bar{b})] \mu(\bar{b})} > 0$$

We have $\lim_{b \rightarrow \bar{b}} g(b) > \lim_{b \rightarrow \bar{b}} h(b)$. If $-1 < \rho \leq 0$, $\mu(0) < +\infty$, which implies $\lim_{b \rightarrow 0} g(b) < \lim_{b \rightarrow 0} h(b)$. However, if $\rho > 0$, $\mu(0) = +\infty$, which implies $g(0) = h(0)$. In this case, the existence of a steady state $b \in (0, \bar{b})$, solving $g(b) = h(b)$, is ensured by $\varepsilon_g(0) < \varepsilon_h(0)$, where:

⁴⁰With our requirements on the $\gamma(b)$ function, \bar{b} is well defined under Assumption 3.

$$\begin{aligned}\varepsilon_g(b) &\equiv \frac{g'(b)b}{g(b)} = \varepsilon_u(c) \frac{1 - \eta_1(b)}{e[1 - \gamma(b)] - b} b + [1 - \eta_1(b)] \varepsilon_v(d) > 0 \\ \varepsilon_h(b) &\equiv \frac{h'(b)b}{h(b)} = \eta_1(b) \frac{[1 - \gamma(b)] [\mu(b) - 1] \{2[1 - \eta_1(b)] - \eta_2(b)\}}{[1 - \eta_1(b)] \{1 - \gamma(b) + \mu(b) [\gamma(b) - \eta_1(b)]\}} \\ &\quad + \frac{\rho}{1 + \alpha} \frac{\mu(b) [\gamma(b) - \eta_1(b)]}{1 - \gamma(b) + \mu(b) [\gamma(b) - \eta_1(b)]}\end{aligned}$$

Under Assumption 4, this last condition is satisfied. A sufficient condition for uniqueness is $\varepsilon_h(b) < \varepsilon_g(b)$ for all $b \in (0, \bar{b})$, or equivalently,

$$\begin{aligned}\varepsilon_v(d) &> \frac{b}{e[1 - \gamma(b)] - b} [\varepsilon_u^s(b) - \varepsilon_u(c)] \\ \text{with } \varepsilon_u^s(b) &\equiv \frac{e[1 - \gamma(b)] - b}{b} \left\{ \frac{\eta_1(b)}{1 - \eta_1(b)} \frac{[1 - \gamma(b)] [\mu(b) - 1]}{1 - \gamma(b) + \mu(b) [\gamma(b) - \eta_1(b)]} \right. \\ &\quad \left. \left(2 - \frac{\eta_2(b)}{1 - \eta_1(b)} \right) + \frac{\rho}{1 + \alpha} \frac{\mu(b) [\gamma(b) - \eta_1(b)]}{1 - \gamma(b) + \mu(b) [\gamma(b) - \eta_1(b)]} \frac{1}{1 - \eta_1(b)} \right\}\end{aligned}\tag{1.49}$$

$$c = e - \frac{b}{1 - \gamma(b)} \text{ and } d = \frac{b}{1 - \gamma(b)}\tag{1.50}$$

□

1.9.4 Proof of Proposition 3

Using the notations of the proof of Proposition 1, we know that $g(\bar{b}) > h(\bar{b})$ and under Assumption 4, $g(b) < h(b)$ for $b > 0$ but arbitrarily close to 0. Since $b^* = 1$ is a steady state, we have: $g(1) = h(1)$. If inequality (1.27) is satisfied, we have $g'(1) < h'(1)$, then by continuity there exist at least two other steady states, b_1 and b_2 such that $b_1 < 1 < b_2$. The number of steady states is generically odd. □

1.9.5 Proof of Lemma 3

Substituting Eq. (1.22) into Eq. (1.21) and differentiating around the NSS with respect to b_{t-1} , b_t and b_{t+1} , we obtain the following linearization⁴¹:

$$\begin{aligned} & \left[\varepsilon_v - \frac{1 - \gamma + \mu^* (\gamma - \eta_1) \frac{1}{1+\alpha}}{1 - \gamma + \mu^* (\gamma - \eta_1)} \right] db_{t+1} + \left[\frac{\varepsilon_u}{e(1-\gamma) - 1} - \eta_1 \varepsilon_v + 1 \right. \\ & \left. - \frac{(1-\gamma)(\mu^* - 1)\eta_1}{1 - \gamma + \mu^* (\gamma - \eta_1)} \left(2 - \frac{\eta_2}{1 - \eta_1} \right) + \frac{\mu^* (\gamma - \eta_1)}{1 - \gamma + \mu^* (\gamma - \eta_1)} \frac{-\alpha - \rho + \eta_1/\gamma}{1 + \alpha} \right] db_t \\ & - \left[\eta_1 \frac{\varepsilon_u}{e(1-\gamma) - 1} + \frac{\mu^* (\gamma - \eta_1)}{1 - \gamma + \mu^* (\gamma - \eta_1)} \frac{\eta_1/\gamma}{1 + \alpha} \right] db_{t-1} = 0 \end{aligned}$$

Lemma 3 follows. □

1.9.6 Bifurcation values of ε_v

ε_v^s is defined by $1 - T(\varepsilon_v) + D(\varepsilon_v) = 0$:

$$\varepsilon_v^s = \frac{1}{e(1-\gamma) - 1} (\varepsilon_u^s - \varepsilon_u),$$

where ε_u^s is given by Eq. (1.36).

ε_v^f is defined by $1 + T(\varepsilon_v) + D(\varepsilon_v) = 0$:

$$\begin{aligned} \varepsilon_v^f = & \frac{\varepsilon_u}{e(1-\gamma) - 1} + \frac{1-\gamma}{1 - \gamma + \mu^* (\gamma - \eta_1)} \left[\frac{2}{1 + \eta_1} - \frac{\eta_1}{1 + \eta_1} (\mu^* - 1) \left(2 - \frac{\eta_2}{1 - \eta_1} \right) \right] \\ & + \frac{2}{1 + \eta_1} \frac{\mu^* (\gamma - \eta_1)}{1 - \gamma + \mu^* (\gamma - \eta_1)} \frac{1 + \eta_1/\gamma - \rho/2}{1 + \alpha} \end{aligned}$$

ε_v^h is defined by $D = 1$:

$$\varepsilon_v^h = \frac{1-\gamma}{1 - \gamma + \mu^* (\gamma - \eta_1)} + \frac{\mu^* (\gamma - \eta_1)^2}{1 - \gamma + \mu^* (\gamma - \eta_1)} \frac{1/\gamma}{1 + \alpha} - \eta_1 \frac{\varepsilon_u}{e(1-\gamma) - 1}$$

⁴¹The arguments of the functions are omitted.

□

1.9.7 Proof of Proposition 5 ($\rho = 0$)

$$\begin{aligned}\frac{d\varepsilon_v^s}{d\alpha} &= -\frac{1}{(1+\alpha)^2} \frac{\mu^*(\gamma - \eta_1)}{1 - \gamma + \mu^*(\gamma - \eta_1)} \frac{\rho}{1 - \eta_1} = 0 \\ \frac{d\varepsilon_v^f}{d\alpha} &= -\frac{1}{(1+\alpha)^2} \frac{2}{1 + \eta_1} \frac{\mu^*(\gamma - \eta_1)}{1 - \gamma + \mu^*(\gamma - \eta_1)} (1 + \eta_1/\gamma - \rho/2) < 0\end{aligned}$$

Proposition 5 follows. □

1.9.8 Proof of Proposition 6

Under Assumptions 1 – 8, we obtain the following derivatives:

$$\begin{aligned}\frac{d\varepsilon_v^s}{d\rho} &= \frac{1}{1+\alpha} \frac{\mu^*(\gamma - \eta_1)}{1 - \gamma + \mu^*(\gamma - \eta_1)} \frac{1}{1 - \eta_1} > 0 \\ \frac{d\varepsilon_v^f}{d\rho} &= -\frac{1}{1+\alpha} \frac{1}{1 + \eta_1} \frac{\mu^*(\gamma - \eta_1)}{1 - \gamma + \mu^*(\gamma - \eta_1)} < 0 \\ \frac{d\varepsilon_v^h}{d\rho} &= 0\end{aligned}$$

Furthermore, ρ^A is defined by $\varepsilon_v^f = \varepsilon_v^s$:

$$\begin{aligned}\rho^A \equiv & (1 + \alpha) \left\{ \frac{\varepsilon_u}{e(1 - \gamma) - 1} \frac{1 - \gamma + \mu^*(\gamma - \eta_1)}{\mu^*(\gamma - \eta_1)} (1 - \eta_1^2) \right. \\ & \left. + \frac{1 - \gamma}{\mu^*(\gamma - \eta_1)} \left[1 - \eta_1 - \eta_1 (\mu^* - 1) \left(2 - \frac{\eta_2}{1 - \eta_1} \right) \right] + \frac{(1 - \eta_1)(1 + \eta_1/\gamma)}{1 + \alpha} \right\}\end{aligned}$$

Proposition 6 follows.

Chapter 2

The stabilizing virtues of fiscal vs. monetary policy on endogenous bubble fluctuations¹

2.1 Introduction

In recent years, asset prices have experienced large fluctuations, and the financial sphere of the economy had strong effects on the real one as illustrating during the last financial crisis. Some empirical contributions shed light on the excessive asset price volatility, and reveal that asset prices fluctuate more than their fundamental value (see Shiller, 1981, 1989, 2000; LeRoy and Porter, 1981; or Campbell 2003). One explanation for this excessive volatility is the existence and the fluctuations of asset bubbles. This paper aims to analyze the stabilizing virtues of fiscal and monetary policies in an economy characterized by bubble fluctuations.

¹This paper is a joint work with Thomas Seegmuller.

A large body of theoretical literature explores the role of credit market imperfections in the existence and dynamics of rational bubbles (Farhi and Tirole, 2012; Martin and Ventura 2012; Wang and Wen 2012; Bosi and Seegmuller, 2010a). Despite the fact that many contributions deal with credit constraints at the level of entrepreneurs, some empirical studies highlight the existence of credit constraints faced by consumers underlying the role of collateral on their behavior (Campbell and Mankiw, 1989; Jappelli, 1990; Iacoviello, 2004; Crook and Hochguertel, 2005). Such types of credit market imperfections may be a source of portfolio choices between different existing assets, but also gaps between their returns. We think that such credit market imperfections may be a main transmission channel between the financial and the real spheres. In this paper, we argue that credit constraints faced by consumer play a crucial role to explain expectation-driven fluctuations of speculative bubbles, as illustrated during the recent subprime crisis. This idea already appears in Bosi and Seegmuller (2010a) and Clain-Chamosset-Yvrard and Seegmuller (2015).² They assume that the share of consumption financed by credit is positively correlated to collateral. Because of this type of credit market distortions, the portfolio choices are no longer constant through time. A change in agents' expectations generates a new trade-off between asset holdings promoting equilibrium indeterminacy, and thus the occurrence of expectation-driven fluctuations. We enrich these contributions by considering the stabilizing role of fiscal policy as well as monetary policy conducted through a Taylor rule.

Economic fluctuations based on consumer credit constraints also open the door to new policy tools for stabilizing issues. A stabilizing policy must dampen the mechanism that makes for indeterminacy. Since our explanation of expectation-

²Only few other contributions have analyzed the existence of bubble fluctuations with an interplay between the real and the financial spheres of the economy (Michel and Wigniolle, 2003, 2005; Kamihigashi, 2008 Bosi and Seegmuller, 2010a; Wigniolle, 2012).

driven fluctuations relies on a trade-off between different assets namely capital, bonds and money, relevant stabilizing policies are those reducing the gaps between their returns. Monetary policy appears to be a natural policy tool, since it affects the opportunity cost of money holdings through the level of the nominal interest rate. In this paper, we analyze the stabilizing role of monetary policies when contrary to most of the literature, bubble fluctuations appear in an economy with production and positive asset prices (Grandmont, 1985, 1986; Bernanke and Woodford, 1997; Woodford, 1999; Benhabib *et al.*, 2001; Sorger, 2005; and Rochon and Polemarchakis, 2006).

As we focus on the interplay between the real and the financial spheres, another relevant policy could be one that alters the capital return. In this perspective, a progressive taxation of capital income could be a good candidate to dampen the gaps between the returns on physical and monetary assets along the dynamic path. While some recent contributions emphasize that nonlinear capital income taxation may be optimal (Saez, 2013; Farhi *et al.*, 2012), we rather focus on its stabilizing virtues. We provide new insights in favor of progressive capital income taxation. Several papers have explored the stabilizing virtues of progressive income taxation, starting with Guo and Lansing (1998). In these models the key mechanism promoting endogenous fluctuations is explicitly to be found in the existence of the so-called wrong slopes in the labor market. The stabilizing tool is per se labor income taxation, and progressivity may rule out these wrong slopes. The stabilizing role of progressive taxation has also been studied in a finance constrained economy *à la* Woodford (1986) (see among others, Dromel and Pintus (2008) and Lloyd-Braga, Modesto and Seegmuller (2008)). They show that capital market distortions play no role in the occurrence of endogenous fluctuations in Woodford (1986) framework.

In our model, progressivity is also stabilizing, but as we will see, the economic mechanism is different.

We consider an otherwise simple overlapping generations (OLG) model with capital accumulation to highlight the role of consumers' credit market imperfections and collateral in an economy characterized by a rational bubble.³ Households save through bonds, money and capital. Bonds are sold by the monetary authority to supply money. Because of a binding cash-in-advance (CIA) constraint, money is held by households to finance a share of their consumption in the second period of their life. Despite the fact that capital is used for the production, it also serves as a collateral: Holding more capital increases the amount of collateral, and thus allows each household to reduce the share of consumption financed through money. It is important to note that the three assets have different returns. Bonds have larger return than capital because this latter is used as a collateral to relax the consumers' credit constraint, and also a larger return than money because we focus on equilibria with binding constraints. As a direct implication, the Fisher relationship is not satisfied.⁴ The violation of this relationship will represent some portfolio choices that promote indeterminacy, and therefore endogenous fluctuations.

We first prove the existence of a steady state characterized by a positive rational bubble on bonds. Unlike Tirole (1985), any bubbly steady states experience over-accumulation of capital in the absence of capital taxation. In contrast to several existing papers (Farmer, 1986; Benhabib and Laroque, 1988; Rochon and Polemarchakis, 2006), expectation-driven fluctuations occur in the neighborhood

³Our work is closed to the framework developed by Rochon and Polemarchakis (2006) who study the existence of endogenous fluctuations in an OLG model. However, our analysis differs in two main points: First, we take into account the role of collateral on the consumption behavior, second we analyze a monetary policy that could fit better the current practices of central banks, instead of an interest rate pegging.

⁴The Fisher relationship means that the gross real interest rate is equal to the gross nominal interest rate deflated by the gross inflation rate.

of a steady state with a positive rational asset bubble under gross substitutability and reasonable values of input substitution, without requiring arbitrarily large increasing returns to scale (Cazzavillan and Pintus, 2005; Azariadis and Reichlin, 1996). This result is obtained when the share of consumption purchased on credit weakly depends on collateral, but is sufficiently concave. Interestingly, this result is connected to a multiplicity of steady states, i.e. a form of global indeterminacy.

Since expectation-driven fluctuations are mainly driven by portfolio choices between capital, money and bonds due to the violation of the Fisher relationship, we consider policy tools that are able to reduce the gaps between the different returns on assets. Hence, we discuss the stabilizing role of monetary and fiscal policies in a second step.

The monetary authority could play an active role in stabilizing the economy by manipulating the nominal interest rates. We consider that the nominal interest rate is determined according to a Taylor rule on expected inflation. In this case, the results are mitigated. A weakly active policy can even promote endogenous fluctuations for some relevant parameter configurations. In addition, such a policy has no effect on the multiplicity of steady states. One explanation is that such a rule does not strongly modify the nominal interest rate, and therefore does not alter so much the portfolio choice.

As discussed earlier, we introduce a progressive taxation of capital income, used to finance useless public spending under a balanced budget. We pursue the analysis investigating whether such a policy can have stabilizing virtues compared to the monetary policy. We show that increasing the degree of marginal progressivity may rule out expectation-driven fluctuations under gross substitutability by reducing the range of parameters for indeterminacy and for the multiplicity of steady states. This fiscal policy is therefore powerful to stabilize endogenous fluctuations. Contrary

to previous contributions (see for instance Guo and Lansing, 1998), progressive capital income taxation is per se stabilizing through a new mechanism based on the portfolio choice between the different assets and on the gaps between their returns. To summarize, in contrast to a Taylor rule, a progressive taxation of capital income is powerful to stabilize the economy which experiences a positive rational bubble. These opposite results are however mitigated by comparative statics on stationary allocations. We show that a Taylor rule can be welfare improving at the stationary equilibrium, while capital taxation may reduce or even rule out over-accumulation of capital.

This paper is organized as follows. In the next section, we present the model. The intertemporal equilibrium is defined in Section 2.3. Section 2.4 is devoted to the steady state analysis. In Section 2.5, we analyze the occurrence of expectation-driven fluctuations when there is a positive bubble. In Section 2.6, we study the stabilizing role of monetary policy and its effect on the stationary welfare. We analyze the stabilizing role of fiscal, and provide a discussion about its effect on the dynamic efficiency in Section 2.7. Concluding remarks are provided in Section 2.8, and all the proofs are gathered in a final Appendix.

2.2 The model

We consider an OLG model with production in discrete time ($t = 0, 1, \dots, +\infty$). This economy consists of identical two period-lived households, firms, a monetary authority and a government.

2.2.1 Households

There is no population growth, and at each date t , a generation of unit size is born and lives for two periods.

A household derives utility from consumption of final good when young (c_t) and old (d_{t+1}). Her preferences are represented by an additively separable life-cycle utility function:

$$u(c_t) + \beta v(d_{t+1}) = \frac{c_t^{1-\varepsilon_u}}{1-\varepsilon_u} + \beta \frac{d_{t+1}^{1-\varepsilon_v}}{1-\varepsilon_v}, \quad \beta > 0 \quad (2.1)$$

$\varepsilon_u > 0$ and $\varepsilon_v > 0$ denote respectively the degrees of concavity of $u(c_t)$ and $v(d_{t+1})$. We further note that $\varepsilon_v < 1$ implies gross substitutability meaning that savings are an increasing function of the global return on portfolio.⁵

In her first period of life, the household is young and supplies one unit of labor inelastically remunerated at the wage w_t . With this wage, she can consume an amount c_t of final good at price p_t , and save through a diversified portfolio of nominal balances M_{t+1} needed for a transaction motive, productive capital per capital k_{t+1} (with rental factor R_{t+1})⁶ and nominal bonds B_{t+1} (with nominal interest rate i_{t+1}). In our framework, bonds denote short-term nominal debts issued

⁵As we will see below, the consumer problem has the following structure:

$$\begin{aligned} \max \quad & \frac{c_t^{1-\varepsilon_u}}{1-\varepsilon_u} + \beta \frac{d_{t+1}^{1-\varepsilon_v}}{1-\varepsilon_v} \\ \text{st.} \quad & c_t + s_t = w_t \\ & d_{t+1} = \tilde{R}_{t+1}s_t + \Delta_{t+1}, \end{aligned}$$

where s_t represents global savings of a household, \tilde{R}_{t+1} the global return on her portfolio, w_t , her labor income and Δ_{t+1} a monetary transfer. From this problem, we obtain:

$$\frac{ds_t}{d\tilde{R}_{t+1}} \frac{\tilde{R}_{t+1}}{s_t} = \frac{1 - \varepsilon_v \tilde{R}_{t+1}s_t / (\tilde{R}_{t+1}s_t + \Delta_{t+1})}{\varepsilon_u s_t / (w_t - s_t) + \varepsilon_v \tilde{R}_{t+1}s_t / (\tilde{R}_{t+1}s_t + \Delta_{t+1})},$$

which is positive for $\varepsilon_v < 1$.

⁶We assume a full capital depreciation within a period.

by the monetary authority in order to inject money in the economy. In contrast to asset papers with no fundamental value considered as freely disposed of, bonds can have a negative nominal value ($B_{t+1} < 0$).

In her second period of life, she is old. She uses her remunerated savings and her monetary transfer Δ_{t+1} received from the monetary authority to purchase an amount d_{t+1} of final good at price p_{t+1} and to pay a tax on her capital income.⁷ Defining $g(R_{t+1}k_{t+1})$ as the after-tax capital income of a household, the first and second-period budget constraints are written as follows:

$$p_t c_t + M_{t+1} + B_{t+1} + p_t k_{t+1} \leq p_t w_t \quad (2.2)$$

$$\begin{aligned} p_{t+1} d_{t+1} &\leq M_{t+1} + (1 + i_{t+1}) B_{t+1} \\ &\quad + p_{t+1} g(R_{t+1} k_{t+1}) + \Delta_{t+1} \end{aligned} \quad (2.3)$$

Considering a progressive taxation of capital income, we assume:

Assumption 9. *Let $y_k \equiv Rk$. The function $g(y_k): [0, +\infty) \rightarrow [0, +\infty)$ is continuous, with $g(0) = 0$, and C^2 on $(0, +\infty)$. In addition, it satisfies $0 < g'(y_k) \leq 1$ and $g''(y_k) \leq 0$ for all $y_k > 0$. We define $\rho_1(y_k) \equiv y_k \frac{g'(y_k)}{g(y_k)} \in (0, 1)$ and $\rho_2(y_k) \equiv -y_k \frac{g''(y_k)}{g'(y_k)} \geq 0$ as the first and second order elasticities of after-tax capital income respectively.*

The after-tax capital income $g(y_k) > 0$ is increasing, concave and satisfies $g(y_k) \leq y_k$. Assumption 9 implies that the tax function $\tau(y_k) \equiv y_k - g(y_k) \geq 0$ is non-decreasing and convex. In addition, the marginal tax rate $\tau_m(y_k) \equiv 1 - g'(y_k) \in (0, 1)$

⁷Here, we implement a taxation on household capital income because they own the capital stock. It can equivalently be interpreted as a taxation of firms profit. Furthermore, we abstract from labor income since taxing labor income for stabilizing issues is meaningless in our framework as the labor supply is exogenous. Finally, to highlight the role of monetary policy, we also abstract the taxation of bond income, since the monetary policy will affect household behavior through the nominal interest rate i_{t+1} .

is increasing with the tax base as $\rho_2(y_k) > 0$ and flat as $\rho_2 = 0$. We note that the elasticity $\rho_2(y_k)$ is a measure of marginal progressivity.⁸

Furthermore, the household has to pay cash a part of the second period consumption d_{t+1} : Her money demand is rationalized by a cash-in-advance (CIA) constraint. We use the constraint introduced by Hahn and Solow (1995), i.e. $\gamma p_{t+1} d_{t+1} \leq M_{t+1}$, but we extend it to capture the role of collateral:

$$\gamma(k_{t+1})p_{t+1}d_{t+1} \leq M_{t+1} \quad (2.4)$$

A binding cash-in-advance constraint means that a share $\gamma(k_{t+1}) \in (0, 1)$ of her second-period consumption has to be paid cash, i.e. with nominal balances M_{t+1} . As underlined in Rochon and Polemarchakis (2006) and Clain-Chamosset-Yvrard and Seegmuller (2015), the household can consume the remaining share $1 - \gamma(k_{t+1})$ of consumption on credit when old, because she holds $B_{t+1} + p_{t+1}k_{t+1}$ in her portfolio when young. Indeed, in addition to the transfer from the monetary authority Δ_{t+1}/p_{t+1} , the household knows that she will have her remunerated savings from bonds and capital $(1 + i_{t+1})B_{t+1}/p_{t+1} + g(R_{t+1}k_{t+1})$ at the next period. As a result, she can consume on credit by borrowing from a bank or a financial institution an amount equal to $(1 + i_{t+1})B_{t+1}/p_{t+1} + g(R_{t+1}k_{t+1}) + \Delta_{t+1}/p_{t+1}$, that she will pay back at the end of her second period of life. In the following, we refer to $1 - \gamma(k_{t+1})$ as the credit share.

Furthermore, we assume that the credit share is increasing with the amount of physical capital held by a household. Through this assumption, we assert that capital acts as a collateral for the household. Since a collateral is by definition an asset that a household offers a bank or a financial institution to secure a loan, we argue that the value of physical capital k_{t+1} can be pledged as a collateral rather

⁸As $\rho_1 = 1$ and $\rho_2 = 0$, no progressive taxation of capital income is implemented.

than the after-tax capital income $g(R_{t+1}k_{t+1})$. If the household fails to repay the loan, the financial institution can seize its physical capital to recover its losses, and thus become the owner of this capital. Since the financial institution takes less risk, it would be easier for the household to obtain credit from the bank or the financial institution by holding more capital in her portfolio, and thus to reduce her need of cash in her second period of life.

In an OLG model with capital accumulation, Bosi and Seegmuller (2010a) assume that all non-monetary savings (physical capital and an asset paper) can be pledged as a collateral. In contrast to Bosi and Seegmuller (2010a), we assume that only capital acts as a collateral asset, even though the agents can also save through bonds. The fact that bonds cannot be used as a collateral relies on three reasons. First, households can save, but also borrow by issuing bonds. The real demand of bonds can be negative ($b_{t+1} < 0$), while the value of capital is always positive ($k_{t+1} > 0$), since it is used for production. Second, a part of bonds are the counterpart of money. If bonds could serve as a collateral, we should disentangle the bonds using to issue money from the bonds using for savings. And, only bonds using for savings could act as collateral. Such a distinction suppose to define a different interest rate for each type of bonds. In fine, this would be equivalent to introduce an other asset. Third, bonds can be potentially riskier than physical assets. For instance, we can quote the case of Greek bonds. Since the 11th of February 2015, Greek bonds are no longer eligible as collateral in Eurosystem monetary policy operations following a decision of the Governing Council of European Central Bank. For these three reasons and to keep the analysis tractable, we do not consider bonds as collateral.

Our assumption is also in accordance with some empirical studies, focusing on U.S data, underlining the negative correlation between money holdings and

wealth (see Wolff, 1998, 2010). In our framework, k_{t+1} can be seen as a proxy of household's wealth.

This assumption is a simple way to introduce credit market imperfections and to capture the role of collateral on consumption behavior of the household as highlighted by empirical studies (among others, Campbell and Mankiw, 1989; Iacoviello, 2004).⁹

Assumption 10. $\gamma(k) \in (0, 1)$ is a continuous function defined on $[0, +\infty)$, C^2 on $(0, +\infty)$, decreasing ($\gamma'(k) \leq 0$). In addition, we define:

$$\eta_1(k) \equiv \frac{[1 - \gamma(k)]' k}{1 - \gamma(k)} = -\frac{\gamma'(k)k}{1 - \gamma(k)} \geq 0, \quad (2.5)$$

$$\eta_2(k) \equiv -\frac{[1 - \gamma(k)]'' k}{[1 - \gamma(k)]'} = -\frac{\gamma''(k)k}{\gamma'(k)} \quad (2.6)$$

Example. The following function satisfies these properties:

$$\gamma(k) = 1 - \frac{a + bk^\epsilon}{1 + ck^\epsilon}, \quad (2.7)$$

with $a \in (0, 1)$, $c > 1$, $b \in (ac, c)$ and $\epsilon > 0$. Using this example, $\eta_1(k)$ and $\eta_2(k)$ are given by:

$$\eta_1(k) = \frac{b - ca}{a + bk^\epsilon} \frac{\epsilon k^\epsilon}{1 + ck^\epsilon} \geq 0 \quad \text{and} \quad \eta_2(k) = 1 + \epsilon \left(\frac{2ck^\epsilon}{1 + ck^\epsilon} - 1 \right)$$

⁹This manner of introducing a collateral effect differs from models with borrowing/collateral constraint à la Kiyotaki and Moore (1997). First, borrowing is typically used to finance investment project in these models with collateral constraint, whereas in our paper borrowing finances consumption. Second, our CIA constraint implies a limit on the borrowing's share of total expenditures instead of the borrowing capacity itself. Indeed, using the second-period budget constraint and introducing $A_{t+1} = (1 + i_{t+1})B_{t+1}/p_{t+1} + g(R_{t+1}k_{t+1}) + \Delta_{t+1}/p_{t+1}$ as the amount of borrowing, we can rewrite our CIA constraint as follows $A_{t+1}/(p_{t+1}d_{t+1}) \leq 1 - \gamma(k_{t+1})$. Finally, our limit is nonlinear and increasing with collateral, whereas the borrowing limit of a standard collateral constraint is exogenous or linear with collateral.

When collateral does not matter ($\eta_1(k_{t+1}) = 0$), and γ tends to 0, money is no longer needed and the credit market distortion disappears, whereas when $\gamma > 0$, there is a need of cash. When collateral matters ($\eta_1(k_{t+1}) > 0$), the households are aware of the credit share function: They are able to relax the CIA constraint by increasing capital holdings.

Using $\pi_{t+1} \equiv \frac{p_{t+1}}{p_t}$ and introducing the real variables $m_{t+1} \equiv \frac{M_{t+1}}{p_{t+1}}$, $b_{t+1} \equiv \frac{B_{t+1}}{p_{t+1}}$ and $\delta_{t+1} \equiv \frac{\Delta_{t+1}}{p_{t+1}}$, the constraints (2.2)-(2.4) can be rewritten as follows:

$$c_t + \pi_{t+1}m_{t+1} + \pi_{t+1}b_{t+1} + k_{t+1} \leq w_t \quad (2.8)$$

$$\begin{aligned} d_{t+1} &\leq m_{t+1} + (1 + i_{t+1})b_{t+1} \\ &\quad + g(R_{t+1}k_{t+1}) + \delta_{t+1} \end{aligned} \quad (2.9)$$

$$\gamma(k_{t+1})d_{t+1} \leq m_{t+1} \quad (2.10)$$

The representative household derives her optimal consumption choice (c_t , d_{t+1}) and her optimal portfolio choice (k_{t+1} , m_{t+1} , b_{t+1}) by maximizing her utility function (2.1) under her budget and cash-in-advance constraints (2.8) – (2.10).

Assumption 11. Let $\tilde{\varepsilon}_u \equiv c \frac{1+i}{\pi} \frac{[i\eta_1(1-\gamma)]^2}{\{\rho_1\rho_2g + [i\eta_1(1-\gamma)]\eta_2d\}(1+i\gamma)^2}$.¹⁰ For all $t \geq 0$, we assume $i_t > 0$, $\eta_2 > 0$ and $\varepsilon_u > \tilde{\varepsilon}_u$.

Since the conditions in Assumption 11 rely on endogenous variables, as i_t , π_t , γ , Assumption 11 can seem quite strong. Nevertheless, as we are interested in the occurrence of fluctuations in the vicinity of a steady state, the set of conditions in Assumption 11 will be supposed satisfied at the steady state.

We can prove the following:

¹⁰For simplicity, the arguments of the functions and the time subscripts are omitted.

Lemma 4. *Under Assumptions 9-11, constraints (2.8)-(2.10) are binding and the second-order conditions are satisfied.*

Proof. See Appendix 2.9.1.

Lemma 4 requires that the function of the credit share $1 - \gamma(k_{t+1})$ is concave: Capital holdings increase, at a decreasing rate, the fraction of second-period consumption purchased on credit. Moreover, the CIA constraint is binding if the nominal interest rate i_{t+1} is strictly positive ($i_{t+1} > 0$).

Note that Assumption 11 does not necessarily imply a positive value of bonds. From Lemma 4, we can rewrite the second-period consumption as follows:

$$d_{t+1} = \frac{(1 + i_{t+1})b_{t+1} + g(R_{t+1}k_{t+1}) + \delta_{t+1}}{1 - \gamma(k_{t+1})} \quad (2.11)$$

A positive second-period consumption does not require a positive value of bonds. An agent can borrow in her first period of life and reimburse in her second-period of life by means of her after-tax capital income and the transfer.

Under Assumptions 9-11, the optimal households' behavior is summarized by the following equations:

$$\frac{u'(c_t)}{\beta v'(d_{t+1})} = \frac{1 + i_{t+1}}{\pi_{t+1}} \frac{1}{1 + i_{t+1}\gamma(k_{t+1})} \quad (2.12)$$

$$R_{t+1}g'(R_{t+1}k_{t+1}) = \frac{1 + i_{t+1}}{\pi_{t+1}} - i_{t+1}\eta_1(k_{t+1}) \frac{1 - \gamma(k_{t+1})}{k_{t+1}} d_{t+1} \quad (2.13)$$

When collateral does not matter ($\eta_1(k) = 0$), and no progressive capital income taxation is implemented ($g'(Rk) = 1$), Eqs. (2.12) and (2.13) rewrite:

$$\frac{u'(c_t)}{\beta v'(d_{t+1})} = \frac{(1 + i_{t+1})/\pi_{t+1}}{1 + i_{t+1}\gamma} \quad (2.14a) \quad \text{and} \quad R_{t+1} = \frac{1 + i_{t+1}}{\pi_{t+1}} \quad (2.14b)$$

We can note that as γ tends to 0, we obtain the intertemporal trade-off found in Diamond (1965) and Tirole (1985), in which there are no credit market distortions in the economy (see Eq. (2.14a)). As $\gamma > 0$, a distortion exists: Old households now have to pay cash γ in order to consume an additional unit of final good, and money entails an opportunity cost. Nevertheless, when collateral does not matter, capital and bonds are perfect substitutes (see Eq. (2.14b)).

When collateral matters ($\eta_1(k) > 0$), capital and bonds are no longer perfect substitutes. Households can now decrease their need of cash by holding more capital in their portfolio. As a consequence, the return on capital is lower than the return on bonds. In contrast, a positive marginal tax rate on capital income has an opposite effect by reducing capital holdings.

The endogeneity of the credit share ensures the portfolio choices to be no longer constant through time. The trade-off between assets is endogenous and depends on the amount of collateral held by the households. A change in expected inflation generates a “portfolio effect”, i.e. a new trade-off between asset holdings. This portfolio effect is the key mechanism through which expectation-driven fluctuations may occur. Since the portfolio choices are the explanation for fluctuations, we focus on stabilizing policies designed to dampen the portfolio effect by modifying the different returns on assets. In this perspective, we consider a taxation of capital income for the fiscal policy and an interest rate rule for the monetary policy.

2.2.2 Firms

A representative competitive firm produces the final good using capital and labor under a constant returns to scale technology $f(K/L)L$. Since we note $k = K/L$, the intensive production function $f(k)$ satisfies:

Assumption 12. $f(k)$ is a continuous function defined on $[0, +\infty)$ and C^2 on $(0, +\infty)$, strictly increasing ($f'(k) > 0$) and strictly concave ($f''(k) < 0$). Defining $\alpha(k) \equiv f'(k)k/f(k) \in (0, 1)$ as the capital share in total income and $\sigma(k) \equiv \left[\frac{f'(k)k}{f(k)} - 1 \right] \frac{f'(k)}{kf''(k)} > 0$ as the elasticity of capital-labor substitution, we further assume $f'(1) < 1$, $\lim_{k \rightarrow 0^+} f'(k) > 1$ and $\sigma(k) > 1 - \alpha(k)$.

The competitive firm takes the prices as given and maximizes the profits $f(K_t/L_t) L_t - w_t L_t - R_t K_t$:

$$R_t = f'(k_t) \equiv R(k_t) \quad (2.15)$$

$$w_t = f(k_t) - k_t f'(k_t) \equiv w(k_t) \quad (2.16)$$

Hence, the interest rate and wage elasticities are respectively equal to $\epsilon_R(k) \equiv R'(k)k/R(k) = -(1 - \alpha(k))/\sigma(k)$ and $\epsilon_w(k) \equiv w'(k)k/w(k) = \alpha(k)/\sigma(k)$. The inequality $\sigma(k) > 1 - \alpha(k)$ involves capital income $R_t k_t$ being increasing with k_t , which is not a restrictive assumption.

2.2.3 Monetary authority

For implementing monetary policy, the monetary authority (central bank) uses open market operations defined as the purchase or sale of bonds in exchange for nominal balances.¹¹ At time t , the central bank creates nominal balances M_{t+1} , which offer liquidity at the next period $t + 1$.¹² The money growth factor $\mu_t = M_{t+1}/M_t$ can be written as follows:

$$\mu_t = \pi_{t+1} \frac{m_{t+1}}{m_t} \quad (2.17)$$

¹¹To study the existence of expectation-driven fluctuations in an OLG model without collateral, Rochon and Polemarchakis (2006) use similar open market operations to issue money in the economy.

¹²Placing a part of their savings in the form of nominal balances in their first period of life, young households have the opportunity to obtain liquidity in their second period of life.

In order to supply M_{t+1} in the economy at $t+1$, the central bank buys bonds from old households, and pays for them in cash through open market operations. Thus, a part of bonds held by the old households \bar{B}_{t+1} corresponds to the counterpart of second period nominal balances M_{t+1} :

$$\bar{B}_{t+1} + M_{t+1} = 0 \quad (2.18)$$

At time t , the central bank sells bonds B_{t+1} and nominal balances M_{t+1} to young households, and buys bonds with interests $(1 + i_t)B_t$ and nominal balances M_t from old households. The profits made by central bank Δ_t at time t are given by:

$$\Delta_t = B_{t+1} + M_{t+1} - (1 + i_t)B_t - M_t \quad (2.19)$$

As bonds are the counterpart of money, we obtain $\Delta_t = i_t M_t$. These profits are distributed as dividends to the old households at time t . Thus, the budget constraint of the monetary authority at time t is written as follows:

$$B_{t+1} + M_{t+1} = (1 + i_t)B_t + M_t + \Delta_t = (1 + i_t)(B_t + M_t) \quad (2.20)$$

or in real terms:

$$\pi_{t+1}(b_{t+1} + m_{t+1}) = (1 + i_t)(b_t + m_t) \quad (2.21)$$

Introducing the variable $\theta_t \equiv (1 + i_t)(b_t + m_t)$, Eq. (2.21) can be rewritten as follows:

$$\pi_{t+1}\theta_{t+1} = (1 + i_{t+1})\theta_t \quad (2.22)$$

Note that if θ_t is positive at the equilibrium, then a part of bonds which are purely unbacked public assets (intrinsically useless) has a positive value. Let $b_t = \bar{b}_t + \tilde{b}_t$, where \bar{b}_t denotes the real counterpart of money, and \tilde{b}_t the real value of unbacked public assets. As $\bar{b}_t + m_t = 0$, $\theta_t = (1 + i_t)(b_t + m_t) > 0$ is equivalent to $\tilde{b}_t > 0$. We can maintain that there is a bubble on bonds when $\tilde{b}_t > 0$. Therefore, $\theta_t > 0$ pertains to a situation in which a positive bubble on bonds exists.¹³ When $\theta_t = 0$, all bonds are the counterpart of money. In this case, all money in the economy corresponds to inside money: No bubbles on bonds exist. When $\theta_t < 0$, there is an excess of households' debt, which could correspond to a negative bubble on bonds.

In addition, the monetary authority chooses the nominal interest rate i_{t+1} as the monetary instrument, and implements the following interest rate rule:

$$1 + i_{t+1} = (1 + i^*) \left(\frac{\pi_{t+1}}{\pi^*} \right)^\phi, \quad (2.23)$$

where $\phi \geq 0$ is a measure of monetary policy responses to expected inflation. Furthermore, i^* and π^* are respectively the stationary values of the nominal interest rate and the inflation of an existing stationary equilibrium chosen as the targets by the monetary authority.

When $\phi = 0$, the central bank decides to fix the level of the nominal interest rate at its stationary level i^* . When $\phi > 0$, Eq. (2.23) depicts an interest rate rule, like a Taylor rule, which responds to expected inflation.¹⁴ For $\phi \in (0, 1)$, the rule weakly reacts to expected inflation. An increase (decrease) in the inflation raises (depresses) the nominal interest rate less than proportionally, involving a decrease

¹³Alternatively, $\theta_t > 0$ corresponds to a situation in which the outside money is positive. A positive outside money indicates that there is fiat money in circulation in the economy. In the literature about rational bubble, fiat money is considered as being a bubble.

¹⁴From Bernanke (2010), a rule which responds to expected inflation is more relevant to describe the US monetary policy than a rule responding to observed inflation. As a consequence, we consider a Taylor rule involving an interest rate response to inflation forecast.

(increase) in the real interest rate. For $\phi > 1$, the rule strongly reacts to expected inflation. An increase (decrease) in the inflation raises (depresses) the nominal interest rate more than proportionally, involving an increase (decrease) in the real interest rate. Following Benhabib et al. (2001), we define a rule with $\phi \in (0, 1)$ as a passive one, and a rule with $\phi > 1$ as an active one.

2.2.4 Government

Taxes on capital income are used to finance wasteful public expenditures G_t . These public expenditures affect neither households' preferences nor the production function. Government budget is balanced at each period, that is:

$$G_t = \tau(R_t k_t) = R_t k_t - g(R_t k_t) \quad (2.24)$$

Note that we could have one institution by introducing a single budget constraint for the government and the monetary authority, i.e. $G_t + (1 + i_t)B_t + M_t + \Delta_t = R_t k_t - g(R_t k_t) + B_{t+1} + M_{t+1}$. This would be perfectly equivalent to our framework. θ_t would still be a non-predetermined variable since it would be deflated by the price.

2.3 Intertemporal equilibrium

At the intertemporal equilibrium, the budget and cash-in-advance constraints of households are given by:

$$c_t + \frac{\pi_{t+1}}{1 + i_{t+1}}\theta_{t+1} + k_{t+1} = w(k_t) \quad (2.25)$$

$$d_{t+1} = \theta_{t+1} + g(f'(k_{t+1})k_{t+1}) \quad (2.26)$$

$$\gamma(k_{t+1})d_{t+1} = m_{t+1} \quad (2.27)$$

The budget constraints of the monetary authority and the government are as follows:

$$\pi_{t+1} = (1 + i_{t+1}) \frac{\theta_t}{\theta_{t+1}} \quad (2.28)$$

$$G_{t+1} = f'(k_{t+1})k_{t+1} - g(f'(k_{t+1})k_{t+1}) \quad (2.29)$$

Substituting Eq. (2.28) into the first-period budget constraint Eq. (2.25), we determine:

$$c_t + \theta_t + k_{t+1} = w(k_t) \quad (2.30)$$

Using Eqs. (2.17), (2.27) and (2.28), we deduce the money growth factor:

$$\mu_t = (1 + i_{t+1}) \frac{\theta_t}{\theta_{t+1}} \frac{\gamma(k_{t+1})}{\gamma(k_t)} \frac{\theta_{t+1} + g(f'(k_{t+1})k_{t+1})}{\theta_t + g(f'(k_t)k_t)} \quad (2.31)$$

Substituting Eqs. (2.25) and (2.30) into Eq. (2.12), then Eqs. (2.26) and (2.28) into Eq. (2.13), the consumers' intertemporal trade-off and the no-arbitrage condition

are respectively given by:

$$\left\{ \begin{array}{l} \theta_t \frac{u'(f(k_t) - f'(k_t)k_t - \theta_t - k_{t+1})}{\beta v'(\theta_{t+1} + g(f'(k_{t+1})k_{t+1}))} = \frac{\theta_{t+1}}{1 + i_{t+1}\gamma(k_{t+1})} \\ \frac{\theta_{t+1}}{\theta_t} = \frac{1 + i_{t+1}}{\pi_{t+1}} = f'(k_{t+1})g'(f'(k_{t+1})k_{t+1})H(k_{t+1}, \theta_t), \end{array} \right. \quad (2.32)$$

$$\text{with } H_{t+1}(k_{t+1}, \theta_t) \equiv \frac{1 + i_{t+1}\eta_1(k_{t+1}) [1 - \gamma(k_{t+1})] [\rho_1(f'(k_{t+1})k_{t+1})]^{-1}}{1 - \theta_t i_{t+1} \eta_1(k_{t+1}) [1 - \gamma(k_{t+1})] / k_{t+1}} \quad (2.33)$$

When collateral does not matter ($\eta_1(k) = 0$), we obtain $H_{t+1}(k_{t+1}, \theta_t) = 1$. Therefore, the Fisher equation ($(1 + i_{t+1})/\pi_{t+1} = f'(k_{t+1})$) holds at the intertemporal equilibrium as soon as there is no progressive taxation on capital income ($g'(f'(k_{t+1})k_{t+1}) = 1$). This means that the return on real asset (capital) is equal to the return on nominal asset (bonds) deflated by the inflation factor. The presence of collateral ($\eta_1(k) > 0$) implies the violation of the Fisher equation ($H_{t+1}(k_{t+1}, \theta_t) > 1$). As capital serves as a collateral, its return becomes lower than the real return on bonds ($f'(k_{t+1}) < (1 + i_{t+1})/\pi_{t+1}$). This violation of the Fisher equation represents some portfolio choices that promote indeterminacy, a source of expectation-driven fluctuations.

Because of credit market distortions (collateral effect), the portfolio choice is the key ingredient for the existence of endogenous fluctuations. Therefore, we will focus on policies designed to counteract these distortions restoring the Fisher relationship. For instance, a progressive taxation on capital income $g'(f'(k_{t+1})k_{t+1}) < 1$ can cancel out the collateral effect (see (2.32)). Interestingly, the level of nominal interest rate can also offset the collateral effect (see Eq. (2.33)). Thus, considering a usual interest rate rule, like a Taylor one, is a priori relevant to study the stabilizing role of monetary policy.

From the budget constraint of the monetary authority Eq. (2.28) and the monetary rule Eq. (2.23), we deduce that:

$$\frac{\pi_{t+1}}{\pi^*} = \left(\frac{\theta_t}{\theta_{t+1}} \right)^{\frac{1}{1-\phi}} \quad (2.34)$$

Substituting Eq. (2.34) into Eq. (2.23), we obtain the nominal interest rate at the equilibrium:

$$i_{t+1} = (1 + i^*) \left(\frac{\theta_t}{\theta_{t+1}} \right)^{a_\phi} - 1, \quad \text{where } a_\phi \equiv \frac{\phi}{1-\phi} \in (-\infty, -1) \cup [0, +\infty) \quad (2.35)$$

Definition 2. Under Assumptions 9-12, an intertemporal equilibrium with perfect foresight is a sequence $(k_t, \theta_t) \in \mathbb{R}_+^2$, $t = 0, 1, \dots, +\infty$, such that (2.32) is satisfied, where i_{t+1} is defined by Eq. (2.35) and $k_0 > 0$ is given.

Taking into account that i_{t+1} is given by Eq. (2.35), we note that k_t is the only predetermined variable of this two-dimensional dynamic system (2.32). The intertemporal sequence of k_t and θ_t enables us to determine all the other variables, namely c_t , d_t , m_t and b_t . In particular, the dynamics of m_t and b_t are given by Eqs. (2.27) and (2.28) respectively.

2.4 Steady state analysis

From the system (2.32), we deduce that two kinds of steady state exist: $\theta = 0$ and $\theta \neq 0$. Since we are interested in fluctuations with a positive bubble, we will focus on steady states with $\theta > 0$. A steady state is a solution $(k, \theta) \in \mathbb{R}_{++}^2$ that satisfies

the following system:

$$\begin{cases} \frac{u'(f(k) - f'(k)k - k - \theta)}{\beta v'(\theta + g(f'(k)k))} = \frac{1}{1 + i^* \gamma(k)} \\ f'(k)g'(f'(k)k)H(k, \theta) = 1 \end{cases} \quad (2.36)$$

$$\text{with } H(k, \theta) = \frac{1 + i^* \eta_1(k) [1 - \gamma(k)] [\rho_1(f'(k)k)]^{-1}}{1 - \theta i^* \eta_1(k) [1 - \gamma(k)] / k} \quad (2.37)$$

Under a constant credit share ($\eta_1(k) = 0$), we see from the system (2.36) that the steady state is unique, and the monetary policy does not affect the production side. Indeed, the second equation of the system (2.36) reduces to $f'(k)g'(f'(k)k) = 1$. Under Assumption 12, this gives a unique stationary solution in k and superneutrality of money holds. When collateral matters ($\eta_1(k) > 0$), the superneutrality of money is canceled. Because of the presence of collateral, the monetary sphere affects the real one.

2.4.1 Existence

From Eq. (2.36), a steady state with $\theta > 0$ is a solution $k \in \mathbb{R}_+$ satisfying:

$$\begin{cases} \frac{u'(c(k))}{\beta v'(d(k))} = \frac{1}{1 + i^* \gamma(k)} \\ \theta = \frac{1 - \psi(k) \{1 + i^* \eta_1(k) [1 - \gamma(k)] [\rho_1(f'(k)k)]^{-1}\}}{i^* \eta_1(k) [1 - \gamma(k)] / k} \end{cases} \quad (2.38)$$

$$\begin{aligned} \text{with } c(k) &= f(k) - k - \frac{k[1 - \psi(k)]}{i^* \eta_1(k) [1 - \gamma(k)]} - [kf'(k) - g(f'(k)k)], \\ d(k) &= \frac{k[1 - \psi(k)]}{i^* \eta_1(k) [1 - \gamma(k)]} \text{ and } \psi(k) = f'(k)g'(f'(k)k). \end{aligned}$$

From these equations, we deduce that $d(k) > 0$ implies $\psi(k) < 1$, and from Eqs. (2.28) and (2.31), $1 + i^* = \pi = \mu > 1$.

Proposition 7. *Let \bar{k} defined by $c(\bar{k}) = 0$ and \underline{k} by $\psi(\underline{k}) = 1$. Under Assumptions 9-12, there exists a steady state characterized by $k^* \in (\underline{k}, \bar{k})$ and $0 < \theta^* < f(k^*) - k^* - f'(k^*)k^*$. A positive bubble $\theta^* > 0$ requires:*

$$[1 - \psi(k^*)] \frac{k^*}{g(f'(k^*)k^*)} > i^* \eta_1(k^*) [1 - \gamma(k^*)] \quad (2.39)$$

Proof. See Appendix 2.9.2.

Proposition 7 indicates that a positive bubble exists at the steady state, and that the second-period consumption is greater than after-tax capital income at the steady state with positive bubble (2.39). We can note that the condition (2.39) is satisfied for η_1 small enough.

When there is no taxation ($g'(f'(k)k) = 1$) and collateral does not matter ($\eta_1(k) = 0$), we can see from Eq. (2.37) that the steady state is at the golden rule ($R(k) = 1$). As the well-known result of Tirole (1985), a positive rational asset bubble crowds out capital. When collateral matters ($\eta_1(k) > 0$), this economy experiences an over-accumulation of capital at the steady state ($R(k) < 1$). The existence of collateral incites households to hold more capital in their portfolio in order to relax the cash-in-advance constraint, and therefore, the capital return decreases. On the contrary, a taxation on capital income incites households to reduce capital accumulation. As long as there are some credit market imperfections, implementing a tax on capital income can counteract the collateral effect, and thus reduce or even rule out over-accumulation of capital.

As regards the monetary policy, we recall that the central bank chooses the stationary values of an existing steady state for its targets. Since, the steady state

k^* persists, we assume that the central bank selects this steady state as target, and therefore, $\pi^* = 1 + i^*$.

2.4.2 Normalized steady state and multiplicity

In order to facilitate the analysis of local dynamics (Sections 5 and 6), we establish the existence of a normalized steady state $k^* = 1$ (NSS). We follow the procedure introduced by Cazzavillan et al. (1998), and use the scaling parameter β to give conditions for the existence of such a steady state.

Assumption 13. *Let $\nu(\eta_1) = i^*\eta_1(1)[1 - \gamma(1)]$, we assume:*

$$f(1) - f'(1) + g(f'(1)) > 1 + \frac{1 - \psi}{\nu(\eta_1)}$$

Assumption 13 ensures that the first period consumption at the normalized steady state is positive (i.e. $c(1) > 0$), and it is satisfied when the productivity is sufficiently large.

Proposition 8. *Under Assumptions 9-13, there exists a unique value $\beta^* > 0$ given by*

$$\beta^* = \frac{u' \left(f(1) - 1 - \frac{1 - \psi(1)}{\nu(\eta_1)} - [f'(1) - g(f'(1))] \right)}{v' \left(\frac{1 - \psi(1)}{\nu(\eta_1)} \right)} [1 + i^*\gamma(1)]$$

such that $k^ = 1$ is a steady state of the dynamic system (2.32). Assumption 13 ensures that $k^* = 1 \in (\underline{k}, \bar{k})$. Moreover, there is a positive bubble ($\theta^* > 0$) if $1 - \psi(1) - g(f'(1))\nu(\eta_1) > 0$.*

Thereafter, we set $\beta = \beta^*$ so that $k^* = 1$. We further note $c^* \equiv c(1)$, $\gamma \equiv \gamma(1)$, $\eta_1 \equiv \eta_1(1)$, $\eta_2 \equiv \eta_2(1)$, $g \equiv g(f'(1))$, $\rho_1 \equiv \rho_1(1)$, $\rho_2 \equiv \rho_2(1)$, $\psi \equiv \psi(1)$, $\alpha = \alpha(1)$

and $\sigma = \sigma(1)$. We can now clarify the conditions for the multiplicity of steady states.¹⁵

Proposition 9. *Under Assumptions 9-13, there exists a value ε_v^s such that there is a multiplicity of bubbly stationary equilibria if the following condition holds¹⁶:*

$$\varepsilon_v < \frac{\Omega}{\varepsilon_{dk}} \left(\frac{\varepsilon_u^s}{c^*} - \frac{\varepsilon_u}{c^*} \right) \equiv \varepsilon_v^s \quad (2.40)$$

Their number is generically odd.

Proof. See Appendix 2.9.3.

As in Clain-Chamosset-Yvrard and Seegmuller (2015), the multiplicity of steady states appears for relatively small degrees of concavity ($\varepsilon_u < \varepsilon_u^s$ and $\varepsilon_v < \varepsilon_v^s$), and this happens only when collateral matters ($\eta_1 > 0$). Therefore, we extend this result to a model with capital accumulation. Because the second-order conditions are satisfied for $\varepsilon_u > \tilde{\varepsilon}_u$, then $\varepsilon_u^s > \tilde{\varepsilon}_u$ must be satisfied.¹⁷ This holds true under the following assumption¹⁸:

Assumption 14. *Let $\bar{\eta}_1 \equiv \min\{\eta_1^{\chi_1}, \eta_1^{\chi_3}, \bar{\bar{\eta}}_1, \underline{\eta}_1, \hat{\eta}_1, \tilde{\eta}_1\}$ and $\bar{\eta}_2 \equiv \frac{\psi}{1-\psi} \frac{(1-\alpha)/\sigma - \rho_2 \left(\frac{1-\alpha}{\sigma} + i^* \gamma \right)}{i^* \gamma}$.*

We assume that

$$\eta_1 < \bar{\eta}_1 \text{ and } \eta_2 > \bar{\eta}_2.$$

¹⁵We note ε_{Jx_i} the elasticity of the function $J(x_1, \dots, x_n)$ with respect to x_i , i.e. $\varepsilon_{Jx_i} = \frac{\partial J(x_1, \dots, x_n)}{\partial x_i} \frac{x_i}{J(x_1, \dots, x_n)}$.

¹⁶The expressions of ε_{dk} , Ω , and ε_u^s are respectively given by Eqs. (2.54), (2.58) and (2.59) in Appendix 2.9.3.

¹⁷The expression $\tilde{\varepsilon}_u$ is given by Eq. (2.60) in Appendix 2.9.3. Furthermore, $\varepsilon_u^s > \tilde{\varepsilon}_u$ is satisfied if and only if $\eta_1 < \frac{1}{i^*(1-\gamma)} \frac{\eta_2(1-\psi)i^*\gamma - \psi \left[\frac{1-\alpha}{\sigma} - \rho_2 \left(\frac{1-\alpha}{\sigma} + i^* \gamma \right) \right]}{1-f'(1) + \left(1 - \frac{1-\alpha}{\sigma}\right)[f'(1) - \psi]} \equiv \bar{\bar{\eta}}_1$ and $\eta_2 > \frac{\psi}{1-\psi} \frac{(1-\alpha)/\sigma - \rho_2 \left(\frac{1-\alpha}{\sigma} + i^* \gamma \right)}{i^* \gamma} \equiv \bar{\eta}_2$.

¹⁸In Appendix 2.9.6, we show the existence of $\eta_1^{\chi_1}, \eta_1^{\chi_3}, \bar{\bar{\eta}}_1, \underline{\eta}_1, \hat{\eta}_1$ and $\tilde{\eta}_1$.

Assumption 14 holds for η_1 sufficiently small enough and η_2 sufficiently large.

This multiplicity implies the existence of several steady states with positive bubble (i.e. $\theta^* > 0$). Moreover, we can check from the second equation of (2.38), that the size of the bubble at the steady state is increasing with the stationary capital stock. Through the collateral effect, an increase in stationary capital stock implies a decrease in stationary nominal balances generating a reallocation of savings towards the bubble.

Furthermore, the multiplicity of steady states can be seen as a form of global indeterminacy. According to their expectations, agents can coordinate on equilibrium paths which converge towards different steady states. This could explain the occurrence of large fluctuations in the economy. Therefore, by considering the role of collateral on consumer behavior, we provide a new and interesting result compared to models without collateral in which the uniqueness of the steady state prevails (Tirole, 1985; and Rochon and Polemarchakis, 2006). We will see in the next sections that this multiplicity is connected with local indeterminacy, and that a sufficiently progressive fiscal policy can remove the multiplicity of steady states.

To convince the reader that all our assumptions leading to our results are compatible, consider the following example:

Example. For desired sets of parameter values $(a, b, c, \epsilon, A, \alpha, \sigma)$, the function $\gamma(k)$ given by Eq.(2.7) in Section 2.1 and the production function $f(k) = A \left(\alpha k^{(\sigma-1)/\sigma} + 1 - \alpha \right)^{\sigma/(\sigma-1)}$ evaluated at the normalized steady state $k^* = 1$ may fit all the requirements imposed by Assumptions 11 – 14 and condition (2.39) when there is neither progressive taxation ($g(Rk) = Rk$) nor implementation of a Taylor rule ($\phi = 0$).

There exist some critical values \bar{b} , $\underline{\epsilon}$, $\underline{\sigma}$, $\bar{\alpha}$, \underline{A} and \bar{A} such that for $a < 1$, $c > 1$, $b \in (ac, \bar{b})$, $\epsilon > \underline{\epsilon}$, $\sigma > \underline{\sigma}$, $\alpha \in (1 - \sigma, \bar{\alpha})$ and $A \in (\underline{A}, \bar{A})$, the function $\gamma(k)$ and the production function $f(k)$ evaluated at $k^* = 1$ satisfy Assumptions 11 – 14, and conditions (2.39) holds.

Proof. See Appendix 2.9.4.

2.5 Expectation-driven fluctuations

In this section, we explain the emergence of business fluctuations in presence of a speculative bubble, considering a framework with an interplay between the financial and the real spheres. The violation of the Fisher relationship and the resulting portfolio choice between bonds, capital and money are the key ingredients to explain these fluctuations. We show that local indeterminacy occurs in the neighborhood of the normalized steady state with a positive bubble under gross substitutability and a not too weak input substitution, because of the credit market distortion. Furthermore, the occurrence of fluctuations under gross substitutability is connected to the multiplicity of steady states, i.e. a form of global indeterminacy.

2.5.1 Local dynamics: Preliminaries

To derive our different results, we start by linearizing the dynamic system (2.32) around the normalized steady state $k^* = 1$ to obtain the characteristic polynomial. As shown in Appendix 2.9.5, we can derive the trace $T(\varepsilon_v)$ and the determinant $D(\varepsilon_v)$ of the associated Jacobian matrix as functions of ε_v ¹⁹: When ε_v varies in $(0, +\infty)$, $D(\varepsilon_v)$ linearly depends on $T(\varepsilon_v)$. As a result, we can apply the

¹⁹See Lemma 6 in Appendix 2.9.5.

geometrical method developed by Grandmont et al. (1998) to discuss the local stability properties of the model.

We study the variations of the trace $T(\varepsilon_v)$ and the determinant $D(\varepsilon_v)$ in the (T, D) plane as ε_v is made to vary continuously in its admissible range $(0, +\infty)$. The locus $\Sigma \equiv \{(T(\varepsilon_v), D(\varepsilon_v)): \varepsilon_v \geq 0\}$ describes a part of a line that we call the Σ -line.

Consider the (T, D) plane (see Figure 2.1). On line (AC) , one eigenvalue is equal to 1 ($D = T - 1$). On line (AB) , one eigenvalue is equal to -1 ($D = -T - 1$). Along segment $[BC]$ ($|T| < 2, D = 1$), the characteristic roots are complex conjugates with modulus equal to 1. These lines and this segment divide the space (T, D) into three different types of regions. Inside the triangle ABC , the steady state is a sink, i.e. locally indeterminate ($|T| < 1 + D$ and $D < 1$). It is a saddle point if (T, D) lies on the right or left sides of both the lines (AB) and (AC) ($|1 + D| < |T|$). It is a source otherwise.

A (local) bifurcation arises when at least one eigenvalue crosses the unit circle, that is, when the Σ -line crosses one of the loci (AB) , (AC) or $[BC]$. According to the changes of the bifurcation parameter, a pitchfork bifurcation (generically) emerges when the Σ -line crosses (AC) , as ε_v goes through ε_v^s .²⁰ A flip bifurcation (generically) occurs when the Σ -line crosses (AB) , as ε_v goes through ε_v^f . Finally, a Hopf bifurcation (generically) arises when the Σ -line crosses the segment $[BC]$, as ε_v goes through ε_v^h .

We locate the Σ -line in the (T, D) plane by analyzing $(T(0), D(0))$, $(T(+\infty), D(+\infty))$ and its slope S (see Appendix 2.9.5). This allows us to analyze the occurrence of local indeterminacy and endogenous cycles.

²⁰Indeed, we have (generically) an odd number of steady states (see Subsection 2.4.2).

2.5.2 Endogenous fluctuations with a bubble

To highlight the occurrence of endogenous fluctuations in an economy characterized by a bubble, we analyze the dynamics when there is neither progressive taxation ($g(Rk) = Rk$) nor implementation of a Taylor rule ($\phi = 0$). Even though we consider capital accumulation, the local dynamics are similar to Clain-Chamosset-Yvrard (2015). Figure 2.1 summarizes the local dynamics of our model in the neighborhood of the normalized steady state. Technical details are relegated to the Appendix 2.9.6.

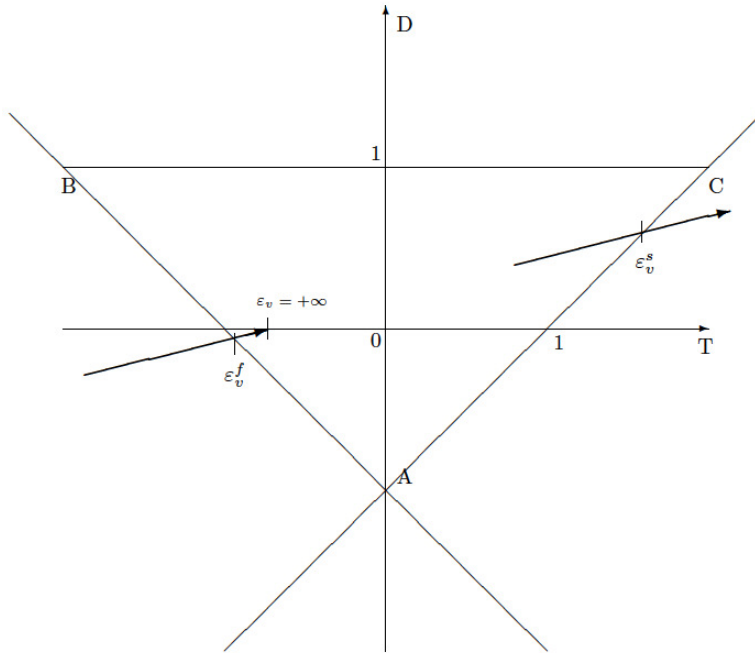


Figure 2.1: Endogenous fluctuations

Proposition 10. *Under Assumptions 9-14, the following generically holds:*

1. When $\varepsilon_u \in (\tilde{\varepsilon}_u, \varepsilon_u^s)$, the steady state is a sink for $\varepsilon_v < \varepsilon_v^s < 1$, undergoes a pitchfork bifurcation for $\varepsilon_v = \varepsilon_v^s$, is a saddle for $\varepsilon_v \in (\varepsilon_v^s, \varepsilon_v^f)$, undergoes a flip bifurcation for $\varepsilon_v = \varepsilon_v^f$, and is a sink for $\varepsilon_v > \varepsilon_v^f$.

2. When $\varepsilon_u > \varepsilon_u^s$, the steady state is a saddle for $\varepsilon_v < \varepsilon_v^f$, undergoes a flip bifurcation for $\varepsilon_v = \varepsilon_v^f$, and is a sink for $\varepsilon_v > \varepsilon_v^f$.

Proof. See Appendix 2.9.6.

Proposition 10 shows the occurrence of persistent endogenous fluctuations around the steady state with a positive bubble under gross substitutability and a not too weak capital-labor substitution. This result extends Bosi and Seegmuller (2010a) and Clain-Chamosset-Yvrard and Seegmuller (2015) to a model with inside money.²¹ In addition, it is important to note that the occurrence of fluctuations under gross substitutability is connected to the multiplicity of steady states (see Proposition 9).

When collateral does not matter ($\eta_1 = 0$), the local stability properties of the model correspond to Proposition 10.2. Our model exhibits endogenous fluctuations and two-period cycles only for large degrees of concavity on $u(c)$ and $v(d)$, i.e. for a significant income effect.²² More interestingly, when collateral matters ($\eta_1 > 0$), local indeterminacy also appears for small degrees of concavity on $u(c)$ and $v(d)$, in particular under gross substitutability ($\varepsilon_v < \varepsilon_v^s < 1$).

The basic mechanism for fluctuations under gross substitutability relies on a portfolio trade-off between the three assets. Because of the difference between the returns on physical and monetary assets, a reallocation between the assets takes place following a modification in agents' expectations.

²¹In these two papers, the stabilizing role of policies is not addressed in the same way as here. Indeed, the fiscal policy is ignored and the monetary authority directly manages the money growth factor, while it fixes the interest rate in our framework.

²²Since $\varepsilon_v > \varepsilon_v^f > 1$, income effects dominate substitution effects. Hence, global savings $(\theta_t + k_{t+1})$ are a decreasing function of their return.

Economic intuition. If households expect an increase in inflation from period t to $t + 1$, the return on bonds becomes less attractive compared to the return on capital. Because of the portfolio effect, households reallocate their savings towards capital. As a consequence, when $\varepsilon_v < \varepsilon_v^s < 1$, the portfolio effect can accelerate capital accumulation. Households consume less by cash (see Eq. (2.27)). The real balances m_{t+1} decrease, entailing a decrease in the return on money. An effective rise in inflation takes place. The initial expectations are self-fulfilling.

Now, we are interested in the stabilizing role of different economic policies. A policy is stabilizing in our framework as soon as it reduces the range of parameters for expectation-driven fluctuations. In the following, we focus on empirically plausible fluctuations, namely fluctuations occurring when savings are increasing with their global return. Therefore, it is relevant to analyze the stabilizing role of fiscal and monetary policies under gross substitutability i.e. when $\varepsilon_v < 1$. Hence, we restrict our attention on the most interesting case, i.e. Proposition 10.1.

Assumption 15.

$$\varepsilon_u < \varepsilon_u^s \text{ and } \varepsilon_v < 1.$$

As expectation-driven fluctuations are mainly driven by portfolio choices between capital, money and bonds, we should consider policy tools that are able to modify the return on assets. In the next sections, we investigate the virtues of monetary and fiscal policies in particular for stabilizing purposes, and compare them. We first consider the monetary policy with an interest rate rule responding to inflation. Second, we examine the role of progressive taxation of capital income. We show that a progressive taxation of capital income is most powerful to stabilize than a Taylor rule.

2.6 The virtues of monetary policy

In this section, we analyze some implications of the monetary policy. We first analyze the stabilizing role of the Taylor rule described in Section 2.2.3 on expectation-driven fluctuations with a bubble, then focus on the effect of monetary policy on the welfare at the steady state.

2.6.1 The stabilizing role of monetary policy ($g(Rk) = Rk$)

Since the portfolio choices and the gaps between asset returns are the main mechanisms that lead to fluctuations, the monetary authority may have a role to play to stabilize the economy by manipulating the nominal interest rate. Therefore, we analyze the stabilizing virtues of a monetary policy based on a Taylor rule ($\phi > 0$) on the occurrence of endogenous fluctuations when a economy experiences a rational bubble. To highlight the role of monetary policy, we consider the model without taxation on capital income ($g(Rk) = Rk$).

More precisely, we examine how local dynamics are modified by the implementation of an interest rate rule given by Eq. (2.23). First of all, enriching the model with an interest rate rule generates new configurations as regards the range of parameter values for which local indeterminacy occurs.

To highlight these new configurations, we conduct the analysis considering the variable $a_\phi = \phi/(1 - \phi)$ instead of ϕ . We note that $a_\phi < -1$ corresponds to an active rule ($\phi > 1$), and $a_\phi > 0$ corresponds to a passive rule ($\phi < 1$). As shown in Appendix 2.9.7, for $a_\phi \in]-\infty, -1] \cup]0, \tilde{a}_\phi]$, local indeterminacy occurs when $\varepsilon_v > \max(\varepsilon_v^f, \varepsilon_v^h, \varepsilon_v^s)$ and when $\varepsilon_v < \min(\varepsilon_v^f, \varepsilon_v^s)$. For $a_\phi \in (0, +\infty)$, local indeterminacy occurs when $\varepsilon_v < \varepsilon_v^s$ (see Figure 2.2).²³ Now, we are interested in the variations of these critical values with respect to a_ϕ in order to analyze the

²³ \hat{a}_ϕ is given by $\varepsilon_v^f = 0$ and the expressions of \tilde{a}_ϕ is given in Appendix 2.9.7.

stabilizing role of a monetary policy under a Taylor rule. See also Figure 2.2 for a qualitative illustration.

Lemma 5. *Under Assumptions 9 – 14, the impact of the monetary policy on the bifurcation values are given by the following derivatives²⁴:*

$$\frac{\partial \varepsilon_v^f}{\partial a_\phi} > 0, \quad \frac{\partial \varepsilon_v^h}{\partial a_\phi} > 0 \quad \text{and} \quad \frac{\partial \varepsilon_v^s}{\partial a_\phi} = 0.$$

Proof. See Appendix 2.9.7.

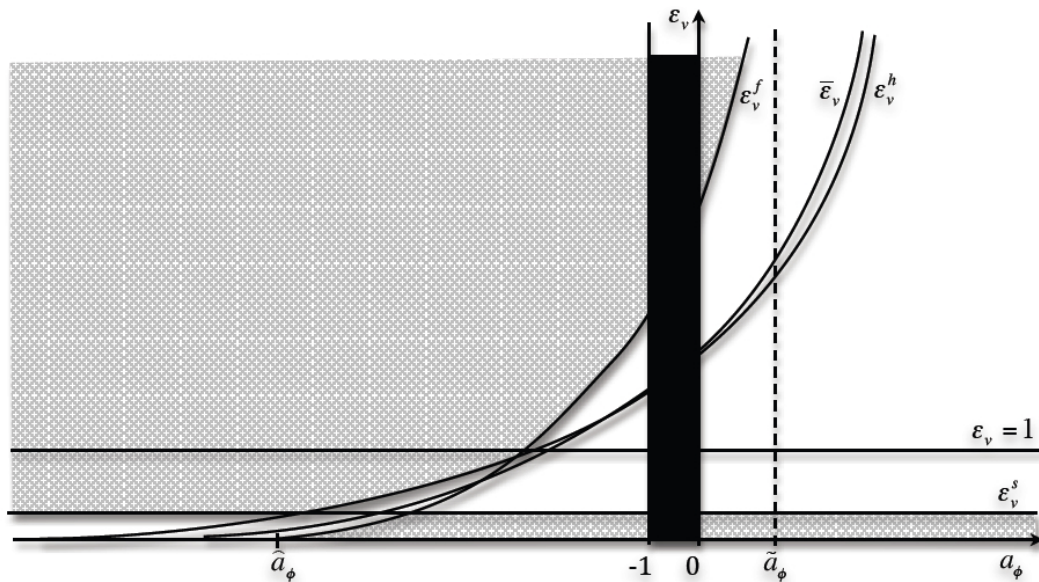


Figure 2.2: Stabilizing role of monetary policy

Appendix 2.9.7 allows us to construct Figure 2.2 that summarizes the stability properties of the economy. Grey areas correspond to local indeterminacy regions. The next proposition summarizes the stabilizing role of this interest rate rule:

Proposition 11. *Under Assumptions 9 – 14, the following generically holds:*

²⁴The functions ε_v^f and ε_v^h are homographics. ε_v^f has a vertical asymptote for $a_\phi = \tilde{a}_\phi$, and ε_v^h has one for $a_\phi = \bar{a}_\phi$.

1. If $a_\phi \in]-\infty, \hat{a}_\phi]$, local indeterminacy is ruled out for $\varepsilon_v < \varepsilon_v^s$ only, i.e. occurs for all $\varepsilon_v \geq \varepsilon_v^s$.
2. If $a_\phi \in (\hat{a}_\phi, -1)$, increasing the degree of responsiveness of the rule with respect to the expected inflation (ϕ) reduces the range of parameters for local indeterminacy when ε_v is large enough, but raises the range of parameters for local indeterminacy when ε_v is small enough.
3. If $a_\phi \in (0, +\infty)$, increasing the degree of responsiveness of the rule with respect to the expected inflation (ϕ) reduces the range of parameters for local indeterminacy when ε_v is large enough, but has no impact on the range of parameters for local indeterminacy when ε_v is small enough.

Proof. See Appendix 2.9.7.

Proposition 11 highlights mitigated results. In contrast to Sorger (2005) and Clain-Chamosset-Yvrard and Seegmuller (2015) in which the monetary policy is however determined by the money growth factor, no clear-cut conclusion is outlined. A weakly passive rule has no impact on fluctuations occurring for $\varepsilon_v < \varepsilon_v^s$, whereas an active rule could stabilize such fluctuations. Nevertheless, an active rule destabilizes promoting indeterminacy for new ranges of parameter values. We can note that under a weakly active rule (i.e. $a_\phi \in]-\infty, \hat{a}_\phi]$), local indeterminacy can occur for $\varepsilon_v > \varepsilon_v^s$.

A Taylor interest rate rule has no clear stabilizing virtues. Our explanation is that the monetary authority manipulates the level of the elasticity of the nominal interest rate with respect to the expected inflation (ϕ) and not directly the interest rate target (i^*). For instance, a weakly active rule has not a huge impact on the nominal interest rate, and therefore does not modify so much the portfolio choices.

Recalling that the multiplicity of steady states occurs for $\varepsilon_v < \varepsilon_v^s$, Lemma 5 allows us to show:

Proposition 12. *Under Assumptions 9 – 15, conducting an interest rate rule given by Eq. (2.23) neither promotes nor rules out the multiplicity of steady states. The range of parameters for the multiplicity of steady states is not altered by such a policy.*

A Taylor rule has no impact on the conditions for which the multiplicity of steady states occurs. The monetary authority cannot restore the uniqueness of the steady state, and thus globally stabilize with a Taylor rule. Such a rule does not allow agents to coordinate their expectations on a particular steady state, since the different steady states are characterized by the same nominal interest rate i^* . Thus, global indeterminacy persists.

2.6.2 The role of monetary policy on welfare at the steady state

Because of credit market imperfections and the existence of collateral, the Fisher relationship is no more satisfied in our framework: The level of the nominal interest rate has real effects on the steady state. Therefore, the superneutrality of money is canceled, and the monetary policy can alter the welfare at the steady state.²⁵

Proposition 13. *Under Assumptions 9 – 15, the effect of the interest rate target on the welfare is the following:*

1. *If $\varepsilon_v < \varepsilon_v^s(k)$, the welfare is an increasing function of the interest rate target i^* .*

²⁵When the Fisher relationship is not satisfied, we can highlight Tobin/anti-Tobin effects. Depending on values on fundamentals, the nominal interest rate enhances or dampens the capital accumulation at the steady state.

2. If $\varepsilon_v > \varepsilon_v^s(k)$, the welfare is a decreasing function of the interest rate target i^* .

Proof. See Appendix 2.9.8.

Proposition 13 indicates that for a sufficiently small degree of utility concavity ε_v , a decrease in the interest rate target deteriorates the welfare at the steady state. For an arbitrarily large degree of utility concavity, we reverse this result.

When collateral does not matter ($\eta_1(k) = 0$), the welfare properties of the model are similar to the second case of Proposition 13. More precisely, the Friedman rule, for which setting the nominal interest rate target at zero is welfare maximizing, is satisfied. In the framework without collateral, distortions are only due to the existence of the CIA constraint. Therefore, by fixing the level of the nominal interest rate at zero, the monetary authority eliminates the distortion associated to money holdings, which involves the highest level of welfare.

When collateral matters ($\eta_1(k) > 0$), the welfare properties depend on the concavity of the utility function. In particular, the Friedman rule does not hold for a small degree of concavity ε_v . Indeed, the monetary authority can improve the welfare by increasing the nominal interest rate. One possible explanation relies on the portfolio choice. An increase in the nominal interest rate target generates a reallocation of savings towards capital. As capital income is increasing with capital, households get a higher capital income and spend less through money holdings (collateral effect). Their welfare improves.

Since our results on the stabilizing virtues of monetary policy are obtained for all value of the nominal interest rate's target i^* , our conclusions are compatible with a target i^* which improves households' welfare at the steady state. Thus, a

destabilizing Taylor rule may be in accordance with welfare improvement at the steady state.

2.7 The virtues of fiscal policy

This section is devoted to the study of the virtues of a fiscal policy both for stabilizing and efficiency purposes. We first analyze the stabilizing role of a progressive capital income taxation on expectation-driven fluctuations with a bubble, then discuss on the effect of fiscal policy on the economic efficiency.

2.7.1 The stabilizing role of fiscal policy ($i_{t+1} = i^*$)

By altering the capital return, a fiscal policy can modify the portfolio choices of the agents, and thus could be more pertinent to stabilize economies with bubble fluctuations.

To keep things as simple as possible, we consider that the monetary authority fixes the interest rate at a constant level, i.e. $i_{t+1} = i^*$. It would be interesting to lead the same analysis when a Taylor rule is implemented. However, such an analysis would be more complicated to realize.

Furthermore, we restrict our analysis to the cases where the marginal tax rate is not too high and/or an elasticity of capital-labor substitution not too small²⁶:

Assumption 16.

$$\sigma \geq \frac{1 - \alpha}{g'(f'(1))} \equiv \hat{\sigma}$$

²⁶Assumption 16 is equivalent to a marginal tax rate sufficiently small, i.e. $\tau'(f'(1)) \leq 1 - \frac{1-\alpha}{\sigma}$.

We conduct the dynamic analysis focusing again on geometrical arguments. We show that under Assumptions 9 – 16, we obtain the same configuration as Proposition 10.1 and the same figure as in Section 2.5.2. The location of the Σ -line is given in Appendix 2.9.6. When $\varepsilon_u < \varepsilon_u^s$, the Σ -line starts inside the triangle ABC , crosses (AC) below C , then (AB) between A and B , and ends inside the triangle ABC at $(T(+\infty), D(+\infty))$ on the horizontal axis. Under Assumptions 9 – 16, local indeterminacy occurs if $\varepsilon_v < \varepsilon_v^s$.

To determine whether a fiscal policy can stabilize expectation-driven fluctuations around the bubbly steady state, we examine how the critical bifurcation value ε_v^s varies as a function of ρ_2 to get a picture of the role of progressivity of capital income taxation on local indeterminacy. See also Figure 2.3 for a qualitative illustration.

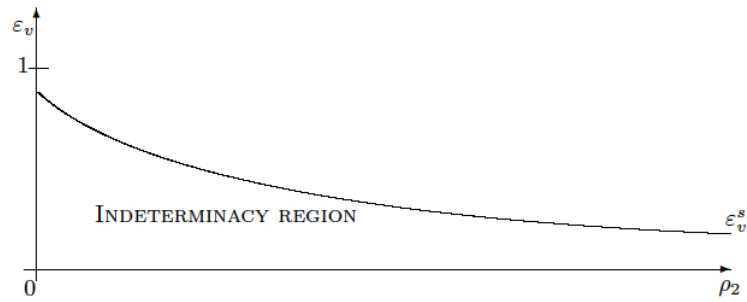


Figure 2.3: Stabilizing role of fiscal policy

Proposition 14. *Under Assumptions 9-16, we have $\partial\varepsilon_v^s/\partial\rho_2 < 0$. Therefore, increasing the marginal progressivity of capital income taxation reduces the range of parameters for local indeterminacy around $k^* = 1$, and the range of parameters for the multiplicity of steady states, i.e. global indeterminacy.*

Proof. See Appendix 2.9.9.

Proposition 14 indicates that increasing the degree of marginal progressivity may rule out expectation-driven fluctuations occurring under gross substitutability, and restore the uniqueness of the steady state.

As underlined in the previous section, the key mechanism generating indeterminacy is the violation of the Fisher equation, characterized by gaps between the returns on physical and monetary assets, due to the presence of collateral.

However, a progressive capital income taxation can reduce the gaps between the returns on different assets along the dynamic path. More precisely, increasing the degree of progressivity of capital income taxation counteracts the collateral effect, and thus mitigates the failure of the Fisher equation (see Eq. (2.13)). Therefore a progressive capital income taxation is powerful to dampen mechanisms that promote endogenous fluctuations.

Closely related, one of the most prominent mechanism for the occurrence of endogenous fluctuations in the neighborhood of a steady state relies on the existence of the so-called wrong slopes in the labor market in one sector Ramsey models with a representative agent. In such models, the stabilizing tool is per se labor income taxation, and progressivity may rule out these wrong slopes Guo and Lansing (1998). In a Ramsey model with heterogeneous agents (Bosi and Seegmuller, 2010b), a larger progressivity in capital income taxation promotes endogenous fluctuations because it makes the after-tax interest rate increasing with capital. In our paper, the mechanism is quite different, and relies on the portfolio choice between the different assets.

We have shown that a fiscal policy is therefore powerful to locally and globally stabilize.

2.7.2 The role of fiscal policy on the efficiency

From a stabilization standpoint, policymakers should resort to a fiscal policy rather than to a standard monetary policy when an economy is characterized by credit market frictions and a rational bubble. However, a stabilizing policy, which eliminates the cost of fluctuations, can have a damaging effect on the efficiency and the stationary welfare.

In our framework, an analysis of the implication of capital income taxation on the welfare at the steady state appears difficult to conduct. However, our analysis of steady states shows that capital income taxation is a distortive instrument which counteracts collateral effect reducing capital accumulation at the steady state. Capital income taxation can rule out the over-accumulation of capital at all steady states, and restore dynamic efficiency.

A progressive capital income taxation proves to be a powerful policy tool both from a stabilization and efficiency perspective. Nevertheless, our conclusions about the destabilizing role of monetary policy made in the previous section should be moderated as long as we consider its implications in term of welfare.

2.8 Concluding remarks

We develop an overlapping generations model with capital accumulation, bonds and money, where the share of consumption purchased on credit depends on the collateral. This allows us to show the existence of expectation-driven fluctuations with a positive rational bubble on bonds. This occurs when the credit share is weakly increasing, but sufficiently concave in the collateral. In addition, endogenous fluctuations are in accordance with gross substitutability and a not too weak substitution between inputs. The basic mechanism for fluctuations relies on a

portfolio trade-off between the three assets due to the violation of the Fisher relationship.

This framework is also used to compare the stabilizing role of fiscal and monetary policies. We show that a progressive taxation of capital income may rule out expectation-driven fluctuations, the multiplicity of steady states, and even over-accumulation of capital. When the monetary policy is fixed according to a Taylor rule on expected inflation, the results are more mitigated depending on the reaction and on the degree of concavity of the utility function. To summarize, in contrast to a Taylor rule, a progressive taxation of capital income is powerful to stabilize the economy which experiences a positive bubble. Nevertheless, our analysis of the welfare at the steady state softens our conclusions concerning the monetary policy.

2.9 Appendix

2.9.1 Proof of Lemma 4

We maximize the Lagrangian function:

$$\begin{aligned}
 \mathcal{L} = & u(c_t) + \beta v(d_{t+1}) \\
 & + \lambda_{1t}(w_t - \pi_{t+1}m_{t+1} - \pi_{t+1}b_{t+1} - k_{t+1} - c_t) \\
 & + \lambda_{2t}(m_{t+1} + (1 + i_{t+1})b_{t+1} + g(R_{t+1}k_{t+1}) + \delta_{t+1} - d_{t+1}) \\
 & + \lambda_{3t}(m_{t+1} - \gamma(k_{t+1})d_{t+1})
 \end{aligned} \tag{2.41}$$

with respect to $(c_t, d_{t+1}, m_{t+1}, b_{t+1}, q_t, \lambda_{1t}, \lambda_{2t}, \lambda_{3t})$.

$$\frac{\partial L(\cdot)}{\partial c} = u'(c_t) - \lambda_{1t} = 0 \quad (2.42)$$

$$\frac{\partial L(\cdot)}{\partial d} = \beta v'(d_{t+1}) - \lambda_{2t} - \gamma(k_{t+1})\lambda_{3t} = 0 \quad (2.43)$$

$$\frac{\partial L(\cdot)}{\partial m} = -\pi_{t+1}\lambda_{1t} + \lambda_{2t} + \lambda_{3t} = 0 \quad (2.44)$$

$$\frac{\partial L(\cdot)}{\partial b} = -\pi_{t+1}\lambda_{1t} + (1 + i_{t+1})\lambda_{2t} = 0 \quad (2.45)$$

$$\frac{\partial L(\cdot)}{\partial k} = -\lambda_{1t} + R_{t+1}g'(R_{t+1}k_{t+1})\lambda_{2t} - \lambda_{3t}\gamma'(k_{t+1})d_{t+1} = 0 \quad (2.46)$$

Since $\lambda_{1t} = u'(c_t) > 0$, $i_{t+1} > -1$ and from Eq. (2.45) $\lambda_{2t} = \lambda_{1t} \frac{\pi_{t+1}}{1+i_{t+1}} > 0$, then the constraints (2.8)-(2.9) become binding. From Eq. (2.44), we obtain:

$$\lambda_{3t} = \lambda_{1t} \frac{i_{t+1}\pi_{t+1}}{1+i_{t+1}} \quad (2.47)$$

The strict positivity of λ_{3t} requires $i_{t+1} > 0$. From Eqs. (2.42)-(2.43) and the expressions of λ_{2t} and λ_{3t} , we get Eq. (2.12). In addition, substituting the expressions of λ_{2t} and λ_{3t} into Eq. (2.46), we obtain Eq. (2.13).

Now, we can compute the Hessian matrix of the Lagrangian function (2.41) with respect to $(\lambda_{1t}, \lambda_{2t}, \lambda_{3t}, c_t, d_{t+1}, m_{t+1}, b_{t+1}, k_{t+1})$ ²⁷:

²⁷For simplicity, the arguments of the functions and the time subscripts are omitted.

$$\mathbf{H}_{88} \equiv \begin{bmatrix} 0 & 0 & 0 & -1 & 0 & -\pi & -\pi & -1 \\ 0 & 0 & 0 & 0 & -1 & 1 & 1+i & Rg'(Rk) \\ 0 & 0 & 0 & 0 & -\gamma & 1 & 0 & -\gamma'd \\ -1 & 0 & 0 & u'' & 0 & 0 & 0 & 0 \\ 0 & -1 & -\gamma & 0 & \beta v'' & 0 & 0 & -\gamma'\lambda_3 \\ -\pi & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ -\pi & 1+i & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & Rg'(Rk) & -\gamma'd & 0 & -\gamma'\lambda_3 & 0 & 0 & R^2g''(Rk)\lambda_2 - \lambda_3\gamma''d \end{bmatrix} \quad (2.48)$$

In order to get a strict local maximum, we need to check the negative definition of \mathbf{H}_{88} over the set of points satisfying the constraints. Let p and n the numbers of constraints and variables. If the determinant of \mathbf{H}_{88} has sign $(-1)^n$ and the last $n - p$ diagonal principal minors have alternating signs, then the optimum is a regular local maximum. In our case $n = 5$ and $p = 3$. Therefore, we need to compute the last two diagonal principal minors, that is $\det \mathbf{H}_{88}$ and $\det \mathbf{H}_{77}$.²⁸ Moreover, we require $\det \mathbf{H}_{88} < 0$ and $\det \mathbf{H}_{77} > 0$.

$$\begin{aligned} \det \mathbf{H}_{88} &= -u''\pi^2(1+i\gamma)^2[R^2g''\lambda_2 - d\gamma''\lambda_3] - \beta v''(1+i)^2[R^2g''\lambda_2 - d\gamma''\lambda_3] \\ &\quad + (1+i)^2(\lambda_3\gamma')^2 \\ &= (\pi\lambda_1)^2 \frac{i\gamma'}{k} \left\{ \frac{\varepsilon_u}{c} \frac{(1+i\gamma)^2\pi}{1+i} \left[\frac{\rho_2\rho_1g}{i\eta_1(1-\gamma)} + \eta_2d \right] \right. \\ &\quad \left. + \frac{\varepsilon_v}{d}(1+i\gamma) \left[\frac{\rho_2\rho_1g}{i\eta_1(1-\gamma)} + \eta_2d \right] - i\eta_1(1-\gamma) \right\} \end{aligned} \quad (2.49)$$

²⁸ \mathbf{H}_{88} is the Hessian matrix, while \mathbf{H}_{77} is the Hessian matrix minus the last column and the last line.

As $\gamma' < 0$, $\det \mathbf{H}_{88} < 0$ if and only if:

$$\frac{\varepsilon_u}{c} \frac{(1+i\gamma)^2 \pi}{1+i} \left[\frac{\rho_2 \rho_1 g}{i\eta_1(1-\gamma)} + \eta_2 d \right] + \frac{\varepsilon_v}{d} (1+i\gamma) \left[\frac{\rho_2 \rho_1 g}{i\eta_1(1-\gamma)} + \eta_2 d \right] - i\eta_1(1-\gamma) > 0 \quad (2.50)$$

While

$$\det \mathbf{H}_{77} = -u'' \pi^2 (1+i\gamma)^2 - (1+i)^2 \beta v'' > 0 \quad (2.51)$$

For $\eta_2 > 0$, $\frac{\varepsilon_u}{c} \frac{(1+i\gamma)^2 \pi}{1+i} \left[\frac{\rho_2 \rho_1 g}{i\eta_1(1-\gamma)} + \eta_2 d \right] - i\eta_1(1-\gamma) > 0$ is a sufficient condition for $\det \mathbf{H}_{88} < 0$. Thus, for $\eta_2 > 0$, the second-order conditions are satisfied if²⁹:

$$\varepsilon_u > c \frac{(1+i)}{\pi} \frac{i\eta_1(1-\gamma)}{\left[\frac{\rho_2 \rho_1 g}{i\eta_1(1-\gamma)} + \eta_2 d \right] (1+i\gamma)^2} \equiv \tilde{\varepsilon}_u \quad (2.52)$$

whatever the value of $\varepsilon_v \geq 0$. □

2.9.2 Proof of Proposition 7

A steady state k is a solution of $h(k) = j(k)$, with:

$$h(k) \equiv \frac{u'(c(k))}{\beta v'(d(k))} \quad (2.53a) \quad j(k) \equiv \frac{1}{1+i^* \gamma(k)} \quad (2.53b)$$

where $c(k) \equiv f(k) - k - \frac{k[1-\psi(k)]}{i^* \eta_1(k)[1-\gamma(k)]} - [k f'(k) - g(f(k)k)]$ and $d(k) \equiv \frac{k[1-\psi(k)]}{i^* \eta_1(k)[1-\gamma(k)]}$, with $\psi(k) \equiv f'(k)g'(f(k))$.

²⁹For simplicity, the arguments of the functions and the time subscripts are omitted.

We start by determining the admissible range of values for k . To ensure $d(k) > 0$, we get at the steady state $\psi(k) < 1$. Under Assumptions 9 and 12, $\psi(k)$ is a decreasing function of k . Hence, $k > \psi^{-1}(1) = \underline{k}$.

Now, we want to determine the range of k such that $c(k) > 0$. The decreasing returns on capital imply:

$$f(\underline{k}) > \underline{k}f'(\underline{k})$$

Since $\psi(\underline{k}) = 1$, one has $f'(\underline{k}) = \frac{1}{g'(f'(\underline{k})\underline{k})}$. Under Assumption 9, $g(y_k)$ is concave, i.e. $g(f'(k)k) > g'(f'(k)k)f'(k)k$. This involves that:

$$g(f'(\underline{k})\underline{k}) > \underline{k}$$

Hence, we deduce that:

$$c(\underline{k}) = f(\underline{k}) - \underline{k}f'(\underline{k}) + g(f'(\underline{k})\underline{k}) - \underline{k} > 0$$

In addition, as $d(k) > 0$ and $\tau(f'(k)k) = kf'(k) - g(f'(k)k) > 0$, we derive the following inequality:

$$\lim_{k \rightarrow +\infty} c(k) < \lim_{k \rightarrow +\infty} f(k) - k = -\infty$$

because $f'(k) < 1$ for k large enough. As a result, there exists one value \bar{k} such that $\forall k < \bar{k}$, $c(\bar{k}) > 0$. By construction, we have $\underline{k} < \bar{k}$, and therefore (\underline{k}, \bar{k}) is a nonempty subset.

To prove the existence of a stationary solution k , we use the continuity of $h(k)$ and $j(k)$. Using Eqs. (2.53a) and (2.53b), we determine the boundary values of $h(k)$ and $j(k)$:

$$\begin{aligned} \lim_{k \rightarrow \underline{k}} h(k) &= \frac{u'(c(\underline{k}))}{\beta v'(0)} = 0^+ & \lim_{k \rightarrow \bar{k}} h(k) &= \frac{u'(c(0))}{\beta v'(d(\bar{k}))} + \infty \\ \lim_{k \rightarrow \underline{k}} j(k) &= \frac{1}{1 + i(\underline{k})\gamma(\underline{k})} \in]0, 1] & \lim_{k \rightarrow \bar{k}} j(k) &= \frac{1}{1 + i(\bar{k})\gamma(\bar{k})} \leq 1 \end{aligned}$$

We have $\lim_{k \rightarrow \underline{k}} h(k) < \lim_{k \rightarrow \underline{k}} j(k)$ and $\lim_{k \rightarrow \bar{k}} h(k) > \lim_{k \rightarrow \bar{k}} j(k)$. Therefore, there exists at least one value $k^* \in (\underline{k}, \bar{k})$ such that $h(k^*) = j(k^*)$. \square

2.9.3 Proof of Proposition 9

Let

$$\varepsilon_{dk} \equiv \frac{d'(1)}{d(1)} = \frac{\psi}{1 - \psi} \left[\frac{1 - \alpha}{\sigma} + \rho_2 \left(1 - \frac{1 - \alpha}{\sigma} \right) \right] + \eta_2, \quad (2.54)$$

$$\nu(\eta_1) = i^* \eta_1 (1 - \gamma), \text{ and} \quad (2.55)$$

$$c^* = f(1) - 1 - \frac{1 - \psi}{\nu(\eta_1)} - [f'(1) - g'(f'(1))] \quad (2.56)$$

To deal with the multiplicity of stationary solutions, we derive the following elasticities at the normalized steady state $k^* = 1$:

$$\begin{aligned} \epsilon_h \equiv \frac{h'(1)}{h(1)} &= \varepsilon_v \varepsilon_{dk} + \frac{\varepsilon_u}{c^*} \left\{ 1 - f'(1) + \frac{1 - \psi}{\nu(\eta_1)} \varepsilon_{dk} \right. \\ &\quad \left. + f'(1) \left(1 - \frac{1 - \alpha}{\sigma} \right) [1 - g'(f'(1))] \right\} \\ \epsilon_j \equiv \frac{j'(1)}{j(1)} &= \frac{\nu(\eta_1)}{1 + i^* \gamma} \end{aligned} \quad (2.57)$$

We can now derive the conditions for the multiplicity of steady states. Let

$$\Omega \equiv 1 - f'(1) + \frac{1 - \psi}{\nu(\eta_1)} \varepsilon_{dk} + f'(1) \left(1 - \frac{1 - \alpha}{\sigma}\right) [1 - g'(f'(1))] \quad (2.58)$$

$$\varepsilon_u^s \equiv c^* \Omega^{-1} \frac{\nu(\eta_1)}{1 + i^* \gamma} \quad (2.59)$$

$$\tilde{\varepsilon}_u \equiv c^* \frac{\nu(\eta_1)^2}{[\rho_2 \psi + \eta_2 (1 - \psi)] (1 + i^* \gamma)^2} \quad (2.60)$$

A sufficient condition for the multiplicity of steady states is $\varepsilon_h < \varepsilon_j$ for $k^* = 1$. This is equivalent to the inequality written in Proposition 9, that is³⁰:

$$\varepsilon_v < \frac{\Omega}{\varepsilon_{dk}} \left(\frac{\varepsilon_u^s}{c^*} - \frac{\varepsilon_u}{c^*} \right) \equiv \varepsilon_v^s \quad (2.61)$$

Note that this condition can be satisfied only if $\varepsilon_u \in (\tilde{\varepsilon}_u, \varepsilon_u^s)$ (see Assumption 11).

Using the notations of the proof of Proposition 7, we know that $h(\bar{k}) > j(\bar{k})$ and $h(\underline{k}) < j(\underline{k})$. Since $k^* = 1$ is a steady state, we have: $h(1) = j(1)$. If the inequality written in Proposition 9 is satisfied, we have $\varepsilon_h(1) < \varepsilon_j(1)$, then by continuity at least two other steady states exist, k_1 and k_2 such that $k_1 < 1 < k_2$. The number of steady states is generically odd. \square

2.9.4 Proof of Example in Section 2.4.2

Let $\bar{b} \equiv \frac{c^2 - 1 + a}{c}$, $\underline{\varepsilon} \equiv \frac{1}{i(\bar{b} - b)} \frac{(1+c)^2}{c} \left(\frac{A\alpha}{1 - A\alpha} \frac{1 - \alpha}{\sigma} - i\gamma \right)$, $\underline{\sigma} \equiv 1 - \frac{1}{2 + \nu(\eta_1)}$, $\bar{\alpha} \equiv \frac{1}{2 + \nu(\eta_1)}$, $\underline{A} \equiv \frac{1 + \nu(\eta_1)}{\alpha + \nu(\eta_1)}$ and $\bar{A} \equiv \frac{1/\alpha}{1 + \nu(\eta_1)}$. For $a \in (0, 1)$, $c > 1$, $b \in (ac, c)$, Assumption 11 is satisfied at the normalized steady state. Assumption 12 requires $A < 1/\alpha$ and $\sigma > 1 - \alpha$. For $A > \underline{A}$, Assumption 13 is satisfied. Moreover, the bubble is positive at the normalized steady state when $A < \bar{A}$. As a consequence, the set (\underline{A}, \bar{A}) must be non-empty. This is true for $\alpha < \bar{\alpha}$ and $\sigma > \underline{\sigma}$. Assumption 14 requires, for

³⁰Under Assumptions 9-12, one has: $\varepsilon_{dk} > 0$, $\Omega > 0$ and $\varepsilon_u^s > 0$.

instance, b closed to ac and ϵ sufficiently large. In addition, if $b \in (ac, \bar{b})$ and $\epsilon > \underline{\epsilon}$, then $\epsilon_u^s > \tilde{\epsilon}_u$. Note that (ac, \bar{b}) is a non-empty set if and only if $a < 1$.

2.9.5 Proofs for Section 2.5.1

2.9.5.1 Linearized dynamic system

We linearize the system (2.32) around a steady state $k^* = 1$ with respect to $(k_t, \theta_t, k_{t+1}, \theta_{t+1})$. Let $\nu(\eta_1) \equiv i^* \eta_1 (1 - \gamma) > 0$ and $a_\phi = \frac{\phi}{1 - \phi}$, we obtain ³¹:

$$\begin{aligned} & \left[\epsilon_v \frac{\nu(\eta_1)\psi}{1 - \psi} \left(1 - \frac{1 - \alpha}{\sigma} \right) + \frac{\epsilon_u}{c^*} - \frac{\nu(\eta_1)}{1 + i^*\gamma} \right] \frac{dk_{t+1}}{k} \\ & \quad + \left[\epsilon_v \frac{1 - \psi - g\nu(\eta_1)}{1 - \psi} - 1 - a_\phi \frac{\gamma(1 + i^*)}{1 + i^*\gamma} \right] \frac{d\theta_{t+1}}{\theta} \\ = & \frac{\epsilon_u}{c^*} f'(1) \frac{1 - \alpha}{\sigma} \frac{dk_t}{k} - \left[1 + \frac{\epsilon_u}{c^*} \frac{1 - \psi - gi^*\eta_1(1 - \gamma)}{\nu(\eta_1)} + a_\phi \frac{\gamma(1 + i^*)}{1 + i^*\gamma} \right] \frac{d\theta_t}{\theta} \\ & - (1 - \psi) \left\{ \frac{\psi}{1 - \psi} \left[\frac{1 - \alpha}{\sigma} + \rho_2 \left(1 - \frac{1 - \alpha}{\sigma} \right) \right] + \eta_2 - \frac{\nu(\eta_1)\psi}{1 - \psi} \left(1 - \frac{1 - \alpha}{\sigma} \right) \right\} \frac{dk_{t+1}}{k} \\ & - \left[\psi + g\nu(\eta_1) + a_\phi (1 - \psi) \frac{1 + i^*}{i^*} \right] \frac{d\theta_{t+1}}{\theta} = - \left[1 + (1 - \psi) a_\phi \frac{1 + i^*}{i^*} \right] \frac{d\theta_t}{\theta} \end{aligned}$$

³¹For simplicity, the arguments of the functions are omitted.

From these above dynamic equations, we derive the characteristic polynomial. Let

$$\chi_1 = \chi_1^a a_\phi + \chi_1^b, \text{ with} \quad (2.62)$$

$$\chi_1^a \equiv -\psi\nu(\eta_1) \left(1 - \frac{1-\alpha}{\sigma}\right) \frac{1+i^*}{i^*}, \quad (2.63)$$

$$\chi_1^b \equiv [1 - \psi - g\nu(\eta_1)] \varepsilon_{dk} - \frac{\psi}{1-\psi} \nu(\eta_1) \left(1 - \frac{1-\alpha}{\sigma}\right), \quad (2.64)$$

$$\chi_2 = \chi_2^a a_\phi + \chi_2^b, \text{ with} \quad (2.65)$$

$$\chi_2^a \equiv -(1-\psi) \frac{1+i^*}{i^*}, \quad (2.66)$$

$$\chi_2^b \equiv -[\psi + g\nu(\eta_1)], \quad (2.67)$$

$$\chi_3 = \chi_3^a a_\phi + \chi_3^b, \text{ with} \quad (2.68)$$

$$\chi_3^a \equiv \frac{(1-\psi)(1+i^*)}{i^*(1+i^*\gamma)} \left\{ \nu(\eta_1) \left[1 + \frac{i^*\gamma\psi}{1-\psi} \left(1 - \frac{1-\alpha}{\sigma}\right) \right] - \varepsilon_{dk} i^*\gamma \right\}, \quad (2.69)$$

$$\chi_3^b \equiv \nu(\eta_1)^2 \frac{g}{1+i^*\gamma} + \nu(\eta_1)\psi \left(1 - \frac{1-\alpha}{\sigma} + \frac{1}{1+i^*\gamma}\right) - (1-\psi) \varepsilon_{dk}, \quad (2.70)$$

with ε_{dk} given by Eq. (2.54) in Appendix 2.9.3.

Lemma 6. *Let*

$$\bar{\varepsilon}_v \equiv -\chi_1^{-1} \left(\frac{\varepsilon_u}{c^*} \chi_2 + \chi_3 \right). \quad (2.71)$$

Under Assumptions 9-14 and $\varepsilon_v \neq \bar{\varepsilon}_v$, the characteristic polynomial, evaluated at the steady state $k^* = 1$, writes $P(X) \equiv X^2 - T(\varepsilon_v)X + D(\varepsilon_v) = 0$:

$$D(\varepsilon_v) = -\varepsilon_u/c^* f'(1) \frac{1-\alpha}{\sigma} \frac{1 + (1-\psi) a_\phi (1+i^*)/i^*}{\chi_1 (\varepsilon_v - \bar{\varepsilon}_v)} \quad (2.72)$$

$$T(\varepsilon_v) = 1 + D(\varepsilon_v) - [1 - \psi - g\nu(\eta_1)] \varepsilon_{dk} \frac{\varepsilon_v - \varepsilon_v^s}{\chi_1 (\varepsilon_v - \bar{\varepsilon}_v)}, \quad (2.73)$$

where ε_{dk} and ε_v^s are given by Eqs. (2.54) and (2.61) in Appendix 2.9.3.

2.9.5.2 Characteristics of the Σ -line

The Σ -line is characterized by a starting point and an endpoint given by:

$$\begin{aligned}
 D(0) &= \varepsilon_u/c^* f'(1) \frac{1-\alpha}{\sigma} \frac{1+(1-\psi)a_\phi(1+i^*)/i^*}{\chi_1 \bar{\varepsilon}_v} \\
 T(0) &= 1 + D(0) - [1-\psi - g\nu(\eta_1)] \varepsilon_{dk} \frac{\varepsilon_v^s}{\chi_1 \bar{\varepsilon}_v} \\
 D(+\infty) &= 0 \\
 T(+\infty) &= -\frac{\psi\nu(\eta_1) \left(1 - \frac{1-\alpha}{\sigma}\right) [1/(1-\psi) + a_\phi(1+i^*)/i^*]}{\chi_1}
 \end{aligned}$$

We further note that the Σ -line has a slope S given by:

$$S = \frac{D'(\varepsilon_v)}{T'(\varepsilon_v)} = \frac{Z_1}{Z_1 + Z_2}, \text{ where} \quad (2.74)$$

$$Z_1 = (\varepsilon_u/c^*) f'(1) \frac{1-\alpha}{\sigma} \left[1 + (1-\psi) a_\phi \frac{1+i^*}{i^*}\right] \quad (2.75)$$

$$Z_2 = [1-\psi - g\nu(\eta_1)] \varepsilon_{dk} (\bar{\varepsilon}_v - \varepsilon_v^s) \quad (2.76)$$

2.9.5.3 Bifurcation values

ε_v^h is defined by $D(\varepsilon_v) = 1$:

$$\varepsilon_v^h = \frac{\Upsilon}{\chi_1} \left(\frac{\varepsilon_u}{c^*} - \frac{\chi_3}{\Upsilon} \right), \text{ where} \quad (2.77)$$

$$\Upsilon = \Upsilon^a a_\phi + \Upsilon^b, \text{ with} \quad (2.78)$$

$$\Upsilon^a \equiv -\chi_2^a - f'(1) (1-\psi) \frac{1-\alpha}{\sigma} \frac{1+i^*}{i^*}, \text{ and} \quad (2.79)$$

$$\Upsilon^b \equiv -\chi_2^b - f'(1) \frac{1-\alpha}{\sigma} \quad (2.80)$$

with χ_1 , χ_2^a , χ_2^b and χ_3 given by Eqs. (2.62), (2.66), (2.67) and (2.68) in Appendix 2.9.5.

ε_v^s is defined by $1 - T(\varepsilon_v) + D(\varepsilon_v) = 0$:

$$\varepsilon_v^s = \frac{\Omega}{\varepsilon_{dk}} \left(\frac{\varepsilon_u^s}{c^*} - \frac{\varepsilon_u}{c^*} \right), \quad (2.81)$$

where Ω and ε_u^s are respectively given by Eqs. (2.58) and (2.59) in Appendix 2.9.3.

ε_v^f is defined by $1 + T(\varepsilon_v) + D(\varepsilon_v) = 0$:

$$\varepsilon_v^f = \frac{\zeta_2}{\zeta_1} \left(\frac{\varepsilon_u}{c^*} - \frac{\zeta_3}{\zeta_2} \right), \quad \text{where} \quad (2.82)$$

$$\zeta_1 = \zeta_1^a a_\phi + \zeta_1^b, \quad \text{with} \quad (2.83)$$

$$\zeta_1^a \equiv 2\chi_1^a, \quad \text{and} \quad (2.84)$$

$$\zeta_1^b \equiv \chi_1^b - \frac{\psi}{1-\psi} \nu(\eta_1) \left(1 - \frac{1-\alpha}{\sigma} \right) \quad (2.85)$$

$$\zeta_2 = \zeta_2^a a_\phi + \zeta_2^b \quad \text{with} \quad (2.86)$$

$$\zeta_2^a \equiv 2(1-\psi) \frac{1+i^*}{i^*} \left[1 + f'(1) \frac{1-\alpha}{\sigma} \right], \quad \text{and} \quad (2.87)$$

$$\zeta_2^b \equiv 2 \left[\psi + g\nu(\eta_1) + f'(1) \frac{1-\alpha}{\sigma} \right] + \Omega [1 - \psi - g\nu(\eta_1)] \quad (2.88)$$

$$\zeta_3 = \zeta_3^a a_\phi + \zeta_3^b, \quad \text{with} \quad (2.89)$$

$$\zeta_3^a \equiv 2\chi_3^a, \quad \text{and} \quad (2.90)$$

$$\zeta_3^b \equiv \chi_3^b + \nu(\eta_1) \left[\psi \left(1 - \frac{1-\alpha}{\sigma} \right) + \frac{1}{1+i^*\gamma} \right] - (1-\psi) \varepsilon_{dk} \quad (2.91)$$

with χ_1^a , χ_1^b , χ_3^a and χ_3^b respectively given by Eqs. (2.63), (2.64), (2.69) and (2.70) in Appendix 2.9.5.1 and ε_{dk} given by Eq. (2.54) in Appendix 2.9.3.

2.9.6 Proof of Proposition 10

In this section, we locate the Σ -line when $\phi = 0$ ($a_\phi = 0$) in the (T, D) plane, using $(T(0), D(0))$, $(T(+\infty), D(+\infty))$ and the value of its slope S when $\phi = 0$ ($a_\phi = 0$).

We recall that we consider η_1 not too large (Assumption 14), since we are interested in the equilibria with a positive bubble ($\theta > 0$).

Before analyzing the location of the Σ -line, we need to determine the sign of χ_1 , χ_2 and χ_3 when $a_\phi = 0$. More precisely, we will show that for η_1 sufficiently small, $\chi_1 > 0$, $\chi_2 < 0$ and $\chi_3 < 0$. First, under Assumptions 9-13 and because $\psi < 1$ at the steady state, we have $\varepsilon_{dk} > 0$, $\Omega > 0$, and $\varepsilon_u^s > 0$.³² There is a threshold $\eta_1^{\chi_1} \in \mathbb{R}_+$ such that $\forall \eta_1 < \eta_1^{\chi_1}$, $\chi_1 > 0$ under Assumptions 9-13. Furthermore, $\chi_2 < 0 \forall \eta_1 \in \mathbb{R}_+$ under Assumptions 9-13. Finally, there is a threshold $\eta_1^{\chi_3} \in \mathbb{R}_+$ such that $\forall \eta_1 < \eta_1^{\chi_3}$, $\chi_3 < 0$ under Assumptions 9-13. Thus, under Assumptions 9-14, one has: $\chi_1 > 0$, $\chi_2 < 0$ and $\chi_3 < 0$, and therefore, $\bar{\varepsilon}_v = -\chi_1^{-1} \left(\frac{\varepsilon_u}{c^*} \chi_2 + \chi_3 \right) > 0$. For the rest of the proof, we consider $\psi < 1$, $1 - \psi - g\nu(\eta_1) > 0$, $\varepsilon_{dk} > 0$, $\Omega > 0$, $\varepsilon_u^s > 0$, $\chi_1 > 0$, $\chi_2 < 0$, $\chi_3 < 0$ and $\bar{\varepsilon}_v > 0$.³³

The starting point and the endpoint of the Σ -line are such that:

$$D(0) = \varepsilon_u / c^* f'(1) \frac{1 - \alpha}{\sigma} \frac{1}{\chi_1 \bar{\varepsilon}_v} > 0 \quad (2.92)$$

$$1 - T(0) + D(0) = \frac{1 - \psi - g\nu(\eta_1)}{\bar{\varepsilon}_v} \varepsilon_{dk} \left(\frac{\varepsilon_u^s}{c^*} - \frac{\varepsilon_u}{c^*} \right) \quad (2.93)$$

$$D(+\infty) = 0 \text{ and } T(+\infty) = -\frac{\psi}{1 - \psi} \frac{\nu(\eta_1)}{\chi_1} \left(1 - \frac{1 - \alpha}{\sigma} \right), \quad (2.94)$$

where ε_u^s and $\bar{\varepsilon}_v$ are respectively given by Eqs. (2.59) and (2.71).

First of all, let us see whether $D(0) < 1$. $D(0) < 1$ is equivalent to the following condition:

$$\chi_3 < \frac{\varepsilon_u}{c^*} \left\{ f'(1) \left[g'(f'(1)) - \frac{1 - \alpha}{\sigma} \right] + g\nu(\eta_1) \right\}$$

³²The expressions of $\varepsilon_{dk} > 0$, $\Omega > 0$, and $\varepsilon_u^s > 0$ are respectively given by Eqs. (2.54), (2.58) and (2.59) in Appendix 2.9.3.

³³From this proof, we deduce that $\chi_1^b > 0$, $\chi_2^b < 0$ and $\chi_3^b > 0$.

This above condition is satisfied under Assumptions 9-14 and 16. Hence, the starting point $(T(0), D(0))$ locates below the segment $[BC]$.

The sign of $1 - T(0) + D(0)$ informs us on which side of (AC) the starting point $(T(0), D(0))$ is located. When $\varepsilon_u < \varepsilon_u^s$, one has $1 - T(0) + D(0) > 0$. Nevertheless, the second-order conditions require that $\varepsilon_u^s > \tilde{\varepsilon}_u$ which is equivalent to³⁴:

$$\eta_2(1 - \psi)i^*\gamma > i^*\eta_1(1 - \gamma) \left\{ 1 - f'(1) + \left(1 - \frac{1 - \alpha}{\sigma} \right) [f'(1) - \psi] \right\} + \psi \frac{1 - \alpha}{\sigma} - \psi \rho_2 \left(\frac{1 - \alpha}{\sigma} + i^*\gamma \right) \quad (2.95)$$

The condition (2.95) holds if and only if

$$\eta_1 < \frac{1}{i^*(1 - \gamma)} \frac{\eta_2(1 - \psi)i^*\gamma - \psi \left[\frac{1 - \alpha}{\sigma} - \rho_2 \left(\frac{1 - \alpha}{\sigma} + i^*\gamma \right) \right]}{1 - f'(1) + \left(1 - \frac{1 - \alpha}{\sigma} \right) [f'(1) - \psi]} \equiv \bar{\eta}_1, \text{ and } (2.96)$$

$$\eta_2 > \frac{\psi}{1 - \psi} \frac{(1 - \alpha) / \sigma - \rho_2 \left(\frac{1 - \alpha}{\sigma} + i^*\gamma \right)}{i^*\gamma} \equiv \bar{\eta}_2 \quad (2.97)$$

Hence, for a small degree of concavity on $u(c)$ and under Assumptions 9-14, the starting point locates on the left-side of (AC) . In addition, when $\varepsilon_u > \varepsilon_u^s$, we get $1 - T(0) + D(0) < 0$. The starting point locates on the right-side of (AC) when $\varepsilon_u > \varepsilon_u^s$.

The endpoint $(T(+\infty), D(+\infty))$ locates on the horizontal axis. Furthermore, one has $T(+\infty) < 0$ under Assumptions 9-14 (see Eq. (2.94)). Let us see now on which side of (AB) is $(T(+\infty), D(+\infty))$, that is if $T(+\infty)$ is lower or greater than -1 . $T(+\infty) > -1$ involves the following condition:

$$[1 - \psi - g\nu(\eta_1)] \frac{1 - \psi}{\nu(\eta_1)} \varepsilon_{dk} - 2\psi \left(1 - \frac{1 - \alpha}{\sigma} \right) > 0,$$

³⁴ $\tilde{\varepsilon}_u$ is given by Eq. (2.60) in Appendix 2.9.3.

which is equivalent to

$$\eta_1 < \frac{(1 - \psi)^2 \varepsilon_{dk}}{g(1 - \psi) \varepsilon_{dk} + 2\psi \left(1 - \frac{1-\alpha}{\sigma}\right) i^* (1 - \gamma)} \frac{1}{i^* (1 - \gamma)} \equiv \hat{\eta}_1$$

Under Assumptions 9-13, this inequality is satisfied for a sufficiently small η_1 . Thus, the endpoint is on the right-side of (AB) under Assumptions 9-14. More precisely, it is inside the triangle (ABC) between $(-1, 0)$ and $(0, 0)$.

Now, we study the slope S of the Σ -line and how $T(\varepsilon_v)$ and $D(\varepsilon_v)$ vary with respect to ε_v . When $\phi = 0$, the slope S of the Σ -line is given by:

$$S = \frac{D'(\varepsilon_v)}{T'(\varepsilon_v)} = \frac{Z_1}{Z_1 + Z_2} \quad (2.98)$$

where

$$\begin{aligned} Z_1 &= (\varepsilon_u/c^*) f'(1) \frac{1-\alpha}{\sigma} > 0 \quad \text{and} \\ Z_2 &= Z_2^a + Z_2^b, \quad \text{with} \\ Z_2^a &= \frac{[1 - \psi - g\nu(\eta_1)] \varepsilon_{dk} \varepsilon_u}{\chi_1 c^*} \left(\chi_1 \frac{\Omega}{\varepsilon_{dk}} - \chi_2 \right), \quad \text{and} \\ Z_2^b &= -\frac{[1 - \psi - g\nu(\eta_1)] \varepsilon_{dk}}{\chi_1} \left[\frac{i^* \eta_1 (1 - \gamma)}{1 + i^* \gamma} \frac{\chi_1}{\varepsilon_{dk}} + \chi_3 \right], \end{aligned}$$

with $\chi_1 > 0$, $\chi_2 < 0$, $\chi_3 < 0$ respectively given by Eqs. (2.62), (2.65) and (2.68) in Appendix 2.9.5, and $\varepsilon_{dk} > 0$, $\Omega > 0$ and c^* by Eqs. (2.54), (2.58) and (2.56) in Appendix 2.9.3.

Under Assumptions 9-14, we have $Z_2^a \geq 0$. Z_2^b can be rewritten as follows:

$$Z_2^b = -\frac{[1 - \psi - g\nu(\eta_1)] \varepsilon_{dk}}{\chi_1} P(\nu(\eta_1)),$$

where $P(\nu(\eta_1))$ is a quadratic polynomial defined on \mathbb{R}_+ such that:

$$P(\nu(\eta_1)) = -\frac{\nu(\eta_1)^2}{1+i^*\gamma} \frac{1}{\varepsilon_{dk}} \frac{\psi}{1-\psi} \left(1 - \frac{1-\alpha}{\sigma}\right) + \nu(\eta_1) \left[\psi \left(1 - \frac{1-\alpha}{\sigma}\right) + \frac{1}{1+i^*\gamma} \right] - (1-\psi) \varepsilon_{dk}$$

Under Assumptions 9-12, $P(\nu(\eta_1))$ is a concave function with $P(\nu(0)) < 0$ and reaches its maximum for $\eta_1 = \eta_{1max} > 0$. As a consequence, there is a threshold $\underline{\eta}_1 \in \mathbb{R}_+$ such that $\forall \eta_1 < \underline{\eta}_1$, $P(\nu(\eta_1))$ is negative. This implies that $Z_2^b > 0$ under Assumptions 9-14, and therefore, $Z_2 > 0$.

As $Z_1 > 0$ and $Z_2 > 0$, we can conclude that the slope S of the Σ -line belongs to $(0, 1)$ under Assumptions 9-14. Moreover, as $D'(\varepsilon_v) = Z_1 / [\chi_1(\varepsilon_v - \bar{\varepsilon}_v)^2]$ and $T'(\varepsilon_v) = D'(\varepsilon_v) + Z_2 / [\chi_1(\varepsilon_v - \bar{\varepsilon}_v)^2]$, we also have $T'(\varepsilon_v) \geq 0$ and $D'(\varepsilon_v) \geq 0$.

To further analyze the slope S , we now show that the Σ -line goes below C . We need to prove that under Assumptions 9-14 (i.e η_1 not too large), we have $\varepsilon_v^s < \varepsilon_v^h$. This is equivalent to the following inequality:

$$\begin{aligned} 0 &> \kappa_1 + \kappa_2, \quad \text{with} \\ \kappa_1 &= -\frac{\varepsilon_u}{c^*} \left\{ \frac{\Omega}{\varepsilon_{dk}} + \frac{g\nu(\eta_1) + f'(1) \left[g'(f'(1)) - \frac{1-\alpha}{\sigma} \right]}{\chi_1} \right\}, \quad \text{and} \\ \kappa_2 &= \frac{\nu(\eta_1)}{1+i^*\gamma} \frac{1}{\varepsilon_{dk}} + \frac{\chi_3}{\chi_1} \end{aligned}$$

Under Assumptions 9-14 and 16, $\kappa_1 \leq 0$. As regards κ_2 , we can rewrite it as follows:

$$\kappa_2 = \frac{P(\nu(\eta_1))}{\chi_1}$$

From the previous analysis about the value of the slope S , we know that $P(\nu(\eta_1)) < 0$ under Assumptions 9-14. Therefore, $\kappa_2 < 0$ under these assumptions. As $\kappa_1 < 0$ and $\kappa_2 < 0$, we can conclude that under Assumptions 9-14:

$$\varepsilon_v^s < \varepsilon_v^h < \varepsilon_v^f \quad (2.99)$$

As a consequence, the Σ -line goes below C .

Note that the proof also holds true when no fiscal policies are implemented (i.e. $g(Rk) = Rk$), and in particular for $\rho_1 = 1$ and $\rho_2 = 0$.

As regards the size of ε_v^s , we can show that when no progressive fiscal policies are implemented (i.e. $\rho_1 = 1$ and $\rho_2 = 0$), $\varepsilon_v^s < 1$. From the condition $\varepsilon_u^s > \tilde{\varepsilon}_u$, we know that³⁵:

$$\nu(\eta_1) < \eta_2 i^* \gamma - \frac{f'(1)}{1 - f'(1)} \frac{1 - \alpha}{\sigma}$$

Note that $\varepsilon_{dk} = \frac{f'(1)}{1 - f'(1)} \frac{1 - \alpha}{\sigma} + \eta_2$ when $\rho_1 = 1$ and $\rho_2 = 0$. Hence, we have

$$\nu(\eta_1) / (1 + i^* \gamma) \frac{1}{\varepsilon_{dk}} < 1.$$

Since

$$\varepsilon_v^s = \frac{\nu(\eta_1)}{1 + i^* \gamma} \frac{1}{\varepsilon_{dk}} - \frac{\varepsilon_u \nu(\eta_1) + \varepsilon_{dk} (1 - f'(1))}{c^* \varepsilon_{dk} \nu(\eta_1)},$$

we can conclude that $\varepsilon_v^s < 1$. Furthermore, we can also show that $\varepsilon_v^h > 1$ for a sufficiently small η_1 when no progressive fiscal policies are implemented (i.e. $\rho_1 = 1$

³⁵The expressions ε_u^s and $\tilde{\varepsilon}_u$ are respectively given by Eqs. (2.59) and (2.60) in Appendix 2.9.3.

and $\rho_2 = 0$). $\varepsilon_v^h > 1$ is equivalent to

$$f'(1)Q(\nu(\eta_1)) > 0,$$

where $Q(\nu(\eta_1))$ is a quadratic polynomial defined on \mathbb{R}_+ such that:

$$\begin{aligned} Q(\nu(\eta_1)) = & -\frac{\nu(\eta_1)^2}{1+i^*\gamma} + \nu(\eta_1) \left[\frac{\varepsilon_u}{c^*} - \left(1 - \frac{1-\alpha}{\sigma} + \frac{1}{1+i\gamma} \right) \right. \\ & \left. + \varepsilon_{dk} + \frac{1}{1-f'(1)} \left(1 - \frac{1-\alpha}{\sigma} \right) \right] + \left(1 - \frac{1-\alpha}{\sigma} \right) \frac{\varepsilon_u}{c^*} \end{aligned}$$

Under Assumptions 9-13, $Q(\nu(\eta_1))$ is a concave function with $Q(\nu(0)) > 0$. As a consequence, there is a threshold $\tilde{\eta}_1 \in \mathbb{R}_+$ such that $\forall \eta_1 < \tilde{\eta}_1$, $Q(\nu(\eta_1))$ is positive.

Therefore, under Assumptions 9-14, we get: $\varepsilon_v^s < 1 < \varepsilon_v^h < \varepsilon_v^f$. \square

2.9.7 Proofs of Lemma 5 and Proposition 11

We study the variations of critical values (ε_v^f , ε_v^h , ε_v^s and $\bar{\varepsilon}_v$) with respect to $a_\phi \neq 0$ when $g(Rk) = Rk$.

The expressions of $1 - T(\varepsilon_v) + D(\varepsilon_v)$, $1 + T(\varepsilon_v) + D(\varepsilon_v)$ and $D(\varepsilon_v)$ can be written as follows:

$$\begin{aligned} 1 - T(\varepsilon_v) + D(\varepsilon_v) &= \{1 - f'(1)[1 + \nu(\eta_1)]\} \varepsilon_{dk} \frac{\varepsilon_v - \varepsilon_v^s}{\chi_1 (\varepsilon_v - \bar{\varepsilon}_v)} \\ 1 + T(\varepsilon_v) + D(\varepsilon_v) &= \frac{\zeta_1 (\varepsilon_v - \varepsilon_v^f)}{\chi_1 (\varepsilon_v - \bar{\varepsilon}_v)} \\ 1 - D(\varepsilon_v) &= \frac{\varepsilon_v - \varepsilon_v^h}{\varepsilon_v - \bar{\varepsilon}_v}, \end{aligned}$$

with $\bar{\varepsilon}_v$, ε_v^s , ε_v^f , ε_v^h , χ_1 and ζ_1 are respectively given by Eqs. (2.71), (2.81), (2.82), (2.77), (2.62) and (2.83) in Appendix 2.9.5, and ε_{dk} by Eq. (2.54) in Appendix 2.9.5.

In this section, we graphically represent the variations of the bifurcation values $(\varepsilon_v^s, \varepsilon_v^f, \varepsilon_v^h)$ and the critical value $\bar{\varepsilon}_v$ in the (a_ϕ, ε_v) plane, and identify the areas in which the normalized steady state is a sink. Note that a_ϕ is defined on $] -\infty, -1[\cup] 0, +\infty[$, since $\phi > 0$.

For the rest of the proof, we recall that under Assumptions 10 – 14, $\chi_1^a < 0$, $\chi_1^b > 0$, $\chi_2^a < 0$, $\chi_2^b < 0$, $\chi_3^a < 0$, $\chi_3^b < 0$, $\zeta_1^a < 0$, $\zeta_1^b > 0$, $\zeta_2^a > 0$, $\zeta_2^b > 0$, $\zeta_3^a < 0$, $\zeta_3^b < 0$, $\Upsilon^a > 0$ and $\Upsilon^b > 0$.

We can see from Eq. (2.61) that ε_v^s does not depend on a_ϕ . On the other hand, the different bifurcation and critical values $(\varepsilon_v^f, \varepsilon_v^h$ and $\bar{\varepsilon}_v)$ are homographic functions of a_ϕ (see Eqs. (2.71), (2.82) and (2.77) in Appendix 2.9.5 with $g'(f'(1)) = 1$ and $\psi = f'(1)$).

ε_v^f has a vertical asymptote at $a_\phi = -\zeta_1/(2\chi_1^a) \equiv \tilde{a}_\phi > 0$ under Assumptions 10 – 14, and that ε_v^h and $\bar{\varepsilon}_v$ have the same vertical asymptote at $a_\phi = -\chi_1^b/\chi_1^a \equiv \bar{a}_\phi > -\zeta_1/(2\chi_1^a)$ under Assumptions 10-14. Note that we consider only the positive values of ε_v^f , ε_v^h and $\bar{\varepsilon}_v$.

Moreover, let introduce the different following points α_ϕ^1 , α_ϕ^2 and α_ϕ^3 to locate ε_v^f , $\bar{\varepsilon}_v$ and ε_v^h in the plane (a_ϕ, ε_v) . α_ϕ^1 is defined by $\varepsilon_v^h = \varepsilon_v^s$:

$$\alpha_\phi^1 = -\frac{(\chi_1^b\Omega + \Upsilon^b\varepsilon_{dk})\varepsilon_u/c^* - (\chi_1^b\Omega\varepsilon_u^s/c^* + \chi_3^b\varepsilon_{dk})}{(\chi_1^a\Omega + \Upsilon^a\varepsilon_{dk})\varepsilon_u/c^* - (\chi_1^a\Omega\varepsilon_u^s/c^* + \chi_3^a\varepsilon_{dk})} \quad (< 0 \text{ under Assumptions 10 – 14})$$

α_ϕ^2 is defined by $\bar{\varepsilon}_v = \varepsilon_v^s$:

$$\alpha_\phi^2 = -\frac{(\chi_1^b\Omega - \chi_2^b\varepsilon_{dk})\varepsilon_u/c^* - (\chi_1^b\Omega\varepsilon_u^s/c^* + \chi_3^b\varepsilon_{dk})}{(\chi_1^a\Omega - \chi_2^a\varepsilon_{dk})\varepsilon_u/c^* - (\chi_1^a\Omega\varepsilon_u^s/c^* + \chi_3^a\varepsilon_{dk})} \quad (< 0 \text{ under Assumptions 10 – 14})$$

α_ϕ^3 is defined by $\varepsilon_v^f = \varepsilon_v^s$:

$$a_\phi^3 = -\frac{(\zeta_1^b \Omega + \zeta_2^b \varepsilon_{dk}) \varepsilon_u / c^* - (\zeta_1^b \Omega \varepsilon_u^s / c^* + \zeta_3^b \varepsilon_{dk})}{(\zeta_1^a \Omega + \zeta_2^a \varepsilon_{dk}) \varepsilon_u / c^* - (\zeta_1^a \Omega \varepsilon_u^s / c^* + \zeta_3^a \varepsilon_{dk})} \quad (< 0 \text{ under Assumptions 10 – 14})$$

Note that $a_\phi^3 < a_\phi^2 < a_\phi^1$ when $f'(1) > 1/(1 + i^* \gamma)$, and $a_\phi^3 > a_\phi^2 > a_\phi^1$ when $f'(1) < 1/(1 + i^* \gamma)$.

Furthermore, we can derive the first derivatives with respect to a_ϕ . Under Assumptions 10 – 14, we obtain:

$$\begin{aligned} \frac{\partial \varepsilon_v^s}{\partial a_\phi} &= 0, \\ \frac{\partial \varepsilon_v^f}{\partial a_\phi} &= \frac{(\zeta_2^a \zeta_1^b - \zeta_2^b \zeta_1^a) \frac{\varepsilon_u}{c^*} - (\zeta_3^a \zeta_1^b - \zeta_3^b \zeta_1^a)}{(\tilde{\zeta}_1^a a_\phi + \tilde{\zeta}_1^b)^2} > 0 \\ \frac{\partial \varepsilon_v^h}{\partial a_\phi} &= \frac{(\Upsilon^a \chi_1^b - \Upsilon^b \chi_1^a) \frac{\varepsilon_u}{c^*} - (\chi_3^a \chi_1^b - \chi_3^b \chi_1^a)}{(\chi_1^a a_\phi + \chi_1^b)^2} > 0 \\ \frac{\partial \bar{\varepsilon}_v}{\partial a_\phi} &= -\frac{(\chi_2^a \chi_1^b - \chi_2^b \chi_1^a) \frac{\varepsilon_u}{c^*} + (\chi_3^a \chi_1^b - \chi_3^b \chi_1^a)}{(\chi_1^a a_\phi + \chi_1^b)^2} > 0 \end{aligned}$$

Lemma 5 follows.

Therefore, we can now derive the dynamic properties of the model when the monetary rule is fixed by a Taylor rule. Two configurations occurs. When $f'(1) < 1/(1 + i^* \gamma)$, the properties of local dynamics are depicted by Figure 2.2. Grey areas in Figure 2.2 correspond to the different regions in which the steady state is a sink, in other words to the indeterminacy regions.

When $f'(1) > 1/(1 + i^* \gamma)$, the indeterminacy regions are qualitatively similar to the first configuration. Since the condition $f'(1) > 1/(1 + i^* \gamma)$ is restrictive enough, we choose to only draw the first configuration. \square

2.9.8 Proof of Propostion 13

We want to determine the effect of the interest rate i^* on the household welfare. At the steady state, the household welfare level is given by Eqs. $W = U(c, d) = u(c) + \beta v(d)$, with c and d given in the proof of Proposition 7 in Appendix 2.9.3.³⁶ The elasticity of the welfare level W with respect to the interest rate i^* is written as follows:

$$\begin{aligned}\varepsilon_{Wi} &= \varepsilon_{Ud}\varepsilon_{di} \left(1 + \frac{U_c}{U_d} \frac{dc/di}{dd/di} \right), \quad \text{where} \\ \varepsilon_{di}(k) &= \varepsilon_{ki}(k)\varepsilon_{dk}(k) - 1, \quad \text{with} \\ \varepsilon_{ki}(k) &= \frac{\varepsilon_v + (\varepsilon_u/c^*(k)) \frac{k[1-f'(k)]}{i^*\eta_1(k)[1-\gamma(k)]} - \frac{i^*\gamma(k)}{1+i^*\gamma(k)}}{(\varepsilon_v - \varepsilon_v^s(k))\varepsilon_{dk}(k)}, \quad \text{and} \\ \varepsilon_{dk}(k) &= \frac{f'(k)}{1-f'(k)} \frac{1-\alpha(k)}{\sigma(k)} + \eta_2(k)\end{aligned}$$

As $dc/di = -\frac{d}{i}[\varepsilon_{ki}i^*\eta_1(1-\gamma) + (\varepsilon_{ki}\varepsilon_{dk} - 1)]$ and $dd/di = \frac{d}{i}[\varepsilon_{ki}\varepsilon_{dk} - 1]$, we obtain:

$$\varepsilon_{Wi} = \frac{\varepsilon_v}{1+i^*\gamma} \frac{\nu(\eta_1) \left(\varepsilon_v - 2\frac{i^*\gamma}{1+i^*\gamma} \right) + \frac{\varepsilon_u}{c^*} [1-f'(k)]k(1+i^*\gamma) + \varepsilon_{dk} \frac{(i^*\gamma)^2}{1+i^*\gamma}}{\varepsilon_{dk}(\varepsilon_v^s - \varepsilon_v)}$$

Because $f'(k) < 1$, $\varepsilon_u > 0$ and $\varepsilon_{dk} > 0$ under Assumptions 9–12, the numerator of ε_{Wi} is positive for a sufficiently large η_2 , because ε_{dk} is increasing with η_2 (see Eq. (2.54)). Hence, ε_{Wi} has the same sign as $\varepsilon_v^s - \varepsilon_v$ under Assumptions 9–14. \square

³⁶For simplicity, the arguments of the functions are omitted.

2.9.9 Proof of Proposition 14

We determine how the bifurcation value ε_v^s varies with respect to ρ_2 when $a_\phi = 0$.

The derivative of ε_v^s with respect to ρ_2 is given by:

$$\frac{\partial \varepsilon_v^s}{\partial \rho_2} = \frac{1}{\varepsilon_{dk}^2} \left[-\frac{\psi}{1-\psi} \left(1 - \frac{1-\alpha}{\sigma} \right) \left\{ \frac{\nu(\eta_1)}{1+i^*\gamma} - \frac{\varepsilon_u}{c^*} \left[1 - f'(1) + \left(1 - \frac{1-\alpha}{\sigma} \right) (f'(1) - \psi) \right] \right\} \right]$$

Assumption 15 is equivalent to:

$$\frac{\nu(\eta_1)}{1+i^*\gamma} > \frac{\varepsilon_u}{c^*} \left[1 - f'(1) + \frac{1-\psi}{i^*\eta_1(1-\gamma)} \varepsilon_{dk} + \left(1 - \frac{1-\alpha}{\sigma} \right) (f'(1) - \psi) \right],$$

As $\varepsilon_{dk} > 0$ under Assumptions 9 – 12 (see Eq. (2.54) in Appendix 2.9.3), we deduce that ε_v^s is a decreasing function of ρ_2 ($\frac{\partial \varepsilon_v^s}{\partial \rho_2} < 0$). Since indeterminacy occurs for $\varepsilon_v < \varepsilon_v^s$, this proves Proposition 14. \square

Chapter 3

International Transmission of Bubble Crashes: Sunspot Equilibria in a Two-Country Overlapping Generations Model¹

3.1 Introduction

The history of financial markets overflows with episodes of asset bubbles (e.g., Kindleberger and Aliber, 2005). Due to global financial integration over the past few decades, financial markets are now highly interdependent across countries (e.g., Tsutsui and Hirayama, 2010; Ehrmann et al., 2011; Madaleno and Pinho, 2012). As a consequence, the bursting of an asset bubble in one country can have significant impacts on financial markets in other countries. This is what may have happened during the global financial crisis of 2007-2008.

¹This chapter bases on a joint work with Takashi Kamihigashi.

Since this event, the macroeconomic literature on asset bubbles has been growing rapidly. Much of the recent literature considers models of bubbles based on financial frictions and examines the real effects of bubbles; see, e.g., Farhi and Tirole (2012), Martin and Ventura (2012), and Miao and Wang (2012a, 2012b).² On the other hand, somewhat surprisingly, very little theoretical work has been done on the international transmission of bubble crashes in highly integrated financial markets.

There have of course been some closely related studies. For example, Ventura (2012) showed that bubbles may comove across countries in an overlapping generations model consisting of multiple countries with different levels of productivity. However, in his model, financial markets in different countries are completely segregated. Tandon and Wang (2003) studied currency substitution in a small open overlapping generations model by analyzing the dynamics of a stochastic bubble, but their analysis was restricted to the deterministic dynamics of the bubble prior to its burst. A recent paper by Martin and Ventura (2015) considers the international transmission of credit bubbles, but in their model the bubbles are affected by a common state variable and assumed to comove.

The purpose of this paper is to analyze the international transmission of bubble crashes in fully integrated financial markets. In other words, we wish to understand the effect of the bursting of a bubble in one country on a bubble in another. For example, if a bubble in one country bursts, then what happens to a bubble in another country when the relevant financial markets are fully integrated? This type of question cannot be answered if the bubbles are assumed to comove at the outset.

²See Miao (2014) for a recent survey. See Kamihigashi (2001, 2008) and the reference therein for discussion on the earlier literature on bubbles.

We construct a two-country version of the overlapping-generations exchange economy developed by Weil (1987).³ The countries, called “home” and “foreign,” are perfectly symmetric in terms of fundamentals. There is a unique consumption good worldwide, and each country has an intrinsically useless asset, or a “bubble.” The good and asset markets are fully integrated internationally; in particular, agents in either country have full access to both asset markets.

In this setting, we consider two types of stationary sunspot equilibria. The first type of equilibrium assumes that only one country receives a sunspot shock, which has no direct influence on the fundamentals of the economy. Alternatively, agents have full confidence in one asset and partial confidence in the other asset. A sunspot shock occurs only once over the infinite horizon, with a constant probability in each period. We assume that if a sunspot occurs in a country, the bubble of this country bursts, and remains at a constant level otherwise. Suppose that only the foreign country receives this sunspot shock causing the bursting of its bubble. How does the bubble in the home country react to the bursting of the foreign bubble when the home bubble is not required to react at all? We show that the home bubble inevitably bursts simultaneously in response to the bursting of the foreign bubble; thus a bubble crash in the foreign country necessarily transmits to the home country.

The second type of stationary sunspot equilibrium assumes that both countries receive sunspot shocks independently. Alternatively, agents have partial confidence in both assets. In each country, a sunspot shock occurs only once over the infinite horizon, with a constant probability in each period. We assume that the foreign bubble bursts if a sunspot shock occurs in the foreign country, and remains at

³See Wigniolle (2014) for a recent extension of Weil’s model based on a rank-dependent utility functions.

a constant level otherwise. Likewise, the home bubble bursts if a sunspot shock occurs in the home country, and remains at a constant level otherwise.

In contrast to the first type of equilibrium, we show the existence of multiple stationary equilibria. There are a unique stationary equilibrium with a bubble after the sunspot event, and a continuum of stationary sunspot equilibria characterized by an economy without bubbles after the sunspot event. More precisely, if the foreign bubble bursts, then the home bubble either bursts simultaneously or jumps to a higher level. Hence, as in the previous case, a bubble crash in one country necessarily transmits to the other country. However, unlike in the previous case, a bubble crash in one country has a positive or negative effect on the bubble in the other country.

The rest of the paper is organized as follows. In Section 3.2 we review the case of a closed economy and show some preliminary results. In Section 3.3 we introduce the two-country economy, define equilibria, and show some basic results. In Section 3.4, we define the two types of stationary sunspot equilibria discussed above, and analyze the international transmission of bubble crashes. In Section 3.5, we provide some concluding remarks and discuss possible extensions. All omitted proofs appear in the Appendix unless otherwise noted.

3.2 A Closed Economy Framework

In this section, we consider a closed economy that is essentially the same as the exchange economy of Weil (1987). The results in this section are presented for later reference; we do not claim originality here.

3.2.1 General Structure

In each period $t \in \mathbb{Z}_+$, a new generation of homogeneous two-period-lived agents are born. They are called young in the first period of their life, and old in the second period. There is no population growth, and the population of each generation is normalized to one. There is a single consumption good, and each agent is endowed with $e_1 > 0$ units of the consumption good when young, and $e_2 > 0$ units when old. There is also an intrinsically useless asset that agents can buy when young, and sell when old. We regard this asset as a bubble whenever its market price is strictly positive.

Each agent born in period $t \in \mathbb{Z}_+$ solves the following maximization problem:

$$\max_{c_t, x_t, d_{t+1} \geq 0} u(c_t) + \mathbb{E}_t v(d_{t+1}) \quad (3.1)$$

$$s.t. \quad c_t + B_t x_t = e_1 \quad (3.2)$$

$$d_{t+1} = e_2 + B_{t+1} x_t \quad (3.3)$$

where c_t is consumption when young, d_{t+1} is consumption when old, $u, v : \mathbb{R}_+ \rightarrow [-\infty, \infty)$ are the utility functions for the first and the second periods respectively, B_t is the price of the asset, x_t is asset holdings at the end of period t as well as at the beginning of period $t + 1$, and \mathbb{E}_t is the expectation conditional on the information set in period t (to be specified below).⁴ An old agent in period 0 simply consumes all his wealth:

$$d_0 = e_2 + B_0 x_{-1} \quad (3.4)$$

⁴Formally, let (Ω, \mathcal{F}, P) be a probability space, and let $\{\mathcal{F}_t\}_{t \in \mathbb{Z}_+}$ be a filtration. The conditional expectation \mathbb{E}_t at time t is defined as the expectation conditional on \mathcal{F}_t . All stochastic processes indexed by $t \in \mathbb{Z}_+$, including the sunspot processes defined below, are assumed to be adapted to this this filtration.

The market-clearing conditions for the consumption good and the asset are as follows:

$$c_t + d_t = e_1 + e_2, \quad \forall t \in \mathbb{Z}_+ \quad (3.5)$$

$$x_t = 1, \quad \forall t \in \mathbb{Z}_+ \quad (3.6)$$

Throughout the paper we assume the following.

Assumption 17. $u, v : \mathbb{R}_+ \rightarrow [-\infty, \infty)$ are continuous functions, C^1 on $(0, +\infty)$, strictly increasing. We define $\varepsilon_u(c) \equiv -cu''(c)/u'(c)$ and $\varepsilon_v(d) \equiv -dv''(d)/v'(d)$ as the degrees of concavity of $u(c)$ and $v(d)$ respectively. Furthermore, we assume $\varepsilon_v(d) \leq 1$ and

$$\lim_{c \rightarrow 0} u'(c) = \lim_{d \rightarrow 0} v'(d) = +\infty, \quad (3.7)$$

Given x_{-1} , an *equilibrium* of this economy is defined as a set of nonnegative stochastic processes $\{c_t, d_t, x_t, B_t\}_{t \in \mathbb{Z}_+}$ such that (i) the vector (d_0, x_{-1}) satisfies (3.4), (ii) for each $t \in \mathbb{Z}_+$, the vector (c_t, x_{t+1}, d_{t+1}) solves the maximization problem (3.1)-(3.3), and (iii) the market-clearing conditions (3.5) and (3.6) hold.

The following result shows that the equilibria of this economy are characterized by the Euler equation for the maximization problem (3.1)-(3.3) along with the budget constraints and the market-clearing conditions.

Lemma 7. *A set of nonnegative stochastic processes $\{c_t, d_t, x_t, B_t\}_{t \in \mathbb{Z}_+}$ is an equilibrium if and only if it satisfies (3.2)-(3.6) and*

$$e_1 - B_t > 0, \quad \forall t \in \mathbb{Z}_+ \quad (3.8)$$

$$u'(e_1 - B_t)B_t = \mathbb{E}_t v'(e_2 + B_{t+1})B_{t+1}, \quad \forall t \in \mathbb{Z}_+ \quad (3.9)$$

Proof. See Appendix 3.6.1.

We call a nonnegative stochastic process $\{B_t\}_{t \in \mathbb{Z}_+}$ an *equilibrium bubble process* if there exist nonnegative stochastic processes $\{c_t, x_t, d_t\}_{t \in \mathbb{Z}_+}$ such that $\{c_t, x_t, d_t, B_t\}$ is an equilibrium. The following result characterizes equilibrium bubble processes.

Lemma 8. *A nonnegative stochastic processes $\{B_t\}_{t \in \mathbb{Z}_+}$ is an equilibrium bubble process if and only if it satisfies (3.8)-(3.9).*

Proof. See Appendix 3.6.2.

3.2.2 Sunspot Equilibria

To define sunspot equilibria of the type studied by Weil (1987), we assume that there is a two-state sunspot process $\{s_t\}_{t \in \mathbb{Z}_+}$ obeying the following:

$$s_t = 0 \Rightarrow s_{t+1} = \begin{cases} 0 & \text{with probability } q, \\ 1 & \text{with probability } 1 - q, \end{cases} \quad (3.10)$$

$$s_t = 1 \Rightarrow s_{t+1} = 1, \quad (3.11)$$

where $q \in (0, 1)$ is a constant. Following Weil (1987), we can interpret state 0 as meaning “no sunspot” and 1 as “sunspots”, though this interpretation is not required for our results. As noted by Weil (1987), the probability q can depict the confidence of agents in the bubble.

Given the sunspot process defined by (3.10) and (3.11), we consider an equilibrium $\{c_t, d_t, x_t, B_t\}_{t \in \mathbb{Z}_+}$ such that for some $b_t > 0$, we have

$$B_t = \begin{cases} b_t & \text{if } s_t = 0, \\ 0 & \text{if } s_t = 1 \end{cases} \quad (3.12)$$

Equation (3.12) means that the bubble persists until a sunspot shock occurs (i.e., s_t switches from 0 to 1), then it collapses to zero and never reappears.

Note from (3.2), (3.3), and (3.6) that under (3.12) we have

$$\begin{aligned} s_t = 0 &\Rightarrow \begin{cases} c_t = e_1 - b_t, \\ d_t = e_2 + b_{t+1}, \end{cases} & (3.13) \\ s_t = 1 &\Rightarrow \begin{cases} c_t = e_1, \\ d_t = e_2 \end{cases} \end{aligned}$$

Suppose that the bubble is valued at time t . It follows from (3.9) and (3.12) that an equilibrium is a sequence $\{b_t\}_{t \in \mathbb{Z}_+}$ which satisfies for all $t \in \mathbb{Z}_+$, the following conditions:

$$e_1 - b_t > 0, \quad \forall t \in \mathbb{Z}_+ \quad (3.14)$$

$$u'(e_1 - b_t)b_t = qv'(e_2 + b_{t+1})b_{t+1} \quad \forall t \in \mathbb{Z}_+ \quad (3.15)$$

We restrict now our attention to a simple type of equilibrium called *stationary sunspot equilibrium* such that for some constant $b > 0$ we have

$$B_t = \begin{cases} b & \text{if } s_t = 0, \\ 0 & \text{if } s_t = 1 \end{cases} \quad (3.16)$$

Equation (3.16) means that the bubble is constant until a sunspot shock occurs (i.e., s_t switches from 0 to 1), then it collapses to zero and never reappears. From (3.15), a stationary sunspot equilibrium is a solution of the following equation:

$$u'(e_1 - b) = qv'(e_2 + b) \quad (3.17)$$

It is easy to see that this equation has a solution $b^* > 0$ if and only if

$$q > u'(e_1)/v'(e_2) \tag{3.18}$$

The solution is unique by strict concavity of u and v .

The preceding discussion together with Lemma 8 establishes the following result.

Proposition 15. *There exists an equilibrium bubble process $\{B_t\}$ satisfying (3.16) if and only if (3.18) holds. Under (3.18), any equilibrium bubble process $\{B_t\}$ satisfying (3.16) satisfies $b = b^*$, where b^* is the unique solution to (3.17).*

Essentially the same result is shown by Weil (1987, Proposition 1). We refer to Weil (1987) for further discussion of asset bubbles in a closed economy. To rule out trivial cases in which no bubble appears, we maintain the following assumption for the rest of this paper.

Assumption 18. *Inequality (3.18) holds.*

3.3 The Two-Country Model

Consider a world economy consisting of two countries indexed by $i \in \{H, F\}$, where H and F stand for “home” and “foreign”, respectively. There is a single consumption good worldwide, while there are two intrinsically useless assets in each country. The markets for the consumption good and the assets are fully integrated, and the countries are entirely symmetric in terms of fundamentals.

Each agent born in period $t \in \mathbb{Z}_+$ in country $i \in \{H, F\}$ solves the following maximization problem:

$$\max_{c_t^i, x_t^{i,i}, x_t^{i,j}, d_{t+1}^i \geq 0} \mathbb{E}_t \left[u \left(c_t^i \right) + v \left(d_{t+1}^i \right) \right] \quad (3.19)$$

$$s.t. \quad c_t^i + B_t^i x_t^{i,i} + B_t^j x_t^{i,j} = e_1 \quad (3.20)$$

$$d_{t+1}^i = e_2 + B_{t+1}^i x_t^{i,i} + B_{t+1}^j x_t^{i,j} \quad (3.21)$$

where c_t^i is consumption when young, d_{t+1}^i is consumption when old, B_t^k with $k \in \{H, F\}$ is the price of the asset in country k , and $x_t^{i,k}$ with $k \in \{H, F\}$ is holdings of the asset in country k , and j is given by $j \in \{H, F\}$ with $j \neq i$; unless otherwise specified, we maintain this definition of j whenever $i \in \{H, F\}$ is given. An old agent in period 0 in country $i \in \{H, F\}$ simply consumes all his wealth:

$$d_0^i = e_2 + B_0^i x_{-1}^{i,i} + B_0^j x_{-1}^{i,j} \quad (3.22)$$

The market-clearing condition for the consumption good is

$$c_t^H + d_t^H + c_t^F + d_t^F = 2e_1 + 2e_2, \quad \forall t \in \mathbb{Z}_+ \quad (3.23)$$

Since each asset is intrinsically useless, its supply can be set to any number without substantially affecting the analysis. For convenience we normalize the supply of each asset to two. Thus the market clearing condition for the asset in country $k \in \{H, F\}$ is

$$x_t^{H,k} + x_t^{F,k} = 2, \quad \forall k \in \{H, F\}, \forall t \in \mathbb{Z}_+ \quad (3.24)$$

Given $x_{-1}^{i,k} \geq 0$ for $i, k \in \{H, F\}$ such that $x_{-1}^{H,H} + x_{-1}^{F,H} = x_{-1}^{H,F} + x_{-1}^{F,F} = 2$, an *equilibrium* of this economy is defined as set of nonnegative stochastic processes $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}_{t \in \mathbb{Z}_+, i \in \{H, F\}}$ such that (i) for each $i \in \{H, F\}$, the vector $(d_0^i, x_{-1}^{i,i}, x_{-1}^{i,j})$ satisfies (3.22), (ii) for each $t \in \mathbb{Z}_+$ and $i \in \{H, F\}$, the vector

$(c_t^i, x_t^{i,i}, x_t^{i,j}, d_{t+1}^i)$ solves the maximization problem (3.19)-(3.21), and (iii) the market-clearing conditions (3.23) and (3.24) hold.

Before we turn to sunspot equilibria, it is useful to exploit the implications of the symmetry of the countries:

Lemma 9. *A set of nonnegative stochastic processes $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ satisfying (3.20)-(3.22), and (3.24) is an equilibrium if and only if for all $t \in \mathbb{Z}_+$, we have*

$$c_t^H = c_t^F = e_1 - (B_t^H + B_t^F) > 0 \quad (3.25)$$

$$d_{t+1}^H = d_{t+1}^F = e_2 + (B_{t+1}^H + B_{t+1}^F) \quad (3.26)$$

$$u'(c_t^i)B_t^k = \mathbb{E}_t v'(d_{t+1}^i)B_{t+1}^k, \quad \forall i, k \in \{H, F\} \quad (3.27)$$

Proof. See Appendix 3.6.3.

For an equilibrium $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ such that the bubble processes $\{B_t^H\}$ and $\{B_t^F\}$ are not perfectly correlated for any $t \in \mathbb{Z}_+$, it is possible to show that $x_t^{H,k} = x_t^{F,k} = 1$ for each $k \in \{H, F\}$. However, since the bubble processes can even be identical, the equilibrium values of $x_t^{i,k}$ for $i, k \in \{H, F\}$ are in general indeterminate. For example, there is an equilibrium in which $B_t^H = B_t^F = 0$ for all $t \in \mathbb{Z}_+$, in which case the values of $x_t^{i,k}$ are essentially irrelevant. Another case in point is an equilibrium in which both $\{B_t^H\}$ and $\{B_t^F\}$ follow an identical stochastic process such as (3.12), in which case the distinction between $x_t^{i,H}$ and $x_t^{i,F}$ is irrelevant for agents. Since we are primarily interested in equilibrium bubble processes, we do not seek to obtain a full characterization of asset holdings $x_t^{i,k}$.

We say that a pair of nonnegative stochastic processes $\{B_t^H, B_t^F\}_{t \in \mathbb{Z}_+}$ is a *bivariate equilibrium bubble process* if there exist nonnegative stochastic processes

$\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}\}$ such that $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ is an equilibrium. We close this section by showing a result that characterizes bivariate equilibrium bubble processes. The following result is a two-country version of Lemma 7.

Proposition 16. *A pair of nonnegative stochastic processes $\{B_t^H, B_t^F\}$ is a bivariate equilibrium bubble process if and only if for all $t \in \mathbb{Z}_+$ we have*

$$B_t^H + B_t^F < e_1, \quad (3.28)$$

$$u'(e_1 - (B_t^H + B_t^F))B_t^k = \mathbb{E}_t v'(e_2 + B_{t+1}^H + B_{t+1}^F)B_{t+1}^k, \quad \forall k \in \{H, F\}. \quad (3.29)$$

Proof. See Appendix 3.6.4.

3.4 Two-Country Sunspot Equilibrium

In this section, we consider two types of sunspot equilibria. The first type assumes that there is a sunspot process only in the country i , and not in the country j with $i, j = \{H, F\}$ and $i \neq j$. The second type assumes that there are sunspot processes in both countries.

3.4.1 Sunspots Only In One Country

In this subsection, we assume that there is a two-state sunspot process $\{s_t^i\}_{t \in \mathbb{Z}_+}$ obeying (3.10) and (3.11) only in the country i . Only the existence of the bubble of the country i is affected by a sunspot event, there is no other source of uncertainty. In other words, we assume that agents have partial confidence in the asset issued by the country i , and full confidence in the asset issued by the country j with $i, j = \{H, F\}$ and $i \neq j$.

Although (3.10) and (3.11) allow for various possibilities, we shall show that the asset bubble of the country j necessarily bursts when the asset bubble of the country i bursts, as long as both asset bubbles are strictly positive and constant until a sunspot shock occurs.

This result seems logical given the nature of the assets (one risky, one safe). Let us provide an intuition. Suppose that only the foreign country receives a sunspot shock, and that asset prices are stationary. Moreover, define the value of an euro (in terms of marginal utility) before a sunspot event by v_{euro} , and the value after the sunspot event by \tilde{v}_{euro} . Note that both assets provide a return equal to 1 until a sunspot shock occurs since the two countries are symmetric in terms of fundamentals. When the sunspot shock occurs in the foreign country, the gross return on the foreign asset is zero, while the home asset provides a return \tilde{R} . *A priori*, \tilde{R} can be strictly positive or zero.

If an agent invests one euro in the foreign bubble in t , she can earn qv_{euro} in $t+1$. If she invests one euro in the home bubble in t , she can get $qv_{euro} + (1-q)\tilde{R}\tilde{v}_{euro}$ in $t+1$. Since the agent is rational, she is indifferent between investing more in the foreign asset or in the home asset at the optimum. Thus, we deduce that \tilde{R} must be zero. In other words, the home bubble must be zero when a sunspot occurs.⁵

We now formalize the above intuition using the two-country version of Weil (1987) described in section 3.3. We consider a sunspot equilibrium such that

$$B_t^i = \begin{cases} b_t^i > 0 & \text{if } s_t^i = 0, \\ 0 & \text{if } s_t^i = 1 \end{cases} \quad (3.30)$$

$$B_t^j = \begin{cases} b_t^j > 0 & \text{if } s_t^j = 0, \\ \tilde{b}_t^j \geq 0 & \text{if } s_t^j = 1 \end{cases} \quad (3.31)$$

⁵Note that if only the home country receives the sunspot shocks, then the foreign bubble bursts, when the home bubble bursts. We can deduce this result by applying the same arguments.

where \tilde{b}_t^j is the value of the bubble of the country j after the occurrence of a sunspot event in the country i at time t with $i \neq j$.⁶

Suppose that the bubble of the country i is valued at time t . It follows from (3.29), (3.30) and (3.31) that an equilibrium is a sequence $\{b_t^i, b_t^j, \tilde{b}_t^j\}_{t \in \mathbb{Z}_+}$ which satisfies for all $t \in \mathbb{Z}_+$, the following conditions:

$$u'(e_1 - (b_t^i + b_t^j))b_t^i = qv'(e_2 + b_{t+1}^i + b_{t+1}^j)b_{t+1}^i \quad (3.32)$$

$$\begin{aligned} u'(e_1 - (b_t^i + b_t^j))b_t^j &= qv'(e_2 + b_{t+1}^i + b_{t+1}^j)b_{t+1}^j \\ &\quad + (1 - q)v'(e_2 + \tilde{b}_{t+1}^j)\tilde{b}_{t+1}^j \end{aligned} \quad (3.33)$$

$$u'(e_1 - \tilde{b}_t^j)\tilde{b}_t^j = v'(e_2 + \tilde{b}_{t+1}^j)\tilde{b}_{t+1}^j \quad (3.34)$$

$$e_1 - (b_t^i + b_t^j) > 0 \quad (3.35)$$

$$e_1 - \tilde{b}_t^j > 0 \quad (3.36)$$

with $i, j = \{H, F\}$ and $i \neq j$. We recall that \tilde{b}_{t+1}^j in (3.33) is the value of the bubble of the country j when a sunspot shock occurs in the country i at $t + 1$ with $i \neq j$. Equation (3.34) governs the dynamics of the bubble of the country j after a sunspot event in the country i with $i \neq j$. Note that the equilibrium dynamics of the bubble of the country j after a sunspot event corresponds to the closed economy developed by Samuelson (1958), namely the model described in section 3.2 with $q = 1$.

To highlight the effect of the bursting of an asset bubble in one country on a bubble in another country, we consider the simplest type of equilibrium, i.e. the stationary sunspot equilibrium:

⁶After the sunspot shock, the economy behaves as the closed economy developed by Samuelson (1958). Therefore, after this event, \tilde{b}_t^j corresponds to an equilibrium value of Samuelson (1958) economy.

$$B_t^i = \begin{cases} b^i > 0 & \text{if } s_t^i = 0, \\ 0 & \text{if } s_t^i = 1 \end{cases} \quad (3.37)$$

$$B_t^j = \begin{cases} b^j > 0 & \text{if } s_t^i = 0, \\ \tilde{b}^j \geq 0 & \text{if } s_t^i = 1 \end{cases} \quad (3.38)$$

Equation (3.37) means that the bubble of the country i bursts when sunspots appear. The question addressed here is: How does this event affect the bubble of the country j ? In (3.38), the bubble of the country j is not forced to react to this event. It is possible *a priori* that $b^j = \tilde{b}^j$, in which case country j 's bubble is not affected by the bursting of country i 's bubble at all. If $0 < \tilde{b}^j < b^j$, then country j 's bubble collapses only partially in response to the bursting of country i 's bubble. If $\tilde{b}^j = 0$, then country j 's bubble collapses totally. It is even possible *a priori* that $b^j < \tilde{b}^j$, in which case bubble j 's country becomes larger when country i 's bubble collapses to zero.

The following result shows that the bubble of the country j necessarily bursts when the bubble of the country i bursts, as long as both bubbles are strictly positive until a sunspot shock occurs.

Proposition 17. *Let $\{B_t^i, B_t^j\}$ be a pair of nonnegative stochastic processes satisfying (3.37) and (3.38) with $b^i, b^j > 0$, $(i, j) = \{H, F\}$ and $i \neq j$. Then $\{B_t^i, B_t^j\}$ is a bivariate equilibrium bubble process if and only if*

$$\tilde{b}^j = 0, \quad (3.39)$$

$$b^i + b^j = b^*, \quad (3.40)$$

b^* is the unique solution to (3.17).

Proof. See Appendix 3.6.5.

Note that (3.39) means that the bubble of the country j bursts when the bubble of the country i bursts. For instance, when only the foreign country receives sunspot shocks, a bubble crash in the foreign country necessarily transmits to the home country, having a devastating effect on the home bubble. Even though there are two bubbles in this economy, there is effectively only one bubble under (3.30) and (3.31) since they burst simultaneously. Not surprisingly, the sum of the two bubbles must be equal to b^* by (3.40), which confirms that the economy here would be equivalent to the closed economy studied in Subsection 3.2.2.

The following result is an immediate consequence of Proposition 17.

Corollary 1. *There exists a continuum of bivariate equilibrium bubble processes satisfying (3.37), and (3.38). In particular, for any $b^i, b^j > 0$ satisfying (3.40), there exists a unique bivariate equilibrium bubble process $\{B_t^i, B_t^j\}$ satisfying (3.37)-(3.39) with $i, j = \{H, F\}$ and $i \neq j$.*

Since the foreign and home bubbles burst simultaneously, one might think that there is no “international transmission of bubble crashes” here, and that both bubbles are simultaneously crashed by the same sunspot event. However, it is the bursting of the foreign (home) bubble, not the sunspot shock, that causes the home (foreign) bubble to burst. This is the message of the following result, which shows that if the bubble of the country i is not valued at all, then the bubble of the country j is not forced to burst even when a sunspot shock occurs in the country i .

Proposition 18. *There exists a bivariate equilibrium bubble process $\{B_t^i, B_t^j\}$ with $i, j = \{H, F\}$ and $i \neq j$ such that $b^i = 0$ and $b^j = \tilde{b}^j > b^*$.*

Proof. See Appendix 3.6.6.

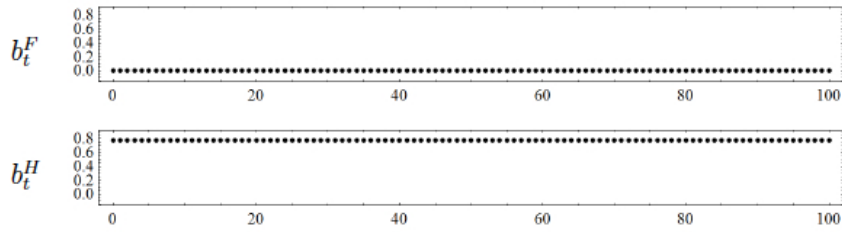


Figure 3.1: A deterministic bivariate equilibrium bubble process with $b^F = 0$ and $b^H > b^*$.

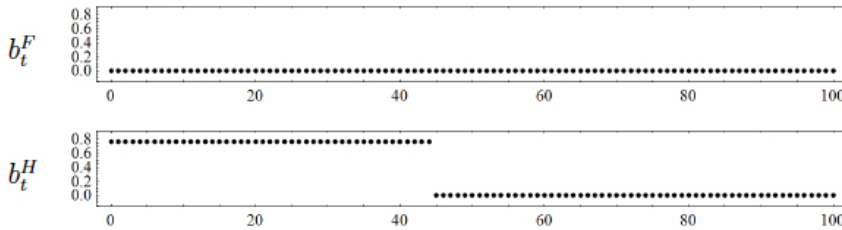


Figure 3.2: The limiting bivariate equilibrium bubble process obtained by letting b^F go to zero ($b^F = 0$ and $b^H = b^*$).

Figure 3.1 shows a bivariate equilibrium bubble process given by Proposition 18 when only the foreign country receives a sunspot shock. The process here is deterministic, and the home bubble never bursts even if a sunspot shock occurs in the foreign country. In this sense, it is the bursting of the foreign bubble, not the sunspot shock, that forces the home bubble to burst in Proposition 17.

It is important to emphasize that the bivariate equilibrium bubble process in Figure 3.1 cannot be obtained as the limit of bivariate equilibrium bubble processes of the type depicted in Proposition 17 by letting b^F go to zero. Indeed, if we let b^i go to zero in Proposition 17, the limiting process is still stochastic. Figure 3.2 depicts the limiting bivariate equilibrium bubble process obtained this way when only the foreign country receives a sunspot shock.

3.4.2 Sunspots In Both Countries

In this subsection, we assume now that sunspots can affect both countries, meaning that both bubbles are now risky assets. In particular, let $\{s_t^H\}$ and $\{s_t^F\}$ be sunspot processes in the home and foreign countries, respectively. We assume that these processes are independent, and obeying (independently) the following stochastic processes⁷:

$$s_t^F = 0 \Rightarrow s_{t+1}^F = \begin{cases} 0 & \text{with probability } q^F, \\ 1 & \text{with probability } 1 - q^F, \end{cases} \quad (3.41)$$

$$s_t^F = 1 \Rightarrow s_{t+1}^F = 1, \quad (3.42)$$

$$s_t^H = 0 \Rightarrow s_{t+1}^H = \begin{cases} 0 & \text{with probability } q^H, \\ 1 & \text{with probability } 1 - q^H, \end{cases} \quad (3.43)$$

$$s_t^H = 1 \Rightarrow s_{t+1}^H = 1 \quad (3.44)$$

We consider sunspot equilibria such that

$$B_t^F = \begin{cases} b_t^F > 0 & \text{if } s_t^H = s_t^F = 0, \\ \tilde{b}_t^F \geq 0 & \text{if } s_t^H = 1 \text{ and } s_t^F = 0, \\ 0 & \text{if } s_t^F = 1 \end{cases} \quad (3.45)$$

$$B_t^H = \begin{cases} b_t^H > 0 & \text{if } s_t^H = s_t^F = 0, \\ \tilde{b}_t^H \geq 0 & \text{if } s_t^H = 0 \text{ and } s_t^F = 1, \\ 0 & \text{if } s_t^H = 1 \end{cases} \quad (3.46)$$

⁷If the two bubbles were not independent, the burst of both bubbles will be correlated, and the transmission will be the consequence of this correlation. Here, we want to highlight the role of financial integration in the transmission of bubble crash. Hence, we abstract the correlation.

In contrast to Section 3.4.1, there are now four states of nature. Table 1 summarizes how the pair (B_t^H, B_t^F) depends on (s_t^H, s_t^F) .

		B_t^H, B_t^F	
		s_t^H	
s_t^F	0	b_t^H, b_t^F	$0, \tilde{b}_t^F$
	1	$\tilde{b}_t^H, 0$	$0, 0$

Table 3.1: Dependence of (B_t^H, B_t^F) on (s_t^H, s_t^F)

Suppose that both bubbles are valued at time t . It follows from (3.29), (3.45) and (3.46) that an equilibrium is a sequence $\{b_t^i, b_t^j, \tilde{b}_t^i, \tilde{b}_t^j\}_{t \in \mathbb{Z}_+}$ which satisfies for all $t \in \mathbb{Z}_+$, the following conditions:

$$\begin{aligned} u'(e_1 - (b_t^F + b_t^H))b_t^H &= q^H q^F v'(e_2 + b_{t+1}^F + b_{t+1}^H)b_{t+1}^H \\ &\quad + q^H(1 - q^F)v'(e_2 + \tilde{b}_{t+1}^H)\tilde{b}_{t+1}^H \end{aligned} \quad (3.47)$$

$$\begin{aligned} u'(e_1 - (b_t^F + b_t^H))b_t^F &= q^F q^H v'(e_2 + b_{t+1}^F + b_{t+1}^H)b_{t+1}^F \\ &\quad + q^F(1 - q^H)v'(e_2 + \tilde{b}_{t+1}^F)\tilde{b}_{t+1}^F \end{aligned} \quad (3.48)$$

$$u'(e_1 - \tilde{b}_t^H)\tilde{b}_t^H = q^H v'(e_2 + \tilde{b}_{t+1}^H)\tilde{b}_{t+1}^H \quad (3.49)$$

$$u'(e_1 - \tilde{b}_t^F)\tilde{b}_t^F = q^F v'(e_2 + \tilde{b}_{t+1}^F)\tilde{b}_{t+1}^F \quad (3.50)$$

$$e_1 - (b_t^F + b_t^H) > 0 \quad (3.51)$$

$$e_1 - \tilde{b}_t^j > 0 \quad (3.52)$$

We recall that \tilde{b}_{t+1}^H and \tilde{b}_{t+1}^F in (3.47) and (3.48) are respectively the value of the home bubble when a sunspot shock occurs in the foreign country at $t + 1$ and the value of the foreign bubble when a sunspot shock occurs in the home country at $t + 1$.

Equations (3.47) and (3.48) govern the dynamics of the economy before the occurrence of a sunspot shock either in country H or in country F . When a sunspot occurs, the dynamics of the economy is given by Equation (3.49) if the sunspot shock occurs in country F , or by Equation (3.50) if the sunspot shock occurs in country H . These two last dynamics are similar to the equilibrium dynamics of the closed economy developed by Weil (1987) described in section 3.2.

As in the previous case, we restrict our attention on a simple type of equilibrium. We study a stationary sunspot equilibrium that has the following properties:

$$B_t^F = \begin{cases} b^F & \text{if } s_t^H = s_t^F = 0, \\ \tilde{b}^F & \text{if } s_t^H = 1 \text{ and } s_t^F = 0, \\ 0 & \text{if } s_t^F = 1 \end{cases} \quad (3.53)$$

$$B_t^H = \begin{cases} b^H & \text{if } s_t^H = s_t^F = 0, \\ \tilde{b}^H & \text{if } s_t^H = 0 \text{ and } s_t^F = 1, \\ 0 & \text{if } s_t^H = 1 \end{cases} \quad (3.54)$$

We do not require the home bubble to react to the bursting of the foreign bubble since it is possible that $b^H = \tilde{b}^H$. Likewise, the foreign bubble is not required to react to the bursting of the home bubble.

The following result shows that under an additional condition, there is a continuum bivariate equilibrium bubble processes following (3.53) and (3.54), provided that both b^H and b^F are strictly positive.

Proposition 19. *Suppose that*

$$q^H \geq q^F \text{ and } q^H q^F > u'(e_1)/v'(e_2). \quad (3.55)$$

Then there exist exactly two types of stationary sunspot equilibria following (3.53) and (3.54) with $b^H, b^F > 0$. In both processes, we have $\tilde{b}^{F*} \leq \tilde{b}^{H*}$.

(a) Let $b^W = b^H + b^F$. In the first type of stationary sunspot equilibria, we have a continuum of stationary equilibria such that $\tilde{b}^H = \tilde{b}^F = 0$ and $b^W < \tilde{b}^{F*}$. In particular, b^W is the unique solution satisfying the following equation:

$$u'(e_1 - b^W) = q^H q^F v'(e_2 + b^W). \quad (3.56)$$

(b) In the other type of stationary sunspot equilibria, we have a unique stationary equilibrium such that $0 < b^{F*} \leq b^{H*}$, $b^{F*} < \tilde{b}^{F*}$ and $b^{H*} < \tilde{b}^{H*}$. In particular, the 4-tuple $(b^{F*}, b^{H*}, \tilde{b}^{F*}, \tilde{b}^{H*})$ is the unique solution of the following system:

$$u'(e_1 - (b^{H*} + b^{F*}))b^{H*} = q^H q^F v'(e_2 + b^{H*} + b^{F*})b^{H*} + q^H(1 - q^F)v'(e_2 + \tilde{b}^{H*})\tilde{b}^{H*} \quad (3.57)$$

$$b^{F*} = \frac{1 - q^H}{1 - q^F} \frac{q^F v'(e_2 + \tilde{b}^{F*})}{q^H v'(e_2 + \tilde{b}^{H*})} \frac{\tilde{b}^{F*}}{\tilde{b}^{H*}} b^{H*} \quad (3.58)$$

$$u'(e_1 - \tilde{b}^{F*}) = q^F v'(e_2 + \tilde{b}^{F*}) \quad (3.59)$$

$$u'(e_1 - \tilde{b}^{H*}) = q^H v'(e_2 + \tilde{b}^{H*}) \quad (3.60)$$

Proof. See Appendix 3.6.7.

Proposition 19 (a) is similar to Proposition 17, which shows that there exists a continuum of asymmetric equilibria if only one country receives a sunspot shock.

Proposition 19 (b) brings a new result: When both countries receive sunspot shocks, the bubble of the country j can persist at a higher level after the crash of the bubble of the country i ($b^{j*} < \tilde{b}^{j*}$). Since $\tilde{b}^i \neq b^i$ in both cases (a) and (b)

above, a bubble crash in either country necessarily transmits to the other country. Its effect is negative in case (a), while it is positive in case (b).

Furthermore, even if bubbles do not have the same probability to persist, the agents hold both assets in their portfolio. This result illustrates the principle of diversification. Being risk averse, the agents will prefer hold more home assets than foreign assets since the home bubble has a higher probability to persist. Before a sunspot event, the value of the home bubble is, thus, higher than the value of the foreign one.

To illustrate Proposition 19, we provide Figure 3.3. In this Figure, we assume that the foreign bursts randomly first, and analyze the effect of its bursting on the home bubble.⁸

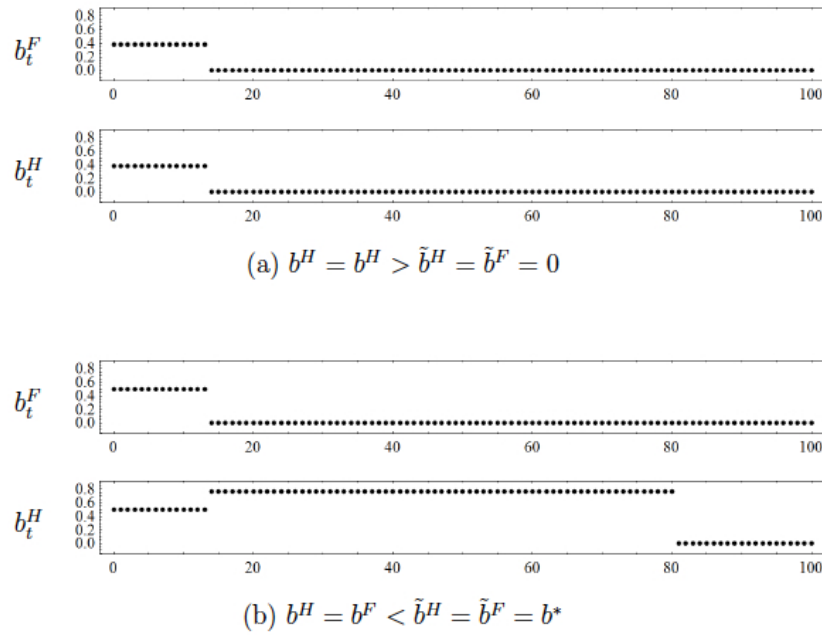


Figure 3.3: Bivariate equilibrium bubble processes satisfying (3.53) and (3.54)

Panel (a) in Figure 3.3 illustrates the bivariate equilibrium bubble process satisfying (3.53) and (3.54) in case (a) of Proposition 19. The intuition behind this

⁸Without loss of generality, we suppose that $q^H = q^F$. Hence, $b^{F*} = b^{H*}$.

result is the following. If agents expect that $\tilde{b}^H = 0$ immediately after the burst of the foreign bubble, then every old agents sell the home bubble and no young want to buy it, the value of the home bubble becomes zero. Beliefs are self-fulfilling.

Since both bubbles survive and burst simultaneously in case (a), there is effectively only one bubble here, and one may expect that the economy is essentially equivalent to the closed economy studied in Subsection 3.2.2 insofar as $q = q^i$ with $i = \{H, F\}$. However, there are two differences between (3.17) and (3.56).

First, while we have q on the right-hand side of (3.17), we have $q^H q^F$ on the right-hand side of (3.56). This is because both bubbles survive here only if neither country experiences a sunspot shock, and this event occurs with probability $q^H q^F$. Second, the size of the aggregate bubble (or the sum of the two bubbles) is b^W in (3.56), while it is b^* in (3.17). We have $b^W < b^*$ because the aggregate bubble survives with probability $q^H q^F < q$ in (3.56), as discussed above. Note also that the left-hand side of (3.55) is $q^H q^F$, while the left-hand side of (3.18) is q . The preceding discussion indicates that the economy here is equivalent to the closed economy with the probability of a sunspot shock reduced to $q^H q^F$.

In case (b) of Proposition 19, the bursting of the foreign bubble causes the home bubble to jump to \tilde{b}^{H*} ; thus a bubble crash in the foreign country has a positive effect on the home bubble. The intuition is the following. If agents expect that $\tilde{b}^H = \tilde{b}^{H*}$ immediately after the burst of the foreign bubble, then they save enough to keep the value of the home bubble constant until the burst of the home bubble. To better understand this, note that after the foreign bubble collapses, there remains exactly one bubble (the home bubble), which means that the economy effectively reverts to the closed economy studied in Subsection 3.2.2 insofar $q = q^H$. The home bubble follows (3.60) and thus must be equal to \tilde{b}^{H*} . Before the bursting

of the foreign bubble, the home bubble b^{H*} satisfies (3.57), which implies that $b^{H*} < \tilde{b}^{H*}$, as shown in the proof of Proposition 19.

Panel (b) in Figure 3.3. illustrates the bivariate equilibrium bubble process in this case. As discussed above, the home bubble jumps to \tilde{b}^{H*} when the foreign bubble bursts, but it collapses on its own later, when a sunspot shock occurs in the home country.

Since the model exhibits two types of stationary equilibria, coordination failure can occur when both countries receive sunspots. Indeed, agents could achieve the socially optimum equilibrium but fail to because they do not coordinate their actions. Note that condition (3.55) implies a dynamically inefficient bubbleless economy. Therefore, we can maintain, from an inefficiency criterion, that the second type of stationary equilibria characterized by a positive bubble after the sunspot event $\tilde{b}^{i*} > 0$ is the socially optimum equilibrium. For a welfare perspective, an economic policy would be useful, since it could coordinate agents' behavior on the more desirable stationary equilibrium.

An analysis of the stability properties of each type of stationary equilibria helps to provide insights about the coordination policy to implement. To do this, we investigate the local dynamic properties of our model. A steady state would be locally indeterminate when there exist multiple equilibrium bubble processes satisfying (3.45) and (3.46) converging to the stationary equilibria. Likewise, it would be determinate, when there is only one equilibrium bubble process satisfying (3.45) and (3.46) converging to the steady state. In other words, local indeterminacy occurs when the stable manifold has dimension greater than the number of predetermined variables. Since there are no predetermined variables in our framework, the steady state will be locally indeterminate when it is a saddle point or a sink, and determinate when it is a source.

We first characterize the local stability properties of the first type of stationary equilibria described in Proposition 19 (a) ($\tilde{b}^H = \tilde{b}^F = 0$), then the local properties of the second one described in Proposition 19 (b) ($0 < \tilde{b}^{F*} \leq \tilde{b}^{H*}$).

Since we have a continuum of stationary equilibria characterized by $\tilde{b}^H = \tilde{b}^F = 0$ and $b^W > 0$, the Jacobian matrix associated to the linearized dynamic system (3.47)-(3.50) evaluated at this stationary equilibria exhibits a unit eigenvalue. The steady state is not-hyperbolic, and, therefore, we cannot apply the Hartman-Grobman theorem. Therefore, we cannot draw a conclusion about the stability properties of the stationary equilibrium from just the linearization. Since the economy for this type of equilibria is essentially equivalent to the closed economy studied in Subsection 3.2.2, we will analyze the dynamics of the stochastic “global” bubble, B_t^W , to overcome this problem. The stochastic global bubble satisfies the following process:

$$B_t^W = \begin{cases} b_t^F + b_t^H > 0 & \text{if } s_t^H = s_t^F = 0, \\ \tilde{b}_t^F \geq 0 & \text{if } s_t^H = 1 \text{ and } s_t^F = 0, \\ \tilde{b}_t^H \geq 0 & \text{if } s_t^H = 0 \text{ and } s_t^F = 1, \\ 0 & \text{if } s_t^H = s_t^F = 1 \end{cases} \quad (3.61)$$

Suppose that the global bubble is valued at time t . From Lemma 9, the dynamics of the global bubble is given by:

$$\begin{aligned} u'(e_1 - b_t^W)b_t^W &= q^H q^F v'(e_2 + b_{t+1}^W)b_{t+1}^W \\ &\quad + q^H(1 - q^F)v'(e_2 + \tilde{b}_{t+1}^H)\tilde{b}_{t+1}^H + q^F q^H v'(e_2 + \tilde{b}_{t+1}^F)\tilde{b}_{t+1}^F \end{aligned} \quad (3.62)$$

$$u'(e_1 - \tilde{b}_t^H)\tilde{b}_t^H = q^H v'(e_2 + \tilde{b}_{t+1}^H)\tilde{b}_{t+1}^H \quad (3.63)$$

$$u'(e_1 - \tilde{b}_t^F)\tilde{b}_t^F = q^F v'(e_2 + \tilde{b}_{t+1}^F)\tilde{b}_{t+1}^F \quad (3.64)$$

$$e_1 - b_t^W > 0 \quad (3.65)$$

$$e_1 - \tilde{b}_t^i > 0, \text{ with } i = \{H, F\}. \quad (3.66)$$

Equation (3.62) governs the dynamics of the global bubble before the occurrence of a sunspot shock either in country H or in country F . When a sunspot occurs, the dynamics of the global bubble is given by Equation (3.63) if the sunspot shock occurs in country F , or by Equation (3.64) if the sunspot shock occurs in country H .

From an analysis of the linearized system evaluated at the stationary equilibrium, we can provide the next proposition:

Proposition 20. *Suppose that the condition (3.55) holds. Under Assumption 17, the following generically holds:*

The stationary equilibrium characterized by $\tilde{b}^H = \tilde{b}^F = 0$ and $b^W > 0$ is a saddle point, and thus locally indeterminate.

Proof. See Appendix 3.6.8.

As regards the second type of stationary equilibrium, we could apply the Hartman-Grobman theorem, and derive the following proposition.

Proposition 21. *Suppose that the condition (3.55) holds. Under Assumption 17, the following generically holds:*

The unique stationary equilibrium characterized by $0 < \tilde{b}^{F} \leq \tilde{b}^{H*}$ is a source, and thus locally determinate.*

Proof. See Appendix 3.6.9.

First, Propositions 20 and 21 shows that the second type of stationary equilibria characterized by a positive bubble after the occurrence of a sunspot shock is also a desirable equilibrium from an economic stability perspective.

Second, we can provide some insights about the policy to implement from Proposition 21. Since the stationary equilibrium is a source, we need a policy which, at the beginning of the economy, will force the agents to coordinate on the asset prices b^{H*} and b^{F*} , then either on \tilde{b}^{H*} when a sunspot occurs in country F , or on \tilde{b}^{F*} when a sunspot occurs in country H .

3.5 Concluding remarks

In this paper, we studied the international transmission of bubble crashes by analyzing stationary sunspot equilibria in a two-country version of Weil's (1987) overlapping generations exchange economy with stochastic bubbles. We considered two types of stationary sunspot equilibria. The first type of equilibrium assumes that only the foreign country receives a sunspot shock, and that the foreign bubble bursts when a sunspot shock occurs. We showed that in this case, the home bubble inevitably bursts when the foreign bubble bursts; thus a bubble crash in the foreign country necessarily transmits to the home country. The second type of equilibrium assumes that both countries independently receive sunspot shocks. We showed that in this case, if the foreign bubble bursts, then the home bubble either bursts simultaneously or jumps to a higher level. In either case, a bubble crash in the foreign country necessarily transmits to the home country again, but its effect on the home bubble can be positive or negative.

There are several ways to extend our analysis by relaxing some of our assumptions. First, we assumed that the countries are symmetric in terms of fundamentals.

Since this assumption greatly simplified the analysis, dropping it would be a non-trivial extension. However, it is important to consider cases in which the countries are asymmetric in various aspects such as preferences, endowments, and population because in reality, no two countries are identical.

Second, we assumed that the probability of a sunspot shock is exogenous and constant over time and across countries. There are various ways to relax this assumption. One may assume that the probability changes over time depending on endogenous variables, or even follows a stochastic process. Such extensions seem to be fairly easy to accommodate especially when the countries are assumed to be symmetric in all the other aspects. One may also consider more general bubble processes such as those studied in Kamihigashi (2008, 2011).

In addition to relaxing some of our assumptions, one may introduce government policies and examine their implications. Of particular interest would be a policy to minimize the effect of a bubble crash in the foreign country on the home bubble. All these extensions are left for future research.

3.6 Appendix

3.6.1 Proof of Lemma 7

If: Let $\{c_t, d_t, x_t, B_t\}$ be a set of nonnegative stochastic processes satisfying (3.2)-(3.6), (3.8), and (3.9). By the definition of an equilibrium, it suffices to show that for each $t \in \mathbb{Z}_+$, the vector $(c_t, 1, d_{t+1})$ solves the maximization problem (3.1)-(3.3).

Note that the problem can be written as

$$\max_{x \geq 0: b_t x \leq e_1} u(e_1 - B_t x) + \mathbb{E}_t v(e_2 + B_{t+1} x). \quad (3.67)$$

For any $x \geq 0$ with $B_t x \leq e_1$, we have

$$u(e_1 - B_t x) + \mathbb{E}_t v(e_2 + B_{t+1} x) - [u(e_1 - B_t) + \mathbb{E}_t v(e_2 + B_{t+1})] \quad (3.68)$$

$$= u(e_1 - B_t x) - u(e_1 - B_t) + \mathbb{E}_t [v(e_2 + B_{t+1} x) - v(e_2 + B_{t+1})] \quad (3.69)$$

$$\leq u'(e_1 - B_t) B_t (x - 1) + \mathbb{E}_t v'(e_2 + B_{t+1}) B_{t+1} (x - 1) \quad (3.70)$$

$$= [-u'(e_1 - B_t) B_t + \mathbb{E}_t v'(e_2 + B_{t+1}) B_{t+1}] (x - 1) = 0, \quad (3.71)$$

where the inequality holds by concavity of u and v , and the last equality holds by (3.9). It follows that it is optimal to choose $x = 1$; i.e., $(c_t, 1, d_{t+1})$ solves the maximization problem (3.1)-(3.3).

Only If: Let $\{c_t, d_t, x_t, B_t\}$ be an equilibrium. Since (3.2)-(3.6) hold by definition, we only need to verify (3.8) and (3.9). Let $t \in \mathbb{Z}_+$. Consider the maximization problem (3.67). Note that (3.8) follows from (3.7). Hence (3.9) is a necessary condition for optimality provided that

$$\left. \frac{\partial \mathbb{E}_v(e_2 + B_{t+1} x)}{\partial x} \right|_{x=1} = \mathbb{E}_t v'(e_2 + B_{t+1}) B_{t+1} \quad (3.72)$$

To see this, note that for any $x > 1$ we have

$$0 \leq \frac{v(e_2 + B_{t+1} x) - v(e_2 + B_{t+1})}{x - 1} \leq v'(e_2 + B_{t+1}) B_{t+1} < v'(e_2) e_1. \quad (3.73)$$

For any $x \in (0, 1)$ we have

$$0 \leq \frac{v(e_2 + B_{t+1}) - v(e_2 + B_{t+1} x)}{1 - x} \leq v'(e_2 + B_{t+1} x) B_{t+1} < v'(e_2) e_1. \quad (3.74)$$

Thus by the conditional dominated convergence theorem, we have

$$\lim_{x \rightarrow 1} \mathbb{E}_t \frac{v(e_2 + B_{t+1}x) - v(e_2 + B_{t+1})}{x - 1} \quad (3.75)$$

$$= \mathbb{E}_t \lim_{x \rightarrow 1} \frac{v(e_2 + B_{t+1}x) - v(e_2 + B_{t+1})}{x - 1} \quad (3.76)$$

$$= \mathbb{E}_t v'(e_2 + B_{t+1}) B_{t+1} \quad (3.77)$$

Now (3.72) follows.⁹ □

3.6.2 Proof of Lemma 8

The “only if” part follows from (7). To see the “if” part, define $\{x_t\}$ by (3.6). Define $\{c_t\}$ and $\{d_t\}$ using (3.2)-(3.4). Then (3.5) holds. Since we already have (3.8) and (3.9), it follows by Lemma (7) that $\{c_t, d_t, x_t, B_t\}$ is an equilibrium. □

3.6.3 Proof of Lemma 9

Only if: Let $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ be an equilibrium. Then (3.20)-(3.24) hold by definition. Fix $t \in \mathbb{Z}_+$. Since u and v are strictly concave, and since $\{c_t^h, x_t^{H,H}, x_t^{H,F}, d_{t+1}^H\}$ and $\{c_t^F, x_t^{F,H}, x_t^{F,F}, d_{t+1}^F\}$ solve the same problem (3.19)-(3.21), we have $c_t^H = c_t^F$ and $d_{t+1}^h = d_{t+1}^F$. Define c_t and d_{t+1} by

$$c_t = c_t^H = c_t^F, \quad d_{t+1} = d_{t+1}^H = d_{t+1}^F \quad (3.78)$$

⁹See Kamihigashi (1998, pp. 112-113) for a more general argument to show (3.72).

Summing (3.20) and (3.21) over $i \in \{H, F\}$ and using the above equations and (3.24), we have

$$2c_t = 2e_1 - 2(B_t^H + B_t^F), \quad (3.79)$$

$$2d_{t+1} = 2e_2 + 2(B_{t+1}^H + B_{t+1}^F) \quad (3.80)$$

Dividing both equations through by 2, we obtain (3.25) and (3.26). The inequality in (3.25) follows from (3.7). To see (3.27), let $t \in \mathbb{Z}_+$ and $k \in \{H, F\}$. By (3.24) there exists at least one $i \in \{H, F\}$ such that $x_t^{i,k} > 0$. Following the “only if” part of the proof of Lemma 7, we see that

$$u'(e_1 - (B_t^H + B_t^F))B_t^k = \mathbb{E}_t v'(e_2 + B_{t+1}^H + B_{t+1}^F)B_{t+1}^k \quad (3.81)$$

Recalling (3.25) and (3.26), we obtain (3.27).

If: Let $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ be a set of nonnegative stochastic processes satisfying (3.20)-(3.22) and (3.24). Suppose that (3.25)-(3.27) hold for all $t \in \mathbb{Z}_+$. Then (3.23) holds. Following the “if” part of the proof of Lemma 7, we see that $(c_t^i, x_t^{i,i}, x_t^{i,j}, d_{t+1}^i)$ solves the maximization problem (3.19)-(3.21) for each $i \in \{H, F\}$. It follows that $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ is an equilibrium. \square

3.6.4 Proof of Proposition 16

We start by preparing the following lemma.

Lemma 10. *Let $\{c_t^i, d_t^i, B_t^H, B_t^F\}$ be a set of nonnegative stochastic processes satisfying (3.25)-(3.27) for all $t \in \mathbb{Z}_+$. Suppose that (3.26) holds for $t = -1$ as well.*

Define $\{x_t^{i,k}\}_{t \in \mathbb{Z}_+, i, k \in \{H, F\}}$ by

$$x_t^{i,k} = 1, \quad \forall i, k \in \{H, F\}, \forall t \in \mathbb{Z}_+ \quad (3.82)$$

Then $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ is an equilibrium.

Proof. Let $\{c_t^i, d_t^i, B_t^H, B_t^F\}$ and $\{x_t^{i,k}\}$ be as given in the statement of the lemma. Then (3.20)-(3.24) hold. Since (3.27) holds for all $t \in \mathbb{Z}_+$ by hypothesis, it follows by Lemma 9 that $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$. \square

To complete the proof of Proposition 16, note that the “only if” part of Proposition 16 follows from Lemma 9. To see the “if” part, let $\{B_t^H, B_t^F\}$ be a pair of nonnegative stochastic processes satisfying (3.28) and (3.29). Define $\{c_t^i\}$ and $\{d_t^i\}$ by (3.25) and (3.26). Then (3.27) holds by (3.29). Define $\{x_t^{i,k}\}$ by (3.82). Then by Lemma 10, $\{c_t^i, d_t^i, x_t^{i,H}, x_t^{i,F}, B_t^H, B_t^F\}$ is an equilibrium. It follows that $\{B_t^H, B_t^F\}$ is a bivariate equilibrium bubble process.

3.6.5 Proof of Proposition 17

Let $\{B_t^H, B_t^F\}$ be a pair of stochastic processes satisfying (3.37) and (3.38) with $b^F, b^H > 0$.

Only If: Suppose that $\{B_t^H, B_t^F\}$ is a bivariate equilibrium bubble process. Let

$$c = e_1 - (b^H + b^F) \quad (3.83)$$

$$d = e_2 + (b^H + b^F) \quad (3.84)$$

$$\tilde{d} = e_2 + \tilde{b}^j, \text{ with } j = \{H, F\} \quad (3.85)$$

By Proposition 16 we have $b^H + b^F < e_1$ and

$$u'(c)b^i = qv'(d)b^i \quad (3.86)$$

$$u'(c)b^j = qv'(d)b^j + (1 - q)v'(\tilde{d})\tilde{b}^j \quad (3.87)$$

with $i, j = \{H, F\}$ and $i \neq j$. Note that from (3.87) that

$$u'(c) = qv'(d) \quad (3.88)$$

This together with (3.87) yields $0 = (1 - q)v'(\tilde{d})\tilde{b}^j = 0$. Hence (3.39) holds. Substituting (3.83) and (3.84) into (3.88) and recalling (3.17), we obtain (3.40).

If: Assume (3.39) and (3.40). Then (3.88) holds by (3.17) with c and d defined by (3.83) and (3.84). Note that (3.88) implies (3.86). We also have (3.87) from (3.88) and (3.39). Since (3.86) and (3.87) imply (3.29), and since $b^* < e_1$, it follows by Proposition 16 that $\{B_t^H, B_t^F\}$ is a bivariate equilibrium bubble process. \square

3.6.6 Proof of Proposition 18

Let $b^i = 0$ with $i = \{H, F\}$. Since $1 > q > u'(e_1)/v'(e_2)$ by Assumption 18, there exists a unique solution $b \in (0, e_1)$ to $u'(e_1 - b) = v'(e_2 + b)$. Since $q < 1$, we have $b > b^*$ (recall that b^* solves (3.17)). Let $b^j = \tilde{b}^j = b$ with $j = \{H, F\}$ and $i \neq j$. Let $\{B_t^i, B_t^j\}$ satisfy (3.37) and (3.38). Then (3.28) and (3.29) hold. Both $\{B_t^i\}$ and $\{B_t^j\}$ are deterministic processes here, and (3.28) and (3.29) trivially hold. Thus $\{B_t^i, B_t^j\}$ with $i, j = \{H, F\}$ and $i \neq j$ is a bivariate equilibrium bubble process by Proposition 16. \square

3.6.7 Proof of Proposition 19

Throughout the proof, we assume (3.55). We also assume that $b^H, b^F > 0$. For $(i, j) = (0, 0), (0, 1), (1, 0), (1, 1)$, let $c_{i,j}$ be the consumption of a young agent in state $(s_t^H, s_t^F) = (i, j)$; let $d_{i,j}$ be the consumption of an old agent in state $(s_t^H, s_t^F) = (i, j)$. From (3.25) and (3.26) we have

$$c_{0,0} = e_1 - (b^H + b^F), \quad (3.89)$$

$$d_{0,0} = e_2 + (b^H + b^F), \quad (3.90)$$

$$c_{0,1} = e_1 - \tilde{b}^H, \quad (3.91)$$

$$d_{0,1} = e_2 + \tilde{b}^H, \quad (3.92)$$

$$c_{1,0} = e_1 - \tilde{b}^F, \quad (3.93)$$

$$d_{1,0} = e_2 + \tilde{b}^F, \quad (3.94)$$

$$d_{1,1} = e_2 \quad (3.95)$$

These equations with (3.27) imply that

$$u'(c_{0,0})b^H = q^H q^F v'(d_{0,0})b^H + q^H(1 - q^F)v'(d_{0,1})\tilde{b}^H, \quad (3.96)$$

$$u'(c_{0,0})b^F = q^F q^H v'(d_{0,0})b^F + q^F(1 - q^H)v'(d_{1,0})\tilde{b}^F, \quad (3.97)$$

$$u'(c_{0,1})\tilde{b}^H = q^H v'(d_{0,1})\tilde{b}^H, \quad (3.98)$$

$$u'(c_{1,0})\tilde{b}^F = q^F v'(d_{1,0})\tilde{b}^F \quad (3.99)$$

Lemma 11. *If $\tilde{b}^H > 0$, then $\tilde{b}^H = \tilde{b}^{H*}$, where \tilde{b}^{H*} is a solution of (3.98). Likewise, if $\tilde{b}^F > 0$, then $\tilde{b}^F = \tilde{b}^{F*}$, where \tilde{b}^{F*} is a solution of (3.99).*

Proof. Suppose that $\tilde{b}^H > 0$. Then from (3.98) we have $u'(c_{0,1}) = q^H v'(d_{0,1})$; i.e.,

$$u'(e_1 - \tilde{b}^H) = q^H v'(e_2 + \tilde{b}^H) \quad (3.100)$$

Recalling (3.17), we know that the equation (3.100) has a unique solution \tilde{b}^{H*} such that $0 < \tilde{b}^{H*} \leq e_1$. Likewise, we have $\tilde{b}^F = \tilde{b}^{F*}$ with $0 < \tilde{b}^{F*} \leq e_1$. \square

Lemma 12. *Suppose that $q^H \geq q^F$. We have $\tilde{b}^{F*} \leq \tilde{b}^{H*}$.*

Proof. Suppose by the way of contradiction that $\tilde{b}^{H*} < \tilde{b}^{F*}$. Then, we have

$$u'(e_1 - \tilde{b}^{H*}) = q^F q^H v'(e_2 + \tilde{b}^{H*}) \quad (3.101)$$

$$> q^F v'(e_2 + \tilde{b}^{H*}) > q^F v'(e_2 + \tilde{b}^{F*}) \quad (3.102)$$

$$= u'(e_1 - \tilde{b}^{F*}) > u'(e_1 - \tilde{b}^{H*}) \quad (3.103)$$

where the second equality in (3.103) holds by (3.99). Since (3.101)-(3.103) lead to a contradiction, we obtain $\tilde{b}^{F*} \leq \tilde{b}^{H*}$. \square

To complete the proof of Proposition 19, dividing (3.96) and (3.97) through by $b^H > 0$ and $b^F > 0$, respectively, we have

$$u'(c_{0,0}) = q^H q^F v'(d_{0,0}) + q^H (1 - q^F) v'(d_{0,1}) \tilde{b}^H / b^H \quad (3.104)$$

$$u'(c_{0,0}) = q^F q^H v'(d_{0,0}) + q^F (1 - q^H) v'(d_{1,0}) \tilde{b}^F / b^F \quad (3.105)$$

From these equations, we get:

$$q^H (1 - q^F) v'(d_{0,1}) \tilde{b}^H / b^H = q^F (1 - q^H) v'(d_{1,0}) \tilde{b}^F / b^F \quad (3.106)$$

Lemma 13. *We have $\tilde{b}^H = 0$ if and only if $\tilde{b}^F = 0$.*

Proof. Suppose that $\tilde{b}^H = 0$. Then, from (3.106) we see that $\tilde{b}^F = 0$. We have shown that $\tilde{b}^H = 0 \rightarrow \tilde{b}^F = 0$. By the same argument, we obtain $\tilde{b}^F = 0 \rightarrow \tilde{b}^H = 0$. \square

Two cases appear: (a) $\tilde{b}^H = \tilde{b}^F = 0$ and (b) $0 < \tilde{b}^F \leq \tilde{b}^H$.

(a). Suppose that $\tilde{b}^H = 0$. Let $b^W = b^H + b^F$. We obtain (3.56) by (3.96). (3.56) admits a unique positive solution under Assumptions 17 and 18.

Suppose by the way of contradiction that $b^W \geq \tilde{b}^{F*}$. Then by (3.56) we have

$$u'(e_1 - b^W) = q^H q^F v'(e_2 + b^W) \quad (3.107)$$

$$< q^F v'(e_2 + b^W) \leq q^F v'(e_2 + \tilde{b}^{F*}) \quad (3.108)$$

$$= u'(e_1 - \tilde{b}^{F*}) \leq u'(e_1 - b^W), \quad (3.109)$$

where the equality in (3.109) holds by (3.99). Since (3.107)-(3.109) lead to a contradiction, we obtain $b^W < \tilde{b}^{F*}$.

(b). Suppose that $\tilde{b}^H > 0$ and $\tilde{b}^F > 0$. Then $\tilde{b}^H = \tilde{b}^{H*}$ and $\tilde{b}^F = \tilde{b}^{F*}$ by Lemma 11.

Using (3.106), we get

$$b^{F*} = \frac{1 - q^H}{1 - q^F} \frac{q^F v'(e_2 + \tilde{b}^{F*})}{q^H v'(e_2 + \tilde{b}^{H*})} \frac{\tilde{b}^{F*}}{\tilde{b}^{H*}} b^{H*} \quad (3.110)$$

(3.110) is equivalent to (3.58).

Substituting (3.110) into (3.96), we get (3.57). Substituting (3.58) into (3.57), (3.57) admits a unique positive solution $b^{H^*} > 0$ under Assumptions 17 and 18. By (3.110), we obtain a unique solution b^{F^*} .

We can show that $b^{F^*} \leq b^{H^*}$. Under Assumption 17, $v'(e_2 + \tilde{b}^j)\tilde{b}^j$ is increasing with \tilde{b}^j with $j = \{H, F\}$. As $\tilde{b}^{F^*} \leq \tilde{b}^{H^*}$ by Lemma 12, we get

$$\frac{v'(e_2 + \tilde{b}^{F^*})\tilde{b}^{F^*}}{v'(e_2 + \tilde{b}^{H^*})\tilde{b}^{H^*}} \leq 1 \quad (3.111)$$

As $q^H \geq q^F$, we get $\frac{1-q^H}{1-q^F} \frac{q^F}{q^H} \leq 1$. Hence, we deduce from (3.110) that $b^{F^*} \leq b^{H^*}$.

Furthermore, we can also show that $b^{H^*} < \tilde{b}^{H^*}$ and $b^{F^*} < \tilde{b}^{F^*}$. Suppose by the way of contradiction that $b^{H^*} \geq \tilde{b}^{H^*}$. Then From (3.96) we have

$$u'(e_1 - (b^{F^*} + b^{H^*})) = q^H q^F v'(e_2 + b^{F^*} + b^{H^*}) + q^H (1 - q^F) v'(e_2 + \tilde{b}^{H^*}) \tilde{b}^{H^*} / b^{H^*} \quad (3.112)$$

$$\leq q^H q^F v'(e_2 + b^{F^*} + b^{H^*}) + q^H (1 - q^F) v'(e_2 + \tilde{b}^{H^*}), \quad (3.113)$$

where the inequality holds since $\tilde{b}^{H^*} / b^{H^*} \leq 1$. Since $b^{F^*} + b^{H^*} > \tilde{b}^{H^*}$, it follows that

$$u'(e_1 - (b^{F^*} + b^{H^*})) < q^H q^F v'(e_2 + \tilde{b}^{H^*}) + q^H v'(e_2 + \tilde{b}^{H^*}) - q^H q^F v'(e_2 + \tilde{b}^{H^*}) \quad (3.114)$$

$$= q^H v'(e_2 + \tilde{b}^{H^*}) = u'(e_1 - \tilde{b}^{H^*}) \quad (3.115)$$

$$\leq u'(e_1 - (b^{F^*} + b^{H^*})) \quad (3.116)$$

where the second equality in (3.115) holds by (3.100). Since (3.114)-(3.116) lead to a contradiction, we obtain $b^{H*} < \tilde{b}^{H*}$. Following the same methodology, we can show that $b^{F*} < \tilde{b}^{F*} \leq \tilde{b}^{H*}$. \square

3.6.8 Proof of Proposition 20

The linearisation of the system given by (3.62)-(3.64) with respect to b_t^W , \tilde{b}_t^H and \tilde{b}_t^F in the neighborhood of a stationary sunspot equilibrium $(b^W, 0, 0)$ gives a triangular Jacobian matrix. As a consequence, the eigenvalues λ_i are given by the diagonal coefficients. Suppose that the condition (3.55) holds. Under Assumption 17, one has:

$$\lambda_1 = \frac{1 + \varepsilon_u(c_{0,0})b^W/c_{0,0}}{1 - \varepsilon_u v(d_{0,0})b^W/d_{0,0}} > 1; \quad (3.117)$$

$$\lambda_2 = \frac{1}{q^H \frac{v'(e_2)}{u'(e_1)}} \in (0, 1) \quad (3.118)$$

$$\lambda_3 = \frac{1}{q^F \frac{v'(e_2)}{u'(e_1)}} \in (0, 1) \quad (3.119)$$

where $c_{0,0} = e_1 - b^W$ and $d_{0,0} = e_2 + b^W$.

Since there are no predetermined variables in our model, we deduce from Blanchard-Kahn conditions (1980) that the stationary equilibrium is locally indeterminate, and the stable manifold has dimension 2.

3.6.9 Proof of Proposition 21

The linearization of the system given by (3.47)-(3.50) with respect to b_t^H , b_t^F , \tilde{b}_t^H and \tilde{b}_t^F in the neighborhood of a stationary sunspot equilibrium $(b^{H*}, b^{F*}, \tilde{b}^{H*}, \tilde{b}^{F*})$

gives the following Jacobian matrix, which is an upper triangular block matrix:

$$\tilde{J} = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix} \quad (3.120)$$

Since J is an upper triangular block matrix, the eigenvalues of J are the eigenvalues of A and C . For the presentation of the proof, we define only A and C . Moreover, we provide a general definition for any stationary equilibrium at this stage.

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} \text{ and } C = \begin{pmatrix} \frac{1 + \varepsilon_u(c_{0,1})\tilde{b}^{H^*}/c_{0,1}}{1 - \varepsilon_v(d_{0,1})\tilde{b}^{H^*}/d_{0,1}} & 0 \\ 0 & \frac{1 + \varepsilon_u(c_{1,0})\tilde{b}^{H^*}/c_{1,0}}{1 - \varepsilon_v(d_{1,0})\tilde{b}^{H^*}/d_{1,0}} \end{pmatrix}$$

with

$$A_{11} = \frac{1}{\alpha} \frac{d_{0,0}c_{0,0}(1 - \varepsilon_v(d_{0,0})b^{F^*}/d_{0,0}) + \varepsilon_u(c_{0,0})d_{0,0}b^{H^*}}{d_{0,0}c_{0,0}(1 - \varepsilon_v(d_{0,0})(b^{H^*} + b^{F^*})/d_{0,0})} \quad (3.121)$$

$$A_{12} = \frac{1}{\alpha} \frac{b^{F^*}(d_{0,0}\varepsilon_u(c_{0,0}) + c_{0,0}\varepsilon_v(d_{0,0}))}{d_{0,0}c_{0,0}(1 - \varepsilon_v(d_{0,0})(b^{H^*} + b^{F^*})/d_{0,0})} \quad (3.122)$$

$$A_{21} = \frac{1}{\alpha} \frac{b^{H^*}(d_{0,0}\varepsilon_u(c_{0,0}) + c_{0,0}\varepsilon_v(d_{0,0}))}{d_{0,0}c_{0,0}(1 - \varepsilon_v(d_{0,0})(b^{H^*} + b^{F^*})/d_{0,0})} \quad (3.123)$$

$$A_{22} = \frac{1}{\alpha} \frac{d_{0,0}c_{0,0}(1 - \varepsilon_v(d_{0,0})b^{H^*}/d_{0,0}) + \varepsilon_u(c_{0,0})d_{0,0}b^{F^*}}{d_{0,0}c_{0,0}(1 - \varepsilon_v(d_{0,0})(b^{H^*} + b^{F^*})/d_{0,0})} \quad (3.124)$$

$$\alpha = q^H q^F v'(d_{0,0})/u'(c_{0,0}) \quad (3.125)$$

Furthermore, $c_{0,0} = e_1 - (b^{H^*} + b^{F^*})$, $d_{0,0} = e_2 + b^H + b^{F^*}$, $c_{0,1} = e_1 - \tilde{b}^{H^*}$, $d_{0,1} = e_2 + \tilde{b}^{H^*}$, $c_{1,0} = e_1 - \tilde{b}^{F^*}$ and $d_{1,0} = e_2 + \tilde{b}^{F^*}$.

For the rest of the proof, suppose that the condition (3.55) and Assumption 17 hold.

As C is diagonal, we can deduce directly its eigenvalues λ_1^C and λ_2^C , which are given by these diagonal elements. One has:

$$\lambda_1^C = \frac{1 + \varepsilon_u(c_{0,1})\tilde{b}^{H^*}/c_{0,1}}{1 - \varepsilon_v(d_{0,1})\tilde{b}^{H^*}/d_{0,1}} > 1 \quad (3.126)$$

$$\lambda_2^C = \frac{1 + \varepsilon_u(c_{1,0})\tilde{b}^{H^*}/c_{1,0}}{1 - \varepsilon_v(d_{1,0})\tilde{b}^{H^*}/d_{1,0}} > 1 \quad (3.127)$$

To determine the eigenvalues of A , we derive the characteristic polynomial associated $P^A(\lambda^A)$ given by:

$$P^A(\lambda^A) = \lambda^{A^2} - T\lambda^A + D \quad (3.128)$$

where T and D are respectively the trace and the determinant of A such that:

$$D = \frac{1}{\alpha^2} \frac{1 + \varepsilon_u(c_{0,0})(b^{H^*} + b^{F^*})/c_{0,0}}{1 - \varepsilon_v(d_{0,0})(b^{H^*} + b^{F^*})/d_{0,0}} > 1 \quad (3.129)$$

$$T = \frac{1}{\alpha} \left(1 + \frac{1 + \varepsilon_u(c_{0,0})(b^{H^*} + b^{F^*})/c_{0,0}}{1 - \varepsilon_v(d_{0,0})(b^{H^*} + b^{F^*})/d_{0,0}} \right) \quad (3.130)$$

Note that $\alpha < 1$ (see (3.96)) and $\frac{1 + \varepsilon_u(c_{0,0})(b^{H^*} + b^{F^*})/c_{0,0}}{1 - \varepsilon_v(d_{0,0})(b^{H^*} + b^{F^*})/d_{0,0}} > 1$. Hence, $D > 1$, meaning that one eigenvalue is greater than 1, and $T > 0$.

From (3.129) and (3.130), we deduce that:

$$P(-1) = 1 + T + D > 0 \quad (3.131)$$

$$P(1) = 1 - T + D = \frac{(\alpha - 1)(1 - \alpha D)}{\alpha} > 0 \quad (3.132)$$

Therefore, we deduce that A has two positive eigenvalues outside the unit circle, and thus \tilde{J} has four eigenvalues outside the unit circle.

Since there are no predetermined variables in our model, we deduce from Blanchard-Kahn conditions (1980) that the stationary equilibrium is locally determinate, and it is a source.

Chapter 4

Asset Price Volatility, Wealth Distribution and Spirit of Capitalism: The Role of Heterogeneity

“Remember, that money is of the prolific, generating nature. Money can beget money, and its offspring can beget more, and so on. Five shillings turned is six, turned again it is seven and three pence, and so on, till it becomes a hundred pounds. The more there is of it, the more it produces every turning, so that the profits rise quicker and quicker. He that kills a breeding sow, destroys all her offspring to the thousandth generation. He that murders a crown, destroys all that it might have produced, even scores of pounds. [...] He that loses five shillings, not only loses that sum, but all the advantage that might be made by turning it in dealing, which by the time that a young man becomes old, will amount to a considerable sum of money.”

— Benjamin Franklin, *Advice to a Young Tradesman, Written by an Old One*, 1748

4.1 Introduction

In his sermon “Advice to a Young Tradesman”, Benjamin Franklin claims that the duty of an individual is to accumulate wealth. Not to fall short of this principle, the individual must adopt a frugal living.

In *The Protestant Ethic and the Spirit of Capitalism*, Max Weber (1905) quotes Benjamin Franklin’ words to illustrate his “Spirit of Capitalism” hypothesis (henceforth, SOC hypothesis) according to which the private accumulation of wealth as an end in itself, and not for consumption purpose was the driver of Industrial Revolution in Europe. The continual desire for wealth accumulation promotes investments, and in the end progress which is synonymous with growth. We can find similar ideas in *The Wealth of Nations* by Adam Smith (1776), in *Capital* by Karl Marx (1867) and in *The Economic Consequences of the Peace* by John Maynard Keynes (1919). For the latter, this Spirit of Capitalism would be a psychological aspect of a capitalist society. Introducing SOC hypothesis in standard optimal growth models through direct preferences for wealth, several contributions (see for instance Kurz, 1968; and Zou, 1994, 1995) confirm that such a hypothesis could explain long-term growth. Furthermore, some empirical studies support the idea that agents have direct preferences for wealth, and underline their importance for explaining saving behavior of agents (Carroll, 2000).

However, in the collective wisdom, the continual desire of wealth accumulation for its own sake by some agents (e.g. financial institutions, hedge funds, banks or traders) is often pointed out as one of the evils of our capitalist society in the times of crisis. For instance, Pope Francis denounced the love of money as responsible for both the recent financial crisis and current social inequalities in his *Evangelii Gaudium* in 2013. This is in accordance with Marx (1867), for who the capital (wealth) accumulation puts all of a country estate in the hands of an

only social class, the bourgeoisie. Moreover, the perpetual lure of profits would generate economic crises, which are inherent in capitalism, from Marx (1867). Most financial crises are closely associated with episodes of excess volatility in asset prices. Through the promotion of savings, the spirit of capitalism is a potential source of such asset price volatility. Several theoretical papers argue that wealth preferences in a standard asset pricing model explain the existence of a bubble on asset prices (Kamihigashi, 2008; Airaudo, 2012; Zhou, 2015), and excess volatility (Bakshi and Chen, 1996; Boileau and Braeu, 2007; and Airaudo, 2012).

The present paper contributes to this literature about the effects of SOC hypothesis in the standard asset pricing models, providing new insights on the role of heterogeneity among agents. The question addressed in this paper is the following: “Would a heterogeneous society, which consists of capitalists and workers, be more likely to experience social inequalities, and financial crisis?” Alternatively, we aim to investigate the role of heterogeneity on the wealth distribution, and the asset price dynamics.

To do this, we extend the continuous time version of infinite-horizon asset pricing model developed by Lucas (1978) to a heterogeneous agent framework with SOC hypothesis. There is a single consumption good and a financial asset generating dividends. For this reason, our paper is close to Kocherlakota (1992), Santos and Woodford (1997), Huang and Werner (2000), and more recently Le Van, Pham and Vailakis (2015), except that these papers consider a framework with borrowing constraints, only. In our paper, we provide an alternative explanation for dynamics of asset prices based on a psychological factor, namely the spirit of capitalism. Furthermore, we consider three sources of heterogeneity: heterogeneity in preferences for wealth, heterogeneity in initial wealth, and heterogeneity in income.

Following Zou (1994, 1995), we introduce SOC hypothesis through preferences for wealth, and consider a non-separable utility function between consumption good and wealth holdings. Since financial wealth is often used as an index to rank individuals in a country, direct preferences for wealth could also be interpreted as preferences for social status. By considering preferences for wealth, our paper also seems close to models with a bequest motive assuming both altruism and warm-glow. Notwithstanding, bequest models and spirit of capitalism models differ on several points as is underlined in Francis (2009): “First, they provide a rationale for all wealthy households to accumulate additional wealth, not only those with an explicit intention to leave their estate to their children. Second, in a model with a bequest motive and no mortality risk, only terminal wealth will appear in the utility function whereas in a model with capitalist spirit preferences wealth appears in the utility function in all periods.[...] An approach that allows households to have a direct preference for wealth, without specifying whether that preference is related to a bequest motive, status, power, or some other motivation, is more flexible and thus better able to match the reasons individuals report for saving [...] as well as their behavior.”

Wealth preferences are a key element in our paper. All agents hold financial assets in the steady state because of preferences for wealth. The spirit of capitalism encourages all agents to accumulate wealth whatever the level of the interest rate. This result contrasts with optimal growth models *à la* Becker (1980) with heterogeneous agents and borrowing constraints in which the most patient agents hold all the wealth of the economy in the steady state. In Becker (1980), agents do not derive preferences for wealth, but have different discount rates. The equilibrium interest rate is given by the discount rate of the most patient agents, i.e. by the lowest discount rate. In such a case, the agents with a discount rate greater than

the equilibrium interest rate reduce their wealth along an equilibrium path until they have zero asset holdings in the steady state, while the most patient agents have positive asset holdings.

Moreover, direct preferences for wealth explain the occurrence of asset price fluctuations, due to self-fulfilling expectations. More precisely, expectation-driven fluctuations are likely to occur when wealth and consumption are Edgeworth-substitutes i.e. when the marginal utility of consumption is decreasing in wealth. A similar result appears in the literature about Money-in-the-Utility-Function since money is a financial asset in these models, but provides no dividends. However, assuming a Edgeworth substitutability between consumption and money is neither empirically plausible (Walsh, 2010) nor consistent with the idea that money serves as a medium exchange, while a negative cross-derivative between wealth and consumption would coherent with the concept of frugality at the root of SOC hypothesis developed by Weber (1905).

Investigating the role of heterogeneity, we show that heterogeneity in preferences matter for two reasons. First, heterogeneity in wealth preferences affects inequalities, asset price level and volatility. We show that a society which consists of different agents with respect to their wealth preferences (e.g. capitalist/workers) is characterized by higher social inequalities and a higher asset price in the long run, and is more likely to experience fluctuations.

Second, heterogeneity in income matters only if preferences for wealth are heterogeneous. We show that if the rich are those with a stronger spirit of capitalism, inequality in income could heighten the asset price level in the long run, and promote the emergence of fluctuations. Therefore, a heterogeneous society which consists of rich capitalists and poor workers, for instance, would be more likely to experience high social inequalities and financial crises.

The paper is organized as follows. In the next section, we present the model. Section 4.3 is devoted to the intertemporal equilibrium. In Section 4.4, we describe the mean-preserving method. Section 4.5 analyzes the existence and uniqueness of the steady state. In Section 4.6, we study local dynamics and the role of heterogeneity on dynamics. Concluding remarks are provided in Section 4.7, while computational details are gathered in Appendix.

4.2 The model

Our starting point is a modified continuous-time version of the exchange economy developed by Lucas (1978). This economy is populated by infinitely-lived households who consume a single non-storable good, and save through a financial asset. Our extension is twofold: First, agents derive direct preferences from consumption, but also from financial wealth (SOC hypothesis). Second, there are three sources of heterogeneity: heterogeneity in initial wealth, in income stream, and in preferences for wealth.

We consider an economy with a constant population of infinitely-lived households. The population size is n . Without loss of generality, we consider two types of households, labeled with $i = 0, 1$. More precisely, there are $n_i > 1$ agents of type i with $n_0 + n_1 = n$. To keep things as simple as possible, we assume that the size of each class of agents are identical.

Assumption 19. $n_0 = n_1 = n/2$.

Individuals derive utility from both consumption $c_i(t)$ and financial wealth $w_i(t)$. Preferences for wealth capture the idea of “Spirit of Capitalism” developed by Max Weber (1905) in *The Protestant Ethic and the Spirit of Capitalism*. They accumulate wealth not only for consumption purpose, but also for its own sake.

The utility function of an agent at time $t = 0$ is the discounted sum of instantaneous utilities

$$\int_0^{+\infty} e^{-\rho t} u_i(c_i(t), w_i(t)) \quad (4.1)$$

where $\rho > 0$ is the common subjective rate of time preference.

Following Smith (2001) and Boileau and Braeu (2007), preferences of a household of type i are summarized by the following non-separable utility function in consumption and wealth:

$$u_i(c_{it}, w_{it}) = \begin{cases} \frac{\{c_i(t)^\alpha w_i(t)^{\beta_i}\}^{1-\varepsilon} - 1}{1-\varepsilon} & \text{if } \varepsilon > 0, \varepsilon \neq 1; \\ \alpha \log c_i(t) + \beta_i \log w_i(t) & \text{if } \varepsilon = 1 \end{cases} \quad (4.2)$$

where $\varepsilon > 0$ and $\alpha \in (0, 1)$ are respectively the common relative risk aversion coefficient and the weight of consumption in the utility function. Heterogeneity in preferences is captured by the parameter $\beta_i > 0$, which measures the weight of wealth in the utility function, as long as $\beta_1 \neq \beta_0$. Without loss of generality, we rule out the case in which agents do not derive wealth preferences, and consider a particular distribution for β_i . We assume that the agents of type 1 have a stronger spirit of capitalism than the agents of type 0.

Assumption 20. $0 < \beta_0 \leq \beta_1$.

To ensure the concavity of the utility function, we assume

Assumption 21. $1 - (\alpha + \beta_i)(1 - \varepsilon) > 0$.

Under this utility function, both consumption and wealth are normal goods. Moreover, when $\varepsilon < 1$, wealth and consumption are Edgeworth complements, mean-

ing that marginal utility of consumption increases with wealth (i.e., $u_{icw}(c_i, w_i) > 0$), and Edgeworth substitutes (i.e., $u_{icw}(c_i, w_i) < 0$) when $\varepsilon > 1$.

At the initial period $t = 0$, individuals are endowed with different shares of the initial stock $s_i(0)$. At time t , each individual i receives a constant dividend π per share and an exogenous income stream of $y_i > 0$ units of final good, trades and buys new shares $s_i(t)$ at price $q(t)$, and consumes $c_i(t)$ units of final good.

Even though there is no production side in our paper, we could interpret y_i as earnings coming from a labor activity. Therefore, the heterogeneity in income y_i depicts the heterogeneity in skills: If all agents face the same wage, a low-skilled agent has a lower income compared to a high-skilled. In contrast to heterogeneity in preferences, we do not impose restrictions neither on the distribution of income y_i , nor on the distribution of initial wealth $s_i(0)$. Since we do not deal with the transitional dynamics of the wealth distribution in this paper, we focus only on two configurations depending on the dispersion of income: $y_0 < y_1$ and $y_0 > y_1$.

In the first case (i.e. $y_0 < y_1$), individuals 0 have both a lower spirit of capitalism and income, while individuals 1 have both a stronger spirit of capitalism and income. The society is thus divided into two broad classes: workers (agents 0) and capitalists (agent 1). This situation ($\beta_0 < \beta_1$ and $y_0 < y_1$) is consistent with several empirical studies on U.S data which show that the individuals working in the financial sector belong to the top income earners (see Kaplan and Rauch, 2009).

In the second case (i.e. $y_0 > y_1$), individuals 1, who have a stronger spirit of capitalism, get the lowest income. This second case could illustrate a society in which the capitalist (an agent of type 1) would be a rentier, namely a person mainly living on capital income.

Throughout the paper, we call agents of type 0, *workers*, and agents of type 1, *capitalists*.

Given an initial level of wealth $w_i(0)$, the household i maximizes (4.1) with respect to $(c_i(t), w_i(t), s_i(t))$ under the following budget and stock constraints:

$$\dot{w}_i(t) = (\dot{q}(t) + \pi)s_i(t) + y_i - c_i(t) \quad (4.3)$$

$$w_i(t) = q(t)s_i(t) \quad (4.4)$$

Let $r(t)$ be the interest rate of the asset defined as follows:

$$r(t) = \frac{\dot{q}(t) + \pi}{q(t)}. \quad (4.5)$$

Under Assumptions 19-21, the optimal behavior of an individual i is summarized by the following Euler equation and the transversality condition:

$$\frac{\dot{c}_i(t)}{c_i(t)} = \frac{1}{1 + \alpha(\varepsilon - 1)} \left(r(t) + \frac{\beta_i}{\alpha} \frac{c_i(t)}{w_i(t)} - \rho - \beta_i(\varepsilon - 1) \frac{\dot{w}_i(t)}{w_i(t)} \right) \quad (4.6)$$

$$\lim_{t \rightarrow +\infty} e^{-\rho t} u_{ic}(c_i(t), w_i(t)) w_i(t) = 0 \quad (4.7)$$

Since, under Assumption 20, all individuals have direct preferences for wealth, and Inada conditions are satisfied both for consumption and wealth, $s_i(t) > 0$ is the only solution satisfying the optimal behavior of a individual i . In infinite-horizon general equilibrium models with heterogeneous agents and borrowing constraints *à la* Becker (1980), households differ in terms of discount rate and initial wealth. In such a framework, an individual would prefer dissaving until $s_i(t) = 0$ as soon as her discount rate ρ is greater than the interest rate $r(t)$. In contrast, the spirit of capitalism makes all agents to accumulate wealth, and thus to hold shares of stock

whatever the interest rate level. In infinite-horizon models with heterogeneous agents and short-sales constraints *à la* Kocherlakota (1992), households differ in term of endowments and initial wealth. Under some conditions, the poor would want to short sell, but they cannot. In such a context, they would sell until $s_i(t) = 0$. In contrast, in our framework, direct wealth preferences encourage all agents to hold a positive stock share $s_i(t) > 0$.

When agents derive utility from wealth, the Euler equation (4.6) has two additional terms compared to the asset pricing model developed by Lucas (1978)¹:

$$\frac{\beta_i c_i(t)}{\alpha w_i(t)} \text{ and } -\beta_i(\varepsilon - 1) \frac{\dot{w}_i(t)}{w_i(t)}$$

The first term corresponds to the marginal rate of substitution of consumption for wealth, through which the preferences for wealth increase the willingness to delay consumption for the future. Through the second term, this willingness to postpone consumption will be reinforced if wealth and consumption are Edgeworth-complements (i.e., $\varepsilon < 1$), or dampened if substitutes (i.e., $\varepsilon > 1$).

4.3 Intertemporal equilibrium

An intertemporal equilibrium is defined as follows:

Definition 3. *Under Assumptions 19-21, an equilibrium of the economy $E = (n, \rho, \pi, (y_i, u_i, s_i(0))_{i=0}^1)$ is an intertemporal path $(q(t), (s_i(t), c_i(t))_{i=0}^1)_{t \geq 0}$ satisfying the optimal behavior of agents (4.3)-(4.7) and the equilibrium condition on the asset*

¹In Lucas (1978), the Euler equation is given by $\frac{\dot{c}_i(t)}{c_i(t)} = \frac{r(t) - \rho}{1 + \alpha(\varepsilon - 1)}$.

market:

$$n \frac{s_0(t) + s_1(t)}{2} = 1. \quad (4.8)$$

Let $\psi = 1 + \alpha(\varepsilon - 1)$ and $\theta_i = \beta_i(1 - \varepsilon)$. From Definition 3, an intertemporal equilibrium is a path $(q(t), c_1(t), s_1(t))_{t=0}^{+\infty}$ satisfying the following three-dimensional dynamic system:

$$\left\{ \begin{array}{l} -\psi \frac{\dot{c}_1(t)}{c_1(t)} + (1 + \theta_1) \frac{\dot{q}(t)}{q(t)} + \theta_1 \frac{\dot{s}_1(t)}{s_1(t)} = \rho - \frac{\beta_1}{\alpha} \frac{c_1(t)}{q(t)s_1(t)} - \frac{\pi}{q(t)} \end{array} \right. \quad (4.9)$$

$$\left\{ \begin{array}{l} \psi \frac{n_1 c_1(t)}{\pi + ny - n_1 c_1(t)} \frac{\dot{c}_1(t)}{c_1(t)} + (1 + \theta_0) \frac{\dot{q}(t)}{q(t)} - \theta_0 \frac{n_1 s_1(t)}{1 - n_1 s_1(t)} \frac{\dot{s}_1(t)}{s_1(t)} \\ = \rho - \frac{\beta_0 \pi + ny - n_1 c_1(t)}{\alpha q(t)(1 - n_1 s_1(t))} - \frac{\pi}{q(t)} \end{array} \right. \quad (4.10)$$

$$\left\{ \begin{array}{l} \frac{\dot{s}_1(t)}{s_1(t)} = \frac{\pi}{q} + \frac{y_1}{q(t)s_1(t)} - \frac{c_1(t)}{q(t)s_1(t)} \end{array} \right. \quad (4.11)$$

where $s_i(0) > 0$ is given. Note that there are one predetermined variable $s_1(t)$ and two non-predetermined variables $q(t)$ and $c_1(t)$.

We shall now study the existence and the uniqueness of the steady state, then local dynamic properties of the economy, while emphasizing the role of the heterogeneity both in preferences and in income.

In order to evaluate the effect of the heterogeneity on the economy, we apply the mean-preserving method. Before starting our analysis of the steady state, we present this method in the next section.

4.4 Mean-preserving approach to heterogeneity

To highlight and understand the role of heterogeneity in preferences and income on the stationary asset price level, wealth distribution, and local dynamics, we impose a mean-preserving spread of distribution both on heterogeneity in preferences and in income. Thus, we fix the midpoints, and obtain under Assumption (19)

$$\beta \equiv \frac{\beta_0 + \beta_1}{2} \quad \text{and} \quad y \equiv \frac{y_0 + y_1}{2}$$

and we define a measure for each source of heterogeneity

$$\sigma_\beta \equiv \sqrt{\frac{(\beta_0 - \beta)^2}{2} + \frac{(\beta_1 - \beta)^2}{2}}, \quad (4.12)$$

$$\sigma_y \equiv \sqrt{\frac{(y_0 - y)^2}{2} + \frac{(y_1 - y)^2}{2}}, \quad (4.13)$$

where σ_β and σ_y are respectively the standard deviations of the distribution of weight of wealth in preferences and the one of the distribution of income.

To keep our analysis as simple as possible, we define two heterogeneity parameters, x and z , given by

$$x = \beta_1 - \beta \quad \text{and} \quad z = y_1 - y \quad (4.14)$$

Under Assumption 1, x is defined on $[0, \beta)$. Since we do not impose any restrictions on the distribution of income, z is defined on $(-y, y)$. When $y_1 < y_0$, one has $z < 0$, and conversely, when $y_1 > y_0$, $z > 0$.

We can now rewrite the standard deviations as functions of x and z :

$$\sigma_\beta = x, \forall x \in [0, \beta) \quad (4.15)$$

$$\sigma_y = \begin{cases} -z, & \text{if } y_1 < y_0, \\ z, & \text{if } y_1 > y_0 \end{cases} \quad (4.16)$$

An increase in x thus depicts an increase in the dispersion of β_i . A x close to β means that individuals have very heterogeneous preferences for wealth. Furthermore, an increase in z in absolute value expresses a raise in the dispersion of income y_i . For instance, a $z < 0$ close to $-y$ would indicate that a worker is very rich compared to a capitalist. Conversely, a $z > 0$ close to y would mean that a capitalist is very rich compared to a worker.

4.5 Steady state analysis

A steady state is an equilibrium where $\dot{s}_1(t) = 0$, $\dot{c}_1(t) = 0$, $\dot{q}(t) = 0$, and $r(t) = r$ for all t .

From (4.5) and (4.8)-(4.11), we deduce that the following equations are satisfied at the steady state:

$$r = \rho - \frac{\beta_1 \pi s_1 + y_1}{\alpha q s_1} \quad (4.17)$$

$$r = \rho - \frac{\beta_0 \pi s_0 + y_0}{\alpha q s_0} \quad (4.18)$$

$$r = \frac{\pi}{q} \quad (4.19)$$

We recall that the second term on the right-hand side of equations (4.17) and (4.18) are respectively the marginal rate of substitution of consumption for wealth of an individual 1 and of an individual 0.

As discussed in Section 4.2, all agents in the economy hold positive shares of stock because of preferences for wealth. This implies that their marginal rates of substitution between wealth and consumption are equal at the steady state. From (4.8), (4.17) and (4.18), we shall get the stationary distribution of wealth and the asset price level, while the interest rate will be given by the dividend-price ratio (see (4.19)).

Therefore, a steady state is a solution (s_1^*, q^*) with $s_1^* \in]0, 2/n[$ and $q^* > 0$ satisfying the following system:

$$\beta_1 \frac{\pi s_1 + y_1}{s_1} = \beta_0 \frac{\pi(1 - n_1 s_1) + n_1 y_0}{1 - n_1 s_1} \quad (4.20)$$

$$q = \frac{\pi}{r(q, s_1)} \quad (4.21)$$

$$\text{with } r(q, s_1) = \rho - \frac{\beta_1 \pi s_1 + y_1}{\alpha q s_1} \quad (4.22)$$

The next proposition proves the existence of an unique steady state, and characterizes it, while bringing the heterogeneity parameters (x and z) out.²

Proposition 22. *Let*

$$\bar{q} = \frac{\pi}{\rho} + \frac{\beta}{\alpha \rho} (\pi + ny). \quad (4.23)$$

Under Assumption 19-21, there exists a unique steady state (s_1^, q^*) such that $s_1^* \in]0, 2/n[$ and $q^* > 0$ are given by*

²In our paper, the uniqueness of steady state is a direct consequence of the class of preferences we consider, namely homothetic preferences. However, we can show that Greenwood–Hercowitz–Huffman preferences $u(c_i(t) + G_i(w_i(t)))$, with $G_i'(\cdot) > 0$ and $G_i''(\cdot) < 0$, ensures a unique steady state.

$$s_1^* = \frac{x\pi - \beta yn/2 - xz/2}{x\pi n} + \frac{\sqrt{(x\pi - \beta yn/2 - xzn/2)^2 + (\beta + x)(y + z)\pi xn}}{nx\pi} \equiv s_1^*(x, z) \quad (4.24)$$

$$q^* = \bar{q} + \frac{1}{\alpha\rho} \frac{[\beta(1 - ns_1^*(x, z)) + x][y(1 - ns_1^*(x, z)) + z]}{2s_1^*(x, z)(1 - ns_1^*(x, z)/2)} \equiv q^*(x, z) \quad (4.25)$$

Proof. See Appendix 4.8.1.

Proposition 22 indicates that the stationary asset price level q^* and the distribution of wealth given by s_1^* ($s_0^* = 2/n - s_1^*$) are affected both by the dispersion of income (z) and the dispersion of preferences (x). The asset price consists of two components: The first term corresponds to stationary asset price level found in the representative agent framework, while heterogeneity affects price through the second term.

Therefore, Proposition 22 shows that if two economies differ with respect to x and z , then they could experience different levels of inequalities, and the price of their financial assets could differ as well. In the following, we first describe the stationary asset price level, and the role of heterogeneity on the latter, then we characterize the steady-state wealth distribution.

4.5.1 Stationary asset price level

Equation (4.19) indicates that the stationary asset price level is equal to dividends per share deflated by the stationary interest level r^* , which is equivalent to

$$\frac{\pi}{r^*} = \pi \int_t^{+\infty} e^{-r^*(s-t)} ds \quad (4.26)$$

The right-hand side of equation (4.26) corresponds to the definition of the fundamental value of an asset, namely the present discounted value of future dividends.

The following proposition characterizes the asset price level at the steady state.

Proposition 23. *Let*

$$\underline{x} \equiv -\beta n z \frac{2\pi + ny + n\sqrt{(y-z)(y+z)}}{4\pi^2 + 4\pi ny + (nz)^2} \quad (4.27)$$

Under Assumptions 19-21, the following generically holds at the steady state³:

23.1 There is no bubble at the steady state;

23.2 The asset price q^ does not depend on σ_y when $x = 0$, is decreasing with σ_y when $x > 0$ and $y_1 < y_0$, and is increasing with σ_y when $x > 0$ and $y_1 > y_0$;*

23.3 For $x > \underline{x}$, the asset price q^ is increasing with x .*

Proof. See Appendix 4.8.2.

Proposition 23.1 shows the non-existence of bubbles at the steady state. The presence of positive dividends explains this result. Indeed, Kamihigashi (2008) and

³When $y_1 < y_0$, we are unable to provide a result for the case $x < \underline{x}$, analytically. However, we could provide a numerical analysis. For this exercise, we just need to provide values for β satisfying Assumption 20, y and d , then let varying x on $(0, \beta)$ and z on $(-y, y)$. Several numerical examples hint that the asset price level is decreasing with x when $x < \underline{x}$ and increasing when $x > \underline{x}$.

Airaudo (2012) obtain the same result in a representative agent framework with direct preferences for wealth. In these two papers, the financial asset generates dividends. In contrast, Zhou (2015) proves that a bubble on a useless asset providing no dividends can exist at the steady state as soon as the bubbleless economy is dynamically inefficient. Notwithstanding, Kamihigashi (2008) shows that a bubble can exist along explosive paths, satisfying the transversality condition in his framework because of preferences for wealth. Since we restrict our attention on the occurrence of expectation-driven fluctuations in the neighborhood of a steady state, our economy does not exhibit bubble.

Note that when agents face same preferences ($x = 0$), heterogeneity in income does not affect the stationary asset price level. We can show from (4.17) and (4.19) that the asset price herein is equal to $\bar{q} = \pi/\rho + (\pi + ny)\beta/(\alpha\rho)$, which corresponds to the stationary asset price level found in the representative agent framework. This result relies on the homothetic property of our preferences. Chatterjee (1994) shows that when preferences are homothetic or quasi-homothetic and agents are only heterogeneous in initial endowments, the aggregate dynamics are exactly the same as in the standard optimal growth model with representative agent. Caselli and Ventura (2000) extend this result to heterogeneity in skills and preferences for public goods.

Interestingly, in contrast to Caselli and Ventura (2000), heterogeneity in preferences matters in our framework, while they are homothetic. First, it affects the stationary asset price level. Second, heterogeneity in income matters only if preferences are heterogeneous. One explanation would rely on the fact we consider preferences for wealth and not for public good.

The intuition behind Proposition 23.2 is quite straightforward. Suppose that $y_1 < y_0$ ($z < 0$). In such a case, workers (agents 0), whose the willingness for

wealth accumulation is the lowest, are those with the greatest income. An increase in the dispersion in income distribution (higher y_0 and lower y_1) urges workers to accumulate more wealth, and capitalist less, since wealth is a normal good. However, the increase in asset demand of workers is not sufficient to counteract the decrease in asset demand of capitalists. Thus, the asset price level declines following an increase in σ_y . We can provide the same rationale for the case $y_1 > y_0$ by considering the reversed mechanism.

As shown by Proposition 23.3, the critical value \underline{x} is determinant in the role of preference heterogeneity for the stationary asset price level. When $y_1 > y_0$ ($z > 0$), the latter is negative ($\underline{x} < 0$), implying that the stationary asset price level increases with the dispersion of β_i distribution. In such case, capitalists (agents 1) have both a stronger spirit of capitalism and a higher income compared to workers (agent 0). Thus, an increase in the dispersion of β_i , that is a higher x , promotes a rise in asset demand involving a rise in asset price. When $y_1 < y_0$ ($z < 0$), \underline{x} is now positive. However, if capitalists are the poor, but their willingness for wealth accumulation is sufficiently high compared to agents 0 ($x > \underline{x}$), the previous mechanism prevails as well.

4.5.2 Wealth and total income distributions at the steady state

Financial wealth and total income are naturally used to rank individuals in a society, and thus provide some insights about social inequalities. For this reason, this subsection aims to characterize the stationary distributions in wealth and total income, and the role of individuals' heterogeneity on these distributions.

In our framework, we define the stationary wealth of an agent i as the real value of assets she holds, namely $w_i^*(x, z) = q^*(x, z)s_i^*(x, z)$, whereas her total

income is given by $R_i^*(x, z) = \pi s_i^*(x, z) + y_i$. The next proposition characterizes the distributions of wealth and total income within the economy at the steady state.

Proposition 24. *Let*

$$\tilde{x} = -\frac{\beta n z}{\pi + n y} \text{ and } \bar{x} = -\frac{\beta z n}{\pi} \quad (4.28)$$

Under Assumptions 19-21, the following generically holds at the steady state⁴:

24.1 The wealth distribution is non-degenerate meaning that capitalists and workers hold capital $s_i^ > 0$;*

24.2 A capitalist holds a greater stock share than a worker (i.e. $s_1^(x, z) > s_0^*(x, z)$) if and only if she has sufficiently strong spirit of capitalism compared to the worker $x > \tilde{x}$;*

24.3 If a capitalist receives a higher income than a worker ($z > 0$), or if she receives a lower income than a worker, but has a sufficiently strong spirit of capitalist compared to the worker ($-\pi < z < 0$ and $x > \bar{x}$), then the capitalist is the richest in terms of total income (i.e. $R_1^ > R_0^*$), and the poorest if she receives a lower income than a worker and the income gap is high ($z < -\pi$), or if she receives a slightly lower income than a worker and she has a not too high spirit of capitalist relatively to a worker ($-\pi < z < 0$ and $x < \bar{x}$).*

Proof. See Appendix 4.8.3.

In an optimal growth framework without wealth preferences, Becker (1980) proves that when the rate of time preference differs across agents, the wealth distribution is degenerate, meaning that the most patient agent holds all the

⁴For simplicity, the arguments of the functions are omitted

financial wealth. As discussed in Section 4.2, preferences for wealth make all agents to accumulate wealth. Our result about the non-degenerate distribution of wealth will be hold true with difference rate of time preference across agents.⁵

Proposition 24 could explain some stylized facts on the saving behavior, namely why the rich accumulate more wealth (see Dynan, Skinner, and Zeldes, 2004). This result is also in accordance with Carroll (2000) who claims that the Spirit of Capitalism can explain the saving choices of the rich.

Let us turn now to the effect of a change in agents' heterogeneity on wealth and total income inequality. After ranking the agents i in $[0, n]$ according to their increasing wealth or total income, we define the Gini coefficients associated with wealth distribution G_w and with total income distribution G_R as follows⁶:

$$G_w = 1 - 2 \frac{\int_0^n W(i) di}{W(n)n} \text{ and } G_R = 1 - 2 \frac{\int_0^1 \Gamma(i) di}{\Gamma(n)n} \quad (4.29)$$

For the definition of Gini coefficients, we suppose that workers are the poor in the economy:

Assumption 22. $y_0 < y_1$.

The stationary aggregate wealth $n_i w_i^*(x, z)$ and aggregate total income $n_i R_i^*(x, z)$ of agents of type i are respectively given by $n_i w_i^*(x, z) = n_i q^*(x, z) s_i^*(x, z)$ and $n_i R_i^*(x, z) = n_i (\pi s_i^*(x, z) + y_i)$. Under Assumptions 19 – 22, we obtain:

$$W(i) = \begin{cases} i w_0^*(x, z), & \text{if } 0 \leq i \leq n_0, \\ n_0 w_0^*(x, z) + (i - n_0) w_1^*(x, z), & \text{if } n_0 < i \leq n \end{cases} \quad (4.30)$$

⁵Suen (2014) obtains also a non-degenerate distributions of capital in a model with time preference heterogeneity and direct preferences for wealth.

⁶To define the Gini coefficient, we apply the same methodology adopted in Bosi and Seegmuller, 2006.

$$\Gamma(i) = \begin{cases} iR_0^*, & \text{if } 0 \leq i \leq n_0, \\ R_0^*(x, z) + (i - n_0)R_1^*(x, z), & \text{if } n_0 < i \leq n \end{cases} \quad (4.31)$$

Under Assumptions 19 – 22, we get two following Gini coefficients evaluated at the steady state:

$$G_w^*(x, z) = \frac{1}{2} - \frac{w_0^*(x, z)}{w_0^*(x, z) + w_1^*(x, z)} = \frac{1}{2} \left(s_1^*(x, z) - \frac{1}{n} \right) \quad (4.32)$$

$$G_R^*(x, z) = \frac{1}{2} - \frac{R_0^*(x, z)}{R_0^*(x, z) + R_1^*(x, z)} = \frac{n \pi (s_1^*(x, z) - 1/n) + z}{2(\pi + ny)} \quad (4.33)$$

with $s_1^*(x, z)$ given by (4.24).

Proposition 25. *Under Assumptions 19-22, the following generically holds at the steady state:*

25.1 *The Gini coefficients $G_w^*(x, z)$ and $G_R^*(x, z)$ are both increasing with σ_x ;*

25.2 *The Gini coefficients G_w^* and G_R^* are both decreasing with σ_y when $y_1 < y_0$ ($z < 0$), and increasing with σ_y when $y_1 > y_0$ ($z > 0$).*

Proof. See Appendix 4.8.4

Since both Gini coefficients are positively correlated, we find that they goes in the same direction when there is a change in preference heterogeneity or in income. Therefore, we can restrict our attention only on one coefficient when we deal with social inequality. Moreover, we recall that agents 0 are the poor to define the Gini coefficients.

The intuition behind the result of Proposition 25.1 is straightforward. A rise in the dispersion of β_i distribution means that the Spirit of Capitalism among

capitalists deepens, and the one of workers reduces. A higher heterogeneity in wealth preferences urges capitalists to save more, and thus to become richer, and workers, to save less. Social inequality increases.

The same argument applies for a rise in income inequality. When $y_1 > y_0$, a rise in the dispersion of income distribution forces agents who are initially rich to save more, and those who are poor to save less. Social inequality increases. When $y_1 < y_0$, the mechanism is reversed. An increase in income heterogeneity means that those who are rich save less, and the poor more. Therefore, the Gini coefficient declines.

4.6 Heterogeneity and Volatility

In this section, we address the following question: "Would a very heterogeneous society be more likely to experience volatility in asset prices, and thus financial crisis, or the opposite?" In other words, we aim to investigate the role of heterogeneity on the existence of expectation-driven fluctuations in the neighborhood of the steady state. Our results are twofold. First, heterogeneity in income plays a role on the asset price volatility only when agents face different preferences. Second, heterogeneity in preferences can destabilize the economy by enlarging the range of parameter values for which fluctuations due to self-fulfilling expectations are likely to occur.

To do this, we analyze the local dynamic properties of our model, and refer to local indeterminacy concept for the existence of expectation-driven fluctuations. Local indeterminacy means that there exist multiple equilibria with the same initial condition which converges to a steady state. Local indeterminacy is a sufficient condition for the occurrence of fluctuations driven by the volatility of agents'

expectations, without requiring shock on the fundamentals i.e. preferences and/or dividends in our model.

From the log-linearization of the 3-dimensional dynamic system (4.9)-(4.11) around the steady state (q^*, s_1^*) , we obtain the characteristic polynomial. As shown in Appendix 4.9., we can derive the trace $T(\varepsilon)$, the sum of the second order principal minor $S(\varepsilon)$ and the determinant $D(\varepsilon)$ of the associated Jacobian matrix as functions of ε_v . The characteristic polynomial of this economy is given by:

$$P(\lambda) = \lambda^3 - T(\varepsilon)\lambda^2 + S(\varepsilon)\lambda - D(\varepsilon) \quad (4.34)$$

Local indeterminacy occurs when the stable manifold has dimension greater than the number of predetermined variables. Since $s_1(t)$ is the only predetermined, the steady state will be locally determinate when the Jacobian matrix has zero or one eigenvalue with negative real part, and locally indeterminate when it has at least two eigenvalues with negative real part.

The next proposition provide the conditions on ε for which local indeterminacy occurs.

Proposition 26. *Let*

$$\underline{\varepsilon}(x, z) \equiv 1 + \frac{\pi + y}{c_1^*(x, z)} \frac{s_1^*(x, z)}{\beta + x} > 1$$

Under Assumptions 19-21, the following generically holds:

26.1 If $\varepsilon \in (0, \underline{\varepsilon}(x, z))$, the steady state is locally determinate.

26.2 If $\varepsilon > \underline{\varepsilon}(x, z)$, the steady state is locally indeterminate.

Proof. See Appendix 4.8.5.

Proposition 26 shows that expectation-driven fluctuations are likely to occur when the coefficient of relative risk aversion ε is sufficiently high, in particular greater than 1. Hence, local indeterminacy occurs only if wealth and consumption are Edgeworth-substitutes (i.e., $u_{cw} < 0$), otherwise the steady state is always determinate.

A similar result appears in the literature about Money-in-the-Utility-Function. Indeed, a necessary condition for local indeterminacy in this kind of model is a negative cross-derivatives of the utility function between consumption and money.⁷ Nevertheless, a negative cross-derivative is neither empirically plausible (Walsh, 2010) nor consistent with the idea that money serves as a medium exchange. In contrast, assuming a negative cross-derivative between wealth and consumption is coherent with the concept of frugality at the root of “Spirit of Capitalism” hypothesis developed by Weber (1905).

More interestingly, Proposition 26 also indicates that the critical value $\underline{\varepsilon}$, for which fluctuations are likely to occur, is a function both of preferences and income heterogeneity (x and z). This means that heterogeneity plays a role in the emergence of expectation-driven fluctuations. In our paper, heterogeneity promotes the occurrence of volatility as soon as it enlarges the range of parameter values for which local indeterminacy occurs. If heterogeneity reduces the range of parameter values, then the heterogeneity has stabilizing virtues.

Before assessing the effect of heterogeneity both in preferences and income, we provide the mechanisms through which expectation-driven fluctuations, and show how heterogeneity modifies these mechanism. Since aggregate consumption is constant along an equilibrium path, and the economy remains near the steady

⁷For an overview about the link between money-in-the-utility function and indeterminacy, the reader could refer to Obstfeld (1984) and Matsuyama (1990).

state, by combining equations (4.9) and (4.10) we obtain

$$\begin{aligned} \frac{1}{n_0 c(t)} \left[\frac{\beta_1}{\alpha} \frac{c_1(t)}{q(t) s_1(t)} c_1(t) + \frac{\beta_0}{\alpha} \frac{c_0(t)}{q(t) s_0(t)} c_0(t) \right] &= \rho - \frac{\dot{q}(t) + \pi}{q} \\ - \frac{1}{n_0 c(t)} \{ c_1(t) [\beta_1(1 - \varepsilon)] + c_0(t) [\beta_0(1 - \varepsilon)] \} &\frac{\dot{q}(t)}{q(t)} \end{aligned} \quad (4.35)$$

with $c(t)$ the aggregate consumption, i.e. $c(t) = n_1 c_1(t) + n_0 c_0(t)$.

Equation (4.35) indicates that the aggregate marginal rate of substitution of consumption for wealth should equal the opportunity cost of wealth holding. We can see that a change in asset price level have an ambiguous effect through the opportunity cost, when $\varepsilon > 1$. $\varepsilon > \underline{\varepsilon}(x, z)$ is equivalent to the following inequality.

$$c_1(t) [1 + \beta_1(1 - \varepsilon)] + c_0(t) [1 + \beta_0(1 - \varepsilon)] < 0 \quad (4.36)$$

Suppose that $\varepsilon > \underline{\varepsilon}(x, z)$. A small drop in asset price from the stationary level q^* lowers the opportunity cost, and (4.35) can be satisfied only if the marginal rate of substitution decreases. This could occur only if the asset price increases. After a small deviation of asset price from its stationary level, the economy will converge monotonically toward its steady state.

Corollary 2. *Under Assumptions 19-21, when preferences for wealth are homogeneous ($x = 0$), heterogeneity in income has no impact on the conditions for the existence of local indeterminacy.*

When preferences are homogeneous, inequality (4.36) is equivalent to $1 + \beta(1 - \varepsilon) < 0$. The critical value $\underline{\varepsilon}$ is given by $1/\beta$. Heterogeneity in income plays, therefore, no role in the emergence of local indeterminacy. As discussed in Section 4.4, this is due to the homothetic properties of preferences.

As $\beta_1 \geq \beta_0$ under Assumption 21, inequality (4.36) is satisfied only if $\varepsilon > 1$ and $[1 + \beta_1(1 - \varepsilon)] < 0$. If $[1 + \beta_1(1 - \varepsilon)] < 0$, inequality (4.36) is equivalent to

$$|c_1(t) [1 + \beta_1(1 - \varepsilon)]| > |c_0(t) [1 + \beta_0(1 - \varepsilon)]| \quad (4.37)$$

Inequality (4.37) means that the effect on the opportunity cost stemming from a change in behavior of capitalists dominates.

Corollary 3. *Under Assumptions 19-21, the following generically holds: for $x > \underline{x}$, a rise in preference heterogeneity destabilizes by enlarging the range of parameter values for which local indeterminacy occurs.*

Proof. See Appendix 4.8.6.

It is not surprising that when $y_1 > y_0$, a rise in the dispersion of β_i distribution promotes the existence of expectation-driven fluctuations. In this case, a rise in x boosts the willingness of capitalists to accumulate wealth, and thus reinforces the effect on the opportunity stemming from capitalists necessary for indeterminacy. When the rich are also the capitalists, an increase in heterogeneity of preferences destabilizes the economy.

From an economic point of view, Corollary 3 claims that a society, in which all agents would have same preferences, would be better from a stabilizing perspective to a society, in which there would be two social classes with the rich capitalists, on one hand, and the poor workers, on the other hand.

Corollary 4. *Under Assumptions 19-21, the following generically holds:*

26.1 When $x > 0$ and $y_1 < y_0$, an increase in income heterogeneity (a higher σ_y) stabilizes, by reducing the range of parameter values for which local indeterminacy occurs;

26.2 When $x > 0$ and $y_1 > y_0$, an increase in income heterogeneity (a higher σ_y) destabilizes, by enlarging the range of parameter value for which local indeterminacy occurs.

Proof. See Appendix 4.8.6.

When $y_1 < y_0$, a rise in the dispersion of income distribution prevents from the occurrence fluctuations. We have shown that indeterminacy are likely to occur only when the effect stemming from a change in behavior of capitalist dominates. However, when $y_1 < y_0$, this effect is dampened by a rise in the dispersion of income distribution. The reverse argument holds when $y_1 > y_0$.

Since we restrict our attention on the dynamics near the steady state, we can claim that the effect of income heterogeneity on the total income distribution along an equilibrium path is similar to the effect at steady state. Thus, we indirectly link the distribution of aggregate income with the occurrence of fluctuations. For instance, we have shown that Gini coefficient is positively correlated with σ_y when $y_1 > y_0$ in Section 4. Therefore, we conclude that an increase in total income inequality destabilizes the economy, when capitalists are also the rich in the society.

To answer our question addresses at the beginning of the section, a heterogeneous society which consists of rich capitalists and poor workers is more likely to experience asset price volatility, and thus financial crisis.

4.7 Concluding remarks

Our model is simple enough since it only adds two ingredients to the well-known asset pricing model developed by Lucas (1978) to inspect the role of consumers' heterogeneity on asset price volatility and social inequality. The first ingredient is

heterogeneity in preferences and income. The second ingredient is preferences for wealth, which captures Spirit of Capitalism hypothesis originated from Max Weber (1905).

Heterogeneous Spirit of Capitalism preferences matter for the occurrence of asset price fluctuation driven by the volatility of agents' expectations. Investigating the role of consumers' heterogeneity, our paper shows that more heterogeneity in preferences, but also in income, could accentuate social inequalities in the long run, and reinforce mechanisms behind asset price volatility in the short-run by promoting local indeterminacy.

By providing new insights about the role of heterogeneity on asset price volatility and wealth inequality, our paper could be used to investigate how a redistribution policy, as a capital income taxation, should be implemented to both reduce social inequalities and stabilize the economy.

4.8 Appendix

4.8.1 Proof of Proposition 22

Let us prove Proposition 22.

A steady state (s_1^*, q^*) is a solution of $q_1(s_1) = q_0(s_1)$, with:

$$\left\{ \begin{array}{l} q_1(s_1) = \frac{\beta_1}{\alpha\rho} \frac{\pi s_1 + y_1}{s_1} + \frac{\pi}{\rho} \\ q_0(s_1) = \frac{\beta_0}{\alpha\rho} \frac{\pi(1 - n_1 s_1) + n_1 y_0}{1 - n_1 s_1} + \frac{\pi}{\rho} \end{array} \right. \quad (4.38)$$

$$\left\{ \begin{array}{l} q_1(s_1) = \frac{\beta_1}{\alpha\rho} \frac{\pi s_1 + y_1}{s_1} + \frac{\pi}{\rho} \\ q_0(s_1) = \frac{\beta_0}{\alpha\rho} \frac{\pi(1 - n_1 s_1) + n_1 y_0}{1 - n_1 s_1} + \frac{\pi}{\rho} \end{array} \right. \quad (4.39)$$

To prove the existence of a stationary solution (q^*, s_1^*) , we use the continuity of $q_1(s_1)$ and $q_0(s_1)$. We recall that s_1 belongs to $(0, 2/n)$ from the equilibrium condition on the asset market (see (4.8)).

Using (4.38) and (4.39), we can determine the boundary values of $q_1(s_1)$ and $q_0(s_1)$:

$$\begin{aligned} \lim_{s_1 \rightarrow 0} q_1(s_1) &= +\infty \\ \lim_{s_1 \rightarrow 2/n} q_1(s_1) &= \frac{\beta_1}{\alpha\rho}(\pi + n_1 y_1) + \frac{\pi}{\rho} > 0 \\ \lim_{s_1 \rightarrow 0} q_0(s_1) &= \frac{\beta_0}{\alpha\rho}(\pi + n_0 y_0) + \frac{\pi}{\rho} > 0 \\ \lim_{s_1 \rightarrow 2/n} q_0(s_1) &= +\infty \end{aligned}$$

We have

$$\lim_{s_1 \rightarrow 0} q_1(s_1) > \lim_{s_1 \rightarrow 0} q_0(s_1), \text{ and } \lim_{s_1 \rightarrow 2/n} q_1(s_1) < \lim_{s_1 \rightarrow 2/n} q_0(s_1).$$

Therefore, there exists at least one value $s_1^* \in]0, 2/n[$ such that $q_1(s_1^*) = q_0(s_1^*)$. As $q_1(s_1) > 0 \forall s_1 \in]0, 2/n[$, we deduce that $q^* = q_1(s_1^*) > 0$.

Furthermore, since $q_1(s_1)$ is strictly decreasing on $]0, 2/n[$ and $q_0(s_1)$ is strictly increasing on $]0, 2/n[$, the solution (q^*, s_1^*) is unique. \square

4.8.2 Proof of Proposition 23

We first prove Proposition (23.1) claiming that the stationary asset price is equal to its fundamental value.

Let $r(t)$ be an interest rate, $q(t)$ the asset price in terms of consumption good at time t and π the dividend in consumption good units generated by the asset. The no-arbitrage condition that governs the evolution of the asset price is given by:

$$q(t) = \frac{\dot{q}(t) + \pi}{r(t)} \tag{4.40}$$

Solving equation (4.40) by iterating forward, we obtain:

$$q(t) = \int_t^{+\infty} e^{\int_t^s -r(i)di} \pi ds + e^{\int_t^{+\infty} -r(i)di} q(+\infty) \quad (4.41)$$

The first term depicts the fundamental value of the asset $v(t)$, while the second term is the definition of a bubble $b(t)$:

$$v(t) = \int_t^{+\infty} e^{\int_t^s -r(i)di} \pi ds \quad (4.42)$$

$$b(t) = e^{\int_t^{+\infty} -r(i)di} q(+\infty) \quad (4.43)$$

At the steady state, $q(t) = q^*$ and $r(t) = r^*$. Therefore, from (4.42), we have

$$v(t) = \pi \int_t^{+\infty} e^{-r^*(s-t)} = \frac{\pi}{r^*} \equiv v^* \quad (4.44)$$

Combining (4.40) evaluated at the steady state with (4.44), one has

$$q^* = \frac{\pi}{r^*} = v^* \quad (4.45)$$

The asset price is equal to its fundamental component at the steady state.

We now prove Propositions 23.2 and 23.3 which discuss the effect of heterogeneity on the stationary asset price level q^* .

We recall that $\sigma_y = -z$ when $y_1 \leq y_0$, and $\sigma_y = z$ when $y_1 \geq y_0$. Hence, $\partial z / \partial \sigma_y \leq 0$ when $y_1 \leq y_0$, and $\partial z / \partial \sigma_y \geq 0$ when $y_1 \geq y_0$. We shall compute the derivatives of $q^*(x, z)$ with respect to σ_y and x , and analyze their signs.

We start with $\partial q^*/\partial\sigma_y$. We get

$$\frac{\partial q^*(x, z)}{\partial\sigma_y} = \frac{\partial q^*(x, z)}{\partial z} \frac{\partial z}{\partial\sigma_y}, \text{ with} \quad (4.46)$$

$$\frac{\partial q^*(x, z)}{\partial z} = \frac{xn^3\pi\beta + xg_1(x, z) - g_2(x, z)}{\alpha\rho d(x, z)^2\pi\sqrt{\Delta(x, z)}} \quad (4.47)$$

where

$$\begin{aligned} g_1(x, z) \equiv & n\left(\frac{n\beta y}{2}\right)^2 + n(\pi x)^2 + xyn^2\left(-\frac{\beta yn}{2} - \beta\pi + \pi x\right) \\ & + xz\frac{n^2}{2}[(\beta - x)(\pi + ny) + \pi(\beta + x)] \end{aligned} \quad (4.48)$$

$$g_2(x, z) \equiv n\left(ny\frac{\beta - x}{2} - \pi x\right)\sqrt{\Delta(x, z)}, \text{ with} \quad (4.49)$$

$$d(x, z) = x\pi - \frac{n\beta y}{2} - \frac{nxz}{2} + \sqrt{(x\pi - n\beta y/2 - nxz/2)^2 + (\beta + x)(y + z)\pi xn} \quad (4.50)$$

$$\begin{aligned} \Delta(x, z) \equiv & \left(\frac{\beta yn}{2}\right)^2 + \left(\frac{nxz}{2}\right)^2 + (\pi x)^2 + \frac{\beta n^2 xyz}{2} - \beta\pi nxy + \beta\pi nxy \\ & + \beta\pi nxyz - \pi nx^2z + \pi nx^2y + \pi nx^2z \end{aligned} \quad (4.51)$$

We can show from (4.47) that $\partial q^*/\partial z = 0$ when $x = 0$.

We analyze now the sign of (4.47) when $x \in (0, \beta)$. First of all, we can note that the denominator of (4.47) is positive $\forall x \in (0, \beta)$ and $z \in (-y, y)$. As a consequence, the sign of (4.47) is given by its numerator, i.e. $g_1(x, z) - g_2(x, z)$.

We can show that $g_1(x, z) > 0 \forall x \in (0, \beta)$ and $y \in (-y, y)$. Since $x \in (0, \beta)$, $g_1(x, z)$ is increasing with $z \in (-y, y)$. Hence,

$$\begin{aligned}
 g_1(x, y) &> n \left(\frac{n\beta y}{2} \right)^2 + n(\pi x)^2 + xy \left(-\frac{\beta y n^3}{4} - \frac{\beta \pi n^2}{2} + \pi x n^2 \right) \\
 &\quad - \frac{n^2 xy}{4} [(\beta - x)(\pi + ny) + \pi(\beta + x)] \\
 &= n \left[\left(\frac{n\beta y}{2} \right)^2 + (\pi x)^2 + \left(\frac{xy n}{2} \right)^2 + \pi x^2 y n - \beta n^2 y^2 x - \beta n y x \pi \right] \\
 &= n \left[\frac{n}{2} y (\beta - x) - \pi x \right]^2 > 0
 \end{aligned} \tag{4.52}$$

Two cases appears when we study the sign of $g_2(x, z)$: case (a) $\frac{ny}{2} (\beta - x) - \pi x < 0$ and case (b) $\frac{ny}{2} (\beta - x) - \pi x > 0$.

Case (a) $\frac{ny}{2} (\beta - x) - \pi x < 0$.

In the case (a), we have $g_2(x, y) < 0$. Therefore, we get $g_1(x, z) - g_2(x, z) > 0$. We can deduce that $\partial q^*(x, z)/\partial z > 0$ in the case (a).

Case (b) $\frac{ny}{2} (\beta - x) - \pi x > 0$.

In the case (b), we have $g_2(x, y) > 0$. As a consequence, $\partial q^*(x, z)/\partial z > 0$ if and only if $g_1(x, z) > g_2(x, z)$.

As $g_1(x, y)$ and $g_2(x, y)$ are both positive, $g_1(x, z) > g_2(x, z)$ is equivalent to

$$g_1(x, z)^2 > g_2(x, z)^2 \tag{4.53}$$

After some algebra, we can show that the inequality (4.53) is equivalent to

$$(\beta - x)(\beta + x) > 0$$

Since $x \in (0, \beta)$, the inequality (4.53) holds true. Hence, we can conclude $g_1(x, z) - g_2(x, z) > 0 \forall x \in (0, \beta)$ and $y \in (-y, y)$. Therefore, $\partial q^*/\partial z > 0$ when $x \in (0, \beta)$ and $\frac{ny}{2}(\beta - x) - \pi x > 0$.

From case (a) and case (b), we can conclude that $\partial q^*/\partial z > 0$ when $x \in (0, \beta)$. As $\partial z/\partial \sigma_y < 0$ when $y_1 < y_0$ and $\partial z/\partial \sigma_y > 0$ when $y_1 > y_0$, Proposition 23.3 follows.

Let prove the last result of Proposition 23. We analyze now the sign of the $\partial q^*(x, z)/\partial \sigma_x$.

We have

$$\frac{\partial q^*(x, z)}{\partial x} = \frac{\partial q^*(x, z)}{\partial x}, \text{ with} \quad (4.54)$$

$$\frac{\partial q^*(x, z)}{\partial x} = \frac{1}{\alpha \rho} \frac{\tilde{g}(x, z)}{d(x, z)^2 \sqrt{\Delta(x, z)}} \quad (4.55)$$

where

$$\begin{aligned} \tilde{g}(x, z) \equiv & \tilde{g}_1(z)x^3 + \left[\tilde{g}_2(z) + \tilde{g}_3(z)\sqrt{\Delta(x, z)} \right] x^2 + \left[\tilde{g}_4(z) + \tilde{g}_5(z)\sqrt{\Delta(x, z)} \right] x \\ & + \tilde{g}_6(z) + \tilde{g}_7(z)\sqrt{\Delta(x, z)}, \text{ with} \end{aligned} \quad (4.56)$$

$$\tilde{g}_1(z) = n^3yz^2 + 4\pi n^2y^2 + 2\pi n^2z^2 + 12\pi^2ny + 8\pi^3 \quad (4.57)$$

$$\tilde{g}_2(z) = 3\beta n^3y^2z - 4\beta\pi n^2y^2 + 10\beta\pi n^2yz + 2\beta\pi n^2z^2 - 4\beta\pi^2ny + 8\beta\pi^2nz \quad (4.58)$$

$$\tilde{g}_3(z) = -n^2yz + 4\pi ny + 4\pi^2 \quad (4.59)$$

$$\tilde{g}_4(z) = 2\beta^2n^3y^3 + \beta^2n^3yz^2 + 2\beta^2\pi n^2y^2 - 2\beta^2\pi n^2yz + 2\beta^2\pi n^2z^2 \quad (4.60)$$

$$\tilde{g}_5(z) = -2\beta(ny)^2 - 2\beta\pi n(y - z) \quad (4.61)$$

$$\tilde{g}_6(z) = \beta^3n^3y^2z \quad (4.62)$$

$$\tilde{g}_7(z) = -\beta^2n^2yz \quad (4.63)$$

As the denominator of (4.55) is positive, the sign of $\partial q^*/\partial x$ will be given by the sign of $\tilde{g}(x, z)$. Since the study of function $\tilde{g}(x, z)$ is not so trivial, we resort to the mathematical software Maple to provide a conclusion.

First of all, the function $\tilde{g}(x, z)$ is defined on $x \in [0, +\infty)$, and $\tilde{g}(0, z) = 0$.

Furthermore, one has:

$$\begin{aligned} \lim_{x \rightarrow +\infty} \tilde{g}(x, z) &= \lim_{x \rightarrow +\infty} x^3(\tilde{g}_1 + \tilde{g}_2 \frac{\Delta(x, z)}{x}) \\ &= [16\pi^3 + 20\pi^2ny + 2n^2(2y^2 - yz + z^2)\pi + n^3yz^2] x^3 > 0, \end{aligned} \quad (4.64)$$

$$\forall z \in (-y, y) \quad (4.65)$$

Therefore, $\tilde{g}(x, z)$ tends towards ∞ when x goes to $+\infty$.

Maple finds that the equation $\tilde{g}(x, z) = 0$ has two solutions in x :

$$x_1 = -zn\beta \frac{yn + 2\pi - n\sqrt{(y-z)(y+z)}}{(nz)^2 + 4\pi ny + 4\pi^2} \quad (4.66)$$

$$x_2 = -zn\beta \frac{yn + 2\pi + n\sqrt{(y-z)(y+z)}}{(nz)^2 + 4\pi ny + 4\pi^2} \quad (4.67)$$

$$(4.68)$$

When $z < 0$, one has $0 < x_1 < x_2$, $x_1 = x_2 = 0$ when $z = 0$, and $0 > x_1 > x_2$ when $z > 0$. Therefore, we assert that $\partial q^*/\partial x > 0$ when $z \geq 0$ and for $x > x_2 \equiv \underline{x}$ (in Proposition 23) when $z < 0$.

Analytically, we are not able to conclude on the sign of $\partial q^*/\partial x$ when $z < 0$, and in particular for the case where $x \in (0, \underline{x})$. A numerical analysis is required.

For this numerical exercise, we just need to fix the values for β satisfying Assumption 20, y and π , then let varying x on $(0, \beta)$ and z on $(-y, y)$.⁸ Several numerical examples hint that the asset price level is decreasing with x when $x < \underline{x}$ and increasing when $x > \underline{x}$. \square

4.8.3 Proof of Proposition 24

Proposition 24.1 is a direct consequence of Proposition 22.

Let prove Proposition 24.2, then Proposition 24.3.⁹ Note that $s_1^* > s_0^*$ is equivalent to $s_1^* > 1/n$. As $q_1'(s_1) < 0$, $q_0'(s_1) > 0$ and $q_1(s_1^*) = q_0(s_1^*)$, where $q_1(s_1)$ and $q_0(s_1)$ are respectively given by (4.38) and (4.39), $s_1^* > 1/n$ if and only if $q_1(1/n) > q_0(1/n)$. This inequality is equivalent to $x > -\beta nz/(\pi + ny) \equiv \tilde{x}$.

⁸There exist few estimations in the literature for the parameter β measuring the weight of wealth in preferences. To match U.S. data, Airaudo (2012) sets β to 0.68, whereas Karnizova (2010) to 0.83.

⁹For simplicity, the arguments of the functions are omitted.

$R_1^* > R_0^*$ is equivalent to

$$s_1^* > (1/n) - z/\pi \quad (4.69)$$

If $z < -\pi/n$, then $1/n - z/\pi > 2/n$. As $s_1^* < 2/n$, the inequality (4.69) cannot hold. Hence, if $z < -\pi/n$, then $R_1^* < R_0^*$. If $z > \pi/n$, then we have $s_1^* > 0 > 1/n - z/\pi$. As a result, when $z > \pi/n$, we get $R_1^* > R_0^*$.

If $z \in (-\pi/n, \pi/n)$, then $2/n > 1/n - z/\pi > 0$. As a consequence, we should analyze under which conditions the inequality (4.69) holds.

As $q_1'(s_1) < 0$, $q_0'(s_1) > 0$ and $q_1(s_1^*) = q_0(s_1^*)$, $s_1^* > 1/n - z/\pi$ if and only if $q_1(1/n - z/\pi) > q_0(1/n - z/\pi)$ which is equivalent to $x > -\beta zn/\pi \equiv \bar{x}$.

Note that for $z > 0$, $\bar{x} < 0$. Therefore, Proposition 24.3 follows. \square

4.8.4 Proof of Proposition 25

First of all, we observe that $G_R^*(x, z)$ is an increasing function of $G_w^*(x, z)$

$$G_R^*(x, z) = \frac{\pi}{\pi + ny} G_w^*(x, z) + \frac{nz/2}{\pi + y} \quad (4.70)$$

As $G_w^*(x, z)$ is a function of $s_1^*(x, z)$, itself function of x and z , we first compute the derivatives of $s_1^*(x, z)$ with respect to x and z , then the derivative of $G_w^*(x, z)$ and $G_R^*(x, z)$ with respect to x and z .

We have

$$\frac{\partial s_1^*(x, z)}{\partial x} = n\beta \frac{y\sqrt{\Delta(x, z)} - (\beta y^2 n/2 + xyzn/2 + 2\pi xy/2 + \pi xz - \pi xyn)}{\pi x^2 n \sqrt{\Delta(x, z)}} \quad (4.71)$$

where $\Delta(x, z)$ is given by (4.51).

Note that if and only if $\beta y^2 n_0 n + x y z n_1 n + 2\pi x y n_1 + 2\pi n_1 x z - \pi x y n < 0$, then $\partial s_1^*(x, z)/\partial x > 0$.

If $\beta y^2 n_0 n + x y z n_1 n + 2\pi x y n_1 + 2\pi n_1 x z - \pi x y n > 0$, we should study the sign of the numerator of (4.71). The numerator is positive if and only if

$$y\sqrt{\Delta(x, z)} > \beta y^2 n/2 + x y z n/2 + \pi x y + \pi x z - \pi x y \quad (4.72)$$

As both sides of (4.72) are positive, we can show after some algebra that inequality (4.72) is equivalent to

$$(y + z)(y - z) > 0 \quad (4.73)$$

Since $z \in (-y, y)$, the inequality (4.72) is always satisfied. Therefore, $\partial s_1^*(x, z)/\partial x > 0 \forall x \in (0, \beta)$.

Applying the chain rule, we get:

$$\frac{\partial G_w^*(x, z)}{\partial \sigma_x} = n_1 \frac{\partial s_1^*(x, z)}{\partial x} \frac{\partial x}{\partial \sigma_x} \quad (4.74)$$

$$\frac{\partial G_R^*(x, z)}{\partial \sigma_x} = \frac{\pi}{\pi + n y} \frac{\partial G_w^*(x, z)}{\partial \sigma_x} \quad (4.75)$$

As $\partial x/\partial \sigma_x > 0 \forall x \in (0, \beta)$, Proposition 25.1 follows.

Furthermore,

$$\frac{\partial s_1^*(x, z)}{\partial z} = \frac{1}{2\pi} \frac{\beta y n/2 + x z n/2 + \pi(\beta + x)/2 + \pi(\beta - x)/2 - \sqrt{\Delta(x, z)}}{\sqrt{\Delta(x, z)}} \quad (4.76)$$

where $\Delta(x, z)$ is given by (4.51). Note that numerator of (4.76) is positive if and only if

$$\beta yn/2 + xzn/2 + \pi(\beta + x)/2 + \pi(\beta - x)/2 > \sqrt{\Delta(x, z)} \quad (4.77)$$

As $x \in (0, \beta)$ and $z > -y$, both sides of inequality (4.77) are positive. We can show that inequality (4.77) is equivalent to

$$(\beta + x)(\beta - x) > 0 \quad (4.78)$$

Since $x \in (0, \beta)$, the inequality (4.77) is always satisfied, and the numerator of (4.76) is always positive. Therefore, $\partial s_1^*(x, z)/\partial z > 0 \forall z \in (-y, y)$.

Applying the chain rule, we get:

$$\frac{\partial G_w^*(x, z)}{\partial \sigma_y} = \frac{n}{2} \frac{\partial s_1^*(x, z)}{\partial z} \frac{\partial z}{\partial \sigma_y} \quad (4.79)$$

$$\frac{\partial G_R^*(x, z)}{\partial \sigma_y} = \frac{1}{\pi + ny} \left(\pi \frac{\partial s_1^*(x, z)}{\partial z} + \frac{n}{2} \right) \frac{\partial z}{\partial \sigma_y} \quad (4.80)$$

As $\partial z/\partial \sigma_y < 0$ when $y_1 < y_0$ and $\partial z/\partial \sigma_y > 0$ when $y_1 > y_0$, Proposition 25.2 follows. \square

4.8.5 Proof of Proposition 26

4.8.5.1 Linearized dynamic system

To conduct our analysis, we log-linearize the dynamic system (4.9)-(4.11) around the steady state (s_1^*, q^*) with respect to (s_{1t}, q_t, c_{1t}) , and define $\hat{x} = \log(x/x^*)$. Let $\psi = 1 + \alpha(\varepsilon - 1)$ and $\theta_i = \beta_i(1 - \varepsilon)$, we obtain¹⁰:

¹⁰For simplicity, the arguments of the functions.

$$\begin{aligned} & \begin{pmatrix} -\psi & 1 + \theta_1 & \theta_1 \\ \psi c_1^*/c_0^* & 1 + \theta_0 & -\theta_1 c_1^*/c_0^* \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \dot{\hat{c}}_1 \\ \dot{\hat{q}} \\ \dot{\hat{s}}_1 \end{pmatrix} \\ &= \begin{pmatrix} -(\rho - \pi/q^*) & \rho & (\rho - \pi/q^*) \\ (\rho - \pi/q^*)c_1^*/c_0^* & \rho & -(\rho - \pi/q^*)\beta_1 c_1^*/(\beta_0 c_0^*) \\ -(\rho - \pi/q^*)\alpha/\beta_1 & 0 & \pi/q^* \end{pmatrix} \times \begin{pmatrix} \hat{c}_1 \\ \hat{q} \\ \hat{s}_1 \end{pmatrix} \end{aligned}$$

with $c_1^* = \pi s_1^* + y + z$ and $c_0^* = \pi(2/n - s_1^*) + y - z$.

4.8.5.2 The Characteristic Polynomial $P(\lambda)$

The characteristic polynomial of this economy is given by:

$$P(\lambda) = \lambda^3 - T(\varepsilon)\lambda^2 + S(\varepsilon)\lambda - D(\varepsilon) \quad (4.81)$$

where

$$T(\varepsilon) = \rho \frac{\varepsilon - \bar{\varepsilon}}{\varepsilon - \underline{\varepsilon}} \quad (4.82)$$

$$D(\varepsilon) = \frac{D_1(\varepsilon)}{\varepsilon - \underline{\varepsilon}} \quad (4.83)$$

$$D(\varepsilon) - S(\varepsilon)T(\varepsilon) = \frac{\delta_0(\varepsilon)}{(\varepsilon - \underline{\varepsilon})^2} [\delta_1(\bar{\varepsilon} - \underline{\varepsilon}) + \delta_2(1 - \varepsilon)(\bar{\varepsilon} - \varepsilon) - \delta_3(\bar{\varepsilon} - \varepsilon)] \quad (4.84)$$

where under Assumptions 19-21,

$$\underline{\varepsilon} = 1 + \frac{c}{c_1^*} \frac{s_1^*}{\beta_1} > 1 \quad (4.85)$$

$$\bar{\varepsilon} = 1 + 2 \frac{c}{c_1^*} \frac{s_1^*}{\beta_1} > \underline{\varepsilon} \quad (4.86)$$

$$D_1 = \frac{\rho}{1 - \alpha(1 - \varepsilon)} \frac{1}{\alpha q^*} \left(\tilde{c}_1^* \frac{y_0}{q^* s_0^*} + \tilde{c}_0^* \frac{y_1}{q^* s_1^*} \right) > 0, \quad \forall \varepsilon > 0 \quad (4.87)$$

$$\delta_0(\varepsilon) = \rho \frac{\rho - \pi/q^*}{1 - \alpha(1 - \varepsilon)} \frac{1}{\beta_1 \beta_0} \frac{s_1^*}{2\beta_1 c_1^*} > 0 \quad \forall \varepsilon > 0 \quad (4.88)$$

$$\delta_1 = \beta_1 \beta_0 \left(c_1^* \frac{y_0}{q^* s_0^*} + c_0^* \frac{y_1}{q^* s_1^*} \right) > 0 \quad (4.89)$$

$$\delta_2 = \beta_1 \beta_0 \left(\beta_1 c_1^* \frac{y_0}{q^* s_0^*} + \beta_0 c_0^* \frac{y_1}{q^* s_1^*} \right) > 0 \quad (4.90)$$

$$\delta_3 = \rho^3 \frac{c}{c_1^*} \frac{s_1^*}{\beta_1} > 0 \quad (4.91)$$

with $c = (c_1^* + c_0^*)n/2 = \pi + ny$.

4.8.5.3 Proof of Proposition 26

We know that

- For $T(\varepsilon) < 0$, if $D(\varepsilon) < 0$ and $D(\varepsilon) > S(\varepsilon)T(\varepsilon)$, there are three eigenvalues with negative real parts;
- For $T(\varepsilon) < 0$ and $D(\varepsilon) > 0$ or for $T(\varepsilon) > 0$, $D(\varepsilon) > 0$ and $D(\varepsilon) > S(\varepsilon)T(\varepsilon)$, there are two eigenvalues with negative real parts;
- For $T(\varepsilon) < 0$, if $D(\varepsilon) < 0$ and $D(\varepsilon) < S(\varepsilon)T(\varepsilon)$ or, for $T(\varepsilon) > 0$, if $D(\varepsilon) < 0$, there is one eigenvalue with negative real part;
- for $T(\varepsilon) > 0$, if $D(\varepsilon) > 0$ and $D(\varepsilon) < S(\varepsilon)T(\varepsilon)$, there is no eigenvalue with negative real part.

By analyzing (4.82) – (4.84), we obtain the following results:

- If $\varepsilon < \underline{\varepsilon}$, then $T(\varepsilon) > 0$ and $D(\varepsilon) < 0$, thus there is one eigenvalue with negative real part.
- If $\varepsilon \in (\underline{\varepsilon}, \bar{\varepsilon})$, then $T(\varepsilon) < 0$ and $D(\varepsilon) > 0$, thus there are two eigenvalues with negative real part.
- If $\varepsilon > \bar{\varepsilon}$, then $T(\varepsilon) > 0$, $D(\varepsilon) > 0$ and $D(\varepsilon) > S(\varepsilon)T(\varepsilon)$, thus there are two eigenvalues with negative real part.

Following Blanchard-Kahn (1980) conditions, we get the following

- Local determinacy when there are zero or one eigenvalue with negative real part;
- Local indeterminacy when there are at least two eigenvalues with negative real part.

□

4.9 Proofs of Corollaries 3 and 4

We have

$$q^* = \frac{\beta + x}{\alpha\rho} \frac{c_1^*}{s_1^*} + \frac{\pi}{\rho} \quad (4.92)$$

$$\underline{\varepsilon} = 1 + (\pi + y) \frac{s_1^*}{c_1^*} \frac{1}{\beta + x} \quad (4.93)$$

From (4.92) and (4.93), we deduce that

$$\text{sign} \frac{\partial \underline{\varepsilon}}{\partial x} = \text{sign} - \frac{\partial q^*}{\partial x} \quad (4.94)$$

$$\text{sign} \frac{\partial \underline{\varepsilon}}{\partial z} = \text{sign} - \frac{\partial q^*}{\partial z} \quad (4.95)$$

Corollaries 3 and 4 follows Proposition 23.

□

Conclusion Générale

Cette thèse est dédiée à l'analyse théorique des interactions financières et réelles. Au regard des crises financières, manifestations extrêmes de ces interactions, l'objectif de cette thèse est d'approfondir nos connaissances théoriques sur la volatilité des prix d'actifs, ainsi que sur l'existence de bulles spéculatives et leur influence macroéconomique, et de prodiguer des recommandations en matière de politiques économiques permettant de stabiliser l'économie dans son ensemble. Contraintes de crédit au niveau des ménages, intégration financière, aspects psychologiques (préférences pour la richesse) et hétérogénéité parmi les agents sont des éléments-clés de notre analyse.

Dans le premier chapitre, nous étudions dans un premier temps l'existence et les fluctuations d'une bulle spéculative rationnelle au sein d'une économie d'échange dans laquelle les ménages réalisent un choix de portefeuille entre un actif sans valeur fondamentale et la monnaie. Nous mettons en exergue le rôle des contraintes de crédit pesant sur les ménages dans l'apparition des fluctuations. Dans un second temps, nous analysons le rôle stabilisateur des politiques monétaires répondant ou non aux prix des actifs. A travers ce chapitre, nous montrons que plusieurs états stationnaires avec bulles existent pour de faibles restrictions, et ce à cause des imperfections financières. Ce résultat est également associé à l'existence de fluctuations endogènes. Dans un tel contexte, un choc sur les anticipations peut

expliquer la volatilité excessive observée sur les marchés financiers. L'existence d'équilibres multiples soulève un problème de la coordination des anticipations. Fort de ce constat, les banques centrales ont un rôle à jouer dans la stabilisation de l'économie. Nos conclusions mitigent celles de Bernanke et Gertler (1999, 2001) dans la mesure où une politique monétaire répondant aux actifs s'avère stabilisatrice dans notre modèle. En intégrant les actifs à sa règle monétaire, l'autorité monétaire pourrait éliminer les fluctuations et permettre aux agents de se coordonner sur un unique état stationnaire.

Dans le deuxième chapitre, nous étendons le chapitre 1 à une économie de production. Les ménages réalisent maintenant un choix de portefeuille entre trois actifs : la monnaie détenue à cause d'une contrainte de liquidité, des bons du trésor servant de contreparties de la monnaie, et du capital physique servant à la fois de collatéral pour les ménages et de facteur de production. Comme dans le chapitre 1, nous montrons l'existence de la multiplicité d'états stationnaires et de fluctuations économiques pour de faibles restrictions du fait des imperfections financières. Nous comparons ensuite les vertus stabilisatrices de la politique monétaire contre la politique fiscale. Pour la politique monétaire, nous considérons une règle de Taylor classique répondant à l'inflation, tandis que pour la politique fiscale, nous optons pour une taxation progressive du capital. Dans une optique de stabilisation de l'économie, une politique fiscale par le biais d'une taxation progressive du capital s'avère être plus favorable, comparée à une politique monétaire régie par une règle de Taylor.

Dans le chapitre 2, nous supposons que l'offre de travail est exogène. Par conséquent, les variables du marché du travail, comme l'emploi total et les heures de travail, ne sont pas affectées par les fluctuations. Cette hypothèse ne semble pas concilier avec l'idée que l'on peut se faire des crises financières. En effet, à la suite

de l'éclatement des bulles d'actifs ces variables du marché du travail ont tendance à baisser. Partant de ce constat, il paraît intéressant d'introduire de l'offre de travail endogène afin de rendre compte de l'influence de la bulle sur ces dernières.

Dans les deux premiers chapitres, nous nous intéressons aux interactions entre sphère financière et sphère réelle au sein d'une économie fermée. Or, les crises financières ont tendance à se propager au reste du monde à cause de la mondialisation financière. Partant de ce constat, le troisième chapitre étudie ainsi la transmission internationale de l'éclatement d'une bulle spéculative. Le but principal de ce chapitre est de construire un cadre de référence pour de futures recherches sur ce thème. Pour ce faire, nous construisons une économie ouverte à deux pays identiques avec intégration parfaite des marchés dans laquelle chaque pays émet sa bulle représentée par un actif sans valeur fondamentale. Lorsque seule la bulle d'un pays a une probabilité non nulle d'exploser, aucune bulle ne persiste au sein de l'économie suite à l'éclatement de cette bulle à cause de la condition d'arbitrage. Lorsque les deux bulles ont une probabilité non nulle d'exploser, l'effet de l'éclatement d'une bulle dans un pays peut avoir un effet positif sur la bulle de l'autre pays. Nous pouvons étendre notre analyse de plusieurs manières en assouplissant certaines de nos hypothèses. Tout d'abord, il paraît important de tenir compte de l'asymétrie entre les pays dans divers aspects comme les préférences, les dotations et la population puisqu'en réalité il n'existe pas deux pays identiques. Deuxièmement, nous supposons dans ce troisième chapitre que la probabilité de la bulle d'exploser est constante dans le temps. Par exemple, nous pourrions relaxer cette hypothèse en supposant des probabilités endogènes dépendant de variables endogènes comme la taille de la bulle. En plus de ces extensions possibles, nous pourrions nous écarter de l'utilité espérée classique comme supposée dans ce chapitre en tenant compte de l'utilité

espérée dépendant du rang. Cette hypothèse aurait pour mérite de formaliser certaines attitudes psychologiques telles que le pessimisme et l'optimisme.

Dans le premier et le deuxième chapitre, les imperfections financières sont au coeur des fluctuations. Dans ce quatrième et dernier chapitre, ce sont l'hétérogénéité associée à des facteurs psychologiques qui sont à l'origine de fluctuations. L'esprit du capitalisme, à savoir le désir d'accumulation de richesse comme une fin en soi par certains agents, pourrait être une des causes des crises financières. L'objectif de ce dernier chapitre est d'apporter des éléments de réponses à la question "Est-ce qu'une société capitaliste au sens de *Max Weber* sera plus exposée aux crises financières si elle est très hétérogène?" Nous mettons en exergue le rôle de l'hétérogénéité des préférences parmi les ménages. Notre étude révèle que l'hétérogénéité des préférences accroît les inégalités sociales et le niveau des prix d'actifs à long terme, d'une part, et favorise l'apparition de fluctuations dans les prix d'actifs, d'autre part. En mettant en lumière le rôle de l'hétérogénéité sur le prix des actifs, la volatilité et les inégalités de richesses, nous pourrions utiliser le cadre de ce chapitre afin d'étudier quelles politiques devraient être mises en place pour atténuer les inégalités et stabiliser l'économie.

Bibliography

- [1] Abel, A., Mankiw, N. G., Summers, L. H. and Zeckhauser, R. J. (1989), “Assessing Dynamic Efficiency : Theory and Evidence,” *The Review of Economic Studies* **56**, 1-19.
- [2] Allais, M. (1947), *Economie et Intérêt*, Imprimerie Nationale, Paris.
- [3] Akerlof, G A., and R. J. Shiller (2009), *Animal Spirits: How Human Psychology Drives the Economy and Why This Matters for Global Capitalism*.
- [4] Airaudo, M. (2012), “Complex stock price dynamics and recurrent bubbles under the spirit of capitalism,” working paper.
- [5] Azariadis, C. (1981), “Self-fulfilling prophecies,” *Journal of Economic Theory* **3**, 380-396.
- [6] Azariadis, C. and R., Guesnerie (1986), “Sunspot and Cycles ”, *The Review of Economic Studies* **53**, 725-737.
- [7] Azariadis, C., and P. Reichlin (1996), “Increasing returns and crowding out,” *Journal of Economic Dynamics and Control* **20** 847-877.
- [8] Bakshi, G.S. and Z. Chen (1996), “The spirit of capitalism and stock-market prices,” *The American Economic Review* **86**, 133–157.

- [9] Barberis, N. and R. Thaler (2003), “A Survey of Behavioral Finance.”, In Handbook of the Economics of Finance, edited by George M. Constantinides, Milton Harris, and René M. Stulz.
- [10] Barsky, R. B. et J. B. De Long (1993), “Why Does the Stock Market Fluctuate?”, *Quarterly Journal of Economics* **108**, p. 291-311.
- [11] Becker, R.A. (1980), “On the long-run steady state in a simple dynamic model of equilibrium with heterogeneous households ,” *Quarterly Journal of Economics* **95**, 375-382.
- [12] Benhabib, J., and R. E. A. Farmer (1999), “Indeterminacy and Sunspots in Macroeconomics,” in Handbook of Macroeconomics, ed. by J. B. Taylor, and M. Woodford. North-Holland.
- [13] Benhabib, J., and G. Laroque (1988), “On competitive cycles in productive economies,” *Journal of Economic Theory* **45**, 145-170.
- [14] Benhabib, J., S. Schmitt-Grohé and M. Uribe (2001), “The perils of Taylor rules”, *Journal of Economic Theory* **96**, 40-69.
- [15] Bernanke, B. (2010), “Monetary Policy and the Housing Bubble by Ben S. Bernanke Chairman” At the Annual Meeting of the American Economic Association, Atlanta, Georgia, January 3.
- [16] Bernanke, B. (2011), “Remarks by Chairman Ben S. Bernanke” At the 56th Economic Conference Federal Reserve Bank of Boston, Boston, Massachusetts, October 18.
- [17] Bernanke, B. and M. Gertler (1989), “Agency Costs, Net Worth, and Business Fluctuations”, *American Economic Review* **79**, 14-31.

-
- [18] Bernanke, B. and M. Gertler (1999), “Monetary policy and asset price volatility”, *Federal Reserve Bank of Kansas City Economic Review* **84**, 17-52.
- [19] Bernanke, B. and M. Gertler M. (2001), “Should central banks respond to movements in asset prices?”, *American Economic Review* **91**, 253-257.
- [20] Bernanke, B. and M. Woodford M. (1997), “Inflation forecasts and monetary policy”, *Journal of Money, Credit and Banking* **29**, 653-684.
- [21] Boileau, M. and R. Braeu (2007), “The spirit of capitalism, asset returns, and the business cycles,” *Macroeconomic Dynamics*, **11**, 214–230.
- [22] Bosi, S. and T. Seegmuller (2006), “Optimal Cycles and Social Inequality: What Do We Learn from the Gini Index?,” *Research in Economics* **60**, 35-46.
- [23] Bosi, S. and T. Seegmuller (2008), “Can heterogeneous preferences stabilize endogenous fluctuations?,” *Journal of Economic Dynamics and Control* **32**, 624–647.
- [24] Bosi, S. and T. Seegmuller (2010a), “On rational exuberance”, *Mathematical Social Sciences* **59**, 249-270.
- [25] Bosi, S., and T. Seegmuller (2010b), “On the role of progressive taxation in a Ramsey model with heterogeneous households,” *Journal of Mathematical Economics* **46**, 977-996.
- [26] Bosi, S. and T. Seegmuller (2013), “Rational bubbles and expectation-driven fluctuations”, *International Journal of Economic Theory* **9**, 69-83.
- [27] Breeden, Douglas T. (1979), “An intertemporal asset pricing model with stochastic consumption and investment opportunities”, *Journal of Financial Economics* **7**, 265–296.

- [28] Brunnermeier, M. K. (2007), “Bubbles,” *S. Durlauf and L. Blume, eds, New Palgrave Dictionary of Economics*, 2nd edn, London: PalgraveMacmillan.
- [29] Brunnermeier, M. K., T. M. Eisenbach and Y. Sannikov (2012), “Macroeconomics with Financial Frictions: A Survey”, NBER Working Papers 18102, National Bureau of Economic Research, Inc.
- [30] Brunnermeier, M. K., and M. Oehmke (2013), “Bubbles, financial crises, and systemic risk,” In: *Handbook of the Economics of Finance*, Amsterdam.
- [31] Bullard, J., E. G. W. and S. Honkapohja (2010), “A Model Of Near-Rational Exuberance,” *Macroeconomic Dynamics* **14**, 166-188.
- [32] Caballero, R. J. (2010), “Macroeconomics after the Crisis: Time to Deal with the Pretense-of-Knowledge Syndrome”, *Journal of Economic Perspectives* **24**, 85-102.
- [33] Caballero, R. and Krishnamurthy, A. (2006), “Bubbles and capital flow volatility: Causes and risk management”, *Journal of Monetary Economics* **53**, 35-53.
- [34] Camerer, C. (1989), “Bubbles and Fads in Asset Prices”, *Journal of Economic Surveys* **3**, 1467-6419
- [35] Campbell, J. (1999), “Asset prices, consumption, and the business cycle”, In: J. Taylor and M. Woodford (Eds.), *Handbook of Macroeconomics*, Chapter 19, 1231-1303.
- [36] Campbell, J. (2001), “Asset pricing at the millennium“, *Journal of Finance* **55**, 1515–1567.

-
- [37] Campbell, J. (2003), “Consumption-based asset pricing”, In: G. Constantinides, M. Harris, R. Stulz (Eds.), *Handbook of the Economics of Finance* Amsterdam: North-Holland) **1B**, 803-887.
- [38] Campbell, J. Y., and J. H. Cochrane (1999), “By force of habit: a consumption-based explanation of aggregate stock market behavior”, *Journal of Political Economy* **107**, 205-251.
- [39] Campbell, J. and G. Mankiw (1989), “Consumption, income and interest rates: reinterpreting the time series evidence”, In: O. Blanchard and S. Fisher (Eds.) *NBER Macroeconomics Annual* (Cambridge MA: MIT Press), 185-216.
- [40] Carlstrom, C. T. and T. S. Fuerst (2007), “Asset prices, nominal rigidities, and monetary policy”, *Review of Economics Dynamics* **10**, 256-275.
- [41] Carroll, C. (2000), “Why Do the Rich Save So Much?,” In *Does Atlas Shrug? The Economic Consequences of Taxing the Rich*, Harvard University Press.
- [42] Caselli, F. and J., Ventura (2000), “A Representative Consumer Theory of Distribution,” *American Economic Review* **90**, 909-926.
- [43] Cass, D., and K. Shell (1983), “Do Sunspots Matter?,” *Journal of Political Economy* **91**, 193—227.
- [44] Cazzavillan, G., T. Lloyd-Braga and P. Pintus (1998), “Multiple steady states and endogenous fluctuations with increasing returns to scale in production ”, *Journal of Economic Theory* **80**, 60-107.
- [45] Cazzavillan, G. and P. Pintus (2005), “On competitive cycles and sunspots in productive economies with a positive money stock,” *Research in Economics* **59**, 137-147.

- [46] Chatterjee, S. (1994), “Transitional dynamics and the distribution of wealth in a neoclassical growth model,” *Journal of Public Economics*, **54**, 97-119.
- [47] Chirinko, R. S. and Schaller, H. (2011), “Fundamentals, Misvaluation, and Business Investment,” *Journal of Money, Credit and Banking*, **43**, 1423–1442.
- [48] Clain-Chamosset-Yvrard, L. and T. Seegmuller (2013), “The stabilizing virtues of fiscal vs. monetary policy on endogenous bubble fluctuations”, AMSE Working Paper 2013 N° 43.
- [49] Clain-Chamosset-Yvrard, L. and T. Seegmuller (2015), “Rational bubbles and macroeconomic fluctuations: the (de-)stabilizing role of monetary policy,” *Mathematical Social Sciences* **75**, 1-15.
- [50] Cochrane, J. H. (2005), *Asset Pricing*, Revised Edition. Princeton University Press.
- [51] Constantinides, G. M. and D. Duffie (1996). “Asset Pricing with Heterogeneous Consumers” *Journal of Political Economy* **104**, 219-40.
- [52] Crettez, B., P. Michel and B. Wigniolle (1999), “Cash-in-advance constraints in the Diamond overlapping generations model: Neutrality and optimality of monetary policy”, *Oxford Economic Papers* **51**, 431-452.
- [53] Crook, J. and S. Hochguertel (2005), “Household debt and credit constraints: Evidence from OECD countries,” Credit Research Center, University of Edinburgh Working Paper Series No. 05/02.
- [54] DeLong, J. B., A. Shleifer, L H. Summers, and R. J. (1990) Waldmann. “Noise trader risk in financial markets” *Journal of Political Economy* **98**, 703-738.

- [55] Diba, B. T. et H. I. Grossman (1987), “On the inception of rational bubbles”, *Quarterly Journal of Economics* **102**, 697-700.
- [56] Diba, B. T. et H. I. Grossman (1988), “Rational inflationary bubbles”, *Journal of Monetary Economics* **21**, 35-46.
- [57] Diamond, P. A. (1965), “National debt in a neoclassical growth model,” *American Economic Review* **55**, 1126-1150.
- [58] Dromel, N. and P. Pintus (2008), “Are progressive income taxes stabilizing?,” *Journal of Public Economic Theory* **10**, 329-349.
- [59] Dynan, K., Skinner, J. and Zeldes, S. (2004), “Do the rich save more?,” *Journal of Political Economy* **112**, 397–444.
- [60] Ehrmann, M., Fratzscher, M., Rigobon, R., (2011), “Stocks, bonds, money markets and exchange rates: measuring international financial transmission”, *Journal of Applied Econometrics* **26**, 948-974.
- [61] Epstein, L. G., and S. E. Zin. (1989), “Substitution, Risk Aversion, and the Temporal Behavior of Consumption and Asset Returns: A Theoretical Framework”, *Econometrica* **57**. 937-69.
- [62] Fama, E. F. (1965), “Random Walks in Stock Market Prices ” *Financial Analysts Journal* **21**, 55-59.
- [63] Fama, E. F. (1970), “Efficient Capital Markets: A Review of Theoretical and Empirical Works ” *The Journal of Finance* **25**, 383–417.
- [64] Farmer, R. E. A. (1999) *The Macroeconomics of Self-Fulfilling Prophecies*. MIT Press, Cambridge, MA, second edn.

- [65] Farhi, E. and J. Tirole (2012), “Bubbly Liquidity”, *Review of Economic Studies*, **79**, 678-706.
- [66] Farhi, E., C. S. I. Werning and S. Yeltekin (2012), “Nonlinear capital taxation without commitment,” *Review of Economic Studies* **79**, 1469-1493.
- [67] Farmer, R. E. A. (1986), “Deficits and cycles,” *Journal of Economic Theory* **40**, 77-88.
- [68] Farmer, R. E. A. (2002a), “Business Cycles with Heterogenous Agents,” UCLA mimeo, Paper prepared for a conference at GREQAM on " New Perspectives of (In)Stability, the Role of Heterogeneity", Marseilles, June 2001.
- [69] Farmer, R.E. A. (2014) “The evolution of endogenous business cycles ” *Macroeconomic Dynamics*, 1-14.
- [70] Farmer, R. E. A., C. Nourry, and A. Venditti (2012), “The Inefficient Markets Hypothesis: Why Financial Markets Do Not Work Well in the Real World” NBER Working Paper 18647
- [71] Francis, J. L. (2009), “Wealth and the capitalist spirit” *Journal of Macroeconomics* **31**, 394–408
- [72] Franklin, B. (1748), *The Complete Works of Benjamin Franklin*. Ed. G. P. Putnam’s Sons.
- [73] Gaballo, G. (2013) “Rational Inattention to News: The Perils of Forward Guidance ”, Working Paper N° 416, Banque de France.
- [74] Gali, J. (1994) “Keeping Up with the Joneses: Consumption Externalities, Portfolio Choice, and Asset Prices,” *Journal of Money, Credit and Banking*, **26**, 1-8.

- [75] Gali, J. (2014), “Monetary policy and rational asset price bubbles”, *American Economic Review* **104**, 721-752.
- [76] Geerolf, F. (2013) “Reassessing Dynamic Efficiency”, working paper.
- [77] Gillman, M., P. Sikles, and J.L. Silver (1997), “Money velocity with costly credit”, *Journal of Economic Research* **2**, 179-207.
- [78] Grandmont, J.-M. (1985), “On endogenous competitive business cycles”, *Econometrica* **53**, 995-1045.
- [79] Grandmont, J.-M. (1986), “Stabilizing competitive business cycles”, *Journal of Economic Theory* **40**, 57-76.
- [80] Grandmont, J.-M., P. Pintus and R. de Vilder (1998), “Capital-labour substitution and competitive nonlinear endogenous business cycles”, *Journal of Economic Theory* **80**, 14-59.
- [81] Greenspan, A. (1996), “Remarks by Chairman Alan Greenspan” At the Annual Dinner and Francis Boyer Lecture of The American Enterprise Institute for Public Policy Research, Washington D.C., December 5.
- [82] Grossman, G. and N. Yanagawa (1993), “Asset bubbles and endogenous growth” *Journal of Monetary Economics* **31** 3-19.
- [83] Grossman, S. and R. Shiller (1981), “The Determinants of the Variability of Stock Market Prices”, *American Economic Review* **71**, 222–227.
- [84] Guo, J. T., and K. Lansing (1998), ”Indeterminacy and stabilization policy,” *Journal of Economic Theory* **82**, 481-490.
- [85] Hahn, F. and R. Solow (1995), *A Critical Essay on Modern Macroeconomic Theory*, Basil Blackwell, Oxford.

- [86] He H. and D. M. Modest (1995), "Market Frictions and Consumption-Based Asset Pricing" *The Journal of Political Economy* **103**, 94-117.
- [87] Hirano, T. and N. Yanagawa (2013), "Asset bubbles, endogenous growth, and financial frictions", Working Paper, CARF-F-223, The University of Tokyo.
- [88] Hirano, T., Inaba M. and N. Yanagawa (2015), "Asset bubbles and bailouts", Working Paper, CARF-F-268, The University of Tokyo.
- [89] Huang, K. and J., Werner (2004), "Implementing Arrow-Debreu equilibria by trading infinitely-lived securities", *Economic Theory*, **24**, 603-622.
- [90] Iacoviello, M. (2004), "Consumption, house prices, and collateral constraints: a structural econometric analysis", *Journal of Housing Economics* **13**, 304-320.
- [91] IMF - International Monetary Fund (2015), WORLD ECONOMIC OUTLOOK (WEO) Uneven Growth: Short- and Long-Term Factors.
- [92] Jappelli, T. (1990), "Who is credit constrained in the US economy?", *Quarterly Journal of Economics* **105**, 219-234.
- [93] Jones, B. (2015) "Asset Bubbles: Re-thinking Policy for the Age of Asset Management ", IMF Working Paper, WP/15/27
- [94] Kamihigashi, T., (1998), "Uniqueness of asset prices in an exchange economy with unbounded utility", *Economic Theory* **12**, 103-122.
- [95] Kamihigashi, T., (2001), "Necessity of transversality conditions for infinite horizon problems ", *Econometrica* **69**, 995-1012.
- [96] Kamihigashi, T., (2008), "The spirit of capitalism, stock market bubbles and output fluctuations ", *International Journal of Economic Theory* **4**, 3-28.

-
- [97] Kamihigashi, T., (2011), “Recurrent bubbles”, *Japanese Economic Review* **62**, 27-62.
- [98] Kamihigashi, T., (2015), “A Simple No-Bubble Theorem, ”, Discussion Paper Series DP2015-03, Research Institute for Economics and Business Administration, Kobe University.
- [99] Kaplan S.N, and J. Rauh., (2010). “Wall Street and Main Street: What Contributes to the Rise in the Highest Incomes?”, *Review of Financial Studies* **23**, 1004-1050.
- [100] Karnizova, L. (2010), “The spirit of capitalism and expectation-driven business cycles,” *Journal of Monetary Economics* **57**, 739–752.
- [101] Keynes, J.M. (1919), *The Economic Consequences of the Peace*.
- [102] Keynes, J.M. (1936), *The General Theory of Employment, Interest and Money*.
- [103] Kydland, F.E., and E. C. Prescott (1982), “Time To Build and Aggregate Fluctuations,” *Econometrica* **50**, 1345-1370.
- [104] Kindleberger, C. P. (1978), *Manias, Panics, and Crashes: A History of Financial Crises*, Palgrave Macmillan, 1st edition.
- [105] Kindleberger, C. P. and R. Z. Aliber (2005), *Manias, Panics, and Crashes: A History of Financial Crises*, Palgrave Macmillan, 5th edition.
- [106] King, I., and D. Ferguson (1993) “Dynamic inefficiency, endogenous growth, and ponzi games”, *Journal of Monetary Economics* **32**, 79-104.
- [107] Kiyotaki, N. and J. Moore (1997), “Credit cycles ”, *Journal of Political Economy* **4**, 211-248.

- [108] Kocherlakota, N. R. (1992), “Bubbles and constraints on debt accumulation”, *Journal of Economic Theory* **57**, 245-256.
- [109] Kocherlakota, N. R. (1996), “The equity premium: It’s still a puzzle”, *Journal of Economic Literature* **34**, 42–71.
- [110] Kocherlakota, N. R. (2008), “Injecting rational bubbles”, *Journal of Economic Theory* **142**, 218-232.
- [111] Kocherlakota, N.R. (2009), “Bursting bubbles: Consequences and cures”, University of Minnesota working paper.
- [112] Krugman, Paul. (1999), “Balance Sheets, The Transfer Problem, and Financial Crises”, Mimeo, MIT.
- [113] Krugman, P. (2009), “How Did Economists Get It So Wrong? ” *The New York Times*, September 6.
- [114] Kurz, M. (1968), “Optimal economic growth and wealth effects,” *International Economic Review* **9**, 348-357.
- [115] Lane, P. R. and Milesi-Ferretti, G.M, (2007). “The external wealth of nations mark II: Revised and extended estimates of foreign assets and liabilities, 1970-2004,” *Journal of International Economics* **73**, 223-250.
- [116] LeRoy, S. (2004) “Rational Exuberance,” *Journal of Economic Literature* **152**, 783-804.
- [117] LeRoy, S. and R. Porter (1981), “The present value relation: Tests based on variance bounds”, *Econometrica* **49**, 555-584.
- [118] Le Van, C., N.S. Pham and Y. Vailakis (2015) “Financial asset bubble with heterogeneous agents and endogenous borrowing constraints ”, Working Paper.

- [119] Lloyd-Braga, T., L. Modesto and T. Seegmuller (2008), “Tax Rate Variability and Public Spending as Sources of Indeterminacy,” *Journal of Public Economic Theory* **10**, 399-421.
- [120] Lucas, R.E., Jr. (1972), “Expectations and the Neutrality of Money,” *Journal of Economic Theory* **4**, 103-124.
- [121] Lucas, R.E., Jr. (1975), “An Equilibrium Model of the Business Cycle,” *Journal of Political Economy* **83**, 1133-1144.
- [122] Lucas, R.E. (1978), “Asset Prices in an Exchange Economy,” *Econometrica* **46**, 1429-1445.
- [123] Lucas, R.E. (1987), *Models of Business Cycles*. Oxford, UK: Basil Blackwell.
- [124] Lucas, R.E. (2009), “In defence of the dismal science,” *The Economist*, August 8.
- [125] Lucas Jr., R.E. and N. Stokey (1987), “Money and interest in a cash-in-advance economy”, *Econometrica* **55**, 491-513.
- [126] Lustig, H. and S. Van Nieuwerburgh (2010), “How much does household collateral constrain regional risk sharing?”, *Review of Economic Dynamics* **13**, 265-294.
- [127] Luttmer E.G.J (1996), “Asset Pricing in Economies with Frictions ”, *Econometrica* **64**, 1439-1467.
- [128] Madaleno, M., Pinho, C., (2012), “International stock market indices co-movements: a new look”, *International Journal of Finance and Economics* **17**, 89-102.

- [129] Martin, A. and J. Ventura (2012), “Economic growth with bubbles bubbles”, *American Economic Review* **102**, 3033-3058.
- [130] Martin, A. and J. Ventura., (2015), “The international transmission of credit bubbles: theory and policy”, mimeo (May 2015).
- [131] Marx, K. (1867), *Capital*.
- [132] Masson, Paul. (1998), “Contagion: Macroeconomic Models with Multiple Equilibria”, *Journal of International Money and Finance* **18**, 587-602.
- [133] Matsuyama, K. (1990), “Sunspot equilibria (rational bubbles) in a model of money-in-the-utility function,” *Journal of Monetary Economics* **25**, 137-144.
- [134] Mehra, R. (2006), “The equity premium puzzle: A review”, *Foundations and Trends R. in Finance* **2**, 1-81.
- [135] Mehra, R. (2012), “Consumption-Based Asset Pricing Models”, *Annual Review of Financial Economics* **4**, 385-409.
- [136] Mehra, R and E.C. Prescott. (1985), “The equity premium: A puzzle”, *Journal of Monetary Economics* **15**, 145-161.
- [137] Miao, J. (2014), “Introduction to economy theory of bubbles”, *Journal of Mathematical Economics* **53**, 130-136.
- [138] Miao, J. and P. Wang (2012a), “Bubbles and total factor productivity”, *American Economic Review: Papers and Proceedings* **102**, 82-87.
- [139] Miao, J. and P. Wang (2012b), “Banking bubbles and financial crisis”, working paper, Boston University and HKUST.

-
- [140] Miao, J., Wang, P. and J. Zhou (2014) “Housing bubbles and policy analysis”, working paper, Boston University.
- [141] Michel, P. and B. Wigniolle (2003), “Temporary bubbles”, *Journal of Economic Theory* **112**, 173-183.
- [142] Michel, P. and B. Wigniolle (2005), “Cash-in-advance constraints, bubbles and monetary policy”, *Macroeconomic Dynamics* **9**, 28-56.
- [143] Minsky, Hyman P. (1986), *Stabilizing an Unstable Economy*. Yale University Press.
- [144] Muth, J. F. (1961), “Rational Expectations and the Theory of Price Movements”, *Econometrica* **29**, 315-335
- [145] Nakajima, T. (2003), “Sunspot fluctuations in asset prices and business cycles in Japan over 1986-1999,” *The Japanese Economic Review* **54**, 253-274.
- [146] Nutahara, K. (2010), “Asset prices and monetary policy in a sticky-price economy with financial frictions”, RIETI Discussion Paper Series 10-E-060.
- [147] Obstfeld, M. (1984), “Multiple stable equilibria in an optimizing perfect-foresight model,” *Econometrica* **52**, 223-228.
- [148] Organisation for Economic Cooperation and Development (2007), OECD Economic Outlook, Volume 1, OECD Publishing, Paris.
- [149] Orléans, A. (1989), “Comportement mimétique et diversité des opinions sur les marchés financiers”, in *BOURGUINAT, H., ARTUS, P., Théorie économique et crises des marchés financiers*, Economica, Paris.
- [150] Piketty, T. (2013), *Capital in the Twenty-First Century*, Harvard University Press.

- [151] Piketty, T. et E. Saez (2006), “The Evolution of Top Incomes: A Historical and International Perspective”, *American Economic Review* **96**, 200-205.
- [152] Poterba, L., Summers, L., (1988), “Mean reversion in stock prices: Evidence and implications ”, *Journal of Financial Economics* **22**, 26-59.
- [153] Ramsey F.P. (1928), “A Mathematical Theory of Saving”, *Economic Journal* **38**, 543-559.
- [154] Reinhart, C.M., and K.S. Rogoff (2009), *This Time Is Different: A Panoramic View of Eight Centuries of Financial Crises*. Princeton University Press.
- [155] Rochon, C. and H.M. Polemarchakis (2006), “Debt, liquidity and dynamics”, *Economic Theory* **27**, 179-211.
- [156] Samuelson, P.A. (1958), “An exact consumption-loan model of interest with or without the social contrivance of money”, *Journal of Political Economy* **66**, 467-482.
- [157] Samuelson, P. A. (1965), “Proof that properly anticipated prices fluctuate randomly”, *Industrial Management Review* **6**, 41–9.
- [158] Saez, E. (2013), “Optimal progressive capital income taxes in the infinite horizon model,” *Journal of Public Economics* **97**, 61-74.
- [159] Saint-Paul G. (1992) “Fiscal policy in an endogenous growth model, ” *Quarterly Journal of Economics* **107**, 1243–1260.
- [160] Santos, M. S., and M. Woodford (1997) “Rational asset pricing bubbles” *Econometrica* **65**, 19-57.
- [161] Saunders, E. M. (1993), “Stock Prices and Wall Street Weather,” *American Economic Review* **83**, 1337–45.

- [162] Shefrin, H. M. (2008), *A Behavioral Approach to Asset Pricing*. Amsterdam: Elsevier.
- [163] Shell, K. (1977), “Monnaie et allocation intertemporelle.” working paper.
- [164] Shiller, R. J. (1981), “Do stock prices move too much to be justified by subsequent changes in dividends?”, *American Economic Review* **71**, 421-436.
- [165] Shiller, R. J. (1984), “Stock Prices and Social Dynamics”, *Brookings Papers on Economic Activity* **2**, 457–98.
- [166] Shiller, R. J. (1989), *Market Volatility*, MIT Press, Cambridge, MA.
- [167] Shiller, R. J. (2015), *Irrational Exuberance*, Princeton University Press, Princeton.
- [168] Shiller, R. J. (2003), “From Efficient Market Theory to Behavioral Finance,” *Journal of Economic Perspectives* **17**, 83-104.
- [169] Shiller, R. J. (2013), “Speculative bubbles don’t just pop- they may deflate and reflate,” *The Guardian Economics blog*, July 19.
- [170] Shiller, R. J. (2014), “Speculative Asset Prices,” *American Economic Review* **104**. 1486–1517.
- [171] Shleifer, A. (2000), *Inefficient Markets: A Survey of Behavioral Finance*. New York: Oxford University Press.
- [172] Schleifer, A., and L. H. Summers (1990), “The Noise Trader Approach to Finance”, *Journal of Economic Perspectives* **4**, 19-33.
- [173] Smith, A. (1776), *The Wealth of Nations*.

- [174] Smith, W. T. (2001), “How does the spirit of capitalism affect stock market prices?,” *Review of Financial Studies* **14**, 1215–1232.
- [175] Sorger, G. (2005), “Active and passive monetary policy in an overlapping generations model”, *Review of Economic Dynamics* **8**, 731-748.
- [176] Stiglitz, J. E. (1970), “A consumption-oriented theory of the demand for financial assets and the term structure of interest rates”, *Review of Economic Studies* **37**, 321-351
- [177] Suen, R. M. H. (2014) “Time preference and the distributions of wealth and income,” *Economic Inquiry* **52**, 364–381.
- [178] Tandon, A., and Y. Wang (2003), “Confidence in domestic money and currency substitution”, *Economic Inquiry* **41**, 407-419.
- [179] Tsutsui, Y., Hirayama, K., (2010), “How fast do Tokyo and New York stockexchanges respond to each other? An analysis with high-frequency data”, *Japanese Economic Review* **61**, 175-201.
- [180] Tirole, J. (1982), “On the possibility of speculation under rational expectations”, *Econometrica* **50**, 1163-1181.
- [181] Tirole, J. (1985), “Asset Bubbles and Overlapping Generations”, *Econometrica* **53**, 1071-1100.
- [182] Trichet, J-C. (2010), “Reflections on the nature of monetary policy non-standard measures and finance theory ”Speech by President of the ECB, Opening address at the ECB Central Banking Conference Frankfurt, 18 November 2010

-
- [183] Ventura, J. (2012), “Bubbles and Capital Flows”, *Journal of Economic Theory* **147**, 738-758.
- [184] Walsh, C. E. (2010), *Monetary Theory and Policy*, Third Edition, MIT Press Books, The MIT Press.
- [185] Wang, P and Y. Wen (2012) “Speculative Bubbles and Financial Crises”, *American Economic Journal: Macroeconomics* **43**, 184-221.
- [186] Weber, M. (1905), “The Protestant Ethic and The Spirit of Capitalism,” Charles Scribner’s Sons, New York.
- [187] Weil, P. (1987), “Confidence and the real value of money in overlapping generations models”, *Quarterly Journal of Economics* **102**, 1-22.
- [188] Weil, P. (1989), “The equity premium puzzle and the risk-free rate puzzle.”, *Journal of Monetary Economics* **24**, 401-421.
- [189] Weinbaum, D. “Investor heterogeneity, asset pricing and volatility dynamics”, *Journal of Economic Dynamics and Control* **33**, 1379-1397.
- [190] Wigniolle, B. (2014), “Optimism, pessimism and financial bubbles ”, *Journal of Economic Dynamics and Control* **41**, 188-208.
- [191] Wilson, C. A. (1981), “Equilibrium in Dynamic Models with an Infinity of Agents, ”, *Journal of Economic Theory*, **24**, 95-111.
- [192] Wolff, E.N. (1998), “Recent Trends in the Size Distribution of Household Wealth,” *Journal of Economic Perspectives* **12**, 131-150.
- [193] Wolff, E.N. (2010), “Recent Trends in Household Wealth in the United States– Rising Debt and the Middle-Class Squeeze–An Update to 2007” Levy Economics Institute of Bard College- Working paper N°589.

- [194] Woodford, M. (1986), “Stationary Sunspot Equilibria In a Finance Constrained Economy,” *Journal of Economic Theory* **40**, 128-137.
- [195] Yellen, J.L. (2009), “A Minsky Meltdown: Lessons for Central Bankers” Presentation to the 18th Annual Hyman P. Minsky Conference on the State of the U.S. and World Economies, New York City, April 16.
- [196] Zheng, Z. (2005), “From Rationality to Bounded Rationality”, *Australian Economic Papers* **44**, 455-474.
- [197] Zou, H-F. (1994), “The Spirit of Capitalism and Long-Run Growth”, *European Journal of Political Economy* **10**, 279-293.
- [198] Zou, H-F. (1995), “The Spirit of Capitalism and Savings Behavior,” *Journal of Economic Behavior and Organization* **28**, 131–145.
- [199] Zhou, G. (2013), “The Spirit Of Capitalism And Rational Bubbles”, MPRA paper 44719.
- [200] Zhou, G. (2015), “The Spirit Of Capitalism And Rational Bubbles”, *Macroeconomic Dynamcis*, forthcoming.

RÉSUMÉ

Cette thèse traite des interactions entre les sphères financière et réelle de l'économie. Au sein de cette problématique générale, nous cherchons à répondre à des questions bien précises concernant des problèmes encore non résolus dans la littérature macroéconomique. La première concerne la volatilité des prix des actifs financiers et l'existence de bulles spéculatives, ainsi que leurs implications sur les fluctuations macroéconomiques. La seconde question se rapporte à la transmission internationale de l'éclatement d'une bulle spéculative. La dernière concerne le rôle des politiques monétaires et fiscales face aux fluctuations endogènes d'une bulle spéculative et de l'activité économique.

Cette thèse se compose de quatre chapitres. Dans les deux premiers chapitres, nous étudions l'existence et les fluctuations d'une bulle spéculative rationnelle, comme source de la volatilité des prix d'actifs, en prenant en compte les imperfections financières dans la modélisation des choix des ménages. Nous soutenons l'idée que l'existence d'un choix de portefeuille et de frictions financières favorisent l'émergence des fluctuations d'une bulle et des cycles économiques endogènes. Dans un tel contexte, nous analysons le rôle stabilisateur des politiques fiscales et/ou monétaires. Dans le premier chapitre, nous montrons qu'une politique monétaire répondant aux prix des actifs permet de stabiliser l'économie dans son ensemble. Dans le second chapitre, nous comparons les vertus stabilisatrices d'un impôt progressif sur revenu de capital à celles d'une politique monétaire régie par une règle de Taylor. Nous montrons qu'un impôt progressif sur le revenu du capital permet de stabiliser l'économie en réduisant la probabilité d'apparition des fluctuations endogènes, alors qu'une règle de Taylor a des vertus stabilisatrices mitigées. Au regard de l'intégration des marchés financiers, nous étudions, dans le troisième chapitre, l'existence de bulles rationnelles dans une économie ouverte à deux pays et la transmission internationale de leur éclatement. L'éclatement de la bulle dans un pays se transmet nécessairement à l'autre pays. Selon les anticipations des agents, cette éclatement peut avoir un effet positif ou négatif sur la bulle émise par l'autre pays. Dans le dernier chapitre, nous analysons le rôle de l'hétérogénéité sur la dynamique des prix d'actifs et les inégalités lorsque les agents économiques ont des préférences pour la richesse. L'hétérogénéité des préférences, mais également du revenu, tend à accroître les inégalités sociales et à augmenter le prix des actifs à long terme et à promouvoir la volatilité des prix des actifs à court terme.

MOTS-CLEFS : Prix d'actifs; Bulle rationnelle; Fluctuations endogènes; Equilibres à taches solaires; Indétermination; Contraintes d'encaisses préalables; Esprit de capitalisme; Taxation progressive; Politique monétaire, Transmission internationale; Intégration financière; Agents hétérogènes; Distribution de richesse.

ABSTRACT

This thesis deals with the interplay between the financial and real sectors of the economy. Within this general issue, we aim to answer specific questions about unresolved issues in macroeconomic literature. The first concerns the volatility of asset prices, the existence of speculative bubbles and their implications on macroeconomic fluctuations. The second question relies on the international transmission of the bursting of a speculative bubble. The latest concerns the stabilizing role of monetary and fiscal policies on fluctuations of a speculative bubble. This thesis consists of four chapters. In the first two chapters, we study the existence and endogenous fluctuations of rational speculative bubbles, as a source of volatility in asset prices, taking into account the financial imperfections at the household level. We argue that the existence of a portfolio choice and financial frictions faced by households promote the emergence of bubble fluctuations and endogenous business cycles. In this context, we analyze the stabilizing role of fiscal and/or monetary policies. In the first chapter, we show that a monetary policy responding to asset prices can stabilize the economy as a whole. In the second chapter, we compare the stabilizing virtues of a progressive taxation of capital income with those of a monetary policy managed by a Taylor rule. We show that a progressive taxation on capital income may rule out endogenous fluctuations. In contrast, a monetary policy under a Taylor rule has a mitigated stabilizing role. Given the financial integration, we study the existence of rational bubbles in a two-country economy, and the international transmission of their bursting in the third chapter. A bubble bursting in a country necessarily transmits to the other country. Depending on agents' expectations, the effect of a bubble crash in one country on the bubble issued by the other country can be positive or negative. In the last chapter, we analyze the role of heterogeneity on the dynamics of asset prices and inequality when economic agents have preferences for wealth. Heterogeneity in preferences, but also in income, could heighten social inequalities and increase the asset price in the long run, and promote asset price volatility in short-run.

KEYWORDS: Asset prices; Rational bubble; Endogenous fluctuations; Sunspot Equilibria; Indeterminacy; Cash-in-advance constraint; Spirit of Capitalism; Progressive taxation; Monetary policy; International transmission; Financial integration; Heterogeneous agents, Wealth Distribution.